

DESIGN OF A RIGID FRAME
CONCRETE HIGHWAY BRIDGE

Thesis for the Degree of B. S.
MICHIGAN STATE COLLEGE
Wayne A. Groesbeck
1941

THESES

Cap. 1

**SUPPLEMENTARY
MATERIAL
IN BACK OF BOOK**

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Design of a Rigid Frame Concrete
Highway Bridge

A Thesis Submitted to

The Faculty of
MICHIGAN STATE COLLEGE

of
AGRICULTURE AND APPLIED SCIENCE

by

Wayne A. Groesbeck
Candidate for the Degree of
Bachelor of Science

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THETA

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INTRODUCTION

The site chosen for this bridge is on U.S.-12 three-fourths of a mile east of Marshall, where the highway crosses Rice Creek. The highway is one of the oldest in the State and as in many cases, the roadway and its appurtenances have lagged behind the ever increasing demands of modern traffic.

The present bridge is of concrete slab-girder construction, has a twenty-four foot roadway, and was built nearly twenty years ago. In the last few years the accidents and fatalities occurring at this bridge have mounted steadily.

National defense, too, will make its demands, since Michigan's Fort Custer is located on U.S.-12. The heavy military traffic which now uses this road requires wider and heavier roads and bridges. Wider roads and bridges mean the discarding of the present bridge and it is for this particular site that the following design is proposed.

The rigid frame concrete bridge was chosen here for two reasons, (1) because of its economy, (2) because it offers design principles different from those covered in any of the courses in concrete design.

The economy afforded by the rigid frame cannot be ignored. It requires only about 60 per cent of the material required of a bridge having members of uniform cross-section. The abutments required by fixed arches of the same span are massive and require, in addition to more concrete,

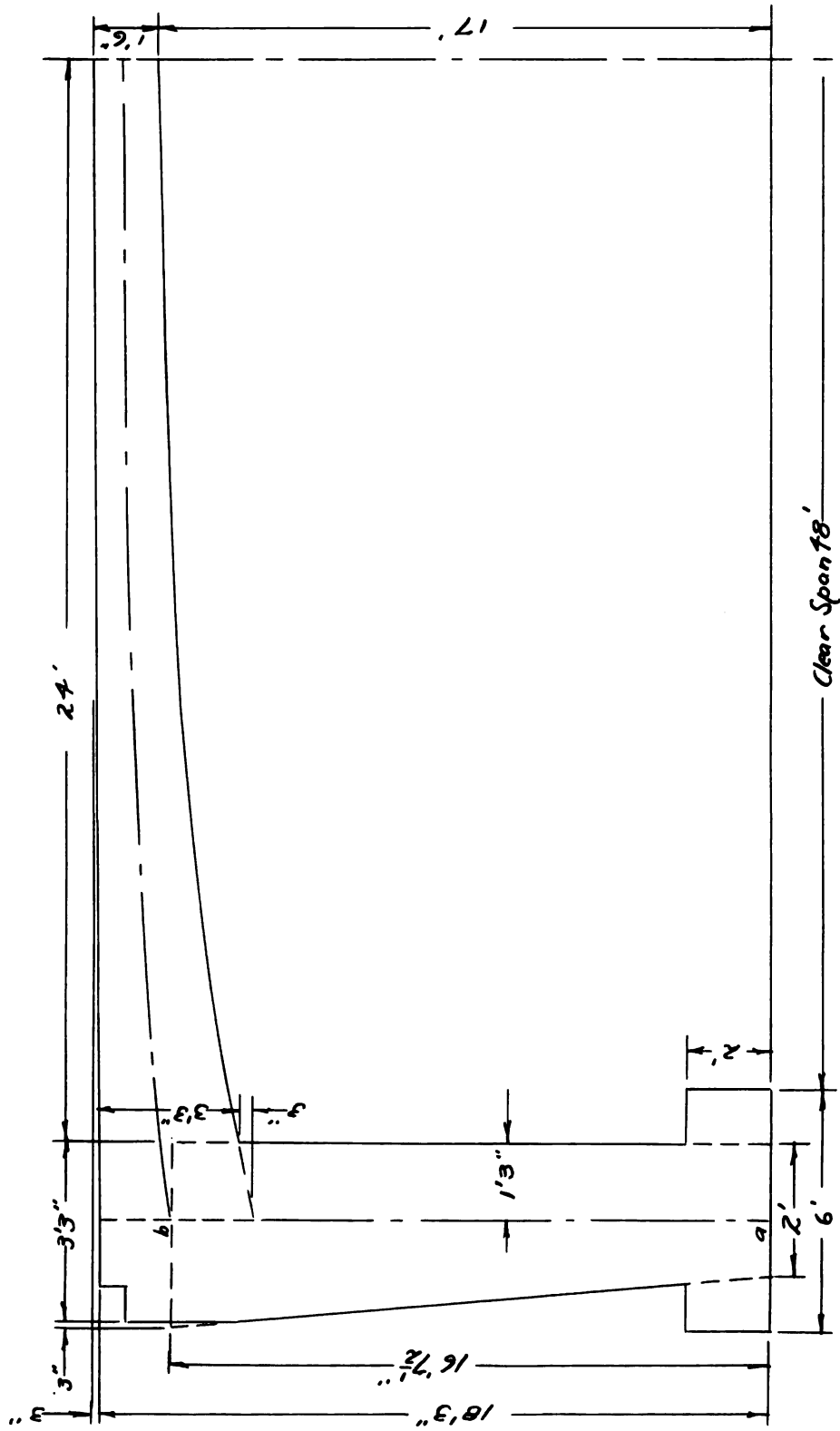
greater excavation. The shallow bridge floor lowers the overpass roadway and thus lowers the grade line. This reduces the amount of approach fill.

Not only are rigid frames low in first cost, but also maintenance and replacement charges are considerably less.

Widening of rigid frame bridges can be done easily, with slight alteration of the existing structure.

ACKNOWLEDGEMENT

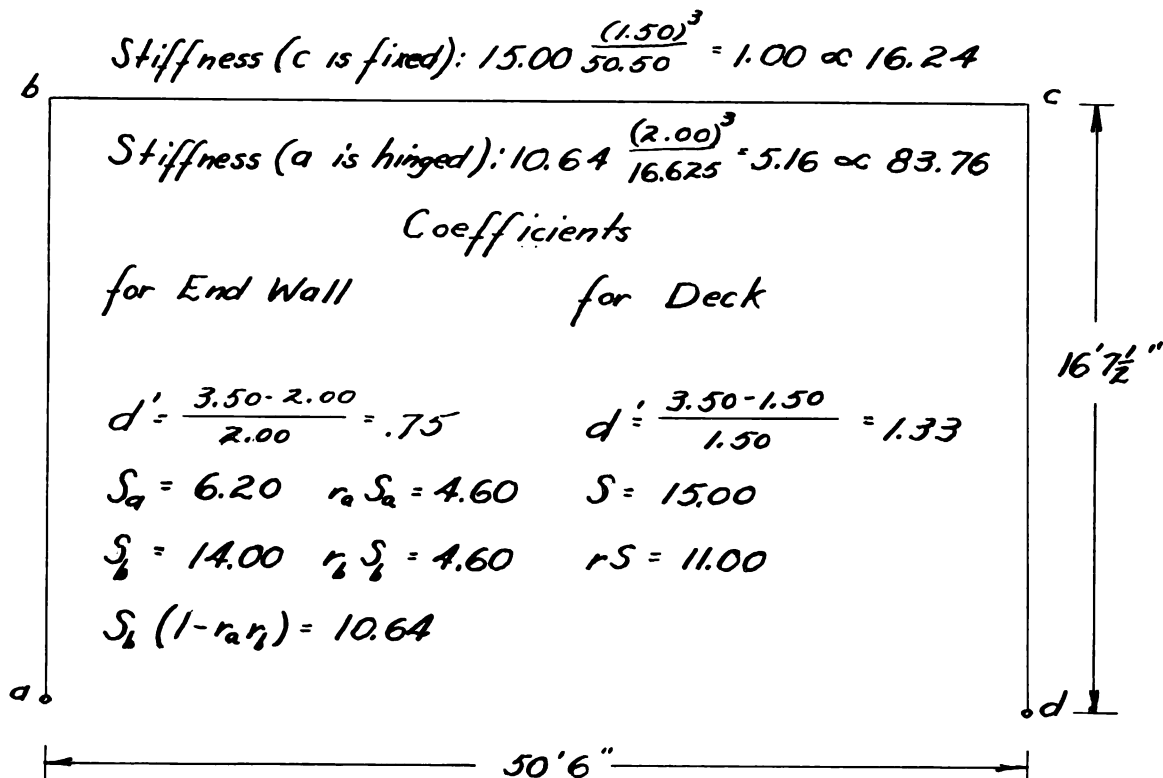
The author wishes to take this opportunity to thank Professor C. A. Miller of the Civil Engineering Department for the valuable assistance and advice he has given. And also, to thank those in the Bridge Design Division of the Michigan State Highway Department, and those in the local office of the Portland Cement Association who have extended their very courteous assistance.



Frame Dimensions

I. DESIGN OF THE FRAME

A. Frame Dimensions, Axes, and Coefficients



Deck Coefficients: -

$$d' = \frac{3.50 - 1.50}{1.50} = 1.33$$

From Chart II

$$s = 15.00$$

$$rs = 11.00$$

$$S \propto \frac{I}{L}, \text{ or proportional to } 15.00 \propto \frac{(1.50)^3}{50.50} = 1.00$$

$$\text{Carry over factor, } r, \text{ equals: } \frac{rS}{S} = \frac{11.00}{15.00} = 0.67$$

End Wall Coefficients: -

$$d' = \frac{3.50 - 2.00}{2.00} = 0.75$$

$$S_a = 6.20, S_b = 14.00, r_a S_a = r_b S_b = 4.60$$

$$S_b \times \frac{1}{L} (1 - r_a r_b) = 14.00 \times \left(1 - \frac{(4.60)^2}{14.00 \times 6.20}\right) \times \frac{1}{L}$$

$$= 10.64 \times \frac{1}{L}$$

or proportional to

$$10.64 \times \frac{(2.00)^3}{16.625} = 5.16$$

The relative stiffness in per cent at "b" is then

$$\frac{1.00}{1.00+5.16} \times 100 = 16.24 \text{ for the deck}$$

$$\frac{5.16}{1.00+5.16} \times 100 = 83.76 \text{ for the wall}$$

The charts used in selecting the coefficients are found in "Analysis of Rigid Frame Concrete Bridges", a publication of the Portland Cement Association.

B. Distribution of Fixed End Moment

	b	16.24	.67	16.24	c
0	83.76	+100.00		0	83.76
-83.76	-16.24	1 { Fixed End Moments		0	0
		2 { Distributed "		0	0
0	0			-10.90	0
0	0	2 { Carry Over "		+1.77	+9.13
		3 { Distributed "			
0	+1.19			0	0
-1.00	-.19	3 { Carry Over "		0	0
		4 { Distributed "			
0	0			-.13	0
0	0			+.02	+.11
-84.76	+84.76	TOTAL MOMENTS		-9.24	+9.24
	a				d

Computations

1st Cycle

$$- 100 \times .1624 = - 16.24 \text{ in bc}$$

$$- 100 \times .8376 = - 83.76 \text{ in ba}$$

Total moments at end of 1st cycle

$$- 83.76, + 83.76 \text{ at "b"; at "c" zero}$$

2nd Cycle

Moment carried from b to c equals

$$- 16.24 \times .67 = - 10.90$$

Distributed Moments at "c" are

$$+10.90 \times .1624 = +1.77 \text{ in bc}$$

$$+10.90 \times .8376 = +9.13 \text{ in cd}$$

Total moment at end of 2nd cycle

- 83.76, + 83.76 at b

+ 9.13, - 9.13 at c

3rd Cycle

+ 1.77 x .67 = + 1.19

- 1.19 x .1624 = -0.19 in bc

- 1.19 x .8376 = - 1.00 in ba

Total moment at end of 3rd cycle

- 84.76, + 84.76 at b

- 9.13, + 9.13 at c

4th cycle

- .19 x .67 = - 0.13

+ .13 x .1624 = + 0.02 in bc

+ .13 x .8376 = + 0.11 in cd

If a fixed end moment of - 100.00 is applied in "ab" at "b" (i.e., in the end wall immediately below the corner joint) and the joints are then released, the final corner moments become: - 100.00 + 83.76 + 1.00 = - 15.24 in "ab" at "b"; + 16.24 - 1.19 + .19 = + 15.24 in "bc" at "b"; + 9.24 and - 9.24 at "c"

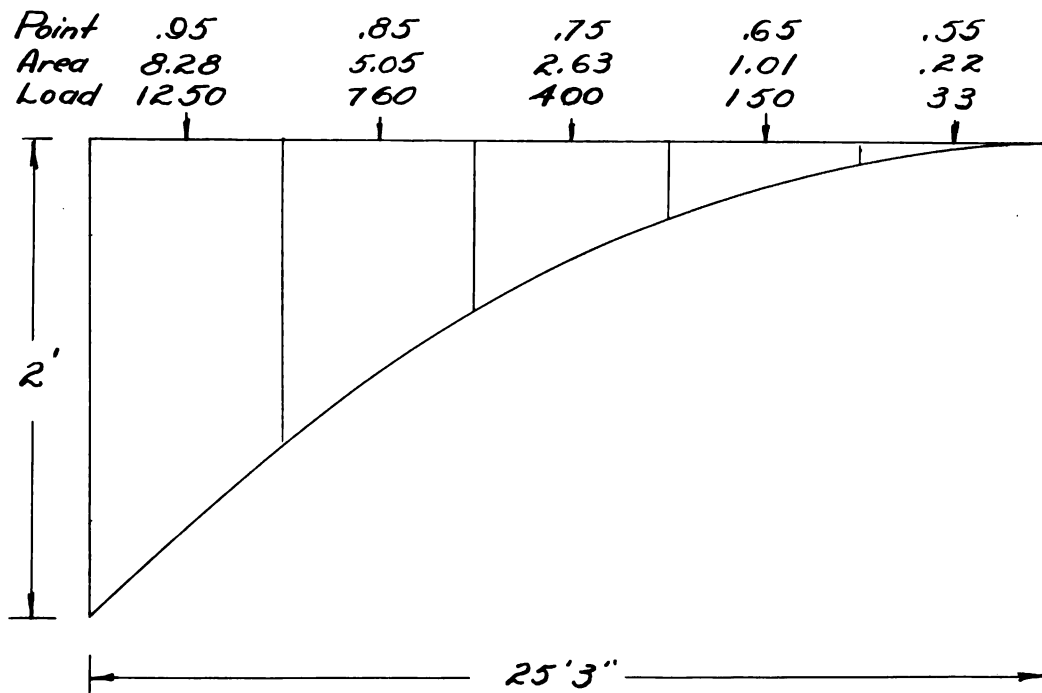
(These relative moment values will be useful in the subsequent analyses by eliminating repetition of computations).

C. Dead Load

Michigan State Highway Department Specifications (1936) call for the following in regard to dead loads: "For structures with concrete slab floors without separate wearing surface, a minimum allowance of 20 #/sq.ft. of roadway shall be made, in addition to the weight of any monolithically placed concrete wearing surface, to provide for future wearing surface."

(The weight of concrete, plain or reinforced = 150#/c.f. In addition to the dead load of the frame an additional concrete thickness of $\frac{1}{2}$ " was allowed for an integral roadway wearing surface.

No moment is created by the weight of the end walls since it is carried directly down to the footings. The longitudinal section through the deck is divided into an area 50 ft. long and depth of 1 ft. $6\frac{1}{2}$ in., weighing 230 p.s.f. The remaining area is subdivided as follows:



Fixed end moment per foot of width

Uniform load:

$$230 + 20 = 250 \text{ p.s.f.}$$

$$250 \times 50.5^2 \times .105 = 66900$$

Equivalent concentrated loads

$$1250 \times 50.5 \times .050 = 3155$$

$$760 \times 50.5 \times .130 = 4980$$

$$400 \times 50.5 \times .185 = 3740$$

$$150 \times 50.5 \times .210 = 1590$$

$$\begin{array}{rcl}
 33 \times 50.5 \times .195 & = & 325 \\
 33 \times 50.5 \times .145 & = & 242 \\
 150 \times 50.5 \times .087 & = & 658 \\
 400 \times 50.5 \times .040 & = & 808 \\
 760 \times 50.5 \times .012 & = & 460 \\
 1250 \times 50.5 \times .002 & = & \frac{126}{82984 \text{ ft.lb.}}
 \end{array}$$

Using the values of 84.76 and 9.24 per cent determined in Part B the numerical values of the corner moments at "b" are

$$82984 \times .8476 \text{ due to F.E.M. at "b"}$$

$$82984 \times .0924 \text{ due to F.E.M. at "c"}$$

The total moment when the deck is straight is:

$82984 \times (.8476 + .0924) = 78000 \text{ ft. lb.}$ and produces tension in the outside of the corner

Correcting for curvature of the deck (rise of deck centerline = $1 \text{ ft. } 1\frac{1}{2} \text{ in.}$) the final corner moment is

$$78000 \times \frac{16.625 + 1.125 \times .5}{16.625 + 1.125} = 75500 \text{ ft.lb.}$$

The crown moment for straight deck centerline can be found by statics. The total positive moment assuming a simply supported deck is:

$$\begin{array}{rcl}
 1250 \times .05 \times 50.5 & = & 3153 \\
 760 \times .15 \times 50.5 & = & 5750 \\
 400 \times .25 \times 50.5 & = & 5050 \\
 150 \times .35 \times 50.5 & = & 2650 \\
 33 \times .45 \times 50.5 & = & 750 \\
 250 \times 25.25 \times .25 \times 50.5 & = & \frac{79600}{96953 \text{ ft. lb.}}
 \end{array}$$

The difference between this moment and the negative corner moment created by the same loading is the moment at the crown of the frame with straight deck and is:

$$96953 - 78000 = 18953 \text{ ft. lb.}$$

Correcting for curvature of deck the final crown moment (tension in bottom of deck) is:

$$18953 \times \frac{16.625 + 1.125 \times .5}{16.625 + 1.125} = 18350 \text{ ft. lb.}$$

Check the final crown and corner moments

	Corner	Crown
From Chart I	-75448	+14140
From Moment Distribution	-75500	+18350

The total dead load of the frame (1 ft. wide) is:

$$\text{Wearing surface: } (20 + 6.25) 54.5 = 1430$$

$$\text{Deck: } 1.50 \times 50.5 \times 150 = 11360$$

$$\text{Deck: } .333 \times 2 \times 50.5 \times 150 = 5040$$

$$\text{Corners: } 2 \times 3.5 \times 2 \times 150 = 2100$$

$$\begin{aligned} \text{walls: } .5 \times (3.25 + 2) \times 14.75 \\ \quad \times 2 \times 150 = 11610 \end{aligned}$$

$$\begin{aligned} \text{Footings: } (5.00 - 2.00) \times 3 \times 2 \\ \quad \times 150 = \underline{2700} \end{aligned}$$

$$34240 \text{ lb.}$$

say, 35000 lb.

The vertical reaction on each footing is

$$.5 \times 35000 = 17500 \text{ lb.}$$

The horizontal thrust at the footing, when the deck is curved, is

$$\frac{75500}{16.625} = 4540 \text{ lbs.}$$

The crown thrust also equals 4540 lbs. since all loads are gravity loads.

D. Live Load

Live load shall be the standard H-20 truck train:

(Michigan State Highway Dept. Spec.)

Impact:

$$I = \frac{L + 20}{6L + 20} = \frac{70.5}{323} = .218$$

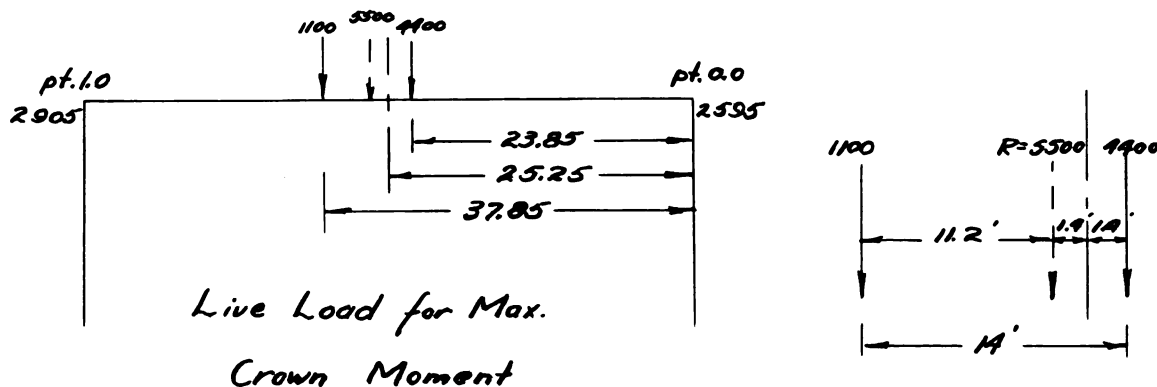
Load per foot of width due to front axle:

$$\frac{8000}{9} \times 1.218 = 1100 \text{ lbs.}$$

Load per foot of width due to rear axle:

$$\frac{32000}{9} \times 1.218 = 4400 \text{ lbs.}$$

Maximum moment is produced at the crown with the concentrated loads at the center of the span.



Fixed end moment per foot of width:

$$\text{at pt. 1.0; } 1100 \times 50.5 \times .187 = 10400$$

$$\text{at pt. 1.0; } 4400 \times 50.5 \times .160 = \underline{35600}$$

46000 ft. lb.

$$\text{at pt. 0.0; } 1100 \times 50.5 \times .04 = 2220$$

$$\text{at pt. 0.0; } 4400 \times 50.5 \times .188 = \underline{41750}$$

43970 ft. lb.

Corner moment after distribution (straight deck):

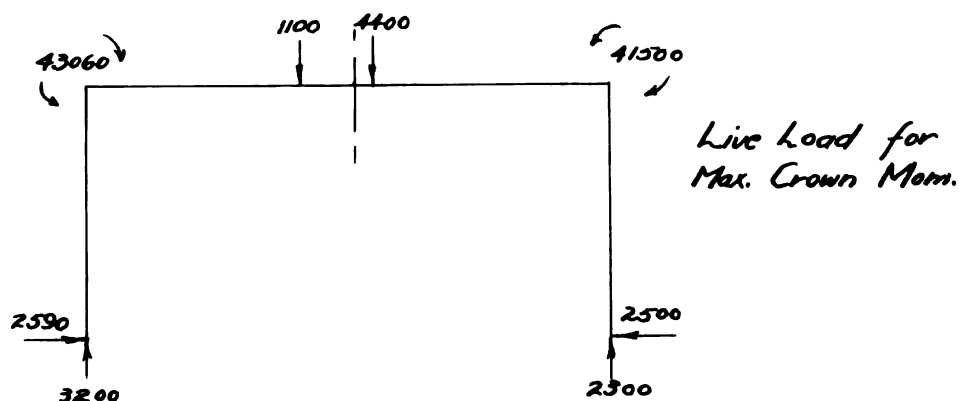
$$\text{at 1.0; } 46000 \times .8476 + 43970 \times .0924 = 39000 + 4060 = 43060$$

$$\text{at 0.0; } 46000 \times .0924 + 43970 \times .8476 = 4250 + 37250 = 41500$$

The corresponding horizontal thrusts (straight deck) are:

$$\text{at 1.0; } \frac{43060}{16.625} = 2590\#$$

$$\text{at 0.0; } \frac{41500}{16.625} = 2500\#$$



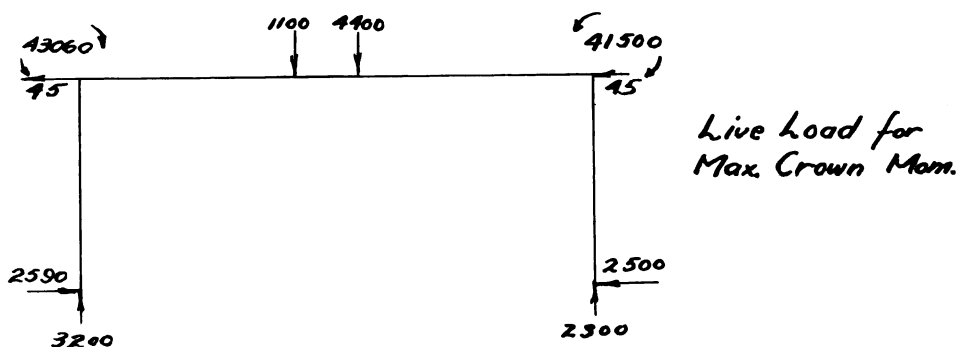
The laws of equilibrium require a force of $2590 - 2500 = 90$ lb.

The vertical reaction at "a" then becomes:

$$4400 \times \frac{23.85}{50.50} + 1100 \times \frac{37.85}{50.50} + 90 \times \frac{16.625}{50.50} = 3200 \text{ lb.}$$

Vertical reaction at "d":

$$4400 \times \frac{26.65}{50.50} + 1100 \times \frac{12.65}{50.50} - 90 \times \frac{16.625}{50.50} = 2300 \text{ lb.}$$



The total positive crown moment assuming a simply supported deck:

$$2905 \times 25.25 - 1100 \times 12.60 = 73300 - 13860 = 59440 \text{ ft. lb.}$$

The crown moment (straight deck) is:

$$+ 59440 - 43060 + 45.0 \times 16.625 = +17128$$

Correcting for curvature the crown moment becomes:

$$17128 \times \frac{16.625 + 1.125 \times .5}{16.625 + 1.125} = 17128 \times .968 = 16600 \text{ ft. lb.}$$

The final corner moment, after curvature correction, is:

$$43060 \times .968 = 41700 \text{ ft. lb.}$$

The horizontal thrust at "a" is:

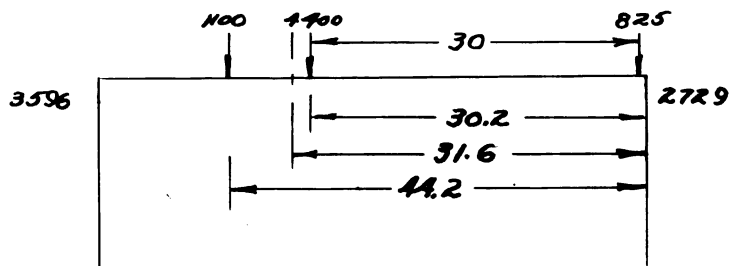
$$\frac{41700}{16.625} = 2510 \text{ lb.}$$

$$\text{or; } 2590 \times .968 = 2510 \text{ lb.}$$

Check the final crown and corner moments:

	Corner	Crown
From Chart I	40680	15042
From Moment Distribution	41700	16600

Maximum moment at the corner is produced when the concentrated loads are at point .625 (pt. 0.0 being at the right hand corner).



*Live Load for Max.
Corner Mom.*

Fixed end moment per foot of width:

$$\text{at pt. 1.0; } 1100 \times 50.5 \times .110 = 6100$$

$$\text{at pt. 1.0; } 4400 \times 50.5 \times .208 = \underline{46150}$$

52250 ft. lb.

$$\text{at pt. 0.0; } 1100 \times 50.5 \times .011 = 610$$

$$\text{at pt. 0.0; } 4400 \times 50.5 \times .119 = \underline{26400}$$

27010 ft. lb.

The values of "k" for the 825 lb. load approach zero so closely that the load was not considered when computing the fixed end moments.

Corner moment after distribution (straight deck):

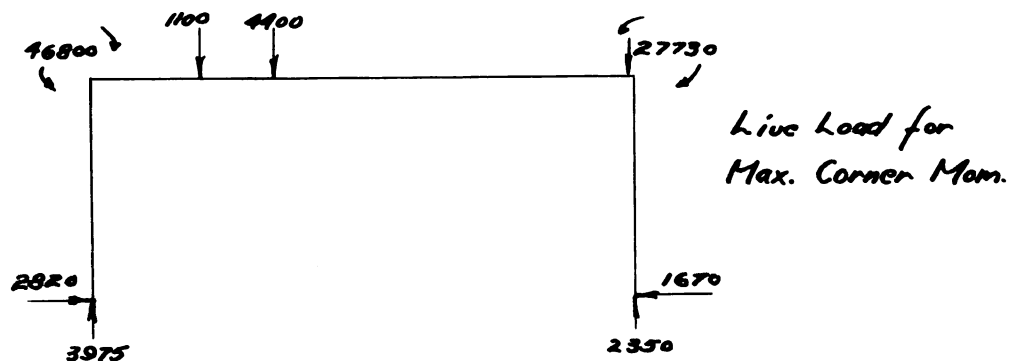
$$\begin{aligned} \text{at 1.0; } 52250 \times .8476 + 27010 \times .0924 &= 44300 + 2500 \\ &= 46800 \end{aligned}$$

$$\begin{aligned} \text{at 0.0 ; } 52250 \times .0924 + 27010 \times .8476 &= 4830 + 22900 \\ &= 27730 \end{aligned}$$

The corresponding horizontal thrusts (straight deck) are:

$$\text{at 1.0; } \frac{46800}{16.625} = 2820 \text{ lb.}$$

$$\text{at 0.0; } \frac{27730}{16.625} = 1670 \text{ lb.}$$



The laws of equilibrium require a force of

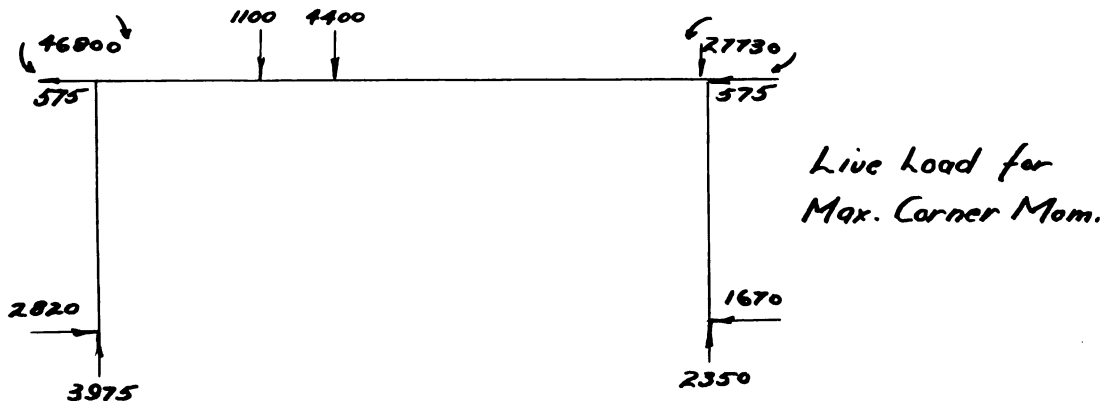
$$2820 - 1670 = 1150 \text{ lb.}$$

The vertical reaction at "a" then becomes:

$$4400 \times \frac{30.2}{50.5} + 1100 \times \frac{44.2}{50.5} + 825 \times \frac{.2}{50.5} + 1150 \times \frac{16.625}{50.5} = 3975 \text{ lb.}$$

Vertical reaction at "d":

$$4400 \times \frac{20.3}{50.5} + 1100 \times \frac{6.3}{50.5} + 825 \times \frac{50.3}{50.5} - 1150 \times \frac{16.625}{50.5} = 2350 \text{ lb.}$$



The total positive crown moment, assuming a simply supported deck:

$$3596 \times 25.25 - 1100 \times 18.95 - 4400 \times 4.95 = 90800 - 42650 = 48150 \text{ ft.lb.}$$

The crown moment, (straight deck) is:

$$+48150 - 46800 + 575 \times 16.625 = 10900 \text{ ft. lb.}$$

Correcting for curvature, the crown moment becomes:

$$10900 \times .968 = 10560 \text{ ft. lb.}$$

The final corner moment, after curvature corrections, is:

$$46800 \times .968 = 45400 \text{ ft. lb.}$$

The horizontal thrust at "a" is:

$$\frac{45400}{16.625} = 2725 \text{ lb.}$$

$$\text{or; } 2820 \times .968 = 2725 \text{ lb.}$$

Check the final crown and corner moments:

	Corner	Crown
From Chart I	46360	10120
From Moment Distribution	45400	10380

E. Change in Length of Deck and Horizontal Displacement

A relative change in length of deck may be either a shortening, due to temperature drop, shrinkage, and outward displacement of footings, or a lengthening due to temperature rise and an inward displacement of the footings.

Assume the frame subject to a deck shortening due to (a) temperature drop of 60° F , (b) shrinkage corresponding to a shrinkage factor of .0002, and (c) outward displacement of the footings equivalent to a contraction coefficient of .0002. The coefficient of thermal expansion is .0000065.

The shortening per unit of length is:

$$60 \times .0000065 + .0002 + .0002 = .00079$$

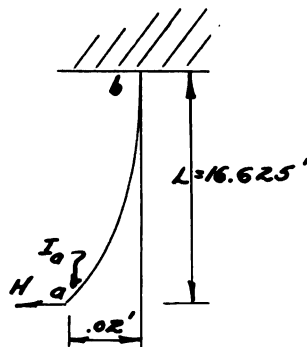
The total shortening in the span of 50.5 feet is:

$$.00079 \times 50.5 = .0399 \text{ ft.}$$

This is equivalent to an outward displacement of .0200 ft. at the footings.

When analyzing the frame by moment distribution begin by locking the joints "b" and "c".

The static conditions from which the fixed end moment at "b" is determined are illustrated by the following sketch.



It can be shown that:

$$\begin{aligned} \text{Fixed end moment at "b"} &= \frac{E \times D}{L} \times S_b \times \frac{I_a}{L} \times (1 - r_a r_b) = \\ &= \frac{(3 \times 10^6 \times 12^2) \times .02}{16.625} \times 14.0 \times \frac{(1/12) \times 2^3}{16.625} \times (1 - \frac{4.6^2}{14.0 \times 6.2}) = \\ &= 264500 \end{aligned}$$

According to Part "B" the formula for both corner and crown moments, when the deck is straight, may be written as:

$$\begin{aligned} \text{Moment} &= 264500 \times I(1.0000 - .8376 - .0100) - .0924 I \\ &= 264500 \times .0600 = 15880 \text{ ft. lb.} \end{aligned}$$

Final moments, correcting for curvature:

$$\text{Corner: } 15880 \times \left(\frac{16.625}{16.625 + 1.125} \right)^2 = 13920 \text{ ft. lb.}$$

$$\text{Crown: } 15880 \times \left(\frac{16.625}{16.625 + 1.125} \right) = 14880 \text{ ft. lb.}$$

Or the moments can be obtained by substitution in the empirical formula:

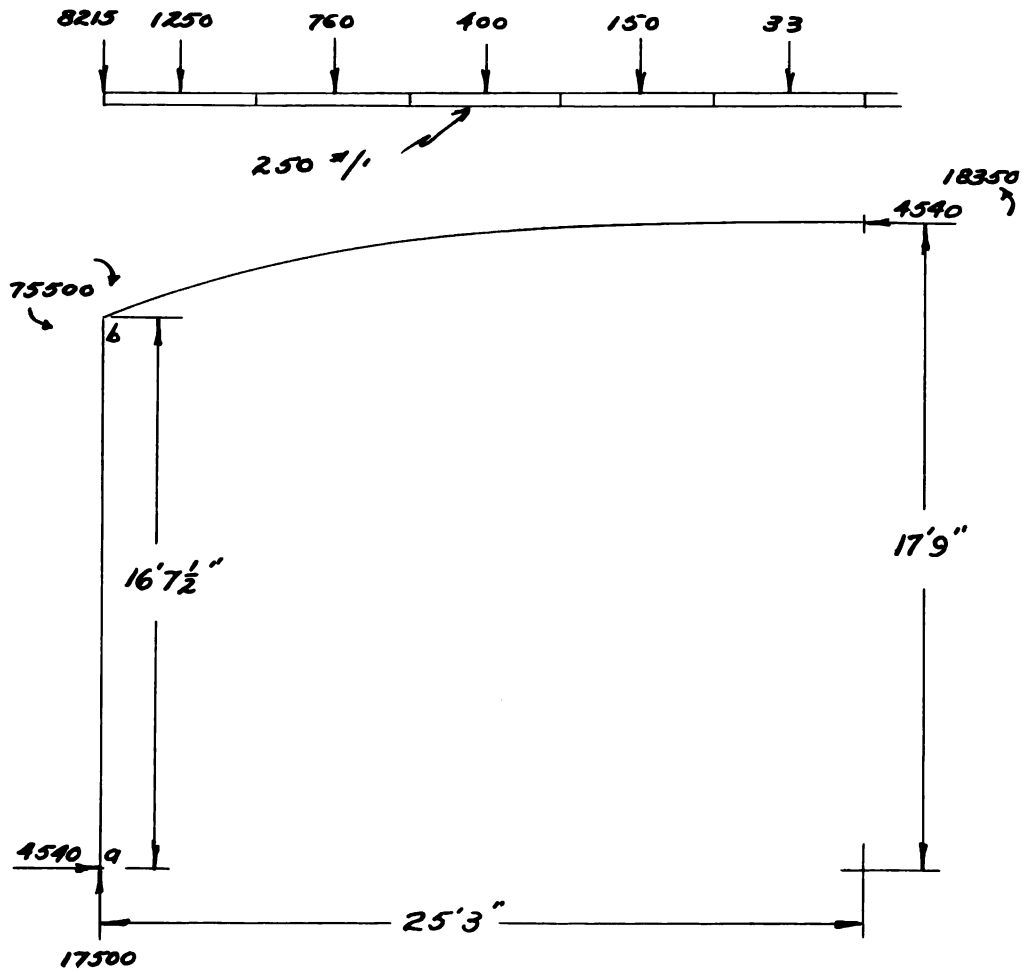
$$\begin{aligned} \text{Corner moment} &= 4.35 \times 10^6 \times \frac{H \times D}{(H + R)^2} \times (\text{Crown thickness})^2 \\ &= 4.35 \times 10^6 \times \frac{16.625 \times .02}{(16.625 + 1.125)^2} \times 1.5^2 \\ &= 10320 \text{ ft. lb.} \end{aligned}$$

$$\text{Crown moment} = 10320 \times \frac{16.625 + 1.125}{16.625} = 11020 \text{ ft. lb.}$$

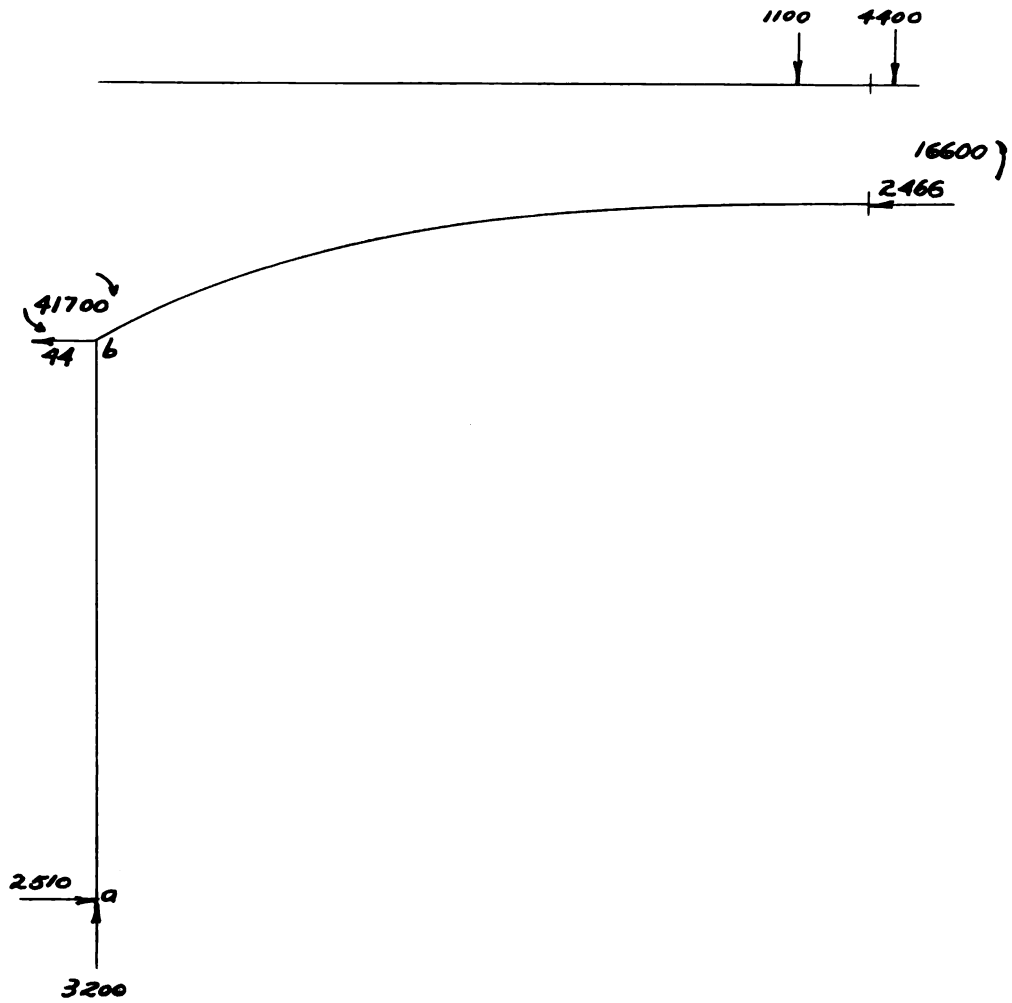
The values obtained by moment distribution will be used.

The horizontal thrust is:

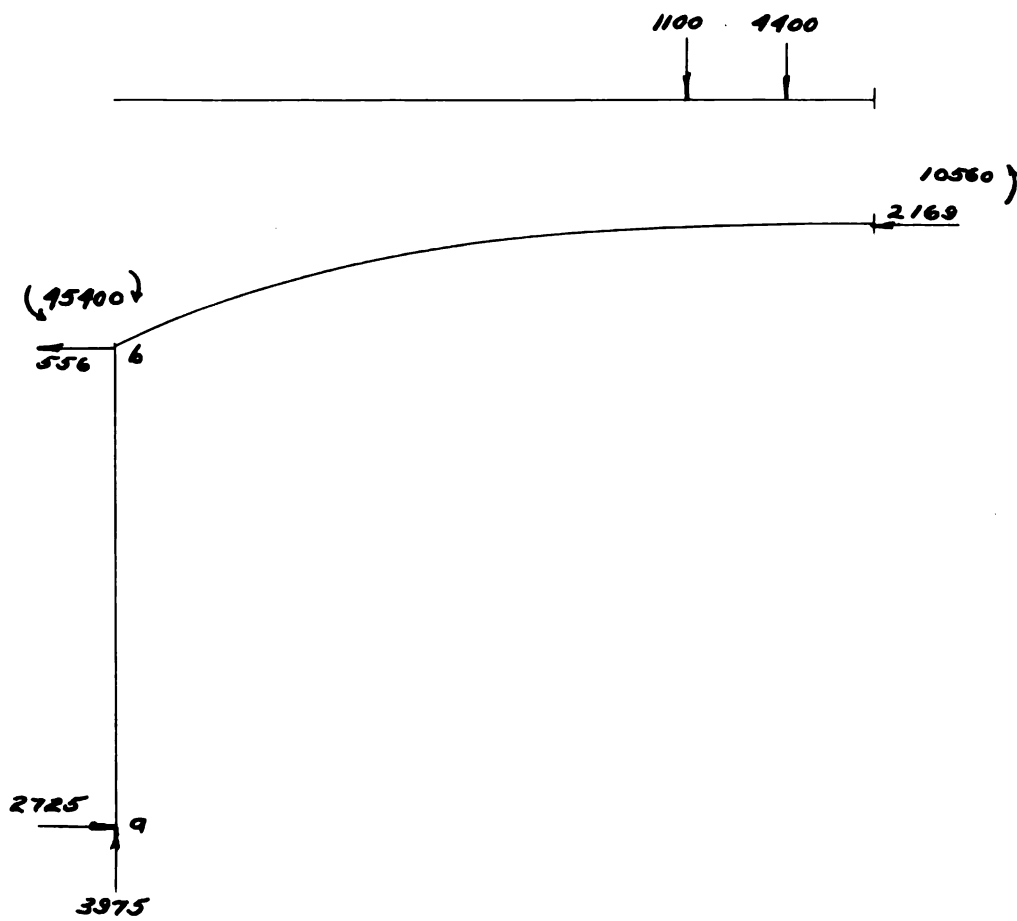
$$\frac{13920}{16.625} = 838 \text{ lb.}$$



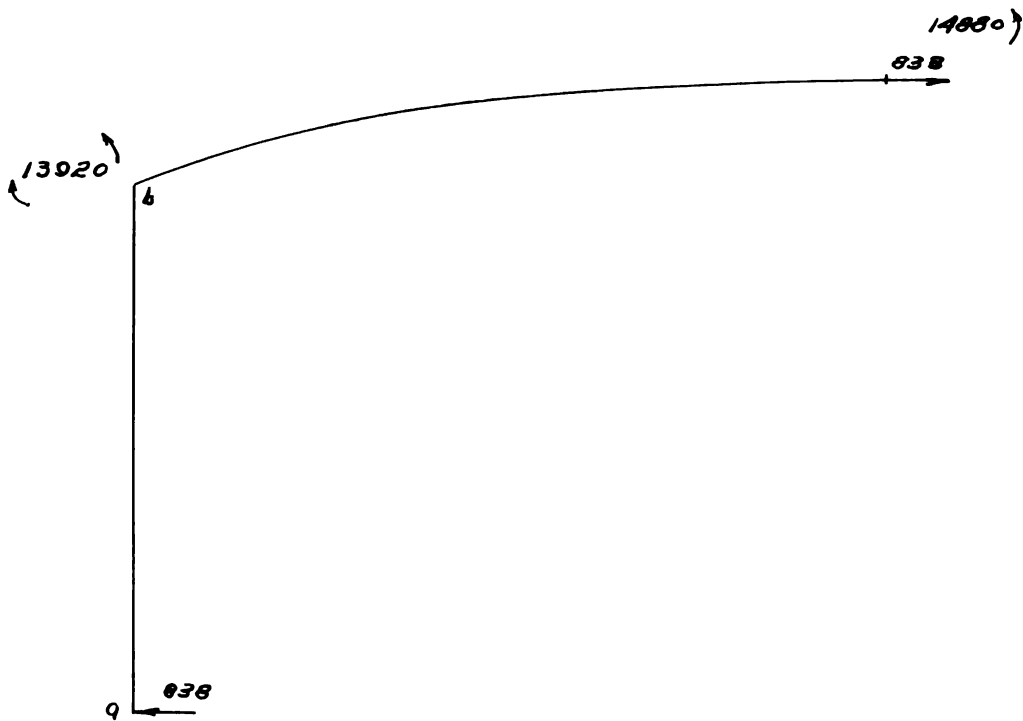
Dead load



Live Load for
Max. Crown Moment
(Sidesway Prevented)



Live Load for
Max. Corner Moment
(Sideways Prevented)



*Deck Shortening and
Footing Displacement*

F. Earth Pressure

Moment distribution gives satisfactory results, for a frame subject to earth pressure, only when the walls are relatively high and slender compared with the deck, (when the wall height is greater than .4 of the span length).

Passive earth pressure can be developed, in addition to active pressure, by flexure under live load. Tests have been run on actual bridges and the results showed that very little passive pressure was developed.

The earth pressure moments are usually a very small percentage of the moments caused by the dead plus the live load and can be disregarded. The dead and live loads create tension in the intrados, while tension is created in the extrados by the earth pressure. It is, therefore, more conservative to disregard the earth pressure moment at the crown.

G. Dissymmetry and Sidesway

Under unsymmetrical loading the moment distribution method gives horizontal thrusts which apparently do not satisfy the statical requirements for equilibrium.

For instance, in Part D, when the live loads are at the center of the span, the corner moments for straight deck are:

at point 1.0: 43060 ft. lb.

at point 0.0: 41500 ft. lb.

The corresponding horizontal thrusts are:

at 1.0: 2590 lb.

at 0.0: 2500 lb.

The algebraic sum of the horizontal forces is not zero, which means a tendency for deck "bc" to move sideways.

If the frame is to remain with the points "b" and "c" vertically above "a" and "d", respectively, sidesway must be prevented. This can be done by the application of a horizontal force of 90 lb. in the line "bc". The force of 90 lb. will increase the vertical reaction at "a" and decrease that at "d" by an amount equal to:

$$90 \times \frac{16.625}{50.50} = 295 \text{ lb.}$$

The condition approached most closely by the ordinary rigid frame highway bridge is that in which there is no sidesway. Greater corner moment is given by the assumption of no sidesway.

The following computations illustrate how the results obtained by moment distribution can be adjusted for the assumption that sidesway is permitted.

The added force of 90 lb. creates the same horizontal thrust at both footings, it being 45 lb.

The total horizontal thrust at both footings is then:

$$2590 - 45 = 2500 + 45 = 2545 \text{ lb.}$$

The corner moments at joints "a" and "b" are:

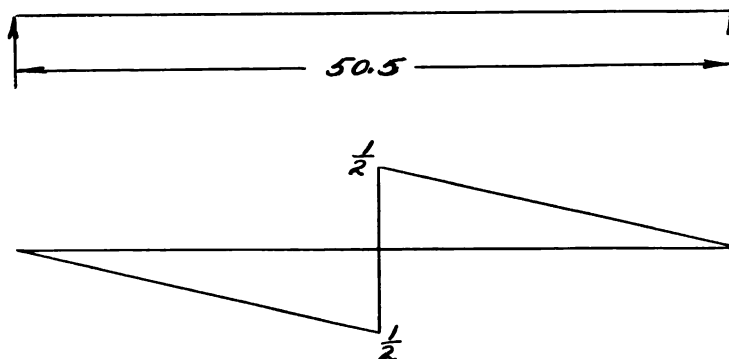
$$2545 \times 16.625 = 42400 \text{ ft. lb.}$$

The maximum negative corner moment when sidesway is prevented is: 43060 ft. lb.

H. Shears

The maximum shear and unit stresses at (1) the crown, (2) the corner, and (3) the top of the footing were computed in the following section.

(1) Crown. The shear due to dead load, deck shortening, and symmetrical earth pressure is zero. Sidesway was assumed to be permitted, making the shears in the deck equal to the shears in a simply supported beam of span equal to 50.5 ft. Shear due to live load was found as follows:



Influence line for Shear at Center

It can be seen by reference to the influence line, that the maximum negative shear will occur with the 4400 lb. load at a "dx" distance to the left of the midpoint. The maximum positive shear will occur with the 1100 lb. load a distance of "dx" to the right of the midpoint.

Maximum positive shear:

$$+1100 \times \frac{1}{2} + 4400 \times \frac{11.25}{25.25} \times \frac{1}{2} = 550 + 920 = 1530 \text{ lb.}$$

Maximum negative shear:

$$1100 \times \frac{11.25}{25.25} \times \frac{1}{2} - 4400 \times \frac{1}{2} = -246 - 2200 = -2446 \text{ lb.}$$

The corresponding unit stress is:

$$v = \frac{V}{bjd} = \frac{2446}{12 \times \frac{7}{8} \times 16} = 14.6 \text{ p.s.i.}$$

Investigation of the shear based on the assumption of side-sway prevented is unwarranted.

(2) Corner. The total dead load from face to face of end walls is:

$$\text{Wearing surface: } (20 + 6.25)48 = 1260$$

$$\text{Deck: } 1.50 \times 150 \times 48 = 10800$$

$$\text{Deck: } .333 \times 2 \times 48 \times 150 = \underline{4800}$$

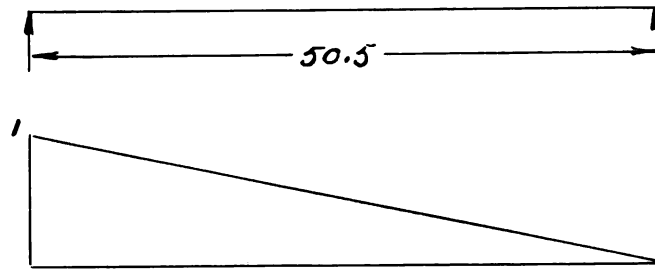
$$16860 \text{ lb.}$$

The maximum shear due to dead load is:

$$\frac{16860}{2} = 8430 \text{ lb.}$$

Deck shortening and symmetrical earth pressure produce no vertical shears.

Shear due to live load was found as follows:



Influence Line for Shear at Left End

It can be seen from the influence line that the maximum shear will occur with the 4400 lb. load a distance "dx" to the right of the left end. The shear will be:

$$+4400 \times 1 + 825 \times \frac{20.5}{50.5} + 3300 \times \frac{6.5}{50.5} = 5160 \text{ lb.}$$

The total dead and live load shear is:

$$8430 + 5160 = 13590 \text{ lb.}$$

The corresponding unit shearing stress is:

$$v = \frac{V}{bjd} = \frac{13590}{12 \times \frac{7}{8} \times 37} = 35 \text{ p.s.i.}$$

(3) Top of footing. The maximum shear equals the horizontal thrust at the support. The dead load thrust is:

$$\frac{75500}{16.625} = 4540 \text{ lb.}$$

Maximum live load thrust is produced by the same load arrangement that produces maximum corner moment, and is:

$$\frac{45400}{16.625} = 2725 \text{ lb.}$$

Since the horizontal thrusts produced by earth pressure and deck shortening counteract the thrusts due to dead and live load, it is more conservative to disregard the earth pressure and deck shortening and take the maximum shear as:

$$4540 + 2725 = 7265 \text{ lb.}$$

The corresponding unit stress is:

$$v = \frac{V}{b_j d} = \frac{7265}{12 \times \frac{7 \times 24}{8}} = 28.8 \text{ p.s.i.}$$

$$\text{Allowable} = 60 \text{ p.s.i.}$$

I. Stresses at Crown and Corner

The tensile and compressive reinforcement was computed using a 3000 lb. concrete, and the corresponding unit stresses were checked to see whether the allowable working stresses, ($f_c = 1200$ p.s.i., $f_s = 20000$) were exceeded.

Moments and thrusts at the midpoint of the deck are:

	Moment	Axial Thrust
Dead Load	+ 18350	+4540
Live Load	+ 16600	+2466
Change in Deck Length	+ <u>14880</u>	- 838
Total	+ 49830	+6168

Eccentricity with respect to centerline:

$$\frac{49830}{6168} = 8.08 \text{ ft.}, \text{ say, } 97 \text{ inches}$$

The tensile steel area for this moment and thrust must be somewhat less than that required when the axial thrust is disregarded; namely:

$$A_s = \frac{M_s}{f_s j d} = \frac{49830 \times 12}{20000 \times \frac{7 \times 16}{8}} = 2.14 \text{ sq. in.}$$

A tensile reinforcement of 1 in. square bars spaced at 6 in. will be chosen.

With this reinforcement, axial thrust still disregarded, the extreme fiber stress in concrete is less than 900 p.s.i. This stress will be raised by the addition of axial thrust, and compressive steel of 1 in. square bars at 12 in. will be chosen.

The depth to the neutral axis in the concrete section equals

$$d (\sqrt{2pn + (pn)^2} - pn) = 5.82$$

when $d = 16$ in., $A' = 2.00$ sq. in., $n = 10$, and E is infinity. Addition of the axial thrust will tend to increase the effective depth to, say, 6.20 in.. The section coefficients with the estimated value of $Z = 6.20$ will be:

Estimated Section Coefficients	Cor- rection	Corrected Value
$A = 12 \times 6.2 + (10-1) \times 1.00 + 10 \times 2.00 = 103.40$	-4.67	98.73
$Q = \frac{1}{2} \times 12 \times 6.2^2 + 9.00 \times 2 + 20.0 \times 16 = 569.00$	-4.67 $\times$ 6.01	540.90
$I = \frac{1}{3} \times 12 \times 6.2^3 + 9.00 \times 2^2 + 20.0 \times 16^2 = 6109.00$	-4.67 $\times$ 6.01 ²	5940.20
$E = 97.0 - .5 \times 18 = 88.0$		

If Z were correctly chosen, it should satisfy the equation:

$$Z = \frac{I + ExQ}{Q + ExA} = \frac{6109.0 + 88 \times 569.0}{569.0 + 88 \times 103.4} = \frac{56209}{9669} = 5.81$$

Using the second value of $Z = 5.81$, correct A , Q , and I used above for the discrepancy in effective concrete area, which equals

$$12 \times (6.20 - 5.81) = 4.67 \text{ sq. in.}$$

the center of which is at a distance of

$$5.81 + \frac{1}{2} (6.20 - 5.81) = 6.01 \text{ in.}$$

below the extreme concrete fiber.

The final value of Z is:

$$Z = \frac{5940.2 + 88 \times 540.9}{540.9 + 88 \times 98.7} = 5.81 \text{ in.}$$

$$g = \frac{Q}{A} = \frac{540.9}{98.7} = 5.48$$

$$e = E + g = 88.00 + 5.48 = 93.48$$

$$I_g = I - \frac{Q^2}{A} = 5940.2 - \frac{540.9^2}{98.7} = 3015.2$$

then:

$$f_c = \frac{P \times e \times Z}{I_g} = \frac{6168 \times 93.48 \times 5.81}{3015} = 1110$$

$$f_s'' = \frac{P \times e \times (z-c)n}{I_g} = \frac{6168 \times 93.48 \times 3.81 \times 10}{3015} = 726$$

$$f_s' = \frac{P \times e \times (d-Z)n}{I_g} = \frac{6168 \times 93.48 \times 10.19 \times 10}{3015}$$

$$= 19500$$

Checking against numerical errors:

$$kd = \frac{f_c d}{f_c + f_s' / n} = \frac{1110 \times 16}{1110 + 1950} = 5.81 = Z$$

Moments and thrusts at the corner are

	Moment	Axial Thrust
Dead Load	-75500	+17500
Live Load	<u>-45400</u>	<u>+3975</u>
Total	-120900	+21475

Eccentricity with respect to center line:

$$\frac{120900}{21475} = 5.63 \text{ ft. or } 67.5 \text{ in.}$$

If the axial thrust is disregarded, the steel area required in tension is:

$$\frac{120900 \times 12}{20000 \times \frac{7}{8} \times 37} = 2.24 \text{ sq. in.}$$

Carry the 1 in square bars at 12 in. from the top of the deck around the corner and add in each interval between these bars one 1-1/8 in. square bar, making a total tensile steel area of 2.27 sq. in. per lin. ft. at the corner. No compression reinforcement will be included in the following stress determination.

The depth to the neutral axis in the concrete section equals

$$d \left(\sqrt{2pn + (pn)^2} - pn \right) = 10.1 \text{ in.}$$

when $d = 37$ in., $A = 2.27$ sq. in., $n = 10$, and E is infinity. Adding the axial thrust will tend to increase the effective depth to, say, 13.5. The section coefficients with the estimated value of $Z = 13.5$ will be:

Estimated Section Coefficients	Correction	Corrected Value
$A = 12 \times 13.5 + 10 \times 2.27 = 184.7$	- 13.8	170.9
$Q = \frac{1}{2} \times 12 \times 13.5^2 + 2.27 \times 37 = 1935.0$	- 13.8 $\times$ 12.93	1756.5
$I = \frac{1}{3} \times 12 \times 13.5^3 + 2.27 \times 37^2 = 40940.0$	- 13.8 $\times$ 12.93 ²	38630.0
$E = 67.5 - .5 \times 37 = 49.0$		

Second value of Z is:

$$Z = \frac{40940 + 49.0 \times 1935}{1935 + 49.0 \times 185} = \frac{135640}{10995} = 12.35$$

Correct A , Q , and I :

$$12 \times (13.5 - 12.35) = 13.8 \text{ sq. in.}$$

$$12.35 + \frac{1}{2} (13.5 - 12.35) = 12.93 \text{ in.}$$

The final value of Z is:

$$Z = \frac{38630 + 49.0 \times 1757}{1757 + 49.0 \times 171} = 12.3$$

$$g = \frac{1757}{171} = 10.27$$

$$e = 49.0 + 10.27 = 59.27$$

$$I_g = 38630 - \frac{1757^2}{171} = 20580$$

$$f_c = \frac{21475 \times 59.27 \times 12.3}{20580} = 761$$

$$f_s' = \frac{21475 \times 59.27 \times 24.7 \times 10}{20580} = 15300$$

Checking the value of Z

$$k_d = \frac{761 \times 37}{761 + 1530} = 12.3 = k$$

The unit stresses of $f_c = 1110$, $f_c = 761$, $f_s = 19500$ and $f_s = 15300$ are conservative, since they include the combined effects of dead load, live load, temperature drop, shrinkage and spreading of footings. They will be reduced by the effect of symmetrical earth pressure, which has been disregarded.

II DESIGN OF THE FOOTINGS

A 3000 lb. concrete was used in the design of the footings. The bearing power of the soil was taken as 2 T per sq. ft. and the net pressure was assumed to be uniform.

Load:

Dead load reaction (Part C) per foot of width	=	17500
Maximum live load reaction " " " "	=	4385
Estimated footing weight	=	<u>1900</u>
Total		23785

$$\text{Area footing} = \frac{23785}{4000} = 5.95 \text{ sq. ft.}$$

Bending Moment: The footing is 6 ft. long and projects 24.0 in. on each side of the frame as a cantilever beam.

$$\text{Net pressure } p = \frac{21885}{6.00} = 3650 \text{ lb./sq. ft.}$$

The section at the edge of the frame is a rectangular beam 1 ft. wide.

$$\text{Maximum moment} = \frac{wl^2}{2} = \frac{3650 \times (24)^2}{12 \times 2} = 87600 \text{ in.lb.}$$

$$d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{87600}{197 \times 12}} = 6.09 \text{ in., say, 6.1 in.}$$

The steel is at the bottom of the footing. Assuming that projecting stones, clay, muck puddles, etc., may spoil 2 in. of concrete, and adding 3 in. for damp proofing, and 12 in. for the key on the bottom of the frame, the minimum depth becomes:

$$h = d + 17 = 23 \text{ in.}$$

Shear:

At the face of the frame:

$$\text{Maximum shear } V = w l = 3650 \times 2.00 = 7300 \text{ lb.}$$

$$\text{Maximum shear stress } v = \frac{V}{b j d} = \frac{7300}{12 \times \frac{7}{8} \times 6.1} = 114$$

Allowable shear $v = .03 f_c' = 90 \text{ p.s.i.}$ (special anchorage of long. bars)

Diagonal Tension: The first failure crack to extend completely through the footing will start at the face of the frame and extend outward at an angle of 45° . The diagonal tension will be computed at a distance of $d = 6.1 \text{ in.}$ from the face of the frame. The shear is due to the net soil pressure on the extension of the footing beyond this section.

$$v = \frac{V}{b j d} = \frac{3650 \times 1.49}{12 \times \frac{7}{8} \times 6.1} = 85$$

Allowable $v = .03 f_c' = 90 \text{ lb./sq. in.}$ (special anchorage)

Weight:

The footing weighs

$$23/12 \times 6.00 \times 150 = 1725 \text{ lb.}$$

$$\text{Assumed wt.} = 1900 \text{ lb.}$$

Steel:

$$A_s = \frac{M}{f_s j d} = \frac{87600}{20000 \times \frac{7}{8} \times 6.1} = .82 \text{ sq. in.}$$

Use 3/4 in. round bars at. 6 in. $A = .88$

Bond at the face of the frame:

$$u = \frac{V}{\phi f_d} = \frac{7300}{4.71 \times 7 \times 6.1} = 290$$

Allowable $u = .080 f_c' = 240 \text{ lb./sq. in. (special anchorage)}$

Change d to 8.0 in. making $h = 25 \text{ in.}$

The new values, using $d = 8 \text{ in.}$, are:

Shear : $v = 87.0$ Allowable = 90 (Special anchorage)

Diagonal Tension: $v = 57.8$ Allowable = 60 (ordinary anchorage)

Weight: $25/12 \times 6.00 \times 150 = 1875 \text{ lb.}$

Steel: $A_s = .627 \text{ sq. in.}$

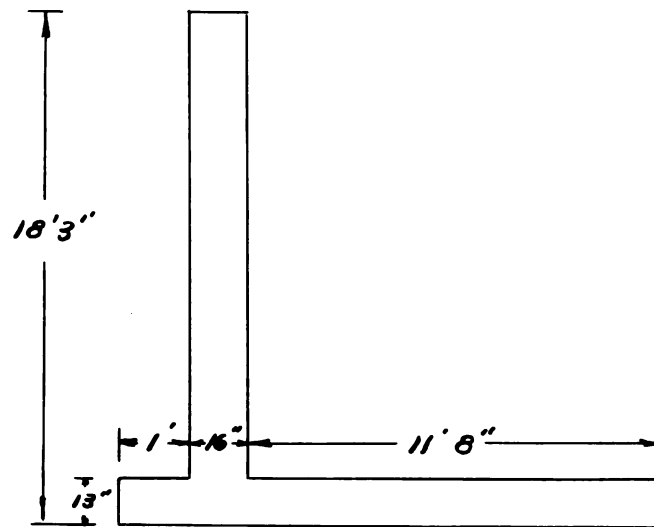
Use 5/8 in. round bars at 5 in. $A = .745$

$u = 222$ Allowable = 240 (special anchorage)

Bars are to be 6 ft. plus 12.5 in.

or 6 ft. 14 in. long.

III Design of the Wingwalls

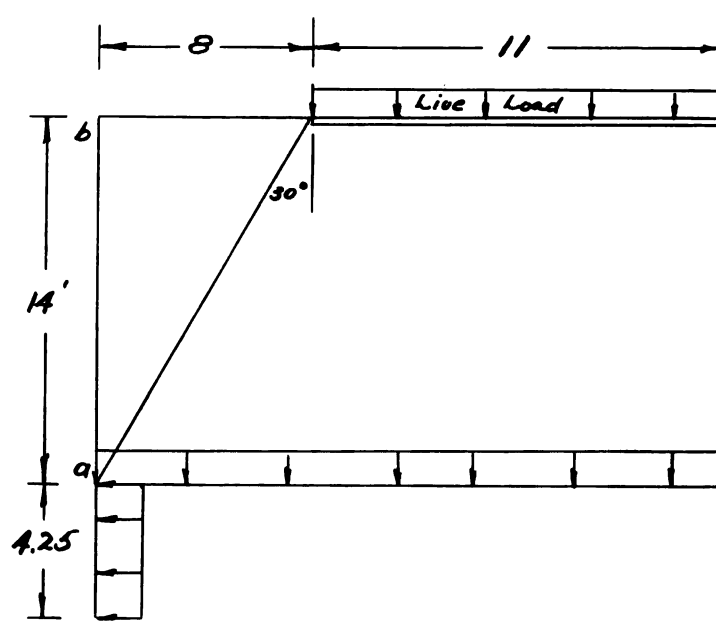


$$f'_c = 3000 \quad f_s = 20000$$

$$p = .0113 \quad K = 197$$

$$j = \frac{7}{8} \quad n = 10$$

Earth Pressure:



$$ab = \frac{8}{\tan 30^\circ} = 13.85 \text{ ft.}, \text{ say } 14 \text{ ft.}$$

$$\frac{32000}{11} \times 1.215 = 3540 \text{ lb./ft. of width (live load)}$$

Combined dead and live load pressure on 11' slab =

$$3540 + 125 \text{ (10 in. slab)} = 3665 \text{ lb./ft.}$$

$$\frac{3665 \times 11}{19} = 2060 \text{ lb./sq. ft. at 14 ft. down}$$

$$\theta = 0^\circ, \phi = 30^\circ, w = 120 \text{ lb./c.f.}$$

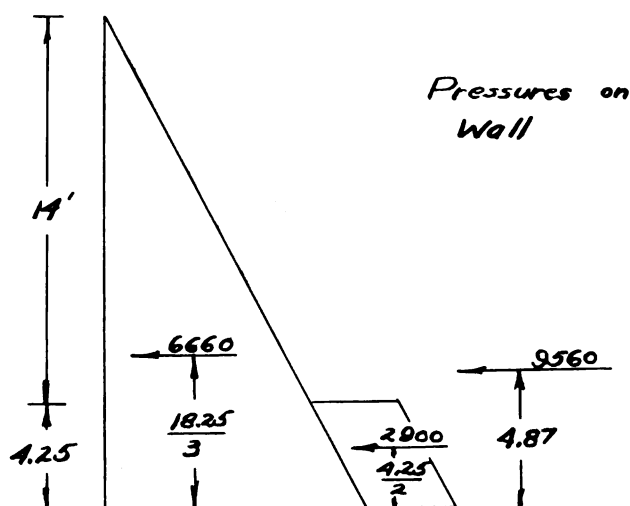
$$c_e = \cos \theta \frac{\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \phi}}{\cos \theta + \sqrt{\cos^2 \theta - \cos^2 \phi}}$$

$$= 1 \frac{1 - \sqrt{1 - .75}}{1 + \sqrt{1 - .75}} = \frac{1 - .5}{1 + .5} = .333$$

$$\text{Side P.} = 2060 \times .333 = 682 \text{ lb./sq. ft.}$$

$$\text{Earth Pressure} = c_e \frac{wh^2}{2} = .333 \frac{120 \times (18.25)^2}{2} =$$

$$= 6660 \text{ lb.}$$



$$9560 \bar{x} = 6660 \times \frac{18.25}{3} + 2900 \times \frac{4.25}{2}$$

$$\bar{x} = 4.87 \text{ ft.}$$

Vertical Forces:

$$\text{wt. of earth} = 11.67 \times 17.17 \times 120 = 24050$$

$$\text{wt. stem} = 1.33 \times 17.17 \times 150 = 3420$$

$$\text{wt. base slab} = 14 \times 1.08 \times 150 = \underline{2270}$$

$$29740 \text{ lbs.}$$

Pt. of application of vert. forces:

$$y = \frac{2270 \times 7 + 3420 \times 1.67 + 24050 \times 8.17}{29740} = \frac{218100}{29740}$$

$$= 7.34 \text{ ft.}$$

$$\frac{x}{9560} = \frac{4.87}{29740} \quad x = 1.56$$

$$\text{Toe dist.} = 7.34 - 1.56 = 5.78 \text{ ft.}$$

$$e = 7.00 - 5.78 = 1.22 \text{ ft.}$$

Soil Pressure:

$$s = \frac{P}{b} \left(1 \pm \frac{6e}{b} \right)$$

$$= \frac{29740}{14} \left(1 \pm \frac{6 \times 1.22}{14} \right)$$

$$= 2125 (1.523) = 3235 \text{ lbs.}$$

$$= 2125 (.477) = 1015 \text{ lbs.}$$

Sliding:

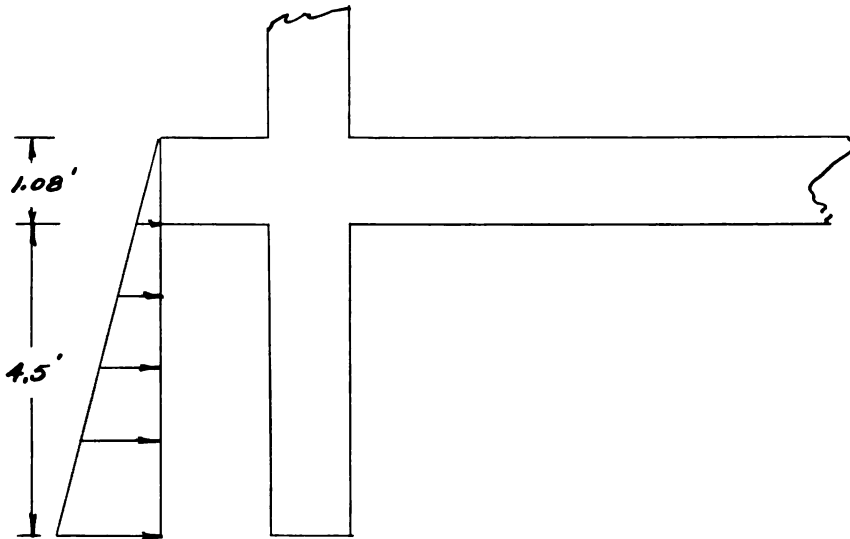
$$f = \frac{29740 \times .33}{9560} = 1.03$$

Overturning:

$$f = \frac{29740 \times 7.34}{9560 \times 4.87} = 4.7$$

To overcome the small factor of safety against sliding a friction key was put on the bottom of the base slab.

Friction Key:



$$C_e = \cos \theta \frac{\cos \theta + \sqrt{\cos^2 \theta - \cos^2 \beta}}{\cos \theta - \sqrt{\cos^2 \theta - \cos^2 \beta}} = \frac{1 + .5}{1 - .5} = 3.00$$

Total Passive Earth Pressure:

$$P = C_e \frac{wh^2}{2} = 3.0 \frac{120 \times (5.6)^2}{2} = 5650$$

Pressure at 1.08 ft.:

$$P = 3.0 \frac{120 \times (1.08)^2}{2} = \frac{210}{5440 \text{ lbs.}}$$

Total Active Earth Pressure:

$$P = .333 \frac{120 \times (22.75)^2}{2} = 10350 \text{ lbs.}$$

Sliding:

$$f = \frac{(29740 + 1 \times 4.5 \times 150) \cdot 33 + 5440}{10350} = 1.50$$

Resultant of passive earth pressures:

$$5650 \times \frac{5.6}{3} = 210 \times 4.9 + 5440\bar{x}$$

$$x = \frac{10550 - 1030}{5440} = 1.75 \text{ ft.}$$

Bending Moment:

$$BM = 5440 \times 2.75 = 15000 \text{ ft. lb.}$$

Effective Depth:

$$d = \sqrt{\frac{M}{Kd}} = \sqrt{\frac{15000}{197}} = 8.73, \text{ say } 9 \text{ in.}$$

$$D = 9 + 3 = 12 \text{ in.}$$

Shear:

$$v = \frac{V}{bjd} = \frac{5440}{12 \times \frac{7}{8} \times 9} = 57.7$$

Allowable = 60 (ordinary anchorage)

Area Steel:

$$A_s = pbd = .0113 \times 12 \times 9 = 1.22 \text{ sq. in.}$$

$$\text{Use } \frac{7}{8} \text{ " round at } 4 \text{ " } A = 1.80$$

This particular size and spacing was used to permit the use of the stem steel as the steel for the key.

Bond:

$$u = \frac{V}{\Sigma ojd} = \frac{5440}{8.25 \times \frac{7}{8} \times 9} = 83.7$$

Allowable = 120 (plain bars, ordinary anchorage)

Stem:

Earth Pressure:

$$= C_e \frac{wh^2}{2} = .33 \times 120 \times \frac{(17.17)^2}{2} = 5840 \text{ lb.}$$

Bending Moment:

$$= 5840 \times \frac{17.17}{3} = 33400 \text{ ft. lb.}$$

Effective Depth:

$$= \sqrt{\frac{33400}{197}} = 13.0$$

$$D = 13 + 3 = 16 \text{ in.}$$

Area Steel:

$$= pbd = .0113 \times 12 \times 13 = 1.76 \text{ sq. in.}$$

$$\text{Use } \frac{7}{8} \text{ in. round at 4 in.} \quad A = 1.80$$

Bond:

$$= \frac{V}{\sum o_j d} = \frac{5840}{8.25 \times \frac{7}{8} \times 13} = 62.2$$

$$\text{Allowable} = 120$$

Shear:

$$= \frac{V}{b_j d} = \frac{5840}{12 \times \frac{7}{8} \times 13} = 42.8$$

$$\text{Allowable} = 60$$

Economy can be effected by using bars of several different lengths.

Cut off 2 bars out of 3. The area remaining is $\frac{1.80}{3}$

= .60 sq. in. This occurs at a distance of 10 ft. from

the top. The bars will end at $10 - 40 \times 7/8 = 7$ ft. 1 in., say, 7 ft. down from the top.

Cut off half of the remaining bars. $A_s = \frac{1.80}{6} = .3$ sq. in. This occurs at a distance of 7 ft. 2 in. from the top. The bars will end at 4 ft. 3 in. from the top. The remaining bars run to the top and are spaced at 2 ft. apart.

Temp. Steel:

In front of wall

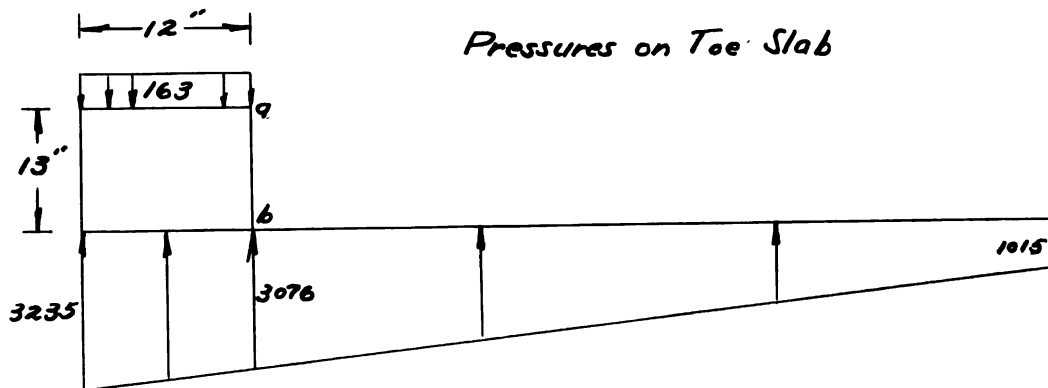
$$A_s = pbd = .0025 \times 12 \times 13 = .39$$

$$\text{Use } \frac{1}{2} \text{ in. round at 6 in. } A = .40$$

In back

$$\text{Use } \frac{1}{2} \text{ in. round at 12 in. } A = .20$$

Toe Slab:



Maximum Bending Moment:

$$\begin{aligned} BM_{ab} &= 3076 \times 1 \times \frac{1}{2} + 159 \times \frac{1}{2} \times \frac{2}{3} + 163 \times 1 \times \frac{1}{2} \\ &= 1509 \end{aligned}$$

Effective Depth:

$$d = \sqrt{\frac{1509}{197}} = 2.76 \text{ use 3 in.}$$

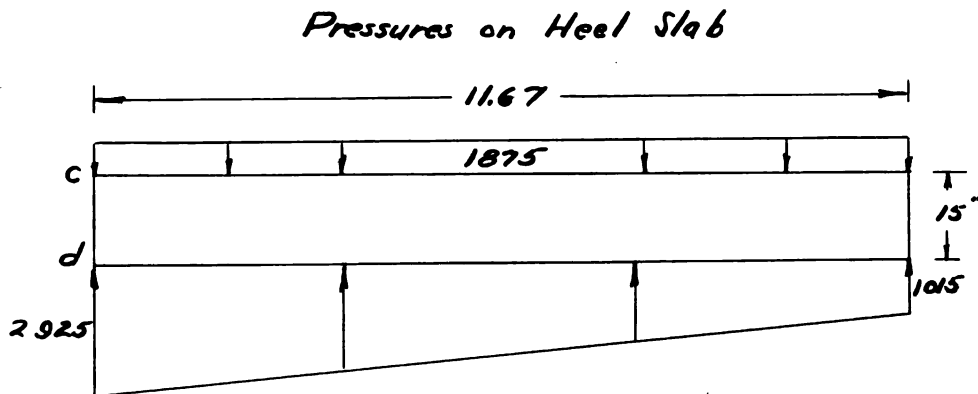
$$D = 3 + 3 + 2 = 8 \text{ in.}$$

Area Steel:

$$A_s = pbd = .0113 \times 12 \times 3 = .41$$

Because the toe slab is so small, the heel slab will be computed and the same d used for the toe slab.

Heel Slab:



Maximum Bending Moment:

$$\begin{aligned} BMed &= 1875 \times 11.67 \times \frac{11.67}{2} - 1015 \times 11.67 \times \frac{11.67}{2} \\ &\quad - 1910 \times \frac{11.67}{2} \times \frac{11.67}{3} = 15300 \end{aligned}$$

Effective Depth:

$$d = \sqrt{\frac{15300}{197}} = 8.8, \text{ say 9 in.}$$

$$D = 9 + 3 + 2 = 14 \text{ in.}$$

Area Steel:

$$A_s = pbd = .0113 \times 12 \times 9 = 1.22 \text{ sq. in.}$$

$$\text{Use } 7/8 \text{ in. round at } 5-3/4 \text{ in. } A = 1.25$$

Shear:

$$\text{Shear due to } 1875 \text{ lb. load} = 1875x$$

$$\text{" " " } 1015 \text{ " " } = 1015x$$

$$\text{" " " } 0 \text{ to } 1910 \text{ lb. load} = 82x^2$$

$$V_{x-x} = 1875x - 1015x - 82x^2$$

$$\frac{dV}{dx} = 860 - 164x$$

$$x = 5.25 \text{ ft.}$$

$$V = 860 \times 5.25 - 82(5.25)^2$$

$$= 4520 - 2260$$

$$= 2260$$

$$v = \frac{V}{bjd} = \frac{2260}{12 \times \frac{7}{8} \times 9} = 23.9$$

$$\text{Allowable} = 80$$

Bond:

$$u = \frac{V}{\phi b j d} = \frac{2260}{5.7 \times \frac{7}{8} \times 9} = 50.3$$

$$\text{Allowable} = 120$$

Toe Slab:

Area Steel:

$$A_s = pbd = .0113 \times 12 \times 9 = 1.22 \text{ sq. in.}$$

$$\text{Use } 7/8 \text{ in. round at } 5-3/4 \text{ in. } A = 1.25$$

Shear:

$$V = 163 \times 1 - 3076 \times 1 - 159 \times \frac{1}{2} = 2993$$

$$v = \frac{V}{b_j d} = \frac{2993}{12 \times \frac{7 \times 9}{8}} = 31.7$$

$$\text{Allowable} = 60$$

Bond:

$$u = \frac{V}{\Sigma o_j d} = \frac{2993}{5.7 \times \frac{7 \times 9}{8}} = 66.7$$

$$\text{Allowable} = 120$$

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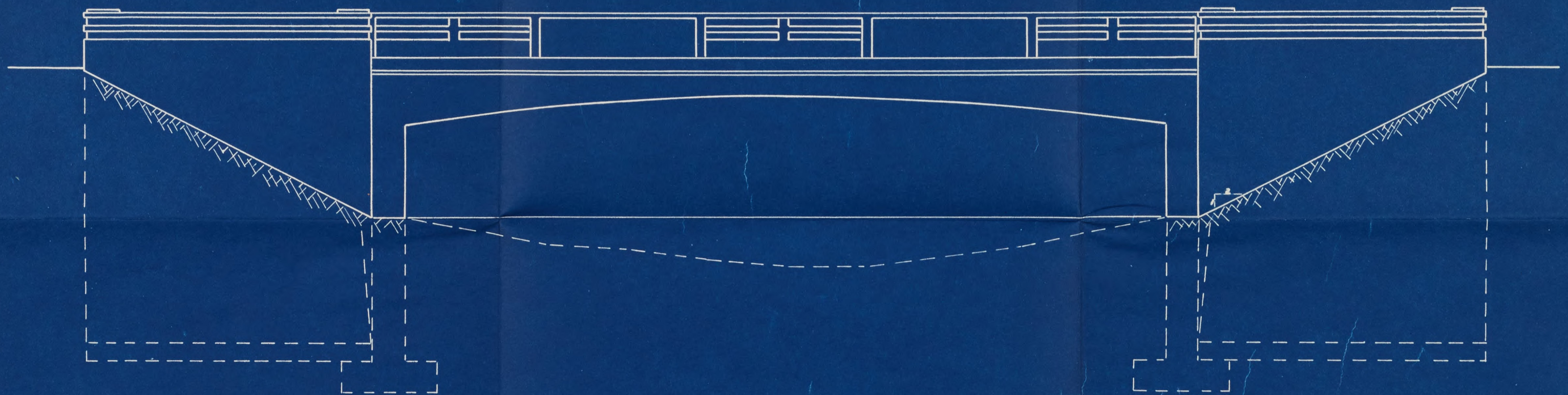
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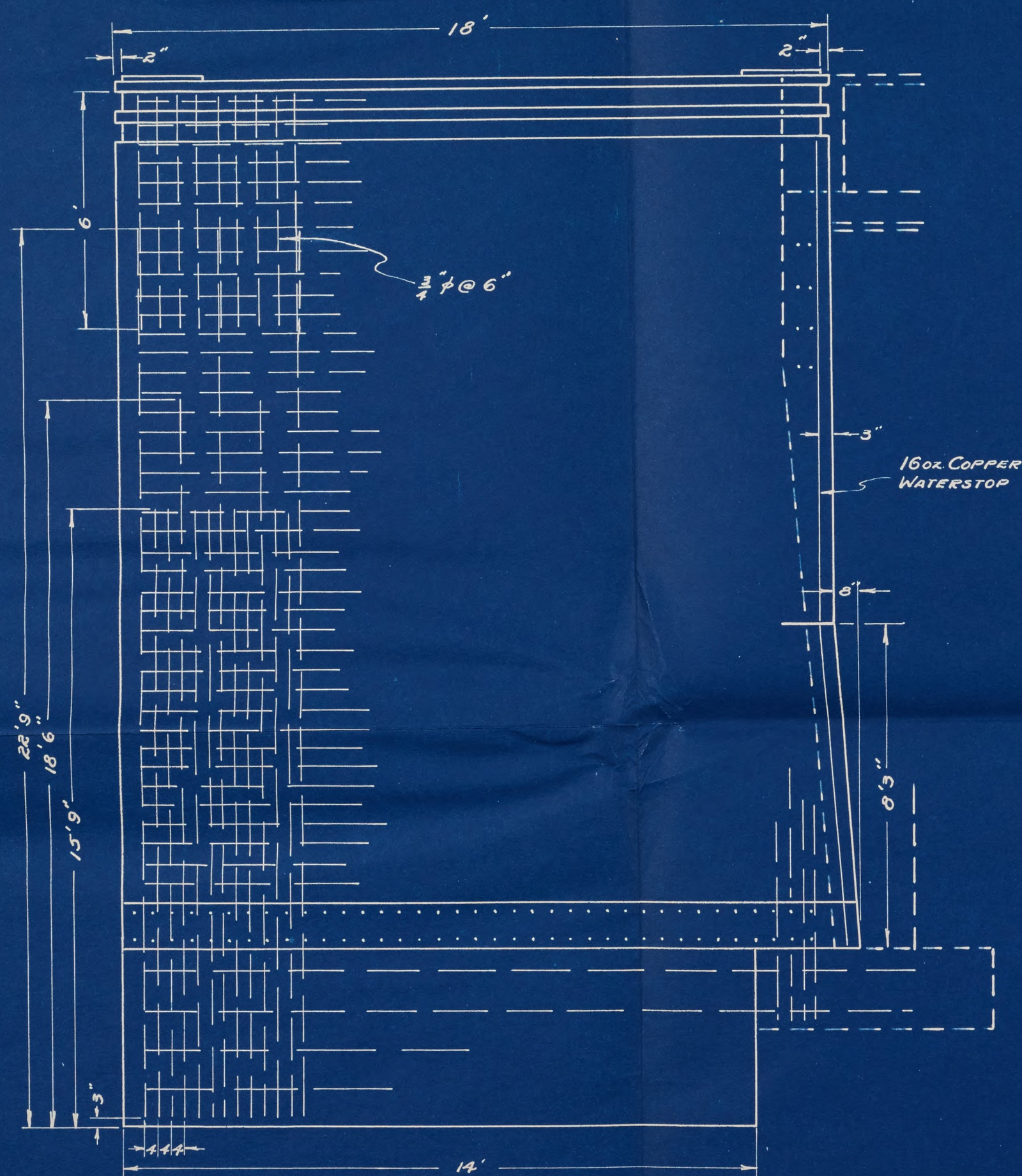
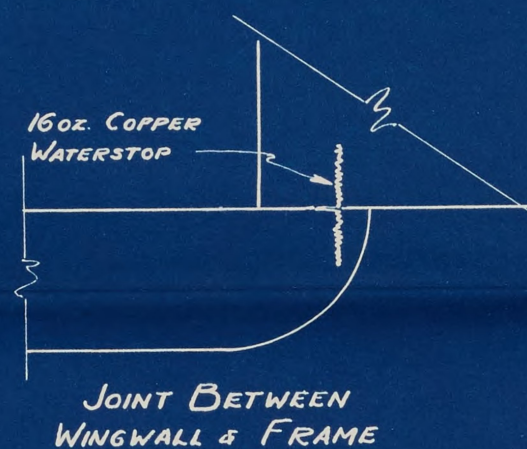
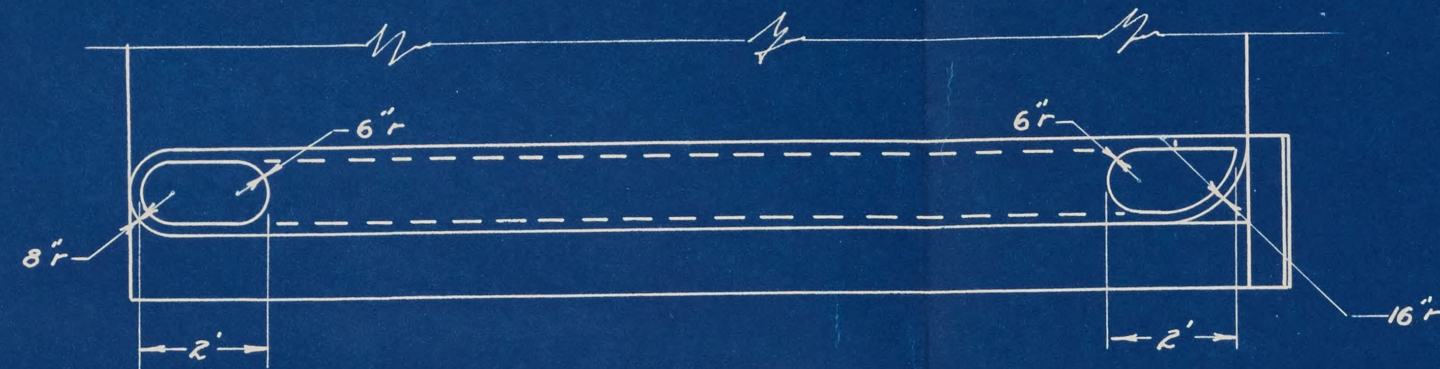
RICE CREEK BRIDGE



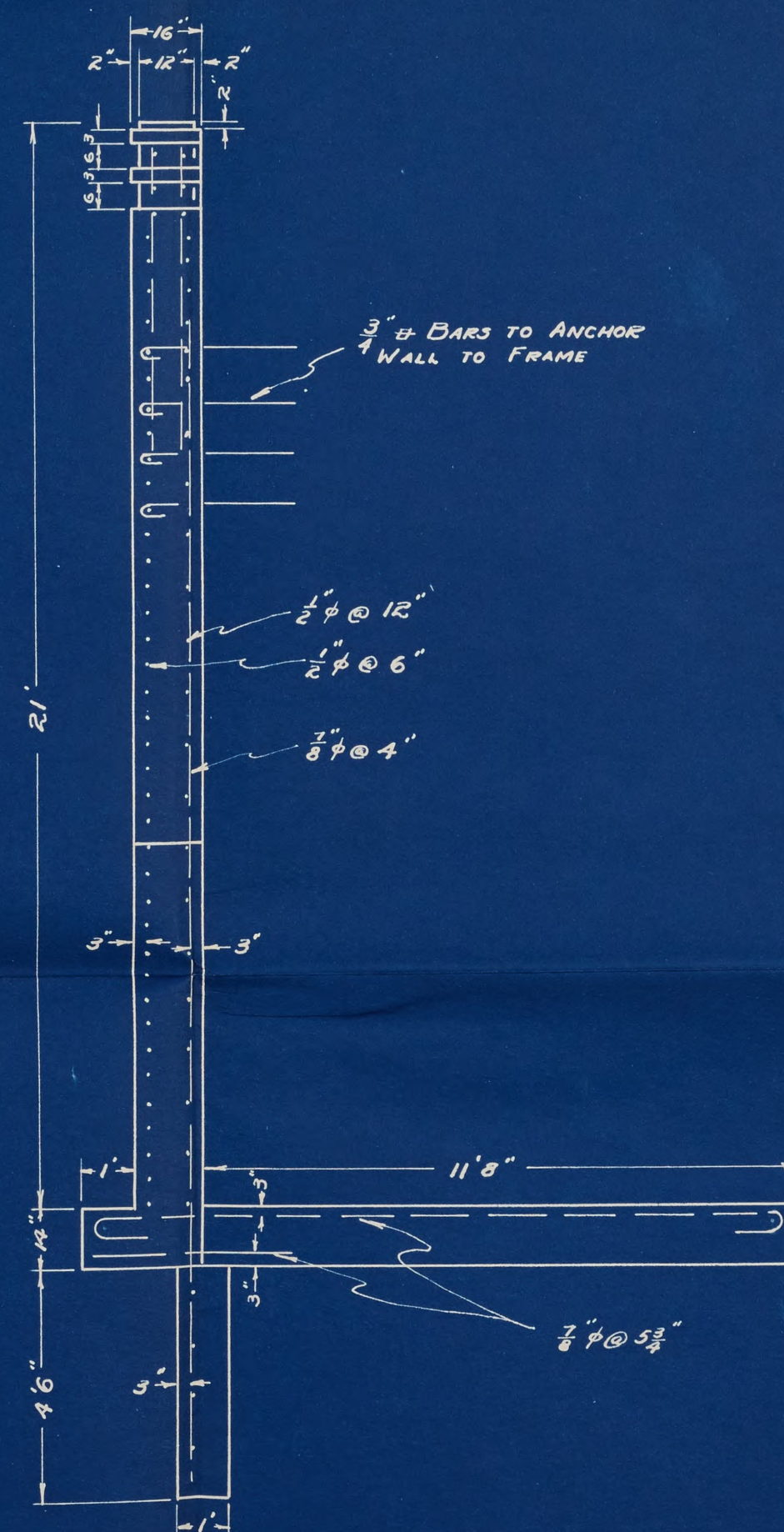
MICHIGAN STATE COLLEGE
DEPT. OF CIVIL ENG.

SIDE ELEVATION

SCALE 1"=6' DATE 5/26/41 GEORGE BECK, W.A.



WINGWALL



MICHIGAN STATE COLLEGE
DEPT. OF CIVIL ENG.

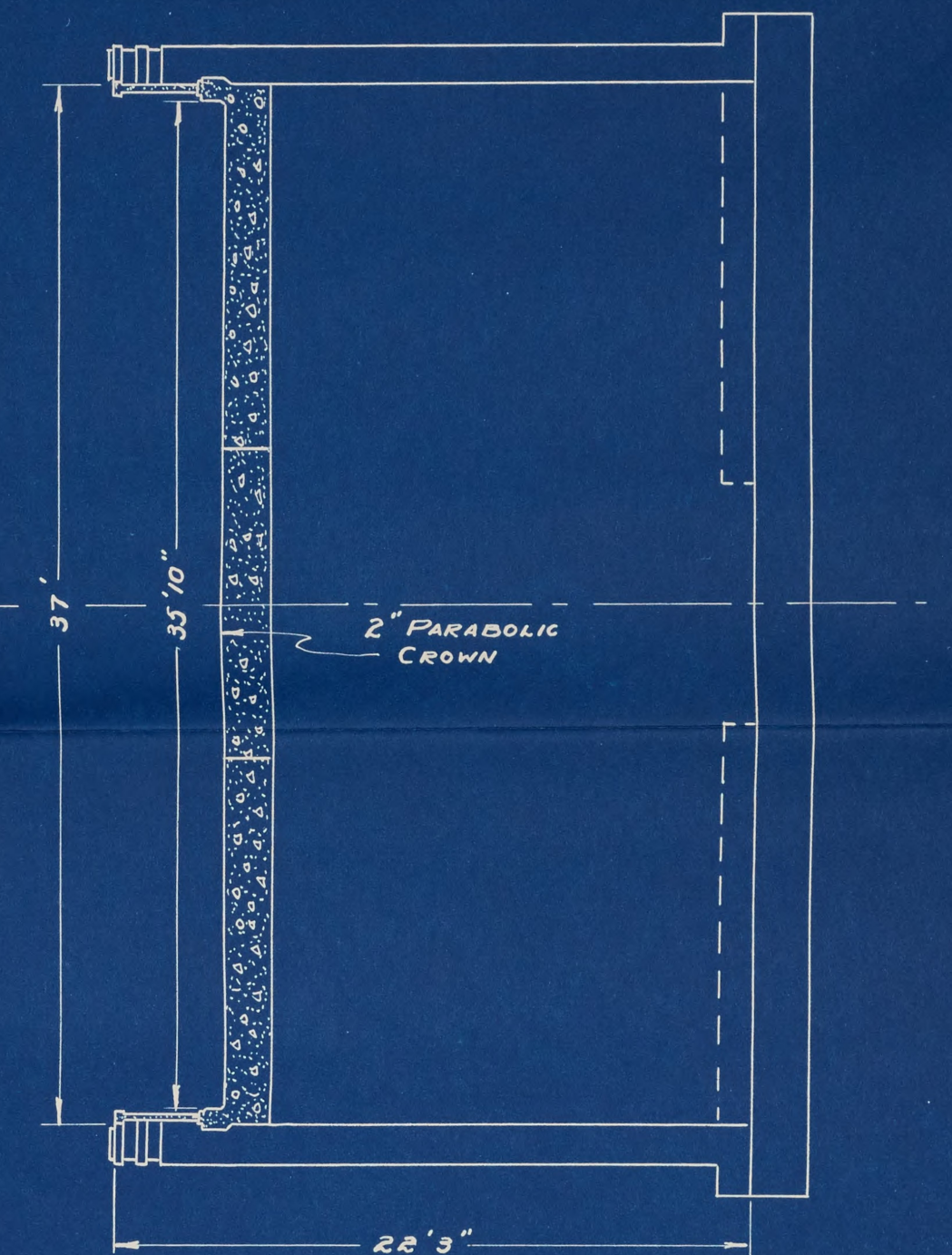
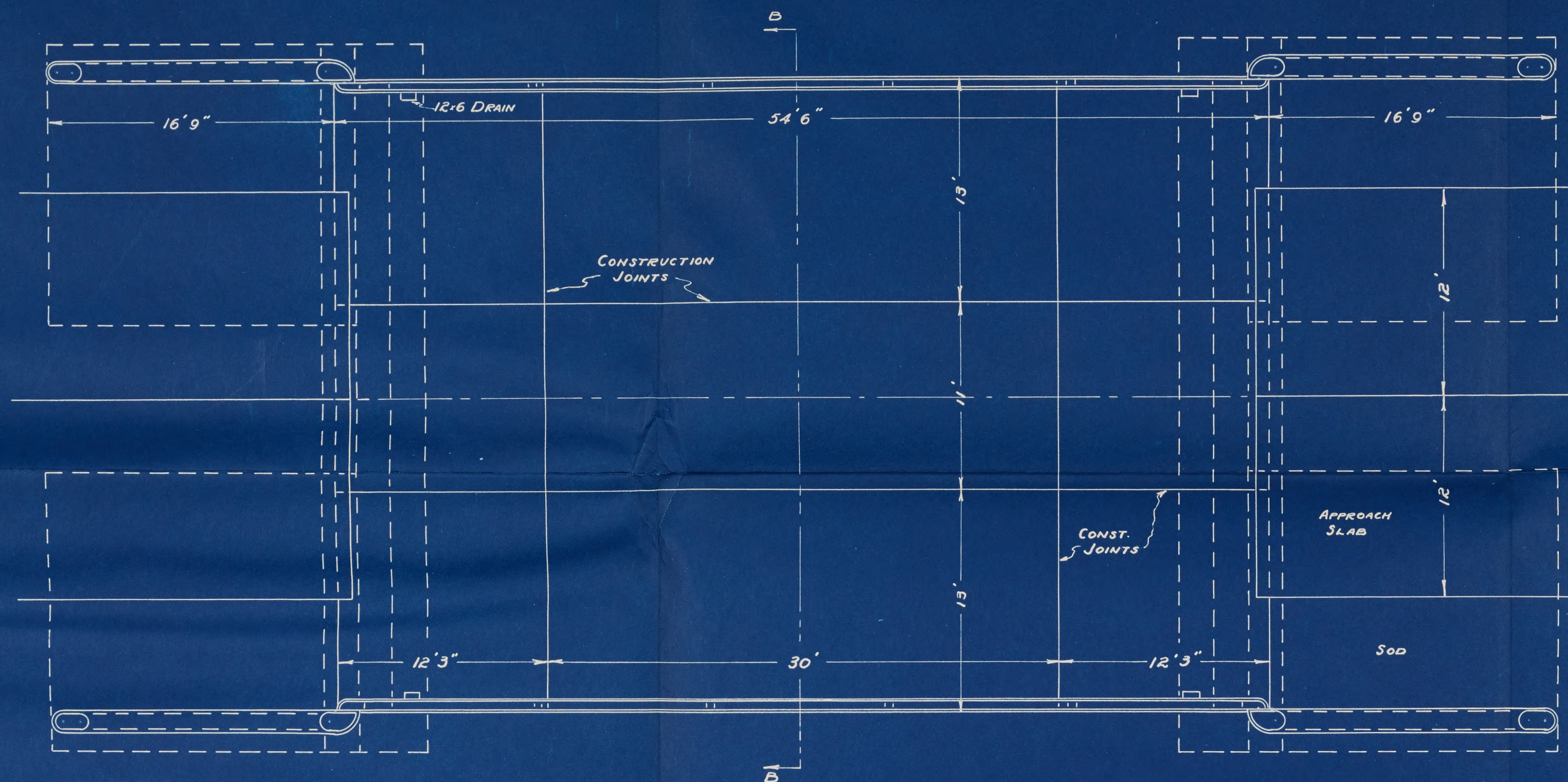
WINGWALL

SCALE 1"=3'

DATE 5/23/41

GROESBECK, HA

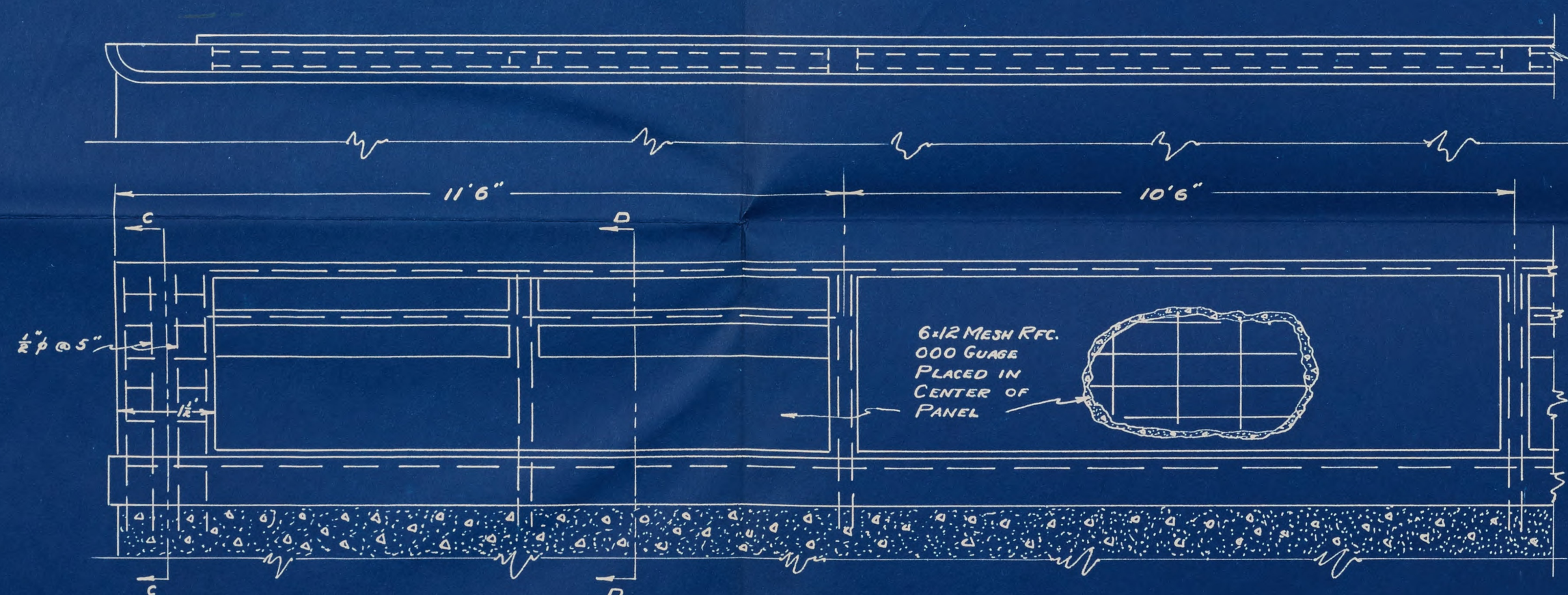
Top & SECTION VIEW OF BRIDGE



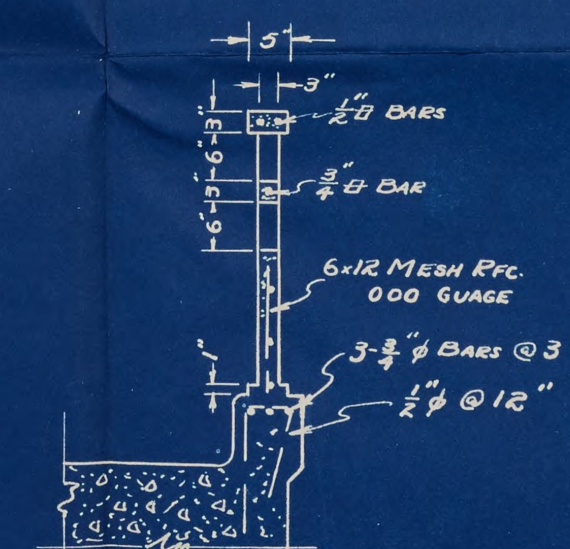
SCALE 1"=6'

SECT. B-B

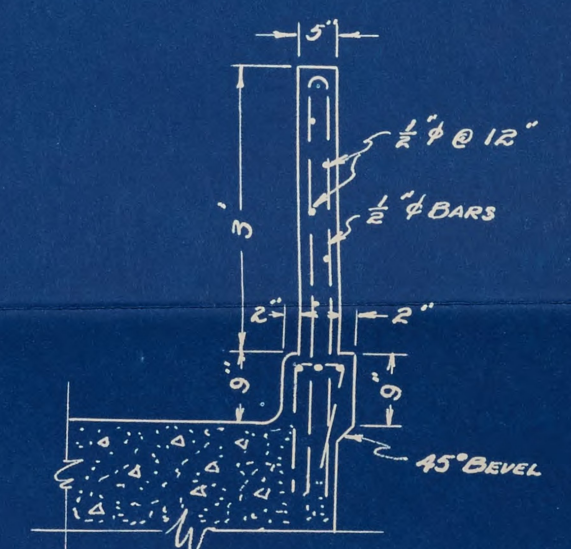
END & ADJOINING PANEL OF RAILING



SCALE 1"=2'



SECT. D-D



SECT. C-C

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BRIDGE LAYOUT & RAILING DETAILS

SCALE AS SHOWN DATE 5/27/41 GROESBECK, M.A.

Pocket has: 5 Supplements

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