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THE DESIGN OF CIRCULAR REINFORCED CONCRETE FOOTING ON ELASTIC FOUNDATIONS

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Yiun-Yuan, Haung

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THESIS

This is to certify that the

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THE DESIGN OF CIRCULAR REINFORCED CONCRETE FOOTING
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Yiun-Yuan Huang

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C L Allen
Major professor

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THE DESIGN OF CIRCULAR REINFORCED CONCRETE FOOTING ON ELASTIC FOUNDATIONS

By

YIUN - YUAN, HAUNG

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The writer is greatly indebted to Professor C. L. Allen, Head of Civil Engineering Department of Michigan State College for his guidance and direction in connection with the study and also to Dr. Richard H. J. Pian for his valuable advices.

Professor Allen is going to retire at the end of June after his thirty years of untiring service at Michigan State College. Fortunately the writer is about to finish his studies toward Master of Science degree at the same time and is proud of being Professor Allen's "The Student Behind the Closing Gate" -- a Chinese proverb which means the last student before the retirement of his mastering professor.

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I. INTRODUCTION

In this paper is presented a practical method for designing a reinforced concrete footing of circular shape resting on elastic foundation based on theories of elasticity. Charts, as the time saver in design work, for solving internal moments under different loading and foundation conditions are prepared.

The most reasonable shape of a single column footing is supposed to be circular, as the foundation pressure under such a footing is symmetrical with respect to its center in every direction when its supporting load is central, which is the most usual case happened to a single footing. High structures like chimney, centrally supported elevated water tank, silo, and etc., which are circular in shape and sensitive to unequal settlement in foundation are specially suitable to have a circular footing underneath.

The conventional design of a single reinforced concrete footing is of square, rectangular or polygonal in shape and the foundation pressure is assumed to be uniform. The moments are investigated at several pair of parallel sections at which bending is assumed to occur. (The method of analyzing moment in such a way is supposed to be based on the Bulletin No. 67 of the University of Illinois Eng. Expt. St. 1913 on the title of 'Reinforced Concrete Wall Footings and Column Footings' by Arthur N. Talbot.) The recent developments in theory of elasticity and soil mechanics prove that those assumptions mentioned above are not near from being true.

Structural engineers are very much interested in the design of a circular reinforced concrete footing on elastic foundation , but high mathematics which is essential for solving the problems concerning the theory of elasticity is too much involved and has kept many of them from being familiar with those theoretical methods. In this paper, high mathematics is eliminated as much as possible in derivation of formulas and in the case of footing carrying a moment load a symmetrical designing load is proposed to replace the actual unsymmetrical one without sacrifice of the reality which makes the complicated problems simple and easy to solve without using high mathematics.

Charts for solving moment in circular footings are prepared in this paper covering the usual loading and foundation conditions. The cut and trial procedure is inevitable in solving every statically indeterminate problem so it is not the inconvenience particularly presented in this paper.

II. FUNDAMENTAL CONSIDERATIONS

1. BASIC ASSUMPTIONS: In this paper the following assumptions are made:

(1) The thickness of the footing is assumed to be small as compared with its radius so that the general theories of thin plates are applicable.

(2) The deflection of the footing is assumed to be small so that the energy method of determining deflection curve is valid.

(3) The center part of the footing directly underneath the column or column capital is assumed to be absolutely rigid.

(4) The intensity of reaction of the foundation, soil or pile, at each point of the bottom surface of the footing is assumed to be proportional to the deflection of the footing at that point.

(5) The Possion's Ratio for concrete is assumed to be constant and equal to 0.2.

(6) The weight of the footing is small as compared with its supporting load, and it is assumed to be uniformly distributed over the base area of the column or column capital in moment analysis.

(7) Those assumptions made in the general theories of reinforced concrete are also to be made in this paper.

2. METHOD OF SUPERPOSITION:

The particular case of loading of a circular footing is such that it carries a downward load from column to which it supports and upward pressures from foundation on which it rests. Both of its surfaces are loaded and its edge is free in all directions. But the basic edge condition upon which the analysis of moments is based in this paper, is simple supported. In each case, the moments, radial and tangential, due to foundation pressure under the basic edge condition, are deducted respectively from those due to column load under the same edge condition which is equal and opposite to and colinear with the resultant of foundation pressure, and the net moments are those under actual free edge condition. In case the column carries a moment load besides a downward load (such as produced by wind load on a chimney footing or eccentricity in a building column footing), an equivalent symmetrical upward foundation pressure is assumed to apply at the bottom surface of the footing and an additional downward column load at top, so that it can maintain equilibrium. The moments in the footing due to moment load are thus computed as in previous way and added to those from real downward column load.

In this paper moments due to downward column load under basic edge condition are called BASIC MOMENTS and those due to foundation pressures are called LOAD MOMENTS.

Thus

Actual Moment = Basic Moment - Load Moment (In any case)

or $M_r = M_r' - M_r''$ (Actual radial moment)

and $M_t = M_t' - M_t''$ (actual tangential moment)

3. NOTATIONS: The following notations are used throughout in this paper.

A, B	Constants in the equations for deflection curve of circular plate
A_s	Steel area in reinforced concrete section
C_1, C_2	Integration constants
D	Flexural rigidity of section
E_c, E	Modulus of elasticity of concrete, etc.
I	Moment of inertia
K	General moment constant in standard notation of R.C.
K_r	Radial moment constant in circular plate
K_t	Tangential moment constant in circular plate
L, L_1, L_2	Total load in each concentric pile circle, or total upward pressure in elastic soil under the footing
M	Moment in general
M_r	Radial moment in circular plate (actual)
M_t	Tangential moment in circular plate (actual)
M'_r	Basic radial moment in circular plate (see definition in 2)
M'_t	Basic tangential moment in circular plate (see definition in 2)
M''_r	Load radial moment in circular plate (see definition in 2)
M''_t	Load tangential moment in circular plate (see definition in 2)
O	Origin

P	Total column load
Q	Unit shearing stress in circular plate
R	Radius of circular plate
S_1, S_2	Areas
U_1, U_2	Strain energy
V	Volume or total shearing force in standard notation of R.C.
X, Y, Z	Rectangular axis
a	Radius of column or column capital
b	Width of section in standard notation of R.C.
e	Eccentricity
f_c	Fibre stress in concrete
f_s	Stress in steel
h	Depth of section
j, k	Constants in standard notations of R.C.
k'	Modulus of reaction of foundation
m	Poisson's ratio
p	Unit load or pressure
r	Radius of circular section
u	Ratio of pressures at rim and center of footing
v	Unit shearing stress in standard notations of R.C.
w.	Deflection
x,y,z	Rectangular coordinates
z (or)	Radius of concentric pile circle

α	Ratio a/R
β	Ratio r/R
μ	Ratio z/R
θ	Angle in polar coordinates in horizontal plane
ϕ	Angle in polar coordinates in vertical plane
$\left.\begin{array}{l}\sigma \\ \epsilon \\ \lambda \\ \Upsilon\end{array}\right\}$	Constants

4. BASIC DIFFERENTIAL EQUATIONS FOR MOMENTS AND EQUILIBRIUM:

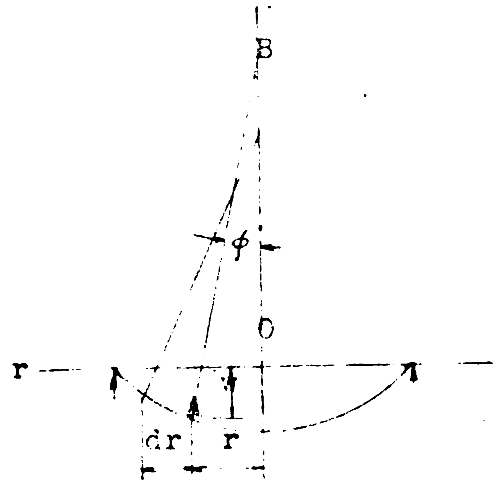


Fig. 2.1 (a)

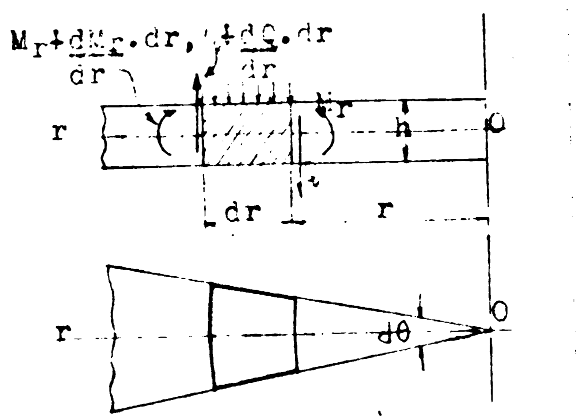


Fig. 2.1 (b)

Figure 2.1a shows a diametrical section of a circular plate under bending and Figure 2.1b an elementary segment of it with forces marked.

The differential equation for moment per unit length arc:

$$M_r = D\left(\frac{d\varphi}{dr} + \frac{m}{r}\varphi\right) = -D\left(\frac{d^2w}{dr^2} + \frac{m}{r}\frac{dw}{dr}\right) \qquad \text{----} \quad (2.1)$$

$$M_t = D\left(\frac{\varphi}{r} + m\frac{d\varphi}{dr}\right) = - D\,\frac{1}{r}\left(\frac{dw}{dr} + \frac{m}{r}\frac{d^2w}{dr^2}\right) \qquad \text{----} \quad (2.2)$$

Considering the element abcd in Figure 2.1b in equilibrium, taking moments about the center O and neglecting the terms containing the small quantities of higher order, the following relation is obtained:

$$M_r + \frac{dM_r}{dr}\, r - M_t + Q_r = 0 \qquad \text{----} \quad (2.3)$$

Substituting the values of M_r and M_t in equation (2.1) and (2.2) the basic differential for equilibrium becomes:

$$D\,\frac{d}{dr}\left(\frac{d\varphi}{dr} + \frac{\varphi}{r}\right) + \frac{dD}{dr}\left(\frac{d\varphi}{dr} + m\,\frac{\varphi}{r}\right) = -Q \qquad \text{----} \quad (2.4)$$

The deflection w in the above equation is considered to be positive when it is downward.

For a plate of constant thickness the Flexural Rigidity D is constant, and equation (2.4) reduces to:

$$\frac{d^2\varphi}{dr^2} + \frac{1}{r}\frac{d\varphi}{dr} - \frac{\varphi}{r} = - \frac{Q}{D} \qquad \text{---- (2.5)}$$

or put into other forms:

$$\frac{d}{dr}\left[\frac{1}{r} \cdot \frac{d(r\varphi)}{dr}\right] = - \frac{Q}{D}, \qquad \text{---- (2.6)}$$

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{Q}{D} \qquad \text{---- (2.7)}$$

If Q is represented by a function of r, these equations can be integrated without difficulty in each particular case.

III. BASIC MOMENTS

The problem is to solve the moments, radial and tangential, in a simple supported circular plate carrying a symmetrical central uniform load with its center portion, direct under the load, considered to be absolutely rigid. Referring to Figure 3.1, the diametrical section of the plate, for sections $r > a$,

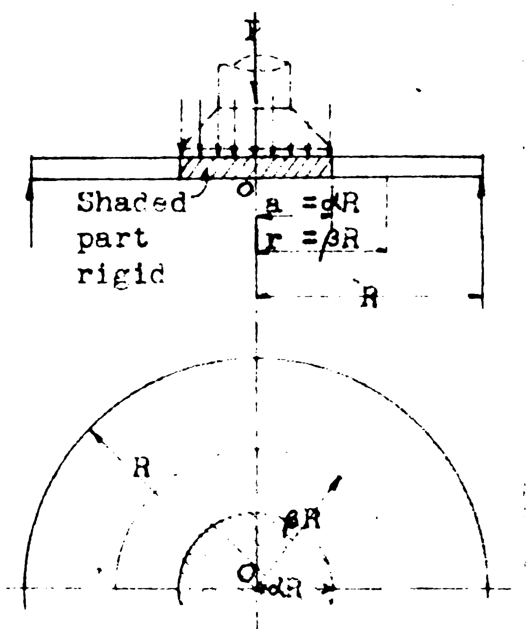


Fig. 3.1

$$Q = \frac{P}{2\pi r}$$

Substituting Q in equation (2.7) and integrating, we have:

$$\frac{dw}{dr} = \frac{Pr}{8\pi D}\left(2 \log \frac{r}{R} - 1\right) - \frac{C_1 r}{2} = \frac{C_2}{r}$$

$$\frac{d^2w}{dr^2} = \frac{P}{8\pi D}\left(2 \log \frac{r}{R} + 1\right) - \frac{C_1}{2} + \frac{C_2}{r^2}$$

From the boundary following conditions,

$$r = a = R, \quad \frac{dw}{dr} = 0$$

$$r = R, \quad M_r = 0, \text{ or } \quad \frac{d^2w}{dr^2} + \frac{m}{r}\frac{dw}{dr} = 0$$

$m = 0.2$ for concrete

we have

$$C_1 = \frac{P}{4\pi D} \times 0.67 \left[1 + \alpha^2 (2 \log\alpha - 1) \right]$$

$$C_2 = \frac{P\alpha^2 R^2}{8 \pi D} \left[2 \log\alpha - 1.67 \right]$$

Thus
$$M'_r = - D \left(\frac{d^2w}{dr^2} + \frac{m}{r}\frac{dw}{dr}\right)$$

$$= P \times 0.0319 \left[\frac{1.5 + \alpha^2(3\log\alpha - 1.5)}{1.5 + \alpha^2} - \left(\frac{3\log\alpha - 2.5}{1.5 + \alpha^2}\right)\frac{\alpha^2}{\beta^2} - 1 - 3\log\beta \right]$$

(moment per unit length) ---- (3.1)

$$M'_t = - D \left[\frac{1}{r} \frac{dw}{dr} + m \frac{d^2w}{dr^2} \right]$$

$$= P \times 0.0319 \left[\frac{1.5 + \alpha^2 (3\log\alpha + 1.5)}{1.5 + \alpha^2} + \left(\frac{3\log\alpha - 2.5}{1.5 + \alpha^2}\right) \frac{\alpha^2}{\beta^2} + 1 - 3\log\beta \right]$$

(moment per unit length) ---- (3.2)

IV. FOOTING CARRYING A VERTICAL LOAD RESTING ON ELASTIC SOIL (1) - THE PRESSURE DISTRIBUTION CURVE

1. ENERGY METHOD OF DETERMINING DEFLECTION CURVE: The exact solution of the deflection curve of a diametric section of a circular plate carrying a central load and resting on elastic soil whose intensity of pressure is proportional to the deflection of the plate at the same point under consideration, is: (refer to Figure 4.1a)

$$w = \sqrt[4]{\frac{D}{k'}}\Big\{ C_1\Big(1-\frac{x^4}{2^2.4^2}+\frac{x^8}{2^2.4^2.6^2.8^2}-\cdots\Big) + C_2\Big(x^2-\frac{x^6}{4^2.6^2}+\frac{x^{10}}{4^2.6^2.8^2.10^2}-\cdots\Big)$$
$$+ C_3\Big[\Big(1-\frac{x^4}{2^2.4^2}+\frac{x^8}{2^2.4^2.6^2.8^2}-.\Big)\log x + \frac{3}{128}x^4 - \frac{25}{1,769,472}x^8+\cdots\Big]$$
$$+ C_4\Big[\Big(x^2-\frac{x^6}{4^2.6^2}+\frac{x^{10}}{4^2.6^2.8^2.10^2}-\Big)\log x + \frac{5}{3456}x^6 - \frac{1054.10^{-4}}{442,368}x^{10}+\ ..\Big]\Big\}^* \quad \text{--- (4.1)}$$

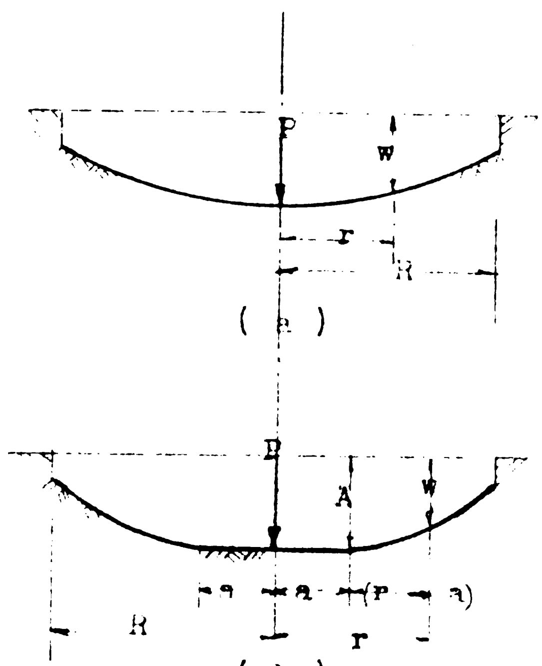


Fig. 4.1

where $x = \frac{r}{D/k'}$.

But for small deflections, the equation of the curve may be expressed on the form

$$w = A + B\,r^2 \qquad \text{--- (4.2)}$$

---

*Timoshinko, Theory of Plates and Shells, p.278

which is obtained from neglecting the terms containing x of higher power than 2 in equation (4.1).

In our particular case, the central portion of the plate being rigid, equation (4.2) becomes: (referring to Figure 4.1b)

$$W = A + B\,(r - a)^2 \qquad \text{--- (4.3)}$$

$$\frac{dw}{dr} = 2Br, \quad \frac{d^2w}{dr^2} = 2B$$

From the above expressions we can conclude that the deflection surface has a constant curvature equal to 2B and the plate is under pure bending.

The strain energy of a plate under pure bending is

$$U_1 = 1/2\ D\ (\text{area of the plate})$$

$$\cdot\ \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + 2\ \frac{m\,\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} \qquad \text{--- (4.4)}$$

In our particular case, $\frac{\partial w}{\partial x} = \frac{\partial w}{\partial y} = \frac{\partial w}{\partial r}$ and the central portion of the plate being rigid,

$$U_1 = D\ (\text{area of elastic portion of plate})\frac{\partial^2 w}{\partial r^2}\ (1+m)$$

$$= 4B^2D\ (1 + m)\ \ R^2\ (1 - \alpha^2) \qquad \text{---(4.5)}$$

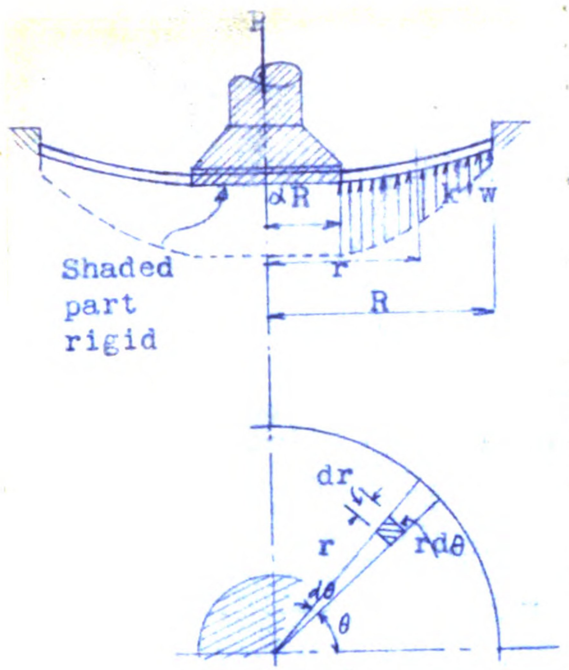


Fig. 4.2

Referring to Figure 4.2, the force exerted on the infinitesimal area of elastic foundation beyond the area direct under the rigid portion of the footing is

$$k'.wr.dr.d\theta$$

and the strain energy stored is

$$1/2.k.w^2.r.dr.d\theta$$

and that within the area direct underneath the rigid portion of the footing is

$$1/2.k.A^2.r.dr.d\theta$$

Total energy stored in the elastic foundation is

$$U_2 = 1/2\int_0^{2\pi}\int_a^R k'w^2\ r\ dr\ d\theta + 1/2\int_0^{2\pi}\int_0^a k'\ A^2\ r\ dr\ d\theta$$

$$= \frac{k'}{2}\int_0^{2\pi}\int_a^R A + B(r-a)^2\ {}^2r\ dr\ d\theta + \frac{\pi a^2}{2}\ k'A^2 \quad \text{--- (4.6)}$$

Integrating and neglecting terms containing α of higher power than 2 (in usual case α is less than 0.3 and α^3 is small as compared with other quantities) we have,

$$U_2 = \pi k'\left[1/2A^2\ R^2 + ABR^4C_1 + B^2R^6C_2\right] \quad \text{--- (4.7)}$$

where $C_1 = \left[1/2 - (4/3)\alpha + \alpha^2\right]$ and $C_2 = \left[1/6-(4/5)\alpha + (3/2)\alpha^2\right]$

The total elastic energy in the whole system is,

$$U = U_1 + U_2 - PA \quad \text{--- (4.8)}$$

where P is the total load from column.

As the total energy of the system in stable equilibrium must be a minimum, substituting (4.5) and (4.7) into (4.8) and putting

$$\frac{\partial U}{\partial A} = 0, \quad \frac{\partial U}{\partial B} = 0, \text{ and } m = 0.2 \text{ for concrete}$$

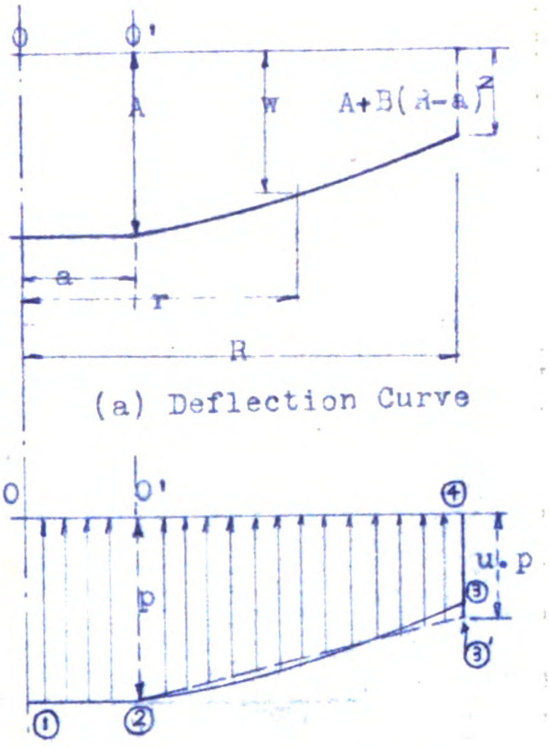
we have

$$A = \frac{P}{\pi k R^2 \left(1 - \dfrac{C_1{}^2 R^4}{9.6(1-\alpha^2)\dfrac{D}{k'} + C_2 R^4}\right)}$$

$$\frac{B}{A} = -\left[\frac{C_1 R^2}{9.6\,(1-\alpha^2)\dfrac{D}{k'} + C_2 R^4}\right] \qquad \text{--- (4.9)}$$

where D is the flexural rigidty of reinforced concrete section (see Appendix I), k' the modulus of reaction and C_1, C_2 as the same meanings as indicated previously. With the constants A and B known, the deflection curves is determined.

2. SHAPE OF PRESSURE DISTRIBUTION CURVE: As we assumed that the intensity of foundation reaction is proportional to the deflection of the footing at that point, the foundation pressure curve has the same shape with that of the deflection curve. Figure 4.3A and b shows the diametrical deflection+pressure curves respectively. The ratio between the pressures at the center and the rim bears the same ratio between the deflections at those points, that is the ratio B/A. As we assumed before, the deflection is small, it is sufficiently accurate to assume straight line variation in deflection or pressure between outer edge of rigid portion of the plate and its rim provided that the area under the curve 2-3' - 4-0' S_2 is making equal to the area 2-3-4-0' (S_1).



(a) Deflection Curve

(b) Pressure Curve

Fig. 4.3

$$S_1 = \int_0^{(R-a)} (A + Br^2)dr \qquad \text{(origin at O')}$$

$$= Ar(1-\alpha) + \frac{B(1-\alpha)^3R^3}{3}$$

$$S_2 = \frac{A(1+u)(R-a)}{2}$$

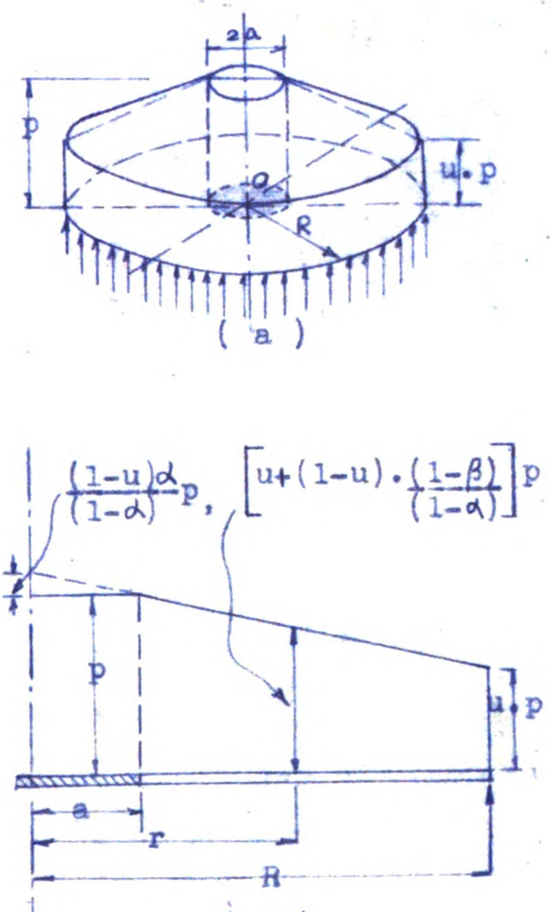
Putting $S_1 = S_2$

$$u = \left[1 + 2/3\ (1 - \alpha)^2 R^2\ \frac{B}{A}\right] \qquad \text{---- (4.10)}$$

where u is the ratio between pressures at point C' and the rim under the new assumption. It is evident that the ratio u is a function of the physical properties of plate, soil, and column which are known (or assumed) before design.

V. FOOTING CARRYING A VERTICAL LOAD RESTING ON ELASTIC SOIL (2) MOMENT ANALYSIS

1. LOAD MOMENTS: The load moments, as defined previously, are internal moments, radial and tangential, of the footing due to foundation pressure when the edge is assumed to be simple supported. Figure 5.1a shows the shape of load exerted on the bottom surface of the footing and Figure 5.1B its diametrical section. Referring to Figure 5.1b,



(b)

Fig. 5.1

The volume of small pressure cone at top is $\frac{1}{3}\ \frac{R^2\alpha^3(1-u)p}{1-\alpha}$, or $\frac{1}{3}R^2\gamma p$, where γ is equal to $\frac{\alpha^3(1-u)}{3(1-\alpha)}$.

Total load on bottom surface $= up\pi R^2 + \frac{1}{3}\pi R^2(1-u)p$

$$- \frac{1}{3}\pi R^2\gamma p$$

$$= \pi R^2 p\left[\frac{2u+1}{3} - \gamma\right] \quad \text{--- (5.1)}$$

Load exerted on a circular area with a radius r =

$$\pi r^2 p\left[u + (1-u)\ \frac{R-r}{R-a}\right] + \frac{1}{3}\pi r^2 p\left[1-u-(1-u)\ \frac{R-r}{R-a}\right]$$

$$- \pi R^2 p\gamma$$

$$= \pi r^2 P\left\{(u-c) + \frac{(1-u)}{3}\left[\frac{3-\alpha}{1-\alpha} - \frac{2r}{R(1-\alpha)}\right]\right\} \quad \text{--- (5.2)}$$

where $c = \tau/\beta^2$.

$$2\pi r Q = (5.2)$$

$$Q = p\left[\frac{(u-c)}{2} + \frac{(1-u)}{6}\,\frac{3-\alpha}{1-\alpha}\right] r - \frac{(1-u)r^2 p}{3(1-\alpha)R}$$

$$= \sigma pr - \epsilon\frac{pr^2}{R}$$

where $\sigma = \left[\frac{(u-c)}{2} + \frac{(1-u)}{6}\,\frac{3-\alpha}{1-\alpha}\right]$, $\epsilon = \frac{(1-u)}{3(1-\alpha)}$.

Since $\frac{d}{dr}\left[\frac{1}{r}\,\frac{d}{dr}\left(r\,\frac{dw}{dr}\right)\right] = \frac{Q}{D}$

Substituting Q into the above equation and integrating, we have,

$$\frac{dw}{dr} = \frac{p\sigma}{8D}r^3 - \frac{p\epsilon}{15RD}r^4 + \frac{C_1 r}{2} + \frac{C_2}{r} \qquad ---(5.3)$$

$$\frac{d^2w}{dr^2} = \frac{3p\sigma}{8D}r^2 - \frac{4p\epsilon}{15RD}r^3 + \frac{C_1}{2} - \frac{C_2}{r^2} \qquad ---\ (5.4)$$

Using the boundary conditions,

$r = a = R$, $\frac{dw}{dr} = 0$

$r = R$, $M_r = 0$, or $\frac{d^2w}{dr^2} + \frac{m}{r}\,\frac{dw}{dr} = 0$

$m = 0.2$ for concrete

and neglecting the terms containing α^4 and α^5 we have

$$C_1 = -\frac{pR^2}{D}\lambda$$

$$C_2 = \frac{pR^4}{2D}\alpha^2\lambda$$

where $\lambda = \frac{\sigma - 0.70\epsilon}{1.5 + \alpha^2}$

Substituting (5.3) and (5.4) with known value of C_1 and C_2 into (2.1) and (2.2) we have,

$$M_r'' = L\left[\frac{(0.6 + 0.4\alpha^2/\beta^2)\lambda - 0.4\sigma\beta^2 + 0.28\epsilon\beta^3}{\pi\left(\frac{2u+1}{3} - \gamma\right)}\right]$$

(moment per unit length) ..(5.5)

$$M_t'' = L\left[\frac{(0.5 - 0.4\alpha^2/\beta^2) - 0.2\alpha\beta^2 + 0.12\epsilon\beta^3}{\pi\left(\frac{2u + 1}{3} - \gamma\right)}\right]$$

(moment per unit length) .... (5.6)

where L is total load exerted on the bottom of the footing which is equal and opposite to the column load P.

2. ACTUAL MOMENTS: Actual internal moments, radial and tangential, in a footing carrying a column load and resting on elastic soil are obtained from deducting the load moments obtained above from the basic moments correspondingly, those are

$$M_r = M_r' \ (3.1) - M_r'' \ (5.5) \qquad \ldots (5.7)$$

$$M_t = M_t' \ (3.2) - M_t'' \ (5.6) \qquad \ldots (5.8)$$

note that P in equation (3.1) and (3.2) is equal to L in equation (5.5) and (5.6).

Equation (5.7) and (5.8) can be put in other forms

$$M_r = K_r P \quad \text{(moment per unit length)} \quad \ldots (5.9)$$

$$M_t = K_t P \quad \text{(moment per unit length)} \quad \ldots (5.10)$$

K's are constants.

Curves for K values in equation (5.9) and (5.10) are plotted with different values of α and u (see P.49-54) so that the designer can readily find the designing moments.

In the particular case u = 1, it becomes the conventional case of assuming the pressure under the footing being uniform.

VI. FOOTING CARRYING A VERTICAL LOAD RESTING ON ELASTIC CONCENTRIC CIRCLES OF PILES

1. DISTRIBUTION OF PRESSURE ON PILES

In investigating the distribution of pressure on piles the same method of minimum energy applied in Section V, is applied again here. As suggested in Section I the piles under a circular footing are to be arranged along the circumferences of concentric circles and in each circle the piles are suggested to be of same modulus of reaction (in practical case, they are of same diameter, length and material.) But piles in different circles may be of different modulus of reactions. Usually, piles in the circumference of inner circles have large modulus than that of outer ones as we can see the pressures over there are larger.

It is assumed that the intensity of reaction is proportional to the deflection of the footing at that point. As the load is symmetrical with respects to center it has equal deflection along the circumference.

Referring to Figure 6.1, z's and k's represent radius and modulus of reaction of different pile circles respectively. In a pile circle the pressure is assumed to be uniformly distributed along its circumference and k values have the unit of force per unit length of circumference per unit deflection. The w's are deflections of the footing at the circumferences.

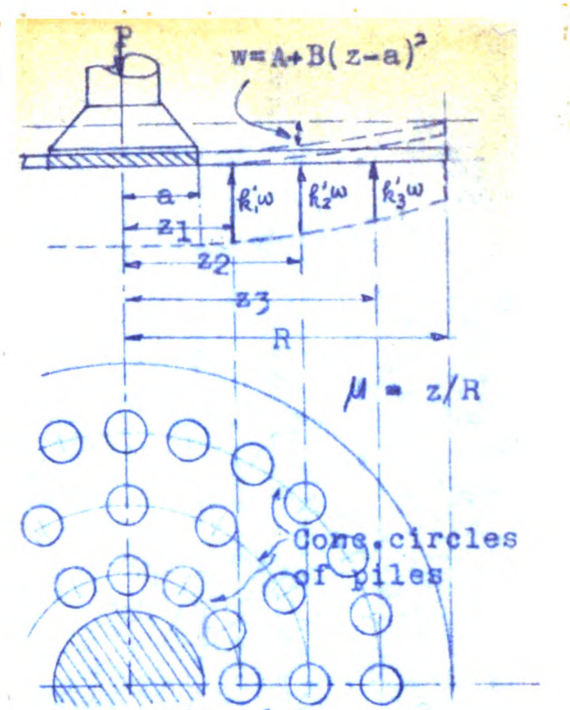


Fig. 6.1

Total load on the circumference of a pile circle with a radius $z_1 = 2\pi z_1 . k'_1 . w_1$

Energy stored in the circle

$$= \frac{\pi}{2} . 2 . z_1 . k'_1 . w_1^2 = \pi z_1 . k'_1 . w_1^2$$

Total elastic energy stored in pile foundation is

$$U_2 = \pi \sum z_1 . k'_1 . w_1^2$$

As assumed before, for small deflections

$$w_1 = A + B\ (z_1 - a)^2$$

$$= A + B\ (\mu_1 - \alpha)^2 R^2$$

where $z_1 = \mu_1 R$

Hence $$U_2 = \pi \sum k'_1 . \mu_1 . R \left[A + B\ (\mu_1 - \alpha)^2 \right]^2 \quad \ldots (6.1)$$

Strain energy stored by the footing is

$$U_1 = 4B^2 D\ (1 + m) \pi R^2\ (1 - \alpha^2) \quad \ldots (6.2) \text{ or } (4.5)$$

Total energy in the system is

$$U = U_1 + U_2 - PA \qquad \ldots (6.3) \text{ or } (4.8)$$

Substituting (6.1) and (6.2) into (6.3) (m = 0.2 for concrete) and putting $\frac{\partial U}{\partial A} = 0$ and $\frac{\partial U}{\partial B} = 0$, we have

$$\frac{B}{A} = -\frac{R \sum \frac{\mu_1(\mu_1 - \alpha)^2}{D/k_1'}}{4.8(1-\alpha^2) + R^3 \sum \frac{\mu_1(\mu_1-\alpha)^4}{D/k_1'}} \qquad \ldots (6.4)$$

The same assumption in Section IV-2 is made here and the shape of diametrical section of pressure curve represented in Figure 6.2 is the same as represented in Figure 4.3b.

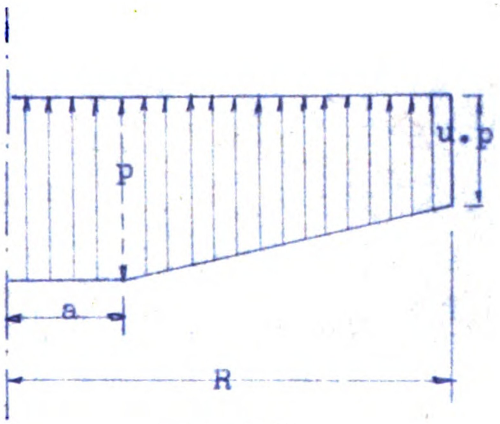


Fig.6.2

In the figure

$$u = \left[1 + \frac{2}{3}(1-\alpha)^2 R^2 \frac{B}{A}\right] \qquad \ldots (6.5) \text{ or } (4.10)$$

The unit pressure exerted by each pile circle of different radius can be proportionated from the ordinate in pressure distribution curve at corresponding radius. The total pressure from each pile circle thus can be calculated from its unit pressure and its circumference.

2. MOMENT ANALYSIS: Considering one pile circle once only, the actual moments, radial and tangential, in a footing are represented by those equations for basic moments, (3.1) and (3.2), except replacing μ, for β and P in those equations representing total pressure exerted on the footing by that pile circle instead of total column load. The total moments in a footing resting on several pile circles can be obtained by summing up the moments due to individual circles correspondingly.

Equations (3.1) and (3.2) can be put in the forms

$M_r = K_r P$ (moment per unit length) . . . (6.6)

$M_t = K_t P$ (moment per unit length) . . . (6.7)

and K values in (6.6) and (6.7) are plot with different values of μ , α and radius of pile circle so that the moments can be obtained readily at desirable points. (see P.55-60).

VII. FOOTING CARRYING A MOMENT LOAD RESTING ON ELASTIC SOIL

1. PRESSURE DISTRIBUTION: On the column, or something otherwise, to which the footing supports, usually carries a moment load besides a vertical one. Like a chimney subjected to wind load and a column subjected to eccentric vertical load are common examples (Figure 7.1). The distribution of foundation pressure in such a case is something like that shown in Figure 7.2a, and its diametrical section in the plane of moment is shown in Figure 7.2b. As the central portion of the footing is rigid, the pressure over there is of linear variation. One side of the diametrical section of pressure distribution curve is magnified in Figure 7.3. If the footing is non-flexible, the pressure distribution would be a straight line throughout and at the rim the pressure is $p = Mc/I$. Now due to the deflection of the footing the

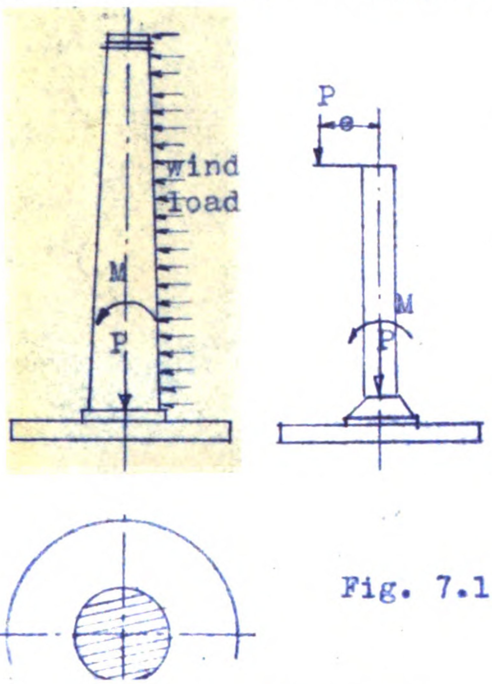


Fig. 7.1

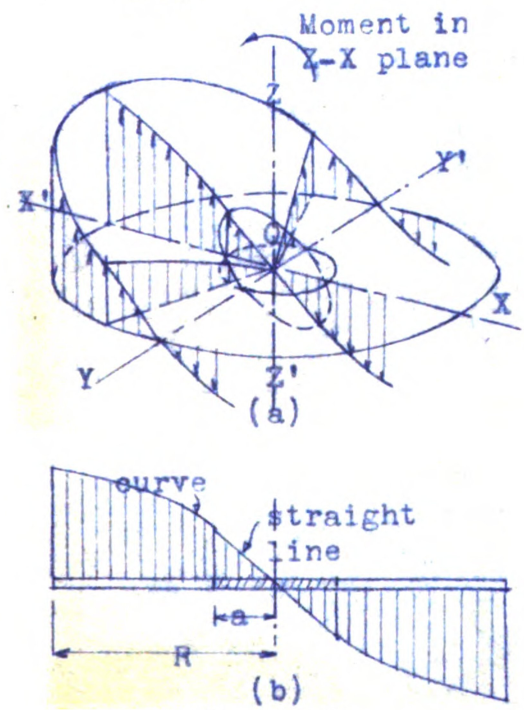


Fig. 7.2

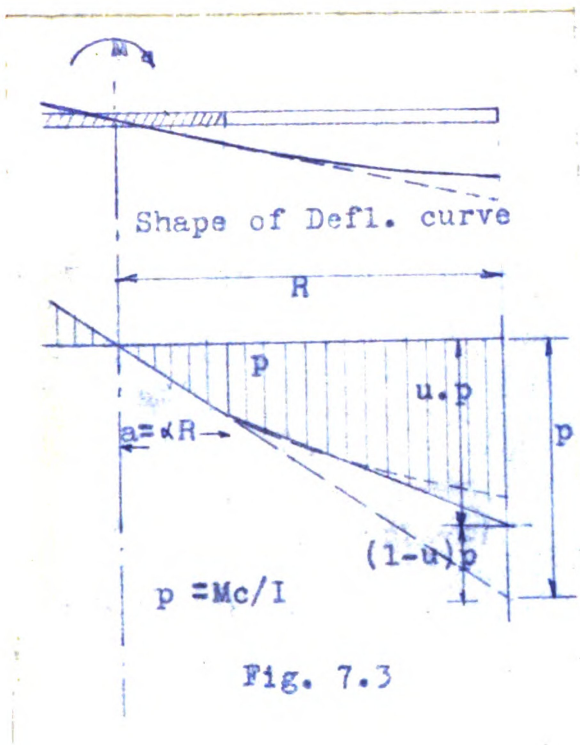


Fig. 7.3

pressure at the rim has been released somewhat becoming less than p, and the distribution of pressure between the rim and the edge of the rigid portion of the footing is along a certain curve. As we assumed first, the deflection is small, the curve is flat, and can be replaced by a straight line, providedly, the areas under those curves are set equal. The pressure at the rim under the assumption is 'up', where u has the same expression as (4.10) and $\frac{B}{A}$ in (4.10) is expressed by (4.9).

2. PROPOSITION OF INTRODUCING A SYMMETRICAL DISTRIBUTION OF PRESSURE IN PRACTICAL DESIGN TO MAKE UNSYMMETRICAL CASE SYMMETRICAL:

The actual case in a circular footing carrying a moment load is of unsymmetrical bending. The exact analysis of such a case is difficult, especially when it rests on elastic foundation. But the moment load, like those produced by wind pressure on a chimney, does not act in a fixed plane and it will revolve in all directions. Thus we have to design the footing in every section according to the maximum pressure distribution curve which is the diametrical curve in the plane of the applied moment. The tension part the distributed pressure under the footing is not interested in design and may be neglected, which is one the safe side. The shape of the introduced symmetrical distribution of foundation pressure making the unsymmetrical case symmetrical is shown in Figure 7.4, where p = Mc/I as before.

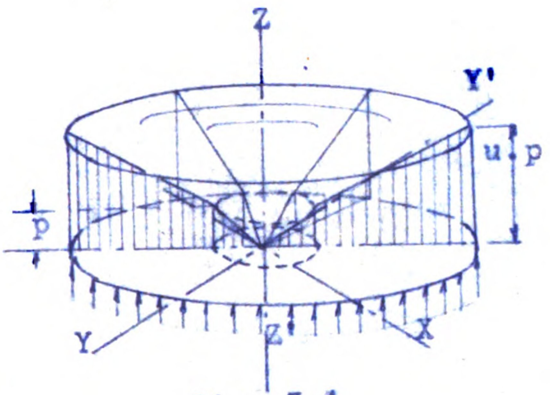


Fig. 7.4

If the design is made according to the proposed pressure it will sustain the moment load from any direction.

In the case of a column carrying an eccentric load which produces a moment in a fixed plane, the footing is suggested to design with the same proposed symmetrical load and just neglect the reinforcements in that part of the footing (semi-circular) which is in the tension region of foundation pressure due to the applied moment.

3. MOMENT ANALYSIS: Figure 7.5 shows the geometrical properties of the diametrical section of proposed symmetrical pressure distribution curve.

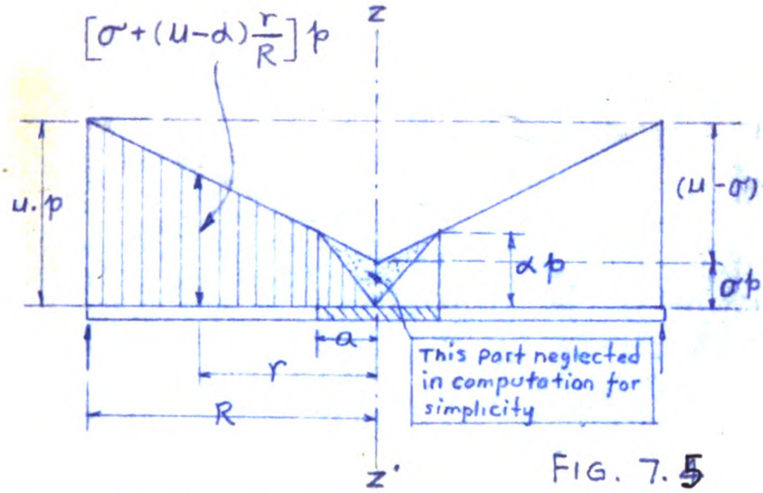


FIG. 7.5

Vol. of Pressure at any Radius r

$$= pr^2\pi\left[\sigma + \epsilon \frac{r}{R}\right] \text{ ----(A)}$$

Total vol. of Pressure

$$= pR^2\pi\left[\sigma + \epsilon\right] \text{ ----(B)}$$

$$\sigma = \alpha\left[1 - \frac{u-\alpha}{1-\alpha}\right]$$

$$\epsilon = \frac{2}{3}\left[\frac{u-\alpha}{1-\alpha}\right]$$

The load moments, or the radial and tangential moments in the footing when it is simple supported are obtained as follows:

$$2\pi r Q = p\pi r^2\left[\sigma + \epsilon\frac{r}{R}\right]$$

$$Q = \frac{p\sigma r}{2} + \frac{p\epsilon r^2}{2R} \qquad \text{. . . (7.1)}$$

where σ and ϵ are constants expressed in Figure 7.5.

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right)\right] = \frac{Q}{D} \qquad \text{. . . (7.2) or (2.7)}$$

Substituting (7.1) into (7.2) integrating, we have

$$\frac{dw}{dr} = \frac{p\sigma}{16D}r^3 + \frac{\epsilon p}{30DR}r^4 + \frac{C_1 r}{2} + \frac{C_2}{r} \qquad \text{. . . (7.3)}$$

$$\frac{d^2w}{dr^2} = \frac{3p\sigma}{16D}r^2 + \frac{4\epsilon p}{30DR}r^3 + \frac{C_1}{2} - \frac{C_2}{r^2} \qquad \text{. . . (7.4)}$$

Using the boundary conditions

$$r = a = R, \quad \frac{dw}{dr} = 0$$

$$r = R, \quad M_r = 0 \text{ or } \frac{d^2w}{dr^2} + \frac{m}{r}\frac{dw}{dr} = 0$$

and putting m = 0.2 for concrete, integrating and neglecting terms containing α^4 and α^5, we have

$$C_1 = -\frac{p\,R^2}{D}\tau \qquad \ldots (7.5)$$

$$C_2 = \frac{p\,R^4}{2D}\tau \qquad \ldots (7.6)$$

where $\tau = \left[\frac{0.50\sigma + 0.35\epsilon}{1.5 + \alpha^2}\right]$

Substituting (7.5) and (7.6) into (7.3) and (7.4) we have

$$M_r'' = -D\left[\frac{d^2w}{dr^2} + \frac{m}{r}\,\frac{dw}{dr}\right]$$

$$= pR^2\pi(\sigma + \epsilon)\frac{(0.60\sigma + 0.40\epsilon) - 0.20\sigma\beta^2 - 0.14\epsilon\beta^3}{\pi(\sigma + \epsilon)} \qquad \ldots (7.7)$$

$$M_t'' = -D\,\frac{1}{r}\,\frac{dw}{dr} + m\,\frac{d^2w}{dr^2}$$

$$= pR^2\pi(\sigma + \epsilon)\;\frac{(0.60\sigma - 0.40\epsilon) - 0.10\sigma\beta^2 - 0.06\epsilon\beta^3}{\pi(\sigma + \epsilon)} \qquad \ldots (7.8)$$

Actual moments, radial and tangential, are obtained by deducting (7.7) and (7.8) from (3.1) and (3.2) respectively, providedly the term $pR^2\pi\,(\sigma + \epsilon)$ which is the total pressure, in (7.7) and (7.8) is set equal to P in (3.1) and (3.2), we have

$$M_r = M_r'\ (3.1) - M_r''\ (7.7)$$

$$M_t = M_t'\ (3.2) - M_t''\ (7.8)$$

or put into other form

$$M_r = K_r pR^2 \text{ (moment per unit length)} \qquad \ldots (7.9)$$

$$M_t = K_t pR^2 \text{ (moment per unit length)} \qquad \ldots (7.10)$$

where $p = Mc/I$ as before and K_r, K_t are constants.

Curves for K values in equation (7.9) and (7.10) are plot with different values of α and u, so that the moments can be readily obtained at desirable points. (see P. 61-66)

VIII. FOOTING CARRYING A MOMENT LOAD RESTING ON ELASTIC CONCENTRIC PILES

Applying the same reasoning in section VI-1, the same shape of pressure distribution curve in section VII-1 will have in a footing carrying a moment load and resting on elastic concentric piles except the B/A term involved in expression for u is the expression (6.4) instead of (4.9).

The unit pressure exerted by each pile circle of different radius can be proportionated from the ordinate in the distribution curve at corresponding radius. The total pressure from each circle can be calculated from its unit pressure and its circumference and the moments can be obtained by repeated use of equation (6.6) and (6.7) if there are several pile circles under the footing.

IX. NUMERICAL EXAMPLES

1. FOOTING CARRYING A VERTICAL LOAD AND A MOMENT LOAD RESTING ON ELASTIC SOIL

Data: Vertical Load . . . . 1,200,000 lbs.

Revolving Moment Load . . . . . 1,500,000 ft-lb

Size of Column Capital. . . . . $\alpha = 0.20$

Allowable Soil Pressure . . . . 6,000 lb/sq.ft.

Modulus of Reaction of

Soil k' . . . . 2,600,000 lb/cu.ft.

Modulus of Elasticity of

Concrete E_c . . . . 3,000,000 lb/sq.in.

$f_c = 1{,}350$ psi, $f_s = 20{,}000$ psi, $n = 10$,

$f'_c = 3{,}000$ psi, $k = 0.400$, $j = 0.866$, $K = 235$

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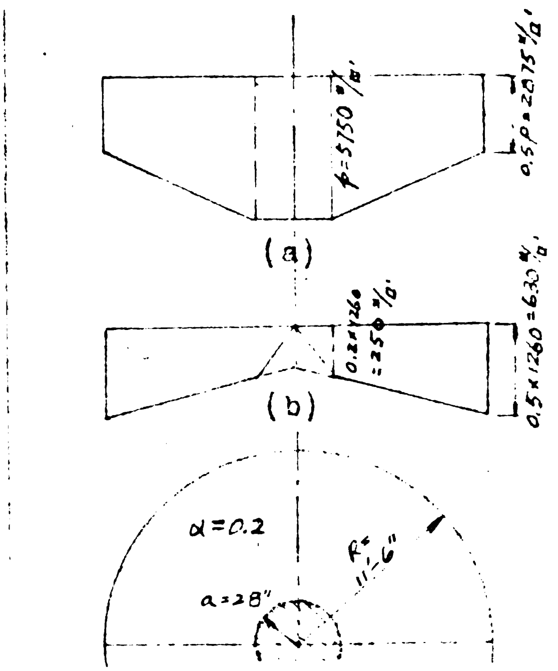
(a) Proportioning the Base Area

Vertical Load =	1,200,000 lbs.
Assumed Wt. of Footing =	208,000 lbs.
Total P	1,408,000 lbs.

Assuming the dia. of footing being 11.5 ft.

$I = AR^2/4 = \pi \times 11.5^4/4 = 13{,}700$ ft.^{4}



p' due to moment = Mc/I

$= \dfrac{1{,}500{,}000 \times 11.5}{13{,}700}$

= 1,260 psf

p'_1 at edge of capital =

$p'\alpha = 0.2 \times 1{,}260 = 250$ psf

Net utilizable pressure for carrying the vertical load

= 6,000 - 250 = 5,750 psf

Assuming $u = 0.4$,

p_2' at the rim of the footing = $u\ p' = 0.4 \times 1,260$

$= 504$ psf.

$504 < 0.4 \times 5750$, (no tension at rim, O.K.)

$$1,400,000 = \pi R^2 \times 5750 \left[\frac{2u + 1}{3} - \gamma \right] \quad \text{. . .(Eq. 5.1)}$$

$$\frac{2u + 1}{3} = 1.8/3 = 0.60, \quad \gamma = \frac{\alpha^3 (1 - u)}{3\,(1-\alpha)} = 0.002$$

$$1,408,000 = \pi R^2 \times 5750 \times (0.600 - 0.002)$$

$$R^2 = \frac{1,408,000}{\pi \times 5750 \times 0.598} = 130, \quad R = 11.4'$$ use 11 ft-6in. as assumed before

$$p'R^2 = 1260 \times 11.5^2 = 167,000 \text{ lbs.}$$

Radius of column capital = $0.2 \times 11.5 = .23$ ft. or 28"

(b) Moment Calculations

Moments at different sections are found from the K-valve-curves corresponding to each particular case.

RADIAL MOMENT TABLE

Section	0.2R	0.4R	0.6R	0.8R	1.0R
K_r	0.17	0.045	0.010	0	0
$M_r = K_rP$ (due to vertical load) (Plate 2)	238,000	65,000	14,000	0	0
K_r	0.21	0.063	0.015	0	0
$M_r = K_r p'R^2$ (due to Moment Load) (Plate 14)	35,100	10,500	2,510	0	0
Total M_r in-lb/in.	273,100	73,500	16,510	0	0

TANGENTIAL MOMENT TABLE

Section	0.2R	0.3R	0.35R	0.4R	0.6R	0.8R	1.0R
K_t	0.041	0.056	0.058	0.054	0.035	0.025	0.0185
$M_t = K_t P$ (due to vertical load) (Plate 6)	57,500	78,500	81,300	75,600	49,000	35,000	25,900
K_t	0.046	0.070	0.068	0.036	0.043	0.034	0.026
$M_t = K_t p' R^2$ (due to moment load) (Plate 1)	7,680	11,700	11,300	11,000	7,180	5,700	4,350
Total M_t in-lb./in.	65,180	90,200	92,600	86,600	56,180	40,700	30,250

(c) Calculation of Depth

$$h = \sqrt{M_r/K} = \sqrt{273{,}100/235} = \sqrt{1160} = 34.0'' \text{ use } 40'' \text{ overall.}$$

Wt. of footing = x 11.5^2 x 40 x 150 = 208,000 lbs.
O.K. assumed.

Flexural Rigidity $D = \dfrac{E_c k^2 j}{2(1 - m^2)} h^3$ (see Appendix)

$$= \frac{3{,}000{,}000 \times 144 \times .375^2 \times .875}{2\,(1 - 0.2 \times 0.2)} \times \left(\frac{34}{12}\right)^3$$

$$= 6.30 \times 10^8 \text{ lb.-ft.}$$

(d) Checking the shearing stress

v_c allowable = 0.3 f'_c = 0.03 x 3000 = 90 psi

$b = (0.2 \times 11.5 \times 12 + 34) \times 2\pi$

$= (28 + 34) \times 2\pi$

= 390 in. (critical section is a circle with r = 62 in.)

Shear due to vertical load

$$V_1 = \left\{1{,}408{,}000 - \left[\text{Equ.}(5.2) \text{ for } r = 62/12 = 5.17'\right]\right\} \frac{1{,}200{,}000}{1{,}408{,}000}$$

$$= \left\{1{,}408{,}000 - \pi\, 5.17^2 \times 5750 \times \left[(0.4 - 0) + \left(\frac{1 - 0.4}{3}\right) \times\right.\right.$$

TANGENTIAL MOMENT TABLE

Section	0.2R	0.3R	0.35R	0.4R	0.6R	0.8R	1.0R
K_t	0.041	0.056	0.058	0.054	0.035	0.025	0.0185
$M_t = K_tP$ (due to vertical load) (Plate 6)	57,500	78,500	81,300	75,600	49,000	35,000	25,900
K_t	0.046	0.070	0.068	0.066	0.043	0.034	0.026
$M_t = K_tp'R^2$ (due to moment load) (Plate 1)	7,680	11,700	11,300	11,000	7,180	5,700	4,350
Total M_t in-lb./in.	65,180	90,200	92,600	86,600	56,180	40,700	30,250

(c) Calculation of Depth

$$h = \sqrt{M_r/K} = \sqrt{273,100/235} = \sqrt{1160} = 34.0'' \text{ use } 40'' \text{ overall.}$$

Wt. of footing = x 11.5^2 x 40 x 150 = 208,000 lbs.
O.K. assumed.

$$\text{Flexural Rigidity } D = \frac{E_ck^2j}{2(1 - m^2)} h^3 \text{ (see Appendix)}$$

$$= \frac{3,000,000 \times 144 \times .375^2 \times .875}{2\ (1 - 0.2 \times 0.2)} \times \left(\frac{34}{12}\right)^3$$

$$= 6.30 \times 10^8 \text{ lb.-ft.}$$

(d) Checking the shearing stress

v_c allowable = 0.3 f'_c = 0.03 x 3000 = 90 psi

b = (0.2 x 11.5 x 12 + 34) x 2π

= (28 + 34) x 2π

= 390 in. (critical section is a circle with r = 62 in.)

Shear due to vertical load

$$V_1 = \left\{1,408,000 - \left[\text{Equ.}(5.2) \text{ for } r = 62/12 = 5.17\right]\right\} \frac{1,200,000}{1,408,000}$$

$$= \left\{1,408,000 - \pi 5.17^2 \times 5750 \times \left[(0.4 - 0) + \left(\frac{1 - 0.4}{3}\right) \times\right.\right.$$

(f) Computation of steel area

A_s1 (for tangential moment) = M_t/f_sjd

$= M_t/20{,}000 \times 0.875 \times 34 = M_t/595{,}000$ sq.in./in.

A_s2 (for radial moment in region of single layer of steel, assuming 1"Ø bars used)

$= M_r/20{,}000 \times 0.875 \times (34 + 1) = M_r/613{,}000$ sq.in./in.

A_s3 (for radial moment in region of double layer of steel assuming 1"Ø bar used and a vertical distance of 3"cc between the layers)

$= M_r/ 20{,}000 \times .875 \times (34 - 2.5) = M_r/552{,}000$ sq.in./in.

RADIAL MOMENT STEEL SCHEDULE

Section	0.2R	0.4R	0.6R	0.8R	1.0R
Moment lb-"/"	273,000	73,500	16,510	0	0
Layer of steel	double	single		single	
As required	0.250 (each layer)	0.119	0.027 (unit-sq.in./in.)	0	0
	1"□ @3½"cc each layer	1"□ @7"cc + ½"Ø @7"cc	½"Ø @7½"cc	0	0
As furnished	1"□ @3½"cc each layer	1"□ @7"cc -½"Ø @7"cc	½"Ø@ 5½"cc	½"Ø @7"cc	½"Ø @8.75"cc

TANGENTIAL MOMENT STEEL SCHEDULE

Section	0.2R	0.3R	0.35R	0.4R	0.6R	0.8R	1.0R
Moment lb-"/"	65,180	90,200	92,600	86,600	65,180	40,700	30,250
As required	0.110	0.150	0.155	0.145	0.095	0.059	0.051
	(single layer, sq.in./in.)						
	1"Ø @7"cc	1"Ø @5"cc	1"Ø @5"cc	1"Ø @5½"cc	1"Ø @6½"	1"Ø @8½"cc	3/4"Ø @8½"cc
As furnished			the same as required				

(g) <u>Checking the bond stress</u>

The critical section for bond is at the edge of the column capital.

Shearing force due to vertical load = V_1

$$= \left[1,408,000 - \left(\frac{28}{12}\right)^2 \pi \times 5750\right] \times \frac{1,200,000}{1,408,000}$$

$= 423,000$ lbs.

Shearing force due to moment load = V_2

$= (\text{Equ. A in Fig. 7.5}) - 0.2 \times 1250 \times \left(\frac{28}{12}\right)^2 \pi$

$= 1250 \times \pi \times 42 - 1250 \times \pi \times 5.42$

$= 1250 \times \pi\ (42 - 1.8) = 159,000$ lbs.

Total shearing force at the critical section =

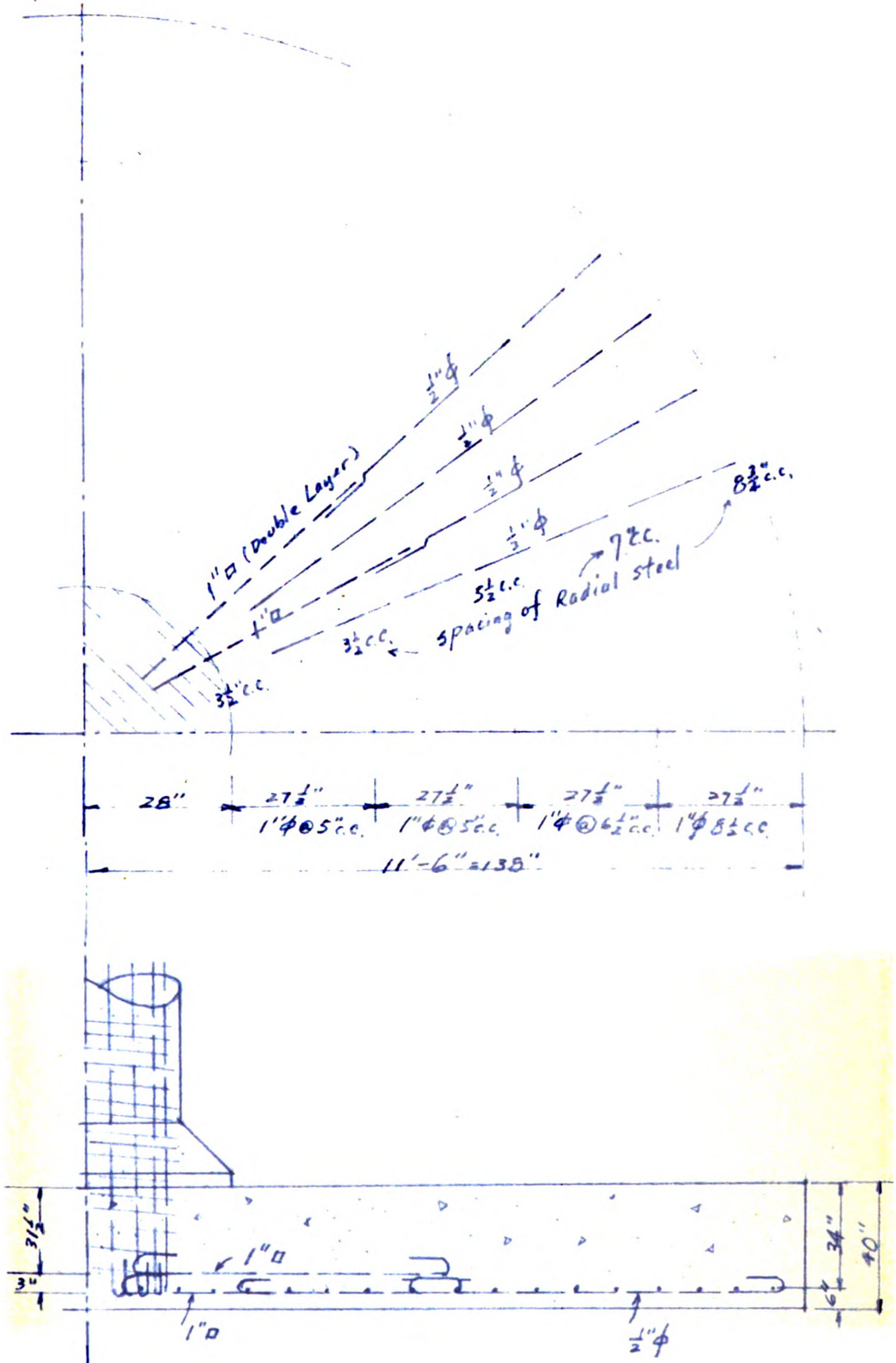
$V = V_1 + V_2 = 582,000$ lbs.

Bond stress $= V/\Sigma o\,jd = 582,000/314 \times .875 \times 36.5 = 85$psi.

where $\Sigma o = \frac{2 \quad \times 28}{3\frac{1}{2}} \times 3.14 \quad \times 2 = 314$ in.

Bond stress allowable $= 0.0375 f'_c = 0.0375 \times 3000$

$= 112$ psi O.K.



1" ☐ (Double Layer)
1/2" φ
1/2" φ
1/2" φ
1/2" φ
1" ☐
8 3/4" c.c.
7" c.c.
5 1/2 c.c.
3 1/2 c.c.
spacing of Radial steel
3 1/2" c.c.
28"
27 1/2"
27 1/2"
27 1/2"
27 1/2"
1" φ @ 5" c.c.
1" φ @ 5" c.c.
1" φ @ 6 1/2" c.c.
1" φ 8 1/2 c.c.
11'-6" = 138"
31 1/2"
3"
1" ☐
1" ☐
1/2" φ
34"
40"
6"

FIG. 9.2

2. FOOTING CARRYING A VERTICAL LOAD AND A MOMENT LOAD RESTING ON ELASTIC PILES

Data: Vertical Load . . . . 1,200,000 lbs

Revolving Moment Load 1,500,000 ft-lbs.

Size of Column Capital $\alpha = 0.3$

Allowable Bearing Capacity of Pile:

Piles in inner circle. . . . . . 30 tons each

Piles in middle circle . . . . . 40 tons each

Piles in outer circle . . . . . 35 tons each

Modulus of Reaction of Piles k':

Pile in inner circle . . . . 26×10^5 lbs/ft each

Pile in middle circle. . . . . . . 34×10^6 lbs/ft each

Pile in outer circle . . . . 30×10^5 lbs/ft each

Modulus of Elasticity of Concrete E_c-3,000,000 psi

$f_c = 1{,}350$ psi, $f_s = 20{,}000$ psi, $n = 10$,

$f_c' = 3{,}000$, $k = 0{,}400$, $j = 0.866$, $K = 235$

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(a) <u>The general arrangement of piles</u>

The arrangement of piles is shown in the figure 9.3 and the diameter of the footing is chosen to be 10 ft.

$z_1 = 3$ ft., $\mu_1 = 0.3$

$z_2 = 6$ ft., $\mu_2 = 0.6$

$z_3 = 9$ ft., $\mu_3 = 0.9$

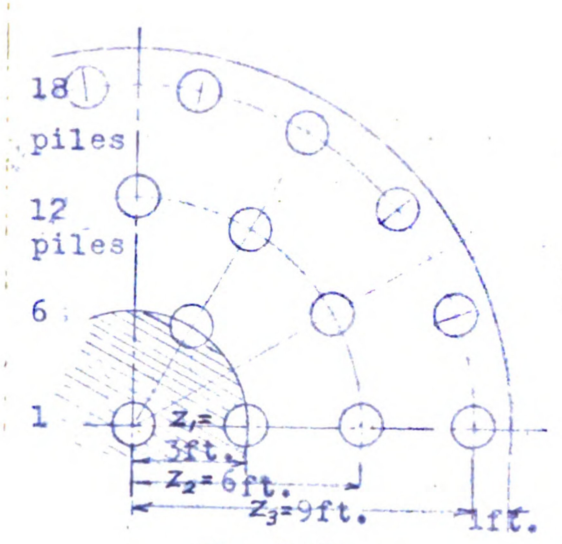


Fig.9.3

(b) Computation of Load on Piles

A. Load on piles due to vertical load

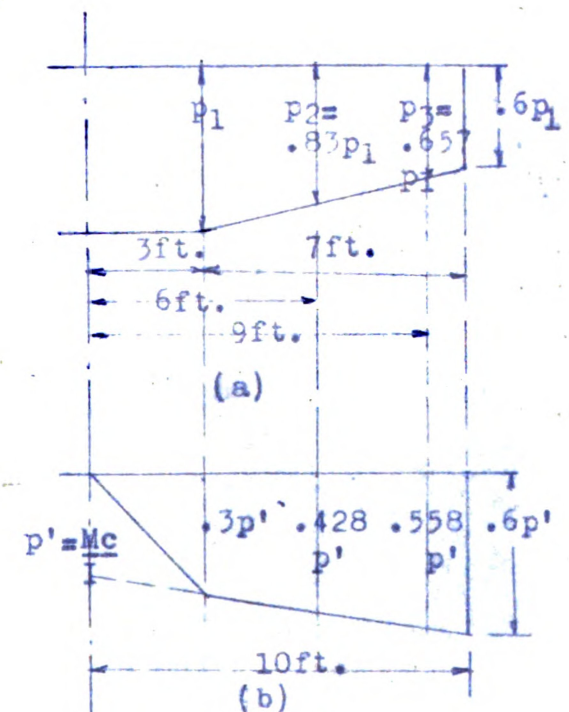


Fig.9.4

Referring to Figure 9.4a, Assuming u= 0.6 we have:

Unit pressure at middle pile circle = p_2

$= [0.6 + 0.4 \times 4/7] p_1$

$= 0.83 p_1$ lb/lin.ft.

Unit pressure at outer pile circle = p_3

$= [0.6 + 0.4/7] p_1$

$= 0.657 p_1$ lb/lin.ft.

Total upward pressure = column load + wt. of footing

$$P = \frac{2\pi \times 3 \times p_1}{6} + 2\pi \times 3 \times p_1 + 2\pi \times 6 \times p_2 + 2\pi \times 9 \times p_3$$

(reaction of center pile) (reaction of inner pile cir.) (reaction of middle pile cir.) (reaction of outer pile cir.)

$= (p_1 + 6p_1 + 12x0.83p_1 + 18x0.657p_1)\pi$

$= 28.8\pi p_1 = 90.5p_1$

Column Load	1,200,000 lbs.
Assumed wt. of footing . . .	160,000 lbs.
Total F	1,360,000 lbs.

$p_1 = 1,360,000/90.5 = 15,000$ lbs/lin.ft.

$p_2 = 0.83 \times 15,000 = 12,500$ lbs/lin.ft.

$p_3 = 0.675 \times 15,000 = 9,900$ lbs/ lin.ft.

Load on entire inner pile circle $= L_1 =$

$15,000 \times 2\pi x3 = 282,000$ lbs.

Load on entire middle pile circle $= L_2 =$

$12,500 \times 2\pi x6 = 472,000$ lbs.

Load on entire outer pile circle $= L_3 =$

$9,900 \times 2\pi x9 = 592,000$ lbs.

B. Load on piles due to moment load

Referring to Figure 9.4b,

$p_1' = 0.3p'$

$p_2' = (0.3 + 0.3x3/7) = 0.428p'$

$p'3 = (0.3 + 0.3x6/7) = 0.558p'$

$p' = Mc/I$, $I = \pi r^3/2$ for each circle (assuming the area of the piles being uniformly distributed along the circumference of each circle)

$I = \frac{1}{2}x\pi(9^3 + 6^3 + 3^3) = \frac{1}{2} x\pi x962 = 1,510$ ft^3

$p' = 1,500,000 \times 10/1,510 = 10,000$ lbs./ft.

$p_1' = 0.3\text{x } 10,000 = 3,000$ lbs./ft.

$p_2' = 0.428\text{x}10,000 = 4,280$ lbs./ft.

$p'3 = 0.558\text{x}10,000 = 5,580$ lbs./ft.

Load on entire inner pile circle $= L_1' =$

$3,000\text{x } 2\pi\text{x}3 = 56,700$ lbs.

Load on entire middle pile circle $= L_2' =$

$4280 \text{ x } 2\pi\text{x}6 = 162,000$ lbs.

Load on entire outer pile circle $= L'3 =$

$5,580 \text{ x } 2\pi\text{x}9 = 316,000$ lbs.

Total load on inner circle $= L_1 + L_1' =$

$282,000 + 56,700 = 338,700$ lbs.

Load on each pile in inner circle $= 337,800/6$

$= 56,300$ lbs.

(< 30 tons O.K.)

Total load on middle circle $= L_2 + L_2' =$

$472,000 + 162,000 = 534,000$ lbs.

Load on each pile in middle circle $= 534,000/12$

$= 44,500$ lbs.

(< 40 tons O.K.)

Total load on outer circle $= L_3 + L_3' =$

$592,000 + 316,000 = 908,000$ lbs.

Load on each pile in outer circle $= 908,000/18$

$= 50,400$ lbs.

(< 35 tons O.K.)

(c) Computation of Moments

$M_r = K_r$ x Total Load on Circle, $M_t = K_t$ x Total load on circle.

MOMENT SCHEDULE

	Section	0.3R	0.4R	0.45R	0.6R	0.8R	1.0R
M_r	$\mu_2 = .3$ K_r(Plate 9)	0.13	0.065	---	0	0	0
	K_rL	69,500	36,600		0	0	0
	$\mu_3 = .9$ K_r(Plate 9)	0.206	0.13	---	0.057	0.014	0
	K_rL	187,000	118,000		51,700	12,700	
	Radial Moment in in-lb/in.	256,500	154,600		51,700	12,700	0

M_t	$\mu_2 = .3K_t$(Plate12)	0.026	0.034	0.032	0.022	0.022	0.022
	K_tL	13,900	18,100	17,100	11,800	11,800	11,800
	$\mu_3 = .9K_t$(Plate12)	0.043	0.065	0.071	0.062	0.048	0.038
	K_tL	39,100	59,000	64,500	56,300	43,600	34,500
	Tangential Moment in in-lb./in.	53,000	77,100	81,600	68,100	55,400	46,300

(d) Calculation of Depth

$$h = \sqrt{M_r/K} = \sqrt{256,000/235} = \sqrt{1,100} = 33.2" \text{ use } 34"$$

and a overall thickness of 40".

Wt. of footing $= \pi \times 100 \times 40 \times 150/12 = 157,000$ lbs. as assumed

Flexural Rigidity of the section = 6.3×10^8 lb.-ft.

(the same as in Example 1.)

(e) Checking the Shearing Stress

The critical section for shear is a circular section at a distance 34" from the edge of the column capital.

$b = (0.3 \times 10 \times 12 + 34) \times 2\pi$

$= 70\pi = 440"$.

$v_c = 0.03f'_c = 0.03 \times 3000 = 90$ psi

Shearing force due to vertical load $= V_1 =$

$(1,360,000 - 282,000 - 282,000/6)\ 1,200,000/1,360,000$

$= (1,360,000 - 329,000) \times 0.883 = 912,000$ lbs.

Shearing force due to moment load = V_2 =

1/2 x (162,000 + 360,000) = 261,000 lbs. (Only 1/2 of the total load is effective in calculating shearing force)

Total shearing force at critical section = V = $V_1 + V_2$

= 912,000 + 261,000 = 1,173,000 lbs.

$$v_c = \frac{1,173,000}{440 \times .875 \times 34} = 90 \text{ psi O.K.}$$

(f) Checking the value of u

Modulus of Reaction of inner pile circle = k'_1

$= 26 \times 10^6 \times 6/2\pi \times 3 = 8.3 \times 10^6 \ \text{lb./ft}^2$

Modulus of Reaction of middle pile circle = k'_2

$= 34 \times 10^6 \times 12/2\pi \times 6 = 10.8 \times 10^6 \ \text{lb./ft.}^2$

Modulus of Reaction of outer circle = k'_3

$= 30 \times 10^6 \times 18/\ 2 \times 9 = 9.5 \times 10^6 \ \text{lb/ft}^2$

$D/k'_1 = 6.3 \times 10^8 / 8.3 \times 10^6 = 76.1 \text{ ft.}^3$

$D/k'_2 = 6.3 \times 10^8 / 10.8 \times 10^6 = 61.0 \text{ ft.}^3$

$D/k'_3 = 6.3 \times 10^8 / 9.5 \times 10^6 = 66.5 \text{ ft.}^3$

α	μ	$\mu(\mu-\alpha)^2$	$\mu(\mu-\alpha)^4$	$\frac{\mu(\mu-\alpha)^2}{D/k'}$	$\frac{\mu(\mu-\alpha)^4}{D/k'}$
0.3	μ_1 = 0.3	0	0	0	0
	μ_2 = 0.6	0.054	0.0048	8.83×10^{-4}	7.86×10^{-5}
	μ_3 = 0.9	0.324	0.0116	48.6×10^{-4}	17.4×10^{-5}
			Σ	57.43×10^{-4}	25.26×10^{-5}

$$\frac{B}{A} = -\frac{R\sum \frac{\mu(\mu-\alpha)^2}{D/k'}}{4.8(1-\alpha^2) + R^3\sum \frac{\mu(\mu-\alpha)^4}{D/k'}} \quad \ldots (5.4)$$

$$= - \frac{10 \times 57.43 \times 10^{-4}}{4.8 \times 0.91 + 1,000 \times 23.28 \times 10^{-3}}$$

$$= -5.74 \times 10^{-2} = - 1.23 \times 10^{-2}$$

$$u = 1 + \frac{2}{3} \quad (1 - \alpha)^2 R^2 \frac{E}{A} \qquad \ldots (3.5)$$

$$= (1 - 0.667 \times 0.49 \times 100 \times 1.23 \times 10^{-2})$$

$$= (1 - 0.4) = 0.6 \text{ (as assumed before O.K.)}$$

(g) The remaining computations are similar to those corresponding ones in Example 1 and they are omitted here.

4. CONCLUSIONS

(1) The moments, both radial and tangential, in a circular footing under any loading and foundation conditions, decrease greatly if the size of the column or column capital increases. A most economical ratio of α at which the sum of the volumes of footing and column capital is a minimum in the particular case under consideration could be obtained from several trial designs.

(2) The tangential moment is always much more smaller than the radial moment at the same point under any loading and foundation conditions.

(3) The maximum radial moment occurs at the edge of the column capital and the maximum tangential moment, in usual case ($\alpha \lesseqqgtr 0.3$), occurs at a section somewhere between the edge of the capital and the mid-point of the remaining part of diametrical section. As those maximum moments do not occur at the same point, the reinforcing steel in the footing will not be over-crowded in the section near column capital, which renders the design practical.

(4) In any case, the radial moment decreases much more rapidly than the tangential moment at the same point along the radius of the footing. It shows only the central part of the footing needs heavy reinforcement.

(5) The ratio u affects both tangential and radial moments due to moment load more than those due to vertical load

on the same foundation, both of them decrease if u decreases.

(6) As we assumed the foundation pressure being proportional to the deflection of the footing at that point, the pressure will be larger at the center than at the rim. Evidently this kind of pressure distribution, which is more or less dealing with fact, needs bigger footing area than that required by conventional design of assuming uniform pressure with the same specified allowable soil or pile bearing capacity. The fact shows the footing design in conventional way has greater tendency of settlement as the central part of the foundation underneath is over-stressed. In other words, with the same specified degree of settlement the allowable bearing capacity of foundation could be raised in design with the new method presented here.

(7) The new design requires thinner section than that required by the conventional design of same specifications as due to both the affect of u in (5) and the affect of considering bending in all directions. (The conventional design assumed the bending only occuring at certain pairs of parallel sections.)

XI. CHARTS FOR DESIGN

Plate 1.	Radial Moment	= 0.10	
Plate 2.	Radial Moment	0.20	
Plate 3.	Radial Moment	0.30	Vertical Load
Plate 4.	Tangential Moment	0.10	Elastic Soil Foundation
Plate 5.	Tangential Moment	0.20	
Plate 6.	Tangential Moment	0.30	
Plate 7.	Radial Moment	0.10	
Plate 8.	Radial Moment	0.20	
Plate 9.	Radial Moment	0.30	Vertical or Moment Load
Plate 10.	Tangential Moment	0.10	Elastic Foundation Pile
Plate 11.	Tangential Moment	0.20	
Plate 12.	Tangential Moment	0.30	
Plate 13.	Radial Moment	0.10	
Plate 14.	Radial Moment	0.20	
Plate 15.	Radial Moment	0.30	Moment Load,
Plate 16.	Tangential Moment	0.10	Elastic Soil Foundation
Plate 17.	Tangential Moment	0.20	
Plate 18.	Tangential Moment	0.30	

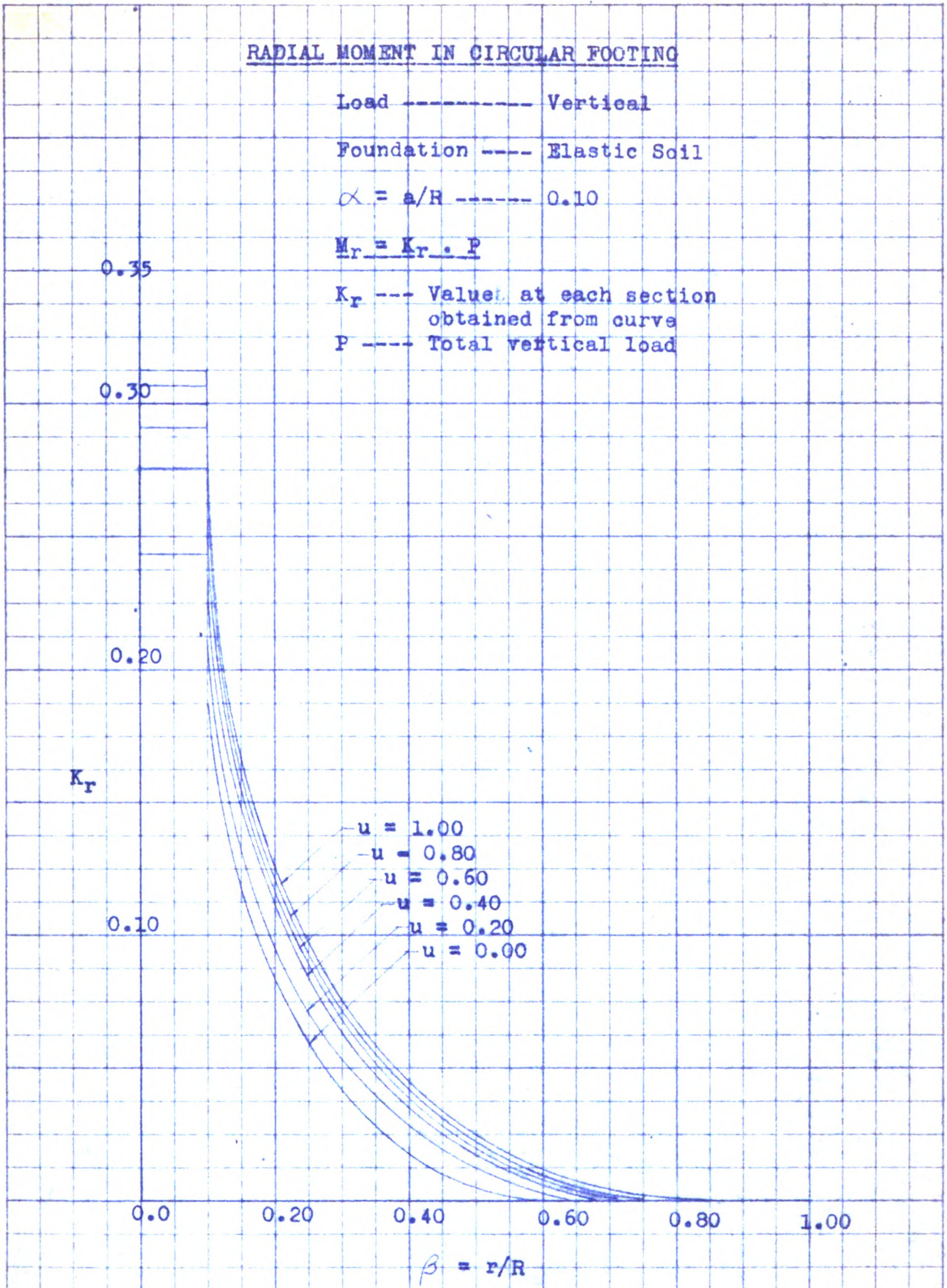

RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical
Foundation ---- Elastic Soil
α = a/R ------ 0.10
Mr = Kr . P
Kr --- Value at each section obtained from curve
P ---- Total vertical load
0.35
0.30
0.20
0.10
Kr
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 1

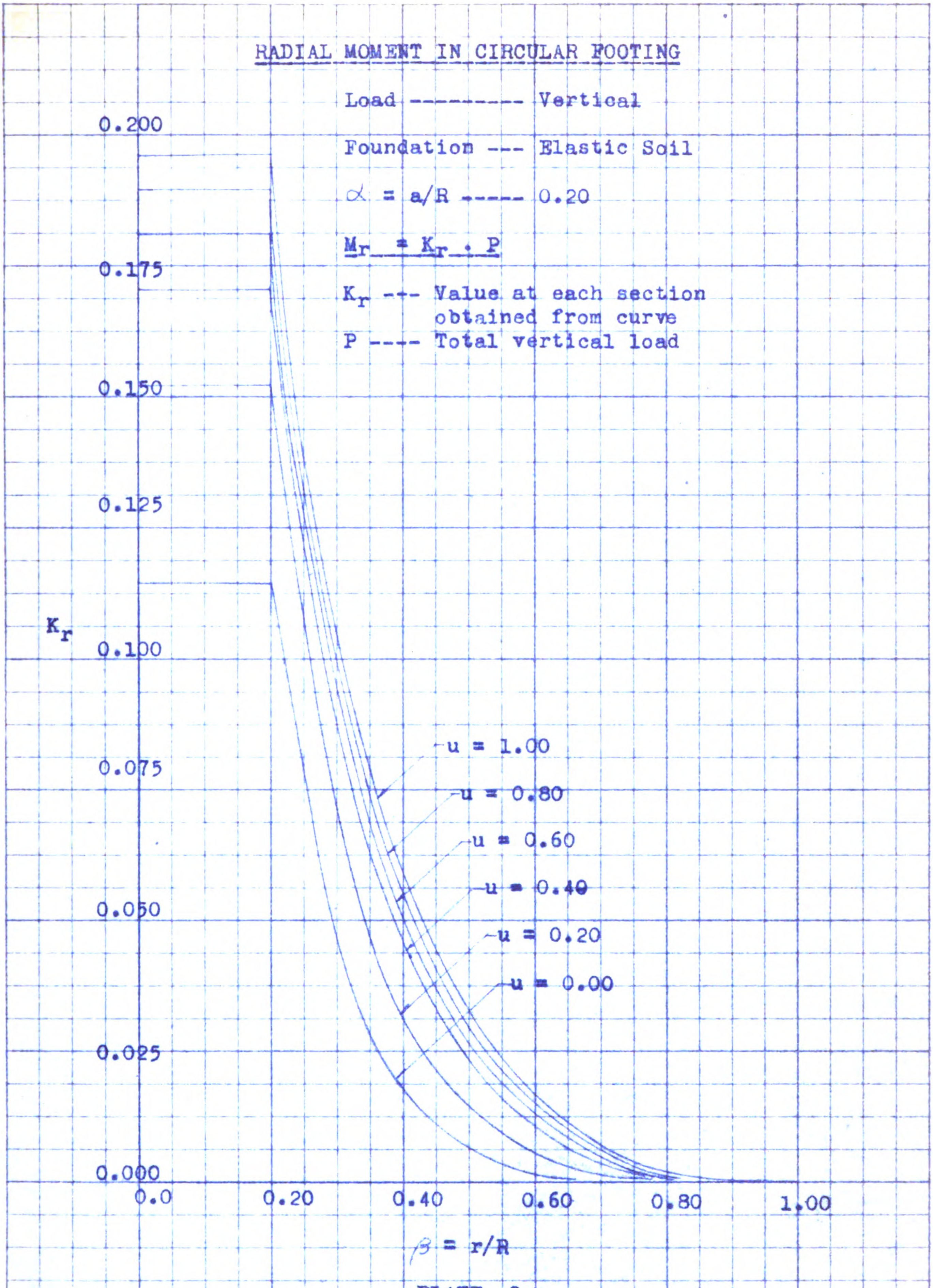

RADIAL MOMENT IN CIRCULAR FOOTING
Load --------- Vertical
Foundation --- Elastic Soil
α = a/R ----- 0.20
$M_r = K_r \cdot P$
K_r --- Value at each section obtained from curve
P ---- Total vertical load
0.200
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.000
K_r
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 2

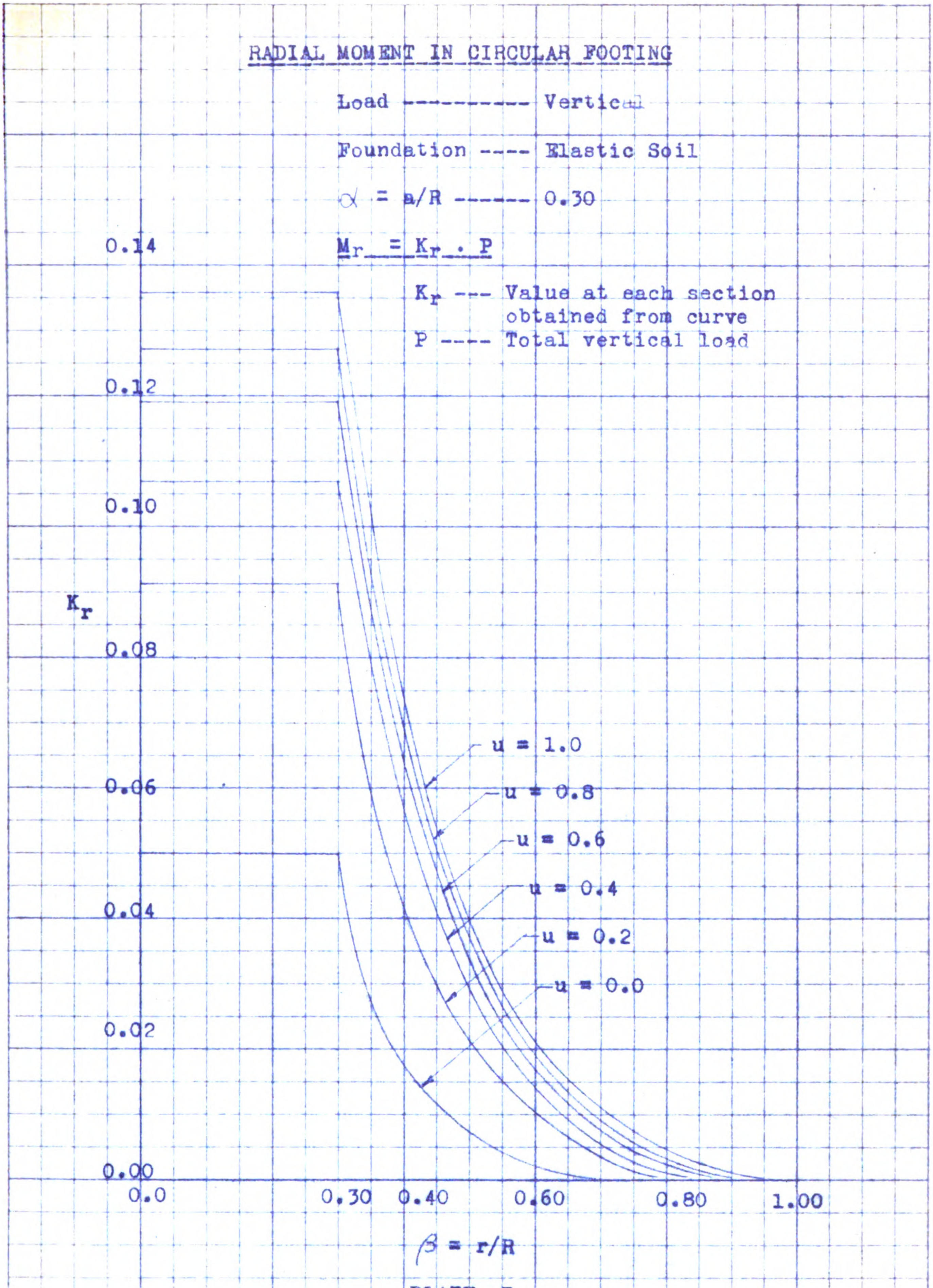

RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical
Foundation ---- Elastic Soil
α = a/R ------ 0.30
$M_r = K_r \cdot P$
K_r --- Value at each section obtained from curve
P ---- Total vertical load
0.14
0.12
0.10
K_r
0.08
0.06
0.04
0.02
0.00
u = 1.0
u = 0.8
u = 0.6
u = 0.4
u = 0.2
u = 0.0
0.0
0.30
0.40
0.60
0.80
1.00
β = r/R

PLATE 3

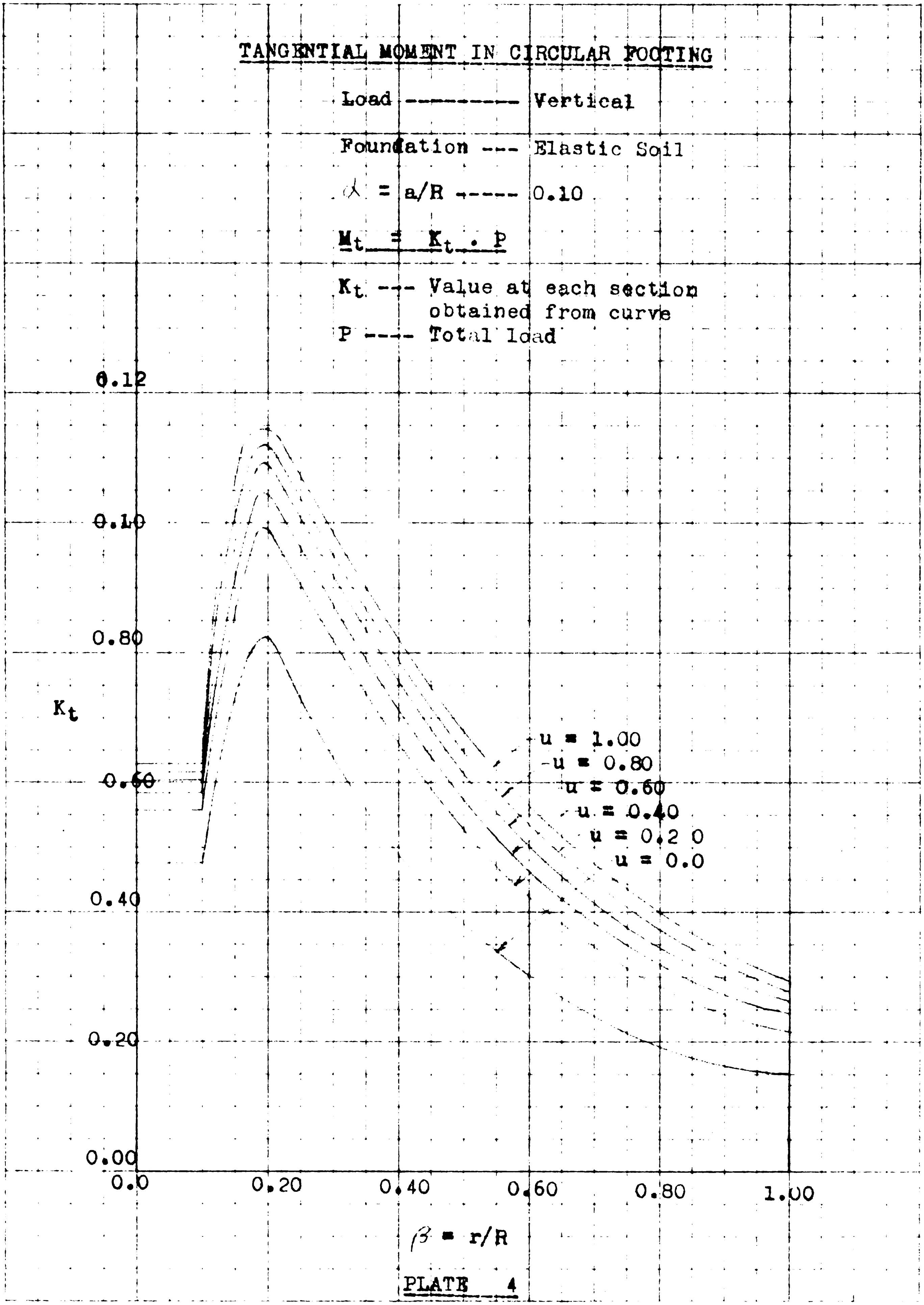

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical
Foundation --- Elastic Soil
α = a/R ----- 0.10
M_t = K_t . P
K_t --- Value at each section obtained from curve
P --- Total load
0.12
0.10
0.80
K_t
0.60
0.40
0.20
0.00
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.2 0
u = 0.0
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 4

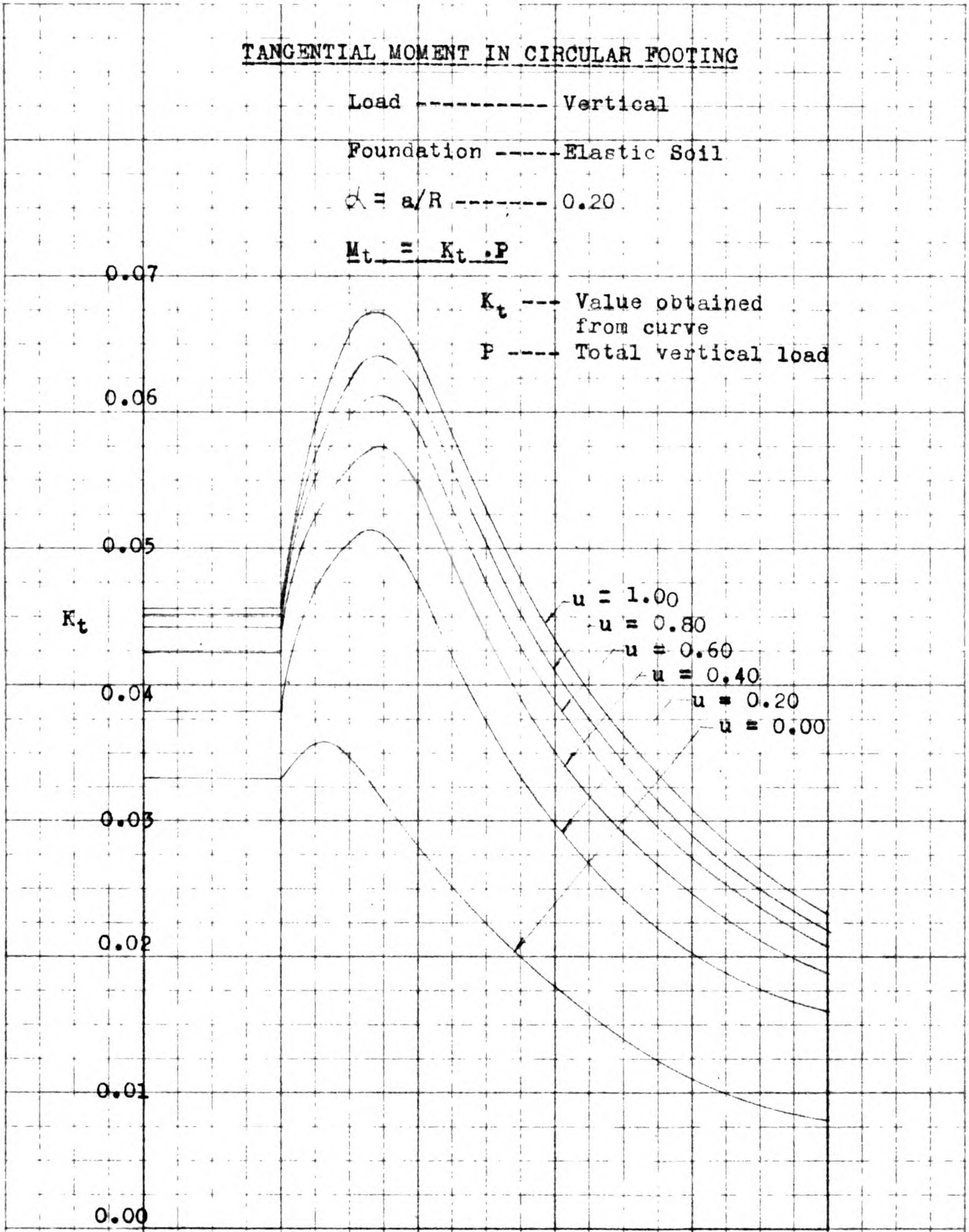

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical
Foundation -----Elastic Soil
α = a/R ------- 0.20
Mt = Kt .P
Kt --- Value obtained from curve
P ---- Total vertical load
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
Kt
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 5

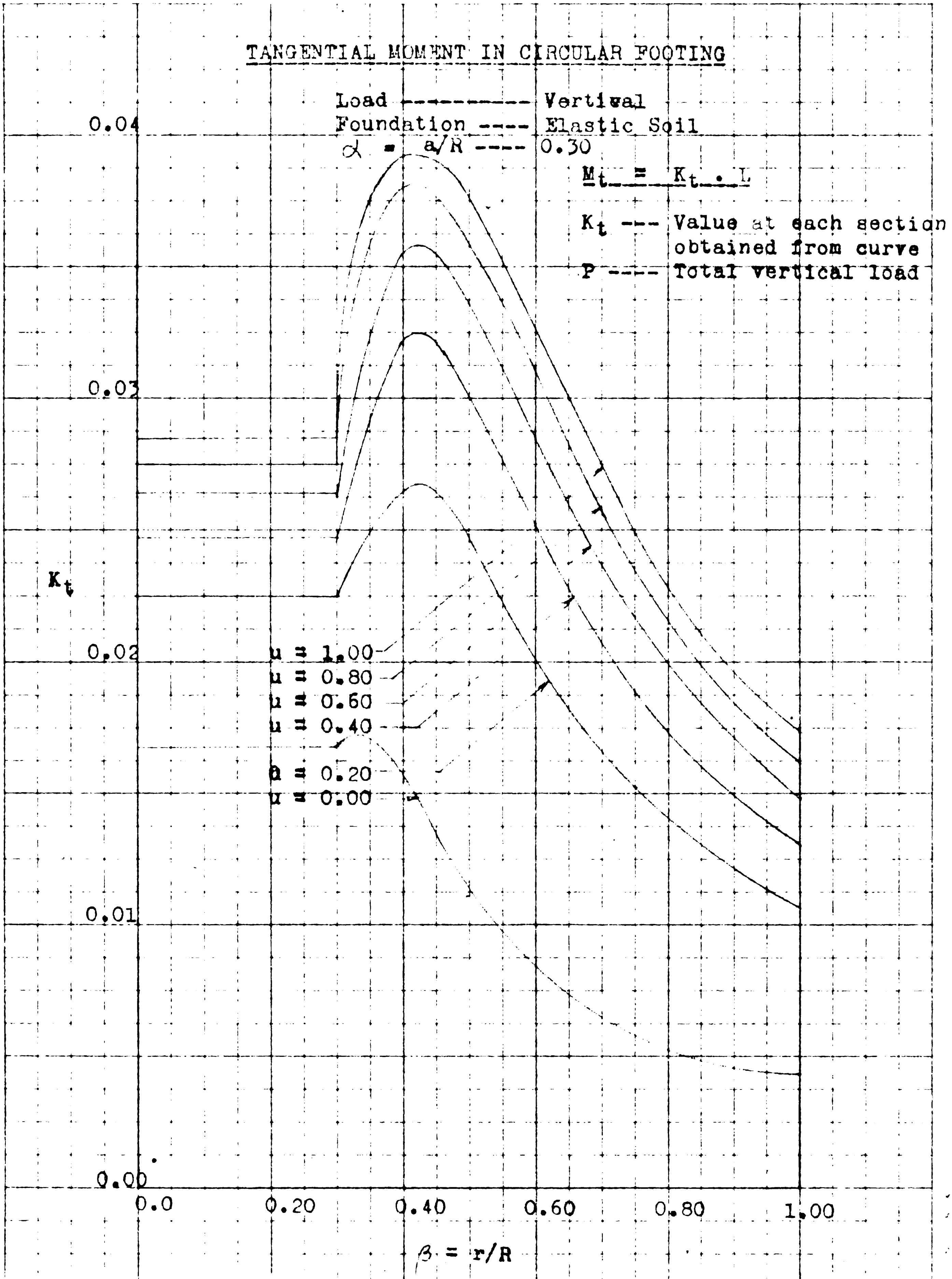

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical
Foundation ---- Elastic Soil
α = a/R ---- 0.30
Mt = Kt . L
Kt --- Value at each section obtained from curve
P ---- Total vertical load
0.04
0.03
0.02
0.01
0.00
Kt
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 6

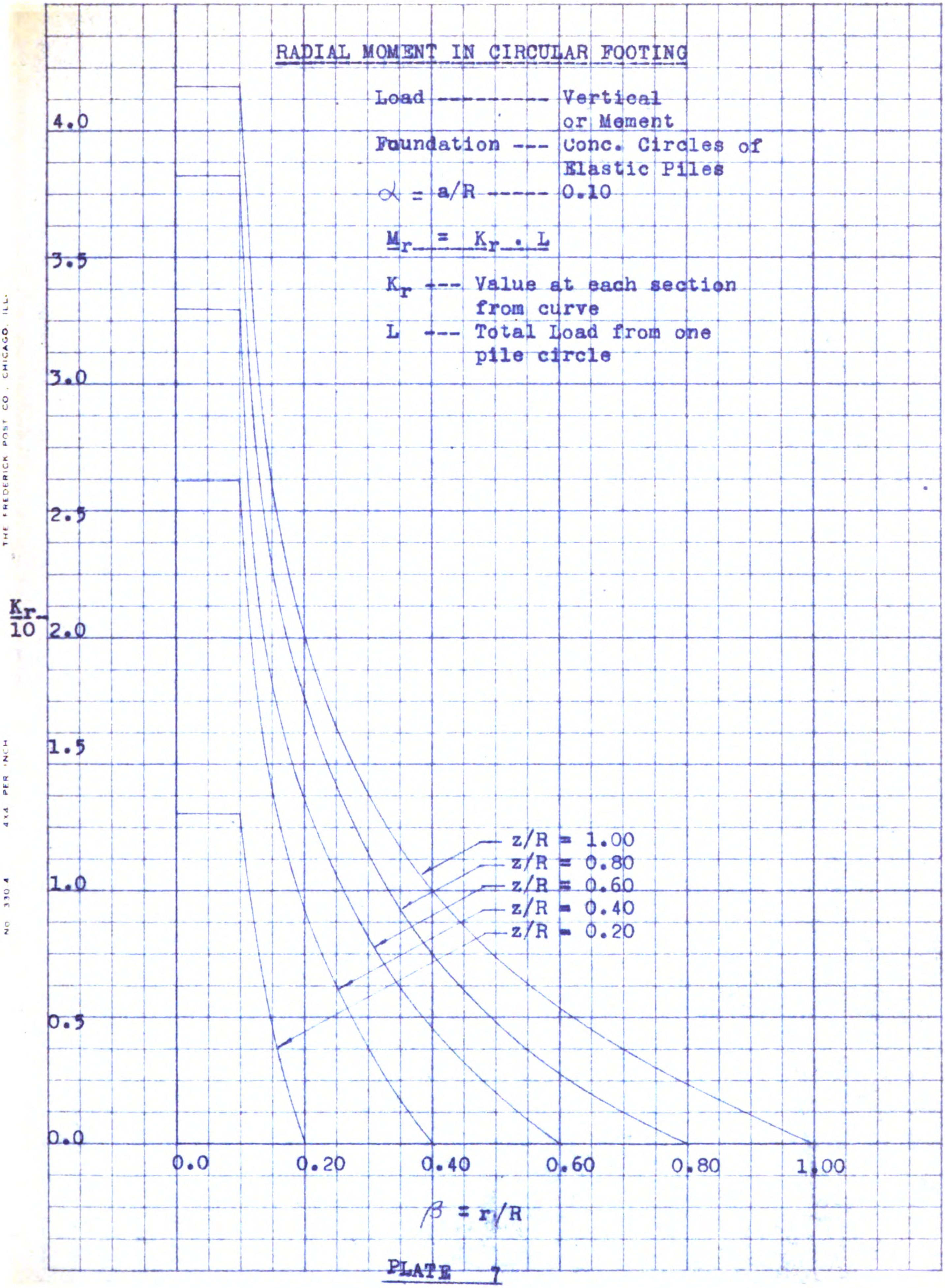

RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical or Moment
Foundation --- Conc. Circles of Elastic Piles
α = a/R ----- 0.10
Mr = Kr . L
Kr --- Value at each section from curve
L --- Total Load from one pile circle
z/R = 1.00
z/R = 0.80
z/R = 0.60
z/R = 0.40
z/R = 0.20
Kr/10
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 7

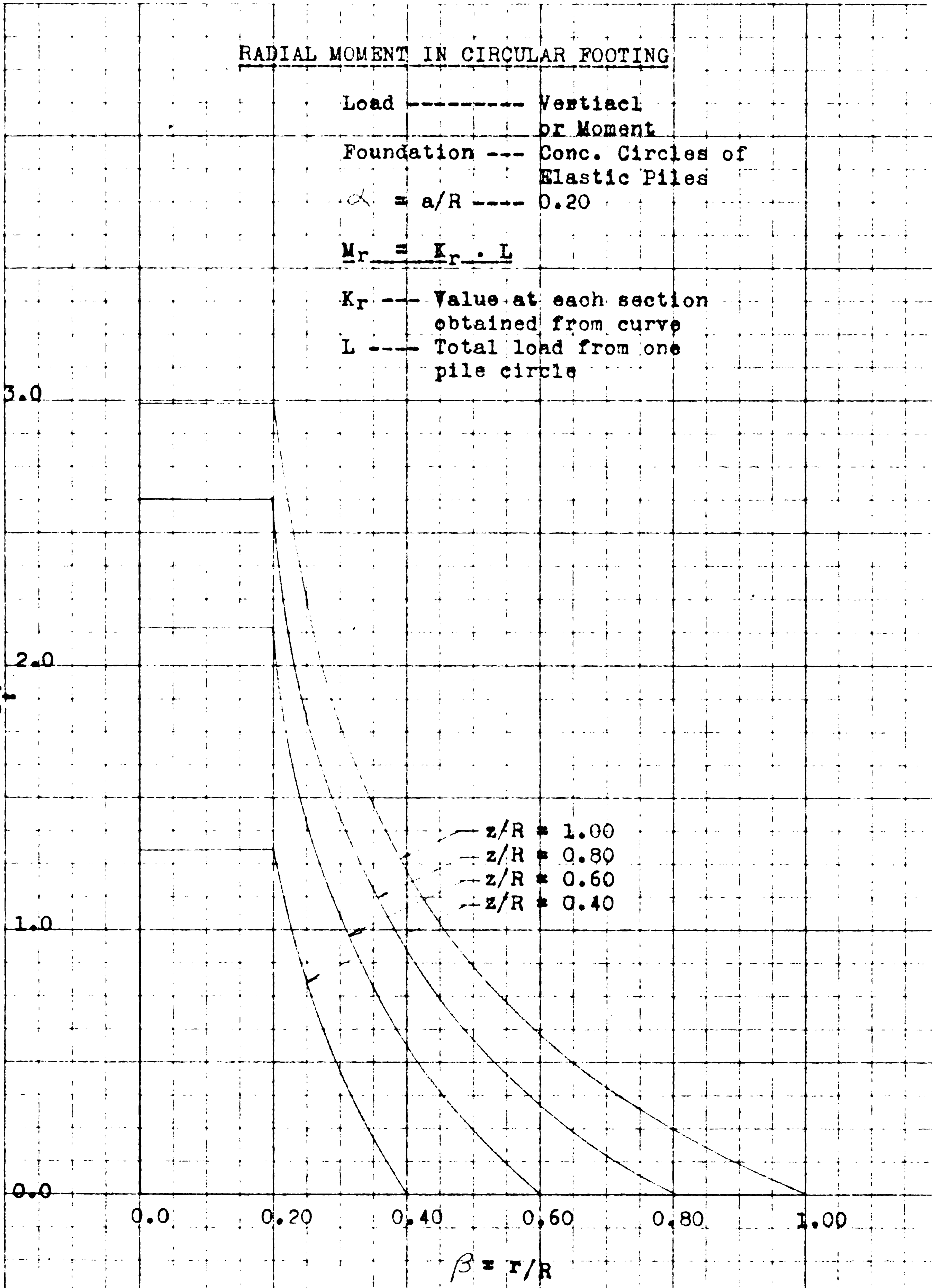

RADIAL MOMENT IN CIRCULAR FOOTING
Load --------- Vertiacl or Moment
Foundation --- Conc. Circles of Elastic Piles
α = a/R ---- 0.20
Mr = Kr . L
Kr --- Value at each section obtained from curve
L ---- Total load from one pile circle
3.0
2.0
1.0
0.0
Kr/10
z/R = 1.00
z/R = 0.80
z/R = 0.60
z/R = 0.40
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 8

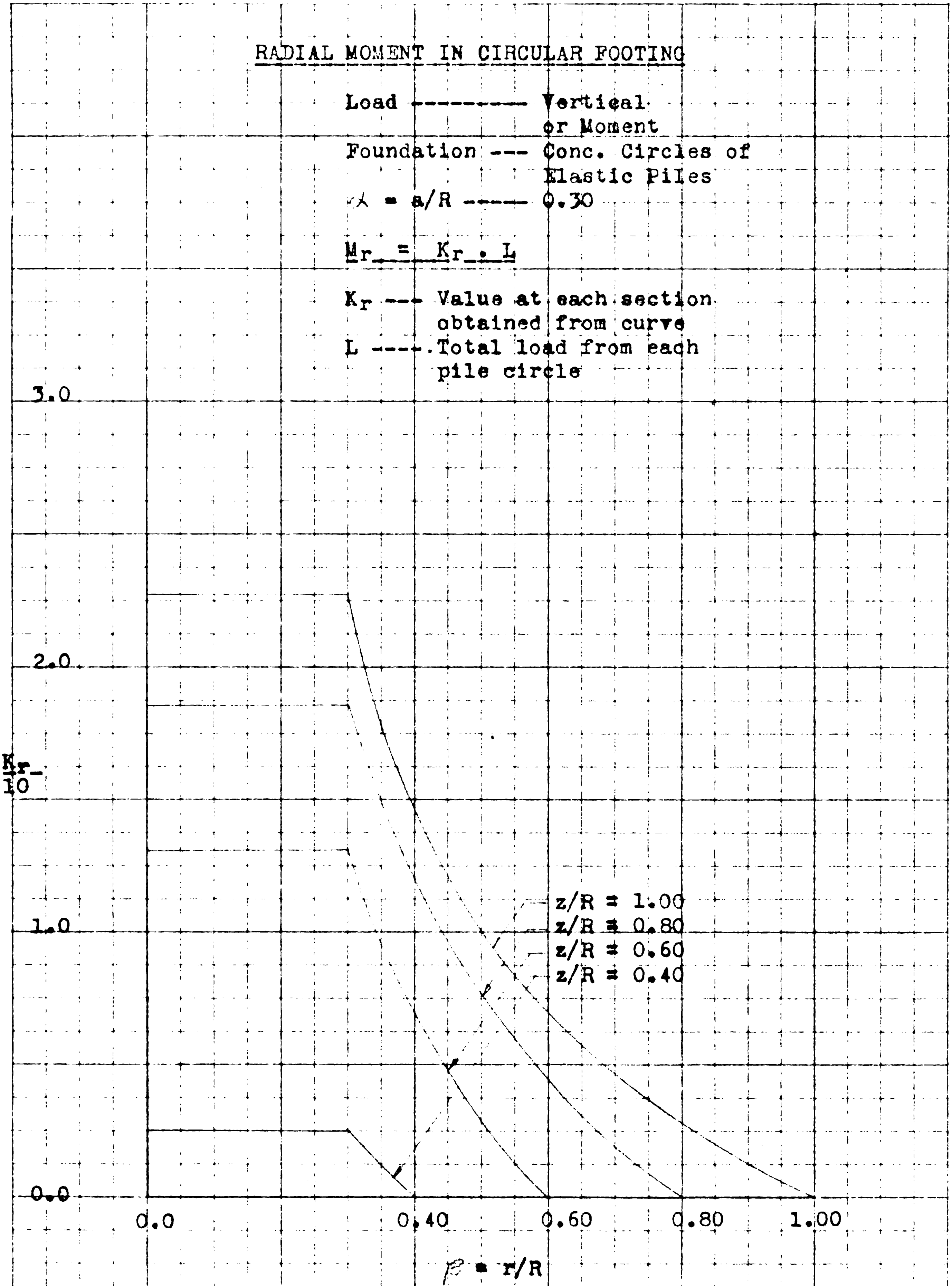

RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical or Moment
Foundation --- Conc. Circles of Elastic Piles
α = a/R ------ 0.30
Mr = Kr . L
Kr --- Value at each section obtained from curve
L ----.Total load from each pile circle
3.0
2.0
1.0
0.0
Kr/10
z/R = 1.00
z/R = 0.80
z/R = 0.60
z/R = 0.40
0.0
0.40
0.60
0.80
1.00
β = r/R

PLATE 9

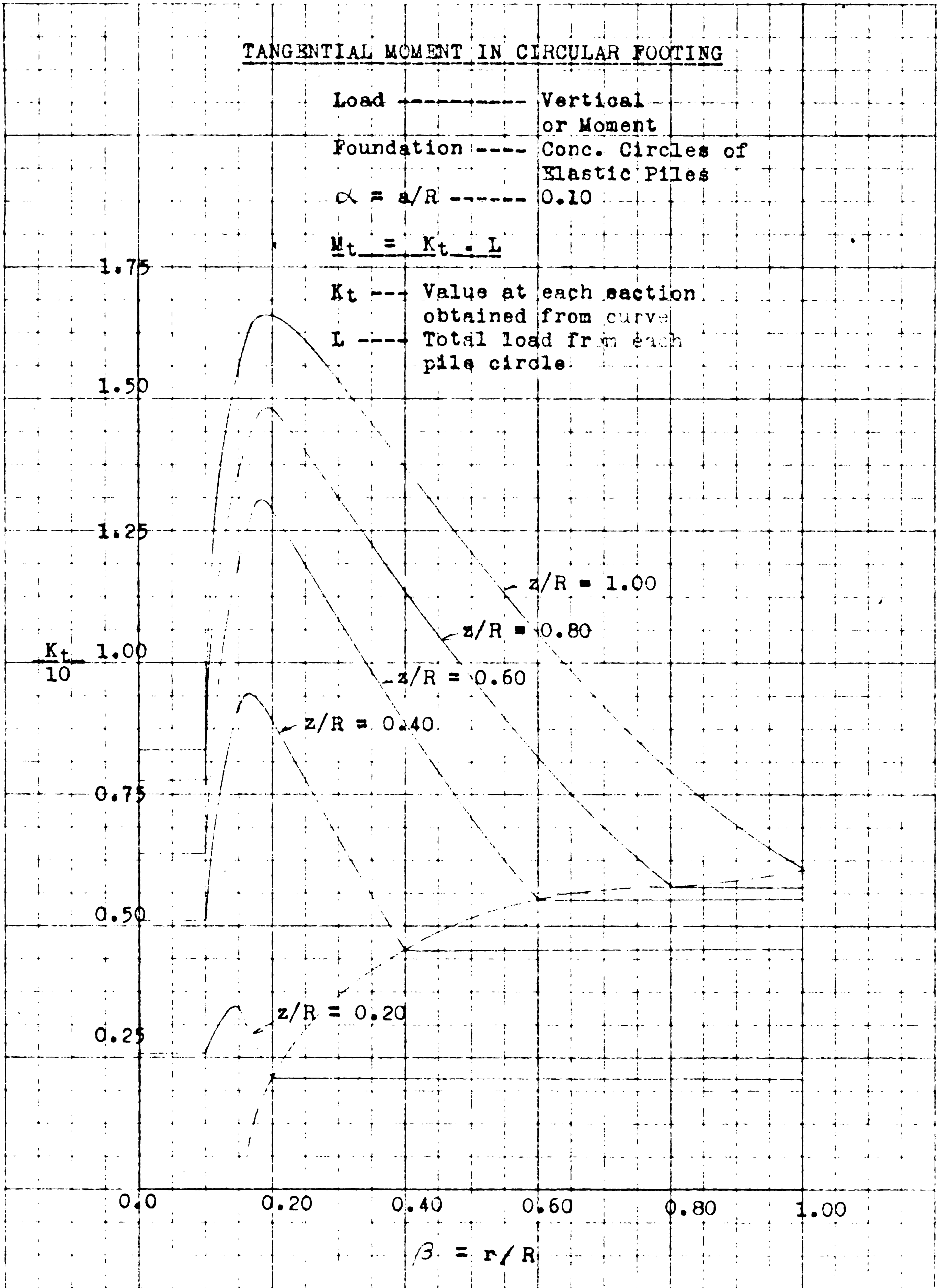

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical or Moment
Foundation ---- Conc. Circles of Elastic Piles
α = a/R ------- 0.10
Mt = Kt . L
Kt --- Value at each section obtained from curve
L ---- Total load fr m each pile circle
z/R = 1.00
z/R = 0.80
z/R = 0.60
z/R = 0.40
z/R = 0.20
Kt / 10
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 10

TANGENTIAL MOMENT IN CIRCULAR FOOTING

Load ---------- Vertical or Moment

Foundation ---- Conc. Circles of Elastic Piles

$\alpha = a/R$ ------- 0.20

$M_t = K_t \cdot L$

K_t = Value at each section obtained from curve

L = Total load from each pile circle

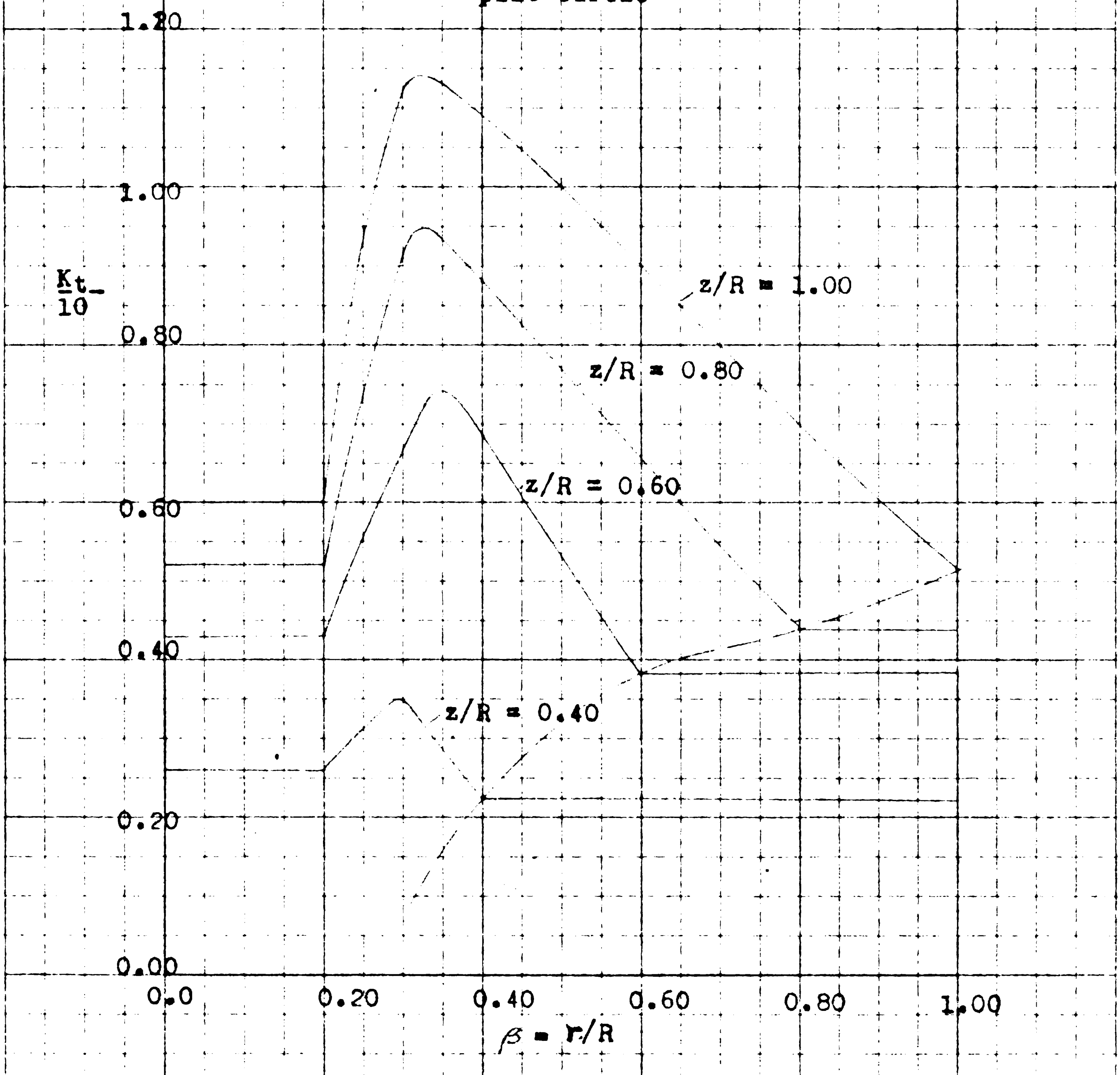


PLATE 11

TANGENTIAL MOMENT IN CIRCULAR FOOTING

Load ----------- Vertical
or Moment

Foundation ---- Conc. Circles of
Elastic Piles

$\alpha = a/R$ ------ 0.30

$M_t = K_t \cdot L$

K_t --- Value at each section
obtained from curve

L ---- Total load from each
pile circle

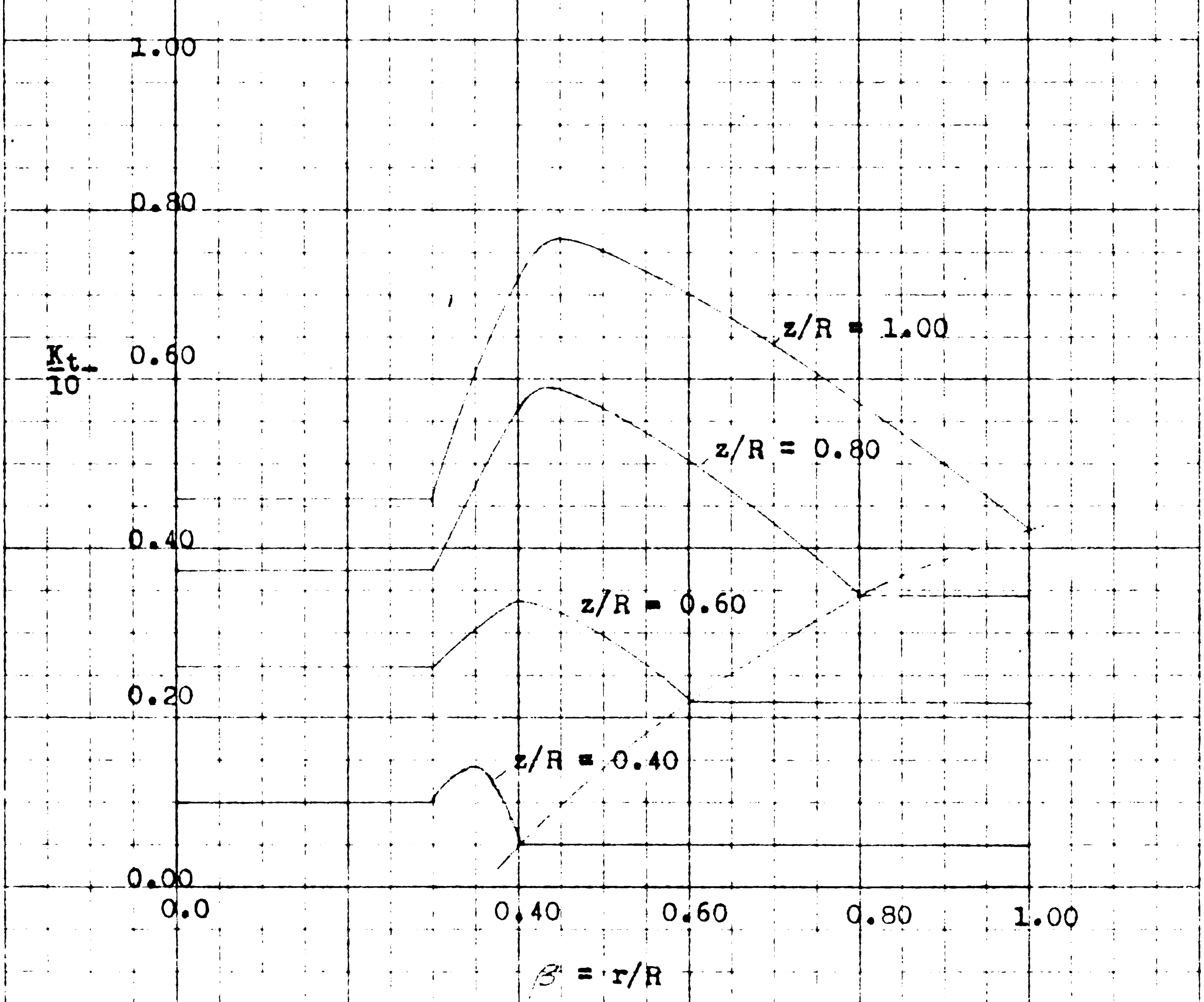


PLATE 12

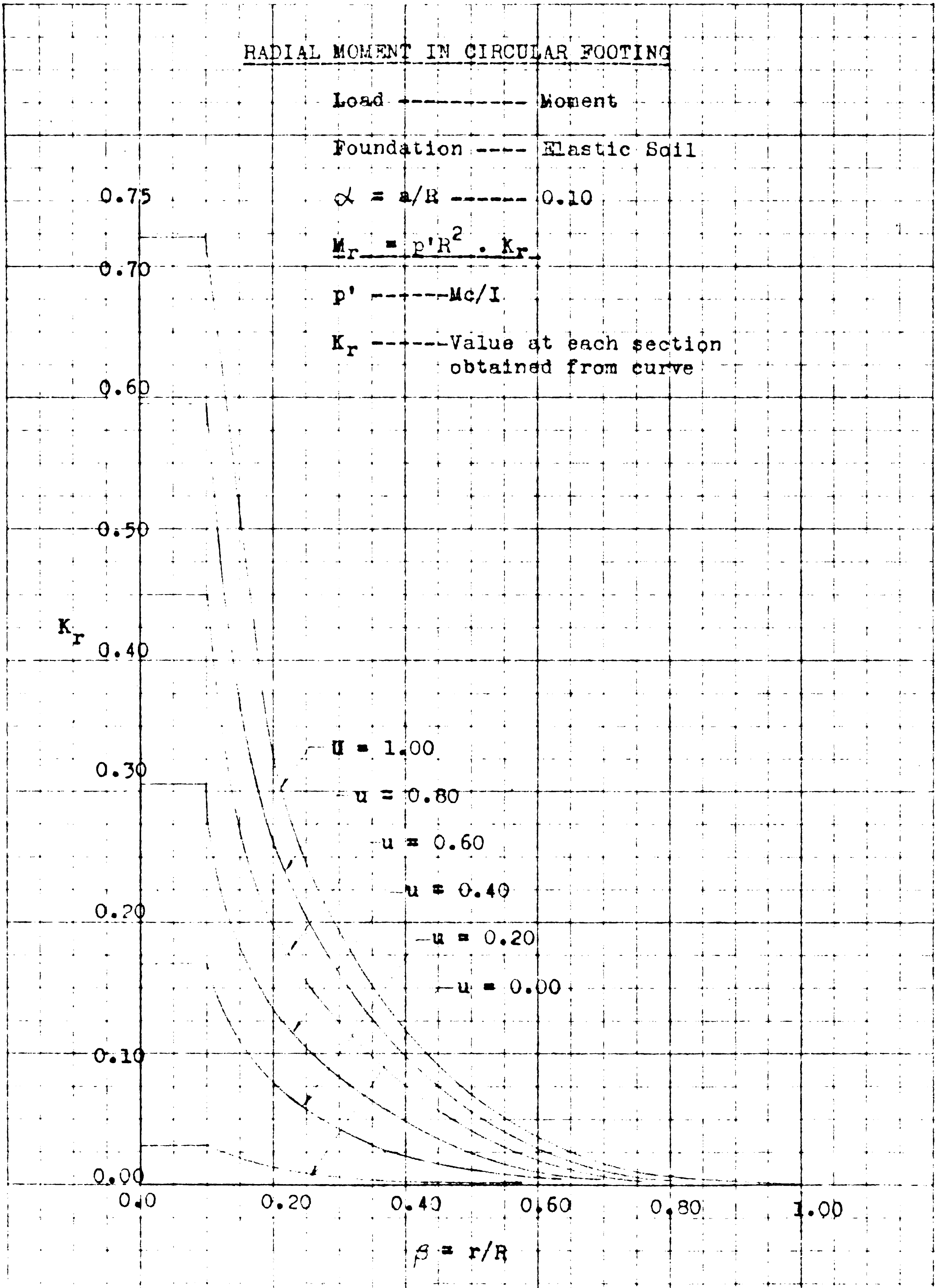
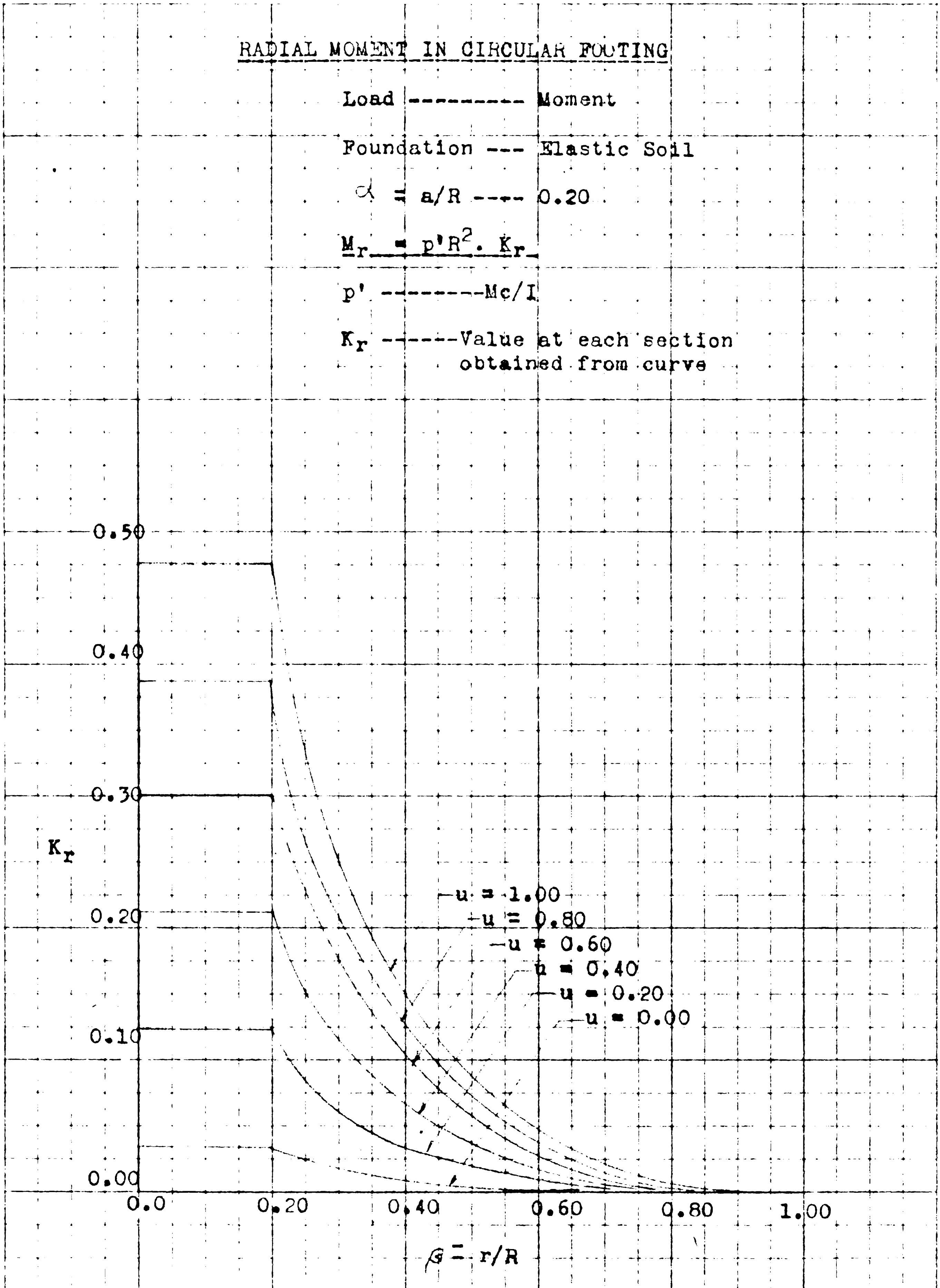

RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Moment
Foundation ---- Elastic Soil
α = a/R ------ 0.10
$M_r = p'R^2 \cdot K_r$
p' ------Mc/I
K_r ------Value at each section obtained from curve
U = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
K_r
0.75
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 13



RADIAL MOMENT IN CIRCULAR FOOTING
Load --------- Moment
Foundation --- Elastic Soil
α = a/R ---- 0.20
M_r = p'R^2. K_r
p' --------Mc/I
K_r ------Value at each section obtained from curve
0.50
0.40
0.30
K_r
0.20
0.10
0.00
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
ρ = r/R

PLATE 14

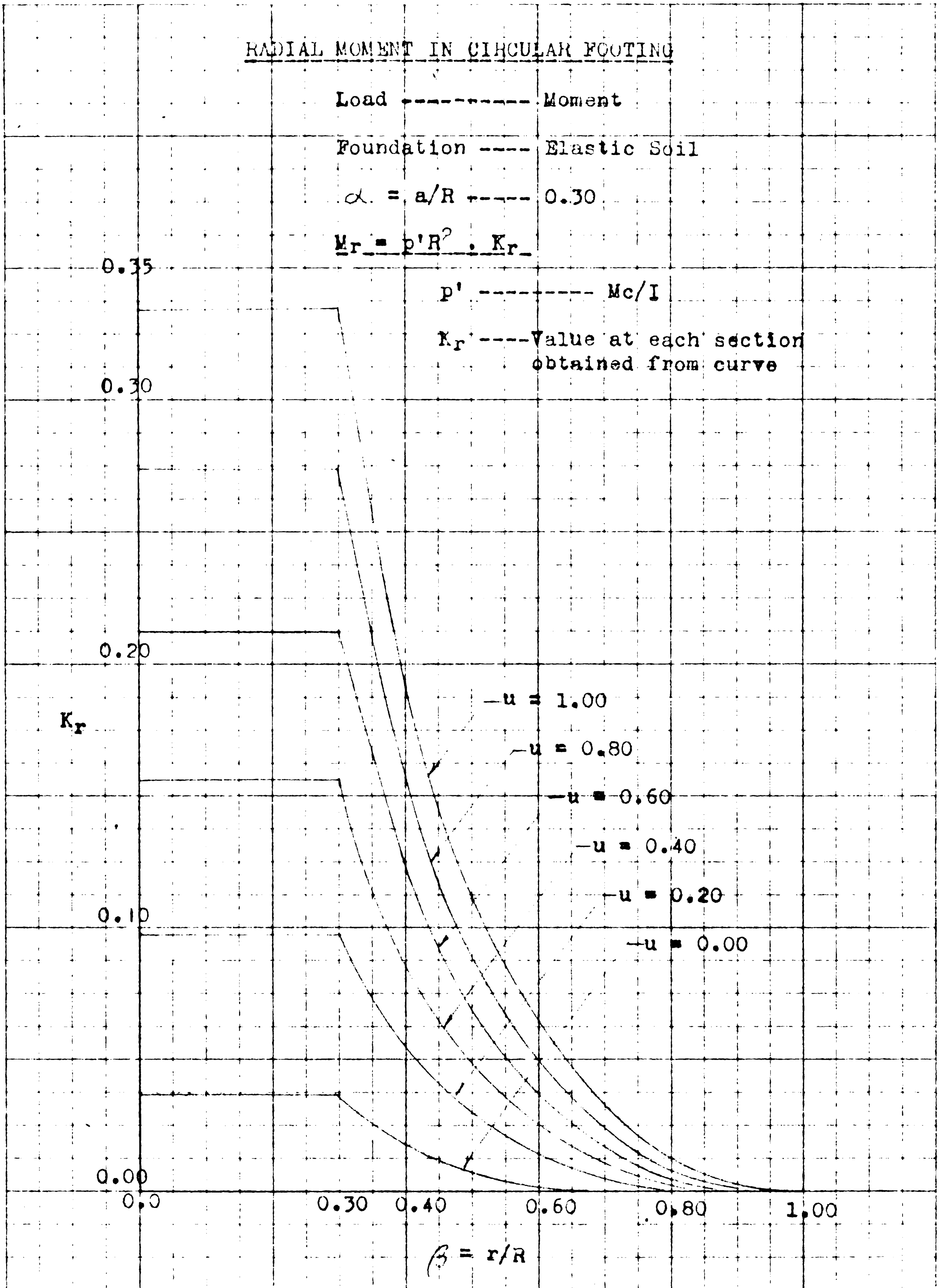

RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Moment
Foundation ---- Elastic Soil
α = a/R ----- 0.30
Mr = p'R² . Kr
p' ---------- Mc/I
Kr ----Value at each section obtained from curve
0.35
0.30
0.20
0.10
0.00
Kr
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.30
0.40
0.60
0.80
1.00
β = r/R

PLATE 15

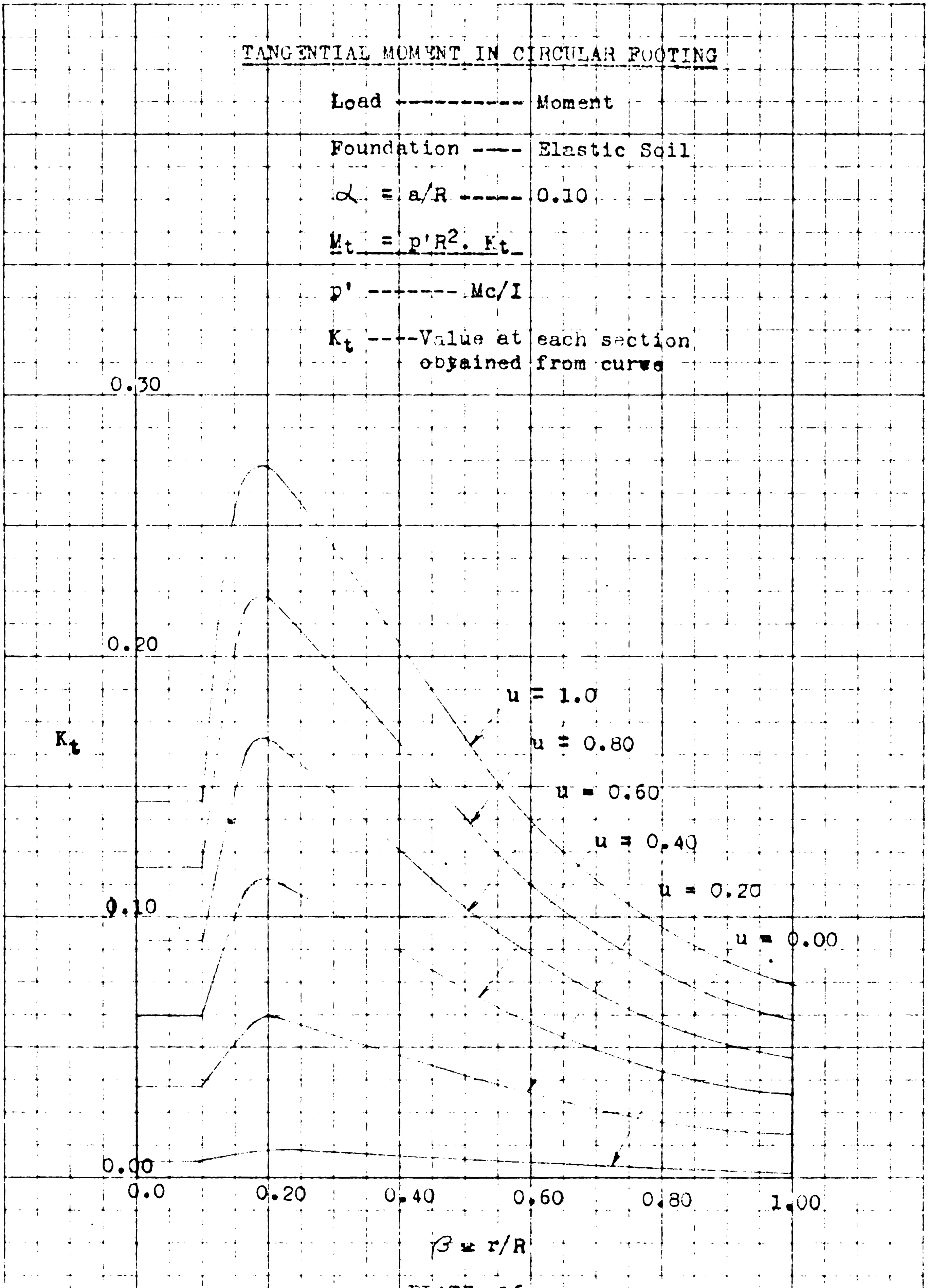

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Moment
Foundation ---- Elastic Soil
α = a/R ----- 0.10
Mt = p'R². Kt
p' -------- Mc/I
Kt ----Value at each section obtained from curve
0.30
0.20
0.10
0.00
Kt
u = 1.0
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 16

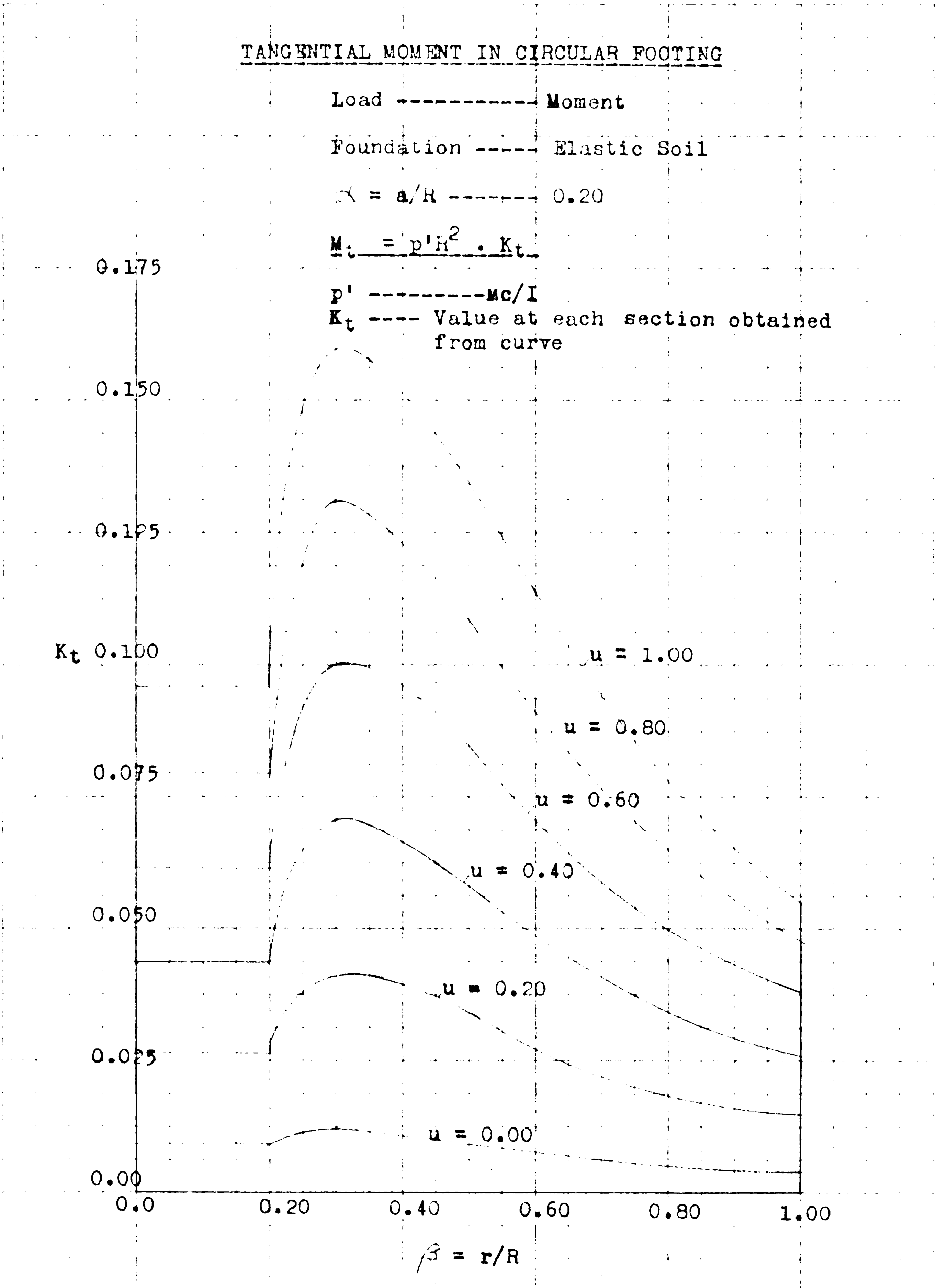

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ----------- Moment
Foundation ----- Elastic Soil
α = a/R ------- 0.20
M_t = p'R^2 . K_t
p' ---------Mc/I
K_t ---- Value at each section obtained from curve
0.175
0.150
0.125
K_t 0.100
0.075
0.050
0.025
0.00
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 17

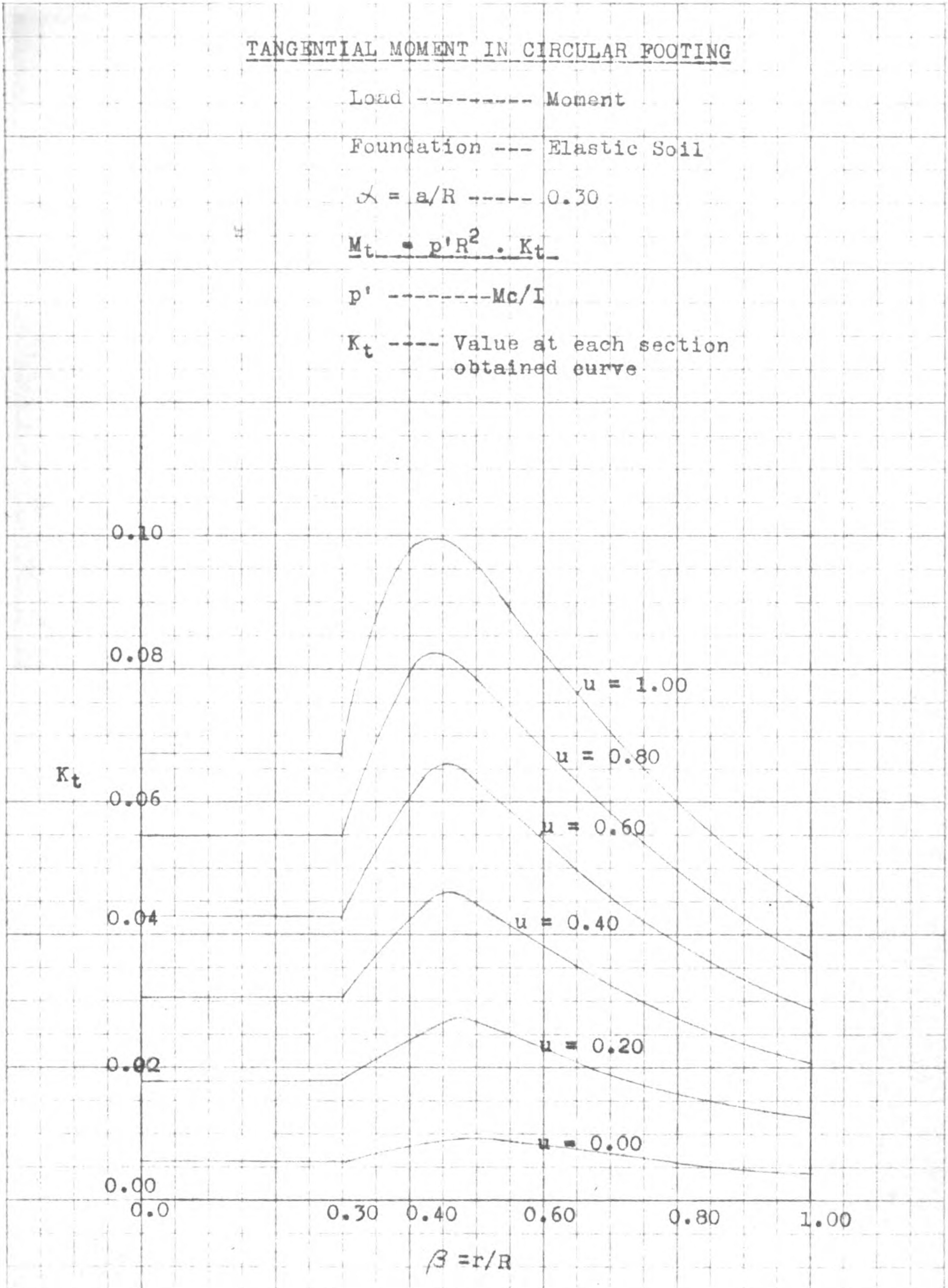

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load --------- Moment
Foundation --- Elastic Soil
α = a/R ----- 0.30
$M_t = p'R^2 \cdot K_t$
p' --------Mc/I
K_t ---- Value at each section obtained curve
K_t
0.10
0.08
0.06
0.04
0.02
0.00
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.30
0.40
0.60
0.80
1.00
β =r/R

PLATE 18

XII. APPENDICES

1. Derivation of Flexural Rigidity of Reinforced Concrete Section

Referring to Figure 11.1,

The fiber stress at a distance z from N.A.

Figure 11.1 in compression region

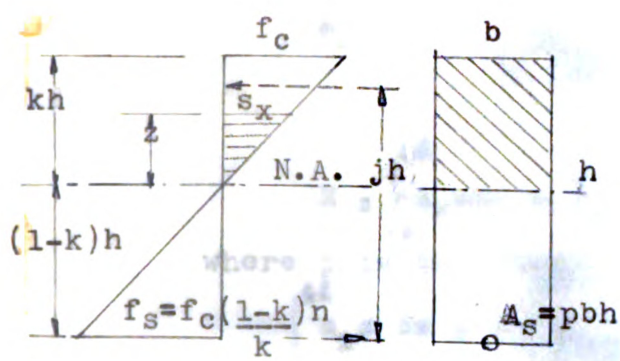


FIG. 11.1

$\frac{{}^2w}{x^2}$ . . . (Timoshenko: Theory of Plate and Shells P.2)

$\underline{k)}$ n.(1-k) h.hp

o of the section.

$- \frac{d^2w}{dx^2} \quad z^2 \; dz$

$\overline{\quad m^2)} \quad h^3 \; \frac{d^2w}{dx^2}$

$$\frac{f_c(1 - k)^2 n.h.hp}{k} = - \frac{E_c kh}{(1 - m^2)} \cdot \frac{(1-k)^2 nph^2}{k} \frac{d^2w}{dx^2}$$

$$= - \frac{E_c \; (1 - k)^2}{(1 - m^2)} (pn)h^3 \; \frac{d^2w}{dx^2}$$

$$M = - \frac{Eh^3}{(1 - m^2)} \; \frac{d^2w}{dx^2} \left[\frac{k^3}{3} + pn \; (1 - k)^2 \right] \quad . . . (1)$$

$kh.\frac{kh}{2} = pn.h. \; (1 - k) \; h$ (taking moment about N.A.)

$$pn = k^2/2 \; (1 - k) \quad . . . (2)$$

Sub. (2) into (1) we have:

$$M = - \frac{E_c k^2 j}{2(1-m^2)} \; h^3 \; \frac{d^2w}{dx^2} = -D \; \frac{d^2w}{dx^2}$$

$$\text{Flexural Rigidity} = D = \frac{E_c k^2 j}{2(1 - m^2)} \; h^3$$

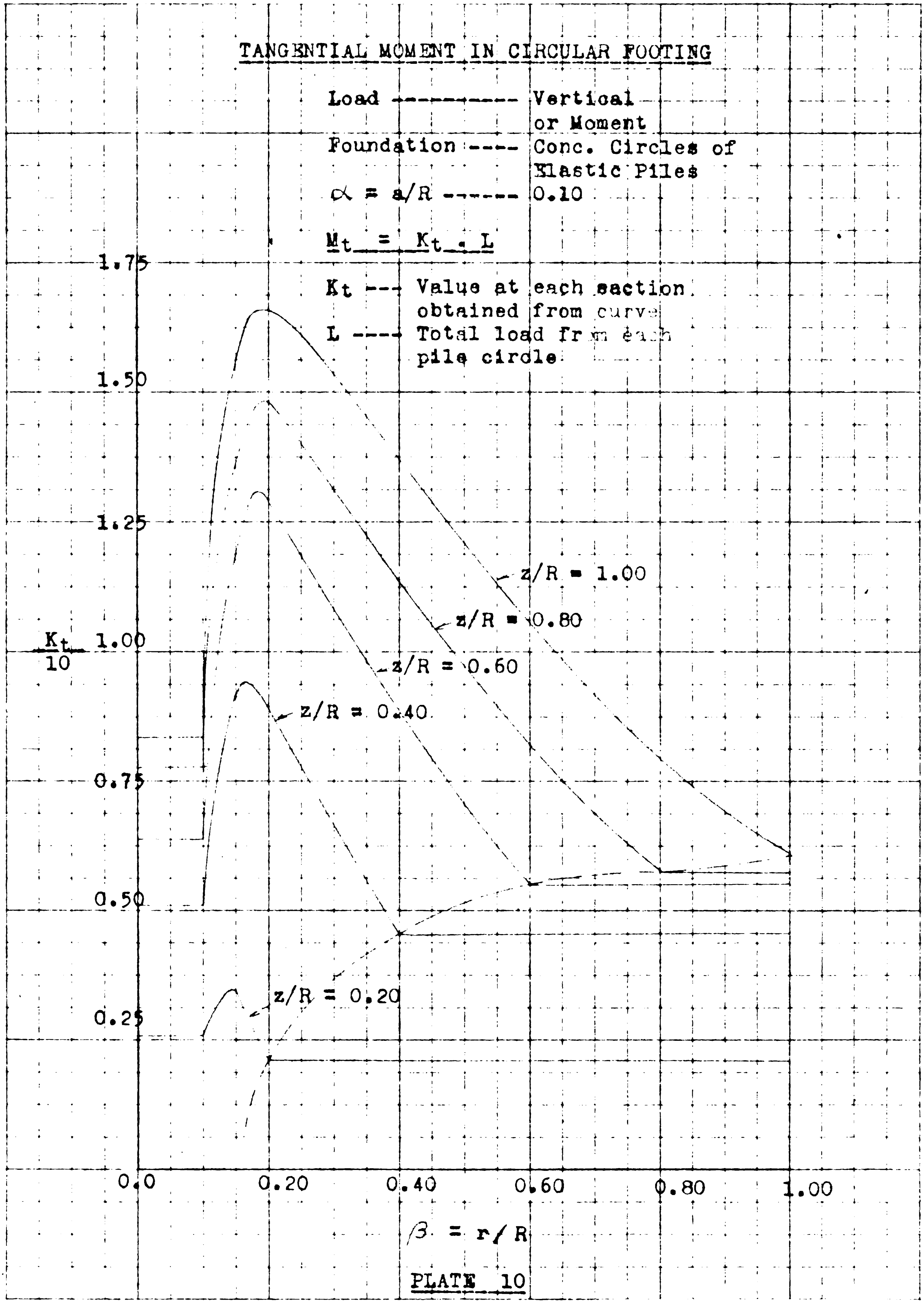
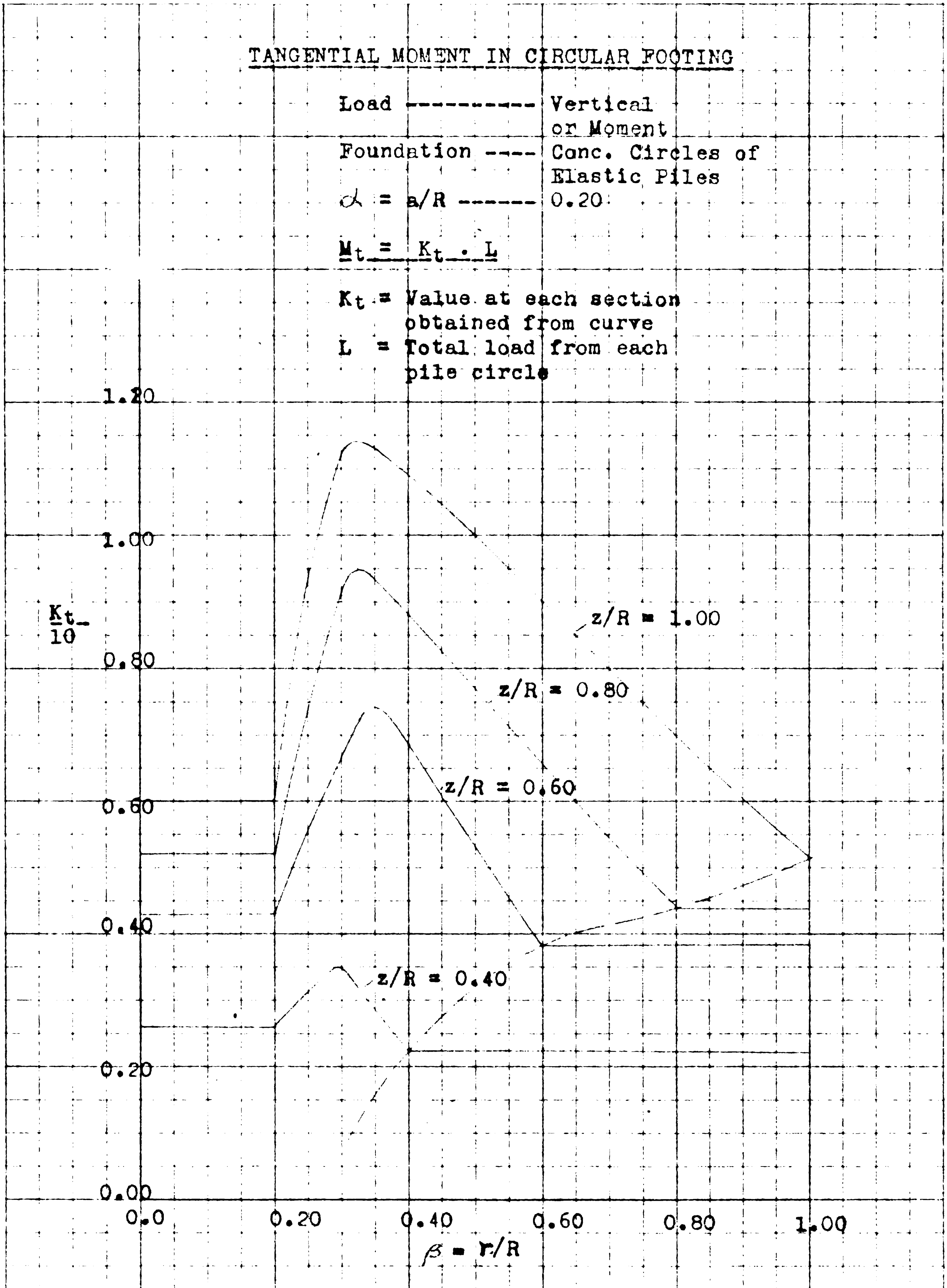

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical or Moment
Foundation ---- Conc. Circles of Elastic Piles
α = a/R ------- 0.10
M_t = K_t . L
K_t --- Value at each section obtained from curve
L ---- Total load from each pile circle
z/R = 1.00
z/R = 0.80
z/R = 0.60
z/R = 0.40
z/R = 0.20
K_t / 10
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 10



TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Vertical or Moment
Foundation ---- Conc. Circles of Elastic Piles
α = a/R ------- 0.20
$M_t = K_t \cdot L$
K_t = Value at each section obtained from curve
L = Total load from each pile circle
$\frac{K_t}{10}$
1.20
1.00
0.80
0.60
0.40
0.20
0.00
z/R = 1.00
z/R = 0.80
z/R = 0.60
z/R = 0.40
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 11

TANGENTIAL MOMENT IN CIRCULAR FOOTING

Load ---------- Vertical or Moment

Foundation ---- Conc. Circles of Elastic Piles

α = a/R ----- 0.30

$M_t = K_t \cdot L$

K_t --- Value at each section obtained from curve

L ---- Total load from each pile circle

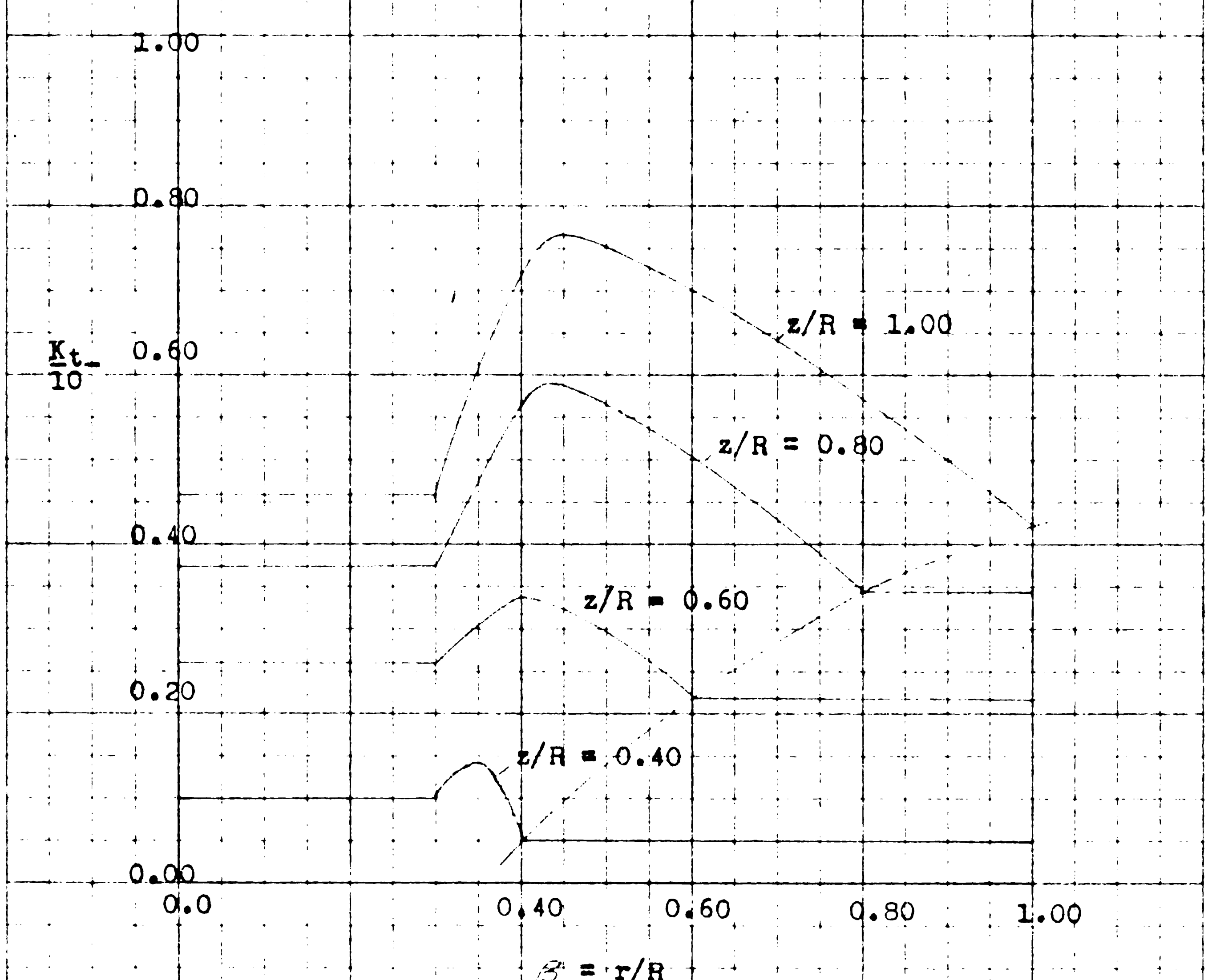


PLATE 12

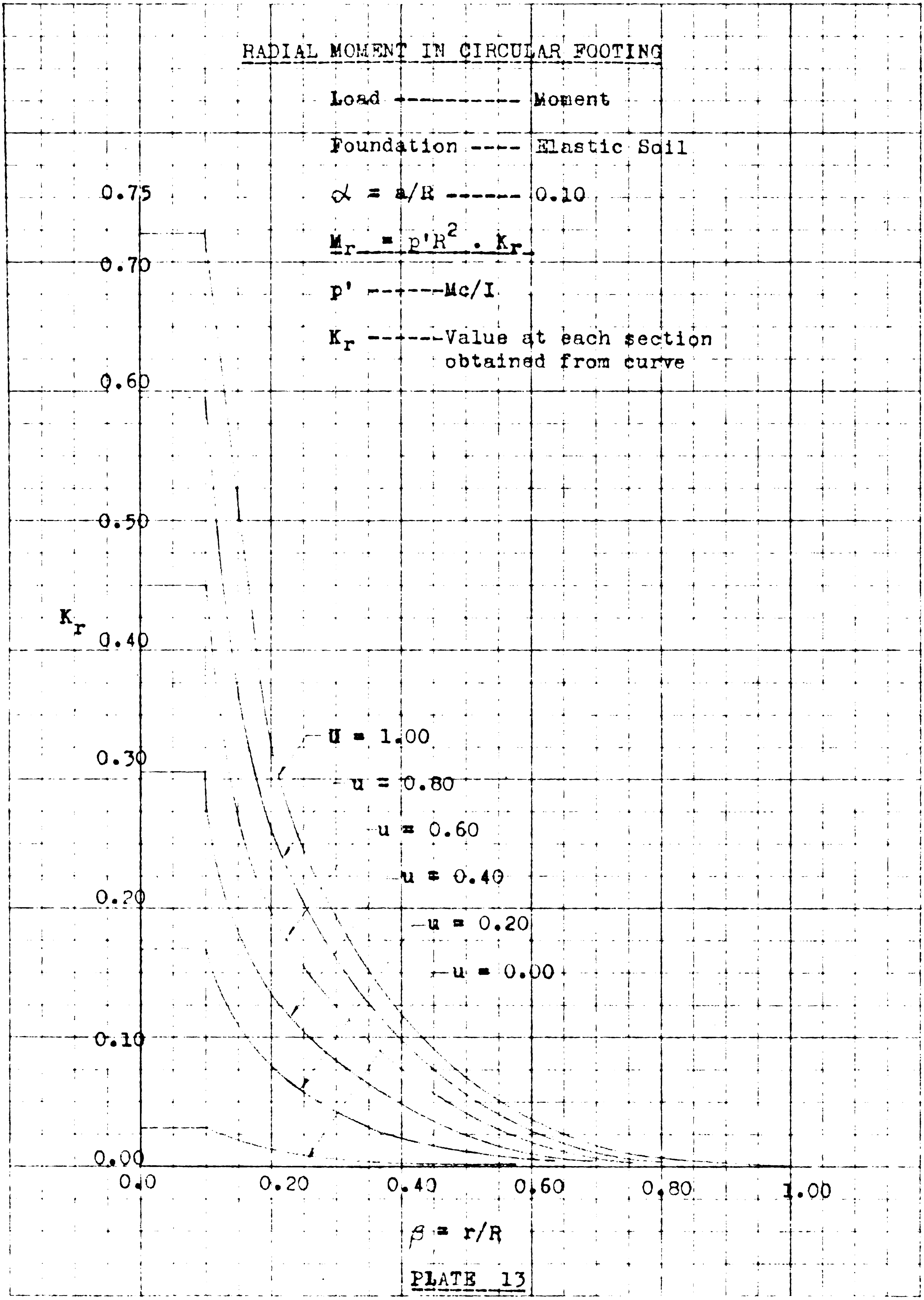

RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Moment
Foundation ---- Elastic Soil
α = a/R ------ 0.10
Mr = p'R2 . Kr
p' ------Mc/I
Kr ------Value at each section obtained from curve
Kr
0.75
0.70
0.60
0.50
0.40
0.30
0.20
0.10
0.00
U = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 13

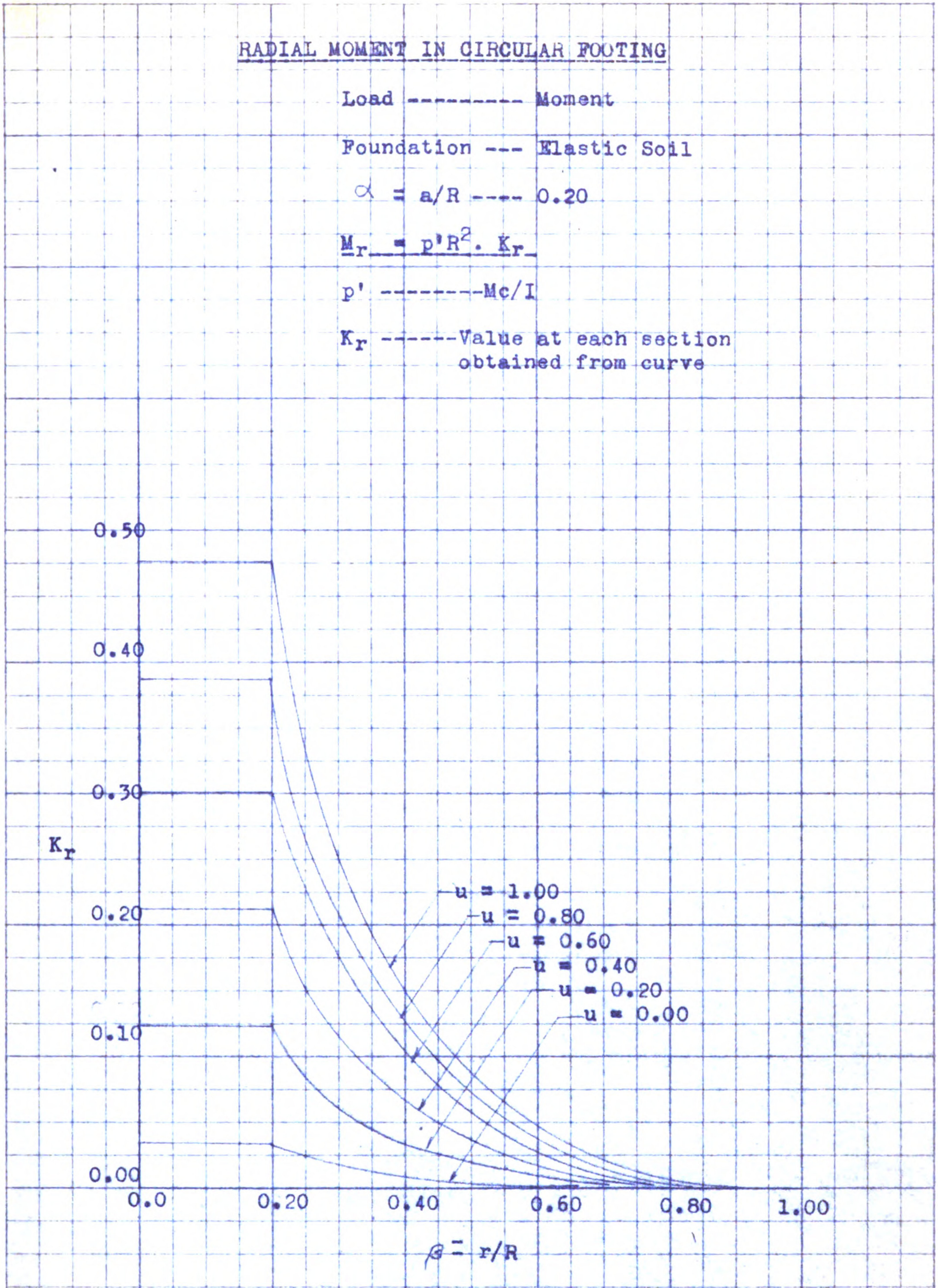
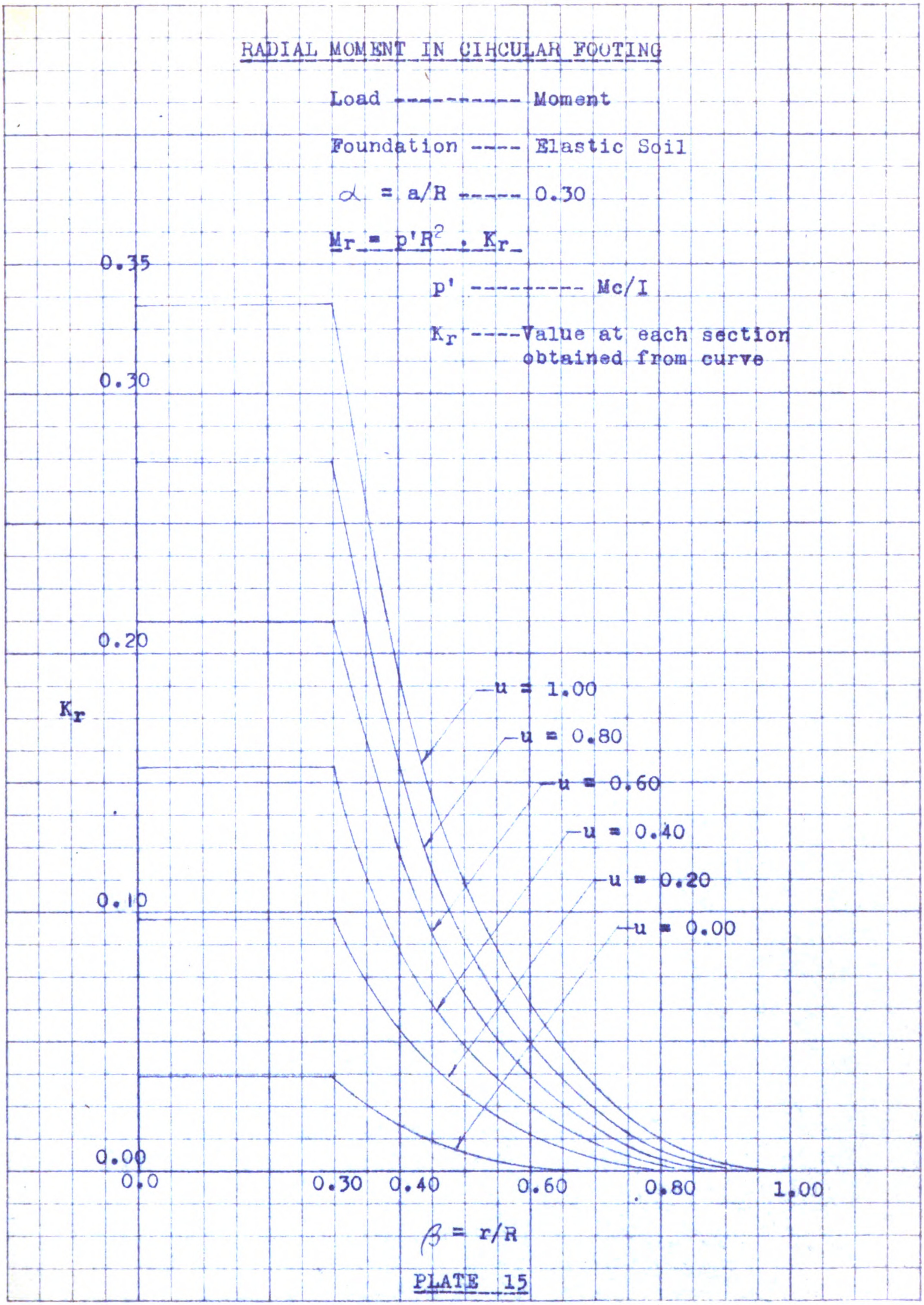

RADIAL MOMENT IN CIRCULAR FOOTING
Load --------- Moment
Foundation --- Elastic Soil
α = a/R ---- 0.20
Mr = p'R2. Kr
p' --------Mc/I
Kr ------Value at each section obtained from curve
0.50
0.40
0.30
0.20
0.10
0.00
Kr
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
β = r/R

PLATE 14


RADIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Moment
Foundation ---- Elastic Soil
α = a/R ----- 0.30
$M_r = p'R^2 \cdot K_r$
p' --------- Mc/I
K_r ----Value at each section obtained from curve
0.35
0.30
0.20
0.10
0.00
K_r
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.30
0.40
0.60
0.80
1.00
β = r/R
PLATE 15

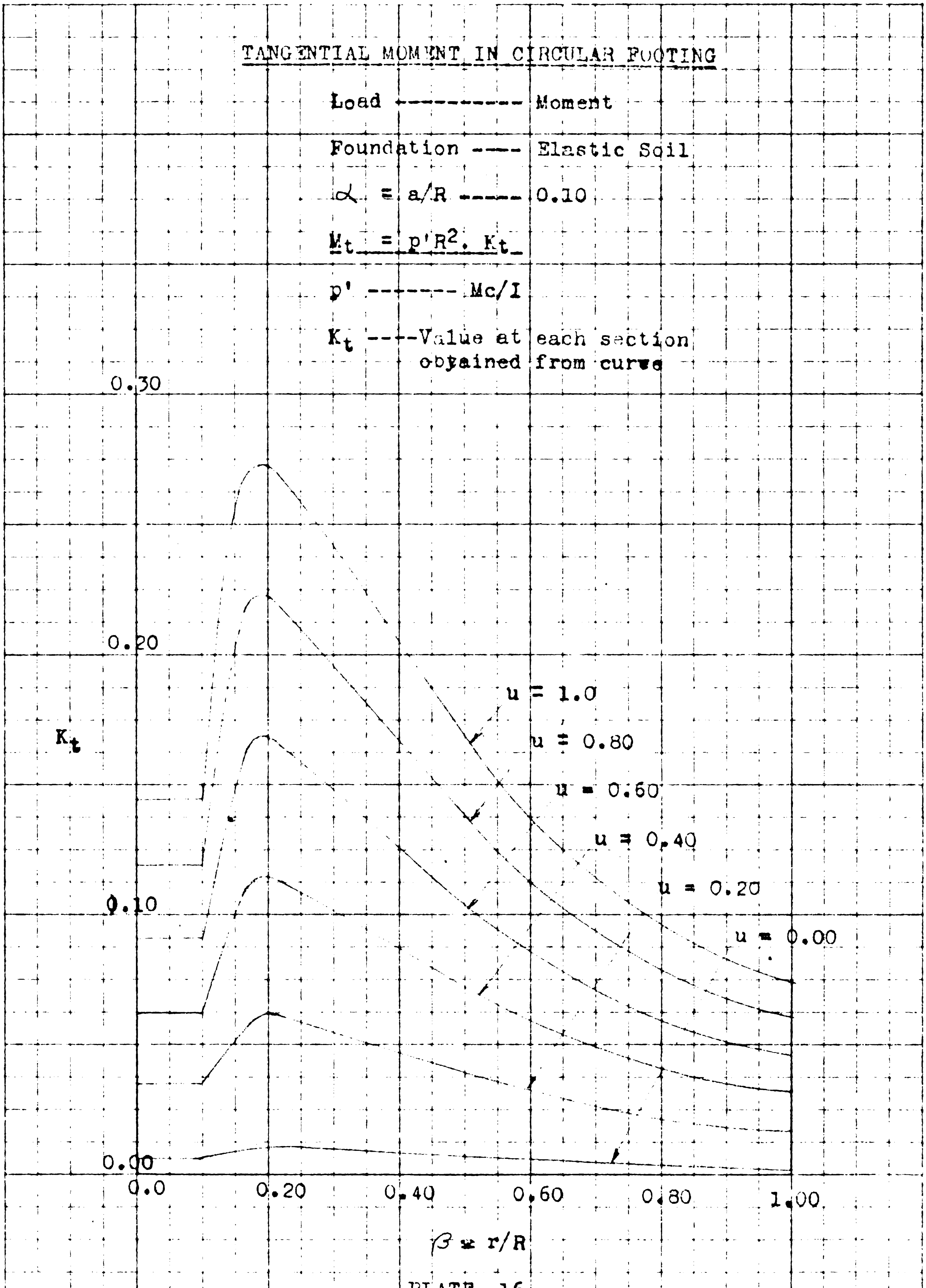

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ----------- Moment
Foundation ---- Elastic Soil
$\alpha = a/R$ ----- 0.10
$M_t = p'R^2 . K_t$
p' ------- Mc/I
K_t ----Value at each section obtained from curve
0.30
0.20
0.10
0.00
K_t
u = 1.0
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
0.0
0.20
0.40
0.60
0.80
1.00
$\beta = r/R$

PLATE 16

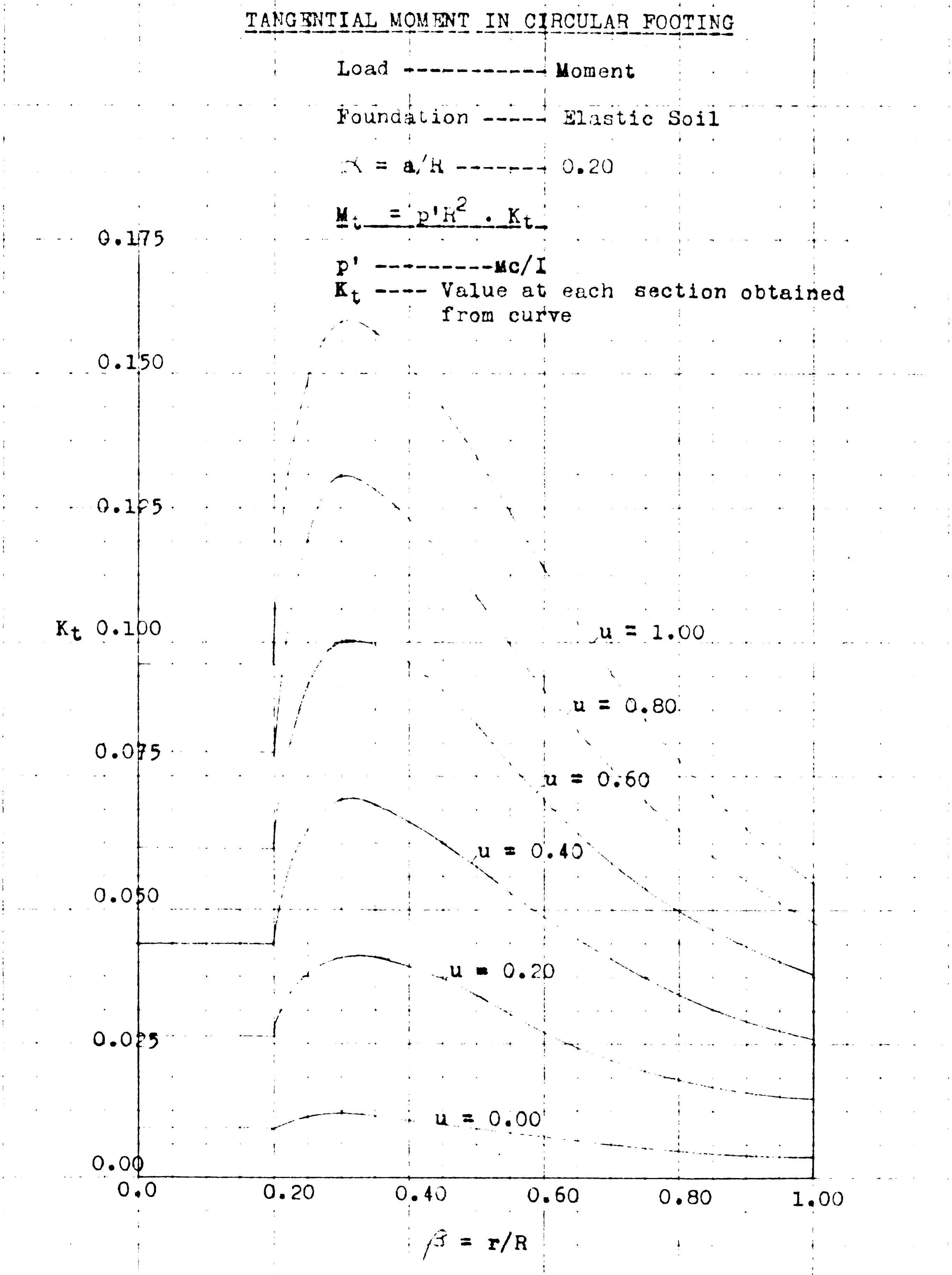

TANGENTIAL MOMENT IN CIRCULAR FOOTING
Load ---------- Moment
Foundation ----- Elastic Soil
$\alpha = a/R$ -------- 0.20
$M_t = p'R^2 \cdot K_t$
p' ---------Mc/I
K_t ---- Value at each section obtained from curve
u = 1.00
u = 0.80
u = 0.60
u = 0.40
u = 0.20
u = 0.00
K_t
0.175
0.150
0.125
0.100
0.075
0.050
0.025
0.00
0.0
0.20
0.40
0.60
0.80
1.00
$\beta = r/R$

PLATE 17

TANGENTIAL MOMENT IN CIRCULAR FOOTING

Load ---------- Moment

Foundation --- Elastic Soil

$\alpha = a/R$ ----- 0.30

$M_t = p'R^2 \cdot K_t$

p' --------Mc/I

K_t ---- Value at each section obtained curve

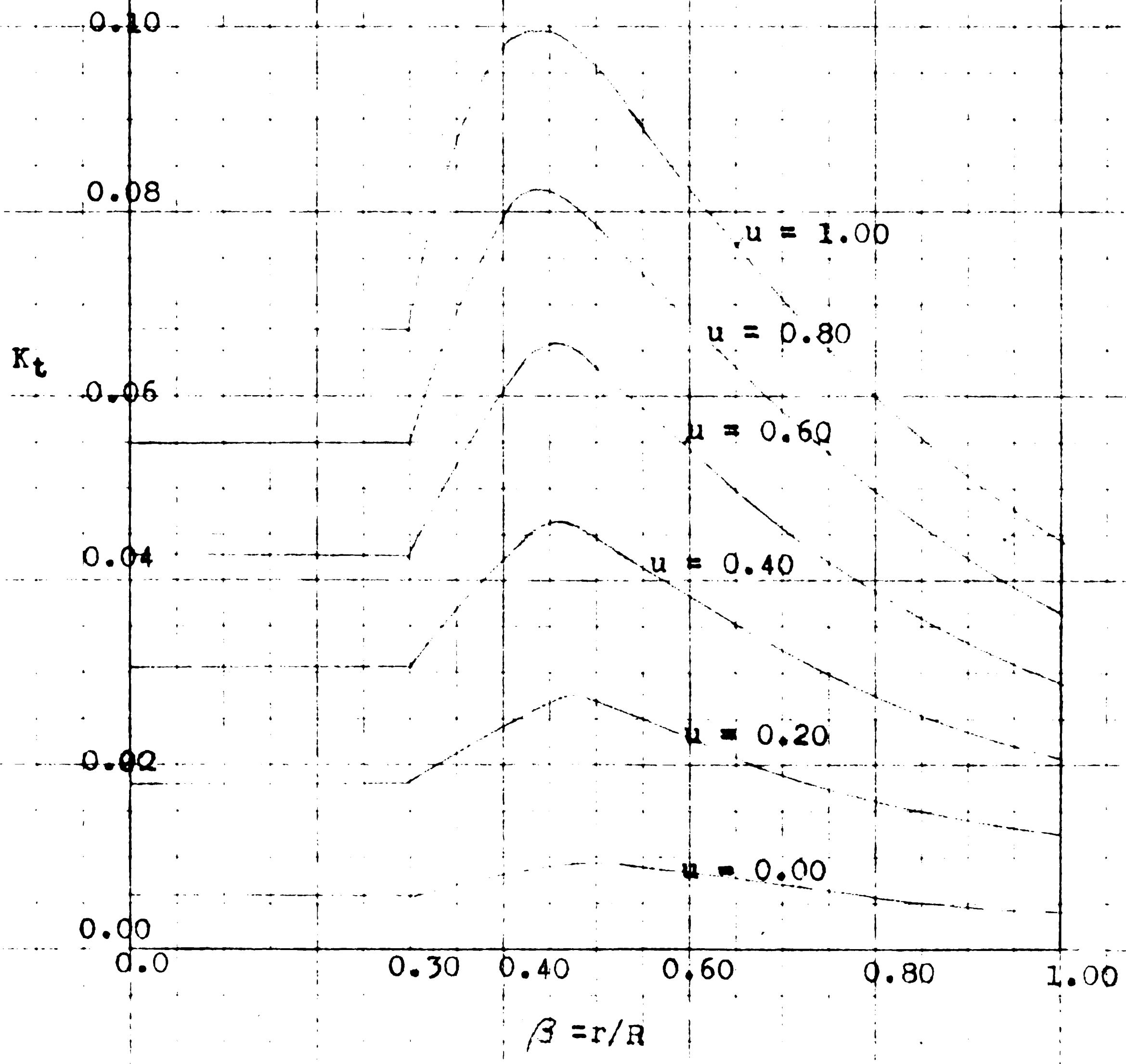


PLATE 18

XII. APPENDICES

1. Derivation of Flexural Rigidity of Reinforced Concrete Section

Referring to Figure 11.1,

The fiber stress at a distance z from N.A.

Figure 11.1 in compression region

is:

$$s_x = - \frac{E_c \; z}{(1 - m^2)} \frac{d^2w}{dx^2} \quad \ldots \text{(Timoshenko: Theory of Plate and Shells P.2)}$$

$$M = \int_0^{kh} s_x z dz + f_c \frac{(1-k)}{k} n.(1-k)\, h.hp$$

where p is the steel ratio of the section.

$$\int_0^{kh} s_x z \, dz = - \int_0^{kh} \frac{E}{(1-m^2)} \frac{d^2w}{dx^2} z^2 \, dz$$

$$= - \frac{E_c \, k^3}{3\,(1-m^2)} h^3 \frac{d^2w}{dx^2}$$

$$\frac{f_c(1-k)^2 n.h.hp}{k} = - \frac{E_c kh}{(1-m^2)} \cdot \frac{(1-k)^2 nph^2}{k} \frac{d^2w}{dx^2}$$

$$= - \frac{E_c \, (1-k)^2}{(1-m^2)} (pn)h^3 \frac{d^2w}{dx^2}$$

$$M = - \frac{Eh^3}{(1-m^2)} \frac{d^2w}{dx^2} \left[\frac{k^3}{3} + pn\,(1-k)^2 \right] \quad \ldots (1)$$

$$kh.\frac{kh}{2} = pn.h.\,(1-k)\,h \quad \text{(taking moment about N.A.)}$$

$$pn = k^2/2\,(1-k) \quad \ldots (2)$$

Sub. (2) into (1) we have:

$$M = - \frac{E_c k^2 j}{2(1-m^2)} h^3 \frac{d^2w}{dx^2} = -D \frac{d^2w}{dx^2}$$

$$\text{Flexural Rigidity} = D = \frac{E_c k^2 j}{2(1-m^2)} h^3$$

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