THE DESIGN OF A RADIO FREQUENCY MECHANICAL FILTER

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This is to certify that the

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THE DESIGN OF A RADIO FREQUENCY MECHANICAL FILTER

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I. INTRODUCTION

One of the first requirements of a good quality commercial, military or amateur communications receiver is a high degree of selectivity. Ordinary receivers generally obtain their selectivity by cascading several intermediate frequency tuned transformer stages, while the better receivers incorporate the much more expensive crystal type bandpass filter. It is doubtful whether the crystal type filter can be surpassed in performance but the high cost of such a filter may make its use impractical.

In recent years interest has been developing around mechanical type filters that will perform almost as good and yet cost considerably less than the crystal filter. This type of filter is composed of a chain of resonators made of a suitable high Q metal with non-resonant metal couplers joining adjacent resonators. The filter is driven electro-mechanically by magnetostriction and is terminated in a similar fashion.

It is the object of this thesis to discuss the analogous relationships between the mechanical filter and a chain of electrical transmission lines; to elaborate on a method suggested by Burns and Roberts[1] for the design of a mechanical filter; to show how methods now in use for the design of transmission line micro-wave filters might be employed, and how the filter can be designed to give the so-called Chebishev response plus several actual filter design examples.

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[1]W.V.B. Roberts and L.L. Burns, Jr., "Mechanical Filters for Radio Frequencies," RCA Rev., Vol. 10, Sept., 1949, pp. 348 - 365.

II. THE TRANSMISSION LINE ANALOGY

A. The Filter Structure

The analysis of a mechanical vibratory circuit would in general involve motion in any one or all three dimensions. Compared to the analysis of a simple electrical circuit with only one independent variable, such as the current, the former could become extremely complex. Due to the endless number of different shapes and modes of vibration possible with a mechanical structure, an endless number of different structures must also exist which could produce filter characteristics.

In order to simplify the analysis as much as possible and to be able to incorporate various electrical-mechanical analogies, only one mode of vibration will be considered for the filter structure; also, only a structure whose symmetry and dimensions are such that one mode of vibration results will be considered.

The type of structure to be discussed in this paper is undoubtedly one of the simplest as far as analysis is concerned. This structure is composed of cylindrical rods, which will be called the resonators, coupled together by cylindrical rods of smaller diameter which may or may not be of the same material as the resonators. These will be referred to as the coupling necks or simply couplers. A diagram of one section of the structure is shown in figure 1.

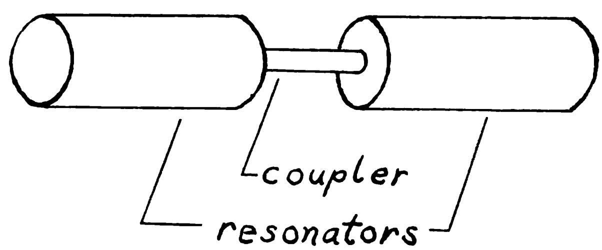
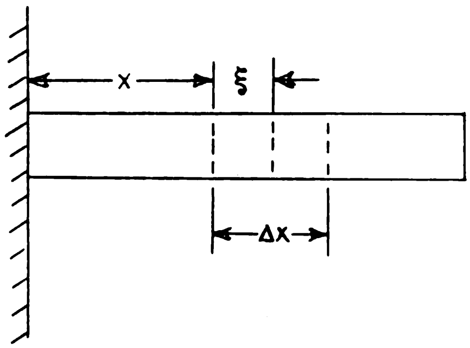


Figure 1.

Due to the relatively slow velocity of radio frequency vibratory waves in the system, it will be necessary to utilize the concept of distributed parameters as in the analysis of high frequency transmission lines. Since it is already known how to design bandpass transmission line filters, it would be convenient to determine if the mechanical structure could be considered an analogous system and hence use the same better known concepts.

B. Longitudinal Mode of Vibration

Consider the vibration of a resonator alone, vibrating in only the longitudinal mode. The situation is shown in figure 2.



x - distance to cross section at rest
ξ - displacement of section from rest position
T - tension on section
S - area of section
e - d /dx = the elongation
E - Young's Modulus
ρ - mass density

Figure 2.

The equation of motion can be easily found as follows.[2] Applying Hooke's Law to the slice bounded by x and $x+\Delta x$ we have

$$E = \frac{\text{stress}}{\text{strain}} = \frac{T_1/s}{d\xi/dx}$$

and $T = SE(d\xi/dx)$, the tension at x.

The tension at $x+\Delta x$ is given by the following equation.

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[2] B. R. Hubbard, "Longitudinal Vibrations in a Loaded Rod," Journ. Accoust. Soc. of Amer., Vol. 2, 1931, p. 372.

$$T_2 = T_1 + d/dx\ (SE\ d\xi/dx)\Delta x$$

or

$$T_2 = SE\ (d\xi/dx + d^2\xi/dx^2 \Delta x)$$

The accelerating force on the slice is T_a $T_2 - T_1$, or

$$T_a = SE\ (d^2\xi/dx^2)\Delta x\ .$$

Equating this to the newtonian equation for the slice gives,

$$SE\ (d^2\xi/dx^2)\Delta x = (\rho S\ x)\ d^2\xi/dt^2, \text{ or}$$

$$d^2\xi/dt^2 = E/\rho\ (d^2\xi/dx^2)\ . \qquad (1)$$

The resonant frequency of the resonator is found to be,

$$f_o = v/2l \qquad (2)$$

where: $v = \sqrt{E/\rho}$ = velocity of longitudinal waves

l = length of the resonator

Equation (1) is exactly like the familiar transmission line wave equation and $\sqrt{E/\rho}$ like $1/\sqrt{LC}$ represents the wave velocity through the medium. By merely extending the well known electrical-mechanical analogies to the case of distributed parameters, the relations in table 1 are obtained.

Electrical		Mechanical	
L	Induc./unit lgth	ρS	Mass/unit lgth
$1/C$	Rec.Cap./unit lgth	ES	Elastance/unit lgth
$1/\sqrt{LC}$	Wave Velocity	$\sqrt{E/\rho}$	Wave Velocity
$\sqrt{L/C}$	Char. Impedance	$S\sqrt{E\rho}$	Char. Impedance
		$\sqrt{E\rho}$	Intrinsic Impedance

Table 1. Electrical - mechanical transmission line analogies for longitudinal vibration.

C. Torsional Mode of Vibration

If the resonator is made to vibrate in the torsional mode, then the analogies for a torsional mechanical system and an electrical circuit can be extended as shown in table 2.

Electrical		Mechanical	
L	Induc./unit lgth	I	Mom.ofIner./unit lgth
$1/C$	Rec.Cap./unit lgth	GJ	Rigidity/unit lgth
$1/\sqrt{LC}$	Wave Velocity	$\sqrt{G/\rho}$	Torsion Wave Velocity
$\sqrt{L/C}$	Char. Impedance	$J\sqrt{G\rho}$	Char. Impedance
		$\sqrt{G\rho}$	Intrinsic Impedance

Table 2. Electrical - mechanical transmission line analogies for torsional vibration.

G in table 2 stands for the shear modulus of elasticity for the material being used. Also, in reference to table 2, for circular cross sections:

$$J = \pi D^4/32 = \text{polar moment of inertia of the cross sectional area}$$

$$I = \rho \pi D^4/32 = J\rho$$

It may happen that the value for G is not known but the value of Poisson's Ratio for the material is known. In this case, the following relation will prove helpful.[3]

$$G = E \,/\, 2(1+\mu) \qquad (3)$$

where μ = Poisson's Ratio for the material.

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[3] L. S. Marks, "Mechanical Engineers' Handbook," McGraw-Hill Book Co., Inc., 1941, p. 444.

III. TRANSMISSION LINES AS BANDPASS FILTERS

A. Bandwidth Equations for Lines of Different Characteristic Impedance in Cascade

One method of obtaining bandpass characteristics with transmission lines is to couple two identical lines by one of different characteristic impedance.[4] The structure in figure 1 could represent this condition. Let the characteristic impedance of the resonators be Z_{01} and their angular length θ_1; the characteristic impedance of the coupler Z_{02} and its angular length θ_2.

We would like to find the over-all characteristic impedance of the section. The equations for a single lossless line in matrix form are,

$$\begin{bmatrix} E_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix} \times \begin{bmatrix} E_2 \\ I_2 \end{bmatrix}$$

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[4] W. P. Mason, "Electromechanical Transducers and Wave Filters," D. Van Nostrand Co., Inc., New York, New York, 1942, p. 75.

where for the resonators,

$$\mathcal{A} = \cos\theta_1 \;,\quad \mathcal{B} = jZ_{01}\sin\theta_1$$

$$\mathcal{C} = j\frac{1}{Z_{01}}\sin\theta_1 \;,\quad \mathcal{D} = \cos\theta_1$$

and for the coupler,

$$\mathcal{A} = \cos\theta_2 \;,\quad \mathcal{B} = jZ_{02}\sin\theta_2$$

$$\mathcal{C} = j\frac{1}{Z_{02}}\sin\theta_2 \;,\quad \mathcal{D} = \cos\theta_2$$

Hence, multiplying the $\mathcal{ABCD}$ matrices in order gives,

$$\begin{bmatrix} \mathcal{A}' & \mathcal{B}' \\ \mathcal{C}' & \mathcal{D}' \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & jZ_{01}\sin\theta_1 \\ j\frac{1}{Z_{01}}\sin\theta_1 & \cos\theta_1 \end{bmatrix} \times \begin{bmatrix} \cos\theta_2 & jZ_{02}\sin\theta_2 \\ j\frac{1}{Z_{02}}\sin\theta_2 & \cos\theta_2 \end{bmatrix} \times \begin{bmatrix} \cos\theta_1 & jZ_{01}\sin\theta_1 \\ j\frac{1}{Z_{01}}\sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\cos\theta_1\cos\theta_2 - \frac{Z_{01}}{Z_{02}}\sin\theta_1\sin\theta_2\right) & j\left(Z_{02}\sin\theta_2\cos\theta_1 + Z_{01}\sin\theta_1\cos\theta_2\right) \\ j\left(\frac{1}{Z_{01}}\sin\theta_1\cos\theta_2 + \frac{1}{Z_{02}}\sin\theta_2\cos\theta_1\right) & \left(-\frac{Z_{02}}{Z_{01}}\sin\theta_2\sin\theta_1 + \cos\theta_1\cos\theta_2\right) \end{bmatrix} \times \begin{bmatrix} \cos\theta_1 & jZ_{01}\sin\theta_1 \\ j\frac{1}{Z_{01}}\sin\theta_1 & \cos\theta_1 \end{bmatrix}$$

Noting that the section is symmetrical, the over-all characteristic impedance is given by $\sqrt{B'/C'}$; hence the multiplication need only be carried far enough to obtain this result. Continuing the multiplication we have,

$$\frac{\mathcal{B}'}{\mathcal{C}'} = \frac{jZ_{01}\sin\theta_1\left(\cos\theta_1\cos\theta_2 - \frac{Z_{01}}{Z_{02}}\sin\theta_1\sin\theta_2\right) + j\cos\theta_1\left(Z_{02}\sin\theta_2\cos\theta_1 + Z_{01}\sin\theta_1\cos\theta_2\right)}{j\cos\theta_1\left(\frac{1}{Z_{01}}\sin\theta_1\cos\theta_2 + \frac{1}{Z_{02}}\sin\theta_2\cos\theta_1\right) + j\frac{1}{Z_{01}}\sin\theta_1\left(\cos\theta_1\cos\theta_2 - \frac{Z_{02}}{Z_{01}}\sin\theta_2\sin\theta_1\right)}$$

Dividing numerator and denominator by $\cos^2\theta_1\cos\theta_2$ and expanding gives,

$$\frac{\mathcal{B}'}{\mathcal{C}'} = \frac{Z_{01}\tan\theta_1 - \frac{Z_{01}^2}{Z_{02}}\tan^2\theta_1\tan\theta_2 + Z_{02}\tan\theta_2 + Z_{01}\tan\theta_1}{\frac{1}{Z_{01}}\tan\theta_1 + \frac{1}{Z_{02}}\tan\theta_2 + \frac{1}{Z_{01}}\tan\theta_1 - \frac{Z_{02}}{Z_{01}^2}\tan^2\theta_1\tan\theta_2}$$

which can be reduced to the form,

$$\frac{B'}{C'} = \frac{Z_{01}^2}{\phi^2} \left\{ \frac{\tan^2\theta_1 - 2\phi \tan\theta_1 \cot\theta_2 - \phi^2}{\tan^2\theta_1 - \frac{2}{\phi} \tan\theta_1 \cot\theta_2 - \frac{1}{\phi^2}} \right\} \qquad (4)$$

where $\phi = Z_{02}/Z_{01}$

Transmission occurs in a filter whenever the characteristic impedance is real or whenever the above expression is positive. Hence, the limits of the transmission bands are given by the roots of both the numerator and denominator of equation 4. In pass bands defined by the numerator roots, the characteristic impedance varies from zero at the band edges to a maximum at the middle of the band. The denominator roots define bands where the characteristic impedance varies from infinity at the band edges to a minimum at mid-band. Hence numerator roots give results similar to the electrical "T" structure and denominator roots give results similar to the "π" structure.

A general solution for the roots of equation 4 would be rather difficult to obtain. However, it is not too difficult to obtain solutions if θ_2 and θ_1 are assumed to have a definite ratio.

Assume $l_1 = l_2 = \frac{\lambda}{4}$ and $Z_{02} \ll Z_{01}$. A pass band will be centered on $\theta = \pi/2$. Solution of the numerator in terms of ϕ gives,

$$\tan^2\theta_1 = \phi^2 + 2\phi \qquad (5)$$

and for the denominator,

$$\tan^2 \theta_1 = \frac{1}{\phi^2} + \frac{2}{\phi} \qquad (6)$$

Since ϕ is very small, equation 6 must be used as the tangent must be large around $\theta = \pi/2$. Equation 6 can be re-written as,

$$\cot \theta = \pm \phi / \sqrt{1+2\phi}$$

Since $\cot \theta$ is near a zero ($\theta = \pi/2$), $\cot \theta \approx (\frac{\pi}{2} - \theta)$. Hence, for sufficiently small ϕ, $\theta \approx \frac{\pi}{2} \pm \frac{\phi}{\sqrt{1+2\phi}}$ or in expanded form

$$\theta \approx \frac{\pi}{2} \pm \left(\phi - \phi^2 + \frac{3}{2}\phi^3 - \cdots\right).$$

Discarding terms higher than ϕ^2 gives for the angular bandwidth,

$$\Delta\theta = 2(\phi - \phi^2)$$

and the fractional bandwidth ($f_2 - f_1 / f_0$) is given by,

$$\frac{f_2 - f_1}{f_0} = \frac{\Delta\theta}{\theta_0} = \frac{4}{\pi}\phi(1 - \phi) \qquad (7)$$

Another case of possible interest is where $\theta_2 = \theta_1 / 2$ and θ_1 is half a wavelength. Then the passband is centered on $\theta = \pi$ and the roots must be determined from the numerator of equation 4 since the tangent must be very small. Near $\theta = \pi$ the following approximations hold ($\theta_1 < \pi$).

$$\tan \theta_1 \approx - (\pi - \theta_1)$$

$$\cot \theta_1/2 \approx \frac{\pi}{2} - \frac{\theta_1}{2}$$

Using these approximations, the numerator of equation 4 becomes,

$$(\pi - \theta_1)^2 + 2\phi(\pi - \theta_1)\left(\frac{\pi - \theta_1}{2}\right) - \phi^2 = 0$$

from which it is found that

$$\theta = \pi \pm \phi/\sqrt{1+\phi}$$

Expanding and discarding terms higher than ϕ^2 gives,

$$\theta = \pi \pm \left(\phi - \frac{\phi^2}{2}\right)$$

The angular bandwidth is

$$\Delta\theta = 2\phi\left(1 - \frac{\phi}{2}\right)$$

and the fractional bandwidth is

$$\frac{f_2 - f_1}{f_0} = \frac{\Delta\theta_1}{\theta_0} = \frac{2}{\pi}\phi\left(1 - \frac{\phi}{2}\right) \tag{8}$$

Obviously, if ϕ is sufficiently small, equations 7 and 8 can be written more simply as

$$\frac{\Delta\theta}{\theta_0} = \frac{4}{\pi}\phi \tag{7a}$$

and

$$\frac{\Delta\theta}{\theta_0} = \frac{2}{\pi}\phi \tag{8a}$$

A third type of structure could consist of half-wave resonators coupled by eighth-wave necks. Again, the numerator of 4 is used to determine the band limits. The approximations to be made for θ slightly greater than π are,

$$\tan\theta \approx \pi - \theta$$

$$\cot\frac{\theta}{4} \approx 1 - (\pi - \theta)/2$$

Substituting the approximations in the numerator of equation 4 gives,

$$(\pi-\theta)^2 - 2\phi(\pi-\theta)\left[1 - \frac{\pi-\theta}{2}\right] - \phi^2 = 0$$

or

$$(\pi-\theta)^2 - \frac{2\phi}{1+\phi}(\pi-\theta) - \frac{\phi^2}{1+\phi} = 0$$

Neglecting the ϕ^2 term gives,

$$\pi - \theta = 2\phi/1+\phi$$

The expression for the fractional bandwidth becomes,

$$\frac{f_2 - f_1}{f_0} = \frac{\Delta\theta}{\theta_0} = \frac{4}{\pi}\phi \qquad (9)$$

B. Bandwidth Equations Found by Using Approximately Equivalent Lumped Elements

Another possible method for designing a filter is one commonly used in designing high frequency transmission line filters and can be found in almost any text on micro-wave theory.[5]

Let Z_o be the characteristic impedance of a length of transmission line. If the line is shorted on the far end, the input impedance becomes,

$$Z_{sc} = j\, Z_o \tan\frac{\omega l}{v} \qquad (10)$$

A plot of X_{sc}/Z_o versus ω is then simply a tangent curve. The input impedance if the line is opened on the far end is

$$Z_{oc} = -j\, Z_o \cot\frac{\omega l}{v} \qquad (11)$$

The plot of X_{oc}/Z_o versus ω is a negative cotangent curve.

By making the slope of one of the curves, near a particular frequency, approximate the slope of a reactance curve near the same frequency, a lumped inductance, capacitance or a series or parallel resonant circuit can be approximated. The type of element or circuit to be approximated

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[5] For example, see G. L. Ragan, "Microwave Transmission Circuits," McGraw-Hill Book Co., Inc., New York, New York, First Edition, 1948, p. 614.

is naturally dependent on the length of the line. The complete procedure will not be given here, however the approximating equations are as follows:

$$L = Z_0 \frac{2\pi l}{\omega \lambda} \text{ : line shorted, } \frac{l}{\lambda} < \frac{1}{4} \tag{12}$$

$$C = \frac{1}{Z_0} \frac{2\pi l}{\omega \lambda} \text{ : line open, } \frac{l}{\lambda} < \frac{1}{4} \tag{13}$$

If the line is an odd number of quarter-waves long, a parallel-tuned circuit can be approximated where,

$$2C = \frac{1}{\omega_0 Z_0} \left(\frac{\pi}{2} + n\pi\right) \text{ : line shorted} \tag{14}$$

or a series-tuned circuit can be approximated where,

$$2L = \frac{Z_0}{\omega_0} \left(\frac{\pi}{2} + n\pi\right) \text{ : line open} \tag{15}$$

If the line is an integral number of half-waves long, a series-tuned circuit can be approximated where,

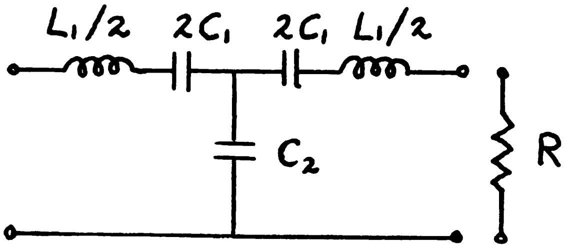
$$2L = Z_0 \frac{n\pi}{\omega_0} \text{ : line shorted} \tag{16}$$

and a parallel-tuned circuit where,

$$2C = \frac{1}{Z_0} \frac{n\pi}{\omega_0} \text{ : line open} \tag{17}$$

The application of these equations to the neck-coupled mechanical structure requires that the resonators be approximately short-circuited and that the thin coupling necks be approximately open-circuited.

One type of structure that can be designed by this method is that composed of half-wave resonators and eighth-wave coupling necks. The analogous electrical circuit is shown in figure 3. This is a simple m-derived band-pass section. For narrow band, C_2 is very much larger than C_1 and the bandwidth ratio with respect to the resonant frequency of the series arm is approximately as shown.



$$L_1 = R/\pi(f_2 - f_1)$$

$$C_2 = 1/\pi(f_2 + f_1)R$$

$$f_1 = 1/2\pi\sqrt{L_1 C_1}$$

$$\frac{f_2 - f_1}{f_1} \approx \frac{2C_1}{C_2}$$

Figure 3.

The parameters of the equivalent mechanical structure can be found as follows. For an open-circuited eighth-wave line, the input impedance is

$$Z_{oc} = -j\, Z_0 \,.$$

Assuming then that the reactance curve of the capacitor C_2 is approximately the same as the negative cotangent curve near $\theta = \pi/4$, the characteristic impedance of the coupling neck can be determined as simply,

$$Z_{02} = 1 / 2\pi f_1 C_2 \tag{18}$$

The characteristic impedance of the half-wave resonators can be found using equation 16 from which it is found that,

$$Z_{01} = \frac{2L_1\omega_1}{\pi} = \frac{2}{\omega_1 C_1 \pi} \tag{19}$$

The bandwidth ratio is thus given by,

$$\frac{f_2 - f_1}{f_1} = \frac{2C_1}{C_2} = \frac{4}{\pi}\frac{Z_{02}}{Z_{01}} . \tag{20}$$

This result is seen to be the same as equation 9 which was found using the first method. The terminating resistance for the section is given by,

$$R = \frac{\sqrt{L_1 C_1}}{C_2} = \frac{1}{\omega_1 C_2} = Z_{02} \tag{21}$$

which of course is equal to the mid-band image impedance of the section. A check of equation 4 for the case $\theta_1 = \pi$ and $\theta_2 = \pi/4$ willgive the same result for the mid-band image impedance.

Another structure that can be treated by either method is that composed of half-wave resonators and half-wave couplers. The fractional bandwidth is found from the second method by comparing it to a constant-K band-pass section. Both methods give the same result for the fractional bandwidth and the mid-band image impedance. These are,

$$\frac{f_2 - f_1}{f_0} = \frac{2\sqrt{2}}{\pi}\sqrt{\frac{Z_{02}}{Z_{01}}} \tag{22}$$

and

$$R_I = \sqrt{2}\sqrt{Z_{01} Z_{02}} \tag{23}$$

These can be readily verified by the reader using the methods given.

This fact, that the image impedances are equal, is stressed because it has been observed that unless this is so, both methods will not give the same results for the same type of structure. This does not come as a complete surprise since in using the second method, a definite lumped electrical equivalent is assumed. In general though, if the mid-band image impedance for a particular structure as found from equation 4, is of the same form as a potentially equivalent electrical structure, then it may be possible to obtain the same result using both methods. Otherwise, the two methods will give rise to two different structures.

IV. RELATED PROBLEMS IN FILTER DESIGN

A. Method of Drive and Takeoff

The fractional bandwidth equations found in the last section will be sufficient for the design of the mechanical structure as will be shown later. The next problem is how to electrically set the structure in vibration; and once this has been accomplished, how to change the vibration energy back to electrical energy.

The possible use of the piezoelectric effect has been investigated,[6] but the best and simplest method for this particular application appears to be electro-mechanical drive by magnetostriction.

Magnetostriction is the phenomenon whereby a material placed in a magnetic field experiences a change in dimension. Some materials undergo an expansion while others a contraction. Of the metals, nickel and some of its alloys undergo the greatest change in length. This change in length is a contraction, and for a constant field is generally not more than one part in 20,000. For a rod of

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[6] Ibid 1

nickel or preferably Ni Span C (a nickel alloy), the resulting motion can be greatly increased by applying a sinusoidally varying field with a frequency equal to the resonant frequency of the rod. In this manner, a fairly efficient conversion of electrical energy to mechanical energy can be achieved.

The basic magnetostriction rod resonator is shown in figure 4. A magnetic bias is required as shown, since

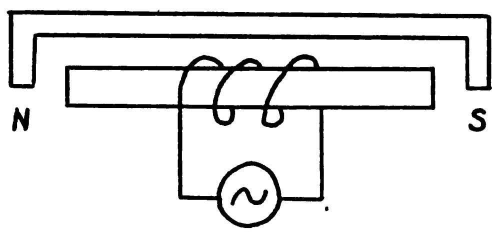

Figure 4.

otherwise the rod would contract on both half cycles of the alternating field producing a double frequency effect. With the bias, the rod undergoes first a decrease and then an increase in length. If the rod is thin enough, it may be possible to remove the biasing magnet with the rod retaining sufficient magnetization to operate. In the diagram, the coil is arranged so as to produce longitudinal vibrations. Naturally, by suitably arranging the coil, torsional vibrations could also be produced and as pointed out by Roberts,[7] the rod could be permanently magnetized in this direction since no free poles would exist.

The magnetostrictive rod can thus be used to drive the mechanical filter and it has the advantage of being one of the filter resonators. The conversion of the mechanical energy back to electrical energy is of course accomplished by the reverse process. A filter structure with the drive and takeoff coils is shown in figure 5.

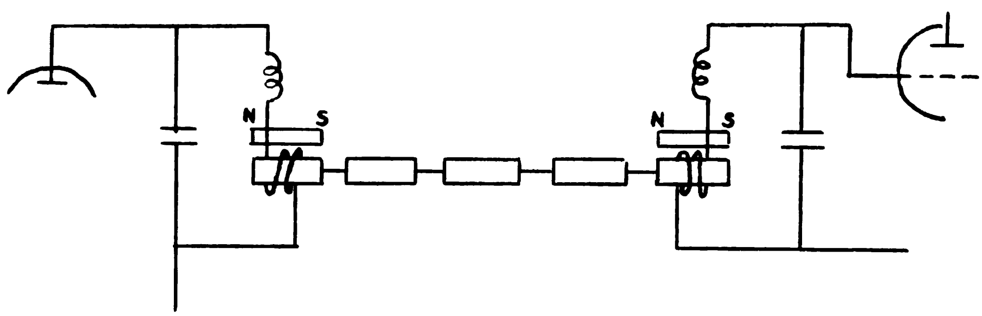

Figure 5.

B. Filter Termination

The type of filter termination to be used should depend on the narrowness of the pass band and whether the

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[7]W. V. B. Roberts, "Some Applications of Permanently Magnetized Ferrite Magnetostrictive Resonators," RCA Review, Vol. XIV, pp. 3-16; March, 1953.

electrical end circuits operate as part of the filter; also on the manner in which the filter is mounted.

For a very narrow band filter, with end resonators made of relatively low Q material, satisfactory results could probably be obtained with no terminating resistance. If the bandwidth is not too narrow, the electrical end circuits might be incorporated as the first and last resonators of the filter and the necessary terminating resistance obtained by adjusting the Q of the electrical circuit. It appears that if some form of external mechanical damping is required, a little cut and try with various methods would be necessary.

One method could be to connect a line which approximates an infinite line, as far as resonator length is concerned, to the end resonators; this line having a characteristic impedance equal to the mid-band image impedance of the filter. For example, the filter structure with half-wave resonators and quarter-wave couplers has a mid-band image impedance equal to the characteristic impedance of the coupler, Z_{o2}. The impedance of an infinite line is simply the characteristic impedance of the line, hence proper termination should be obtained if the infinite line has a characteristic impedance equal to Z_{o2}.

The image impedance of the filter just mentioned varies from zero at the band edges to a maximum of Z_{o2} at mid-band, hence a compromise termination as suggested by Guillemin[8] would probably give better results. The termination he suggests would be equal to .707 times the mid-band image impedance.

C. Materials

For the magnetostrictive end resonators, two materials stand out. The choice at which is dependent on the specific design requirements of the filter.

The first is Ni Span C which has already been mentioned. It is highly magnetostrictive and has a low temperature coefficient of expansion making its use desirable where frequency stability is of prime importance.

The second material is more in the field of ceramics than metals. This is the so-called ferrite material. By using a suitable ferrite for the driving resonator, the Q of the coil surrounding it may be greatly increased;[9] whereas if Ni Span C were used, the coil Q would be lowered due to eddy currents, etc. Ferrites have a very high resistivity and hence eddy currents are almost non-exis-

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[8]E. A Guillemin, "Communication Networks," Vol. 2, John Wiley & Sons, Inc., New York, New York, 1935, p. 309.

[9]Ibid 7

tent. Also, the mechanical Q of the ferrite may be of the order of a thousand or greater, whereas a nickel resonator would have a Q of only several hundred. The determining factor in whether to use the Ni Span C or ferrite lies in the temperature stability requirements of the filter since ferrite has a considerably higher temperature coefficient of expansion.

The materials to be used for the interior resonators and couplers is again determined by the desired Q and temperature characteristics and also by the machinability of the material. It probably should be mentioned here that the mechanical Q is determined as the number of cycles of vibration required for the amplitude to die down 4.32 percent of its original amplitude after the driving force has been removed.[10]

Table 3[11] gives the characteristics of some common materials.

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[10]W. V. B. Roberts, "Magnetostriction Devices and Mechanical Filters for Radio Frequencies," Q. S. T., July, 1953, p.28.

[11]Ibid 1.

Material	Density gm/cm^3	Long.vel.in thin rod x 10^{-5} cm/sec	Intrinsic Impedance x 10^{-3} gm.sec/cm^3	Q
1. Aluminum	2.73	5.11	1.39	4000
2. Brass(hard)	8.54	3.64	3.11	2500
3. Brass(soft)	8.50	3.52	2.99	2000
4. Dural	2.81	5.07	1.43	8000
5. Ferrite	4.46	5.58	2.49	1250
6. Nickel	8.88	4.94	4.39	450
7. Ni Span C	7.99	4.80	3.83	900
8. Steel(drill rod)	7.86	5.13	4.03	900
9. Steel(stainless)	7.94	4.97	3.95	1500

Table 3. Properties of Filter Materials.

V. PRE-DESIGN CONSIDERATIONS

A. Filter Parameters and Assumptions

As a preliminary to the actual design examples, a brief review of the assumptions used and the constants to be used for the designs will now be given.

It seems that in discussing mechanical filters, the best and most fundamental constants to consider are:

1. f_0 = resonant frequency of each resonator
2. $k_{r(r+1)}$ = coefficient of coupling between the r th and (r+1)th adjacent resonators
3. $d_r(\frac{1}{Q_r})$ = decrement of the resonator

The main assumption made is that the interior resonators have zero decrement or infinite Q. This leads to the second assumption which is; that the coefficient of coupling between adjacent resonators is numerically equal to the fractional bandwidth when the two resonators are considered alone. For example, consider the structure made up of half-wave resonators and quarter-wave coupling necks. The coefficient of coupling between adjacent resonators is given by equation 8 or 8a.

Using equation 8 we have,

$$k_{r(r+1)} = \frac{2}{\pi}\,\phi\,(1 - \phi/2)$$
$$\frac{2}{\pi}\,\frac{Z_{02}}{Z_{01}}\left(1 - \frac{Z_{02}}{2Z_{01}}\right)$$

For longitudinal vibration,

$$k_{r(r+1)} = \frac{2}{\pi}\left(\frac{D_c^2\,\rho_c\,v_c}{D_R^2\,\rho_R\,v_R}\right)\left(1 - .5\left(\frac{D_c^2\,\rho_c\,v_c}{D_R^2\,\rho_R\,v_R}\right)\right)$$

and if resonator and coupler are of the same material,

$$k_{r(r+1)} = \frac{2}{\pi}\,\frac{D_c^2}{D_R^2}\left(1 - .5\,\frac{D_c^2}{D_R^2}\right)$$

For torsional vibration,

$$k_{r(r+1)} = \frac{2}{\pi}\left(\frac{D_c^4\,\rho_c\,v_c'}{D_R^4\,\rho_R\,v_R'}\right)\left(1 - .5\left(\frac{D_c^4\,\rho_c\,v_c'}{D_R^4\,\rho_R\,v_R'}\right)\right)$$

and for resonator and coupler of the same material,

$$k_{r(r+1)} = \frac{2}{\pi}\,\frac{D_c^4}{D_R^4}\left(1 - .5\,\frac{D_c^4}{D_R^4}\right)$$

A summary of the symbols used is as follows:

E	modulus of elasticity
G	modulus of rigidity (shear modulus)
μ	poisson's ratio
ρ	mass density
I	moment of inertia per unit length = ρ J
J	moment of inertia of cross section area
S	cross section area
D	diameter
l	length

R	subscript denoting resonator
c	subscript denoting coupler
n	total number of resonators in filter; this may include the electrical end circuits
r	resonator number in filter chain. Input resonator is no. 1.
Z_{o1}	characteristic impedance of resonator (see table 1 or 2)
Z_{o2}	characteristic impedance of coupler
ϕ	Z_{o2}/Z_{o1}
v	longitudinal wave velocity
v'	torsional wave velocity
V_p	voltage output at peak of response curve
V_v	voltage output at valley level
F	$f_2 - f_1 / f_o$
F_v	$\Delta f_v / f_o$
Δf_v	bandwidth at valley level

B. Equations For Obtaining Chebishev Response

The types of filters described up to this point are those made up of a chain of identical sections. This type of filter may have a fairly flat response around mid-band but ripples become quite pronounced toward the band edges, especially as the number of filter sections is increased as shown in figure 6a.

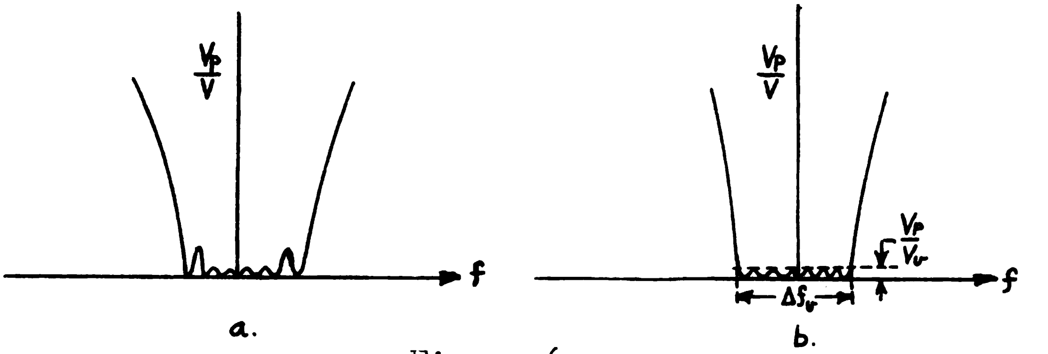

Figure 6.

When the unloaded Q of the elements to be used is high enough for them to be considered non-dissipative, a fairly simple method worked out by Dishal[12] can be incorporated to design a filter which will have the optimum attenuation shape.

This method as described by Dishal, is based on the allowable pass-band insertion loss and is particularly valuable when there are stringent requirements on the pass-band tolerance and rate of cut-off of the filter. The attenuation shape is like that shown in figure 6b. where ripples occur throughout the entire pass band but the peaks are all equal and the peak to valley ratio can be made as small as desired. However, according to Dishal, it is best to design for the maximum allowable ripple as this will give the greatest rate of cut-off.

The equation for the optimum response curve of figure 6b. is,

$$\left(\frac{V_P}{V}\right)^2 = 1 + \left[\left(\frac{V_P}{V}\right)^2 - 1\right]\cosh^2\left\{n \cosh^{-1}\frac{\Delta f}{\Delta f_v}\right\} \quad (24)$$

This equation was arrived at by the method of approximating a constant by means of Chebishev polynomials and hence

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[12] M. Dishal, "Two New Equations for the Design of Filters," Electrical Communication, Vol. 30, Dec., 1953, pp. 324 - 337.

the term "Chebishev Response" is used. The response shape is optimum in the sense that for a given allowable ripple in the pass band, it produces the maximum possible rate of cutoff for a given number of elements.

Dishal has worked out two design equations to obtain the shape of equation 24. One applies for the symmetrical filter with damping at both ends, and the other applies for the unsymmetrical filter with damping at one end only. These equations are shown in figure 7.

---

a. Symmetrical Filter
(Resistive Gen., Resistive Load)

$$\frac{Q_{1,n}}{f_0/\Delta f_v} = \frac{2 \sin\theta}{S_n} \qquad Q_{2 \to (n-1)} = \infty$$

$$\left[\frac{k_{r(r+1)}}{\Delta f_v/f_0}\right]^2 = \frac{(S_n^2 + \sin 2r\theta)}{4\{\sin (2r-1)\theta\}\{\sin (2r+1)\theta\}}$$

---

b. Unsymmetrical Filter
(Resistive Gen., Reactive Load) or vice versa

$$\frac{Q_1}{f_0/\Delta f_v} = \frac{\sin\theta}{S_n} \qquad Q_{2 \to n} = \infty$$

$$\left[\frac{k_{r(r+1)}}{\Delta f_v/f_0}\right]^2 = \frac{(S_n^2 + \sin^2 r\theta)}{\sec^2(r\theta)\{\sin (2r-1)\theta\}\{\sin (2r+1)\theta\}}$$

$$\theta = \frac{90^\circ}{n} \qquad S_n = \sinh\left\{\frac{1}{n}\sinh^{-1}\left[\left(\frac{V_P}{V}\right)^2 - 1\right]^{-\frac{1}{2}}\right\}$$

Figure 7.

VI. EXAMPLES OF FILTER DESIGN

A. Campbell Type - Longitudinal Mode

The campbell type filter is basically that shown in figure 8. By constructing the equivalent T for a section

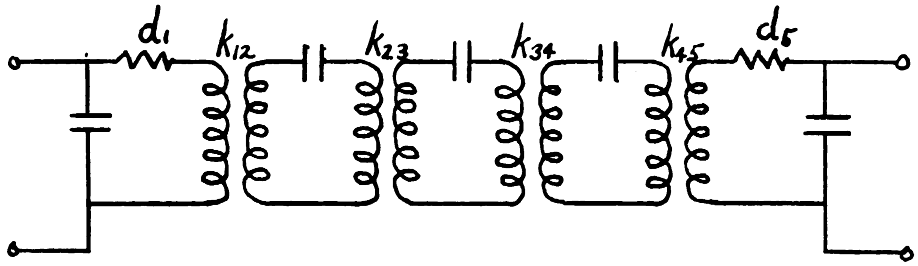

Figure 8.

of this filter, the mid-band image impedance, for $M \ll L$, is found to be $Z_{I0} = j\omega M$. The mechanical structure with half-wave resonators and quarter-wave couplers has a mid-band image impedance $Z_{I0} = Z_{02}$. We might expect, since these forms are the same, that the same design criteria could be used for the mechanical structure. That is, the coefficients of coupling between each end resonator and its adjacent interior resonator should be .707 times the fractional bandwidth and the interior coefficients of coupling should be .5 times the fractional bandwidth. The decrement of each end circuit should be equal to the fractional bandwidth.

Let the specifications for the mechanical filter be as follows:

1. $f_o = 110$ kc
2. $\Delta f = 6$ kc
3. $F = .0545$
4. Resonators - half-wave length; .25" dia., Ni Span C for temperature stability
5. Couplers - quarter-wave, Ni Span C
6. n = five resonators

The resonator lengths are,

$$l_R = v/2f = \frac{1.89\times10^5}{2\times110\times10^3} = .86''$$

The coupler lengths are,

$$l_c = l_R/2 = .43''$$

The interior coefficients of coupling should be,

$$k_i = F/2 = .0273$$

Thus, the coupler diameters are given by,

$$D_{ci} = D_R \left(1 - \sqrt{1 - \pi \times k}\right)^{\frac{1}{2}}$$

$$= .25 \left(1 - \sqrt{1 - \pi \times .0273}\right)^{\frac{1}{2}}$$

$$= .0512''$$

By making the end resonators contain half the energy of the interior resonators, the proper coefficient of coupling is obtained and the first and last coupling necks can be made the same diameter as the interior couplers. The cross sectional area of the end resonators will then be made equal to half the area of the interior resonators giving for the diameter of the end resonators,

$$D_{R1,5} = .707\ D_{Ri}$$
$$= .177''$$

An infinite line termination consisting of a few feet of copper wire can be used with characteristic impedance equal to .707 Z_{02}. The diameter of this wire is given by,

$$D_L = D_c\ (.707\psi)^{\frac{1}{2}}$$

$$\text{where } \psi = \frac{\rho v \text{ of coupler}}{\rho v \text{ of copper}} = 1.15$$

$$\text{hence, } D_L = .0462''$$

which is about the size of no. 16 or 17 wire. The lines can be coiled in some manner to make them more compact and coated with a viscous substance to adjust for the exact amount of damping required. The complete structure is shown in figure 9.

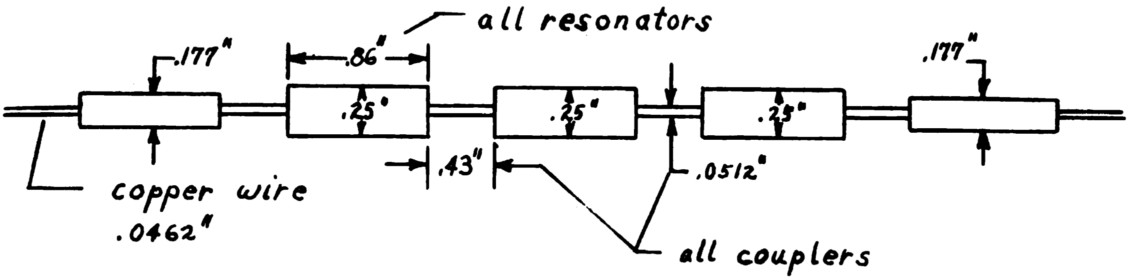

Figure 9.

B. Campbell Type - Torsional Mode

Let the specifications be as follows:

1. $f_0 = 100$ kc
2. $\Delta f = 4$ kc
3. $F = .04$
4. half-wave resonators, quarter-wave couplers
5. Use electrical circuits as end resonators and .25" dia. ferrite for magnetostrictive resonators and .25" dia. dural rod for other resonators.
6. Couplers - dural rod
7. $n = 7$, including electrical circuits

The Q of the electrical end circuits should be,

$$Q = \frac{1}{.707 \times .04} = 35.3$$

The coefficient of coupling between the electrical circuits and the ferrites should be,

$$k_{1,2} = k_{6,7} = .028$$

The others should be,

$$k_i = .02$$

Using equation 8a, the coefficient of coupling between adjacent resonators is,

$$k_{r(r+1)} = \frac{2}{\pi}\left(\frac{D_c}{D_R}\right)^4$$

Although this is not strictly correct for $k_{2,3}$ and $k_{5,6}$, the error introduced is not great. The diameter of the coupling necks is thus found to be,

$$D_C = D_R \left(\frac{\pi k l}{2}\right)^{\frac{1}{4}}$$

$$= .25 \left(\frac{\pi \times .02}{2}\right)^{\frac{1}{4}}$$

$$= .105''$$

The aluminum resonators and couplers can be turned out from a single quarter-inch dural rod with extensions for attaching the ferrite resonators.

The resonator lengths are:

Ferrite: $$l_R = \frac{1}{2f}\sqrt{\frac{G}{\rho}} = \frac{1}{2 \times 10^5}\sqrt{\frac{4.96 \times 10^8}{4.46/980}} \text{ cm.}$$

$$= 1.65 \text{ cm.} = .65''$$

Dural: $$l_R = \frac{1}{2 \times 10^5}\sqrt{\frac{2.6 \times 10^8}{2.81/980}} \text{ cm.}$$

$$= 1.505 \text{ cm.} = .593''$$

The length of the couplers is:

$$l_c = .5 \times .593 = .2965''$$

A schematic of the filter is shown in figure 10.

The coils surrounding the ferrite resonators must be wound so as to provide a toroidal field (not as shown), and their positions over the ferrites adjusted to give the proper coefficient of coupling.

By drilling holes thru the ferrites as shown in the diagram, they can be permanently magnetized with a toroidal field by passing a current carrying wire through the hole. Thus the biasing magnets can be eliminated.

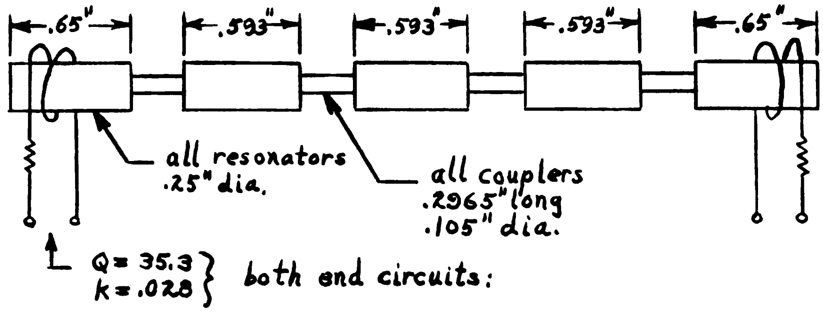

Figure 10.

C. Chebishev Type - Longitudinal Mode

The specifications are as follows:

1. f_o = 200 kc
2. Δf = 8 kc
3. F = .04
4. V_p/V_r = 1.05
5. Symmetrical type filter using electrical end circuits as resonators, n = 7.
6. Half-wave resonators, quarter-wave couplers
7. .25" dia. ferrite and .25" dia. dural rod as in previous example

The lengths of the resonators are,

Ferrite: $l_R = .55''$

Dural: $l_R = .503''$

The coupler length is,

$$l_c = .2565''$$

The first set of equations in figure 7 is now used to find the required end circuit Q's and the required coefficients of coupling.

$$\theta = 90^o/n = 12.86^o \text{ and } S_n = .27$$

The required coefficients of coupling are,

$$k_{1,2} = k_{6,7} = .0290$$

$$k_{2,3} = k_{5,6} = .0220$$

$$k_{3,4} = k_{4,5} = .0213$$

For $k_{2,3}$ and $k_{5,6}$, the coupler diameter is,

$$D_c = .0465''$$

and for $k_{3,4}$ and $k_{4,5}$,

$$D_c = .0457''$$

The filter structure is shown in figure 11.

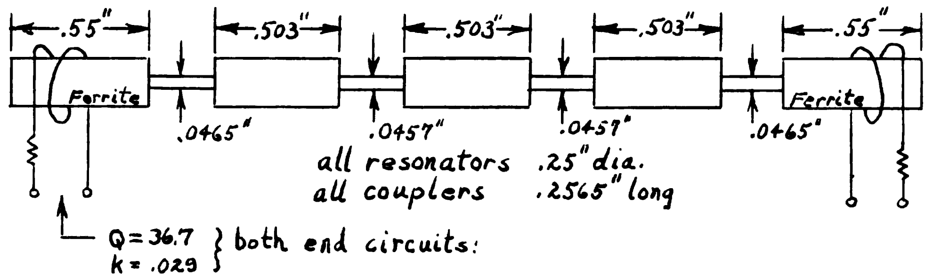

Figure 11.

D. Chebishev Type - Torsional Mode

Use the same specifications as for filter A, except for torsional mode of vibration and $V_p/V_r = 1.05$. The velocity of torsion waves for Ni Span C is approximately 3×10^5 centimeters per second.

The length of the resonators is,

$$l_R = \frac{3 \times 10^5}{2.2 \times 10^5 \times 2.54} = .538''$$

The coupler length is,

$$l_c = .5 \times .538 = .269''$$

The required end Q's are,

$$Q_{1,n} = \frac{2 \sin \theta}{S_n \Delta f_v / f_o} = \frac{2 \sin 18^\circ}{.382 \times .0545} = 29.7$$

The required coefficients of coupling are,

$$k_{1,2} = k_{4,5} = .038$$

$$k_{2,3} = k_{3,4} = .031$$

The coupling neck diameter for $k_{1,2}$ and $k_{4,5}$ is,

$$D_C = .25 \left(\frac{\pi}{2} \times .038\right)^{\frac{1}{4}} = .124$$

and for $k_{2,3}$ and $k_{3,4}$,

$$D_C = .25 \left(\frac{\pi}{2} \times .031\right)^{\frac{1}{4}} = .117''$$

Enough damping must be added to the end resonators to reduce their Q to 29.7. Using the infinite copper line termination, its characteristic impedance can be found from the expression,

$$Z_{0L} = \frac{\pi Z_{01}}{2Q}$$

and the line diameter is found to be,

$$D_L = .25 \left(\frac{\pi}{2 \times 29.7} \times \frac{8.0}{8.95} \times \frac{4.8}{3.7}\right)^{\frac{1}{4}}$$

$$= .25\ (.0615)^{\frac{1}{4}}$$

which gives $D_L = .125''$.

A comparison of this design with the first shows that less disparity is required between the resonator and coupler diameters when the torsional mode of vibration is used. A diagram of this structure is shown in figure 12.

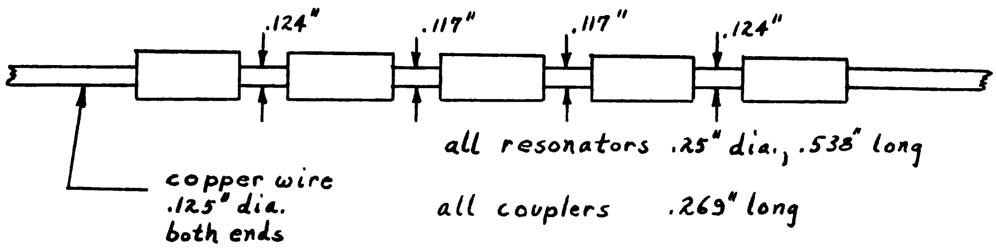

Figure 12.

E. Chebishev Type - Torsional (unsymmetrical)

This filter will be unsymmetrical; i.e., with damping at one end only. The second set of equations in figure 7 will be used. The specifications are as follows:

1. $f_o = 100$ kc
2. $\Delta f = 1$ kc
3. $F = .01$
4. $V_P/V_r = 1.03$
5. All Ni Span C construction; .25" dia. half-wave resonators, quarter-wave couplers.
6. Torsional mode of vibration
7. $n = 4$

The required Q for the first resonator is,

$$Q = \frac{\sin 22.5^\circ}{.555 \times .01} = 69$$

The required coefficients of coupling are,

$$k_{1,2} = .01$$

$$k_{2,3} = .0072$$

$$k_{3,4} = .00725$$

The coupler diameters are,

$$D_{c_{1,2}} = .25 \left(\frac{\pi}{2} \times .01\right)^{\frac{1}{4}} = .0883''$$

$$D_{c_{2,3}} = .25 \left(\frac{\pi}{2} \times .0072\right)^{\frac{1}{4}} = .0815''$$

$$D_{c_{3,4}} = .25 \left(\frac{\pi}{2} \times .00725\right)^{\frac{1}{4}} = .0818''$$

The length of the resonators is .59" and the coupler length is .295".

Enough damping must be added to the first resonator to reduce its Q to 69. For the infinite line termination, the diameter of copper wire needed is,

$$D = .25 \left(\frac{\pi}{2 \times 69} \times \frac{24}{19.7}\right)^{\frac{1}{4}}$$

$$= .102''$$

This is about the size of no. 10 wire.

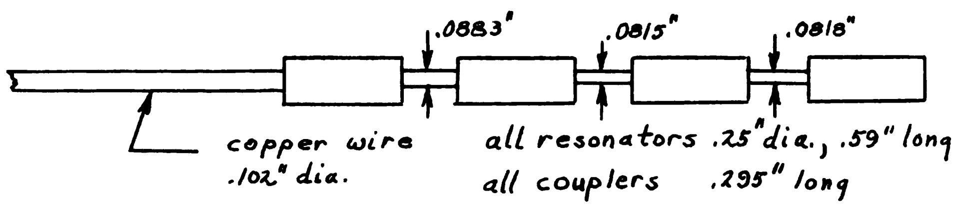

Figure 13.

A diagram of the complete filter structure is shown in figure 13.

VII. CONCLUSIONS

This paper has discussed a problem of prime interest in the communications field today. Namely, that of designing a cheap, practical bandpass filter with a selectivity curve closely approximating that for the ideal filter. The mechanical filter with its high Q elements has been proposed as a possible answer to this problem.

It has been shown how the valuable concept of the electrical-mechanical analogy has enabled a simple design procedure to be worked out. Although far from rigorous; from a practical viewpoint, it provides a simple, workable method of analysis. Two closely related methods were described both of which compared the mechanical filter to a cascaded chain of electrical transmission lines.

The various structural considerations were discussed and equations were given whereby the filter could be designed to produce the Chebishev response or maximum rate of cutoff. Several examples were given for both the longitudinal and torsional modes of vibration.

Naturally, the particular filter structure described has some drawbacks. One doesn't have to raise the frequency very much higher than in the examples before the elements become too small to handle. Flimsiness of the structure may also result for an extremely narrow bandwidth. This may be overcome by using the torsional mode of vibration; however, if the frequency is very high, then the lengths of the elements may be too small since the torsional mode requires shorter elements than the longitudinal. These problems can undoubtedly be overcome in some cases by changing the element shapes and vibration modes, etc.

It is regretted that the actual construction and testing of the mechanical filter could not be included in this paper, but time would not permit such a complete treatment.

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