

A STUDY OF BRIDGE STRESSES BY PHOTO-ELASTICITY

Thesis for the Degree of B. S. R. A. Briggs
1937

THESIS

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A Study of Bridge Stresses By Photo-Elasticity

A Thesis Submitted to

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of

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Candidate for the Degree of
Bachelor of Science

THESIS

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INTRODUCTION

When it was mentioned that a subject for a thesis should be considered Photo-Elasticity was one of the first things thought of. This was probably brought about by the fascination experienced in the study of Polarized Light in a Physics course. This led to an investigation of the possibility of such a subject. It was immediately apparent, however, that this topic was impossible for two reasons.

- (1) The cost of Photo-Elastic apparatus was found to be anywhere from \$300.00 up.
- (2) There were no experiments or investigations published or available in this subject that could be readily understood without preliminary study.

However, in the latter part of 1936 two events resurrected the hopes and crystallized the possibility of Photo-Elastic investigations.

- (1) A new efficient method of Polarization by means of an inexpensive piece of apparatus.
- (2) By the publication of "Treatise of Photo-Elasticity" by Coker & Filon, two Englishmen who for some years have done extensive study on this subject.

Using this new Polarizing method and by studying the Coker & Filon Treatise it was decided the apparatus could be built at a small cost and investigation carried on in the five or six weeks allotted during the Spring term for thesis work. By using the apparatus built for this thesis simple problems in tension and

compression could be easily determined. It takes only a slight addition to this apparatus to solve more intricate problems, such as tracing the lines of principal stress. However, investigation of this kind will be left to some other student.

CHAPTER I

General Theory of Photo-Elasticity

"Photo-Elasticity is the science which deals with the effects of stress upon light traversing transparent material." In applying this subject to engineering we determine stresses from the optical effects caused by Polarized Light traversing these transparent materials. It is apparent, therefore, that we understand what Polarized Light is. We cannot jump headlong into a definition without discussing the essentials of light itself.

Light is a disturbance, believed to be electromagnetic in character, which is propagated in empty space, or in transparent materials, with a finite velocity. An elementary source of light is one that may be treated as a point from which a disturbance traverses outward. If the medium is homogeneous the velocity of the light will depend only in the direction in which it is moving. If, however, the medium is Isotropic the wave-velocity will be the same in all directions and the wave-surface becomes a sphere. Examples of this are water, glass, vacuum, etc. If, however, the medium happens to be a crystal the wave-surface is two-sheeted, that is, the light wave splits up into two rays both of which become polarized. Examples of this can be seen in Iceland Spar, Mica, etc. It is easily realized that as the light leaves the elementary source it is vibrating in all Azimuths this is known as common or unpolarized This now brings us to a definition of Polarized Light. light.

Polarized Light is light in which the light-vector is throughout parallel to a fixed direction, that is, the light is all vibrating in the same plane and this plane is called the Plane of Polarization. The angle that it makes with a fixed plane through the wave-normal is the Azimuth of Polarization.

It was mentioned before that as light traverses a crystal there is two distinct wave-fronts. The waves corresponding to these two wave-fronts, which travel with different velocities, are always Plane-Polarized in definite directions depending on the directions of the wave-normal and the nature of the crystal. These two directions are always perpendicular and are known as the Polarizing Axes.

It was discovered by Sir David Brewster in 1816 that as a transparent Isotropic material is subjected to stress it behaves like a temporary crystal. Since his day the laws of this phenomenon have been investigated by many men who instigated the following laws.

- (1) The directions of polarization corresponding to a given wave-normal are along the directions of principal stress in the wave-front.
- (2) If, P, Q are the principal stresses in the wave-front, np, nq the refractive indices of the waves polarized in the directions of P, Q respectively no the refractive index of the unstrained materials, and R the normal stress across the wave-front.

$$N_p - N_0 = c_1 Q + c_2 (P + R)$$

$$N_q - N_o = C_1 P + C_2 (Q + R)$$

The coefficient C_1 , C_2 are termed the Stress Optical coefficients. If P = 0, so that the plate is under a simple tension Q, the wave is then polarized in the directions of the tension.

It is also found that these laws hold for considerable amount of stress, generally exceeding the elastic limit.

Since polarized light is the essential in this study it must be understood that there are various methods of producing this kind of light.

- (1) Glass Spar Prism
- (2) Nicol's Prism
- (3) Reflection
- (4) Transmission Polarizer (Pile of Plates)
- (5) By use of the Polaroid (A trade name)

The last one of these is the method that will be used in the apparatus. A brief discussion might be in order on the physical properties of the Polaroid.

It was discovered that if a certain liquid chemical compound was allowed to evaporate, long slender crystals were formed. These crystals were microscopic in size, in fact, they could not be seen at a magnification of 1100 times. Using these facts as a bases the following things were then done. The crystals were forms on a membrane of gelatine and were then automatically parallel sligned by merely stretching this gelatine in opposite directions. The crystals were then held in this position on the gelatine by clamping the entire mass between two plates of glass. This piece of apparatus then has the property of passing polarized light only, and is better than 98% efficient. The Polarizing Axes of these polarizers are plainly marked by the manufacturer.

We now come to what is known as a Polariscope. If we take two of these Polaroids and place them in a beam of light with their polarizing axes parallel the light would pass completely through both (except the small amount that is lost by reflection, absorption, etc.)

but as their axes are turned so as to become perpendicular the field becomes dark and no light can be seen except that which is passed by the translucency of the crystals. In this position the two Polaroids become a Plane-Polariscope and is the type of Polariscope that is used in this thesis. If light is polarized by one of these Polaroids and viewed by another as in the above case the first one is known as a Polarizer and the second is known as an Analyzer. In the case of such a Plane-Polariscope the kind of light is relatively unimportant. It would be, however, if we were solving more intricate problems, such as tracing the lines of principal stress in which case it would be necessary to use Mica Quarter Wave Plates but as we are solving problems dealing only with simple tension and compression a white source of light is all that is necessary. Much better results, however, are obtained in these simple problems by making the light rays as nearly parallel as possible.

As to the choice of transparent material to be used in our investigations there are certain considerations which have to be born in mind:

- (1) It must be easily worked and shaped.
- (2) It must be sensitive.
- (3) It must be fairly rigid.
- (4) It must have only a small amount of creep.
- (5) It must be free from initial double refraction.
- (6) It must be transparent and absent of color.
- (7) It must not be too expensive.

Of the four materials that could be used namely glass, celluloid, bakelite and gelatine the second one holds nearest to the desired oualities. It is available in large sheets and can be bought for \$1.20 per lb. at the time of writing. The 1/8 inch material used in this thesis was found to be very satisfactory.

In this thesis the method of actually making a measurement of tension or compression in a piece of transparent material is by a system first suggested by Werthein but was actually developed by E.G.Coker.

A tension test piece is cut from the same plate as the specimen. This piece is subjected to a pull in such a manner that the tension in the material can be determined. This test piece shell be referred to as a Coker compensator. The method of actually mounting and determining the tension in this test piece will be explained later in the paper. It is first necessary, however, to understand what happens in such a system. If the specimen is subjected to a tension Q in any direction the compensator is set with its Plane parallel to that of the specimen, and its axes is set perpendicular to the line of stress. The Polarizing Axes of the specimen and the compensator are then in the same directions, but the wave which is accelerated in one case is retarded in the other. If then the compensator and the specimen are placed in a Plane-Polariscope the stress in the specimen will be the same as the stress in the compensator when extinction in the light field is obtained.

Since the tension in the compensator is known this.is, therefore, a direct means of determining stress. It may be, though, that the

specimen happens to be in compression; in this case the axes of the compensator must be set parallel, instead of perpendicular, to the line of stress. If the specimen is under two perpendicular P, Q, the compensator axes must be parallel to that stress (tension being positive) which is algebraically least. In the case of simple tension or compression the stress will be measured by the formula $Q = T/b_0 \ d_0$.

T - The total pull on the test piece.

bo . The width of the central part of the test piece.

 d_{O} = The thickness of the central part of the test piece. Knowing the dimensions of the specimen, the total stress in it can be determined.

The advantages of such a compensator are very great. In the first place, it gives the stress, or the stress difference, directly, and does not require a preliminary determination of the optical coefficient of the material. In the second place, the observations are independent of wave-length, or of Photo-Elestic dispersion, and may be taken in any kind of light.

In solving for the stresses P and Q it will appear at once by trial in which of the two possible directions extinction can be obtained. This last is to be used only if the specimen is under two perpendicular stresses.

CHAPTER II

Description of Apparatus

The idea for this apparatus was obtained in Coker & Filon's Treatise but as the book gave only a general explanation the details had to be worked out as they arose. The apparatus more or less is built around the compensating piece which will be described below.

A celluloid strip was cut to a general shape by means of a coping saw, which, incidentally, was found to adapt itself very well to the material. The final and desired dimensions were obtained by the use of sandpaper and a micrometer. Holes were then drilled approximately six and a half inches apart. on the centerline of the piece. This last is very important because if the holes are not on the centerline, uneven stresses would be introduced and the final results untrue. As .093 inch celluloid was used and as the strip was made one inch wide, the cross sectional area was .093 square inches. This area is capable of standing a tension of 150# without undue deformation.

Logically then, this leads us to the holders for this piece which will be described on the next page.

This paragraph will describe the holder on one end and will refer to illustration #1.

Holes (B) were drilled in the plates (C) the same size as the hole in the celluloid. Dowels were then fitted to these holes so that they could be removed by tapping them lightly with a hammer. The size of the plates are approximately 1-1/2" x 1-1/4" x 1/8" and are separated by the bar (D). Holes were drilled in the plates and the bar so they could be held together by nuts and bolts (E) as shown. Before assembling a hole was drilled and tapped at (F), the purpose of which will be explained later.

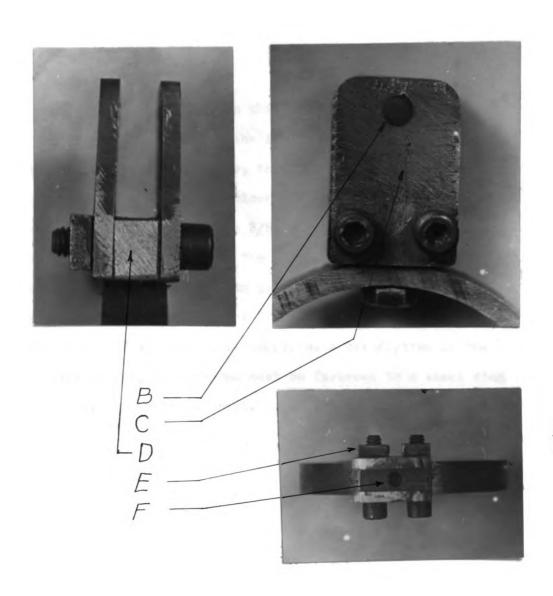
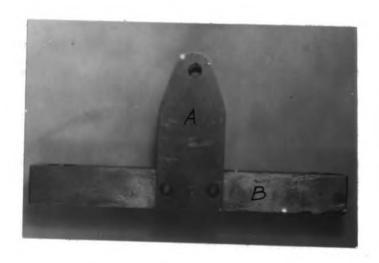


Illustration #1

The references made in this description will refer to illustration #2, which is the fastening of the other end of the test piece. In this, the plates (A) are doweled tightly to the separating block (B). This block was drilled and tapped at (C) to take a $3/8^n$ belt, also holes were drilled at (D) $9/16^n$ in diameter. The purpose of the holes at (C) and (D) will be explained on page 14. The dimensions of the plates (A) are $3^n \times 1-1/4^n \times 1/8^n$ while those of block (B) are $6^n \times 3/4^n \times 3/4^n$. This completes a description of the holders proper, but the one must be fastened to a steel ring which will be explained next.



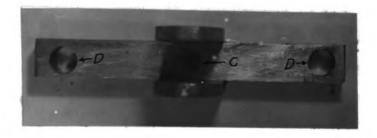


Illustration #2

Two 5/16" holes were then drilled diametrically opposite on a steel ring (A), illustration #3, the dimensions of which are the following: internal diameter 3", 1/2" wide and 1/8" thick. A bolt (B) was then inserted in the one hole and screwed into the holder described on page. This then explains the purpose of the hole in the holder mentioned on that page. In the other hole another bolt (B) was placed and by the use of a nut the ring was tightened securely to the block (D), in which a hole was drilled for this purpose. The dimensions of this block are 7" long and 3/4" square. Holes were drilled at (E) 3/16" in diameter for a purpose that will be explained on the next page. Two of these blocks were made of the same size and shape.



Illustration #3

In order to form a single unit of all the parts described on the preceding pages it was necessary to make two more pieces in the machine shop. Two 1/2" drill rods 18" long were drilled and tapped 1" deep on each end so as to take a 3/16" bolt.

The parts so far described were then assembled and mounted on an oak board by means of the bolts at (A) illustration #4. The block and holder at (B) are free to move along the rods which act as guides. The purpose of the bolt (C) is to enable the user to screw the holder downward and thus create a tension in the test piece. A hole was cut in the mounting board so as to allow the light to pass through the test piece. It is necessary to rotate the apparatus around this hole for purposes which will be explained later. To do this a cast iron ring 6" in diameter was obtained and by drilling three holes equally spaced and parallel to the axes of the ring, it is bolted to the back of the mounting board and around the hole. Illustration #5 shows this ring in its proper place. Before a support for the apparatus is explained it is necessary to complete the details of the apparatus itself which will be done next.





Illustration #4

Illustration #5

Illustration #6 shows one of the most important parts of the apparatus and upon what the accuracy of the determinations depend. This is the "scale" or the "balance" whose operation works on the deformation of the steel ring. wheels in this "belance" were obtained from an old clock while the rest of the parts were made. A multiplying arm (A) was cut out roughly, filed to the final shape, and mounted on an axle. A connecting arm (B) was then cut out and fitted between the multiplying arm and the wheel (C). This wheel also multiplies the movement through the wheel (D) on which a pointer will later be placed. A spring (E) holds the multiplying arm against the bolt head and thus always makes the pointer return to zero. This mechanism was mounted between two metal plates (bottom one shown) which are held apart at the proper distance by the four brass columns (F). It is held apart from the ring, except at the point where it touchs the bolt head, by clamping it under the block (G). Illustration #7 shows how the apparatus was clamped in a vice and arranged with a hanger so the balance could be calibrated by applying 5# weights one at a time. This completes the apparatus and the support will be described.

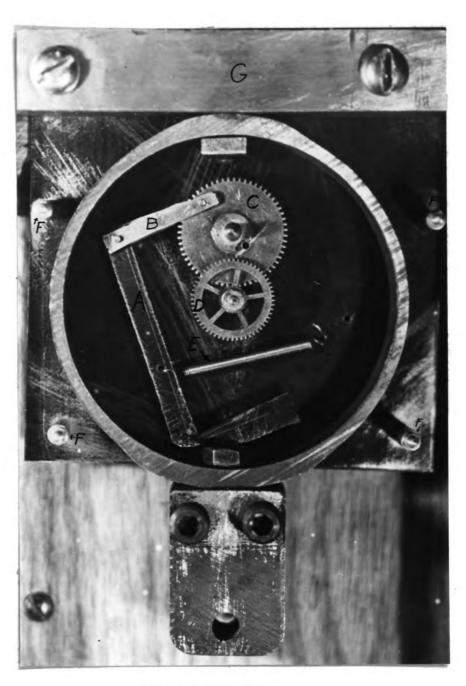


Illustration #6

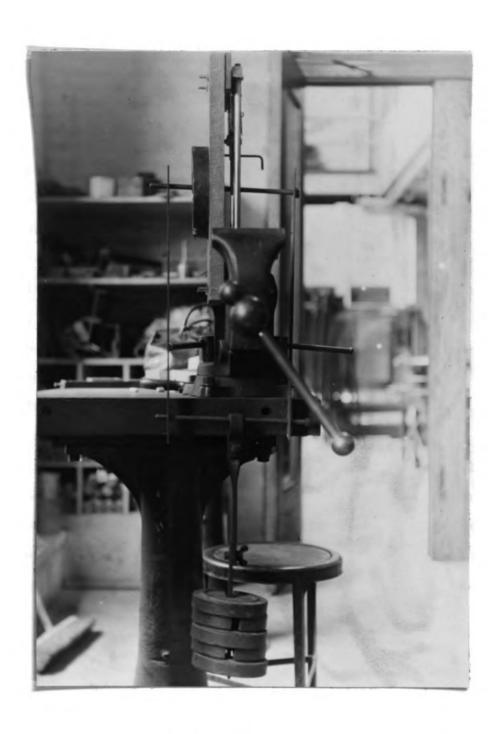


Illustration #7

Illustration #8 shows the support for the compensator apparatus. The wood blocks, around the hole in the middle upright, were put there so the cast iron ring, mentioned on the preceeding page, could revolve on them. The uprights are held in place by angle irons obtained from the local hardware store. The purpose of the two outside uprights are to form a support for the 360° calibrated metal discs which are holders for the five centimeter Polaroids.

It might be appropriate at this time to mention where these Polaroids were obtained. They were purchased from the Polarizing Instrument Company, 8 West 40th Street, New York City for the price of five dollars each (\$5.00). Quarter Wave Plates for more intricate investigations can be obtained from the same firm at the same price and size. Illustration #9 shows the two Polaroids with their axes perpendicular.



Illustration #8





Illustration #9

Illustration #10 shows the complete apparatus set up and ready for operation. The flash-light is operated from the regular power line by the use of a small transformer. A flash-light with a focusing beam was used so a beam of light as nearly parallel as possible could be obtained. As the apparatus is set the Polaroid nearest the light is the Polarizer and the other the Analyzer. All that is needed now is a model structure in which to determine the stresses, this will be discussed in Chapter III.

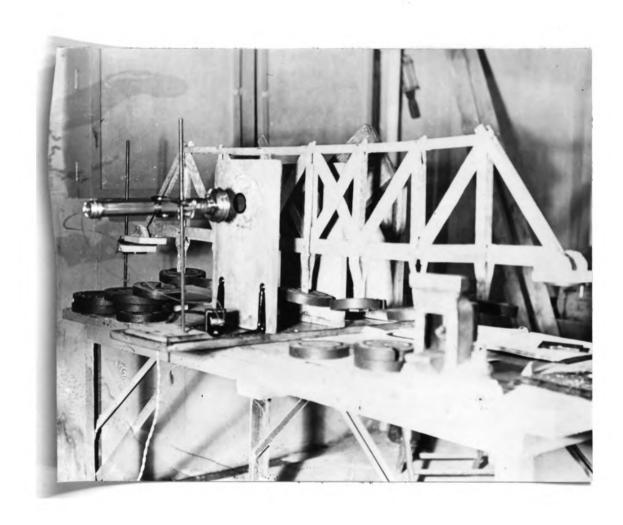


Illustration #10

CHAPTER III

STRESS INVESTIGATIONS

In order to get a structure in which there were only tension and compression members when loaded at the panel points, it was necessary to use one that was Pin-connected. After investigations along this line a Pin-connected Pratt Truss Highway Bridge, as designed in Thomson's "Bridges", was used. The specifications used in designing the model are as follows:

Length C to C of bearings 120 ft. = 8 panels of 15'
Depth C to C of chords = 20 ft.

Length of diagonal web-members = $\sqrt{20^2 - 15^2}$ = 25 ft.

Using these values and the reasonable scale $1/2^n = 1$ ft. the specifications of the model are as follows:

Length C to C of bearings 60 inches = 8 panels of 9-1/2"
Depth C to C of chords = 10 inches

Length of diagonal web-members = 12-1/2 ft.

At first it seems necessary to make the entire bridge of celluloid if it is to be investigated by Photo-Elasticity, but with a little reasoning it will be understood that only the ber that is to be investigated need be of this material. Acting on this principal the bridge was completely made of 24 gauge sheet metal. The tension members were simply strips of the material with pin holes drilled at the proper distances. This type of bar would not hold compression though, so channels were made with the pin holes drilled on the neutral axes and were found completely satisfactory. These bars were made 1° across the top and the flange 3/4°, while 1/8° cotter-pins were used holding the bridge together. Hangers of the same material were made for applying the loads to the panel points. In the investigation it was not necessary to test every bar as the

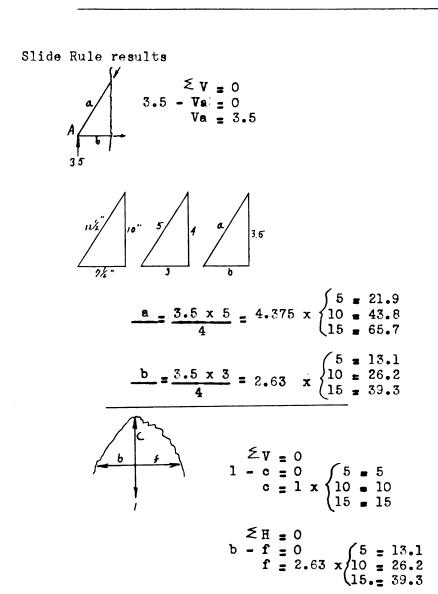
bridge was symmetrical. In the actual tests all the tension members were examined first. The loads were applied 5# at a time to a total of 15#, beyond this it was found that the cotterpins would not stand up. The following procedure was followed in the investigation; the apparatus was so placed that the light could shine through the celluloid member and the Polaroids crossed so the field was dark. If it was a tension member the compensating piece was set with its axes perpendicular to the bridge member or rather perpendicular to the line of stress. Then as the bridge was loaded it was noticed that the field was no longer dark, which proved there was a stress in the celluloid piece. By means of the screw a tension was applied to the compensator until the field again became dark. The "scale" was read at this point and was a direct measurement of the stress in the member. If, however, the member was in compression the compensator was set with its axes parallel to the line of stress and the procedure repeated. Illustration #11 shows the apparatus set up with the bridge model in position, ready to make an investigation of the celluloid tension member. All the celluloid bars were made of the same width and thickness as the compensating piece which was $1^n \times .093^n$. If desired the unit stress could have been easily computed from the formula $Q = \frac{T}{b d}$. The following table gives the computed stresses obtained by solving the model with the different loads on it. Method of Moments & Shear were used. The table also gives the stresses obtained by Photo-Elasticity. It is noticed that the values found by this method are several pounds more than the computed stresses, this was noticed at the time the readings were made and by looking at the member through

the Polariscope without the bridge being loaded the field was found light instead of dark like it should have been, proving that there were stresses produced in the bar by the weight of the bridge. Unfortunately the "scale" could not be read accurately under such a low stress so an average was taken and it was found that the stresses obtained are about 4# more than actual. If this is taken account of the values will be fairly close.

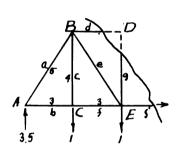
M et	3.5 hod of Loading (1) (2) (3)	B d D C F E 1 1 5 5 10 10	1 1 5 5 10 10	1 1 1 5 5 10 10	1 3.5 5
		15 15	15 15		15
Bar		thod of oading	Measured Stress	Stress by Computation	
8	Compression	(1)	25	21.9	
8	M M M M M M M M M M M M M M M M M M M	$\binom{1}{2}$	48	43.8	
a	Ħ	(3)	68	65.7	
ъ	Tension	(1)	16	13.1	
ъ	n	(2)	29	26.2	
Ъ	Ħ	(3)	43	39.3	
c	Tension	(1)		5.0	
C	n n	(2)	14	10.0	
C	"	(3)	19	15.0	
đ	Compression	(1)	26	22.5	
d	n n	(2) (3)	4 9	45.0	
d		(3)	73	67.5	
e	Tension	(1)	19	15.6	
•	tt tt	(2) (3)	33	31.2	
•		(3)	5 2	46.8	
f	Tension	(1)	18	13.1	•
f	n n	(2)	27	26.2	
f	"	(3)	43	39.3	
ģ	Compression	(1)	12	7.5	
g g	- n	(2)	20	15.0	
g	W .	(3)	27	22.5	
h	Compression	(1)	3 2	28.2	
h	- n 11	(2)	60	56 .4	
h	π	(3)	90	84.6	
i	Tension	(1)	15	9.4	
i	11	(2)	22	18.8	
i	11	(3)	32	28.2	

Ber j j j	Tension "	Method of Loading (1) (2) (3)	Measured Stress 25 48 71	Stress by Computation 22.5 45.0 67.5
k k k	Compression	1 (1) (2) (3)	12	2.5 5.0 7.5
1 1 1	Compression #	1 (1) (2) (3)	35 64 95	30.0 60.0 90.0
m m m	Tension " "	(1) (2) (3)	13	3.1 6.2 9.3
n n n	Tension	(1) (2) (3)	only take ten	ecause bar could sion. Did not eversal of stress.
o o o	Tension "	(1) (2) (3)	33 6 2 89	28.2 56.4 84.6
p p p		(1) (2) (3)	0 0 0	•0 •0 •0

Results found by this method can be transferred to a large scale structure very easily. In every case the load applied to the model must bear a constant ratio of to the corresponding load applied to the full-size. Then the total tension in any bar of the model is in the ratio of to the total tension of the corresponding bar of the full-size. It is also possible to work on another basis and that is to make the width of each bar of the model proportional to the cross-section of the corresponding bar of the full-size. In such a case the relative retardations in the model are proportional to the stress-intensities in the fullsize, this then, determines the safety conditions. It can also be shown that in a great number of problems, such as the one investigated here, the final results are independent of the difference between the values of the Moduli of the material. As the object of investigation of stresses in transparent models is to obtain information about the corresponding stresses in a fullsize structure we have then been shown that such a method is accurate with even such a crude apparatus as described here, and that with good apparatus the results would be very dependable. Since the loads increased by even increments the bridge was loaded (theoretically) at the panel point with 1# weights and the stresses figured with this load. To get the stresses with the 5, 10, and 15# loads all that was necessary was to multiply the stress obtained with unit loads by these figures.



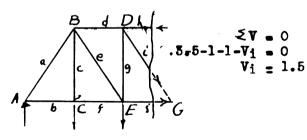
$$3.13 \times \begin{cases} 5 = 15.6 \\ 10 = 31.2 \\ 15 = 46.8 \end{cases}$$

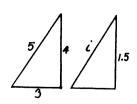


$$\xi_{\text{H}} = 0$$

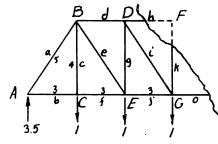
$$d = 4.5 = 0$$

$$d = 4.5 \times \begin{cases}
5 = 22.5 \\
10 = 45.0 \\
15 = 67.5
\end{cases}$$

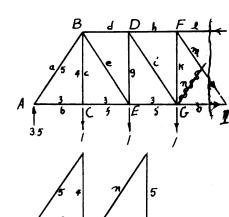




$$i = \frac{1.5 \times 5}{4} = 1.88 \times \begin{cases} 5 = 9.40 \\ 10 = 18.8 \\ 15 = 28.2 \end{cases}$$



$$\xi$$
H = 0
2.82-h = 0
h = 5.64 x $\begin{cases} 5 = 28.2 \\ 10 = 56.4 \\ 15 = 84.6 \end{cases}$



so we use the other diagonal

$$M = \frac{.5 \times 5}{4} = .625 \begin{cases} 5 = 3.1 \\ 10 = 6.2 \\ 15 = 9.3 \end{cases}$$

CONCLUSION

The purpose of this thesis was not to make an investigation of a full size bridge by Photo-Elastic methods but to show that it could be done and to build the apperatus necessary to do so. It has been shown in this thesis that the results obtained in structure investigation by Photo-Elasticity are reasonably accurate and fairly rapid compared to solving for stresses by calculation. It is easy to imagine how much quicker the stresses could be found in complicated frameworks, rigid frames and indeterminate structures by this method. Not saying, however, that these problems could be solved with the apparatus built here, but with apparatus that could be purchased for a not too large sum of money. One instrument in particular can be bought for about one hundred dollars (\$100.00), this apparatus is called a Soleil-Babinet Compensator and with it Lines of Stress can be traced, stress differences can be measured and many other things determined. With such a compensator stresses in a complicated framework can be read with surprising speed; Coker & Filon have read as many as one hundred stresses in an hour that would take a week to determine by calculation. It is obvious then that the old adage "Good in theory only" is untrue if reference is made to such a method. All the other apparatus (except the Polaroids and the size used in this thesis are plenty large) can be built very easily.

The theory of Photo-Elasticity has been known for many years but it is only in the last few that the engineer has began to realize its practical possibilities, and it won't be very long before results from it will be a dominant factor in structure design. Many schools

already have recognized the need for instruction along this line if they were to be up to date in their engineering courses. It has one drawback, however, in that the many Photo-Elastic investigations require much preliminary study that is very complicated. The result is that to fully understand the subject extra courses in Mathematics, Physics and Strength of Materials had to be incorporated. This then limits Photo-Elastic study to graduate students or part time instructors but even used by this limited group of students it keeps the school up to date and a leader in engineering studies.

ROOM USE ONLY

Jul 24'39





