DESIGN CURVES FOR REINFORCED CONCRETE COMBINED FOOTINGS ON SOIL

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

Avram Kazez

1952

This is to certify that the

thesis entitled

DESIGN CURVES FOR REINFORCED CONCRETE

COMBINED FOOTINGS ON SOIL

presented by

AVRAM KAZEZ

has been accepted towards fulfillment
of the requirements for

M. S. degree in CIVIL ENGINEERING

Major professor

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By

AVRAM KAZEZ

A THESIS

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

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CONTENTS

INTRODUCTION

1. Introductory Statement

This study describes a simple graphical procedure for designing reinforced concrete combined footings.

The design curves presented in this study can be used by a designer or checker.

Following the derivation of equations, the curves are drawn and a typical example of their use is given to make the method of procedure clear.

Derivation of the acale equations used to draw the charts are given in the Appendix.

It is hoped that the use of the charts herein will eliminate calculations thereby saving time.

2. Scope

With the use of the charts,the length and width of the footing, the effective depth, the steel, bond required and the shear stress are found.

This study is not concerned with the selection of the main or web reinforcement bars, nor in their distribution.

3. Acknowledgement.

The author wishes to express his indebtedness to Dr. R. H. J. Pian for his valuable advises and suggestions, and also to acknowledge his thanks to Mr. W. A. Bradley who read the study.

ASSUMPTIONS, NOTATIONS AND DEFINITIONS

1. Assumptions

Adaptation of the nomograms will be made to design a combined footing, which will meet the following requirements:

a) An area great enough to give the safe allowable soil pressure.

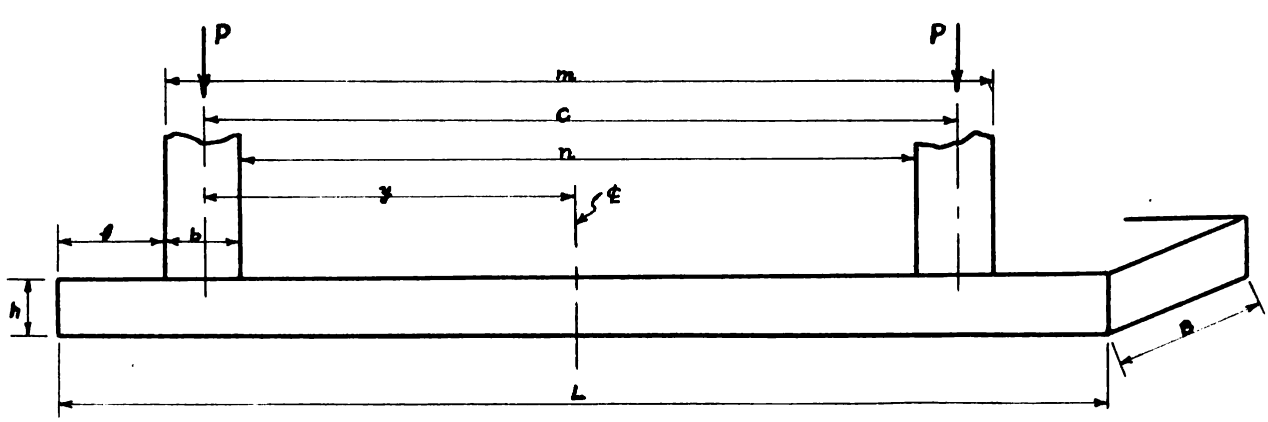
b) A shape such that a uniform net soil pressure is given.

c) The two loads acting on each column are equal.

d) The footing will have a constant depth.

2. Notations and Definitions

The symbols in this study, defined below conform essentially to Letter Symbols for Structural Analysis prepared by the American Standards Association, with ASCE participation, and approved by the Association in 1949. For their significance, where not conforming to the standard, is made clear by Fig. 1.



where

P = load on each column in kips.

p = allowable soil pressure in lb./ft.sq.

W = weight of footing in lb.

w = soil pressure against base of footing resisting column loads in lb./ft. of length.

d = effective depth of the footing in inches.

B = width of the rectangular footing in feet.

h = height of base of footing in inches.

L = length of footing in feet.

c = distance **between** column loads in feet.

f = distance between outside edge of column and width of footing in feet.

b = size of column along the length of footing in inches.

m = distance between outside edge of the columns in feet.

y = distance between center line of footing and column load in feet.

Typical Shear and Moment Diagrams.

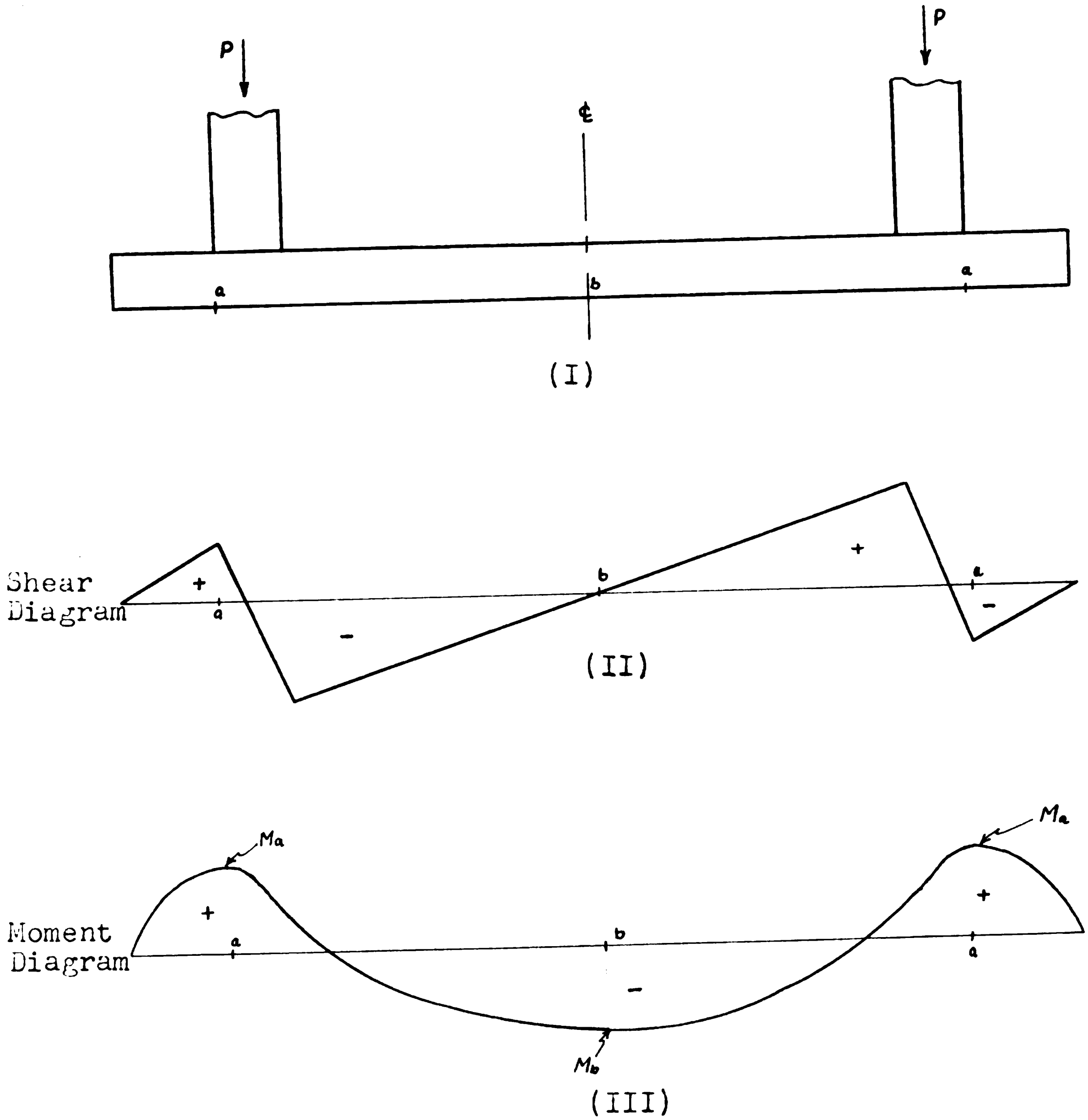


Fig. 2.

CONDITION FOR MINIMUM DEPTH OF FOOTING

From the moment diagram in Fig.2(II), the moment at point a;

$$M_a = \frac{f^2P}{L} = BKd^2$$

but, $A = BL$

therefore, $f^2P = AKd^2$ . . . . (a)

Keeping P, A and K constant, the variation of "f" with "d" will be investigated. Then

$d = c_1f$ where c_1 is a constant.

The equation above represents a family of straight lines through the origin with positive slopes. Plotting "d" as ordinate and "f" as abscissa, the family of lines below is obtained.

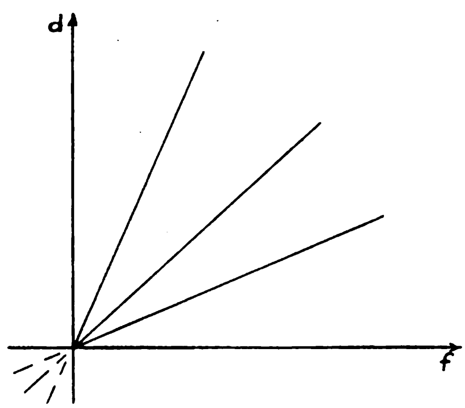


Fig. 3.

Similarly the expression for the moment at point b will be

$$M_b = Py - \frac{L^2w}{8} = BKd^2$$

Similarly, the expression for the moment at point b can be written;

$$M_b = Py - L^2w/8 = BKd^2$$

Simplifying terms and substituting, B = A/L

$$4yL - L^2 = 4AKd^2/P \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

but, $y = (c-b)/2$ and $L = 2f+b+c$

therefore,

$$2(c-b)(2f+b+c) - (4f^2+4fb+4fc+b^2+2bc+c^2) = 4AKd^2/P$$

simplifying and transposing terms,

$$-c^2+4bf+2b^2+4f^2+4fb+b^2+2bc+4AKd^2/P = 0$$

Keeping P, A, K, b, and c constant the variation of "f" with "d" will be investigated. Then

$$k_1d^2 + 4f^2 + 6bf + k_2 = 0$$

where k_1 and k_2 are constants.

The equation above represents a family of ellipses with center at (-h, 0). Plotting "d" as ordinate and "f" as abscissa the family of ellipses below is obtained:

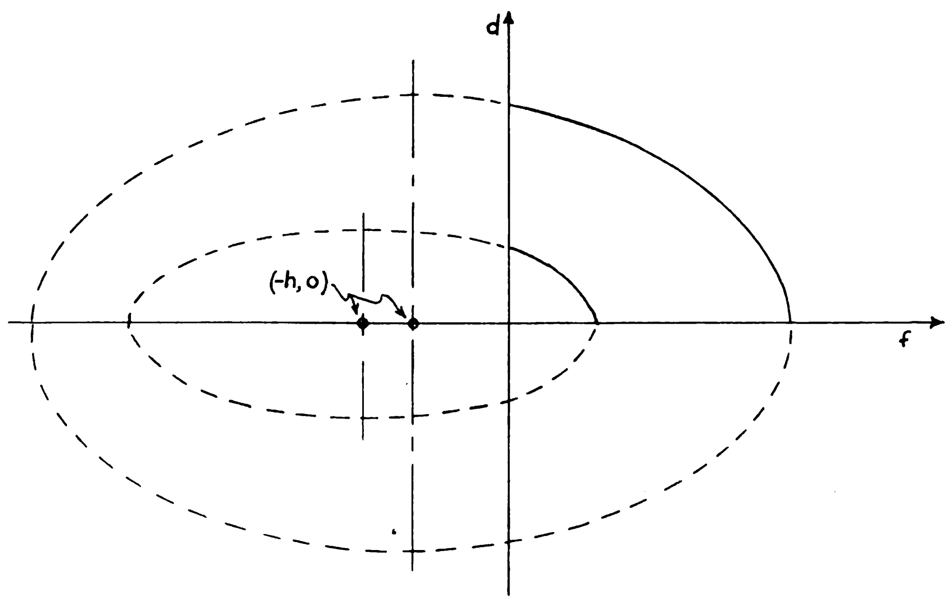


Fig. 4.

Superimposing a typical curve of each family shown above,

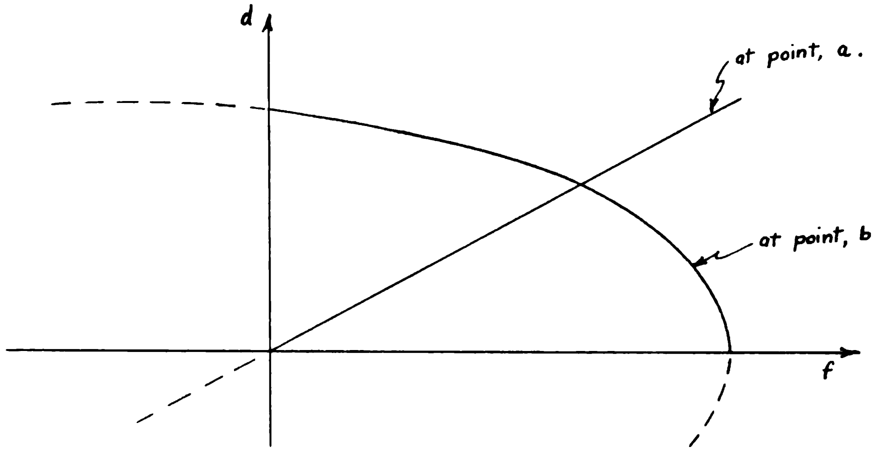


Fig.5.

Investigating the variation of "d" with "f" in Fig. 5 it is concluded that; the depth "d" of the footing is to be designed for the largest value of "d" required at any point. Therefore the smallest value of "d" required for the footing will be when the required depth at points "a" and "b" are equal.

From equations (a) and (b), and Fig. 2(III), it is obvious that for the above condition $|M_a| = |M_b|$.

DERIVATION OF EQUATIONS FOR CURVES

1. Length of Footing "L"

An expression for the length of the footing "L" will be derived for the condition $|M_a| = |M_b|$.

From the moment diagram in Fig. 2(II); the following equation can be written.

$$|M_a| = \frac{f^2 w}{2}$$

and $$|M_b| = \frac{P(n+b)}{2} - \frac{L^2 w}{8}$$

hence $$\frac{wf^2}{2} = \frac{P(n+b)}{2} - \frac{L^2 w}{8}$$

or $$\frac{wf^2}{2} + \frac{L^2 w}{8} = \frac{P(n+b)}{2}$$

from Fig. 1 $$y = \frac{n+b}{2}$$

therefore $$\frac{wf^2}{2} + \frac{wL^2}{8} = Py$$

also $$w = \frac{2P}{L}$$

therefore $$\frac{2Pf^2}{2L} + \frac{2PL^2}{8L} = Py$$

simplifying terms $$4f^2 + L^2 = 4yL$$

From the above equation it is concluded that the length "L" of the base is independent of the loads "P" on the column.

Substituting $$f = \frac{L-b-c}{2}$$

and $2y = c$ in the above equation

$$(L-b-c)^2 + L^2 = 2cL$$

$$(L^2+b^2+c^2-2Lb-2Lc+2bc) + L^2 = 2cL$$

$$2L^2 + (b^2+2bc+c^2) - 2Lb - 4cL = 0$$

$$L^2 + \frac{(b+c)^2}{2} - \frac{L(2b+4c)}{2} = 0$$

$$L^2 - (b+2c)L + \frac{(b+c)^2}{2} = 0$$

Solving the quadratic equation above for "L" in terms of "b" and "c",

or, $$L = \frac{b + 2c + \sqrt{2c^2 - b^2}}{2} \quad . \quad . \quad . \quad . \quad (1)$$

also, $$f = \frac{L - b - c}{2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

2. Effective depth of footing "d"

The value "d" is usually calculated by assuming the weight of the footing, then checking the weight.

An expression for "d" in terms of the already known values will be derived, thus eliminating the ambiguous method of trial and error.

As before, $M = f^2w/2$

but; $w = 2P/L$

hence $M = f^2P/L$

also; $M = BKd^2$

where, B is in feet.
f is in feet.
and d is in inches

For a balanced design; $f^2P/L = BKd^2$

or, $BL = f^2P/Kd^2$ . . . . (3)

Two expressions for the weight of the footing "W" can be written,

$W = BpL - 2P$ . . . . (4)

and, $W = \frac{150(d+4)LB}{12} = 12.5(d+4)LB$ . . (5)

where, L is in feet.
P is in lbs.

Equating equations (4) & (5);

$BpL - 2P = 12.5(d+4)LB$ . . (6)

Substituting equation (3) in (6);

$$\frac{pf^2.P}{Kd^2} - 2P = \frac{12.5(d+4)f^2P}{Kd^2}$$

multiplying by $\frac{Kd^2}{Pf^2}$;

$$p - \frac{2Kd^2}{f^2} = 12.5(d+4)$$

or,

$$\frac{2Kd^2}{f^2} + 12.5(d+4) = p \quad . \quad . \quad . \qquad (7)$$

"d" can be calculated from equation (7), as all other variables are know.

3. Width of footing "B"

The value of "d" once known, the width of the footing "B" will be evaluated.

As before $M = f^2P/L$

also, $M = KBd^2$

for a balanced design, $KBd^2 = M = f^2P/L \quad . \quad . \quad . \quad . \qquad (11)$

"B" can be calculated from equation (11), as all other variables. are known.

4. Area of steel required "A_s"

The area of steel "A_s" will be calculated from equation below

$$M = jf_s dA_s \quad . \quad . \quad . \quad . \quad . \qquad (12)$$

5. Bond "Σ_o"

The shear force "V", at the outside edge of the column from Figs. 1 & 2 is: $V = 2Pf/L$ . . . . . . (13)

and at the inside edge is:

$$V = 2Py/L = P(c-b)/L \quad . \quad . \quad . \qquad (14)$$

the larger value of V above is of interest,

$$\frac{P(c-b)}{L} \neq \frac{2Pf}{L} \quad . \quad . \quad . \quad . \quad . \qquad (15)$$

dividing both sides by P/L and substituting equation (2) for "f"

$$c-b \neq L-b-c$$

or,

$$c \neq L-c \quad . \quad . \quad . \quad . \quad . \quad . \qquad (16)$$

from chart A the following approximate equation can be written

$$L = \left(\frac{44-9}{20}\right)c = 1.75c$$

substituting the latter value of L into equation (16);

$$c \neq 1.75c-c$$

$$c \neq 0.75c$$

hence;

$$c > 0.75c$$

therefore

$$(c - b)/2 > f \quad . \quad . \quad . \quad . \quad . \quad . \qquad (15a)$$

The shear force "V" expressed by equation (14) will have the larger value, which will be used to evaluate "Σ_o"

also,

$$V = \frac{7}{8} \cdot d \cdot u \cdot \Sigma_o \quad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

equating equations (14) and (17),

$$2Py/L = V = \frac{7}{8} \cdot d \cdot u \cdot \Sigma_o \quad . \quad . \quad . \quad . \qquad (18)$$

6. Shear "v"

The shear "v" will be calculated from equation below

$$V = vjdB \qquad (19)$$

EXPLANATION OF CURVES

1. Length of footing "L"

The first curve will be drawn to evaluate the length of the footing "L" by equation;

$$L = \frac{b+2c + \sqrt{2c^2-b^2}}{2} \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

Owing to the fact that the variables in the above equation cannot be separated, a value of "b" will be assigned and thus eliminating one variable. For the same value of "b" different values of "c" will be given and the corresponding values of "L" calculated.

The L- and c- scales will be at right angles to each other and will have uniform scales. Hence the nomogram will have the form shown below:

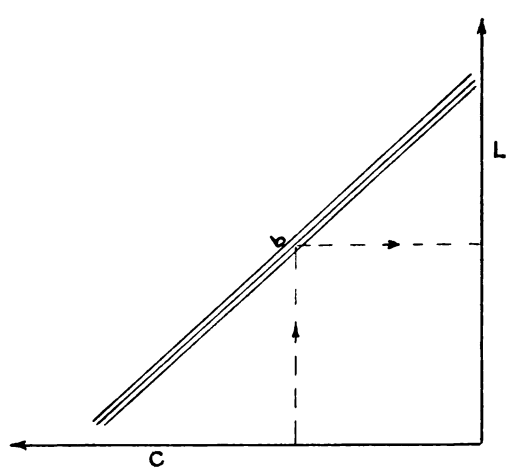


Fig.6.

c will vary from 5 to 25 ft. Its scale: 5 ft. = 1 inch.

b will vary from 12 to 36 inches.

l will vary from 9 to 45 ft. Its scale: 10 ft. = 1 inch.

(See chart A.)

The second curve will be drawn to evaluate "f", by equation

$$L = 2f + b + c \quad . \quad . \quad . \quad . \quad . \quad (2)$$

alignment charts will be used.

let $$L - c = T$$

then $$L - c = T = 2f + b$$

Let the modulus of L, m_L be equal to 1/10, which is the same value as before.

The scale length of "c" will be about 4 inches.

$$m_c = \frac{4''}{25-2} = 1/5$$

hence, $$m_T = \frac{m_L \cdot m_c}{m_L + m_c}$$

$$= 1/15$$

consider equation $$T = 2f + b$$

$$m_b = \frac{1''}{3} = 1/3$$

$$m_f = \frac{m_T \cdot m_b}{m_b - m_T}$$

$$= 1/12$$

The relative distances of the scales to each other will be calculated:

$$m_L/m_c = 5/10 = 1/2$$

similarly $$m_f/m_c = 3/12 = 1/4$$

and the scale equations will then be:

$X_L = L/10$ (L is measured in feet)

$X_c = c/5$ (c is measured in feet)

$X_b = b/3$ (If b is measured in feet)

$= b/36$ (If b is measured in inches)

and $X_f = f/6$ (f is measured in feet.)

The general form of the Alignment Chart for the equation,

$L = 2f + b + c$, is shown below:

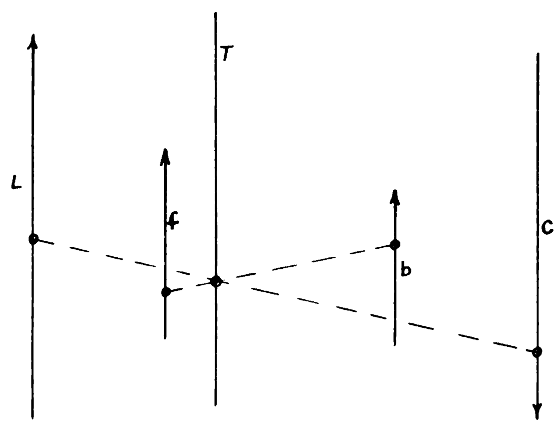


Fig. 7.

Since the modulus of L is the same in both charts, Fig. 6 and 7 will be combined.

The complete chart thus, and drawn to scale is drawn on next page, (chart A).

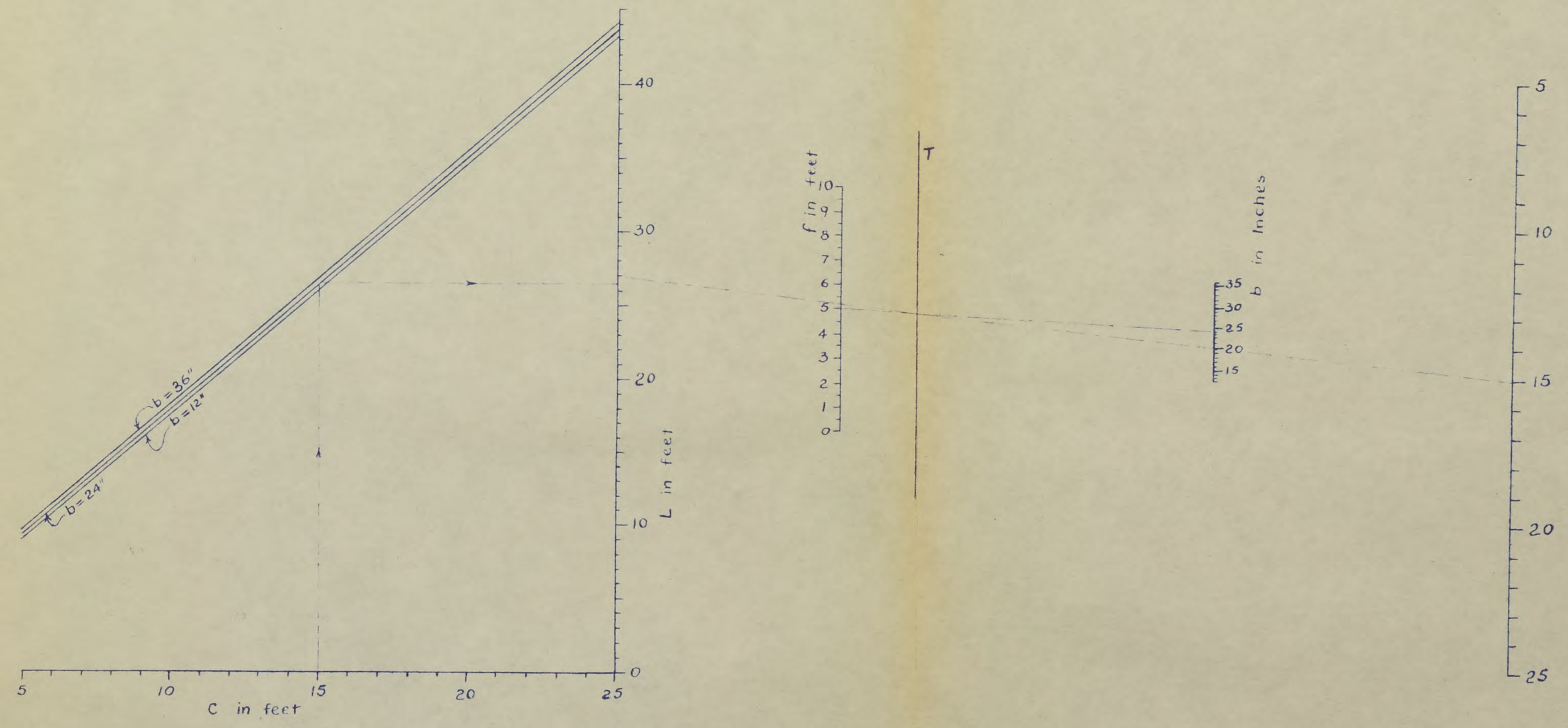


Chart A

2. Effective depth of footing "d"

The value of "d" will be found from equation

$$\frac{2Kd^2}{f^2} + 12.5(d+4) = p \quad . \quad . \quad . \quad . \quad . \qquad (7)$$

let $$z = \frac{2K}{f^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (8)$$

substituting equation (8) in (7);

$$zd^2 + 12.5(d+4) = p \quad . \quad . \quad . \quad . \quad . \qquad (9)$$

First a curve for equation (8) will be drawn. The values of "K" can be taken from "Reinforced Concrete Design Handbook".

f varies from 1 to 10 feet.

K varies from 125 to 423.

therefore z varies from 5 to 500.

The scale lengths are to be approximately 5 inches.

equation(8) can be written in the form;

$$\log 2K = 2 \log f + \log z$$

$$-2 \log f = - \log 2K + \log z$$

then, $$m_K = \frac{6''}{\log 846 - \log 250} = 11.3$$

A more convenient value $m_K = 10$ will be used.

and, $$m_z = \frac{6''}{\log 500 - \log 5} = 3$$

therefore, $$m_f = \frac{m_K \cdot m_z}{m_K + m_z} = \frac{3x10}{3+10} = \frac{30}{13}$$

The relative distances of the scales to each other will be calculated:

$$\frac{m_K}{m_z} = \frac{10}{3}$$

the scale equations will then be;

$$X_K = 10\ (\log 2K - \log 250) = 10 \log (K/125)$$

$$X_z = 3\ (\log z - \log 5) = 3 \log (z/5)$$

$$X_f = \frac{30x2 \log f}{13} = 4.62 \log (f)$$

The general form of the Alignment Chart for the equation,

$z = \frac{2K}{f^2}$, is shown below:

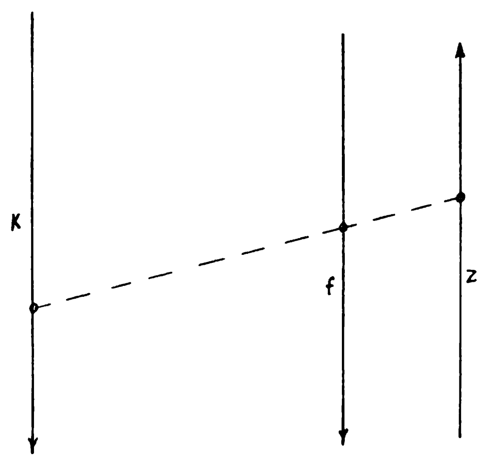


Fig. 8.

The value of "d" will now be evaluated. substituting (8) in (9);

$$zd^2 + 12.5(d+4) = p$$

or;

$$zd^2 + 12.5d + (50-p) = o$$

solving for d;

$$d = \frac{-12.5 + \sqrt{(12.5)^2 - 4.z\ (50 - p)}}{2.z} \quad . \qquad . \ (10)$$

A value of "p" will be assigned and the corresponding values of "d" will be calculated for different values of "z".

Same procedure will be repeated for different values of "p" assigned.

The general form of the nomogram is shown below:

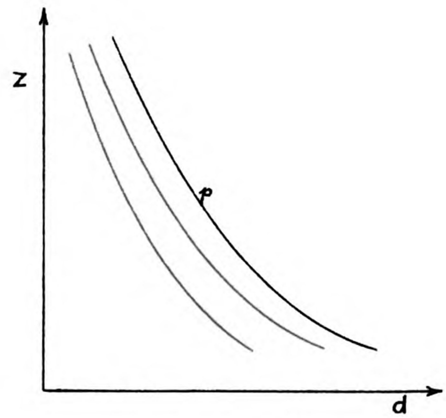


Fig. 9.

The modulus of "z" will have the same value and hence the latter two curves will be combined to give the complete nomogram drawn to scale on next page.

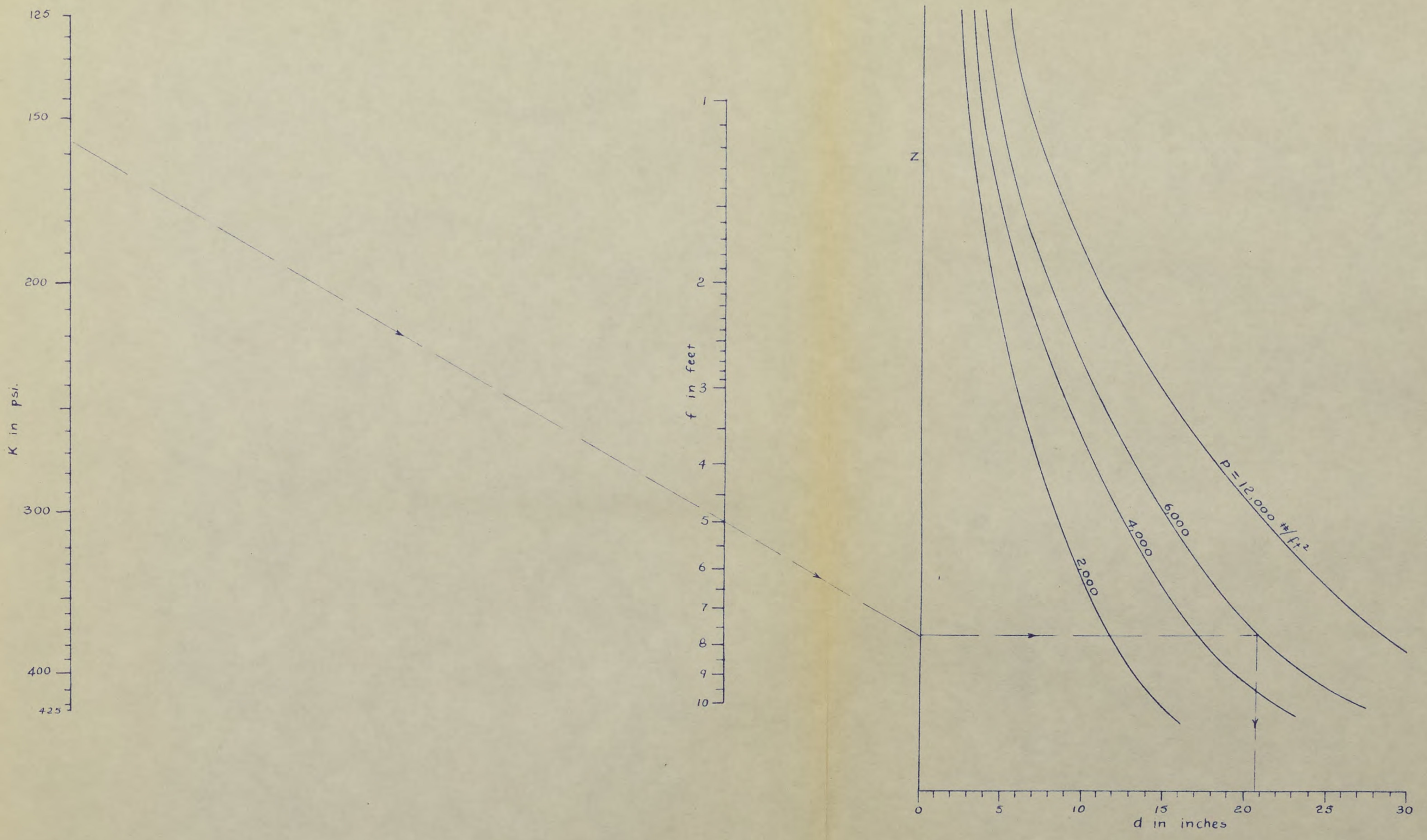


Chart B

3. Width of footing "B".

First the curve for equation, $M = f^2P/L$ will be drawn

or,
$$\frac{f^2}{M} = \frac{L}{P}$$

f varies from 0 to about 10 feet.

L varies from 0 to about 50 feet.

P varies from 0 to about 1300 kips.

therefore, M varies from 0 to about 3000 ft-kips.

$$m_f = \frac{5''}{10x10-0} = 0.05$$

$$m_L = \frac{5''}{50-0} = 0.1$$

$$m_P = \frac{5''}{1,300-0} = 0.00385, \quad \text{Use } m_P = 0.004$$

therfore,
$$m_M = \frac{m_f \cdot m_P}{m_L} = 0.002$$

The scale equations are;

$$X_f = 0.05f^2$$
$$X_L = 0.1L$$
$$X_P = 0.004P$$
$$X_M = 0.002M$$

The general form of the chart for the equation $M = f^2P/L$ is shown below:

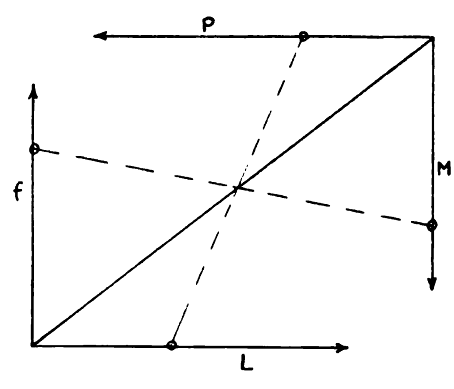


Fig. 10.

The value of "B" will be evaluated next from, $M = KBd^2$

or, $\frac{M}{d^2} = \frac{B}{1/K}$

M varies from 0 to 3,000,000 ft-lb.

d varies from 0 to 30 inches.

K varies from 125 to 423 psl.

therefore B varies from 0 to 10 feet

$$m_M = \frac{6''}{3,000,000} = 1/500,000$$

$$m_d = \frac{4''}{900} = 0.004,44$$

$$m_K = \frac{6.4''}{1/125} = 800$$

then, $$m_B = \frac{m_M . m_K}{m_d} = 0.36$$

the scale equations are:

$$X_M = M/500,000$$

$$X_d = 0.004,44d^2$$

$$X_K = 800/K$$

$$X_B = 0.36B$$

The general form of the chart for the equation $M = KBd^2$ is shown below:

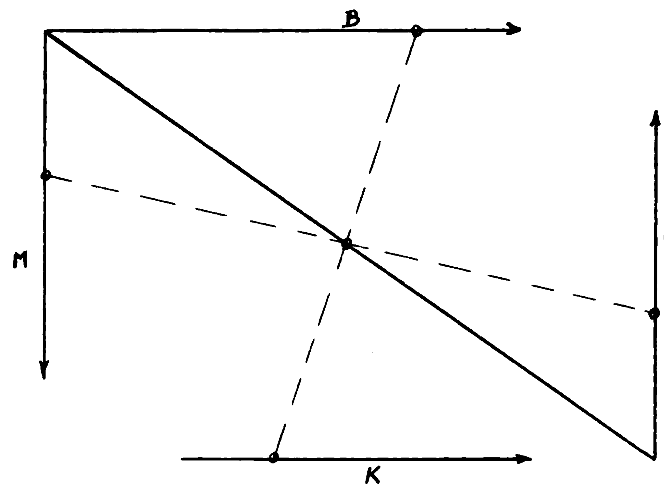


Fig. 11.

The M - scale lengths are the same in both equations and hence the two charts for equation (11), will be combined. (See chart B)

4. Area of steel required "A_s"

The value of "A_s" will be evaluated next from,

$$M = \frac{7}{8} \cdot f_s \cdot d \cdot A_s \text{ lb.-in.} \quad . \quad . \quad . \quad . \qquad (12)$$

or,

$$M = \frac{1}{12} \times \frac{7}{8} \cdot f_s \cdot d \cdot A_s \text{ lb.-ft.}$$

or,

$$\frac{M}{d} = \frac{7A\ /96}{1/f_s}$$

M varies from 0 to 3,000,000 ft.- lb.

d varies from 0 to 30 in.

f_s varies from 8,000 to 30,000 psi.

A_s varies from 0 to 100 in.

$$m_M = \frac{6''}{3,000,000} = 1/500,000$$

$$m_d = \frac{4.2''}{30} = 0.14$$

$$m_{f_s} = \frac{6''}{1/8000} = 48,000$$

$$m_{A_s} = \frac{48,000 \times 1/500,000}{0.14} = 24/35$$

the scale equations are:

$$X_M = M/500,000$$

$$X_d = 0.14d$$

$$X_{f_s} = 48,000/f_s$$

$$X_{A_s} = A_s/20$$

The general form of the chart for the eqution $M = \frac{1}{96} . A_s . f_s . d$ is shown below:

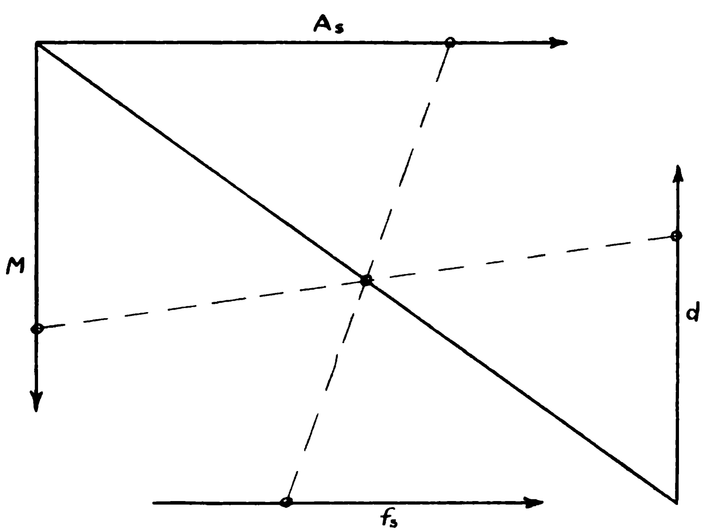


Fig. 12.

The modulus of "M" will have the same value and hence the latter three curves will be combined to give the complete nomg-gram drawn to scale on next page.

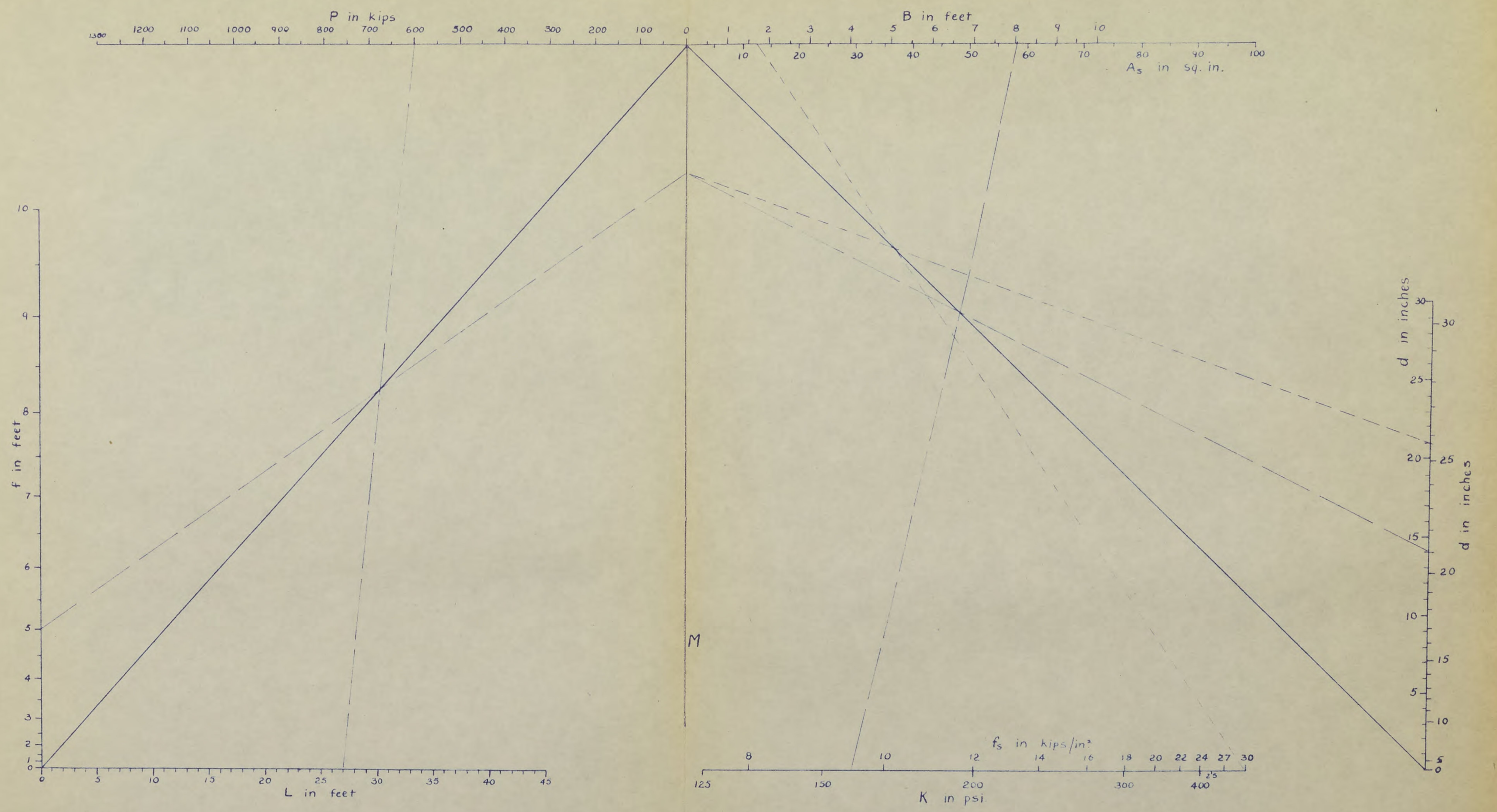

P in kips
B in feet
A_s in sq. in.
f in feet
d in inches
d in inches
M
f_s in kips/in²
L in feet
K in psi

Chart C

5. Bond "Σ_o"

The value of "V" will be evaluated from equation $V = \frac{2Py}{L}$. .(14)

or, $$\frac{V}{y} = \frac{2P}{L}$$

y varies from 0 to 12 feet.

L varies from 0 to 50 feet.

P varies from 0 to 1,300,000 lb.

$$m_y = \frac{6''}{12} = 1/2$$

$$m_L = \frac{5''}{50} = 1/10$$

$$m_P = \frac{5.2''}{2,600,000} = 1/500,000$$

therefore $$m_V = \frac{m_P \cdot m_y}{m_L} = 1/100,000$$

the scale equations are:

$$X_y = y/2$$

$$X_L = L/10$$

$$X_P = P/250,000$$

$$X_V = V/100,000$$

The general form of the chart for the equation V = 2Py/L is shown below:

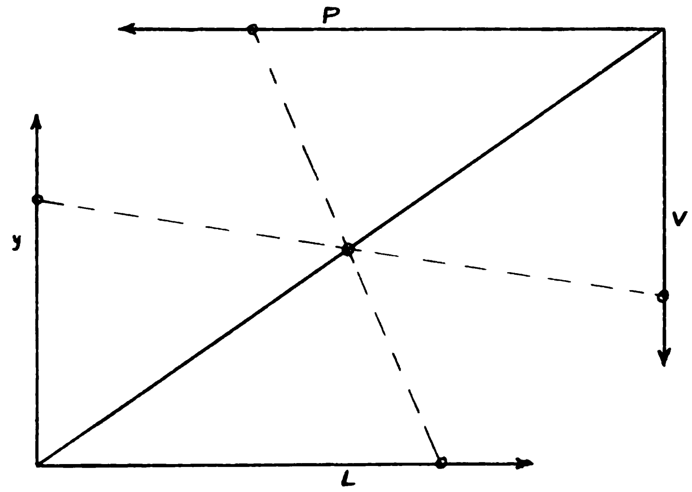


Fig. 13.

The value of "Σ_o" will be evaluated next, from,

$$V = \frac{7ud\Sigma_o}{8} \qquad . \quad . \quad . \quad . \quad . \quad . \qquad (17)$$

or,

$$\frac{V}{7d/8} = \frac{\Sigma_o}{1/u}$$

d varies from 0 to 30 inches.

u varies from 140 to 350

$$m_v = 1/100{,}000$$

$$m_d = \frac{2.5''}{30 \times 7/8} = \frac{4}{21} \quad \text{use, } m_d = 1/7$$

$$m_u = \frac{5''}{1/140} = 700$$

therefore

$$m_{\Sigma_o} = \frac{700 \times 1/100{,}000}{1/7} = 0.049$$

The scale equations are:

$$X_V = V/100{,}000$$

$$X_d = d/8$$

$$X_u = 700/u$$

therefore $$X_{\Sigma_o} = 0.049\,\Sigma_o$$

The general form of the chart for equation $V = \frac{7ud\Sigma_o}{8}$ is shown below:

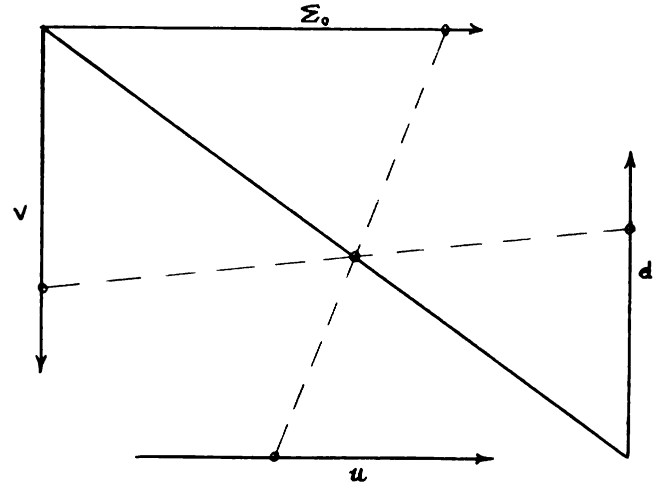


Fig. 14.

The V - scale length are the same in both equations and the two charts for equation (18) will be combined. (See chart D)

6. Shear "v"

The value of "v" will be evaluated from equation

$$V = vBjd \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (19)$$

or, $$\frac{V}{7d/8} = \frac{12.B}{1/v}$$

$$m_V = 1/100{,}000$$

$$m_d = \frac{4'' \times 8}{30 \times 7} = \frac{2}{15} \qquad \text{use, } m_d = 1/7$$

$$m_B = \frac{2.5}{10 \times 12} = 1/48$$

$$m_v = \frac{m_B \cdot m_d}{m_V} = 297.5$$

The scale equations are:

$$X_V = V/100{,}000$$

$$X_d = d/8$$

$$X_B = B/4$$

$$X_v = 297.5/v$$

The general form of the chart for the equation V = vBjd is shown below:

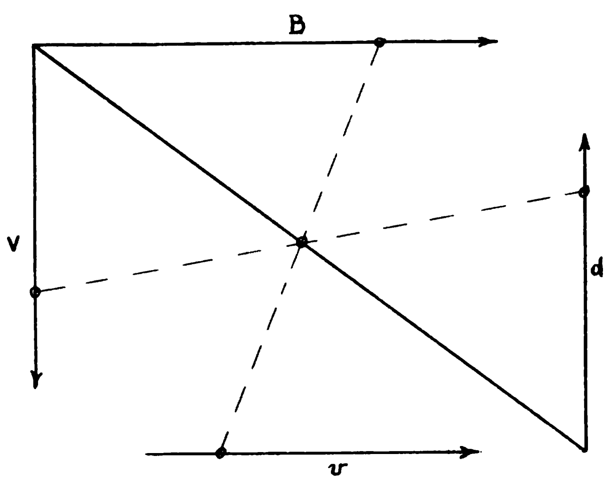


Fig. 15.

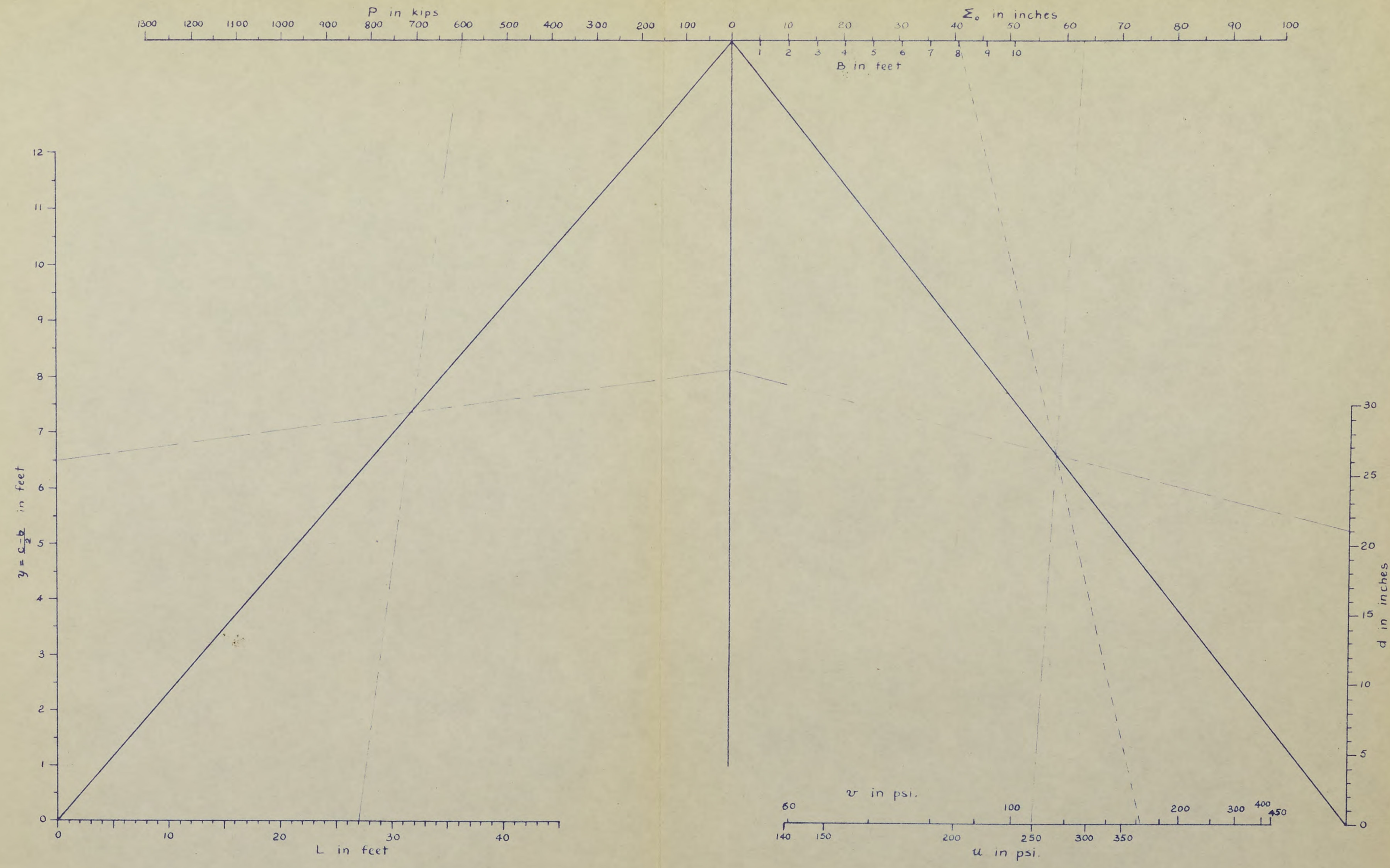

P in kips
Σ_o in inches
B in feet
$y = \frac{c-b}{2}$ in feet
L in feet
v in psi.
u in psi.
d in inches

Chart D

NUMERICAL EXAMPLE

An example of the use of the charts will be given to make the method of procedure clear.

Given:

P = 600 kips

c = 15 ft.

b = 24" = 2 ft.

f_s = 30,000 psi.

f'_c = 2500 psi.

p = 6,000 #/ft.^{2}

u = 250 psi. (deformed bars, A. C. I. 1951 Code)

Solution:

From chart A. and with values of c and b:

L = 27 ft. and f = 5 ft.

K = 156 (Reinf. Conc. Handbook)

From chart B and values of K and f: d = 21 in.

From chart c and values of P, f, L, d, K, f_s:

B = 8.1 ft. and A_s = 12.1 in^2.

From chart D and values of P,y,L,d,u,v,B,

Σ_o = 63 in. and v = 163 psi.

SUMMARY

The curves are more flexible and cover a wider range of design conditions than the most comprehensive tables. Their use entails a minimum of calculation and shows clearly the design procedure and the relation between the design conditions.

The resultant design will be within the practical limits of loading and soil bearing determination.

APPENDIX

An alignment chart in its simplest form consists of three parallel scales so graduated that a straight line cutting the scales will determine three points whose values satisfy the given equation.

In general, alignment charts may consist of three or more straight-line scales, of curved scales, or of combinations of both.

ALIGNMENT CHARTS FOR EQUATIONS OF THE FORM

$$f_1(u) + f_2(v) = f_3(w)$$

Suppose we have three parallel scales (Figure 16), A, B, and C, so graduated that lines (isopleths) 1 and 2 cut the scales in values which satisfy the equation $f_1(u) + f_2(v) = f_3(w)$. Now,

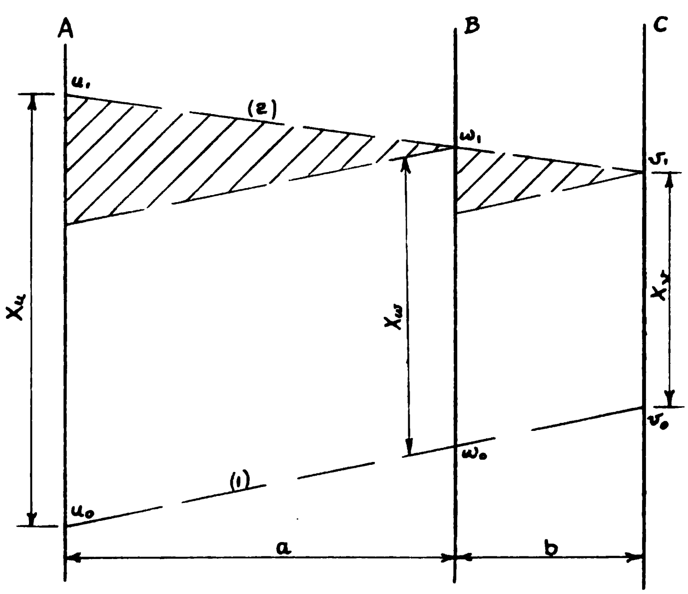


Fig. 16.

$$X_u = m_u \quad f_1(u_1) - f_1\ (u_0)$$

$$X_v = m_v \quad f_2(v_1) - f_2(v_0)$$

$$X_w = m_w \quad f_3(w_1) - f_3(w_0)$$

If u_0, v_0, w_0, represent zero values of the function, and if line 2 is any line, we may drop the subscripts and write simply

$$X_u = m_u f_1(u) \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$X_v = m_v f_2(v) \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$X_w = m_w f_3(w) \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Let us agree further that the spacing of the scales is in the ratis a/b. If we graduate the scale of $f_1(u)$ and $f_2(v)$ in accordance with their scale equations (1) and (2), respectively, what will be the modulus for the scale equation of $f_3(w)$ and what will the ratio a/b equal if the chart satisfies this relation $f_1(u) + f_2(v) = f_3(w)$?

In figure 16 draw lines through points w_1 and v_1 parallel to line u_0v_0. The shaded triangles are similar by construction, hence,

$$\frac{X_u - X_w}{X_w - X_v} = \frac{a}{b}$$

$$X_u b + X_v a = X_w(a + b)$$

$$\frac{X_u b}{ab} + \frac{X_v a}{ab} = X_w \frac{(a + b)}{ab}$$

or
$$\frac{X_u}{a} + \frac{X_v}{b} = \frac{X_w}{\frac{ab}{a+b}}$$

since
$$X_u = m_u f_1(u)$$

$$X_v = m_v f_2(v)$$

$$X_w = m_w f_3(w)$$

$$\frac{m_u f_1(u)}{a} + \frac{m_v f_2(v)}{b} = \frac{m_w f_3(w)}{\frac{ab}{a+b}}$$

If $f_1(u) + f_2(v) = f_3(w)$, then

$$m_u = a; \quad m_v = b$$

therefore
$$\frac{a}{b} = \frac{m_u}{m_v}$$

and
$$m_w = \frac{ab}{a+b} = \frac{m_u m_v}{m_u + m_v}$$

PROPORTIONAL CHARTS OF THE FORM

$$\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

This equation can be solved in a manner similar to that used in the preceding problem. This simply means transforming the above equation to the form

$$\log f_1(u) - \log f_2(v) = \log f_3(w) - \log f_4(q)$$

In many cases, however, where the functions are linear, the proportional type alignment chart has a distinct advantage in that the scales

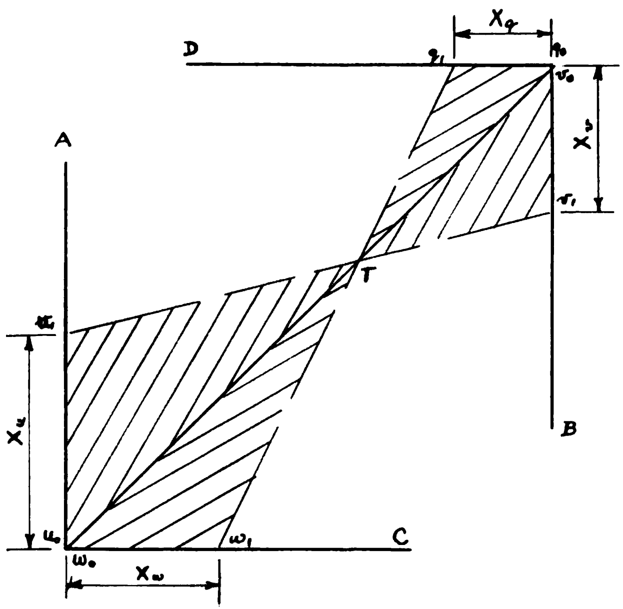


Fig. 17.

are uniform thus permitting more accurate readings and also simplifying the construction of the scales.

Consider the figure shown in Figure 18.

Scales A and B are parallel to each other, and graduated in accordance with the scale equations:

$$X_u = m_u f_1(u) \text{ and } X_v = m_v f_2(v), \text{ respectively}$$

In a similar manner, scales C and D are parallel to each other, and are graduated in accordance with the scale equations:

$$X_w = m_w f_3(w) \text{ and } X_q = m_q f_4(q)$$

The angle between scales A and C may be of any convenient magnitude.

Triangles $u_1 u_0 T$ and $v_1 v_0 T$ are similar, hence

$$\frac{X_u}{X_v} = \frac{u_0 T}{v_0 T}$$

Likewise, triangles $w_0 T w_1$ and $q_0 T_q 1$ are similar, hence

$$\frac{X_w}{X_q} = \frac{w_0 T}{q_0 T}$$

But lenghts $u_0 T = w_0 T$; and $v_0 T = q_0 T$. Therefore

$$\frac{X_u}{X_v} = \frac{X_w}{X_q}$$

or

$$\frac{m_u f_1(u)}{m_v f_2(v)} = \frac{m_w f_3(w)}{m_q f_4(q)}$$

Since

$$\frac{f_1(u)}{f_2(v)} = \frac{f_3(w)}{f_4(q)}$$

it follows that $\frac{m_u}{m_v} = \frac{m_w}{m_q}$

This means that three moduli may be determined from the given data, but the fourth modulus will be dependent upon the first three.

BIBLIOGRAPHY

1. "Reinforced Concrete Structures" by Dean Peabody, Jr.

2. "Reinforced Concrete Design Handbook" of the American Concrete Institution..

3. The ACI Building Code 1951. (ACI 318-51)

4. "Recommended Practice and Standard Specifications for Concrete and Reinforced Concrete" Joint Committee on Standard Specifications for Concrete, and Reinforced Concrete, 1940.

5. "Nomography" by A. S. Levens.