VARIABLES AFFECTING RATIO FACTORS FOR ESTIMATING 305-DAY PRODUCTION FROM PART LACTATIONS

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Robert C. Lamb

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THESIS

VARIABLES AFFECTING RATIO FACTORS FOR ESTIMATING 305-DAY PRODUCTION FROM PART LACTATIONS

By

Robert C. Lamb

AN ABSTRACT

Submitted to the College of Agriculture
Michigan State University of Agriculture and
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ABSTRACT

Complete lactation records for 12,561 Holsteins, 2,262 Guernseys, 990 Jerseys, and 459 Brown Swiss compiled in Michigan DHIA-IBM from June 1954 through July 1957 were analyzed to ascertain the relative effects of five variables on the relationship of total to part milk production, and then were used to derive ratio factors for extending partial records to 305 days.

The ratio of total milk produced on ten monthly test days to milk produced on each test day was used as the measure of relationship between total and part production. Components of variance of ratios indicated that lactation number had a larger influence on the total to part relationship than did age at freshening. Season of freshening also exerted an influence on the ratio of total to part, but to a slightly less degree than either lactation number or age. The effect of herd on the total to part relationship was small and unimportant. Breeds were analyzed separately, but a visual inspection of the components of variance for the different breeds suggested that differences between breeds existed in the total to part relationship.

Ratio factors for extending records from each of ten monthly test days and from cumulative test-day

production were computed for different ages, lactation numbers, and seasons of freshening for each of the four breeds. Although no interactions between the variables were important, ratio factors adjusting for breed, age, and season and for breed, lactation, and season were presented in a combined form.

Only small differences exist between the factors which adjust for ages and those which adjust for lactation number, indicating that either set of factors should adequately extend incomplete records. However, factors based on age are more useful in extending those records in which age at freshening and lactation number do not coincide. In practice breed, age, and season of freshening should be considered in extending partial records to 305 days.

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INTRODUCTION

More than one-fifth of the lactations started in DHIA are not completed because the cows die or are sold (2, 18). Seldom are the records terminated early in lactation used in programs to improve dairy cattle, and what information they contain is lost or ignored. One method of salvaging the information which is lost when a cow leaves the herd prior to completing a lactation is to extend the incomplete record to estimate from her production before leaving the herd what the cow would have produced had she remained.

The most important use for extended incomplete records is to make possible the inclusion in a sire's progeny test a record for every daughter that comes into production. At present it is permissible to remove poor producers from the herd prior to completion of the first lactation and thereby prevent including these low records in the sire proof. Varying intensities of selecting daughters among bulls will result in unfairness in comparing progeny tests unless the information contained in the incomplete records can be used. In addition, unselected daughters may unavoidably leave the herd early due to injury or disease.

Their terminated lactations also may supply worthwhile information to be included in the progeny test.

Another purpose for extending incomplete records is to project what a cow will produce in a given lactation while that lactation is still in progress. This early information may be valuable to the frequent decisions of selection which must be made prior to the completion of a lactation, particularly the first lactation. Selection of a bull by the performance of his offspring is also delayed until several of his daughters have completed at least one lactation. Projection of these records while still in progress would reduce the time needed to prove a sire by up to nine months. The gain in genetic progress due to shortening the generation interval by this amount may more than offset the possible decrease in reliability of the incomplete records.

An additional obstacle to obtaining satisfactory progeny tests is that only about 10% of the dairy cattle are on some form of a testing program (29). However, if a cow could be tested less frequently, each test extended to a 305-day basis, and then some form of a weighted average of these extended records taken as actual production, more herds could be tested without greatly increasing the cost or the labor involved. For measuring the transmitting ability of a bull the increased number of records under different management conditions should tend to offset the decreased accuracy of estimating any one lactation.

Several workers have reported multiplicative and regression factors for estimating complete lactation yield from various portions of the lactation. Their studies have shown that some non-genetic variables should be adjusted for in extending incomplete lactations. Extension factors which will correct for the effects of all of these variables are not available, nor are they practical. Nevertheless, the variables which contribute a relatively larger portion of the total variation in the total to part relationship do need to be corrected for in extending short-time records. Age, number of previous calvings, breed, season of freshening, milking frequency, gestation, length of previous dry period, body weight, herd environment, and many other variables have been shown to influence the quantity of milk and butterfat produced during the lactation. However, the influence of each of these variables upon the various stages of the lactation or upon the relationship of complete to part lactations is not fully understood.

The objectives of this study are (a) to estimate the relative effects of breed, age, lactation number, herd, and season of freshening on the relationship between total milk produced in 305 days and milk production in each month, and (b) to compute factors, based on these findings for extending incomplete lactations to 305 days.

REVIEW OF LITERATURE

Variables Affecting Relationship of Total to Part Production

Production records of dairy cows are influenced by many variable factors such as age, weight, frequency of milking, length of lactation, gestation, and general herd management. Numerous studies have shown that these and other factors influence the amount of milk and butterfat produced by a dairy cow. However, most of these studies have been concerned with the practical effects of these variables upon milk and fat production during the entire lactation, and relatively little is known of effects of these same variables on the relationship of total production to various portions of the lactation. Yet the relative effect of each variable upon the whole to part relationship determines what adjustments are needed to predict lactation yields from short-time records.

Age and lactation number

A difference in the shape of the lactation curve, in which first-calf heifers reach a lower peak in production but decline less rapidly than do older cows, suggests that different factors are needed for extending records made by cows of different ages. The question which arises at this point is whether this difference in the shape of the lactation curve is strictly a function of age, whether it depends upon the number of previous lactations, or whether both age and lactation number are important. If the influence of one of these variables on the whole to part relationship is greater than the influence of the other one, then that particular variable should be adjusted for in extending incomplete records. However, if there is little difference in their relative influence then extension factors based upon either of them should suffice, or one could choose to adjust for both of them.

To ascertain the effect of age on the relationship of total to part production, Eldridge and Atkeson (6) developed regression factors for estimating total yield from one day's test for 12 different age groups. Separate factors appeared necessary, especially for younger cows. By regrouping the early ages into first and second lactations, they found that factors based upon lactation number more accurately extended the part record to a complete lactation.

Using 599 lactation records of Holstein cows in the Iowa State College herd, Madden _et al_. (23) studied the relationship between total production and various parts of the lactation. They found that age correction factors, which had been derived by Kendrick (19) from 305-day lactation totals, were fairly suitable for correcting the center months but unsuitable for the first and last months. This was especially true for records initiated at less than three years of age. By plotting separately the average monthly production for records started prior to three years of age and those begun at three years and over, the lactation curve for the older cows was shown to reach a higher peak but drop more rapidly during the last months of the lactation. Ratio factors for extending cumulative monthly production for both milk and fat were so dissimilar for the two age groups that it was concluded that separate factors are needed for at least the first 150 days. Factors for ages above three years did not differ from each other enough to warrant additional separate factors. Later, Madden _et al_. (24) confirmed these results while developing ratio factors from 6,495 Holstein HIR records.

In a more recent paper, Madden _et al_. (25) reported on the influence of age at calving in connection with the influence of milking frequency and level of production on the relationship between production for single months, cumulative part, and total milk production. When monthly milk production of 6,715 Holstein-Friesian HIR records was plotted by year of age, the lactation curves for one and two year olds were parallel, the curves for ages three and older were also parallel, but differed from those for younger ages by achieving a larger maximum but declining more rapidly.

In compiling ratio factors for extending short-time Ayrshire records, Kendrick (20) investigated the influence of age, season of freshening and level of production on these factors. Records were separated into three age groups, under 30 months, 31-44 months, and over 45 months, corresponding to first lactations, second lactations, and all later lactations, respectively. Differences between factors for each age group supported the findings in Holsteins (23) that separate factors are needed for different age groups. After the sixth month age did not significantly affect the factors for extending incomplete records.

Harvey (13) in supplementing Kendrick's (20) work with Ayrshire data also broke the records down into three age groups, under 33 months, 33 to 46 months, and 47 months or older. This grouping gave essentially the same results as reported by Kendrick.

Using Michigan DHIA records, Fritz (9) studied the influence of breed, herd, lactation number, season, and age of freshening on the relationship between test day production and production for the entire lactation. In order to study environmental influences, the data were grouped into two seasons of freshening (October-March and April-September), three lactation numbers (first, second, and third and above), and eight age groups (under 24 months, 60 months and over, and at 6-month intervals in between). Linear regression equations were used to measure the relationship between cumulative part production and total production. The relative influence of age and lactation number on the total to part relationship was measured by analysis of variance of the intra-herd regression coefficients for ages within the three lactation groups. This technique not only failed to show the relative influence of age and lactation number, but also failed to show whether any differences exist.

Season of freshening

Numerous investigations have shown that season of freshening influences milk and butterfat production. Cows freshening during the fall and winter months produce about 10% more milk than those freshening during the summer months. Several authors (17, 26, 27, 30) have noted that the influence of season on total milk production varies from herd to herd, and often from year to year. Whether herd-season and year-season interactions have any influence on the relationship of total to part production is not known.

Direct influences of season of freshening on the whole to part relationship have been studied by Kendrick (20) who concluded that season of freshening should be considered for all age groups in extending short-time Ayrshire records. However, after the sixth month of lactation the factors are not affected by season of calving. In contrast, Eldridge and Atkeson (6) considered season of freshening but found its influence insignificant. In studying the effect of season on the whole-part relationship for both milk and fat, Fritz (9) noted a significant effect of season only in the first month for fat and a significant herd-season effect only in the first and seventh cumulative months for fat and the seventh cumulative month for milk. Therefore, season of freshening was not considered an important factor influencing the relationship between total and part production.

Herds

Harvey (13) suggested that differences between herds in part-whole relationships should be studied.

Fritz (9) found in his study of Holstein data that variation in the total to part relationship due to herd differences is significant only for the first month of milk production and not at all for fat. This indicates that herd differences are not an important factor influencing the relationship of total to part production.

Breeds

Possible breed differences in the total to part relationship for milk and butterfat production have been reported (4, 9). Cannon *et al*. (4), using data from five breeds, plotted percentage relationships of each month's yield to first month yield for each breed separately. The shapes of these curves were so similar that breeds were not considered separately in calculating the extension factors. Although Fritz (9) studied breeds separately, no conclusions were drawn as to possible breed differences. However, a visual comparison of the extension factors, which he presents for both milk and fat for four different breeds, indicates that there are breed differences in the relationship of total to part production.

Level of production

Madden *et al*. (25) fitted quadratic regression equations of total on cumulative part production for various groups of ages and milking frequencies to determine if different extension factors are required for low and high producing cows. The small differences noted indicate that the same relationship between part and total production exists for both low and high producers. This is in complete agreement with the findings of Harvey (13) in a study of 2,867 Ayrshire Herd test records. It differs somewhat from the observations of Kendrick (20) who found that for mature cows there was a slight difference in factors, at least for the first few months, due to level of production. For cows freshening in the 31-44 age group there was a significant difference due to level of production, while for records begun at 30 months or less there were insufficient data to reach any conclusion.

Frequency of milking

Madden et al. (25) grouped 6,715 Holstein HIR records according to milking frequencies (2X and 3X) within age groups (under 3 years and 3 years and over). The regression factors for the 2X and 3X frequencies differed significantly for the first month for young cows. No other significant differences were found between extension factors for the 2X and 3X frequencies, indicating that milking frequency is not of major importance in extending part-time records.

Ways to Use Incomplete Records

Selecting during first lactation

Factors to extend incomplete first lactations are of practical importance because the information which these records contain may be all one has on which to judge the performance of first calf heifers. Yet, the question of how much reliance can be placed on initial records has led to several investigations as to whether production during the first lactation can be used as a basis for selection. In 1940 Johansson and Hansson (17) reported that among the first three records of a cow the second lactation yield was the poorest indicator of the cow's ability to produce, and that the first record was the best indicator.

Kennedy and Seath (21) investigated the value of incomplete first records as a basis for culling and progeny testing. Their results, based on 80 Holstein and 80 Jersey records, indicate that the production of first-calf heifers during the first four months is a good index of the complete first lactation and also gives some indication as to the probable production for the second lactation. They found a correlation of .78 between cumulative production for four months and the complete (305-day) first lactation. The correlation between the same four months of the first lactation and the complete (305-day) second lactation was .45 for Jerseys and .30 for Holsteins. The correlation between complete first and second lactations was .54.

Hickman and Henderson (15), in studying whether selection of sires based on daughters' production in the first lactation will adversely affect lifetime production, concluded that selection on the basis of production in the first lactation will favor increased lifetime production.

In a study of 4,912 dam-daughter pairs of Swedish cattle, Johansson (16) found that records for the first lactation showed the least variation and those for the second lactation the largest, the difference between the standard deviations of the two lactations being about 10%. The heritability of butterfat yield estimated from the same data was .33 for the first lactation, .10 for the second lactation, and .24 for the third lactation. He concluded that the first lactation record is significantly superior to the second, and slightly superior to the third as an indicator of the cows inherent capacity to produce. In addition, the heritability of average yield for the first and second lactations was .14, which is less than the heritability for the first lactation; thus nothing is gained by including the second record.

This conclusion has been supported by Rendel <u>et al</u>. (30) who measured heritabilities of 70-day milk yield and 305-day milk and fat yield and found them to be .36, .43, and .43, respectively, for first lactations and .09, .24, and .42, respectively, for second lactations. Because of the low heritability of production in the second lactation, the average yield of the first four lactations proved to be of no more value as an indicator of production of the daughter than did production in the first lactation alone. Robertson and Khishin (31) present data which indicate that the genetic regression of increase in production with age on yield in first lactation is close to zero. Therefore, selection based on first lactations alone should not change the increase in yield with age and should lead to improvement in later lactations.

<u>Predicting total production from parts</u>

Associations between production at various stages in the lactation and total production suggest that certain periods in the lactation may be more indicative of production for the entire lactation than are other periods. Gaines (10), in studying milk and fat percentage data from the Holstein and Guernsey Advanced Registries, found that a one or two day test conducted during the fourth month of the lactation gave the best indication of what the cow would produce during the lactation.

Cannon _et_ _al_. (4) cite work conducted in India which also indicates that a short-time test taken during the fourth month gives the most accurate prediction of 305-day production. However, this differs slightly from their own results from 400 Iowa State College Holstein records and 1,289 Iowa DHIA lactation records. The low standard error of estimate for the fifth month for both sets of data indicates that 305-day production is most accurately predicted from the fifth month. Tests made during the sixth month gave the next most accurate prediction, followed closely by those in the fourth and seventh months. Prediction of a cow's lactation yield from a single test is least accurately made in the first and last months of the lactation.

In order to ascertain which single test days are most closely associated with total production, Madden _et_ _al_. (25) computed correlations between milk produced on a single test day and the total milk produced for the first ten test days. Correlations were largest for the fifth month followed closely by the sixth, fourth, and seventh months, which agrees with the results of Cannon _et_ _al_. (4). Regressions of total production on production for a single test day also were largest during the fourth to the seventh months. Correlations between cumulative test day production and the sum of production for ten test days became larger as the cumulative part became a larger part of the total production and were .9 by the fourth month of cumulative production. This supports Kennedy and Seath's (21) earlier findings that the first four cumulative months are at least as valuable as any single month for predicting total lactation production.

In 1957 Voelker (32) extended two-year old records in progress in the South Dakota State College Herd. The correlations between the actual 305-day production and the records extended from 1-9 months increased from .68 for 1 month to .99 for 9 months with a correlation of .89 by the end of 5 cumulative months. Rendel et al. (30) observed a correlation of .8 between 70-day yield and production for the total lactation while studying 3,109 production records of six main dairy breeds in Great Britain. The results of these studies, plus those of Gifford (12) and Harvey (13), have shown that incomplete records of five months or less will provide a reasonably accurate indication of a cow's ability to produce.

Deriving extension factors

Basically two methods have been used to derive factors for estimating total lactation production from either a single test or from cumulative production. The simplest method is to obtain the ratio of total production to part production. This ratio, when multiplied by actual part production equals estimated 305-day production. Symbolically this is expressed as $\hat{Y} = cX$ where $\hat{Y}$ is predicted lactation yield; c, the ratio of total to part production; and X, the actual part production.

The other method is to obtain the regression of total production on partial production. Both linear and quadratic regression equations have been used, but Harvey (13) found the curvilinearity of accumulated production and stage of lactation to be small enough that the linear regression equation will provide a satisfactory means of extending part production. The linear equation has the general form $\hat{Y} = a + bX$, where $\hat{Y}$ is estimated 10-month production; a, the point where the regression line intercepts the Y axis; b, the regression coefficient measuring the average change in Y for each unit change in X; and X, the actual part production.

The choice between the two methods depends on the purpose for which the extended records are to be used, the ease and simplicity of use, and the comparative accuracy of the methods. The ratio method is by far the simplest and easiest to derive, to use, and to be understood by the dairy farmer. The comparative accuracy of the two methods has been reported previously (13, 24, 25). The ratio method appears to underestimate total production of low-producing cows and overestimate total production of high-producing cows (24) since it takes into account only the incompleteness of the lactation. The regression method corrects for this incompleteness as well as for the incomplete repeatability of parts of the lactation. Harvey (13) has shown the difference in total production estimated by the two methods to be $(b - c)(X - \bar{x})$, which is the amount which must be added to cX to obtain $a + bX$. ($\underline{b}$ equals linear regression of total on part, $\underline{c}$ represents the ratio of total to part, X is actual part production, and $\bar{x}$ is the mean of part production). The difference $(b - c)$ is negative and largest $(c > b)$ during the early (1-3) cumulative months, least $(c \geq b)$ during the center months (4-6), and positive and small during the longer (7-9) cumulative period (25).

Although Harvey (13) has shown that 10-month records for butterfat estimated from records of one to three months duration are 12 to 47 per cent more variable when estimated by the ratio method as compared to the linear regression equation, Madden et al. (25) pointed out that the variation in total production estimated by ratio is more nearly like the variation in actual total production, whereas total production estimated by regression is less variable than actual production. If extended part records are used for culling females within a herd or for proving sires, particularly in artificial insemination, extending by means of the ratio method should result in culling the same females or in selecting the same sires as extending by regression since extending by ratio does not change the order of records but merely spreads them out. Since records extended by regression differ less than actual, this tendency to group the records more closely about the mean may make selection decisions more difficult.

Reducing costs of testing

In recent years interest has grown in the possibility of less frequent testing as a means of lowering the cost of testing. Such a plan is contingent upon the accuracy of lactation records made from less frequent tests as compared to that under the present system. Several investigators have reported on the accuracy of bi-monthly and quarterly tests versus monthly tests. Bayley et al. (3)

found that the differences in variability between monthly, bi-monthly, and quarterly tests are small for both milk and fat. The average differences in yield are significant but unimportant. Because of the frequency with which large errors were observed, it was concluded that records based on bi-monthly and quarterly tests should be satisfactory for sire provings and population studies, but that they may be unsatisfactory when used as individual lactation records. Several other investigators (1, 5, 11, 28) have arrived at the same conclusions regarding bi-monthly and quarterly testing.

Erb *et al.* (7) found that the estimation of milk yield is not grossly affected by testing at 60-day intervals using the centering date method of calculation. However, they found with this system that fat yield is, on the average, overestimated when the cow is first tested early in lactation and underestimated when the cow is first tested toward the end of the second month of lactation. In a later publication (8) it was reported that bi-monthly testing using the centering date method is nearly as accurate as the calendar month method of monthly testing.

In view of the results with bi-monthly and quarterly testing it appears possible that the use of extension factors to describe the lactation curve with less frequent testing may be a way of obtaining accurate records at a lower cost. The next step, which is beyond the scope of this study, is to see how accurately production can be predicted from three or four tests in a herd per year, combining the separate estimates from the same lactation of a cow to estimate that lactation.

SOURCE OF DATA

The data were 16,272 complete lactation records from four major dairy breeds obtained from the Michigan DHIA-IBM program for the period June 1954 through July 1957. Each of these records conformed to the following specifications: (a) 2X milking, (b) less than 50 days production calculated from a single test day, (c) first test day within 34 days of freshening, and (d) 10 consecutive monthly tests. All records were terminated at 10 months. The number of records for each breed were: 12,561 Holstein, 2,262 Guernsey, 990 Jersey, and 459 Brown Swiss.

Each record identified the cow and the herd in which the record was made, and contained information on date of freshening (month and year), breed, age at freshening (in months), lactation number, and milk and fat production on test day. Milk produced on test day was recorded to the nearest one-tenth lb., and fat production was recorded to the nearest one-hundredth lb.

METHODS AND RESULTS

Grouping of Data

The data were analyzed separately for each breed. In order to consider the postulated environmental influences, the data were grouped into four seasons of freshening, four ages, and three lactation numbers.

Most studies of the effect of season of freshening on production have grouped seasons in such a manner as to take into account differences in systems of feeding, housing, and climatic conditions for the particular area. The same principle was used in this study. Records were grouped by month of freshening into four groups, October-December, January-March, April-June, and July-September. This grouping should place between groups differences between periods when dry roughage is normally fed and when cows are on pasture. Table 1 shows the distribution of records by season and breed.

First lactation records were separated from later lactations because the shape of the lactation curve differs between first and later lactations, and because of the important need for extending first lactations. Since the information from second lactations is also helpful in estimating a cow's producing ability, second lactation

TABLE 1

DISTRIBUTION OF RECORDS BY BREEDS IN EACH SEASON

Breed	Season of Freshening							
	Jan.-Mar.		April-June		July-Sept.		Oct.-Dec.	
	No.	%	No.	%	No.	%	No.	%
Brown Swiss	100	21.8	119	25.9	136	29.6	104	22.7
Guernsey	473	20.9	327	14.5	747	33.0	715	31.6
Holstein	2,474	19.7	1,805	14.4	4,790	38.1	3,492	27.8
Jersey	163	16.5	151	15.3	396	40.0	280	28.3
Total	3,210		2,402		6,069		4,591	
Percentage[a]		19.7		14.8		37.3		28.2

[a]Percent of records in each season for all breeds combined.

records were also separated, while all later lactation records were grouped together. Table 2 shows the distribution of records by breed and lactation number.

Several studies have shown that age in months should be considered in extending incomplete records. Although Madden and co-workers (23, 24, 25), Fritz (9), Harvey (13), and Kendrick (20) used different groupings of age, all agree that separate factors are needed for extending records made at younger ages. Consequently, records for two-, three-, and four-year olds were studied separately

TABLE 2

DISTRIBUTION OF RECORDS BY BREEDS IN EACH LACTATION

Breed	Lactation Number					
	1		2		3 and over	
	No.	%	No.	%	No.	%
Brown Swiss	113	24.6	70	15.3	276	60.1
Guernsey	697	30.8	467	20.6	1,098	48.6
Holstein	4,110	32.7	2,759	22.0	5,692	45.3
Jersey	305	30.8	222	22.4	463	46.8
Total	5,225		3,518		7,529	
Percentage[a]		32.1		21.6		46.3

[a]Percent of records by lactation number for all breeds combined.

from those of older cows. The groups contained those records initiated prior to 36 months, 36 through 47 months, 48 through 59 months, and 60 months and over. Table 3 shows the distribution of records for each age by breed.

Measure of Relationship

The ratio of total milk produced on ten monthly test days to milk produced on each test day was used as the measure of relationship between total and part production. The ratio of total to part was used in preference to using linear regression coefficients (9) because

TABLE 3

DISTRIBUTION OF RECORDS BY BREEDS IN EACH AGE

Breed	Age (in months)							
	<36		36-47		48-59		>59	
	No.	%	No.	%	No.	%	No.	%
Brown Swiss	103	22.4	70	15.3	67	14.6	219	47.7
Guernsey	677	29.9	447	19.8	352	15.6	786	34.7
Holstein	3,939	31.3	2,709	21.6	2,023	16.1	3,890	31.0
Jersey	326	32.9	208	21.0	125	12.6	331	33.5
Total	5,045		3,434		2,567		5,226	
Percentage[a]		31.0		21.1		15.8		32.1

[a]Percentage of records by ages for all breeds combined.

(a) the ratio is a direct measure of the relationship of total to part production, whereas the regression coefficients also adjust for the incomplete repeatability of various portions of the lactation. Such correction may be useful in extending incomplete records but confuses the study of the relationship of total to part production. (b) The regression coefficients have the disadvantage that each regression coefficient may have a different variance which would require more complex analysis. (c) Single observations within any classification can be used with the ratio method; hence, all records are usable. In contrast, the regression method needs at least two observations within a classification. Because of insufficient records within each intra-herd, -season, -lactation, -age group, approximately 30% of the data were lost in the study of the total to part relationship by Fritz (9). (d) Ratios are simpler and easier to compute.

The ratios of total to part production were calculated to the nearest one-hundredth lb. for each of the ten individual months in the lactation.

Estimating Components of Variance

The estimation of the components of variance is a means of apportioning the total variance among a group of contributing elements. The method used to obtain the components of variance from non-orthogonal data is described in detail by Henderson (14). In this method the sums of squares of ratios are computed as in the standard analysis of variance. These sums of squares are equated to their expectations, as obtained under the assumptions of the model, and the resulting equations are solved simultaneously for the unknown variances.

Because of the disproportionate distribution of the data in this study, ages and lactation numbers were studied on a within-herds basis. Analysis on this basis set aside any herd differences in ratios which would have entered into the age or lactation component if, as is suspected, the age distribution varied for herds.

Let y_{ijklm} denote the ratio of total to part production for the m^{th} record made during the i^{th} season in the k^{th} age group and the l^{th} lactation group in the j^{th} herd. Then the linear model representing this ratio is

$$y_{ijklm} = u + s_i + h_j + a_{jk} + l_{jl} + sh_{ij} + sa_{ijk} + sl_{ijl} + al_{jkl} + r_{ijklm},$$

where u is the unknown population mean and common to all observations, s_i is an effect common to all records made in the i^{th} season, h_j is an effect peculiar to all records in the j^{th} herd, a_{jk} is a common effect of all records made at the k^{th} age in the j^{th} herd, l_{jl} is an effect common to all records made during the l^{th} lactation in the j^{th} herd, sh_{ij} is peculiar to all records in the i^{th} season in the j^{th} herd, sa_{ijk} is peculiar to all records made at the k^{th} age in the j^{th} herd during the i^{th} season, sl_{ijl} is common to all records made in the l^{th} lactation in the j^{th} herd during the i^{th} season, al_{jkl} is peculiar to all records made in the j^{th} herd at the k^{th} age during the l^{th} lactation, and r_{ijklm} is a random element peculiar to each record.

In using this method an assumption is made that, except for the constant u, all elements of the model are uncorrelated variables with means zero and variances S, H, A, L, SH, SA, SL, AL, and R. In this case a positive correlation between age and lactation number is known to exist. Lush (22) has pointed out that a correlation between any two elements a and b makes each include in its between-class variance all of their covariance and part (r^2_{ab}) of the variance directly caused by the other. Therefore the interaction sum of squares is biased to seem too small. However, since in this case only the larger sources of variation are being sought, what bias does exist is not as important as if precise estimates of components were being sought. Henderson (14) presents another method for estimating components of variance which yields unbiased estimates, but it is computationally prohibitive. The results of the analysis of variance are presented in Tables 4, 5, 6, and 7.

A represents the variance due to differences in ratios between cows freshening at different ages within the same herd, while L is the variation between ratios for cows in different lactations within the same herd. H represents the variance caused by differences between herds, while S represents the variation brought about by cows

TABLE 4

COMPONENTS OF VARIANCE FOR THE RATIO OF TOTAL PRODUCTION TO MONTHLY PRODUCTION--BROWN SWISS

Source	Month																			
	1		2		3		4		5		6		7		8		9		10	
	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%
A	2.53	27	1.34	21	-.19	0	-.01	0	-.04	0	-.17	0	.28	8	-.20	0	.90	5	12.07	6
L	1.27	13	.95	15	.13	8	-.06	0	.03	1	-.26	0	.27	8	.69	11	3.72	19	32.94	18
H	-.41	0	-.34	0	-.01	0	-.15	8	.04	2	.14	6	-.21	0	-.02	0	-1.08	0	-16.84	0
S	.02	0	.15	2	.24	15	.32	18	.35	20	.28	11	.64	18	1.00	16	2.96	15	7.12	4
SH	.55	6	.43	7	.06	4	-.51	0	.12	7	-.18	0	.44	12	.20	3	1.64	8	42.58	23
SA	-1.23	0	-1.03	0	.18	11	.25	14	.05	3	-.09	0	-.05	0	-.15	0	.63	3	-7.18	0
SL	-.42	0	-.24	0	-.12	0	.37	21	-.13	0	.44	17	-.39	0	.21	3	-3.14	0	-38.24	0
AL	-3.21	0	-1.87	0	.40	25	.18	10	.13	8	.27	11	-.01	0	-.17	0	-1.36	0	-20.81	0
R	5.16	54	3.64	55	.60	37	.54	29	1.02	59	1.40	55	1.90	54	3.99	67	9.66	50	93.51	49
Total[a]	9.52		6.51		1.62		1.81		1.75		2.53		3.55		6.09		19.50		188.22	

[a]Negative components were considered zero for computing totals and percentages.

TABLE 5

COMPONENTS OF VARIANCE FOR THE RATIO OF TOTAL PRODUCTION TO MONTHLY PRODUCTION--GUERNSEY

Source	Month 1		2		3		4		5		6		7		8		9		10	
	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%
A	.23	10	.18	12	-.57	0	.84	28	.22	8	.00	0	-.26	0	2.08	16	4.70	12	15.51	8
L	.39	16	.17	11	-.11	0	-.89	0	.19	7	.04	1	.21	5	1.30	10	6.75	17	3.75	2
H	-.02	0	.02	1	.02	1	-.02	0	-.08	0	.02	1	.05	1	.14	1	-.26	0	-3.52	0
S	.13	5	.17	11	.25	13	.37	12	.43	16	.05	2	.82	19	2.17	17	5.64	14	16.34	8
SH	.11	5	-.06	0	-.04	0	.04	1	.36	13	.20	7	.08	2	.17	1	1.04	3	19.12	9
SA	-.21	0	-.13	0	.39	21	.01	0	-.11	0	.55	18	-.20	0	.44	3	2.23	6	-.43	0
SL	.17	7	.16	10	.06	3	.04	1	-.34	0	-.48	0	.36	8	-.26	0	-5.04	0	-24.04	0
AL	-.32	0	-.14	0	.99	53	.69	23	-.34	0	-.21	0	.12	3	-2.37	0	-6.46	0	14.86	7
R	1.40	57	.82	55	.14	8	.97	35	1.50	56	2.14	71	2.65	62	6.62	52	18.99	48	133.95	66
Total[a]	2.44		1.51		1.85		2.96		2.70		3.00		4.29		12.92		39.34		203.54	

[a]Negative components were considered zero for computing totals and percentages.

TABLE 6

COMPONENTS OF VARIANCE FOR THE RATIO OF TOTAL PRODUCTION TO MONTHLY PRODUCTION--HOLSTEIN

Source	Month																			
	1		2		3		4		5		6		7		8		9		10	
	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%
A	.19	7	.04	3	.46	20	.23	13	.56	17	.42	14	-.46	0	.09	1	2.27	9	-43.55	0
L	.50	18	.22	15	-.04	0	.23	13	.20	6	.27	9	1.69	21	.86	13	4.16	17	17.32	6
H	-.13	0	-.09	0	-.15	0	-.16	0	-.15	0	-.22	0	-.43	0	-.28	0	-1.06	0	3.67	1
S	.13	5	.10	7	.13	6	.17	9	.16	5	.18	6	.37	5	.78	12	1.92	8	6.30	2
SH	.18	6	.03	2	.20	9	.25	14	.24	8	.38	12	1.17	14	.65	10	1.41	6	-1.50	0
SA	.19	7	.02	1	-.44	0	-.06	0	-.44	0	-.18	0	3.82	46	.31	5	1.14	5	76.88	28
SL	-.22	0	.13	9	.19	8	-.13	0	.01	0	-.24	0	-2.89	0	-.48	0	-1.94	0	-37.38	0
AL	-.30	0	.07	5	-.10	0	-.21	0	-.53	0	-.33	0	- .98	0	.31	5	-2.06	0	38.71	14
R	1.61	57	.84	58	1.33	57	.91	51	1.94	64	1.82	59	1.10	14	3.58	54	13.21	55	130.76	49
Total[a]	2.80		1.45		2.32		1.80		3.06		3.05		8.15		6.59		24.11		273.64	

[a]Negative components were considered zero for computing totals and percentages.

TABLE 7

COMPONENTS OF VARIANCE FOR THE RATIO OF TOTAL PRODUCTION TO MONTHLY PRODUCTION--JERSEY

Source	Month																			
	1		2		3		4		5		6		7		8		9		10	
	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%
A	.15	4	0.00	0	.24	15	.13	8	-.53	0	.39	11	.47	6	4.21	16	4.28	14	11.46	4
L	.58	17	.08	6	.14	9	-.01	0	-.03	0	.48	13	.74	10	1.13	4	1.96	7	47.22	17
H	.09	3	.08	6	.09	5	.11	7	.07	2	-.07	0	.28	4	3.38	13	4.01	14	3.42	1
S	.08	2	.18	14	.21	13	.27	16	.39	13	.53	14	1.21	17	1.70	7	5.63	19	14.81	5
SH	.19	5	0.00	0	.04	3	.06	4	.23	8	.21	6	.46	6	-.89	0	-.16	0	30.50	11
SA	1.15	33	-.12	0	-.09	0	-.16	0	.38	13	.02	0	-.56	0	-5.13	0	1.26	4	20.45	7
SL	-.85	0	.11	8	0.00	0	.02	1	-.58	0	-.16	0	-.13	0	.97	4	-1.44	0	-34.13	0
AL	-.55	0	.06	4	-.19	0	-.01	0	.70	23	-.75	0	-.79	0	-5.19	0	-4.76	0	-61.41	0
R	1.28	36	.85	62	.86	55	1.11	64	1.25	41	2.09	56	4.08	57	14.41	56	12.44	42	157.62	55
Total[a]	3.52		1.36		1.59		1.70		3.02		3.72		7.24		25.80		29.59		285.47	

[a]Negative components were considered zero for computing totals and percentages.

freshening in different seasons of the year. SH can be interpreted as the variation between herds with respect to the relationship of total to part production for records initiated during different seasons. The next three components also represent variance due to interactions. SA estimates the variation between total and part production for records initiated during the different seasons by cows in the various age groups within the same herd. SL estimates the variation between total and part production for records started during the different seasons by cows in the same herd with different numbers of previous calvings. AL is an intra-herd estimate of the variation in total to part production between different lactation groups with respect to records begun at the different ages. And finally, R is the residual component. It consists mainly of unanalyzed differences between individual ratios, many of which are possibly genetic. Several minor interactions have also been combined into this component.

Residual variance

Considerable variation between months of the lactation and between breeds is observed for all components. A few extreme departures from the general trend occur; however, these appear to be sampling rather than identified sources of variation. Differences attributable to R are by far the largest, ranging from 8-71% of the variance. Since such a large portion of the variance is unidentified and, therefore, will not be adjusted for, some of the extended individual lactations will vary considerably from what they normally would be, but large numbers of extended records should average about what they would under normal conditions.

Age and lactation number

On an intra-herd basis differences due to age contribute 0-28% of the variation in the relationship of whole to part production, while those differences due to the number of previous calvings contribute 0-21%. However, there is less fluctuation from month to month in the contribution of lactation number, and overall it appears to be the larger of the two components. Lactation number furnishes a larger portion of the variation in all breeds except Guernseys, and in all months except the third through the sixth.

Season

Differences due to season of freshening contribute a relatively larger portion of the variance than was expected, the proportion ranging from 0-20%. There are some indications of breed differences in this component with Holsteins being least influenced by season of calving. Month to month differences in this component are important. In Tables 4-7 the actual variance due to season of freshening increases with each succeeding month in the lactation, with the largest increase occurring during the tenth month. This indicates that the season in which a cow freshens has an increasingly larger effect on the relationship of total to part production as the lactation progresses. Although the actual variance due to season of freshening increases greatly in the tenth month, variance attributable to the other sources increases even more so that the relative effect of season drops during the tenth month.

Herd

The small size of H indicates that differences between herds account for almost no variation in the relationship of total to part production. The contribution does vary some between breeds, with Jerseys being the only breed seemingly influenced by herd differences. Month to month variation due to herd differences is almost nonexistent.

Season-Herd

The size of SH, ranging from 0-23%, also indicates a larger contribution to the variation than had been expected, particularly in view of the almost non-existent overall effect attributed to herds. This indicates that although herds do not vary appreciably in their effect on the total to part relationship, there is a difference between herds as to the influence season of freshening has upon this relationship. It is noted that while S is smaller for Holsteins, SH is larger for this breed. Evidently season of freshening has approximately the same amount of influence on the total to part relationship for all breeds, but in Holsteins this influence varies more from herd to herd. A large amount of month to month variation is evident in SH, making it more difficult to determine the exact influence exerted by this interaction. In Holsteins the size of SH is small for the first and last months and large during the more stable center months, but in all other breeds there is considerable fluctuation from month to month.

Season-Age

Except for the first month for Jerseys and the seventh and tenth months for Holsteins, the size of SA is negligible.

If the wide difference in these three months is due to sampling, then the age of a cow and the season in which she freshens act independently of each other.

Season-Lactation

The differences due to the interaction of season and lactation number, with the exception of the fourth and sixth months in Brown Swiss, are of no importance.

Age-Lactation

Some month to month variation between breeds exists for AL. Abnormally large deviations for the third and fourth month in Guernseys are the only indications of any interaction between age and lactation number. However, a lack of interaction cannot, in this case, imply an independent effect of the variables in question since age at freshening and lactation number are to a large extent simply different measures of the same variable. On the other hand it does indicate that there are certain aspects of lactation number, such as condition of the cow and condition of her udder, which are unique to lactation number and not age and, therefore, exert an independent effect on production. It must also be recalled from earlier discussion that due to the correlation between age and lactation number most of the variation due to interaction of these variables is expected to show up in the individual components. Therefore, there may be more age-lactation number interaction than is indicated by the analysis.

In view of these findings, three new linear models were constructed and applied to the Holstein data to see if additional information could be obtained, particularly regarding age and lactation number. These new models are hereafter designated as linear models two, three, and four. In model two the non-significant interactions are assigned to the residual component. The other two models were constructed so as to place in one case the age affects and in the other case the lactation affects with the residual component, thereby allowing the other component to express itself unhindered.

Using the same definition of terms as in the original model, model two is expressed as:

$$y_{ijklm} = u + s_i + h_j + a_{jk} + l_{jl} + sh_{ij} + r_{ijklm}.$$

To fit the data to this model, the residual term was recalculated to include all of the classifications dropped from the previous model. Therefore, the mean squares and expectations for the s, h, a, l, and sh classifications are the same as in model one, and only the residual term is changed. The results of this analysis are presented in Table 8. Only changes of a minor nature are noted between this table and the results of model one for Holsteins as presented in

TABLE 8

COMPONENTS OF VARIANCE FOR THE RATIO OF TOTAL PRODUCTION
TO MONTHLY PRODUCTION
LINEAR MODEL TWO--HOLSTEIN

Source	Month																			
	1		2		3		4		5		6		7		8		9		10	
	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%
A	.06	2	.10	7	.24	14	.07	5	0.00	0	.13	6	.17	5	.40	7	1.28	6	8.42	5
L	.34	15	.28	19	0.00	0	.13	10	.06	3	.09	4	.46	12	.78	13	2.96	15	15.23	8
H	-.10	0	-.11	0	-.13	0	-.13	0	-.09	0	-.15	0	-.38	0	-.26	0	-.84	0	-.56	0
S	.13	6	.10	7	.13	8	.17	12	.16	8	.18	8	.37	10	.78	13	1.92	10	6.30	3
SH	.15	7	.08	6	.14	8	.18	13	.11	6	.23	10	1.16	31	.56	9	.98	5	6.59	3
R	1.58	70	.91	61	1.19	70	.82	60	1.69	83	1.60	72	1.60	42	3.57	58	12.75	64	155.23	81
Total[a]	2.25		1.47		1.72		1.37		2.03		2.23		3.76		6.09		19.88		191.78	

[a]Negative components were considered zero for computing totals and percentages.

Table 6. The sum of the variance components for each month are smaller under model two. Both the age and the lactation components contribute a smaller percentage of the total variation, with A taking a decidedly larger drop. Differences due to season of freshening and herd-season interaction are larger in this analysis and appear to be about equal to those of lactation number. Absolutely no variance is exhibited by H, reemphasizing the observation that herds alone have no influence on the relationship being studied.

Model three is expressed symbolically as:

$$y_{ijklm} = u + s_i + h_j + a_{jk} + sh_{ij} + sa_{ijk} + r_{ijklm},$$

again using the same definition of terms. Since models three and four are best interpreted when presented together, the linear model for the latter is:

$$y_{ijklm} = u + s_i + h_j + l_{jl} + sh_{ij} + sl_{ijl} + r_{ijklm}.$$

The results are presented in Tables 9 and 10, respectively. Again the sum of the variance components for each month is less for each of these models than under the original model (Table 6), but the sums for models two, three, and four are in quite close agreement with each other. As was expected, age contributes a larger portion of the variance when lactation number is disregarded in the analysis, and likewise L is larger when age is disregarded. When the

TABLE 9

COMPONENTS OF VARIANCE FOR THE RATIO OF TOTAL PRODUCTION TO MONTHLY PRODUCTION LINEAR MODEL THREE--HOLSTEIN

Source	Month																			
	1		2		3		4		5		6		7		8		9		10	
	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%
A	.27	12	.25	18	.35	17	.21	14	.19	8	.32	12	-.17	0	.92	15	3.30	17	.19	0
H	-.04	0	-.05	0	-.16	0	-.12	0	-.12	0	-.17	0	-.12	0	-.12	0	-.31	0	7.27	4
S	.13	6	.10	7	.13	6	.17	11	.16	6	.18	7	.37	10	.78	13	1.92	10	6.30	3
SH	.14	7	.06	4	.24	12	.23	15	.24	10	.33	13	.62	17	.56	10	1.07	5	-8.75	0
SA	.04	2	.11	8	-.31	0	-.15	0	-.44	0	-.35	0	1.86	51	.01	0	-.16	0	52.96	26
R	1.60	73	.90	63	1.34	65	.90	60	1.90	76	1.78	68	.81	22	3.68	62	13.23	68	133.23	67
Total[a]	2.19		1.41		2.06		1.50		2.50		2.60		3.67		5.94		19.52		199.96	

[a]Negative components were considered zero for computing totals and percentages.

TABLE 10

COMPONENTS OF VARIANCE FOR THE RATIO OF TOTAL PRODUCTION TO MONTHLY PRODUCTION LINEAR MODEL FOUR--HOLSTEIN

Source	Month																			
	1		2		3		4		5		6		7		8		9		10	
	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%	Var.	%
L	.41	17	.30	20	.17	10	.23	15	.15	6	.30	11	.76	17	1.13	18	4.18	20	17.00	9
H	-.12	0	-.09	0	-.13	0	-.15	0	-.12	0	-.19	0	-.45	0	-.28	0	-.93	0	1.14	1
S	.13	5	.10	7	.13	7	.17	11	.16	7	.18	7	.37	9	.78	12	1.92	9	6.30	3
SH	.19	8	.03	2	.18	10	.25	16	.21	9	.37	14	1.37	31	.67	10	1.47	7	2.76	2
SL	-.10	0	.14	10	-.08	0	-.16	0	-.26	0	-.35	0	-.55	0	-.29	0	-1.24	0	9.94	5
R	1.64	70	.88	61	1.30	73	.90	58	1.79	78	1.78	68	1.87	43	3.81	60	13.65	64	154.07	80
Total[a]	2.37		1.45		1.78		1.55		2.32		2.62		4.37		6.39		21.22		191.21	

[a]Negative components were considered zero for computing totals and percentages.

two tables are compared, lactation number still contributes a larger portion of variation than does age. The effect of season still appears to be an equally important contributor to the variation, although about half of the effect is combined with a difference between herd-season classes.

Computing Extension Factors

Results of the analysis of variance components indicate that lactation number has an added influence on the total to part relationship to that of age at calving, and therefore, will predict the complete lactation more adequately in extending part-time records. However, under the first linear model, age exerts a larger influence during the third through sixth months than does lactation number. Since these same months are among those with the lowest total variance (Tables 4-7), the most accurate results in predicting lactation production from a single monthly test will be obtained during these months. Under this situation, extension factors based on age appear to be the most desirable.

Season of freshening is also an important contributor to the variation between whole and part production, and, therefore, should be adjusted for when extending records. Since herd-season interactions appear to be important only in Holsteins, and since separate factors to adjust for herds are so limited in application and yet so costly and laborious to derive, only factors adjusting for season were computed.

Factors for extending records from each of ten individual monthly tests were computed for different ages, lactation numbers, and seasons of freshening for each of the four breeds by averaging the ratio of total milk produced on 10 test days to milk production for each test day for all records in that particular group.

A set of ratio factors for each breed for use in extending short-time first lactation records and another set for extending all later lactations are presented in Table 11. Separate factors were computed for first, second, and all later lactations; however, those for second lactations were so similar to those for lactations three and above that the data for second lactations were incorporated with that of later records and one set of combined factors obtained. This agrees with the earlier reports of Madden and co-workers (23, 24, 25), who found that a difference existed between factors for heifers and older cows, but that one set of factors are satisfactory for all older ages.

TABLE 11

NON-CUMULATIVE RATIO FACTORS BY LACTATION NUMBER AND BREED

	Breed							
	Brown Swiss		Guernsey		Holstein		Jersey	
	Lactation		Lactation		Lactation		Lactation	
Month	1	≥2	1	≥2	1	≥2	1	≥2
	(113)[a]	(346)	(697)	(1565)	(4110)	(8451)	(305)	(685)
1	8.82	7.74	7.97	7.03	8.40	7.48	7.80	7.10
2	8.81	7.80	8.18	7.33	8.41	7.56	7.96	7.39
3	9.12	8.29	8.87	8.23	8.98	8.26	8.80	8.24
4	9.57	9.05	9.65	9.21	9.49	9.05	9.66	9.28
5	9.85	9.84	10.24	10.13	9.97	9.82	10.33	10.20
6	10.37	10.57	10.73	11.09	10.43	10.62	11.01	11.04
7	10.68	11.50	11.30	11.97	10.86	11.52	11.54	12.03
8	11.32	12.69	11.67	13.38	11.29	12.77	11.83	13.44
9	12.05	14.62	12.60	15.81	12.33	15.13	12.76	15.52
10	15.27	19.27	14.62	22.00	15.25	22.12	14.92	21.30

[a]Indicates the number of records averaged to obtain the ratio factors.

Factors for extending records based upon age at freshening are presented in Table 12. A comparison of Tables 11 and 12 points out a high degree of similarity between ratio factors for ages and those for lactations. This could be expected since Madden _et al_. (25) have shown that approximately 10% of the first lactations are

TABLE 12

NON-CUMULATIVE RATIO FACTORS BY AGE AT FRESHENING AND BREED

Month	Brown Swiss Age[b] <36 (103)[a]	Brown Swiss Age[b] ≥36 (356)	Guernsey Age <36 (677)	Guernsey Age ≥36 (1585)	Holstein Age <36 (3939)	Holstein Age ≥36 (8622)	Jersey Age <36 (326)	Jersey Age ≥36 (664)
1	8.83	7.76	7.91	7.07	8.36	7.52	7.79	7.08
2	8.77	7.84	8.14	7.36	8.38	7.59	7.94	7.38
3	9.09	8.32	8.85	8.25	8.97	8.30	8.77	8.24
4	9.53	9.08	9.62	9.23	9.47	9.07	9.63	9.28
5	9.82	9.85	10.24	10.07	9.96	9.82	10.29	10.21
6	10.40	10.55	10.75	11.02	10.43	10.62	11.00	11.05
7	10.73	11.47	11.32	11.95	10.88	11.50	11.54	12.05
8	11.38	12.63	11.71	13.34	11.33	12.72	11.83	13.49
9	12.16	14.52	12.70	15.73	12.44	15.03	12.91	15.54
10	15.33	19.14	14.81	21.82	15.66	21.80	15.16	21.38

[a]Indicates the number of records averaged to obtain the ratio factors.

[b]Age in months at time of freshening.

initiated after 36 months of age, while 12% of the second lactations are started prior to 36 months. Fritz (9) reported less overlapping of the two lactations with 7% of the first lactation records initiated after 36 months of age and 5% of the second lactations initiated prior to 36 months.

Cumulative ratio factors based upon age at freshening are given in Table 13. These factors differ from those presented in earlier tables in that they are calculated so as to extend cumulative milk production rather than production for a single test day. Since most testing programs report cumulative production for a lactation, and since

TABLE 13

CUMULATIVE RATIO FACTORS BY AGE AND BREED

Test Day	Brown Swiss		Guernsey		Holstein		Jersey	
	<36	≥36	<36	≥36	<36	≥36	<36	≥36
1	8.83	7.76	7.91	7.07	8.36	7.52	7.79	7.08
2	4.40	3.90	4.01	3.61	4.19	3.78	3.93	3.61
3	2.96	2.65	2.76	2.51	2.85	2.60	2.72	2.51
4	2.26	2.05	2.14	1.97	2.22	2.02	2.12	1.98
5	1.84	1.70	1.77	1.65	1.80	1.67	1.76	1.66
6	1.56	1.46	1.52	1.44	1.53	1.45	1.51	1.44
7	1.36	1.30	1.34	1.28	1.34	1.28	1.34	1.29
8	1.22	1.18	1.20	1.17	1.20	1.17	1.20	1.17
9	1.11	1.09	1.10	1.09	1.10	1.08	1.10	1.09

several investigators have shown cumulative production to be at least as valuable, if not more valuable, for use in extending short-time records, these are the more useful type of factor.

Cumulative ratio factors were obtained from individual monthly factors in the following manner. The reciprocals of the monthly ratio factors for the first two months were added and the sum then reciprocated to produce the factor for extending the cumulative production for the first two months. The reciprocal of the third monthly factor was added to the sum of the reciprocals for the first two months and then reciprocated to obtain the third cumulative factor. Factors for succeeding months were obtained in the same manner.

Since age-season-breed and lactation-season-breed interactions are unimportant, corrections for all three in each case can be made in the same set of factors. In order to reduce further the number of sets of factors, the number of seasons was reduced from four to two. The non-cumulative factors for Jan.-March are of the same magnitude as those for Oct.-Dec. for five months and as those for April-June for the remaining five months. From this it is evident that the original grouping of seasons does not adequately fit the data, therefore seasons were regrouped into two new seasons (November-April and May-October) which more adequately fit the climatic and management systems in Michigan.

Non-cumulative ratio factors based on breed, age, and season of freshening are presented in Table 14, while Table 15 presents cumulative factors which will adjust for the same three variables. These factors are the most practical for actual use, since all three variables have been shown to influence the whole to part relationship in milk production and, therefore, need to be taken into account in extending part-time records to a 305-day basis.

Since factors which adjust for lactation number can be used to extend incomplete records, and since occasions may arise in which lactation number but not age may be known about a particular record, factors which adjust for breed, lactation number, and season of freshening are presented in Table 16. Factors which adjust only for lactation number and breed can be obtained by combining the factors for seasons within a lactation number, each one weighted according to the proportion of records in that season. Factors which adjust only for season and breed can be obtained from either Table 15 or Table 16 by combining seasons over age or lactation number as the case may be.

TABLE 14

NON-CUMULATIVE RATIO FACTORS BY BREED, AGE, AND SEASON OF FRESHENING

	Brown Swiss				Guernsey			
	<36		≥ 36		<36		≥ 36	
Test Day	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May. Oct.	Nov.-Apr.	May Oct.
	(48)[a]	(55)	(159)	(197)	(249)	(428)	(709)	(876)
1	9.42	8.32	8.02	7.55	8.06	7.83	7.36	6.83
2	9.08	8.51	7.96	7.74	8.00	8.22	7.38	7.34
3	8.98	9.19	8.27	8.36	8.51	9.05	8.10	8.37
4	9.18	9.84	8.80	9.30	9.24	9.84	8.84	9.55
5	9.40	10.19	9.29	10.30	9.68	10.57	9.43	10.59
6	9.88	10.85	9.92	11.06	10.22	11.06	10.32	11.59
7	10.29	11.11	11.08	11.78	11.08	11.47	11.56	12.27
8	11.59	11.20	13.02	12.31	12.47	11.27	13.84	12.94
9	13.03	11.40	15.44	13.73	14.12	11.87	17.47	14.32
10	17.45	13.47	20.90	17.72	17.19	13.43	25.39	18.94

	Holstein				Jersey			
	<36		≥ 36		<36		≥ 36	
Test Day	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.
	(1326)	(2613)	(3632)	(4990)	(102)	(224)	(266)	(398)
1	8.59	8.22	7.81	7.33	7.95	7.72	7.32	6.92
2	8.31	8.40	7.62	7.56	7.75	8.03	7.59	7.22
3	8.69	9.10	8.13	8.42	8.49	8.90	8.20	8.24
4	9.17	9.63	8.81	9.26	9.27	9.79	9.01	9.46
5	9.59	10.16	9.44	10.10	9.52	10.64	9.61	10.62
6	10.02	10.65	10.19	10.92	10.37	11.29	10.12	11.62
7	10.64	11.10	11.76	11.73	11.09	11.69	11.09	12.49
8	11.72	11.43	13.31	12.52	12.47	11.53	13.61	13.36
9	13.50	11.97	16.03	14.50	14.70	12.09	16.91	14.62
10	17.14	14.49	23.94	20.36	17.41	14.13	23.81	19.13

[a]Indicates the number of records averaged to obtain the ratio factors.

TABLE 15

CUMULATIVE RATIO FACTORS BY BREED, AGE, AND SEASON OF FRESHENING

	Brown Swiss				Guernsey			
	< 36		≥ 36		< 36		≥ 36	
Test Day	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.
	(48)[a]	(55)	(159)	(197)	(249)	(428)	(709)	(876)
1	9.42	8.32	8.02	7.55	8.06	7.83	7.36	6.83
2	4.62	4.21	4.00	3.82	4.01	4.01	3.68	3.54
3	3.05	2.89	2.69	2.62	2.73	2.78	2.53	2.49
4	2.29	2.23	2.06	2.05	2.11	2.17	1.97	1.97
5	1.84	1.83	1.69	1.71	1.73	1.80	1.63	1.66
6	1.55	1.57	1.44	1.48	1.48	1.55	1.41	1.45
7	1.35	1.37	1.28	1.31	1.31	1.36	1.25	1.30
8	1.21	1.22	1.16	1.19	1.18	1.22	1.15	1.18
9	1.11	1.10	1.08	1.09	1.09	1.10	1.08	1.09

	Holstein				Jersey			
	< 36		≥ 36		< 36		≥ 36	
Test Day	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.
	(1326)	(2613)	(3632)	(4990)	(102)	(224)	(266)	(398)
1	8.59	8.22	7.81	7.33	7.95	7.72	7.32	6.92
2	4.22	4.15	3.86	3.72	3.92	3.94	3.73	3.53
3	2.84	2.85	2.62	2.58	2.68	2.73	2.56	2.47
4	2.17	2.20	2.02	2.02	2.08	2.13	1.99	1.96
5	1.77	1.81	1.66	1.68	1.71	1.78	1.65	1.65
6	1.50	1.55	1.43	1.46	1.47	1.54	1.42	1.45
7	1.32	1.36	1.27	1.30	1.30	1.35	1.26	1.30
8	1.18	1.21	1.16	1.17	1.17	1.21	1.15	1.18
9	1.09	1.10	1.08	1.09	1.09	1.10	1.08	1.09

[a]Indicates the number of records averaged to obtain the ratio factors.

TABLE 16

CUMULATIVE RATIO FACTORS BY BREED, LACTATION NUMBER, AND SEASON OF FRESHENING

Test Day	Brown Swiss				Guernsey			
	Lactation 1		Lactation ≥2		Lactation 1		Lactation ≥2	
	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.
	(53)[a]	(60)	(154)	(192)	(261)	(436)	(697)	(868)
1	9.35	8.36	8.00	7.51	8.11	7.89	7.33	6.79
2	4.61	4.23	3.98	3.80	4.03	4.04	3.68	3.52
3	3.05	2.90	2.68	2.61	2.74	2.79	2.53	2.47
4	2.29	2.24	2.06	2.04	2.11	2.18	1.96	1.96
5	1.84	1.84	1.68	1.70	1.73	1.81	1.63	1.66
6	1.55	1.57	1.44	1.47	1.48	1.55	1.40	1.45
7	1.34	1.38	1.28	1.31	1.31	1.37	1.25	1.30
8	1.20	1.23	1.16	1.18	1.18	1.22	1.15	1.18
9	1.10	1.11	1.08	1.09	1.09	1.10	1.08	1.09

Test Day	Holstein				Jersey			
	Lactation 1		Lactation ≥2		Lactation 1		Lactation ≥2	
	Nov.-Apr.	May-Oct.	Nov.-Apr.	May Oct.	Nov.-Apr.	May-Oct.	Nov.-Apr.	May-Oct.
	(1412)	(2698)	(3546)	(4905)	(97)	(208)	(271)	(414)
1	8.64	8.28	7.78	7.27	8.03	7.69	7.30	6.96
2	4.24	4.18	3.84	3.70	3.97	3.93	3.71	3.55
3	2.85	2.87	2.61	2.57	2.71	2.73	2.55	2.48
4	2.18	2.21	2.01	2.01	2.10	2.13	1.99	1.97
5	1.77	1.82	1.66	1.68	1.72	1.78	1.65	1.66
6	1.51	1.55	1.43	1.45	1.47	1.54	1.42	1.45
7	1.32	1.36	1.27	1.29	1.30	1.36	1.26	1.30
8	1.19	1.21	1.15	1.17	1.18	1.22	1.15	1.18
9	1.09	1.10	1.08	1.08	1.09	1.11	1.08	1.10

[a]Indicates the number of records averaged to obtain the ratio factors.

DISCUSSION

Variables Affecting Relationship of Total to Part Production

From a standpoint of ease and efficiency, one set of general factors for extending all part time records is optimum. Unfortunately the solution is not that simple. Several workers have shown that there are environmental influences which should be adjusted for in extending incomplete records. One of the purposes of this study is to ascertain the relative influence of five of these variables on the relationship between total milk production and production during various portions of the lactation in order that factors may be obtained which adjust for the most important variables.

Age and lactation number

Age and lactation number have been studied concurrently to determine if one is exerting a greater influence on the relationship of total to part production than is the other. Even though they are to a large extent merely different measures of the same variable, they are not entirely synonymous. First-calf heifers do not decline in production as rapidly during the last months of the lactation as do older cows, and several workers have shown that this difference should be compensated for in extending part-time records. On the surface this appears to be a difference in parities, although Madden and co-workers (23, 24, 25), Kendrick (20), and Harvey (13) have all used age in months as the measure of this variable. On the other hand Eldridge and Atkeson (6) have pointed out that differences in the total to part relationship between first- and second-calf heifers appear to be due to lactational differences rather than age differences. This study supports earlier reports that the total to part relationship varies between heifers and older cows; in addition it substantiates the findings of Eldridge and Atkeson (6) that number of the lactation is more important than age. Although the component for lactation number is not uniformly larger than the one for age, it does account for more of the variability in the total to part relationship than does age alone, particularly for the first and last months of lactation. However, the slightly larger component for ages during the center months of the lactation indicates that factors based on age should be used to extend records from a single test taken during these months.

The ratio factors themselves indicate no practical differences between factors for age and lactation number. The differences between non-cumulative age and lactation factors are small and unimportant, while those for cumulative factors are small for the first four cumulative months and almost non-existent thereafter. Cumulative Holstein factors for both ages and lactations are very similar to the cumulative age factors obtained by Madden et al. (25) from Holstein HIR data.

Age factors should be used to extend first lactation records initiated after 36 months of age, since first lactation factors will overestimate production and thereby favor an undesirable situation. Age factors should also be used to extend second lactations started prior to 36 months, since using lactation factors in this case will underestimate production and thereby penalize a desirable breeding practice. The remaining 90% of the time either age or lactation factors will work equally well. However, since in actual practice all incomplete records can be extended using age factors irrespective of lactation number, while lactation factors are as equally usable as age factors only 90% of the time and should be used in combination with age in the remaining situations, age factors are preferable to lactation factors for extending incomplete records.

At this point concern over the bias introduced into the analysis of variance components by the correlation between age and lactation number appears to be only of minor importance. As mentioned previously, it is thought that this correlation causes each variance between classes to include all of the covariance between the two variables as well as part of the variance caused directly by the other. Hence, the interaction component is biased to seem too small. This interaction is not important in this case since only one measure of the correlated variables is to be adjusted for, and particularly since there appears to be no real differences between them. Since both models three and four show that some measure of age is important in apportioning the variation between the total to part relationship, it is evident that the correlation in model one did not bias the relationship of the components for age and lactation number to the other components being analyzed.

Season of freshening

The observation that season of freshening is almost as great a source of variation in the whole to part relationship as is age is not in complete agreement with the literature. Eldridge and Atkeson (6) and Fritz (9) all considered the effect of season of freshening on this relationship but concluded that it was insignificant. On the other hand, Kendrick (20) concluded that season of freshening should be considered for the first six months in extending cumulative short-time Ayrshire records. In this study the large differences between factors for each of the different seasons indicates that season of freshening does influence the total to part relationship and should be adjusted for in extending records.

Herds

The results of the analysis of the components of variance show that herd to herd differences do not influence the total to part relationship for milk production, and, therefore, separate extension factors are not required for each herd. This is in complete agreement with the earlier observations made by Fritz (9).

Breeds

Because of the large difference in the number of records available for each breed, the various breeds were studied separately. In the results of the analysis of the components of variance, considerable variation between breeds is observed. The average ratio factors also point out a difference between breeds. Guernsey and Jersey factors tend to be more like each other, and Holstein and Brown Swiss factors also tend to be more alike, with factors for Guernseys and Jerseys being more nearly alike than those for Holsteins and Brown Swiss. Irrespective of whether the ratio factors are dependent upon age, lactation number, or season of freshening (second grouping), the non-cumulative factors for Brown Swiss and Holsteins are larger than those for Guernseys and Jerseys for the first three months, and then smaller for the remaining months of the lactation. This difference is large for the first month, diminishes until the fourth month, and then steadily increases through the tenth month. The factors for Brown Swiss are larger the first months and smaller the last months than those for Holsteins, with the point of crossover occurring at the fifth month.

Because of the differences between breeds, records extended with factors for a different breed will be biased, particularly if Guernsey or Jersey records are extended with Brown Swiss or Holstein factors, and vice versa. This bias will cause Guernsey or Jersey records to be overestimated when extended from a single test during the first three months using Holstein or Brown Swiss factors, and underestimated if Holstein or Brown Swiss factors are used during any of the remaining months. Jersey records, in particular, will be more seriously overestimated early in first lactations and then more seriously underestimated later that same lactation.

A difference between breeds is also evident in the cumulative factors. Brown Swiss factors, when applied to cumulative production for any of the other breeds will overestimate 305-day production. On the same basis, Holstein factors will overestimate Guernsey and Jersey production, while Guernsey factors overestimate Jersey production for the first lactation, but underestimate it for all later lactations. These overestimations in all cases are larger during early lactation, decreasing as production for each succeeding month is added to the cumulative total, and becoming almost negligible after the eighth cumulative month.

The fact that breed differences are the same irrespective of age, lactation number, or season of freshening indicates that there is no interaction between breeds and any of these other variables.

In view of the results obtained and the points discussed in preceding paragraphs, age, breed, and season of freshening should be taken into consideration in extending part records to a 305-day basis.

Use of the Factors

Two basic types of ratio factors have been presented, cumulative and non-cumulative. The cumulative ratio factors are more useful and more widely applicable since they utilize all of the test day information which is available rather than production information from only one test day. Since most testing programs report total production to date, an even more practical means of using cumulative extension factors is to interpolate them so that cumulative production for any number of days can be extended to 305-days by multiplication by a single factor.

The obvious way to develop factors for cumulative production for any number of days would be to obtain the ratio of total production to production for that number of days. However, this is not a practical approach since (a) production is not reported for every day of the lactation. The cumulative production which is reported for a given number of days is calculated from test day production. (b) This method would place the available records into approximately 30.5 times as many groups, thereby requiring a larger volume of data or risking random errors in the factors due to small samples. In the present study even the Holstein data do not approach large enough numbers to use this method. (c) The amount of time and labor required also makes this method less useful than the method of interpolation.

The simplest means of interpolation is to reduce the factors for each succeeding day between two test days by the difference between the test day factors divided by 30.5. The difficulty with this is that it assumes that changes in production between test days is linear. A more realistic approach is to interpolate using the first and second differences between test day production for adjacent months. This method describes the change in production throughout the lactation as a curve rather than a series of linear changes.

The cumulative factors given in tables 13, 15, and 16 can be interpolated using the following method:

Let X be total milk production of 10,000 pounds. An arbitrary level can be used here since the factors are independent of level of production (13, 25). Let m_i be the non-cumulative ratio factors, i = 1, ---, 10. Then define a_i to be X/m_i or production for the i^{th} month, b_i to be $a_i/30.5$ or i^{th} test day production for the 1st diff., and c_i equal $(b_i - b_{i+1})/30.5$ or the 2nd difference.

At this point the average number of days prior to each test day needs to be established. Madden _et al_. (25) reported that Michigan DHIA-IBM Holstein data averaged 33.5

days for the first test period. Since the centering date method is used in calculating DHIA records, this indicates that on the average the first test day falls around the 17-18 countable day of production. Succeeding test days should then come on the average at 30.5 day intervals. On this basis, b_1 is the production for the 17.5 day of the lactation, b_2 for the 48th day, b_3 for the 78.5 day, etc.

Daily production for each day (18-305) is now found by starting at b_1 and reducing production for each succeeding day by c_i. In cases where first test day production is lower than production for the second test day, take b_2 as the starting point and work in both directions, subtracting c_i to obtain succeeding days production. Once daily production is established it is then cumulated, obtaining a cumulative total for each day. Total cumulative production is divided by each day's cumulative production to obtain the extension factor for that day. It is expected that because of rounding errors, total cumulative production will not equal X, the original total production; therefore, the total production obtained by summing daily production should be used as the dividend in deriving the extension factors.

Interpolated cumulative ratio factors are simple and easy to apply. Cumulative production, as reported by the testing program, is multiplied by the factor for the number of days involved to obtain estimated 305-day production.

Uninterpolated cumulative factors may be used in cases where only the production for several test days is known. Cumulative test day production is multiplied by 30.5, the average number of days in a month, to give cumulative monthly production which is then multiplied by the factor for the number of test days involved to obtain estimated 305-day production.

The non-cumulative factors are more useful than cumulative factors for comparing one set of factors with another for all months, since with the cumulative factors any differences in early months tend to mask differences in later months. From a practical standpoint, monthly factors will be used only when cumulative production is not available. To use these factors, multiply individual test day production by 30.5 to get monthly production which is multiplied by the factor for that particular month to obtain estimated 305-day production.

Implications for Future Investigations

The purpose of this investigation has been to determine what variables need to be adjusted for in extending part records, and to compute factors, based on these findings, for extending incomplete lactations to

305 days. This is merely the initial step in determining the role which part records may play in selection and evaluation of dairy cattle in the future. The present study has pointed out a need for further investigations into the following aspects of the overall picture:

(1) Whether the same variables influence the relationship of total to part production of butterfat and solids-not-fat as influence this relationship for milk.

(2) Whether the same factors can be used to extend butterfat and solids-not-fat records as are used for milk records.

(3) Whether different factors are needed for extending records terminated by uncontrollable factors such as disease, injury, etc., for extending records terminated by culling for low production, and for predicting production while the lactation is still in progress.

(4) The accuracy of lactation records extended from a single test during the lactation and from a weighted average of three or four extended single tests during the lactation need to be compared with the accuracy of present testing methods to determine if these are feasible ways of reducing the cost of testing.

(5) To determine the heritability and genetic correlations for various portions of the lactation. These statistics will help answer the questions of how much genetic progress can be expected from mass selection on the basis of part records, and to what extent the same genes operate during various portions of the lactation.

(6) The general applicability of the ratio factors obtained in this study need to be verified for other populations, while the factors for Brown Swiss, Guernseys, and Jerseys need to be verified using a larger number of records.

SUMMARY AND CONCLUSIONS

Complete lactation records for 12,561 Holsteins, 2,262 Guernseys, 990 Jerseys, and 459 Brown Swiss compiled in Michigan DHIA-IBM from June 1954 through July 1957 were analyzed to ascertain the relative effects of five variables on the relationship of total to part milk production, and then were used to derive ratio factors for extending partial records to 305 days.

The ratio of total milk produced on ten monthly test days to milk produced on each test day was used as the measure of relationship between total and part production. Components of variance of ratios indicated that lactation number had a larger influence on the total to part relationship than did age at freshening. Season of freshening also exerted an influence on the ratio of total to part, but to a slightly less degree than either lactation number or age. The effect of herd on the total to part relationship was small and unimportant. Breeds were analyzed separately, but a visual inspection of the components of variance for the different breeds suggested that differences between breeds existed in the total to part relationship.

Ratio factors for extending records from each of ten monthly test days and from cumulative test-day production were computed for different ages, lactation numbers, and seasons of freshening for each of the four breeds. Although no interactions between the variables were important, ratio factors adjusting for breed, age, and season and for breed, lactation, and season were presented in a combined form.

Only small differences exist between the factors which adjust for lactation number, indicating that either set of factors should adequately extend incomplete records. However, factors based on age are more useful in extending those records in which age at freshening and lactation number do not coincide. In practice breed, age, and season of freshening should be considered in extending partial records to 305 days.

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