DEVELOPMENT AND TEST OF A MODEL OF CONFLICT IN A TRUEL

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ABSTRACT

DEVELOPMENT AND TEST
OF A MODEL OF
CONFLICT IN A TRUEL

BY

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Pure conflict (called uelative conflict) was investigated through the use of a three participant experimental gaming paradigm called the truel. By developing a mathematical model of the truel, some conclusions about uelative conflict were developed and tested.

The major results of the study indicated that no single, simple process operates in a simple conflict situation. The assumption that the participants in a conflict situation attack their strongest attack choice was not consistently verified. The strongest participant attacked his strongest attack choice as a function of the type of power structure the triad was in. The remaining two power positions quite consistently attacked their strongest attack choice.

The mathematical model was based on these assumed preferred attack choices of the participants. Because the real choices were not entirely consistent between or within power structures the model was unable to account for some of the data. The points of bad fit were indicated and alternative preferred attack choices suggested. The value of a mathematical model for the testing of hypotheses was indicated.

The final point of the study indicated some implications for Caplow's classification of triad power structures for conflict

situations. The major result being a preference on the part of the triads in the study for type 1 and type 2 power structures.

Approved by James L. Phillips

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Introduction

The present study was performed to investigate the problem of people in conflict over goals. With its widespread occurrence within and between nation-states, conflict, and its reduction, have become major areas of research. Although war is the most severe type of conflict it is by no means the only type. The cold war, confrontations on college campuses, and political conventions are also types of conflict and give just as much impetus to this research as do the wars in the Middle East and Vietnam.

With the exception of the internation simulation game (Guetzkow, 1962; Burgess and Robinson, 1969; and Hermann and Hermann, 1969), the research on conflict and cooperation has generally involved the use of very abstract, and relatively simple gaming paradigms (prisoner's dilemma, chicken, parchesi, political convention, and the Deutsch and Krauss trucking game). The underlying premise of the research using these simple, rather artificial laboratory games is that it is necessary to understand the basis of conflict in its most elementary form before it is possible to explain and control conflict in the real world.

For this paper, it is assumed that conflict exists whenever at least two participants are in a situation in which only one can fully achieve his goal. A more detailed discussion of conflict is presented below but for the present this rough definition will suffice to categorize situations as cooperative or conflictive. The former are situations in which all parties can achieve their goals simultaneously, and the latter are situations in which, at best, each party achieves only a portion of his goal and, at worst, no party achieves any part.

In the real world, situations vary in the degree to which they manifest conflict. These conflict situations range along a continuum from cooperative (negotiated settlement of differences, formation of a coalition against a third party, etc.) to pure conflictive (war). The various experimental paradigms used to study conflict and cooperation reflect these degrees of conflict. Presented below is a selected summary of these paradigms that starts with games at the cooperative end of the continuum and ends with the games at the conflictive end.

Those types involving the least amount of conflict are the parchesi game (Vinacke and Arkoff, 1957; Vinacke, Crowell, Dien, and Young, 1966; and Vinacke, Lichtman, and Cherulnik, 1967), the political convention (Chertkoff, 1966; Nitz, 1969; and DeYoung and Phillips, 1970) and the internation simulation game mentioned previously. The first two types were designed primarily to study coalition formation, and therefore little conflict is generated in either game. The last type was designed to simulate nation-state interaction and, thus, conflict was a possible result rather than a necessary condition of the situation.

The parchesi game, used most extensively by Vinacke, presents subjects with the opportunity to form a coalition which will insure them of winning the game and thus sharing the payoff. The three participants are assigned a certain amount of power, if no coalition is formed the player given the most power will win. This paradigm forces the participants to form a coalition if they want to win. Although the subjects are in conflict over the goal, they are presented an opportunity to cooperate and divide the payoff (partial fulfillment).

The political convention game is very similar to the parchesi game.

The three or more participants are assigned a certain amount of power. For any one to win he must possess a majority of the power in the game. Generally no one player has a majority and, therefore, for anyone to win he must form a coalition. This is in contrast to the parchesi game in which the strongest participant can win alone if no coalition is formed. The difference between the two paradigms lies in the reason for forming a coalition; in the parchesi game the only rational strategy for two of the three players is to form a coalition, while in the political convention they are required to form a coalition.

These two paradigms mirror the cooperative end of the real world cooperation-conflict range described previously. Two of the three participants are required to form a coalition to receive a share of a divisible payoff, thus partial, simultaneous achievement of the goal. The amount (degree) of conflict is small while cooperation is high.

The internation simulation game differs from the parchesi and political convention paradigms, and all subsequent paradigms, in the unstructuredness of the situation. The participants can produce situations which cover the full range of conflict type situations from cooperative to completely conflictive. The advantage of producing all possible outcomes, however, limits the analysis and therefore the conclusions that can be drawn from the data. The situation does, however, offer a starting point since it can be used to formulate hypotheses which later can be tested in more rigidly controlled experimental settings.

Most of the research in the area of conflict has centered on the mixed-motive situation, using various types of games as experimental paradigms. Of those games that have been most widely used to study some aspect of conflict, the prisoner's dilemma (Bixenstine, Potash, and Wilson, 1963; Bixenstine and Wilson, 1963; Lave, 1965; Oskamp and Perlamn, 1965; Radlow, 1965; Rapoport and Chammah, 1965; Sampson and Kardush, 1965; and Evans and Crumbaugh, 1966), the Deutsch and Krauss trucking game (Deutsch and Krauss, 1962), and the chicken game (Scodel and Minas, 1960; Sermat and Greyovich, 1966; and Ells and Sermat, 1968) have been the most widely used. All three of these paradigms employ two people, and provide an opportunity for the participants to cooperate with each other. This is accomplished by giving them a choice of performing one of two actions, with differential payoffs for each combination of choices made by the two participants. The general type of payoff matrix for the prisoner's dilemma and the chicken game is presented in Table 1. Since the trucking game does not involve simultaneous choice by the participants, a payoff matrix is quite difficult to construct and therefore is not presented.

Matrix 1 in Table 1 is the general matrix, with each cell identified by a capital letter to allow for easier identification. Matrix 2 in the same table presents the general payoff matrix for the two paradigms, the numbers 1 and 2 designate the two alternatives. These two alternatives and the relative values of the "high" and "low" payoffs differ between the two games. Following is a brief characterization of each experimental paradigms' payoff matrix. A more detailed presentation of these types of payoff matrices has been made by Rapoport (1963, 1968) and Rapoport and Guyer (1966).

In the prisoner's dilemma, alternative "1" is the choice of cooperating with the other player, alternative "2" is the choice of

Table 1. General and Particular Payoff Matrices for the Prisoner's Dilemma and Chicken Games

Matrix 1. A General Matrix with Each Cell Identified by a Capital Letter

A	B
C	D

Matrix 2. A General Payoff Matrix for the Prisoner's Dilemma and Chicken Games

		Y 1	Y 2
X	1	high / high	high / low
X	2	low / high	low / low

Matrix 3. A Particular Payoff Matrix for the Prisoner's Dilemma

		Y 1	Y 2
X	1	+10¢ / +10¢	+15¢ / -10¢
X	2	-10¢ / +15¢	-5¢ / -5¢

Matrix 4. A Particular Payoff Matrix for the Chicken Game

		Y 1	Y 2
X	1	+10¢ / +10¢	+15¢ / -10¢
X	2	-10¢ / +15¢	-100¢ / -100¢

defecting to the police. In cell A, both players cooperate and receive +10¢; in cells B and C, one player cooperates and the other defects, resulting in -10¢ for the cooperator and +15¢ for the defector; in cell D, both players choose alternative 2 and receive a -5¢. The preference structure for player X is B > A > D > C, while player Y's preference structure is C > A > D > B. Where the preference structure is the order in which the players desire the outcomes in the particular cells. Given the above structure, player X prefers the payoff in cell B to any other payoff, and prefers any other cell's payoff to the payoff of cell C. The point of interest, in the choice structures, is the congruence of the second preferences. It is this congruence that allows for cooperation between the participants.

In the chicken game alternative "1" is the cooperative choice and alternative "2" is the noncooperative choice. Thus, in cell A, both players cooperate and receive +10¢; in cells B and C, one player cooperates, while the other does not, resulting in the cooperator receiving -10¢ and the noncooperator receiving +15¢; and in cell D, neither cooperates and both receive the higly negative outcome of -100¢. The preference structures for the two players are as follows: X: B > A > C > D; and Y: C > A > B > D. The difference between the prisoner's dilemma and the chicken game is the ordering of the last two preferences for the two players. In prisoner's dilemma the payoff in cell D is preferred by both players over the payoff in B for player Y, and C for player X, while in chicken payoff D is the least preferred outcome for both players.

In terms of the payoffs, the difference between the prisoner's dilemma and the chicken game is the relative sizes of the low payoffs in the two low cells for each player. In the chicken game the low payoff in the low-low combination (cell D) is much lower than the low payoff in the two high-low combinations (cells B and C). This is in contrast to the prisoner's dilemma where the low payoff in the high-low combination (cells B and C) is much lower than the payoff in the low-low combination (cell D).

In general the mixed-motive payoff matrix (matrix 2 Table 1) can be described in the following manner; one combination of choices results in a low payoff for both participants (cell D), one combination in a moderate payoff for both participants (cell A), and the remaining two combinations result in a low payoff for one participant and a high payoff for the other, with the receiver of the high payoff reversed in the two high-low conditions (cells B and C). Thus the payoff matrix allows for cooperation, by providing a cell in which both participants receive a moderate payoff, and conflict, since any movement from the cooperative cells results in a lower payoff for at least one participant.

Most of the studies performed with these two paradigms, as with the parchesi and political convention paradigms, have been concerned with the dimensions of cooperation rather than the exploration of conflict. Little research has been aimed at the interpersonal process that arises when people are placed in a situation in which they have no opportunity to cooperate and, therefore, must compete. This type of pure conflict (no cooperation between participants), represented in the real world by nuclear war and the duel to the death, has been labeled pure uelative conflict by Cole and Phillips (1969) and Cole, Phillips, and Hartman (in preparation). Uelative conflict is defined as a _n_ participant system in which there is a single, indivisible payoff for all participants. This means that at most one participant can receive the payoff (achieve his goal) and all may lose. Note that it is not a zero-sum game since there is the possibility that all parties may lose, but it does contain a constant sum condition in which when one person wins, all others lose.

This type of conflict falls at the extreme end of the cooperation-conflict continuum defined previously. No cooperation between participants is possible because the payoff for each participant is not divisible. This is an important aspect of the situation; it is the indivisibility of the payoff which distinguishes pure conflict from the mixed-motive, or cooperative type situations (Boulding, 1963; and Schelling, 1969). No player is able to achieve a partial goal; either he achieves his total goal or he achieves nothing.

An experimental game paradigm, the truel, has been designed to study this extreme conflict situation. Introduced by Shubik (1954) and subsequently employed by Willis and Long (1967) and revised by Cole (1969, 1970), the truel is a game involving three players, each of whom begins the game with a particular number of points. Prior to the start of each game, the experimenter assigns a certain number of points and a label to each player. The game itself consists of a number of moves. On each move each player must destroy a point belonging to one of the other players. This is accomplished by each player secretly indicating his attack choice to the experimenter, who, when all three players have indicated their choices, announces who attacked whom and removes a point from the attacked player's total. This procedure allows each player to make his choice independently, without the knowledge of who is going to attack him on that move.

The game continues until only one player has points remaining. He is the winner of the game. If the two players remaining, when the first player is eliminated, have the same number of points, or if all of the players are eliminated on the same move, the game is a tie with no player declared the winner.

The payoff matrices for the three person prisoner's dilemma (matrix 1), a three person chicken game (matrix 2), and a truel in which all three players have one point (matrix 3) are presented in Table 2.[1] The values presented in each cell represent the payoffs to the participants. The first component in the vector in each cell represents the payoff for player X, the second for player Y and the third for player Z. The cells are labeled from A through H and the preference structures for the three players are listed below each matrix.

As the three preference structures listed below matrix 3 indicate, each player has two types of outcomes; a most preferred and a least preferred. For the three players there are no points of congruence for the most preferred cells and only two points of congruence for the least preferred cells. Player X prefers cells D and H to all other outcomes while player Y prefers cells E and F to all others and player Z prefers cells A and C to all others. It is the dichotomous preference structures and the noncongruence of the first preferences caused by the indivisibility of the positive payoff, that

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[1]Note that the truel is a three person game, while the other two paradigms have generally been two person. The payoff matrix for the truel is a 2 x 2 x 2, while the payoff matrix for the other two paradigms is usually 2 x 2. However, for the purpose of comparison, the prisoner's dilemma and the chicken game were expanded to a three participant system and, thus, their payoff matrices were expanded to a 2 x 2 x 2.

Table 2. The Normal Payoff Matrices for the Three Person Prisoner's Dilemma, Chicken, and Truel Games

Matrix 1. Prisoner's Dilemma

	Player Z	1		2	
	Player Y	1	2	1	2
X	1	(+10,+10,+10) A	(-10,+15,-10) B	(-10,-10,+15) C	(-10,-5,-5,) D
	2	(+15,-10,-10) E	(-5,-5,-10) F	(-5,-10,-5) G	(-5,-5,-5) H

Player X: E > A > F = G = H> B = C = D
Player Y: B > A > D = F = H> C = E = G
Player Z: C > A > D = G = H> B = E = F

Alternative 1 = cooperate
2 = not cooperate

Matrix 2. Chicken Game

	Player Z	1		2	
	Player Y	1	2	1	2
X	1	(+10,+10,+10) A	(-10,+15,-10) B	(-10,-10,+15) C	(-10,-100,-100) D
	2	(+15,-10,-10) E	(-100,-100,-10) F	(-100,-10,-100) G	(-100,-100,-100) H

Player X: E > A > B = C = D >F = G = H
Player Y: B > A > C = E = G >D = F = H
Player Z: C > A > B = E = F >D = G = H

Alternative 1 = cooperate
2 = not cooperate

Matrix 3. Truel

	Player Z	X		Y	
	Player Y	X	Z	X	Z
X	Y	(-10,-10,+15) A	(-10,-10,-10) B	(-10,-10,+15) C	(+15,-10,-10) D
	Z	(-10,+15,-10) E	(-10,+15,-10) F	(-10,-10,-10) G	(+15,-10,-10) H

Player X: D = H > A = B = C = E = F = G
Player Y: E = F > A = B = C = D = G = H
Player Z: A = C > B = D = E = F = G = H

makes this situation one of pure conflict.

A comparison of the payoff matrices for the two mixed motive paradigms with the payoff matrix for the truel reveals the differences between the three types of paradigms.

In both matrix 1 and matrix 2 the three person prisoner's dilemma and chicken game, respectively, alternative "1" is the cooperative choice and alternative "2" the noncooperative one. As was pointed out in the discussion of the 2 x 2 payoff matrix, there is a common second choice for all of the preference structures. The difference between the chicken game and the prisoner's dilemma is the ordering of the last two sets of preferences. These orderings can be compared in Table 2. Those preferences of the same value in the matrix are treated in the preference structure as being of equal desirability. The difference between the mixed-motive situation and the pure conflict situation is the absence in the latter's preference structures of a common point in the orderings for the three players.

The truel game has the basic requirements for uelative conflict: an inability of the participants to achieve their goals (winning) simultaneously.[2] In the actual playing of the game the motivation for winning is assumed to be instilled by a monetary reward for doing so. With motivation established, and an indivisible payoff structure, with at most one winner, the truel satisfies all of the requirements for uelative conflict.

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[2]It might be argued that the basis of all conflict is the belief of the participants that they cannot achieve their respective goals simultaneously.

This paradigm has two important properties in that it can be extended to any number of players, and it characterizes the pure conflict situation, which has been ignored to the present time. By breaking the conflict situation down to its most basic elements, however, the paradigm overlooks factors that affect conflict situations. Some of the factors that are ignored are: (1) the effect of secondary goals, (2) the formation of coalitions, and (3) the ability of the participants to avoid the conflict situation. Despite these and other limitations that introduce some degree of artificiality into the situation, it seems an excellent starting point for the investigation of pure conflict.

The Investigation of Conflict through the Truel

One aspect of pure conflict that is quite easily investigated through the use of the truel is the effect of power distributions upon the conflict process. If it is assumed that the number of points possessed by each of the players represents the power of that player, then any distribution of points can be classified according to the eight types of power distribution defined by Caplow (1956, 1959, and 1968) and presented in Table 3. Although Caplow only analyzed the consequences of each of the distributions for the cooperative type of situation, an extension to the pure conflict situation may prove profitable. In an attempt to discover some of the consequences of the various power distributions, several of them representing Caplow's type 2, type 3, and type 5 were constructed. The exact distributions are also presented in Table 3 and are discussed in more detail later.

Another aspect of conflict that can be investigated through the use of the truel is the determinants of attack choice, i.e., the reasons why a player will choose to attack a given other player on each move of the game. Since in a truel, each player is forced to make just such a choice, and he must do so with a limited number of cues available

Table 3. Caplow's Classification of Types of Power Distributions and the Distributions Used in the Experiment.

Types	Power Distributions	Distributions of Points
1	A = B = C	NONE
2	A > B = C (A > (B + C))	(24,9,9)
3	A B = C	(19,19,4)
4	A B = C (A (B + C))	NONE
5	A B C (A (B + C))	(22,9,8) (20,9,7) (18,8,6) (16,9,5) (18,17,4) (17,15,4) (16,13,4) (15,11,4) (14,9,4)
6	A B C (A (B + C))	NONE
7	A B C (A = (B + C))	NONE
8	A B = C (A = (B + C))	NONE

it is possible to present a limited set of possible bases for this decision. When persons engage in uelative conflict, they have many cues upon which to make their decision of whom to attack. Within the truel this set of cues is reduced to a finite number, with those cues being immediately present or from several moves previous. Several of the possible attack strategies that may be used are presented below.

A player may attack the player who attacked him last, or he may attack a player who had not been attacked in several moves. A player also may alternate his attacks from one player to another or continually attack the same player. Several such heuristics may be used by the participants in selecting an attack choice. Although these heuristics do not seem to be the most rational approach they still may be used by the players to make their decision. However, if it is assumed that the participants in a truel are rational, then strategies of play rather than heuristics of attacks would be the most likely to be employed.

Phillips, Hartman, and Klein (1970) presented three strategies of play that might be used by participants in a truel: (a) the fair play strategy; (b) the threat minimization strategy; and (c) the dyadic competition strategy. The fair play strategy assumes that a player attacks the stronger of his two attack choices. This means that the strongest player attacks the second strongest and the second and third strongest players attack the strongest. In the threat minimization strategy, a player makes the attack that minimizes the threat to his survival. It is not the case that this is always the stronger of the two attack choices. It has been demonstrated by Cole and Phillips (1967) that the strongest player is in a position where he is likely to be the object of the other two player's attacks. The strongest player attacks the weakest player because this player does just as much damage and is removed from the game more quickly than is the second strongest player. By eliminating the weakest player and thus reducing the number of attacks made on him, the strongest player minimizes the threat to his survival in the fewest number of moves. The threat minimization strategy for the other two power positions results in the same attacks as in the fair play strategy with both players attacking the strongest player.

In the dyadic competition strategy the players attack that player who is closest to them in the power structure. Both the weakest and the strongest players would attack the middle power position. The person in the middle position would attack the player--sometimes the strongest, sometimes the weakest--whose power was closer to his own.

Shubik (1954) pointed out that when the participants in a three person duel differ with respect to power, the more rational strategy is to attack the stronger of one's two attack choices (the fair play strategy). With respect to the truel, the power of a player is the number of points he possesses and thus, for each player, a more rational strategy is to attack that player of his two attack choices who has the greater number of points. The assumption, that all three players employ the fair play strategy, was used to build the mathematical model of the truel (and therefore of uelative conflict) presented below.

A Model of Attack Choices

A one parameter model is proposed to account for the interpersonal process that operates when three persons engage in a truel. The model generates all predictions using the estimated probability (P) of attacking the stronger of each player's two attack choices. If the two attack choices of a player have the same number of points (power), a probability of attacking each player is set at .50. The set of three numbers representing how many points each of the three players has is called the distribution of points. This distribution is represented by a three component vector, with the numbers arranged in a decreasing order of magnitude. As an example, if one player had 13 points, another 7 points and a third 20 points, the vector reoresenting the distribution would be (20,13,7).

A characteristic of the process that is of interest is the pattern of choices made by the participants on each move of the game.

Since on every move each player is required to attack one of the other two players, it is possible to characterize combinations of attacks within the truel in terms of who received an attack. The set of three attacks made on any one move is called the attack vector, with each number in the vector being the number of times a particular player received an attack on that move. The order within the vector is one of decreasing strength, i.e., the player with the most points is listed first in the vector and the player with the fewest points is listed last. For example, if the distribution of points was (10,9,8) an attack vector of (2,1,0) would indicate that the player with 10 points had been attacked twice, the player with 9 points had been attacked once and the player with 8 had not been attacked at all. After this combination of attacks the distribution would be reduced to (8,8,8) because two points were taken from the player with 10 points, one point from the player with 9 points and none from the player who had 8 points. In the truel there are seven such attack vectors and they are presented in Table 4.

Table 4. The Seven Possible Attack Vectors

I	J	J-1	K	K-1	L	L-1
1	2	0	1	1	0	2
1	1	1	0	2	2	0
1	0	2	2	0	1	1

There are two characteristics of interest in each distribution of points. They are called the disparity of relative strengths and the projected level of equality. The disparity of relative strengths is discussed at length here, and the explanation of the projected level of equality follows.

The disparity of relative strengths (DRS) is a term used to describe how far apart the players are in the number of points they possess. The term can be quantified by using the differences between the players as the index of disparity. For example, the (10,9,8) distribution has a lower disparity of relative strengths than does the (15,9,3) distribution. Not only is this evident from visual inspection, but the index of the DRS is (1,1) for the first distribution and (6,6) for the second. The index was calculated by subtracting the second component from the first and then subtracting the third from the second. These two differences were then placed in a two component vector to give the DRS for each of the distributions. Further, it is possible to sum the two components to obtain a scalar quantity that gives a rough index of the DRS for any distribution. Later, an easier method for determining the DRS is described.

To simplify referencing, each attack vector is given a single letter label. The letter I is assigned to the (1,1,1) attack vector to indicate that this attack vector, when applied to a distribution of points, maintains the differences that exist between each of the three players. In other words, the I attack vector maintains the disparity of relative strengths.

Of the six remaining attack vectors, the vector (2,1,0) is assigned the letter J. J^{-1} designates the (0,1,2) vector since it is the only vector that, when applied to a distribution of points, returns the disparity of relative strengths to the level held prior to the application of the J vector. The sum of a J and a J^{-1} attack vector is (2,2,2) or two I vectors, which by definition maintains the disparity of relative strengths. The remaining attack vectors are assigned letters by the same process. A vector is chosen and assigned a letter and the attack vector, that when added to it maintains the DRS, is given the inverse of that letter. All of the vectors and their labels are listed in Table 4.

The projected level of equality (PLE), the second characteristic of the distribution of points, describes the nearest point at which all of the players will have the same number of points. The PLE can only be calculated for distributions in which the sum of the components is divisible by three. It is only these distributions that can be reduced to equality through combinations of three attacks and therefore by the application of attack vectors. An example of a distribution that cannot be reduced to equality by attack vectors is (5,3,2). No combination of attacks can reduce this distribution to another distribution in which all the players have an equal number of points. The attack vector which brings it nearest to equality is the (2,1,0), or J vector. After the application of this vector the distribution is reduced to (3,2,2), and after the second application of this vector, and the necessary arrangement, the distribution is reduced to (2,1,1). At this point the application of any attack vector will end the game, since any additional attack must eliminate either one or two (but not all three) of the players. Since three points must be taken away from the triad on any move, and there are four points remaining, it is not possible to reach an all equal state before one of the players is eliminated. The projected level of equality of a distribution is that distribution which is tne nearest and the attack vector, that when added to it maintains the DRS, is given the inverse of that letter. All of the vectors and their labels are listed in Table 4.

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The distribution of points which any triad begins a truel is called the initial distribution. If this distribution is divisible by three then it can be described by a sequence of attack vectors. Any subsequent distribution (those caused by attacks made within the triad) is then described by the sum of the attack vectors used by the triad and the sequence of attack vectors used to describe the initial distribution. Thus any distribution of points whose sum is divisible by three, can be described by a sequence of attack vectors.[3] This sequence is composed of the attack vectors used by the triad in the playing of the game and the attack vectors used to describe the initial distribution. This sequence is called the decomposed distribution of points. The sequence with the I attack vectors deleted is called the deleted decomposed distribution of points. Later these two sequences will be shown to be composed of at most two attack vectors. This reduction results from the relationships between attack vectors that are discussed next.

There are certain relationships that exist between attack vectors. If two attack vectors, a and b, when applied to a distribution, have the same effect as another attack vector, c, added to the I vector and then applied to the same distribution, then the two vectors, a and b, are defined as reducing to the third vector, c. The reduction is based on the fact that the two vectors, a and b, have the same effect

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[3]The necessity of having the distribution divisible by three is discussed later.

on the disparity of relative strengths as does the single attack vector, c. The application of the single attack vector to a distribution does not result in the same distribution as the application of the two vectors; it is only the differences between the players that are the same in the two distributions. If the I vector is applied to the distribution of the single vector, c, then the number of points controlled by each of the players would be the same as if the two vectors, a and b, had been applied. The rule for reducing attack vectors is given in terms of their labels. Any two vectors raised to the same power (1, or -1) and of different letter will reduce, when added together, to the vector of the remaining letter raised to the inverse of their common power. An example of a reduction is: $J + K = L^{-1}$, or $(2,1,0) + (1,0,2) = (1,1,1) + (2,0,1)$ or $I + L^{-1}$. Since the I vector has no effect on the disparity of relative strengths, the difference between the disparity of relative strengths of the original distribution and the distribution resulting from the application of the J and K vectors is equivalent to the difference that the L^{-1} vector alone would produce.[4] The resulting distribution of points, after the application of the L^{-1} vector and the I vector, is the same as the distribution resulting from the application of the J + K vectors. All of the attack vectors and their relationships are presented in Table 5.

With the definition of the attack vectors, the definition of the disparity of relative strengths, and the rules for the combination and reduction of attack vectors, it is now possible to describe whole

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[4]The equal sign indicated that the elements on each side are equal with respect to the DRS.

Table 5. The Relationships Between Attack Vectors

Combinations:[1]

$$\begin{matrix}2\\1\\0\end{matrix} + \begin{matrix}0\\1\\2\end{matrix} = \begin{matrix}1\\1\\1\end{matrix} + \begin{matrix}1\\1\\1\end{matrix} \qquad \begin{matrix}0\\2\\1\end{matrix} + \begin{matrix}2\\0\\1\end{matrix} = \begin{matrix}1\\1\\1\end{matrix} + \begin{matrix}1\\1\\1\end{matrix} \qquad \begin{matrix}1\\0\\2\end{matrix} + \begin{matrix}1\\2\\0\end{matrix} = \begin{matrix}1\\1\\1\end{matrix} + \begin{matrix}1\\1\\1\end{matrix}$$

$$J + J^{-1} = I + I \qquad L + L^{-1} = I + I \qquad K + K^{-1} = I + I$$

Reductions:[1] — **Summary**

$$\begin{matrix}2\\1\\0\end{matrix} + \begin{matrix}1\\0\\2\end{matrix} = \begin{matrix}2\\0\\1\end{matrix} + \begin{matrix}1\\1\\1\end{matrix} \qquad \begin{matrix}0\\1\\2\end{matrix} + \begin{matrix}1\\2\\0\end{matrix} = \begin{matrix}0\\2\\1\end{matrix} + \begin{matrix}1\\1\\1\end{matrix}$$

$$J + K = L^{-1} + I \qquad J^{-1} + K^{-1} = L + I$$

Summary: $J + K = L^{-1}$, $J^{-1} + K^{-1} = L$

$$\begin{matrix}2\\1\\0\end{matrix} + \begin{matrix}0\\2\\1\end{matrix} = \begin{matrix}1\\2\\0\end{matrix} + \begin{matrix}1\\1\\1\end{matrix} \qquad \begin{matrix}0\\1\\2\end{matrix} + \begin{matrix}2\\0\\1\end{matrix} = \begin{matrix}1\\0\\2\end{matrix} + \begin{matrix}1\\1\\1\end{matrix}$$

$$J + L = K^{-1} + I \qquad J^{-1} + L^{-1} = K + I$$

Summary: $J + L = K^{-1}$, $J^{-1} + L^{-1} = K$

$$\begin{matrix}1\\0\\2\end{matrix} + \begin{matrix}0\\2\\1\end{matrix} = \begin{matrix}0\\1\\2\end{matrix} + \begin{matrix}1\\1\\1\end{matrix} \qquad \begin{matrix}1\\2\\0\end{matrix} + \begin{matrix}2\\0\\1\end{matrix} = \begin{matrix}2\\1\\0\end{matrix} + \begin{matrix}1\\1\\1\end{matrix}$$

$$K + L = J^{-1} + I \qquad K^{-1} + L^{-1} = J + I$$

Summary: $K + L = J^{-1}$, $K^{-1} + L^{-1} = J$

All Possible Types of States[2]

$(2J^{-1},K)$	$(2J^{-1},L)$	$(2K^{-1},J)$	$(2K^{-1},L)$	$(2L^{-1},J)$	$(2L^{-1},K)$
$(2J,K^{-1})$	$(2J,L^{-1})$	$(2K,J^{-1})$	$(2K,L^{-1})$	$(2L,J^{-1})$	$(2L,K^{-1})$

[1] The numbers represent the components of the attack vectors.

[2] If the two components are equally represented, for example $(2J^{-1},2L)$, row one represents all possible types of states.

distributions of points in terms of two attack vectors. Since it is not possible to add more than two vectors together without two of them being of the same letter and therefore combinable, or of the same power and therefore reducible, a two component vector will suffice to characterize any distribution of points. Because the attack vectors are combined on the basis of their effect on the disparity of relative strengths, this two component vector defines the minimum number of attack vectors necessary to reduce the distribution of equality. The sum of these two components is a simple index of the DRS for any distribution. This two element vector is called the state vector.

When, a given a distribution of points, it is possible to determine the state vector of this distribution and the constituent attack vectors. The state in which all the players have the same number of points is defined as the (0,0) state vector. With this definition it is now possible to describe how to calculate the state vector of a distribution. If one starts from an all equal state, the state vector of any subsequent distribution is the reduced combination of all the attack vectors used to produce that distribution. A few examples of distributions and their state vectors should clarify the concept. If the first five moves after an initial state of (0,0) were; J, K, K^{-1}, L and K, then the state vector of the resulting distribution would be (0,0). The above result is arrived at by the following sequence of reductions; $J + K = \underline{L}^{-1}$, then $\underline{L}^{-1} + K^{-1} = \underline{J}$, $J + L = \underline{K}^{-1}$, $\underline{K}^{-1} + K = I$. It is through a sequence of reductions such as this that one determines the state of a distribution. Further examples are given in Table 6.

Table 6. Examples of the Determination of the State of a Distribution with an Initial State of (0,0)

	Initial State (0,0)
	Attack Vectors: L, L, K^{-1}, J, J^{-1}
Reductions:	$(0,0) + L = (L,0)$
	$(L,0) + L = (2L,0)$
	$(2L,0) + K^{-1} = (2L,K^{-1})$
	$(2L,K^{-1}) + J = (2K^{-1},L)$
	$(2K^{-1},L) + J^{-1} = (2L,K^{-1})$
	Resulting State $(2L,K^{-1})$
	State Vector (2,1)
	Initial State (0,0)
	Attack Vectors: J, J^{-1}, L, K
Reductions:	$(0,0) + J = (J,0)$
	$(J,0) + J^{-1} = (0,0)$
	$(0,0) + L = (L,0)$
	$(L,0) + K = (J^{-1},0)$
	Resulting State $(J^{-1},0)$
	State Vector (1,0)

If one does not start from an all equal distribution the problem of finding the state of the distribution is complicated by the fact that the state of the initial distribution is not known. Since it is impossible to calculate the state of any subsequent distribution without knowing the state of the initial distribution, it is necessary to have a method by which this initial state may be determined. One such method is to add to the distribution the attack vector which produces the maximum reduction in the disparity of relative strengths.

The state of any initial distribution is the reduced sequence of attack vectors, which when added to the initial distribution produces the nearest all equal distribution from which the initial distribution is reachable. Again an example should clarify the point. If the initial distribution is (10,9,8), the attack vector which reduces the disparity of relative strengths the most is the (0,1,2), or J^{-1} vector. When the J^{-1} vector is added to the (10,9,8) distribution, the resulting distribution is (10,10,10), the nearest all equal distribution from which the (10,9,8) distribution is reachable. The state of the initial distribution is $(J^{-1},0)$. (Further examples of the determination of initial states are presented in Table 7.)

It is possible to find the state of a distribution only if the sum of the three components of the distribution is divisible by three, since only distributions which meet this restriction have at least one all equal point. Only these distributions will be considered in this paper.

All distributions subsequent to the initial distribution are described by the reduced combined sum of the initial state and all the attack vectors produced by the triad. (If the sum were not reduced

Table 7. Examples of Obtaining the Initial State of a Distribution

Initial Distribution of (10,10,7)

(10,10,7)	+	(0,1,2)	=	(10,11,9)
		J^{-1}	=	$(J^{-1},0)$
(10,11,9)	+	(1,0,2)	=	(11,11,11)
$(J^{-1},0)$	+	K	=	(J^{-1},K)

Initial State is (J^{-1},K)

State Vector is (1,1)

Initial Distribution of (13,8,6)

(13,8,6)	+	(0,1,2)	=	(13,9,8)
		J^{-1}	=	$(J^{-1},0)$
(13,9,8)	+	(0,1,2)	=	(13,10,10)
$(J^{-1},0)$	+	J^{-1}	=	$(2J^{-1},0)$
(13,10,10)	+	(0,1,2)	=	(13,11,12)
$(2J^{-1},0)$	+	J^{-1}	=	$(3J^{-1},0)$
(13,11,12)	+	(0,2,1)	=	(13,13,13)
$(3J^{-1},0)$	+	L	=	$(3J^{-1},L)$

Initial State is $(3J^{-1},L)$

State Vector is (3,1)

it would be the decomposed distribution of points discussed earlier.) An example of the determination of a state when the initial state was not (o,o) follows. If the initial state was $(2J^{-1},K)$ and the attack vectors produced by the triad were J, K^{-1} and L the resulting state would be determined by the following reduction sequence; $(2J^{-1},K) + J = (J^{-1},K)$, $(J^{-1},K) + K^{-1} = (J^{-1},0)$, $(J^{-1},0) + L = (J^{-1},L)$.[5] Thus the resulting state would be (J^{-1},L). More examples of the determination of a state when the initial state is not (0,0) are presented in Table 8.

Table 8. Examples of the Determination of the State of a Distribution with an Initial State Other Than (0,0).

Initial States	$(J^{-1},0)$	$(2J^{-1},L)$
Attack Vectors	J, K, L	L^{-1}, J, L
	$(J^{-1},0) + J = (0,0)$	$(2J^{-1},L) + L^{-1} = (2J^{-1},0)$
	$(0,0) + K = K$	$(2J^{-1},0) + J = (J^{-1},0) =$
	$(K,0) + L = (J^{-1},0)$	$(J^{-1},0) + L = (J^{-1},L)$
Resulting State	$(J^{-1},0)$	(J^{-1},L)
State Vector	(1,0)	(1,1)

The resulting reduced sum of attack vectors constitute the state of the distribution. This state will be composed of a single type of attack vector, or some combination of two types of attack vectors. These two types will be any two vectors of different letter and power.

[5] Again the equal sign indicates equality of DRS.

(All possible states are listed at the bottom of Table 5.)

The number of each type of attack vector in the state of the distribution, when listed without the type of vector, is designated the state vector. The state vector, then, is the pair of numerical components of the state of the distribution. By convention the larger of the two components is listed first in the vector. There are only two types of state vectors: a pure state in which only one type of attack vector is needed to characterize the distribution, and a mixed state in which two types are needed.

At this point it is necessary to make an assumption about the rearrangeability of the power positions within the triad. It is assumed that it is of no importance which player is in which power position over the course of the game. If, during the game, a power position switches from one player to another, the states are treated as if the same player was in that position throughout the game. This results in a distribution of points always being listed in a decreasing order, regardless of which player is in each position. The rearrangeability assumption allows for the considerable simplification of the model which follows.

With the stipulation that the distribution of points always be listed in decreasing order of magnitude, all state vectors have J^{-1} as a basis. It is clear why this is so when one looks at the J^{-1} attack vector, (0,1,2), and considers the rearrangeability assumption. When the distributions are arranged in descending order, any pure state will be some number of J^{-1} vectors. For instance a $4K^{-1}$ state representing a distribution of (5,13,9) when rearranged as (13,9,5) is a $4J^{-1}$ state (four times (0,1,2) equals (0,4,8) added to the distribution results in a (13,13,13), the nearest all equal state.

All mixed states also have J^{-1} as a base, again due to the rearrangeability assumption. For example, a (3,1) state vector composed of $(3L^{-1},K)$ could represent a (12,20,15) distribution. Upon rearrangement it becomes a (20,15,13) distribution or a $(3J^{-1},L)$ state. The state was determined in the same manner as all previous states have been, by adding the attack vector which produces the greatest reduction in the disparity of relative strengths until an all equal state is reached. Any distribution can be rearranged so as to have J^{-1} as a basis. As a result of the rearrangeability assumption, only three types of attack vectors are used in the mixed states $(J^{-1},K$ or $L)$ while only one is used in the pure states (J^{-1}).

Probability Equations for Each Attack Vector and Possible Transitions

Each attack vector is produced by a unique combination of attacks, with the exception of the I vector which is produced by two such combinations. From these unique combinations of attacks, equations predicting the probability of any attack vector occurring are derived by constructing a probability tree, with P being assigned to the probability of attacking the stronger from any decision point. The tree is shown in Figure 1. The letter designates which player is making the decision. It is assumed that $x > y > z$.

Because there are only seven attack vectors, each state vector can be transformed to a maximum of seven different states. If (i,j) designates any arbitrary state vector, then the seven possible states after the application of each of the attack vectors listed in Table 4 are: (i,j), $(i+1,j)$, $(i-1,j)$, $(i,j+1)$, $(i,j-1)$, $(i-1,j+1)$, and $(i+1,j-1)$.

Which transition is produced is determined by the types of attack

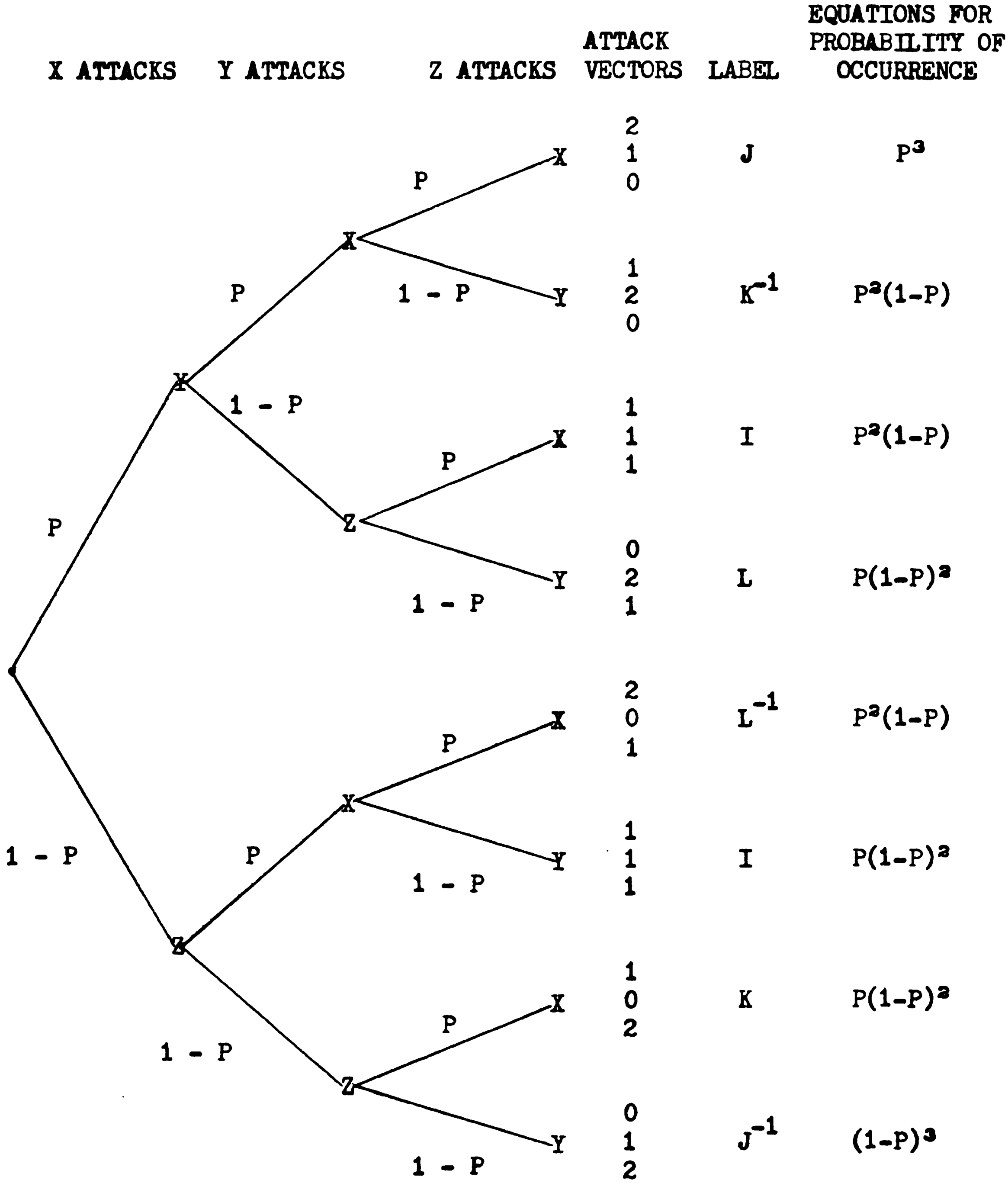

Figure 1. Probability Equations and Labels for Each Attack Vector.

vectors which make up the state, and the attack vector which is applied to it. Different attack vectors have differential effects on different states. For instance a state vector composed of $(2J^{-1},K)$ will go to a (2,0) state vector with the application of a K^{-1} attack vector, but a $(2J^{-1},L)$ will go to a (2,1) with the application of the same K^{-1} attack vector.

If the state has a zero as the second component (pure state) then at most five states are reachable from it, since both transitions resulting in a state with j-1 as the second component are, by definition, nonexistent. No attack vector when applied to such a state, can reduce the second component (0) to a j-1 or -1. A -1 has no meaning, since it implies that a negative of an attack vector is needed to describe a distribution.

The all equal state is the exception to the above transition states. Only two states are reachable from the (0,0) state and they are the (0,0) and (1,0) states. It is quite clear why this is so, since no transition state which has a i-1, or a j-1 as an element is possible. As stated above, no attack vector when applied to a (0,0) state, can produce a -1 in either position. The application of any attack vector, except the I vector, increases the state of the distribution from (0,0) to (1,0). The sum of any attack vector added to 0 is the attack vector. The (1,0) state also has a restricted transition range with the (i-1,j+1) state being underfined, due to the restriction that the largest element be listed first in the vector.

As was mentioned previously, the all equal state has transition probabilities that are independent of the estimate of P. Since all of the players have the same number of points in this state, they are assumed, for attack purposes, to be indistinguishable from one another. Since they are indistinguishable, the probability of attacking either of the attack choices for each player is .50. As was stated above there are only two states reachable from the all equal or (0,0) state: the same (0,0) state or the (1,0) state. The (0,0) state is reachable only through the application of an I attack vector, of which there are two (see Figure 1). The probability of each I vector is .125 (.50 for each of the three decision points in the tree). The total probability of remaining in the (0,0) state is the sum of the two I attack vectors, or .25. The probability of going to the only other reachable state, the (1,0) state, is one minus the probability of remaining in the (0,0) state, or .75.

The equation for any transition from a state is determined by the attack vector or vectors which produce that transition. The attack vectors which produce each transition and the resulting equations are shown in Table 9. With J^{-1} being the basis of all of the states only this single set of equations is needed to predict all possible transitions. Although the K, L, K^{-1}, L^{-1} attack vectors produce different transitions on mixed states depending on which attack vector (K or L) is the second component of the state vector, the resulting equations are the same due to the fact that the K and L vectors have the same equations, as do the K^{-1} and L^{-1} vectors.

This transition table offers a general framework within which data can be analyzed. All of the types of states encountered in a set of data would be listed in the first column of the table and the frequency of each transition from these states would be indicated in each cell. The cells could then be compared with respect to the

Table 9. **Transitions Produced by Each Attack Vector from Each Type of State With the Corresponding Equations**

State	Type		(i,j)	(i-1,j)	(i+1,j)	(i+1,j-1)	(i,j-1)	(i,j+1)	(i-1,j+1)
(0,0)	Pure	Vectors:	I	---	$J^{-1}+J+K^{-1}+K+L^{-1}+L$	---	---	---	---
		Equations:	.25	---	.75	---	---	---	---
(1,0)	Pure	Vectors	$K^{-1}+L^{-1}+I$	J	J^{-1}	---	---	$K+L$	---
		Equations:	$-2P^3+P^2+P$	P^3	$(1-P)^3$	---	---	$2P(1-P)^2$	---
$(i,0)^1$	Pure	Vectors:	I	J	J^{-1}	---	---	$K+L$	$K^{-1}+L^{-1}$
		Equations:	$P(1-P)$	P^3	$(1-P)^3$	---	---	$2P(1-P)^2$	$2P^2(1-P)$
$(i,i)^1$	K	Vectors:	I	---	$J^{-1}+K$	$L+L^{-1}$	$K^{-1}+J$	---	---
		Equations:	$P(1-P)$	---	$(1-P)^2$	$P(1-P)$	P^2	---	---
	L	Vectors:	I	---	$J^{-1}+L$	$K+K^{-1}$	$L^{-1}+J$	---	---
$(i,j)^2$	K	Vectors:	$I+L^{-1}$	J	J^{-1}	L	K^{-1}	K	---
		Equations:	$P(1-P^2)$	P^3	$(1-P)^3$	$P(1-P)^2$	$P^2(1-P)$	$P(1-P)^2$	---
	L	Vectors:	$I+K^{-1}$	J	J^{-1}	K	L^{-1}	L	---
$(i,j)^3$	K	Vectors:	I	J	J^{-1}	L	K^{-1}	K	L^{-1}
		Equations:	$P(1-P)$	P^3	$(1-P)^3$	$P(1-P)^2$	$P^2(1-P)$	$P(1-P)^2$	$P^2(1-P)$
	L	Vectors:	I	J	J^{-1}	K	L^{-1}	L	K^{-1}

[1] $i > 1$ if pure states, $i > 0$ if mixed K or mixed L
[2] $i - j = 1$
[3] $i - j > 1$

observed and expected frequencies using a chi-square goodness of fit test. The expected values are calculated from the predicted probability of the transition (calculated using the estimate of P from the data and the transition equations presented in Table 9) and the marginal frequency for each state.

The Model Axiomatized

Now that the model has been described, it is axiomatized to present its logical organization.[6]

Definition 1. The three numbers representing the number of points each player has is called the distribution of points. This distribution of points is the three component vector D.

Definition 1a. If the distribution of points is arranged so that the three components are in a decreasing order of magnitude, the vector will be called the ordered distribution of points D^*.

Definition 1b. The differences which exist between the three components in the vector D^* are called the disparity of relative strengths and is represented by a two component vector (i,j) where i is the difference between the first two components, and j is the difference between the last two.

Definition 2. The attacks on any single move are represented by a three component vector called the attack vector, V, with each component being the number of times that a particular player was attacked on that move. The complete list of attack vectors is given in Table 4.

Lemma 1. Any distribution of points with a sum that is divisible by three can be represented by the sum of a sequence of attack vectors.

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[6]A simpler axiomization of this model was formulated by Phillips, Klien, and Hartman (1970).

Definition 3. The sequence of attack vectors that constitute a distribution of points is called the <u>decomposed distribution of points</u>.

Definition 4. If all I attack vectors are removed from the decomposed distribution of points, the residual attack vectors constitute the <u>deleted decomposed distribution of points</u>.

Theorem 1. The I attack vector is the only vector which maintains the disparity of relative strengths. Given any distribution of points D^*, with components (j,k,L), and a disparity of relative strengths vector of the form (m,n) where $j - k = m$, and $k - l = n$ then the application of any arbitrary attack vector with the components (x,y,z) results in the distribution (j-x, k-y, L-z) and the DRS vector would be formed by $(j-x) - (k-y) = j - x - k + y$ and $(k-y) - (L - z) = k - y - L + z$. For the DRS of the distribution to be maintained the following must be true.

1 $j - x - k + y = m = j - k$

2 $j - k - x + y = j - k$

3 $- x + y = 0$

4 $x = y$

5 and; $k - y - L + z = n = k - L$

6 $k - L - y + z = k - L$

7 $- y + z = 0$

8 $y = z$

From 4 and 8, $x = y = z$ is the only way the DRS is maintained, the only attack vector for which this is true is the I attack vector with (1,1,1), all other attack vectors have as components a 1, a 0, and a 2, definitely not equal.

Lemma 2. For a given distribution of points, D^*, the corresponding decomposed distribution of points and the corresponding deleted decomposed distribution of points are said to be equivalent with respect to the disparity of relative strengths (DRS) of that distribution.

This follows from the definitions of the I attack vector and the disparity of relative strengths, definitions 1b and theorem 1.

Definition 5. If two vectors are equivalent with respect to DRS, that is, the DRS vectors for the two vectors have the same values in the corresponding positions, the equivalence will be denoted $V_i = V_j$.

Lemma 3. The identity rule:

$I + V = V$ where V is any attack vector. This result follows directly from Theorem 1 and Definition 5.

Lemma 4. The complementation rule:

$V + V^{-1} = I$ where V is any attack vector. This result follows from the definitions of attack vectors in Table 1.

Lemma 5. The combination rule:

$nV + V = (n+1)V$ where n is any positive integer and V is any attack vector. This result follows from definition 5.

Lemma 6. The reduction rules:

$J + K = L^{-1}$	$J^{-1} + K^{-1} = L$
$J + L = K^{-1}$	$J^{-1} + L^{-1} = K$
$K + L = J^{-1}$	$K^{-1} + L^{-1} = J$.

These rules follow from the definitions of attack vectors in Table 1, from the laws of addition for vectors, and from Lemma 3.

Theorem 2. A deleted decomposed distribution of points has, at most two distinct non-zero attack vector components.

Proof:

Let T and R be two distinct attack vectors such that $T \neq R^{-1}$, and such that they are not reducible under Lemma 6. Consider a deleted decomposed distribution of points, D, such that $D = nT + mR$.

Let us add an attack vector to D that is distinct from T, that is not T^{-1}, and that is not reducible in combination with T under Lemma 5. We call this vector S.

If T is a member of the set J, K, L , then S must be a member of the set J^{-1}, K^{-1}, L^{-1} in order to meet the above conditions.

However, R must also be a member of the set (J^{-1}, K^{-1}, L^{-1}) in order to meet the conditions that have been placed on R.

Therefore, R and S must be reducible under Lemma 6. Similarly if T is a member of (J^{-1}, K^{-1}, L^{-1}), then both R and S must be members of (J, K, L) and must be reducible under Lemma 6. Thus the theorem is proved.

Definition 6. The positive integers associated with the two distinct vectors in a deleted decomposed distribution of points constitute the components of a two component vector which is called the state vector S. If all players have the same number of points, $S = (0,0)$. A pure state has a state vector in which at most one component is non-zero. A mixed state has a state vector in which both components are non-zero.

Definition 7. In each state the largest component is always listed first.

Definition 8. Those two attack vectors which are in the deleted decomposed distribution of points are those attack vectors which when added to the vector D^*, produce the nearest all-equal state. In other words these attack vectors reduce the disparity of relative strengths to 0 in the fewest number of steps.

Theorem 3. Each deleted decomposed distribution of points has J^{-1} as its largest component.

This follows from the definition of the J^{-1} attack vector, definitions 1z, 1b, and 8, and from theorem 2. Since the distribution of points D^* has its components in a decreasing order of magnitude, the attack vector which reduced the DRS to 0 in the fewest number of steps is the J^{-1} or (0,1,2) attack vector.

Theorem 4. The second component of the deleted decomposed distribution is a 0, a K or an L. It follows that if it is a pure state the second component is a 0. If it is not 0, then , from Lemma 6 and Theorems 2 and 3, it must be in the set (K,L).

Theorem 6. Each state can be transformed to a maximum of seven different states. Given an arbitrary state (i,j) the reachable states are (i,j), (i-1,j), (i+1,j), (i+1,j-1), (i,j-1), (i,j+1), and (i-1,j+1).

This follows from the fact that there are seven attack vectors.

Axiom 1. On any given move, the probability of a player attacking either of his attack choices is independent of previous moves.

Axiom 2. Two players with the same number of points are indiscriminable to the third player, and thus each will be attacked with probability .50.

Definition 9. Let S be a strategy that identifies, for each triad member, his more preferred attack choice (MPAC). The sole basis for the choice of MPAC is the number of points associated with each player so this identification holds only if both players are distinguishable.

Axiom 3. Each player attacks his MPAC independently of the other players' attacks and with probability P. Also each player attacks his less preferred attack choice (LPAC) independently of the other players' attacks and with a probability of 1-P.

Axiom 3a. P is greater than 1-p. (P is greater than .50.)

Lemma 7. P is invariant over power positions.

This follows directly from Axiom 3.

Lemma 8. P is invariant over games.

This follows directly from Axiom 1.

Axiom 4. P is invariant over all distributions of points.

Lemma 9. In the all-equal distribution the players are indistinguishable from one another, and the probability of each player attacking either of his choices is .50.

This follows directly from Axiom 2.

Theorem 7. For any strategy S, each player attacks his MPAC independently and with probability P on each move of the game.

This follows directly from definition 9 and axioms 1 and 3.

Theorem 8. The theorem on rearrangeability:

Any distribution of points, D, can be rearranged so as to obtain an ordered distribution of points D^*, after each move, without affecting any player's MPAC, or his probability of attacking the MPAC.

By Axiom 1, each move is equivalent to an initial move, and by definition 9 the choice of MPAC depends only on the distribution of points. Hence the MPAC will not be changed by rearranging the distribution of points. From theorem 3, it is apparent that if the MPAC is not changed, the probability of attacking him will not be changed and the theorem is proved.

Axiom 5. The probability of the occurrence of each attack vector is the joint probability of the occurrence of the attacks represented by the three components of the vector.

Definition 10. P(i) is the probability of attack vector i, where the three components of the vector are (j,k,l).

Lemma 10. P(i) = PR(X) PR(Y) PR(Z)

Where PR(X) = P if player X attacks him MPAC
1-P if player X attacks his LPAC

PR(Y) = P if player Y attacks his MPAC
1-P if player Y attacks his LPAC

PR(Z) = P if player Z attacks his MPAC
1-P if player Z attacks his LPAC

This follows from definitions 10 and 11, axiom 5, and theorems 3 and 4.

Axiom 6. The probability of any transition from a state is the sum of the probabilities of the attack vectors which produce that transition.

Previous Test of the Model

Hartman and Phillips (1969) applied this model to a limited set of data that consisted only of transitions from states that had a zero as the second component (pure states). The results of the test were inconclusive. The model fit the data only if the data points from the first move of every game were excluded. It did not fit the first move data alone, nor all of the data with the first move data included.

Hartman and Phillips (1969) proposed that the bad fit of the first move data was due to the procedure used in the experiment. The crucial point of the procedure was that the subjects were in a face to face situation that allowed them to know which of the other players had won the previous game. This generated the possibility that the subjects were responding on the first move to who had won the previous game, rather than on the distribution of points for that game. It was to eliminate the interference of previous games and to produce a larger variety and number of data points that the present experiment was designed.

Method

Subjects. Forty-three groups, each composed of three male undergraduates were used in the experiment. The subjects were obtained through a subject pool maintained by the Cooperation/Conflict Research Group at Michigan State University. The subjects had been originally recruited through a newspaper advertisement offering to pay subjects for participating in motivational research. The pool had been collected to provide a group of subjects who were highly motivated to participate in competitive game experiments. Since uelative conflict assumes a desire on the part of the participants to achieve their goal (winning in the truel), these subjects appeared better suited for the experiment than the usual subjects obtained through introductory psychology cources.

The subjects were called one week prior to the beginning of the experiment and asked to participate. If they consented to participate, a time convenient for all parties (the three subjects and the experimenter) was arranged.

When the subjects were called they were told that the experiment was a three person game in which they could win up to three dollars for the one hour. Only if they asked, were they told that the minimum was 75¢. Due to the importance of having everyone appear at the agreed upon time, the importance of fulfilling the obligation was stressed.

Setting and materials: The experiment was conducted in a small room with a rectangular table in the center. On top of the table was a wooden partition which divided the table into four sections.

Figure 2 is a diagram of the partition and table.

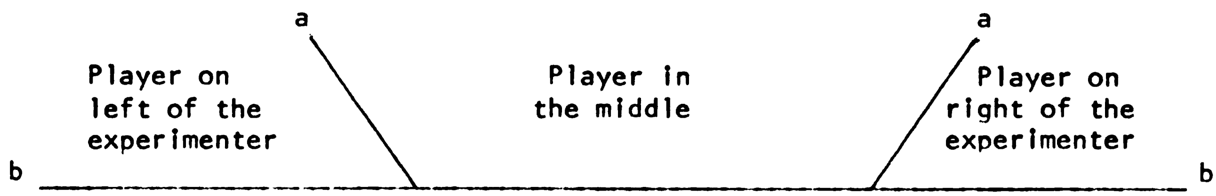

Figure 2. The Diagram of the Table Partitions: Top View.

The partition was constructed so that subjects were able to see the experimenter but not each other. The panels between the subjects (labeled "a" in the diagram) were 30 inches high and 24 inches wide. The panel between the subjects and the experimenter (labeled "b" in the diagram) was 10 inches high but varied in width for the three different positions. The center position had a 30 inch opening while the two end positions had 15 inches each.

To standardize the experimental situation, it was necessary to give each of the three subjects a label, with the entire set of three labels remaining constant over all groups. In previous gaming research the labels A-B-C and VAF-ZEJ-YOV had been used for this purpose. To find the least reactive label set, a pilot study was performed (Hartman, 1970). The most important result was that the label set ARGON-BORON-KRYPTON appeared to have essentially no response biases for subjects. It was this set that was used to represent the three players in all of the games in the experiment.

To allow each player to indicate which other player he wished to attack on each move of the game, three cards were placed in every cubical with one label of the set appearing on each card. Each player also had a wooden card holder on top of the panel separating him from the experimenter into which the experimenter inserted a card with the label of that player. This card was in full view of the subject and the experimenter but out of sight of the other two subjects. This allowed the experimenter to know the complete distribution of labels, and each player to know only his own label.

An abacus like arrangement was used as a scoreboard. It was mounted above and behind the experimenter in full view of all of the subjects. The labels were listed in a vertical line on the left side of the scoreboard with the points for each label listed to the right of it. The points were represented by circular discs mounted on a horizontal rod. The points taken away from each player were placed on the right side of the scoreboard and covered by a wooden shield. Thus, only the points still possessed by a player were visible, with all others concealed behind the shield. Figure 3 is a diagram of the shield and the scoreboard.

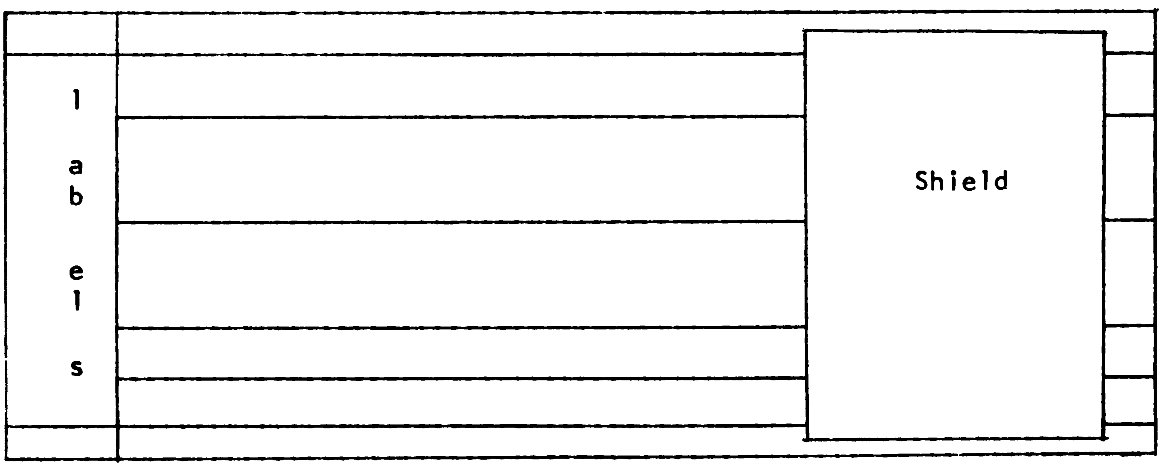

Figure 3. The Diagram of the Scoreboard.

Procedure: In order to produce a large number of data points, eleven different initial states were used, ten of which had a non-zero second component (mixed states). All of the distributions and their corresponding state vectors and orders of presentation are listed in Table 10. Because each state vector from (5,0) to (5,5) represented different disparities of relative strengths, ranging from 5 to 10, two orders of presentation were constructed for the experiment. One order increased the DRS over the six games played, i.e., the triad began with the (5,0) state and ended with the (5,5). The other order presented the games in a decreasing order of DRS, beginning with the (5,5) state and ending with the (5,0)

Table 10. The Eleven Initial States and the Distribution for the Two Orders of Presentation.

Game Number	Order 1	1	2	3	4	5	6
	Order 2	6	5	4	3	2	1
State Vector		(5,0)	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)
Distributions	Type K	14	15	16	17	18	19
		9	11	13	15	17	19
		4	4	4	4	4	4
	Type L	14	16	18	20	22	24
		9	9	9	9	9	9
		4	5	6	7	8	9

Each group of subjects received one of the four conditions appearing in Table 10, with ten groups in each condition. The four conditions were created by two types of initial states and two orders of presentation.

These four conditions were numbered from one to four. In condition one, type K states were presented in order one. In condition two, type K states were presented in order two. In the third condition, type L states were presented in order one, and in the fourth condition, type L states were presented in order two. The conditions were presented in a constant 1, 2, 3, 4, order over the first 40 groups, i.e., the first group received condition one, the second condition two, the third condition three and the fourth condition four, with this sequence repeated for the remainder of the 40 groups[1].

Since the labels were listed in a vertical line on the scoreboard there was a possibility of a response bias due to the label position on the scoreboard. To eliminate this possibility the labels were listed on the scoreboard in the three different sequences presented in Table 11. For the ten groups in each condition the first label sequence was presented four times, while the second and third sequences were presented three times each. This procedure counterbalanced any effect due to label position on the scoreboard.

Table 11. The Three Sequences of Labels on the Scoreboard Used for the Ten Groups in Each Condition.

SEQUENCE 1	SEQUENCE 2	SEQUENCE 3
ARGON	BORON	KRYPTON
BORON	KRYPTON	ARGON
KRYPTON	ARGON	BORON

[1]Three groups had to replace thus the total of 43 groups mentioned previously. More will be said about the replacement of groups in the section with the presentation of the results.

To eliminate possible individual biases of the subjects for a particular label, each player was represented by each of the labels once in each half of the experiment (once in the first three games and once in the last three). Within each group of subjects each label represented each power position (the most points twice, the fewest twice, and the middle number twice) once in the first half and once in the second half of the six games played.

At the beginning of the experiment, the subjects were told that they would play several games (an unspecified number), and that one of these games would be chosen at random to determine payment. The winner of that game would receive the \$3.00, the other two players each would receive 75¢. Further they were told that if the game chosen had no winner (a tie) then all three would receive 75¢.[2]

A cylindrical urn, four inches high and two and one half inches in diameter was used to obtain the random draw. The six games played were represented by the numbers one through six pasted on six poker chips. The subjects were told that the six chips were placed in the urn, but the experimenter, out of sight of the subjects, put only those chips into the urn which represented a game with a winner.

To give each player an equal opportunity to win the three dollars every player was assigned each power position twice. These assignments were distributed such that the sum of the disparity of relative strengths for the two assignments of each of the power positions for

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[2]Although they were told ties resulted in all three receiving 75¢, in fact it was not true. Since many of the subjects knew other people in the subject pool, it was felt that one of the subjects must receive the \$3.00 in order to maintain the credibility of the reward, and to make recruitment of subsequent subjects easier.

each of the players was 15.[3] As an example, a player would be in the strongest power position in the game with an initial state of (5,0) (DRS of 5) and in the game with an initial state of (5,5) (DRS of 10), thus a total of 15 for the two games in which he was in the strongest power position. This means that the sum of the disparity of relative strengths for the two presentations of each power position was equal for all subjects.

After the subjects were seated at the table, the experimenter read them the instructions (presented in Appendix), and all questions regarding the playing of the game were then answered. Each game was begun by designating the distribution of points for that game and placing these points on the left side of the scoreboard. The players were then given their labels for that game and the subjects indicated the player they wished to attack on the first move of the game. On each move of the game all players indicated their choice by holding up the card with the label of the chosen player. The card was held so it was below the top of the panel separating the subjects but above the panel separating the subjects from the experimenter. This procedure allowed for simultaneous, concealed attacks. The game continued until one player was eliminated (ran out of points); the player with the most points at this time was the winner, but if no player had a plurality of points the game was a tie.

When each group finished the six games, the winner of the three dollars was determined by the experimenter shaking the urn and drawing

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[3]The value of the DRS is taken as the simple index which is calculated by adding the two components in the state vector.

out one of the chips. The number on the chip represented a game, the winner of which received the three dollars. The subjects were then questioned as to their knowledge of the labels of the other players during the games and also asked to verbalize their strategies in playing the game. Finally the subjects were told the purpose of the experiment and thoroughly debriefed.

Results

Of the forty-three groups recruited for the experiment only 40 were used in the data analysis. Group 13 was discarded because one of the subjects had participated in a pilot study performed nine months previously. This subject won the first three games played and three out of the four games which had a winner. The other subjects felt he had an advantage and therefore the $4.50 was divided equally between the players. This group was replaced by group 41. The 22nd and 36th groups had to be discarded due to an error made by the experimenter in (at least) one of the games. (The second game in group 22 was started with the wrong distribution of points and group 36 was presented with the wrong sequence of labels.) Group 22 was replaced by group 42 and group 36 by group 43.

Although the major thrust of this section is to present the results of the test of the fit of the model to the data, a large portion of the section is devoted to an extensive examination and analysis of the estimated probability of attacking the stronger of each player's two attack choices. Estimates of P were obtained for the three power positions in every game played, resulting in 720 estimates (three power positions, six games per group and 40 groups). The estimates were analyzed from two different points of view. The first approach was to determine the effect of the initial state, order of presentation, type of state, and power position on the estimated value of P. The second analysis focused on the possible effects of game number, order of presentation, type of state, and power position on the estimate of P. Because all distributions have J^{-1} as a base, the different types of states are referred to as pure, K, or L type states.

Results of the Analysis on the Estimates of P[1]

Before the results of these analyses are presented, it is necessary to explain some procedural difficulties in the determination of an estimate of P for each game and power position. Although each game began as a definite state, it did not remain in that state throughout the entire game. This would not be a problem if the type of state remained the same throughout the game, for instance if the game began as a mixed K and remained a mixed K. However, several times throughout the experiment the type of state switched from one type to another (either K to L, or L to K). To solve this problem of nonhomogenous types of states within each group, the estimates of P were analyzed according to the type of state from which the attacks originated rather than according to the type of state the group began with. In other words, attacks made while a group was in a K type state would be analyzed within the K type factor, even if the initial state had been a type L state.

The first group of estimates, classified as a function of initial state, power position, type of state, and order of presentation (increasing or decreasing) is presented in Table 12. These are the combined estimates from all ten groups in each condition. The estimates

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[1]It was necessary to separate the analysis of initial state from the analysis of game number because the initial states were confounded with game number. This meant that each game number was one of two initial states and each initial state appeared in two different game numbers. For example, the (5,0) initial state appeared in the first and last game numbers and no others, while the first and last games represented only initial states of (5,0) and (5,5). This confounding made it impossible to analyze for either effect directly.

Table 12. Estimates of P for Each Initial State, Game Number, Power Position, Type of State, and Order of Presentation

Initial State	Game Number Order 1 (Inc)	Power Position						Game Number Order 2 (Dec)
		1		2		3		
Type K		Inc	Dec	Inc	Dec	Inc	Dec	
(5,0)	1	.39	.54	.90	.92	.81	.91	6
(5,1)	2	.61	.71	.91	.94	.87	.86	5
(5,2)	3	.90	.76	.71	.87	.89	.84	4
(5,3)	4	.66	.79	.82	.96	.80	.83	3
(5,4)	5	.76	.70	.92	.89	.82	.74	2
(5,5)	6	.69	.75	.81	.73	.85	.70	1
Type L								
(5,0)	1	.21	.53	.76	1.00	.62	.63	6
(5,1)	2	.33	.61	.77	.96	.67	.87	5
(5,2)	3	.19	.66	.94	.93	.66	.95	4
(5,3)	4	.45	.56	.99	.91	.79	.90	3
(5,4)	5	.31	.37	.90	.92	.79	.89	2
(5,5)	6	.48	.22	.96	.81	.90	.68	1
Column Numbers		1	2	3	4	5	6	

indicate that there are few differences between orders, or consistent trends over initial states. The exceptions to this are the first and third power positions for the type L states, where the decreasing order had consistently higher estimates than the increasing order.

The estimates for both orders were then combined, resulting in a single estimate for each initial state, type of state, and power position. The estimates are presented in numerical form in Table 13, and, to make for easier comparisons, in graphic form in Figures 4 to 8. The first two figures compare the three power positions for each type of state (K or L) as a function of initial state. For state type L (Figure 5) there was a consistent difference between the three power positions, with the second position having the highest estimate, the third position the second highest, and the first position the lowest estimate of P. The same pattern held for the type K states (Figure 4) with the exception of the (5,2) state where the ordering of power positions was changed to a 3 > 1 > 2. The differences between the power positions for the type K states were not as large as for the type L states, but they were consistently in the same direction.

Table 13. The Combined Estimates of P Over Orders of Presentation for Each Power Position of Each Initial State and Type of State

Type of State	K			L		
	Power Position			Power Position		
Initial State	1	2	3	1	2	3
(5,0)	.47	.91	.87	.38	.89	.63
(5,1)	.66	.92	.87	.49	.88	.79
(5,2)	.82	.79	.87	.45	.94	.83
(5,3)	.74	.85	.82	.50	.95	.85
(5,4)	.73	.91	.78	.34	.91	.81
(5,5)	.72	.77	.77	.37	.90	.80

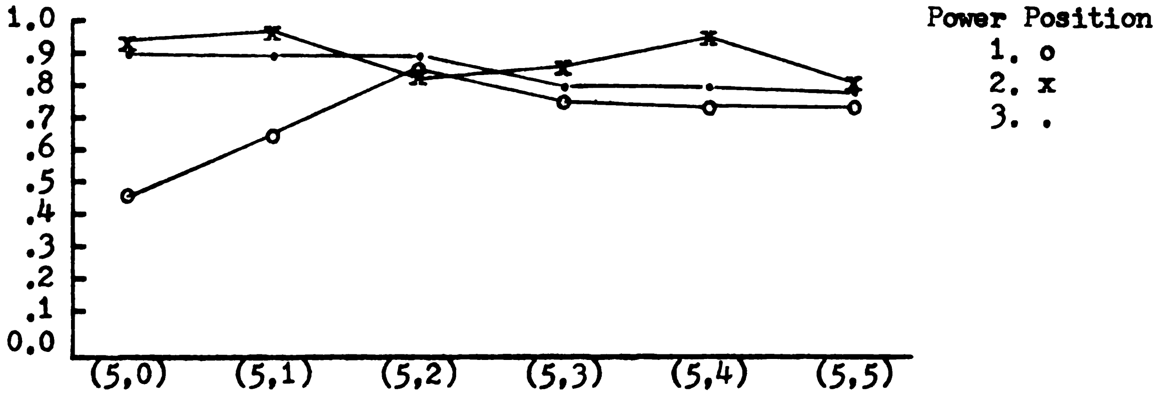

Figure 4. The Estimates of P for the Three Power Positions for Type K States as a Function of Initial State.

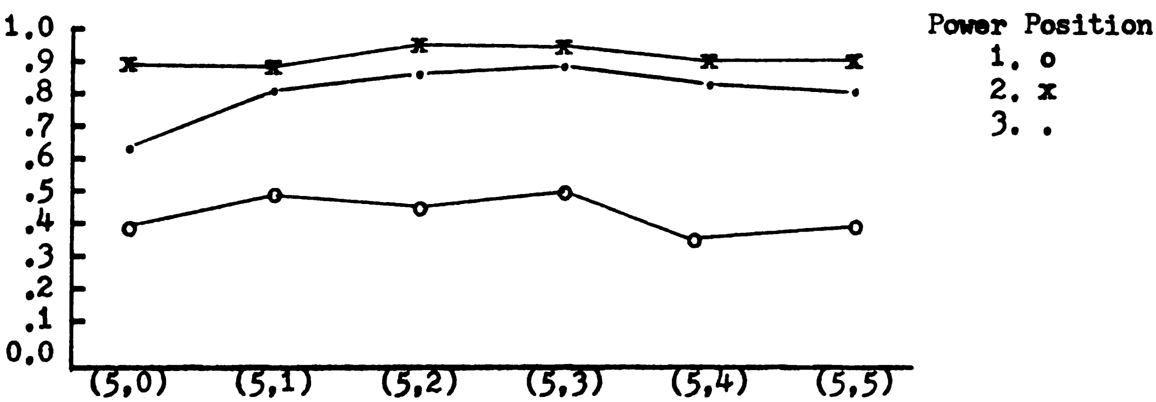

Figure 5. The Estimates of P for the Three Power Positions for Type L States as a Function of Initial State.

The last three figures of this set (Figures 6 to 8) compare the estimates for each power position of type K states to each power position of type L states. The differences for power positions two and three were quite small, with neither type having estimates consistently higher than the other. The first power position for type K, however, had consistently higher estimates than the corresponding power position for type L states. In none of the comparisons in this set were there consistent differences over initial states.

To determine if the differences displayed in the figures were significant, an analysis of variance was performed on the 720 estimates of P (three for each of the six games played by the 40 groups). The data were analyzed for type of state, order of presentation, power position, and initial state. This resulted in a 2 x 2 x 3 x 6 design with repeated measures on the last factor and ten observations per cell. A summary of the results of this analysis are shown in Table 14. The table indicates there was a significant main effect for power position, as Figures 4 and 5 indicated. Significant main effects were also identified for type of state, order of presentation, and initial state (labeled DRS because each initial state had a different DRS, ranging from 5 for the (5,0) initial state to 10 for the (5,5) initial state). Significant effects were also identified for the first order interaction of power position with state type, and the first order interaction of order of presentation with initial state (DRS). All of these findings, with the exception of the significant results for initial state, and the interaction of initial state with order of presentation, were indicated previously in the tables comparing the estimates.

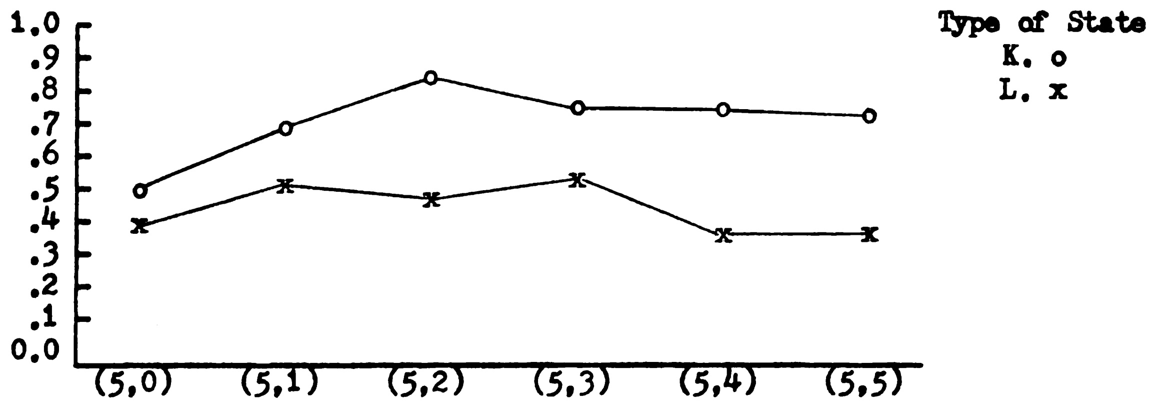

Figure 6. The Estimates of P for the First Power Position for Each Type of State as a Function of Initial State.

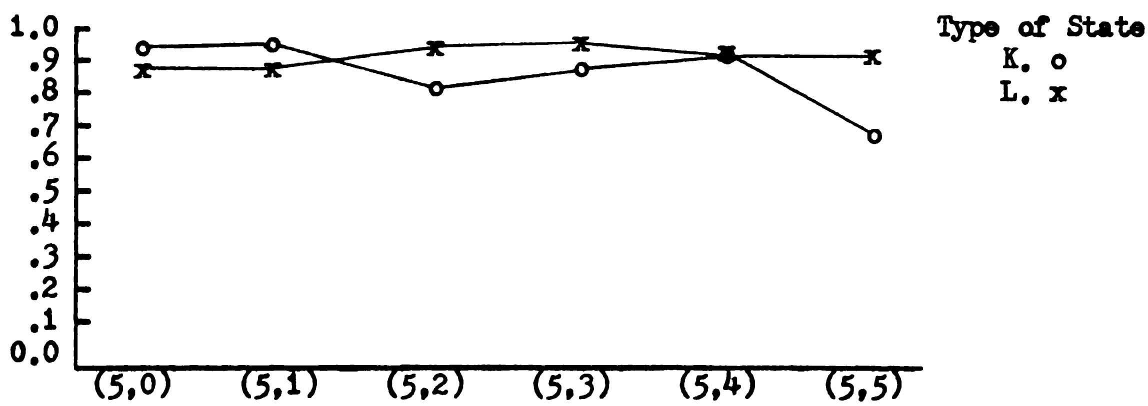

Figure 7. The Estimates of P for the Second Power Position for Each Type of State as a Function of Initial State.

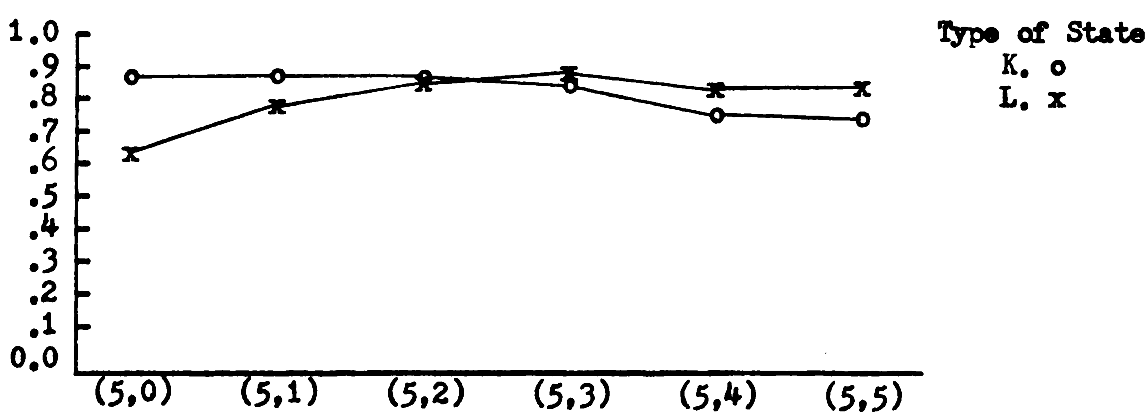

Figure 8. The Estimates of P for the Third Power Position for Each Type of State as a Function of Initial State.

Table 14. Analysis of Variance on the Order of Presentation, Type of State, Power Position, and Initial State (DRS)

Source	df	MS	F
<u>Between Subjects</u>			
Power Position (A)	2	9.431	126.412*
Type of State (B)	1	1.835	24.598*
Order of Presentation (C)	1	0.579	7.764**
A X B	2	1.724	22.113*
A X C	2	0.132	
B X C	1	0.266	
A X B X C	2	0.069	
Subjects within Groups	108	0.075	
<u>Within Subjects</u>			
Initial State (DRS)	5	0.249	5.209*
A X D	10	0.101	
B X D	5	0.054	
C X D	5	0.414	8.649*
A X B X D	10	0.070	
A X C X D	10	0.035	
B X C X D	5	0.089	
A X B X C X D	10	0.049	
D X Subjects within Groups	540	0.048	
Total Error	719	0.091	

* $p < .0005$

** $p < .01$

The estimates of Table 12 were then arranged in terms of game number (first game played, second played, etc.) for each type of state, power position, and order of presentation. The estimates in the first, third, and fifth columns (those labeled increasing) of Table 12 are identical for the two dimensions of game number and initial state (DRS). The estimates in the decreasing columns (the second, fourth, and sixth) are reversed for the two dimensions, with initial state (DRS) increasing going down the table while game number decreases. This was caused by the confounding of initial state with game number. This confounding results in the comparison of the last game in the decreasing order with the first game in the increasing order.

The estimates of the two different orders of presentation for each power position of the type K states presented as a function of initial states were previously presented in Table 12. The two orders of presentation can be compared as a function of game number using the same table by comparing the estimates for the (5,0) state of order one (increasing) with the (5,5) state of the decreasing or number two order of presentation, and then comparing the (5,1) state of order one with the (5,4) state of order two, continuing until the final comparison is made between the (5,5) state of order one and the (5,0) state of order two. The comparisons indicated that there were no consistent differences between orders, nor a consistent trend over game number for any power position.

The same comparison procedure that was used for the K type states was also used for the L type. As opposed to the K type states the L type states indicated consistent differences between the two orders. In power position one the decreasing order had consistently higher estimates of P than did the increasing order. In power position two the decreasing order had higher estimates in four of the six games, and in the third power position the decreasing order had higher estimates in five of the six games played. Again no consistent differences were indicated across game numbers.

The two orders of presentation (increasing and decreasing) were again combined producing 36 estimates of P (one for each power position, game number, for each type of state). These combined estimates are presented in Table 15. Because the graphs of these estimates did not differ from the graphs of the estimates when presented as a function of initial state, they are not presented here (see Figures 4 to 8).

Figures 4 and 5 compared the three power positions for each type of state as a function of initial state, however, they also illustrate the relationship between the power positions for each type of state as a function of game number. The three power position for type K are presented in Figure 4 and the three for type L in Figure 5. Table 15 indicates that in the type K states, the same $2 > 3 > 1$ ordering held for all but the third and sixth games. In the third game the ordering was changed to $3 > 1 > 2$, and in the sixth game to $3 > 2 > 1$. It will be noticed, however, that the reversed estimates in both cases differed by less than .06. In general the $2 > 3 > 1$ ordering of power positions held for the type K states. Across games there was a consistent trend for all estimates to increase as game number increased.

Table 15. The Combined Estimates of P Over Orders of Presentation for Each Power Position of Each Game Number and Type of State

Type of State	K			L		
	Power Position			Power Position		
Game Number	1	2	3	1	2	3
1	.59	.80	.76	.22	.80	.66
2	.66	.90	.80	.36	.87	.82
3	.83	.80	.86	.39	.92	.80
4	.72	.85	.83	.55	.96	.87
5	.74	.93	.84	.44	.92	.79
6	.61	.87	.89	.49	.97	.82

Figure 5 presented the differences between the estimates for the three power positions for state type L as a function of initial state but it also illustrates the differences found between these estimates as a function of game number. These differences are indicated in Table 15. Here again the same 2 > 3 > 1 ordering of power positions was found for all games but with no points of reversal as were found for the type K states. As was found for the type K states, there was a slight trend for the estimates to increase over game numbers for all power positions.

Table 15 presents the estimates of each power position for the type K states and the corresponding estimates for the type L states, as a function of game number. As for the initial state analysis (see Figures 6 to 8), power position one showed the only consistent differences between the two types, with the estimates for the other two power positions varying little from each other. The estimates of the first power position for type K were consistently larger than the corresponding estimates for type L. The differences between the two types ranges from .12 to .44.

To determine the significance of these trends an analysis of variance was performed on the factors of order of presentation, state type, power position, and game number. This analysis resulted in a 2 x 2 x 3 x 6 design with repeated measures on the last factor and ten observations per cell. The results of the analysis are presented in Table 16. Significant main effects were found for all four factors. The factors of power position, state type, and game number were all significant beyond the .0005 level. The main effect for order was significant beyond the .01 level. One first order interaction was indicated with power position interacting with state type at the .005 level of significance.

As was mentioned previously, there seemed to be a consistent increase in the estimates of P over game number, with the largest differences coming between the first three games. To test the possibility that all of the differences were located in the first two games, the same analysis of variance was performed on the last four games separately. The design was thus reduced to a 2 x 2 x 3 x 4, with repeated measures on the last factor and ten observations per cell. A summary of the results of this analysis appears in Table 17. This analysis indicated a significant main effect for power position, state type, and order of presentation. The main effects for power position, and type of state were significant beyond the .0005 level, while the main effect for order was significant beyond the .01 level. A significant first order interaction was also indicated for the interaction of power position with type of state, which had a probability of less than .0005. No main effect nor interaction effects with game number were indicated. This indicates that most of the variance was

Table 16. Analysis of Variance for Order of Presentation, Type of State, Power Position, and Game Number for All Games

Source	df	MS	F
Between Subjects			
Power Position (A)	2	9.431	126.412*
Type of State (B)	1	1.835	24.598*
Order of Presentation (C)	1	0.579	7.764**
A X B	2	1.724	22.113*
A X C	2	0.132	
B X C	1	0.266	
A X B X C	2	0.069	
Subjects within Groups	108	0.075	
Within Subjects			
Game Number (D)	5	0.606	12.662*
A X D	10	0.048	
B X D	5	0.068	
C X D	5	0.057	
A X B X D	10	0.041	
A X C X D	10	0.089	
B X C X D	5	0.075	
A X B X C X D	10	0.078	
D X Subjects within Groups	540	0.048	
Total Error	719	0.091	

* $p < .0005$

** $p < .01$

Table 17. Analysis of Variance for Order of Presentation, Type of State, Power Position, and Game Number for Last Four Games

Source	df	MS	F
Between Subjects			
Power Position (A)	2	5.473	97.009*
Type of State (B)	1	0.821	14.545*
Order of Presentation (C)	1	0.444	7.871**
A X B	2	1.083	19.198*
A X C	2	0.037	
B X C	1	0.111	
A X B X C	2	0.150	
Subjects within Groups	108	0.056	
Within Subjects			
Game Number (D)	3	0.024	
A X D	6	0.048	
B X D	3	0.073	
C X D	3	0.080	
A X B X D	6	0.014	
A X C X D	6	0.071	
B X C X D	3	0.105	
A X B X C X D	6	0.011	
D X Subjects within Groups	324	0.041	
Total Error	479	0.075	

* $p < .0005$

** $p < .01$

due to the first two games played. This interpretation if further emphasized by the relative size of the error terms of the six and four game analysis. The error term for the last four games is almost one half as large as the error term for all of the games. Thus one third of the data produced almost one half of the error variance.

To test this interpretation a multiple comparison test was performed on the estimates of P for the six game numbers. The test was designed by Scheffe' (1955) and described in Edwards' (1960). The results of this test, presented in Table 18, partially support

Table 18. The t' Values of the Multiple Comparisons of the Estimates of P for the Six Game Numbers

Estimate of P	Game Number	1	2	3	4	5	6
.597	1	---	3.79*	5.90**	6.79**	6.10**	5.62**
.704	2	---	---	2.12	2.65	2.33	1.83
.764	3	---	---	---	.88	.21	.28
.789	4	---	---	---	---	.67	1.16
.770	5	---	---	---	---	---	.49
.756	6	---	---	---	---	---	---

Mean Square within Subjects = .0283

* $p < .05$

** $p < .001$

the hypothesis that all of the variance was coming from the first two games. As the table indicates, the difference between the first two games was significant beyond the .05 level while all other differences between the first game and the last four were significant beyond the .001 level. The second game had no significant differences with any of the last four games. These tests indicated that the differences were generated by the first game, with little of the variance coming from the remaining games. Although the tests indicated that the second game was not significantly different from the last four games, its lower estimate of P and its significant difference with the first game at only the .05 level cast doubt on the assumption that it was not played differently from the last four games.

To test the hypothesis that the subjects had no response biases for the labels used in the experiment, the number of attacks made on each label was counted. By counting only attacks in which the attacker had a choice between two players of equal power (points), and therefore indiscriminable except for their labels, a who to whom matrix of attacks was constructed (Table 19). These data were not completely independent since each player could contribute more than one attack for each label. Because only a few subjects contributed more than one attack for each label, the non-independence of these few data points, out of a total of 800, would have a negligible effect on a chi square goodness of fit test. Therefore, the test was performed on the data, with the assumption that the probability of any label attacking either of his choices was .50. The chi square for the entire table was 2.52 with three degrees of freedom. The observed, expected, and chi square values are presented in Tables 19 to 21 respectively. The fact that the chi square was less than the degrees of freedom indicates that no response biases were present in the data set.

Table 19. The Observed Attacks Made by Each Label on Each of Its Choices

	Argon	Boron	Krypton	Sum
Argon	---	155	131	286
Boron	136	---	127	263
Krypton	123	130	---	253

Table 20. The Expected Values for the Number of Attacks on Each Label's Choices, Assuming Random Behavior

	Argon	Boron	Krypton	Sum
Argon	---	143.0	143.0	286
Boron	131.5	---	131.5	263
Krypton	126.5	126.5	---	253

Table 21. The Chi-Squares for Each Label's Two Attack Choices

	Argon	Boron	Krypton	Sum
Argon	---	1.007	1.007	2.014
Boron	.154	---	.154	.308
Krypton	.097	.097	---	.194

Sum for whole table is 2.52 with 3 degrees of freedom

Test of the Model

Although many data points were generated within the experiment, several cells in the following analyses had expected values of less than one and several more had expected values less than five. These low values were caused by very low predicted probabilities for these cells. These cells violated the condition for the chi-square goodness of fit test, that no cell have an expected value less than one and only 20% of the cells be less than five (Hays, 1963). To guard against accepting the model by using spurious data, due to the few cells with values less than one, the data were collapsed over particular types of states. Which states were collapsed together was determined by the number of transitions possible from the state. For each of the following tests, all pure states, excluding the (0,0) and (1,0) states, were collapsed to form one transition state, the mixed states with equal components ((1,1) to (5,5)) were collapsed, those differing by one ((2,1) to (5,4)) were collapsed, and those differing by more than one ((3,1) to (5,3)) were also collapsed. Each of these three types of mixed states, and each of the three types of pure states is characterized by a particular number of transitions which are possible from it. The (0,0) state has two possible transitions, the (1,0) state has four possible, and all other pure states have five possible transitions. The three types of mixed states are characterized by four possible transitions for the equal component state, six possible for the state whose components differ by one, and seven possible for the states having components differing by more than one.

The chi-square goodness of fit test was then applied to these collapsed transition states, none of which had a cell with an expected value less than one. This was done for all of the tests that are presented in this section. The results of the analysis on the collapsed data were not different from the full analysis. Due to the similarity of the results and the fact that the full table provides a more detailed picture of the results, only the results for the complete table and its chi-square are presented. By presenting the entire table rather than the collapsed data, it is possible to determine more precisely the cells in which the model does not fit.

Despite the invalidity of many of the assumptions of the model, the test of the model was performed. The data were analyzed in the general table discussed previously. Because so few data points occurred beyond the (5,5) state only transitions between (0,0) and (5,5) were used in any of the following analyses. The estimate of P was obtained by dividing the frequency of attacks on the stronger by the total number of attacks.

The frequency of all of the transitions in the data set are presented in Table 22, the expected values in Table 23, and the chi-squares in Table 24. The estimate of P for the entire data set was .77. The chi-square for the entire table was 524.26 with 89 degrees of freedom. The degrees of freedom were produced by 111 cells, 21 rows with marginal constraints, and one estimated parameter. Because the model did not fit, the data were broken down in several ways in an attempt to find a set of data that the model did fit.

As the analysis of the estimate of P consistently indicated, there were differences between the three power positions, between the

Table 22. Observed Frequencies for All Transitions from States (0,0) to (5,5) for the Entire Data Set

(i,j)	(i,j)	(i-1,j)	(i+1,j)	(i+1,j-1)	(i,j-1)	(i,j+1)	(i-1,j+1)	Sum
(0,0)	28	---	98	---	---	---	---	126
(1,0)	82	106	0	---	---	5	---	193
(2,0)	2	22	1	---	---	0	12	37
(3,0)	5	13	0	---	---	2	9	29
(4,0)	1	15	0	---	---	1	11	28
(5,0)	16	18	0	---	---	6	59	99
(1,1)	17	---	1	12	95	---	---	125
(2,1)	35	102	1	2	16	2	---	158
(3,1)	6	41	1	0	8	1	15	72
(4,1)	15	31	1	1	10	2	45	105
(5,1)	7	20	2	6	43	5	18	101
(2,2)	18	---	2	12	81	---	---	113
(3,2)	38	91	0	5	19	1	---	154
(4,2)	7	33	0	0	12	0	14	66
(5,2)	18	23	0	2	42	3	12	100
(3,3)	12	---	4	7	54	---	---	77
(4,3)	26	55	2	3	16	2	---	104
(5,3)	16	33	1	3	35	0	12	100
(4,4)	7	---	2	7	34	---	---	50
(5,4)	38	31	2	2	34	3	---	110
(5,5)	14	---	8	5	32	---	---	59

Table 23. **Expected Values for All Transitions from States (0,0) to (5,5) for the Entire Data Set**

(i,j)	(i,j)	(i-1,j)	(i+1,j)	(i+1,j-1)	(i,j-1)	(i,j+1)	(i-1,j+1)
(0,0)	31.50	---	94.50	---	---	---	---
(1,0)	87.59	86.75	2.47	---	---	16.19	---
(2,0)	6.63	16.63	.47	---	---	3.10	10.16
(3,0)	5.20	13.03	.37	---	---	2.43	7.96
(4,0)	5.02	12.59	.36	---	---	2.35	7.69
(5,0)	17.75	44.50	1.27	---	---	8.31	27.19
(1,1)	22.41	---	6.84	22.41	73.34	---	---
(2,1)	50.01	71.01	2.02	6.63	21.69	6.63	---
(3,1)	12.91	32.36	.92	3.02	9.89	3.02	9.89
(4,1)	18.82	47.19	1.35	4.40	14.42	4.40	14.42
(5,1)	18.10	45.40	1.29	4.24	13.87	4.24	13.87
(2,2)	20.25	---	6.19	20.25	66.30	---	---
(3,2)	48.75	69.22	1.97	6.46	21.14	6.46	---
(4,2)	11.83	29.66	.85	2.77	9.06	2.77	9.06
(5,2)	17.92	44.95	1.28	4.19	13.73	4.19	13.73
(3,3)	13.80	---	4.22	13.80	45.18	---	---
(4,3)	32.92	46.74	1.33	4.36	14.28	4.36	---
(5,3)	17.92	44.95	1.28	4.19	13.73	4.19	13.73
(4,4)	8.96	---	2.74	8.96	29.34	---	---
(5,4)	34.82	49.44	1.41	4.61	15.10	4.61	---
(5,5)	10.58	---	3.23	10.58	34.62	---	---

Table 24. The Chi-Squares for All Transitions from States (0,0) to (5,5) for the Entire Data Set

(i,j)	(i,j)	(i-1,j)	(i+1,j)	(i+1,j-1)	(i,j-1)	(i,j+1)	(i-1,j+1)
(0,0)	0.39	---	0.13	---	---	---	---
(1,0)	0.36	4.27	2.47	---	---	7.75	---
(2,0)	4.39	1.73	0.59	---	---	3.11	3.33
(3,0)	0.22	0.00	0.37	---	---	0.08	0.14
(4,0)	4.13	0.46	0.36	---	---	0.77	1.43
(5,0)	1.22	15.78	1.27	---	---	0.65	37.23
(1,1)	3.47	---	4.99	8.02	6.39	---	---
(2,1)	4.50	13.50	0.52	3.24	1.49	3.24	---
(3,1)	3.69	2.31	0.01	3.02	0.36	1.35	2.64
(4,1)	0.77	5.56	0.09	2.64	1.35	1.32	64.85
(5,1)	6.81	14.21	0.39	0.73	61.20	0.14	1.23
(2,2)	0.25	---	3.83	3.36	3.26	---	---
(3,2)	2.37	6.85	1.97	0.33	0.22	4.62	---
(4,2)	1.97	0.37	0.85	2.77	0.95	2.77	2.69
(5,2)	0.00	10.70	1.28	1.15	58.21	0.35	0.22
(3,3)	0.23	---	0.01	3.35	1.72	---	---
(4,3)	1.45	1.46	0.34	0.43	0.21	1.28	---
(5,3)	0.21	3.18	0.61	0.34	32.95	4.20	0.22
(4,4)	0.43	---	0.18	0.43	0.74	---	---
(5,4)	0.29	6.88	0.25	1.48	23.64	0.57	---
(5,5)	1.11	---	0.20	7.03	2.94	---	---

types of states and between the first two games and the last four. These differences directed a series of changes in the model, beginning with a change from one parameter to three parameters, progressing to splitting the data by type of state, and ending with only the data from the last four games. The results of these tests are presented in Table 25. For each disection of the data, the chi-square goodness of fit, and correlation coefficient were calculated. The chi-squares, their degrees of freedom, the correlation coefficient, and the estimates of P used in calculating the predicted probability of each attack vector and thus each transition, are presented in Table 25.

Table 25. The Summary Results of the Four Tests of the Model

Data Set	P for Power Position 1	2	3	Chi-square	df	r	Ratio
All Data	.77	.77	.77	524.26	89	---	5.89
K States	.69	.88	.83	197.67	87	.91	2.27
L States	.42	.91	.81	257.72	87	.86	2.96
K Types without Pure states	.70	.88	.82	158.05	68	.91	2.32
L Types without Pure States	.36	.90	.79	194.47	68	.84	2.86
Pure States	.55	.91	.85	52.75	17	.93	3.10
K Last Games	.72	.89	.86	136.52	68	.90	2.05
L Last Games	.40	.93	.82	189.73	68	.85	2.78
Pure Last Games	.63	.93	.87	31.57	17	.93	1.85

The introduction of a three parameter model produced some problems for the transition equations. The transition table had previously been simplified by the fact that the equations for the crucial K and L vectors were the same as were the equations for the crucial K^{-1} and L^{-1} vectors. As was mentioned earlier, these are crucial vectors because they produce differential transitions depending on the type of state of the triad. Because rearrangeability causes J^{-1} to be the basis of all states, the J, J^{-1}, and I vectors produce the same transitions regardless of the type of state they are applied to. When three parameters are introduced, the equations for the crucial vectors are no longer the same. Because the equations for the vectors are not the same, the equations for the transitions produced by these vectors are not the same. The new equations for the attack vectors are presented in Table 26, and the new equations for the transitions are presented in Table 27.

Table 26. The Equations for the Attack Vectors for the Three Parameter Model

Attack Vectors	Equations
I	PR + Q - QP - QR
J	PQR
J^{-1}	(1-P)(1-Q)(1-R)
K	R(1-P)(1-Q)
K^{-1}	PQ(1-R)
L	P(1-Q)(1-R)
L^{-1}	QR(1-P)

P = probability of strongest player attacking his MPAC
Q = probability of middle player attacking his MPAC
R = probability of weakest player attacking his MPAC

Table 27. The Transition Equations for the Three Parameter Model

State	Type	(i,j)	(i-1,j)	(i+1,j)	(i+1,j-1)	(i,j-1)	(i,j+1)	(i-1,j+1)
(0,0)	Pure	.25	---	.75	---	---	---	---
(1,0)	Pure	Q+RP-2PQR	PQR	(1-P)(1-Q) x (1-R)	---	---	R(1-P)(1-Q)+ P(1-Q)(1-R)	---
(i,0)[1]	Pure	PR+Q-QP-QR	PQR	(1-P)(1-Q) x (1-R)	---	---	R(1-P)(1-Q)+ P(1-Q)(1-R)	PQ(1-R)+ QR(1-P)
(i,i)[1]	K	PR+Q-QP-QR	---	(1-P)(1-Q)x (1-R) + Rx (1-P)(1-Q)	P(1-Q)x (1-R) + (1-P)QR	PQR + (1-R)PQ	---	---
	L	PR+Q-QP-QR	---	(1-P)(1-Q)x (1-R) + P x P(1-R)(1-Q)	R(1-P)x (1-Q) + PQ(1-R)	PQR + (1-P)QR	---	---
(i,j)[2]	K	PR+Q-QP-QR + (1-P)QR	PQR	(1-P)(1-Q) x (1-R)	P(1-Q) x(1-R)	(1-R) x QP	R(1-Q) x(1-P)	---
	L	PR+Q-QP-QR + (1-R)PQ	PQR	(1-P)(1-Q) x (1-R)	R(1-Q) x(1-P)	(1-P) x QR	P(1-Q) x(1-R)	---
(i,j)[3]	K	PR+Q-QP-QR	PQR	(1-P)(1-Q) x (1-R)	P(1-Q) x(1-R)	(1-R)QP	R(1-Q) x(1-P)	(1-P)QR
	L	PR+Q-QP-QR	PQR	(1-P)(1-Q) x (1-R)	R(1-Q) x(1-P)	(1-P)QR	P(1-Q) x(1-R)	(1-R)QP

[1] $i > 1$ if pure state, $i > 0$ if mixed K or L

[2] $i - j = 1$

[3] $i - j > 1$

The estimates of the three parameters for each of the separations are listed in Table 25. These estimates show no consistent trend over the various splits of the data. After the estimates were made for each power position separately in the first disection of the data (rows 4, 5, and 6 of Table 25), the estimates remained fairly constant for each type of state. The only consistent differences between the estimates for the various disections of the data were the higher estimates for all power positions for each state for the last four games than the same estimates but for the entire data set.

The data were first split by separating all transitions from mixed L states from transitions from mixed K states. The pure states were divided on the basis of the type of state the remainder of the initial states were for that group. For example, a transition from a pure state would be placed in the K data group if the initial states of the remaining games were K, or in a L data group if the remaining games were type L's.

As Table 25 shows, the model again failed the test of having a chi-square less than the degrees of freedom, with the type L data differing considerably more from the model than the type K data. The chi-square for the type K was 197.67 and for the type L 257.72, both with 87 degrees of freedom.

The second splitting of the data, rows 4, 5, and 6, was done by type of state (pure, mixed K or mixed L). Again the model did not fit, with a chi-square of 52.75 with 17 degrees of freedom for the pure states, a chi-square of 158.05 for the mixed K and 194.47 for the mixed L, both with 68 degrees of freedom.

The data were finally reduced to only transitions which occurred in the last four games played by each group (rows 7, 8, and 9). The data were left in the three state form and all transitions which occurred in the first two games were removed. The chi-square for the pure states was 31.57 with 17 degrees of freedom, for the mixed K it was 136.52 and for the mixed L 189.73, the later two had 68 degrees of freedom.

The results presented in Table 25 are listed in a decreasing order of generality. As the data were split, and made less general, the chi-squares were reduced, however, as the chi-squares were decreased so were the corresponding degrees of freedom. These nonconstant degrees of freedom made the interpretation of the decreases in the chi-squares difficult to make. To solve this difficulty the ratio of the chi-square to its degrees of freedom was calculated for each test of the model. These ratios are presented in the last column of Table 25. In general these ratios decreased as the data were split and made less general. The only disection of the data that produced no reduction in the ratios was the separation of the pure states from the mixed K and mixed L states (rows 4, 5, and 6). These decreases indicate that the fit of the model is improved with continued separation of the data, but the fit does not improve enough to allow acceptance of the model.

In all of the tests of the model the chi-squares were larger than their degrees of freedom. After the initial splitting of the data the transitions from each state became so meager that any test of the model would be meaningless. For this reason the search for a fit of the model to the data was abandoned.

Discussion

The extensive analysis of the estimates of P revealed that many of the assumptions of the model were invalid. The invariance of P over power positions, state type, initial state, and order of presentation were all called into question.

The main effect for power position was found in all of the analyses performed. The significant effect for power position shown in the analyses of variance and illustrated in Figures 4 and 5 indicated a 2 > 3 > 1 ordering of the estimates of P for the three power positions.

The relatively low estimates for the first power position was due to the fact that each of the two weaker power positions could take away one point, and therefore were equally threatening to the strongest player. The weakest player had a slightly lower estimate of P than did the middle palyer. Although it was to the weakest player's advantage to attack the strongest player, both of the other two players were stronger than he and, therefore threatening to him. The threat of the strongest player, however, was considerably greater than that of the second strongest and therefore the weakest player attacked him more often than he did the second strongest. The second power position had the highest estimate of P in almost all of the games played. This high estimate was caused by the fact that the strongest player was by far the greatest threat to the second strongest player. As the estimates show, the second power position attacked the strongest player in over 90% of the attacks made.

One of the most interesting results of the experiment was the implication that at least the first game and possibly the first two games were played differently from the remaining games. The Scheffe' multiple comparison test showed that the difference between the first game and the last four were significant at the .001 level and the difference between the first and second game was significant at the .05 level. Although the second game did not differ significantly from the last four games, the t' values of the Scheffe' test for those differences were all much larger than the t' values for the differences between any of the last four games. These t' values suggest that the last four games were played differently from the first two.

The error variance that is produced by the first two games is approximately one half as large as the error variance for the entire data set, and the large differences between the first two games and the last four indicate that this effect was very likely due to a learning effect that was concentrated in the first two games. The small t' values for the differences between the last four games indicate that the estimates of P stablized after the second game. This stabalization indicates that any effect due to learning was eliminated after the second game.

The main effect for initial state is difficult to locate. None of the figures indicated any consistent trends over initial state for any of the power positions. Because there was a significant effect for game number, and game number was confounded with initial state, it is possible that the significnat effect for initial state was an artifact of game number. The confounding of initial state with game number caused the (5,0), (5,1), (5,4), and (5,5) initial states to have lower estimates of P than the (5,2) and (5,3) states. It was the former set of initial states that occurred in the first and second game numbers for the two orders of presentation, and for this reason had lower estimates of P than did those states which occurred in the third and fourth games only ((5,2) and (5,3)).

The two orders of presentation were originally inserted into the design to counter any effects due to game number. The assumption had been that the effect of game number would be linear, thus a lower estimate of P for an initial state in an earlier game would be countered by a higher estimate when that initial state appeared in a later game. The fact that the effect of game number was not linear after the second game caused the two orders of presentation to be ineffectual. Because the first two games, and thus the four initial states mentioned previously, had lower estimates of P in one order of presentation and because the estimates of P asymptote at the third game and thus change little after that, the initial low estimate for the four initial states cannot be compensated for by placing them in a later game. The elimination of the significant effect for game number by the removal of the first two games, and the significant differences between the first game and the remaining five, lend support to this interpretation.

The interaction between initial state and order of presentation was significant at the .0005 level. This effect would also seem to be caused by the significant effect of game number. In the increasing order the (5,0) and (5,1) states were in the first two game positions, thus both had low estimates of P, however in the decreasing order of presentation these same states were in the fifth and sixth game numbers and therefore had high estimates of P. The states (5,5) and (5,4) were in the same situation except they appeared in the earlier games in the decreasing order and in the later positions in the increasing order. Thus different orders of presentation produced different estimates of P for different initial states. Some initial states increased their estimates from one order to another, while others decreased their estimates making the same transition, and still others maintained their estimates for both orders of presentation. From the results of the analyses performed it seems that it is this kind of process that caused the significant order by initial state interaction.

In general the order of presentation was significant at the .01 level. No interpretation of this result is possible since there seemed to be no consistent differences between the orders. More substantial evidence is needed before any meaningful explanation can be given.

The significant interaction for type of state with power position is apparently due to the difference between the two first power positions. The estimate of P for the first power position for state type L was much lower than its counterpart for the K type states. The estimate of P for this power position for the type L states is much lower than the estimates of P for either of the other two power positions for either state type. As opposed to the first power positions' estimates there seemed to be little difference between the estimates of P for the two lower power positions (see Figures 4 and 5).

The significant main effect for state type is clouded by its highly significant interaction with power position. The large difference between the first power position of state type K and the same position for state type L could cause the main effect for state type. The extremely low estimate of P (.49) for the first power position of state type L drives down the estimate of P for the entire state type. Since this reduction did not occur in the type K states, the first power position could very well have caused the significant effect.

These analyses of the significant effects for state type illustrate a fundamental difference between the type of play in the two states. In the type K states the estimate of P indicates a predominant tendency for all players to employ the fair play strategy. The type L states, however, differ from the type K states with respect to the action of the first power position. This power position seemed to fluctuate between the fair play or the dyadic competition strategies and the threat minimization strategy, with the latter slightly favored over the former. The reason for this difference between states is obvious when one looks at the power structure of each state type. As pointed out previously, the type L state is characterized by a power structure of one stronger and two weaker players, and the power structure for the type K states is characterized by two stronger players and one weaker player. In type L states the two weaker players are equally threatening to the strongest player and therefore the strongest player predominantly employs the threat minimization strategy. The K type distributions, however, have two stronger players, each of whom is the greatest threat to the other. Neither of the two stronger players can afford to attack the weakest for an indefinite number of moves since the weakest player will attack him in retaliation. This internal constraint against attacking the weakest in the K type states produces different transitions and estimates of P from those produced by the type L distributions.

The power structure (distribution) of the two types of states seemed to produce different types of strategies for the first power position, with the type K states tending to produce the fair play strategy and the type L the threat minimization strategy. The two strategies are indiscriminable for the two lower positions because they result in the same attacks. This indicates that two processes are involved in the truel, one for each type of state. Ignoring these differences between the types of states the simple model was tested on the entire data set.

The test of the model showed that there was no single process occurring in the truel. Therefore, the simple model proposed did not, in any way, capture the interpersonal process within the truel nor did it mirror behavior in pure conflict situations. Contrary to expectation, the participants did not blindly attack the stronger of their attack choices.

Based on the results of the analyses on the estimate of P, the data were segregated in various ways and the model was changed from one having one parameter to one having three. Estimates were made for the three parameters, and transition equations generated for each of the state types. These separations of the data generally resulted in a reduction of the chi-squares, but the reduction was not substantial enough in any of the cases to permit acceptance of the model. These negative results indicated that even within types of states there was no simple, single process operating.

If the players had used the strategies the analyses of the states types indicated, then the fair play model should have at least fit the K type states where this type of strategy was the most prevalent. As the analyses of the model indicated, even the data from the K type of states were not reproducible by the model. In general, the results of the tests of the model indicated that at least one type of process was operating within each of the three types of states defined (pure, mixed K, and mixed L). These processes were not being captured by the fair play model nor its three parameter variate. All attempts to select particular types of states which produced correspondence between the model and the data proved fruitless. Although there was a tendency for higher levels of DRS to produce larger chi-squares, it did not hold consistently enough to produce any change in the fit of the model when those states were excluded from the analyses.

Since neither the proposed model nor its post hoc variation fit the data, an additional examination of the results through a visual representation of the subjects response was performed. A geometric representation of the state component system was previously developed by Phillips, Hartman, and Klein (1970). Because all state vectors can be represented by a pair of numbers it is possible to represent the state component system in a two dimensional coordinate system. Such a representation is presented in Figure 9.

Figure 9 shows some arbitrary state, (i,j), and the six possible transitions from that state. Each of these changes corresponds to one given attack vector. For example the change from (i,j) to (i-1,j) is along the axis labeled J and corresponds to a J attack vector. A transition in the opposite direction, toward a (i+1,j) state, is along the J axis but toward the J^{-1} end. This transition is caused by the J^{-1} attack vector. Similarly movements along the other axes are caused by the corresponding attack vectors. The seventh attack vector, I, results in no transition from any state vector.

All possible state vectors can be represented in this coordinate system. The state (0,0) falls at the origin of this system and any

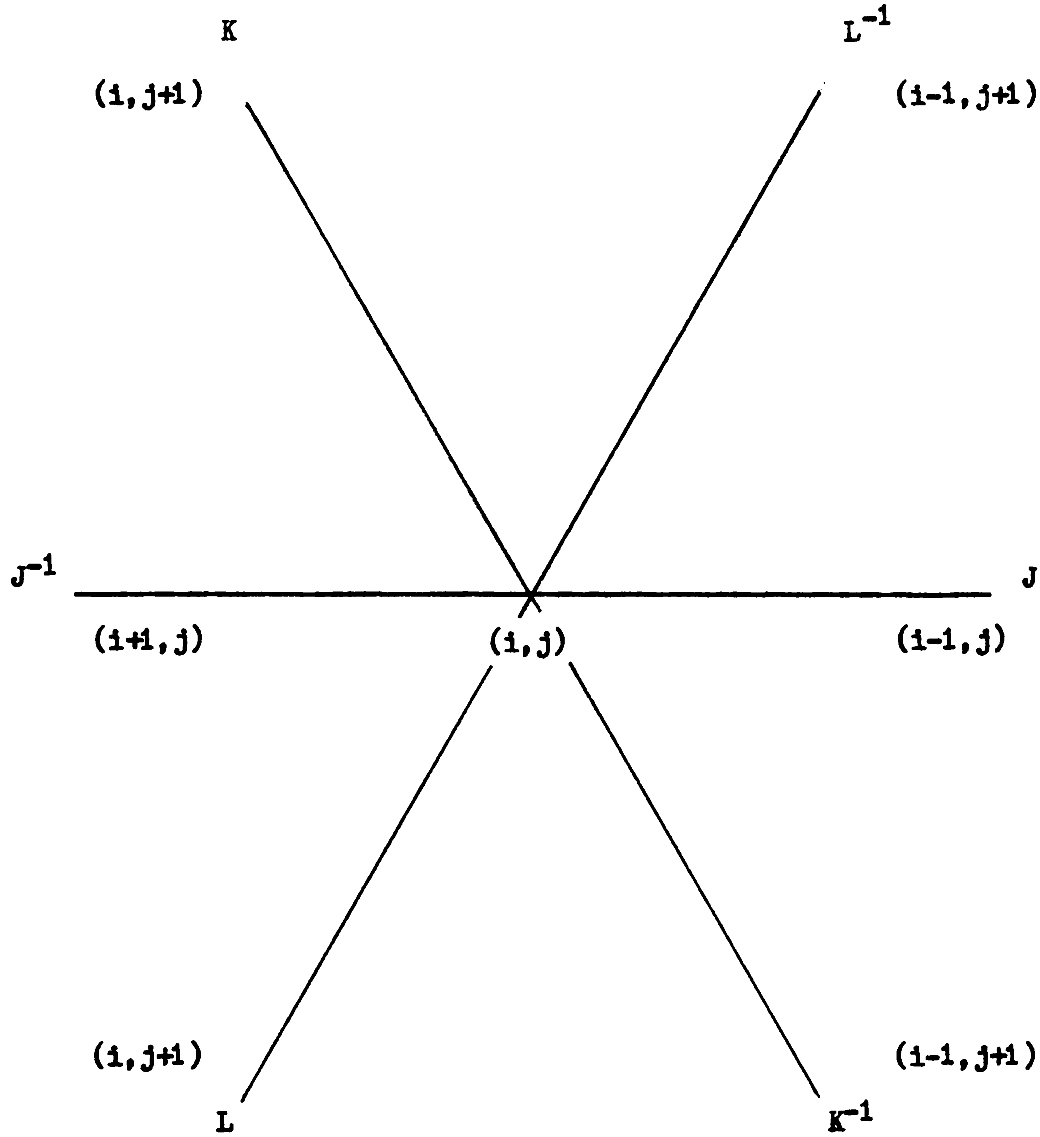

Figure 9. Representation of the State Component System.

attack vector moves the system in one of the six possible directions. Because it has been stipulated that the distribution of points be listed in a decreasing order of magnitude, only a limited area of this coordinate system is needed. With each state having J^{-1} as a base, only a 60 degree wedge of the entire coordinate system is needed to represent the state component system. Thus, all possible states correspond to points within the region bounded by the dashed lines in Figure 10. Those states falling directly on the J axis are pure states, those falling above this axis are mixed K, and those falling below it are mixed L.

Since the boundaries represent states of the form (iJ^{-1}, iK) or (iJ^{-1}, iL) only moves which result in states of the form $(i+1,i)$, $(i,i-1)$, $(i+1,i-1)$, or (i,i) are possible. This is due to the fact that only these transition states maintain the decreasing order in the distribution of points. As was proven in the development of the model, only states which have the J^{-1} component larger than the second component are listed in a decreasing order of points. Movements beyond these boundaries result in states of the form $(i-1,i)$, $(i,i+1)$, or $(i-1,i+1)$ and therefore violate the restriction that the points be listed in a decreasing order of magnitude, and, thus, these boundaries are impermeable.

The impermeability of the boundaries results in a reduced number of possible transitions from those states that lie on or near to them. Those states which lie on the boundaries have only four possible transitions. For example the point below the J axis labeled (3,3) has only three other reachable states (besides remaining at that state): (4,3): (4,2): and (3,2). Since all attack vectors are still

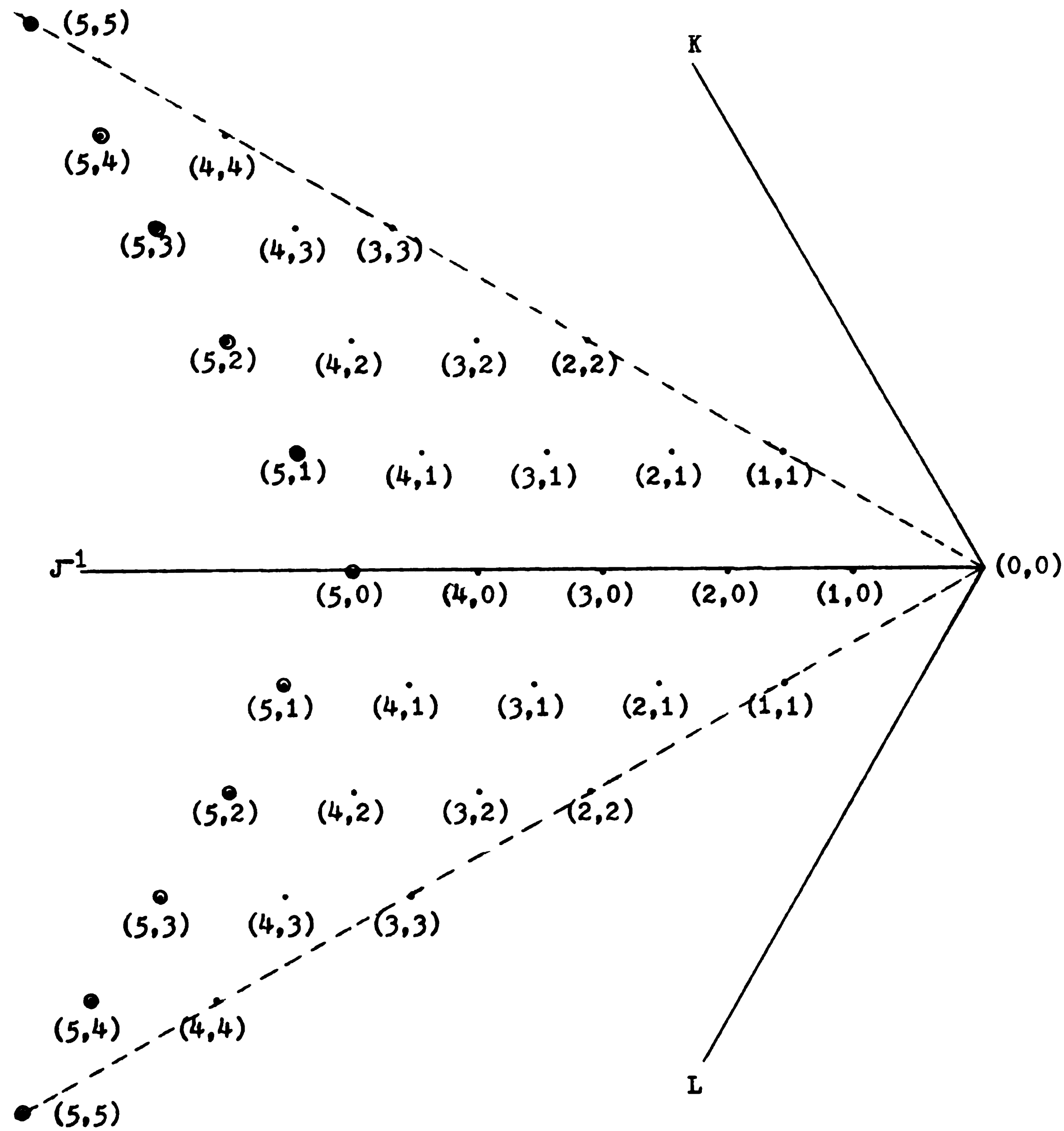

Figure 10. **Reduced Co-ordinate System for the Representation of State Vectors.**

possible, it is necessary to introduce the term <u>directional vector</u> to deal with the collapsing of attack vectors into one transition. The transition from (3,3) to (3,2) is in a direction parallel to the L axis and in the L^{-1} direction along that axis. Thus, this transition will be referred to as an L^{-1} or an L^{-1} directional vector. This directional vector would occur whenever an L^{-1} or J attack vector occurred. This can be verified by noting that the lower (3,3) state represents a $3J^{-1}$, 3L state. Thus, an L^{-1} attack vector changes the (3,3) state to a $3J^{-1}$, 2L state, while the J attack vector changes the (3,3) state to a $2J^{-1}$, 3L state. By virtue of rearrangeability, those two states are equivalently (3,2).

Those states which fall adjacent and parallel to the boundaries also have restricted transition ranges. The (i-1,j+1) transition violates the restriction that the points be in a descending order of magnitude since the second component is larger than the first. This follows from the fact that all states on this line are of the form (i,i-1) and a transition of the form (i-1,j+1) would result in states of the form (i-1,i) in which the first component is smaller than the second. When this state is rearranged a transition of the form (i,i-1) is produced, the same transition that is produced by the l attack vector.

The (1,0) and (0,0) states also have restricted transition ranges. The (1,0) state has five possible transitions with the (1,-1) and (2,-1) transition states being undefined. The (0,0) state has two possible transitions, the (1,0) state and the (0,0) state: all other transition states are undefined.

In each of the states presented above, the type of directional vector is determined by the axis to which the vector is parallel. For those states which have no restrictions on the transition range, the directional vectors are isomorphic to the attack vectors and, therefore, take on the label of the attack vector that produced the transition. These attack vectors correspond to the three axes, and the sign of their exponents corresponds to direction.

For the entire data set, a mean directional vector from each state was calculated by the following method. A resultant directional vector was calculated for each of the axes intersecting a state. This vector was represented by the lower case letter corresponding to its axis (j, k, or ℓ). The vectors were computed by subtracting the probability of the inverse directional vector from the probability of the directional vector. As an example the resultant directional vector for the J axis was the probability of the J directional vector minus the probability of the J^{-1} directional vector. Each directional vector was calculated by adding the probabilities of all those attack vectors which contributed to it. The directional vectors and those attack vectors which contribute to their formation are presented in Table 28. Following are the equations for the resultant directional vectors.

$$j = J' - J^{-1'}$$

$$k = K' - K^{-1'}$$

$$\ell = L' - L^{-1'}$$

From the three resultant directional vectors a mean directional vector was calculated for each state. By using the parallelogram law, two coordinates were determined from the three resultant directional vectors. The x coordinate lies on a line which is parallel to the

Table 28. The Attack Vectors Which Contribute to Each Directional Vector for Each Type of State

		Directional Vectors							
State	Type	I'	J'	J^{-1}'	K'	K^{-1}'	L'	L^{-1}'	
(0,0)	Pure	I	$J + K^{-1} + K + L^{-1} + L$	---	---	---	---	---	ATTACK VECTORS
(1,0)	Pure	$K^{-1} + L^{-1} + I$	J	J^{-1}	K	---	L	---	
$(i,i)^1$	K	I	---	$J^{-1} + K$	---	$K^{-1} + J$	$L^{-1} + L$	---	
	L	I	---	$J^{-1} + L$	$K^{-1} + K$	---	---	$L^{-1} + J$	
$(i,j)^2$	K	$L^{-1} + I$	J	J^{-1}	K	K^{-1}	L	---	
	L	$K^{-1} + I$	J	J^{-1}	K	---	L	L^{-1}	
$(i,j)^3$	K and L	I	J	J^{-1}	K	K^{-1}	L	L^{-1}	

[1] $i > 0$

[2] $i - j = 1$

[3] $i - j > 1$

J axis and intercepts the state from which the mean directional vector was being calculated. Type y coordinate lies on the line that is orthogonal to the x axis and intercepts that state.

$$x = j - (\text{COS. } 60 \text{ degrees})\ (k + \ell)$$

$$y = (\text{COS. } 30 \text{ degrees})\ (k - \ell)$$

The j resultant directional vector contributes nothing to the y coordinate because it is orthogonal to the line which in which it lies, while the k and ℓ resultant vectors contribute to both coordinates. These two coordinates describe the mean directional vector for each state for which they were calculated. The mean directional vectors were calculated for all of the states presented in the wedge in Figure 10 and are presented in Figure 11.

This visual representation shows the diverse processes that are operating when people engage in a truel. The strategies mentioned previously can be identified by directional vectors. The fair play strategies mentioned previously can be identified by directional vectors. The fair play strategy would be represented by the J directional vector and the threat minimization strategy by the L^{-1} directional vector. These vectors are represented by dashed lines from each state, thus showing how closely each state's mean directional vector corresponds to each type of strategy. As the figure shows, the fair play strategy was employed at the boundries while the threat minimization strategy was employed in the inner regions of the wedge. An exception to this was the slight preference for the former strategy in the lower pure states. At the extreme states, the threat minimization strategy was employed more frequently than the fair play strategy, while this tendency reversed for those less extreme states.

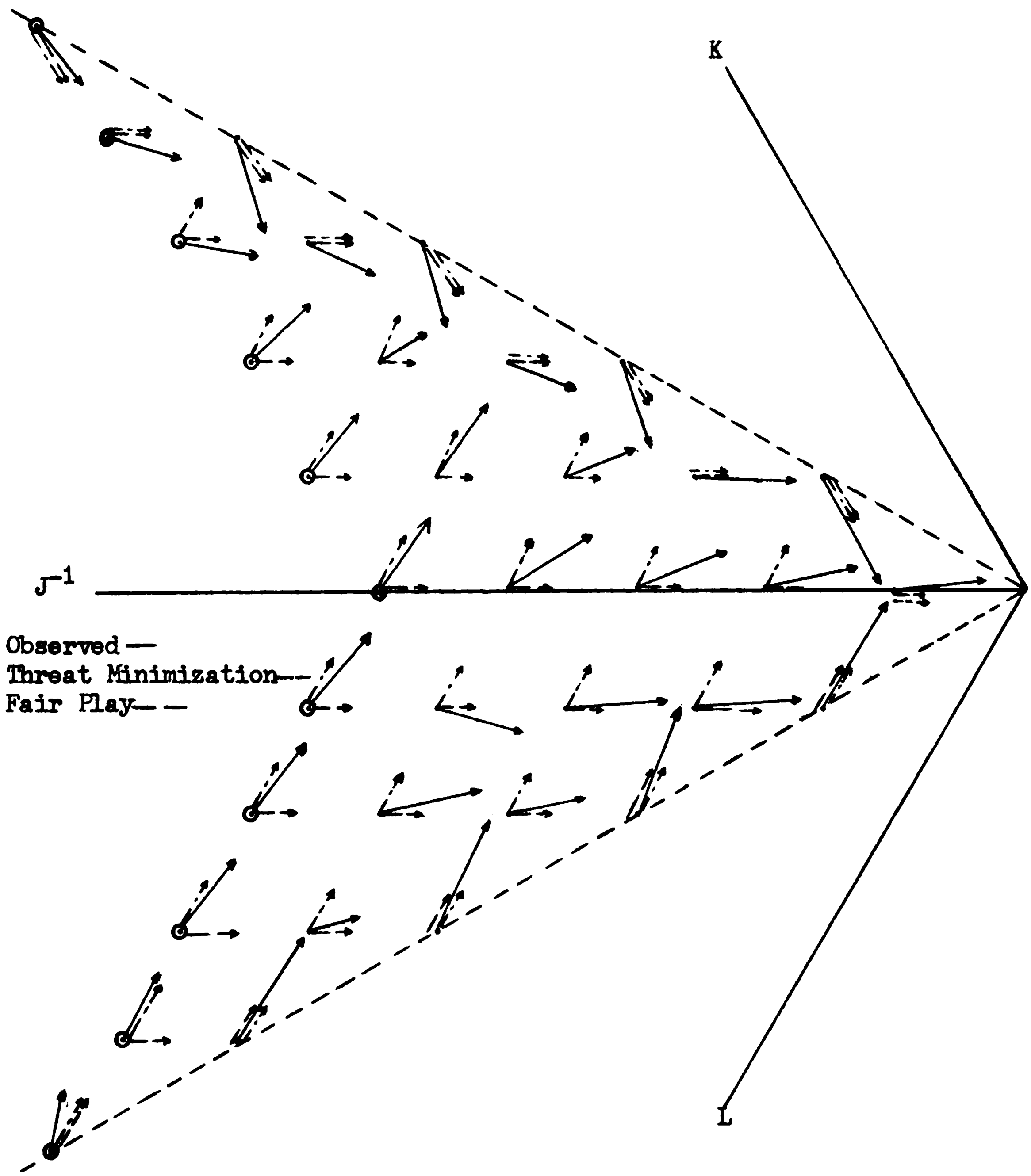

Figure 11. The Mean Observed Directional Vectors and the Directional Vectors for the Fair Play and Threat Minimization Strategies.

These analyses indicate that at least two processes are involved in the playing of the truel. The fact that the processes were not separated along state type lines, but rather by proximity to boundaries and extreme states, illustrates why separation of the data by state type did not produce an acceptable fit of the model.

An interesting point about the results was the fact that all initial states of type L and the three lower initial states of type K have threat minimization as the predominant stragegy, whereas the initial states of $(5J^{-1},3K)$, $(5J^{-1},4K)$, and $(5J^{-1},5K)$ have the fair play strategy as the most predominant. The boundary of the K type states has two stronger players of equal strength and therefore the fair play strategy is that movement which is parallel to the K axis but in the K^{-1} direction. Whether the threat minimization strategy was employed at the extreme states because they were initial states or because they represent some kind of threshold for the strongest player is impossible to determine from the data.

The threat minimization strategy and the fair play strategy have particular consequences for the distribution of points. The first strategy results in the increasing of the difference between the two weaker players and a decreasing of the difference between the two stronger players. This indicates a movement from two weaker players to two stronger or a type K state. This tendency for a preference for the type K states is clearly seen in Figure 11. The second movement results in the simultaneous decrease in both differences and therefore toward the all equal distribution.

Using the above analysis of the processes involved, some implications for Caplow's types of power structures can be formulated.

All states which fall on the lower boundary represent the type three distribution of Caplow. These distributions have one strong player and two weak players, with the weak players equal in strength. Those states which fall on the upper boundary represent Caplow's type two distribution, with two strong players of equal strength and one weaker player. All of those states which fall between these two boundaries represent Caplow's type five structure.

Although the interior distributions are technically type five's they also resemble either a type two or a type three structure, with the exception of the states on the J axis which resemble neither. Which type of distribution a state resembles depends on the relative size of the differences between the first and second power positions and the second and third power positions. If the former difference is larger than the latter, the distribution resembles a type three structure; if the latter difference is larger, then the distribution resembles a type two structure.

The pure states are the exception to the above discussion because the differences between the three power positions are equal. The pure states are, thus the clearest type five power structure. Those distributions which are found between the two boundaries in Figure 10 and 11 form a continuum of Caplow's type five power structure. These states span the range from his type two structure to his type three.

As Figure 11 indicates there are several different types of movements within each type of power structure. At the type three boundary there are two types of movements; one toward the type one distribution (all equal) and one toward the type two distribution. The latter movement occurred when there were extreme differences between the power of the three participants. These points include the initial states for the type L states. The former movement predominated on the boundary and on the type five structures between the boundary and the J axis. In the type two power structures the predominant movement was toward the type one or all equal state. The exceptions to this were the states with extreme differences between the participants, with these states moving toward a pure type two power structure.

An interesting result of this visual representation was to indicate that the triads in the pure type five distributions preferred to move toward a type two power structure than to a type three structure. This tendency to prefer type two distributions decreased as the DRS of the state decreased, and the power structure approached the type one or all equal state.

The implications for Caplow's types and for theories about group processes in general are quite clear. Given the opportunity to choose between a type two distribution, characterized by two strong players and one weak player, and a type three distribution, characterized by one strong player and two weaker players, the triads, in this experiment, preferred the former to the latter. In other words, groups prefer distributions of points in which there are two strong players of relatively equal strength and one weak player with considerably less strength, to distributions of one stronger and two weaker players.

In summary, although the model did not fit the data, it afforded the opportunity to test hypotheses about conflict situations. There was much evidence to indicate that more than two processes were involved in these situations. Through the test of the model and the visual representation generated by the model insights into the interpersonal process occurring within the truel were obtained. The test of the model allowed for the rejection of the single, simple assumption that all participants attack their stronger attack choice, and additional examination of the data explored more complex alternatives.

It is through this type of quantification, and axiomatization of psychological processes, that allows for the acceptance or rejection of theories. The results of this experiment indicate to what extent even negative results can advance knowledge. Despite the inability of the model to predict the interpersonal processes of the truel, it served well the function of validating and testing the assumptions about the processes. It offered the framework within which it was possible to define particular processes that were in operation within the truel and it gave the opportunity to test other predictions about the processes involved.

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APPENDIX

Instructions

This is an experiment in decision making. The experiment is a game consisting of several moves and I will keep track of each move that is made.

We will play several games. To begin each game, each player will be assigned a specified number of points. These points will be displayed on the scoreboard behind me. For the first game player Argon will have _____ points, player Boron will have _____ points, and player Krypton will have _____ points.

The game consists of moves, each move consisting of each of you taking a point away from one of the other players. You are required to take a point away on each move, but you may choose from which other player. You may not take a point from yourself. When a point is taken away from a player it belongs to no one and is taken out of the game. When a player looses all of his points he is out of the game. The game is ended when only one player has points remaining, he is the winner. It is possible for no one to win, i.e. two or more players may run out of points on the same move.

At the end of the experiment a number will be chosen at random from this glass. This number represents a game and the winner of that game wins the three dollars. This number corresponds to the order in which the games were played, for instance if the number three were chosen, the winner of the third game would win the three dollars. The other two players will receive 75¢. If the number chosen represents a game in which there was no winner, i.e. a tie, then all three of you will receive 75¢.

The front of your cubical is open so you may see the scoreboard and so you may communicate with me. In your cubicals there are three cards with the three names that will be used in the experiment. It is with these cards that you will indicate to me which of the other two players you wish to attack. On each move of the game hold up the card with the name of the player you choose. After you have indicated who you wish to attack I will record your choice and then tell you to put your cards down. I will then read who took a point from whom and remove the point from the board.

Some people like to keep track of which games they have won. It is for this reason that the paper and pencils have been placed in your cubicals.

The purpose of the partitions is to keep you from knowing which of the players the other names on the board represent, so please do not talk or attempt to communicate, in any way, with the other players. Noises also make you identifiable to the other players, so please refrain from making noises of any kind during or between games.

Are there any questions? If not hold up the card with the name of the player you wish to attack on the first move of the game.