A STUDY OF THE STURGIS HYDRO-ELECTRIC PLANT

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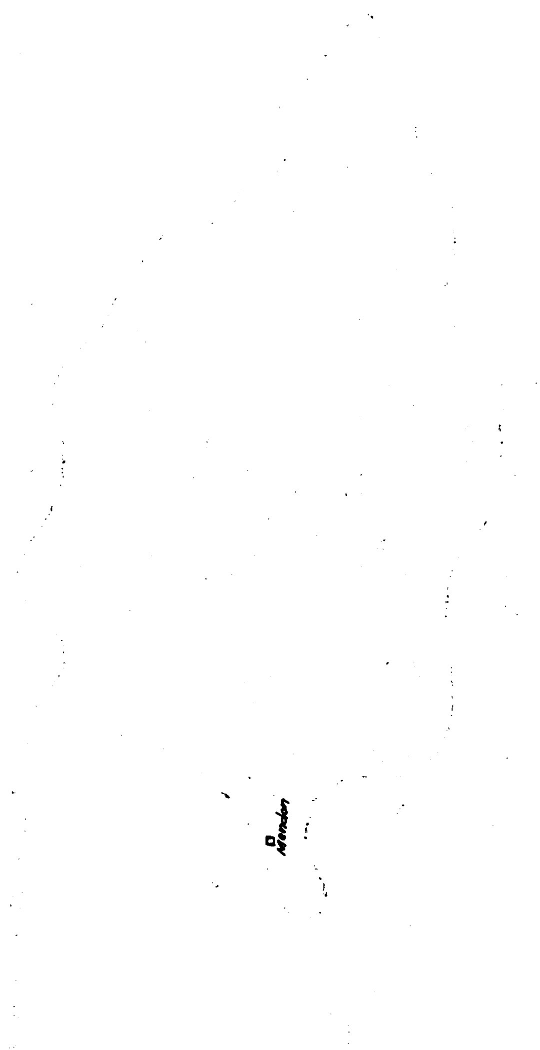
PREFACE

The writer wishes to take this opportunity to acknowledge indebtedness to Ayres, Lewis, Norris, and May, a firm of engineers in Ann Arbor, for their willingness in supplying plans of the dam and also to C. A. Miller, C. M. Cade, and other members of the Civil Engineering Department of Michigan State College for the assistance rendered by them in the writing of this paper.

A Study of the Sturgis Hydroelectric Plant

Sturgis is an industrial city of approximately 8000 inhabitants located in the southeastern part of St. Joseph County in southwestern Michigan. Previous to the completion of its municipal hydro-electric plant on the St. Joseph River in 1911, its electrical power was supplied by a small steam-operated generating plant. Due to the growth of the city it became necessary to either enlarge the generating plant or obtain electricity from some other source. The St. Joseph River was already being used by several cities further downstream for power purposes so it was decided that the feasibility of a hydro-electric plant should be investigated. Accordingly, the services of the firm of engineers, Ayres, Lewis, Norris, and May of Ann Arbor and Gardner S. Williams, consulting engineer, were obtained to determine whether or not a dam would be economical. It was found that not only could much more electrical power be developed, but that it could be delivered at a smaller unit cost, so a hydro-electric plant was erected. The capacity of the plant is 1100 kw. while that of the old steam plant was only 200 kw. The machinery consists of two 550 kw. generators directly connected to 844 H.P. turbines and a 40 kw. exciter with a 67 H.P. turbine.

There is approximately 870 square miles of the watershed above the dam. This area contains a large number of lakes and ponds which increase the storage capacity of the area, thus making large floods less frequent, but which, at the same time, decrease the ultimate runoff because of the large amount of evaporation. The river's headwaters are in the central part of Hillsdale county. The portion of the watershed in Hillsdale and eastern Branch counties is very hilly.


Mendon

In the central part of Branch county and in the southern part of Calhoun county, the land is much smoother, but becomes quite hilly again in western Branch and Calhoun counties and in the eastern part of St. Joseph county. The distance from the ground surface to bed rock varies between 100 and 200 feet over most of the watershed. The soil is chiefly sand, sandy loam, and gravel making the amount of surface runoff small.

Rainfall over the watershed is moderately heavy since the area lies in the path of most of the storms going eastward over the southern portion of Lake Michigan. Rainfall data is available from Wasepi station (see map) for the years of 1880 to 1920 inclusive and from Coldwater station for the years of 1898 to 1920 inclusive. Precipitation records since 1920 have been printed only in annual bulletins and several of these bulletins are not available. The first eighteen years of the rainfall curve represents the annual precipitation at Wasepi alone while the remainder of the curve represents the averages of the annual values. These values will be found in a table at the end of this paper. Since the mean annual precipitation at Wasepi is 39.60 inches and at Coldwater is 35.92 inches, it is reasonable to expect a mean annual rainfall of 37 inches on the watershed. The curve of annual rainfall shows that the yearly precipitation is quite uniform and that dry years are consequently infrequent.

There has been but one stream gaging station on the St. Joséph River upstream from the Sturgis dam and the records from this station are very unsatisfactory. These records were taken in 1903, 1904, and 1905 at the Marantette Bridge in Mendon (see map) and can be found in U.S. Geological Survey Water Supply Papers Nos. 97, 129, and 170.

These records are of little use since the discharge measurements were all taken at practically the same gage height and hence do not determine either the shape or the position of the discharge curve. The measurements were taken at various times by different parties and the discharge obtained by one party at a certain gage reading varies greatly from the discharge obtained by another party at practically the same gage reading. These records were made even less useful by the fact that no readings were taken in February 1904 and early in March, 1904 there was an ice gorge at the bridge, immediately followed by the failure of a dam above the station keeping the gage reading far above normal throughout the month. A part of these records, however, may be used to obtain a bare estimate of the percentage of the precipitation which runs off. The daily gage readings from October 1, 1904, to September 30, 1905, will be found at the end of this paper.

Month	Ave. Gage Reading (ft.)	Ave. Discharge c.f.s	Total Runoff million cu.ft.	Precipitation (in.) Coldwater	Waseon
Oct. '04	1.31	520	1,392	4.57	3.19
Nov. '04	.93	370	959	.10	.01
Dec. '04	1.00	410	1,097	1.87	2.22
Jan. '05	1.34	540	1,448	2.27	2.59
Feb. '05	1.32	530	1,282	1.50	1.41
Mar. '05	2.20	1230	3,295	3.64	3.12
Apr. '05	2.08	1100	2,850	3.54	3.20
May '05	3.01	2400	6,440	7.51	7.41
June '05	2.05	1100	2,850	3.50	2.56
July '05	1.94	950	2,540	4.61	4.31
Aug. '05	1.71	770	2,060	8.03	3.54
Sept '05	1.77	880	2,142	4.00	3.18
			28,355	45.14	35.74

Assume that an average of 40 inches of rain fell on watershed above gaging station from October 1, 1904 to September 30, 1905.

Then $\frac{40}{12}$ x 5280 x 5280 x 844 = 78,500 million cu. ft. of rain fell since the drainage area above the station is 844 square miles. $\frac{28355}{78500} = .361$ or 36.1% of the precipitation run off in the year.

In a paper printed in the Transactions, A.S.C.E., Vol. 77, Page 346, Joel D. Justin advances the following formula which shows the relationship between precipitation and runoff.

$$C = .934\ S^{.155}\ \frac{R^2}{T}$$

Where C is annual runoff in inches

R is annual precipitation in inches

S is slope = elevation of highest point minus elevation of lowest point ÷ $\sqrt{\text{area}}$

T is mean annual temperature.

Substituting $C = .934 \left[\frac{(1150-827)12}{\sqrt{844x5280x5280x12x12}}\right]^{.155} \frac{37x37}{47.8} = 10.27$ inches

Then $\frac{10.27}{37} = .278$ or 27.8% of the precipitation ran off.

From data taken for a sanitary survey obtained from F. R. Theroux of the Civil Engineering Department of Michigan State College, the average flow over a dam in St. Joseph River at Mishawaka, Indiana, for a period of 12 years was found to be 1200 c.f.s. According to an article in the Engineering Record, July 14, 1906, which describes the dam at Mishawaka, the area of the watershed above this point is 3000 square miles. Then the average runoff per square mile is $\frac{1200}{3000} = 0.4$ c.f.s. The runoff at the Sturgis dam would then be $\frac{.4x3600x365x1728}{5280x5280x144} = 5.425$ in. per year which is only 14.7% of the precipitation. A larger percentage of the rainfall than this can be expected to run off from the portion of the watershed above the Sturgis dam since the slope of the watershed from the dam to the headwaters is greater than it is from Mishawaka to the Sturgis dam, thus decreasing the chance for evaporation. One of the reasons why the percentage of runoff is so low at Mishawaka is that the 1670 square miles of the watershed in northern Indiana contains a much larger percentage of water surface than the portion of the watershed in Michigan, due to the large number of lakes in northeastern Indiana.

Considering the three estimates and giving each its proper weight it seems reasonable to expect that about 25% of the precipitation will run off on an average year, that is, any year with temperatures close to normal and not preceded by an unusually wet or dry year. Using this percentage as a basis, the runoff curve was drawn on the same sheet with the rainfall curve which shows the variation that can be expected in annual runoff. Of course not all of this runoff will be available for power purposes since nearly one-fourth of it runs off during the spring floods and some of this must go over the spillway.

Due to the lack of stream flow records it is impossible to draw a mass diagram so that the power available can only be estimated. Assuming that one-eighth of the flow goes over the spillway in the course of a year, there will still be 37x.25x7/8 or 8.08 inches of rainfall available for power purposes which is equivalent to 520 c.f.s. Assuming an effective head of 16' then the average power available would be $\frac{520x62.4x16}{550} = 945$ H.P.

The size of the spillway is determined by the maximum flood that can be expected during the life of the structure. In some cases where the difference in the cost of making a spillway large enough to accommodate a 500 year flood and the cost of one to take care of a 100 year flood plus the interest on this difference of cost is more than the cost of the structure and the damage that would be caused by the failure of it, it is more economical to design an spillway for the 100 year flood and expect it to fail sometime between 100 and 500 years in the future. In this case, however, the capacity of the spillway can easily be increased by lengthening it, thus decreasing the necessary length of the earth dam at the south bank. An adequate spillway is especially necessary in this case since the failure of the dam would undoubtedly cause the failure of one or more of the several dams below it, the first of which is at the upstream side of Three Rivers, a distance of six miles.

Of the various formulas for intensity of flood flow the one which gives the most reasonable results is found in "Elements of Hydrology" by Meyer on page 369. This formula, $Q = 100\ A^{0.6}$ in which Q is the maximum flood in c.f.s. to be expected once in 25 years and A is the area of the watershed in square miles, is designed for use under Minnesota conditions. This must be multiplied by a coefficient depending on soil, slope, lakes, and other features affecting flood runoff which is .45 in the case of the St. Joseph watershed. Precipitation in Minnesota is only about 25" annually so correction must be made for this also. Then $Q = 100 \times .45 \times \frac{37}{25} \times 870^{0.6} = 3590$ c.f.s. is the maximum flood to be expected once in 25 years. According to Pickels in his "Drainage and Flood Control Engineering", the maximum flood that can be expected once in 500 years is 1.70 as large as the maximum flood to be expected once in 25 years. Then the spillway should be designed for $3590 \times 1.70 = 6110$ c.f.s.

The length of the spillway required will be found by the formula on page 131 of Creager's and Justin's "Hydro-electric Handbook"

$$Q = C\ l\ h^{1.5}$$

Where Q = total discharge in c.f.s.
C = coefficient of discharge depending on shape of crest and head on crest.
l = effective length of crest
h = actual head on crest.

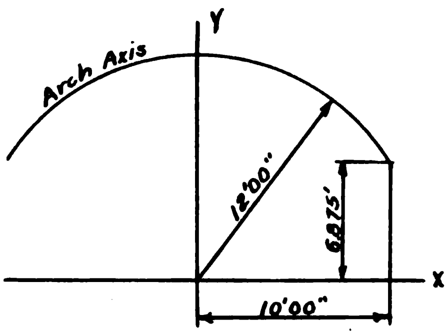
The value of C from Fig. 77 of same handbook is 3.94, using $\frac{h}{h'} = 1$ where h' is the head used to determine the shape of the crest.

Transposing $L = \frac{Q}{c\ h^{1.5}}$

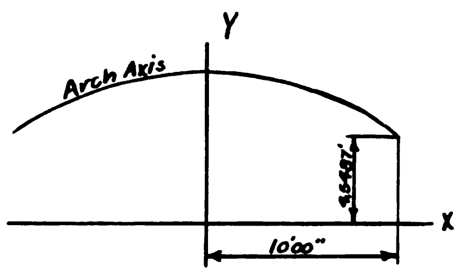
Then $L = \frac{6110}{3.94 \times 3^{1.5}} = 298$ feet

The actual length of the spillway is 308 feet but the vertical T-rails, which hold crest gates and walk at intervals of 10 feet decrease the effective length to some extent.

The spillway is of the multiple-arch type, composed of 15 arches, 13 being 20 feet long and the end arches 23 feet long. A horizontal section through the face of the dam is a segment of a circle with a 12 foot radius, hence a right section is elliptical.



Horizontal Section



Normal Section

In the horizontal section through the arch when $x = +$ or -10, $y = 6.875$. Since the slope of the face of the dam is $\frac{15}{17}$, when $x = +$ or -10, $y = 6.875 \times \frac{15}{\sqrt{15^2 + 17^2}} = 4.5487$ in section normal to face of the dam and the semiminor axis of the ellipse is $12 \times \frac{15}{\sqrt{15^2 + 17^2}} = 7.9395$. The general formula for an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Then $x^2 b^2 + y^2 a^2 = a^2 b^2$

Substituting $10^2 \times 7.9395^2 + 4.5487^2 \times a^2 = 7.9395^2 a^2$

From which $a^2 = 148.5100$

Then $7.9395^2 x^2 + 148.5100\ y^2 = 7.9395^2 \times 148.5100$

And $y = \frac{\sqrt{9361.4318 - 63.0357\ x^2}}{12.1864}$

From this equation a section perpendicular to the face of the dam can be plotted for the purpose of analysis.

Since the buttresses are relatively thin it looks logical that the arch should be analyzed in the same manner as the arches of a two span bridge with an elastic pier. For this analysis, a section one foot long is taken at AB (See section through spillway) since this is the lowest full arch and hence has the most pressure on it. According to the theory of elastic piers the system is considered free to move along section GH, (see Analysis Sheet No.1) thus making three cantilevers out of the system--the two arches fastened to the outside buttresses, but free to move along lines ab and cd, and the buttress which is fastened to the foundation but free to move along section GH. The trapezoidal section on the buttress above GH and between ab and cd needs to be considered only in finding the resultant thrusts upon the buttress. Each arch is divided into an even number of sections such that S/I is constant for all sections, where S is the length of the section measured along the arch axis and I is the moment of inertia at the section including steel. Since the cross-sectional area is the same throughout the arch, I is also constant and the arch between skewbacks can be divided into 12 equal sections, 1.60 feet long. The head on each rectangular section was then found under the assumption that the maximum elevation of the water surface is 825.00 feet, or 3 feet above the crest. Now the total water pressure on each section and the normal component of the weight can be computed. These are shown for only half of one arch since all others are the same.

Section	Water Load --lbs.	Wt. of Sect.-Lbs.	Norm. Comp. of Wt.
1-2	62.4x1.6x21.03 = 2100	150x1.60x1 = 240	180
2-3	62.4x1.6x20.23 = 2020	240	180
3-4	62.4x1.6x19.63 = 1960	240	180
4-5	62.4x1.6x19.20 = 1917	240	180
5-6	62.4x1.6x18.86 = 1883	240	180
6-7	62.4x1.6x18.78 = 1876	240	180

The resultant of the water pressure and the normal component of the weight can easily be found graphically. Only the vertical component of this resultant needs to be considered since for a given value of y, there are two equal and opposite collinear horizontal components which have no moment about 0. The following nomenclature is used in the analysis when buttresses are considered elastic (see also Analysis Sheet No. 1).

X_L Y_L = coordinates of any point on axis of left arch referred to 0 as origin

X_R Y_R = Same for right arch

m_L, m_R = Moment at any point on axis of left arch and right arch respectively of all external loads between point in question and top of buttress.

n_L, n_R = Number of S/I divisions in left arch and right arch

C_L, C_R = Values of S/I for left arch & right arch respectively.

H_1, V_1 = Horizontal and vertical components of the thrust from the left arch on the top of the buttress.

M_1 = Moment at section GH due to thrust from left arch.

H_2, V_2 = Horizontal and vertical components of the thrust from the right arch on the top of the buttress.

M_2 = Moment at section GH due to thrust from right arch.

Cantilever Moments

Point	X	Diff. of X's	Loads	Sum of Loads	Increment of Moment	Moment
13	1.00		0	0	0	0
P12	1.68	.68	1960	1960	0	0
12	2,38	.70			1372	1372
P11	3.10	.72	2020	3980	1412	2784
11	3.83	.73			2905	5689
P10	4.57	.74	2030	6010	2945	8634
10	5.32	.75			4508	13142
P 9	6.08	.76	2040	8050	4569	17710
9	6.85	.77			6198	23908
P 8	7.63	.78	2045	10095	6279	30187
8	8.41	.78			7874	38061
P 7	9.20	.79	2050	12145	7975	46036
7	10.00	.80			9716	55752
P 6	10.80	.80	2050	14195	9716	65468
6	11.59	.79			11214	76682
P 5	12.37	.78	2045	16240	11072	87754
5	13.15	.78			12667	100421
P 4	13.92	.77	2040	18280	12505	112926
4	14.68	.76			13893	126919
P 3	15.43	.75	2030	20310	13710	140529
3	16.17	.74			15029	155558
P2	16.90	.73	2020	22330	14826	170384
2	17.62	.72			16078	186462
P 1	18.32	.70	1960	24290	15631	202093
1	19.00	.68			16517	218610
Total			24290		218610	

Pt.	X	Y	X^2	Y^2	XY	m	mX	mY
1	19.00	.82	361.00	.67	15.58	281610	4153590	179260
2	17.62	1.64	310.46	2.69	28.90	186462	3285460	305798
3	16.17	2.30	261.47	5.29	37.19	155558	2515373	357783
4	14.68	2.78	215.50	7.73	40.81	126819	1861703	352557
5	13.15	3.13	172.92	9.80	41.16	100421	1320536	314318
6	11.59	3.34	134.33	11.16	38.71	76682	888744	256118
7	10.00	3.40	100.00	11.56	34.00	55752	557520	189557
8	8.41	3.34	70.73	11.16	28.09	38061	326093	127123
9	6.85	3.13	46.92	9.80	21.44	23908	163770	74832
10	5.32	2.78	28.30	7.73	14.79	13142	69915	36535
11	3.83	2.30	14.67	5.29	8.81	5689	21789	13085
12	2.38	1.64	5.66	2.69	3.90	1372	3265	2250
13	1.00	.82	1.00	.67	.82	0	0	0
Tot.	130.00	31.42	1728.96	86.24	314.20	1002476	15161758	2209216

From the results of this table the six equations used in analyzing arches with elastic buttresses can be solved for the 6 unknowns.

$$C_L(M_1\Sigma y_L - H_1\Sigma y_L^2 + V_1\Sigma x_L y_L - \Sigma m_L y_L) = -C_R(M_2\Sigma y_R - H_2\Sigma y_R^2 + V_2\Sigma x_R y_R - \Sigma m_R y_R)$$

Since arches are of the same thickness and symmetrical

$$2\Sigma y\, M_1 - 2\Sigma y^2 H_1 + 2\Sigma xy\, V_1 - 2\Sigma my = 0$$

Then $62.84\, M_1 - 172.48\, H_1 + 628.40\, V_1 - 4{,}418{,}432 = 0$ (1)

$$M_1\Sigma x_L - H_1\Sigma x_L y_L + V_1\Sigma x_L^2 - \Sigma m_L x_L = 0$$

$$M_2\Sigma x_R - H_2\Sigma x_R y_R + V_2\Sigma x_R^2 - \Sigma m_R x_R = 0$$

Adding the two equations, dividing by 2, and substituting,

$130\, M_1 - 314.20\, H_1 + 1722.96\, V_1 - 15{,}161{,}758 = 0$ (2)

$$C_L(n_L M_1 - H_1\Sigma y_L + V_1\Sigma x_L - \Sigma m_L) = -C_R(n_R M_2 - H_2\Sigma y_R + V_2\Sigma x_R - \Sigma m_R)$$

Transposing, dividing by 2, and substituting,

$12\, M_1 - 31.42\, H_1 + 130\, V_1 - 1{,}002{,}476 = 0$ (3)

From (3), $M_1 = 2618\, H_1 - 10833\, V_1 + 83{,}539.7$

Substituting this value of M_1 in (1),

$7.97\, H_1 + 52.35\, V_1 - 831{,}200 = 0$ (4)

Substituting same value of M_1 in (2)

$26.14\, H_1 + 314.67\, V_1 - 4{,}301{,}601 = 0$ (5)

From (4), $H_1 = -6.56838\, V_1 + 104{,}291.1$

Substituting this value of H_1 in (5)

$142.97\, V_1 - 1{,}575{,}442 = 0$

And $V_1 = V_2 = 11{,}019.4$#

Then $H_1 = H_2 = 31{,}912.5$#

And $M_1 = M_2 = 47.714$ foot-lbs.

The thrust from the left arch acts M_1/V_1 feet to the right of point O. Since the force polygon drawn from this point 4.33 feet to the right of O does not follow the arch axis, it is obvious that the arches were not designed by a theory considering the buttresses to be elastic.

By observing the manner in which the horizontal components of the water pressure from two adjacent arches equalize each other at the buttress, thus causing little or no horizontal displacement of the buttress, it appears that the buttresses were probably assumed to be inelastic and the arches designed in the same manner as a single span, symmetrical arch bridge. In this method the arch is considered to be cut in the middle and half of it acting as a cantilever as shown in Analysis Sheet No. 2. The arch was taken from the same elevation in the face of the dam so that the sections and loads will be the same as they were in the preceding case.

The following nomenclature is used in this analysis.

S = length of a division measured along axis of arch.

n_h = number of divisions in one-half of the arch

l = length of span

C_a = average unit compression in concrete of arch ring due to thrust

t_c = coefficient of linear temperature expansion

t_o = number of degrees rise or fall of temperature

E_c & modulus of elasticity of concrete

H_c, V_c, M_c = thrust, shear, and moment respectively at crown

N = normal thrust on radial section

X_o = eccentricity of thrust on section, or distance of N from arch axis

t = thickness of section

I = moment of inertia of section including steel = $I_c + nI_s$

A = area of section including steel

P_o = steel ratio for total steel at section

d' = embedment of steel from either upper or lower surface

M = moment = $N X_o$

m = moment at any point on left half of arch axis of all external

Pt.	X	Diff. of X's	Loads	Sum of Loads	Increment of m	m
P6	.80	.80	2050	2050	0	0
6	1.59	.79			1620	1620
P5	2.37	.78	2045	4095	1600	3220
5	3.15	.78			3194	6414
P4	3.92	.77	2040	6135	3153	9567
4	4.68	.76			4663	14230
P3	5.43	.75	2030	8165	4601	18831
3	6.17	.74			6042	24873
P2	6.90	.73	2020	10185	5960	30833
2	7.62	.72			7333	38166
P1	8.32	.70	1960	12145	7130	45296
1	9.00	.68			8259	53555
			12145		53555	

From these moments the computations on Analysis Sheet No. 3 were made. The values of k and l used in finding unit stresses were found in diagrams in Hool's Vol. 1 "Reinforced Concrete Construction", on pages 362, 369, and 370.

For the computation of the stresses in the buttfesses Analysis Sheet No. 4 was drawn. The arch ring thrust normal to the face of the buttress at D is 12,150#. Since there is an arch on each side the total normal thrust per foot at D is 2x12,150 = 24,300#. The total pressure can be determined by a pressure triangle as shown with but a small error. The resultant of the water pressure and weight of the buttress were combined and their resultant extended until it met the base of the buttress. Then the unit pressures at the toe and heel of the dam were determined.

$$p \text{ at } B = V \times \frac{4\,AB - 6\,BC}{d(AB)^2} = \frac{(4 \times 40 - 6 \times 16) \times 450000}{2 \times 1600} = 9{,}000\# \text{ per sq.ft.} = 62.5\# \text{ per sq.in.}$$

$$p \text{ at } A = V \times \frac{4AB - 6AC}{d(AB)^2} = \frac{(4 \times 40 - 6 \times 24) \times 450000}{2 \times 1600} = 2250\# \text{ per sq.ft.} = 15.6\# \text{ per sq. in.}$$

The unit stresses found to be present in this structure were all rather low so that the conclusion can be arrived at by this investigation is that the dam is very stable structurally. Of course the purpose of this investigation was to find as near as possible the method used by the designers as well as to check the stresses in the structure, so that

ANALYSIS SHEET
NO. 3

Pt	x	y	x^2	y^2	m	mx	my	$H_c y$	M	N (scaled)	Temperature $H_c y$	Temperature M	Temperature N	Rib Short. M	Rib Short. N	c	P_o	Totals M	Totals N	$\frac{k_o}{t}$	K	L	Unit Stresses f_c	Unit Stresses f_s
C									+1121	20678	-1680	+1790	-1681	+2151	-2020	1.0	.00182	+5072	16978	.299	.632	.0968	364	1735
1	9.00	2.58	81.000	6.656	53555	481995	138172	53,349	+415	21160	-4334	-2544	-1375	-3050	-1650	1.0	.00182	-4679	18135	.258	.744	.0982	331	595
2	7.62	1.76	58.064	3.098	38166	290825	67172	36393	-652	21080	-2957	-1167	-1480	-1400	-1775	1.0	.00182	-3219	17825	.181	.200	—	250	all comp.
3	6.17	1.10	38.089	1.210	24873	153466	27,360	22746	-1006	20920	-1848	-58	-1550	-70	-1860	1.0	.00182	-1134	17510	.065	.135		164	—
4	4.68	.62	21.902	.384	14230	66596	8883	12820	-289	20840	-1042	+748	-1610	+900	-1930	1.0	.00182	+1359	17300	.079	.143		172	
5	3.15	.27	9.923	.070	6414	20204	1732	5583	+290	20720	-454	+1336	-1665	+1600	-1980	1.0	.00182	+2226	17075	.130	.173	—	204	—
6	1.59	.06	2.528	.004	1620	2576	97	1241	+742	20700	-101	+1689	-1675	+2030	-2010	1.0	.00182	+4461	17015	.262	.732	.0982	315	662
	32.21	6.39	211.486	11.425	138,858	1,015,662	243,416																	

Water Pressure

$$H_c = \frac{n_h \Sigma (m_L + m_R) y - \Sigma (m_R + m_L) \Sigma y}{2[n_h \Sigma y^2 - (\Sigma y)^2]}$$

Since $m_L = m_R$

$$H_c = \frac{6 \times 2 \times 243,416 - 2 \times 138,858 \times 6.39}{2[6 \times 11.425 - 6.39^2]}$$

$$= 20,678^{\#}$$

$$V_c = \frac{\Sigma (m_L - m_R) x}{2 \Sigma x^2} = 0 \text{, since } m_L - m_R = 0$$

$$M_c = \frac{\Sigma(\Sigma m_L + m_R) - 2 H_c \Sigma y}{2 n_h}$$

$$= \frac{2 \times 138858 - 2 \times 20678 \times 6.39}{2 \times 6}$$

$= +1121$ foot-lbs.

Temperature

$$H_c = \frac{I}{S} \frac{t_c t_o \ell n_h E_c}{2[n_h \Sigma y^2 - (\Sigma y)^2]}$$

$$= \frac{(\frac{1}{12} \times 12 \times 1728 + 15 \times 2 \times 131 \times 16) \times .000006 \times (50) \times 30 \times 2,000,000 \times 144}{16 \times 144 \times 144 \times 2[6 \times 11.425 - 6.39^2]}$$

$$= -1680^{\#}$$

$V_c = 0$

$$M_c = -\frac{H_c \Sigma y}{n_h} = -\frac{-1680 \times 6.39}{6} = +1790 \text{ foot-lbs}$$

Rib Shortening

Assume $c_a = 120^{\#}$/sq in.

$$H_c = -\frac{I}{S} \frac{c \ell n_h}{2[n_h \Sigma y^2 - (\Sigma y)^2]}$$

$$= \frac{1791 \times 120 \times 20 \times 6}{1.6 \times 144 \times 144 \times 55.44}$$

$$= -2020^{\#}$$

$$c_a \text{ (at crown)} = \frac{20,678 - 1680 - 2020}{1 + .027} = 16,500^{\#}/\text{sq ft}$$

$$c_a \text{ (pt 4)} = \frac{20840 - 1610 - \frac{610}{1680} 2020}{1 + .027} = 15,870^{\#}/\text{sq ft}$$

$$c_a \text{ (pt 1)} = \frac{21160 - 1375 - \frac{1375}{1680} 2020}{1 + .027} = 17,650^{\#}/\text{sq ft}$$

3) 50020

$16,673^{\#}$/sq ft (ave)

$$\frac{16673}{144} = 116^{\#}/\text{sq in} \quad \text{OK}$$

$$\text{Ratio } \frac{H_c \text{ for Rib shortening}}{H_c \text{ for Temperature}} = \frac{2020}{1680} = 1.20$$

$$M_c = -\frac{H_c \Sigma y}{n_h}$$

$$= -\frac{-2020 \times 6.39}{6} = +2151.3 \text{ foot-lbs}$$

Compression over entire section - $f_c = \frac{N}{bt} K$

Tension over part of section - $f_c = \frac{M}{L b t^2}$, $f_s = n f_c \left(\frac{d}{kt} - 1\right)$

ANALYSIS SHEET
NO 4

of the many things learned by the writer in the studying of this project, perhaps the most important is that in a structure of this nature the buttresses can be considered inelastic.

Miscellaneous Tables

Annual values of rainfall in inches used in plotting rainfall curve.

Year	Wasepi Station	Coldwater Station	Ave.	Year	Wasepi Station	Coldwater Station	Ave.
1880	46.75			1900	35.95	35.72	35.8
1881	53.88			1901	33.87	37.30	35.16
1882	52.05			1902	39.49	35.58	37.5
1883	55.27			1903	40.65	35.75	38.12
1884	38.21			1904	40.61	33.78	37.2
1885	53.99			1905	39.26	46.21	43.3
1886	40.28			1906	37.34	37.89	37.6
1887	40.90			1907	43.59	40.19	41.3
1888	31.67			1908	41.38	36.50	38.9
1889	40.22			1909	46.49	39.05	42.7
1890	46.35			1910	34.18	28.40	31.3
1891	34.22			1911	42.23	31.24	36.7
1892	37.61			1912	35.60	30.35	33.0
1893	42.65			1913	33.83	29.10	31.4
1894	30.74*			1914	38.64	31.80	35.2
1895	30.68			1915	33.93	29.84	31.8
1896	38.60			1916	42.29	35.65	39.1
1897	30.33			1917	37.71	29.03	33.3
1898	37.64	42.89	40.2	1918	42.20	34.63	37.4
1899	34.76	36.46	35.6	1919	37.53	31.61	34.5
				1920	30.17	34.06	32.1

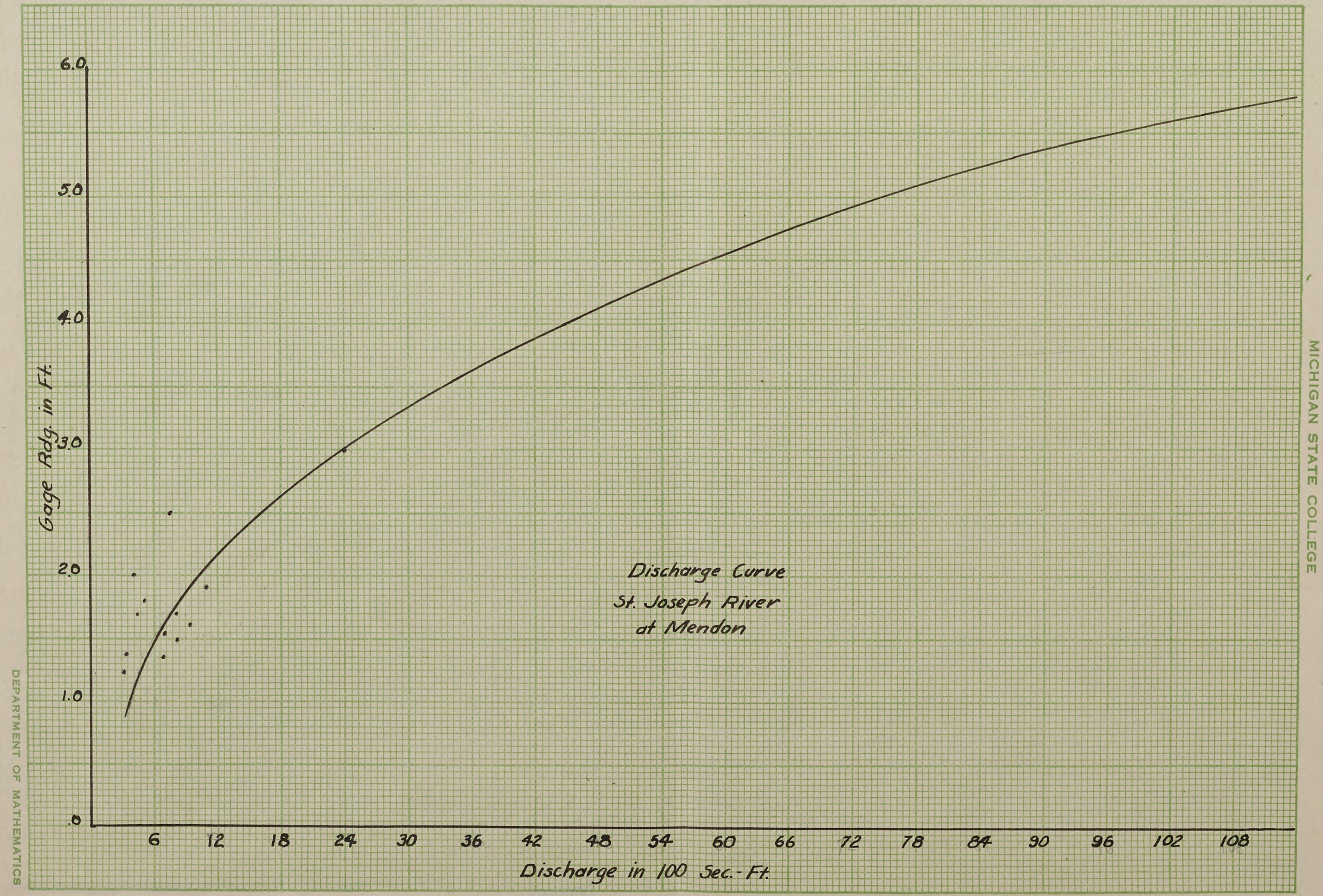
*Precipitation for March 1894 is not in records so this value was found by adding to the precipitation in the other eleven months of 1894 the mean monthly precipitation for March at the station.

Discharge measurements used for plotting discharge curve

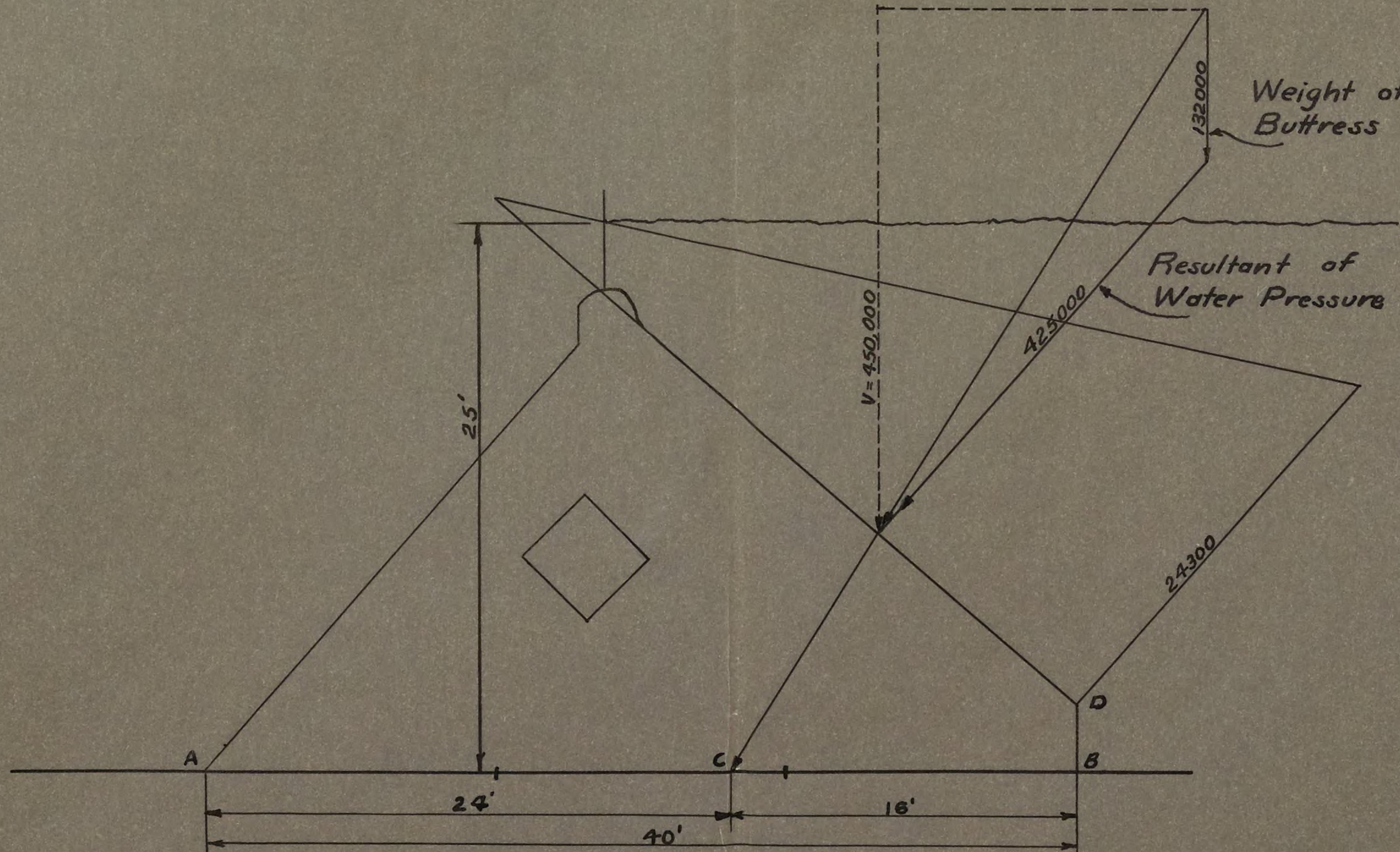
Date	Area of Sect.(sq.ft)	Mean Velocity	Discharge c.f.s.	Gage Height
Mar. 20, 1903			2396	3.00
May 11, 1903			949	1.62
July 3, 1903			530	1.80
July 6, 1903			467	1.70
Aug. 4, 1903			436	2.00
Aug. 31, 1903			777	2.50
June 3, 1904	468	1.73	810	1.50
June 7, 1904	455	1.53	697	1.35
Sept. 9, 1904	387	.83	322	1.38
Sept. 22, 1904	389	.79	306	1.24
June 2, 1905	512	2.13	1093	1.90
Nov. 7, 1905	481	1.50	710	1.53
Nov. 9, 1905	500	1.63	818	1.70

Gage Readings from Oct. 1, 1904 to Sept. 30, 1905 inclusive

Day	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.
1	1.90	1.20	.80	1.50	1.20	1.60	2.60	1.75	1.95	1.50	1.70	1.60
2	1.75	1.20	.75	1.50	1.20	1.60	2.60	1.70	1.90	1.50	1.70	1.70
3	1.65	1.20	.75	1.50	1.20	1.55	2.45	1.70	1.95	1.50	1.70	1.80
4	1.60	1.20		1.50	1.20	1.50	2.40	1.65	1.80	1.50	1.70	1.90
5	1.50	1.20		1.50	1.20	1.50	2.25	1.90	1.80	1.55	1.70	2.10
6	1.50	1.10		1.45	1.20	1.45	2.05	2.00	1.80	1.60	1.72	2.00
7	1.40	1.05	.85	1.45	1.20	1.40	1.90	2.00	1.90	1.70	1.75	2.00
8	1.35	1.00	.90	1.35	1.20	1.40	1.90	2.00	2.05	1.80	1.75	1.95
9	1.30	1.00	.90	1.25	1.20	1.40	1.75	2.00	2.20	1.90	1.72	1.90
10	1.30	1.00	.95	1.20	1.20	1.40	1.70	2.00	2.35	2.05	1.60	1180
11	1.20	1.00	1.00	1.20	1.200	1.50	1.70	2.75	2.40	2.35	1.60	1.80
12	1.20	1.00	1.00	1.25	1.25	1.30	1.70	3.85	2.35	2.50	1.60	1.70
13	1.20	.95	1.00	1.30	1.30	1.30	1.70	5.20	2.30	2.70	1.50	1.60
14	1.20	.85	.95	1.35	1.30	1.30	1.70	5.85	2.30	2.60	1.50	1.60
15	1.20	.80	.90	1.40	1.30	1.45	1.70	5.75	2.30	2.60	1.65	1.50
16	1.20	.80	.90	1.40	1.40	1.50	1.60	5.50	2.20	2.50	1.70	1.50
17	1.20	.80	.90	1.40	1.40	1.60	1.55	5.00	2.20	2.35	1.80	1.60
18	1.20	.80	.90	1.40	1.50	2.10	1.50	4.70	2.35	2.25	1.80	1.80
19	1.20	.80	.90	1.40	1.40	2.60	1.50	4.25	2.55	2.00	1.80	1.90
20	1.20	.80	.90	1.35	1.40	3.10	1.50	4.70	2.15	2.00	1.80	1.90
21	1.20	.80	.90	1.30	1.40	3.55	1.95	3.45	2.00	1.90	1.80	1.90
22	1.30	.90	.90	1.30	1.40	3.80	2.25	3.05	1.90	1.80	1.80	1.90
23	1.30	.85	1.00	1.30	1.400	3.75	2.80	2.70	1.80	1.80	1.90	1.90
24.	1.25	.80	1.10	1.30	1.40	3.65	2.80	2.45	1.80	1.70	1.90	1.90
25.	1.20	.80	1.20	1.30	1.50	3.55	2.80	2.40	1.80	1.70	1.80	1.80
26.	1.20	.80	1.40	1.30	1.50	3.40	2.80	2.40	1.80	1.70	1.80	1.80
27.	1.20	.80	1.40	1.30	1.55	3.15	2.60	2.35	1.80	1.70	1.70	1.70
28	1.20	.80	1.40	1.30	1.60	2.95	2.50	2.15	1.75	1.70	1.70	1.60
29	1.20	.80	1.40	1.25		2.75	2.40	2.05	1.60	1.70	1.60	1.50
30.	1.20	.80	1.40	1.20		2.65	1.95	2.00	1.55	1.70	1.60	1.50
31	1.20		1.40	1.20		2.60	1	2.00		1.70	1.60	

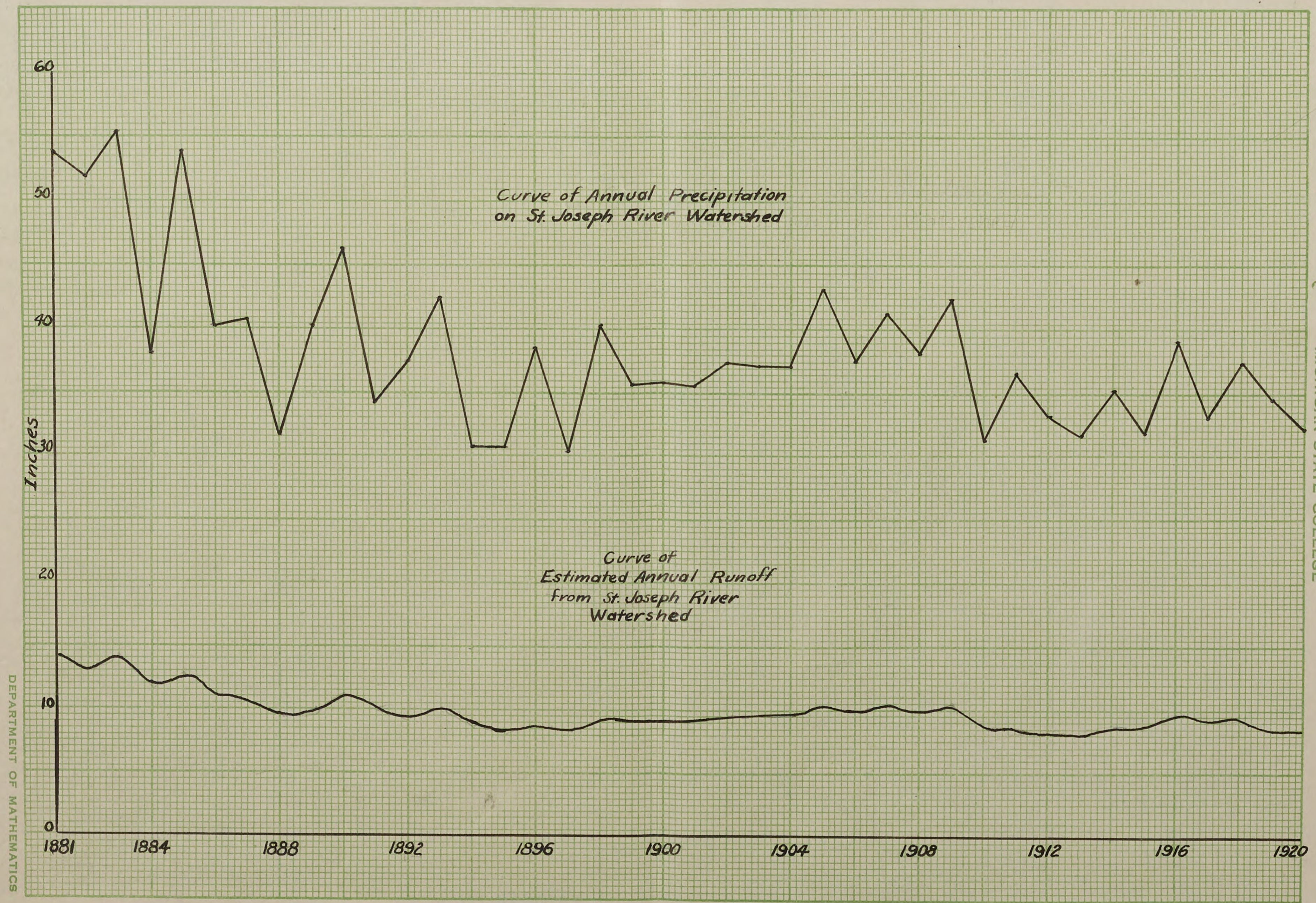


6.0
5.0
4.0
3.0
2.0
1.0
.0
Gage Rdg. in Ft.
6
12
18
24
30
36
42
48
54
60
66
72
78
84
90
96
102
108
Discharge in 100 Sec.-Ft.
Discharge Curve
St. Joseph River
at Mendon
MICHIGAN STATE COLLEGE
DEPARTMENT OF MATHEMATICS



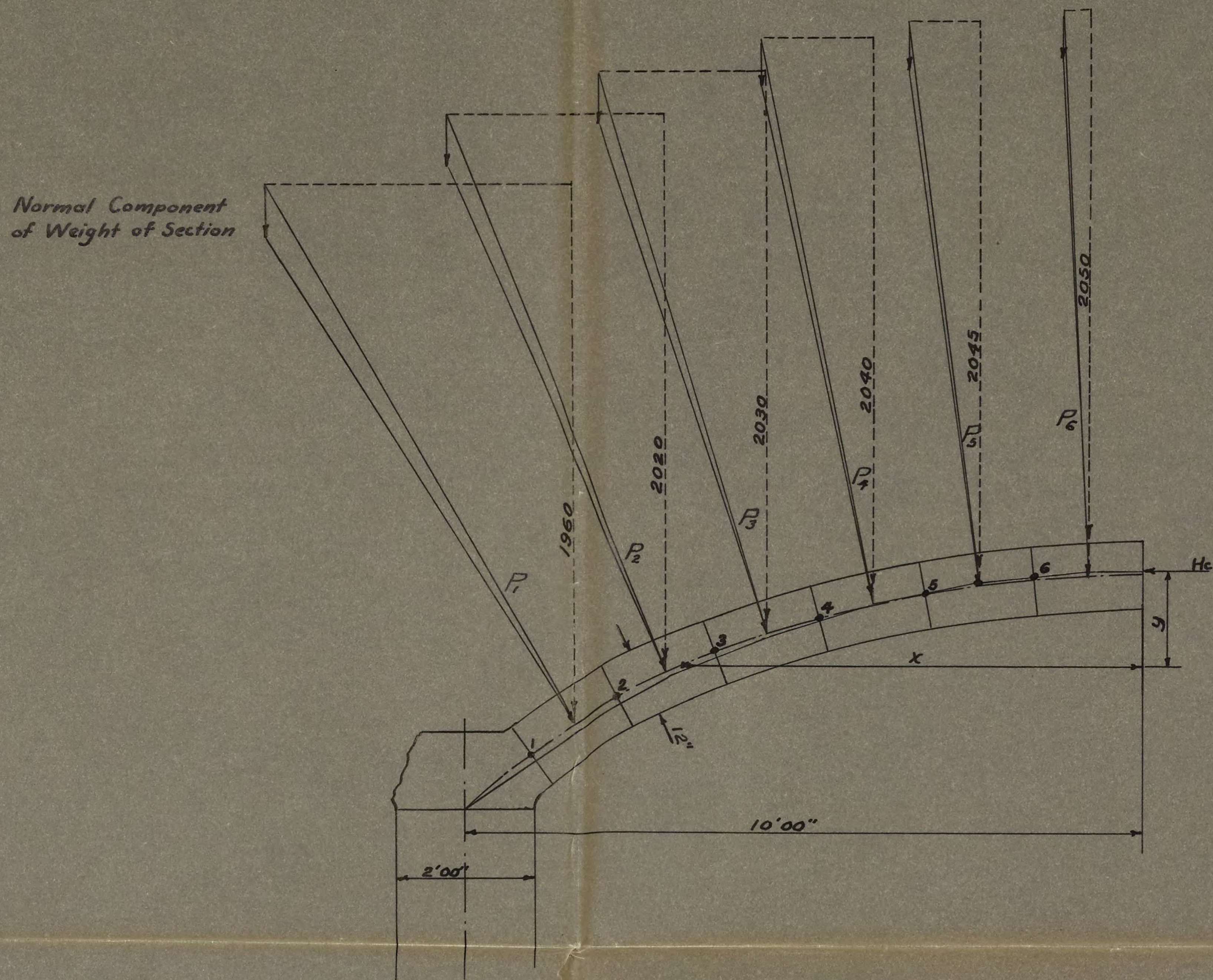
d = width of buttress at base = 2'

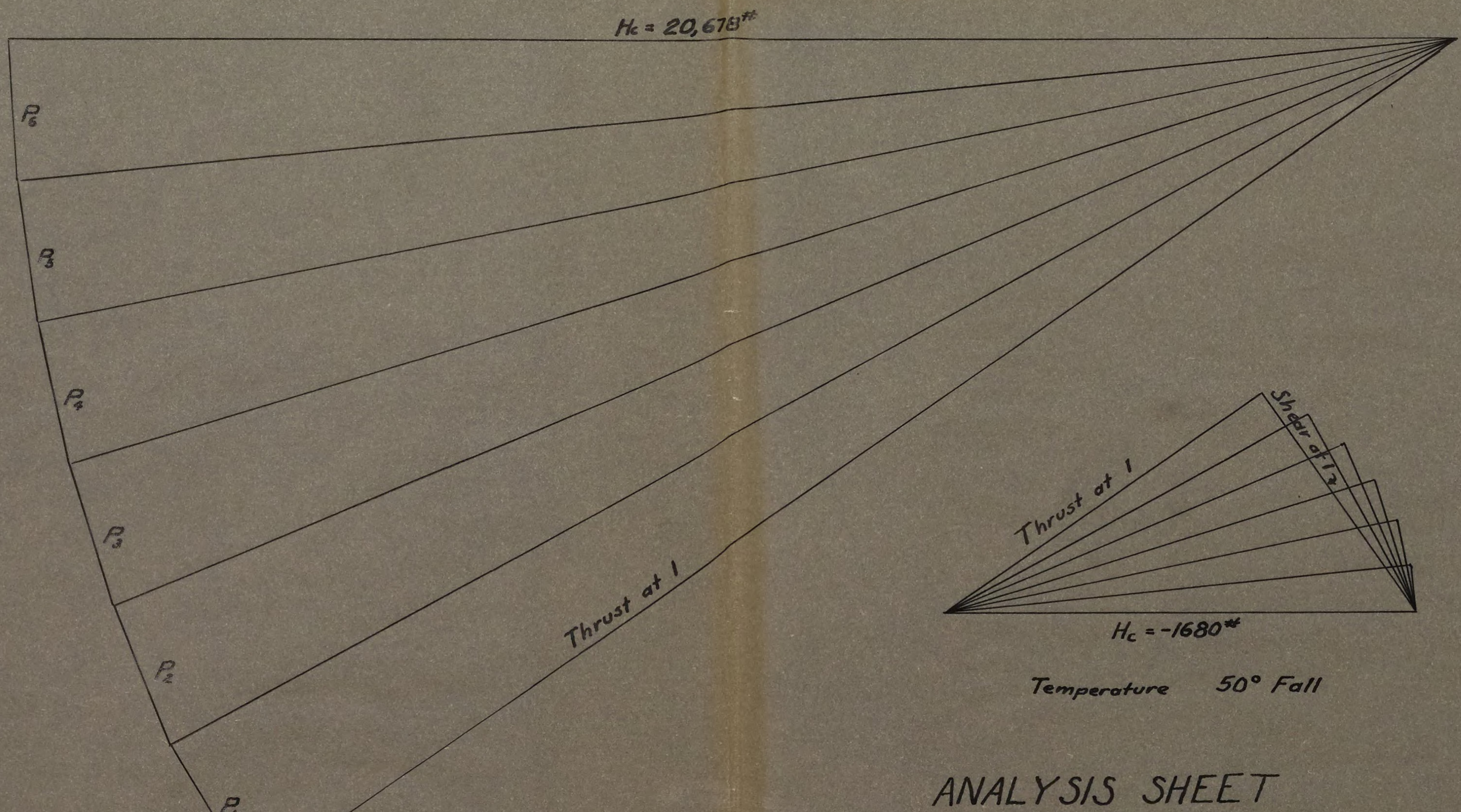
ANALYSIS SHEET NO 4



Curve of Annual Precipitation on St. Joseph River Watershed
Curve of Estimated Annual Runoff from St. Joseph River Watershed
Inches
60
50
40
30
20
10
0
1881
1884
1888
1892
1896
1900
1904
1908
1912
1916
1920
MICHIGAN STATE COLLEGE
DEPARTMENT OF MATHEMATICS

MAP OF THE PORTION OF THE WATERSHED ABOVE THE DAM





ANALYSIS SHEET NO. 2

ANALYSIS SHEET
NO. 3

Pt	x	y	x^2	y^2	m	mx	my	$H_c y$	M	N (scaled)	Temperature			Rib Short.		t	Po	Totals		$\frac{y_o}{t}$	K	L	Unit Stresses	
											$H_c y$	M	N	M	N			M	N				f_c	f_s
Cr.									+1121	20678	-1680	+1790	-1680	+2151	-2020	1.0	.00182	+5072	16978	.299	.632	.0968	364	1735
1	9.00	2.58	81.000	6.656	53555	481995	138172	53,349	+915	21160	-4334	-2544	-1375	-3050	-1650	1.0	.00182	-4679	18135	.258	.744	.0982	331	595
2	7.62	1.76	58.064	3.098	38166	290825	67172	36393	-652	21080	-2957	-1167	-1480	-1400	-1775	1.0	.00182	-3219	17825	.181	2.00	—	250	all comp.
3	6.17	1.10	38.089	1.210	24873	153466	27,360	22746	-1006	20920	-1848	-58	-1550	-70	-1860	1.0	.00182	-1134	17510	.065	1.35	—	164	—
4	4.68	.62	21.902	.384	14230	66596	8883	12820	-289	20840	-1042	+748	-1610	+900	-1930	1.0	.00182	+1359	17300	.079	1.43	—	172	—
5	3.15	.27	9.923	.070	6414	20204	1732	5583	+290	20720	-454	+1336	-1665	+1600	-1980	1.0	.00182	+2226	17075	.130	1.73	—	204	—
6	1.59	.06	2.528	.004	1620	2576	97	1241	+742	20700	-101	+1689	-1675	+2030	-2010	1.0	.00182	+4461	17015	.262	.732	.0982	315	662
	32.21	6.39	211.486	11.425	138,858	1,015,662	243,416																	

Water Pressure

$$H_c = \frac{n_h \Sigma (m_L + m_R) y - \Sigma (m_R + m_L) \Sigma y}{2 [n_h \Sigma y^2 - (\Sigma y)^2]}$$

Since $m_L = m_R$

$$H_c = \frac{6 \times 2 \times 243,416 - 2 \times 138,858 \times 6.39}{2[6 \times 11.425 - 6.39^2]}$$

$$= 20,678^{\#}$$

$$V_c = \frac{\Sigma (m_L - m_R) x}{2 \Sigma x^2} = 0 \text{ , since } m_L - m_R = 0$$

$$M_c = \frac{\Sigma (\Sigma m_L + m_R) - 2 H_c \Sigma y}{2 n_h}$$

$$= \frac{2 \times 138858 - 2 \times 20678 \times 6.39}{2 \times 6}$$

$$= +1121 \text{ foot-lbs.}$$

Temperature

$$H_c = \frac{I}{S} \frac{t_c t_o \ell n_h E_c}{2[n_h \Sigma y^2 - (\Sigma y)^2]}$$

$$= \frac{(\frac{1}{12} \times 12 \times 1728 + 15 \times 2 \times 131 \times 16) \times .000006 \times (60) \times 30 \times 2,000,000 \times 144}{1.6 \times 144 \times 144 \times 2[6 \times 11.425 - 6.39^2]}$$

$$= -1680^{\#}$$

$$V_c = 0$$

$$M_c = -\frac{H_c \Sigma y}{n_h} = -\frac{-1680 \times 6.39}{6} = +1790 \text{ foot-lbs}$$

Rib Shortening

Assume $C_a = 120^{\#}/\text{sq. in.}$

$$H_c = -\frac{I}{S} \frac{C_a \ell n_h}{2[n_h \Sigma y^2 - (\Sigma y)^2]}$$

$$= \frac{1791 \times 120 \times 20 \times 6}{1.6 \times 144 \times 144 \times 55.44}$$

$$= -2020^{\#}$$

$$C_a \text{ (at crown)} = \frac{20,678 - 1680 - 2020}{1 + .027} = 16,500^{\#}/\text{sq. ft}$$

$$C_a \text{ (pt. 4)} = \frac{20840 - 1610 - \frac{1610}{1680} 2020}{1 + .027} = 15,870^{\#}/\text{sq. ft.}$$

$$C_a \text{ (pt. 1)} = \frac{21160 - 1375 - \frac{1375}{1680} 2020}{1 + .027} = 17,650^{\#}/\text{sq. ft}$$

3) 50020

$16,673^{\#}/\text{sq. ft (ave.)}$

$$\frac{16673}{144} = 116^{\#}/\text{sq. in.} \quad \text{O.K.}$$

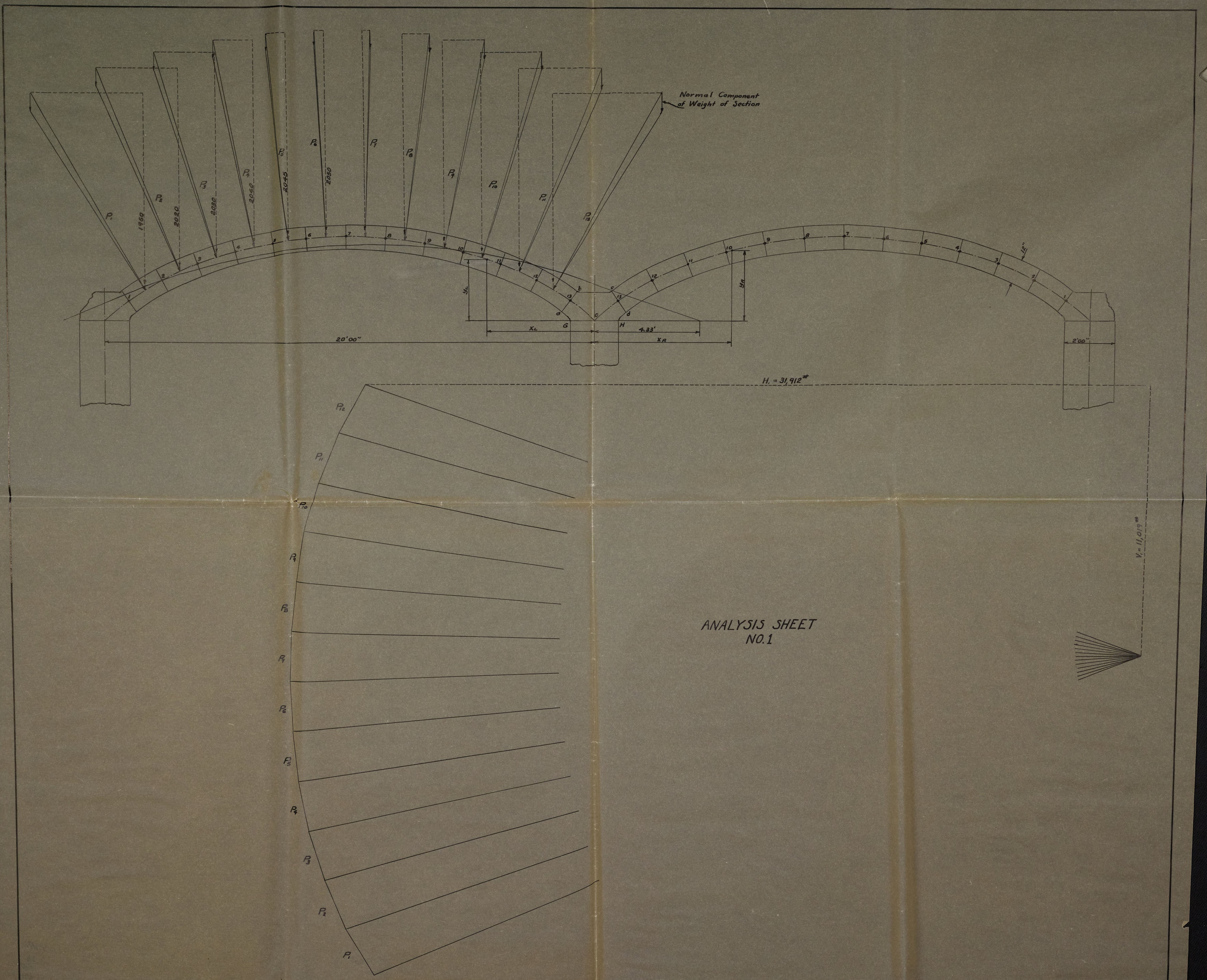
$$\text{Ratio } \frac{H_c \text{ for Rib shortening}}{H_c \text{ for Temperature}} = \frac{2020}{1680} = 1.20$$

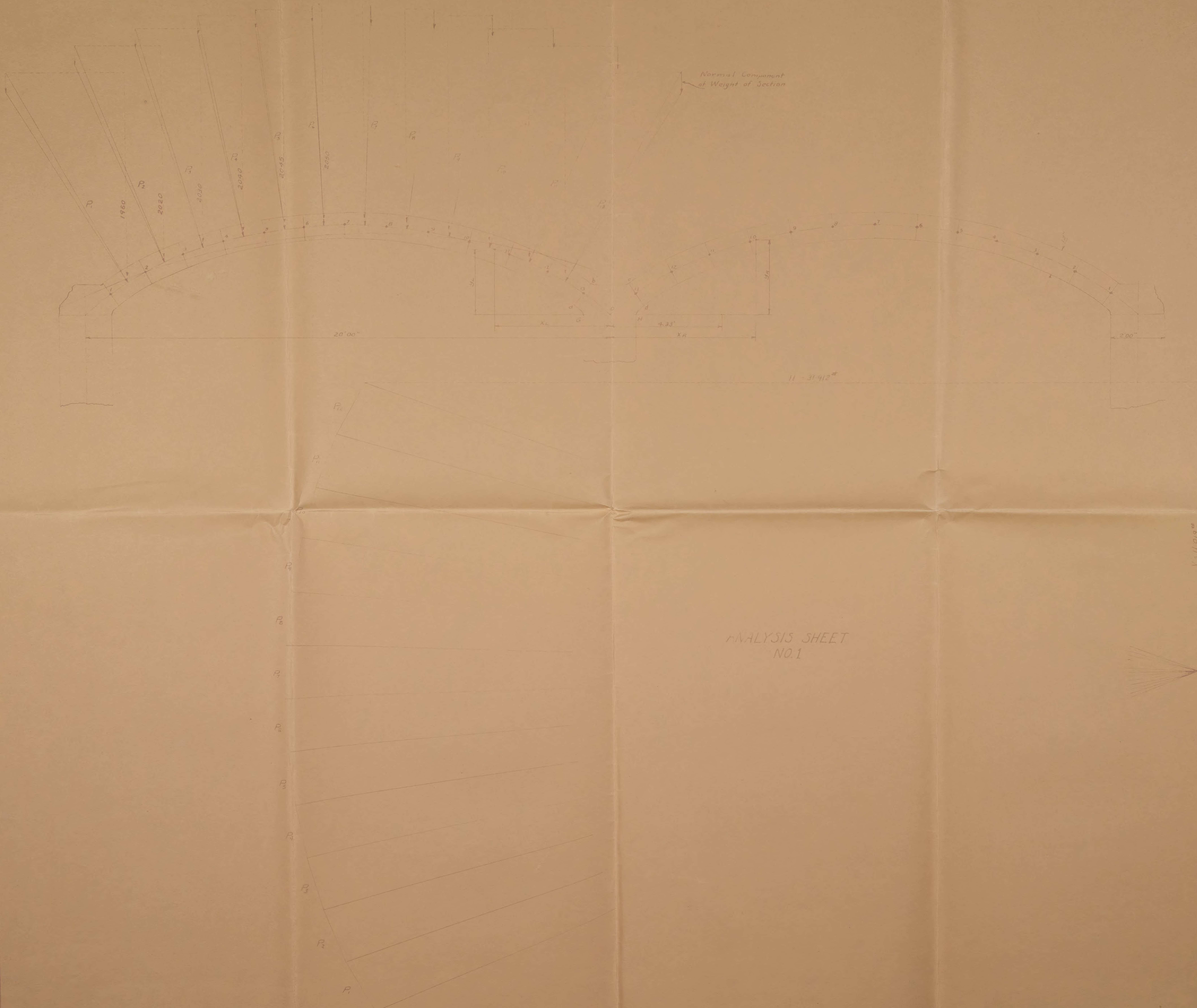
$$M_c = -\frac{H_c \Sigma y}{n_h}$$

$$= -\frac{-2020 \times 6.39}{6} = +2151.3 \text{ foot-lbs.}$$

Compression over entire section - $f_c = \frac{N}{bt} K$

Tension over part of section - $f_c = \frac{M}{L b t^2}$, $f_s = n f_c \left(\frac{d}{kt} - 1\right)$


Normal Component
of Weight of Section
1950
2020
2030
2050
2045
2050
20'00"
2'00"
4.33'
H. = 31,912#
V = 11,019#
ANALYSIS SHEET
NO. 1



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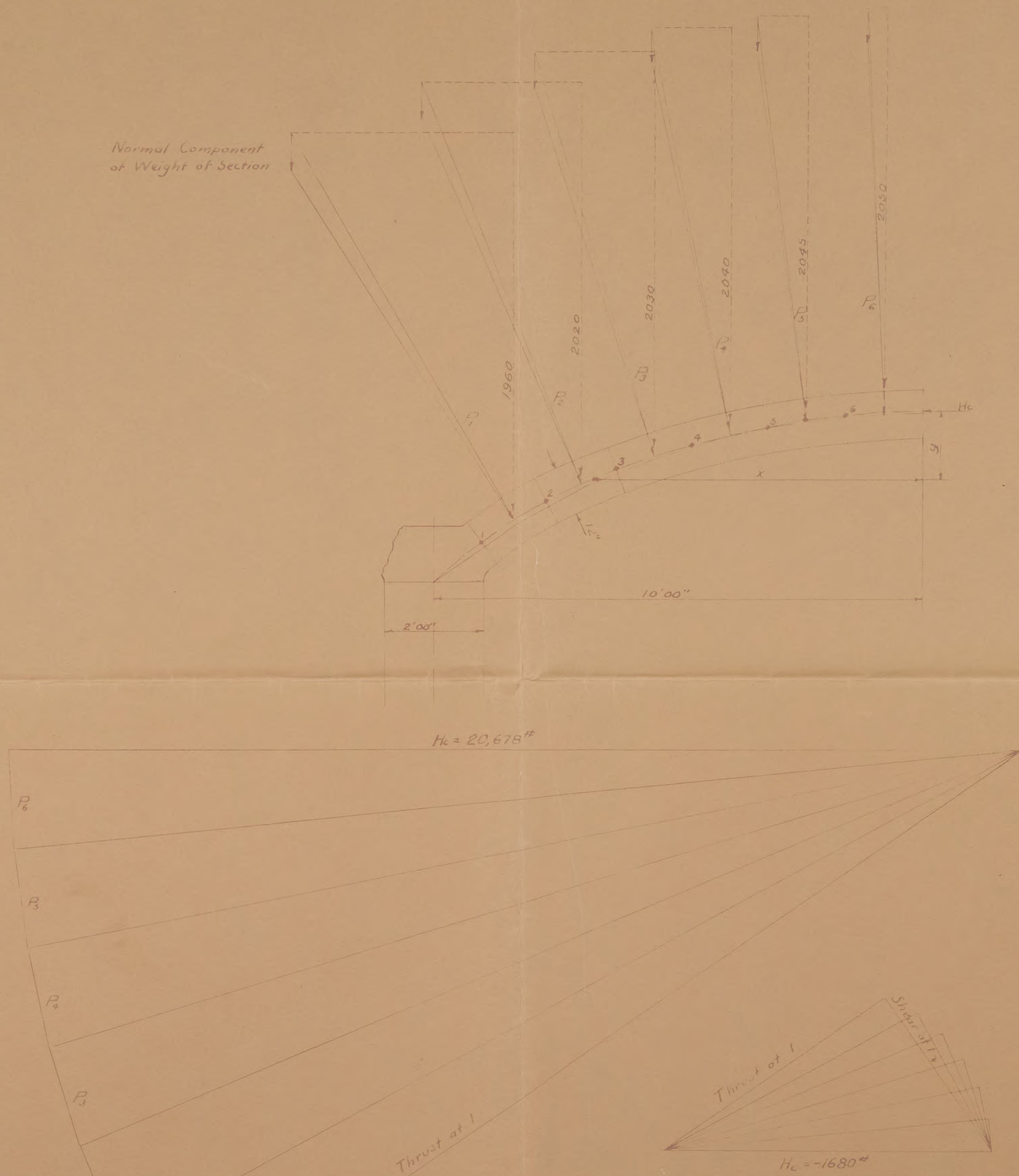
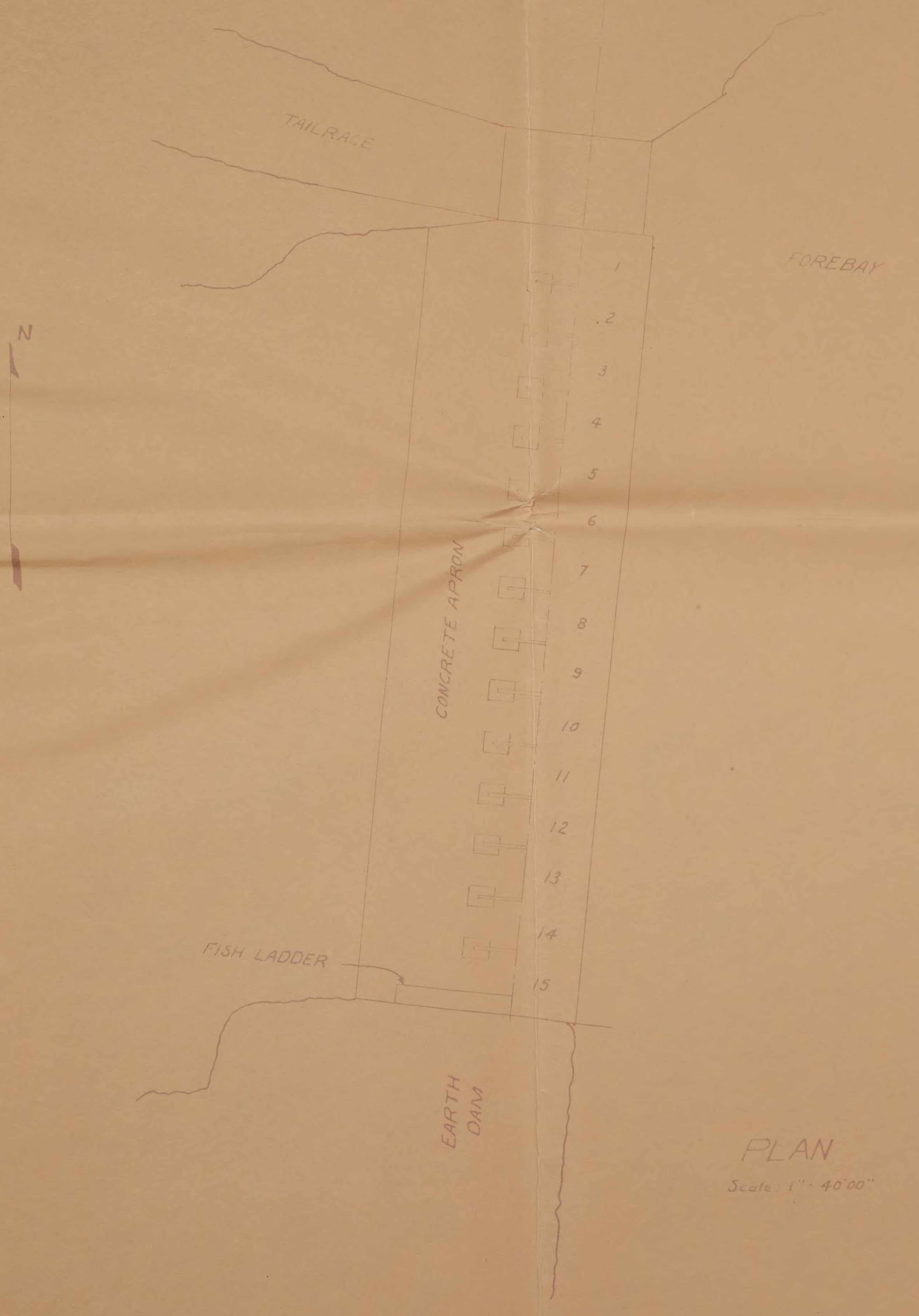
NO.1

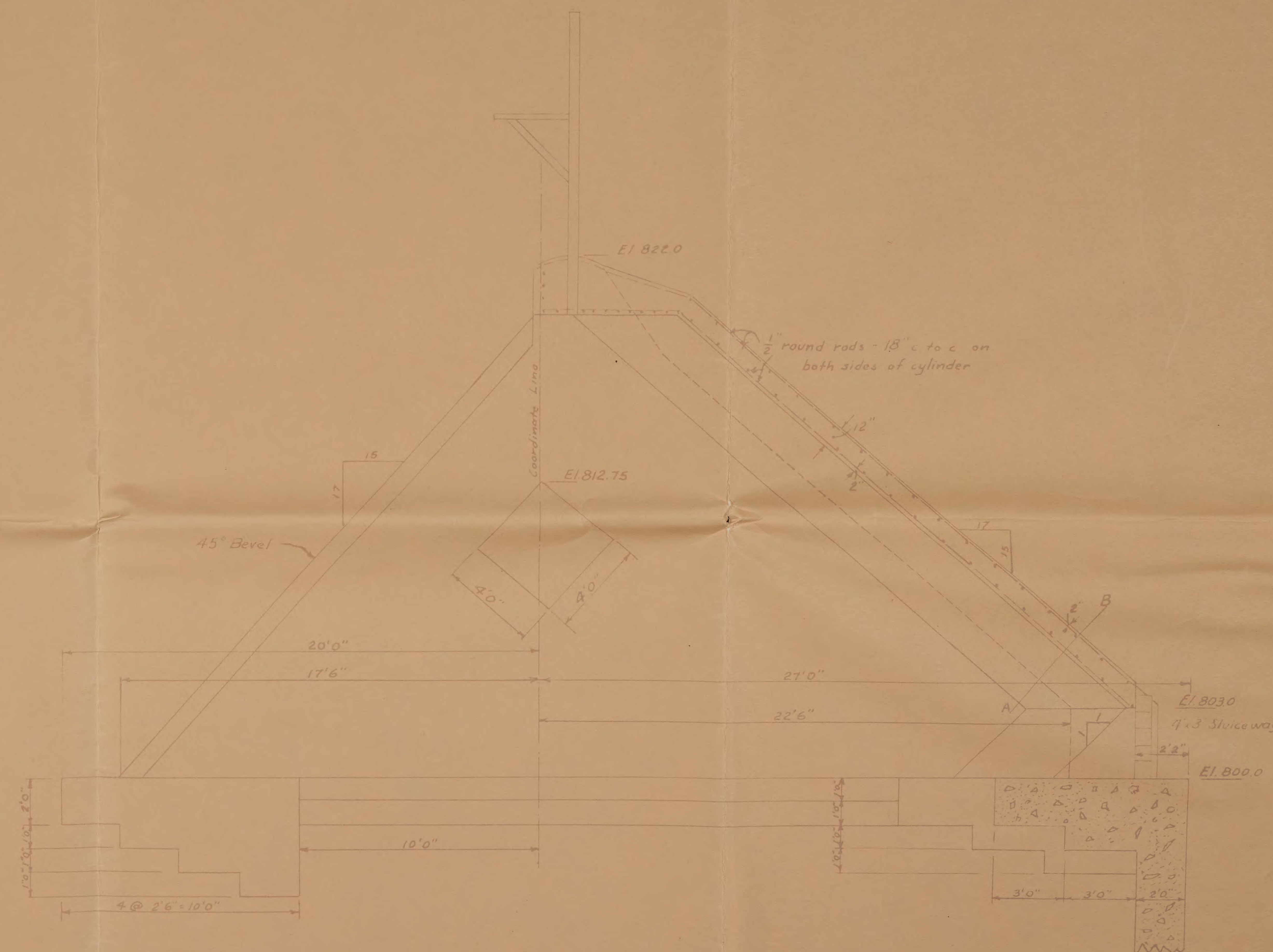


ANALYSIS SHEET
NO. 2



PLAN

Scale 1" = 40'0"



SECTION AT CROWN OF ARCHES 2-14

SUPPL
MA