ECONOMIC OPTIMA FROM AN EXPERIMENTAL CORN-FERTILIZER PRODUCTION FUNCTION, CAUCA VALLEY, COLOMBIA, S. A., 1958

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ABSTRACT

ECONOMIC OPTIMA FROM AN EXPERIMENTAL CORN-FERTILIZER PRODUCTION FUNCTION, CAUCA VALLEY, COLOMBIA, S.A., 1958

by Enrique Delgado C.,

Latin America has a large number of farms where fertilizers are needed. For those farms where use of fertilizers is a common practice, little attention has been paid to economic optima.

Michigan State University has initiated fertilizer experiments in the Cauca Valley in Colombia, South America. These have been multiple purpose experiments. This thesis reports an analysis of data produced by an experiment designed to permit determination of the most profitable amounts of fertilizers to use.

The crop studied was corn and the variable nutrients were nitrogen, phosphorus and potassium. In addition, three different plant populations were studied and half of the 120 plot observed were irrigated.

A production function was fitted to the data obtained from the field experiment. The function used was as an incomplete second degree polynomial of the form:

$$Y = a + b_1N + b_2P + b_3K + b_4N^2 + b_5P^2 + b_6NK + b_7I + b_8S + b_9S^2 + b_{10}NS + b_{11}PS.$$

This function was found adequate by inspection of the distribution of unexplained residuals for the analysis of the data according to standard statistical tests. The unexplained residuals for the experiment and function fitted were graphically studied.

High profit points were computed under different assumptions of price for both inputs and output. It was observed that when the price of corn was low (between $1.00 and $1.30 Bu.) and the price of nitrogen was fixed at $ .068 KG., the use of nitrogen was not profitable. When the price of corn was $1.60 Bu., however, the use of 42.7 lbs. of nitrogen per acre became the most profitable quantity to use. The increase in the price of corn showed that the optimum quantity of phosphorus to use is also increased although slightly (from 47.2 lbs/acre up to 51.6 lbs/acre when the price of corn is changed from $1.00 to $1.60). This study showed that use of estimated HPP quantities of fertilizers increased profits about $24.75 per acre over the use of no fertilizer.

The experiment was performed at only two levels of irrigation; this limited inferences about changing marginal productivity of irrigation. No irrigation cost data were available. Yields averaged 18.340 bushels per acre higher on those plots which received irrigation.

The data here analyzed represent observations from one year only. The promising results obtained under economic analysis should encourage continuation of this kind of research. Valuable experience was also gained.

The extension of results from this kind of economic analysis to the farm level may provide positive assistance in the general effort to increase productivity and standard of living in agricultural sectors of several Latin American countries.

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By

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CHAPTER I

INTRODUCTION

The main objective of this thesis was to estimate the quantities of fertilizer inputs which would have maximized net returns in the production of corn on a fertilizer experiment in Colombia, South America. However, the author has in mind additional objectives such as summarization and translation of this work into Spanish to be published in the near future.

It is well known that in general in Latin America the research on and the teaching of agriculture at the University level has been carried out for many years under unsatisfactory conditions. Lack of funds has been one of the most serious handicaps. This scarcity has been reflected in a shortage of buildings, equipment, laboratories and other facilities. On other occasions when requirements for physical facilities were fulfilled, a shortage of instructors became a serious problem. There were also cases where expensive facilities and instructors were available but where there existed a shortage of students.

Almost all the twenty Latin American countries have at least one school of agriculture. Some of these schools are as old as in the United States of America. For example, Chapingo in Mexico, is more than one hundred-eighty years old having been founded around 1776. There are several countries with schools of agriculture founded at the end of the last century. European influence is great in a large number of our colleges. The bachelor degree in agriculture is called "INGENIERO AGRONOMO" in resemblance of the title given by the Institute Agronomique du Paris, France. Colleges in Chile reflect a strong French influence. In fact, by 1875 several agronomic experiments were carried out for French professors specially contracted to promote agricultural teaching. Other Latin American countries had similar experiences.

For at least eighty years, experiments on fertilizer use have been carried out in different Latin American countries under the control of some College of Agriculture. In addition, private organizations have developed their own experimental stations. The results of these experiments have been, in many cases, poorly extended to the farmers. The main reason for this limitation may be found in the organization of schools of agriculture; in many instances, extension programs were forgotten. Most of these experiments were done by able professional people, some even at personal sacrifice, but unfortunately the real value of the work remained almost unknown in many instances.

Since 1930, important rearrangements in the experimental stations were made in many Latin American countries. During the decade of the 40's, the experiments were generally worked out using statistical analysis. Of course the statistical approach was used much earlier in some experiment stations.

Very useful agronomic results were obtained from experiments on fertilizer use. However, the economic analysis was often neglected, badly used or not used at all.

This thesis is written with the intention of showing systematically an approach to the economic analysis of a fertilizer experiment. It is hoped that this work will eventually be printed in Spanish. As the explanation is in a simple form, it is hoped that it will serve as a basic reference in those places of Latin America where such a reference is needed.

A. General background.

Although the results obtained from an experiment in fertilizer use can be significant from an agronomic standpoint, the results may have additional economic significance.

For the agriculturist and economist working in a team, higher yields do not always mean higher profits. The additional use of fertilizer becomes optimum (most profitable) when the cost of the last unit of fertilizer used is equal to the value of the additional output or, in the vocabulary of the economist, when marginal factor cost equals marginal value product.

In order to find the economic optimum in the use of fertilizer, a number of approaches can be used. Some of these approaches will be reviewed in Chapter II. With these approaches, the analysis becomes straightforward for meeting the minimum necessary conditions. In some of the well-established experiment stations of today, fertilizer experiments are being controlled by a group of people with several different interests in the results. In many instances, experiments are designed to provide the necessary data for a variety of interests. The soil specialist, for example, tries to find the best <u>kind</u> of fertilizer for a given type of soil while the agricultural economist may be interested in the high profit <u>amount</u> of that fertilizer to use.

It seems pertinent to say here that the economic analysis that is going to be presented in this thesis can be applied to fields other than the use of fertilizers, as for instance, livestock feeding or any experiment where a group of inputs are combined in the production of a product.

B. Fertilizers in Latin America.

In order to compare fertilizer production and consumption in South America, Central and North America with Europe, the following figures are given from a F.A.O. report:[1]

Table 1. Production, consumption and net balance of various fertilizers by regions of the world, 1958.

	Production	Consumption	Balance
	(Thousand metric tons)		
South America			
Nitrogen	291	132	+ 159
P_2O_5	85	159	- 74
K_2O	16	91	- 75
North and C. America			
Nitrogen	2,358	2,272	+ 86
P_2O_5	2,398	2,259	+ 139
K_2O	1,797	1,764	+ 33
Europe			
Nitrogen	4,712	3,293	+ 1,419
P_2O_5	3,833	4,175	- 342
K_2O	5,074	4,221	+ 853

[1] Annual Review of World Production and Consumption of Fertilizer, Food and Agriculture Organization of the United Nations, FAO, Nov., 1958.

Two conclusions are suggested for South America: (a) with the exception of nitrogen, the other two fertilizers are in a serious deficit production position, and (b) considering that, in general, the intensity of fertilizer use is still low, it can be concluded that South America must find additional sources of phosphorous and potassium fertilizer in order to increase crop productivity.

On taking into consideration the 1958-59 fertilizer consumption in five South American countires, the following is found, according to the same F.A.O. information:

Table 2. Fertilizer consumption in selected South American countries, 1958-59.

Country	N	P_2O_5	K_2O
		(Thousand metric tons)	
Colombia	7,000	17,500	5,400
Argentina	6,000	3,400	3,400
Brasil	33,000	73,000	59,000
Chile	36,000	33,000	9,000
Perú	36,000	14,300	4,000

Perhaps the most surprising facts shown by the figures above is that Argentina, one of the world's top wheat and corn producers, is the lowest consumer of fertilizers.

C. The source of data for this thesis.

The data here analysed were provided by an experiment carried out at Florida, Cauca Valley, Colombia, South America. Personnel from Michigan State University have been working cooperatively with the Faculty of Palmira at the Cauca River Valley. Full details on these data will be given in Chapter IV of this work.

D. Approach of this thesis.

This thesis will first present the theoretical basis for using production functions. This will be developed in Chapter II, the main sub-heading of which will deal with a short historical review of the use of production functions from earlier days until now. Then, the mathematical meaning will be explained briefly. At the same time, actual uses of production functions will be summarized. The final part of Chapter II will deal with the problem of selecting appropriate mathematical expressions for production function analysis.

In Chapter III, the hypothesized model will be presented. At the same time a brief review will be made of the general agronomic conditions under which fertilizer experiments are performed. Certain uses of mathematical functions will be illustrated with an example.

In Chapter IV, characteristics of the data used in this thesis will be presented including the available information on the Colombian experiment, the quality of the data collected and the relevant characteristic of the experimental design.

In Chapter V, data from the field experiment will be fitted to the hypothesized model.

In Chapter VI, an analysis of the results obtained from the previous chapter will be made. High profit points (HPP) and predicted yields will be found for optimum quantities of fertilizers at various price levels of inputs and output.

Finally, Chapter VII will be devoted to an evaluation of the approach used and the results obtained.

CHAPTER II

THEORETICAL BASIS OF THE PRODUCTION FUNCTION

Use of input-output analysis has increased greatly during the last several decades. This approach is of paramount importance to the economic analyst dealing with fertilizer experiments. Mathematical functions have been employed to explain and predict input-output relationships.

This chapter discusses the underlying concepts and use of mathematical functions to describe relationships between plant nutrients and crop yields.

A. Definition.

A function can be defined as the relationship between two variables. One of the variables is dependent on the others. For the usual production function, a dependent variable Y (which can be translated as output per unit of time) and $X_1, \ldots, X_n$, which are inputs per unit of time or independent variables. In this case the production function can be written as follows:

$$Y = f(X_1, \ldots, X_n)$$

which means that output (Y) is a function of or depends on the inputs $X_1, \ldots, X_n$. The inputs involved in the production function can be classified into two groups, variable and fixed. The usual notation for this is $X_1 | X_2$ where the bar ($|$) means "given X_2 fixed." Relationships between variables can be shown by word description, tabulation or graph.[1]

B. A brief historical review.

The famous German scientist Justus von Liebig may be considered the first person to devise the production function concept. In his famous "law of the minimum" he held that the yield of any crop is determined by changes in the quantity of that factor which appears in lowest amount; this factor is called the minimum factor. If this minimum factor is increased, the yield of the product is increased in proportion to that factor until another nutrient becomes limiting. If another factor--not at the minimum--is increased, the yield of the product does not change. In other words, Liebig intended to show that the yield of any product is a linear function of the minimum factor.

Liebig's law was formulated about 1860. It has had

[1]The function can be continuous or discrete. In developing a function in production economics, cardinal numbers (1, 2, 3, etc.) are used in contrast with the ordinal numbers (1st, 2nd, 3rd, etc.) used in consumption economics.

a tremendous influence on agricultural scientists for almost a hundred years, especially on agronomists and on farm economists.

The well known graphic illustration of a water barrel with the lowest stave representing the minimum factor that, in turn, shows the limit of water (or profits in the case of farm business) was widely used in presenting Liebig's concept. Under this circumstance, Liebig recognized constant returns to the limiting factor but denied the presence of factor substitution. He did not consider substitution of resources and that farmers, for example, on considering the factor-product price ratio can add the minimum factor profitably as long as the marginal factor cost (MFC) is greater than or equal to the marginal value product (MVP).

Researchers have rejected Liebig's formulation for two main reasons: (a) factors of production are seldom perfect complements and a given crop yield can be produced with different quantities and combinations of nutrients (such as P_2O_5, N and K_2O), moisture, heat, etc., providing the necessary minimum amount of each is present, and (b) successive additions of factors limiting crop yields do not necessarily result in linear additions to crop yields but result, instead, in diminishing additions to crop yields and eventually decreases in total yield.

Lawes and Gilbert at the Rothamsted Experiment Station, England, demonstrated that the law of diminishing returns operates in fertilizer use.[1] Ewald Wollny at the end of the last century (1897-98) conducted an experiment to test Liebig's law of the minimum. He concluded that additional amounts of nutrients cause a rise in production of a plant which first is progressive and eventually becomes smaller and smaller arriving at a limit where further additions of nutrients provoke yield reduction.[2]

Jethro Tull thought that yield was an increasing function of inputs which varied with manure or tillage. He claimed that as more tillage was used on a crop, it became less expensive and that crop yield increased. Most of the people have rejected Mr. Tull's conclusion, in part, because he was an inventor and manufacturer of tillage machinery[3] and more importantly because it does not always meet the test of experience.

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[1]E. J. Russell, Plant Nutrition and Crop Production (Berkeley: University of California Press, 1926).

[2]W. J. Spillman and E. Lang, The Law of Diminishing Returns and the Law of the Soil (New York: World Book Co., 1924), p. 100. (Quoted from Ewald Wollny's "Untersunchungen uber den Einfluss der Wachstumsfaktoren auf das Produktronsvermogen der Kulturpflantzen," Forschungen auf dem Gebiete der Agrikulturophysik 1897-98, p. 105-06).

[3]Jethro Tull, Horse Hoeing Husbandry (London: William Cobbett, 1829).

Hellriegel thoroughly studied the application of nutrients to a plant in 1880. He did his experiment using nitrogen (N) on a barley crop. In the beginning, additional amounts of N gave increments in yield followed, as the N was increased in amount, by greater increments in yield--greater even than those shown by Liebig's law. Finally, when still more N was applied, a declining incremental yield response was observed and the well known sigmoid yield curve became evident. This experiment was repeated several times and the yield curves were always similar.[1]

E. A. Mitscherlich at the Experiment Station of Koenigsberg, Germany, observed that the sigmoid yield curve could be represented by a mathematical equation in order to quantify the manner in which crop yields were related to plant nutrients. His principal assumption was that maximum yields would be obtained under ideal conditions unless any essential growth factor were shown to be limiting.[2]

Baule, on enlarging Mitscherlich's ideas, suggested that the final yield is the result of all factors working together. The ideas of Baule can be summarized as follows:

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[1]E. J. Russell, <u>Soil Conditions and Plant Growth</u> (New York: Longmans, Green and Company, 1950).

[2]J. Redman and S. Q. Allen, <u>Journal of Farm Economics</u>, XXXVI (August, 1954), p. 457.

yield responses will be determined by the level of use of the fixed factors. If only one factor is variable and the others fixed, the response to this unique factor is lower than when two or more factors are used as variables.[1]

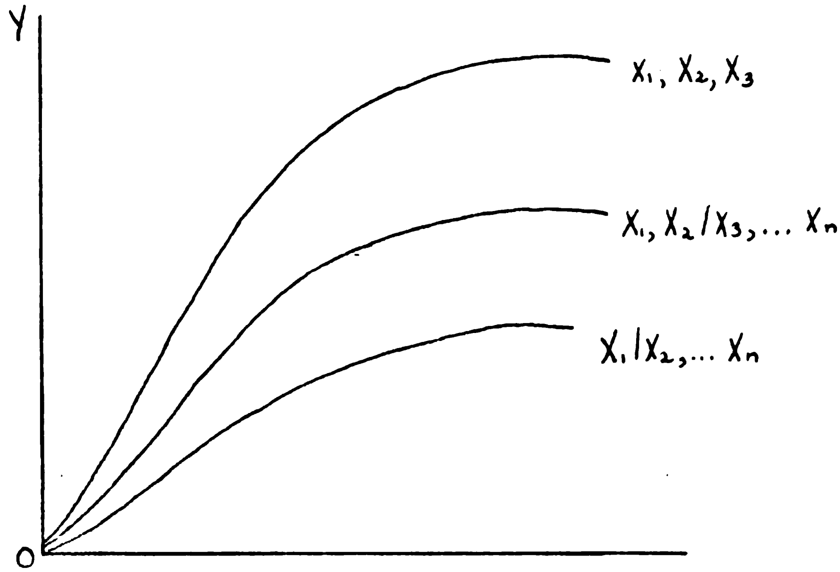

Figure 1. Baule units of growth factors.

In Figure 1 "a Baule unit" effect is illustrated. A Baule unit is the amount of a yield influencing factor necessary to produce one half the maximum yield when other factors are at their optima.[2]

W. J. Spillman in the USA, a contemporary of Mitscherlich in Germany, developed an equation dealing with the growth

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[1]Spillman and Lang, op. cit., p. 147.

[2]Redman and Allen, op. cit., p. 458 fn.

factor's effect on plant yields.

From the early 30's, when Mitscherlich and Spillman developed their equations, until now, a number of other equations and mathematical functions have been used including the Cobb-Douglas, Carter-Halter-Hocking and a large number of less specialized polynomial equations. Most of these will be explained later on in this thesis.

Heady and others[1] have used two types of polynomials in a prediction equation for corn. Johnson, when working with a production function, has placed particular emphasis on the distribution of "residuals" (u's) generated by uncontrolled factors[2] in selecting mathematical functions to represent production responses.

C. Mathematical forms.

When trying to find the mathematical form of the most suitable function for the analysis of available fertilizer data, physiological and biological growth have to be

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[1] E. O. Heady, J. Pesek, and W. Brown, "Crop Response Surfaces and Economic Optima in Fertilizer Use," Research Bulletin 424, Iowa Agricultural Experiment Station (1955).

[2] G. L. Johnson, "Interdisciplinary Considerations in Designing Experiments to Study the Profitability of Fertilizer Use," Economic Analysis of Fertilizer Use Data, edited by Baum and others (Ames, Iowa, 1956), p. 26.

considered. When all the growth factors except one are fixed, the expected increase in yield is influenced by the varying proportions between the variable and the fixed factors. Here the law of diminishing returns comes into the picture. When the level of the fixed nutrients is extremely low, the yield of the product may decrease as successive additional amounts of the variable factor are applied. The action of N under low moisture is a good example of the preceding statement.[1] On the other hand, when the fixed nutrient factors are near the physical optimum, the total physical product (TPP) should be expected to increase first at an increasing rate and then at a decreasing rate until a maximum (theoretical) is reached beyond which output shall be expected to decrease. Experiments done in Nebraska, Oregon and Washington confirm this assertion.[2]

The results of an input-output experiment using fertilizers as inputs are not only subject to variations because of controlled forces but also because of uncontrolled forces. More details on this topic will be given later on.

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[1] Heady, _et al._, _op. cit_. In Iowa, N was applied at different levels of P_2O_5 in a corn-N-P experiment; the fixed level of moisture was found very low, p. 330.

[2] J. L. Paschal and B. L. French, "A Method of Economic Analysis Applied to Nitrogen Fertilizer Rate Experiments on Irrigated Corn," _USDA Bulletin 1141_ (1956).

Attempts to describe input-output relationships have included use of the following functions:

1) Exponentials

2) Other polynomials:

   i) quadratic and

   ii) square root

3) Cobb-Douglas or power function[1] (a special case of the exponential Carter-Halter-Hocking function) and

4) the exponential Carter-Halter-Hocking function.

An early mathematical formulation was due to Mitscherlich. His function that he called "law of diminishing soil yield" intended to show that a maximum yield is obtained when one of the essential growth factors is limited.
This can be expressed as

$$Y = A(1 - e^{-cx})$$

where,

A = maximum possible yield

e = a constant

c = a constant

x = input

Mitscherlich believed that additional yields brought about

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[1] E. O. Heady and J. Dillon, Agricultural Production Functions (Iowa State Press, 1961).

by one factor had no effect on the productivity of the other nutrient factor. This belief was modified by Baule who used the production function:

$$Y = A(1 - 10^{c_1 X_1})(1 - 10^{c_2 X_2}) \ldots (1 - 10^{c_n X_n})$$

where,

A = maximum yield

c_n = effect factors

X_n = variable growth factors not considered by Mitscherlich.

1) <u>Exponential functions</u>.

An exponential function was developed by Spillman in analyzing fertilizer data of a tobacco experiment.[1] His production function was:

$$Y_j = M - A\,R^X$$

where,

Y_j = is the yield

M = maximum yield possible to obtain theoretically

A = the increase in yield between the yield with no application of fertilizer and the theoretical maximum

R = the constant ratio of successive increments in yield

X = the variable.

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[1] W. J. Spillman, "Use of the Exponential Yield Curve in Fertilizer Exponents," <u>USDA Technical Bulletin 348</u> (1933).

Spillman found in the Baule and, consequently, Mitscherlich's formulations a good deal of inspiration for his own formula. The main difference between Mitscherlich and Spillman is that the former claimed that the ratio of successive increments in yield, given a unit increase of a given growth factor, is the same for all crops and all soils, providing no other factors are limiting.

Spillman's exponential production function has been found useful for a number of input-output relationships but, at the same time, it has been pointed out that it is unable to give estimates of those segments where the data under analysis show negative marginal products (or diminishing total returns). On the other hand, it is said that the exponential function gives a good answer providing that A (the constant) has a positive sign and the crop yield becomes asymptotic to M. In this case M is a minimum, not the maximum, as originally assumed in the function.

The Spillman function, expressed for general cases, can be written as follows:

$$Y = M\,(1 - R_1^{X_1})\,(1 - R_2^{X_2}) \ldots (1 - R_n^{X_n}).$$

2) Polynomial functions.

It is possible to conceive of an infinite number of polynomial functions. However, those equations involving a third or higher degree have not been used much because they have been considered unnecessary to describe data used.

i) Quadratic polynomial functions.

It is usual to speak about the family of polynomial functions. One widely used member of this family is the quadratic, being represented as follows:

$$Y = a + b_1 X + b_2 X^2$$

The quadratic function can be fitted using the method of least squares and has the favorable characteristic of permitting terms to be added or subtracted giving a new pattern to the function. When using this quadratic function, a negative marginal product can be shown when, for example, fertilizer application produces a restriction on the growth of the crop under experiment.

ii) Square-root polynomial functions.

A type of square root polynomial function was used by Heady, Pesek and Brown[1] as a prediction equation for corn:

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[1] Heady, et al. op. cit.

$$Y = a + b_1\sqrt{N} + b_2\sqrt{P} + b_3 N + b_4 P + b_5\sqrt{NP}$$

The square root seemed to provide better estimates of relationship than the quadratic function.

3) Cobb-Douglas or power function.[1]

This function has been widely used by researchers for the analysis of the input-output relationships. This function has the form:

$$Y = a\, X_1^{b_1}\, X_2^{b_2} \ldots X_n^{b_n}$$

This function becomes a linear function when the dependent and the independent variables are transformed to logarithms. Estimation of parameters by least squares is easy because of the linearity characteristic.

Two of the chief restricting assumptions of this function is continuously increasing yields if $0 < b_i$ and a constant factor elasticity of production. With this function, yield can increase at an increasing, constant or diminishing rate. However, the response curve can only show one of these stages, not a combination. This is one of the limitations of the Cobb-Douglas function. Another characteristic is that it takes on a zero value where any input is zero.

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[1] Heady and Dillon, op. cit.

4) Carter-Halter-Hocking function.[1]

This is an exponential function of the form

$$Y = a\, N^{b_1} c_1^{N}\, P^{b_2} c_2^{P}\, K^{b_3} c_3^{K}$$

Before being fitted by least squares, this function also requires a logarithmic transformation. Carter and Halter (formerly from M.S.U.) and Hocking worked out this function as a more general form of the Cobb-Douglas.

The chief advantage of this function is that it is more flexible than the Cobb-Douglas. However, it requires two parameters for each variable (i.e., for variable N, parameters b_1 and c_1 are needed). Greater complexity in locating various economic optima is another disadvantage.

In the Carter-Halter-Hocking function, the b_i's and c_i's have to be positive and less than one in order for it to reflect the law of diminishing returns. This function, under this condition, will show a total product increasing at a decreasing rate and eventually decreasing. Preliminary results in fitting this function have shown that it is able to describe the three stages of production satisfactorily.

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[1] Journal of Farm Economics, Vol., XXXIX, No. 4 (Nov., 1957).

For a long time, it was the general belief that one functional form could be found to describe the relationship between plant nutrient and yields. No such single function has been found yet. Researchers have to select, from among the vast number of functions conforming to all or part of the law of diminishing returns, one which does a reasonably good job of describing their data.

D. Actual use.

During the last twenty years, increasingly wide use of production functions has been made in the analysis of data coming from different controlled fertilizer experiments.

A number of researchers making economic analyses of fertilizer data have been using the production function approach. A large number of studies could be mentioned but perhaps the most illuminating are those performed at Universities like Iowa, Michigan State, Oklahoma, California and those made by technical departments in the TVA and USA.[1]

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[1] H. Bertolotto, "Economic Analysis of Fertilizer Input-Output Data from the Cauca Valley, Colombia" (unpublished Masters Thesis, Department of Agricultural Economics, Michigan State University, East Lansing, Michigan, 1959); G. I. Trant, "Implications of Calculated Economic Optima in the Cauca Valley, Colombia, S. A.," Journal of Farm Economics, XL (Feb., 1958).

Other workers dealing with dairy problems have also made use of production function in their analytical approach to the study of available research data.[1] Production functions have been as widely used for whole farms as for individual enterprises and the results have been at least as reliable.[2]

E. Selecting mathematical forms.

There is no definitive criteria for selecting a production function to describe an input-output relationship. However, Johnson has discussed several problems which should be taken into consideration in designing an experiment and in selecting functions.[3] In discussing selection of appropriate functions on the basis of objective statistical tests, he states, "Perhaps a fruitful approach would be to test the degree to which the alternative functions individually meet the usual assumptions with respect to the distribution

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[1]E. Jensen, *et al.*, "Input-Output Relationships in Milk Production," *USDA, Technical Bulletin 815* (1942).

[2]G. L. Johnson, "Sources of Incomes on Upland Marshall County Farms," *Progress Report 1*, Kentucky Agricultural Experiment Station (Lexington, Kentucky, 1952); G. Tintner, O. H. Brownlee, "Production Functions Derived From Farm Records," *Journal of Farm Economics*, Vol. 26 (1944); E. O. Heady, "Production Functions from a Random Sample of Farms," *Journal of Farm Economics*, Vol. 28 (1946).

[3]G. L. Johnson, "Discussion: Economic Implications of Agricultural Experiments," *Journal of Farm Economics*, Vol. XXXIX, No. 2 (May, 1957), p. 391.

of unexplained residuals. If this were done, all functions failing to meet these assumptions could be rejected on an objective basis without the use of subjective judgment." The usual assumptions regarding the residuals are: that the U_i residuals are independently and normally distributed with zero mean and constant variance. If these assumptions are met then the usual tests for statistical significance can be applied in choosing between alternative functions.

These tests, however, may fail to reveal statistically significant differences between alternative functions. Johnson recognized this possibility and concluded that in such cases, "One would be forced to turn to 'experimenters judgment', expert opinion and independent information as a basis for judgment." The advantage of this approach rests on the fact that objective statistical tests are first used in arriving at a decision regarding the appropriate function. Failing this, subjective concepts and judgments are used in arriving at a decision.

In general, the function selected must provide a good fit of the data in the region where: 1) yield is increasing at decreasing rate and, 2) total yield is decreasing. The "goodness of fit" can be finally determined with the help of statistical measures. The most important

CHAPTER III

THE HYPOTHESIZED MODEL

The agricultural economist who designs a model for use in producing and analyzing experimental fertilizer data, should have in mind not only correlations and mathematical manipulation of the data, but also the agronomic aspects of what he is doing.

Early in this chapter, a brief review is made of the general agronomic conditions under which a fertilizer experiment is performed. After this, experimental design, mathematical manipulation and fitting of functions are discussed and illustrated in some instances with elementary examples.

A. Agronomic model.

A general mathematical model for a production function dealing with input-output relationships in using fertilizers could be written as follows:

$$Y_c = f(N,P,K,I,S \mid C,M,L,W,R,FS) + u$$

The above formula can be interpreted as follows: the yield of corn, Y_c, is a function of the controlled variables nitrogen, phosphorus, potassium, irrigation (I), and number statistical tests are:

a) the coefficient of multiple correlation and multiple determination which show and compare variance explained by regression with the total variance of the yield data; and

b) the standard error of the parameters in the prediction equation.

Another measure which can be determined is the magnitude of residual variance not associated with the regression. The plotting of alternative functions using at the same time a scatter diagram of the experimental observations may also be useful. Or, by following logical expectations, the technical relationships of the variables represented in the function can be checked.

As Sundquist and Robertson[1] have pointed out, the pragmatic test of whether or not a particular production function formulation is found in its ability to predict over time. This kind of test "can be applied only by prediction, further observation and further prediction."

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[1]W. B. Sundquist and L. S. Robertson, "An Economic Analysis of Some Controlled Fertilizer Input-Output Experiments in Michigan," Technical Bulletin 269, Agricultural Experiment Station, Department of Soil Science, Michigan State University.

of plants per unit of area (S) and of the fixed conditions such as soil condition (C), management of the soil (M), animal manure (L), climate and soil moisture (W), and systems of farming (FS), plus a number of unexplained variations (u's) which generate the unexplained variations (u) in Y.

It is advantageous to review the agronomic conditions which are often fixed but which affect productivity of fertilizers and, consequently, the final yield of the crops.

1) Soil conditions.

By considering the chemical composition, texture, topography, slope and natural drainage of soil, we have a starting point to assess fertilizer use. Of course, the most definite way to establish fertilizer responses is field experimentation.

2) Management of the soil.

Previous management practices (including crop rotation, manuring, fertilizing, cultural practices, irrigation and water conservation) are capable of producing significant alterations in soil properties.[1] Examples of this kind are

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[1] G. R. Anderson, "An Economic Evaluation of Three Soil Nitrogen Tests," (unpublished Masters Thesis, Department of Agricultural Economics, Michigan State University, East Lansing, 1958).

found all over the world where two fields with the same type of soil are receiving substantially different fertilizer treatment. Greater response to nitrogen than to other nutrients is generally shown by pasture grasses and cereals. Sugar cane and corn, for instance, need large quantities of nitrogen. Heavy application of nitrogen on pasture grasses usually result in a greater response with adequate soil moisture. However, when grasses are being grown in mixture with legumes, the required amount of nitrogen fertilizer is reduced. On the other hand, the need for minerals such as phosphorus, potassium or calcium may be increased.

Sometimes small amounts of "minor" mineral nutrients are enough to produce crop response. Zinc, boron, sulphur and iodine are good examples of the so-called "minor elements." Potatoes, beets, corn and legumes are especially responsive to potassium fertilizers.

3) Animal manure.

Natural organic fertilizer is usually applied on the more valuable crops with good responses. Manure is able to improve both soil structure and biological activity in the soil. Sugar beets, potatoes, turnips, corn and oilseeds respond well to manure. Cereals and legumes usually respond less well because legumes can utilize atmospheric nitrogen and cereals have lower potassium and phosphorus requirements than other crops. Diminishing returns hold for manure as for other inputs.

4) <u>Climate and soil moisture</u>.

These factors can seriously affect the optimum amount of fertilizers to use. In dry climates, fertilizer application without provision for adequate moisture is often almost useless while under irrigation there may be a profitable response. In rainy regions, water control with soil conservation practices can improve soil moisture conditions making possible better crop responses from the use of fertilizer.

5) <u>Systems of farming</u>.

It is important to consider the effect of farm enterprises on fertilizer requirements. On livestock farms, fertilizer use can be quite different from that on farms where livestock is not kept and no manure is used. Furthermore, the kind of livestock and its management is another point to be considered.

Usually, farming systems are closely related to crop, soil, climate, etc. Fertilizer use has no unique pattern for all crops and for all conditions. For instance, potash and some phosphates have slow solubility and in many soils remain near the surface out of the upward reach of plant roots. On the other hand, nitrogen, especially "Salitre de Chile" ($Na\ NO_3$ and $K\ NO_3$), is extremely soluble under good and even relatively poor soil moisture conditions. For arid regions, experiments have shown that deep placement of fertilizer may increase yield. In these cases, deep rooted crops are usually grown. The advantage of deep fertilizer placement depends on textural, drainage and aeration conditions. This method of fertilizer placement is less useful in heavy soils.

B. Empirical work.

Use of a mathematical model permits a more definite statement of an unspecified agronomic model and is helpful in understanding the scope of the latter.

But before discussing mathematical functions further, it seems necessary to establish here those statistical aspects essential in planning experiments.[1]

i) Statistical planning of experiments.

In the first place, the data to be obtained from the

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[1] "The Economics of Fertilizer Application," Conference Proceedings, Farm Management Research Committee of the Western Agricultural Economics Research Council, Corvallis, Oregon, 1956.

experiment must be relevant to the existing problem and be suitable for statistical analysis. Within a certain margin of error, one (or more) questions must be answerable.

In planning controlled experiments, the chief points to be considered are as follows:

1. The researcher should know the problem. If any hypotheses is going to be tested, it should be clearly stated. Usually a mathematical formulation is necessary to express a production function.

2. All the factors or variables to be studied in the experiment must be defined. As we have already seen in the previous section, the independent variables are classified in two groups: controlled and not controlled. The uncontrolled factors are randomized to permit averaging out their effects (see 5 below).

3. The range of variation and number of levels of each controlled variable to be investigated must be stipulated. Of course the <u>fixed</u> controlled variables have no range and, hence, only one level of use.

4. The number of replications of the experiment under each set of conditions should be determined. Though the number of replications should be the same for each level of input to simplify statistical computations, other considerations may offset this advantage.

5. A method of assigning treatments to the experimental units must be established in accordance with some scheme of randomization so that the "experimental error" due to uncontrolled variables may be independent of the independent variables.

6. The correlations between the independent variables should be minimized. Ideally the $r_{X_iX_j}$ should equal zero where $r_{X_iX_j}$ is the correlation coefficient of the X_i and X_j independent variables.

7. Some functional form should be used to represent the distribution of experimental errors. The usual form used is the normal distribution though other assumptions are frequently made and with justification.

8. The possible outcomes should be considered. Provision should be made to be certain that a sufficient proportion of these will answer the problem. If not, a new plan is required.

9. Assuming that the experiment has been carried out as stipulated in the preceding steps and that the assumptions made were justified, the type of statistical analysis should be specified in the plan. This point has to be closely related to item 8 above and section E of the last chapter.

10. The plan should also indicate the manner in which the conclusion are to be presented so that they may be understood

readily by those persons who are not well trained in statistical theory and methods.

ii) <u>The straight line function.</u>[1]

Let us consider a simple linear function in two variables

$$Y = a + bX + u$$

We can describe this function as follows:

Y = a variable dependent on X

$\hat{Y} = (Y - u)$ = the predicted Y

a = a constant

b = a constant

X = the independent variable

u = an unexplained residual generated by a set of uncontrolled variables which are independent of X.

The previous equation is called the <u>equation of the function</u>. It describes a straight line between two variables. In order to determine such a mathematical relationship between X and Y in a set of data, the constants a and b must be estimated.

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[1] This and the following section is based on M. Ezekiel's <u>Method of Correlation Analysis</u> (New York: John Wiley and Son, 1953; 2nd edition).

The symbol Y in the equation simply represents the number of units of the variable designated as Y. These can be acres, pounds, dollars, etc.; similarly X represents the number of units designated as X.

The meaning of the constants a and b of the formula will be explained first in a simple graphic way and then with the aid of mathematics.

In Figure 2 we have the graphical representation of the function $\hat{Y} = a + bX$.

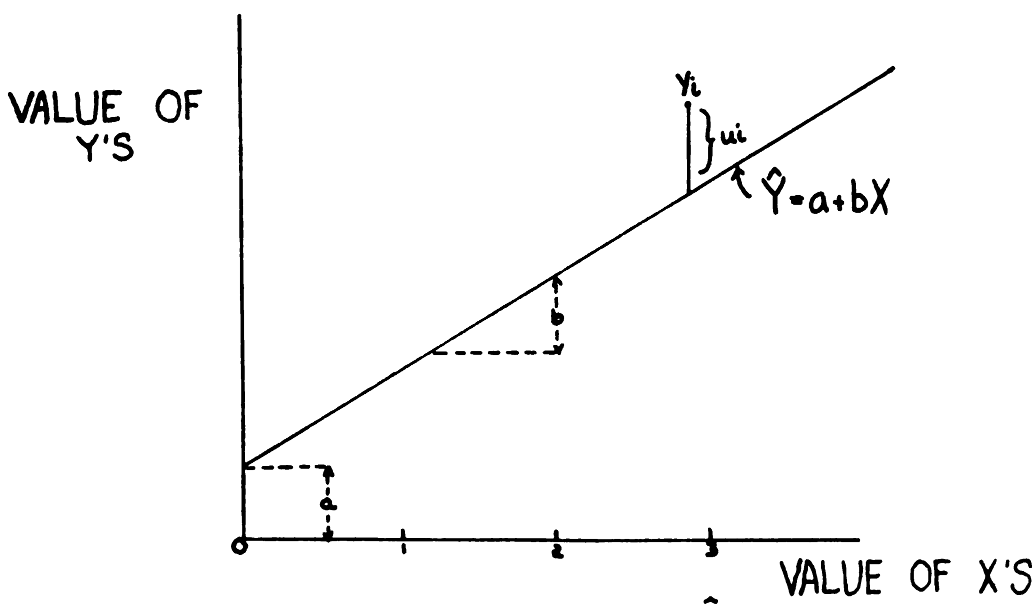

Figure 2. Graph of the function $\hat{Y} = a + bX$

When the value of X is 0, b times X is 0 (b.0 = 0) and therefore $\hat{Y} = a + 0 = a$. The constant a gives, then, the height of the line in terms of Y or vertical units at the point where X is zero. We can see this in the lower left part of Figure 2.

In the same equation,[1] every time X increases one unit, $\hat{Y}$ increases b times one unit, since $\hat{Y}$ is computed as a plus b times X. Therefore, the difference of the height of the vertical line measured in Y units between two adjacent values of X, say from 1 to 2, is b units of Y as indicated in the graph. This is true for every unit change of X whether 0 to 1, or from 25 to 26 or 99 to 100.

The difference between a point in the above diagram, Y_i, and the regression line is the residual variation, $u_i = Y_i - \hat{Y}_i$.

Fitting the line by "least squares." A mathematically determined straight line can be obtained from experimental data by the method of least squares. In this process for determining values of the constants a and b, all the observations are considered and given equal weight.

The pattern of computation from the total number of X and Y observations for determination of the straight line by "least squares" is as follows:

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[1]The discussion of properties of a linear function presented in this section is of course, well known by production economists and is fully discussed in elementary algebra and statistical text books. It is reviewed here primarily because it serves as a basis for further discussion of curvilinear functions of the type used in this study.

Table 3. Hypothetical data demonstrating the computational procedures of least squares regression.

Observations of X	Observations of Y	X^2	XY
4	3	16	12
2	7	4	14
6	4	36	24
5	6	25	30
3	8	9	24
$\Sigma X = 20$	$\Sigma Y = 28$	$\Sigma X^2 = 90$	$\Sigma XY = 104$

All the observations of X and Y are listed under headings "X" and "Y". Then each X observation is squared and entered below "X^2". Each X observation is multiplied by the corresponding Y observation and entered in the XY column. The summation of each column is represented by ΣX; ΣY; ΣX^2 and ΣXY. This means the sum of all X's, sum of all Y's and so on.

With these values calculated, we can proceed to find the values of a and b, with the aid of the following formulas:

$$b = \frac{\Sigma XY - n\Sigma X\Sigma Y}{\Sigma X^2 - n\ (\Sigma X)^2}$$

$$a = My - bMx$$

Where

n = number of observations (five in our example)

M_x = arithmetic average of X's (four in our example)

M_y = arithmetic average of Y's (5.60 in our example)

Using the values in our example we can solve for b as follows:

$$b = \frac{104 - 5 \cdot 20 \cdot 28}{90 - 5\ (20)^2} = \frac{104 - 2800}{90 - 2000} = \underline{1.41}$$

$$a = 5.60 - (+1.41)\ (4) = 5.60 - 5.64 = \underline{-.04}$$

Therefore the equation of the straight line determined by all the observations is

$$Y = -.04 + 1.41\ X$$

This line is called the <u>line of best fit</u>. The nature of this best fit is explained by the fact that if the differences between each of the <u>real</u> observations and the estimated values given by this equation are computed, squared and summed, this sum will be smaller than it would be if any other straight line were used. Because with this method the line with the smallest possible sum of the squared deviations is determined, this line is known as the "least squares" line of regression and the process of its computation is called the "method of least squares." In addition, a normal distribution of the residuals or u's produced by uncontrolled variables is assumed.

The determination of the constants for the linear equation, when certain data or observations are given, is known as 'fitting' the equation to the data. The linear equation is popular because of its simplicity in "fitting." However, since it describes only a straight line, its use is limited.

iii) <u>The curvilinear functions</u>.

Curvilinear relations may also be fitted to a set of data. The number of curves that can be described with equations is infinite.

In statistical analysis, some useful and familiar equations for describing different kind of curves or types of functional relationships are shown below:

1) $Y = a + b X + c X^2$

2) $\log Y = a + b X$

3) $\log Y = a + b \log X$

$y = a + b \log X$

4) $Y = \dfrac{1}{a + bX}$

5) $Y = a + bX + cX^2 + dX^3$

6) $Y = a + bX + c \left(\dfrac{1}{X}\right)$

Equation 1) is found to be the equation of a parabola:

1) $Y = a + bX + cX^2$

Assuming values of the unknown constants $a = 1$; $b = 0.5$ and $c = -0.1$, and entering these in the formula, we have,

$$Y = 1 + 0.5\,X - 0.1\,X^2.$$

Now if X takes different values, we can work out the value of Y. For instance, if Y is computed at successive values of X from 0 to 6, we have the following results:

$X = 0$	$Y = 1 + 0.5(0) - 0.1(0^2) = 1$
$X = 1$	$Y = 1 + 0.5(1) - 0.1(1^2) = 1.4$
$X = 2$	$Y = 1 + 0.5(2) - 0.1(2^2) = 1.6$
$X = 3$	$Y = 1 + 0.5(3) - 0.1(3^2) = 1.6$
$X = 4$	$Y = 1 + 0.5(4) - 0.1(4^2) = 1.4$
$X = 5$	$Y = 1 + 0.5(5) - 0.1(5^2) = 1$
$X = 6$	$Y = 1 + 0.5(6) - 0.1(6^2) = .4$

When the above values are graphed, we have a parabola.

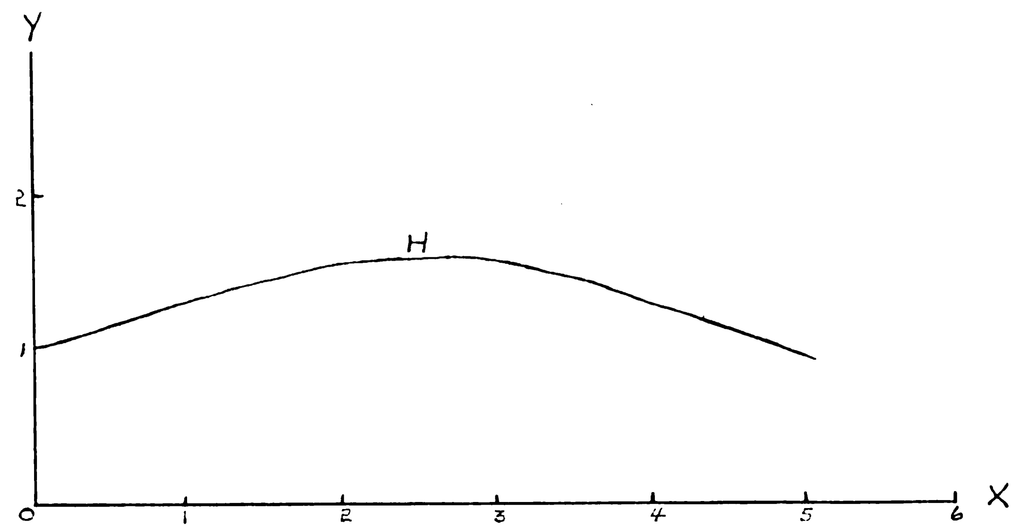

Figure 3. Graph of the function $Y = a + bX + cX^2$ with given values for a, b and X.

One of the characteristics of the curve in Figure 3 is that it is always symetrical on both sides with respect to the highest point (H in our case).

If the value of b were negative and the value of c were positive, the curve would be concave from above instead of convex and would be symetrical with respect to its lowest point.

This curve has great flexibility in that many other curves with different shapes can be represented by this parabola or by some of its segments. On the other hand, the parabolic shape is so simple that the real relationship between the variables may not be describable.

When log of Y is used instead of Y, our curves are mathematically modified. For example, a straight line function like $Y = a + bX$ can be transformed into a curvilinear form by using $\log Y = a + bX$.

In using $\log Y = a + bX + cX^2$ instead of 1) $Y = a + bX + cX^2$ the top of the bend is lengthened if b is positive. The botton of the dip flattens out if b is negative. When considering a cubic parabola the results are pretty much the same. The above logarithmic equation is graphed in Figure 4.

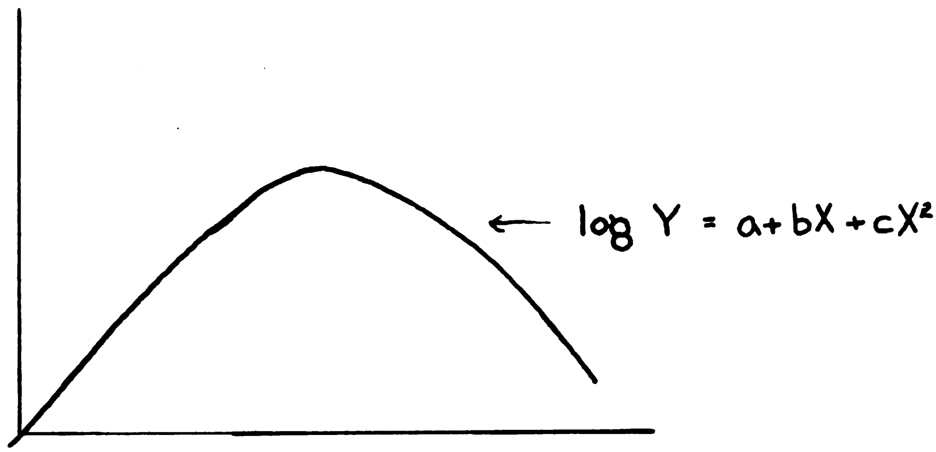

Figure 4. Graph of the function log $Y = a + bX + c\ X^2$ when $a = 0$, $b > 0$, $c < 0$.

If X is replaced by log X in the straight line formula $Y = a + bX$, we have $Y = a + b \log X$ that becomes convex from above if b is positive and concave from above when b is negative; this is shown in Figure 5.

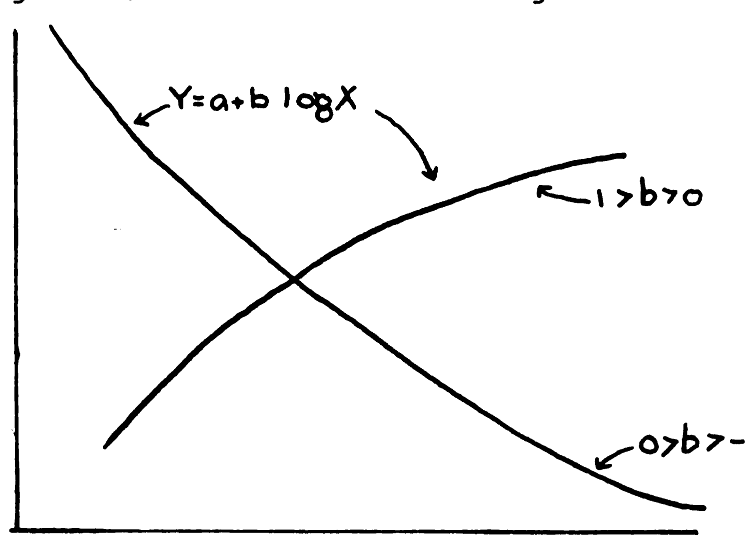

Figure 5. Graph of the function $Y = a + b \log X$.

A third case is found when logarithms are for both Y and X (the Cobb-Douglas case). The curve log Y = a + b log X is concave or convex when b is positive, being always concave from above if b is negative;[1] this can be seen in Figure 6.

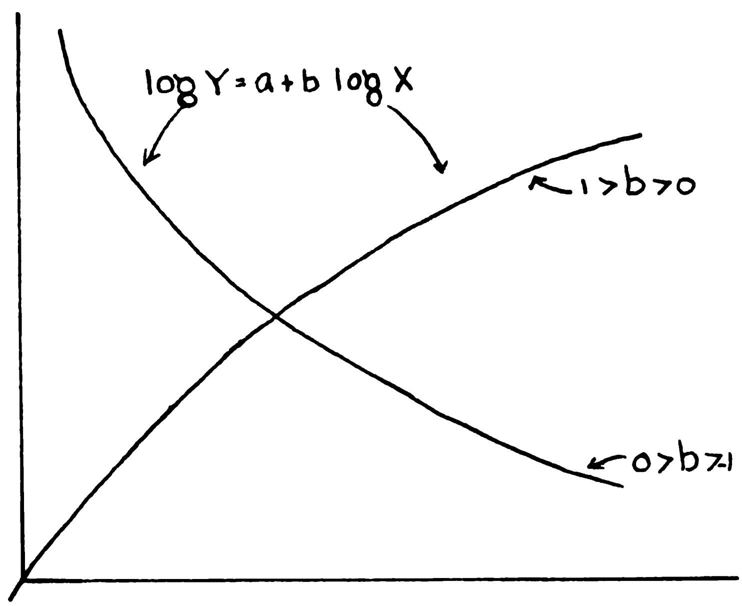

Figure 6. Graph of the function log Y = a + b log X when $b < 1$.

It can be stated that the curves described by logarithmic equations maintain certain characteristics similar to equations without logarithms. For example, a and b are constants in both forms.

[1]When $b > 1$ the curve is concave from above. When $b < 1$ the curve is convex from above and when $b = 1$ the curve becomes a straight line.

It can be stated that a formula with logarithms of observations can only be used when zero and negative values are absent from the observations. Unlike other functions which are able to show both positive and negative values, the logarithmic curves described by the formula tend to but do not necessarily become asymptotic to a constant value for Y as X approaches infinity. In other words, they tend to become parallel with the X axis for extremes positive values for X.

CHAPTER IV

CHARACTERISTICS OF THE DATA HERE USED

In this chapter an experiment on fertilizer use performed in Colombia will be described. In the first place, an ecological description will be drawn and then irrigation, plant population, fertilizer rates used and corn yields obtained will be presented.

The Colombian Experiment

The experiment on fertilizer use analyzed in this thesis was performed in the Cauca Valley, of Florida, Colombia, South America. Because of certain characteristics of the Cauca Valley, it seems worthwhile to discuss, in brief, some of its details.

It has been said that much of the Cauca Valley area is one of the most fertile pieces of land in Latin America comparable with the "Pampa" of Argentina or the Central Valley of Chile, to mention only a few other fertile lands.

The Cauca Valley is located in the southern part of Colombia, running between the western and central ranges of the Andes cordillera (mountains). Its total area is about .8 million acres (320,000 hectars) covering an area 100 miles long and 8 to 20 miles wide. This Valley is an old lake bed, fairly flat with some broad terraces and a general altitude of 3000 feet.

The weather conditions associated with wet and dry seasons, make it possible to obtain two crops, corn and beans. When irrigation is present, sugar cane is a permanent crop. The years are divided into nearly two periods each with 3 months dry and 3 months wet. About 40 inches (1000 mm) of precipitation is the annual average. The average temperature is around 75°F (24°C) in the wet periods and 77°F (25°C) in the dry months from July to September and January to March.

The above description provides a general background on the area in which the experiment here under analysis was performed.

In March of 1958, Lawton and Patiño[1] started an experiment to study the effects on corn yield of irrigation, number of plants per unit of area and different fertilizers. The soil chosen was a well drained, loam to clay loam with pH 6.5, with 3 to 4 percent organic matter, located two miles east of Florida. The available phosphorus was 16 Kgs.

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[1] Kirk Lawton, Ph.D., Professor of Soil Science at M.S.U.; Edgardo Patiño, M.Sc., Colombian agriculturist in Florida, Valle del Cauca, Colombia.

per hectar (about 14 lbs/acre) and 218 Kgs. of exchangeable potassium per hectar (about 194 lbs/acre).

The following variables were studied:

1) Irrigation.

Irrigation was given to half of the area where the experiment was conducted. The need for water was determined by a soil moisture test, plant appearance and the frequency of rainfall. The experimental plots were designed in such a way that water could not pass from irrigated to non-irrigated plots.

2) Plant populations.

Three level of plant populations were chosen: low, medium and high, or 11,200, 16,000 and 18,500 plants per acre respectively (about 27,500; 39,300 and 45,500 plants per hectar).

The corn seed used was a hybrid called Diaco H-203 (with yellow grain). The seed was sown four inches deep because moisture was scarce near the surface. Each plot had four rows 16.35 yards long separated by a 1 yard inter-lane (15 mts. x .90 mts.). One hundred twenty plots were treated including 6 check plots (3 with and 3 without irrigation).

3) <u>Fertilizer used</u>.

Three fertilizers were used: nitrogen, N; phosphorus, P; and potassium, K. The N was applied as sulfate of ammonium, the P as concentrated superphosphate and the K as muriate of potash.

The amounts used were 50, 100 and 150 kilos (kgs.) of N per hectar; 50 and 100 kilos of P_2O_5 per hectar and 50 kilos of K_2O per hectar. The combinations used were as follows:

Table 4. Use of N, P, and K on irrigated and non-irrigated experimental plots with three levels of plant population in the Cauca Valley of Florida, Colombia, 1958.

Number of plots	N (Kgs/Há)	P_2O_5 (Kgs/Há)	K_2O (Kgs/Há)	Irrigation: Yes + No -	Plant population per acre: 1 = 11,200 3 = 16,000 4 = 18,500
3	0	0	0	-	1,3,4
3	0	0	0	+	1,3,4
3	50	0	0	-	1,3,4
3	50	0	0	+	1,3,4
3	0	50	0	-	1,3,4
3	0	50	0	+	1,3,4
3	0	0	50	-	1,3,4
3	0	0	50	+	1,3,4
3	0	50	50	-	1,3,4
3	0	50	50	+	1,3,4

Table 4.--Continued.

Number of plots	N (Kgs/Há)	P_2O_5 (Kgs/Há)	K_2O (Kgs/Há)	Irrigation: Yes + No -	Plant population per acre: 1 = 11,200 3 = 16,000 4 = 18,500
3	50	50	0	-	1,3,4
3	50	50	0	+	1,3,4
3	50	0	50	-	1,3,4
3	50	0	50	+	1,3,4
3	50	50	50	-	1,3,4
3	50	50	50	+	1,3,4
3	100	0	0	-	1,3,4
3	100	0	0	+	1,3,4
3	0	100	0	-	1,3,4
3	0	100	0	+	1,3,4
3	100	100	0	-	1,3,4
3	100	100	0	+	1,3,4
3	100	0	50	-	1,3,4
3	100	0	50	+	1,3,4
3	0	100	50	-	1,3,4
3	0	100	50	+	1,3,4
3	100	100	50	-	1,3,4
3	100	100	50	+	1,3,4
3	150	0	0	-	1,3,4
3	150	0	0	+	1,3,4
3	150	50	0	-	1,3,4
3	150	50	0	+	1,3,4
3	150	0	50	-	1,3,4
3	150	0	50	+	1,3,4

Table 4.--Continued.

Number of plots	N (Kgs/Há)	P_2O_5 (Kgs/Há)	K_2O (Kgs/Há)	Irrigation: Yes + No -	Plant population per acre: 1 = 11,200 3 = 16,000 4 = 18,500
3	150	100	0	-	1,3,4
3	150	100	0	+	1,3,4
3	150	50	50	-	1,3,4
3	150	50	50	+	1,3,4
3	150	100	50	-	1,3,4
3	150	100	50	+	1,3,4
120 plots in total					

4) Corn yield.

The yield of each plot was computed by the researchers on the basis of shelled corn with 15 percent moisture.

When the corn was cropped without irrigation the highest average yield of 75.8 bu/acre (equivalent to 20.3 "cargas/fanegada"[1] or 4,757 Kgs/Há.) was obtained from the medium level plant population (16,000 plants/acre).

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[1]A special appendix is presented with the weight and measure equivalents used in this thesis.

With irrigation, the highest average yield of 94.8 bu/acre (25.4 cargas/fanegada or 5,953 Kgs/Há.) came again from the medium plant population.

In both cases, irrigated and non-irrigated, all the plots under experiment were considered.

CHAPTER V

FITTING THE DATA TO THE HYPOTHESIZED MODEL

In this chapter, data from the field experiment are fitted to the hypothesized model and the results evaluated from a statistical standpoint. The function used was an incomplete second degree polynomial.

A. Seeking the estimated production function.

In the present work, the hypothesized model is one which treats the yield of the corn crop as a function of the amount of fertilizer used (in this case N, P and K measured separately) irrigation and number of plants per acre.

In addition, the interaction effects of nitrogen and potassium (NK), nitrogen and number of plants per acre (NS) and phosphorus and number of plants per acre (PS) were also considered.

I^2 was not included because of the nature of the data, i.e., only two levels of I were used. It was decided not to study the interaction terms IN, IP and IK. Use of I^2 variable implies a curvilinear relation between I and Y_c. To test the hypothesis that a curvilinear relation existed between Y_c and I (by use of I^2) at least three levels of I were required. When only two points are drawn on a two-dimensional diagram, one and only one straight line of regression fits these data perfectly. No residual error is possible if the line fitted minimizes the squared unexplained residuals. However, an infinite number of vastly different curved lines of regression would also fit these two observations with no unexplained residuals. Thus, in order to obtain even a gross measure of the curvilinearity which may exist between two variables, at least three sets of observations are required. More would be desirable since the reliability would thus be increased.

NK, NS and PS were included because it was decided to study interaction effect of these variables. N, N^2, P, P^2, K, S, and S^2 were included in the analysis in order to study their effects on Y.

The final unspecified function used (two previous ones were tested) was of the following type:

$$\hat{Y} = f\ (N, P, K, N^2, P^2, NK, I, S, S^2, NS, PS)$$

The above formula was specified as a second degree polynomial in order to estimate the parameters as follows:

$$\hat{Y} = a + b_1N + b_2P + b_3K + b_4N^2 + b_5P^2 + b_6NK + b_7I + b_8S + b_9S^2 + b_{10}NS + b_{11}PS$$

Where:

$\hat{Y}$ = estimated yield

a = a constant, representing here a yield independent of the effect of the input variables

b_1 to b_{11} = parameters to be estimated

N, P, K, S and I = symbols for nitrogen, phosphorus, potassium, number of corn plants per acre, and irrigation, respectively.

With the help of the electronic computer (called MISTIC at Michigan State University), the a and b constants were calculated, giving the following results:

$$\hat{Y} = 35.1085 + \underset{(.0463)}{.047638\ N} + \underset{(.06365)*}{.247890\ P} + \underset{(.03926)}{.062468\ K}$$

$$- \underset{(.000254)}{.0001341\ N^2} - \underset{(.000542)*}{.001698\ P^2} - \underset{(.0004082)}{.00042\ NK} + \underset{(.6145)*}{9.17084\ I}$$

$$+ \underset{(3.356)*}{29.265\ S} - \underset{(.6637)*}{5.18672\ S^2} + \underset{(.00819)}{.00948\ NS} - \underset{(.01188)}{.009273\ PS}$$

* = significant difference from 0 at the .05 percent level of significance;

Numbers in parentheses are standard errors of the estimated coefficients.

B. Some statistical remarks.

For this experiment and equation, the variables N, P, K, S, I were associated with approximately 79 percent of the sample variance. This is shown by the coefficient of multiple determination, $R^2 = .79$.

The standard error of estimate was 6.73. This measures the error about the fitted regression line. In addition, the location of the regression line is subject to a related error, part of which is indicated by the standard error of the b's.

The coefficient of multiple determination corresponds to the square of the coefficient of multiple correlation; this latter coefficient may be measured by dividing the standard deviation of the estimated values by that of the original values.

By the standard t-test, the regression coefficients of P, P^2, I, S and S^2 were significantly different from 0 at the .05 percent level. For N, K, N^2, NK, NS and PS, the hypotheses that the coefficients were different from 0 was not accepted at the same level of significance. However, there is strong a priori evidence that changes in these terms are, in fact, associated with changes in crop yields.

The marginal physical productivity of an input **X** is found by taking the partial derivative of the production function with respect to that input. The reliability of this estimate of $MPP_{X(Y)}$ is, of course, influenced by the standard errors of the estimated coefficients contained in the partial derivative. However, the standard error associated with the estimated $MPP_{X(Y)}$ is some undefined (in this thesis) linear combination of the standard errors of the b_i's. Thus, we cannot conclude that non-significant b-coefficients imply non-significant $MPP_{X(Y)}$ estimates.

In this experiment, the following equation describes how expected yield changes with small change in N. Numbers in parentheses refer to associated standard errors of the coefficients:

$$MPP_{N(Y)} = \frac{\delta \hat{Y}}{\delta N} = \underset{(.0463)}{.0476} - \underset{(.000254)}{2(.000134)}N - \underset{(.000408)}{.00042}K + \underset{(.00819)}{.00948}\,S$$

The partial derivative of the production function with respect to P is:

$$MPP_{P(Y)} = \frac{\delta \hat{Y}}{\delta P} = \underset{(.06365)}{.24789} - \underset{(.000542)}{2(.001698)}P - \underset{(.01188)}{.009273}\,S$$

The partial derivative of the production function with respect to S is:

$$MPP_{S(Y)} = \frac{\delta \hat{Y}}{\delta S} = \underset{(3.356)}{29.265} - \underset{(.6637)}{2(5.18672)}S + \underset{(.00819)}{.00948}N - \underset{(.01188)}{.009273}P$$

It is impossible to make concrete statements regarding the significance of the above MPP estimates. Future work will be necessary in order to test the hypothesis that the individual MPP's are or are not significantly different than zero.

C. Analysis of the Residuals.

The residual figures are actual yields minus the predicted yields obtained from the polynomial formula presented above. The residuals $(Y - \hat{Y}) = u$ represent the effects of uncontrolled variables. The u's are assumed to be randomly and independently distributed in relation with the variables under study. Assuming these circumstances are met, the effects of the uncontrolled and unstudied variables which generate the u's can be averaged out with statistical procedures.

Other requirement is that the u's be small enough to make the estimates of Y_c usable.

Unexplained residuals in experimental data are themselves partial functions of uncontrolled variables such as between-plot variation in soils, insects, disease, experimental errors, hail, weeds, past soil treatment, etc.

It is important that experiments be designed to insure that unexplained residuals are reasonably random with respect to the inputs treated as experimental variables.

The factors that cause unexplained residuals can be divided into three main types:[1]

1) Errors in recording the data.

2) Those due to the omission of certain variables. This may be because the analyst failed to think of them, because no data were available or, perhaps, because they were so minor as not to be worth including in the study. This is the kind of random error normally assumed in a least squares analysis. It is the type of error allowed for in simultaneous-equation "shock" models.

3) Those resulting from the use of wrong types of curves, incorrect lags, and similar factors.

Johnson[2] relates the importance of the unexplained residuals and farmer's estimate of uncertainty as follows: "Both the size of unexplained residuals in experimental data and the correspondence between the causes of unexplained residuals under experimental and farm conditions are crucial as farmers form their subjective estimates of the uncertainty

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[1] USDA, Agricultural Handbook No. 146, Agricultural Marketing Service, 1958, p. 172.

[2] G. L. Johnson, "Discussion: Economic Implications of Agricultural Experiments," Journal of Farm Economics, XXXIX, (May, 1957), p. 394.

involved in using experimental results. Large subjective uncertainties relative to the objective uncertainties involved slow up adoption of experimental results unduly. Biased estimates of yields and of partial derivatives mislead farmers. Similarly, inaccurate adoption results if subjective uncertainty is less than objective uncertainty. The problem is to help bring a farmer's estimates of expected yields and the derivatives of uncertainty into line with those he actually faces."

The occurrence of u's may be reduced, in part at least, by (1) using procedures able to reduce errors in measuring X_j and Y_c, (2) better control on non-studied inputs and factors, and (3) randomization of the incidence of unstudied and uncontrolled variables in the experiment.

Finally, some assumptions[1] commonly made about unexplained residuals may be summarized as follows:

1. u's are random variables.
2. The variance of u's is constant over time.
3. The u's are normally distributed.
4. The u's are not correlated with any predetermined variable.

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[1]S. Valvanis, Econometrics, An Introduction to Maximum Likelihood Methods (New York: McGraw-Hill Book Co., Inc., 1959); also see p. 24 f. this thesis.

In order to study the characteristics of unexplained residuals for the experiment and fit here analyzed, a number of graphs were drawn. In these graphs the actual and the predicted yields at different levels of inputs were compared.

The result of this graphic analysis indicated that in the majority of cases observed no correlation existed between the residuals and the independent variables in the equation. The conclusion was that the functional form used in this analysis was adequate and that the statistical tests used above were valid.[1]

The actual and predicted yields plus the calculated residuals for each one of the 120 plots in the experiment are given in Appendix A.

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[1]Again, se p. 24 f. this thesis.

CHAPTER VI

ECONOMIC ANALYSIS OF THE DATA AND STATISTICAL ESTIMATES

Because one of the chief purposes of the farmer is to maximize profit on crops, one of the first answers that the farm economist can give to him on using a production function such as above is the profit maximizing amounts of fertilizer and plants per acre for different prices of corn, N, P, K and S.

In order to evaluate profit maximizing alternatives under varying price levels, seven different prices for corn, three different price levels for N and P, and four for S were set. The amount of K was maintained constant.

The combinations of prices used to compute high profit amounts of the inputs are shown in Table 5.

A. High profit point analysis under different price assumptions.

Knowledge of the input-output coefficients, such as those obtained from estimation of the parameters in the production function from the previous section, permits determination of optimum fertilizer inputs under varying assumptions regarding input-output price ratios.

Table 5. Different price levels of corn, N, P, and S; the amount of K being constant.

			P_Y(bu)	P_N(Kgs)	P_P(Kgs)	P_S(bu)	K(Kgs/Há)
			$	$	$	$	
	item	(1)	1.00	.068	.045	3.90	40
	"	(2)	1.10	"	"	"	"
	"	(3)	1.20	"	"	"	"
Vary	"	(4)	1.30	"	"	"	"
P_Y	"	(5)	1.40	"	"	"	"
	"	(6)	1.50	"	"	"	"
	"	(7)	1.60	"	"	"	"
	"	(8)	1.30	.054	.045	3.90	40
Vary	"	(9)	"	.068	"	"	"
P_N	"	(10)	"	.082	"	"	"
	"	(11)	1.30	.068	.036	3.90	40
Vary	"	(12)	"	"	.045	"	"
P_P	"	(13)	"	"	.054	"	"
	"	(14)	1.30	.068	.036	4.20	40
Vary	"	(15)	"	"	"	4.50	"
	"	(16)	"	"	"	4.80	"
P_S	"	(17)	"	"	"	5.10	"

Optimum inputs are determined by the profit maximizing principle:

$$\frac{MVP_{X_1}}{P_{X_1}} = \frac{MVP_{X_2}}{P_{X_2}} = \cdots = \frac{MVP_{X_i}}{P_{X_i}} = \cdots \frac{MVP_{X_m}}{P_{X_m}} = 1$$

where,

X_i = the X_ith input

MVP_{X_i} = the marginal value product of the X_ith input.

For expository purposes let us consider the profit maximizing principle given one variable input X in the production of a single commodity Y. The quantity of X at the HPP (high profit point) in this simple case is given by solving the following equation for X.

$$\frac{MVP_X}{P_X} = 1$$

The MVP_X in this case is given also by multiplying the marginal physical product, MPP, of a unit of X by the unit price of the product P_Y.[1] Hence, we may rewrite the above formula as:

$$\frac{MPP_X \cdot P_Y}{P_X} = 1 \text{ or } MPP_X = \frac{P_X}{P_Y} .$$

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[1]Providing that unit price of the product is constant throughout all levels of output.

In words, this principle says that maximum net profit is attained where the addition to total product forthcoming from an added unit of the X input is equal to the input-output price ratio.

For the case of several variables as shown previously, the quantities of the input at the HPP are given by equating $(MPP_{X_i}) \frac{P_Y}{P_{X_i}}$ to one, and solving simultaneously for the X_i's, where,

$i = 1, \ldots, m.$

Thus both price ratios and the marginal physical products derived from the production function are involved in estimating optima or high profit combinations and amount of inputs and of production.

Actually the procedure described above to determine the HPP levels of the several inputs in this study follows a mathematical principle. This principle says that given a functional equation in several variables having a maximum but no minimum in a specified range, the values of the several variables which maximize the functional value in that range can be determined by: 1) taking the partial derivatives of the function with respect to each independent variable; 2) setting these derivatives equal to zero and 3) solving

these equations simultaneously for the unknown values which fall in the stated range.

Thus, in order to maximize profits we may follow the procedure outlined above on the profit function which is

$$\pi = P_Y\, Y - P_N\, N - P_P\, P - P_S\, S - P_K\, K - P_I\, I - FC$$

for our polynomial function.

A change in N is known to affect the output Y by the amount given by the marginal physical product of N or $MPP_{N(Y)}$. Hence, the partial derivative of the profit equation with respect to N is:

$$\frac{\delta\pi}{\delta N} = P_Y\,(MPP_N) - P_N = P_Y\,[.047638 - 2\,(.000134)\,N - .00042\,K + .00948\,S] - P_N = 0$$

Similar reasoning leads to the two other equations:

$$\frac{\delta\pi}{\delta S} = P_Y\,(MPP_P) - P_p = P_Y\,[.24789 - 2\,(.001698)\,P - .009273\,S] - P_P = 0$$

$$\frac{\delta\pi}{\delta S} = P_Y\,(MPP_S) - P_S = P_Y\,[29.265 - 2\,(5.18672)\,S + .00948\,N - .009273\,P] - P_S = 0$$

These equations were set equal to zero and solved simultaneously for N, P and S with the use of MISTIC.[1] The several different sets of input-output prices presented on page 57 were used and the analysis rerun in each case. The results follow in Tables 6 and 7.

[1] Michigan State Integral Computer.

Table 6. HPP quantities of N, P and S, at various prices of corn P_Y and P_N (values for P_P, P_S and K are held fixed).

P_Y	P_N			
$ Bu	$ .054 (Kg)		$ .068 (Kg)	$ .082 (Kg)
1.00	-	item (1)	* - 56 N 53 P 2.346 S	-
1.10	-	item (2)	- 31 N 54 P 2.402 S	-
1.20	-	item (3)	- 10 55 2.499	-
1.30	* 49 (8) 56 2.527	items (4) = (9)	8 56 2.489	* - 34 (10) 56 2.451
1.40	-	(5)	23 56.6 2.523	-
1.50	-	(6)	36 58 2.554	-
1.60	-	(7)	48 58 2.578	-

P_P = $ .045

P_S = $ 3.90

K = 40 Kgs/Há.

* = HPP's quantities with data given in Table 5 from the different items ( ).

1) <u>The effect of various prices of nitrogen, P_n, and of P_Y, on optimum quantities of N to use.</u>

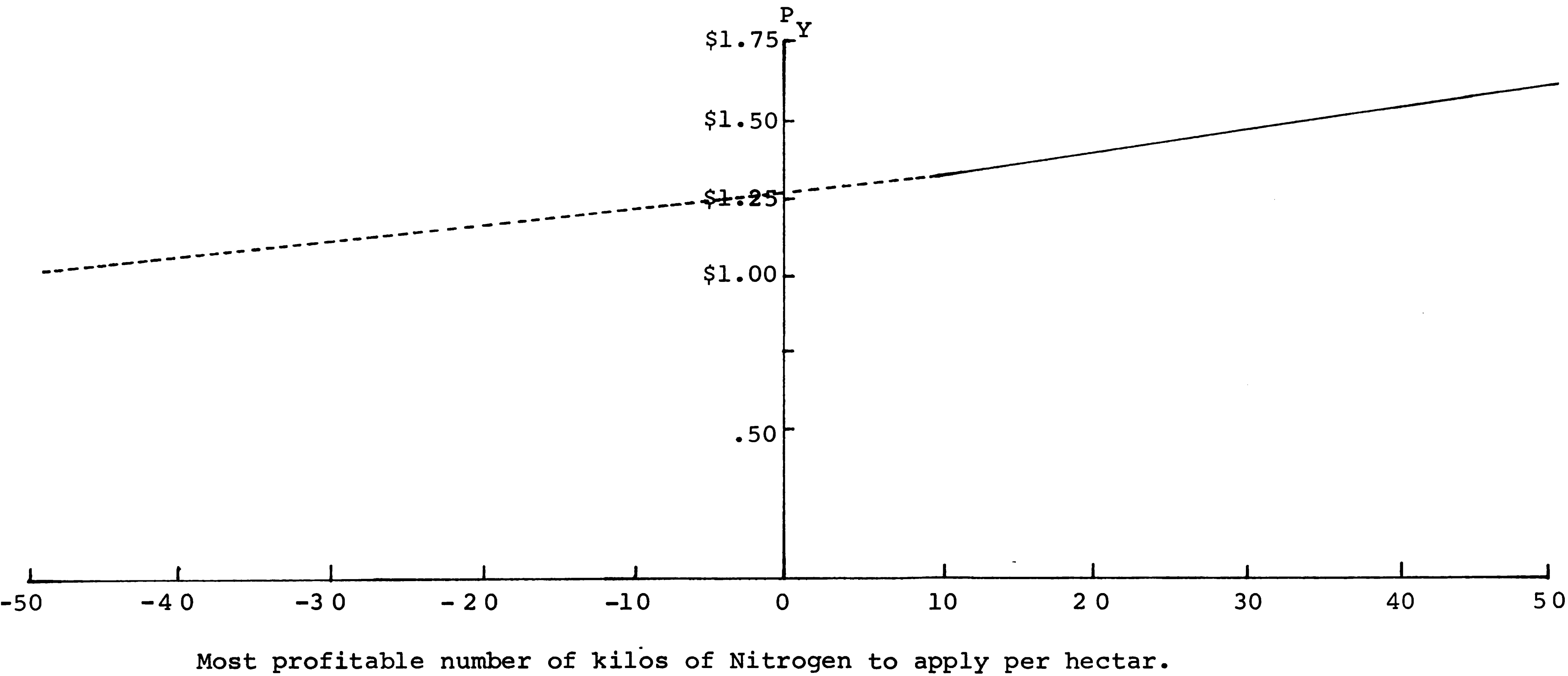
From Table 6 we can see that three price levels of N have been chosen: \$ .054 Kg., \$ .068 Kg., and \$ .082 Kg.

Here the prices of corn, P_Y, take on values of \$1.00; \$1.10; \$1.20; \$1.30; \$1.40; \$1.50 and \$1.60 per Bu. Fixed values were given for P_P at \$ .045 Kg.; P_S at \$3.90 Bu. and K at 40 Kgs/Há.

From Table 6, it is possible to observe that when P_N is maintained at \$ .068 Kg., with P_Y running from \$1.00 to \$1.20, the use of N was not profitable and only negative N's quantities were obtained as HPP (- 56; - 31 and - 10 Kgs/Há). However, when P_Y became \$1.30 per bushel, the use of 8 Kgs/Há. of N was most profitable.

When P_Y = \$1.40 Bu., 23 Kgs/Há. of N was most profitable;
" " = \$1.50 " , 36 " " " "
" " = \$1.60 " , 48 " " " " .

Thus, between corn prices of \$1.20 and \$1.60, for every increase of \$ .05 per bushel of corn, profits would have been maximized by increasing nitrogen applications about 6.5 Kgs/Há. providing prices of other inputs remained fixed at the levels shown. These results can be visualized in Figure 7.



When:

$P_N = \$ .068$
$P_P = .045$
$P_S = 3.90$
$K = 40$ Kgs/Há.

Figure 7. Effects of various P_Y on optimum levels of N.

2) The effect of various prices of nitrogen, P_N, on optimum quantities of N to use.

If P_Y is maintained at $1.30 per Bu. while P_P, P_S and K remain fixed as above, but P_N is changed, the following results are observed:

When P_N = $ .054 Kg., 49 Kgs/Há. of N was most profitable;
" " = .068 " , 8 " " " " ;
" " = .082 " , the most profitable amount of N to use becomes negative, - 34 Kgs/Há. In other words, at the high P_N, application of N is not recommended for economic reasons. The decrease of the amount of N which maximizes profits is consistent with increase of P_N. In general, this means that for an increase in P_N of $ .05 the quantity of N should be decreased around 15 Kgs/Há. in order to maximize profits. In Figure 8 this result is shown.

3) The effect of various prices of corn, P_Y, on optimum quantities of P to use.

From Table 6, we observe the effects of changes in price of P_Y the optimum quantity of P to use when P_P is fixed at $ .045 Kg/Há. It can be noticed that the optimum quantity of P per Há. goes up when P_Y goes up from $1.00 to $1.60; however, when P_Y is $1.30 and $1.40, the resulting

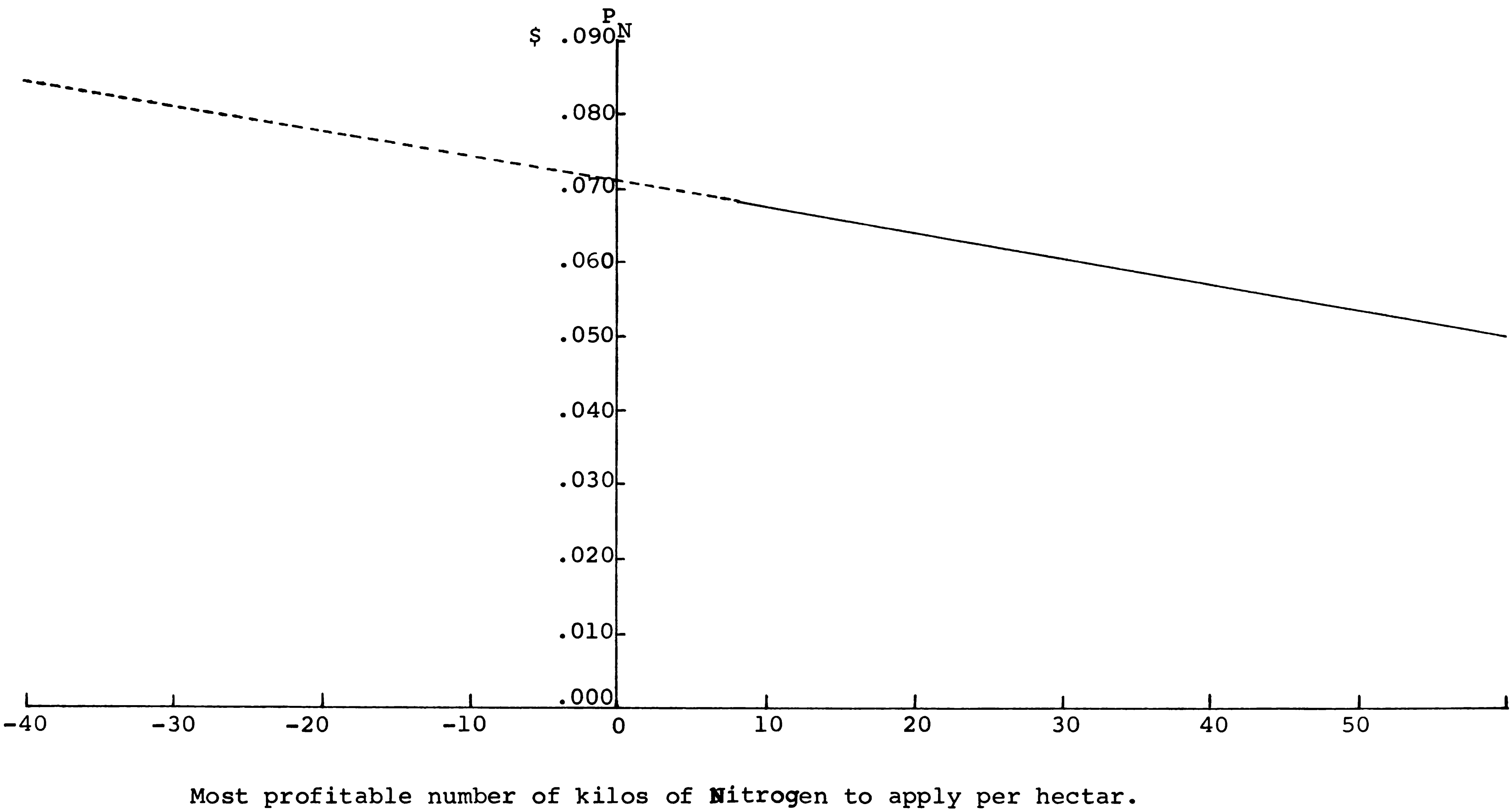


Figure 8. Effects of various P_N on optimum levels of N.

When:

P_Y = \$ 1.30

P_P = .045

P_S = 3.90

K = 40 Kgs/Há.

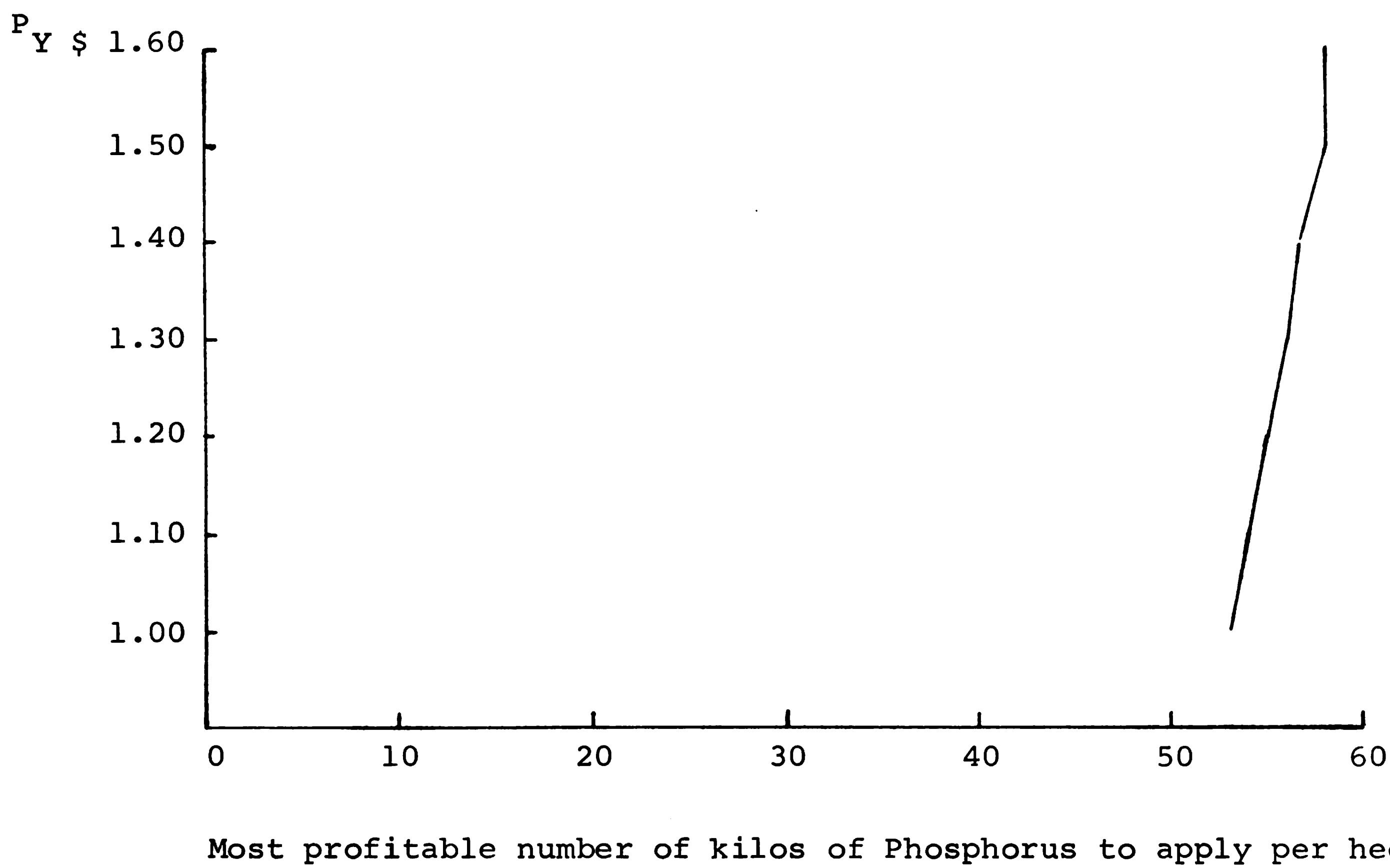
optimum amounts of P is 56 and 56.6 Kgs/Há., respectively. When P_Y is \$1.50 and \$1.60 the quantity of P use remains at 58 Kgs/Há. Figure 9 shows this result.

4) The effect of various prices of corn, P_Y, on optimum quantities of seed, S, to use.

In regard to the different number of plants per acre, different amounts were found to be associated with the HPP's for the various price levels of P_Y, P_N, P_P and P_S. This is shown in Table 7.

Table 7. High profit number of plants per acre at various prices of corn and with P_N, P_P, P_S and amount of K held at specified levels.

Coded plants/ acre	Actual plants/ acre	P_Y \$ per Bu.	P_N \$ per Kg.	P_P \$ per Kg.	P_S \$ per Bu.	K Kgs/ acre
2.346	16,865	1.00	.068	.045	3.90	40
2.402	17,005	1.10	"	"	"	"
2.449	17,123	1.20	"	"	"	"
2.489	17,223	1.30	"	"	"	"
2.523	17,308	1.40	"	"	"	"
2.554	17,385	1.50	"	"	"	"
2.578	17,445	1.60	"	"	"	"
2.527	17,317	1.30	.054	.045	3.90	40
2.451	17,127	1.30	.082	.045	3.90	40
2.457	17,143	1.30	.068	.036	4.20	40
2.435	17,087	"	"	"	4.50	"
2.408	17,020	"	"	"	4.80	"
2.386	16,965	"	"	"	5.10	"



When:

P_N = $ .068

P_P = .045

P_S = 3.90

K = 40 Kgs/Há.

Figure 9. Effects of various P_Y on optimum levels of P.

The original data were coded as follows:

S = 1 = 11,200 plants/acre;

S = 3 = 16,000 " ;

S = 4 = 18,500 " .

The optimum quantity of S to use goes up when P_Y increases from \$1.00 to \$1.60 Bu. Taking into consideration the different values of S from Table 7, we obtain a range of 16,865 to 17,445 plants per acre.

5) <u>The effect of various prices of seed, P_S, on optimum quantities of N, P and S to use</u>.

Four price levels (\$4.20, \$4.50, \$4.80 and \$5.10 Bu) of hybrid seed corn were used. The result can be expressed as follows: When P_S is varied under the assumed conditions all figures for N are negative suggesting that no N be used; under other assumptions positive amounts of N would have been profitable. When the price of seed went up, the optimum number of plants per acre decreased.

B. <u>Comparison of some predicted yields, $\hat{Y}$, with and without irrigation under varying prices of corn and fixed prices of N, P, S, and a fixed quantity of K.</u>

In order to obtain some predicted yields, $\hat{Y}$, from given prices and quantities of fertilizers, S, P_Y and K, a number of computations were worked out with the help of the estimated production function given on page 54. As this production function gives the value of Y under irrigation, in order to get the result without irrigation a simple transformation was made as follows:

$$\hat{Y} = (a \pm 9.17084I) + .047638N + .247890P + .062468K - .0001341N^2 - .001698P^2 - .00042NK + 29.265S - 5.18672S^2 + .00948NS - .009273PS.$$

where

$I = +1$, irrigation

$I = -1$, no irrigation.

1) The first computation of $\hat{Y}$ was worked out using price data from Table 5, item (1) and using quantity data from Table 6, item (1).

Given:	Using HPP quantities:
P_N = \$ .068 Kg.	-56N
P_P = \$ .045 "	53P
P_S = \$3.90 Bu.	2.346S
P_Y = \$1.00 Bu.	
K = 40 Kgs/Há.	

we obtain,

$\hat{Y}$ = 90.710 Bu/acre with irrigation

$\hat{Y}$ = 72.370 " with no irrigation.

It can be observed that the effect of irrigation is positive giving a higher $\hat{Y}$. Unfortunately the net profit from this increase of about 20 percent in yield cannot be compared with that for non-irrigated yield because of the lack of cost data on irrigation. However, the difference in total revenue between irrigation and no irrigation can be determined by computing $Y \cdot P_Y$. This difference is the amount that is available to cover the cost of irrigation and, in this example, amounts to \$18.34.

2) If we take P_Y at \$1.10 Bu. but P_N, P_P, P_S and K are fixed as above, the HPP quantities of fertilizer given in Table 6, item (2), are as follows:

$N = -31$; $P = 54$ and $S = 2.402$.

With this information we have the following result:

$\hat{Y} = 93.917$ Bu/acre with irrigation

$\hat{Y} = 75.577$ " with no irrigation.

Again the use of irrigation gives a higher yield of corn with our estimated production function.

3) When P_Y moves to \$1.20, other things equal as 2) above, from Table 6, item (3) we get:

$N = -10$; $P = 55$ and $S = 2.449$.

The predicted yields are:

$\hat{Y}$ = 94.034 Bu/acre with irrigation

$\hat{Y}$ = 75.694 " with no irrigation.

4) If P_Y goes to \$1.30, other things equal as in 2) above, from Table 6, item (4) and (9) we obtain:

N = 8; P = 56 and S = 2.489.

The predicted yields were:

$\hat{Y}$ = 95.177 Bu/acre with irrigation

$\hat{Y}$ = 76.837 " with no irrigation.

5) When P_Y is \$1.40, from Table 6 item (5) we get:

N = 23; P = 56.6 and S = 2.523.

The predicted yields are:

$\hat{Y}$ = 96.032 Bu/acre with irrigation

$\hat{Y}$ = 77.692 " with no irrigation.

6) At P_Y \$1.50, N = 36; P = 58 and S = 2.554.

The associated predicted yields were:

$\hat{Y}$ = 96.788 Bu/acre with irrigation

$\hat{Y}$ = 78.448 " with no irrigation.

7) If P_Y is \$1.60, the highest price that we have assumed in this thesis for corn, from Table 6, item (7) we obtain:

N = 48; P = 58 and S = 2.578.

The predicted yields obtained are:

$\hat{Y}$ = 97.375 Bu/acre with irrigation

$\hat{Y}$ = 79.035 " with no irrigation.

The recommended amounts of fertilizer increases with the increases of P_Y. The same thing is true with the recommended quantity of S.

CHAPTER VII

EVALUATION, RESULTS AND IMPLICATIONS

A. Evaluation.

In the first place, a general evaluation of "goodness" of the experiment here analyzed shows that it was well done. For the purposes of this experiment, 120 plots were considered adequate. Three replications of each treatment were performed. However, it is necessary to keep in mind that this experiment contains observations from only one particular year, for a particular type of soil under particular environmental conditions.

The experiment, even with the limitations listed above, is a very promising starting point. In addition, the experience obtained will permit still better future experiments to be planned. Future work should involve more fertilizer combinations and levels of irrigation to take into account the further problems of the area. With a more complete set of data it would be possible to use more flexible functions to give better answers to a number of fertilizing problems.

It should be recognized that those elements "fixed" in an experiment (such as soil type, management of the soil and slope) may not be controllable by the farmer. Important barriers between plot experiment and commercial farms include (1) the difference in levels at which controlled variables are fixed and (2) the exercise of controls by experimenters which cannot be maintained by farmers. Unique characteristics sometimes associated with an experiment can be very important in determining the results from an experiment. On applying such results to a large number of farms, several reservations would be required. Though reduction of variance in experimental results is highly desirable, the experimental situation should be made similar to those on the farms expected to use the results.

When the characteristics of the unexplained residuals for the experiment here analyzed were studied graphically, no correlation was revealed between residuals and the independent variables in the equation. Under these circumstances, it was possible to assume that the function used fitted well enough to justify use of the common statistical tests.

From a general statistical viewpoint, the experiment has shown a high coefficient of multiple determination ($R^2 = .79$) that can be taken as a measure of confidence in the application of results. The standard error of predicted yield (6.73) can be considered satisfactory and the average yield of 76.87 Bu/acre is a realistic result. Here again, the limitations discussed above are important to consider if an extension program is eventually worked out.

B. Results.

Coming now to the results, the optimum quantities of fertilizer and plants per acre used were found to increase when price of corn was increased. However, when the fertilizer and seed prices also increase different answers were obtained:

a) When the price of nitrogen went up the optimum quantity of N to use decreased. In general, it was estimated that for an increase in P_N of \$ .05, the optimum amount of N which should have been used to maximize profits decreased about 13.3 lbs/acre.

b) When the price of seed corn went up the number of plants per acre, S, tended to decrease. However, the influence of changing seeding rates on profits was slight and of little practical importance so long as around 17,000 plants were maintained on an acre. It may be added here that "better seed corn" is much more important than changes in corn seed prices as seed costs per acre are low.

The predicted optimum yields when P_N, P_P, P_S and K were fixed, but P_Y was increasing, increased, as indicated above, while the optimum quantities of N, P and S increased. This was shown in Chapter VI, item B. In more detail, computations indicated that if the price of corn increases, while fertilizers and seed prices are fixed, the optimum amount to use of these imputs should be increased in order to obtain the higher most profitable yields. For example, when corn price, P_Y, was \$1.00 Bu., and the HPP quantities for N, P and S were - 49.8 lbs/acre, 47.2 lbs/acre and 16,865 plants/acre, respectively, other things equal; this results assumes P_N = \$ .068, P_P = \$ .045, and P_S = \$3.90. The predicted corn yield was 90.710 and 72.370 Bu/acre with irrigation and non-irrigation, respectively. But when P_Y increased to \$1.60 Bu., HPP quantities were 42.7 lbs/acre for N, 51.6 lbs/acre for P and 17,445 plants/acre; the predicted corn yield was 97.375 and 79.035 Bu/acre for irrigation and non-irrigation, respectively.

If we remember, for instance, that the best corn yield reported from the experiment results was obtained from medium level plant population (16,000 plants/acre), for both irrigated and non-irrigated plots, we can say that farmers still have wide possibilities for higher yields by increasing the amount of plants per acre from around 14,000 plants per acre, which is common in Colombia, almost regardless of seed corn prices.

Because the experiment was performed at only two levels of irrigation, any statistical inference regarding changing marginal returns to irrigation would be inaccurate. Increased yields were obtained on irrigation plots. However, three or more irrigation levels are needed to measure the marginal influence of irrigation on yield, fertilizer-irrigation interaction and hence, optimum rates of irrigation.

C. Implications.

In general for Colombia and the rest of South America, more experiments such as analyzed would help farmers find the most profitable combinations and amounts of fertilizer to apply in producing different crops under varying price levels. In addition such experiments have methodological value. These methodological values direct researchers in attaining better estimates to help farmers maximize profits. The development of workable solutions to practical problems is one of the ultimate goals of fundamental researchers. Fundamental research dealing with more efficient use of fertilizer may provide, eventually, the best help on practical recommendations for individual farmers.

In Latin America, the fertilizer information received by farmers often indicates that the best fertilization level is that at which the highest yield per hectar is obtained. It is almost forgotten that maximum profits obtained from a given fertilizer application are seldom found at the highest yield. It seems highly desirable that more emphasis be placed in profit maximization than on maximizing yields. The data here analyzed have shown that in the particular year under study, the estimated HPP quantities of fertilizers varied with prices and exceeded common rates of application.

Although one year's data are limited, experiments over a longer period would remedy this difficulty.

Fertilizer recommendations should be based on experimental data for a period of years. Such data would permit an average production function to be derived which would average out between year variations. It would also be possible to estimate probable deviations from expected returns as well as expected deviations from the recommended amounts of fertilizer to use as it has been shown above. With this information farmers may adjust fertilization programs in relation with particular capital levels in order to minimize the risk and uncertainty involved.

Similar conclusions may be drawn concerning irrigation data. In this study irrigation plots have proved much more productive than non-irrigated plots.

It seems important to stress that the economic optimum conditions are related both to physical function relationship and the input-output prices in a particular period of time. Input-output price changes implies a new optimum amounts and combinations of fertilizer inputs to use. This point has a practical implication when recommendations are being extended to farmers.

From the point of view of farm planning, it sounds logical to suggest that a farmer attempting to make the best use of limited resources in spending money on fertilizers, should use fertilizer inputs until a point is reached where a greater return can no longer be obtained from fertilizer than elsewhere in the business. If it were possible to obtain reliable information on the returns farmers in the Cauca Valley are earning from other than fertilizer inputs in their businesses, the opportunity cost principle could be used in making marginal productivities comparisons to maximize profits. Still better decisions could then be reached. A well designed farm management survey or record keeping system would be suitable means of obtaining some of the necessary information on farm business returns.

It should also be remembered that general management levels seems important in determining economic use of fertilizer. "Intangible as management measures are, fertilizer is apparently more productive under superior management which includes efficiency in timeliness of operations, choice of varieties, and other recommended cultural practices. Increased use of fertilizer is most effective on many farms only if improved cultural practices are used at the same time. Management considerations also involve adjustments to risk."[1]

The use of fertilizer is badly needed in most of the Latin American farms. The research reported here on levels of fertilization to maximize profits should be of great interest among farmers, providing these kinds of results can be transmitted in such a way that farmers can understand their real meaning and benefit.

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[1]R. C. Woodworth, "Organizing Fertilizer Input-Output Data in Farm Planning," Economic Analysis of Fertilizer Use Data, edited by Baum and others (Ames, Iowa, 1956), p. 158.

APPENDIX A

Fertilizer use experiment, Florida, Colombia, 1958: actual yields, predicted yields and calculated residuals.

Number of plot	Actual yield Y	Predicted yield $\hat{Y}$	Calculated residual u
1	57.6	50.015	+ 1.584
2	66.5	67.052	- .552
3	47.8	60.010	-12.210
4	59.0	68.357	- 9.357
5	86.0	85.393	+ .606
6	75.0	78.351	- 3.351
7	53.9	52.536	+ 1.363
8	68.3	70.520	- 2.220
9	63.0	63.952	- .952
10	69.7	70.878	- 1.178
11	102.0	88.862	+13.137
12	90.5	82.294	+ 8.205
13	63.6	57.700	+ 5.899
14	77.0	73.809	+ 3.190
15	66.0	66.303	- .303
16	67.2	76.042	- 8.842
17	78.3	92.151	-13.851
18	93.7	84.645	+ 9.054
19	61.7	53.139	+ 8.560
20	73.2	70.175	+ 3.024

Appendix A.--Continued

Number of plots	Actual yield Y	Predicted yield $\hat{Y}$	Calculated residual u
21	65.4	63.133	+ 2.266
22	58.5	71.481	-12.981
23	90.3	88.517	+ 1.782
24	92.3	81.475	+10.824
25	65.1	60.823	+ 4.276
26	83.0	76.932	+ 6.067
27	67.9	69.427	- 1.527
28	77.4	79.165	- 1.765
29	95.0	95.274	- .274
30	84.3	87.768	- 3.468
31	63.6	60.221	+ 3.378
32	72.1	77.278	- 5.178
33	55.1	70.246	-14.746
34	76.3	78.562	- 2.262
35	109.8	95.619	+14.180
36	93.7	88.588	+ 5.111
37	55.1	54.609	+ .490
38	72.3	72.594	- .294
39	58.5	66.026	- 7.526
40	74.4	72.951	+ 1.448
41	88.2	90.935	- 2.735
42	90.0	84.367	+ 5.632
43	54.8	62.294	- 7.494
44	83.6	79.351	+ 4.248
45	88.2	72.319	+15.880
46	75.7	80.636	- 4.936

Appendix A.--Continued

Number of plots	Actual yield Y	Predicted yield $\hat{Y}$	Calculated residual u
47	92.6	97.693	- 5.093
48	94.2	90.661	+ 3.538
49	64.5	54.386	+10.113
50	73.0	73.318	- .318
51	55.4	67.224	-11.824
52	65.2	72.728	- 7.528
53	91.3	91.660	- .360
54	85.2	85.566	- .366
55	68.0	56.892	+11.107
56	78.1	72.074	+ 6.025
57	50.8	64.105	-13.305
58	73.3	75.234	- 1.934
59	94.3	90.416	+ 3.883
60	97.1	82.446	+14.653
61	61.1	61.263	- .163
62	72.9	78.341	- 5.441
63	73.1	71.319	+ 1.780
64	74.1	79.605	- 5.505
65	95.5	96.682	- 1.182
66	95.9	89.661	+ 6.238
67	46.6	55.410	- 8.810
68	71.7	74.342	- 2.642
69	70.2	68.248	+ 1.951
70	74.2	73.751	+ .448
71	96.8	92.683	+ 4.116
72	82.7	86.586	- 3.889

Appendix A.--Continued

Number of plots	Actual yield Y	Predicted yield $\hat{Y}$	Calculated residual u
73	75.1	60.016	+15.083
74	79.0	75.197	+ 3.802
75	60.2	67.228	- 7.028
76	70.0	78.357	- 8.357
77	88.7	93.539	- 4.839
78	70.5	85.570	-15.070
79	63.1	62.286	+ .813
80	78.4	79.364	- .964
81	71.9	72.343	- .443
82	83.0	80.628	+ 2.371
83	96.3	97.706	- 1.406
84	91.7	90.684	+ 1.015
85	62.8	55.566	+ 7.233
86	71.5	75.446	- 3.946
87	57.2	69.826	-12.626
88	80.6	73.908	+ 6.691
89	99.5	93.788	+ 5.711
90	91.5	88.168	+ 3.131
91	65.1	63.251	+ 1.848
92	77.0	82.203	- 5.203
93	79.0	76.120	+ 2.879
94	78.3	81.592	- 3.292
95	94.0	100.545	- 6.545
96	100.0	94.461	+ 5.538
97	63.9	55.539	+ 8.360
98	70.8	75.419	- 4.619

Appendix A.--Continued

Number of plots	Actual yield Y	Predicted yield $\hat{Y}$	Calculated residual u
99	70.4	69.799	+ .600
100	75.2	73.881	+ 1.318
101	89.0	93.761	- 4.761
102	94.6	88.141	+ 6.458
103	59.0	62.443	- 3.443
104	81.1	80.468	+ .631
105	70.9	73.921	- 3.021
106	70.2	80.785	- 1.585
107	109.0	98.810	+10.189
108	91.5	92.263	- .763
109	63.0	63.224	- .224
110	80.4	82.177	- 1.777
111	82.5	76.093	+ 6.406
112	78.9	81.566	- 2.666
113	100.6	100.518	+ .081
114	92.3	94.435	- 2.135
115	63.1	62.416	+ .683
116	81.1	80.442	+ .657
117	72.5	73.894	- 1.394
118	80.0	80.758	- .758
119	91.7	98.783	- 7.083
120	97.0	92.236	+ 4.763

APPENDIX B

Weights and measures: metric equivalents used in this thesis.

1 centimeter	=	0.3937	inches
1 meter	=	39.37	inches
1 hectar (Há)	=	2.47	acres
1 acre	=	0.4047	hectar
1 sq. kilometer	=	0.386	sq. mile
1 sq. mile	=	2.59	sq. kilometer (Km.)
1 sq. hectar	=	10,000	sq. meters
1 bushel (Bu)	=	0.3524	hectoliter
1 hectoliter	=	2.8375	Bu.
1 kilógramo (Kg.)	=	2.2046	lbs.
1 pound (lb.)	=	0.4536	Kg.
1 "carga"	=	150	Kgs.
1 "fanegada"	=	6,400	sq. meters

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