STRESS DISTRIBUTION AND ARCHES

THESIS FOR THE DEGREE OF B. S.

Wilho R. Keturi

1930

THESIS

Strains + stresses

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Civil engineering Bridges & roofs

STRESS DISTRIBUTION AND ARCHES

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INTRODUCTION

Considerations

A concrete arch constructed with fixed ends is statically indeterminate and is analyzed by taking into account the elasticity of the material. The arch is merely a curved beam following, generally in bridges, the curve of a parabola, ellipse, segmental or a semicircle. The selection of a curve in most cases is matter of economy.

In arches of stones it is found that the line of pressure passes through the middle third to avoid the tendency of the joints to open. In the plain concrete arch the line of pressure should also pass through the middle third, due to the plastic material used in the arch. As there will always be some tension produced in the arch it is necessary that the arch be monolithic to withstand the tension and allow the line of pressure to pass outside the middle third. But due to the low strength of concrete, reinforcing is used to permit a greater variation of this line which is of little account theoretically since the direct compression in the arch usually controls to such an extent that the allowable stress in the concrete permits of but a small unit tensile stress in the steel. It is obvious that the steel adds greatly to the reliability of the construction and makes possible a higher working stress in the concrete than could properly be employed in the design of the plain concrete structure. The higher working stress produces a thinner ring and lighter abutments.

In considering the action of any structure under a load whether it is statically determinate or indeterminate it is necessary that the geometric requirements imposed on the structure be fulfilled, such as having constant span length, etc. The sum of the components in any direction of all forces acting on the whole structure or any part of the structure be equal to zero, and the sum of the moments of the whole structure or any part be equal to zero. The ratio of stress to strain in the differential parts of the structure must be so related to satisfy the properties of the materials.

Purpose

The analysis of an arch by the elastic theory consists in the finding the thrust, shear, and bending moment at the crown and at intermediate sections in the arch ring or arch rib, and then finding the stresses resulting therefrom.

It is evident that many materials of construction, especially concrete during the first few months after fabrication do not follow the laws upon which the theory of elasticity is based. Some of the factors in the analysis vary, as the yield of a stressed body with time and the ratio of stress to strain. For this need L.G. Straub has proposed methods of analysis termed as the "theory of plasticity" and the "proposed theory of elasticity." The methods provide a general statement of the displacements and deformations in structures to fit a greater number of properties of materials. The investigations made by L.G. Straub are based on experiments of precise and accurate measurements. The new theories are developed with the idea of supplementing the order of theory of elasticity.

Obviously, the new theories tend to make the old inadequate for studying stresses in statically indeterminate structures. They undoubtedly make for a more scientific study of arches and will eventually result in better economy in saving material, etc. The construction of arches by the older theory has not been inadequate but has been made by the use of reinforcement.

It is clear that the strength of materials of arches and structures is much discussed. By graphical methods the lines of thrusts can be shown for various loadings in arches. The graphical methods show quite accurately the distribution of loads through the arches, but not exactly. Dr. Coker's experimental arrangement using polarized light for stress distribution and determination by the use of transparent models can be used conveniently in illustrating what occurs in celluloid model arches.

Dr. Coker's photo-elastic method affords a means of direct measurement of stress in members of celluloid. This means has been used to advantage in the mechanical lines for standard test pieces, gears, etc. Due to our crude arrangements and lack of time we are unable to get quantitative results, but a study of the stresses can be had and the information gained from these polarization tests afford a better conception of a structure when placing reinforcement.

Analysis of Arches

The analysis of arches by the elastic theory method is the most common and will be briefly outlined. A concrete arch with fixed ends is statically indeterminate. The analysis consists in finding the thrust, shear and moment at the crown and intermediate points. There are three unknowns at each support and three unknowns for the deflections of the arch making six in all. At the supports, the magnitude, the direction and the point of application are the first three unknowns and are determined by the principle of statics. The other three consist of the displacement horizontal, vertical and angular. The horizontal is the change that occurs in the span length and is denoted by $\triangle x = 0$. The vertical is the relative displacement of

one end to the other end is denoted by $\Delta y = 0$ and the angu-
1
lar displacement is that which takes place at the two ends where the angle between the tangents to the arch axis remains unchanged and is denoted by $\Delta k = 0$.

The method of procedure followed in the arch-ring design as given by Holl is as follows:

1. Assume a thickness of arch ring at the crown and springing.

2. Lay out the curve assumed for the introdos.

3. Lay out a curve for the extrodos to give as nearly as possible the assumedring thickness at the springing.

4. Draw arch axis between the extrodos and introdos.

5. Divide the arch axis into an even number of divisions such that the ratio S/I is constant for all.

6. Compute the dead and live loads.

7. Compute Hc, Vc and Mc at the crown for the different conditions of loading.

8. Draw the force polygons for the different conditions of loading and the corresponding equilibrium polygons, or line of pressure.

9. Determine the thrusts, shears, bending moments and eccentric distances at the center of the S/I division of the arch ring for the different conditions of loading.

10. Compute thrust and moment for variation in temperature.

11. Compute thrust and moments for rib-shortening.

12. Combine the thrusts, shears, and moments due to the different condition of loading with the thrusts, shears, moments due to temperature and rib-shortening.

13. Compute the maximum stresses — compression in the concrete and tension in the steed — due to the thrusts and moments.

No attempt is here made to develop formulae deduced in textbooks. The equations are the same as those given by Heel.

Deflection formulae are first given because of their use in the arch theory. The material considered is reinforced concrete. The formulae apply to curved beams with a radius of curvature large compared to the depth. The arch is assumed with one end fixed.

The angular change at the crown as represented by Figure 1 is denoted by k or $k = \sum_A^B \frac{Ms}{EcI}$

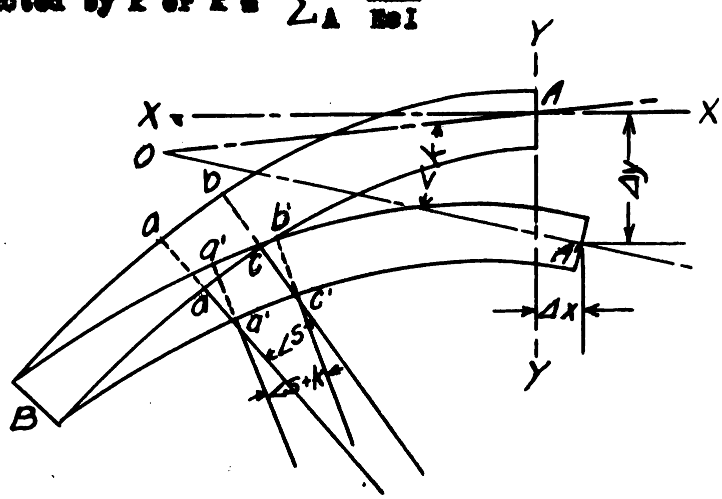


Figure 1

Figure 2 shows the same bending with the components of deflection given, namely, the horizontal and vertical movements. Horizontal movements = $\Delta x = \sum_A^B \frac{Mys}{E_c I}$

Vertical movement = $\Delta y = -\sum_A^B \frac{Mxs}{E_c I}$

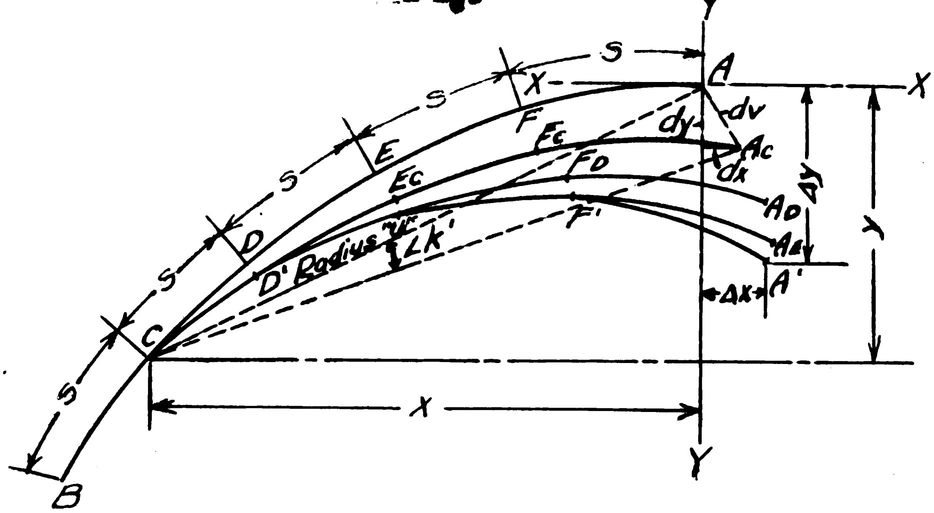


Figure 2

Formulas for Thrust, Shear, and Moment

Figure 3 represents a symmetrical arch cut at the crown to show the acting forces at the half section.

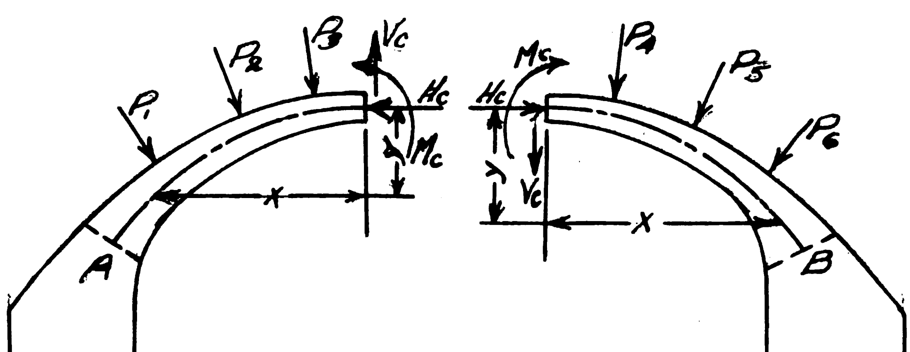


Figure 3

$$Hc = \frac{Nh\sum(m_L + m_R)y - \sum(m_L + m_R)\sum y}{2\left[Nh\sum y^2 - (\sum y)^2\right]}$$

$$Vc = \frac{\sum(m_L - m_R)x}{2\sum x^2}$$

$$Mc = \frac{\sum(m_L + m_R) - 2Hc\sum y}{2Nh}$$

The values of the thrust, shear, and moment being found from the above equations, the moment of any section of the left cantilever is $M = Mc + Hcy + Vcx - m_L$ and, at any section to the right cantilever,

$$M = M_c + Hcy - Vcx - m_R$$

<u>Formulae of Thrusts, Shears and Moments Due to a Change of Temperature</u>

$$Hc = \frac{I}{S} \cdot \frac{tc\varepsilon_p lNhEc}{2\left[Nh\sum y^2 - (\sum y)^2\right]}$$

$$Mc = -\frac{Hc\sum y}{Nh}$$

The bending moment at any point is

$$M = Mc + Hcy$$

<u>Formulae of Thrusts and Moments Due to Rib Shortening</u>

$$Hc = \frac{-I}{S} \cdot \frac{e_s lNh}{2\left[Nh\sum y^2 - (\sum y)^2\right]}$$

<u>and for Temperature Stresses</u>

$$Mo = -\frac{Hc \Sigma y}{Nh}$$

$$M = Mo + Hcy$$

MODELS

Construction of the Models

So far, an analytical method is given by which the arches are designed. The knowledge of the existing conditions of stress distribution is still vague and so to supplement this, three typical arches were made of transparent material for the purpose seeing the existing conditions in the loaded arches. At this point recourse is made to experiment with polarized light. Such experiments were made with the eliptical, parobolic and the circular arch.

The arches were cut from a rough piece of celluloid plate. The plate was sandpapered with three grades of sandpaper to smoothness. The shape of the arches were first made on ordinary paper and then pasted on to the plate from the three models were cut off with a coping saw. The rough surfaces were smoothened and the arches polished, first on the canvas and then on broad-cloth for the finishing touches. Supports were made of wood and spandrel walls were constructed over the arches as shown in the pictures. The loading from the walls were concentrated to five points equi-distant from each other. This was done by soldering tin-lead to the shape of a lump at each point of concentration to a strip of tin that was nailed to the curved surface of the wall. These points were then scraped to size such that from each point an equal load would be distributrd. The accuracy of loading from these points is difficult to determine. However, the points are filed by comparing the bands produced by polarized light.

Various other difficulties are encountered which make computing the exact stresses in this case practically impossible. One difficulty encountered is the sizes of the arches. They are as follows:

<u>Parabolic</u>

span	5 inches
rise	2-1/2 inches
thickness at crown	3/8 inches
springing	3/4 inch

<u>Elliptical</u>

span	5-1/2 inches
rise	1-1/2 inches
thickness at crown	1/2 inch
springing	1/2 inch

<u>Circular</u>

span	5-1/2 inches
rise	15/8 inches
thickness at crown	1/3 inch
springing	2/3 inch

The thickness of the material in all is about 1/4 inch. This shows immediately that the arches, even though small, are exaggerated greatly in thickness. So the strains shown by the polarized light will also be greatly exaggarated through an area that might other wise show a variation in the lines of stresses where one line would coincide with another and balancing the results to some extent and perhaps leaving no stress at the particular point. Another thing that enters arches and is not shown in the models is the reinforcing which in itself takes up stress. However, this experiment will help to visualize what happens inside the arch. While in the construction another difficulty arose but was remedied. It was noticed that initial stresses were produced at certain spots such as, for instance, at the springing line. Each of the arches had these initial stresses at that end of the arch. From this was inferred that the original plate from which the arches were cut an initial stress was produced. It may be possible that in the course of construction that some parts were unequally balanced by temperature heat, hence causing a stress in such a portion. The stresses were eliminated by placing the celluloid models in water of rising temperature to a point near the plastic stage of the material or near 180 degrees. The models were allowed to cool in this water for a period of some five or six hours, thereby bringing the material slowly back to its nomal state leaving practically no strains in the materials.

Polarizing

A simple box is used for experimental arrangements that contains the source of light and the reflectors. A few rows of blue incandescent lamps placed above a diffusing screen reflecting plane polarized light at a black mirror. The mirror is set at 45° to the diffusing screen or to the horizontal. From this mirror the plane polarized light is passed through the transparent model to an analyzer and to the eye of the observer. The analyzer is another black mirror set at 45 degrees to the vertical. The image that appears on the analyzer appears colored according to the strained conditions in the arch. An unstrained part will appear black and other parts appear in different tinted colors according to the amount and kind of strain, whether the strain is tension or compression. This arrangement is much simpler than that used by Dr. Coker in his experiments.

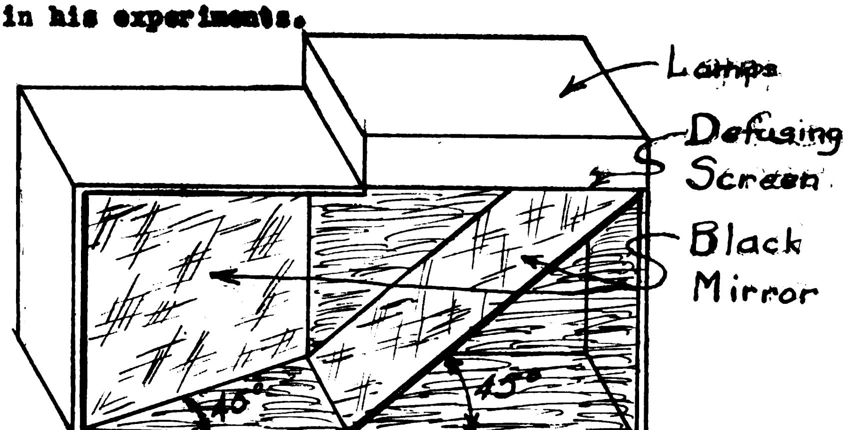


Figure 4

Statement of the Theory

Light, according to physics, is an electromagnetic disturbance. The light waves travel in all directions. Plane polarised light travels only in one direction in one plane. It may be clearly understood as passing through a narrow slit, immersing with a single beam. If another plane containing a similar slit is placed in the beam of light coming from the first plane and is made parallel to it, it will then allow the same beam to pass through without interference, but if the second plane is placed perpendicular to the first it will allow no light to pass through. The coloring effect produced in the models may be understood as rotating the second plane 90 degrees, being black when the slits are at right angles and passing through the colors until it coincides with the beam and giving the white ray of light.

Plane stress in a homogeneous medium produces no color, as the incoming rays of polarised light coincide with the lines of stress.

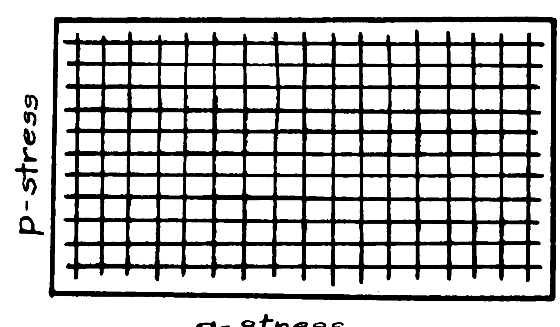


Figure 5

All systems of plane stress lie in one plane and are represented by double system of curves intersecting at right angles at all points. These lines may be called Principal Stress Lines and are denoted by p and q. There is also a three dimensional stress but in our study we consider only plane stress. Tangent at every point on the line shows direction of the principal stress.

As stated by Dr. Coker, "Light is thought of as being separated into two polarized components with vibrations at right angles and the direction of the vibrations coinciding with the principal stress. If principal stresses are unequal one of the vibrations is retarded with respect to the other; this relative retardation being proportional to the principal stress difference at the point considered." Mathematically expressed:

$$c(p-q)t$$

when

$p=$ one principal stress

$q=$ other principal stress

$t=$ thickness of specimen

$c=$ optical constant for material used

The colors produced in the specimen may be an order of colors or a series of colors. As p and q differ more and more this series of colors produced may pass through more than one order. And as the relative shift of different colors becomes greater, there is a consequent dimming of colors. The white results from a superposition of all colors of equal intensity. As p and q increase (p-q) colors pass through definite series. In celluloid when p-q = 0 or p=q=black. With thedifference increasing the colors follow in this order: straw, orange, red, blue, green, and again straw, orange, red, blue, green, etc. In celluloid these colors represent a certain stress for each color. When the amount of stress for each color is found that color will stand for that stress in celluloid of any shape and form, except where the model might be of such a shape as round and the colors become too confused.

INVESTIGATIONS

Thus far, a method of arch analysis has been given and a statement of the theory of photo-elasticity. A knowledge of the magnitude of the loadings and deflections is required for the approximate computations of the magnitude of stress in the various points of the arch. As mentioned before, we are unable to make quantitative measurements and our time does not permit, so conclusions are drawn by inspecting the color fringes. It might be well to state two methods used in determining stresses. The simplest but less accurate method is the comparison of colors known in a stressed specimen to the specimen in question. The other method is the compensation method.

Various photographs were taken of the arch when placed in the polarized light. The photographs were taken with Eastman Kodak Company's panchromatic cut films. The film differs from ordinary films in that it will show a greater contrast of colors. The printing was done on contrast paper. The photographs show bands of uniform darkness and brightness in the arch and has this disadvantage, that black does not represent the same stress; neither does gray nor white.

The first photograph, Figure 7, was taken of the parabolic arch, inserted in the instrument with a sixteen pound weight hanging out on an arm as in Figure 6.

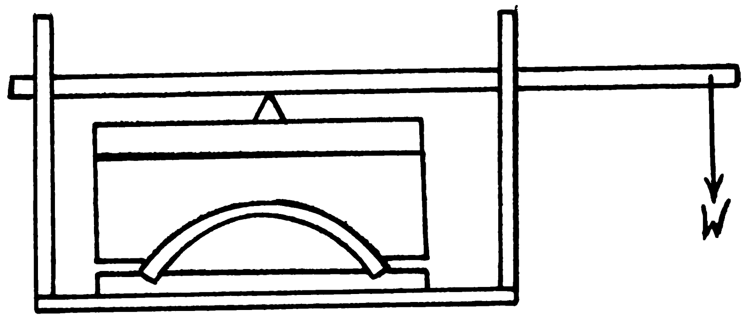


Figure 6

The loading was attempted in such a manner that it would be concentrated equally at each of the five points. However, this was not accomplished. A rough estimate of the loading shows that our loads are somewhat more than the proportional loads generally used.

From the photograph in Figure 7 it will be noticed that the maximum shear occurs at the points of application. The stress produced in the arch is mainly a compression stress. With the varying cross-section the compression also varies. The compression stress is disturbed at each of the five points where there is a concentration of stress. The maximum load at these points reveals that there is a reversal of stress and the spots in the arch are spots of high stress concentration. The stress is reversed into a tension stress so that at the bottom of the arch at each point, to the form of the curved band, tension is produced, being maximum when directly beneath the point. The amount of tension then follows the curve to a minimum and changes over to a compression stress. It may be illustrated as follows, with tension on the inside of the arch and compression in the arch and outside as shown in Figure 8.

Figure 7

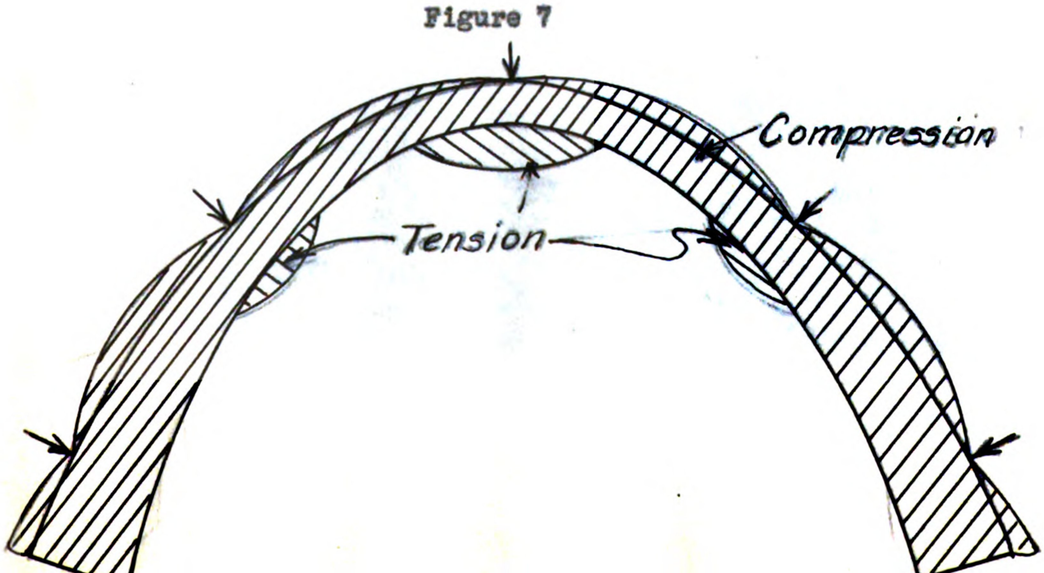


Figure 8

It can readily be inferred that at these points, stress concentration becomes serious due to the alternating stress. Due to stress reversals, as in this case, a crack may start which gradually spreads and produces failure. Between the points the compression stress seeks a higher level producing tension near the lower surface. At the base of the arch on both ends another spot of high stress concentration is found. Here we find no reversal of stress but the same compression stress highly concentrated in a band appearing black in the photograph. The band is parallel to the base.

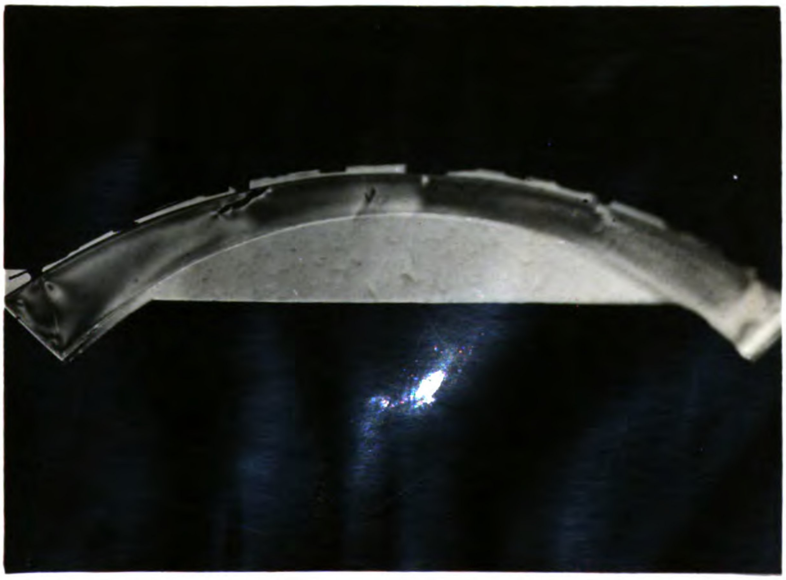


Figure 9

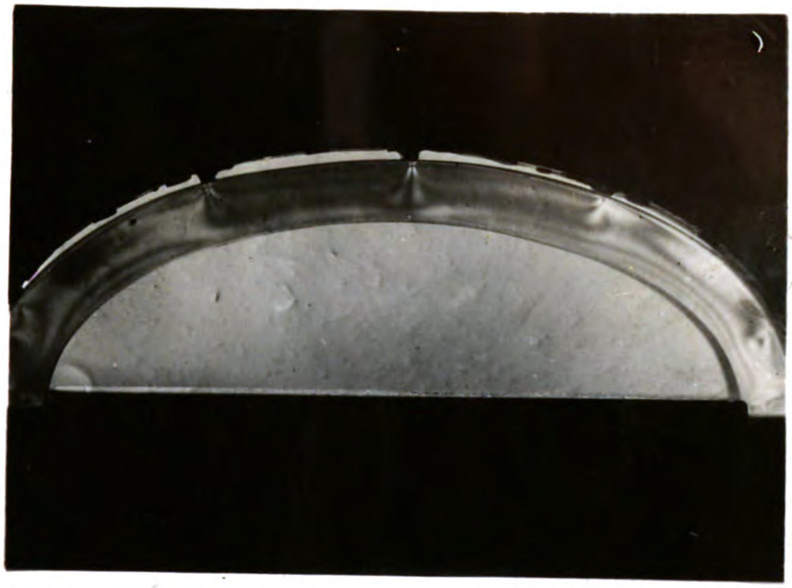


Figure 10

Figure 11

Figure 11 shows this up well. The loading is all from one point at the crown. The black band shades off into the dark bands showing clearly at the base a maximum stress for a short distance and distributed more or less evenly. In Figure 11 the reversal of stress is displayed over a greater area and reveals a sounder proof of what has already been said of alternating stresses.

In order to get a better conception of the colors in their relative magnitudes of stress a table of colors devised by R.V. Baud in his experiments might be used. The table is made from celluloid gear tests making most things equal. Mr. Baud, in his tests, used an ordinary black and white film having an orthochromatic emulsion. So we may safely use the table with probably only a little variation, at least, not a very great difference.

TABLE OF COLORS

Order	Color	Black and White Shade as on Photograph	Band No.	Stress or Stress Difference (p-q) in pounds per square inch
1	Black	Black	0	0
	Straw	Grey		0-x
	Orange	White		x-y
	Red	Grey		y-z
	Dark green	Black	1	etc.
	Light green	Grey		

Order	Color	Black and White Shade as on Photograph	Band No.	Stress or Stress Difference (p-q) in pounds per square inch
2	Orange	White		
	Red	Grey		
	Green	Black	2	
3	Orange	White		
	Red	Grey		
	Green	Black	3	
4	Orange	White		
	Red	Grey		
	Green	Black	4	

The three arches described were subjected to the same load conditions and very much the same stress display was produced. So then, the same load was tried from one point, the center point at the crown. This, as already described, shows in more detail the reversal of stress. The iso-clinic lines are more pronounced showing the series of orders of colors. The strain evidently being greatest near the center line of the arch and lessening towards both edges until it becomes zero.

The third trial was made by subjecting the load over the first point which seemed to give some marked results, Figure 12. The load was distributed into the arch from three points to the greatest amount, the first, the second or the point at the crown, and the third point. The stresses at the first point act rather peculiarly from what has already been seen. A greater amount of tension near the base opposite the point is produced. The approximate compression condition is shown in Figure 13 in the same manner as before.

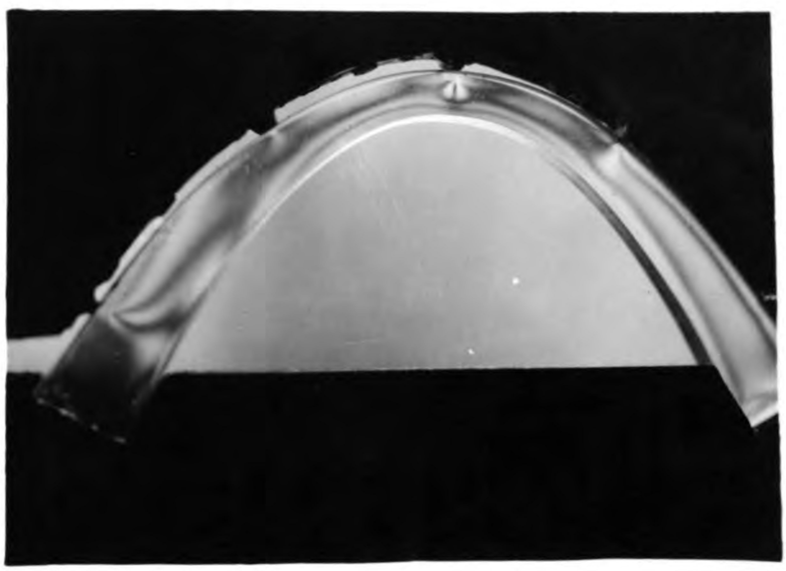


Figure 12

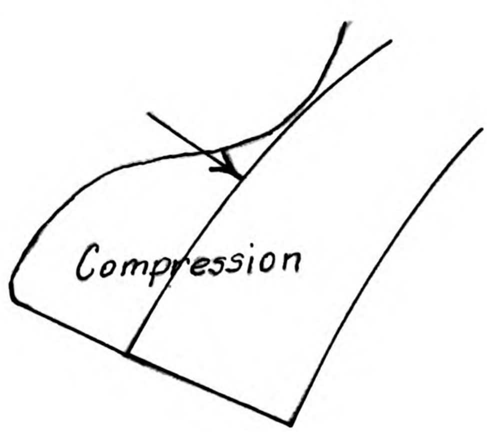


Figure 13

The figure also reveals that the load when over the first point pulls down on the other two points therefore causing tension in the spandrel wall, from the first to the second and third points. The same condition seemed to hold true for the other two arches. In the arch very little tension is found (as nearly as one can tell from inspection). At the first point there is very little if any tension. Tension at the crown and end point is produced as illustrated in Figure 13. Another noticeable thing to take into account is the way stress behaves through that half of the arch. Between the first and second points the full width of the arch is stressed to about an equal amount and does not follow the common rule of the pressure line passing through the middle third. Fortunately the stress is all compression so that if the cross-section is isotropic the pressure exerted will deform the arch at all points on the cross-section equally or nearly so. Then, if used in actual practice, the material used would be designed for high slippage resistance. From the second point to the crown compression passes through the middle third.

The investigations so far seem to tell us that all the arches tend to act in the same manner. The loads are distributed in very much the same manner and in each case follow through the middle third. Where the load is from one end the same rule holds true. These conclusions verify the graphical methods for pressure lines. One thing, so far not mentioned and found in each trial, is the manner in which the load applied acts in the arch. At each point the line of shear acts perpendicular to the arch axis. This is equally true of the case where the load is applied from the end. The load applied to the arch from any direction acts normal to the arch axis.

In the last trial a uniform load over a part of the arch was tried which did not work out successfully, Figure 14. The uniform load seems to distribute the stresses quite evenly producing no alternations. The strains follow the curve of the arch symmetrically. They are not as strong or distinct near the introdos in the upper half of the arch as they are through the middle and nearer the extrodox. Near the base they spread out ending in the width of the arch. It would seem plausible to say that practically no tension is produced in an arch where the spandrel wall covers the entire arch at all points. The load is not distributed qt a single p oint or small area but over a greater area. The action of a load over a spandrel wall of this nature may be illustrated as in Figure 15. The same may be shown in Figure 16 with a load from one end. Here the tendency could be for the arch to act somewhat like the one in Figure 12.

Figure 14

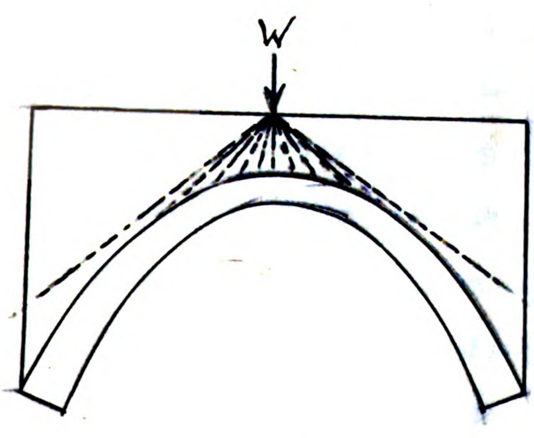


Figure 15

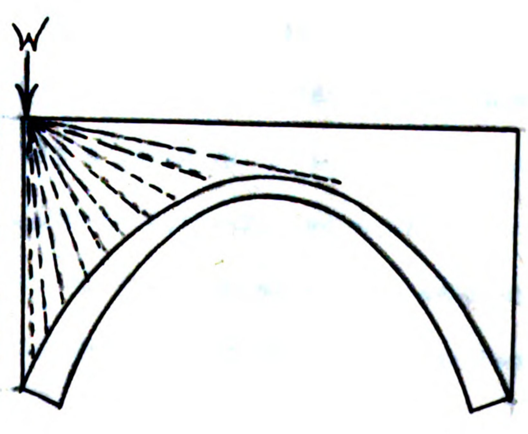


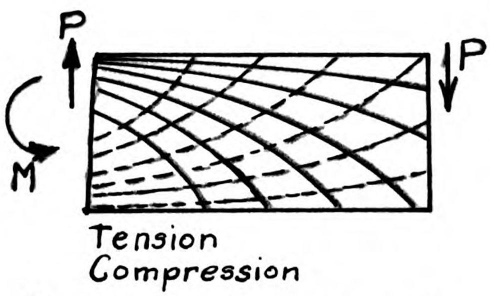
Figure 16

The compression and tension lines help to visualize the amount of stresses. If quantitative results at this time were possible the exact amount of compression and tension could be computed. However, knowledge of the manner in which stresses are distributed has been gained. The photo-elastic method also gives complete information of the properties of materials. Localized stresses are very complicated and without the use of this method and becomes practically impossible to solve. All this leads us to believe that in design of most any kind the photoelastic method is a great aid.

Experiments with models indicate stress differences (p-q) in material and the shear is proportional to this stress. The shear stress can be determined for most any body capable of being represented by models. Where failure is likely to occur the location can easily be determined. "Since breakdown of the structure shows itself by a clouding of the color bands," states Dr. Coker, "and the ultimate formation of a black patch in the over-stressed area which on the removal of the load shows its presence by its persistence when stress intensity has been carried far enough to break down the internal structure ver much. A small overstress experienced the removal of the load causes these places to appear as white patches."

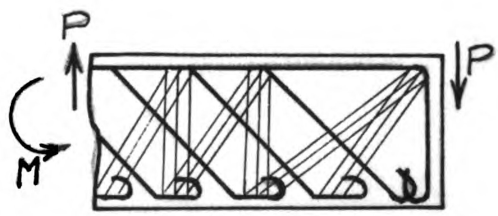
Obviously, a clear picture of stress distribution is a quick means of determining whether a failure is due to faulty material or poor design.

"It is well known that concrete is weaker in tension than compression and can be strengthtened by reinforcing bars," states Mr. Baud. "To do this the direction and magnitude of the tension at all points of the structure must be known. This is precisely the information gained from the polarization tests. This would be too expensive to follow exactly the lines of stress show but an approximation as to shape and location is usually made."



Beam showing principal stress direction.

Figure 17



Same beam reinfored

Figure 18

CONCLUSIONS

I.

All forces applied to an arch act perpendicular to the arch axis.

II.

Loads acting on an arch through arch ribs give rise to tension; a reversal of stress may occur or a high concentration of stress.

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