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An Analysis of a Continuous

Beam Bridge

A Thesis Submitted to

The Faculty of

MICHIGAN STATE COLLEGE

of

AGRICULTURE AND APPLIED SCIENCE

Ъу

M. L. <u>Deimling</u> Candidate for the Degree of

Bachelor of Science

June 194**9**

ACKNOWLEDGEMENT

*

I wish to acknowledge my appreciation for the assistance given to me by Mr. Takashi Nakamura without which this Thesis might never have been written.

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INDEX

- I Abbreviations & Symbols used in Thesis
- II Allowable Stresses
- III Introduction
 - IV Design of Railing
 - A Specifications
 - B Bolts
 - C Straps
 - D Railing Channels
 - E Railing Posts
 - V Design of Sidewalk & Curbs
 - A Specifications
 - B Loading
 - C Reinforcing Steel
 - VI Design of Floor Slab
 - A Specifications
 - B Loading
 - C Reinforcing Steel
- VII Design of Diaphragm
 - A Specifications
 - B Design

VIII Design of Girder

- A Specifications E Maximum Moments
- B Moment Distribution
- F Design of Girder

- C Reactions
- D Influence Diagrams
- IX Conclusions

I. ABBREVIATIONS AND SYMBOLS USED IN THESIS

M.S.H.D. - Michigan State Highway Department

A.A.S.H.O.- American Association of State Highway Officials

# – pounds	p.s.i #/sq. in.
sq square	p.s.f #/sq. ft.
in inch	u - bond stress
ft foot	v - unit shear
A - area	p. – page
I - Moment of Inertia	Art article
Z - Section Modulus	Lft lineal feet
c - Extreme fiber distance	Σ - summation
b - Base dimension	< - less than
h - Height	/ - per

II. ALLOWABLE STRESSES

f _c ' - 3,000 #/sq. in.	n - 10
$f_c4 f_c' #/sq. in.$	j 867
f _s - 18,000 #/sq. in.	k4
Loading - H20-S16-44	r - 209

v- 60 #/sq. in.; u- 150 #/sq. in. with no special anchorage v- 90 #/sq. in.; u- 300 #/sq. in. with special anchorage Shear- 13,500 p.s.i. for power driven rivets Shear- 10,000 p.s.i. for unfinished bolts Piles- 20 tons/sq. ft. supported.

III. INTRODUCTION

This thesis was primarily written to increase my own knowledge, to crystalize half formed ideas, to better understand the work of the designer, and most of all, to integrate many of our individual courses such as reinforced concrete, indeterminate structures, and contracts. Upon completion of this thesis I understood much better how these courses related to and depended upon one another.

Secondarily, this thesis was written to check the design of the superstructure of the Michigan State Highway Department bridge Bl of 32-23-13 on which I was an inspector last summer.

The bridge is a 3-span continuous I-beam bridge having two end spans of 42 feet each and a center span of 57 feet, giving a total length of 141 feet. This bridge is located in Sebewaing, Michigan over the Sebewaing River on highway 51. The abutments and paers are supported by 12" H 74# bearing piles. The clear distance of the roadway is 38'-0" curb to curb.

Continuous girder bridges are best proportioned when the interior spans are from 1.3 to 1.4 times the length of the end spans. The interior span of this bridge is 1.36 times the end spans and therefore is of good design in this respect.

During the spring thaws, ice had been piling and jamming on the old bridge causing a hazard to the structure by backing up water like a dam thereby causing undue pressure. The piers of this bridge were designed as ice breakers by slanting the upstream ends about 6 feet at water level and protecting this slanted pier nose with $6"x6"x\frac{1}{2}"$ steel angles. It was thus hoped that the ice flows would break up on these and flow under the bridge without stress or strain to the bridge.

All the above mentioned conditions were taken into consideration both in the analysis and design of the bridge. IV. DESIGN OF THE RAILINGS

Roadway railings shall be designed to resist a lateral horizontal force of 150#/lineal ft. together with a simultaneous vertical force of 100#/lineal ft. applied at the top of the railing. When curbs are 10" or less in height, the lower rails shall be designed to resist a lateral horizontal force of 300#/lineal ft.

M.S.H.D. Spec. Art.35 p.14

RAILINGS:

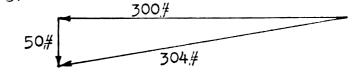
(The bottom rail carries the maximum load and will be investigated first; if it proves satisfactory, the top rail will be also).

1. Bolts 3/4"\$\u03c6 in single shear.
Capacity=(3.14)(3/8)²(10,000)=4,420#
Load =(300)(7.874)=2 =1,310#
Factor of Safety= 4,420=1,310=338%

2. Straps:

Shear Capacity = (1 3/4 - 13/16)(5/8)(10,000) = 5,860 #

Load: (Using 50#/Lft. as the dead weight of the railing).



Load=(304)(7.874)=1,330#

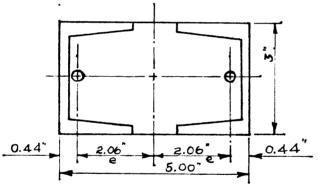
Factor of Safety=5860-1330=4.41

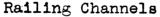
Bolts control as they have the minimum capacity.

3. Bending Moment in Strap:

Distance of strap load application from post is 2" f = Mc/I = M/Z (where $Z = bh^2/6$) $f_{hor} = \frac{(1310)(2)(6)}{(.625)(1.75)^2} = 8,200 \text{ p.s.i.}$ $f_{vert} = \frac{2}{(1310)(2)(6)} = 3,830 \text{ p.s.i.}$ $f_{total} = 12,030 \text{ p.s.i.}$ actual $f_{allow} = 18,000 \text{ p.s.i.}$ allowable 0.K. Safety factor = 1.5 Bending Moment is controlling factor.



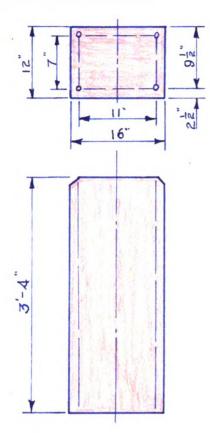




(Only the two side channels will be used and if they give enough strength, it won't be necessary to add the top channel).

a. Length=
$$(7.874) + \frac{(2x \ 1 \ 5/8)}{12} = 7.60 \ \text{ft.}$$

b. $M = \frac{w1^2}{8} = \frac{(300)(7.60)^2(12)}{8} = 26,100 \ \text{in.lbs.}$
c. $A= (2)(1.46) = 2.92 \ \text{sq. in.}$
c = 2.5 e = 2.06" $I_g = 2x.25 = 0.50$ "
d. $I = Ae^2 - I_g = 12.89 \ \text{in.}^3$
f. $f = \frac{Mc}{I} = \frac{(26,000)(2.5)}{(12.89)} = 5,060 \ \text{p.s.i.}$ actual
18,000 p.s.i. allowable
Factor of safety = 356% (without top channel).



b = 16"; d= $9\frac{1}{2}$ " A_s = 4 x 3/4 "\$ bars = 1.76 \div 2 =.88 sq.in.

$$k = \frac{nfc}{f_{g} + nfc} = 0.40$$

$$j = (1 - k/3) = 0.867$$

$$K = \frac{1}{2}(f_{c}kj) = 208$$

Bending Moments:

a. 150(7.874 + 1.333) 32 = 44,220 in.-lbs. <u>300(7.874 + 1.333) 5 = 13,830 in.-lbs.</u> Total B.M. = 58,050 in.-lbs.

$$d = \sqrt{\frac{58,050}{\frac{1}{2}(1200)(.4)(.867)(16)}} = 4.2" \text{ required}$$

9.5" furnished O.K.

c.
$$f_{g} = \frac{M}{Ajd} = \frac{58,050}{.88(.867)(9.5)} = 8,020 \text{ p.s.i. actual}$$

18,000 p.s.i. allowed

0.K.

The large safety factor (45%) is desireable for safety in case of accidents. Another reason for the posts being overdesigned is for aesthetic reasons. The massive architectural design lends beauty to the structure. The modern bridge is certainly an object of beauty.

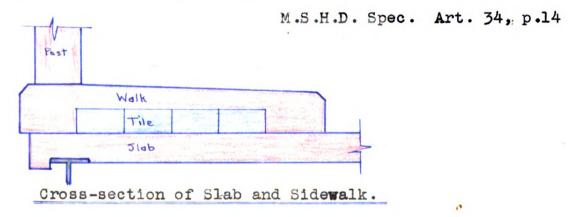
V. CURB AND SIDE WALK

Substantial curbs shall be built on each side of the roadway and they shall have a width not less than 6" and a height of not less than 9" measured above the wearing surface at a point adjacent to the curb.

M.S.H.D. Spec. Art. 20, p. 8

Curbs shall be designed to resist a minimum force of not less than 500 #/Lft. of curb applied at top of curb. M.S.H.D. Spec. Art. 36, p. 14

Sidewalk floors, stringers and their immediate supports shall be designed for a live load of not less than 100 pounds per square foot of sidewalk area.



The sidewalk and curb are fully supported and therefore could fail only by crushing. But its strength by crushing is more than adequate. Therefore temperature steel is the only steel necessary. The hooked structural steel bars serve to keep the curb in place. Bending moments shall be calculated by the following methods. A.A.S.H.O. Spec. Art. 3.32, p.138-140

In this bridge the main reinforcing steel is perpendicular to the center line of roadway.

Distribution of wheel loads:

E = 0.65 + 2.5

Bending moment for freely supported spans:

 $M = 0.25 \times P/E \times S(100 + I + 10\%)$ for longitudinal

forces.)

Bending moment for continuous spans:

 $M = 0.2 \times P/E \times S(100 + I + 10\%)$

In the above formulas, the symbols used are:

- E = Width of slab over which wheel load is distributed.
- I = Impact coefficient as determined by the formula:
- $I = \frac{L + 20}{6L + 20} = M.S.H.D. Spec. Art. 37, p.14$
- L = Span length
- M = Bending moment
- **P** = Load on one wheel (maximum)
- S = Distance between flanges of girders plus one-half
 width of girder flanges. (S = 4.397')

The forces due to traction or sudden braking of vehicles shall be considered as longitudinal forces having a magnitude of 10% of the gross live load that can be placed in one traffic lane. This load shall be assumed as acting in the direction of traffic movement and applied at the top of the pavement.

Design of Slab:

a. E = (.6 x 4.397) + 2.5 = 5.14' (P = 16,000, S = 4.397) b. $I_1 = \frac{42 + 20}{(6 x 42) + 20} = 22.8\%$ $I_2 = \frac{57 + 20}{(6 x 57) + 20} = 21.3\%$ $I_{O_1} = I_1 + 10\% = 32.8\%$ $I_{O_2} = I_2 + 10\% = 31.3\%$ c. M = (0.2)(16,000/5.14)(4.397)(1.328) = 43,600 in.-lbs. d. d = $\sqrt{M/Rb} = \sqrt{43,560/208 \times 12} = 4.2$ in.

Specifications for slabs: the distance from the surface of the concrete either top or bottom, to the center of the nearest bar shall be not less than one and one-half times the diameter of the bar nor less than one and one-half inches.

Required thickness:

• = 1.5 x 3/4 = 1.1 1.5 therefore use 1.5"

t = 4.2 + 1.5 = 5.7" required; 7.0" furnished. O.K. therefore d = 7.0 - 1.5 = 5.5"

 $e \cdot A_{s} = M/f_{s} jd = 43,600/18,000 \times 0.875 \times 5.5$

= 0.504 sq. in. required; 0.612 sq.in. furnished.

f. Steel at bottom of slab for lateral distribution: Per cent of main steel required = 100/ S[‡]

= 100/1/4.397 = 47.5% A_s = .504 x .475 = .24 sq.in./ft. required. 0.K. A.A.S.H.O. Spec. Art. 3.2.2, p.140 g. Bond and Shear:

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Slabs designed for bending moment in accordance with the foregoing shall be casidered satisfactory in bond and shear.

A.A.S.H.O. Spec. Art. 3.2.2, p.140

VII. DIAPHRAGM DESIGN

Specifications:

The forces due to wind and lateral vibrations shall consist of a horizontal moving load equal to 30 pounds per square foot on $l\frac{1}{2}$ times the area of the structure as seen in elevation, including the floor system and railing and on l/2 the area of all trusses and girders in excess of 2 in the span.

M.S.H.D. Spec. Art.38, p.15

Rivets:

- a. Size: Rivets shall be of the size specified but generally shall be 3/4 or 7/8 inch diameter.
- b. Pitch of Rivets: The minimum distance between centers of rivets shall be three times the diameter of the rivet but preferably shall be not less than the following:

For 3/4 " diameter rivets --- $2\frac{1}{2}$ "

M.S.H.D. Spec. Art.92, p.39-40

Diaphragms shall be provided at the third points of all I-beam spans of forty or more feet.

M.S.H.D. Spec. Art.124, p.48

Design: Lateral, longitudinal, and transverse bracing shall be composed of angles or other shapes and shall have riveded connections.

M.S.H.D. Spec. Art.123, p.48

VII. DIAPHRAGM DESIGN (Con't.)

Area of structure as seen in elevation: 2.5 x 137 Beam = 342 Railings 1.5 x 137 = 205 Sidewalk 1.0 x 137 = 137 Int. Posts $14(1.3 \times 3.7) = 67$ Ext. Posts $2(4.0 \times 8.3) = 66$ Piers $2(2.5 \times 9.0) = 45$ Abut. $2(4.15 \times 11.2)$ = 93 Total Area = 955 sq.ft. Total Effective Area = 1.5 x 955 = 1433 sq.ft. Wind Force = 30 p.s.f. x 1433 = 42,990 # Area required = 42,990/18,000 = 2.4 sq.in. Area furnished by $4" \times 4" \times 3/8$ " angles:

A = 5.2 sq.in. 0.K.

The intermediate diaphragms meet all the necessary specifications provided by the M.S.H.D. for depth of web, size of angles, pitch of rivets, depth of hitchangles, and number of stiffeners. VIII. GIRDER DESIGN

Specifications:

Main trusses and girders shall be spaced a sufficient distance apart center to center to be secure against $over\frac{1}{2}$ turning by the assumed lateral and other forces.

M.S.H.D. Spec. Art.76, p.36

For the calculation of stresses, span lengths shall be assumed as follows: Beams and girders --- distance between centers of bearings.

M.S.H.D. Spec. Art.77, p.36

For structures with concrete slab floors without separate wearing surface, a minimum allowance of 20 p.s.f. of roadway shall be made in addition to the weight of any monolithically placed concrete wearing surface, to provide for future wearing suface.

M.S.H.D. Spec. Art.30, p.12

When provision is made for three or more lanes of traffic, the design shall provide for the following percentages of the simultaneous maximum loading of all lanes: For four or more traffic lanes ----- 80%.

M.S.H.D. Spec. Art.33, p.13 & 14

Use H20 - S16 - 44 loading from appendix A.

A.A.S.H.O. Spec. p.229

Design of Girder:

Span AB --- $M_{ba} = \frac{P a b (a + L)}{2 L^2}$ where L = 42', P = 1^k

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8	M _{ba} (ftkips)
0	0
6	2.94
12	5.51
18	7•35
23	8.04
24호	8.075
24	8.08 max.
25	8.05
27	7.91
29	7.58
30	7.38
36	4.77
42	0
21	7.88

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Spa	n BC M	$bc = \frac{P a_2 b^2}{L}$	(a = distance to) rt. of B)
	8	M _c (ftkips)	M _{cb} (ftkips))
	0	0	0	
	6	-4.80	0.57	
-	12	-7 •46	1.99	
	17	-8.36		
	18	-8.41	3.89	
-	19	-8.43_max.	4.22	
	20	-8.42		
	21	-8.36		
	24	-8.04	5.84	
-	28.5	-7.13	7.13	
	30	-6.74		
	33	-5.84	8.04	
	39	-3.89	8.41	
	45	-1.99	7.46	
	51	-0.57	4.80	
	57	0	0	

,

Moment Distribution:

Opposite end of member hinged: C = 3 I = 4461 in.⁴ Opposite end of member fixed: C = 4 $L_1 = 504"$ $K_1 = I''/L'' = 8.84$ Cin. $K_2 = I''/L'' = 6.52$ Cin. $(L_2 = 684")$ $CK_{ba} = 3 \times 8.84 = 26.5$ $CK_{bc} = 4 \times 6.52 = 26.1$ $CK_{cb} = 4 \times 6.52 = 26.1$ $CK_{cd} = 3 \times 8.84 = 26.5$ $CK_{cd} = 3 \times 8.84 = 26.5$ $CK_{cd} = 3 \times 8.84 = 26.5$ $CK_{cd} = 3 \times 8.84 = 26.5$ $CK_{cd} = 26.5/52.6 = 0.504$
$$r_{bc} = r_{cb} = 26.1/52.6 = 0.496$$

At a = 12, $M_{ba} = 5.51$ ft.-kips

•504	•496		•496 •504
5.51			
-2.78	-2.73		-2.73
	1.36		1.36 1.37
-0.69	-0.67		-0.67
	0.33		0.33 0.34
-0.17	-0.16		-0.16
	0.08		0.08
-0.04	-0.04		-0.04
	0.02		0.02 0.02
<u>-0.01</u>	-0.01		
1.82	-1.82		-1.81 1.81

. . .

At a = 21, $M_{ba} = 7.88$ ft.-kips

	-	-	-		
•504	•496		•496	•504	
7 •88					Å
-3.97	-3.91		-3.91		
	1.94	◄	1.94	1.97	
-0.98	-0.96	>	-0.96		
	0.48	◄	0.48	0.48	
-0.24	-0.24	***** *	-0.24		
	0.12	4	0.12	0.12	
-0.06	-0.06		-0.06		
	0.03		0.03	0.03	
0.02	0.01				
2.61	-2.61		-2.60	2.60	
I					

At a = 24, $M_{ba} = 8.08$ ft.-kips

•504	•496		•496	•504	
8.08	↑				4
-4.07	-4.01	>	-4.01		
	1.99		1.99	2.02	
-1.00	-0.99		-0.99		
	0.49		0.49	0.50	
-0.25	-0.24	>	-0.24		
	0.12		0.12	0.12	
-0.06	-0.06		-0.06		
	0.03		0.03	0.03	
0.02					
2.68	-2.68		-2.67	2.67	

At a = 30, $M_{ba} = 7.37$

•504	•496		.595	•504
7 •37				Á
-3.71	-3.66	>	-3.66	
	1.82	◄	1.82	1.84
-0.92	-0.90	>	-0.90	
	0.45		0.45	0.45
-0.23	-9.22		-0.22	
	0.11		0.11	0.11
-0.06	-0.05		-0.05	
	0.02	◄	0.02	0.03
_0.01	-0.01	>		
2.44	-2.44		-2.43	2.43

•504	•496		•496	•504
	-7.46		1.99	
3.76	3.70		3.70	
	-2.83		-2.83	-2.86
1.43	1.40	>	1.40	
	-0.69		-0.69	-0.71
0.35	0•34	>	0.34	
	-0.17	◄	-0.17	-0.17
0.09	0.08		0.08	
	-0. 04		-0.04	-0.04
_0.02	0.02	>	0.02	
5.65	-5.65		<u>-0.01</u> 3.79	<u>-0.01</u> -3.79

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At a = 12, $M_{bc} = -7.46$, $M_{cb} = 1.99$ ft.-kips

At A = 19, $M_{bc} = -8.43$, $M_{cb} = 4.22$ ft.-kips

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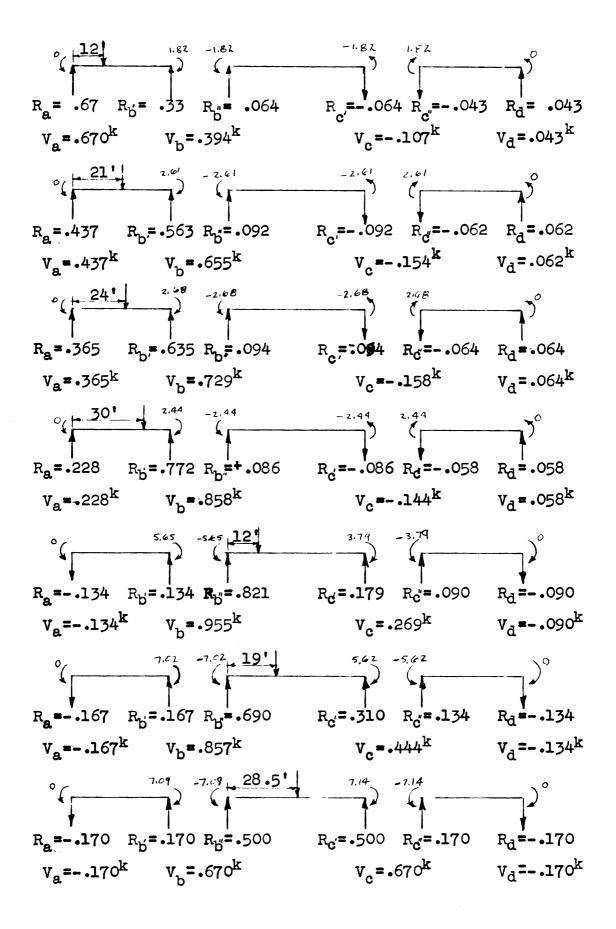
HU A - 1/9	•• DC	· · · · · · · · · · · ·			
	•504	•496		•496	•504
	Å	-8.43		4.22	
	4.24	4.19	>	4.19	
		-4.17		-4.17	-4.24
	2.10	2.07	>	2.07	
		-1.03		-1.03	-1.04
	0.52	0.51		0.51	
		-0.25	-	-0.25	-0.26
	0.13	0.12		0.12	
		-0.06	◄	-0.06	-0.06
	0.03	0.03		0.03	
				-0.01	-0.02
	7.02	-7.02		5.62	-5.62
•					

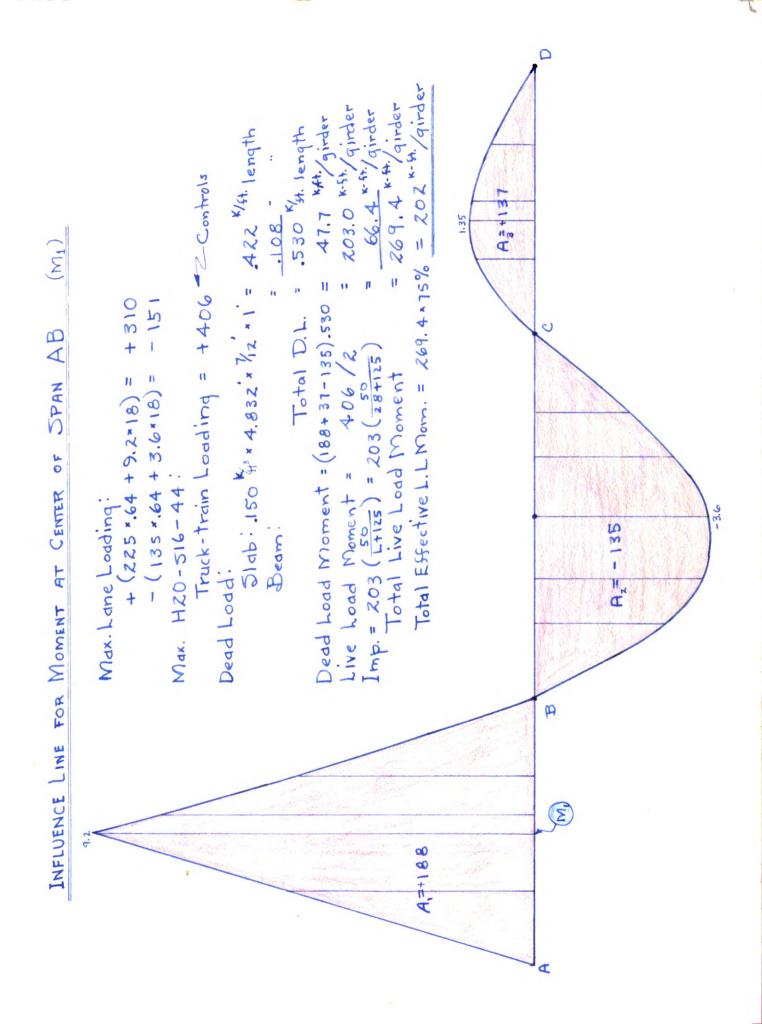
At a = 28.5, $M_{bc} = -M_{cb} = -7.13$ ft.-kips

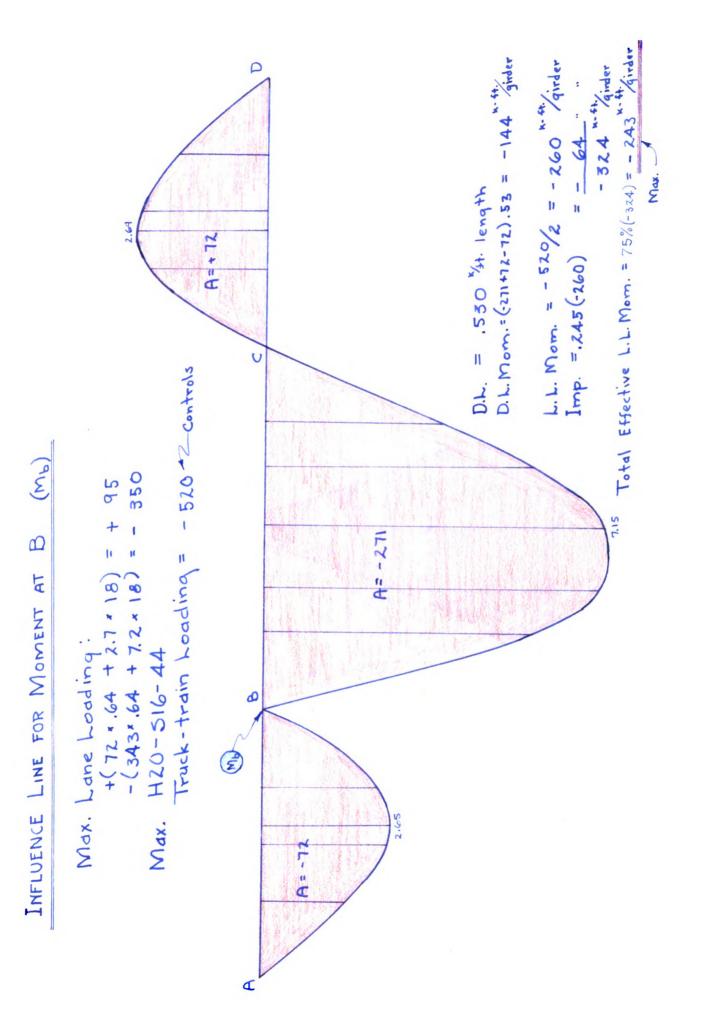
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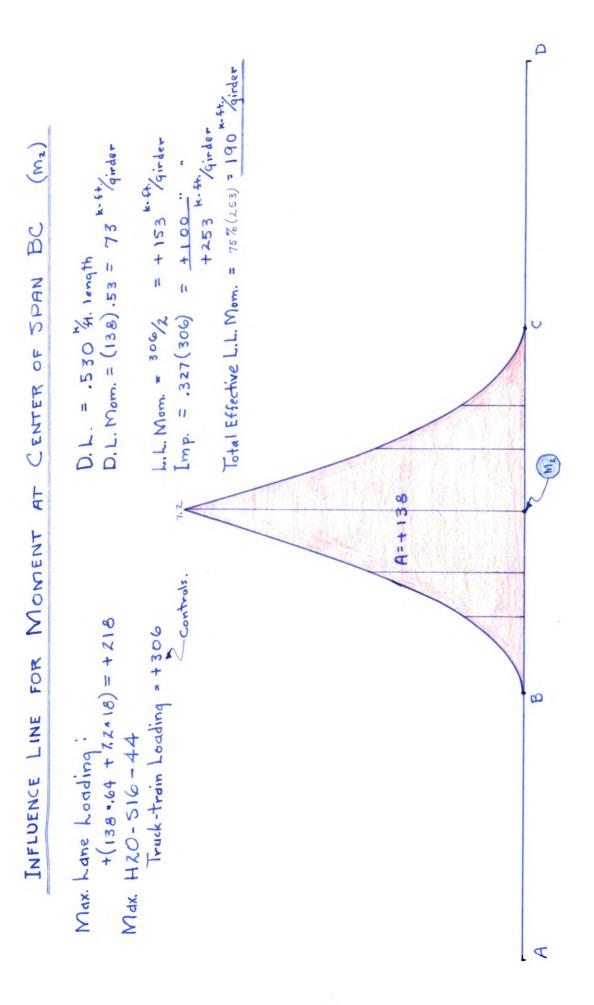
•504	•496		•496	•504
	-7.13		7.13	
3•59	3.54		3•54	
	-5.28		-5.28	-5.39
2.63	2.63		2.63	
	-1.31	◄	-1.31	-1.32
0.66	0.65		0.65	
	-0.32	◄	-0.32	-0.33
0.16	0.16		0.16	
	-0.08		-0.08	-0.08
0.04	0.04		0.04	
	-0.02		-0.02	-0.02
0.01	0.01			
7.09	-7.09		7.14	-7.14
	I			1

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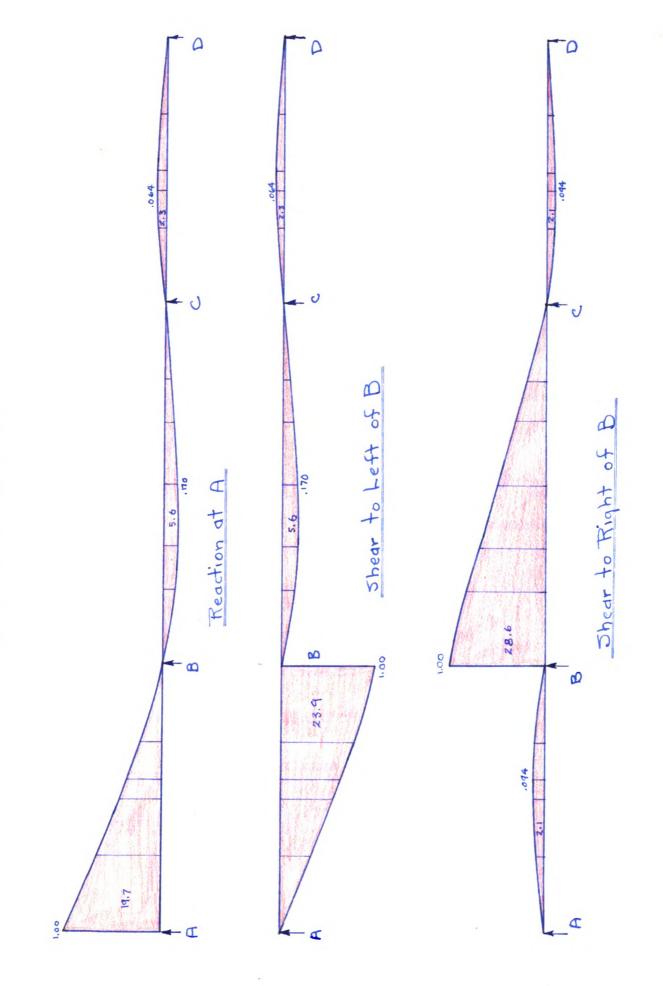








INFLUENCE DIAGRAMS



BEAM FLEXURE

L.L.Mom.=(-243)+(-144)= 387 ft.-kips = 4,644,000 in.-lbs. I = 446l in.⁴, c = 29.82/2 = 14.91", f_{max}= Mc/I f_{max}=(4,644,000 x 14.91) / 446l = 15,550 p.s.i. actual (18,000 p.s.i. allowable, beam 0.K. in flexure.)

FLANGE BUCKLING

flange width = b = 0.872"ft. L = 19 ft.

$$f_{B} = \frac{22,500}{1 + (L^{2} / 1800b^{2})} = \frac{22,500}{1 + (19^{2} / 1800x0.872^{2})} =$$

$$f_{B} = 17,800 \text{ p.s.i. actual} (18,000 \text{ p.s.i. allowable. 0.K.})$$

VERTICAL BUCKLING AT REACTION

$$\mathbf{f}_{s} = \frac{R}{(a + d/4)t} = \frac{90,000}{(11 + 29.82/4)0.548} = \frac{8,900 \text{ p.s.i.}}{\text{actual}}$$
(18,000 p.s.i. allowable. 0.K.)

DIAGONAL WEB BUCKLING

$$h/t = 28.3"/0.548" = 51.6 > 50$$

 $s_{g} = 15,000 - 100 h/t = 9,840 p.s.i. actual
(11,000 p.s.i. allowable. 0.K.)$

VERTICAL WEB CRIMPLING AT REACTION

$$f_{g} = \frac{R}{(a + k)t} \text{ where } k = \text{flange thickness} = 0.760"$$

$$f_{g} = \frac{90,000}{(11 + .760)0.548} = 14,000 \text{ p.s.i. actual}$$

$$(18,000 \text{ p.s.i. allowable. 0.K.})$$

IX. CONCLUSIONS

The superstructure of this bridge was very well designed both for stress and architectural beauty. It is neither over-designed nor under-designed, but designed closely to the allowable specification stress limits. This means that the design costs and material costs have been kept to a minimum.

It was a pleasure to work with the Michigan State Highway Department's plans and specifications.

