DESIGN OF A REINFORCED CONCRETE OPEN SPANDREL BRIDGE

Thesis for the Degree of B. S.
MICHIGAN STATE COLLEGE

Henry G. Dunkelberg

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Design of a Reinforced Concrete

Open Spandrel Bridge

A Thesis Submitted to

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AGRICULTURE AND APPLIED SCIENCE

by

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THESIS

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INTRODUCTION

The importance of bridges is unquestionable when one reads the history of the world. Important battles have been fought at bridge sites, weary miles of travel have been saved by the spanning of turbulent rivers and impassable canyons.

The very lives of many people have been altered by bringing within easy reach routes to heretofore inaccessible places of market and travel.

From the ancient draw bridges over castle moats to the magnificent engineering structures spanning the San Francisco Bay and the Golden Gate, bridges have been essential to civilization.

In the beginning, if a bridge was structurally sound enough to support the traffic upon it, it served its purpose.

With the development of civilization, however, and the building of arch bridges, the requirements for a bridge grew. No longer was structural soundness sufficient. The structure must also present a pleasing appearance. And some of the ancient arch bridges which have stood through the years with their graceful, pleasing, spans surpass many of the modern attempts at artistic design.

With the advent of reinforced concrete as a building material, a new type of structural beauty has come into existence. The modern builder with his modern materials and greatly accelerated construction methods, must now vie with the ancient day builders for that illusive gracefulness and artistic orginality in bridge design.

It is with these thoughts in mind that the author presents this following arch design. First the bridge must be structurally sound and then it must present a pleasing appearance in harmony with the setting.

The ground profile used in the design, while only assumed, approaches very nearly the conditions to be met with the building of a new bridge at Berrien Springs, Michigan, to span the St. Joseph river. The greatest difference in the design site and the actual site is that three or more spans would be needed to cross the St. Joseph river. It is the author's opinion that the following arch design could be used as one of the spans for this bridge.

Design of floor slab, floor beam

and spandrel column.

For references see "The Design of Reinforced Concrete Structures" by Dean Peabody, Jr. The pages will be noted below.

<u>Notations and Equations</u>.

b = width of section.

h = total depth of section.

d = distance from extreme fiber in compression to the center of gravity of the steel in tension.

jd = moment arm of internal couple.

kd = distance from extreme fiber in compression to neutral axis.

E_c = modulus of elasticity of concrete.

E_s = modulus of elasticity of steel.

n = ratio of moduli, or $\frac{E_s}{E_c}$.

f_c = maximum intensity of fiber stress in concrete.

f_s = average intensity of fiber stress in steel.

M = external moment at the section.

M_p = positive moment of resistance exressed in concrete terms

M_n = negative moment of resistance expressed in concrete terms

A_s = area of steel

A_p = area of positive tension steel

A_n = area of negative steel

v = unit shear

R = least radius of gyration

N = axial column load

$M_p = \frac{f_c}{2} j k b d^2$ page 24

$\text{Min. } d = \sqrt{\frac{M}{Kb}}$ page 24

$A_p = \frac{M_p}{f_s\, jd}$ page 24

$v = \frac{V}{b\, j\, d}$ page 36

$R = \sqrt{\frac{I}{A}}$ page 227

$N = fe \times A\,(\,1 + (n-1)\,p\,)$ page 224

$A_s = p\,A$ page 228

Design of Floor Slab

(pages 22-25)

Span between center of supports is 13'-4".

Designing slab as simply supported for positive moment and using $\frac{wl^2}{1L}$ for negative moment is on the safe side.

Assume 7" slab

Live load	=	200 lbs.	per	sq.	ft.	
Dead "	=	88	"	"	"	"
Total		288	"	"	"	"

Using 2500 lb. concrete

$fc = 1200$

$fs = 18000$ lbs. per sq. in.

$n = 12$

$k = .4$

$j = 1 - \frac{1c}{3} = .867$

$$M_p = \frac{wl^2}{8} = \frac{288 \times (13.33)^2}{8} = 76{,}800 \text{ in lbs.}$$

$$M_p = \frac{f_c}{2}\, j\, k\, b\, d^2 = 76{,}800 \text{ in lbs.}$$

$$d^2 = \frac{76,800 \times 2}{1200 \times .867 \times .4 \times 12} = 5.21$$

Minimum d = 2.29

Assuming 7/8" steel, 1-1/2" covering, 1/2" wearing surface on top from Michigan State Hiway Specifications.

h = d + clearances

= 2.29 + 1/2 + 1-1/2 + 1/2 = 4.79

Hence 7" slab satisfactory

Commercial d = 7 - 2.5 = 6.5

Positive steel

$$A_p = \frac{M_p}{f_s\ j\ d} = \frac{76,800}{18,000 \times .867 \times 6.5} = .756 \text{ sq. in.}$$

Use 2 - 3/4" round bars 6" on center

Plot 5, page 436 gives point of inflection for positive bending moment at 10% of clear span or 144 x .1 = 14".

Point of inflection for negative bending moment is at 21.5% of clear span or 31". Hence steel is bent up between these points.

Design of Floor Beam

(pages 56-60)

A beam with two equal overhangs, supporting a uniform distributed load. The most favorable condition is when the bending moment at the center equals the bending moment at the supports. If the distance between the supports = d and the length of the beam = L, equal bending moments are obtained at the center and supports when d = .5x6 L. In this design L = 45' giving d = .586x45 = 26.4'.

d is taken as 27' the distance between the centers of the arch ribs.

Assume a 16" x 32" beam.

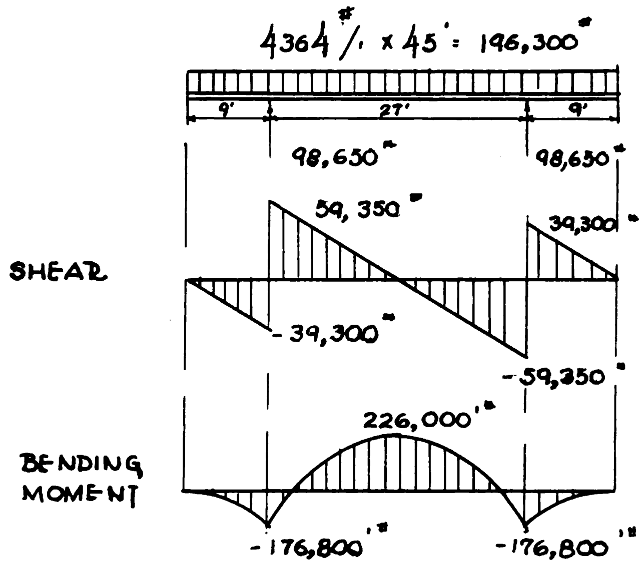
Live load = 200 lbs. per sq. ft.

Dead load = 88 lbs. per sq. ft.

Total floor load = 288 x 13.3 = 3830 lbs. per lin. ft.

Wt. of beam = $\frac{16 \times 32}{144} \times 150$ = 534 " " " "

Total uniform load = 4364 lbs. per lin. ft.



2500 lb. concrete

fc = 1000

f_s = 18000

K = 185 plot page 434

$$\text{Minimum } d = \sqrt{\frac{M}{Kb}} = \sqrt{\frac{226{,}000 \times 12}{185 \times 16}} = 30.3''$$

Assume one row of 1-1/4 " bars.

Min. clearance to stirrup	2.00
Stirrup diameter	.50
Distance to center of bars	.63
Total	3.13

Hence use 16"x34" beam.

$$A_p = \frac{M_p}{f_s\ j\ d} = \frac{226,000 \times 12}{18,000 \times .867 \times 30.3} = 5.74 \text{ sq. in.}$$

Use 4 1-1/4" square bars.

$$A_n = \frac{M_n}{f_s\ j\ d} = \frac{176,800 \times 12}{18,000 \times .867 \times 30.3} = 3.21 \text{ sq. in.}$$

Use 4 one inch square bars.

Shear

$V = 59,350$

$$v = \frac{V}{bjd} = \frac{59.350}{16x.867x32} = 134 \text{ lbs. per sq. in.}$$

Allowable v using web reinforcement without ordering anchorrage of longitudinal steel = 150 lbs. per sq. in.

Hence beam is satisfactory.

Design of Spandrel Column

(pages 223-228)

The longest unsupported column will be 18'.

Assume column 16"x18" x 18' long.

$$R = \sqrt{\frac{I}{A}} = \sqrt{\frac{18 \times (16)^3}{12 \times 18 \times 16}} = 4.63$$

Limiting length = h = 40 R = 185".

Actual length = 18x12 = 216" giving a long column with radius of gyration of 4.63.

Column load =	98,650 lbs.	
Wt. of col. =	5,400 "	Col. $\frac{18x16}{144}$ x 150x18
Total = N =	104,050 lbs.	

By article 306, page 401 for column with lateral ties, $f_c = 563$.

By article 1108 b

$$\frac{f_e A}{f_c A} = 1.33 - \frac{h}{120 R}$$

$$f_e = 563 \left(1.33 - \frac{18x12}{120 \times 4.63}\right) = 563.$$

$N = f_e \times A (1 \times (n-1)p) = f_e \times A (1 \times 11p)$

$104{,}050 = 530 \times 18 \times 16 (1+11p)$

$p = .029$

$A_s = p A = .029x288 = 8.35$ sq. in.

Use 12-1" round bars and 3/8" ties 12" on center.

Design of the Arch Rib

For references and derivations of formulae used see "Bridge Engineering" Vol. I by J. A. L. Waddell, pages 783-877. The diagrams noted are to be found within these pages.

Notations

The Calculation of Stresses in Arch Ribs with Fixed Ends.

L = length of span of arch,

r = rise of arch,

x, y = co-ordinates of any point with reference to the crown C as an origin, y being positive when measured downward, and x being positive in each half when measured from the crown toward the springing,

a = angle of inclination of the axis at any point, Fig. 37t

B = angle of inclination of the axis at the springing,

L = length of rib, measured along the axis (= 2   ds),

b = width of rib at any point,

b_o = width of rib at crown,

b_s = width of rib at springing,

h = thickness of rib at any point, measured normally to axis,

h_o = thickness of rib at crown, measured normally to axis,

h_s = thickeness of rib at springing, measured normally to axis,

A = area of rib at crown, or area of concrete plus n times area of steel,

A_o = area of rib at crown, or area of concrete plus n times area of steel,

I = moment of inertia or rib at any point, or moment of inertia of concrete plus n times moment of inertia of steel,

I_o = moment of inertia of rib at crown, or moment of inertia of concrete plus n times moment of inertia of steel, Fig. 37t

I_s = moment of inertia of rib at springing, or moment of inertia of concrete plus n times moment of inertia of steel,

E = coefficient of elasticity of the concrete, to be taken as 2,000,000 when dimensions are in inches, and as 288,000,000 when they are in feet,

w = coefficient of linear expansion of concrete (= 0.000006),

t = change of temperature in degrees Fahrenheit, positive when the temperature rises, negative when it falls,

P_1, P_2, P_3, etc., = any loads on the arch, acting in any direction,

p_o = equivalent uniform load at crown,

p_s = equivalent uniform load at springing,

$u = \frac{p_s}{p_o}$,

p = equivalent uniform load at any point,

H_o, V_o, M_o = thrust, shear, and moment at crown,

H_a, M_a = thrust and moment at crown from arch shortening,

H_t, M_t = thrust and moment at crown from temperature change,

C_t = coefficient of temperature thurst, Fig. 37ff

y_o = vertical distance from crown to plane of contraflexure for arch shortening and temperature change, Fig. 37ff

T_1 = normal thrust at any section in left half of rib, positive when compressive,

T_r = similar quantity for right half of rib,

T = Normal thrust in general,

T_s = normal thrust at the springing,

M = Actual bending moment in general,

M_s = actual bending moment at springing,

C_m = moment coefficient Fig. 37ee

Equations

$$\sec B = \sqrt{1 + \left(\frac{4r}{L}\right)^2}$$

$$h = h_o + \frac{2x}{L}\left(\frac{h_s}{\sec. B} - h_o\right) \sec a$$

$$u = \frac{p_s}{p_o}$$

Thrust due to loading

$$H_o = \frac{p_o L^2}{48r}(u + 5)$$

$$T = H_o \sec a$$

$$T_s = H_o \sec B$$

Thrust due to arch shortening

$$H_a = -\left(1 + 0.3\frac{I_s}{I_o}\right)\frac{H_o h_o^2}{r^2 \sec B}$$

Moment at or near a spandrel col. due to loading

$$M = +\frac{p_o L^2}{1500}$$

Moments at other points due to loading

$$M = +\frac{p_o L^2}{3000}$$

Positive and negative live load moments (exclusive of the effect of arch shortening) at the crown, springing and various intermediate points.

$$M = C_m pL^2$$

Moments due to arch shortening

$$M_a = -H_a y_o$$

$$M = H_a (y-y_o)$$

$$M_s = H_a (r-y_o)$$

Moments due to temperature changes

$$M_t = -H_t y_o$$

$$M = H_t (y-y_o)$$

$$M_s = H_t (r-y_o)$$

The following procedure is used to calculate the arch rib.

1. Assume values of h_o, h_s and calculate p_o, p_s, and u, for dead load plus one half live load.

2. Compute I_o, I_3 $\frac{I_3}{I_o}$ and $\frac{r}{L}$

3. Figure the dead and live load stresses at the crown and springing and at one point in the haunch. Use Fig. 37ee

4. Compute the stresses for arch shortening at the same sections. Use Fig. 37ff.

5. Compute the stresses from temperature changes at the same section. Use Figs. 37 ff and 37 gg.

6. Test the various sections for the calculated stresses.

Calculations of Arch Rib

r = rise = 50'

L = span = 120'

$$\frac{r}{L} = \frac{50}{120} = .416$$

$$\sec B = \sqrt{1 + \left(\frac{4r}{L}\right)^2} = 1.94$$

Assume each rib 3'-0" thick x 4'-4" wide at crown

6'-0" " X 4'-4" " " springing

$$h = h_o + \frac{2x}{L} \frac{(h_s - h_o)}{\sec B} \sec a$$

$$= 36 + \frac{24}{120} \frac{(72-36)}{1.94} \sec a = 36 + .309 \times \sec a$$

p_o and p_s for dead load + 1/2 live load

p_o for one rib

D. L. floor - paving, spandrel cols, etc.	6,100#/lin ft.
Arch rib 3'-0" x 4'-4" x 150 lbs.	1,950 " "
Live Load - 60' span -200#/sq ft including impact	
1/2 (L. L.) 200 x 22.5 x 1/2	2,250 " "
Total	10,300#/lin ft.

p_s for one rib

D. L. Floor - paving spandrel cols, etc.	7,275#/lin. ft.
Arch rib 6'-0" x 4'-4" x 150 lbs. X 1.94	7,540
1/2 (L. L.) as for p_o	2,250
	17,065#/lin. ft.

$$\therefore u = \frac{p_s}{p_o} = \frac{17,065}{10,300} = 1.66$$

Assume reinforcement as 0.5% in each face throughout located at a distance of $\frac{h}{10}$ from the surface.

$$\therefore I_o = bh^3 - \text{From tables } d = .106$$

$$= .106 \times 4.33 \times 3 \times 3 \times 3 = 12.4$$

$$I_s = .106 \times 4.33 \times 6 \times 6 \times 6 = 99.25$$

$$\frac{I_s}{I_o} = \frac{99.25}{12.4} = 8$$

Stresses from dead load + 1/2 live load

Crown

$$H_o = \frac{p_o l^2}{48r} (u+5) = \frac{10,300 \times \overline{120}^2}{48 \times 50} (6.67) = +408,000\#$$

$$M_o = \frac{p_o l^2}{1500} = \frac{10,300 \times \overline{120}^2}{1500} = 98,000 '\#$$

Haunch--say at a spandrel col. located at a distance of $\frac{l}{10}$ from the crown

$$T = \text{say} \qquad +408,000\#$$

$$M = \qquad +98,000 '\#$$

Springing

$$T_s = 408,000 \times 1.94 = \qquad +792,000\#$$

$$M_s = \frac{p_o \times l^2}{3000} = \frac{10,300 \times 14,400}{3000} = \qquad \pm 49,000 '\#$$

Live Load Moments

Crown

$M_o = \pm Cmpl^2$ - Cm from tables at crown haunch

springing

$$M_o = \pm .0043 \times 4500 \times \overline{120}^2 = \qquad \pm 278,500 '\#$$

Haunch

$$M_o = \pm .007 \times 4500 \times \overline{120}^2 = \qquad \pm 454,000 '\#$$

Springing

$$M_o = \pm .023 \times 4500 \times \overline{120}^2 = \qquad \pm 1,490,000 '\#$$

Stresses from Arch Shortening

Crown

$$H_a = -(1 + .3 \frac{I_s}{I_o}) \frac{H_o h_o^2}{r^2 \sec B} = -1+(.3x8) \frac{3^2 H_o}{50^2 x 1.94}$$

$$= -.00445 H_o = -1820\#$$

$\frac{y_o}{r}$ = .225 when $\frac{I_s}{I_o}$ = 8 from tables

y_o = .225 x r = .225 x 50 = 11.25

M_a = $-H_a\ y_o$ = 1820 x 11.25 = + 20,450'#

Haunch

T = say - 1,800#

M = $H_a\ (y-y_o)$ = -1820 (6-11.25) = + 10,470'#

Springing

T_s = H_a x coo = H_a + su B = -1820 + 1.94 = - 940#

M_s = H_a (r-y) = -1820 (50-11.23) = - 70,500'#

Stresses for Temperature Changes

Crown	50° Fall	30° Rise
$H_t = C_t\ \frac{w\ t\ E\ I_o}{r^2}$; C_c from tables		
$= \frac{-30x.000006\ x\ 50\ x\ 288,000\ x\ 12.4}{50^2}$	= -12,850#	+ 7,700#
$M_t = -H_t\ y_o$ = 12,850 x 11.25	=+144,500'#	- 86,500'#
Haunch		
T = say	- 12,850'#	+ 7,500#
M = 12,550 x 5.25	+ 67,500'#	+ 40,400'#
Springing		
$T_s = H_t$ coo = - 12,850 x $\frac{1}{1.94}$	= - 6,620#	3,970#
$M_s = H_t\ (r-y_o)$ = - 12,850 (50-11.25)	= -497,500'#	-298,000'#

The Three Sections are now Tested as Follows:

Section at Crown

Thrust	Temperature not Considered	Temperature Considered
Dead + 1/2 live	+ 408,000	+ 408,000
Arch Shortening	- 1,820	- 1,820
Temperature - 50° Fall	------	- 12,850
Total	+ 406,180	+ 393,330
Moment		
Dead + 1/2 live	+ 98,000	+ 98,000
Live Load - Positive	+ 278,500	+ 278,500
Arch Short	+ 20,450	+ 20,450
Temp. 50° Fall	------	+ 144,500
Total	+ 396,950 1#	+ 541,450 1#
Or	4,760,000 "#	6,500,000 "#

e = Mom ÷ Thrust = 11.72" 16.51"

Dimensions h x b = 36"x52" = 1872 sq. in.

$\frac{h}{e} = \frac{3.6}{11.72} =$ 3.6 = 2.18

$R/fc = \frac{4,760,000}{52 \times 36^2 \times 600} =$ .118 $\frac{5,500,000}{52 \times 36^2 \times 780} =$ .1236

$\frac{d'}{h} = \frac{3.6}{36} =$ .1   .1

p per face Fig. 37g.

This is satisfactory.

Section in Haunch

Thrust	Temperature not Considered	Temperature Considered
Dead + 1/2 Live	+ 408,000 #	+ 408,000 #
Arch Short	- 1,800 #	- 1,800 #
Temp. 50° Fall	------	- 12,850
	406,200 #	393,350 #

	Temperature not Considered	Temperature Considered
Moment		
Dead + 1/2 Live	+ 98,000 '#	+ 98,000 '#
Live Load Positive	+ 454,000	+ 454,000
Arch Short	+ 10,470	+ 10,470
Temperature 50° Fall	------	+ 67,500
	+ 562,470 '#	+ 7,560,000 '#
or	6,750,000 "#	7,560,000 "#
$e = \frac{Mom}{Thrust}$	= 16.6"	19.25"

Thickness Haunch Section = $h = h_o + .309 \times \sec d$

$\sec = \sqrt{1 + (\frac{2r}{L})^2} = 1.3$

$h = 36 + .309 \frac{L}{10} \times 1.3 = 36 + 52 \quad 41"$

Dimensions $41 \times 52_t = 2132$ sq. in.

$\frac{h}{e} = \frac{41}{16.6} = 2.47 \qquad \frac{41}{19.25} = 2.13$

p per face $.55 \times \frac{36}{41}$ = .483% .483%

R/fc Fig. 37g

$fc = \frac{R}{bh^2 \times .128} = \frac{6,750,000}{52 \times 41^2 \times .128} = 604 \qquad \frac{2,560,000}{52 \times 41^2 \times .129} = 671$

Section at Springing

Thrust

	Temperature not Considered	Temperature Considered
Dead + 1/2 Live	+ 792,000	+ 792,000
Arch Short	- 900	- 940
Temp. 50° Fall	-----	- 6,620
Total	+ 791,060	+ 784,440

		Temperature not Considered	Temperature Considered
Moment			
Dead + 1/2 Live		- 49,000	- 49,000
Live Load - Negative		-1,490,000	-1,490,000
Arch Short		- 70,500	- 70,500
Temp. 50° Fall		-----	- 298,000
		-1,609,500 '#	-1,907,500 '#
	or	19,300,000 "#	22,860,000 '#
$e = \frac{Mom}{Thrust}$	=	24.4"	29.2'

Dimensions hxb = 72x52 = 3744 sq. in.

$\frac{h}{e}$ = $\frac{72}{24.4}$ = 2.95 $\frac{72}{29.2}$ = 2.465

R/fc = $\frac{19,300,000}{52x72^2x600}$ = .1193 $\frac{22860,000}{52x72^2x780}$ = .1086

$\frac{d'}{h}$ = $\frac{3.6}{72}$ = .05 .05

p per face Fig. 37g .3% .21%

Steel for the arch rib

Crown

Dimensions hxb = 36" x 52" = 1872 sq. in.

p per face = .55%

Use 12 - 1" sq. bars in each face

making p = $\frac{12x1}{1872}$ = .64%

Haunch

Dimensions hxb = 41" x 52" = 2132 sq. in.

p per face = .483%

Use 12 - 1" sq. bars in each face making

$$p = \frac{12x1}{2132} = .56\%$$

Springing

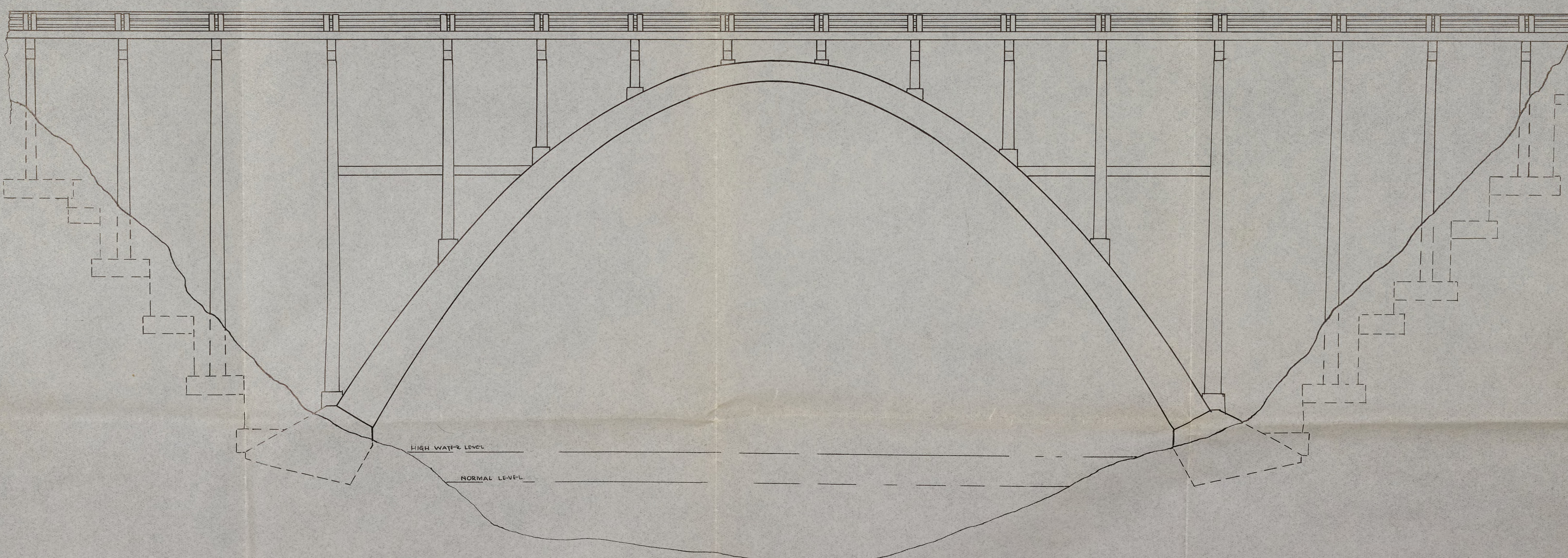
Dimensions hxb = 72' x 52" = 3744 sq. in.

p per face = .3%

Use 12 - 1" sq. bars in each face making

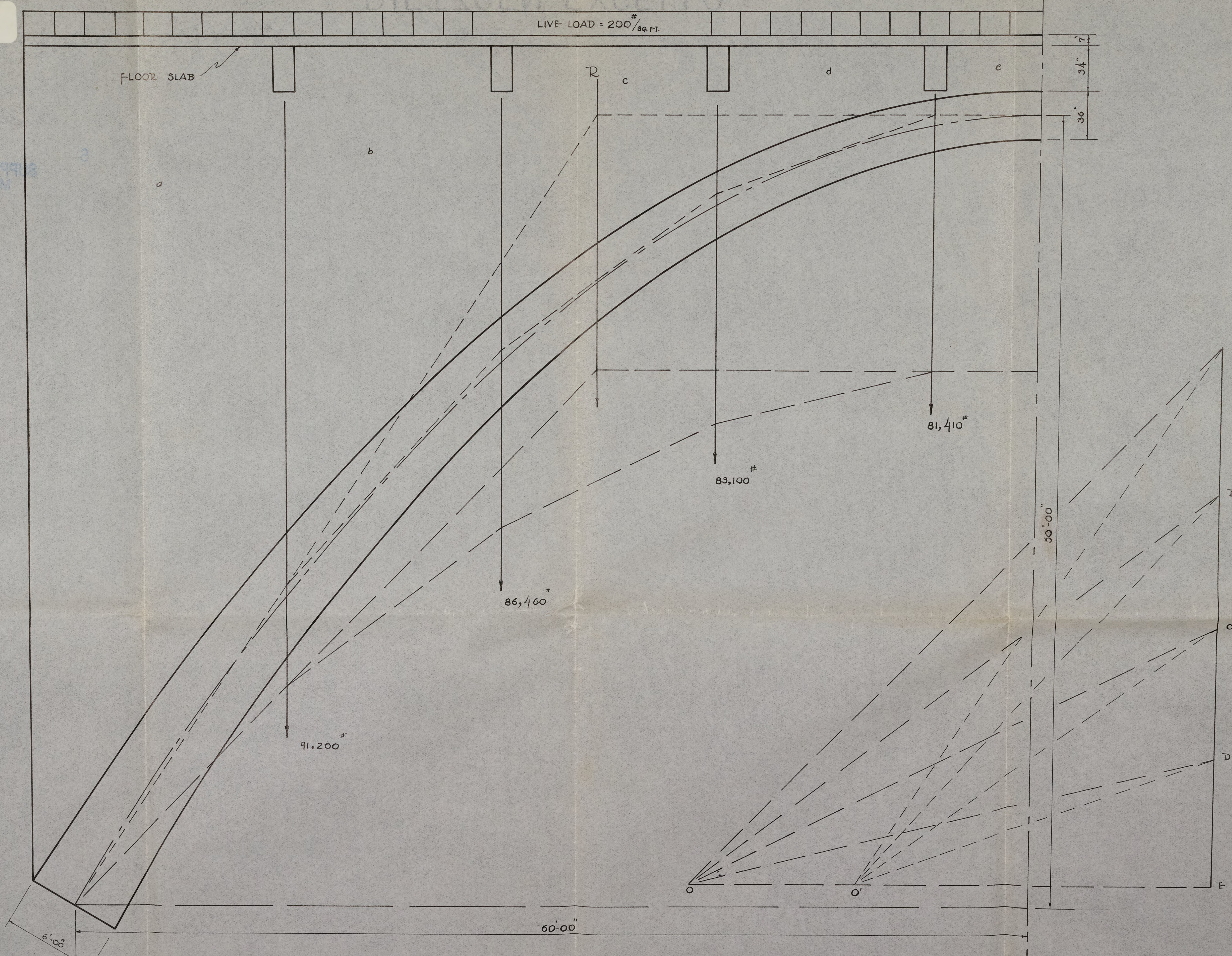
$$p = \frac{12x1}{3744} = .32\%$$

Pocket has
4 Suppls



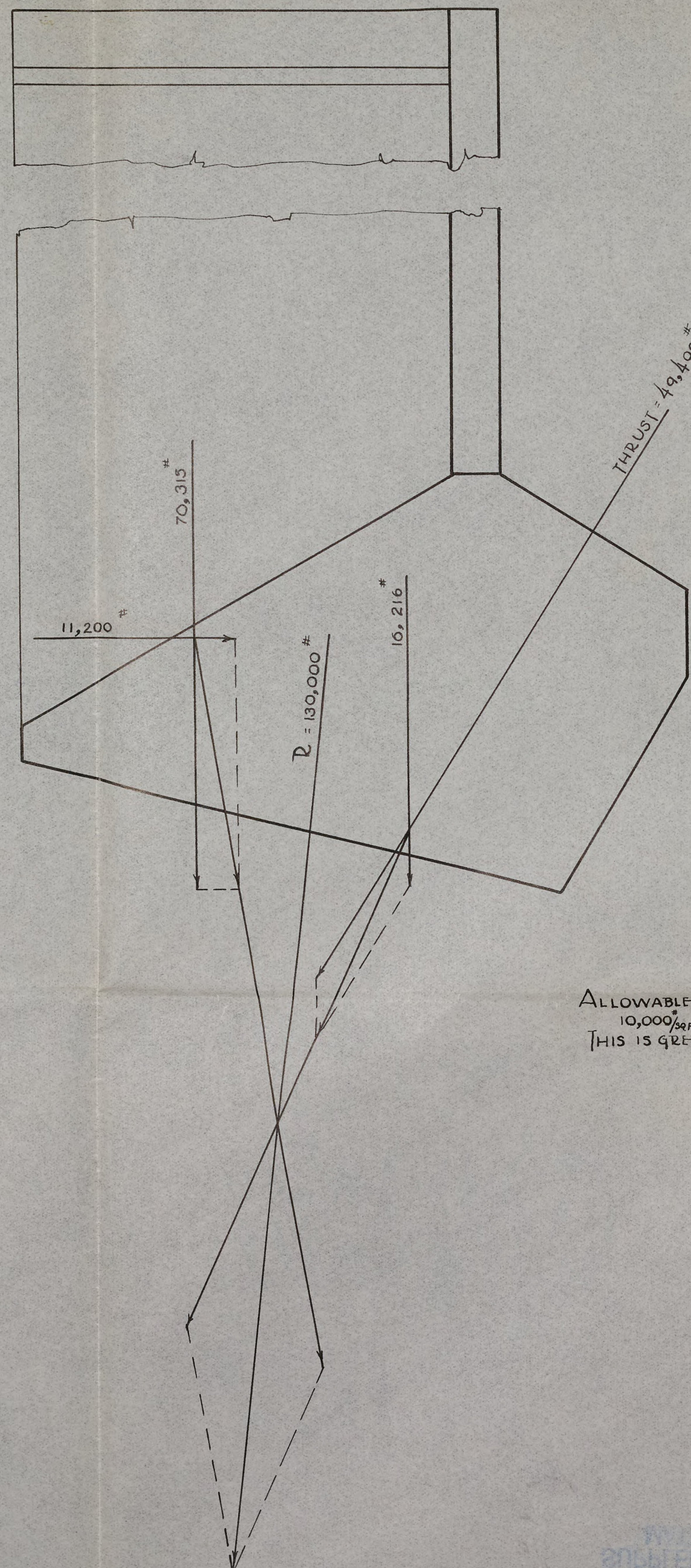
OPEN SPANDREL BRIDGE
GENERAL PLAN

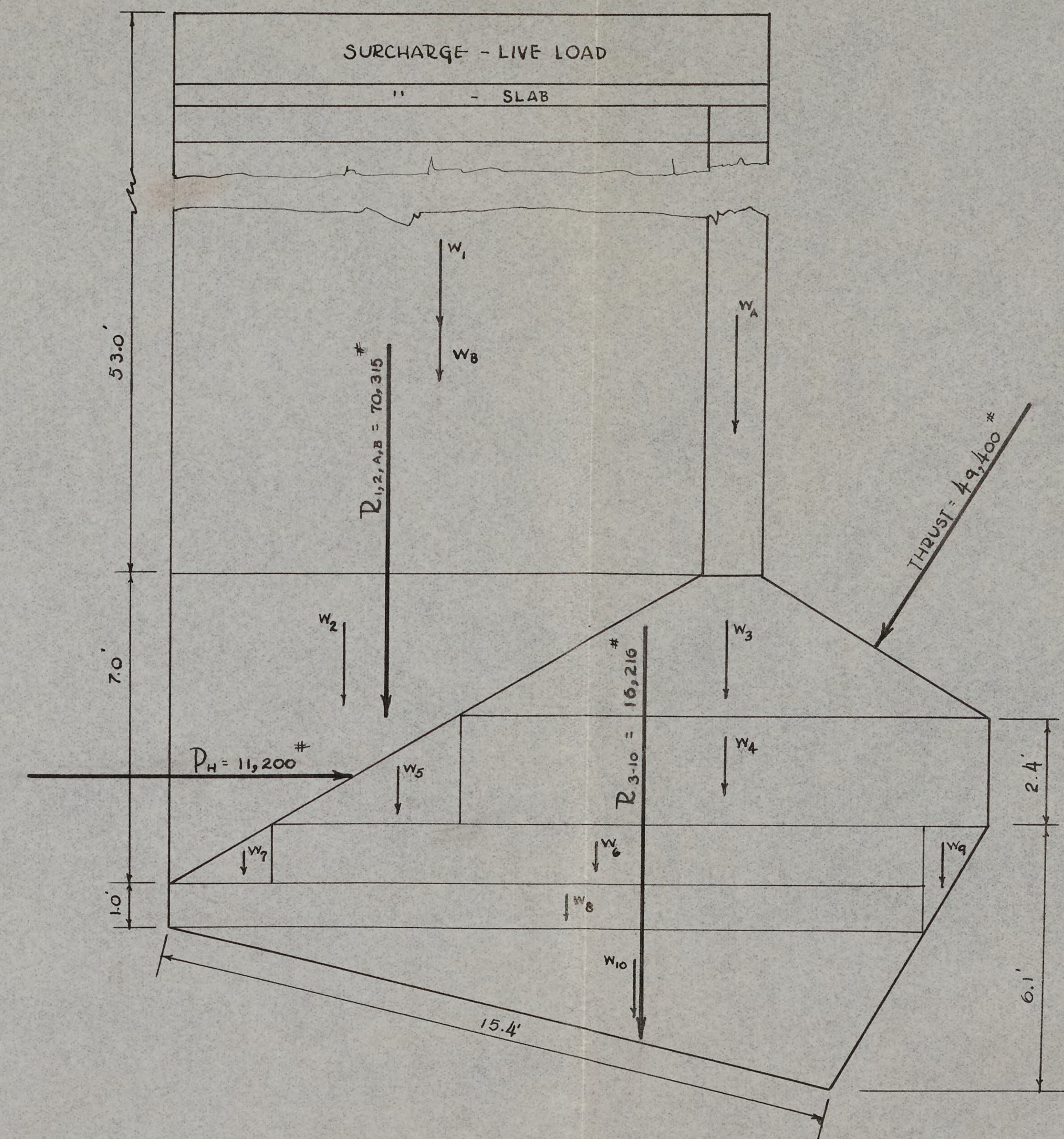
DESIGNED BY H.G.D. DRAWN BY H.G.D.	JUNE 1943 SHEET 1



OPEN SPANDREL BRIDGE
ARCH LOADING

DESIGNED BY H.S.D.	JUNE 1943
DRAWN BY H.S.D.	SHEET 2



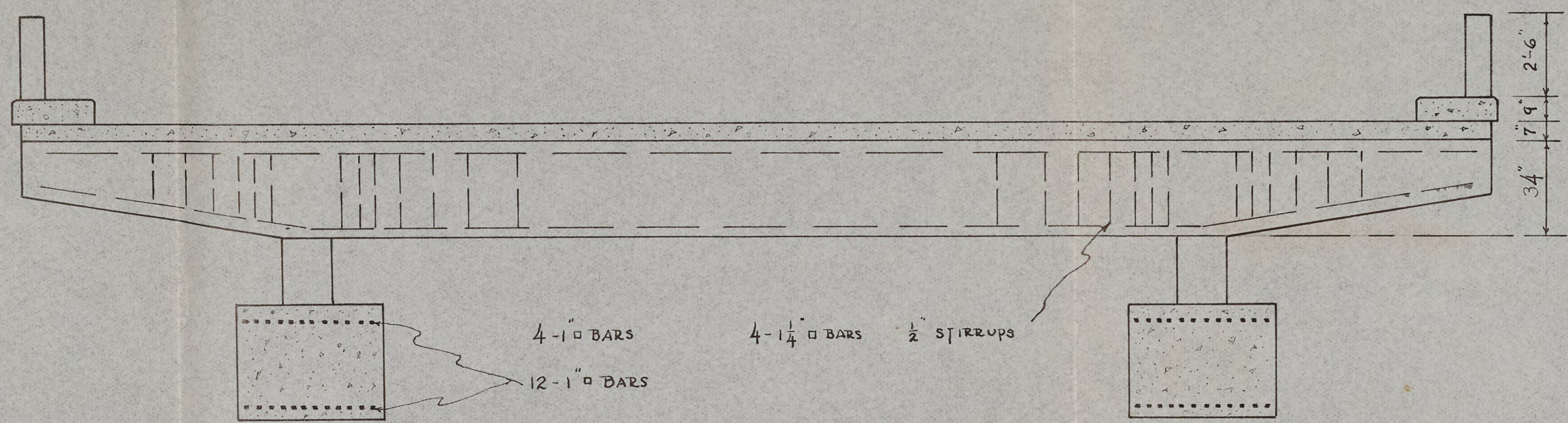


ALLOWABLE SOIL PRESSURE = 10,000#/SQ.FT.
10,000#/SQ.FT. × 15.4' = 154,000#
THIS IS GREATER THAN R, HENCE ABUTMENT IS SATISFACTORY

NOTE :-
W_A = WEIGHT OF FRONT WALL
W_B = " " WING WALLS
ALL FORCES SHOWN CONSIDERED AS ACTING ON A SECTION OF ABUTMENT 1' WIDE

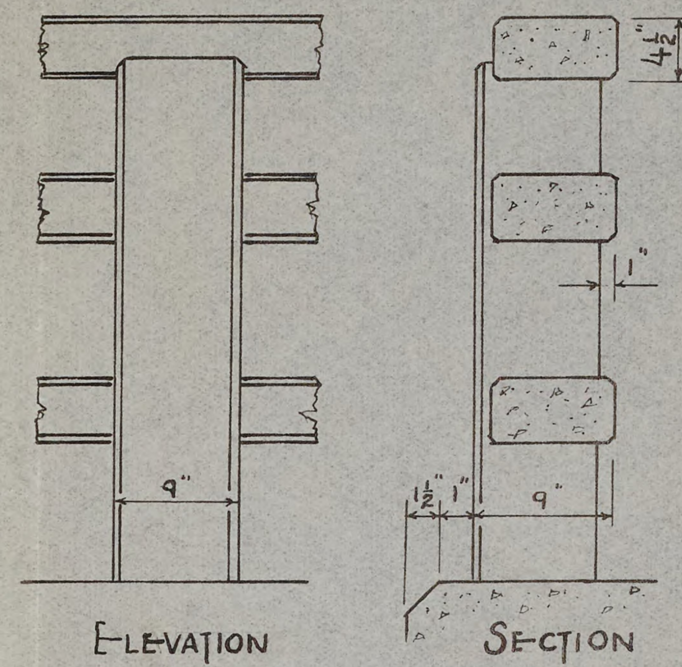
OPEN SPANDREL BRIDGE ABUTMENT

DESIGNED BY H.G.D.	JUNE 1943
DRAWN BY H.G.D.	SHEET 3

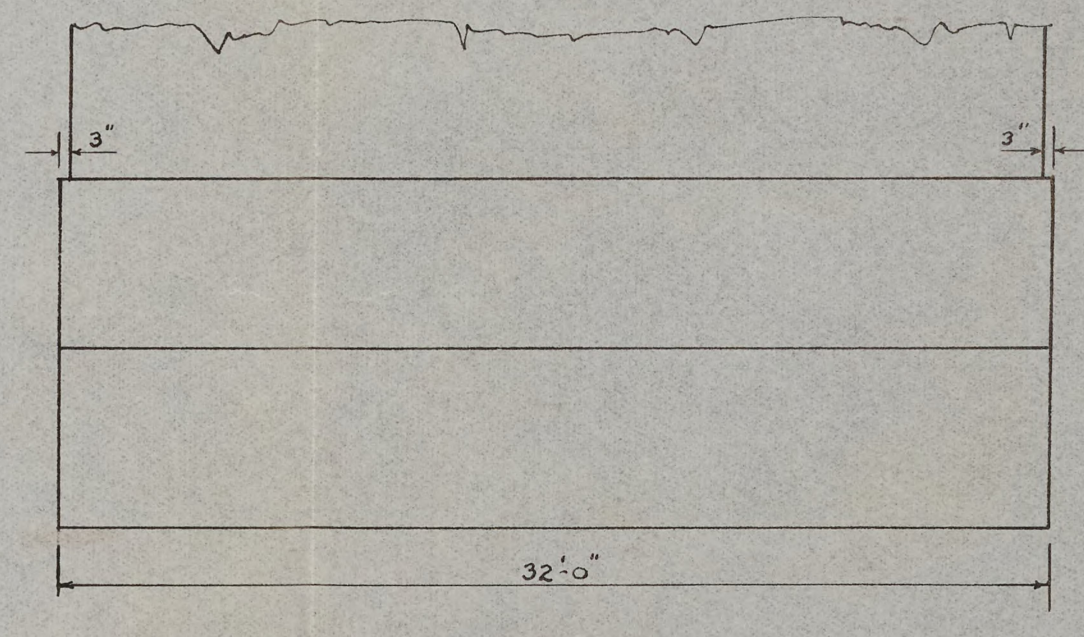


TYPICAL TRANSVERSE SECTION

SCALE 1"=4'

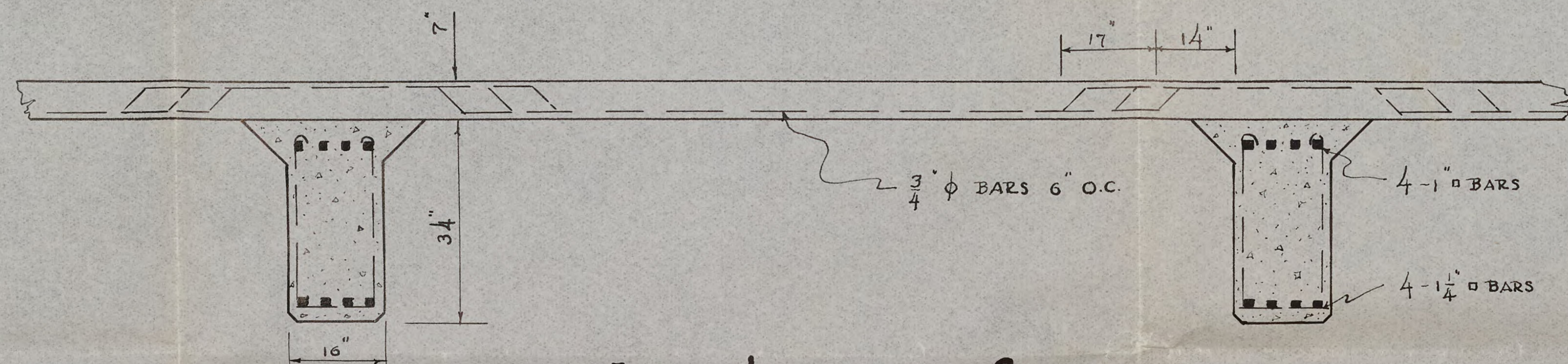


HANDRAIL



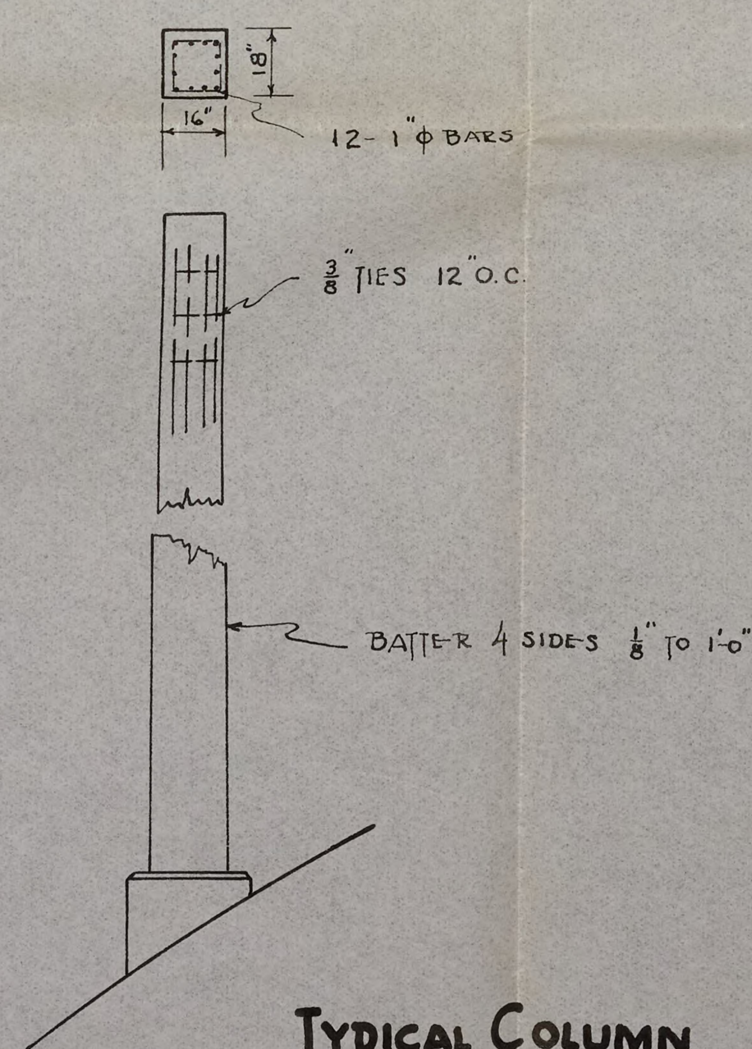
ABUTMENT ELEVATION

SCALE 1"=6'



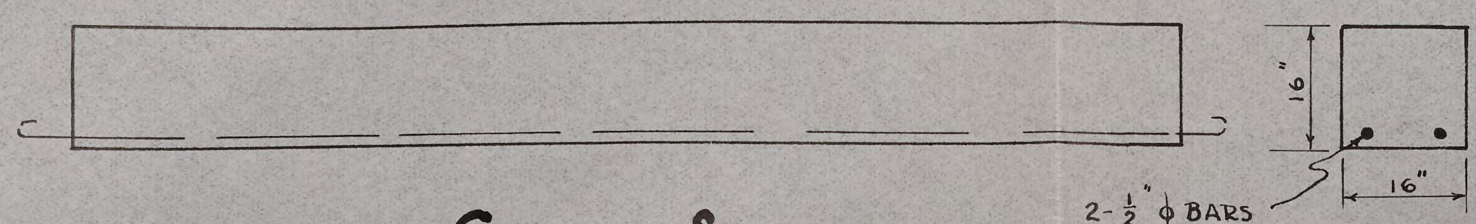
TYPICAL LONGITUDINAL SECTION

SCALE 1"=2'



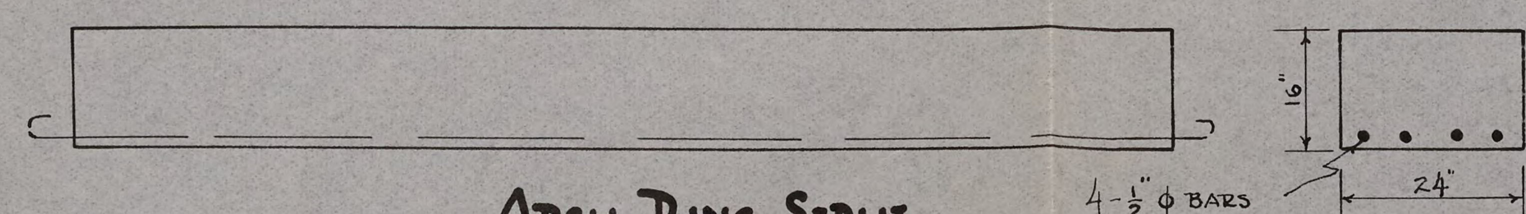
TYPICAL COLUMN

SCALE 1"=4'



COLUMN STRUT

SCALE 1"=2'



ARCH RING STRUT

SCALE 1"=2'

OPEN SPANDREL BRIDGE DETAILS	
DESIGNED BY H.S.H. / DRAWN BY H.S.H.	JUNE 1943 / SHEET 4