

ULTIMATE DESIGN OF REINFORCED
CONCRETE

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

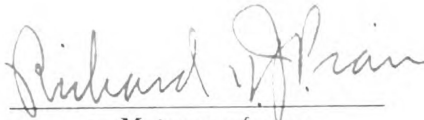
Lyle Clinton Davis

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This is to certify that the
thesis entitled
Ultimate Design of Reinforced Concrete

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Major professor

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Ultimate Design of Reinforced Concrete

By

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INTRODUCTION

Although present day specifications emphasize stresses at working loads (theory of elasticity) and present day calculations pretend to calculate these stresses, laboratory tests of reinforced concrete beams show that actual deformations and stresses at working loads only faintly resemble our calculated values. The theory of elasticity, as applied to design of sections, is too inflexible and inaccurate to be entirely satisfactory and give the most efficient use of materials. Actual stresses under working loads are affected by so many conditions that they are quite indeterminate. It has been proved that shrinkage and plastic flow entirely upset stress computations based on the elastic theory but they have little effect on ultimate strength because before failure the strains are great enough to cause redistribution of stress.

There are several advantages that the ultimate design theory has over the elastic theory but only a few of the more important ones will be listed here. (1) The application of the ultimate design theory is much simpler, easier to use and is better supported by tests than the elastic theory. (2) Due to local buckling, load and stress are not always directly related, therefore it would be more

consistent to apply the safety factors to the load (ultimate design theory) rather than the stress (elastic design theory). The useful strength of each member would have approximately the same relationship to the strength of the entire structure.

The load factors can be varied to comply with the accuracy of predicting the applied loads. Dead loads are definite and constant, therefore a small safety factor may be used. T. Y. Lin and R. C. Reese suggested the value of 1.2. Live loads being variable and highly indeterminate, demand a higher factor of safety. R. C. Reese suggested the value of 2.4 and T. Y. Lin, the value of 2.0.

This paper deals only with the flexural computations of the various theories.

DERIVATIONS OF THE ULTIMATE THEORY

The ultimate theory of reinforced concrete as presented by C. S. Whitney is presented here. The derivations will be limited to flat slabs and rectangular beams with tension steel.

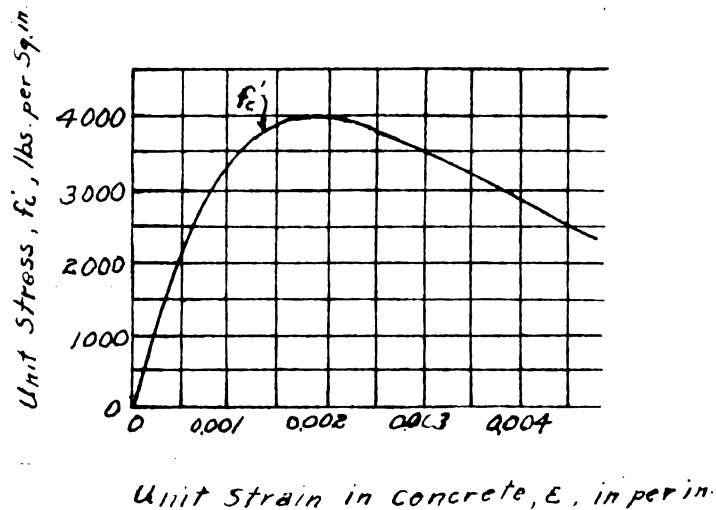


Fig. 1: Idealized Stress-Strain Curve
for concrete cylinder.

1. "Plastic theory of Reinforced Concrete Design" by Charles S. Whitney, Trans. Am. Soc. of Civ. Engrs., Vol. 107, Dec. 1942, pp. 251-326.

The stress distribution in a concrete beam at failure may be assumed to have the same shape as the cylinder stress-strain curve shown in Fig. 1. The total compression, C , is the area bounded by the curve and the line of action of C passes through the center of gravity of this area. For simplicity, the actual irregular stress block is replaced by a rectangular stress block of equal size. If the width of the rectangular stress block is $0.85 f'_c$ and the depth equal to a the location of the center of gravity of this rectangle will correspond closely with that of the actual area. The rectangle is an equivalent replacement and does not mean that the theory is based on a rectangular stress distribution.

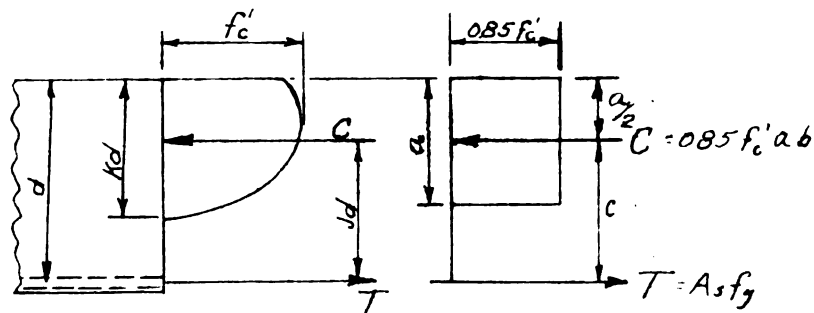


Fig. 2: Stress distribution on the cross-section of a rectangular beam.

It is worthy to note in Fig. 2 that the depth a is less than the distance to the neutral axis kd , and bears no definite relation to it.

The equilibrium of the internal forces gives:

$$T = C$$

therefore

$$a = \frac{A_s f_y}{0.85 b f'_c} \text{-----} 1$$

in which: A_s = area of tensile steel; f_y = yield point stress of steel; b = width of beam; f'_c = standard concrete cylinder strength.

UNDER-REINFORCED DESIGN. The ultimate resisting moment of the beam as controlled by steel failure can be written:

$$M = Tc = A_s f_y \left(d - \frac{a}{2}\right) = A_s f_y \left(d - \frac{A_s f_y}{1.70 b f'_c}\right) \text{--} 2$$

also:

$$M = Cc = 0.85 f'_c ab \left(d - \frac{a}{2}\right) \text{-----} 3$$

$$0.85 f'_c b (a^2 - 2ad) = - 2M \text{-----} 4$$

$$a^2 - 2ad + d^2 = - \frac{2M}{0.85 f'_c b} + d^2 \text{-----} 5$$

$$a - d = \pm d \sqrt{1 - \frac{2M}{0.85 f'_c b d^2}} \text{-----} 6$$

Since a must be less than d , use the negative sign with the square root term.

$$\frac{a}{d} = \left[1 - \sqrt{1 - \frac{2.35M}{f'_c b d^2}} \right] \text{-----} 7$$

The foregoing expressions are relatively simple and are independent of n , (the modulus ratio). It should be

noted that all of the equations apply only to ultimate load conditions and do not predict stresses under working loads.

The required steel area is

$$A_s = \frac{M}{c f_y} \text{ ----- } 9$$

BALANCED DESIGN: It has been shown by beam tests reported by Slater and Lyse that for balanced design, a equals 0.537d and $c = 0.732d$.

Substituting these values in equation 2,

$$M = \frac{f'_c}{3} b d^2 \text{ ----- } 10$$

and

$$M = T_c = f_y A_s (0.732d) \text{ ----- } 11$$

or

$$0.732d f_y A_s = \frac{f'_c}{3} b d^2 \text{ ----- } 12$$

therefore

$$A_s = 0.456 \frac{f'_c}{f_y} b d \text{ ----- } 13$$

The following graphs were drawn to simplify the computations involved in checking or designing some types of reinforced concrete members. In design problems, f'_c , f_y and the bending moment including dead load with proper safety factors applied are usually known.

The following curves are based on the preceding derivations.

The curves in fig. 3 were drawn using equation 10. Being all one way slabs are designed as rectangular beams

12 inches wide, the only variable which M is dependent upon are f'_c and d^2 . If we draw a family of curves with a value of f'_c constant for each curve, M is dependent on d^2 . This plots as a straight line on logarithmic paper. The curves in fig. 4-7 were plotted using equation 13. A_s is dependent upon three variables ($b = 12$ in.). One variable f_y was eliminated by drawing four graphs, one for each of four common values of f_y . If a value of f_y is encountered other than one of the values used, the value of A_s may be interpolated from the two adjacent graphs.

The curves of fig. 8 were drawn using equation 4, letting $N = 2 ad - d^2$. The curves of fig. 9 were drawn using the earlier substitution of $N = 2 ad - d^2$. The curves of fig. 10-13 were drawn using equation 1.

The curves pertaining to beams are very similar to those of one way slabs except the width of the beam b is a variable. The curves of fig. 14 were drawn using equation 10. The curves of fig. 15-18 were drawn using equation 11.

The curves of fig. 19-26 were drawn using equation 8, and the curves of fig. 27-30 were drawn using equation 9.

To design one-way slabs using balanced design procedure: enter fig. 3 with the bending moment and f'_c to obtain, d , the effective depth of the slab. With d and f'_c enter

either fig. 4, 5, 6 or 7 which ever corresponds to the given f_y and get, A_s , the area of the reinforcing steel.

To design one-way slabs using under-reinforced design procedure; enter fig. 3 with the bending moment and f to get a value N . Now enter fig. 9 to get values of d and a . In some cases there may be a choice of values for these quantities. Enter fig. 10, 11, 12 or 13 corresponding to the given value of f_y , with f'_c and a , of each choice, to arrive at values of A_s . Use the most economical set of values for the final design.

To design rectangular beams using balanced design procedure: enter fig. 14 with the bending moment and f'_c to obtain bd^2 . Compute values of d with assumed values of b . Now enter fig. 15, 16, 17 or 18 with the bending moment and values of d to get values of A_s . Use the most economical set.

To design rectangular beams using under-reinforced design procedure; enter fig. 19-26 depending on f'_c , with the bending moment and bd^2 , that you have chosen, to get $\frac{c}{d}$. bd^2 will be a difficult value to arrive at until experience is gained in this procedure. The cross section will be very nearly the same as that gotten by elastic design, so experience in other design procedures will be helpful. Keep in mind when picking bd^2 that the under-reinforced design will dictate failure of the reinforce-

ment. Therefore a theoretical limit would be a steel beam incased in concrete. The steel varies inversely as the concrete area, so the designer should make an attempt at an economical balance between the two materials providing there are no other restrictions. Compute c from the values of $\frac{M}{b d^2}$ and f . Now enter fig. 27, 28, 29 or 30 with the bending moment and c to obtain the reinforcing steel area A_s . The flexure computations are finished now and it may be necessary to check the design for shear, bond, diagonal tension and deflection.

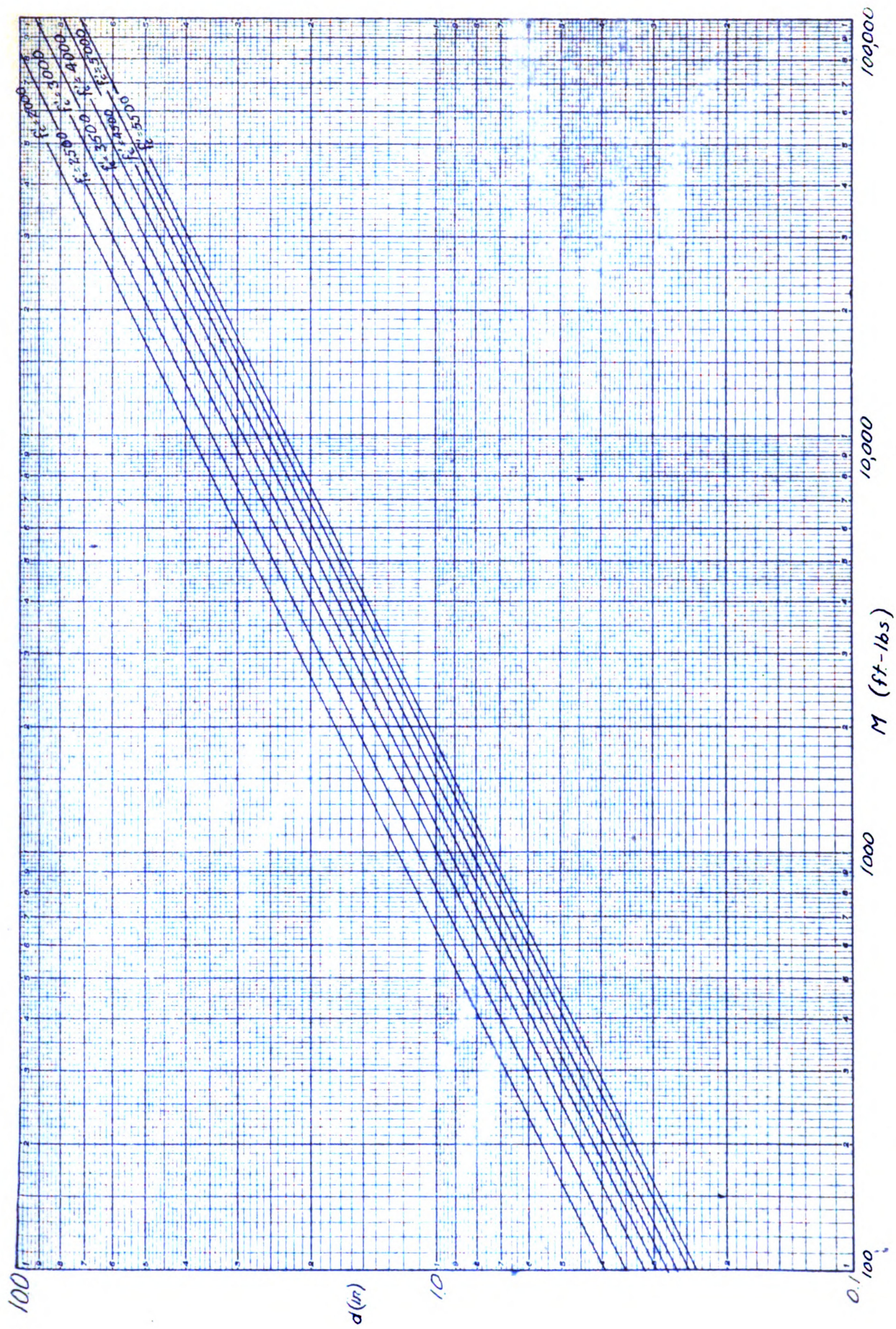
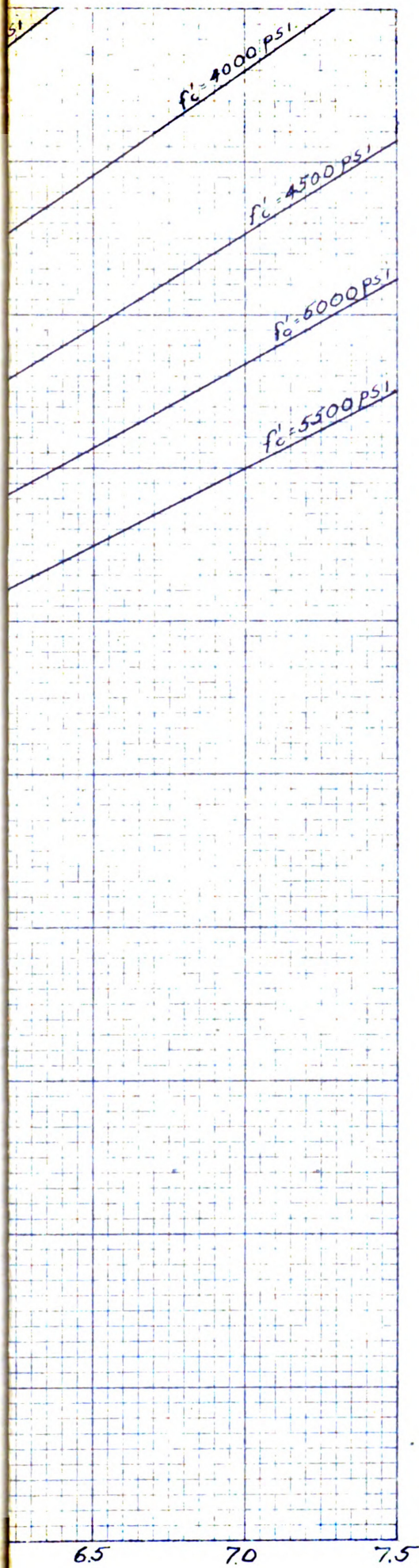
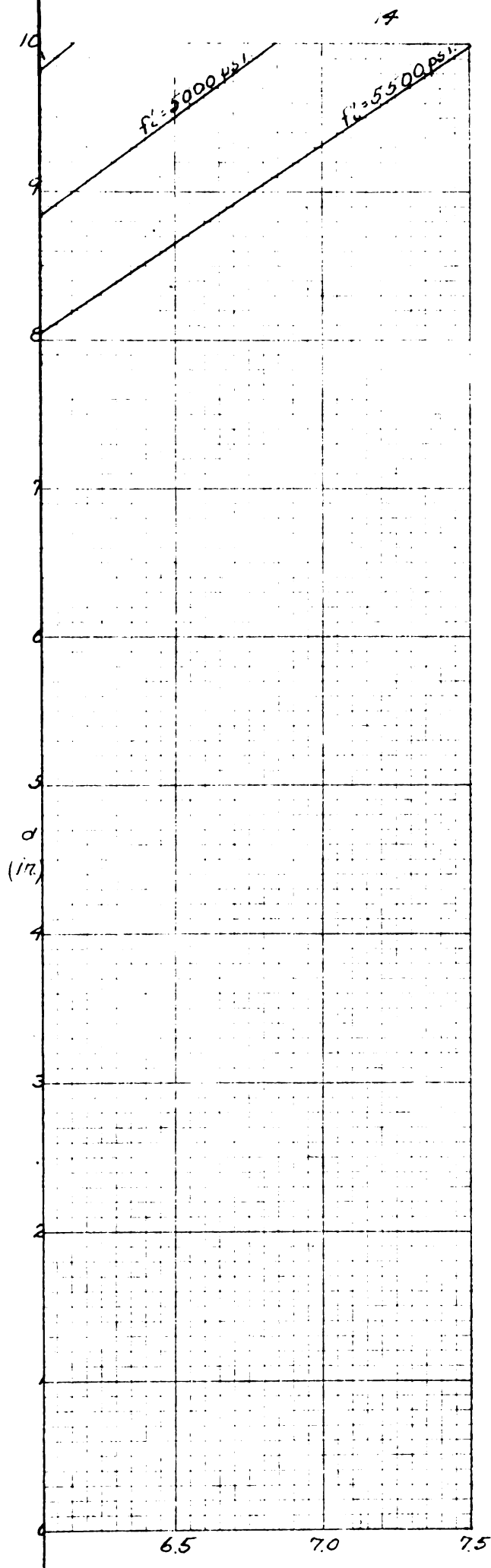
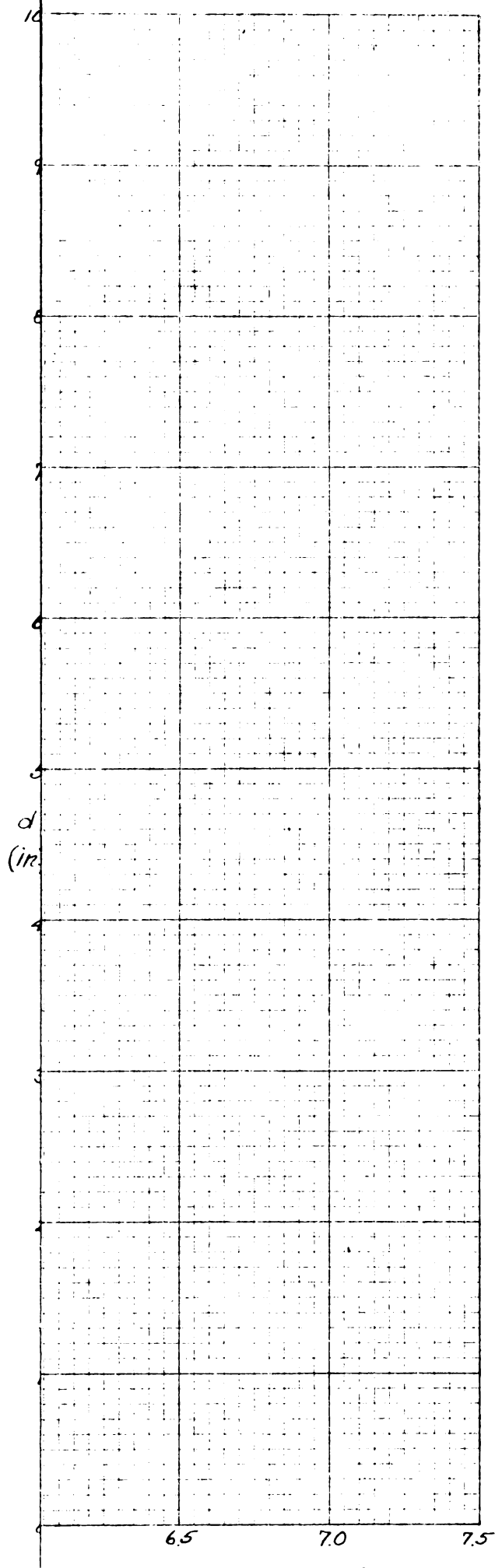
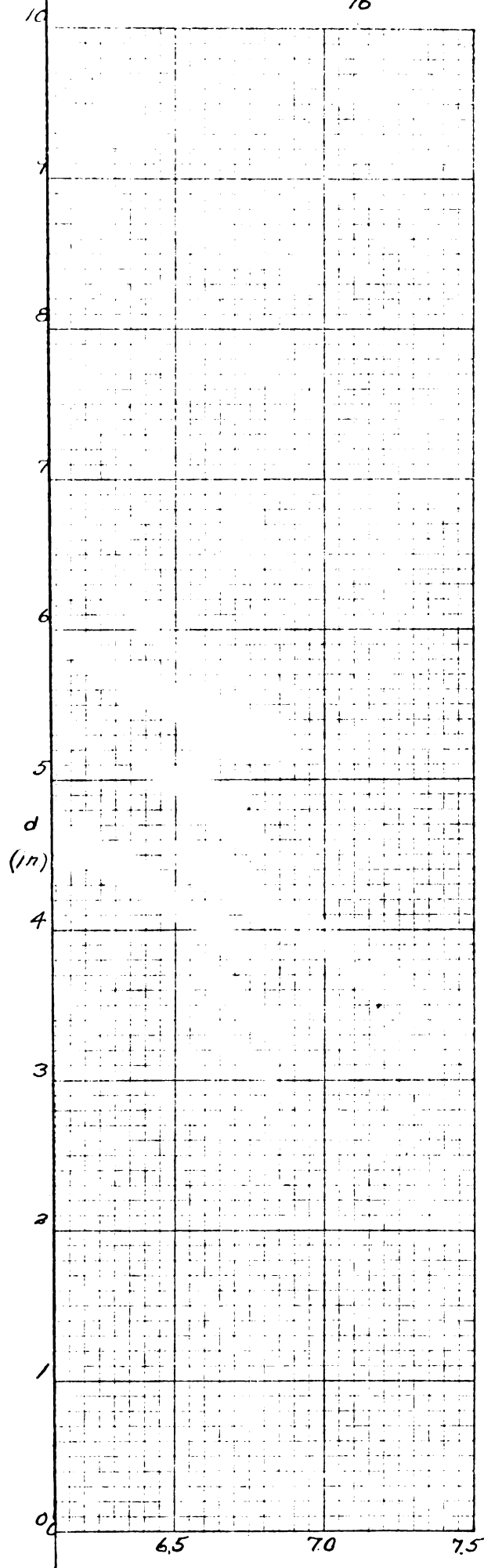


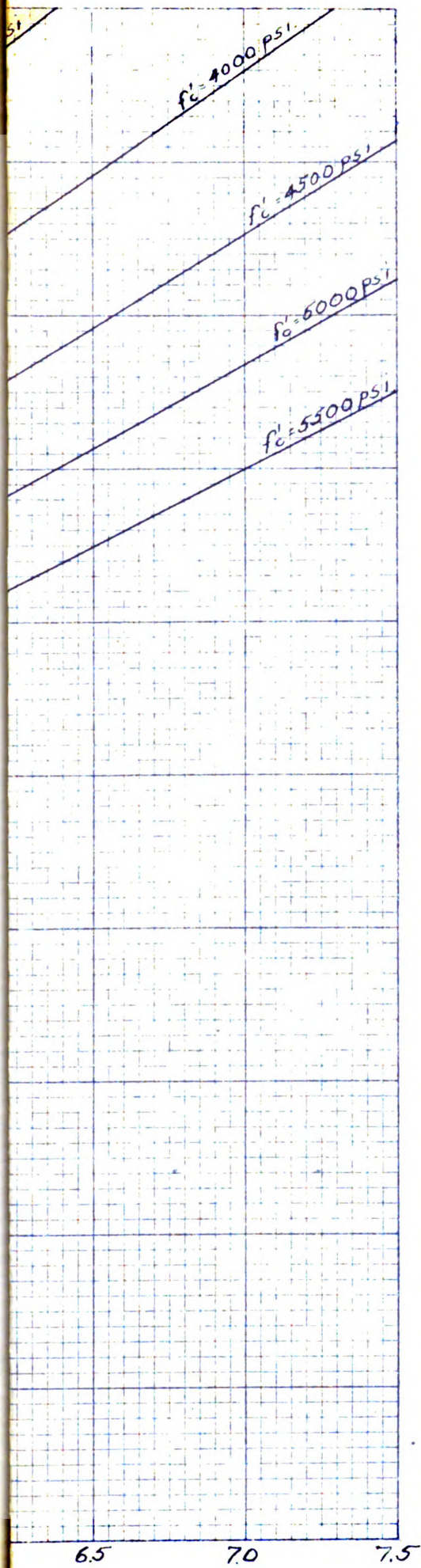
Fig. 3 : One-Way Slabs using Balanced Design

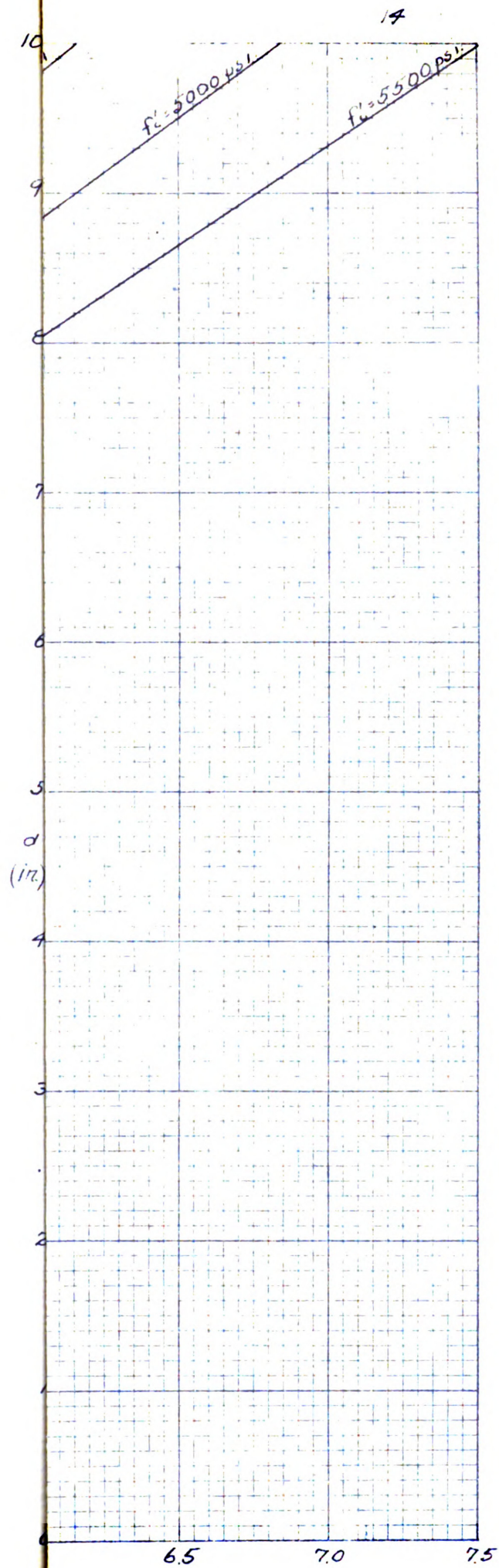


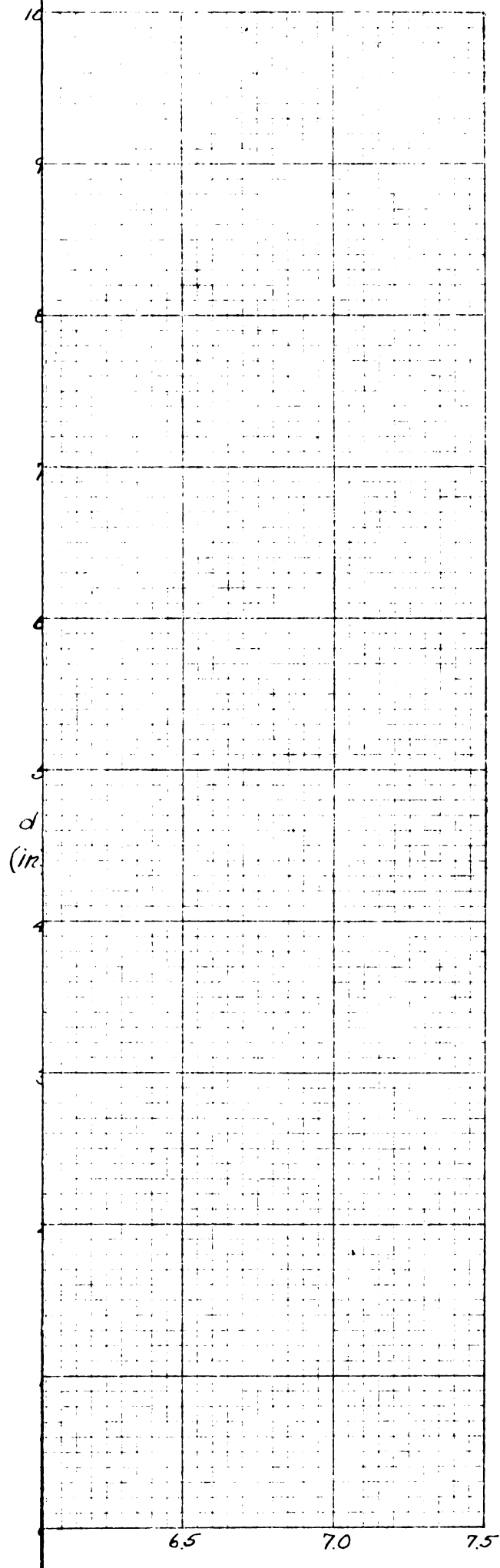


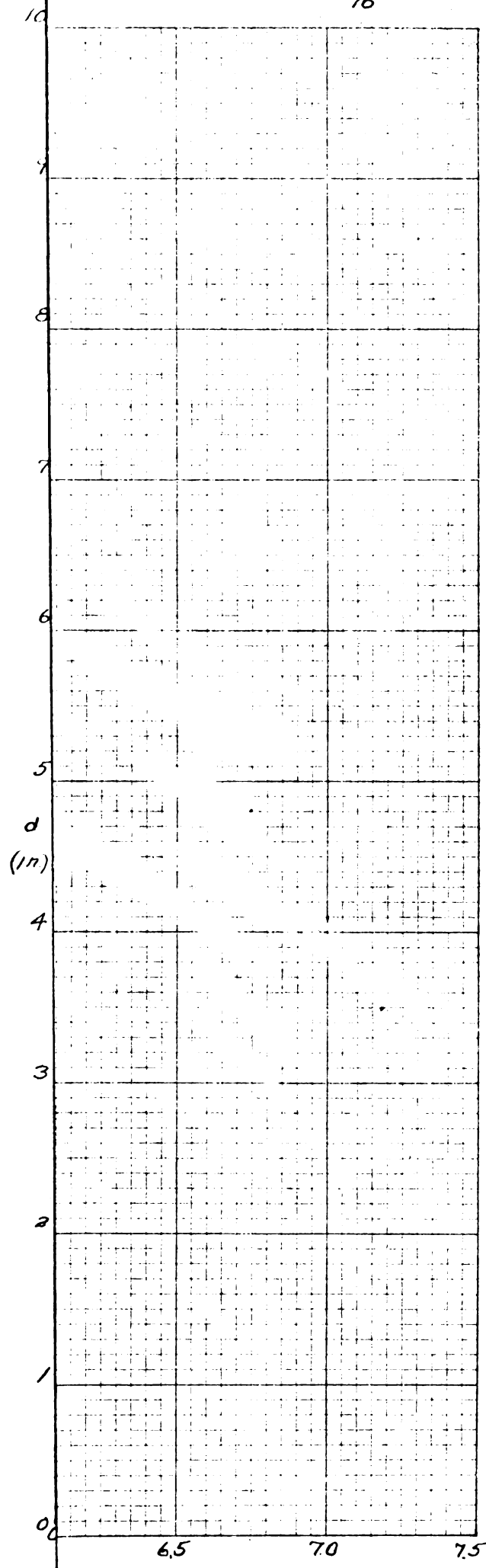


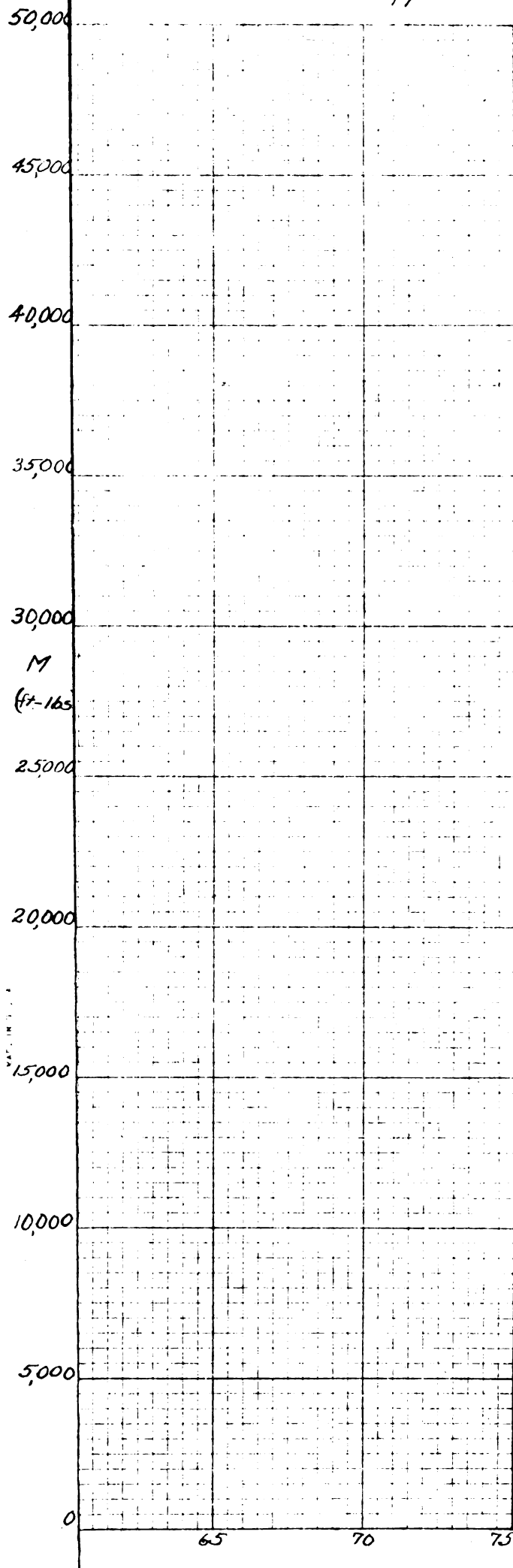


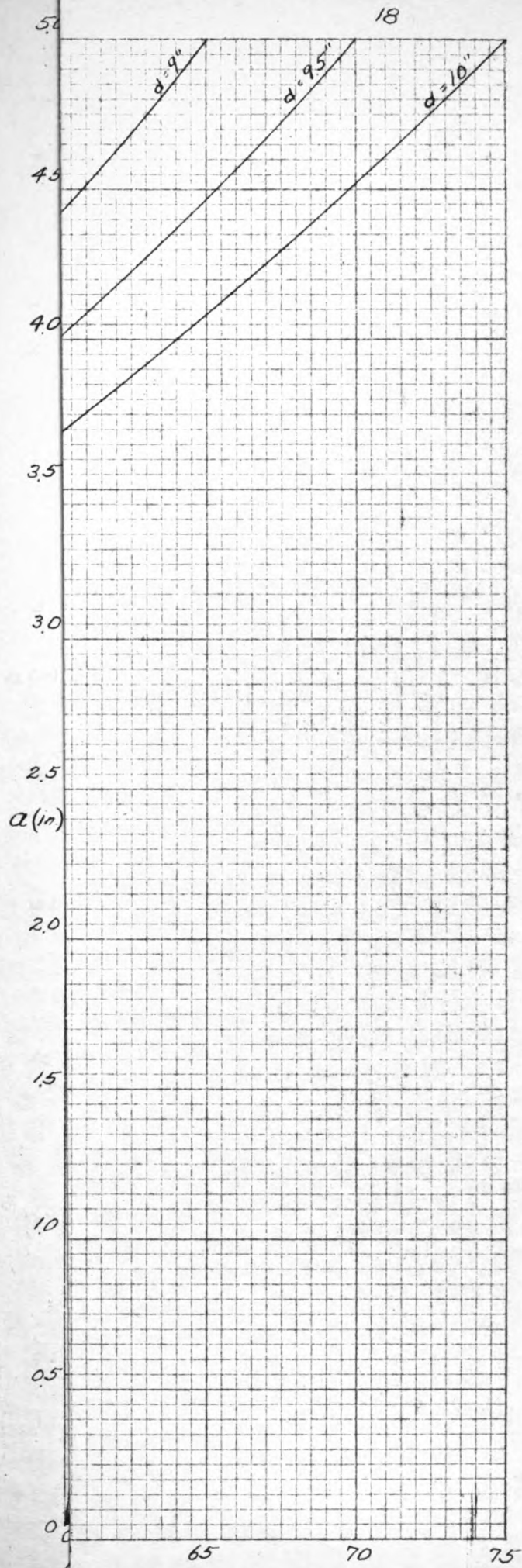


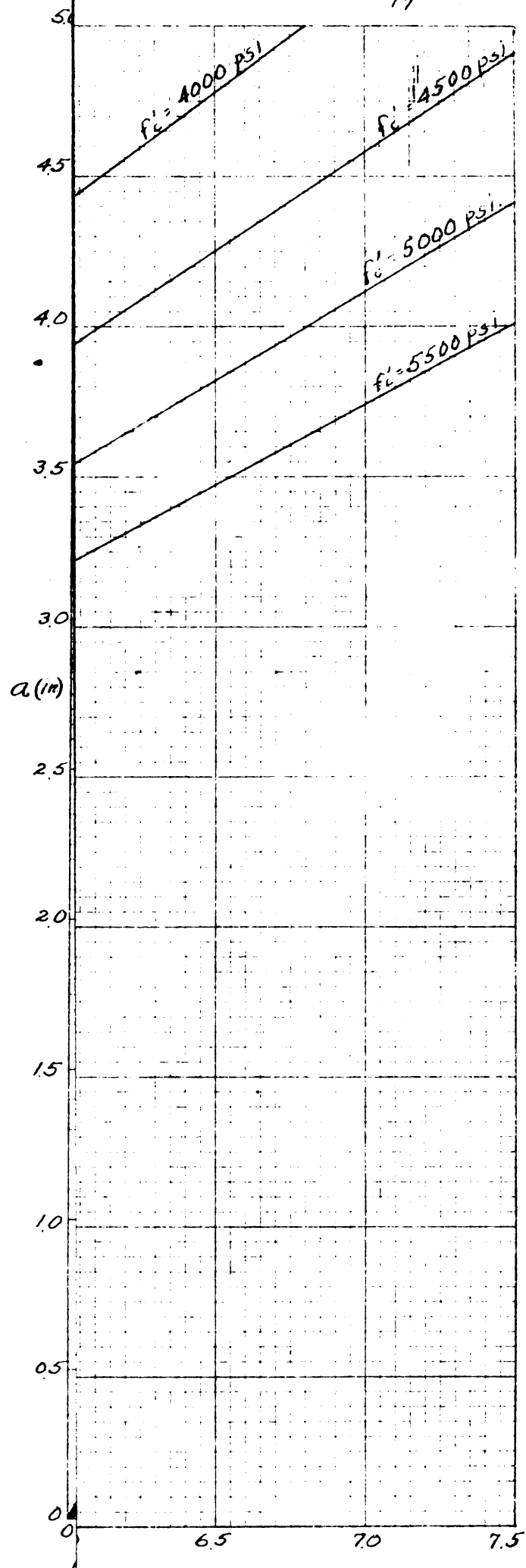


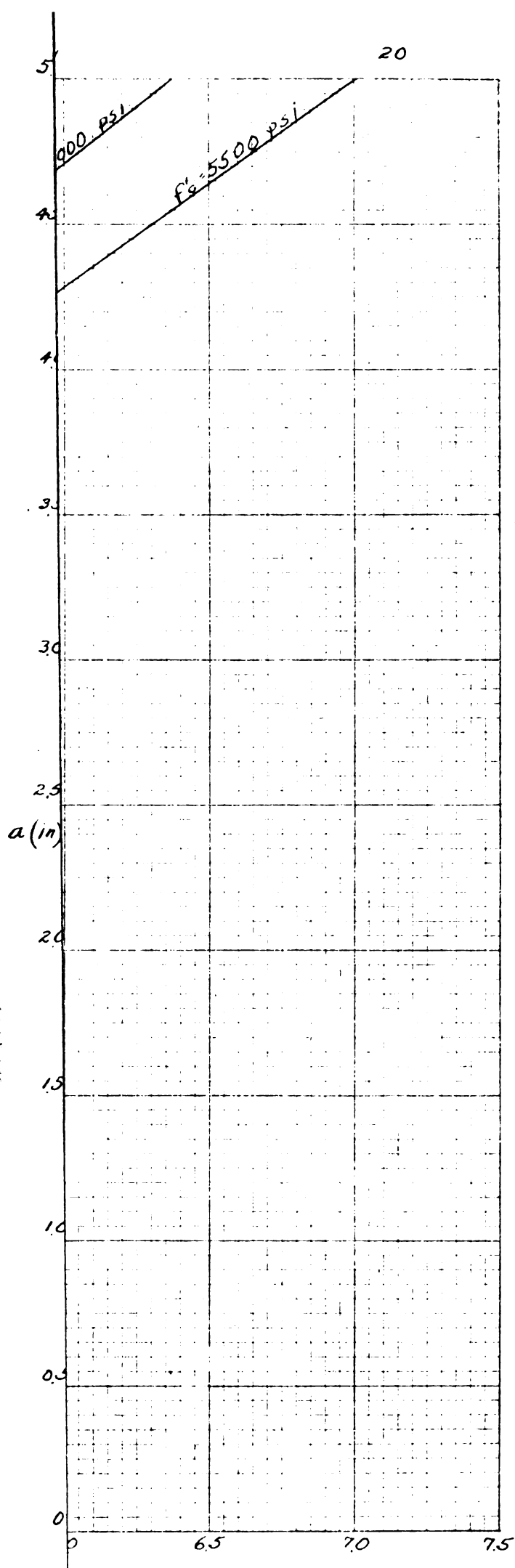


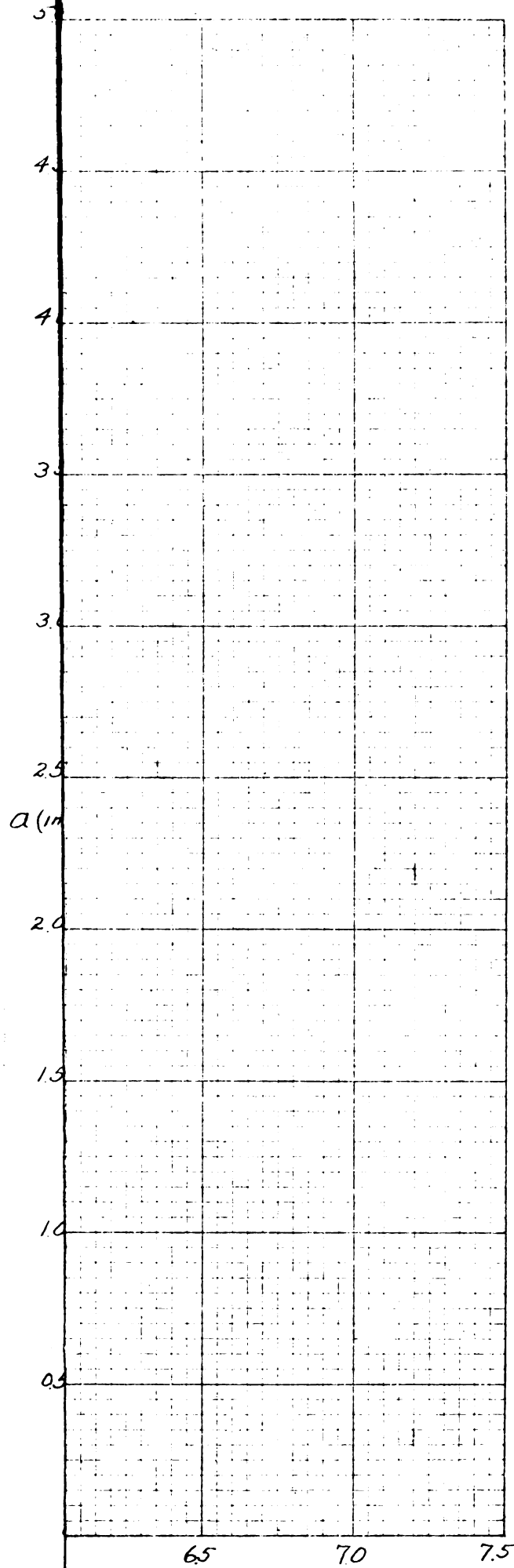


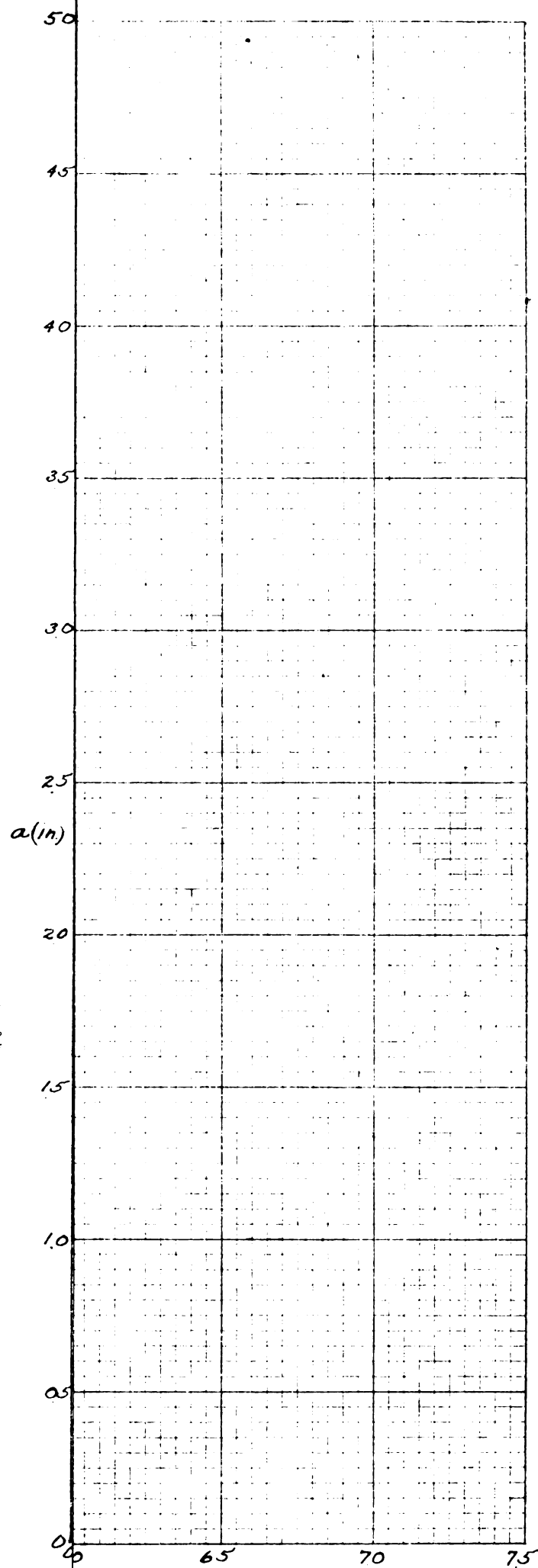




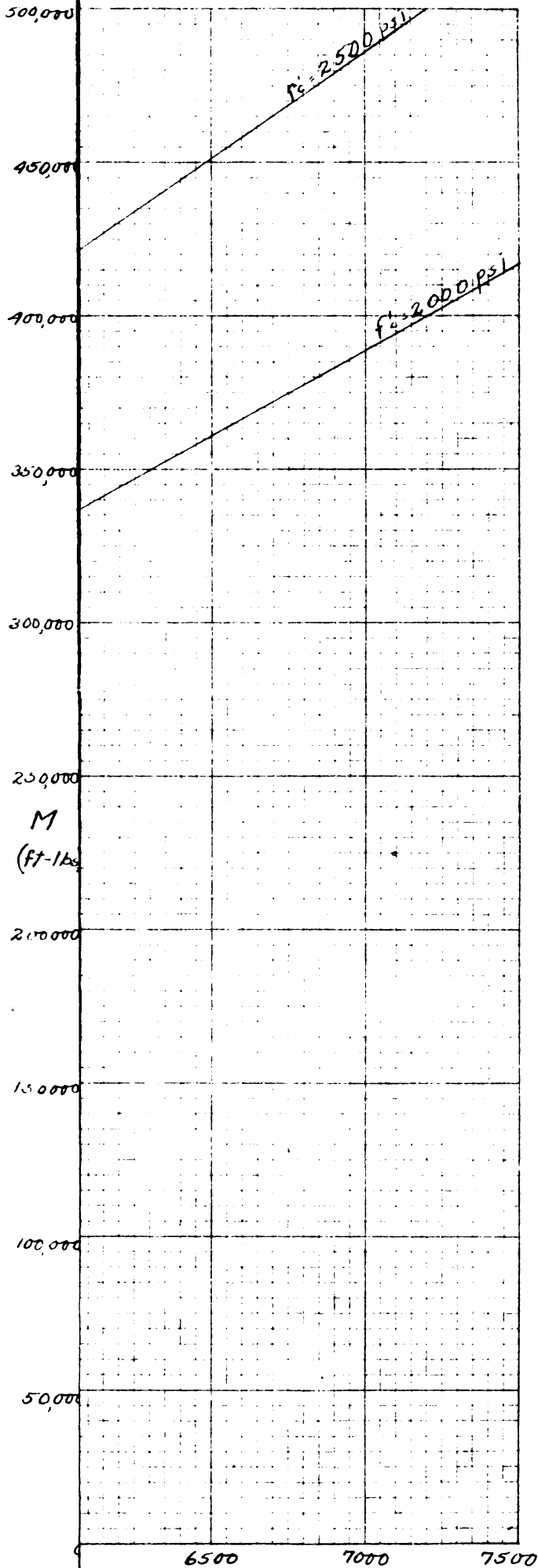


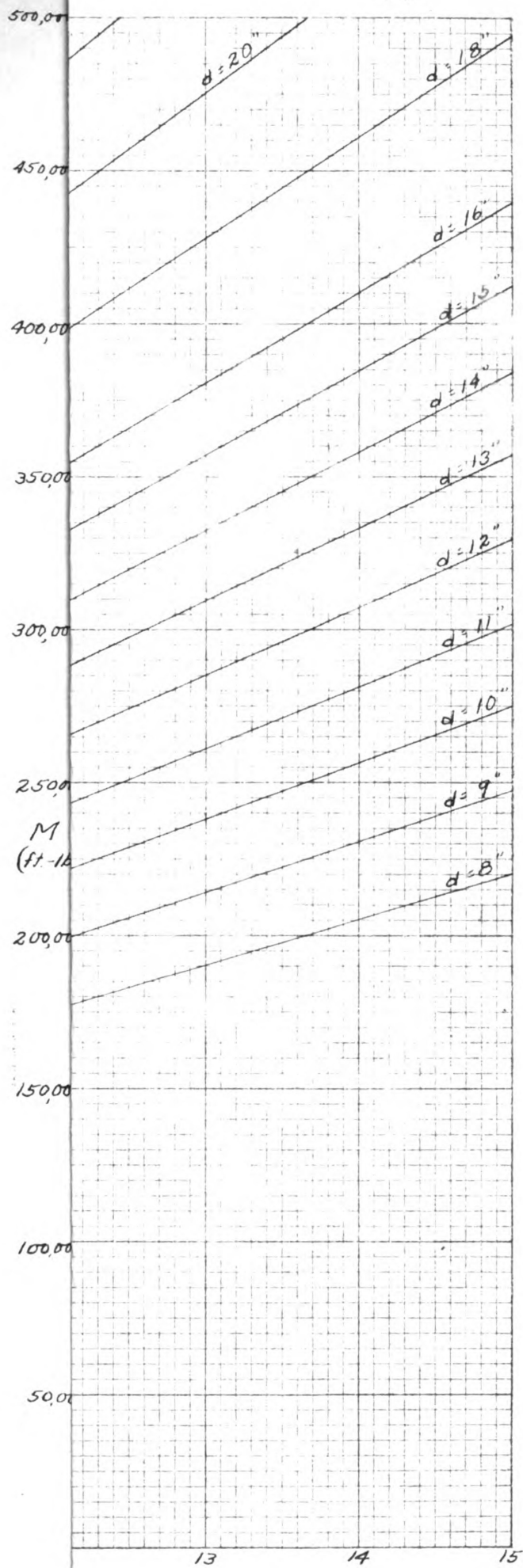


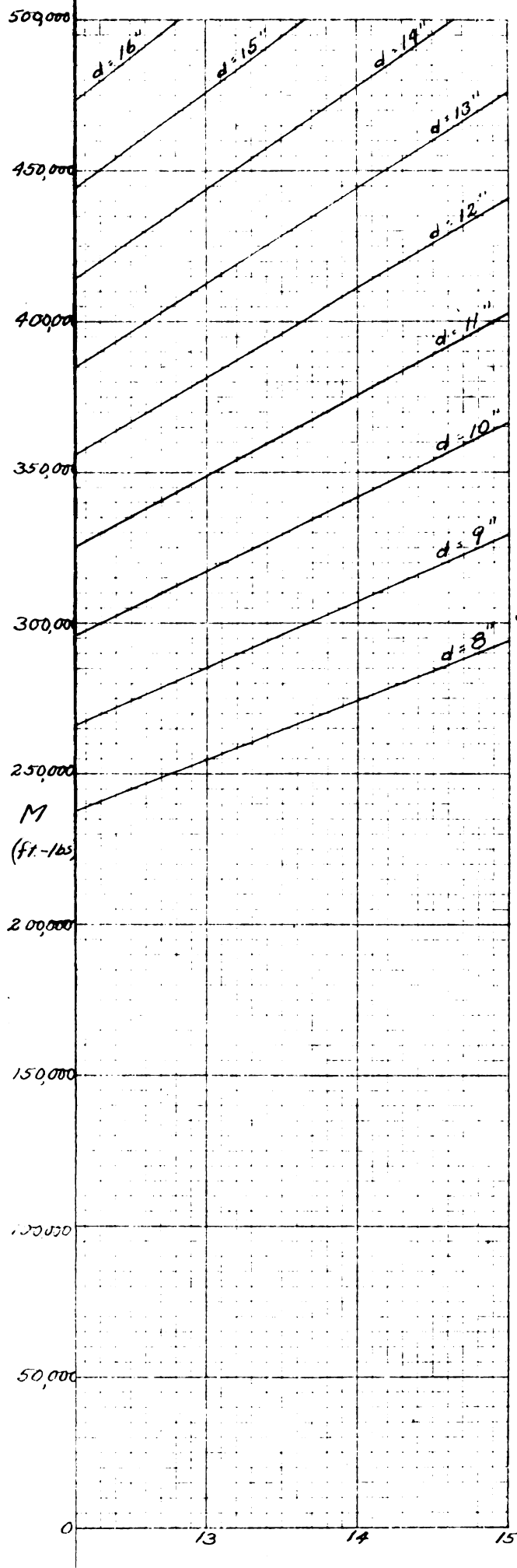


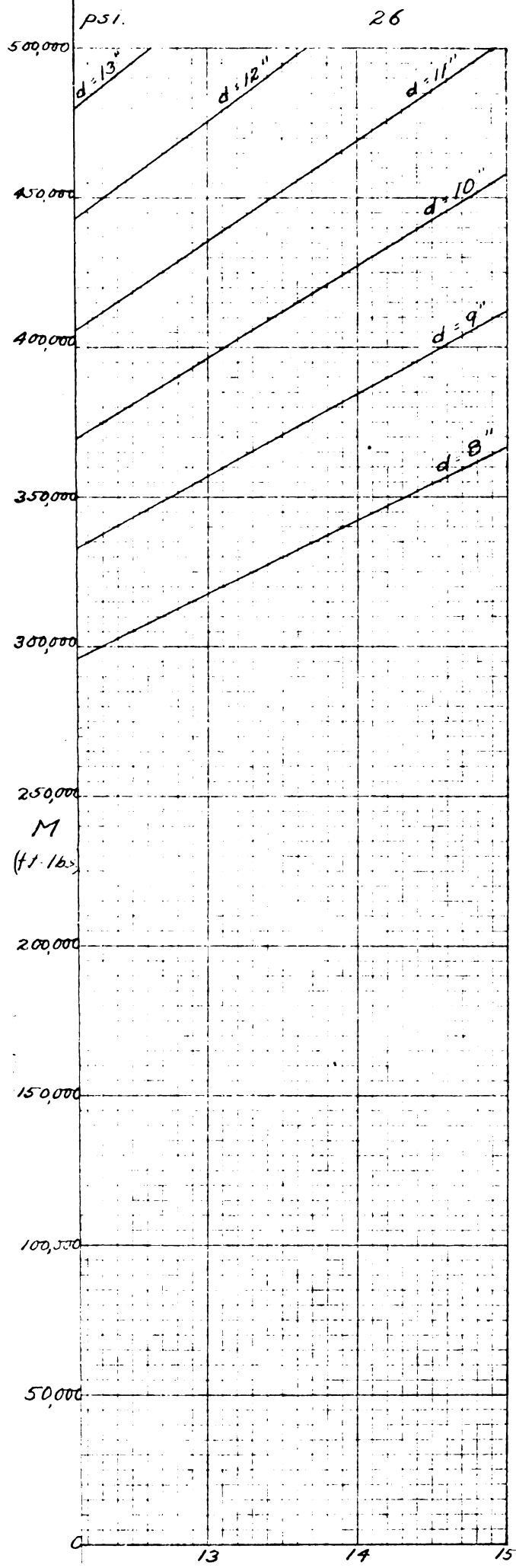


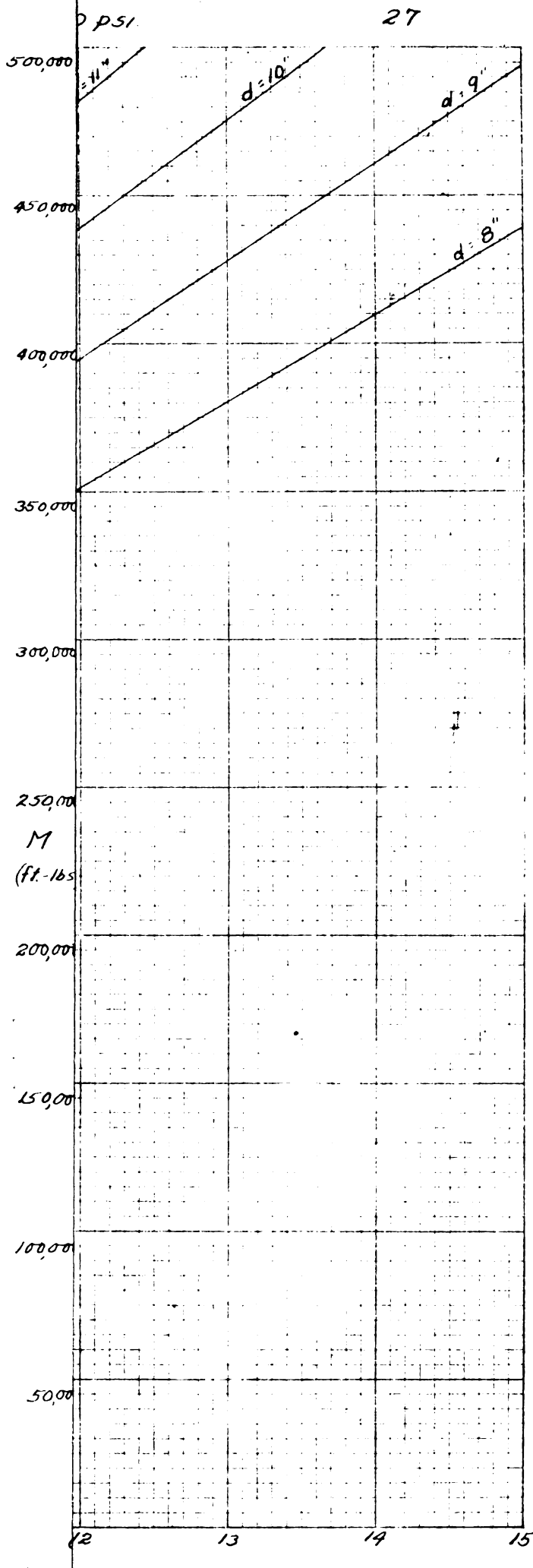
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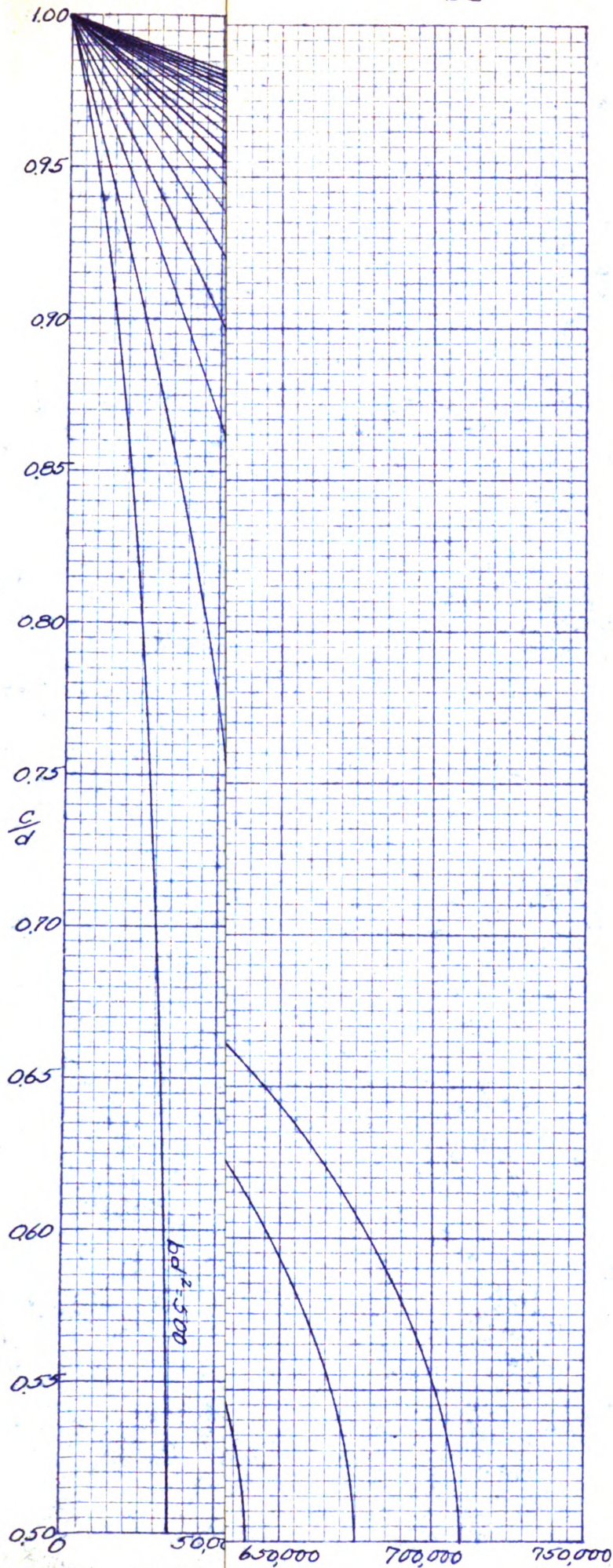


Fig. 19: Red

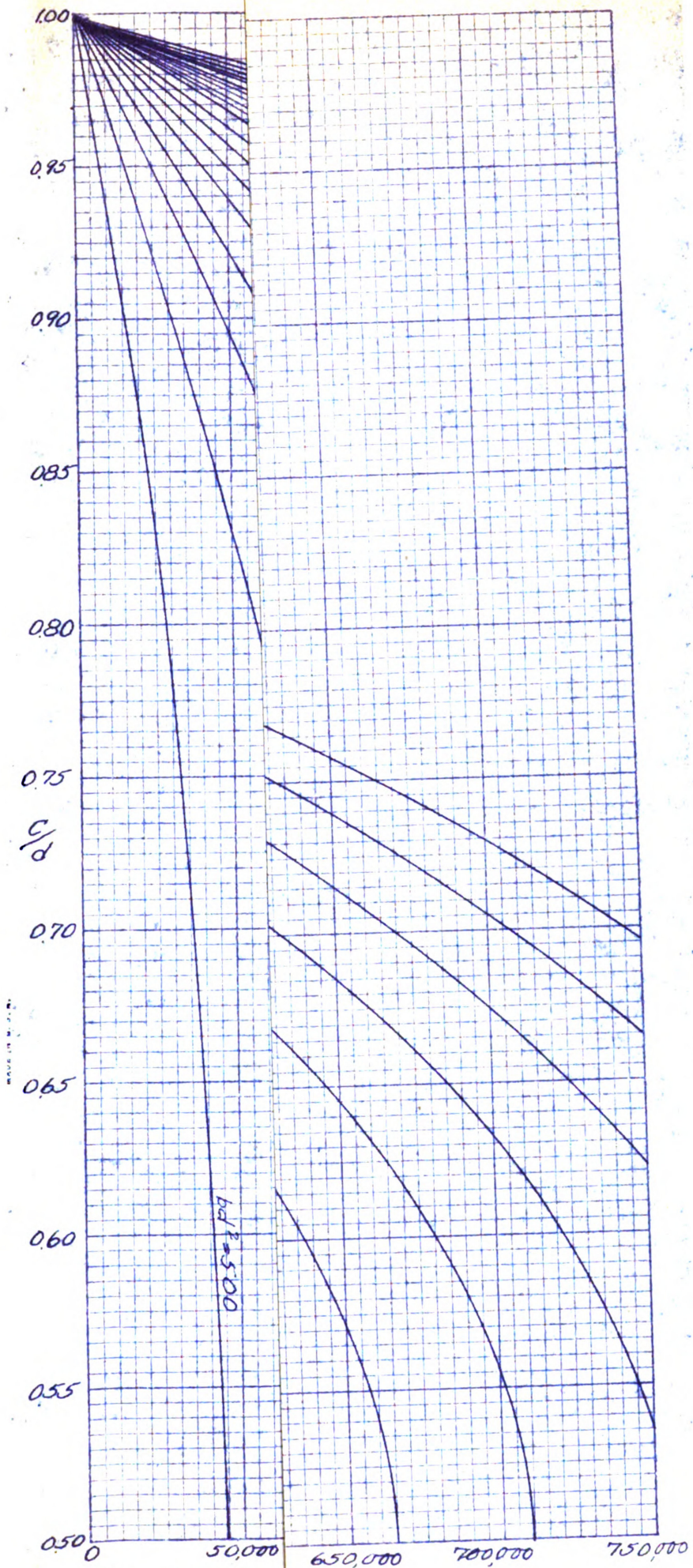


Fig. 20: Rec

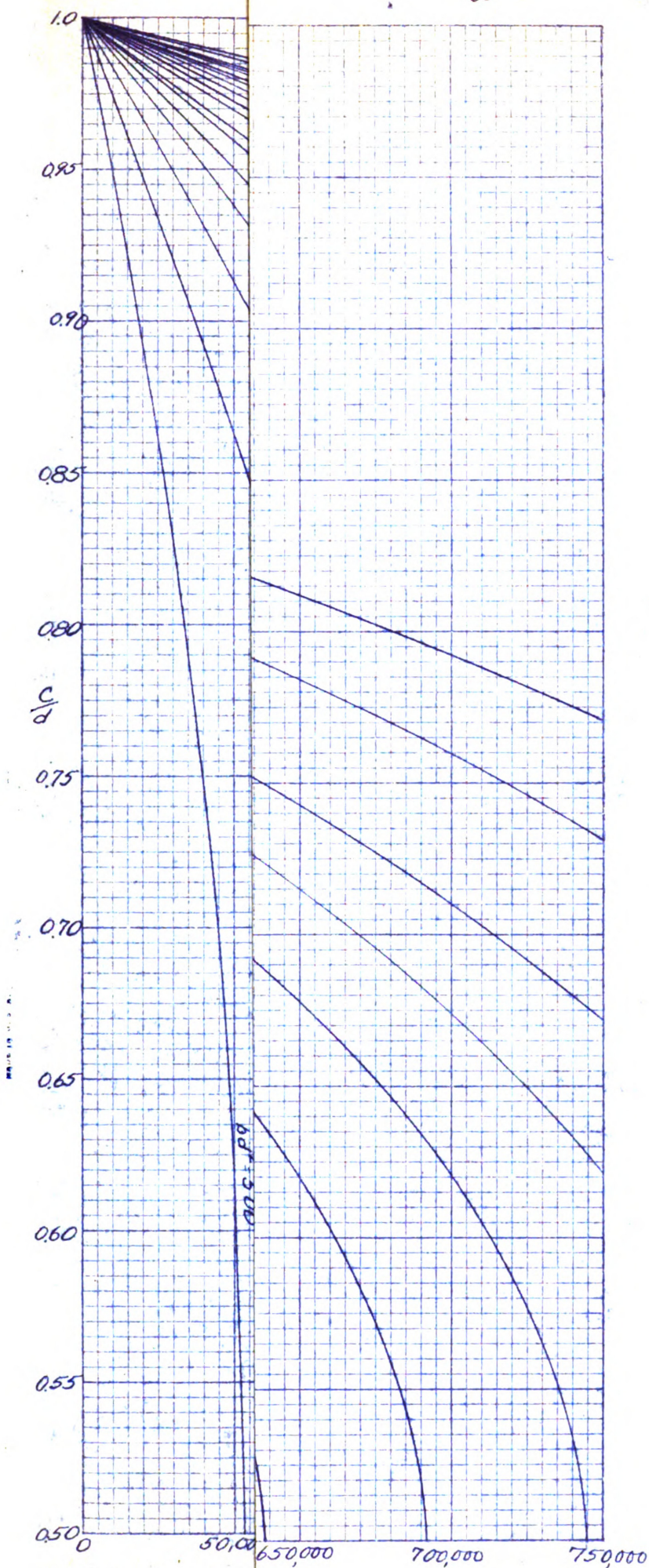


Fig. 21 Rect

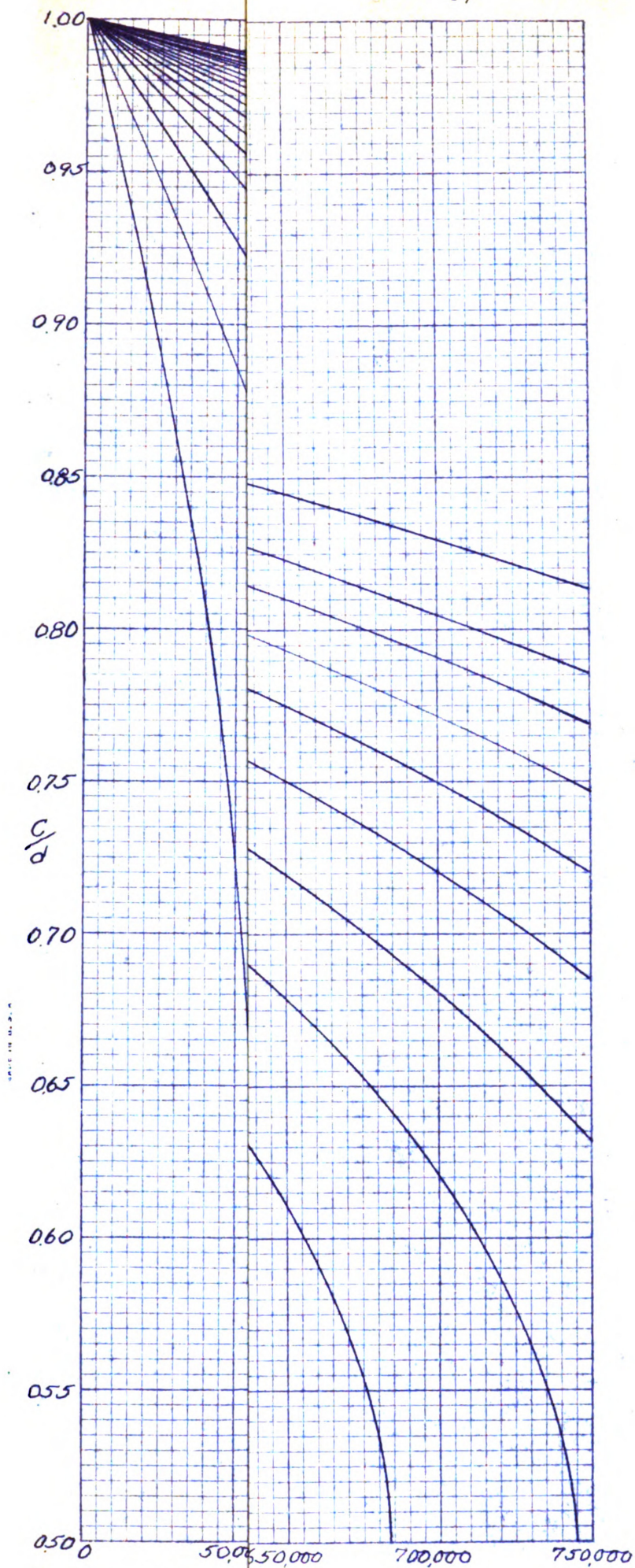


Fig. 22: Re

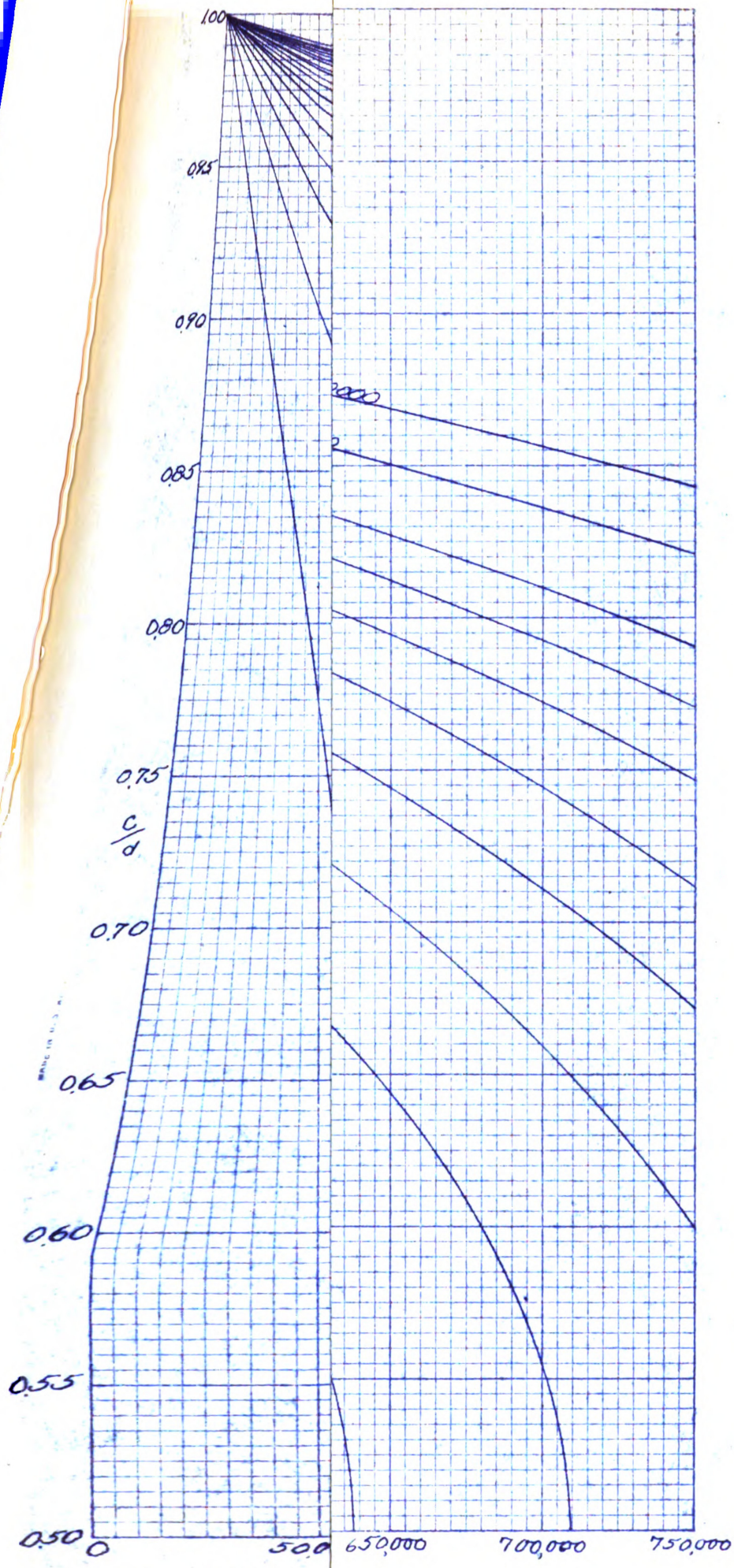


Fig. 23 : R

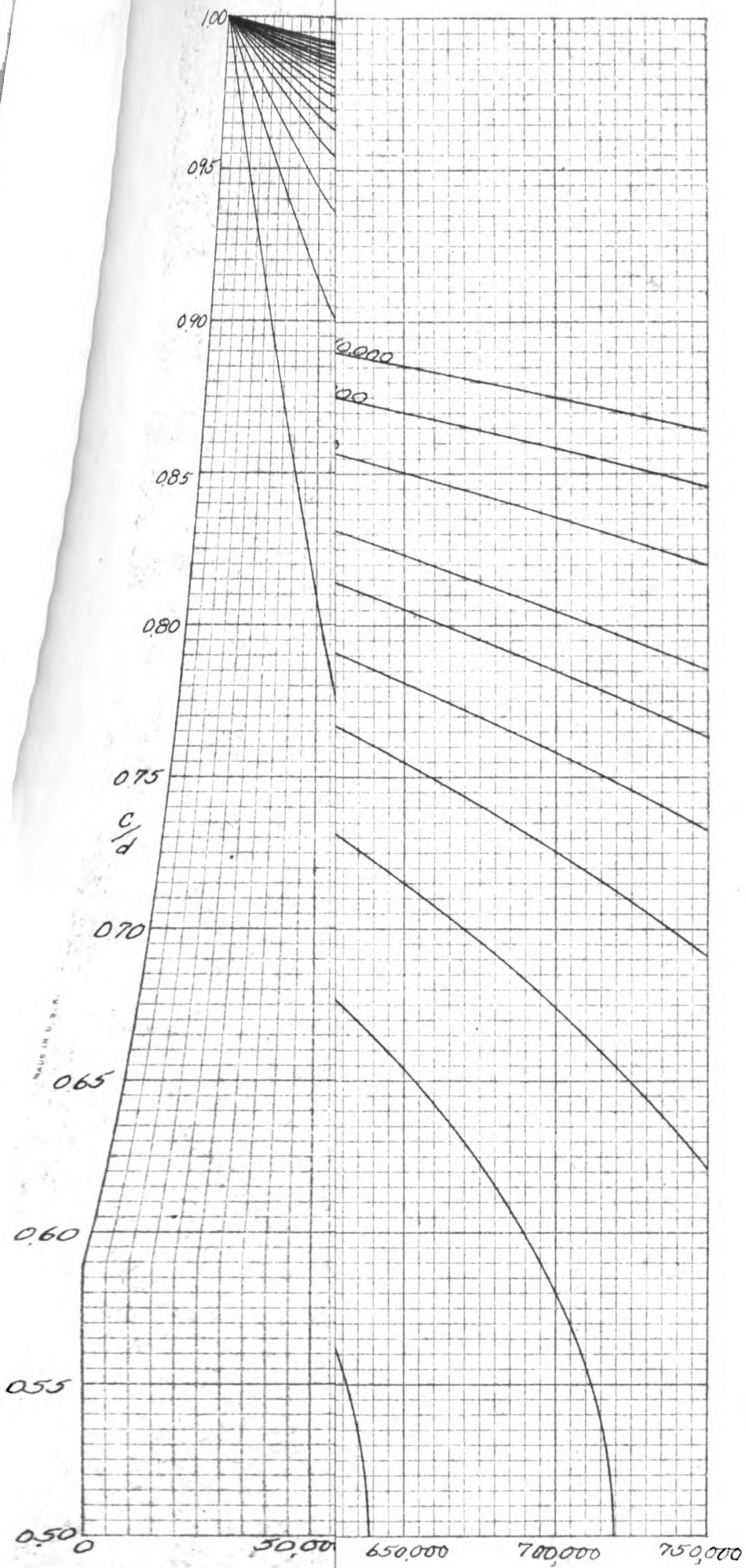


Fig. 24: R

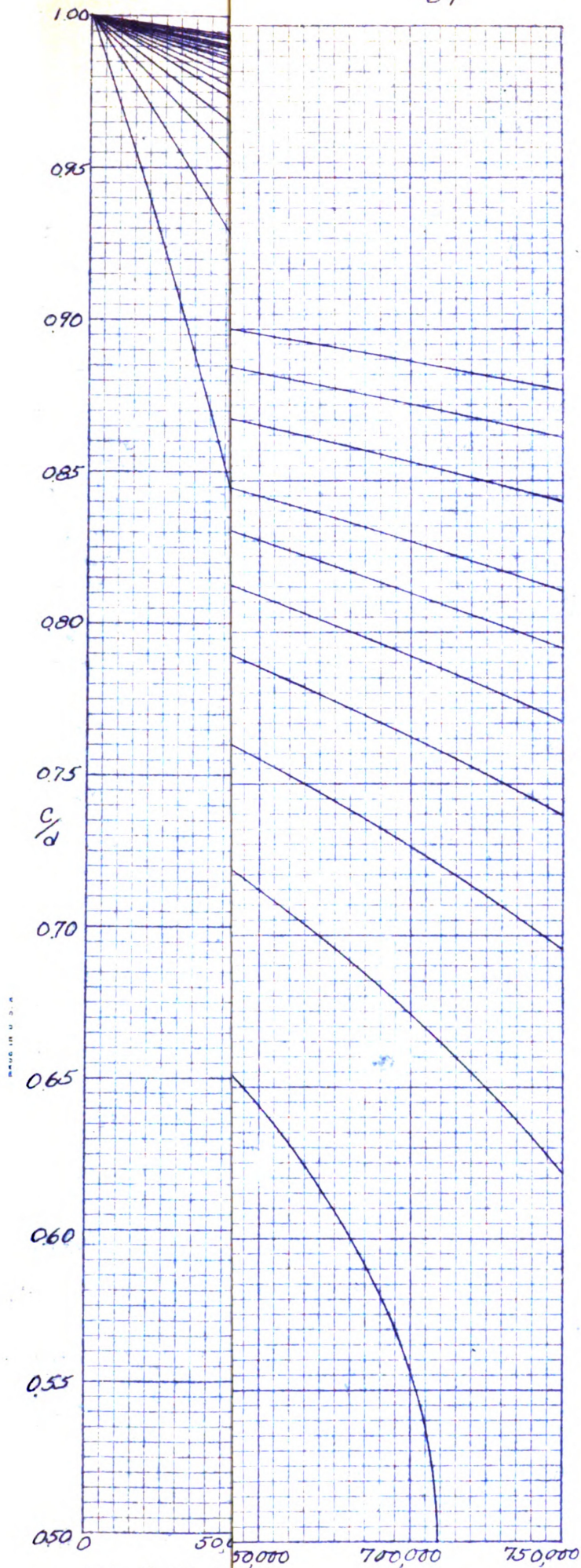
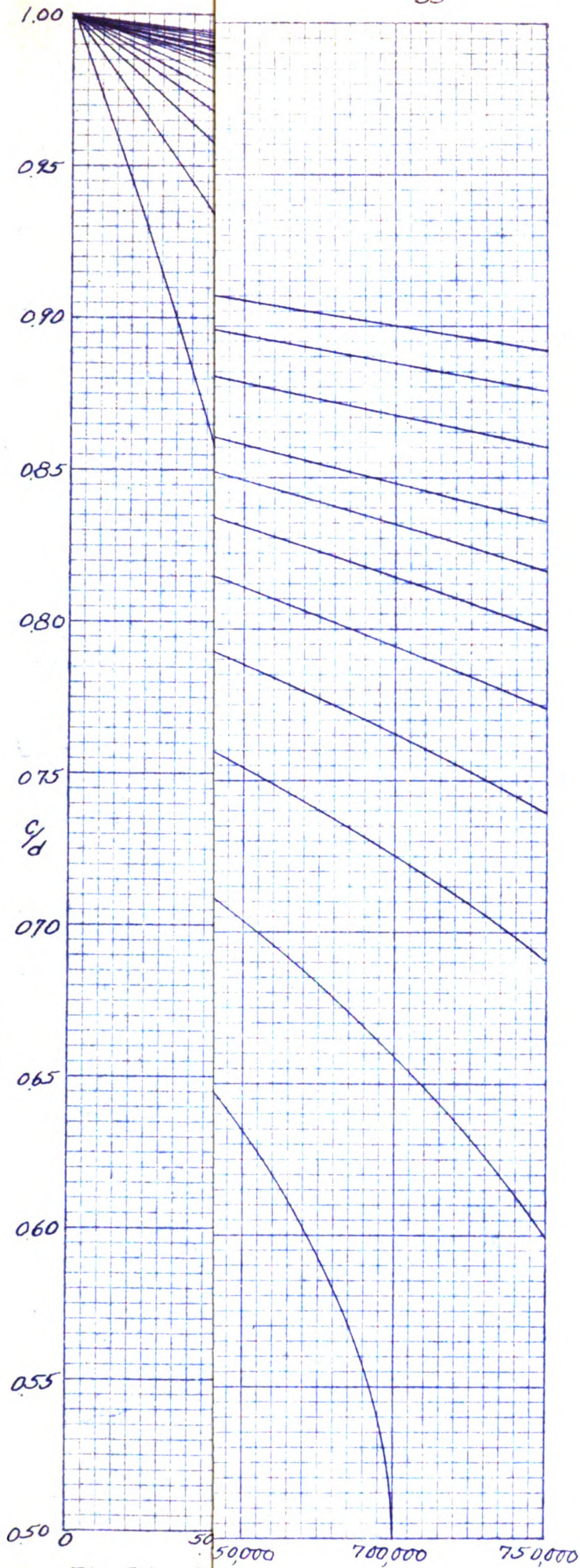


Fig. 2.5: Re

35



0 psi

36

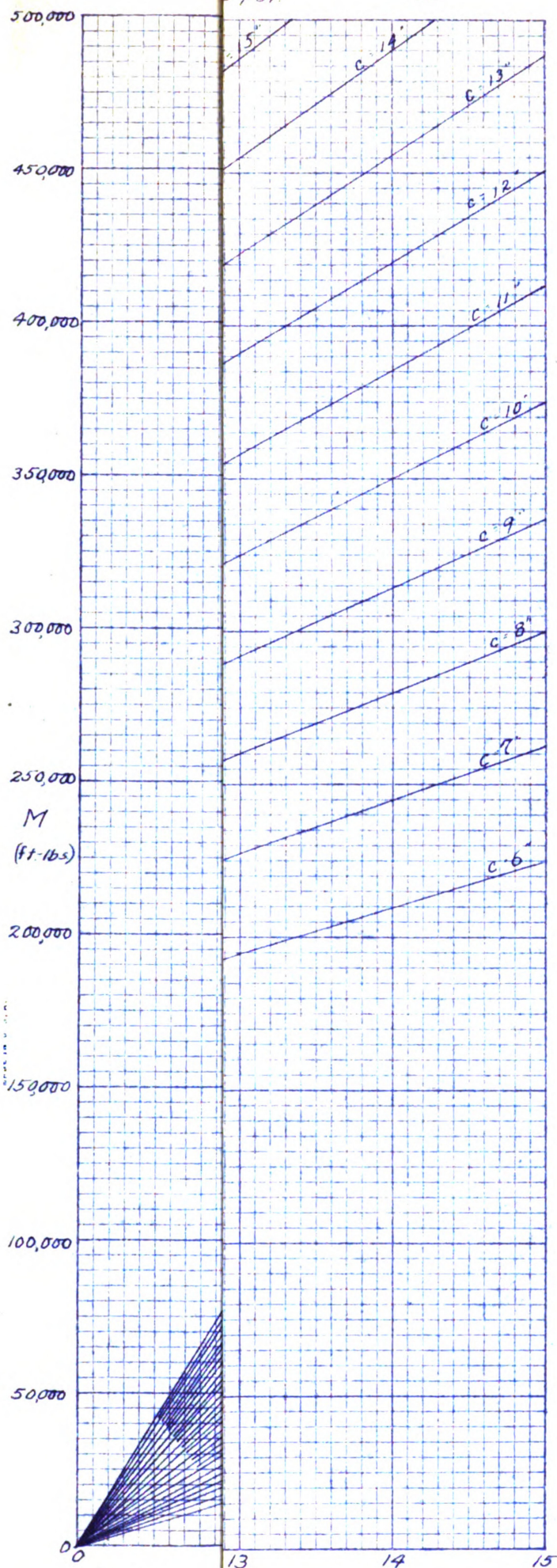


Fig. 27: 100

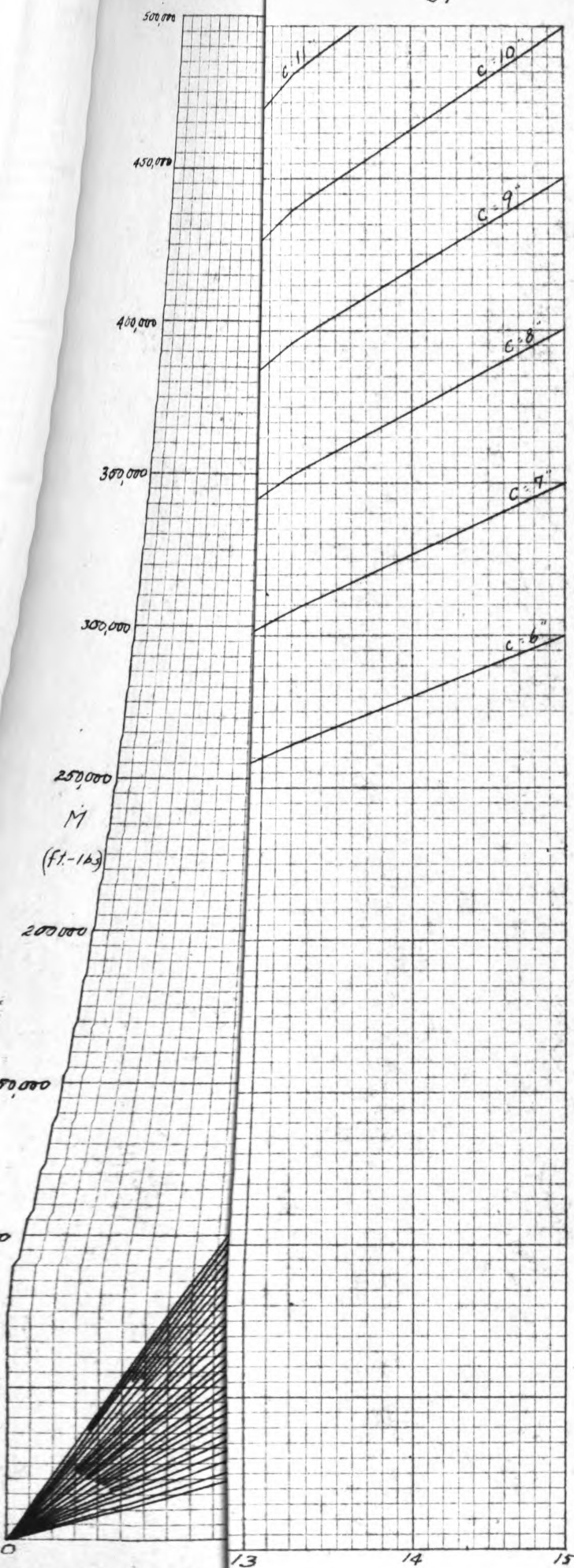


Fig. 28 : Re

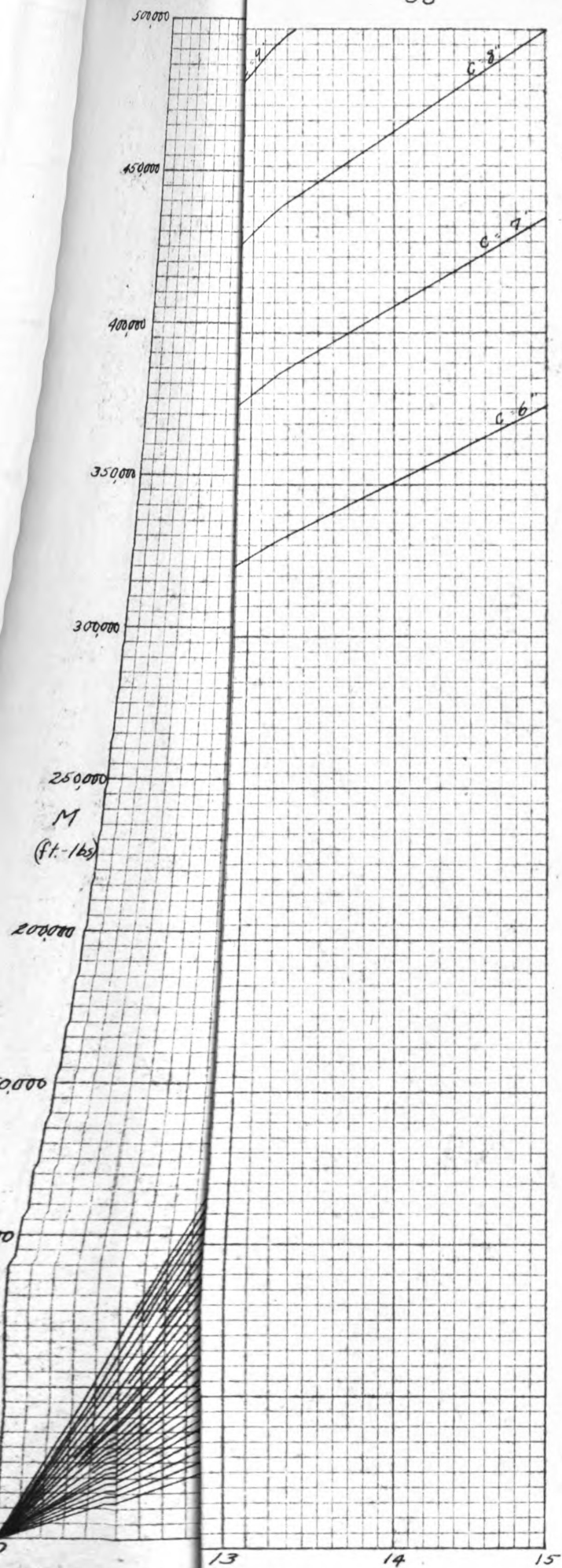


Fig. 29: R

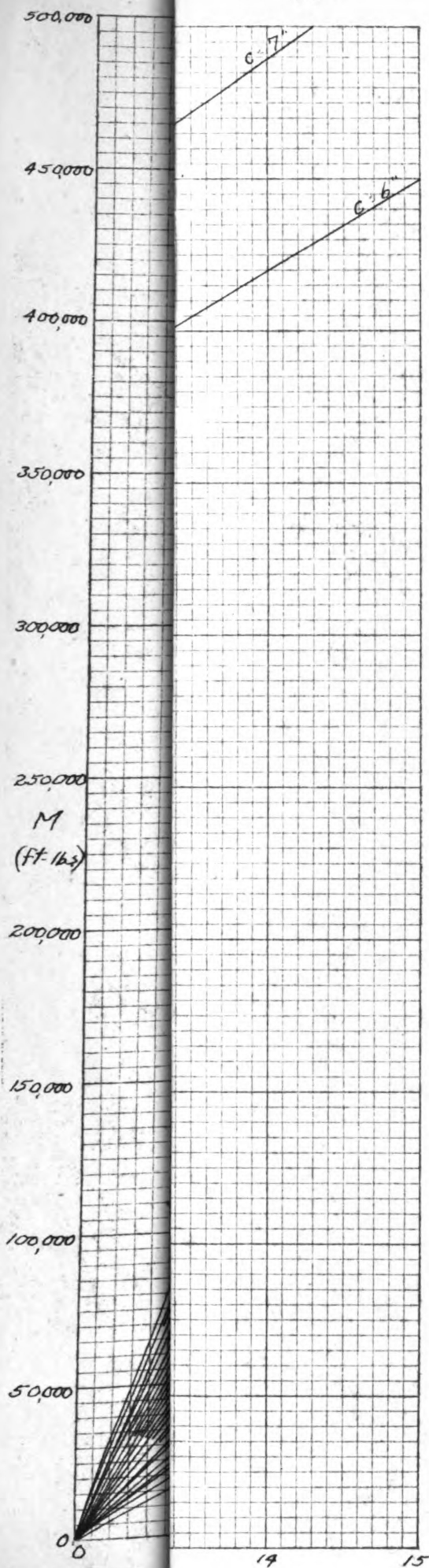


Fig.

EXAMPLE PROBLEMS

Two illustrative problems are included, involving one way slabs and rectangular beams, to summarize the comparative qualities of the elastic theory, and the ultimate theory with balanced design and under-reinforced design.

PROBLEM 1 - Design a one-way slab with the given conditions:

$f'_c = 3,000$ psi, $f_s = 20,000$ psi., $f_y = 40,000$ psi.,
clear span = 19' - 0", L. L. = 200 lbs./ft., $+M = \frac{wl^2}{16}$,
 $-M = \frac{wl^2}{11}$, and safety factors of 1.2 and 2.4.

A. Elastic Design

7.5 in. slab.

$$D. I. = 7.5 \times 12 \times \frac{1}{12} \times 150 = 93.7 \text{ lbs./ft.}$$

$$L. L. = 200 \text{ "}$$

$$T. L. = 293.7 \text{ lbs./ft.}$$

$$+M = \frac{(293.7) (19)^2 (12)}{(16)} = 79,500 \text{ in.-lbs.}$$

$$-M = \frac{(293.7) (19)^2 (12)}{(11)} = 115,600 \text{ in.-lbs.}$$

$$j^2 = \frac{M}{Kb} = \frac{(115,600)}{(235) (12)} = 40.8 \text{ sq. in.}$$

$$j = 6.39 \text{ in., use } d = 6.5 \text{ in.}$$

$$A_s = \frac{M}{f_s j d} = \frac{(115,600)}{(20,000) (0.866) (6.5)} = 1.03 \text{ sq. in.}$$

$$A_s = \frac{(79,500)}{(20,000) (0.866) (6.5)} = 0.706 \text{ sq. in.}$$

B. Ultimate theory - Balanced design 5.5 in. slab

$$D. L. = 5.5 \times 12 \times 1/12^2 \times 150 \times 1.2 = 82.5 \text{ lbs./ft.}$$

$$L. L. = 200 \times 2.4 = \underline{480} \quad "$$

$$T. L. = 562.5 \text{ lbs./ft.}$$

$$+M = \frac{(562.5)(19)^2}{(16)} = 12,700 \text{ ft.-lbs.}$$

$$-M = \frac{(562.5)(19)^2}{(11)} = 18,500 \text{ ft.-lbs.}$$

for negative moment:

$$\text{from fig. 3; } M = 18,500 \text{ ft.-lbs., } f'_c = 3,000 \text{ psi.}$$

$$d = 4.3 \text{ in. use } d = 4.5 \text{ in.}$$

$$\text{from fig. 5; } d = 4.5 \text{ in., } f'_c = 3,000 \text{ psi.}$$

$$A_s = 1.85 \text{ sq. in.}$$

for positive moment:

to eliminate the necessity of keeping 1.85 sq. in.

of steel or varying the depth d to maintain a

balanced design in view of the smaller moment,

it would be more practical to revert to under-

reinforced design.

$$\text{from fig. 8; } M = 12,700 \text{ ft.-lbs., } f'_c = 3,000 \text{ psi.}$$

$$N = 9.96$$

$$\text{from fig. 9; } N = 9.96, d = 4.5 \text{ in.}$$

$$a = 1.29$$

$$\text{from fig. 11; } a = 1.29, f'_c = 3,000 \text{ psi.}$$

$$A_s = 0.99 \text{ sq. in.}$$

C. Ultimate theory - under-reinforced design 7.5 in. slab

$$D. L. = 7.5 \times 12 \times 1/12^2 \times 150 \times 1.2 = 112.5 \text{ lbs./ft.}$$

$$L. L. = 200 \times 2.4 = 480 \text{ "}$$

$$T. L. = 592.5 \text{ lbs./ft.}$$

$$+M = \frac{(592.5)(19)^2}{(16)} = 13,400 \text{ ft.-lbs.}$$

$$-M = \frac{(592.5)(19)^2}{(11)} = 19,500 \text{ ft.-lbs.}$$

for negative moment:

$$\text{from fig. 8; } M = 19,500 \text{ ft.-lbs., } f'_c = 3,000 \text{ psi.}$$

$$N = 15.30$$

$$\text{from fig. 9; } N = 15.30$$

$$d = 9, 8, 7, 6.5, 6 \text{ in.}$$

$$a = 0.89, 1.02, 1.10, 1.30, 1.45 \text{ in.}$$

$$\text{from fig. 11; } A_s = 0.63, 0.78, 0.91, 1.00, 1.11 \text{ sq. in.}$$

$$\text{probably use } d = 6.5 \text{ in. and } A_s = 1.00 \text{ sq. in.}$$

for positive moment:

$$\text{from fig. 8; } M = 13,400 \text{ ft.-lbs., } f'_c = 3,000 \text{ psi.}$$

$$N = 10.51$$

$$\text{from fig. 9; } d = 6.5", N = 10.51$$

$$a = 0.37 \text{ in.}$$

$$\text{from fig. 11; } a = 0.37, f'_c = 3,000 \text{ psi.}$$

$$A_s = 0.67 \text{ sq. in.}$$

A brief summary of the results would be as follows:

	$+A_s$	$-A_s$	d
Elastic theory	0.71	1.03	6.5
Ultimate theory-balanced design	0.99	1.35	4.5
" " under-reinforced design	0.67	1.00	6.5

PROBLEM 11 - Design a simply supported rectangular beam

with the given conditions: $f'_c = 3,000$ psi., $f_s = 20,000$ psi.,
 $f_y = 40,000$ psi., clear span = 18' - 0", L. L. = 2,940 lbs./ft.,
 $M = \frac{wl^2}{8}$, and safety factors of 1.2 and 2.4.

A. Elastic design

$$D. L. = \frac{(12)(25)(150)}{144} = 310 \text{ lbs./ft.}$$

$$L. L. = 2940 \text{ "}$$

$$T. L. = 3250 \text{ lbs./ft.}$$

$$M = \frac{(3250)(18)^2(12)}{(8)} = 1,580,000 \text{ in.-lbs.}$$

$$bd^2 = \frac{M}{K} = \frac{1,580,000}{236} = 6700$$

$$\text{use } b = 12", d = 24"$$

$$A_s = \frac{M}{f_s jd} = \frac{1,580,000}{(20,000)(0.866)(24)} = 3.80 \text{ sq. in.}$$

B. Ultimate theory - balanced design

$$D. L. = 310 \times 1.2 = 370 \text{ lbs./ft.}$$

$$L. L. = 2940 \times 2.4 = 7060 \text{ "}$$

$$T. L. = 7430 \text{ lbs./ft.}$$

$$M = \frac{(7430)(18)^2}{(8)} = 301,000 \text{ ft.-lbs.}$$

from fig. 14; $M = 301,000$ ft.-lbs., $f'_c = 3,000$ psi.

$$bd = 3610 \text{ in.}$$

use $b = 10$ in., $d = 20$ in.

from fig. 16; $M = 301,000$ ft.-lbs., $d = 20$ in.

$$A_s = 6.17 \text{ sq. in.}$$

C. Ultimate theory - under-reinforced design

$$D. L. = 310 \times 1.2 = 370 \text{ lbs./ft.}$$

$$I. L. = 2940 \times 2.4 = \underline{7056} \text{ "}$$

$$T. L. = 7430 \text{ lbs./ft.}$$

$$M = \frac{(7430)(13)^2}{(8)} = 301,000 \text{ ft.-lbs.}$$

$$\text{try } b = 12 \text{ in., } d = 24 \text{ in.; } bd^2 = 6910 \text{ in.}^3$$

from fig. 21; $M = 301,000$ ft.-lbs., $bd^2 = 6910 \text{ in.}^3$

$$\frac{c}{d} = 0.834$$

$$c = (0.834)(24) = 21.2 \text{ in.}$$

from fig. 26; $M = 301,000$ ft.-lbs., $c = 21.2$ in.

$$A_s = 4.26 \text{ sq. in.}$$

A brief summary of the results are as follows:

	A_s	A_c
Elastic design	3.80	238
Ultimate theory - balanced design	6.17	200
" " under-reinforced design	4.26	233

CONCLUSIONS

The ultimate theory, under-reinforced procedure tends to give approximately the same cross-sectional dimensions as the elastic theory but the balanced design procedure gives a smaller beam section with more steel. The economical values of the materials involved indicate that the balanced design procedure is often more costly than the others when limited to an investigation of the member only. If we take into consideration the entire structure, other factors are involved. The reduction in dead load throughout the structure, due to a smaller d in slabs and beams, will allow smaller supporting beams, columns and footings; thus a possible reduction in the total expenditure of the structure. If a tall building is involved, a smaller d will shorten the floor to floor height and consequently the height of the building. Also there is an advantage in using less steel than called for by balanced design, in that incipient failure will be noticed, by increased deflection and cracking of the concrete, in time to reduce the load or strengthen the beam.

One way slabs are designed as rectangular beams to carry the applied bending moment. Unusually heavy loads for short spans require a check for shear stresses. Slabs

with very long spans should be checked for deflection.

Ultimate design theory has been used in Europe more extensively than in the United States. Although our city building codes are quite comprehensive, they vary a great deal and are often based on imnerical formulas. The design procedure must keep pace with the improvements found in the methods of evaluating service loads.

The graphs are presented to improve the usefulness of the ultimate design theory. With the proper size of scales and arrangement of linework, they are as accurate and much easier to use than tables and slide rule computations.

The ultimate design theory of reinforced concrete as used here is not a perfected standard but is a good basis for research leading toward modernization of standard practice. An attempt was made to levelop a practical design procedure that was consistent with all of the pertinent facts that have been discovered regarding the nature of the materials, and not with a few incomplete characteristics selected for convenience.

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