A STUDY OF FACTORS AFFECTING THE STABILITY AND STRENGTH OF SHIPS

Thesis for the Degree of B. S.

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A STUDY OF FACTORS AFFECTING THE STABILITY AND STRENGTH OF SHIPS

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FOREWORD

It is the purpose of this paper to compile a study of some of the basic principles involved in the design of surface ships from a viewpoint of their stability and strength. Since the design and construction of ships is in itself a branch of engineering, the fundamental laws of physics, mathematics and mechanics upon which it is based should be a matter of interest to the civil engineer who, at some time in his career may be introduced to the field of naval architecture.

It is the hope of the author that the reader, with no previous knowledge of the subject matter will be able to find herein the information he desires with a minimum of confusion. No effort has been spared to condense this information to within the limits of the paper and yet give a clear explanation of all the problems discussed. All factors affecting ship stability have by no means been covered in this thesis. However, the main factors with which the designer and damage control expert are mainly concerned have been discussed. It would be to the advantage of the civil engineer to familiarize himself with the topics taken up in the following pages in the event that he is called upon to consult with the naval architect.

All curves used in this paper for purposes of illustration have been drawn from similar ones contained in the Navy Damage Control Handbook with the exception of Figs. 13 and 14 which have been partially reproduced from "Naval Architecture" by Peabody.

The author wishes to express his gratitude for the helpful comment and criticism made during the preparation of this thesis by Mr. Leo Nothstine of the Civil Engineering Department of Michigan State College.

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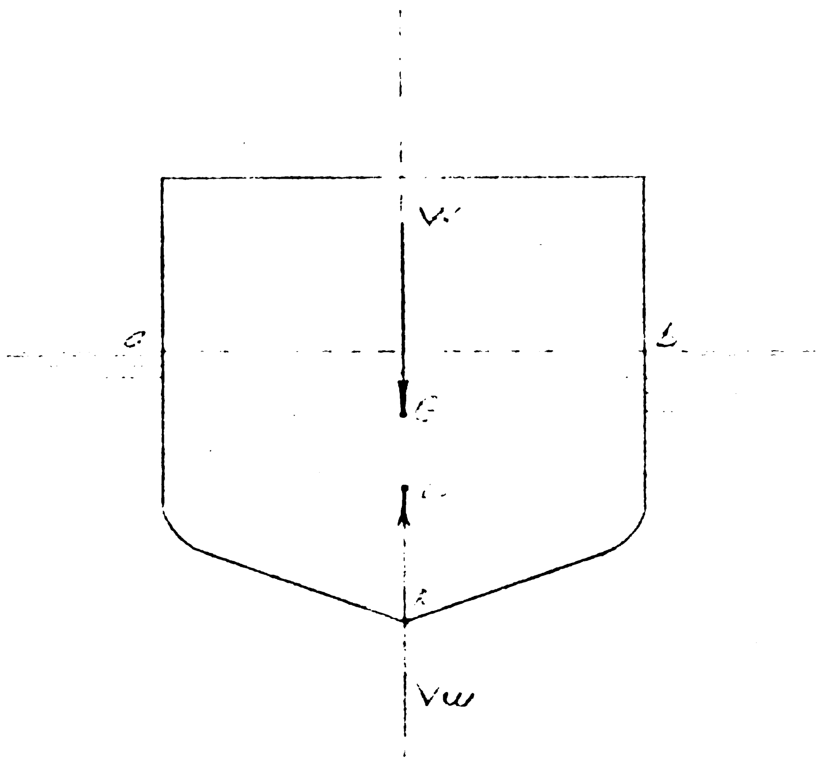
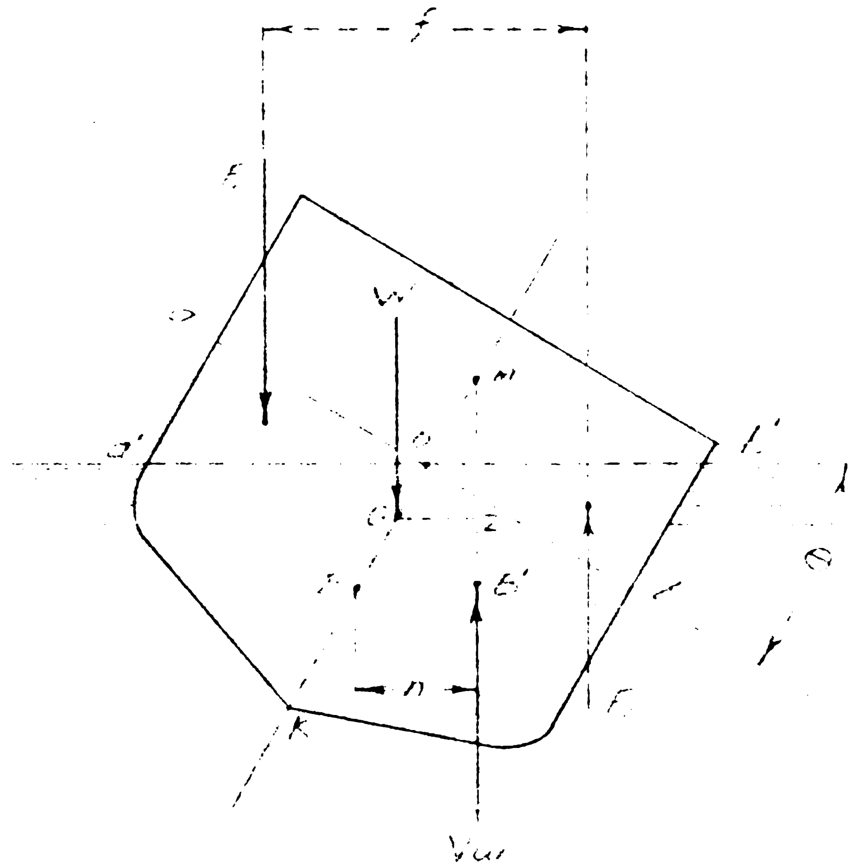
PART I

BUOYANCY AND STABILITY

There are two basic properties of a ship which are of the utmost importance during both its design and operation stages. The first of these is buoyancy or the ability of a ship to stay afloat and the second is stability which is defined as the ability of a vessel to remain in an upright position. Why does a ship stay afloat? This is best explained by Archemedes' principle which states that the weight of a body, in our case a ship, is decreased by an amount equal to the weight of the displaced liquid. In other words the effective weight of any floating or immersed body is equal to the actual weight of the body less the buoyant force of the liquid acting upward and if the weight of the object is less than the weight of an equal volume of the liquid the object will float. This property of a liquid is known as its buoyant effect. The actual buoyancy of a liquid on any body either wholly or partially immersed in it is the resultant upward pressure of the liquid on the wetted surface of that body while the resultant horizontal pressure is equal to zero since the horizontal pressures are in equilibrium. The buoyant force acts through the center of gravity of the underwater portion of the body or if the body is totally immersed it acts through the center of gravity of the body itself if that body is homogeneous. This point of application of the buoyant force is known as the center of buoyancy and is required in calculations for trim or fore-and-aft inclination and for calculations of stability. The volume of the water-



(a)



(b)

Fig. 1

tight portion of a ship above the waterline is called its reserve buoyancy and is an index to the safety of a ship.

This leads to the question of what the static forces acting on a ship are composed of. A ship at rest is under the influence of two equal and opposite forces: the downward force of gravity and the upward buoyant force as shown in Fig. 1a and for a ship in an upright position these forces are co-linear which is obviously the case for symmetrical loading. The downward force acts through the center of gravity, G and the upward or buoyant force through the center of buoyancy, B. Since B is at the geometric center of the underwater portion of the ship it will rise with respect to the keel, K as the draft increases. Now when a horizontal force such as a wind load acts on the vessel causing an inclining moment the ship heels over to some angle θ changing the shape of the under-water volume and thus moving the center of buoyancy to a new position, B' as shown in Fig. 1b. In order for the ship to remain in equilibrium the sum of all moments must be zero, thereby requiring a moment equal and opposite to the inclining moment which originally caused the heel. This equal and opposite moment is known as the ship's righting moment the value of which is the product of the righting arm, GZ and the upward force or weight of the ship, Vw. When the angle of heel becomes extremely large G will fall to the right of B' and an upsetting or capsizing moment will exist. Assuming that the ship is in stable equilibrium, that is a righting moment exists, it is seen from Fig. 1b that the line of action of the buoyant force cuts the vertical centerline at a point, M above the center of gravity. For large angles of heel the location of this point varies considerably but as θ approaches zero its cosine approaches one and the point M approaches a limiting position above G which is known as the metacenter. The distance from G to the metacenter is known as the metacentric height (GM). The distance BM is termed the metacentric radius. For angles of heel up to about 10° these distances are assumed to be constant for all practical purposes. It is customary to determine and record for each ship the metacentric height for the erect position and the normal displacement and trim. It is this initial position of the metacenter which is most useful in studying the stability characteristics of a ship and this is what is always meant by metacentric height unless otherwise specified.

This relative value of the initial stability is a basis for subsequent changes in the position of the metacenter caused by unsymmetrical loading, flooded compartments due to battle damage, filling or emptying of fuel tanks, etc. If the metacentric height (hereafter the term will refer to the initial GM) is large, large righting arms develop at small angles of heel which is a desireable property for any ship. The larger the GM therefore, the greater the stability of the ship.

In figure 1b, letting the two equal forces F_1 and F_2 represent the loss and gain of buoyancy respectively and taking moments about B we have:

$(Vw)n$ = moment of couple $F_1F_2 = F_2f$

$n = F_2f \,/\, Vw$

where F_2 is equal to the **buoyant** effect of the displaced water, obb' the length of which is the length of the ship and whose width ob varies.

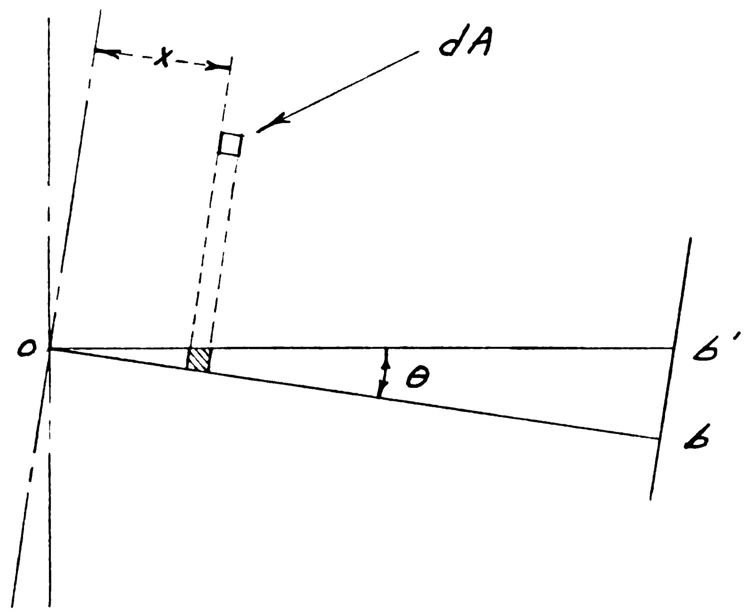


Fig. 2

The volume of the elementary prism, Fig. 2, is then equal to:

$$x \tan \theta \, dA$$

and its buoyant effect is:

$$wx \tan \theta \, dA$$

for small angles of θ. The moment of this force about o is then:

$$wx^2 \tan \theta \, dA$$

and the moment of the couple, F_1F_2 is:

$$F_2f = 2w \tan \theta \int x^2 \, dA \quad \text{(for both halves)}$$

$$= wI \tan \theta.$$

Therefore:

$$n = \frac{wI \tan \theta}{Vw} = \frac{I \tan \theta}{V}.$$

From Fig. 1b:

$$n = BM \sin \theta$$

and

$$BM = \frac{I \tan \theta}{V \sin \theta}.$$

Since for small angles of heel the sine and tangent of θ may be assumed equal the value of BM becomes merely:

$$BM = \frac{I}{V} \tag{1}$$

Or:

$$GM = KM - KG.$$

In the above formula I is the moment of inertia of the ship's waterplane about the longitudinal centerline. This can be found by dividing the area into small rectangles and finding the moment of inertia, i, for each rectangle. The total moment of inertia is then:

$$I = i_1 + i_2 + i_3 \text{ --------- etc.}$$

Up to this point it has been shown that ships float because they displace a certain amount of water which has a buoyant effect and that they have a reserve of buoyancy. A vessel's ability to remain upright is due to its righting moment which is a measure of stability. Metacentric height is also an index to stability if the angle of heel is less than 10° and if the displacement of the ship remains constant.

PART II

CURVES OF FORM AND THE INCLINING EXPERIMENT

We have seen that in order to study the stability characteristics of a ship the vertical center of gravity must be known not only during the design stage but also upon completion of the ship. The experienced designer can forecast the location of this point closely enough for reference during design. The procedure for estimating the various dead and live loads is similar to that for the design of a bridge or of an ordinary floor slab. The dead load consists of the ship itself including machinery, tanks, etc., while the live load is composed of stores, fuel, the crew, etc. Moments of these various loads are then taken about the keel and the sum divided by the total load gives a fairly good approximation of the location of the center of gravity which will vary of course during the construction phase.

Before discussing a more accurate method of finding G it is a good idea to discuss a topic which is very important in the study of ship stability and which will come up frequently in the subsequent discussion - namely the Curves of Form (Fig. 3). This is a set of curves which shows the various geometric properties of a ship's underwater body. Although some of these curves are self-explanatory it is best to touch on each one. The displacement curves obviously give the displacement of a ship in salt or fresh water for a known mean draft. The displacement of the ship at the designed draft in salt water is obtained by dividing the vol-

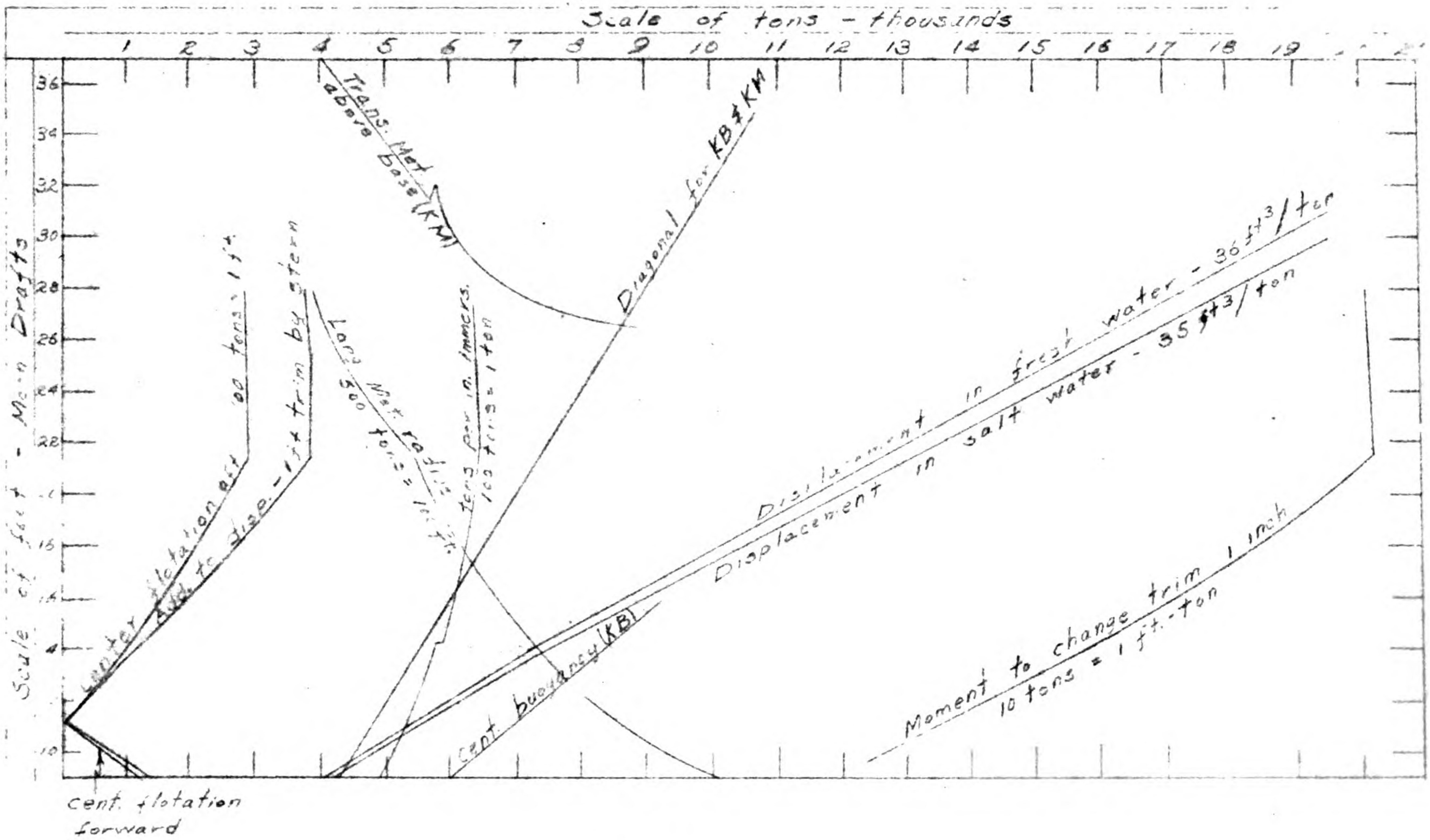


Fig. 3
CURVES OF FORM

ume in cubic feet by 35, since 35 cubic feet of sea water weigh one ton. The displacement in fresh water is divided by 36 since 36 cubic feet of fresh water weigh one ton. The volume of the ship is determined by finding the areas of transverse sections up to the waterline and applying the prismoidal formula.

The tons per inch immersion curve shows the change in displacement accompanying a one inch change in mean draft at various waterlines or the change in mean draft due to a weight change. If a weight change causes an increase in draft of one inch or 1/12 of a foot the change in volume will be equal to the area at the waterline in square feet times 1/12 and the change in displacement is equal to this value divided by 35 cubic feet per ton. This value is known as the "tons per inch immersion" and is denoted by T where:

$$T = \frac{A}{12 \times 35} = \frac{A}{420}, \tag{2}$$

where A denotes the area at the waterline plane. To determine this value from the curves enter with the mean draft prior to the weight change and read the value on the horizontal (displacement) scale. This value divided by the scale factor, 100 tons, gives the tons per inch immersion.

In order to locate the center of buoyancy or transverse metacenter a diagonal is laid off at a convenient angle, usually 45°, and this is used instead of a scale factor. Some curves are plotted without this diagonal and the desired values are read directly from the mean draft scale. However, for our set of curves the desired value of KB or

KM is found by entering with the mean draft and proceeding to the diagonal, then vertically upward or downward to the KB or KM curve as the case may be and horizontally to the vertical scale where the desired value may be read directly.

The longitudinal metacentric radius, BM', is found in the same manner as the tons per inch immersion except that in this case the scale factor converts tons to feet. Likewise the required moment to change the trim (difference between drafts fore and aft) one inch is obtained from the indicated curve while the scale factor converts tons to ton-feet.

The curve of the center of flotation is often labeled "center of gravity of the waterplane forward or aft of the mid-perpindicular." In order to read the curve enter with the mean draft, proceed horizontally to the indicated curve and then up to the scale of tons making the necessary scale factor correction. For a mean draft of 20 feet (Fig. 3) the center of flotation is 26 feet aft of midships. These curves do not apply to a seriously damaged ship but under ordinary circumstances may be read accurately enough when the degree of accuracy of the draft readings is taken into consideration.

We are now in a position to determine quite accurately the exact position of a ship's center of gravity which, as may be seen later, is of primary importance in the computation of a vessel's stability. Therefore an inclining experiment is performed on a completed ship to obtain a precise value of the distance KG. If the ship's draft is known and since the value of GM can be accurately measured then:

$$KG = KM \text{ (from curves of form)} - GM.$$

The problem at hand then is to determine the value of GM and this is done in the following manner. A large weight is moved from the centerline of the vessel outward until a small angle of heel (not over 5) is reached by a moment of known amount, and, since a position of equilibrium in the inclined position is reached, the righting moment inherent in the ship must equal the inclining moment. Referring to Fig. 4, the inclining moment is nearly:

$$wd$$

and the righting moment is nearly:

$$(W + w)(GM) \tan \theta.$$

Equating the two moments we have:

$$wd = (W + w)(GM) \tan \theta$$

$$GM = \frac{wd}{(W + w) \tan \theta}. \tag{3}$$

The angle of inclination is measured by use of two suspended plumb bobs, one forward and one aft. The length of these plumb lines is chosen so that with the inclinations expected the displacement will be at least two inches since it is difficult to obtain accurate results with smaller deflections. Before the experiment the length of the plumb line must be accurately measured.

The following precautions should be taken to insure satisfactory results:

a) The ship must be in quiet water.

b) There should be little wind.

c) All lines from ship to shore must be cast off or made slack.

d) There must be no free liquid, i.e., there must be no free surface in boilersor tanks and the bilges must be pumped dry.

e) All loose objects must be lashed down.

f) The crew must be either removed from the ship or kept on the centerline of the ship.

All data obtained from the inclining experiment including the curves of form should be compiled and kept on board ship at all times for reference in case of damage fueling, etc.

PART III

STATICAL AND DYNAMIC STABILITY

It has been shown that at any angle of heel the value of the righting moment is equal to W(GZ) where

$$GZ = GM \sin \theta. \qquad (4)$$

W denotes the displacement of the ship in long tons (2240 pounds). For small angles of heel this righting moment is a measure of the stability but as θ increases beyond 10° GM varies and it becomes necessary to determine the length of the moment arm graphically at various angles of heel and these values then become the index to the stability of any given vessel. It is now convenient to express the results in the form of a curve aptly termed the curve of statical stability (Fig. 5). Values of the righting arm are plotted along the ordinate scale while the corresponding angles of heel are plotted horizontally. In Fig. 5 the curve crosses the horizontal axis at about 72° indicating that for this particular vessel an upsetting moment exists at that angle of heel. It may also be noted that the maximum righting arm occurs at about 40°.

The curve in Fig. 5, however, is plotted for only one displacement. Obviously this is never the case for freighters or naval ships and it now becomes necessary to investigate the changes in the statical stability curve for various values of displacement. This variation is best illustrated by the Cross Curves of Stability (Fig. 6) which are obtained in the following manner.

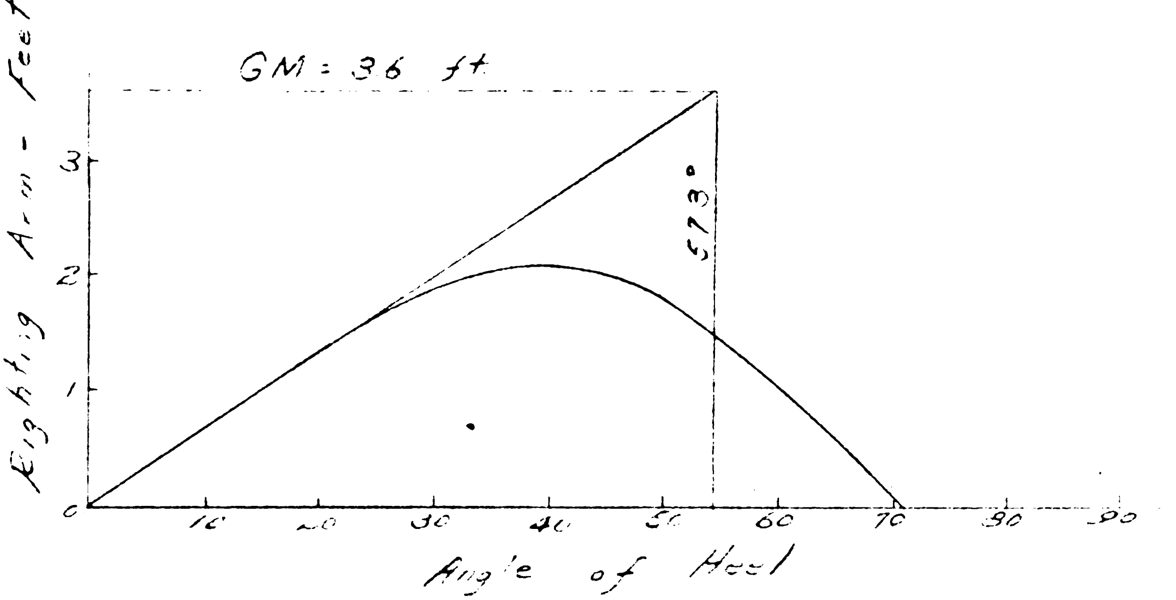

GM = 3.6 ft
57.3°
Righting Arm - Feet
Angle of Heel
0
1
2
3
10
20
30
40
50
60
70
80
90

Fig. 5

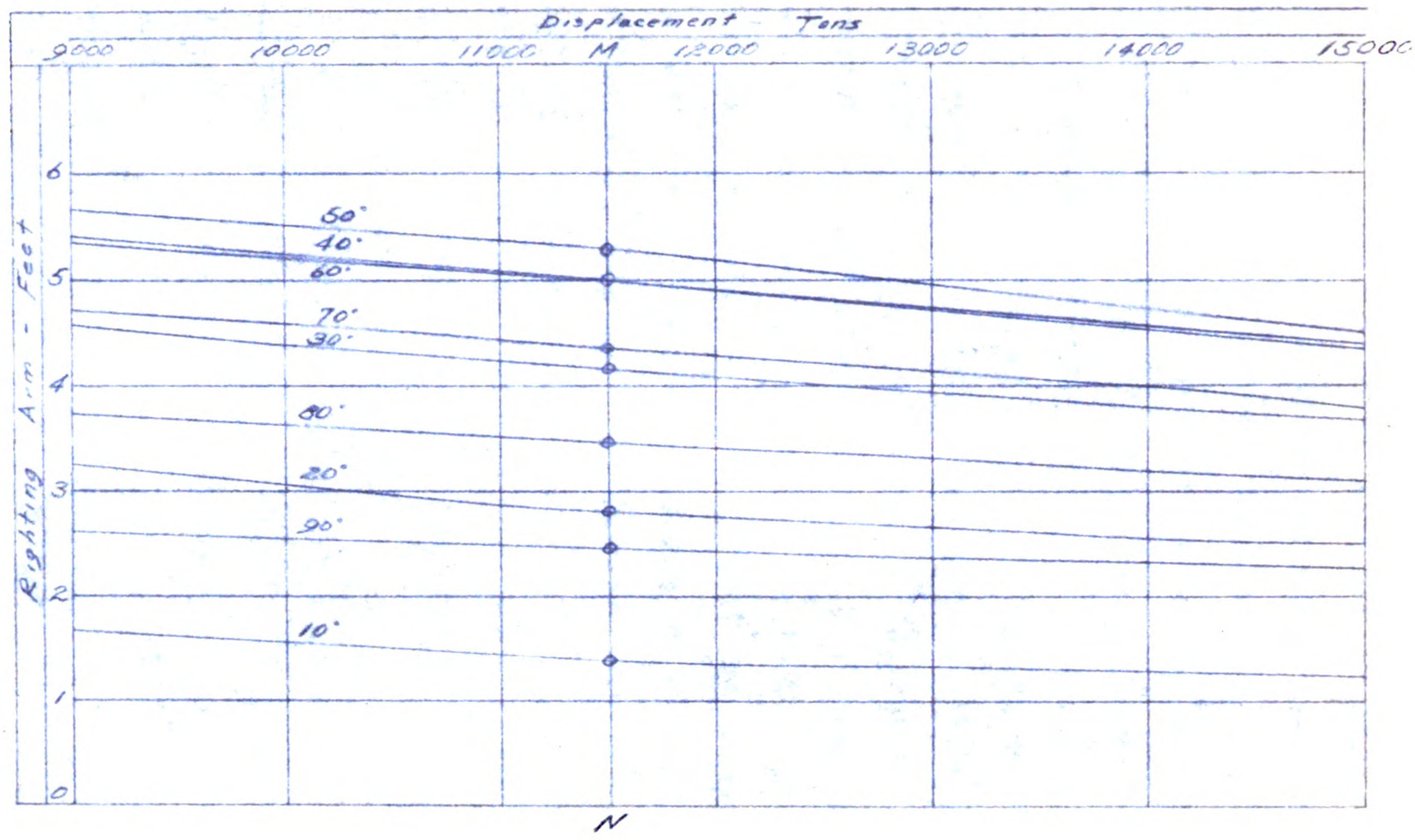


Fig. 6
Cross Curves of Stability

A cross-sectional view of the ship in question is plotted to scale at a given angle of heel, say 10°. This cross-section is taken through the longitudinal center of gravity. As the displacement of the vessel changes the corresponding waterline must also change. These various waterlines are drawn in at one-foot intervals on the inclined cross-section and the corresponding centers of buoyancy are then determined graphically as before while the center of gravity is assumed to be at a fixed point. In this manner a series of righting arms for successive drafts is obtained and the cross-curve of stability for a heel of 10° is plotted. This procedure is followed at intervals of 10° of heel up to 90° and the result is a set of curves similar to that of Fig. 6.

It is now possible to easily plot the statical stability curve for any given displacement of the ship. Assuming that the displacement of the vessel represented by Fig. 6 is 11,500 tons a vertical line, MN, may be passed through the curves as indicated. Values of the righting arm for different angles of heel are then simply read off and the statical stability curve is obtained.

Now that we are in a position to plot the curve of statical stability for a given vessel of what value is it? We have seen that the curve will indicate the range of stability and the maximum righting arm (or moment) as well as at what angle this righting arm occurs. This brings us to a new property of stability termed dynamic stability. Dynamic stability is defined as the amount of work done in inclining a ship to a given angle. We know from physics that work is equal to moment times the angle through which that moment acts. This amount of work may be seen to equal the area under the statical stability curve where the unit of the ordinates is converted to righting moment units or ton-feet and the abcissas are expressed in radians. The total dynamic stability is the amount of work required to capsize a ship and is represented by the entire area under the curve whereas the dynamic stability required to heel the ship to a given angle is equal to the area bounded by the curve to the left of the angle, the horizontal axis and a vertical line drawn through the value of the given abcissa. Expressed mathematically:

$$WK = M\int d\theta \qquad (5)$$

$$= W(GZ)\ \theta. \qquad (6)$$

This is a cumbersome and impractical method, however, and a planimeter may be used to determine the area and thus the work done in terms of foot-tons. Since the numerical value of this area has little meaning it is more useful to compare by inspection the areas under the two curves for the ship under different conditions of loading. This gives a relative resistance to capsizing and the total dynamic stability is therefore another index to the safety of a ship. A convenient method of comparing the dynamic stability of a ship for various angles of heel at a given displacement is to plot a curve of dynamic stability using values obtained at these various angles. This curve is also shown in Fig. 5

but the proper scale factor must be applied.

The statical stability curve of Fig. 5 is useful in another manner in that the metacentric height may be determined directly from the curve. We have seen that the general expression of the righting arm, here referred to as R, at any inclination may be given as:

$$R = GM \sin \theta.$$

The slope of the curve of righting arms is obtained by differentiating R with respect to θ. Thus:

$$\frac{dR}{d\theta} = GM \cos \theta.$$

But when $\theta = 0$ and $\cos \theta = 1$, we have:

$$\frac{dR}{d\theta} = GM.$$

Since we wish to read the value of GM directly from the righting arm scale it is necessary for the above expression to become:

$$dR = GM$$

which is possible only if $d\theta$ is equal to unity. Thus if a vertical line is drawn through a value for θ of one radian (57.3°) we will have met this requirement and GM may be read directly from the righting arm scale.

We now have seen that the factors defining a vessel's transverse stability are:

a) Metacentric height (GM).

b) Maximum righting arm (or moment).

c) Angle at which the maximum righting arm occurs.

d) Range of stability.

e) Total dynamic stability.

Although the initial stability or metacentric height of a particular ship is an indication of its stability it serves as no basis for comparison with a different type of vessel. The metacentric height of a battleship may be three times that of a destroyer while its range of stability may be only two-thirds that of the same destroyer. Of more importance than the range of stability, however, are the angle at which the maximum righting arm occurs and the value of the maximum righting arm. In order to insure greater safety the maximum righting arm should be large and should occur at a large angle of inclination.

PART IV

EFFECTS OF WEIGHT SHIFTS AND WEIGHT CHANGES ON TRANSVERSE STABILITY

Thus far in the development of curves and formulas for stability it has been assumed that the ship has been symmetrically loaded or that the center of gravity has been on the vertical centerline. If the position of the center of gravity is shifted by means of shifting weights already on board ship the transverse stability will be influenced.

The shift in the position of the center of gravity of a system of weights caused by the movement of any one weight in the system equals the product of the weight moved and the distance it is moved divided by the total weight of the system. In other words if a large weight is shifted, G will be shifted a distance of:

$$GG_1 = \frac{ws}{W} \tag{7}$$

where w is the magnitude of the weight, s is the distance moved and W is the total weight of the vessel. Furthermore, the shift of the center of gravity of the system will be on a line parallel to the line of movement of the single weight.

A weight can be moved in three directions but since we are dealing only with transverse stability at present the following discussion will be limited to a two-dimensional analysis, namely vertical and athwartship. It should be noted that moving a weight a weight already on board ship will cause no change in displacement and the metacenter will remain in a fixed position.

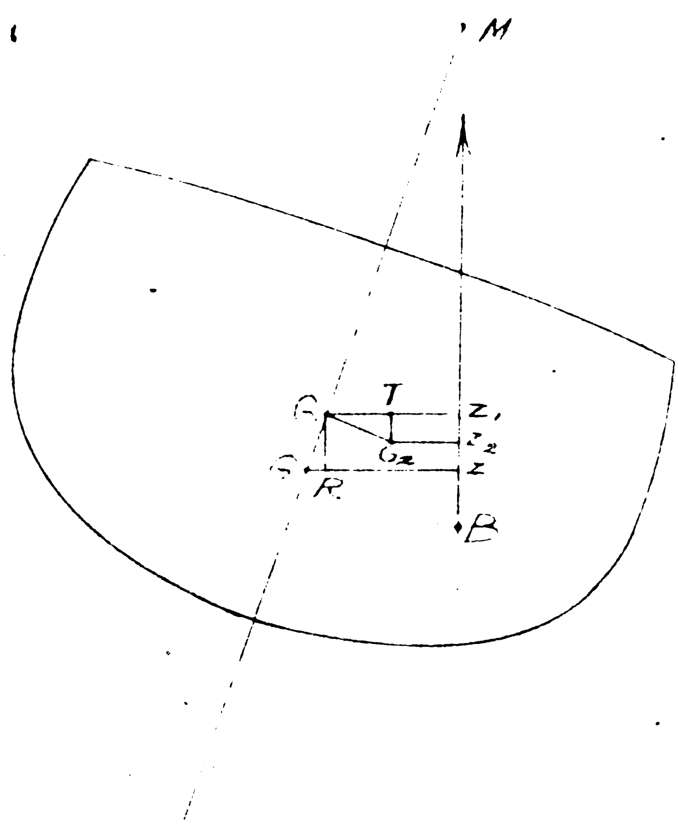
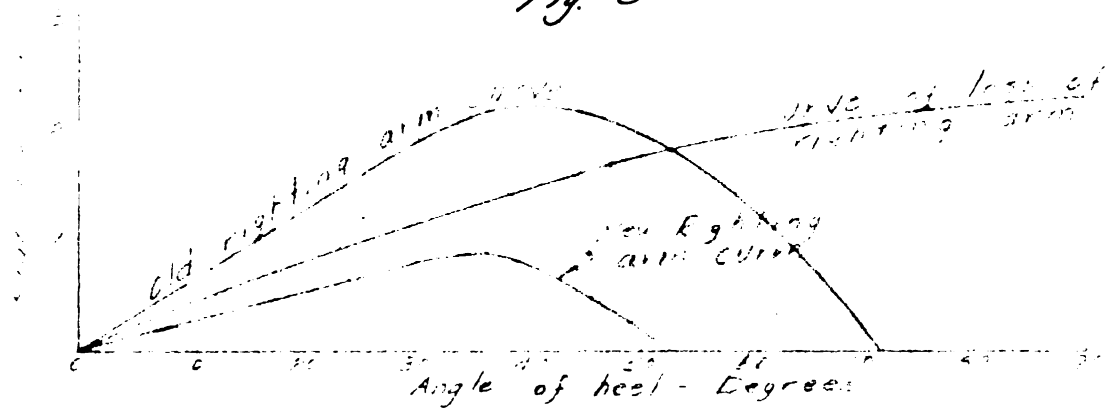


Fig. 7

Fig. 8



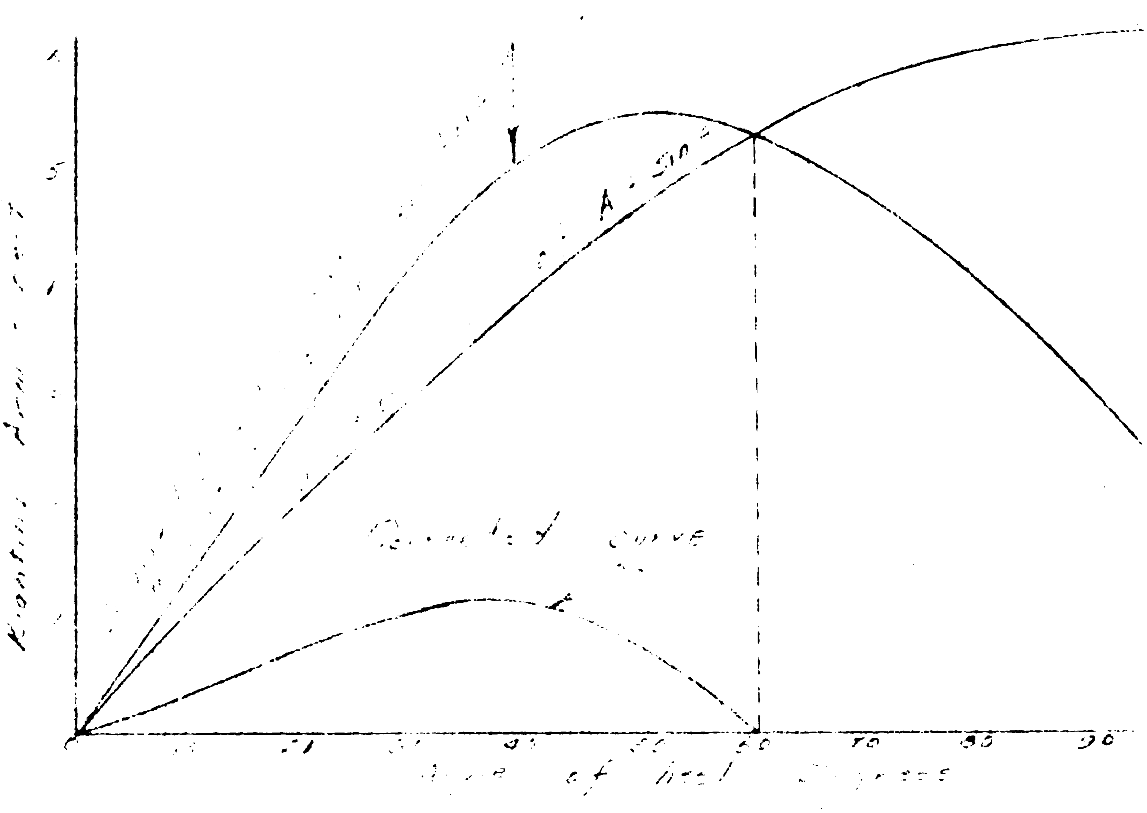


Fig. 9

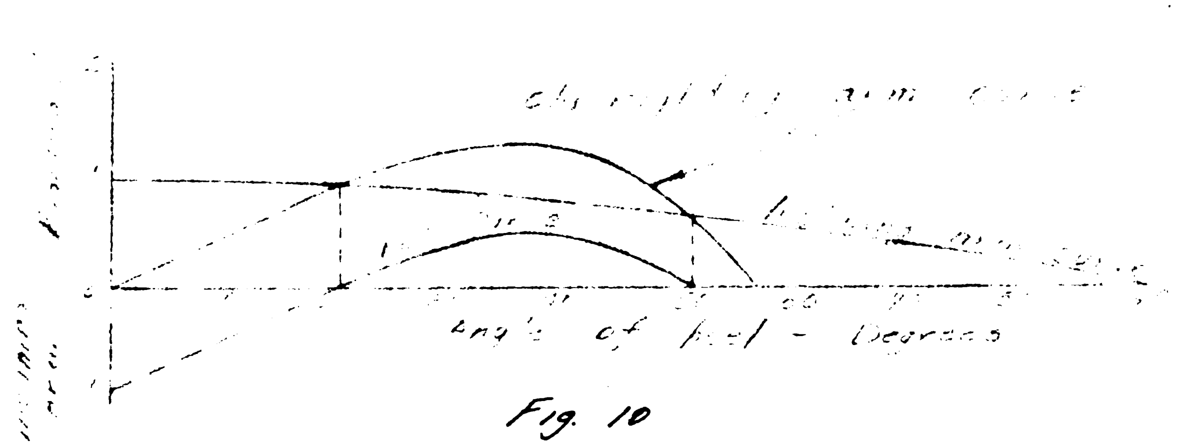


Fig. 10

Let us first consider vertical weight shifts. If a weight is moved vertically upward a certain amount, G will be moved by the amount:

$$GG_1 = \frac{wz}{W} \quad \text{(Fig. 7)}$$

and the new metacentric height will be:

$$G_1M = GM - GG_1. \tag{8}$$

If the vessel is at any angle of heel the loss of righting arm in Fig. 7 will be equal to:

$$\begin{aligned} GR &= GG_1 \sin \theta \\ &= GZ - GZ_1. \end{aligned} \tag{9}$$

This loss of righting arm may be subtracted graphically from the original statical stability curve as shown in Fig. 8 giving a new righting arm curve.

It has been previously mentioned that for the cross-curves of stability the center of gravity was assumed to be at a fixed point which for the purpose of this discussion we shall denote as A. We can now correct the statical stability curve obtained from the cross-curves in the same manner as shown in Fig. 9. The correction in this case will be equal to AG sin θ, where AG is the distance between point A and the actual center of gravity, G.

After the vertical shift of the center of gravity is determined the horizontal shift remains to be considered. The horizontal component of the movement of a weight will shift the point G_1 a horizontal distance of:

$$G_1G_2 = \frac{wd}{W}.$$

Since G_1 was originally on the vertical centerline, B and G are no longer in the same vertical line and an upsetting moment exists at zero inclination. This causes a permanent list to the angle where B is vertically under G. The further loss of righting arm may also be determined from Fig. 7. This loss is equal to:

$$G_1T = G_1Z_1 - TZ_1$$
$$= G_1G_2 \cos \theta. \qquad (10)$$

This value is known as the ship's inclining arm and when various values are plotted a cosine curve results. This curve is subtracted from that of Fig. 9 and the final righting arm curve is obtained as shown in Fig. 10. The residual maximum righting arm, AB, is now about 0.4 feet and develops at an angle of 37°. The new range of stability is from 20° to 50° and the vessel will have a permanent list of 20° which is the point where the length of the inclining arm is equal to the length of the righting arm.

A weight may now be shifted diagonally or a shift may be accomplished in two steps, first by a vertical shift of one weight and a horizontal shift of another. A diagonal shift should be treated in two steps as outlined above. It should be obvious to the reader that for the purposes of illustration extremely large weight shifts were made and that a curve such as that of Fig. 10 will very seldom actually become a reality except perhaps in the event of severe battle or storm damage.

After the effect of weight shifts on the location of the center of gravity has been determined the changing of weights on a ship, that is the addition or removal of weights, should be investigated. The shift in the position of the center of gravity of a system of weights caused by the addition or removal of a single weight, equals the moment of the added weight about the center of gravity of the original system, divided by the total weight of the final system. The movement of the center of gravity of the system will be along the line connecting the center of gravity of the original system and the center of gravity of the added or removed weight and will be toward the weight if added and away from the weight if removed.

Let us assume that a weight is added to the ship and that it is placed on the vertical centerline directly on the center of gravity. The new displacement being known, the curves of form (Fig. 3) may be entered and the new mean draft obtained. Then assume that the weight is moved to its final height above the keel and on the centerline and find GG_1 as previously shown in the discussion of weight shifts. We now know that:

$$KG_1 = KG \pm GG_1, \tag{11}$$

depending upon whether the weight is moved upward or downward. The next step is to find the height of the new metacenter above the keel. Entering the curves of form with the new mean draft determine the new value of KM. The new metacentric height is then:

$$G_1M_1 = KM_1 - KG_1. \tag{12}$$

A new uncorrected curve of statical stability is then taken from the cross-curves for the new displacement but this must be corrected for the new height of the ship's center of gravity above the base line. This is done as follows:

$$AG_1 = KG_1 - KA. \tag{13}$$

Values of $AG_1 \sin \theta$ are subtracted from the curve to give the corrected curve. Since we now have the corrected curve for the vertical shift in weight, all that remains to be determined is the effect of the horizontal shift the procedure for which has already been outlined. Actually the problem of weight changes is the same as that of weight shifts after the initial step of finding the new metacentric height is performed.

PART V

THE EFFECTS OF LOOSE WATER

The total weight of a ship is made up of liquid matter such as fuel as well as solid matter. If the solid matter is lashed down so that it will not move as the ship rolls or pitches its center of gravity will remain in a constant position relative to the ship. This is also the case for liquid matter if it competely fills its container such as the water in a completely filled water tank. If, however, this condition doed not exist we have a free surface and the liquid will change its location and shape as the ship changes position. This free surface tends to lessen the metacentric height thus decreasing the vessel's statical stability as well as her initial stability. If a tank or compartment is full and no free surface exists then this is merely treated as a solid weight and it does not effect a ship's stability beyond weight effect. However, for a partly filled tank or compartment the center of gravity of the liquid moves from side to side as the ship rolls and it acts like a suspended or moveable body.

As an illustration of free surface effect upon a ship's stability consider Fig. 11 showing a tank partially filled with sea water where G is the ship's center of gravity after the water is in the ship. When the vessel is upright the center of gravity of the water in the tank is at point D, but as the ship inclines to some angle, θ, a wedge of the liquid will shift from fcf_1 to s_1cs, g and g_1 being the centers of grav-

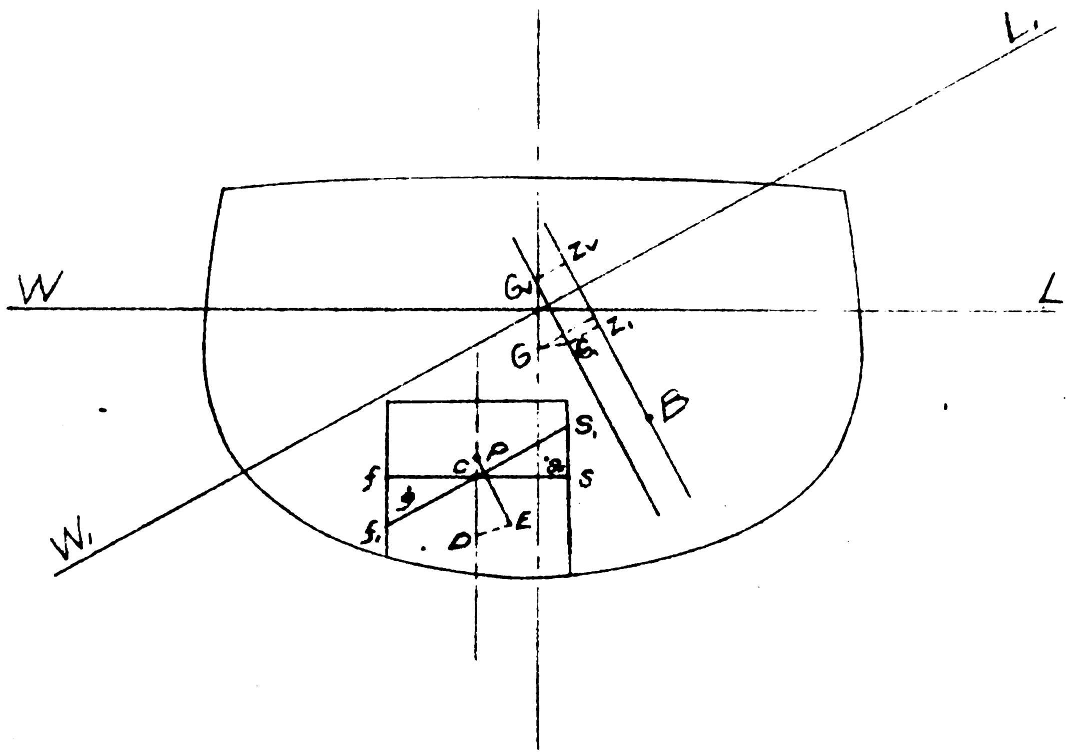


Fig. 11

ity of the left and right wedges respectively and the shift of this weight being along the line gg_1. When the ship is upright the weight or force due to the water will act vertically through D and when the ship is inclined the line of action of the force due to the water will be through point E, the new center of gravity of the liquid. However, we know from Fluid Mechanics that the force per unit volume of the temporarily inclined liquid must act through the center of gravity of the liquid perpendicular to its surface. These two lines of action when drawn are seen to intersect at point p (Fig. 11) which will always be the case, no matter what the inclination. The water is behaving as though its center of gravity were at p rather than at D or E creating the same effect as a pendulum. Point p is therefore referred to as the virtual center of gravity of the loose water. The distance of this point above D can be shown to equal i/v in the following manner where i is the moment of inertia of the free surface about a longitudinal axis through c and v is the volume of liquid in the tank. The volumes of the two wedges are equal, i.e.

$$fcf_1 = s_1cs = A.$$

By mechanics:

$$v\,(DE) = A\,(gg_1)\int_0^z dz \qquad (14)$$

where dz is an increment of length of the tank, the total length of which is Z.

For very small angles of heel we can assume that

$$gc = g_1c = \frac{2h}{3},$$

where h is equal to half the width of the tank. The area of A is:

$$A = \tfrac{1}{2}h\ (h\theta),$$

since for small angles of heel the numerical value of θ may be used. The moment of one wedge is equal to:

$$\int_0^z \tfrac{1}{2}h\ (h\theta)\left(\frac{2h}{3}\right)(dz).$$

The total moment is:

$$\int_0^z \frac{2h^3}{3}(dz)(\theta).$$

The total moment is also equal to:

$$v(DE)$$

and

$$DE = Dp \sin\theta = (Dp)\theta.$$

Therefore:

$$v(Dp)\theta = \theta\int_0^z \frac{2h^3}{3}dz.$$

But:

$$\int_0^z \frac{2h^3}{3}\ dz = i$$

And:

$$Dp = \frac{i}{v}. \tag{15}$$

In the same manner the virtual rise of the center of gravity of the entire vessel can be shown to equal:

$$GG_v = \frac{i}{V}, \tag{16}$$

where V equals the existing volume of displacement of the ship.

The effect of free surface on a ship's initial stability is the same as though an equal weight were raised through a distance of Dp and the reduction of GM is equal to the quantity i/v. The loss of righting arm or reduction of GZ (Fig. 11) is equal to:

$$\frac{i}{V} \sin \theta = GG_V \sin \theta. \tag{17}$$

It may be noted that neither the depth of water nor the depth of free surface in the ship influences the free surface effect on the initial stability. However, the breadth of the tank or compartment in question has a considerable influence on the free surface effect since the moment of inertia of the surface about its longitudinal axis varies as the cube of the breadth. Therefore narrow compartments should be designed if possible.

Another means by which loose water may occur in a ship is by the opening up of a large hole in the side plating either wholly or partially below the waterline due to battle damage or perhaps a collision. If the compartment is completely below the waterline the only effect will be added weight. However, if it is partially below the waterline water will flow into it up to the level of the external waterline and a free surface will exist. A free surface may also occur in the former case in the event that an air bubble is entrapped in the compartment. However, in all probability the vessel will be rolling and pitching to some extent and the amount of the flooding water will continually vary. This condition is known as free communication effect and it always tends to reduce the metacentric height of a vessel.

PART VI

LONGITUDINAL STABILITY

Up to this point only transverse stability has been considered, that is, angles of heel have been our main concern. But the stability of a ship in the fore-and-aft direction, called the longitudinal stability, is also an important factor in the design of any ship. Trim, the longitudinal inclination of a ship is measured by the difference in drafts foreward and aft rather than in degrees as for the angle of heel since the longitudinal angles of inclination are so small. Actually, the principle of longitudinal inclination is the same as that of transverse inclinations and its equations involve GM', the longitudinal metacentric height, instead of GM.

When a vessel trims it inclines about an axis through the centroid of the waterplane area, this centroid being known as the ship's center of flotation. This point point may be found from the curves of form for any particular vessel as previously explained. This brings us to another of the curves of form which has not yet been mentioned, that being the curve of the initial displacement for a one foot trim by the stern. This curve is plotted for a vessel with no trim thereby necessitating a correction in displacement for trim. When a ship trims by the stern her displacement is increased but when it trims by the bow it is decreased by the amount shown on the curve.

Longitudinal stability is represented by the tendancy of a ship to resist a change in trim. Changes in trim are produced by moving weights from aft of the center of flotation to forward or vice versa. In order to insure that a ship will have the proper trim it is necessary to make calculations for the weight of the hull and its contents and for the position of the center of gravity. From the plans and specifications the size and weight of every important member of the ship and of the machinery, fittings and contents of the ship are known. The center of gravity may be found by taking moments about any convenient axis. If this center of gravity is not directly over the longitudinal center of buoyancy the ship will be out of trim. The effect of this discrepancy is determined by the aid of the moment required to change trim one inch which is the moment in foot-tons which will produce such a longitudinal inclination that the change in trim is one inch. If the discrepancy is large some of the weights on board ship must be shifted until a satisfactory trim is obtained. The necessary change in trim may be calculated by dividing the trimming moment by the moment to change trim one inch. The moment to change trim one inch may be found in the following manner. When the trim is one inch the longitudinal inclination is equal to:

$$\tan^{-1}\frac{1}{12} \div L = \tan^{-1}\frac{1}{12\,L} .$$

Now visualizing the vessel of Fig. 4 to be inclined longitudinally rather than transversly we have:

$$W(GZ') = wd \cos \theta$$

$$GZ' = GM' \sin \theta.$$

Therefore:

$$W(GM') \sin \theta = wd \cos \theta$$

$$wd = W(GM') \tan \theta.$$

Letting C equal the moment to change trim one inch we have:

$$C = \frac{W(GM')}{12\ L} \tag{18}$$

The longitudinal metacentric height, designated as GM', may be found from the following formula:

$$GM' = KB + BM' - KG \tag{19}$$

where KB and KG are the same as for transverse stability computations and BM' is the distance from the center of buoyancy to the longitudinal metacenter. It is obtained from the following formula:

$$BM' = \frac{I'}{V} \tag{20}$$

where I' is the moment of inertia of the waterline plane about a transverse axis through the center of flotation and may be assumed to be the sum of the moments of inertia of a series of small rectangles as in the previous case for the computation of the transverse metacentric radius, BM.

Many smaller vessels are designed with a trim by the stern, this trim being known as drag. It is the difference between drafts aft and foreward when the ship is floating at the designer's waterline. In the case of ships designed with drag, the designer's waterline is inclined to the base line. It is essential that sufficient propeller tip immersion be obtained. Therefore a trim by the bow is not a desired characteristic of any ship although too much trim by the stern reduces a ship's maximum speed thereby requiring more fuel oil and these factors must all be taken into consideration by the naval architect while the vessel is in the design stage.

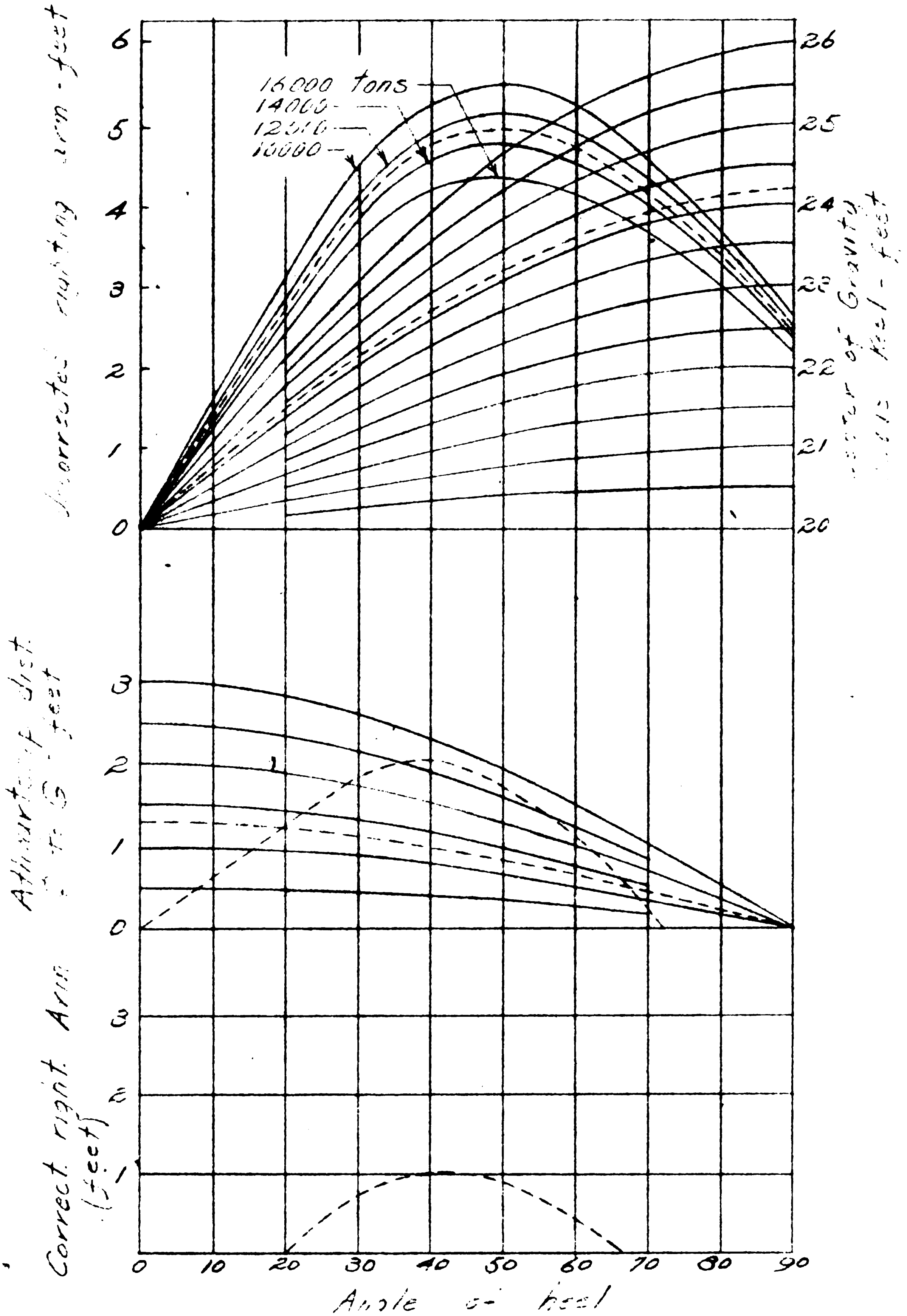
Trim also has an effect on transverse stability since the curves of form are based upon a vessel having no trim. For most ships these curves are applicable if the trim does not exceed about 1% of the length. When a ship trims by the stern the transverse metacenter will be a bit higher than indicated by the KM curve because both KB and BM increase due to a large waterplane and thus a larger moment of inertia of the trimmed waterplane. The reverse is usually true in the event that the vessel is trimmed by the bow. Longitudinal stability, therefore, is a very important factor in the design of a ship and the designer should give it his careful investigation in an effort to reduce excessive trim as much as possible.

PART VII

THE GENERAL STABILITY DIAGRAM AND IMPAIRED STABILITY

Most of the important factors affecting ship stability have been discussed up to this point. We have seen that stability characteristics may be shown by certain properties of the stability curves and that weight additions or removals change the stability characteristecs of any given vessel. If a large change of weight does occur it has been seen that the changes in stability may be accounted for by first taking a new righting arm curve from the cross- curves. This, however, must first be corrected for the final vertical height of the center of gravity above the keel by means of the sine curve correction and secondly for the final off-center position of the center of gravity by means of a cosine curve correction. This may be done in separate steps as previously explained but it is a very long process and is not of much value to the designer or to the engineering or damage control personnel on board ship. A more expeditious may be used, therefore, by which the desired purpose may be accomplished more rapidly and just as accurately. In order to do this use is made of a set of curves called the general stability diagram (Fig. 12) and a supply of large sheets similar to this diagram should be provided with the stability and inclining experiment data furnished any completed ship.

As can be seen from the figure, the general stability



16000 tons
14000
12310
10000
Uncorrected righting arm - feet
Center of Gravity above keel - feet
Athwartship dist. C. to G. feet
Correct. right. Arm (feet)
Angle of heel
0
1
2
3
4
5
6
20
21
22
23
24
25
26
10
30
40
50
60
70
80
90

Fig. 12

diagram consists of three sections. The upper section is composed of uncorrected righting arm curves and a family of superimposed sine curves for correcting the original righting arm curve. The former are plotted from the cross-curves of stability of the vessel for a given displacement and the latter are plotted for sucessive heights of the center of gravity.

The middle section of the diagram contains a family of cosine curves for correction due to off-center position of the center of gravity. These curves are plotted for various distances of the center of gravity from the longitudinal centerline and the values of heel may best be found by use of a small model and the performance of an inclining experiment.

The bottom section of the general stability diagram is left blank and this space is used to plot the resulting curve of residual righting arms.

The method for using this diagram is a fairly simple process. Assuming that the ship represented by Fig. 12 has a displacement of 13,000 tons with the center of gravity 24.2 feet above the keel and 1.3 feet to starboard of the longitudinal centerline its final statical stability curve is shown as a dotted line in the lower section of the general stability diagram. The first step in arriving at this curve is to sketch in by eye the proper displacement and sine curves as shown in the upper section of Fig. 12. The sine curve is then subtracted graphically from the displacemint curve and plotted in the middle section by means of a pair of dividers.

We now have a righting arm curve corrected for the vertical position of the center of gravity. The proper cosine curve corresponding to an athwartship offset of the center of gravity of 1.3 feet is then sketched in. Finally the corrected statical statical stability curve can be represented by the curve in the lower section of the general stability diagram and is obtained by the graphical difference between the curves of the middle section.

By comparing the properties of the original and final righting arm curves we are now in a position to investigate the changes in the stability of the ship in question due to unsymmetrical loading which may have been caused by flooding or by one of several other reasons. One index to the stability of a ship is its dynamical stability which is represented by the area under the statical stability curve. It is very obvious in this case by inspection that the vessel's dynamic stability has been decreased considerably. The metacentric height has also decreased, it now being negative as compared to about 7.2 feet (not shown) when the ship was in its original condition as shown by the curve in the middle section. We can also see that the ship will assume a permanent angle of list of 20° to starboard about which it will roll to various angles of heel and that the ship may be expected to capsize at an angle of heel of approximately 67°. In this case we have a negative metacentric height indicating that an inclining moment rather than a righting moment exists.

Thr Three things must be known about any particular vessel before use can be made of its general stability diagram. They are: its displacement, the height of its center of gravity above the keel and either the distance of G from the fore-and-aft centerline or the angle of heel. In order to produce a noticeable change in a ship's stability it is usually necessary to move several weights on board ship and it is necessary to compile these changes in tabulated form in order to facilitate calculations. These weight changes may be due to loading or unloading of cargo or ammunition, to fueling, flooding or to the formation of large amounts of ice on the weather decks. In the tabulation of these changes columns should be provided for weight changes, added or removed vertical moments, change in the vertical position of G due to free surface effect and for starboard and port inclining moments. These results may then be summed up and the final computations made in the following manner using the value $W \pm w$ for the new displacement. First, the net change in vertical moment is determined by summing up the values of wh, where w is the added or removed weight and h is its height above the keel. The new vertical height of the center of gravity above the keel is then found from the formula:

$$KG_1 = \frac{W(KG) \pm wh}{(W \pm w)} \tag{21}$$

which consists of nothing more than taking moments about the keel in the same manner as is used in mechanics to find the centroid of a composite area.

Following this step the next step is to determine the net effect of free surface if such a surface exists. The various values for i/v for all compartments that may be flooded are summed up and this value is added to KG_1 to obtain the position of the virtual center of gravity above the keel (KG_v). The upper section of the general stability diagram may now be entered since we know the new KG and the new displacement.

Before we can proceed to the middle section of Fig. 12 it remains for us to determine the athwartship distance from the centerline to the center of gravity. This may be determined by first finding the athwartship moments due to all added or removed weights and summing up these values of wd where d is the distance of each individual weight from the centerline. When the values are summed up the inclining moment, port or starboard is then known and the value of the inclining arm or distance of G from the centerline may be found from the formula:

$$GG_1 = \frac{wd}{(W \pm w)}. \tag{22}$$

We may now use the middle portion of the general stability diagram after which the final statical stability curve may be drawn in.

This method may seem like a long and laborious procedure but actually it is a simple process if some form of tabulated sheet is provided on board ship along with a simple form for performing the computations such as may be found in many engineering offices for various types of work. If, as in the case of a large ship, a large amount of personnel is available it is a good idea to have one or two persons checking the stability of the ship in this manner in case of damage or unusually heavy flooding so that an overall picture of the vessel's stability may be known at any time.

The results of the general stability diagram are as accurate as the data used to enter the diagram but if added weight due to loose water resulting from free surface is present an approximation is introduced. However, the method outlined above is accurate enough for most practical purposes. The net effect of added high weight is always a reduction of stability and this is also the case for the removal of low weights such as removing machinery or fuel or making alterations on the ship itself.

Another way by which a ship's stability may be impaired is by her loss of reserve buoyancy. Reserve buoyancy is that characteristic of a ship which prevents it from sinking bodily. It may be used up either by the addition of weight or by the rupture of boundaries. Heavy armor plating, of course, lowers the possibility of this type of rupture but nevertheless, the underwater body of a ship should be divided into several large watertight compartments so that in the event of underwater flooding the water may be confined to one relatively small section of the ship.

PART VII

WEIGHT AND STRENGTH

It has been shown that in order for a ship to have sufficient stability, buoyancy and trim it is necessary to make calculations for the weight of the hull and its contents and for the position of the center of gravity. Naturally the size and weight of every structural member of the ship is known from the plans but it is still important that a vessel be weighed during the process of construction. All materials added to the ship are weighed as is all refuse and scrap as it is removed. An accurate record should be kept of all these weights by some responsible person in order that the exact weight of the completed vessel may be determined. Even for the hull of a large ship it is possible to set up some type of system to calculate for weight, center of gravity and trim. In the early stages of design, however, the final weight of the ship and the location of her center of gravity are esttimated from those of actual ships already in operation, and the geromtric and stability characteristics calculated on that assumption.

In order for the weights going on to a ship under construction to be tabulated, it is best to have a definite system of compiling these weights so that a final value of the displacement will be known. The various weights should be divided into groups such as the hull itself, hull fittings, armor protection, machinery, water, ordnance, equipment, fuel and any other classifications which the desigher may deem adviseable. A margin or safety factor may be allowed for in the final sum of these weights during design to allow for any changes during construction. The close estimate of these weights is essential not only for the displacement and stability calculations but also for the determination of the required horsepower. As the various weights going on board ship are recorded it is also to record both the vertical and athwartship positions of their centers of gravity so that the final position of the center of gravity of the entire system or completed vessel may be located by the same method as outlined in the previous chapter on impaired stability.

Upon completion of the design of the vessel, its strength must be determined to within a reasonable degree of accuracy in order to ascertain whether or not it is in a seaworthy condition. Calculations are first made for the ship resting in quiet water where stresses are likely to be comparatively small and then they are made for the ship on a wave the same length of the ship. It is not certain that the ship will encounter such a wave in actual service nor is the calculated maximum stress expected to be equal to the actual maximum stress. But by assuming such a wave to occur the maximum possible bending moment on the vessel will be known and designed for thus providing a factor of safety as well as giving relative results which will give the designer a good idea of the strength of the ship as compared to others already in service.

To begin with, there are two types of stresses occuring in a ship, both of which must be taken into account during the calculations for strength. The first of these is a sagging stress which means that the top portion of the ship is in compression while the bottom is in tension, a condition similar to that of a simply supported beam with a uniformly distributed load. The amount of deflection occuring at the middle of the ship is knowm as the amount of sag. This type of stress may occur with a very large load in the center of the ship such as engines and boilers or it may occur when the vessel is supported by waves with the bow and stern riding the crests and the midship region in the trough. The reverse of this condition is the tendancy of the ship to bend upward in the middle and the stresses caused by this tendancy are called hogging stresses where the top of the ship is in tension and the bottom is in compression. A vessel may be in a hogging condition when it is riding the crest of a wave or it may occur in still water with large concentrated loads, such as gun turrets, fore and aft. When a ship has atendancy to hog the deflection at the ends is known as the amount of hog.

The first step in the calculation of the stresses in a ship is to draw the curve of weights which consists of nothing more than assuming the ship to be a beam and plotting at regular intervals along its length the weights acting downward. If the location of all weights going on board a vessel under construction are properly recorded, the task of plotting a curve of weights is a simple one. A typical curve of weights for a battleship is shown in Fig. 13 by an irregular line. Since all vertical forces acting on a ship are desir-

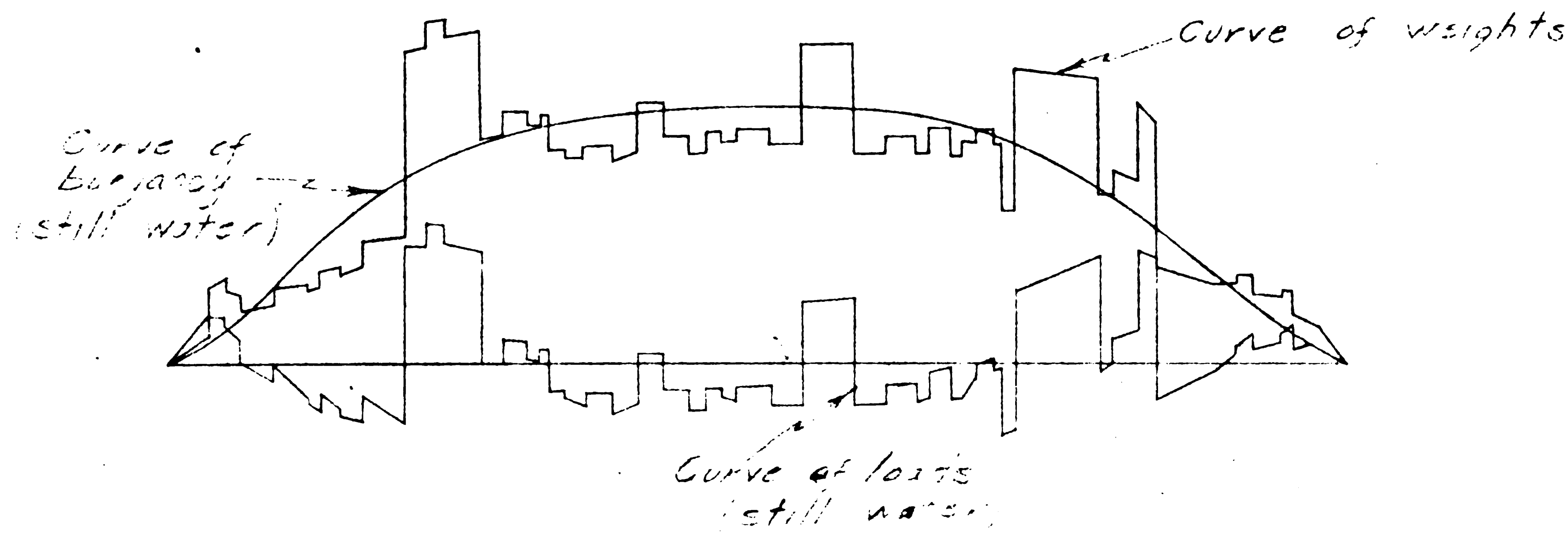

Curve of weights
Curve of buoyancy (still water)
Curve of loads (still water)

Fig 13

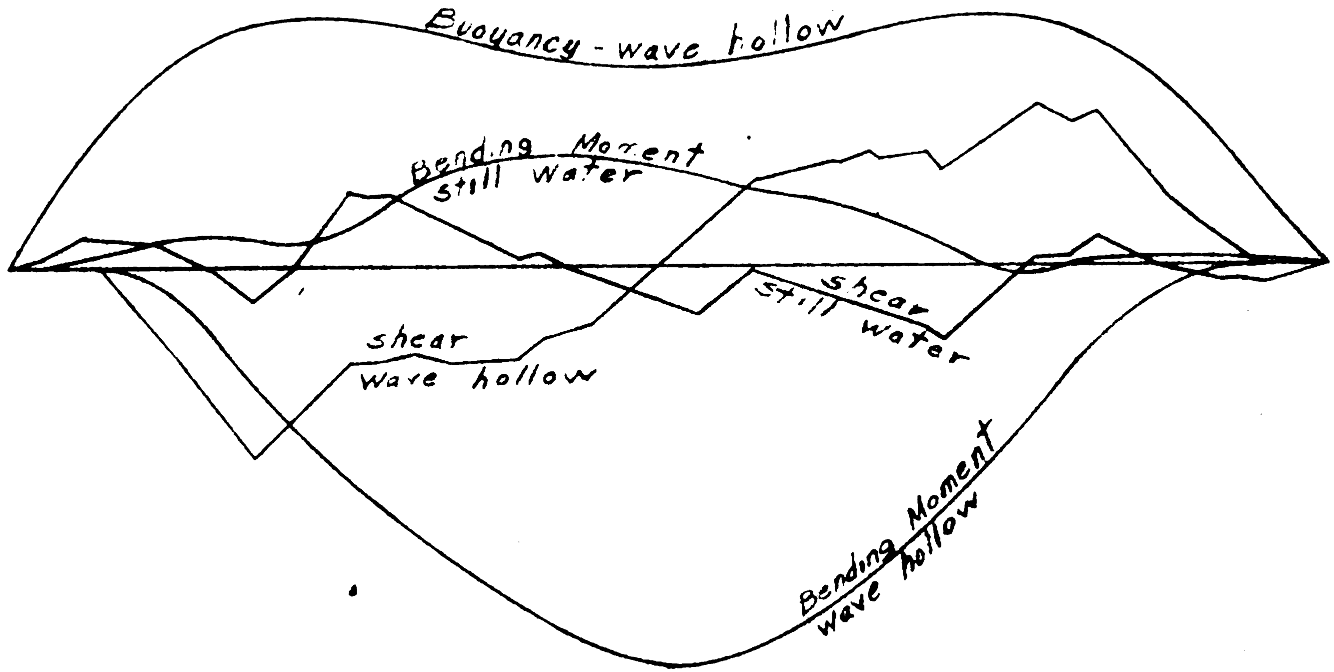

Buoyancy - wave hollow
Bending Moment
still water
shear
still water
shear
wave hollow
Bending Moment
wave hollow

Fig. 14

ed, the buoyant effect of the water must be taken into consideration and this is best done by drawing a curve so that the ordinate at any point shows the upward force of buoyancy at that point. Although the curve of weights will never change while a ship is at sea, except for the amount of fuel in the tanks, the curve of buoyancy is constantly changing. This may easily be seen by comparing the curve of buoyancy for still water in Fig. 13 with that for the vessel in a wave hollow as indicated in Fig. 14. Now since we have the buoyant force acting upward and the weight acting downward, the graphical difference between these two curves will give a curve, any ordinate of which represents the effective load per foot of length at that point. The load per unit length indicated by this curve deals with infinitesimal lengths but the values indicated by the curve at any point are as accurate as necessary and any strength calculations may be made from these values. Since the total upward force must equal the total downward force in order for the vessel to remain in equilibrium, the area under the curve of weights must equal the area under the curve of buoyancy. Otherwise the ship would sink. If the ordinate for the curve of weights is the larger, the ordinate for the curve of loads is considered to be positive. This curve crosses the base line at points where the weight per unit length is equivilent to the buoyancy per unit length. The curves of Fig. 13 are plotted for a vessel floating in still water. The curve of loads for a vessel in a wave hollow is ommitted from Fig. 14 for purposes of clear-

ness, but it obviously is continually changing since the curve of buoyancy is continually changing.

The next step in finding the maximum stresses is the determination of the shearing forces acting on the hull and for this purpose it is convenient to assume half the ship to be a cantilever for which the shearing force at any section may be determined all the loads between that section and the end of the cantilever. This is true since the difference between the load and the buoyancy at any point leaves a free force which tends to move that section of the ship up or down alongside adjacent sections. In order to find the shear at any section a planimeter may be used to measure the area under the curve of loads. We know from Strength of Materials that the area under this curve beginning at the end of the ship and extending to any particular section is proportional to the shearing force at that abcissa. These areas for various sedtions at certain intervals are found and the shear is computed by dividing the area by the value of the abcissa. These values may then be plotted to obtain the shear curves shown in Fig. 14. Shear curves are shown both for a ship in still water and for a ship in a wave hollow or sagging condition. Knowing the probable maximum shear which will occur at any point in the vessel, the proper structural members may now be designed for shear.

Shear, however, is not the only internal stress which must be considered in the design of a ship since it will also undergo a bending moment. In order to find the bending moment at any section of the ship the same principle as for the case of a simple beam is again used. The curves of bending moment as shown in Fig. 14 are obtained from the curves of shearing forces by integrating. Or, expressed mathimatically:

$$M = \int s \, dL. \qquad (23)$$

Since this is an impossibility the areas under the shear curves must be found by means of a planimeter. Starting with the bending moment at one end of the ship as zero, the change in bending moment up to any section in question is merely equal to the area under the shear diagram between the end of the ship and that particular section. Various areas are then found at sections at different intervals and the curve of bending moments is obtained. It may be noted that, as in the case of a simple beam, the maximum bending moment occurs at the point where the shear curve crosses the base line. In the determination of maximum bending moments and shear these curves must also be plotted for the ship on the crest of a wave or in a hogging condition. These curves are not shown in Fig. 14.

The shear or bending moment curves for a ship in a hogging or sagging condition are found in the same manner as outlined above for a ship in still water. They should also be found for a ship coming off the ways when she is being launched. In order to plot the curve of buoyancy, the displacement in tons per foot of length is calculated by finding the area of several sections of the ship in square feet, multiplying by 1 for a 1 foot length and dividing by 35 cubic feet per ton of salt water to obtain the buoyancy in tons per foot of length. These quantities are laid off and a curve drawn through them giving the curve of buoyancy.

Since the largest probable bending moment occuring in a ship as well as its location are now known from the bending moment curves it becomes necessary to find the moment of inertia of that section of the ship before the beam flexure formula may be applied. The first step is to cut the ship at the desired section and take moments of the areas of all structural members, machinery, etc. at this section about the keel and divide by the total area to obtain the distance of the center of gravity or neutral axis from the keel. Then, to find the moment of inertia about this neutral axis, multiply the area of each section by the square of its distance from the neutral axis and sum the results to find the total moment of inertia. If, however, the moment of inertia of any individual member about its centroid is large, then this amount must be added to the total moment of inertia of the section. In making calculations for moment of inertia deductions for rivet holes should be made whether the section is in either tension or compression.

After the bending moment at a given section and the moment of inertia for that section have been determined the maximum stresses at the top or bottom of the ship, as the case may be, may be found from the familiar formula:

$$s = \frac{Mc}{I} \quad (24)$$

In the use of this formula care must be taken to see that the proper units are used. Moments are usually given in ton-feet and should be reduced to pound-feet giving a final answer of pounds per square foot for the stress, which may then be converted to pounds per square inch if so desired. These stresses vary in different vessels and for large ones a stress of 20,000 pounds per square inch is considered safe for a standard wave length with the bow and stern supported. If an excessive stress is indicated members must be added near the member which has the greatest stress.

The weather deck or main deck is often referred to as the strength deck for this is the highest continuous deck on the ship and consequently undergoes the largest stress in the same manner as does the upper chord of a bridge girder. If the second or third deck is made continuous as in the case of large ships these will take some of the stress although not as great a share as the main deck. These stresses are considered in the design of lower continuous decks and should not be overlooked after severe structural damage because if the main deck is damaged then the second deck becomes the strength deck. Decks above the main deck are usually discontinuous and therefore are not strength decks. Since they are not designed to take the load of the strength deck, expansion joints are provided to avoid buckling of deck houses and superstructure.

The shear and bending moment curves are of value not only in finding the maximum stresses occuring in a vessel but also in finding the stress that may be expected at any depth in the ship. It is mainly by the use of these curves that the size of all structural members may be properly proportioned to the stress carried and an economical design may be effected. It is, therefore, good design to reduce the size of structural members beyond the middle half-length of the ship. The members at the bottom of the ship must not be overlooked since they too will undergo a maximum stress for the portion of the ship below the neutral axis. These include the top row of plating called the sheer strake and the curved plating at the turn of the bilge called the bilge strake. The proper thickness of this plating must be determined by consideration of the maximum stresses occuring at the bottom of the ship.

Another type of stress encountered by a ship is that due to hydrostatic pressure. Although the horizontal pressures of water exerted on each side of the ship cancel each other the force still acts upon the hull. Now if the hull of the ship is ruptured below the waterline and flooding follows, these hydrostatic pressures will be transmitted to the bulkheads. These bulkheads will, therefore, require stiffening as well as thickening as their depth in the ship increases. Besides hydrostatic pressure the weight of every object on the ship, solid or liquid, rests at some point on a deck and the load of these various must be supported and transmitted to the hull of the vessel where it is resisted by hydrostatic pressure. In order to prevent excessive stress these loads must be distributed over a wide area by means of structural bulkheads and girders. These factors must all be taken into consideration in the design of these members.

In addition to these static loads the ship may be subjected to dynamic or impact loads due to pitching and pounding, wind pressures, collisions, gunfire recoil, and several other types of forces. Some conditions of loading may put a torsional stress on the structure but in well designed ships these stresses are seldom serious in magnitude. During design allowance should also be made for a ship in dry-dock supported only by blocking under its bottom where the transverse bulkheads may come under excessive stresses due to longitudinal shear. Any or all of the above mentioned stresses may result in excessive stress on certain members and if such is the case, additional structural members may be required in these regions.

These various types of stress to which a vessel is subjected require strong bulkheads, the framing of which should consist of Z bars or other rolled or built up forms. These should be adequately secured at their ends to the framing of the ship.

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