BRIEF STUDY OF THE THEORY OF STRESSES IN NON-PARALLEL CHORD TRUSS AND TRUSS WITH SECONDARY WEB SYSTEM

Thesis for the Degree of B. S.

Steven Antonoff

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Brief Study of the Theory of Stresses in Non-Parallel Chords Truss and Truss with Secondary Web System

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CONTENTS

I The purpose of this thesis is not a dissertation upon the stresses in various members of steel bridges; nor is it an attempt to amplify the literature on the subject. Many excellent books and magazine articles have already been published in which the subject of stresses in bridges is covered thoroughly. My only sorrow was, as I, occasionaly, looked over some of the literature on this subject, that a four year course in engineering, as practiced by most colleges in the country, is a very short course in which sufficient time and effort can be devoted to the subject of the theory of stress in structures. This subject is broad, very important and interesting.

It was then my endeavor to find some way which would enable me to study the subject a little deeper than it is generally possible for an undergraduate student in his regular course. This thesis then is, as far as its present writing is concerned, an attempt to write down those fundamentals which I could learn in this short time that was allotted to me. It was nevertheless, my intention in deriving formulas, to write down every step, in order to make the work simple and comprehensible. I did this with special amphasis on the free body method, because the free body method principles as they were taught by the men in our department in courses of Mechanics, Strangth of Materials and Theory of Structures, are really the same principles upon which the subject of stresses in bridges is based. It is in no way based on any highly intricate or complex formulas.

II When the length of the span exceeds certain destance (about 175 ft.) it is then considered advisable to use variable height of truss in order to secure greater economy. Curved upper chord in such a case is generally called upon to answer the purpose. The most ideal type of truss with curved upper chord appears to be the one in which, for uniform loading the panel points would fall on the respective points of the moment diagram curve for that loading. But bridges are also subjected to partial loading. This condition produces an entirely different effect on the stresses. From zero stresses in diagonal web members and equal horizontal components in top chord produced by uniform loading, the partial loading subjects all diagonals to reversal of stress. Such condition calls for counters in every panel. The practice, however, taught engineers to use flatter top chords and thus avoid using counters except the center panels. The use of flatter top chord not only eleminates some counters but it also contributes considerably to the aesthetical effect making the structure, at the same time, reasonable economical.

The stress in any top chord of non-parallel chord truss, as shown in digure I, may be determined by the general formula,

$$S = \frac{M}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad I$$

where S = stress in top chord

M = moment at a point opposite that chord

r = perpendicular distance from the top chord to the moment center.

Figure I represents a common , eight panels, curved top chord truss. (See "The Theory of Structures" by Charles M. Spofford page 184, figure 146.)

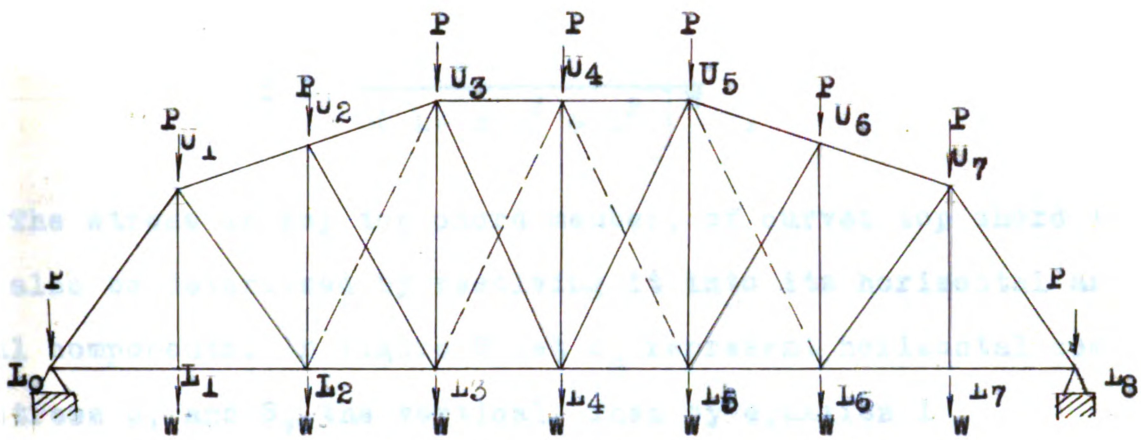

Figure I

Consider top chord U_1U_2. By general formula (I) the stress inthis member is,

$$S = \frac{M}{r}$$

where M = moment at panel point L_2, and $r = L_2b$ (fig. 2) perpendicular distance from the moment center to the member, the lever arm. The value of r may be determined by proportions from similar triangles U_1U_2a and U_2L_2 b, from which we have,

$$r : l :: h : s.$$

Solving for r we have,

$$r = \frac{l \times h}{s}$$

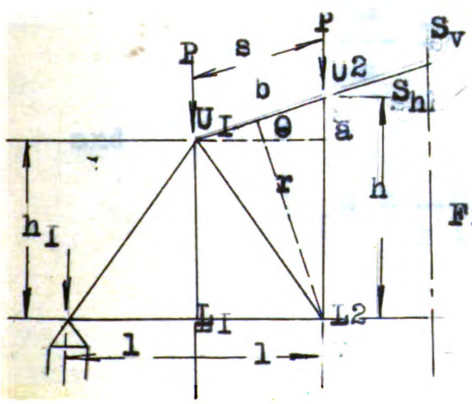

Fig.2

It must be noted that triangle U_1U_2a is right angled triangle and the value of s as obtained therefrom is,

$$s = \left[(h - h_I)^2 + l^2 \right]^{\frac{1}{2}}$$

thus making r, when this value of s is substituted in preceeding equation equal to,

$$r = \frac{l \times h}{\left[(h - h_I)^2 + l^2 \right]^{\frac{1}{2}}} .$$

The stress in any top chord member, of curved top chord truss, may also be determined by resolving it into its horizontal and vertical components. In figure 2 let S_h represent horizontal component of stress S, and S_v the vertical. Then by equation I

$$S_h = \frac{M}{h} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 2$$

and

$$S = S_h \operatorname{Sec} \theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 3$$

where θ denotes the angle between chord member and horizontal.

Both equation I and equation 3 give the same results.

$$S_h = \frac{M}{h}$$

and $$S_h = \frac{S}{\operatorname{Sec} \theta} \quad \text{(from equation 3)}$$

Solving these two equations simultaneously we obtain,

$$\frac{M}{h} = \frac{S}{\operatorname{Sec} \theta}$$

and

$$S = \frac{M}{h \cos \theta} = \frac{M}{r}$$

To determine the stress in any diagonal web member as U_IL_2 figure 3, cut section I-I, as shown in the figure, and consider the part to the left of the section as free body. By general formula

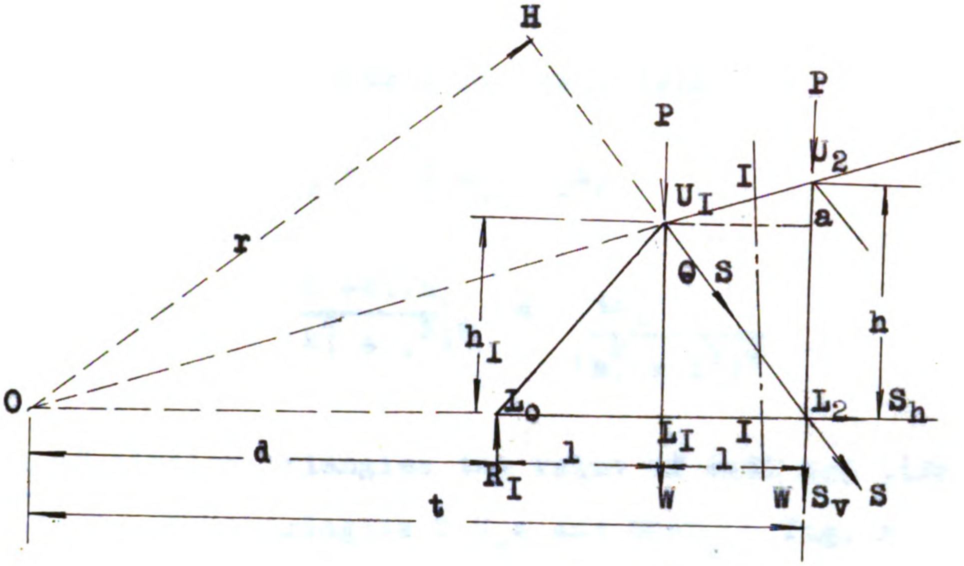

Figure 3

the stress in U_IL_2 is

$$S = \frac{M_o}{r} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 4$$

where M_o is the moment of the applied loads to the left of section I-I taken about point O, and r is the perpendicular distance from O to the line of action of U_IL_2. The value of M_o may be obtained by considering, in figure 3, part to the left of section I-I as free body and taking moments of the applied loads about point O, (clockwise moments are negative and counter-clockwise are positive), thus,

$$R_I d - P(d + l) - W (d + l) - Sr = 0$$ and solving for

$$S = \frac{P_I(d + l) + W (d + l) - R_I d}{r} \quad \ldots\ldots\ldots\ldots 5$$

but $P_I(d + l) - W(d + l) - R_I d = M_o$

substituting this value in equation 5 we have equation 4, namely

$$S = \frac{M_o}{r}$$

The value of r may be calculated from similar triangles OHL_2 and $U_I L_I L_2$, from which, by proportions, we obtain,

$$OH : OL_2 :: U_I L_I : U_I L_2.$$

Solving for OH ,

$$OH = r = \frac{(d + 2l) h_I}{(h_I^2 + l^2)^{\frac{1}{2}}} = \frac{t h_I}{(h_I^2 + l^2)^{\frac{1}{2}}} .$$

By reason of similar triangles the value of d+2l may likewise be determined. Consider triagles $U_I U_2 a$ and $OU_2 L_2$ (Fig. 3) in which

$$OL_2 : U_2 L_2 :: U_I a : U_2 a$$

and

$$OL_2 = d + 2l = t = \frac{hl}{(h - h_I)}$$

Another method by which the stress in diagonal web member can be determined is in terms of its vertical component. Thus, resolving the stress S into its S_v and S_h (vertical and horizontal) components and applying general equation the vertical component of the stress in $U_I L_2$ may be expressed by equation,

$$S_v = \frac{M_o}{t} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 6$$

where M_o is the moment of all the loads applied to the left of section I-I taken about point O, and t is the distance from O to the panel point L_2.

$$S = S_v \sec\theta \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 7$$

where θ is the angle between diagonal and vertical. Taking moments about point O the results may be obtained in the following form.

$$M_o = R_I d - W(d + l) - P(d + l)$$

$$= (R_I - W - P)d - (W + P)l.$$

But

$$R_I - (W + P) = V$$

is the shear on section I-I, and $(W + P)l$ is the moment M_{Lo} about panel point L_o. Arranging the terms of the above stated equations the value of M_o may be expressed by the following equation.

$$M_o = Vd - M_{Lo} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots 8$$

Note: For further proof and criterion of this case see article 34, page 5I of " The Theory of Structures" by Charles M. Spofford.

To produce maximum tensile stress in the diagonal web member $U_I L_2$ full panel loads must be applied to the right of the section I-I and no loads to the left of this section. Considering the conventional method of loading, the value of M_o in equation 8 must be positive and as great as possible. Any load to the left of the section I-I will produce negative moment and therefore reduce the positive value of M_o.

In present case let n = number of panels

m = number of panels not loaded

W = weight per panel on lower chord.

(Upper chord loads will now be omitted in order facilitate the work and simplify the formulas. It is my belief that this step may be taken

without any detrimental effect to this work. Because the upper chord loads may easily be taken care of by the formulas for the lower chord loads and the combined effect thus determined.)

Returning back to the problem and taking moments abount point O, figure 5, we have,

$$M_0 = R_I d - Sr \ldots\ldots\ldots\ldots\ldots\ldots\ldots 9$$

From equation 7

$$S = S_v \operatorname{Sec}\theta$$

and

$$\operatorname{Sec}\theta = \frac{(h_I^2 + l^2)^{\frac{1}{2}}}{h_I}$$

Combining these three equations and solving for S_v we obtain,

$$S_v = \frac{R_I d h_I}{r(h_I^2 + l^2)^{\frac{1}{2}}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots IO$$

The value of R_I can be obtained by taking moments R_2 (right reaction) of all the loads applied on the truss.

Using notations stated above in this work the equation is

$$R_I = \frac{W}{2nl}(n - m + I)(n - m) \ldots\ldots\ldots\ldots\ldots\ldots II$$

Substituting value of R_I, as obtained in equation II, in equation IO, and remembering that

$$r = \frac{th}{(h_I^2 + l^2)^{\frac{1}{2}}}$$

equation IO becomes

$$S_v = -\frac{Wd}{2nl}(n - m + I)(n - m) \ldots . I2$$

The diagonal web members are subjected to compressive stress as well as to tensile. For maximum compression in diagonal $U_I L_2$ the moment M_0 must be negative and as large as possible. Such condtion can be obtained when the negative moment due to loads applied to the left of the section I-I is greater than the positive moment produced by left reaction R_I. It is that placing loads on all panels to the left of the section I-I and no loads to the right of that section will produce maximum moment M_0.

As before, consider n = number of panels in whole truss

m = number of panels loaded

W = uniform load per panel

the maximum compressive stress by equation (I) is (vertical com.)

$$S_v = \frac{-M_0}{t}$$

By application of the same method of analysis as was used in case of tensile stress it is possible to express the vertical component of compressive stress in this diagonal by the following equation.

$$S_v = -\frac{wm}{2nt}(m - I)(d + ln) \quad \ldots\ldots\ldots\ldots I3$$

Since equations I2 and I3 express the value of vertical components of stresses in diagonal members, it is than possible, by application of these two formulas, to obtain stresses in any vertical as U_2L_2. For all loads to the right of panel point L_2, the vertical member U_2L_2 is in compression and equation /2

applies, while for all loads to the left of the panel point U_2 the member is in tension and equation I3 applies. Since, in these equations, the positive sign indicates tension and negative compression the application of these equation to the vertical members involves interchange of signs in the right hand member of both equations.

III

Stresses in Members of Truss Containing Secondary Web System.

For spans of considerable length the maximum economy is secured by means of subdividing panels and adding secondary diagonals and verticals. The method of determining stresses in truss of this type becomes somewhat complicated, especially when dealing with secondary system. The application of ordinary methods of joints, moments and shear require a little modification. In some members however the stress can be obtained directly by one of these methods. The methods of determining stresses in various members of truss with secondary web system will now be given.

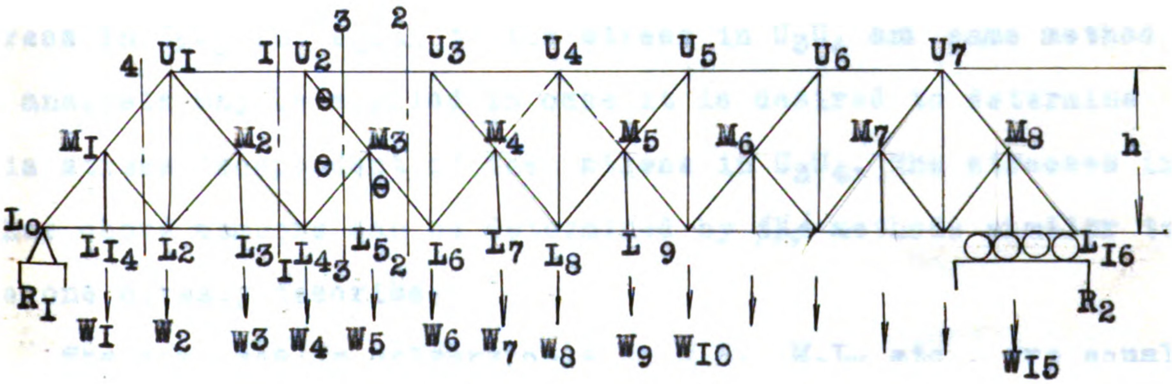

Fig. 4.

Figure 4 represents one type of truss with secondary web system.

The upper chord stresses in a truss similar to the one shown in Fig. 4 may be found by using general equation I. Consider the upper chord U_IU_2. By general methods of moments the stress S is,

$$S = \frac{M_{L4}}{h} \text{ ....} I4$$

where M_{L_4} is the moment at panel point L_4 due to the applied loads and h is the height of the truss.

The stresses in lower chord members can be found likewise by taking moments abount the upper chord points. Thus, for example, the stress in L_3L_4 is foung by passing section I-I Fig. 4, and taking moments about point U_I. The resulting equation may be stated in following terms,

$$2 X 1R_1 - W_I X 1 + W_3 X 1 - S X h = 0$$

Solving for S, we have,

$$S = \frac{1 (W_I - W_2 - 2R_I)}{h} = \frac{M_{U_I}}{h} \text{ .... } I5$$

(1 = length of panel)

Stress in U_2U_3 is equal to the stress in U_3U_4 and same method of analysis may be applied in case it is desired to determine this stress independent of the stress in U_3U_4. The stresses in other chord members can be determined by methods similar to the one already describe.

The stresses in subverticals as M_IL_I, M_2L_3 etc,. are equal to the loads applied at their respective panel points. For example, the stress in M_IL_I is equal to W_I. The member is in tension.

The stresses in sub-diagonals as L_2M_2 may be obtained by resolving its stress S into horizontal and vertical components and finding value of vertimal component by method of moments.

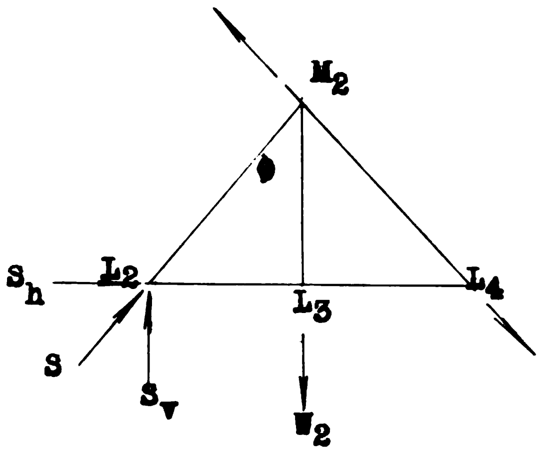

Fig. 5

In figure 5, consider the forces acting on joint M_2. An equation of moments about point L_4 gives,

$$W_2 \times l - S_v \times 2l = 0$$

Solving for S_v we have,

$$S_v = 1/2W_2 \text{ .... } 16$$

Making use of eqaution 3 previously derived we find S to be,

$$S = S_v \sec\theta = 1/2W_2 \sec\theta \qquad \text{(compression)}$$

The stresses in lower ends of diagonal web members are equal to the product of shear in their respective panels and secant of angle between member and the vertical. Stress in M_3L_6 may be obtained as follows. The shear on section 2-2 in Fig. 4 is,

$$V = R_1 - mW \text{ .... } 17$$

where m = number of panels from left reaction to the left end

of panel containing section 2-2, and R_I and W as before. Solving for R_I,

$$R_I = \frac{W}{2}(n - I). \qquad (n = \text{number of panels})$$

Substituting this value of R_I in equation I7, we have,

$$V = \frac{W}{2}(n - 2m - I) \text{ .... I8}$$

Therefore the stress in M_3L_6 is,

$$S = \frac{W}{2}(n - 2m - I)\text{Sec}\theta \text{ .... I9}$$

where θ = angle between member M_3L_6 and the vertical M_3L_5.

Stresses in upper ends of diagonal web members can be determined by method of shears. (Diagonals as U_2M_2, U_2M_3, etc.) Consider diagonal U_2M_3. Pass section 3-3 Fig. 4, and consider the portion of the structure to the left of this section as free body. The shear on this section may be given by equation

$$V = R_I - (W_I + W_2 + W_3 + W_4)$$

This shear is distributed between U_2M_3 and M_2L_2. It is evident then, that vertical component of stress S in U_2M_3 may be expressed by equation,

$$S_v = V - S_v^1 \text{ .... 20}$$

where V = shear on section 3-3, and S_v^1 = vertical component of stress in sub-diagonal M_3L_4.

Equation 20 makes possible to determine the stresses due to uniformly distributed dead load. For as explained before,

$$S = S_v\text{Sec}\theta$$

For maximum live load stresses the position of loads must first be considered before equation can be applied. Either the method of moving up the loads or the average load method may be used. These two methods are given in detail in chapter III of "The Theory of Structures" by Charles M. Spofford. Equation 3 may be applied to determine the maximum live load stresses after the value of V and S'_v in equation 20 had been determined for the position of loads producing maximum stress at that section.

Another method which may be employed to determine stress in U_2M_3 is the method of moments. This method will not be given here because the reader can find at a glance the value of S_v by taking moments about points L_5 and L_6 respectively.

Stress in verticals as U_2L_4 is readily seen to be eaqual to the vertical companent of U_2M_3 plus the load at joint U_2. Thus, the stress in vertical U_IL_2 can be determined by the method of joints.

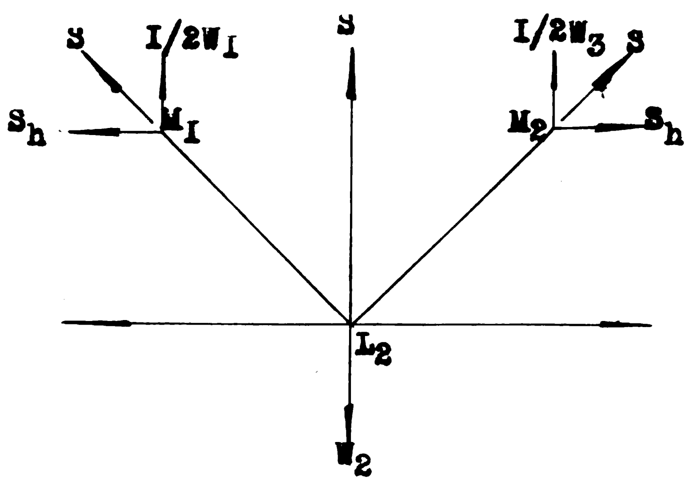

Fig.6

Consider joint L_2, Fig. 6 as a free body. Resolve stresses S in sub-diagonals M_IL_2 and M_2L_2 into their vertical and horizon-

tal components. Applying principles of equation I6, it is readily seen that the value of vertical component of stress in $M_I L_2 = I/2W_I$. Likewise S_v in $M_2 L_2 = I/2W_3$. The total stress in vertical $U_I L_2$ is thesum of these two components, plus load W_3 whish is at joint L_2. The above result may be expressed by equation,

$$S = I/2W_I + I/2W_2 + I/2W_3 \text{ .... } 2I$$

For equal loads equation 2I becomes,

$$S = 2W \text{ .... } 22$$

Equation 22 gives value of stress in $U_2 L_2$ produced by uniformly distributed dead load. In cases where the stress is due to concentrated live loads this equation must be slightly modified before its application can successfully be made for correct resutls.

Let M_I', M_2', M_3' and M_4', in Fig. 4, represent moments at points L_I, L_2, L_3, and L_4 respectively due to the applied loads to the left of those points. Let l = length of panel. Then by general moment equation,

$$W_2 = \frac{M_I' - 2M_2' + M_3'}{l} \text{ .... } (a)$$

$$W_3 = \frac{M_2' - 2M_3' + M_4'}{l} \text{ ....} (b)$$

$$W_I = \frac{-2M_I' + M_2'}{l} \text{ ....} (c)$$

Substituting equations (a), (b), and (c) in equation 22 we have,

$$S = \frac{M_4' - 2M_2'}{2\,l} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad 23$$

Note: For complete discussion of the above principles see "Stresses in Framed Structures" by Hool and Kinne, article 71, page 138.

The stress in upper end of end post L_0U_1 can be determined by passing section 4-4, Fig. 4, and considering that portion as free body. Applying general equation I we have,

$$S = \frac{M_{L2}}{r}$$

where M_{L2} = moment at point L_2, and r = perpndicular distance from point L_2 to the line of action of L_0U_1.

IV

CONCLUSION

It was my great sorrow that limited time did not permit me to inlude into the present writing of my thesis all that I originally planned to write. It was my endevor to write, in addition to this work, on the methods of stress analysis in trusses with multple web systems and on the methods of determining stresses in cantilever bridges. The subject, after all, proved to be too broad to attempt in my present work. Because, knowing a subject and being able to write on the subject so that others may know it, are two different things. To be able to describe the short and comprehensive form a subject so broad involves a necessity, on my part, to study the subject thoroughly. It would also add at least twenty pages of written matter to the present work. This conditions are a total impossibility for just now. I must conclude this thesis ast it is being content with the fact that I had a splendid opportunity to become somewhat familiar with the subject of stresses in advanced types of structures and be able to appreciate the vast amount of work and the difficulty one encounters in attempting to write on the principles of stress analysis.

I do, however, feel that so far as I was able to write , I have fully fulfilled my expectations. I have summarized a few fundamental principles which, if properly applied, will enable one to determine stresses in any statically determinate type of bridge truss.

V

SUMMARY OF BOOKS USED FOR STUDY AND REFERENCE.

I The Theory of Structures, by Charles M. Spofford,

2 Stresses in Framed Structures, by Hool and Kinne.

3o Modern Framed Structures. by Johnson, Bryan, and Turneaure.

(vols. I and II)

4 Handbook for Engineers and Architects. by HÜTTE

(Foreign Language book.)