A COMPARISON OF METHODS FOR COMPUTING SEASONALS USED IN THE ANALYSIS OF TIME SERIES

THESIS FOR THE DEGREE OF M. A.

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Mathematics

A COMPARISON OF METHODS

FOR COMPUTING SEASONALS USED

IN THE ANALYSIS OF TIME SERIES

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To Mr. S. E. Crowe, who has
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my work

CHAPTER I

Introduction

With the analysis of time series in business and economic statistics comes the problem of the determination and elimination of the seasonal factor. As is already known, there are in all four factors which enter into the analysis of any time series. These factors are secular trend (the long time tendency), cyclical variation (the wave-like movement superimposed on secular trend), seasonal variation (a variation within the year due to seasonal influences) and residual variation (due to forces unforeseen).

The problem here is the analysis, comparison and application of the various methods used in the measurement of seasonal variation. The important characteristics of a seasonal variation are that it is a periodic change with a period of one year and that each year it exactly repeats itself. In the analysis of such a variation, is there any way of determining the reliability of the different methods and, if so, which is the most reliable? In answer to this question a hypothetical set of data has been constructed such that if the method of monthly means is used the true seasonals are known. It is the pur-

pose here to show how close the other methods will come to the true results. Also, these same methods will be applied to the analysis of motor bus and truck production for a period of seven years.

But before going into the study of the main problem, a review of the various methods for the determination of seasonal variation will be given.

CHAPTER II

Methods Used in Determining Seasonals

A short description in outline form of the methods used in the determination of seasonal variation will be given in this chapter. The methods are as given below:

I. Method of monthly means

1. Compute an arithmetic mean for each month and express these averages in terms of percent.
2. Correct averages for secular trend.
3. Change the corrected averages so their average will equal 100 percent.

II. Link relative method

1. Express each monthly figure as a percent of the figure for the previous month. These are called the link relatives.
2. Determine the median link relative for each month.
3. Compute the chained relatives using January as a constant base.
4. Correct the chain relatives for error.
5. Adjust these corrected chain relatives so their average will equal 100 percent.

III. Method of moving averages

1. Compute the moving averages.
2. Determine the moving average ratios by dividing the actual items by the corresponding moving averages.
3. Compute a suitable average ratio for each month.
4. Adjust the average ratios so that their average will equal 100 percent.

IV. Ratio-to-trend

1. Fit a suitable line of trend to the yearly averages to determine the annual increment and trend values.
2. Express the actual item as a percent of the trend value. These are called the trend ratios.
3. Determine a suitable monthly average of these ratios.
4. Adjust so the average equals 100 percent.

V. Method of first differences (using trend ratios)

1. Ratios of the original data to trend ordinates computed.
2. First differences of the ratios next determined.
3. Compute a suitable average of these differences for each month.

4. Adjust so the sum of the first difference averages is equal to zero.
5. With January as a base, compute the chained first differences.
6. Adjust so that the average of the chained first differences is equal to 100 percent.

VI. Method of first differences (using moving average ratios)

1. Compute ratios by dividing the original data by the corresponding moving averages.
2. Compute the first differences of these ratios.
3. Get a suitable average of these first differences for each month
4. Adjust so the sum of the first difference averages is equal to zero.
5. With January as base, compute the chained first differences.
6. Adjust so the average is equal to 100 percent.

VII. Thirteen-months-ratio-first-difference method

1. Compute a monthly mean for each year.
2. Obtain percent ratios by dividing ac-

tuals of each year by the monthly average for that year. Also include the ratio of the following January - although the following January is not used in the determination of the monthly average for the year.

3. Leaving the January values as they are, compute the first differences of the above ratios from February through the following January.
4. Select an average for each of the thirteen months.
5. Cumulate the first difference averages to the January average.
6. The annual trend increment is found by subtracting the two January values.
7. Correct for trend making the two January values equal.
8. Adjust the twelve monthly values so obtained so as to make the average equal to 100 percent.

VIII. Detroit Edison method

A more detailed discussion of the development and results will be given in regard to this method and the one following (which is a modification of the Detroit Edison method).

Any time series is made up of the following four factors:

secular trend $f(x)$

cyclical variation $c\ (x)$

seasonal variation $s\ (x)$ and

residual errors e_x

Let $_0y_x = f(x) \quad c(x) \quad s(x) + e_x$ where $_0y_x$ represents the xth term of the time series. The standard error will be equal to $\sqrt{\frac{e_x^2}{n}}$. If the standard error is to be a minimum then the e^2 must be a minimum value. Values for $s(1)$, $s(2)$, ..., $s(12)$ can be found that will minimise the standard error by taking the partial derivative of e^2 with respect to $s(1)$, $s(2)$,......$s(12)$ and putting this partial derivative equal to zero. The result is that

$$s(i) = \frac{\sum^{i}{}_0y_x \cdot f(x) \cdot c(x)}{\sum^{i} f^2(x) \cdot c^2(x)}$$

If $\psi(x) = f(x) \cdot c(x)$, then $s(i) = \frac{\sum^{i}{}_0y_x}{\sum^{i}\psi x}$

Now if T_{i-3}, T_{i-2} . .....T_{i+2} . T_{i+3} represents the total production for seven years and if a sixth degree parobola is fitted in such a way that the areas under the curve for the seven equidistant points is equal to T_{i-3} . T_{i-2} . ..., T_{i+3} . we have the following result:

$$\sum^{1}\psi(x) = c_{1:1}T_1 + c_{2:1}T_2 + c_{3:1}T_3 + c_{4:1}$$

$$(T_4 + T_5 + \ldots\ldots + T_{n-3}) + c_{5:1}T_{n-2} + c_{6:1}$$

$$T_{n-1} + c_{7:1}T_n \quad .$$

This gives a formula for the determination of $\sum^{1}\psi x$ so that the seasonal factor can easily be determined. The values of the coefficients in the above equation have been worked out and put in table form The table of values will not be given here but can be found by referring to the list of references.

IX. A second Detroit Edison method

This method differs from the above in that a third degree curve is used instead of one of the sixth degree. The theory underlying this method will, therefore, be the same as the above. The two methods will differ, though, when it comes to determining a formula for the $\sum^{1}\psi(x)$. In this case it is done by using a curve of the type $y = a + bx + cx^2 + dx^3$.

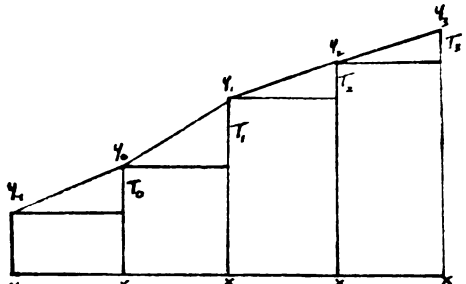

Figure I.

Figure 1 is an accumulation curve representing the total production up to each successive year.

$T_0 = y_0 - y_{-1}$ = total production for the first year.

$T_1 = y_1 - y_0$ = total production for the second year.

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If the values of x are substituted in the equation $y = a + bx + cx^2 + dx^3$ we obtain when

$$x = -1,\ y_{-1} = a - b + c - d$$

$$x = 0,\ y_0 = a$$

$$x = 1,\ y_1 = a + b + c + d$$

$$x = 2,\ y_2 = a + 2b + 4c + 8d$$

These equations are solved for a, b, c, and d. The results are:

$$a = y_0$$

$$c = \frac{T_1 - T_o}{2}$$

$$d = \frac{T_2 - 2T_1 + T_o}{6}$$

$$b = \frac{2T_o + 5T_1 - T_2}{6}$$

As it is necessary to express the results in terms of monthly production, let M_1 = January production, M_2 = February production, etc. Then:

$$M_1 = \frac{b}{12} + \frac{c}{12^2} + \frac{d}{12^3}$$

$$M_2 = \frac{b}{12} + \frac{3c}{12^2} + \frac{7d}{12^3}$$

$$M_3 = \frac{b}{12} + \frac{5c}{12^2} + \frac{19d}{12^3} \qquad \text{etc.}$$

By differencing the results

dM $\qquad d^2M$

CHAPTER III

An Application of These Methods to a Hypothetical Set of Data

A hypothetical set of data has been worked out by W. L. Hart, of the University of Minnesota, such that if the method of monthly means is used the exact seasonals will be obtained.

Suppose we have a series of monthly items from which the secular trend has been removed and the items are for a period of K years. If $f(t)$ represents the item t months from January of the first year and if an arithmetic average of the monthly items is taken, then the results obtained by averaging is the best approximation to $f(t)$.

Now the question arises as to how to determine a function $P(t)$, which is periodic, that will be the best approximation to $f(t)$. The two theorems now given will be an answer.

Theorem 1: If $f(t)$ actually is a periodic function whose period is one year, the monthly entries obtained by the method of monthly means are exactly the value of $f(t)$ at the corresponding months.

If $P(t)$ represents the periodic function with the period one year, whose value for all the Januarys, Februarys, etc., are the corresponding monthly means

we have:

Theorem 2: Let $f(t)$ be any function of time t known from $t = 0$ to $t = 12K$, that is, over a period of K years. Then, the sum of the squares of the residuals $(f(t) - P(t))$ for all values of t is smaller in value than it would be if any other periodic function with a period of one year were used in place of $P(t)$.

Another theorem will be stated because it gives the proper criterion of applicability of the above theory.

Theorem 3: The method of monthly means gives us the actual monthly values of the seasonal variation in case $f(t)$ is made up of the following component parts:

A - A seasonal variation, strictly periodic throughout the period of years under consideration.

B - A long term variation which consists of certain independent pieces, each extending over a whole number of years, where each piece represents a whole number of complete oscillations of a corresponding periodic function whose period is an integral number of years (two or more).

C - A second, third, etc., long term variation having the characteristics specified in B.

In order to obtain a hypothetical set of data so

$$\frac{2c}{12^2}+\frac{6d}{12^3}$$

$$\frac{6d}{12^3} \quad \text{(constant)}$$

$$\frac{2c}{12^2}+\frac{12d}{12^2}$$

If the differencing process is carried out in more detail than is given here, the second difference will remain a constant value $\frac{6d}{12^3}$.

Expressing the M's in terms of T and using the fact that the second difference is constant the results as given below are obtained:

$$M_1 = 253\ T_0 + 754\ T_1 - 143\ T_2$$

$$M_2 = 187\ T_0 + 814\ T_1 - 137\ T_2$$

$$M_3 = 127\ T_0 + 862\ T_1 - 125\ T_2$$

$$M_4 = 73\ T_0 + 898\ T_1 - 107\ T_2 \quad \text{etc, where}$$

$$D^2M = 6T_0 - 12T_1 + 6T_2$$

Summing all the January values gives:

$$\text{Jan.}\sum\psi = \frac{1}{6\cdot 12^3}(253\ T_0 + 1001\ T_1 + 864(T_2 + \ldots + T_{n-1}) + 611\ T_n - 143\ T_{n+1}).$$

Summing all the February values gives:

$$\text{Feb.}\sum\psi = \frac{1}{6\cdot 12^3}(187\ T_0 + 1001\ T_1 + 864(T_2 + \ldots + T_{n-1}) + 677\ T_n - 137\ T_{n+1}). \qquad \text{etc.}$$

Now if the two substitutions $T_0 = 2T_1 - T_2$ and $T_{n+1} = 2T_n - T_{n-1}$ be made so as to cut off the first and last terms we will have a set of formulas which will readily enable one to solve for $\overset{1}{\sum}\psi$. These results are:

Jan. $\sum\psi = \frac{1}{6\cdot 12^3}(1513T_1 + 611T_2 + 864(T_3 + \cdots + T_{n-2}) + 1007T_{n-1} + 325T_n).$

Feb. $\sum\psi = \frac{1}{6\cdot 12^3}(1375\ T_1 + 677\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 1001T_{n-1} + 403\ T_n).$

Mar. $\sum\psi = \frac{1}{6\cdot 12^3}(1243\ T_1 + 737\ T_2 + 864(T_3 + \cdot\cdot + T_{n-2}) + 989\ T_{n-1} + 487\ T_n).$

Apr. $\sum\psi = \frac{1}{6\cdot 12^3}(1117\ T_1 + 791\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 971\ T_{n-1} + 577\ T_n).$

May $\sum\psi = \frac{1}{6\cdot 12^3}(997\ T_1 + 839\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 947\ T_{n-1} + 673\ T_n).$

June $\sum\psi = \frac{1}{6\cdot 12^3}(883\ T_1 + 881\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 917\ T_{n-1} + 775\ T_n).$

July $\sum\psi = \frac{1}{6\cdot 12^3}(775\ T_1 + 917\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 881\ T_{n-1} + 883\ T_n).$

Aug. $\sum\psi = \frac{1}{6\cdot 12^3}(673\ T_1 + 947\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 839\ T_{n-1} + 997\ T_n).$

Sept. $\sum\psi = \frac{1}{6\cdot 12^3}(577\ T_1 + 971\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 791\ T_{n-1} + 1117\ T_n).$

$$\text{Oct.}\sum\psi = \frac{1}{6\cdot 12^3}(487\ T_1 + 989\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 737T_{n-1} + 1243\ T_n).$$

$$\text{Nov.}\sum\psi = \frac{1}{6\cdot 12^3}(403\ T_1 + 1001\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 677T_{n-1} + 1375\ T_n).$$

$$\text{Dec.}\sum\psi = \frac{1}{6\cdot 12^3}(325\ T_1 + 1007\ T_2 + 864(T_3 + \ldots + T_{n-2}) + 611T_{n-1} + 1513\ T_n).$$

By using the first three of these equations for the $\sum^1\psi(x)$ and differencing, formulas for $D'\psi$ and $D''\psi$ are obtained:

$$D'_1\psi = \frac{1}{6\cdot 12^3}(-138\ T_1 + 66\ T_2 - 6\ T_{n-1} + 78\ T_n)$$

$$D'_2\psi = \frac{1}{6\cdot 12^3}(-132\ T_1 + 60\ T_2 - 12\ T_{n-1} + 84\ T_n)$$

$$D''\psi = \frac{1}{12^3}(T_1 - T_2 - T_{n-1} + T_n)$$

These formulas for $D'\psi$ and $D''\psi$ will simplify the computation work considerably because of the fact that the second difference is a constant. The only value for $\sum^1\psi(x)$ which needs to be directly computed from the equations is the value of Jan. $\psi(x)$.

that the above conditions would hold, the following equations were used:

$$f(t) = 15 + \sin t\,(30^\circ) + 4 \sin t\,(10^\circ) \text{ from } t = 0 \text{ to } t = 36.$$

$$f(t) = 15 + \sin t\,(30^\circ) + 6 \sin t\,(10^\circ) \text{ from } t = 36 \text{ to } t = 72.$$

$$f(t) = 15 + \sin t\,(30^\circ) + 2 \sin t\,(10^\circ) \text{ from } t = 72 \text{ to } t = 108.$$

In the equations, $15 + \sin t(30^\circ)$ gives the seasonal variation and the long term variations are given by $4 \sin t\,(10^\circ)$, $6 \sin t\,(10^\circ)$, and $2 \sin t\,(10^\circ)$. From these equations the data of Table I ~~was~~ were computed.

TABLE I

	1	2	3	4	5	6	7	8	9	y_0
J	15.000	18.464	11.536	15.000	20.196	9.804	15.000	16.732	13.268	135.000
F	16.196	18.564	11.740	16.544	20.096	9.860	15.848	17.032	13.620	139.500
M	17.234	18.438	11.926	17.918	19.724	9.956	16.550	17.152	13.896	142.794
A	18.000	18.000	12.000	19.000	19.000	10.000	17.000	17.000	14.000	144.000
M	18.438	17.234	11.926	19.724	17.918	9.956	17.152	16.550	13.896	142.794
J	18.564	16.196	11.740	20.096	16.544	9.860	17.032	15.848	13.620	139.500
J	18.464	15.000	11.536	20.196	15.000	9.804	16.732	15.000	13.268	135.000
A	18.260	13.804	11.436	20.140	13.456	9.904	16.380	14.152	12.968	130.500
S	18.074	12.766	11.562	20.044	12.082	10.276	16.104	13.450	12.848	127.206
O	18.000	12.000	12.000	20.000	11.000	11.000	16.000	13.000	13.000	126.000
N	18.074	11.562	12.766	20.044	10.276	12.082	16.104	12.848	13.450	127.206
D	18.260	11.436	13.804	20.140	9.904	13.456	16.380	12.968	14.152	130.500
=	212.564	183.464	143.972	228.846	185.196	125.958	196.282	181.732	161.986	

For comparison purposes this set of data was used to test out the accuracy of the methods for computing seasonal variation and the results are given in Table II.

Table II.

1 - Actual Indices as Worked Out by Method of Monthly Means

2 - Link Relative Method

3 - Detroit Edison Method

4 - Second Detroit Edison Method

5 - First Difference Method

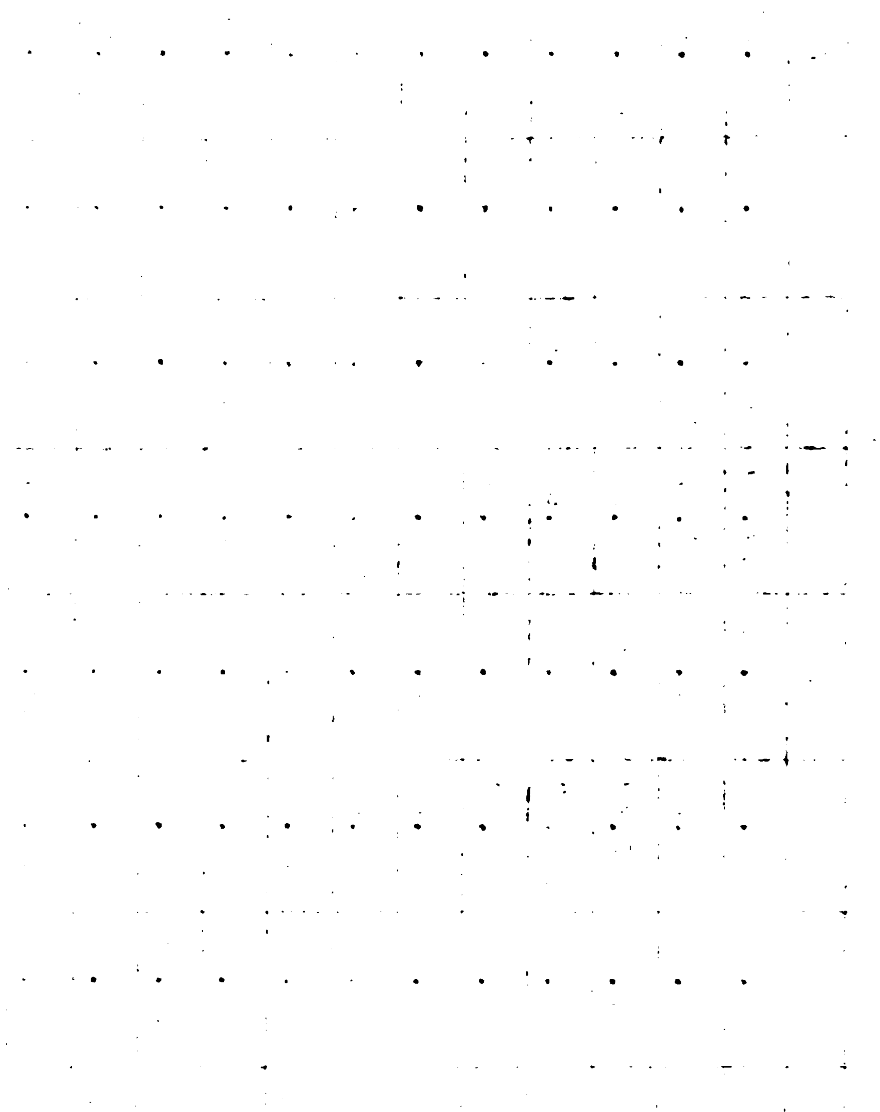
6 - Thirteen-Months-Ratio-First-Difference Method

7 - Method of Moving Averages (a twelve-month moving average, centered, adjusted by a two-month moving average, centered, was used)

TABLE II

TABLE OF SEASONAL INDICES

	1	2	3	4	5	6	7
J	100.00	100.00	98.46	97.89	100.00	101.27	100.27
F	103.33	101.80	102.02	101.55	103.33	104.15	103.46
M	105.80	103.47	104.72	104.36	105.80	106.20	105.81
A	106.67	104.07	105.90	105.66	106.67	106.76	106.60
M	105.80	103.47	105.31	105.18	105.80	105.56	105.54
J	103.33	101.80	103.18	103.16	103.33	102.87	102.91
J	100.00	100.00	100.14	100.22	100.00	99.38	100.08
A	96.67	97.93	97.09	97.25	96.67	96.00	96.70
S	94.20	96.93	94.92	95.16	94.20	93.65	94.23
O	93.33	96.53	94.29	94.61	93.33	93.02	93.34
N	94.20	96.87	95.47	95.88	94.20	94.33	94.26
D	96.67	97.93	98.22	98.73	96.67	96.86	96.79



The method which is the easiest to use - the first difference method - seems to lead to the most accurate results. However, in the application of this method to a practical problem its accuracy will depend on the accuracy in eliminating the trend. As the moving average method also gives fairly accurate results it is probably a better method to use as the elimination of trend is in this case perhaps as accurate as possible. If the first difference method is used, I would suggest that instead of eliminating the trend by fitting a straight line or curve and using the ratio-to-trend values the trend be removed by the moving average method and that the ratios of the actual data to the corresponding moving average be used in obtaining the first differences.

While the link relative method means a lot of work and seems to lead to rather inaccurate results its use in a practical problem could not necessarily be relied upon.

The thirteen-months-ratio-first-difference method is superior to the link relative method. This is probably because it takes the average for the year as its base and therefore yields a better criterion for comparison.

The two Detroit Edison methods do not appear to be as good as might be expected. Although the re-

sults are in general better than the link relative method, the extension of either of the two Detroit Edison methods to a practical problem might be questioned.

CHAPTER IV

Application of Methods to Motor Bus and Truck Production

The set of figures given on the following page represents the monthly production of motor busses and trucks in the United States for a period of seven years. (These figures are given in terms of a thousand).

The equation of the line of trend which was fitted to the yearly averages of this data is $y = 43.54857 + 3.78536\ X$. This gives an annual increment of 3.78536, a monthly increment of .3154 and a semi-monthly increment of .158 .

Chart I represents a graph of the original data and a comparison of the straight line and moving average as a means of obtaining trend values.

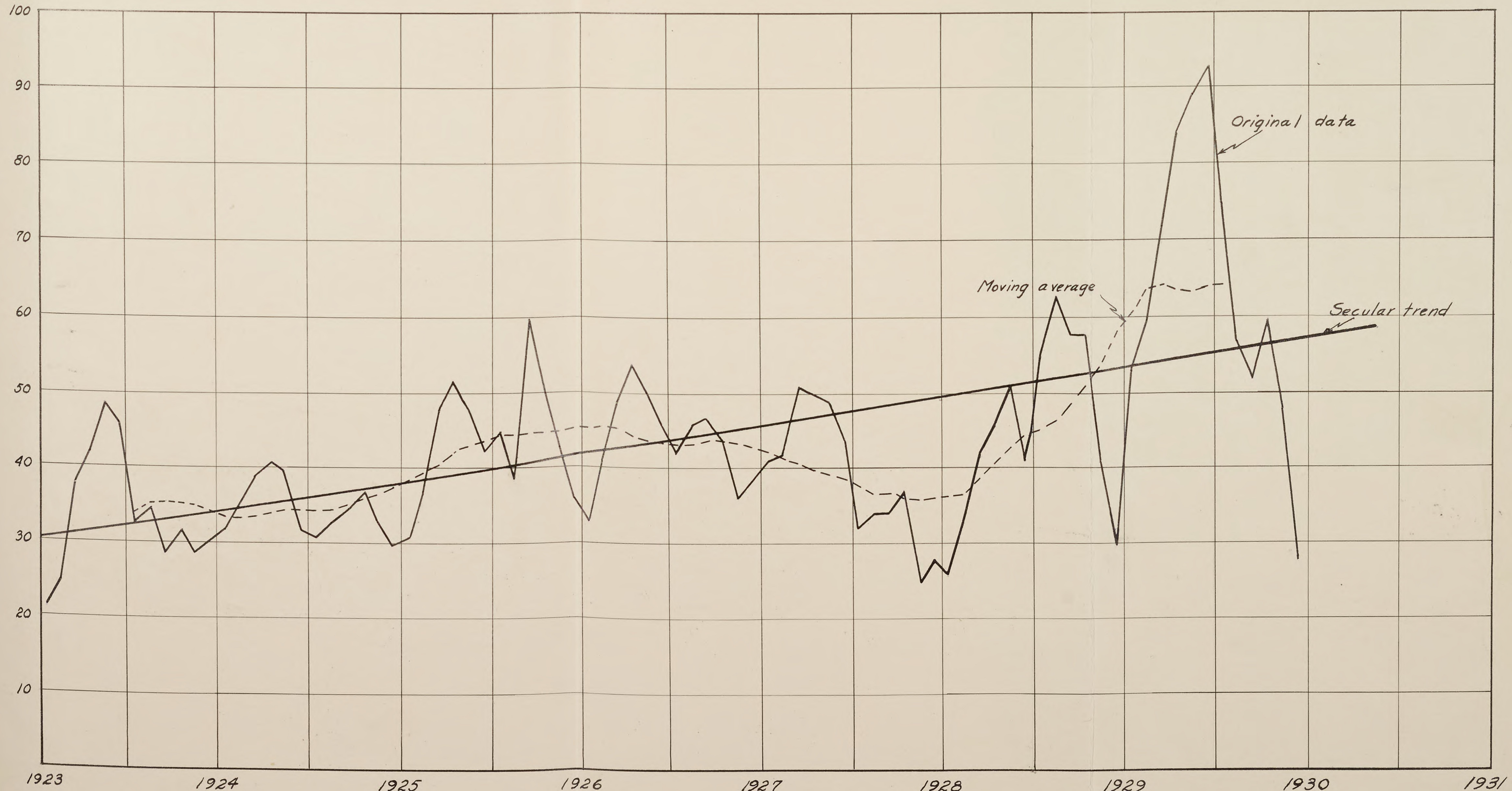
The frequency tables in Charts II, III and IV give a general idea of the type and amount of seasonal variation.

The various methods for computing seasonals have been worked out for this problem and the results are given in Table IV. The results vary considerably. If the conclusions drawn before were accurate, it would be expected that the moving average method would check up with the first difference method using

TABLE III

	1922	1923	1924	1925	1926	1927	1928	1929	1930	Av.
J		22	32	31	33	41	26	53	37	34.00
F		25	35	37	41	42	33	60		39.00
M		38	39	48	49	51	42	72		48.43
A		42	41	52	54	50	46	84		52.71
M		49	40	48	50	49	51	89		53.71
J		46	32	42	46	44	41	93		49.14
J		33	31	45	42	32	55	75		44.71
A		35	33	38	46	34	63	57		43.71
S		29	35	60	47	34	58	52		45.00
O		32	37	50	44	37	58	60		45.43
N		29	33	43	36	25	41	48		36.43
D	22	30	30	36	30	28	30	28		30.29
Av.		34.17	34.83	44.17	43.17	38.92	45.33	64.25		

(These figures are given in terms of a thousand)



100
90
80
70
60
50
40
30
20
10
Original data
Moving average
Secular trend
1923
1924
1925
1926
1927
1928
1929
1930
1931

CHART I

CHART II

FREQUENCY TABLE OF LINK RELATIVES

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
55 - 59.9												1
60 - 64.9												
65 - 66.9				1							1	
70 - 74.9							11				1	1
75 - 79.9								1				
80 - 84.9						11	1	1	1	1	11	11
85 - 89.9						11					11	1
90 - 94.9	11				1	11	1		11	1	1	
95 - 99.9				1	11		1					
100 - 104.9	11	1				1			11	1		1
105 - 109.9	1	1		111	1		1	1111	1	11		
110 - 114.9		11	1	1	1			1		1		1
115 - 119.9		1	1	1	1					1		

CHART II

(continued)

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
120 - 124.9		1	11									
125 - 129.9		1	11									
130 - 134.9							1					
135 - 139.9	1											
140 - 144.9												
145 - 149.9					1							
150 - 154.9			1									
155 - 159.9									1			
160 - 164.9												
165 - 169.9												
170 - 174.9												
175 - 179.9	1											

CHART III

FREQUENCY TABLE OF RATIOS TO TREND

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
45 - 49.9												1
50 - 54.9	1										1	
55 - 59.9												11
60 - 64.9												
65 - 69.9		1					1					1
70 - 74.9	1								1			
75 - 79.9	1							1		1	1	1
80 - 84.9	1	1	1			1					1	
85 - 89.9	1					1	1		1		111	11
90 - 94.9	1	1		1		1		11	1			
95 - 99.9	1	11					1		1	111		
100 - 104.9		1			11		1	11			1	
105 - 109.9				1		11	1	1	1	1		

CHART III

(continued)

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
110 - 114.9		1	11		1		1		1	1		
115 - 119.9			1	1	1							
120 - 124.9			11		1			1		1		
125 - 129.9				1								
130 - 134.9			1	11								
135 - 139.9							1					
140 - 144.9						1						
145 - 149.9									1			
150 - 154.9					1							
155 - 159.9				1								
160 - 164.9					1							
165 - 169.9						1						
170 - 174.9												

CHART IV

FREQUENCY TABLE OF FIRST DIFFERENCES

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
-50 - -45.1												
-45 - -40.1							1					
-40 - -35.1												1
-35 - -30.1							1	1			1	
-30 - -25.1							1			1	1	
-25 - -20.1						11					1	1
-20 - -15.1						1		1	1		11	1
-15 - -10.1					11	111			1		1	1
-10 - -5.1	11						1		1	1	1	1
-5 - -.1	1			1	11		1		1	1		
0 - 4.9	1	1		1		1		11	11	1		1
5 - 9.9	1	11		11	11		1	11		11		1
10 - 14.9		11	1	11				1		1		

CHART IV

(continued)

	Jan	Feb	Mar	Apr	May	June	July	Aug	Sept	Oct	Nov	Dec
15 - 19.9		11	111									
20 - 24.9	1		1	1	1							
25 - 29.9			1				1					
30 - 34.9												
35 - 39.9												
40 - 44.9	1		1									
45 - 49.9												
50 - 54.9									1			
55 - 59.9												
60 - 64.9												

TABLE IV

1 - Median Link Relative Method

2 - Moving Average Method

3 - Detroit Edison Method

4 - Second Detroit Edison Method

5 - Ratio-to-Trend Method

6 - First Difference Method Using Ratio-Trend Values

7 - Thirteen-Months-Ratio-First-Difference Method

8 - Method of First Differences Using Moving Average Ratios

9 - Method of Monthly Means

TABLE IV

	1	2	3	4	5	6	7	8	9
J	77.51	82.87	80.93	81.54	83.9	79.37	80.70	82.84	81.34
F	89.20	94.36	91.98	92.68	95.6	90.82	91.69	94.21	92.54
M	109.32	113.53	113.17	113.99	117.2	113.29	113.91	113.21	114.35
A	117.30	121.30	122.07	122.86	126.1	122.46	122.62	120.85	123.83
M	122.82	119.73	123.26	123.93	117.7	124.23	124.35	119.16	125.48
J	109.99	107.12	111.77	112.20	102.4	111.81	111.52	106.36	113.78
J	98.27	97.58	100.81	100.99	102.4	100.96	100.46	98.22	102.44
A	104.18	100.82	97.71	97.65	101.2	91.11	99.88	101.35	99.28
S	101.91	104.73	99.74	99.39	98.8	102.29	102.47	105.14	101.63
O	106.51	102.40	99.86	99.17	102.4	102.34	102.67	102.70	101.89
N	87.73	81.86	95.02	94.00	84.4	82.93	81.25	82.07	79.62
D	75.24	73.82	65.53	64.55	68.0	70.44	68.45	73.91	63.92

the moving average ratios. The two methods do give fairly close results. When the first difference method is used with the ratio-trend values the results are quite different This result might very well be expected as a straight line is not the best representation of trend and would lead to quite an error.

As one might expect from the problem, previously worked, using a hypothetical set of data, the link relative method and the two Detroit Edison methods vary considerably from the moving average method and the first difference method using moving average ratios. The thirteen-months-ratio-first-difference method appears to be better than any of these three methods.

The method of monthly means doesn't agree with the moving average method as closely as would be expected. This is probably due to the inaccuracy of trend elimination.

Although there is no way of knowing what the true seasonal indices are in this practical problem, it should be noticed that if the first difference method using moving average ratios is used as a basis of comparison our results check up as accurately as could be expected with the results from the hypothetical set of data. From this one would be led to believe that the method of moving averages or the method of first differences using moving average ratios could be fair-

ly well relied upon in the determination of seasonal variation.

CHAPTER V

The Determination of Cyclical Variation

The trend has already been removed. It is now necessary to remove the seasonal variation in order to have left a measure of the cycle. The seasonal indices obtained by the method of moving averages will be used, basing the choice upon the conclusions of Chapter IV. A detailed method as to how this is done can be found in Mill's text book on "Statistics".

The final graph, after the elimination of secular trend and seasonal variation, is shown in Chart V. It represents the cyclical variation of motor bus and truck production over a period of seven years (1923-1930). It should be noticed that in only one case is the variation greater than plus or minus three standard units.

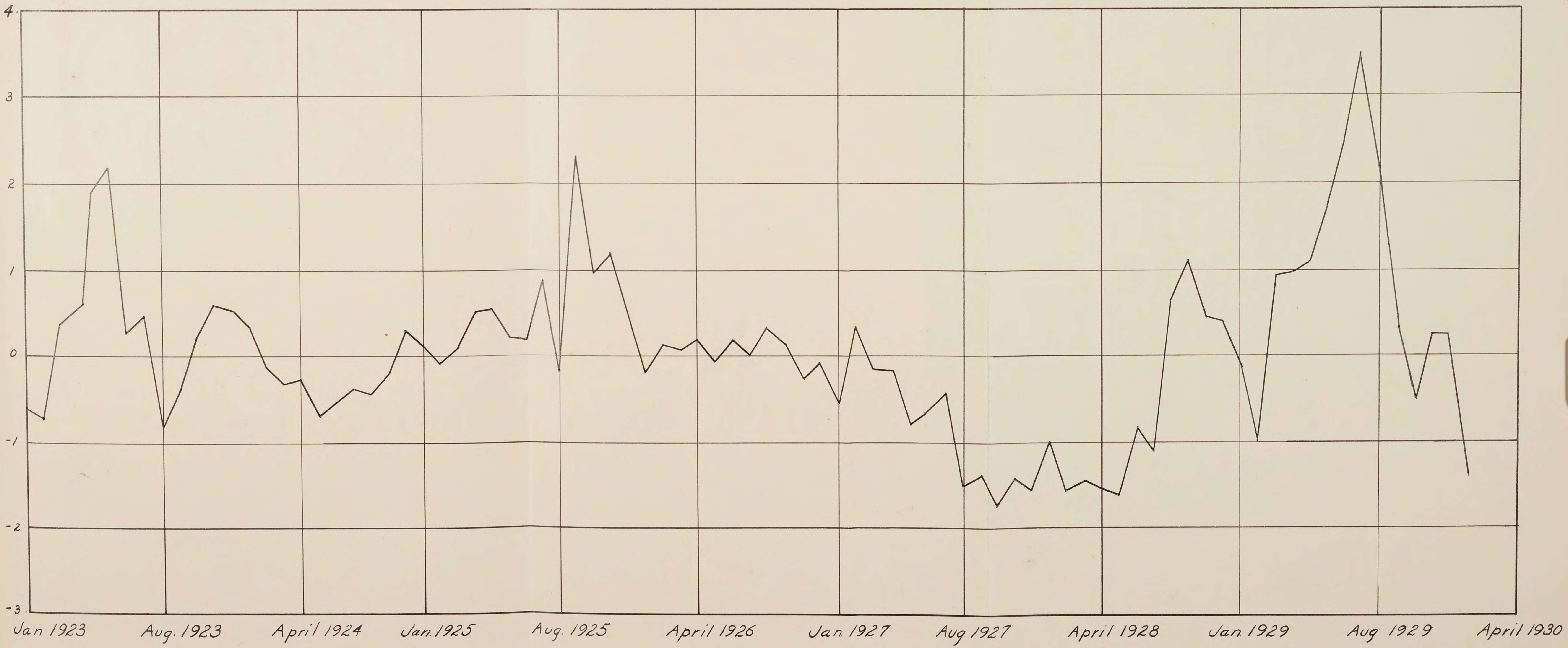

4
3
2
1
0
-1
-2
-3
Jan 1923
Aug. 1923
April 1924
Jan. 1925
Aug. 1925
April 1926
Jan. 1927
Aug 1927
April 1928
Jan. 1929
Aug 1929
April 1930

CHART V

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