DESIGN, CONSTRUCTION, AND CALIBRATION OF A SIXTY-INCH INTEGRATING SPHERE PHOTOMETER

THESIS FOR THE DEGREE OF M. S.

Walter Alfred Hedrich

1932

THESIS

Photometry

Title: Integrating sphere photometer

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Electrical engineering

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OF A SIXTY-INCH

INTEGRATING SPHERE PHOTOMETER

A Thesis

Submitted to the Faculty

of

Michigan State College

of

Agriculture and Applied Science

by

Walter Alfred Hedrich

Candidate for the Degree

of

Master of Science

June 1932

THESIS

ACKNOWLEDGMENT

The writer is indebted to the members of the Department of Electrical Engineering and Engineering Shops for their worthy cooperation in this project. He also wishes to express his gratitude to members of the Department of Mathematics whose services have been invaluable in the preparation of the theoretical treatment of the subject. For the generous opinions, advice, and criticisms contributed by Dr. J. E. Powell and Dr. W. S. Kimball of this department I am particularly grateful.

My gratitude is especially due to Professor L. S. Foltz of the Department of Electrical Engineering for helpful criticisms and generous attention in checking the manuscript.

W. A. H.

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INTRODUCTION

This thesis comprises the design, construction, and calibration of a sixty-inch integrating sphere photometer. Unless otherwise indicated, the expression "sphere photometer" in this work shall invariably refer to the sphere as an auxiliary unit in the photometry of light sources.

If the sphere photometer is to constitute part of the photometric equipment of a laboratory, it is desirable to design and dimension the sphere in accordance with the principles of laboratory precision and commercial practice. The features incorporated in the design of this photometer afford the possibility of photometering over a wide range of luminous intensities as associated with various types of commercial lamps and lighting units. The sphere must provide for rapid handling of lamps in quantity tests. As a mechanical unit, it should be sufficiently flexible to permit ready replacement of parts, ease of conveyance, and should combine lightness with stability.

It is the purpose of this work to attain a desirable balance in the design, construction, and calibration of a sphere intended to meet the requirements of laboratory and commercial tests.

GENERAL CONSIDERATIONS

Historical Development

Early determinations of total luminous flux from a light source were made without the use of integrating devices. The point by point method, consisting of the measurement of candle power of the source at different angles in a vertical plane, was then employed. From these candle power values the total luminous output was obtained by summation of the various zonal fluxes previously computed. The measurement of candle power at various angles in a vertical plane through the source is still in vogue today, largely for the purpose of determining the distribution of luminous intensity. The unsymmetrical nature of the source makes it necessary to rotate the lamp about its vertical axis, or to conduct measurements in a number of vertical planes through the source. Very accurate results can be obtained by the point by point method. Equally reliable results obtained through the use of integrating devices have emphasized the laborious aspect of the point by point method, and unquestionably justified the use of the integrating sphere photometer.

A number of photometric appliances were devised and used for obtaining the mean spherical candle power of a lamp. They were designed to give the M. S. C. P. in a single reading. Disadvantages inherent to these forms of integrators and their practical limitations lead early investigators in this field to turn their attention to the hollow sphere, which was first investigated by Dr. Ulbricht in the year 1900. At that time Dr. Ulbricht was uninformed of an earlier publication on the dif-

fusion of light, which appeared in 1893 in the Phil. Mag. and was written by Dr. Sumpner. His photometric calculations revealed the striking conclusion that the illumination on the inner wall of the hollow sphere is everywhere the same, due to diffusely reflected light. Obtained under the hypothesis of Lambert's cosine law of emission, this result was the theoretical foundation for the development of the sphere photometer. Apparently Dr. Sumpner did not make specific use of this important deduction, however, as applied to photometric considerations and particularly to the problem of measuring the M. S. C. P. by means of the hollow sphere. Contemporary investigators who did much to promote valuable experimental work on spheres of different sizes were Bloch, Corsepius, Marchant, Dyhr, and Monasch. There has been very little change in, or addition to the theory of the integrating sphere beyond that embodied in the beginning treatment of the subject. Much has been done, however, to make the sphere better suited to practical application. In recent years the sphere has served remarkably well in connection with other methods of photometry which are quicker and more reliable than the long established visual comparison process.

Description of the Sphere Photometer

General.

The integrating sphere is, as its name implies, an integrating device. It integrates the illuminating effect of any source of light mounted or suspended within its enclosing spherical wall. The latter must be as nearly perfectly diffusing as possible. A small window of diffusion glass replaces the equivalent surface area on the inner wall of the sphere. According to the theory of the sphere, if a source of light be placed anywhere within the enclosure, every point on the inner surface will receive both direct and reflected light. If a white, diffusely reflecting, and opaque screen be interposed between the source of light and the window in such a manner that the latter is shielded from direct radiation only, the illumination on the window will be due to diffusely reflected light, and will be directly proportional to the mean spherical candle power of the source. Since the total luminous output of the source is proportional to the M. S. C. P., the brightness of the window is a measure of the total lumens emitted by the source. Probably the most general use of the sphere photometer is confined to the measurement of this total luminous output, although it serves a number of other practical purposes which will be mentioned later.

The mathematical development of the theory assumes an empty sphere with continuous inner surface obeying Lambert's cosine law of emission. In order to realize a practical development, certain variations are unavoidable. They are caused by the necessary use of screens, imperfect diffuse reflection from sphere surface, selective absorption by paint, presence of non-luminous bodies within the sphere, imperfect diffusion of window, position of lamps and luminaires in the sphere, inconstancy of wall paint, and difference in window and wall absorption factors. These are some of the most important sources of error in the sphere. While it is possible to minimize these errors through the selection of suitable methods of measurement or the application of general corrective measures to overcome such departures, it is nevertheless essential to investigate the order and magnitude of some of these sources of error inasmuch as they may become appreciable in certain photometric measurements. This quantitative treatment of the matter of errors is discussed in the section on theoretical considerations.

Screens.

The screen, which is necessary for the proper functioning of the sphere photometer, reduces the illumination on the window in two ways. All direct light from the source intercepted by the screen must first be reflected from the latter before it can reach the diffusing wall of the sphere. Hence this flux is diminished by an amount equal to the screen absorption before it illuminates the sphere, from whence it is reflected to the window. The screen acts as a secondary source of lower intensity. The screen also obscures a portion of the surface of the sphere from the window. Light from this hidden area must be reflected to some other portion of the surface which re-reflects it to the window. Such light flux reaching the window has suffered greater absorption and in consequence the window illumination due to this component is lower. Since the screen and the inner surface of the sphere have the same diffusing quality, that side of the screen facing the window will reflect light from the sphere to the window and thus compensate for some of the loss. By increasing the size of the sphere, the screen error may be sufficiently reduced to render its effect negligible. The ratio of screen surface to sphere surface is less. The screen absorption is less in the same proportion. Two other important considerations in reducing the screen error are its dimensions and relative position in the sphere. This matter is given quantitative attention in the discussion under "Theoretical Considerations."

Non-luminous Bodies within the Sphere.

Included in this classification are screens and their fittings, lamp supports, lamp fittings, shades and reflectors. All of these will absorb part of the direct light from the source. The error involved due to the presence of these bodies is materially reduced by giving all non-luminous bodies except the source and accessories integral to it, a matte white finish like that used on the inner wall of the sphere. Non-uniform diffuse and specular reflection due to lamp shades, reflectors, etc., constitute a departure from ideal conditions. This and other errors resulting from causes heretofore outlined are minimized by following the substitution method in photometering.

Sphere Paint.

The material used to render the inner surface of the sphere diffusely reflecting must closely approximate theoretical requirements. The latter demand non-selective absorption, perfect diffusing quality, and a uniform and minimum absorbing power. Departure from these specifications must be expected. Furthermore, the diffusing paint should be permanent over a long period of time and not appreciably affected by moisture or moderate changes in temperature. From time to time the surface must be refinished, as it becomes soiled due to collection of dust and foreign matter. Refinishing should not be done over the soiled surface. A sphere coating made up of a permanent oil base should form the foundation for the finishing paint. The latter should be readily removable when soiled, by the application of a suitable and quick-acting solvent. It is obvious that such a finishing paint will expedite the refinishing process. It is not warranted, however, unless it possesses to a high degree the qualities prescribed by theoretical requirements.

Window.

The window consists of an appropriate diffusion glass, which may be located almost anywhere on the inner surface of the sphere. It is desirable to have the window flush with the inner surface in order to minimize the error that otherwise would exist through the discontinuity. In practice, the offset is small. In order to facilitate measurements with a bar photometer, the window is generally located so that the line through its center and normal to the surface lies in the equatorial plane. This position of the window affords greater flexibility in obtaining measurements with various types of photometric devices. The window opening is small compared to the inner surface of the sphere. Glass used for this purpose must be uniformly and highly diffusing as well as non-selective in absorbing power. It is important that the interior surface be matte, preferably of the same diffusing quality as that of the sphere paint. The window error can be minimized by careful workmanship and selection of glass.

Method of Obtaining Photometric Balance.

Two general methods of obtaining a photometric balance are customarily classified as the "Direct" and "Substitution" procedures. The former, sometimes called the "Direct Comparison" method, involves the direct comparison of the test lamp with a calibrated standard lamp. The method is simple, but open to objection owing to the possibility of errors which cannot be eliminated from the measured results. Although the errors inherent to the integrating sphere become less as the diameter of the latter is increased, the substitution method should be used to insure maximum accuracy under the most unfavorable conditions. This method involves the use of a third light source known as a comparison lamp. It must be a constant source but need not be a calibrated standard. A standard lamp and a test lamp are interchanged as sources of illumination within the sphere. Conditions within the sphere will then have the same effect on the window illumination in both cases. This illumination in each case is compared with the illumination produced on the photometric screen from the comparison lamp. The readings obtained from the settings for photometric balance in each case are evaluated by the application of the Law of Inverse Squares, whereby the candle power of the test lamp is obtained in terms of that of the standard lamp. In some cases the scale on the photometer bar is graduated according to the Inverse Square Law or in lumens. This is done to simplify the work. The use of either method for the purpose of determining the M. S. C. P. or total luminous output of the source, assumes a knowledge of the constant of the sphere as determined by calibration.

When the candle power of the test lamp is much in excess of that of the comparison lamp, the quantity of light transmitted from the window to the photometer head is reduced by means of an iris diaphragm. This is, in effect, a variable aperture for control of the light flux from the window.

Frequently the light from the test source and that from the comparison lamp show slight difference in color. In making a photometric balance this color difference is a disturbing factor, often preventing satisfactory comparison. Colored glass plates, known as color filters, of known transmission coefficients for the ranges of spectrum over which they are to be used, are interposed either between the window and the photometric head or between the latter and the comparison lamp. Color filters and iris diaphragms serve to obtain a well defined and reliable photometric balance.

THEORETICAL CONSIDERATIONS

Theory of the Sphere

Illumination Due to Diffusely Reflected Light.

Let r represent the radius of the sphere. Assume inner surface of sphere to give perfectly diffused and non-selective reflection, and let L be a source of light of mean spherical candle power I_o. Let ϕ represent the total lumens flux emitted by L.

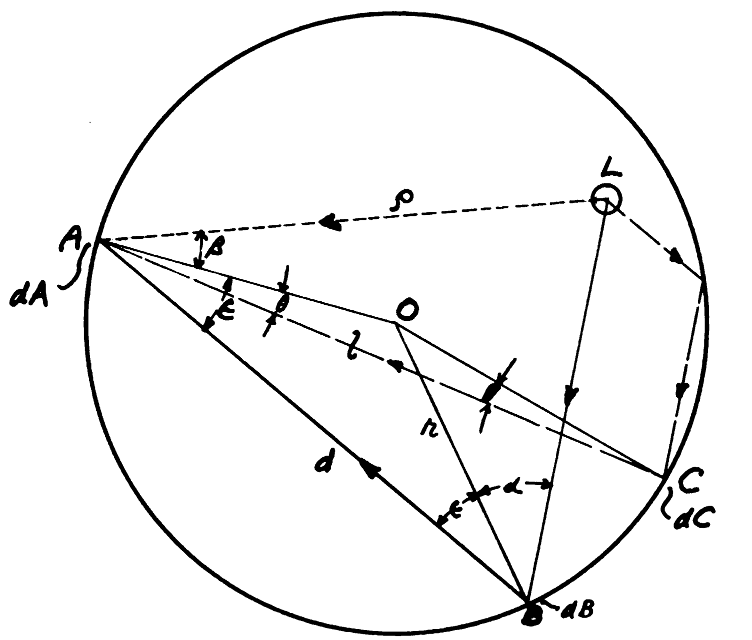


Fig. 1

Then $\phi = 4\pi I_o$ regardless of the candle power distribution of L. It is desired to investigate the illumination at any point A on the surface of the sphere.

Let B represent any other point on the surface, and consider the differential element of area dB at B. Since ϕ is the total flux from L, let $d\phi$ represent the lumens flux incident on dB. The illumination at B is $E_B = \frac{d\phi}{dB}$ lumens per square centimeter (phots). By hypothesis there is no specular reflection, hence let k represent the diffuse reflection factor, where k is defined as the ratio of light flux diffusely reflected from a surface to that incident on it. Then $\frac{d\phi}{dB} \cdot k$ represents lumens per sq. cm. reflected from dB. Since the surface of the sphere gives a reflected flux distribution following Lambert's Cosine Law of Emission, this quantity represents the brightness of the surface at B.

$b = \frac{d\phi}{dB} \cdot k$ lamberts (emitted lumens per sq. cm.) and is the same in all directions from within the sphere. Then

$b = \frac{1}{\pi} \frac{d\phi}{dB} \cdot k$ is the brightness in candles per sq. cm.

$k \cdot d\phi = d\phi_r$ gives the reflected flux from dB, and the brightness at B may also be formulated according to

$b = \frac{1}{\pi} \frac{d\phi_r}{dB}$ candles per sq. cm.

The luminous intensity of dB in direction BO is

$$I_{BO} = \frac{1}{\pi} \frac{d\phi}{dB} \cdot k \cdot dB = \frac{1}{\pi} d\phi \cdot k \quad \text{candles.}$$

Luminous intensity of dB in direction BA is

$$I_{BA} = \frac{1}{\pi} \cdot d\phi \cdot k \cos \varepsilon \quad \text{candles.}$$

The Inverse Square Law gives the illumination at A:

$$E_{A_1} = \frac{d\phi \cdot k \cdot \cos\varepsilon \cdot \cos\varepsilon}{\pi d^2} \quad \text{lumens per sq. cm. (phots),}$$

but $d = 2r\cos\varepsilon$,

therefore $E_{A_1} = \dfrac{d\phi \cdot k \cdot \cos^2\varepsilon}{4\pi r^2 \cos^2\varepsilon} = \dfrac{d\phi \cdot k}{4\pi r^2} = \dfrac{d\phi \cdot k}{S}$,
where $S = 4\pi r^2$ is the area of the sphere.

E_{A_1} is the illumination at A due to the flux $d\phi$ incident on B and once reflected. The expression for E_{A_1} is obviously independent of ε , and hence E_{A_1} is the same for all positions of A. It follows that <u>all elements of area on the surface of the sphere are equally illuminated by diffusely reflected light emitted from any surface element on the sphere and obeying Lambert's Cosine Law of Emission.</u>

That this is true for twice and multiply reflected light may be seen by considering the illumination at A due to twice reflected light. Consider once reflected light at any point C on the surface of the sphere. Some light flux from C will be reflected diffusely along the path CA. From the above discussion, the illumination at C due to once reflected light is

$$E_C = E_{A_1} = \frac{d\phi \cdot k}{4\pi r^2}$$

Brightness of dC is $b = \dfrac{d\phi \cdot k^2}{4\pi r^2}$ lamberts.

Expressed in candles per sq. cm., $b = \dfrac{d\phi \cdot k^2}{4\pi^2 r^2}$.

Intensity of illumination (candle power) of dC in direction CA is

$$I_{CA} = \frac{d\phi \cdot k^2}{4\pi^2 r^2} \cdot \cos\theta \cdot dC$$

Illumination at A due to twice reflected light is

$$E_{A_2} = \frac{d\phi \cdot k^2 \cos\theta \cdot \cos\theta \, dC}{4\pi^2 r^2 \cdot l^2} \text{ phots.}$$

Since $l = 2r\cos\theta$, therefore

$$E_{A_2} = \frac{d\phi \cdot k^2 \cdot \cos^2\theta \cdot dC}{4\pi^2 r^2 \cdot 4r^2\cos^2\theta} = \frac{d\phi \cdot k^2 dC}{(4\pi r^2)^2} = \frac{d\phi \cdot k^2 dC}{S^2} \text{ phots.}$$

Since θ does not appear in the final expression for E_{A_2}, the illumination at A due to twice reflected light is independent of the position of A. E_{A_2} is the illumination at A due to twice reflected flux $d\phi$ from dC. An extension of the above to multiply reflected light leads to the following important result:

<u>All points on the surface of a sphere are equally illuminated by diffusely reflected light emanating from every infinitesimal element of that surface.</u>

The total illumination at A due to all light flux from L once reflected is readily obtained as

$$_tE_{A_1} = \int_0^{\phi} \frac{d\phi \cdot k}{S} = \phi\frac{k}{S} \qquad \text{phots.}$$

The total illumination at A due to all flux from L twice reflected is

$$_tE_{A_2} = \int_0^{\phi}\int_0^{S} \frac{k^2 d\phi \, dC}{S^2} = \int_0^{\phi} \frac{k^2 S}{S^2} d\phi = \frac{k^2}{S} \cdot \phi$$

Similarly,

$$_tE_{A_3} = \frac{k^3}{S}\phi$$

$$_tE_{A_4} = \frac{k^4}{S}\phi$$

— — — — — — —

$$_tE_{A_n} = \frac{k^n}{S}\phi$$

The resultant illumination at A due to an infinitude of reflections is

$$ {}_{R}E_{A} = \frac{k\phi}{S} + \frac{k^2\phi}{S} + \frac{k^3\phi}{S} + \cdots + \frac{k^n\phi}{S} + \cdots $$

$$ {}_{R}E_{A} = \frac{k\phi}{S}\left(1 + k + k^2 + k^3 + \cdots + k^n + \cdots\right) $$

It is known that the infinite series within the parenthesis converges for all values of k within the interval $-1 < k < 1$ and defines the function $f(k) = \frac{1}{1-k}$ for all values of k within this interval. In practice, k will always lie in the interval $0 < k < 1$;

therefore, $ {}_{R}E_{A} = \frac{k\phi}{S}\cdot\frac{1}{1-k} = \frac{\phi}{4\pi r^2}\cdot\frac{k}{1-k}$ phots.

Discussion of Results.

Introducing the direct illumination from source L, the result is

$$ {}_{R}E'_{A} = \frac{I_{\beta}\cos\beta}{\rho^2} + \frac{\phi}{4\pi r^2}\cdot\frac{k}{1-k} \quad \text{phots.} $$

The illumination at A due to direct rays from L is governed entirely by the photometric distribution of light from the source and its relative location within the sphere.

The significance of the expression $ {}_{RA}E = \frac{\phi}{4\pi r^2}\cdot\frac{k}{1-k}$ is at once apparent. The illumination at any point on the surface of the sphere due to diffuse reflected light is directly proportional to the total lumens emitted from the source. (Perfect Diffusion)

We may write $ {}_{RA}E = \frac{4\pi I_0}{4\pi r^2}\cdot\frac{k}{1-k} = \frac{I_0}{r^2}\cdot\frac{k}{1-k}$ phots from which it is seen that the illumination is proportional to the mean spherical candle power of the source. If at the point A a small window of diffusion glass is inserted and shielded from the direct rays of light coming from L, the illumination of the window will be directly proportional to the total luminous output of the source. Thus by measuring the illumination at the window, either the mean spherical candle power or the total luminous output of the source can be readily calculated. It is likewise significant to note that ${}_{R}E_{A}$ is the same regardless of the position of the source L within the sphere. In practice, however, the source should not be made to approach the wall of the sphere too closely; otherwise, the error introduced in consequence of later considerations may become appreciable. A single measurement of ${}_{R}E_{A}$, carefully executed, is sufficient to give a very fair value of the M. S. C. P. or total luminous output of the light source under consideration. Certain refinements in construction and methods serve to minimize the error within the limits commensurate with the size of sphere employed.

The following consideration will serve to clarify the simple relations involved in the direct and reflected illumination. The average illumination due to direct light from L is

$$E_{AV} = \frac{\phi}{4\pi r^2} = \frac{I_0}{r^2} \qquad \text{lumens per sq. cm. (phots)}$$

The average total illumination is

$${}_{T}E_{AV} = E_R + E_{AV} = \frac{I_0}{r^2}\cdot\frac{k}{1-k} + \frac{I_0}{r^2} = \frac{I_0}{r^2}\cdot\frac{1}{1-k} \quad ,$$

in which E_R is synonymous with ${}_{R}E_{A}$ as employed in the above.

Assuming an average k, for example, k = 0.80,

$$E_R = \frac{I_0 \cdot 0.80}{r^2(1-0.80)} = 4\frac{I_0}{r^2} \qquad \text{lumens per sq. cm.}$$

$${}_TE_{Av} = 4\frac{I_0}{r^2} + \frac{I_0}{r^2} = 5\frac{I_0}{r^2} \qquad \text{lumens per sq. cm.}$$

The average total illumination would be five times as great as the illumination of the wall if the latter were black with a zero reflection factor. Again by placing the source L at the center of the sphere, the average direct illumination would be $E_{Av} = \frac{I_0}{r^2}$ and in the event L is a source of uniform intensity in all directions, the illumination on the wall would be the same at all points and equal to

$$E_U = \frac{I_0}{r^2} \qquad \text{lumens per sq. cm.}$$

Barring invisible radiation, the source supplies energy in the form of visible radiation sufficient to balance the loss due to absorption. If a is the absorption factor for the inner surface of the sphere, the total absorbed light flux is ϕ , and the total reflected flux is $\phi \cdot \frac{1-a}{a}$. These deductions follow from elementary considerations.

	Light flux absorbed		Light flux reflected
First absorption	$a\phi$	First reflection	$(1-a)\phi$
Second "	$a(1-a)\phi$	Second "	$(1-a)^2\phi$
Third "	$a(1-a)^2\phi$	Third "	$(1-a)^3\phi$
nth "	$a(1-a)^n\phi$	nth "	$(1-a)^n\phi$

Total absorbed flux = $a\phi + a(1-a)\phi + a(1-a)^2\phi + \cdots + a(1-a)^n\phi + \cdots$

" " " = $a\phi\left[1 + (1-a) + (1-a)^2 + \cdots + (1-a)^n + \cdots\right]$

Let S_n represent the sum of the first n terms of the series in the parenthesis. Then $S_n = \frac{1}{a} - \frac{(1-a)^n}{a}$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{1}{a} - \lim_{n\to\infty}\frac{(1-a)^n}{a}$$

Since $(1-a) < |1|$, $\lim_{n\to\infty}\frac{(1-a)^n}{a} = 0$,

Therefore, $\lim_{n\to\infty} S_n = \frac{1}{a}$

Hence, total absorbed flux = $a\phi \cdot \frac{1}{a} = \phi$

Similarly,

total reflected flux = $(1-a)\phi + (1-a)^2\phi + \cdots + (1-a)^n\phi + \cdots$

" " " = $(1-a)\phi\left[1 + (1-a) + (1-a)^2 + \cdots + (1-a)^n + \cdots\right]$

" " " = $\phi\frac{1-a}{a}$

The source supplies energy to overcome the loss ϕ, which is constant as long as the quality of the reflecting medium and the luminous output of the lamp remain the same.

The window in the wall of the sphere may have an area of A sq. cm. The flux incident on the window is $\phi_w = E_R \cdot A$.

$$\phi_w = \frac{\phi A}{4\pi r^2}\cdot\frac{k}{1-k} = \frac{\phi A}{S}\cdot\frac{k}{1-k} \text{ lumens}$$

Brightness of window as viewed from outside is b_w.

$b_w = E_R \cdot J$ lamberts, in which J is the coefficient of transmission for diffusion glass.

$b_w = \frac{E_R \cdot J}{\pi}$ candles per sq. cm.

$$b_w = \frac{\phi}{S}\cdot\frac{Jk}{\pi(1-k)} = \phi\frac{Jk}{4\pi^2 r^2(1-k)} = K\phi \text{ candles per sq. cm.}$$

K is the so-called constant of the sphere. The constant of the sphere as determined by calibration can be formulated mathematically in a number of ways, depending upon whether the brightness or the luminous intensity of the diffusion glass window is measured. It is neither necessary to know or to determine the values of k and Υ, as these quantities are embodied in K as determined by calibration.

$K = \frac{b_w}{\phi} = \frac{b_w}{4\pi I_o}$, in which b_w is expressed in candles per sq. cm., ϕ represents total luminous output, and I_o is the M. S. C. P. of light source.

Again, since $I_w = b_w \cdot A_w$, $K = \frac{I_w}{A_w \phi} = \frac{I_w}{4\pi A_w I_o}$, in which A_w is the area of the window in sq. cm., and I_w is the normal candle power of the window.

However, the constant of the sphere may be calculated to conform with the simple ratios $\frac{b_w}{\phi}$, $\frac{b_w}{I_o}$, $\frac{I_w}{\phi}$, $\frac{I_w}{I_o}$; whence it should be observed that the K's are essentially different. K may also be expressed as the reciprocal of these ratios. Having obtained K by measuring the brightness of the window as given by a source of known total lumens ϕ, the luminous output of a test source may be readily obtained as ϕ_t, in which $\phi_t = K b_t$, where b_t is the brightness of the window due to the test lamp, expressed in candles per sq. cm.

Effects of Non-Luminous Bodies within the Sphere upon Formulations Deduced under Ideal Conditions

The presence of non-luminous bodies in the sphere such as the screen, lamp fixtures, supporting devices for lighting units, etc., must have an effect on the uniform illumination produced on the diffusing surface within, for these bodies absorb some of the direct as well as reflected light flux. Point sources of illumination must be excluded from the discussion, for all practical sources of light have finite dimensions. There follows a discussion of the effect of non-luminous bodies within the sphere upon diffusely reflected light.

Non-Luminous Bodies within the Sphere and their Effect upon Diffusely Reflected Light.

Consider a non-luminous diffusely reflecting body N screened from all direct rays of light, and let the surface area of N be U. Its reflection factor is K_u . If N occupies any position in the sphere such that it receives only reflected light, it will absorb a small amount of light from each of the infinite reflections intercepted by its outer surface. See Fig. 2. Let dU represent a differential element of area on the surface U. A simple photometric calculation will disclose that any surface whose dimensions are negligible in comparison with the distance from the source of light to the surface will have an illumination $E' = \pi b$, regardless of the orientation or position of this element of surface dU within the sphere. The quantity b represents the uniform brightness of the source; namely, the interior surface of the sphere. It may be well to establish the relation $E' = \pi b$ now, and then proceed with the discussion.

Referring to Fig. 2, consider a sphere of inner radius r, and whose inner surface has a uniform brightness b. The brightness of any element of area on the inner surface of the sphere will, therefore, be the same when viewed from every direction, and its luminous intensity

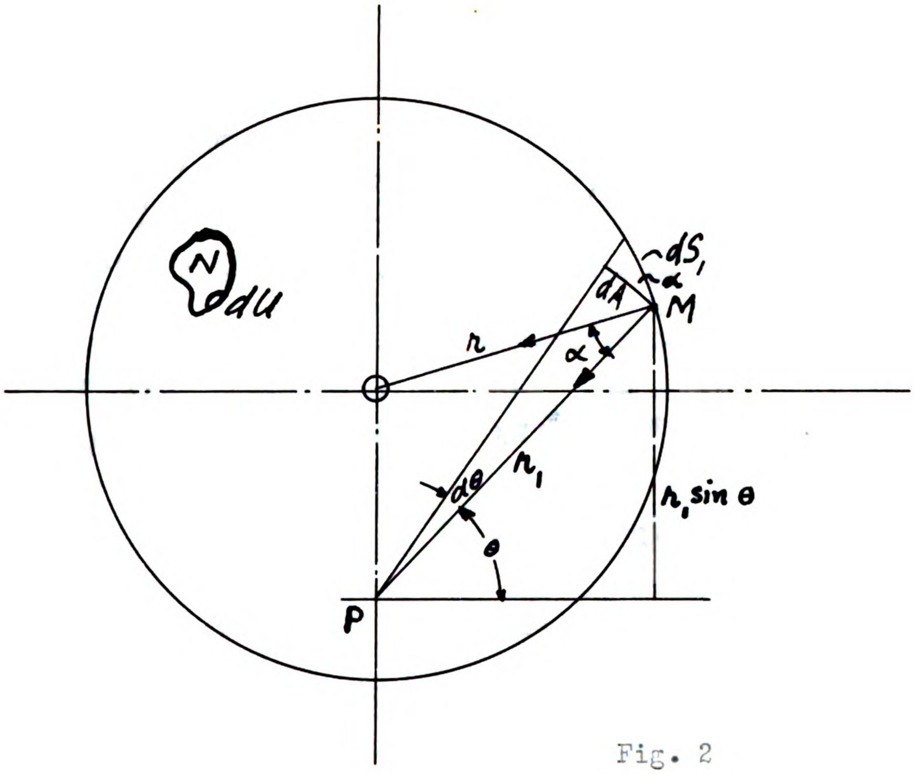


Fig. 2

will vary as the cosine of the angle of emission. Mathematically, $I_\alpha = b \cdot dA$, where I_α is the candle power in any direction α from the normal to the surface dS_1, and dA is the projected area $dS_1 \cdot \cos\alpha$. Let P be any point within the hollow sphere and dP a differential element of area free to assume any fixed position at P. Since $bdS_1 = I_n$ is the normal candle power of dS_1 in direction MO, $bdS_1 \cos\alpha = I_\alpha$ is the candle power of dS_1 in direction MP. But $dS_1 \cos\alpha = dA$, therefore, $bdA = I_\alpha$, and

$\frac{b \cdot dA \cdot \cos\theta}{r_1^2} = dE_p'$ is the illumination or incident flux density at P due to dS_1.

Since $dA = r_1 d\theta \cdot r_1 \sin\theta \, d\phi = r_1^2 \sin\theta \, d\theta \, d\phi$, therefore

$$\frac{b \cdot dA \cdot \cos\theta}{r_1^2} = \frac{b \cdot r_1^2 \sin\theta \cos\theta \cdot d\theta \cdot d\phi}{r_1^2} = b \cos\theta \sin\theta \, d\theta \, d\phi = dE_p'$$

Hence $dE_p' = b \cos\theta \cdot d\omega$, where $\frac{dA}{r_1^2} = d\omega = \sin\theta \, d\theta \, d\phi$ is the solid angle subtended by dA. If E_p' represents the total illumination received on the right side of dP, then

$$E_p' = \int dE_p' = \int_0^{\frac{\pi}{2}}\int_0^{2\pi} b \sin\theta \cos\theta \, d\theta \, d\phi = 2\pi b \int_0^{\frac{\pi}{2}} \sin\theta \cos\theta \, d\theta = 2\pi b \left[\frac{\sin^2\theta}{2}\right]_0^{\frac{\pi}{2}}$$

or $E_p' = \frac{1}{2} \cdot 2\pi b = \pi b$ incident lumens per sq. cm.

The incident flux density on the opposite side of dP is

$$E_p' = \int_{\frac{\pi}{2}}^{\pi}\int_0^{2\pi} b \sin\theta \cos\theta \, d\theta \, d\phi = -2\pi b \int_{\frac{\pi}{2}}^{\pi} \sin\theta \cos\theta \, d\theta = -2\pi b \left[\frac{\sin^2\theta}{2}\right]_{\frac{\pi}{2}}^{\pi}$$

$$E_p' = \pi b \quad \text{lumens per sq. cm.}$$

Thus the flux density is the same at every point within the sphere, and hence the illumination or incident flux density will be the same at every point on the outer surface of a body N immersed in the flux within the sphere. This result also holds for radiation of any wave length λ. It is interesting to note the reciprocal case, where the flux emitted per sq. cm. area of uniform brightness b is $E' = \pi b$. This accounts for the existence of the conversion factor π whereby brightness expressed in candles per sq. cm. can be converted to lamberts (emitted lumens per sq. cm.).

Now, if E represents the total uniform incident illumination on the inner surface of the sphere, then

$\frac{Ek}{\pi} = b$ candles per sq. cm. uniform brightness of sphere.

Therefore, $\pi b = Ek = E'$ is the illumination or incident density in lumens per sq. cm. at every point on surface of N,

and $\pi b U = EkU = \phi_N$ is the total flux incident on N in lumens.

Of this, N absorbs $\phi_N(1-k_u) = EUk(1-k_u)$ lumens.

Total flux incident on the sphere to give it an illumination E would be $\phi = 4\pi r^2 E$ lumens. Of this, there is absorbed a flux equal to $4\pi r^2 E(1-k)$ lumens. The total loss of flux due to absorption within the enclosed sphere must be equal to ϕ.

Hence, $4\pi r^2 E(1-k) + EUk(1-k_u) = \phi$. Solving for E,

we have $E = \dfrac{\phi}{4\pi r^2(1-k) + Uk(1-k_u)}$ lumens per sq. cm.

The average direct illumination is $E_{AV} = \dfrac{\phi}{4\pi r^2}$. Therefore,

$$E_R = E - E_{AV} = \frac{\phi}{4\pi r^2(1-k) + Uk(1-k_u)} - \frac{\phi}{4\pi r^2},$$

where E_R is the illumination on the surface of the sphere due to diffuse reflected light. Removing the body N from the sphere, the illumination due to diffuse reflected light would then be

$$E_R = \frac{\phi}{4\pi r^2} \cdot \frac{k}{1-k}$$

Thus the illumination on the wall of the sphere and therefore at the window has been diminished by an amount ΔE_R due to the presence of N.

$$\Delta E_R = \frac{\phi}{4\pi r^2} \cdot \frac{k}{1-k} - \frac{\phi}{4\pi r^2(1-k) + Uk(1-k_u)} + \frac{\phi}{4\pi r^2}$$

$$\Delta E_R = \frac{\phi}{4\pi r^2}\left[\frac{k}{1-k} + 1 - \frac{4\pi r^2}{4\pi r^2(1-k) + Uk(1-k_u)}\right]$$

$$\Delta E_R = \frac{\phi}{S}\left[\frac{k}{1-k} + 1 - \frac{S}{S(1-k) + Uk(1-k_u)}\right]$$

$$\Delta E_R = \left[\frac{1}{(1-k)S} - \frac{1}{S(1-k) + Uk(1-k_u)}\right]\phi$$

It was assumed that b represents the uniform brightness of the sphere, which in practice is not true because no matte surface obeys Lambert's Cosine Law exactly. Nevertheless, the ratio of diffuse reflected light to direct light in the sphere is so large as to render the error due to assumed uniform brightness, negligible. The wall of the sphere can be made to meet the requirements of perfect diffusion to a degree satisfactory for all laboratory and practical measurements.

The last expression above for ΔE_R can be written

$$\text{(1)} \qquad \Delta E_R = \frac{\phi}{(1-k)S}\left[1 - \frac{1}{1 + \frac{Uk(1-k_u)}{(1-k)S}}\right]$$

and shows at once the effect on the window illumination with changes in U. Accessories within the sphere such as screens, fittings, supports, etc., which are not a part of the lamp or lighting unit, are customarily coated with the same diffusing paint as employed for the inner surface of the sphere. For these parts then, $k_u = k$, and the above expression simplifies to

$$\text{(2)} \qquad \Delta E_R = \frac{\phi}{(1-k)S}\left[1 - \frac{1}{1 + \frac{Uk}{S}}\right] \qquad \text{lumens per sq. cm.}$$

It follows that the constant of the sphere K decreases with increasing values of U, since K is proportional to E_R. $K = \frac{b_w}{\phi} = \frac{E_R J}{\phi}$.

The expression (1) for ΔE_R would indicate that the reduction in window illumination due to the presence of a non-luminous body N depends entirely upon the surface area and absorption factor of N, and is independent of its position in the field of reflected light. Very accurate experimental work would undoubtedly disclose some variation in ΔE_R with change of position of N. To account for this change which, it is reasonable to believe, must be well within the allowable limit of error, greater refinement is necessary in the analysis.

The hypothesis underlying the derivation of ΔE_R assumed the resultant illumination E due to direct and reflected light to be uniform over the inner surface of the sphere. Likewise, it assumed a diffusely reflecting body N with no mention relative to the degree of diffusion. All foreign bodies in the sphere except polished fittings, glass, porcelain, metal, and similar surfaces strong in specular reflection are more or less diffusely reflecting or transmitting, but to a varying and lesser degree than the inner surface of the sphere. Further considerations will disclose the importance of rendering all bodies within the sphere except the lamp or lighting unit including reflectors, shades, and integral accessories, diffusely reflecting, preferably using the same diffusing paint employed for the inner surface of the sphere.

The body N in the preceding discussion may be termed an isolated body. The term "isolated" refers here to all objects within the sphere which are wholly distinct from the source and its auxiliary equipment. It is important to investigate the effect of an isolated body N upon ΔE_R under the assumption that N is again diffusely reflecting and the inner surface of the sphere is pronouncedly non-uniformly lighted.

Effect of Isolated Bodies within the Non-Uniformly Illuminated Sphere upon Diffusely Reflected Light.

The total illumination at any point P on the inner wall of the sphere varies for different positions of P except those screened from direct light, and was found to be

$$E_A = \frac{J_\beta \cos\beta}{\rho^2} + \frac{\phi}{4\pi r^2}\,\frac{k}{1-k} \qquad \text{phots.}$$

In the following, consider the isolated diffusely reflecting body N within the sphere. When the distribution of luminous intensity from the source is very irregular, it becomes necessary to separate the losses at N due to direct and reflected light received on the sphere. If ϕ_t is the loss of flux due to absorption by N, then

$$\phi_t = \phi_R + \phi'$$

where $\phi_R = E_R\, k(1-k_u)\,U$ represents the flux absorbed by N as a result of the uniform illumination E_R, and ϕ' is that absorbed resulting from the non-uniform component $\frac{I_\beta \cos\beta}{\rho^2}$. The sum of the losses in the sphere must equal ϕ.

Therefore, $E_R\, k(1-k_u)U + \phi' + (1-k)\phi + 4\pi r^2 E_R(1-k) = \phi$

from which $E_R\left[k(1-k_u)U + 4\pi r^2(1-k)\right] = k\phi - \phi'$

and $$E_R = \frac{k\phi - \phi'}{k(1-k_u)U + 4\pi r^2(1-k)} .$$

Hence $$\phi_R = \frac{k\phi - \phi'}{1 + \frac{4\pi r^2(1-k)}{k(1-k_u)U}} \qquad \text{lumens.}$$

If ϕ' is negligible compared to $k\phi$,

$$\phi_R = \frac{k\phi}{1 + \frac{4\pi r^2(1-k)}{k(1-k_u)U}} \qquad \text{lumens.}$$

The loss ϕ_R is a function of U, but remains practically constant for all positions of N in the sphere. This would no longer hold when N is sufficiently close to the wall of the sphere to cast a pronounced shadow, whereby a portion of its surface would not be exposed to the component of uniform illumination E_R.

The absorption ϕ' will vary considerably, depending upon the

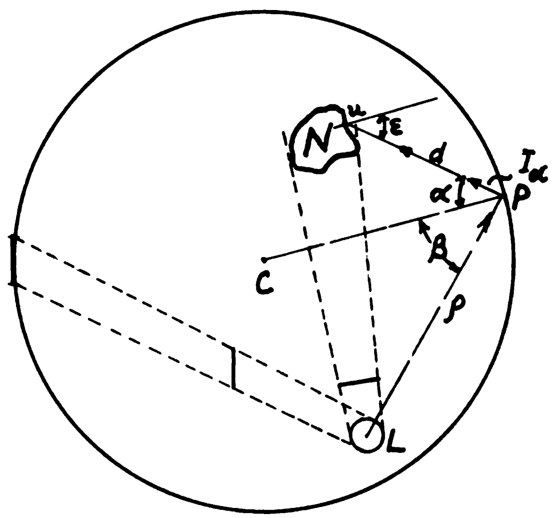


Fig. 3

constants k_u and U for the body N as well as in a large measure upon the proximity of N to strongly illuminated parts of the sphere. This relationship is apparent from the following differential notation. See Fig. 3. If dE_u represents the illumination at point u on surface of N due to the uniformly bright source at P, then

$$dE_u = \frac{I_\alpha \cos \varepsilon}{d^2} \qquad \text{lumens per sq. cm. (phots)}$$

$$d\phi' = \frac{I_\alpha \cos \varepsilon}{d^2}(1 - k_u)\, dU \qquad \text{lumens,}$$

where dU represents a differential element of surface at U, and $d\phi'$ the light absorbed by N at u as the result of the reflected component I_a of non-uniform illumination at P.

Since E_R depends upon both ϕ' and ϕ_R, N should not be allowed to approach the more intensely illuminated portions of the sphere. As previously noted, ϕ_R is practically constant for all positions of N within the sphere.

When N is located at the center of the sphere, ϕ' is constant for all variations in candle-power distribution of the source as long as the M. S. C. P. remains the same. The average illumination is unchanged with I_o = const., and hence the $\int d\phi'$ will give the same absorption ϕ'_c for different candle power distributions.

The decrease in ΔE_R (window illumination) due to the loss $(\phi'_c + \phi_R)$ is accounted for when calibrating the sphere. The error is offset by operating the standard lamp with bodies such as N in the same relative positions which they occupy when lighting the test source. The effect in both cases on the window illumination will be the same.

Screen Effect and Screen Errors

Effect of Screens upon Diffusely Reflected Light.

The screen is usually a flat, opaque body interposed between the source of light and the window in order to exclude all direct rays from the latter. When properly dimensioned and located within the sphere, it permits only diffused light to reach the window of which the brightness is a measure of the total luminous output of the source. The screen is an isolated body exposed to direct and reflected light, and in consequence lowers the window illumination due to absorption of both components of light flux. The following discussion is initially confined to the effect of the screen upon diffusely reflected light.

Let the screen be a plane surface provided with diffusing paint of the same quality as that employed on the inner surface of the sphere. If A_s represents the area of one face of the screen in square centimeters, then $U = 2A_s$. Its reflection factor is k. The light absorbed by the screen is readily calculated to be ϕ_s, where

$$\phi_s = \pi b \cdot 2A_s(1-k) \qquad \text{lumens,}$$

and b is the uniform brightness of the inner wall of the sphere in candles per sq. cm. The brightness b is the result of the total uniform illumination on the inner surface, which is

$$E_{Tot.} = E_R + E_D = \frac{\phi k}{4\pi r^2(1-k)} + \frac{\phi}{4\pi r^2} = \frac{\phi}{4\pi r^2(1-k)} \qquad \text{phots,}$$

in which E_R is the wall illumination due to diffusely reflected light, and E_D is the average direct illumination on the sphere. Then

$$b = \frac{\phi}{4\pi r^2(1-k)} \cdot \frac{k}{\pi} \qquad \text{candles per sq. cm.}$$

and
$$\phi_s = \frac{\pi \cdot \phi}{4\pi r^2(1-k)} \cdot \frac{k}{\pi} \cdot 2A_s(1-k) \quad ;$$

that is,
$$\phi_s = \frac{kA_s}{2\pi r^2} \cdot \phi = \frac{2kA_s}{r^2} \cdot I_o \qquad \text{lumens.}$$

The absorption of diffusely reflected light is a linear function of the total lumens output of the source. With ϕ_s lumens absorbed by the screen, the window illumination suffers a reduction. When expressed as a fraction of the original window illumination, the reduction is

$$\delta = \frac{E_R - {}_sE_R}{E_R}$$

where ${}_SE_R$ is the reduced wall illumination due to diffusely reflected light which obtains in the presence of the screen.

$$\delta = \frac{\dfrac{\phi k}{4\pi h^2(1-k)} - \left(\dfrac{\phi}{4\pi h^2(1-k) + k(1-k)2A_s} - \dfrac{\phi}{4\pi h^2}\right)}{\dfrac{\phi k}{4\pi h^2(1-k)}}$$

$$\delta = \frac{\dfrac{\phi k A_s}{4\pi h^2(1-k)[2\pi h^2 + kA_s]}}{\dfrac{\phi k}{4\pi h^2(1-k)}} = \frac{A_s}{2\pi h^2 + kA_s}$$

Likewise,

$$\delta' = \frac{E_R - {}_sE_R}{E} = \frac{\dfrac{\phi k A_s}{4\pi h^2(1-k)[2\pi h^2 + kA_s]}}{\dfrac{\phi}{2(1-k)[2\pi h^2 + kA_s]}}$$

gives the reduction in window illumination as a fraction of the final wall illumination, from which

$$\delta' = A_s \frac{k}{2\pi h^2}$$

In the above, E is the reduced <u>total</u> wall illumination which obtains in the presence of the screen.

Similarly,

$$\delta'' = \frac{E_R + E_D - ({}_sE_R + E_D)}{E_R + E_D} = \frac{E_R - {}_sE_R}{E_{Tot.}}$$

$$\delta'' = \frac{\dfrac{k\phi A_s}{4\pi h^2(1-k)[2\pi h^2 + kA_s]}}{\dfrac{\phi}{4\pi h^2(1-k)}}$$

$$\delta'' = \frac{k A_s}{2\pi r^2 + k A_s}$$

where δ'' gives the reduction in wall illumination as a fraction of the original total wall illumination.

The quantities δ, δ', and δ'' are essentially constants. Comparing δ and δ'', it is apparent that the per cent drop in window illumination is somewhat greater than that on the wall. The error incurred as the result of δ is completely eliminated when calibrating the sphere. Since its evaluation depends only upon the constants A_s, k, and the radius of the sphere, the error will be the same when calibrating the sphere with a standard lamp. In other words, the reduction in window illumination due to the presence of the screen in diffusely reflected light will be the same for the standard and test lamps. Thus, when determining the luminous output of a lighting unit, all necessary screening should be adjusted to accommodate both standard and test lamps. With this arrangement unchanged, concurrent readings are taken, once with the standard lamp lighted, followed by the latter extinguished and the test lamp lighted. This procedure leaves the measurements unaffected by the absorption of diffusely reflected light.

Direct Light Absorbed by Screen.

The screen is at all times exposed to direct light rays from the source, and must absorb part of this incident flux before it can be diffusely reflected, eventually to reach the window. It is proposed to determine the resultant reduction in illumination at the window.

Referring to Fig. 4, let L be a finite source of light, and consider the screen parallel to the window. Since the size of the screen and its position relative to the source must combine to exclude all direct light from the window, let a proper balance be obtained without allowing the screen to approach either the source or the window too closely. The line WL joining lamp and window centers makes an angle

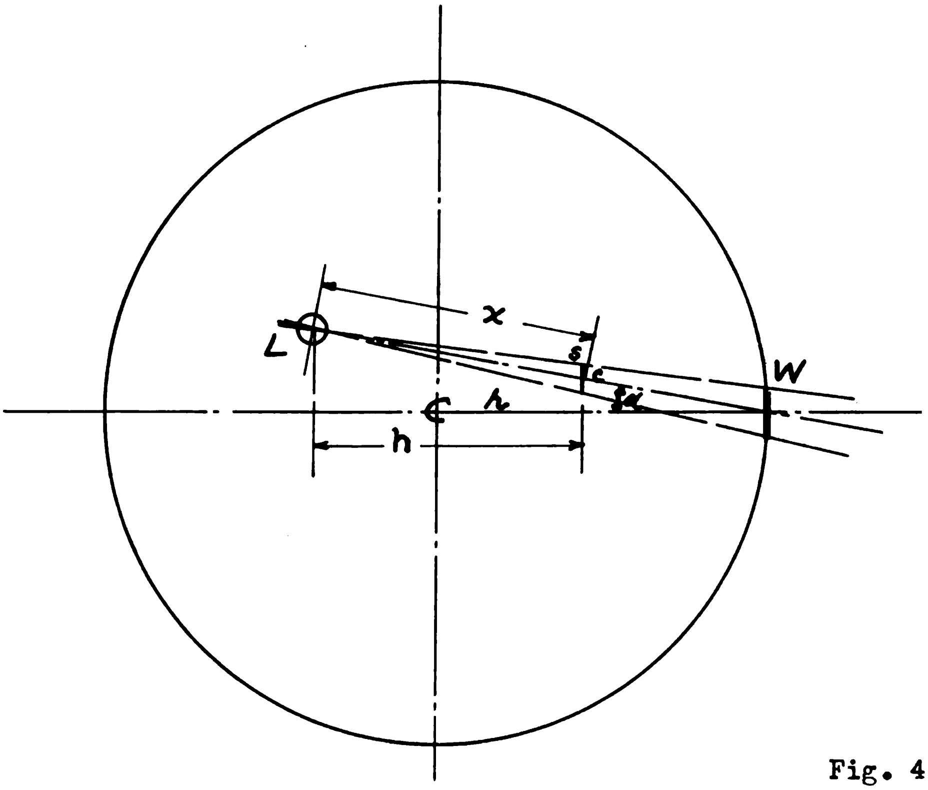


Fig. 4

α with the diameter through W. Consider a uniform distribution of luminous intensity emitted by L. It is within the limit of allowable error to assume the illumination on the screen to be uniform. This may be seen from the following consideration.

Since the candle power of L is the same in all directions, we may replace L by a point source of equivalent candle power, and compare the uniform and average values of illumination on the screen for a normally convenient position of the source as shown in Fig. 5. From Fig. 4 it is apparent that the illumination at the center c of the screen is

$$E_D = \frac{I_0 \cos\alpha}{x^2} = \frac{\phi \cos\alpha}{4\pi \cdot x^2} \qquad \text{phots.}$$

If this illumination were uniform over the area A_s of one side of the screen, the direct flux absorbed by A_s would be

$$\phi_s = E_u \cdot a \cdot A_s \qquad \phi_s = a \frac{\phi}{4\pi} \cdot \frac{\cos\alpha}{x^2} \cdot A_s$$

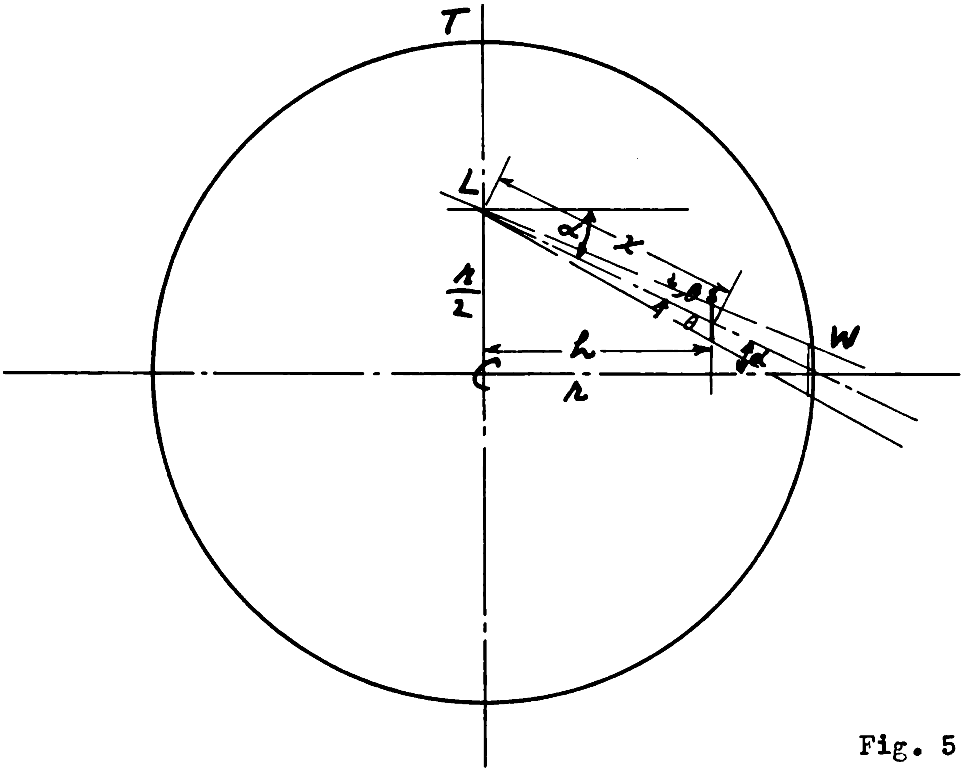


Fig. 5

where a is the absorption factor of the screen, and $E_u = E_D$. In practice, the window area is small compared to the area of the sphere, ranging from 0.06 to approximately 0.17 per cent. The angle 2θ, which is largely determined by the relative positions of L and s, does not become excessive even in cases where L may represent a fairly large lighting unit.

Consider L vertically suspended from the top of the sphere at a convenient distance $\frac{h}{2}$ from the center C. Fig. 5. The illumination at the center of the screen due to L was found to be

$$E_u = \frac{I_0 \cos\alpha}{x^2} = \frac{I_0 \cos^3\alpha}{h^2} \quad .$$

Since $\tan\alpha = \frac{1}{2}$, and $\alpha = 26^\circ\ 34'$,

$$E_u = \frac{I_0}{h^2}\cos^3(26^\circ 34') = 0.71547\frac{I_0}{h^2}$$ lumens per sq. cm.

The maximum illumination on the screen, which will be assumed circular for purposes of illustration, is

$$E_{max} = \frac{I_0}{h^2}\cos^3(\alpha - \theta) = \frac{I_0}{h^2}\cos^3(16^\circ 34') \quad .$$

Therefore $E_{max} = 0.88059\frac{I_0}{h^2}$, where θ was chosen 10°. Minimum illumination on the screen is

$$E_{min} = \frac{I_0}{h^2}\cos^3(\alpha + \theta) = \frac{I_0}{h^2}\cos^3(36^\circ 34') \quad ,$$

or $E_{min} = 0.51809\frac{I_0}{h^2}$.

The average illumination on the screen will be proportional to the average value of the cosine-cube function over the interval whose limits are $\alpha - \theta = \alpha_1$ and $\alpha + \theta = \alpha_2$.

The average value of $y = \cos^3\alpha$ over the interval α_1 to α_2 is

$$\frac{\int_{\alpha_1}^{\alpha_2}\cos^3\alpha\, d\alpha}{\alpha_2 - \alpha_1} = \frac{\int_{\alpha_1}^{\alpha_2}(1 - \sin^2\alpha)\cos\alpha\, d\alpha}{\alpha_2 - \alpha_1} =$$

$$\frac{1}{\alpha_2 - \alpha_1}\left[\int_{\alpha_1}^{\alpha_2}\cos\alpha\, d\alpha - \int_{\alpha_1}^{\alpha_2}\sin^2\alpha\cos\alpha\, d\alpha\right] =$$

$$\frac{1}{\alpha_2 - \alpha_1}\left[\sin\alpha - \frac{\sin^3\alpha}{3}\right]_{\alpha_1}^{\alpha_2} = \frac{1}{\alpha_2 - \alpha_1}\left[\frac{1}{3}\sin\alpha(\cos^2\alpha + 2)\right]_{\alpha_1}^{\alpha_2}$$

Noting that $\alpha_1 = 16^\circ\ 34'$, and $\alpha_2 = 36^\circ\ 34'$, also $\alpha_2 - \alpha_1 = 20^\circ = 0.349$ radians, the last expression, when evaluated, gives the average value of the cosine-cube function as 0.7102. Hence,

$$E_{AV} = 0.7102\ \frac{I_0}{h^2}$$

This also compares very favorably with the arithmetic average of $E_{max.}$ and $E_{min.}$, which is 0.6993. The close agreement is due to the relatively small value of 2θ. However, it is obvious that E_u may replace $E_{AV.}$ in this consideration. As α approaches zero, the difference between $E_{AV.}$ and E_u becomes extremely small, and vanishes for $\alpha = 0$. Again, α is limited to values appreciably below 45, owing to the centrally located lamp structure which extends some distance below T. It may be shown that the error involved is also negligibly small for any position that L may occupy in the sphere when making measurements. Thus, <u>the illumination on the screen may be considered uniform and equal</u> to

$$E_D = \frac{I_0 \cos\alpha}{X^2} = \frac{\phi}{4\pi}\ \frac{\cos\alpha}{X^2} = \frac{I_0}{h^2}\cos^3\alpha$$

The error caused by the screen absorbing direct light from the source may be expressed as

$$\epsilon_s = \frac{{}_SE_R - {}_SE'_R}{E}$$ where

${}_SE_R$ represents illumination on sphere.

(screen present and absorbing diffuse light only)

${}_SE'_R$ represents illumination on sphere.

(screen present and absorbing diffuse and direct light)

E represents total uniform illumination on sphere.

(screen present and absorbing diffuse light only)

E' represents total uniform illumination on sphere.

(screen present and absorbing diffuse and direct light)

E_D represents average illumination on sphere due to direct light.

Since $\qquad {}_SE_R = E - E_D$, and ${}_SE'_R = E' - E_D$,

$$\epsilon_s = \frac{E - E'}{E}$$

An expression for E was obtained in previous considerations; namely,

$$E = \frac{\phi}{2(1-k)\left[2\pi h^2 + k A_s\right]} = \frac{\phi}{2a\left[2\pi h^2 + (1-a) A_s\right]}$$

E' is obtained at once from the following equation, which states that the sum of all the flux absorbed must be equal to the total lumens emitted by the source.

$$4\pi h^2 E' a + E'(1-a)2aA_s + \frac{\phi \cos\alpha}{4\pi x^2} \cdot a \cdot A_s = \phi$$

$$E'\left[4\pi h^2 a + 2a(1-a)A_s\right] = \phi\left(1 - \frac{\cos\alpha}{4\pi x^2} \cdot a A_s\right)$$. Solving for E',

we obtain

$$E' = \frac{\phi\left(4\pi h^2 - aA_s \cos\alpha\right)}{8\pi x^2 a\left[2\pi h^2 + (1-a)A_s\right]} .$$

Since $\qquad \cos\alpha = \dfrac{l}{2h}$,

$$E' = \phi\,\frac{\left(8\pi x^2 h - aA_s l\right)}{16\pi x^2 a h\left[2\pi h^2 + (1-a)A_s\right]} ,$$

and, $\quad \epsilon_s = 1 - \dfrac{8\pi h x^2 - aA_s l}{8\pi x^2 h} = \dfrac{aA_s l}{8\pi x^2 h}$

$$\epsilon_s = \frac{A_s \cdot a}{\pi h} \cdot \frac{l}{8x^2} = \frac{A_s \cdot a}{\pi h^2} \cdot \frac{h l}{8x^2}$$

Since E' is proportional to I_0, an irregular distribution of candle power from the source will affect E' in the ratio $\frac{I_s}{I_0} = \xi_s$, where I_s is the intensity in the direction Lc toward the screen. The error, that is, the reduction in window illumination, becomes

$$\epsilon_s = \frac{A_s a}{\pi h^2} \xi_s \cdot \frac{h l}{8x^2} \quad ,$$

and varies inversely as the square of the distance from the source to the screen.

Occlusion of Luminous Flux Reflected

from Portion of Sphere Obscured by the Screen.

That portion of the inner surface of the sphere obscured from the window by the screen cannot reflect the direct light received from the source to the window without the aid of some other portion of the spherical surface. This lessens the window illumination. Fig. 6.

The flux from JV is readily obtained if the area represented by JV can be expressed in terms of known quantities.

$$\frac{A_s \cos\alpha}{A_{MN}} = \frac{(l-d-x)^2}{l^2}$$

$$A_{MN} \cong A_{JV} \cdot \cos\alpha \quad .$$

$$A_{JV} = A_s \frac{l^2}{(l-d-x)^2}$$

, where A_{JV} represents the approximate area on the sphere.

Extending the analysis as above for ϵ_s, we find

$$\epsilon_w = \frac{A_s a}{\pi h^2} \cdot \frac{l^3 h}{8(l-x-d)^2 d^2}$$

If I_W represents the intensity along Lw, then $\frac{I_w}{I_0} = \rho_w$

$$\epsilon_w = \frac{A_s a}{\pi h^2} \rho_w \frac{l^3 h}{8(l-x-d)^2 d^2}$$

The errors ϵ_s and ϵ_w are functions of x. However, if we consider the dimensions of the screen to remain fixed, the source L may be effectively screened from W by modifying either d or x while the

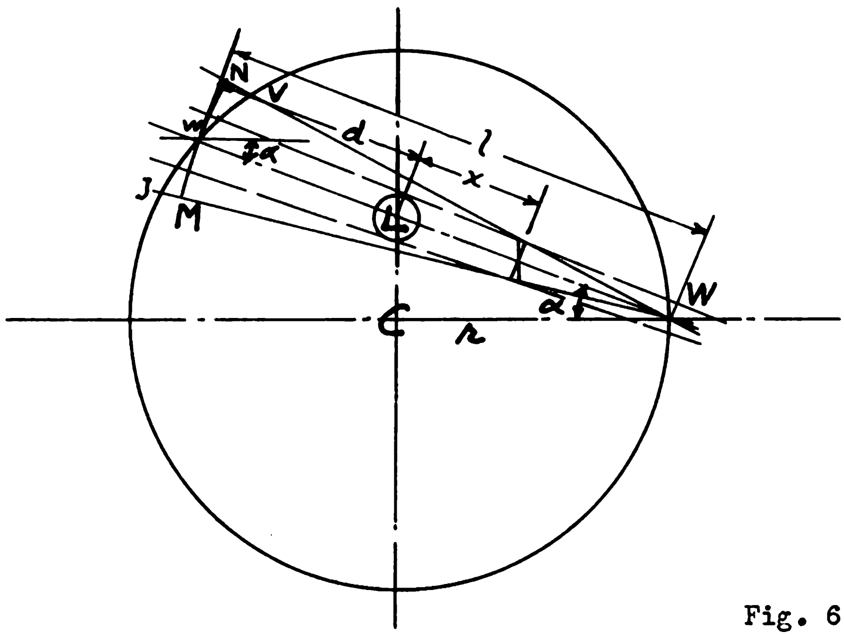


Fig. 6

other quantity is held constant. Since both errors are a function of either x or d, the minimum total error for variable x and constant d is obtained by differentiating the sum $\epsilon_s + \epsilon_w$ with respect to x, and allowing the resulting function to vanish.

$$\frac{\partial}{\partial x}\left(\epsilon_s + \epsilon_w\right) = -\frac{A_s a}{8\pi h^2} \rho_s h l \cdot \frac{2}{x^3} + \frac{A_s a}{8\pi h^2} \rho_w \frac{l^3 h \cdot 2}{d^2 (l-x-d)^3} = 0$$

$$\sqrt[3]{\rho_s d^2}\,(l-x-d) = \sqrt[3]{\rho_w l^2} \cdot x$$

$$x = \frac{(l-d)\sqrt[3]{\rho_s d^2}}{\sqrt[3]{\rho_s d^2} + \sqrt[3]{\rho_w l^2}} = \frac{l-d}{1+\sqrt[3]{\frac{\rho_w}{\rho_s}\cdot\frac{l^2}{d^2}}}$$

When x = const., and d is permitted to vary, we have

$$\frac{\partial}{\partial d}\left(\epsilon_s + \epsilon_w\right) = \frac{A_s l^3 h}{8\pi h^2}\rho_w\left\{\frac{(l-x-d)^2\cdot 2d - 2d^2(l-x-d)}{(l-x-d)^4 d^4}\right\} = 0$$

from which, $d = \dfrac{l-x}{2}$.

Eliminating x, we have $l - 2d = \dfrac{l-d}{1+\sqrt[3]{\frac{\rho_w}{\rho_s}\cdot\frac{l^2}{d^2}}}$,

from which, $\dfrac{(l-2d)^3}{d^5} = \dfrac{\rho_s}{\rho_w}\cdot\dfrac{1}{l^2}$.

The quantity d may be obtained from the last equation for known values of the constants involved, and when substituted in the expression solved explicitly for x will give a value of x consistent with a minimum total error $\epsilon_s + \epsilon_w$. In practical work, d is frequently fixed by the manner in which the source must be suspended or mounted, in which case the distance from the source to the screen may be obtained directly by evaluating

$$x = \frac{l-d}{1+\sqrt[3]{\frac{\rho_w}{\rho_s}\cdot\frac{l^2}{d^2}}} .$$

At this time it will be well to investigate the upper and lower limits of x. These results have a practical bearing on the design of the sphere.

Screen Location.

The screen must always be situated so that it excludes any direct rays that would otherwise reach the window. Unless an adjustable screen is provided, one or possibly two diffusing screens must fulfill this requirement for the entire range of lighting units to be accommodated by the sphere. Heretofore, the area of the screen was considered constant, and with a definite location of lamp, the distance between source and screen, denoted by x, assumes a definite value consistent with the minimum error $\epsilon_s + \epsilon_w$. Practical luminous sources, however, vary considerably in size, so that for the customary alignment along the vertical axis of the sphere, a large lamp will require a larger screen at the same distance x than a smaller lamp similarly located. Again, if we permit x to vary, a larger lamp will be effectively screened by a smaller screen as x increases. Excluding from this discussion a variable screening device created to accommodate different sized lamps at a more or less fixed distance from the source, let us consider the effect of variation in screen upon the x for minimum total error $\epsilon_s + \epsilon_w = \epsilon_t$. Practical considerations will dictate the working limits essential in arriving at an adequate screen-to-source separation.

Referring to Fig. 7, the window opening at W may be assumed to approximate a circular disc of diameter d_w. L represents the light source with center at C_1 on the vertical axis through C. As before, A_s is the area of one side of the screen. Let A_v represent the area of the base of the shadow on VV'. Then

$$\frac{A_s}{A_v} = \frac{(l-d-x+p)^2}{(l-d+p)^2}\,; \quad A_s = A_v\frac{(l-d-x+p)^2}{(l-d+p)^2}.$$

Substituting this expression for A_s in ϵ_s and ϵ_w, gives rise to the following two equations:

$$(1)\quad \epsilon_s = \frac{A_v(l-d-x+p)^2}{(l-d+p)^2}\,\frac{a}{\pi h^2}\cdot\xi_s\cdot\frac{r\,l}{8x^2}$$

$$(2)\quad \epsilon_w = \frac{A_v(l-d-x+p)^2}{(l-d+p)^2}\,\frac{a}{\pi h^2}\cdot\xi_w\,\frac{r\,l^3}{8(l-x-d)^2 d^2}$$

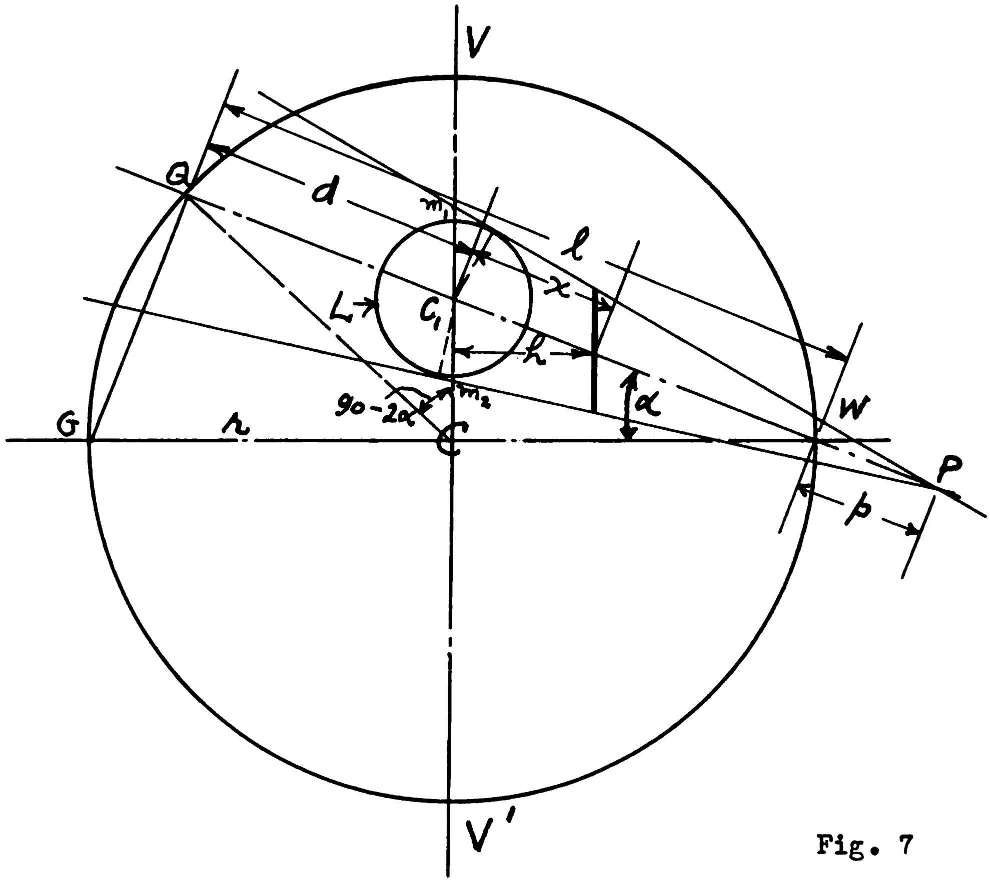


Fig. 7

The geometry of Fig. 7, triangle CQG , requires that

$$\frac{d}{r\tan\alpha} = \frac{\sin(90-2\alpha)}{\sin\alpha} = \frac{\cos 2\alpha}{\sin\alpha}$$

$$d = \frac{r\cos 2\alpha}{\cos\alpha}\,.$$

Also, $l = 2r\cos\alpha$ $\qquad l-d = \dfrac{r}{\cos\alpha}$

and $$\frac{p}{l-d+p} = \frac{d_w}{d_v} = \frac{1}{n}$$, where $d_v = m_1m_2$ is the distance intercepted by the boundary rays on the vertical axis VV'. It will be noted that this relation holds only approximately, since for different values of α, the tangent lines and the line through C , and center of window, will not converge to a common point. For all practical purposes, whether the lighting unit is spherical in form or otherwise, the tangent lines to the maximum dimension of the luminous source as viewed from the window will meet in a point P when passing the circumference of the window opening, and a line drawn from P through center of window will, in general, not pass through C_1 , but the angle made by this line with the horizontal GW will differ little from α especially if α ranges only from 0 to 30°.

It follows that

$$\frac{p}{l-d+p} = \frac{1}{n}$$

$$p(n-1) = l-d$$

$$p = \frac{l-d}{n-1} = \frac{h}{(n-1)\cos\alpha}$$

Substitution in Eq. (1) and (2) gives

$$\mathcal{E}_t = \frac{A v a}{\pi h^2}\frac{(n-1)^2}{n^2}\cos^3\alpha\left\{\frac{\rho_s}{4}\frac{\left(\frac{h}{\cos\alpha}-x+\frac{h}{(n-1)\cos\alpha}\right)^2}{x^2} + \rho_w\frac{\left(\frac{h}{\cos\alpha}-x+\frac{h}{(n-1)\cos\alpha}\right)^2\cos^4\alpha}{\left(\frac{h}{\cos\alpha}-x\right)^2\cos^2 2\alpha}\right\}$$

Differentiating partially with respect to x , simplifying, and equating to zero, we obtain

$$\frac{\partial \mathcal{E}_t}{\partial x} = \rho_s \cos^2 2\alpha \left(\frac{h}{\cos\alpha} - x\right)^3 - \frac{4\rho_w x^3 \cos^4\alpha}{n} = 0$$

Solving for x , we have

(3) $$x = \frac{h}{\cos\alpha\left(1 + \sqrt[3]{\frac{\rho_w}{\rho_s} \cdot \frac{4\cos^4\alpha}{\cos^2 2\alpha} \cdot \frac{1}{n}}\right)}$$

This gives the distance x for minimum total error, and is a function of α . The limits of x for a typical case of a source suspended vertically along VV' with $\alpha = 30$, obtain as follows. Let $\frac{\rho_w}{\rho_s} = 1$ and $n = 1$. Then the shadow is bounded by parallel rays, and the expression (3) reduces to $x = 0.37488r = 0.325(1-d)$. For $\frac{\rho_w}{\rho_s} = 1$ and $n = 9$, we have $x = 0.57735r = 0.500(1-d)$.

Lamps and lighting units tested in spheres of commercial sizes, 40 inches in diameter or larger, will seldom permit $d_v < d_w$. For all practical purposes $\frac{d_w}{d_v} \geqq \frac{1}{9}$; that is, $d_v \leqq 9 d_w$. For a 60" sphere $n = \frac{d_v}{d_w}$ will rarely exceed 5. Note that d_v is the maximum dimension of an equivalent luminous source in the vertical plane through the lamp center, and may be greater or less than the maximum dimension at the base of the luminous cone intercepted by the actual light source. See Fig. 8. Nearly all practical illuminants will disclose symmetry about a single axis, and many types display symmetry about a vertical axis. Although d_v does not represent the maximum dimension of luminosity, it serves very well to establish an approximate upper limit for x . Thus, to a first approximation, x may be said to lie within the limits

$$x = 0.3(1-d) = 0.346 \cdot r$$

$$x = 0.5(1-d) = 0.577 \cdot r$$

for $\alpha = 30^\circ$, $\xi_w = \xi_s$, and covering values of n from less than unity to and including $n = 9$. If we were to use an average of these values, then

$$x = 0.4(l-d) = \frac{0.4r}{\cos\alpha} = \frac{0.4r}{\cos 30^\circ} \quad \text{and } h = 0.4r$$

This gives the distance of the screen from the vertical axis of the sphere ($\alpha = 30^\circ$) for minimum error due to occlusion and absorption of

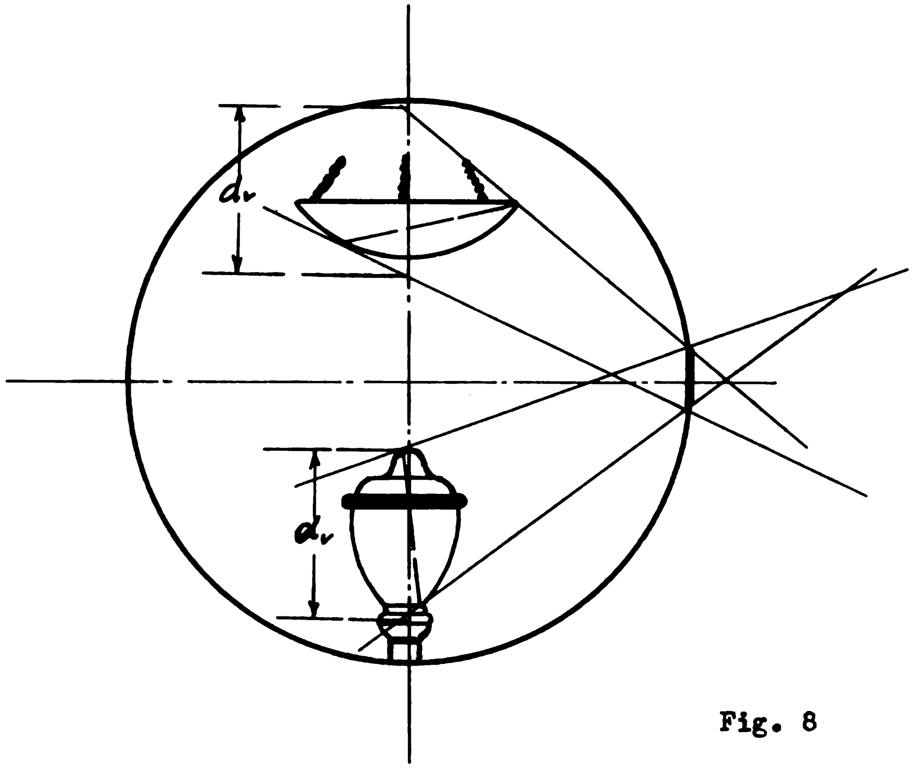


Fig. 8

light, and agrees very well with figures quoted in general discussions on the sphere photometer.

Referring to a 60" sphere, it was stated that n rarely exceeds 5. This follows almost immediately from Fig. 7, in which $CC_1 = 30 \cdot \tan 30^\circ$,

$$CC_1 = 17.32" , \qquad d_v = 5d_w = 5 \cdot 3.5 = 17.5"$$

This would still permit a convenient position of the source with reference to V without incurring any serious error due to proximity to the wall. An average value for x may be obtained from the following tabulated values:

α	n	$\frac{\rho_w}{\rho_s}$	x
30°	5	0.5	$0.58749r = x_1$
0°	5	0.5	$0.57577r = x_2$
30°	1	1	$0.37488r = x_3$
0°	1	1	$0.38648r = x_4$

The average of smallest and largest x is $x_{AV} = 0.4811r$.

$$h_{AV} = 0.4811\, h \frac{\sqrt{3}}{2} = 0.416\, h$$

Again, 0.4r comes very close in fulfilling requirements for minimum screen error.

It is important to gain an impression of the order and magnitude of the minimum total error ${}_m\epsilon_t$ for a given set of conditions, and equally essential to show that ${}_m\epsilon_t$ exhibits a minimum. There follows the calculations for minimum total screen error (${}_m\epsilon_t$) due to absorption and occlusion of light for a 60" sphere under the following conditions:

$$n = 5,\ \alpha = 30°,\ r = 30'',\ a = 0.2,\ \frac{\rho_w}{\rho_s} = \frac{1}{2}\ .$$

The x for minimum error is $x = 0.58749r$.

$${}_m\epsilon_t = \epsilon_s + \epsilon_w = \frac{A_v}{8}\,\frac{(l-d+p-x)^2}{(l-d+p)^2}\cdot\frac{a}{\pi h^2}\,\rho_s\left\{\frac{l}{x^2} + \frac{1}{2}\,\frac{l^3}{(l-d-x)^2 d^2}\right\}$$

Evaluating ${}_m\epsilon_t$, we have

$${}_m\epsilon_t = \frac{1.2850\, A_v\, \rho_s}{4500\,\pi}$$

For a 60" sphere, $r = 30"$, $d_w = 3.5"$,

$$d_v = n \cdot d_w = 5 \cdot 3.5 = 17.5" \,.$$

Hence $A_v = \frac{\pi}{4}(17.5)^2 = 240.528 \; in^2$, from which

$$_m\epsilon_f = 0.02186 \, \xi_s$$

Thus, for $\xi_s = \frac{I_s}{I_0} = 1$, $_m\epsilon_f \cong 2.1\%$;

for $\xi_s = \frac{1}{2}$, $_m\epsilon_f \cong 4.3\%$.

To show that $x = 0.58749r$ represents the screen distance for minimum error under above conditions, it is sufficient for practical purposes to calculate the error for a slight variation in x to the right and left of the critical point. The usual mathematical criteria may be applied to show that

$$X = \frac{h}{\cos\alpha\left(1 + \sqrt[3]{\frac{\xi_w}{\xi_s}\,\frac{4\cos^4\alpha}{\cos^2 2\alpha}\,\frac{1}{n}}\right)}$$

represents a minimum for all practical values of n and $\frac{\xi_w}{\xi_s}$.

Let x <u>increase</u> from $0.58749r$ to $0.60000r$.

$$\Delta X = 0.01251\,h \cong 1.25\%\,h \cong 0.37"$$

$$\epsilon_f = \frac{1.2863}{4500\pi} A_v \xi_s = 0.02188\,\xi_s$$

$$\epsilon = \frac{\epsilon_f - {}_m\epsilon_f}{{}_m\epsilon_f} = \frac{0.00002}{0.02186} \cong 0.1\%$$ increase in min. error.

Let x <u>decrease</u> $1.25\%r$, then $X - \Delta X = 0.57498\,h$

$$\epsilon_f = \frac{1.2866}{4500\pi} A_v \xi_s = 0.02189\,\xi_s$$

$\epsilon = 0.1\%$ increase in minimum error.

Screen Error Influenced by Angle.

Up to this point it was merely stated that x is a function of α. The expression for total error may be written

$$\epsilon_t = f(x, \alpha)$$

from which ϵ_t is a function of x and α. It is important to investigate what general effect the position of the source will have upon the screen error ϵ_t and in particular to determine the angle α and x for minimum total screen error.

We have

$$\frac{\partial}{\partial x}\epsilon_t = \frac{A_v a}{\pi h^2}\frac{(n-1)^2}{n^2}\frac{\partial}{\partial x}\left\{\frac{\rho_s}{4x^2}\left(\frac{h}{\cos\alpha} - x + \frac{h}{(n-1)\cos\alpha}\right)^2\cos^3\alpha\right.$$

$$\left. + \rho_w\left[\frac{\frac{h}{\cos\alpha} - x + \frac{h}{(n-1)\cos\alpha}}{\frac{h}{\cos\alpha} - x}\right]^2 \cdot \frac{\cos^7\alpha}{\cos^2 2\alpha}\right\} = 0$$

substitutions having been made for the quantities p, 1-d, 1, and d in terms of n and functions of α. See page 41. The solution of this equation for x was found to be

$$x = \frac{h}{\cos\alpha\left(1 + \sqrt[3]{\frac{\rho_w}{\rho_s}\cdot\frac{4\cos^4\alpha}{\cos^2 2\alpha}\frac{1}{n}}\right)}$$

In order to simplify the simultaneous solution of $\frac{\partial}{\partial x}\epsilon_t = 0$ and $\frac{\partial}{\partial \alpha}\epsilon_t = 0$, we can, without loss of generality, let r = 1. We confine the discussion to a definite set of conditions for $\frac{\rho_w}{\rho_s}$ and n. The quantity r, although essential to any particular solution for x, would not appear in the solution of $\frac{\partial}{\partial \alpha}\epsilon_t = 0$. Let n = 5, and $\rho_s = 2\rho_w$. Then

$$x\cos\alpha\left(1 + \sqrt[3]{\frac{2}{5}\frac{\cos^4\alpha}{\cos^2 2\alpha}}\right) = 1$$

$$x\cos\alpha + x\cos\alpha\sqrt[3]{\frac{2}{5}\frac{\cos^4\alpha}{\cos^2 2\alpha}} = 1$$

Transposing $x\cos\alpha$ and combining,

$$x^3\cos^3\alpha \cdot \frac{2}{5}\,\frac{\cos^4\alpha}{\cos^2 2\alpha} = (1-x\cos\alpha)^3 = 1-3x\cos\alpha+3x^2\cos^2\alpha-x^3\cos^3\alpha$$

$$2x^3\cos^7\alpha = (5-15x\cos\alpha+15x^2\cos^2\alpha-5x^3\cos^3\alpha)(2\cos^2\alpha-1)^2$$

Expanding and combining in powers of x,

$$x^3(22\cos^7\alpha-20\cos^5\alpha+5\cos^3\alpha)+x^2(-60\cos^6\alpha+60\cos^4\alpha-15\cos^2\alpha)$$
$$+x(60\cos^5\alpha-60\cos^3\alpha+15\cos\alpha)+(-20\cos^4\alpha+20\cos^2\alpha-5)=0$$

Now, differentiating ϵ_t partially with respect to α, we have

$$\frac{\partial\epsilon_t}{\partial\alpha}=\left[\frac{nr-(n-1)x\cos\alpha}{(n-1)(r-x\cos\alpha)}\right]^2\left(\frac{8\sin\alpha\cos^8\alpha-7\sin\alpha\cos^6\alpha\cos2\alpha}{\cos^3 2\alpha}\right)$$
$$-2\left[\frac{nr-(n-1)x\cos\alpha}{(n-1)(r-x\cos\alpha)}\right]\frac{r(n-1)x\sin\alpha}{(n-1)^2(r-x\cos\alpha)^2}\cdot\frac{\cos^7\alpha}{\cos^2 2\alpha}$$
$$+\frac{\cos^3\alpha}{x^2}\left[\frac{rn-x(n-1)\cos\alpha}{(n-1)\cos\alpha}\right]\left(\frac{x(n-1)\sin\alpha\cos\alpha+[rn-(n-1)\cos\alpha]\sin\alpha}{(n-1)\cos^2\alpha}\right)$$
$$-\frac{3\sin\alpha\cos^2\alpha}{2x^2}\left[\frac{rn-x(n-1)\cos\alpha}{(n-1)\cos\alpha}\right]^2=0$$

Substituting ω^2 $n = 5$, $r = 1$,

$$\left[\frac{5-4x\cos\alpha}{4(1-x\cos\alpha)}\right]^2\left(\frac{8\sin\alpha\cos^8\alpha-7\sin\alpha\cos^6\alpha\cos2\alpha}{\cos^3 2\alpha}\right)-\omega\,\frac{x\sin\alpha\cos^7\alpha}{2(1-x\cos\alpha)^3\cos^2 2\alpha}$$
$$+\frac{\cos^3\alpha}{x^2}\cdot\frac{5-4x\cos\alpha}{4\cos\alpha}\,\frac{5\sin\alpha}{4\cos^2\alpha}-\frac{3}{2}\,\frac{\sin\alpha\cos^2\alpha}{x^2}\left(\frac{5-4x\cos\alpha}{4\cos\alpha}\right)^2=0$$

Multiplying both members by $16\cos\alpha\cos^3 2\alpha(1-x\cos\alpha)^2x^2\neq0$,

$$x^2\cos\alpha(5-4x\cos\alpha)^2(8\sin\alpha\cos^8\alpha-7\sin\alpha\cos^6\alpha\cos2\alpha)$$
$$-2x^3\cos\alpha\cos2\alpha\,\frac{5-4x\cos\alpha}{1-x\cos\alpha}\sin\alpha\cos^7\alpha$$
$$+5\sin\alpha\cos\alpha\cos^3 2\alpha(1-x\cos\alpha)^2(5-4x\cos\alpha)$$
$$-\frac{3}{2}\sin\alpha\cos\alpha\cos^3 2\alpha(1-x\cos\alpha)^2(5-4x\cos\alpha)^2=0$$

Clearing of fractional expressions,

$$2x^2\cos\alpha(1-x\cos\alpha)(5-4x\cos\alpha)^2(8\sin\alpha\cos^8\alpha-7\sin\alpha\cos^6\alpha\cos2\alpha)-$$

$$4x^3\cos^8\alpha\cos 2\alpha\,(5-4x\cos\alpha)\sin\alpha$$
$$+10\sin\alpha\cos\alpha\cos^3 2\alpha\,(1-x\cos\alpha)^3(5-4x\cos\alpha)$$
$$-3\sin\alpha\cos\alpha\cos^3 2\alpha\,(1-x\cos\alpha)^3\cdot(5-4x\cos\alpha)^2=0$$

Division by $(5\text{-}4x\cdot\cos\alpha)$ is permissible in view of the fact that $0\leqq x\leqq\sqrt{2}$. For if $5\text{-}4x\cdot\cos\alpha = 0$, then for $\alpha = 0°$, $x = \frac{5}{4}$. But, physical conditions of the problem require $0<x<1$ for $\alpha = 0$. Therefore, $5\text{-}4x\cdot\cos\alpha \neq 0$. Likewise for $\alpha = 45°$, to satisfy the above condition, $x = \frac{5}{4}\sqrt{2}$. But, for $\alpha = 45°$, x must be at the most equal to or less than $\sqrt{2}$. Hence, again $(5\text{-}4x\cdot\cos\alpha)\neq 0$.

Thus,
$$\frac{2x^2\cos\alpha(8\sin\alpha\cos^8\alpha-7\sin\alpha\cos^6\alpha\cos 2\alpha)}{\frac{1}{(1-x\cos\alpha)(5-4x\cos\alpha)}}$$
$$-4x^3\cos^8\alpha\cos 2\alpha\sin\alpha+10\sin\alpha\cos\alpha\cos^3 2\alpha(1-x\cos\alpha)^3$$
$$-3\sin\alpha\cos\alpha\cos^3 2\alpha\,(1-x\cos\alpha)^3(5-4x\cos\alpha)=0$$

which simplifies to
$$\frac{2x^2\cos\alpha(8\sin\alpha\cos^8\alpha-7\sin\alpha\cos^6\alpha\cos 2\alpha)}{\frac{1}{(1-x\cos\alpha)(5-4x\cos\alpha)}}$$
$$-4x^3\cos^8\alpha\cos 2\alpha\sin\alpha-\sin\alpha\cos\alpha\cos^3 2\alpha(1-x\cos\alpha)^3(5-12x\cos\alpha)=0$$

Therefore, $\sin\alpha\cdot\cos\alpha\cdot U = 0$

$\sin\alpha = 0$; $\alpha = 0$ is one solution.

$\cos\alpha = 0$; $\alpha = \pm\frac{\pi}{2}$ solution ruled out.

Thus,
$$2x^2(5-9x\cos\alpha+4x^2\cos^2\alpha)(8\cos^8\alpha-7\cos^6\alpha\cos 2\alpha)$$
$$-4x^3\cos^7\alpha\cos 2\alpha-\cos^3 2\alpha(1-x\cos\alpha)^3(5-12x\cos\alpha)=0$$

Expanding and rearranging in powers of x, we have

$$x^4(-144\cos^{10}\alpha+200\cos^8\alpha-72\cos^6\alpha+12\cos^4\alpha)$$
$$+x^3(428\cos^9\alpha-614\cos^7\alpha+246\cos^5\alpha-41\cos^3\alpha)+$$

$$x^2(-468\cos^8\alpha + 682\cos^6\alpha - 306\cos^4\alpha + 51\cos^2\alpha)$$
$$+ x(216\cos^7\alpha - 324\cos^5\alpha + 162\cos^3\alpha - 27\cos\alpha)$$
$$+ (-40\cos^6\alpha + 60\cos^4\alpha - 30\cos^2\alpha + 5) = 0$$

This equation together with the one derived from the explicit solution for x emanating from $\frac{\partial}{\partial x} \epsilon_t = 0$, may be reduced to give the following simultaneous set:

$$x^3(22\cos^7\alpha - 20\cos^5\alpha + 5\cos^3\alpha)$$
$$+ x^2(-60\cos^6\alpha + 60\cos^4\alpha - 15\cos^2\alpha)$$
$$+ x(60\cos^5\alpha - 60\cos^3\alpha + 15\cos\alpha)$$
$$+ (-20\cos^4\alpha + 20\cos^3\alpha - 5) = 0$$

and,
$$x^2(-38.4\cos^4\alpha + 51.2\cos^2\alpha)$$
$$+ x(96\cos^3\alpha - 120\cos\alpha)$$
$$+ (-60\cos^2\alpha + 70) = 0$$

One solution of these is $\alpha = 0$, $x = 0.57577r$.

To effect the solution of these equations for x and α for other solutions, if any, one may turn to a dialytic process of elimination.

We write these equations in the form

$$f(x) = a_0x^3 + a_1x^2 + a_2x + a_3 = 0$$

and
$$g(x) = b_0x^2 + b_1x + b_2 = 0 \quad .$$

Then, if $f(x)$ and $g(x)$ have a common root, and if the first equation be multiplied by x, and the second by x^2 and x in turn, the resultant of $f(x)$ and $g(x)$ is the determinant

$$\begin{vmatrix} a_o & a_1 & a_2 & a_3 & 0 \\ 0 & a_o & a_1 & a_2 & a_3 \\ b_o & b_1 & b_2 & 0 & 0 \\ 0 & b_o & b_1 & b_2 & 0 \\ 0 & 0 & b_o & b_1 & b_2 \end{vmatrix} = 0$$

which is zero if the two equations $f(x)$ and $g(x)$ have a common solution. Expanding this determinant, we get

$$\begin{aligned} &a_0(a_0 b_2^3 - a_1 b_1 b_2^2 - a_3 b_1^3 + 2 a_3 b_0 b_1 b_2 + a_2 b_1^2 b_2 - a_2 b_0 b_2^2) \\ &+ b_0(a_1^2 b_2^2 + a_1 a_3 b_1^2 - 2 a_1 a_3 b_0 b_2 - a_1 a_2 b_1 b_2 - a_0 a_2 b_2^2 + a_0 a_3 b_1 b_2 \\ &+ a_2^2 b_0 b_2 + a_3^2 b_0^2 - a_2 a_3 b_0 b_1) = 0 \end{aligned}$$

in which

$$a_o = 22 \cos^7\alpha - 20 \cos^5\alpha + 5 \cos^3\alpha$$

$$a_1 = -60 \cos^6\alpha + 60 \cos^4\alpha - 15 \cos^2\alpha$$

$$a_2 = 60 \cos^5\alpha - 60 \cos^3\alpha + 15 \cos\alpha$$

$$a_3 = -20 \cos^4\alpha + 20 \cos^3\alpha - 5$$

$$b_o = -38.4 \cos^4\alpha + 51.2 \cos^2\alpha$$

$$b_1 = 96 \cos^3\alpha - 120 \cos\alpha$$

$$b_2 = -60 \cos^2\alpha + 70$$

Substituting the values of a's and b's and simplifying, we obtain

$$-1{,}007{,}769.6\, x^{20} - 9{,}555{,}148.8\, x^{19} - 9{,}566{,}899.2\, x^{18} + 81{,}330{,}291.2\, x^{17} + 27{,}844{,}646.4\, x^{16} - 227{,}126{,}476.8\, x^{15} + 27{,}734{,}956.8\, x^{14} + 262{,}624{,}870.4\, x^{13} - 101{,}777{,}612.8\, x^{12} - 108{,}795{,}494.4\, x^{11} + 55{,}141{,}632\, x^{10} + 220{,}774.4\, x^9 - 221{,}379.2\, x^8 + 43.2\, x^6 = 0$$

in which $x = \cos\alpha$, where x was chosen to represent $\cos\alpha$ to simplify the expressions, and has no relation to the x employed to represent the distance from source to screen.

From the last equation it appears that $x^6 = \cos^6\alpha$ is a factor which may be discarded, since $\cos\alpha \neq 0$. We have finally

(1) $f(x) = 62985.6\,x^{14} + 597196.8\,x^{13} + 597931.2\,x^{12} - 5083143.2\,x^{11}$
$- 1740290.4\,x^{10} + 14195404.8\,x^9 - 1733434.8\,x^8 - 16414054.4\,x^7$
$+ 6361100.8\,x^6 + 6799718.4\,x^5 - 3446352\,x^4 - 13798.4\,x^3 + 13836.2\,x^2$
$- 2.7 = 0$.

The arccosines of the roots of this equation may be critical points for the function $\epsilon_t = f(x,\alpha)$. Any possible real roots in the closed interval $\alpha = \pm\frac{\pi}{6}$ are of importance; roots outside this interval are of little or no practical significance.

According to Budan's Theorem, successive derivatives $f'(x)$,

x	f	f'	$f^{(2)}$	$f^{(3)}$	$f^{(4)}$	$f^{(5)}$	$f^{(6)}$	$f^{(7)}$	$f^{(8)}$	$f^{(9)}$	$f^{(10)}$	$f^{(11)}$	$f^{(12)}$	$f^{(13)}$	$f^{(14)}$	Variations
1	+	−	−	−	−	−	+	+	+	+	+	+	+	+	+	2
$\frac{\sqrt{3}}{2}$	+	+	−	+	+	−	−	+	+	+	+	+	+	+	+	4
0	−	0	+	−	−	+	+	−	+	+	−	−	+	+	+	7

$f''(x)$, $f'''(x)$, ... $f^{(n)}(x)$ were obtained for $f(x)$ and evaluated for three significant values of the argument given in the above tabulation.

It appears that the number of real roots between $x = 0$ and $x = 1$ is either five, three, or one. Between $x = \frac{\sqrt{3}}{2}$ and $x = 1$, there may be either two real roots or no real root. The result is ambiguous and reveals nothing more than the fact that there can be not more than two real roots between $\frac{\sqrt{3}}{2}$ and 1. It will be noted that $f(0)$ and $f(1)$ have opposite signs, hence $f(x) = 0$ has at least one real root between these values of the argument.

It is possible to determine in this problem whether there are any real roots of $f(x) = 0$ between $\frac{\sqrt{3}}{2}$ and 1 by noting the following. An upper limit to the number of real roots of $f(x) = 0$ between a and b is obtained, if we set

$$x = \frac{a+by}{1+y}\ , \quad \text{from which } y = \frac{x-a}{b-x}\ ,$$

and multiply $f(x) = 0$ by $(1+y)^n$, whereupon Descartes' rule can be applied to the resulting equation in y. For, when $x = a$, $y = 0$, and when $x = b$, $y \rightarrow \infty$. Thus, the upper limit to the number of real roots is obtained for y ranging from zero to infinity.

We proceed to expand $f(x)$ in powers of $(x-1)$, which is equivalent to synthetic division applied to $f(x)$. The origin is thereby transferred to the point $(1,0)$. We obtain a second equation, which is $f(x_1 + h) = f(x)$, in which $x_1 = 1$ and $h = x - x_1$. The resulting equation has the following form:

$$\begin{aligned}(2)\quad & 62{,}958.6\, x^{14} + 1{,}478{,}617.2\, x^{13} + 14{,}090{,}713.2\, x^{12} + 71{,}590{,}267.0\, x^{11} \\ & + 215{,}628{,}347.0\, x^{10} + 401{,}803{,}228.0\, x^{9} + 462{,}626{,}364.8\, x^{8} + 308{,}906{,}029.4\, x^{7} \\ & + 87{,}810{,}107.8\, x^{6} - 26{,}948{,}817.2\, x^{5} - 38{,}147{,}888.6\, x^{4} - 20{,}023{,}876.4\, x^{3} \\ & - 5{,}600{,}732.6\, x^{2} - 138{,}508.2\, x + 197{,}066.9 = 0\ .\end{aligned}$$

Subsequent substitutions are simplified by considering roots between 0.8 and 1 in the original function, which correspond to those between -0.2 and 0 in (2). Again, $a = 0$ and $b = 0.2$ may be conveniently used in $x = \frac{a+by}{1+y}$, provided (2) is modified for $x = -x$. The resulting equation is:

(3) $62{,}958.6\,x^{14} - 1{,}478{,}617.2\,x^{13} + 14{,}090{,}713.2\,x^{12} - 71{,}590{,}267\,x^{11} + 215{,}628{,}347\,x^{10} - 401{,}803{,}228\,x^{9} + 462{,}626{,}364.8\,x^{8} - 308{,}906{,}029.4\,x^{7} + 87{,}810{,}107.8\,x^{6} + 26{,}948{,}817.2\,x^{5} - 38{,}147{,}888.6\,x^{4} + 20{,}023{,}876.4\,x^{3} - 5{,}600{,}732.6\,x^{2} + 138{,}508.2\,x + 197{,}066.9 = 0$.

Since $x = \dfrac{0.2y}{1+y}$, substituting in (3) and multiplying by $(1+y)^n$, we obtain the following equation, which is significant in the uniformity of the signs of its coefficients.

(4) $+ 111{,}182.36\,y^{14} + 1{,}683{,}495.92\,y^{13} + 772{,}347.24\,y^{12} + 50{,}398{,}117.82\,y^{11} + 147{,}579{,}811.15\,y^{10} + 312{,}700{,}908.21\,y^{9} + 494{,}370{,}840.91\,y^{8} + 592{,}328{,}500.50\,y^{7} + 540{,}317{,}707.20\,y^{6} + 373{,}256{,}922.54\,y^{5} + 192{,}101{,}766.37\,y^{4} + 71{,}364{,}918.88\,y^{3} + 18{,}069{,}179.91\,y^{2} + 2{,}786{,}638.24\,y + 197{,}066.9 = 0$.

By Descartes' rule, there are no positive real roots of (4), consequently no real roots between -0.2 and 0 in (2), and hence no real roots between 0.8 and 1 in the original function (1). For an angle $(-\alpha)$, the screen is symmetrical with respect to the equatorial plane, and the analysis holds for negative angles within the same range. Thus, there are no real maxima or minima for ϵ_t in the intervals

$$0 < \alpha \leqq \frac{\pi}{6}; \quad -\frac{\pi}{6} \leqq \alpha < 0$$

The solution, $\sin\alpha = 0$, (p. 48) is the only critical value within the above limits, and as yet may represent a maximum, minimum, or point of inflection in ϵ_t. That $\sin\alpha = 0$ renders ϵ_t a minimum, may be most conveniently shown by evaluating it for α somewhat greater than and less than zero. To satisfy any doubt in the mind of the reader, this conclusion may be confirmed by applying the customary mathematical criteria to the above analysis.

Calculations for minimum total error ${}_{min}\epsilon_t$ are given for reference.

Since $\alpha = 0$ represents the angle for <u>minimum error</u>, ${}_{min}\epsilon_t$ is determined for the conditions $\alpha = 0$, $n = 5$, $\frac{\xi_w}{\xi_s} = \frac{1}{2}$. The value of x corresponding to $\alpha = 0$ is $x = 0.57577r$. Evaluating ${}_{min}\epsilon_t$, we have

$$ {}_{min}\epsilon_t = 1.02768 \, \frac{A_v \xi_s}{4500\pi} $$

Since $A_V = 240.528$ sq. in. for $n = 5$, therefore

$$ {}_{min}\epsilon_t = 0.01748 \, \xi_s $$

This value of ϵ_t is the <u>least</u> value for <u>all</u> angles from -30° to $+30^\circ$. Similar calculations for $\alpha = \pm 30^\circ$, $n = 5$, and $\frac{\xi_w}{\xi_s} = \frac{1}{2}$ afford

$$ \epsilon_t = 0.02186 \, \xi_s $$

For $\alpha = \pm 5^\circ$, $\frac{\xi_w}{\xi_s} = \frac{1}{2}$, and $n = 5$, the following values of x and ϵ_t obtain:

$$ x = 0.57672 \, h $$

$$ \epsilon_t = 1.02831 \, \frac{A_v \xi_s}{4500\pi} = 0.01749 \, \xi_s $$

Thus, the position for minimum error ${}_{min}\epsilon_t$ corresponds to the source located at the center of the sphere. The largest ϵ_t will obtain at $\pm 30^\circ$ with no intermediate real critical points. These calculations based on the analysis cover the conditions $n = 5$, $\frac{\xi_w}{\xi_s} = \frac{1}{2}$ only. A like procedure may be extended to cover all practical values of n and $\frac{\xi_w}{\xi_s}$, if desired. For a given set of conditions, the per cent error will depend largely upon $\xi_s = \frac{I_s}{I_0}$, whereas the variation in ϵ_t is small over the entire range of α for given ξ_s.

Error Due to Absorption of Reflected Light from Screen by Lamp Assembly.

A reduction in window illumination is also caused by reflected light from the screen being absorbed by the lamp assembly. The error due to this absorption will vary with the surface exposed to reflected light, and will depend upon the mean absorption factor for parts of the assembly exposed to such reflected light. Although it will modify to some extent the distance x for minimum total error $\epsilon_s + \epsilon_w + \epsilon_L$, this error will not permit x to fall below $0.4(l - d)$. The practical range for x will lie between $0.4(l - d)$ and $0.5(l - d)$; that is, $h = 0.4r$ to $h = 0.5r$. The 60" sphere was constructed with allowance for $h = 0.416r = 12.5"$. Again referring to Fig. 7, it is well to keep in mind that within the range $x = 0.4(l - d)$ and $x = 0.5(l - d)$ with $\alpha = 30^\circ$, ϵ_w will be from four to nine times as effective compared to ϵ_s. Consequently, whenever possible, it will be necessary to guard against strongly illuminating any portion of the sphere such as in the neighborhood of Q which is hidden from the window opening by the screen.

It is not permissible to allow the screen to approach the window too closely even though the errors ϵ_w and ϵ_s vary inversely with x^2. As the screen approaches the window, the illuminated part of the wall hidden from view by the screen increases very rapidly especially when the screen is near the window. Thus, a large part of light from the first reflection cannot reach the window except in the course of successive reflections. The resultant decrease in window brightness introduces an error which may not be negligible.

When determining the luminous output of a source, the screen errors discussed in this work do not affect the final results in the measure indicated. For, in calibrating the sphere for a given lamp assembly, similar screen errors obtain. The actual error in any case will be the difference of the two obtained for successive operation of standard and test sources. Although this difference is quite close to zero in many instances, it is nevertheless essential to know what errors are involved and to what extent they may affect the final results in questionable cases.

DESIGN AND CONSTRUCTION

Body of Sphere

The sphere, constructed in accordance with the requirements of this thesis, is built of cast aluminum and made in sixteen sections as per Detail No. 1. The master pattern made of aluminum is fashioned after a wood pattern constructed on the basis of the dimensions on the above detail. The aluminum used is alloyed with 12% copper to give rigidity to the construction. These castings were poured with allowance for one-sixteenth inch finish on surfaces indicated on the blue print. One casting of Detail No. 1 is converted into a door large enough to admit any commercial lighting unit consistent with the size of the sphere, which measures sixty inches on the inside diameter. The castings forming the body of the sphere were machined, filed, and when assembled, were ground along the inner contours to insure proper alignment. When bolted together, they form the quarter-inch spherical hull.

Cover Plates

Upper and lower cover plates were cast of the same material as per Detail No. 3, in order to provide for lamp assemblies normally suspended and mounted vertically upright. The plates, cast in halves, were bolted together on a finished surface. The cover plate surfaces exposed to light from the sphere, conform to the curvature of the latter. The interior surface of castings forming the body and cover plates was given a light sand blast to prepare them for the inside finish described in a subsequent statement.

Supporting Framework

The sphere rests on a supporting framework made of standard one-half and one inch black wrought iron pipe welded together as per Detail No. 12. Flange plates welded to the risers and bent to conform to the curvature of the sphere, are bolted to the body. The risers are provided with swivel bearing casters for easy movement of the photometer. The pipe framework is fastened to the body in two similar constructions, permitting the photometer to be separated into vertical hemispheres when it is desired to move it from one room to another. Although vertical, the risers are amply spaced to give stability when the photometer is divided.

Window

The window opening of three and one-half inches diameter, counter bored to receive the diffusion glass, is formed by the alignment of two semi-circular openings in an upper and lower section respectively. The center of the window lies in the horizontal plane through the center of the sphere and midway between the meridian flanges. This location of the window, although theoretically non-essential, makes it convenient to use the sphere in connection with almost any auxiliary photometric devices. The center of the window approximates 52 inches from the floor level, a convenient height for the eye of the observer in conducting measurements. A diffusion glass cut to fit the counter bore is held in place by a framework of steel bolted to the castings. Thin Vellumoid gaskets protect the glass from slight irregularities in the bearing surfaces. It is essential that the surface of the glass exposed to incident light from the sphere be matte. A surface disclosing any visible specular reflection would transmit variable amounts of equal incident fluxes, depending upon the angle of incidence. Thus, light incident on the window would not be transmitted equally for all angles of incidence between zero and ninety degrees. A matte surface exposed to incident flux of uniform brightness will, if homogeneously translucent, appear as a uniformly bright disk from the outside. Such a surface differs somewhat in diffusing qualities from the sphere paint, but will transmit light more nearly equally for different angles of incidence.

Screens and Screen Supports

In order that the incident flux on the window produce a surface of uniform brightness, it is necessary to screen off all direct rays of light from the source to the glass. The location of screens for minimum total error was discussed at length in the theoretical treatment. A screen rod of 5/16" cold rolled stock rests in a bearing on the under side of the sphere, and fits into a similar device located on a vertical line above. Screens of sheet metal are provided and held in place on the rod by setscrews.

The screen dimensions were obtained by actual measurement and observation in the use of the largest bare lamp and lighting unit consistent with the size of the sphere. The largest screen will effectively obscure any direct light from a unit of 18" maximum dimension. Normally, no light assemblies exceeding 16" maximum dimension will be under consideration for test. Three screens are provided for use with light sources. The small one with transverse members serves for measurements on bare lamps. The remaining two may be used when lamp assembly tests are to be conducted. Flexibility obtains through rotation and axial adjustment.

Assemblies for Light Sources and Circuits

A side assembly made up almost entirely of standard equipment provides for measurement of bare lamps with base down or up. Two standard Mogul receptacles with adapters for medium base lamps are connected to a length of curved conduit by means of a suitable T unilet. The whole assembly is held rigidly in place by locknuts drawn against steel face plates on the sphere. The upper and lower assemblies, also constructed of standard equipment except for the wood bushing and steel ring in the upper cover plate casting, make it possible to use the photometer for testing lighting units. Current leads project from the upper assembly and are provided with insulated clips for ready connection to lamp terminals. Each assembly comprises a current and potential circuit brought out through standard fittings and flexible conduit to a terminal board mounted on the supporting framework. The potential leads are tapped near the lamp receptacle in each case, in order to minimize the drop in potential through the power circuit. The latter is amply dimensioned with No. 12 stranded copper wire to take care of loads as high as 16 amperes. All connections to lamp receptacles and junctions are soldered to insure good contact.

Interior Finish

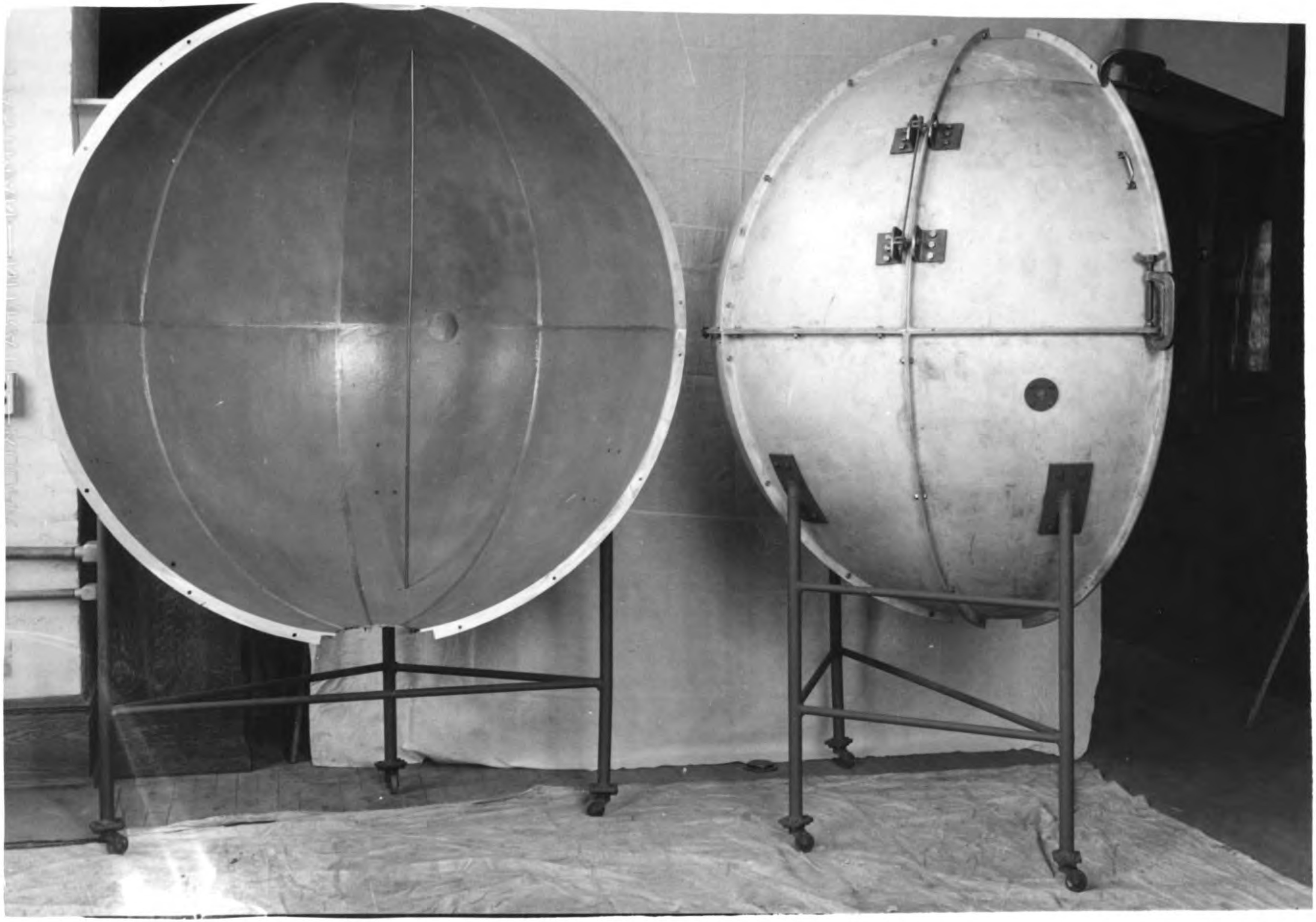
When ready to assemble, the inner surface of the sphere was washed and cleaned with carbon tetrachloride to remove any remaining grease or oil through handling and machining. Two coats of white shellac were applied, allowed to dry, and sanded. Four coats of flat white interior paint form the base for the finishing material, which is a special sphere paint used in the photometric laboratory of the United States Bureau of Standards. Two coats of sphere paint were applied with good results. The best results may be obtained by spraying. All permanent parts of the sphere such as screens and conduit assemblies are likewise finished in the manner described. Determination of the mean reflection factor k of the finished interior is a problem which should be of interest to those conducting experiments with this photometer. In addition to the applications, it may be well to determine the reflection factor of the sphere paint and the diffusion glass constant.

Technical Data

The following specifications are listed for ready reference:

Material of Body	Cast Aluminum (12% Cu)
Weight complete with Accessories	525 pounds
Inside Diameter	60 inches
Wall Thickness	1/4 inch
Center of Window to Floor Level	52 inches
Door Measurements	Width, 23 inches; Length, 25-1/4 inches

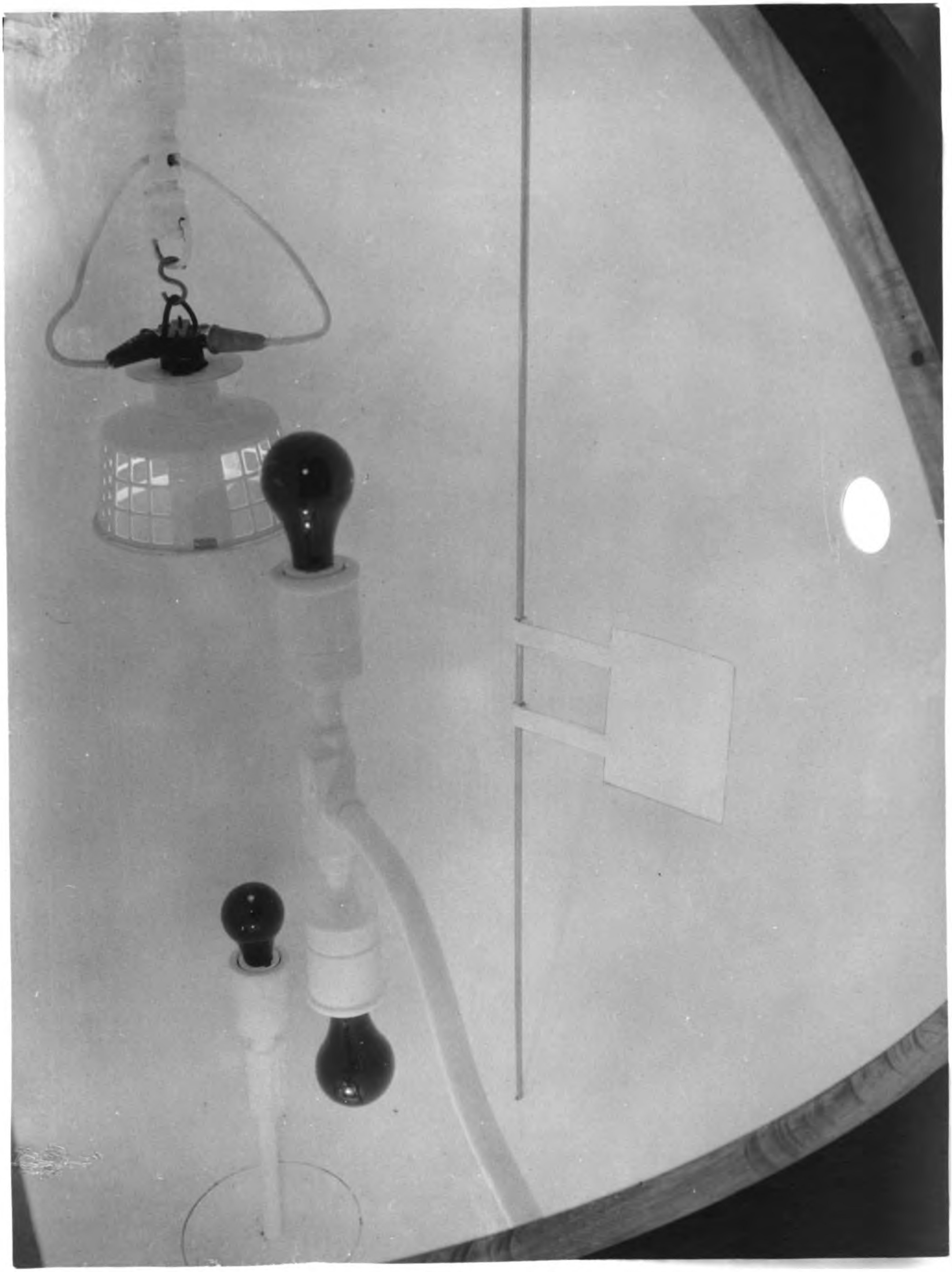
Maximum Height	8 feet, 2 inches
Maximum Diameter	66 "
Diameter of Diffusion Glass	4 "
Diameter of Window	3-1/2 "
Wall Paint	Sonneborn Flat White Interior Paint
Sphere Paint	Special Reflecting White Paint Manufactured by Benjamin Moore & Company, New York City



SIXTY-INCH CAST ALUMINUM SPHERE PHOTOMETER IN PROCESS OF CONSTRUCTION

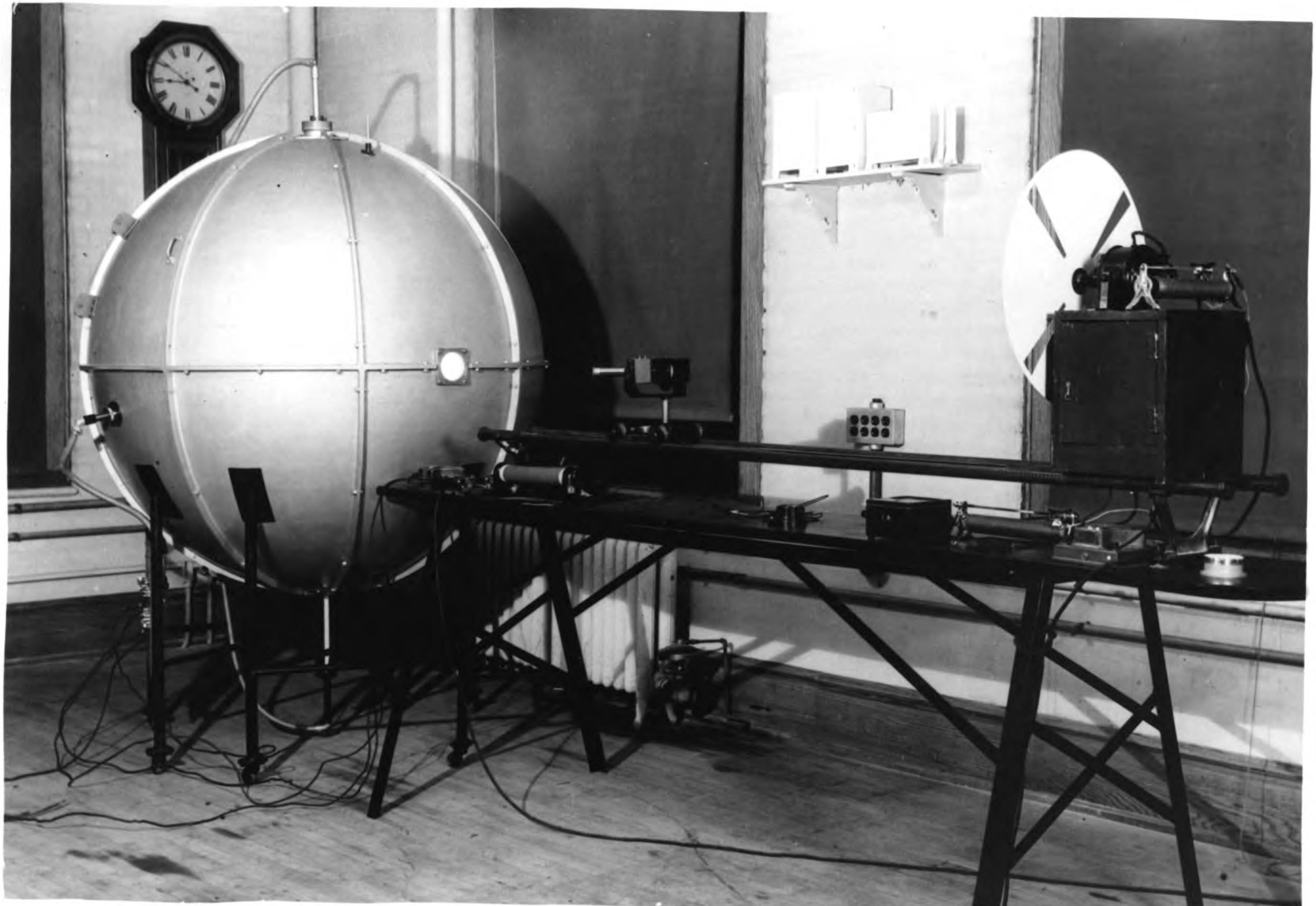
HEMISPHERES SEPARATED

SPHERE PHOTOMETER WITH DOOR OPEN
WINDOW TO THE RIGHT



VIEW OF INTERIOR FROM DOOR ENTRANCE SHOWING CONDUIT ASSEMBLIES, SCREEN SUPPORT, SCREEN, AND WINDOW

ASSEMBLED SPHERE WITH CLOSE VIEW OF SUPPORTING STRUCTURE AND TERMINAL PANEL FOR CURRENT AND POTENTIAL LEADS



SPHERE PHOTOMETER WITH AUXILIARY EQUIPMENT

PHOTOMETRIC TESTS AND METHODS

The method employed in measuring the luminous output of a lamp or lighting unit will depend upon the relative size and reflection coefficient of the test unit compared to these quantities when referred to the sphere. Commercial illuminants may be subdivided into two general classes in accordance with the above; viz., bare lamps and lighting units, the latter including all lamp assemblies for reflecting, refracting, and diffusing media exemplified by the variety of semi- and totally inclosing types on the market.

Bare Lamps

All present commercial bare lamps are small compared to the 60" sphere; consequently, one routine type of measurement will, in the majority of cases, satisfy requirements. The mean reflection factor of such lamps is quite high. It was shown that the illumination at the window is diminished by an amount ΔE_R due to the presence of a non-luminous diffusely reflecting body N screened from all direct light, where

$$\Delta E_R = \frac{\phi}{(1-k)S}\left[1 - \frac{1}{1 + \frac{Uk(1-k_u)}{(1-k)S}}\right]$$

Although the lamp represents the source of light, the enclosing glass acts as a non-luminous body for diffusely reflected light. If we interpret k_u as the mean reflection factor of the glass, the analysis may be extended to cover this case. The window illumination decreases in the ratio

$$1 + \frac{1}{(1-k)}\,\frac{1}{\frac{S}{(1-k_u)U} - 1} : 1$$

and the constant of the sphere, if expressed to vary inversely as E_R, will increase in the reciprocal ratio. The per cent increase in K is given by

$$\Delta K = \frac{1}{(1-k)\left(\frac{S}{(1-k_u)U} - 1\right)} \cdot 100\%$$

When this is less than unity, the effect of the lamp is negligible, for the error in K is less than one per cent. The ratio $\frac{S}{U}$ is sufficiently large for a 60" sphere to render $\Delta K < 1$. An arrangement for photometer-

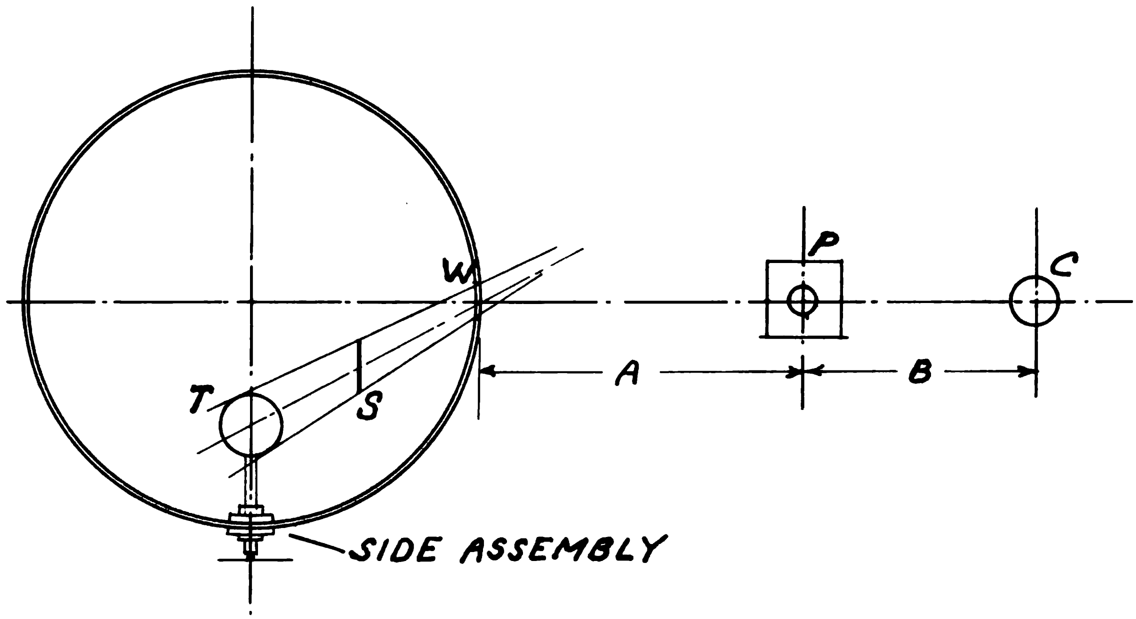


Fig. 9

ing bare lamps is shown in Fig. 9. The side conduit assembly provides for bare lamp measurements. It is advisable to use the substitution method whereby the calibrated lamp, which may be either a primary or secondary standard, is mounted in the side assembly in the same position as that demanded by the test lamp for normal operation. A reading is obtained with the comparison lamp on the photometer bench. The calibrated lamp is then replaced by the test lamp, and a second reading is obtained for balance of test lamp against comparison lamp. It is not necessary to know the candle power of the comparison lamp, but it is essential to operate this lamp at a constant voltage either equal to or somewhat less than its rated voltage. Operating it at values in excess of its rated voltage causes undue filament deterioration and attendant variations in luminous intensity. The position of the screen remains fixed throughout this procedure. The formulation whereby the luminous output of the test lamp obtains is as follows:

$$\phi = \phi_s \frac{A_1^2 \cdot B^2}{B_1^2 \cdot A^2}$$

This result is readily deduced by referring to Fig. 9. Let I_S and I denote the intensities of standard and test sources respectively; in this case, the candle powers of the window as viewed from P. Let A and B be as shown in the figure when the standard lamp is lighted. Then A_1 and B_1 will conveniently represent the distances involved when the standard lamp is replaced by the test source. We have

$$\frac{I_s}{I_c} = \frac{A^2}{B^2} \qquad \text{and} \qquad \frac{I}{I_c} = \frac{A_1^2}{B_1^2}.$$

Therefore, $$\frac{I}{I_s} = \frac{A_1^2}{B_1^2} \cdot \frac{B^2}{A^2}$$

Since $I = K\phi$ and $I_s = K\phi_s$,

hence $$\phi = \phi_s \frac{A_1^2}{B_1^2} \cdot \frac{B^2}{A^2} .$$

In addition to following the "Substitution Method" in order to reduce the probable errors in sphere photometry, the procedure to be described under "Lamp Assemblies" may be applied to bare lamp measurements with even greater reduction in error and less handling of lamps. When smaller spheres are used for testing bare lamps, it becomes necessary to have available a large supply of standardized lamps of various sizes and types similar to those which are to be submitted to test. For instance, if it were desired to conduct tests on frosted lamps in one of the smaller spheres, standardized frosted lamps must be provided. Clear gas-filled 150 watt lamps would require a clear gas-filled 150 watt standardized lamp. This impracticability is removed by conducting tests with spheres 30 inches in diameter or greater. The error involved by using one or probably two standardized lamps to cover all commercial sizes and types, is negligible for a sixty-inch sphere. The standard and test lamps can occupy positions in the sphere simultaneously. They can be operated with base up or down according to design. The standard remains in the sphere, thus requiring no further handling. Lamps for testing are introduced into the sphere in succession, and readings taken. Absorption due to glass and lamp bases is compensated for in this procedure. The test lamp output is again calculated as described above; namely,

$$\phi = \phi_s \frac{A_1^2}{B_1^2} \frac{B^2}{A^2}$$

When the set of lamps under test are alike, the absorption is practically constant, hence $\frac{B^2}{A^2}$ is a constant, and it will not be necessary to check this ratio for each lamp. When the lamps tested in succession differ widely, readings for A and B are followed by those for A_1 and B_1 for each lamp. No further handling of the standardized lamp is necessary after insertion in its place in the sphere, until tests are completed.

Lamp Assemblies

Nearly all lamp assemblies absorb sufficient light to render $\Delta K \leqq 1$. Calibration and tests on lamp assemblies cannot be conducted in the manner described under "Bare Lamps" without introducing an appreciable error. When dealing with a lighting unit whose M. S. C. P. or total luminous output is desired, it is customary to insert the unit and standard lamp in the sphere simultaneously. Consider, for example, a suspension type which would normally be hung from the ceiling. It is conveniently located along the vertical axis of the sphere and directly over the calibrated lamp mounted along the same axis through the opening at the bottom. With the test unit on open circuit, the sphere is calibrated first. Under these conditions, the brightness of the window is a measure of the relative absorption in the sphere when the test unit is lighted. Opening the standard lamp circuit and closing that of the test lamp, another brightness reading is obtained for the window. The two readings thus obtained, plus the known luminous output of the standard lamp, enable one to compute readily the luminous output of the test unit. The arrangement is shown in Fig. 10. Separate screens are provided for both test and standard lamps. The screens are located parallel to the vertical axis of the sphere. This simplifies the adjustment, since both screens are conveniently raised or lowered along a common screen support. Fig. 10 also discloses a small screen s' placed so as to intercept the rays of light from L_1 to L. This refinement should be considered when the intensity of illumination of L_1 in the direction of L_1L is reasonably pronounced, and when the surface area of L exposed to rays from L_1 is large. Whether the auxiliary screen s' shall be employed in testing lamp assemblies will depend upon the distance L_1L, the intensity of L_1 in the direction L_1L, the surface area of L exposed to direct light from L_1, and the coefficient of absorption of L. Careful consideration of these factors will in each case determine the advisability of using s'. Generally, however, when the diameter of the sphere is 60" or greater, the use of s' will be limited.

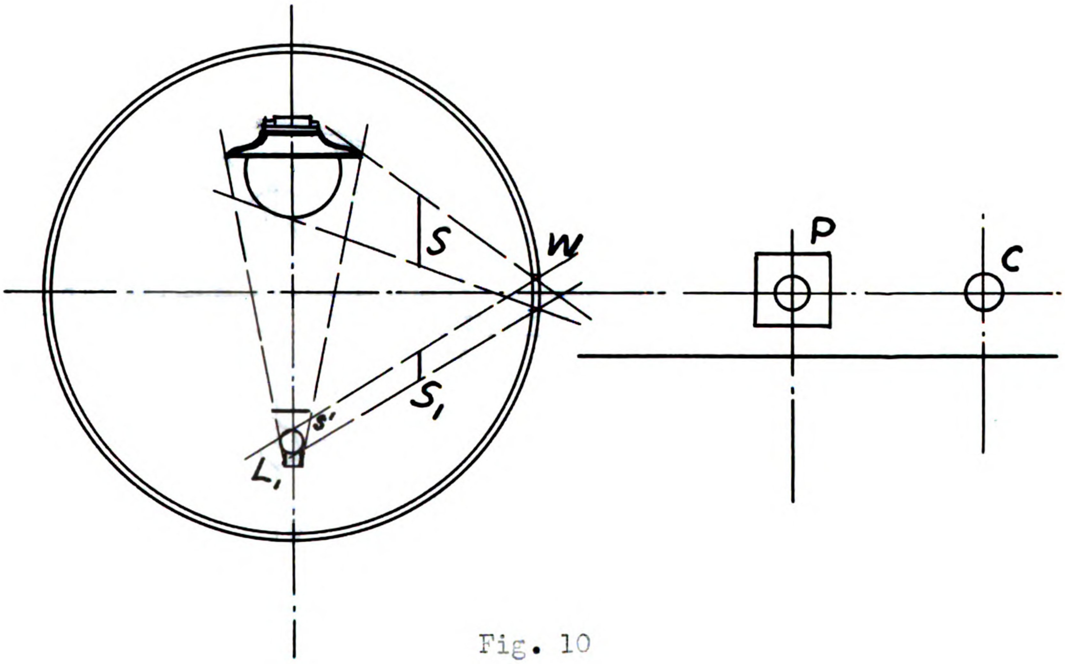


Fig. 10

In the avent that s' should be called into service, it will be necessary to determine the luminous output of the aggregate; namely, L_1 provided with s'. This is accomplished by comparing the aggregate with a well calibrated bare lamp, which is equivalent to calibrating the combination L_1s'. Knowing the luminous flux emitted by L_1s', tests are made in the manner described above. Equations (1) and (2) page 40 serve as a check on the probable screen errors ϵ_s and ϵ_w in any particular lamp assembly test.

Use of Sphere without Diffusion Glass Window

It is desirable to have one or more diffusing glasses available to replace an impaired or broken glass. To meet the contingency in the event breakage occurs during a period of continued tests, it is possible to operate the sphere without the diffusing glass. A slight, temporary change in design will readily solve the problem. Remove the existing screen rod with its screens. The resulting recesses in the wall of the

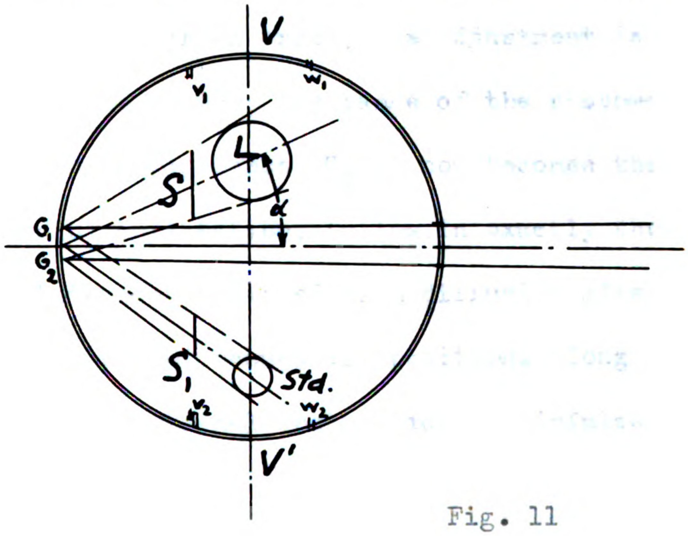


Fig. 11

sphere may be filled with dry, white blotting paper tightly wrapped to fit the holes. Locate points v_1 and v_2 symmetrically with respect to w_1 and w_2, and press rubber vacuum bushings coated with sphere paint into place with centers at v_1 and v_2 as indicated in Fig. 11. Insert screen rod into the rubber bearings. The rubber bushings must be sufficiently rigid to hold rod and screens in normal vertical position; hence, the rubber should be only mildly flexible. When determining the luminous output of lamp units in this manner, it is self-evident that a screen must be employed whose dimensions will not obstruct the bright spot on the wall of the sphere diametrically opposite the window opening. The screen should be as small as possible in keeping with minimum screen absorption, and yet sufficiently large so that no portion of the field of view in the photometer head is illuminated by once reflected light coming from the wall opposite the window. In other words, for all possible positions of the photometer head over the operating range, the field must be illuminated by diffusely reflected light emanating from that portion G_1G_2 of the sphere screened from the direct rays of the source. The screen support has a fixed location relative to VV' for minimum screen error; the adjustment is attained by modifying the size of the screen, the range of the photometer head, or both. The area of uniform brightness G_1G_2 now becomes the test source. Measurements, including calibration, follow in exactly the same manner as outlined for the photometer equipped with diffusing glass. It is at once apparent that L cannot occupy all positions along VV' from $\alpha = \frac{\pi}{6}$ to $\alpha = -\frac{\pi}{6}$. This constructional restriction is minimized when photometering bare lamps.

Measurement of Transmission and Reflection Coefficients for Diffusing Media

Diffusing media play an important role in a variety of lamp accessories used to overcome glare and produce the soft shadows and generally pleasing effect characteristic of many types of direct and semi-indirect units. The latter not only diminish the light, but scatter the flux in such a manner as to produce much the same effect as obtained from a diffusely reflecting surface. The sphere photometer can be used conveniently to determine the coefficient of transmission of diffusing media, since the sphere can be relied upon to integrate the effect produced by these substances. We begin by measuring the total lumens of a source without the diffusing unit. Then the source is enclosed by the diffusing glass, and the readings obtained for balance of window and comparison lamp enable one to obtain the total lumens of source with diffusing bowl. The ratio of flux thus obtained to the bare lamp flux is the coefficient of transmission. In case the bowl does not completely surround or enclose the source, the above ratio still represents the average transmission factor, since $\phi' = I_0 \omega = \frac{\phi}{4\pi} \omega$.

In order to obtain the diffuse reflection factor, one may utilize the circular opening at the top of the sphere to admit light from an exterior source. If this flux be measured with the aid of a convenient standard lamp, and then allowed to fall upon a plane specimen of the diffusely reflecting material arranged at any desired angle within the sphere, a second measurement will enable one to compute the flux reflected and the coefficient of diffused reflection for the given angle of incidence. The reflecting medium here functions simultaneously as source and screen.

Reflection Factor of Sphere Paint and Transmission Coefficient of Diffusion Glass Window

It was shown in the discussion of the theory of the sphere that the constant is $K = \frac{b_w}{\phi} = \frac{I_w}{A_w \phi}$. If ϕ is a known flux from a carefully calibrated lamp, and I_W is balanced against a known intensity I_c of a comparison lamp, then K, the constant of the sphere, is determined. The constant may be expressed in a somewhat different form.

$$K = \frac{b_w}{\phi} = \frac{E_R \tau}{\pi \phi}$$

$$K' = KA_w = \frac{I_w}{\phi} = \frac{E_R A_w \tau}{\pi \phi}$$

Since $$E_R = \frac{\phi}{4\pi r^2} \cdot \frac{k}{1-k}, \quad K' = \frac{\tau k \rho^2}{S(1-k)}$$

$$k = \frac{K'S}{\tau\rho^2 + K'S} = \frac{I_w S}{\tau\rho^2\phi + I_w S}$$

$$k = \frac{I_w S}{\phi\rho^2 + I_w S} \quad \text{for} \quad \tau = 1 \qquad \text{(no diffusion glass)}$$

Thus, the reflection factor of the sphere paint may be obtained from measurements of sphere constant K' without diffusion glass and the radii r and ρ of sphere and window respectively. The source of known lumens ϕ must be screened so that no direct rays will fall upon the part of the sphere wall opposite the window and visible to the observer stationed at the photometer head. If this precaution is not taken, the brightness of that portion of the sphere will be non-uniform due to the component of direct illumination from the source which follows the photometric distribution curve. The reflection factor thus obtained, however, will not be the true coefficient for the wall paint, since the inner surface of the window will generally reflect differently than the paint. The result for k will be sufficiently accurate for practical considerations. Since k is independent of τ, measurement of K' may be made with the window in place and τ computed from known values of the constants.

Auxiliary Equipment

The sphere photometer can be used with any of the modern equipment designed to measure luminous intensity, brightness, or illumination.

The Sharp-Millar photometer, the Macbeth illuminometer, and the Weber photometer are well adapted to sphere photometry. The Weston Photronic Cell, if calibrated to read in foot-candles, may be used to measure the illumination produced on the sensitized disc by the window opening in the sphere. Due to the fact that certain parts of the spectrum of the illumination produced by the window affect the cell and eye differently, the cell can be relied upon only when used to measure light of the same character for which the cell was standardized. This limitation may be overcome by the use of color filters, whereby a given source of mild color tone may be made to register the illumination on the disc which in turn is proportional to the luminous output of the source.

Advantages of the Sphere Photometer

To derive all of its possible advantages, certain care in handling the sphere must be closely observed. Since all parts of the spherical shell are more or less in tension, it is recommended not to separate the sphere into upper and lower hemispheres, in order to preserve the alignment at the window. When necessary to separate the sphere into hemispheres preparatory to making a change in location, the contour along the plane of separation should be refinished with sphere paint after assembly. It is advisable to check the constant of the sphere before and after moving, using the same standard both times. The sphere paint coating will become soiled and discolored with age and should be renewed. If soiled, the finish should be cleaned with carbon tetrachloride and given a light coat of sphere paint. If clean but discolored, it is sufficient to apply one coat of sphere paint. The upper and lower conduit assemblies may be refinished outside the sphere. The side assembly can be removed and painted, but a simpler procedure is to refinish it in position, taking care to protect the adjacent surfaces from stray paint.

Some advantages in the use of the sphere photometer are:

1. Simplicity in construction.
2. Limited adjustment of parts.
3. Absence of flicker due to rotation of lamps.
4. More flexibility in arc lamp tests.
5. Elimination of breakage due to rotation of lamps.
6. Use not confined to dark room.
7. Greater accuracy than other types of integrating photometers.

CALIBRATION

As previously described, the constant of the sphere will depend upon the nature of the interior finish, and is affected by the presence of light absorbing surfaces.

It was shown that the constant may be expressed in various forms. It is convenient, when using the bar photometer, to standardize the

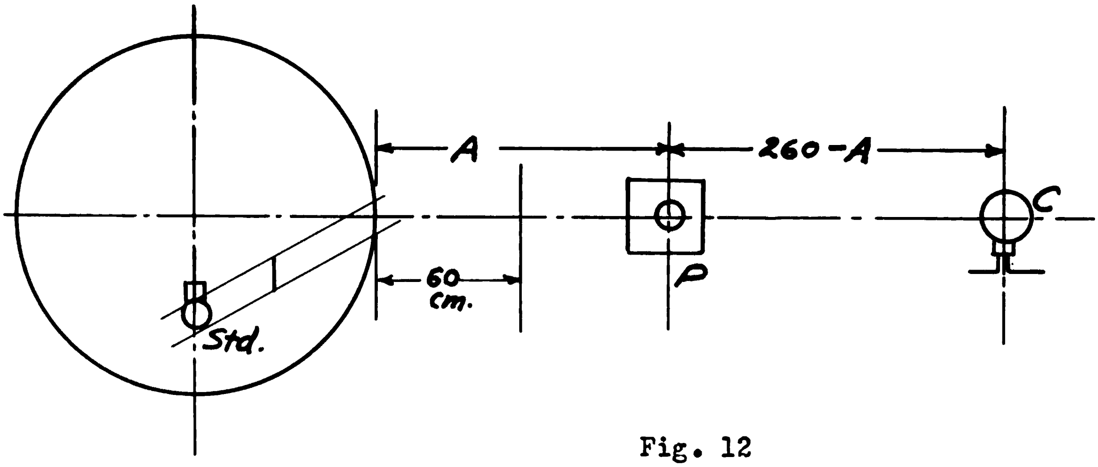


Fig. 12

sphere in terms of candles per lumen; that is, candle power of window per lumen output of source.

The 60" sphere was standardized using a two meter bar photometer. A fixed point near the left end of the latter was set at a distance of 60 centimeters from the surface of the window. See Fig. 12.

Notes and Data

Standard Lamp. Std.

Tungsten Filament, B. S. No. 3264, standardized base up by the United States Bureau of Standards.

Volts	105.0
Amperes	4.227
Lumens	8040

Comparison Lamp. C.

Vacuum Tungsten Standard Lamp, B. S. No. 4772, E.T.L. 5452, standardized by the United States Bureau of Standards tip up and stationary.

Volts	109.0
Amperes	0.350
Candles	33.8

The above figure, 33.8, represents the candle power in the direction normal to the screen in the Lummer and Brodhun photometer head.

The sphere was standardized with upper conduit assembly removed, top closed, lower conduit assembly in place, and wall finished with one coat of sphere paint. The diffusion glass was used as furnished by Leeds and Northrup Company.

Photometer Bench Readings.

A	A	Std.	C
82.7 cm.	82.8 cm.	E = 105 Volts	E = 109 Volts
84.1 "	81.8 "	I = 4.24 Amps.	I = 0.358 Amp.
81.7 "	81.5 "		
81.6 "	82.6 "		
82.3 "	81.0 "		
82.4 "	81.0 "		
82.6 "	82.6 "		
82.5 "			

The mean value for A from the table is A = 82.21 cm.

Calculations

$$\frac{I_w}{I_c} = \frac{A^2}{(260-A)^2}$$

$$I_w = I_c \frac{A^2}{(260-A)^2}$$

$$K' = \frac{I_w}{\phi} = \frac{I_c}{\phi} \cdot \frac{A^2}{(260-A)^2}$$

$$K' = \frac{33.8\,(82.21)^2}{8040\,(260-82.21)^2} = 0.00089$$ candles per lumen.

$$b_w = \frac{I_w}{A_w} = \frac{7.22}{62.07} = 0.116$$ candles per sq. cm.

b_w = 0.36442 lamberts = 364.42 millilamberts

$$\text{Std. } I_o = \frac{\phi}{4\pi} = \frac{8040}{4\pi} = 639.8 + \text{ c.p.}$$

The discrepancy in the current readings is due in part to the current taken by the voltmeters, $i_v = \frac{E}{R_v}$, as well as to slight variations in effecting the scale readings.

The constant of the sphere for the conditions above described is K' = 0.00089 candles per lumen.

Calibration curves for the sphere may be obtained with various sizes and types of bare lamps. As pointed out in a previous discussion, however, the variation in K' will be negligible when conducting tests on such lamps in the larger spheres. It is advisable to check K' when measuring the luminous output of lighting units having an appreciable absorption.

BIBLIOGRAPHY

Transactions of Illuminating Engineering Society.

Vol. III, No. 7, October 1908, p. 502.

Text in Illumination, by Kunerth.

Illuminating Engineering, by F. E. Cady and H. B. Dates, 1925.

Light Photometry and Illuminating Engineering, by W. E. Barrows, 1925.

Illuminating Engineering Practice.

Lectures on Illuminating Engineering delivered at the University of Pennsylvania in 1917 under Joint Auspices of the University and the Illuminating Engineering Society.

The Integrating Photometric Sphere, by E. B. Rosa and A. H. Taylor.

Transactions of Illuminating Engineering Society,

Vol. XI, 1916, p. 453.

Sphere Photometer, by R. Ulbricht, 1920.

Lehrbuch der Photometrie, by Uppenborn-Monasch, 1912.

Advanced Calculus, by Woods.

Theory of Equations, by Dickson.

Handbook for Electrical Engineers, by Pender.

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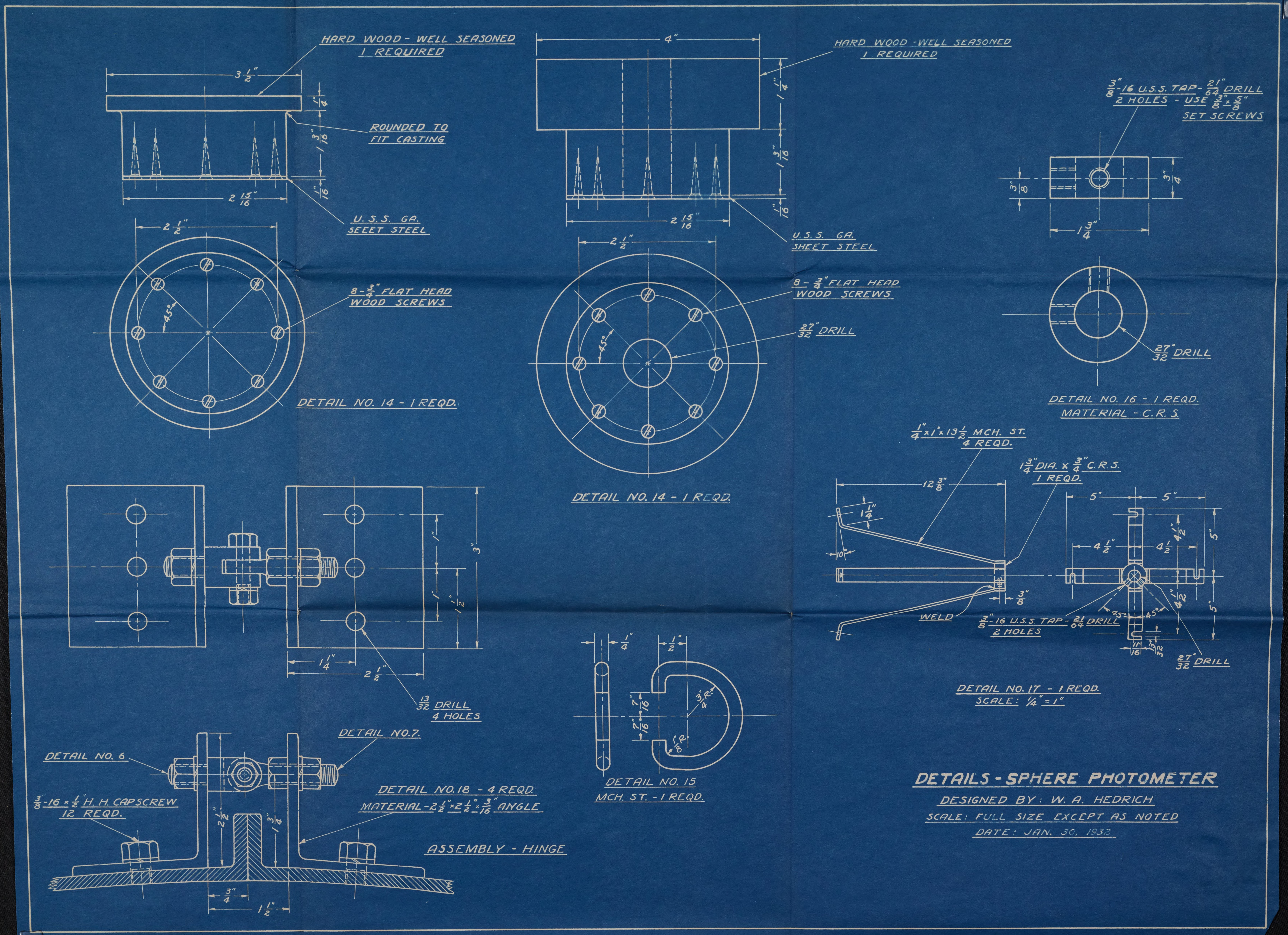
HARD WOOD - WELL SEASONED
1 REQUIRED
3 1/2"
1/4"
ROUNDED TO
FIT CASTING
1 3/16"
2 15/16"
1/16"
U. S. S. GA.
SEEET STEEL
2 1/2"
45°
8 - 3/4" FLAT HEAD
WOOD SCREWS
DETAIL NO. 14 - 1 REQD.
4"
HARD WOOD - WELL SEASONED
1 REQUIRED
1 1/4"
1 3/16"
2 15/16"
1/16"
U. S. S. GA.
SHEET STEEL
2 1/2"
8 - 3/4" FLAT HEAD
WOOD SCREWS
27/32" DRILL
45°
DETAIL NO. 14 - 1 REQD.
3/8"-16 U.S.S. TAP - 21/64" DRILL
2 HOLES - USE 3/8" x 5/8"
SET SCREWS
3/8"
3/4"
1 3/4"
27/32" DRILL
DETAIL NO. 16 - 1 REQD.
MATERIAL - C.R.S.
1" 1/4" x 1" x 13 1/2 MCH. ST.
4 REQD.
1 3/4" DIA. x 3/4" C.R.S.
1 REQD.
12 3/8"
1 1/4"
10°
5"
5"
4 1/2"
4 1/2"
4 1/2"
5"
3/8"
WELD
3/8"-16 U.S.S. TAP - 21/64 DRILL
2 HOLES
45°
45°
4 1/2"
5"
11/16
13/32
27/32" DRILL
DETAIL NO. 17 - 1 REQD.
SCALE: 1/4" = 1"
1"
1"
1 1/2"
3"
1 1/4"
2 1/2"
13/32 DRILL
4 HOLES
1/4"
1/2"
7/16"
7/16"
3/4" R.
1/8" R.
DETAIL NO. 15
MCH. ST. - 1 REQD.
DETAIL NO. 6
DETAIL NO. 7.
3/8"-16 x 1/2" H. H. CAPSCREW
12 REQD.
DETAIL NO. 18 - 4 REQD.
MATERIAL - 2 1/2" x 2 1/2" x 3/16" ANGLE
2 1/2"
1 3/4"
ASSEMBLY - HINGE
3/4"
1 1/2"
DETAILS - SPHERE PHOTOMETER
DESIGNED BY: W. A. HEDRICH
SCALE: FULL SIZE EXCEPT AS NOTED
DATE: JAN. 30, 1932.

Pocket has

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Blueprint 2