THE EVALUATION OF EFFECTS OF TORSIONAL VIBRATIONS

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This is to certify that the

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Geo Hobbs
Major professor

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THE EVALUATION OF EFFECTS OF TORSIONAL VIBRATIONS

By

GORDHANBHAI FULABHAI PATEL

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THE EVALUATION OF EFFECTS OF TORSIONAL VIBRATIONS

There are two ways in which the change of position of the centre of gravity or motion of mass may occur.

(1) Due to variations in speed of revolution of crankshaft.

(2) By the distortion of the members of a mechanism due to elastic properties of the material of which they are made.

There is distortion of crankshaft due to the forces acting upon them: The first are serious because of their effects on other parts, while the second are serious because of their effects upon themselves.

Torsional vibration in an engine may likewise be due either to the change of speed of rotating parts or to the elastic distortion of those parts. The force impulses on the pistons cause varying torque impulses on the crankshaft and flywheel. The speeding up and slowing down of these parts within a cycle are due mainly to the forces within the engine and, therefore, the engine as a whole supplies the reacting torque. If it is not rigidly mounted, its inertia may be small enough to permit a noticeable rocking vibration. The elastic distortion of crankshaft is the most important, however, in that at certain speeds, the rate of application of the torque impulses may coincide with the natural frequency of the crankshaft or with some small multiple of fraction of this frequency in which the impulse adds additional energy to vibrating member causing it to oscillate with an increasing amplitude. This may continue until the part fails.

<u>Forces stimulating torsional vibration are due to</u>:

(1) Pressure forces in working cylinder.

(2) Inertia forces of reciprocating parts.

(3) Gravity forces of reciprocating and unbalanced rotating parts.

(4) Uneven absorption of power at the driven machinery.

<u>The damping forces are due</u>:

(1) Elastic hysteresis.

(2) Driven machinery.

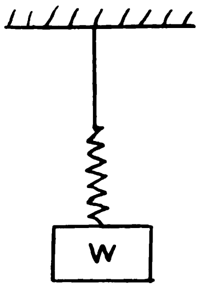
(3) Slight slippage at the union of the two shafts line-up.

(4) Surface friction between the moving and stationary parts.

(5) Energy absorbed by oil film around bearings.

(6) Energy transmitted by the side thrust of crankshaft.

Since vibration cannot arise under the action of constant forces only, the force creating and sustaining a vibration is always a fluctuating one. Fluctuating forces may vary in magnitude only, and are then usually called reciprocating forces, or they may vary in direction only and are then usually called rotating forces. Fluctuating force that causes vibration is called an exciting force, a disturbing force or a shaking force. Spring stiffness or spring constant K is given as the force necessary to stretch or compress the spring one unit of length.



<u>Free Vibration</u>: When displaced from position W goes down and force upward becomes greater, so goes up, due to momentum it continues to go up from center, and then due to downward force motion slows down and comes back. This is free vibration because there is no external fluctuating force.

<u>Restoring Force</u>: The force which tries to take W back to its original position.

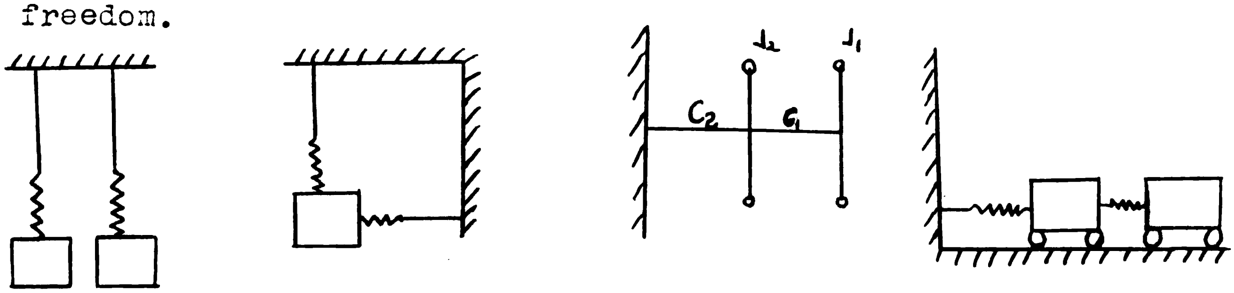
<u>Friction force</u>, which may be of complex form is called damping.

Number of cycles completed in one unit of time is called frequency. Frequency of free vibration is independent of amplitude but it will increase with increasing spring stiffness and with decreasing weight of vibrating mass. This frequency of free vibration of a system is called its <u>natural frequency</u> and it increases as sq. root of spring stiffness K and inversely as sq. root of weight W.

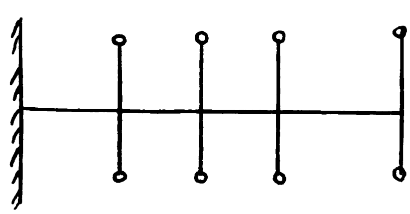
Suppose now that weight is shaken by external force. In this case, frequency of forced vibration depends only on that shaking force. Amplitude depends both upon fluctuating force and on the ratio of its frequency to the natural frequency of the system, and when this ratio becomes unity, the amplitude of vibration may build up to a dangerous value. This condition is called resonance, and the purpose of most vibration investigation is to avoid its occurence. The above mentioned vibrating system is of simplest type in-so-far as only one co-ordinate is necessary to specify the motion of mass. This is called system of one degree of freedom.

Inertia is merely a body possessing a mass moment of inertia J about a particular axis, and torsional stiffness, 'C', of the shaft takes place of spring constant, 'K'.

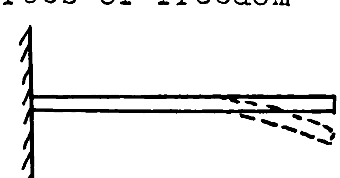
A rigid body restrained to move in two directions or to rotate about two axis is said to have two degrees of freedom.



Two degrees freedom system.



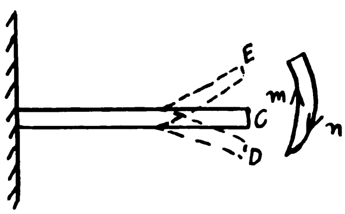
Four degrees of freedom



Infinite number of degress of freedom.

Any irregular motion of a particle about some fixed position of equilibrium may be called a vibration.

If the hammer blows occur when the vibrating rod is at _C_ and moving in direction m, vibration will obviously be damped, but if the blows are timed to occur at n', the force of blows will aid in continuing and amplifying the vibration. Such a case is called synchronous vibration or resonance.



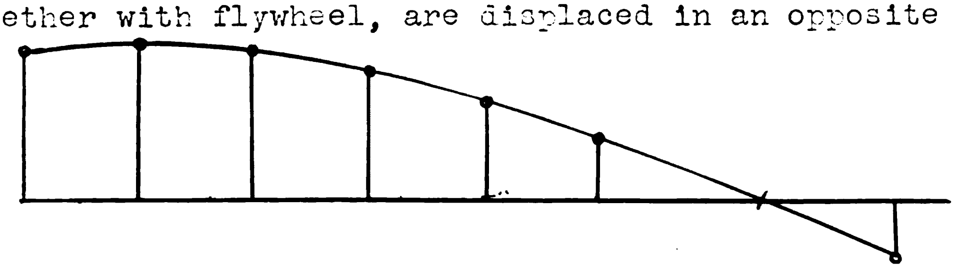
Every elastic body has a natural period of vibration, i.e. time per cycle of movement which depends upon its mass, moment of inertia, and stiffness. Crankshaft is more complex than simple rod, and its natural periods of vibration are harder to predict, but the basic idea is the same.

Excessive torsional vibrations in an engine cause noise or wear on gears and auxiliary drives, and in worst cases result in a broken crankshaft. Just as a pendulum has a natural period of swing, so the moving parts of an engine mounted on the crankshaft, i.e. pistons, connecting rods, flywheel, have a natural period of torsional oscillation. The irregular turning effort diagram of an engine is made up of a large number of sine curves known as harmonics, having varying magnitude and frequencies. Should the period of the main forcing torque or of the various harmonic orders synchronize with the natural period of oscillation of the shaft system, excessive vibration of shaft will occur. This state is called resonance, and is avoided wherever possible.

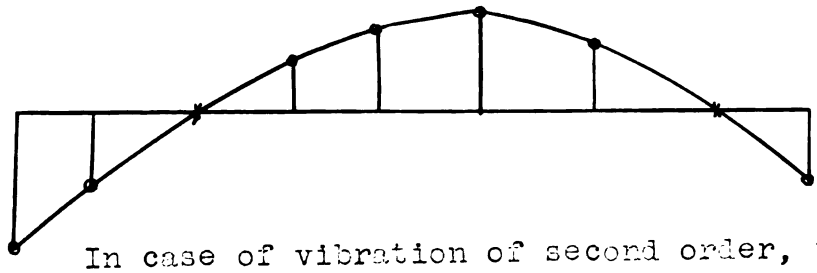
Where a state of resonance occurs in the running speed, the amplitudes of twist are kept down to a safe limit by stiffening of parts where possible, and sometimes by vibration damper, but the damper cannot be used successfully to enable the engine to run under the load at criticals. Its function is to take the engine through criticals.

A crankshaft with a flywheel at one end forms a compound torsional pendulum and vibrates as such. Torsional vibration may be of two kinds, forced and free. When subject deflected from normal position and is then released, it executes what is known as free vibration. If the pendulum is dealt with a rapid succession of blows, it is forced to vibrate at the rate of these impulses, and this is called forced vibration. Such forced vibration occurs in crankshafts, but their amplitudes are small and therefore they cause trouble.

First order of vibration when there is a single nodal point between flywheel and crank unit nearest to it. The nodal point does not vibrate but rotates at uniform speed. All parts of crankshafts ahead of the nodal point are then displaced in the same direction, while parts to the rear, together with flywheel, are displaced in an opposite direction.



First order torsional vibration of six throw crankshaft.



In case of vibration of second order, two nodal points are formed.

Fourier's Series: Any periodic or recurrent function of this kind can be accurately represented by a constant mean value and a series of harmonics (sine curve function) of which the first has same period as the basic function, and the following have the periods equal to 1/2, 1/3, 1/4, etc., that of basic function. Amplitudes of succeeding harmonics (the coefficient of suceeding terms) decrease in a general way, although each one is not smaller than the preceding one.

From this explanation we see that the gas pressure torque impresses upon crankshaft not only a succession of harmonic forces of the same frequency as its own, but also a series of harmonic forces twice, thrice, four times, etc., this frequency. The method of resolving an irregular periodic function into its component harmonics is known as harmonic analysis. Torsional vibration will depend upon the firing order of the engine. Frequency is between 12000 and 16000 cycles per minute. Sixth harmonic is the lowest which can cause torsional vibration of the first order in a six cylinder engine. Such vibration in crank, having frequency of free vibration of 15000 per minute, occurs at

2x 15000/6 = 5000 R.P.M.

This is beyond operating speed. The next harmonic that may give trouble is 9th, which is in resonance at an engine speed of

$$2 \times 15000/9 = 3333 \text{ r.p.m.}$$

Resonance occurs when $q = \frac{nN}{2}$

q= natural frequency.

n = nth. harmonic.

N = speed per min.

This vibration can be suppressed by the choice of a suitable firing order.

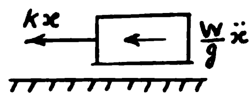
The next important harmonic is 12th, which is in resonance at 2500 r.p.m. The third harmonics are the lowest, causing torsional vibration of the first order, and since their amplitude is only about 1/16 that of first harmonics, inertia torque is not a very important factor in causing torsional vibration in multi-cylinder engines. In aircraft engine, since a useful speed range is much less than in an automobile, it is usually possible to avoid the most troublesome critical torsional speeds by designing the shaft so that it does not have a natural period of severe vibration within the desired operating range.

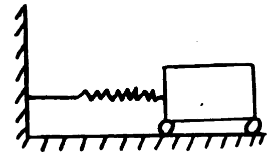
The simplest form of a periodic motion is a harmonic one, i.e. sine or cosine function. And most vibrating systems have motions that are nearly harmonic and may be written as

$x = X\cos wt.$ X = amplitude.

wt = angle in radians.

Free Vibration:





$\frac{W\ddot{x}}{g}$ is the inertia force that resists acceleration $\ddot{x}$ (+ to right) and therefore is -ive. for -ive. value of x. This becomes +, and acts to right:

$$\frac{W}{g}\ddot{x} + Kx = 0$$

$$x = A\cos(\omega_n t - \gamma)$$

From which acceleration found and $\omega_n = \sqrt{\frac{Kg}{W}}$

Natural frequency of the system is given by:

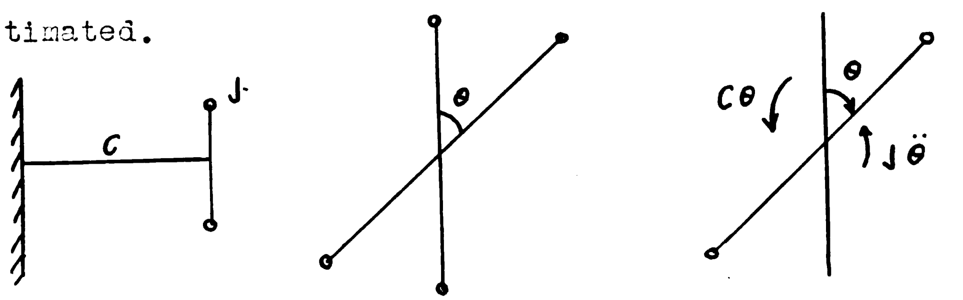
$$f_n = \frac{1}{2\pi}\sqrt{\frac{Kg}{W}}$$

If the spring shown in figure were stretched by a force equal to the weight W of vibrating mass, resulting deflection would be:

$$\delta = \frac{W}{K}$$

$$\& \quad f_n = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \qquad \delta = \text{deflection}$$

This is easy, since deflection can be found or estimated.



Because of twist of the shaft, a counter-clockwise torque c is exerted, and because of angular acc. there will be a resisting inertia torque (acting in -ive. direction) of J .

for dynamic equilibruim

$$J\ddot{\theta} + C\theta = 0$$

from which

$$f_n = \frac{1}{2\pi}\sqrt{\frac{C}{J}} \qquad T_n = 2\pi\sqrt{\frac{J}{C}} \qquad \text{---- (A)}$$

By measuring natural period of oscillations, the value of J of the moment of inertia about the suspension axis may be found from equ. (A). This gives

$$J = \frac{t_n^2 C}{4\pi^2} \quad .$$

After free vibration has died out, there remains only forced vibration, which is called steady state vibration. Steady state forced vibration is harmonic, and has the same frequency as the shaking force. Free vibration is the sum of several harmonic motions of different frequencies.

Torsional System

Torsional vibration of crankshaft produces vibration of the reciprocating parts, and the polar inertia of crankshaft itself is very small compared with the inertia at each cylinder, due to the motion of these reciprocating parts.

Torsional Analysis

Steps: (1) Calculate the torsional rigidity of the shaft between each rotating member.

(2) Calculate the moments of inertia of all the reciprocating and rotating masses in the system.

(3) Calculate the natural frequencies of torsional oscillation of the system.

(4) Calculate amplitude of vibration and the resulting stresses due to resonance.

Torsional Rigidity

Let

C = torsional rigidity of actual shaft.

G = modulus of rigidity.

I_p= polar moment of inertia or second moment or area.

L = actual length of shaft.

D = actual dia. of shaft.

C_e= torsional rigidity of equivalent shaft.

L_e= equi. length of shaft.

D_e= " dia. of shaft.

$$c = \frac{GI_p}{L} = \frac{G}{L} \times \frac{\pi D^4}{32}$$

$$\text{and } C_e = \frac{G}{L_e} \times \frac{\pi D_e^4}{32} \qquad \text{But } C = C_e$$

$$\frac{G}{L} \times \frac{\pi D^4}{32} = \frac{G}{L_e} \times \frac{\pi D_e^4}{32}$$

$$\therefore \quad L_e = L \left(\frac{D_e}{D}\right)^4$$

L_e, then, is the length of shaft of dia. D_e which when subjected to a certain torque, will twist through an angle equal to that produced by same torque on a shaft of length L and diameter D.

<u>Crank Shaft Stiffness</u>:

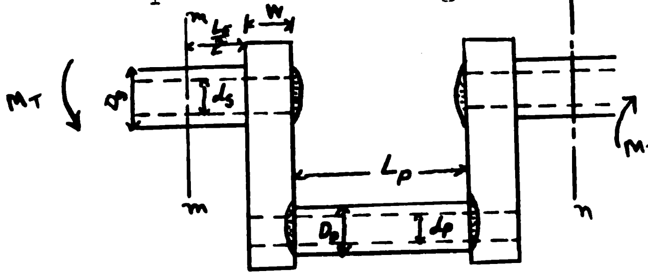
Reduce crankshaft to an equivalent length of parallel shafting diameter of crankshaft journal.

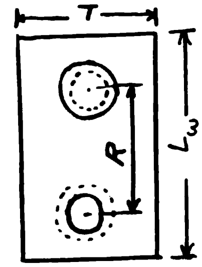
$$L_e = 2L_s + .4W + 1.096\, L_p \frac{(D^4 - d^4)}{(D_p^4 - d_p^4)} + 1.284R \left(\frac{D^4 - d^4}{WT^3}\right)$$

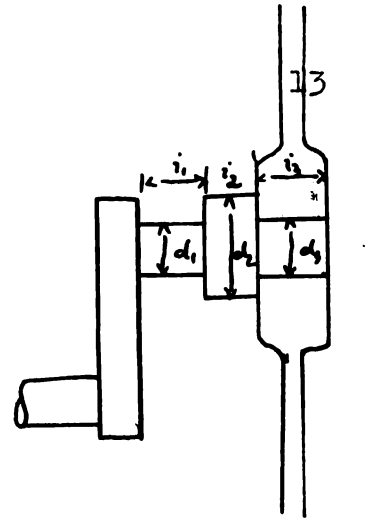
Having found the equivalent length of crank through, it is necessary to find the length from the flywheel end cylinder to the flywheel. Total equivalent length from end cylinder to the last iron flywheel

$$= \frac{L_e}{2} + l_1 \frac{(D_e)^4}{d_1^4} + l_2 \frac{(D_e)^4}{D_2^4} + \frac{l_3}{4} (D_e)^4 + \frac{l_3}{4} \left(\frac{D4}{d_3^4}\right)^4 \times \frac{11.8}{6.3}$$

The shaft is considered free to twist to a length of a quarter the diameter of the bore, and is then considered as integral with the boss, which is taken to twist to half the depth of the flange.







For solid crank pin and journal

$$L_e = (L_s + .8w) + \frac{3}{4}\frac{L_p D_s^4}{D_p^4} + \frac{1.5RD_s^4}{wt^3}$$

By Carter's formula

$$L_e = D\left(\frac{L_s + .8W}{D_s^4} + \frac{.75L_p}{D_p^4} + \frac{1.5R}{WT^3}\right)$$

Torsional rigidity one crank depends upon the condition of constraint at the bearing. Assuming that the clearances in the bearings are such that free displacements of the cross section m-m and n-n during twist are possible, the angle of twist produced by torque moment M can be easily obtained. This angle consists of three parts:

(1) Twist of journal

(2) Twist of crank pin

(3) Bending of web.

Let C_1 be $= \frac{D_s^4 G}{32}$ torsional rigidity of journal.

$C_1 = \frac{D_p^4 G}{32}$ torsional rigidity of crank pin.

$B = \frac{W\ T^3}{1\ 2} E$ torsional rigidity of web.

In order to take into account local deformation in the web in the regions shaded in the figure due to twist, the lengths of the journal and of the pin are taken equal to:

$$L_s = L_s + .9w$$

and

$$L_p = L_p + .9w \quad \text{respy.}$$

The angle of twist θ of the crank produced by a torque moment M will then be

$$\theta = \frac{L_s M_T}{C_1} + \frac{L_p M_T}{C_2} + \frac{2\,R\,M_T}{B}$$

In calculating the torsional vibration of a crank-shaft, every crank can be replaced by an equivalent shaft of uniform cross section of a torsional rigidity C. The length of equivalent shaft will be found from

$$\frac{M_T l}{C} = \theta \qquad \theta \text{ as calculated above.}$$

Then length of equivalent shaft be

$$L = C\left\{\frac{L_s}{c_1} + \frac{L_p}{c_2}\left(1 - \frac{R}{K}\right) + \frac{2R}{B}\left(1 - \frac{R}{2K}\right)\right\} \quad \text{---- (B)}$$

in which

$$K = \frac{\dfrac{R(L_p + W)^2}{4\,c_3} + \dfrac{L_p R^2}{2\,c_2} + \dfrac{L_p^3}{24\,B_1} + \dfrac{R^3}{3\,B} + \dfrac{1.2}{G}\left(\dfrac{L_p}{2\,F} + \dfrac{R}{F_1}\right)}{\dfrac{L_p R}{2\,c_2} + \dfrac{R^2}{2\,B}}$$

in which again

$$c_3 = \frac{c^3 w^3 G}{3.6\,(c^2 + W^2)}$$ which is the torsional rigidity of the web as a bar of rectangular cross section with sides W and T.

$$B_1 = \frac{\pi D_p^4 E}{64}$$ flexible rigidity of crankpin

F and F_1 cross sectional areas of crank pin and of the web respectively.

By taking $L_p = L_s$ and $c_1 = c_2$, the complete constraint as it is seen from above equation dminishes the equivalent length of shaft in the ratio

$$\frac{1}{\left(1 - \frac{R}{2K}\right)}$$

<u>Another question to be considered is the calculation of inertia of moving masses</u>.

Mass m of connecting rod is replaced in two masses $m_1 = \left(\frac{I}{\ell^2}\right)$ at crank pin and $m_2 = m - \left(\frac{I}{\ell^2}\right)$ at cross head, where I denotes Moment of Inertia of connecting rod about the centre of cross head. All other moving masses are concentrated in the same two points, so that finally only two masses, M and M_1 are taken.

For torsional considerations, all reciprocating motion must be reduced to equivalent rotating motion. Half the weight of reciprocating parts may be considered as acting at crank pin.

Piston = weight of it is added to reciprocating part.

Rod = partly 2/3 revolving and partly reciprocating.

W = 1/2 weight of reciprocating parts of connecting rod and piston + weight of revolving parts of rod.

$W_k{}^2 = W_t R^2$

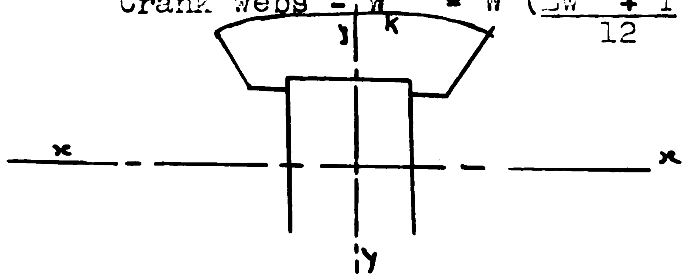
Journal $W_k{}^2 = \frac{W D_p{}^2}{8}$ for solid

$= \frac{W D_p{}^2 + d_s{}^2}{8}$ for hollow

Crank pin $W_k{}^2 = W \left(\frac{D_p{}^2}{8} + R^2 \right)$ for solid

$= W \left(\frac{D^2{}_p + D_s{}^2}{8} + R^2 \right)$ for hollow

Crank webs $= W_k{}^2 = W \left(\frac{Lw^2 + T^2}{12} + d^2 \right)$ for solid



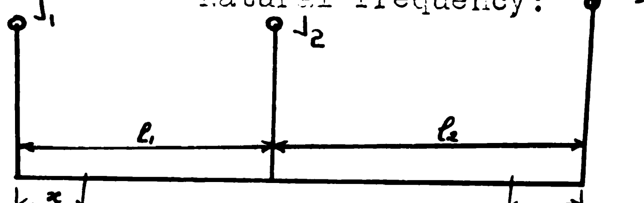
Balance weight symmetrical about Y-Y.

$I = I_{x-x} + I_{y-y}$

I = rotational moment of inertia.

I_{x-x} I_{y-y} are found by constructing the first and second derived figures about both axis and finding radius of gyration for each.

Natural frequency:



Three mass system

J_1, J_2, J_3, M. of I. of 3 masses.

l_1, l_2 lengths of connecting shafts.

x be the distance of node N_1 from J_1.

y be the distance of node N_2 from J_3

The node is the point of reversal of twist. The period torsional oscillation of J_1 about its node is given by the expression:

$T = 2\pi\sqrt{\frac{J_1}{C}}$ C = torsional rigidity of x length

Frequency F of mass J is

$F = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{C}{J_1}}$ but $C = \frac{G\ I_p}{L}$ where G is modulus of rigidity which will take as 11.8×10^6 #/ square inch

I_p is polar moment of inertia of the shaft in inches and L is the length of the shaft in inches.

If J_1 is expressed in Lb. in.^{2}

$$F_1 = \frac{60}{2\pi}\sqrt{\frac{11.8 \times 10^6 \times 386 \times I_p}{J_1\ x}}$$

$$= 644000\sqrt{\frac{I_p}{J_1\ x}} \quad \text{vibration per minute} \quad \text{-- (1)}$$

and

$$F_2 = 644000\sqrt{\frac{I_p}{J_2}\left(\frac{1}{l_1 - x} + \frac{1}{l_2 - y}\right)} \quad \text{-- (2)}$$

$$F_3 = 644000\sqrt{\frac{I_p}{J_3 y}} \quad \text{-- (3)}$$

since $F_1 = F_2 = F_3$ equating the three expressions give as

$$\frac{1}{J_1 x} = \frac{1}{J_2}\left(\frac{1}{l_1 - x} + \frac{1}{l_2 - y}\right)$$

$$= \frac{1}{J_3 y}$$

Expressing Y in terms of X, we have

$$Y = \frac{J_1 x}{J_3}$$

by substituting

for Y in equation (2) and equating (2) and (1) we get two values of x.

Substituting it in (1) we get two frequencies, representing the oscillation of J_1 J_2. The highest figure represents the oscillation of J_2 against J_2 and J_3 causing a two node vibration. For multi-cylinder it becomes difficult, but the system can be reduced to three mass system and results can be obtained. In a multi-cylinder engine with flywheel, total inertia of the cylinder masses is considered to act at the centre of the engine for close approximation. Frequency is obtained by multiplying the total ($W R^2$) of the cylinder masses by .85 and considering this acting at centre.

Example:

Inertias - each cylinder = .518#.in. sec.^{2}

flywheel = 32.7 #. in. sec.^{2}

generator= 13.25 #. in. sec.^{2}

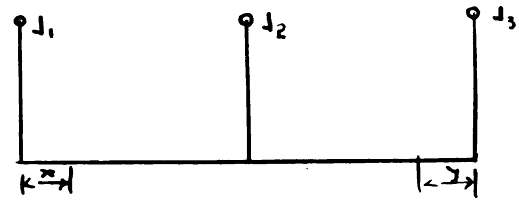
First reduce to equivalent three mass system. Consider the inertia as acting at the centre, i.e. between cylinder 3 and 4. Apply correction factor .85. Effective inertia of the cylinder masses:

$= .85 \times 6 \times .518 = 2.64$ #. in. sec.^{2}

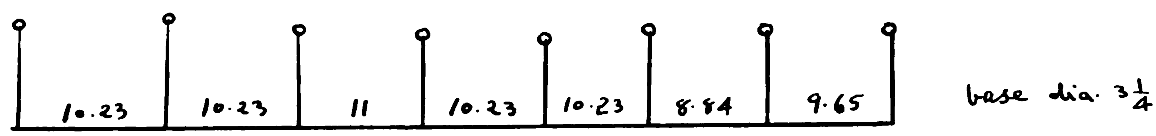
$= 1020$ #. in.^{2}

Equivalent length from this mass to the flywheel

$= \frac{1}{2} \times 11 + (2 \times 10.23) + 8.84 = 34.80$ in.



Let x be the distance of one node from the effective cylinder mass and Y the distance of the other node from generator.



Single node normal elastic (~~Below~~). PAGE 6.

Then $J_1 X x = J_3 X Y$

$$\therefore \qquad Y = \frac{J_1}{J_2} x = \frac{2.64}{13.25} x = 0.1995x \text{ say } .2x.$$

Also

$$\frac{1}{J_1 X} = \frac{1}{J_2} \left(\frac{1}{34.8 - x} + \frac{1}{9.65 - y} \right)$$

Substituting for Y

$$\frac{1}{2.64x} = \frac{1}{32.7} \left(\frac{1}{34.8 - x} + \frac{1}{9.65 - .2x} \right)$$

from which $x = 23.75$ in. or 47.45 in.

Natural frequency $= 344000 \sqrt{\frac{I_p}{J_1 x}}$

where J_1 is the effective WR^2 of the cylinder masses #. in. sec^2.

one node $F = 644000 \sqrt{\frac{10.9}{1020 \times 47.45}}$

$= 9560$ V.P.M.

Two nodes normal elastic curve. (SEE PAGE 7)

Two node $F = 644000 \sqrt{\frac{10.9}{1020 \times 23.75}}$

$= 13650$ V.P.M.

The approximate figures for the two frequencies having been obtained, the exact figure must now be found by the torque summation method.

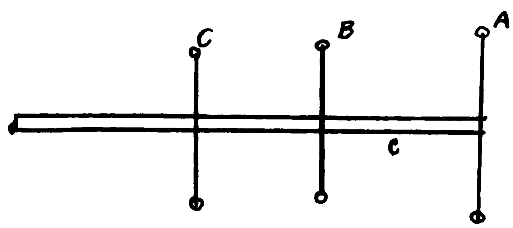
Method for calculation of natural frequencies of torsional vibrations.

In the engine with a small number of cylinders and slow speed, the torsional vibration can be avoided by making shaft diameter large.

(1) One type of equivalent shaft arrangement is obtained by considering each cylinder, flywheel, and alternator as concentrated masses connected by elastic shafts having no mass.

(2) Another type of equivalent shaft arrangement is obtained by averaging the various masses distributed along the shafting of installation into a number of connected uniform shafts, having both mass and elasticity.

Heavy flywheel may be considered concentrated at certain points of uniform shafts. There are three masses and two shafts as compared with the eight masses and seven shafts when a concentrated mass system is used. The masses may be reduced from the beginning to the end, or backward from end to beginning, or from both ends up to some section. A natural frequency is obtained when the sum of the masses reduced from both ends up to any section added to mass at that section is zero.

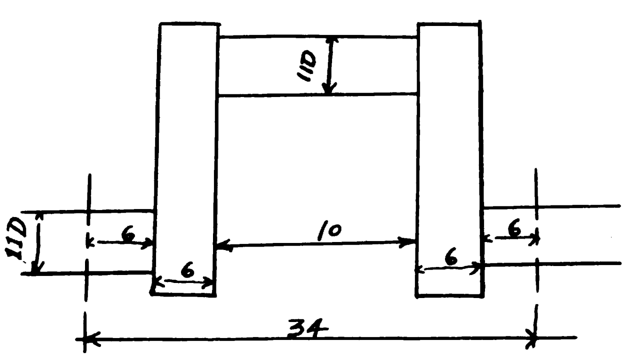


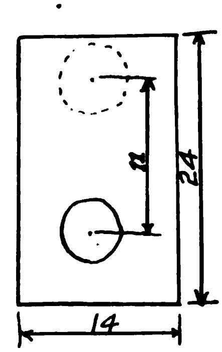
The effect of A, acting upon B, through elastic shaft C can be obtained at B by adding fraction of A to B.

This sum can again be considered as the first mass, the effect of which is reduced to next concentrated mass. At this frequency the elastic forces in the shafts and the inertia forces of masses are in a state of equilibrium, so that the system when once set into motion at this frequency, will continue to vibrate indefinitely (assuming no existence of damping influence).

Example:

Consider six cylinder, two cycle connected to a flywheel and the alternator.





Weight of piston and pin, complete including cooling gear and oil = 1000 pounds.

Weight of connecting rod = 500 pounds.

Center of gravity of rod occurs at 0.38 times the centre line length of the rod from the crank pin center line.

$$\frac{\text{crank}}{\text{Connecting rod}} = \frac{+1}{5}$$

I = factor of journals is that of a solid cylinder about its own center line.

$$\frac{Ld^4}{36} \quad \#.\text{In.}^2$$

L = length.

d = diameter.

weight of steel = .2830 #/ square inch.

I = factor for crank pin is that of solid cylinder about an axis at a distance r, from its own center line, thus

$$\frac{L\,d^2}{4.5}\left(\frac{d^2}{8} + r^2\right) \quad \#.\ \text{in.}^2$$

L = length of pin.

d = diameter.

r = crank radius.

A close approximation for inertia factor for webs, considering each as rectangular parallelopiped, having the same width and thickness and height to give same cross section area is given by:

$$\frac{b\ d\ h}{3.534}\left\{\left(\frac{b^2 + h^2}{12} + \left(\frac{r}{2}^2\right)\right)\right\} \quad \#.\ \text{in.}^2$$

b = breadth.

d = thickness.

h = height of equi. rectangular section.

r = crank radius.

$\therefore$ I - factor are

$$\text{journal} = 12 \left(\frac{11}{33}\right)^4 = 4880 \ \#.\ \text{in.}^2$$

$$\text{pin} = 10 \left(\frac{11}{4.5}\right)^2 \left\{ \left(\frac{11}{8}\right)^2 + (11)^2 \right\} = 33600$$

$$\text{Two webs } \frac{2\text{x}14\text{x}6\text{x}22.5}{3.534} \left\{ \frac{(14)^2 + (22.5)^2}{12} + (5.5)^2 \right\} = 94950$$

Piston and connecting rod

$$11^2 \left\{ .62 \times 600 + .5050 \ (.38 \times 600 + 1000) \right\} = 120,050$$

Total 256,480 # in.^{2}

$$J = \frac{256480}{34} = 7544 \ \#\ \text{IN.}^2 \text{ per inch}$$

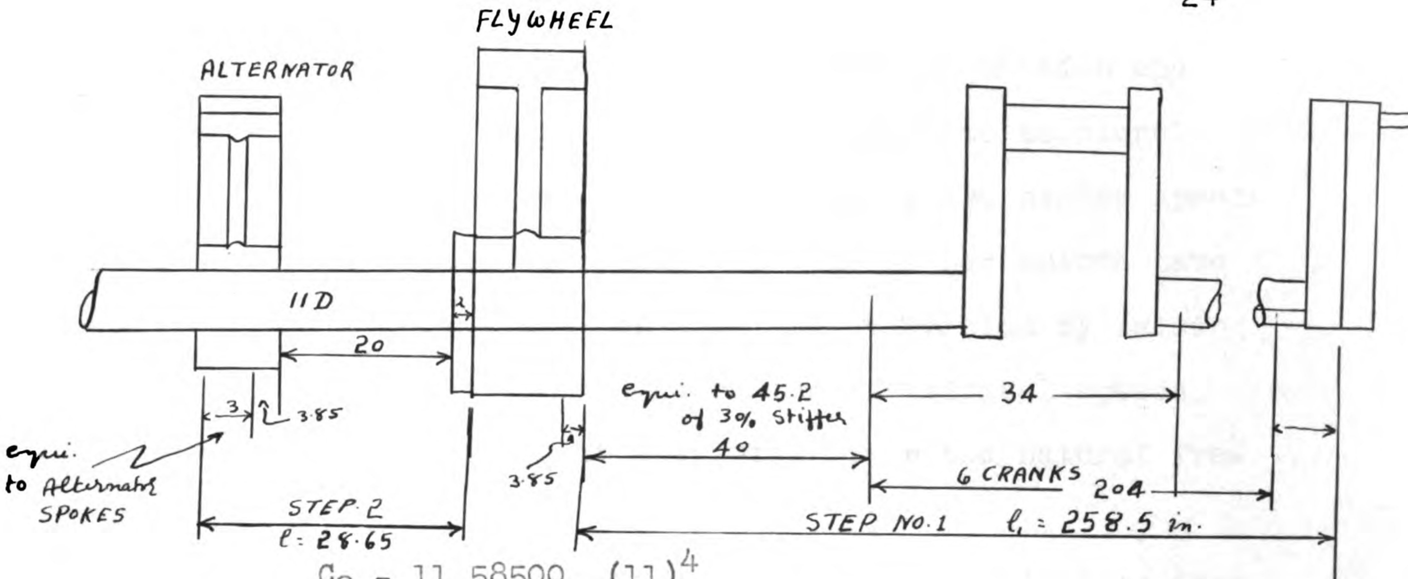
Considering the crank three percent stiffer than a uniform shaft having the same section as the journal, the value of C L, may be calculated from equation.

$$CL = 115800 \ (11)^4 \quad 1.03$$

$$= 17470 \times 10 \ \#.\ \text{in.}^2$$

In the same way inertia factor for crank was calculated. The total inertia factor for the scavenge pump, including the shaft up to center line of the first bearing is 70000 #. in.

$\therefore \dfrac{70000}{7544}$ = 9.3 in. corresponds to the increase in length of main cranks due to scavenging pump.



$$C_2 = 11\ 58500\ \frac{(11)^4}{28.85}$$

$= 588,000,000$ #. in.

$L_1 = 259.5$ in.

$j_1 = 258.5 \times 7544 = 1,950,000$ #. in.^{2}

$C_1 = 17470 \times 10^6/258.5 = 67,580,000$ #. in.

$$0 = 18.325\ \frac{J_1}{C_1} = 3.113$$

Resulting equi. system.

$C_2 = 588 \times 10^6$ #. in.

$= 0.09221$

$L_1 = 258.5$ in.

$j_1 = 1950,000$ #. in.^{2}

$C_1 = 67580000$

$0 = 3.113$

All present day trends in the application and design of these engines have been conducive to torsional vibration difficulties. Tendencies are toward higher speeds and a greater number of cylinders. The higher speeds have increased the possibility of torsional vibration by raising the running speeds up into the region of critical speeds. Increase in mass or flexibility will lower the natural frequency.

The Holzer method is based on the principle that the sum of the inertia torques, developed in a system because of the vibrations, must equal zero if the vibration is zero.

Example:

A 4 cylinder engine with a flywheel runs at 1200 r.p.m. and is connected with a generator by a flexible coupling. The mass moments of inertia in in. # sec.^{2} are as follows:

Cylinder $J_1 = J_2 = J_3 = J_4 =$		.55
Flywheel	J_5	= 25.6
Coupling hub	J_6	= 0.5
Generator	J_7	= 8.75

The equivalent lengths referred to a 3 inch diameter steel shaft in inches are the following:

Cranks $L_{e_1} = 5$

$L_{e_2} = 5.5 \qquad L_{e_3} = L_{e_4} = 5$

Flexible Coupling $L_{e_5} = 45.5$

Hub to generator $L_{e_6} = 41$

To determine the lowest natural frequency with the aid of Holzer method.

Before setting up the table, it is desirable to convert equivalent length to spring scale.

$$\text{since } K_t = \frac{G\pi d_e^4}{32 L_e} \quad \text{and} \quad d_e = 3 \text{ in.}$$

$$= \frac{12\ (10)^6 \pi\ 3^4}{32\ Le}$$

$$= \frac{95.6\ (10)^6}{Le}$$

The values of the spring scales, then are

$$K_{t_1} = K_{t_3} = K_{t_4} = 19.12\ (10)^6$$

$$K_{t_2} = 17.37\ (10)^6$$

$$K_{t_5} = 2.1\ (10)^6$$

$$K_{t_6} = 2.33\ (10)^6 \text{ in. \#. per radian}$$

Notes on calculation procedure:

<u>How to assume frequency</u>:

A fair trial value can be obtained by grouping together the masses that have a short equi. shaft length or large spring scale between them (neglect coupling mass as small).

$$\text{Then } J_1^1 = J_1 + J_2 + J_3 + J_4 + J_5$$

$$= 27.8$$

$$J_2^1 = J_7 = 8.75$$

$$\text{and } L_e = L_{e_5} + L_{e_6} = 86.5 \text{ in.}$$

$$K_t = 1.105\ (10)^6$$

$$f_n = \frac{60}{2\pi}\sqrt{\frac{K_t(J_1 + J_2)}{J_1 J_2}}$$

$$= \frac{60}{2\pi}\sqrt{\frac{1.105\ (10)^6\ (27.8 + 8.75)}{27.8 \times 8.75}}$$

$$= 3890 \text{ c.p.m.}$$

$$= \left\{\frac{(3890 \times 6.28)}{60}\right\}^2 = 166000 = 0.165\ (10)^6$$

assumed.

1 Item	2 J	3 $JW^2(10^5)$	4 β	5 $JW^2\beta(10^5)$	6 $\Sigma JW^2\beta(10^5)$	7 $K_t(10^5)$	8 $\frac{\Sigma JW^2\beta}{K_t}$
1	.55	0.0908	1.000	0.0908	0.0908	19.12	0.0047
2	.55	0.0908	.9953	0.0905	0.1813	17.37	0.0104
3	.55	0.0908	.9849	0.0894	0.2707	19.12	0.0142
4	.55	0.0908	.9707	0.0881	0.3588	19.12	0.0188
5	25.60	4.2240	.9519	4.0208	4.3796	2.10	2.0855
6	.50	0.0825	-1.1345	-0.0937	4.2859	2.33	1.8394
7	8.75	1.4438	-2.9740	-4.2939	-0.0080		

Refer to above table:

Second column, inertia as given.

Seventh column, spring scales (calculated)

The value of W_n^2 = .165 $(10)^5$ will be assumed as the natural frequency. Then fill up the table. The amplitude of the first disk (β_1) is always assumed to be one radian.

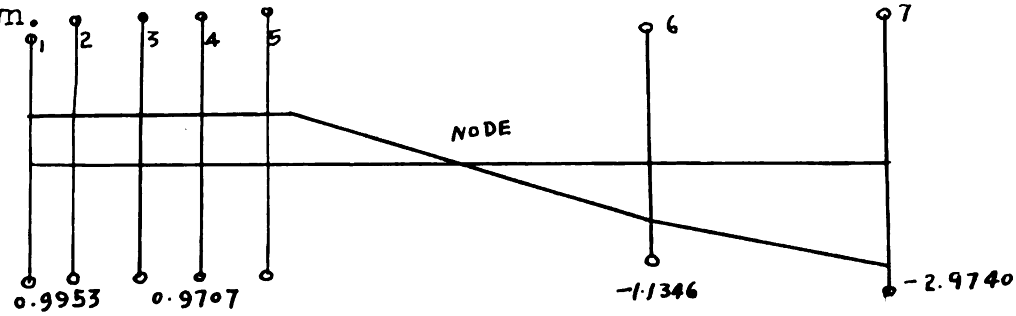
By dividing 6 column/ 7 column, we get angle of twist. $= \Delta\beta_2$ Radians = 0.0047

$$\beta_2 = \beta_1 - \Delta\beta_2 = 1 - .0047$$
$$= 0.9953$$

which is placed in second line of column 4.

In this example torque remainder in column (6) is -0.0080 in. #. If the correct frequency had been chosen, this remainder would be zero.

The deflexion curve may be plotted along the shaft by using the values of β in column 4, for the various disks as shown.



It will be seen that there is only one node (located between disks 5 and 6). Hence the value of $\omega_n^2 = .165\ (10)^6$ and f_n = 3880 c.p.m. is very close to the first natural frequency. For torsional vibration the mode of the vibration is the same as the number of nodes, i.e. the first critical has one node, the 2nd - two, and so on.

Calculations:

Holzer tabulation procedure is a widely accepted method for calculation of torsional vibration characteristics. This method can be simplified by application of the distributed mass concept to some parts of the equivalent mass elastic system.

Since with present equipment it is impractical to measure all the amplitudes and torques in entire system under all operating conditions. Calculation, laboratory testing, and engine measurement should therefore be considered as mutually dependent techniques for guiding development.

Fundamental calculation methods:

Torsional vibration involves the response of a 'mechanical network' of inertia, coupled by shaft flexibilities, to a series of complex periodic exciting torque, applied at several points and with different phase angles. A 'cut and dry' method is used.

General methods:

(A) Fundamental simplifications

(1) Equivalent system: Distributed factors lumped into equivalent concentrated factors.

(a) Inertias: By normal calculations, mechanical integrator, or test.

(b) Elasticities

(2) Exciting torque: Actual single cylinder torque vibration curves expressed as fourier series.

(a) Harmonic analysis of gas torque diagrams constructed from indicator card.

(b) Mathematical expression for inertia torque vibration, expressed as fourier series

(3) Damping factors: verycomplex and variable, depend on the shaft material and stress, bearing, piston and ring friction, coupling absorption, electrical eddy

currents, propeller, etc. The best guide is previous experience on comparable engines.

Calculations aids: Techniques and methods

(1) Use of Holzer table

(2) Disregard of damping to avoid out of phase components.

(3) Judicious reduction of complex systems by the use of the distributed mass with tables and curves, showing performance of simplified systems.

(4) Formation of algebraic equations for balancing Holzer tables

(5) Use of the standard calculating machines

(6) Use of special G.M.R. slide rule calculating board

(7) Mechanical vibrating models

(8) Electrical oscillating network

(9) Specialized calculating machines

(10) The vectorscope

(11) Harmonic analyzer

Methods of calculating torsional vibration:

First, it requires the determination of a suitable mass elastic system expressed in numerical terms, having approximately equivalent torsional vibration characteristics. This is called 'Equivalent system', 'mass elastic system', etc.

It requires numerical evaluation of the inertia factors and stiffness factors for all the moving parts of the installation. After that, it requires the calculations for:

(1) natural frequency

(2) peak amplitudes or stresses at synchronism between natural frequency and stimulating impulses

(3) forced vibration amplitudes or stresses at various frequencies of stimulating impulses

Equilibrium system is obtained by considering the moving masses at each cylinder, flywheel, alternator and other large compact parts as concentrated masses connected by elastic shafts having no mass. This is called 'concentrated mass system'. Another is obtained by averaging various masses distributed along the shafting of the installation into a number of connected uniform shafts, having both mass and elasticity. This is called 'uniform shaft system'. A third type uses any combination of concentrated masses, elastic shafts without mass and elastic with mass called 'combination equivalent system'.

A natural frequency is defined as the frequency at which a sustained vibration of the system may occur if no damping exists, and stimulating impulses are removed. In using this Holzer table for natural frequency calculation, the initial value of unity is assumed for θ for mass # 1, and the rest of the values are completed. The requirement for natural frequency is $\sum_{1}^{e} I P^2 \theta = 0$. Various values of P^2 are tried until this condition is realized. Columns 3 and 5 give the relative amplitudes and moments of vibration at natural frequency when this occurs, and the relative stresses due to vibration at this frequency can be calculated.

Then if the actual amplitude of natural vibration at any mass is known accurately by test or calculation, the actual vibration stresses may be computed.

Calculation for peak amplitudes due to several impulses acting at various masses may be made by equating energy input = energy absorbed by damping. If the impulses are $m_s \cos(pt - \varphi_s)$ acting at the masses indicated by s, and the amplitude at the first mass is $\theta_1 \sin(pt - \varphi)$. The energy input equation is

$$E = \pi \theta_1 \sum m_s \bar{\theta}_s$$

where $\bar{\theta}_s$ is the relative amplitude at the various masses given in column 3.

$$\sum m_s \bar{\theta}_s = \sqrt{(\sum m_s \bar{\theta}_s \sin \varphi_s)^2 + (\sum m_s \theta_s \cos \varphi_s)^2}$$

$$\tan\varphi = \frac{\sum m_s \bar{\theta}_s \sin \varphi_s}{\sum m_s \theta_s \cos \varphi_s}$$

If the magnitude of impulses are all the same such that $m_s = m$. The energy equation is

$$E = \pi \theta_1 m \sum \bar{\theta}_s$$

where $\sum \bar{\theta}_s = (\sum \bar{\theta}_s \sin \psi_s)^2 + (\sum \bar{\theta}_s \cos \psi_s)^2$

$$\tan \phi = \frac{\sum \bar{\theta}_s \sin \psi_s}{\sum \bar{\theta}_s \cos \psi_s}$$

Using 0, $\psi_2 - \psi_1$, $\psi_3 - \psi_1$ ..... in place of ψ_1, ψ_2, ψ_3, the phase angles all refer to the instant at which the maximum value of the first impulses occurs in this case,

$$\psi = \psi_1 + \tan^{-1} \frac{\sum \bar{\theta}_s \sin (\psi_s - \psi_1)}{\sum \bar{\theta}_s \cos (\psi_s - \psi_1)}$$

Numerical values of m are obtained by harmonic analysis of the applied torque curves or torque / AR curves, where A is piston area and R is crank radius. If m the magnitude of harmonic curve

m_1 torque / A R curve

Then $m = ARm_1$

Curves for m_1 due to gas pressure are appended.

The energy loss due to a marine propeller may be stated as:

$$K = \pi p \rho \bar{\theta}_e^2 \quad \theta_1^2$$

where $\bar{\theta}_e$ is the relative amplitude of vibration at the propeller as given by Holzer.

Average value of ρ is

$$\rho = 35 \frac{Q}{\text{r.p.m.}}$$

Q = mean shaft torque at the r.p.m.

at which the variation occurs r.p.m. of shaft speed at which vibration occurs. Assuming torque varies with the square of r.p.m.

$$\rho = 35 \text{ r.p.m}_f \frac{Q}{(\text{r.p.m}_f)^2}$$

where Q_f and r.p.m_f refer to the values at full speed.

An empirical formula for the damping along the steel shafts, due to internal absorption of energy and other unknown effects is

$$K = \frac{70\,(d_2^{4.3} - d_1^{4.3})\, \ell \bar{M}^{2.3}}{10^{10}\,(d_2^4 - d_1^4)^{2.3}}\,\theta_1^{2.3}$$

$$= \frac{70\, \ell \bar{M}^{2.3}\, \theta_1^{2.3}}{10^{10}\, d^{4.9}} \qquad \text{if } d_1 = 0$$

where L is the length of shaft.

d_2 and d_1 outer and inner dia.

$\bar{M}$ vibration twisting moment in the shaft corresponding to the relative amplitude in Holzer table.

If E_o = input energy for $\theta_1 = 1$

K_o = damping energy along the shafts for $\theta_1 = 1$

The amplitude is obtained by equating the input and damping energy as follows:

$$E\ \theta_1 = K_o\, \theta_1^{2.3}$$

$$\theta_1 = \sqrt[1.3]{\frac{E_o}{K_o}}$$

If K_ρ propeller damping energy for $\theta = 1$

Amplitude we get by

$$E_o\, \theta_1 = K_\rho\, \theta_1^2 + K_o\, \theta_1^{2.3}$$

$$E_o = K_\rho\, \theta_1 + K_o\, \theta_1^{1.3}$$

from which value of θ_1 may be determined. In this case $K_\bullet$ is taken as zero. $\theta_1 = E_\bullet / K_\rho$

Holzer table can be used to compute undamped forced vibrations at any frequency by using two tables, one for sine components, and one for cosine components of impulses. At a natural frequency the amplitude of vibration along the system for a unit amplitude at some designed section are called the relative amplitude of natural vibration. Column 3 gives relative amplitude when $\Sigma I\, P^2\, \theta = 0$.

The solution for uniform shaft equivalent system may be carried out graphically by using Lewis polar diagram. In this case if calculations at resonance are made, the energy equation for equal impulses along the uniform shaft is the same as previously stated. Where θ_1 is the amplitude at the beginning of the system and $\bar{\theta}_s$ are the amplitudes on the relative amplitude curve at various cylinders. The values of $\bar{\theta}_s$ may be measured from the polar diagram. Lewis term $\Sigma\beta$ is the same as $\Sigma\theta_s$ given here. The empirical formula for the damping along steel shafts considered with distributed mass:

$$K = \frac{7\,0\,(d_2^{4.3} - d_1^{4.3})\,\theta_1^{2.3}}{10^{10}(d_2^{4} - d_1^{4})\,2.3} \int_{x_1}^{x_2} \bar{M}^{2.3}\,dx$$

X = variable length along the shaft

M = is the vibration twisting moment along the shaft corresponding to the relative amplitude curve

Solution for combination type equivalent system may be carried out by reduction method. This method reduces the masses from one end to the other or from both ends to some section in any manner convenient.

Solution for the combination type equivalent system.

Reduction method of calculation makes use of following idea.

FIGURE PAGE 21.

A and B are the concentrated masses.

C shaft without mass.

Then at a given frequency, the effect of A acting on B through the elastic shaft C can be obtained at B by adding a certain fraction of A to B. This sum can again be considered as the first mass, the effect of which can be reduced to the next concentrated mass. Elastic shafts are called steps of line up. Concentrated masses are placed between steps.

C = stiffness factor # in/ radian of shaft twisting moment M(#.in.) and deflection θ (radian)

M = $C\theta$

g = in./sec^2 = 32.2 ft./sec.^{2}

= 386.088 in./ sec.^{2}

G = shearing modulus of elasticity (#/ square inch)

J = inertia factor or weight polar moment of inertia (#.in.^{2})

J = sectional polar moment of inertia of cross section of a shaft (in.^{4})

J = weight polar moment of inertia per unit length of a step with mass (#. in.^{2}/in.)

K = number of typical step

= length of step (in.)

M = twisting moment (#. in.)

N = frequency of vibration in vib./sec.

P = frequency constant = $2\pi n$

$T(\phi n) = \frac{180}{\phi \pi n} \tan \phi n$

$T_1(\phi n) = \frac{\phi n \pi \tan \phi n}{180}$

X = length variable for step (in/)

α = constant in degrees applying to step with mass.

γ = constant in degress applying to step.

θ = amplitude of vibration in radians.

θ' = amplitude of vibration in degrees.

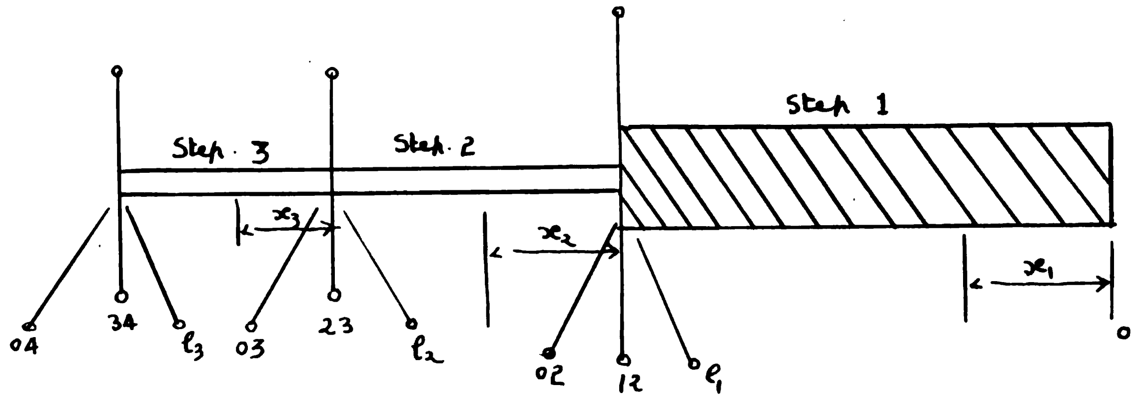
$$\phi = 360\sqrt{\frac{J}{gc}} = 18.3\sqrt{\frac{J}{C}}.$$

Single subject means that the symbol applies to the step having number of subscript. Subscript made of two figures means:

01, 12, 23, between steps whose numbers are included.

01, 02, 03, beginning of step whose number is given.

L1, L, L2, L3, end of the step whose number is given.



Thus $\theta_{\ell_1} = \theta_{o2} = \theta_{12}$

$\theta_{\ell k} = \theta_{o,k+1} = \theta_{k,k+1}$ (general)

$\theta_{\ell e} = \theta_{o,\ell+1} = \theta_{e,e+1}$ (e number of end step)

A prime mark (') means that the masses are being reduced from the beginning. ('') means masses are being reduced from end.

$J'^{1}_{o1} = J_{o1}$

$J'^{1}_{o2} = J'^{1}_{\ell} + J$

$J'^{1}_{o3} = J'^{1}_{\ell 2} + J_{23}$

- - - - - - - - - - - -

- - - - - - - - - - - -

$J'^{1}_{ok} = J'^{1}_{k} + J_{k,k+1}$

$J''_{\ell e} = J_{e,e+1}$

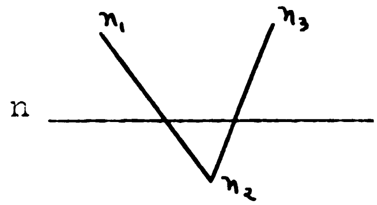
$J''_{\ell,e-1} = J''_{oe} + J_{e-1,e}$

(end of e-1) =)

- - - - - - - - -

$J''_{\ell,k-1} = J''_{ok} + J_{k-1,k}$

Interpolation formula for natural frequency n.



n_1 = assumed value of n.

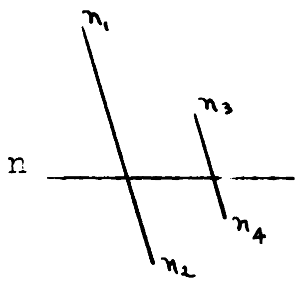
n_2 = first approximation.

n_3 = 2nd approximation.

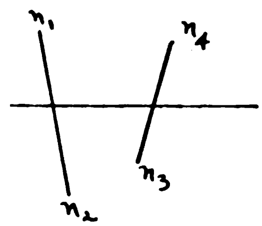
$$n = n_2 + \frac{\Delta_1 \Delta_2}{\Delta_1 + \Delta_2}$$

$\Delta_1 = n_1 - n_2$

$\Delta_2 = n_3 - n_2$



or



n_1 = first assumed value

n_2 - first approximation

n_3 = 2nd assumed

n_4 = 2nd approximation

$$n = n + \frac{(n_4 - n_2)\Delta_1}{\Delta_1 + \Delta_2}$$

$\Delta_1 = n_1 - n_2$

$\Delta_2 = n_4 - n_3$

Relative vibration twisting moments

(1) for steps without mass

$$M = \frac{c_K}{57.30} \left(\theta^{\circ}_{0,K+1} - \theta^{\circ}_{0,K} \right)$$

(2) with mass

$$M = -\left(\frac{\pi}{180}\right)^2 c\,\phi\, n\,\alpha \sin\left(\frac{\phi\, n\, x}{\ell} + \gamma\right)$$

Formulas for the reduction method of calculation:

(1) For reducing the inertia factors from the beginning toward the end

(a) For steps without mass:

$$J'_{eK} = \frac{J'_{oK} \; 1}{1 - \frac{J'_{oK}}{C_K} \frac{p^2}{g}}$$

(b) For steps with mass:

(i) If the first step has no concentrated mass at the beginning

$$J'_{e1} = \frac{180}{\pi} J_1 \frac{\tan \phi N}{\phi_1 \; n} = J_1 T (\phi n)$$

(ii) If first step has concentrated mass at the beginning or for any other step

$$\tan Y'_K = \frac{\phi_K n \; J'_{oK}}{\frac{180}{\pi} \; J_K}$$

$$J'_{eK} = \frac{180}{\pi} J_K \frac{\tan (\phi_K + Y'_K)}{\phi_K \; n}$$

(2) For reducing inertia factors from end toward the beginning:

(a) For steps with mass:

(i) If the last step has no concentrated mass at the end

$$J''_{oe} = -\frac{180}{\pi} J_e \frac{\tan \phi_e n}{\phi_e \; n} = J_e T (\phi_e n)$$

(ii) If the last step has a concentrated mass at the end or for any other step

$$\tan (\phi_K n + Y''_K) = -\frac{\phi_K n \; J''_{eK}}{\frac{180}{\pi} \; J_K}$$

$$J''_{oK} = -\frac{180}{\pi} J_K \frac{\tan Y''_K}{\phi_K n}$$

(b) For steps without mass:

$$J''_{oK} = J''_{\ell K} \frac{1}{1 - \frac{J''_{\ell K} p^2}{C_K g}}$$

(3) At natural frequency:

$$J'_{\ell, K-1} + J''_{oK} + J_{K-1,K} = 0,$$

or $$J'_{\ell K} + J_{o,K+1} + J_{K,K+1} = 0$$

which is the same as

$$J'_{oK} + J''_{oK} = 0, \quad J'_{\ell K} + J''_{\ell K} = 0,$$

or $$\frac{J'_{oK}}{J'_{\ell K}} = \frac{J''_{oK}}{J''_{\ell K}}$$

(a) For step without mass:

$$\frac{p^2}{g} = \ell_K \left(\frac{1}{J'_{oK}} + \frac{1}{J''_{\ell K}}\right)$$

(b) For steps with mass $Y'' - Y' = 0$

If the first step has mass and no concentrated mass at the beginning. $T_1 (\phi_1 n) = - \frac{J''_{e1}}{C_1} \cdot \frac{p^2}{g}$

(A) <u>Relative amplitude curve</u>:

(1) For steps without mass:

$$\frac{\theta_{\ell K}}{\theta_{oK}} = \frac{J'_{oK}}{J'_{\ell K}} = \frac{J''_{oK}}{J''_{\ell K}} = 1 - \frac{J'_{oK}}{C_K} \frac{p^2}{g}$$

A node occurs in the Kth step when $\frac{\theta_{\ell K}}{\theta_{oK}}$ is -ive.

(2) For steps with mass:

$$\dot{\theta}_K = \alpha_K \cos \left(\frac{\theta_K n}{\ell_K} x + Y_K\right)$$

where $\dot{\theta}_K = \alpha_K \cos Y_K$

$\dot{\theta}_{\ell K} = \alpha_K \cos (\phi_K n + Y_K)$

A node occurs in the Kth. step when

$$\frac{\phi_K n x}{\ell_K} + Y_K = 90^\circ,\ 270^\circ,\ ----$$

(B) Vibration twisting moment:

(1) For steps without mass:

$$M = \ell_K(\theta_{o,K+1} - \theta_{oK}) = \frac{\pi}{180} c_K (\theta^\circ_{o,K+1} - \theta^\circ_{oK})$$

(2) For steps with mass:

$$M = -\left(\frac{\pi}{180}\right)^2 c\ \phi\ n\alpha \sin\left(\frac{\phi\ n x}{\ell} + Y\right)$$

$$= M_o \sin\left(\frac{\phi n x}{\ell} + Y\right)$$

$$M_o = -\left(\frac{\pi}{180}\right)^2 c\ \phi\ n\alpha$$

(C) Vibration stress in any uniform circular portion of a step is:

$$S = \frac{16\ d_2 M}{\pi\,(d_2^4 - d_1^4)}$$

Where d_2 and d_1 are the outer and inner diameters of the section

(D) Peak amplitudes:

Energy input equation due to a number of equal impulses of different phases is given by

$m \cos(pt - \psi_s)$ acting at various points ξ_s from beginning of a step with masses is

$$E = \frac{\pi^2 m}{180} \theta^\circ_{o1} \bar{\alpha} \Sigma \beta$$

or if the impulses are unequal in magnitude

$$E = \frac{\pi^2 \theta^\circ_{o1}}{180} \bar{\alpha} \Sigma m \beta$$

where $\bar{\alpha}$ is the constant for relative amplitude curve for $\theta^\circ_{o1} = 1$

$$\Sigma\beta = \sqrt{A^2 + B^2}$$

$$A = \Sigma\vartheta_s \sin\psi_s$$

$$B = \Sigma\vartheta_s \cos\psi_s$$

$$\vartheta = \cos\left(\frac{\phi n \xi_s}{\ell} + Y\right)$$

The motion at the beginning of the system is $\dot{\theta}_{01} \cos(Pt - 90 - \psi)$

$$= \dot{\theta}_{01} \sin(Pt - \psi)$$

where $\psi = \tan^{-1} \frac{A}{B}$

Using 0, $\psi_2 - \psi_1$, $\psi_3 - \psi_1$ - - - - in place of ψ_1, ψ_2, ψ_3 - - the phase angles all refer to the instant at which the maximum value of the first impulse occurs.

In this case

$$\psi = \psi_1 + \tan^{-1} \frac{A}{B}$$

Where $A = \Sigma\vartheta_s \sin(\psi_s - \psi_1)$

and $B = \Sigma\vartheta_s \cos(\psi_s - \psi_1)$

If the impulses having the same magnitude but different phases occur in two different steps, U and V, both with mass,

$$E = \frac{\pi^2 m}{180} \dot{\theta}_{01} \Sigma\bar{\alpha}\beta$$

$$\Sigma\bar{\alpha}\beta = \sqrt{A^2 + B^2}$$

$$A = \bar{\alpha}_u \Sigma\vartheta_{us} \sin\psi_{us} + \bar{\alpha}_v \Sigma\vartheta_{vs} \sin\psi_{vs}$$

$$B = \bar{\alpha}_u \Sigma\vartheta_{us} \cos\psi_{us} + \bar{\alpha}_v \Sigma\vartheta_{vs} \cos\psi_{vs}$$

The energy loss due to marine propeller is

$$K = 2 \left(\frac{\pi}{180}\right)^2_{\rho} P \, (\bar{\dot{\theta}}_e)^2 \, (\dot{\theta}_{01})^2$$

Where $\bar{\dot{\theta}}_e$ is the relative amplitude at the propeller for $\dot{\theta}_{01} = 1$ and average value of ρ is

$$\rho = +35 \frac{Q}{\text{r.p.m.}}$$

The accuracy of the input energy calculations and undamped forced vibration calculations depends upon the accuracy of values of the stimulating impulses, including the magnitude and phase angles. The impulses are obtained by harmonic analysis from torque curves at the various cylinders. Lower harmonics can be obtained with fair accuracy. The higher probably have appreciable variations even from one combustion cycle to the next. Harmonic series known as simplest Fourier's series may be stated as:

$$f(X) = \frac{b_0}{2} + \sum_{r=1}^{p-1} b_r \cos r x + \frac{b_p}{2} \cos p x + \sum_{r=1}^{p-1} a_r \sin r x$$

$$a_r = \frac{1}{p} \sum_{s=1}^{2p-1} f(s\Delta) \sin r\Delta, \quad (r = 1,2,3,\text{--}\ p-1)$$

$$b_r = \frac{1}{p} \sum_{s=0}^{2p-1} f(s\Delta) \cos r\Delta, \quad (r = 0,1,2,\text{--}\ p)$$

P = half the number of equi. distant ordinates

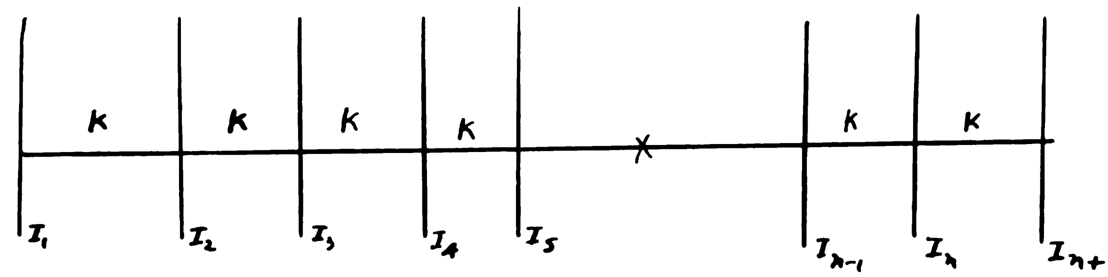
$\Delta = \pi / p$

r = Harmonic number.

s = ordinate number.

<u>Simplified method for torsional vibration calculation as used at the Chrysler Corporation</u>.

Torsional vibration system considered as a number of discs connected in series through torsional springs. All springs are equal and all discs end ones are equal.



$I_1 = I_2 = I_3 = \text{-------} \; I_N = I_0$

n springs are equal and each has stiffness K.
All discs have equal moment of inertia.

If the first disc is given a sinusoidal angular displacement, a torsional disturbance will travel along the system from left to right. It is legitimate to consider this disturbance as a wave of torsional deflection, which will travel back and forth through the system with reflection at both ends.

Assume that in progressing from one disc to the next, the change in phase of the wave is ϕ. When the wave arrives at I_{n+1}, its relative phase will be $n\phi$, if upon reflection of I_{n+1}, there is a change in phase of $2\phi_{n+1}$. Then when the wave arrives at I_1 the relative phase will be $n\phi + \phi_{n+1} + n\phi$.

If reflection at I_1 causes a phase change, $2\theta_1$, the relative phase after one complete transit of the system will be:

$$2n\phi + 2\theta_{n+1} + 2\theta_1$$

Now if relative phase is equal to zero or some multiple of 360°, a so called 'stationary' or 'standing' wave pattern will result. Under these conditions the system is vibrating at one of its natural frequencies and we can write at a natural frequency.

$$2\,n\,\phi + 2\,\theta_{n+1} + 2\theta_1 = 0 \text{ or multiple of } 360^\circ \quad \text{-- (A)}$$

$$n\,\phi + \theta_{n+1} + \theta_1 = 0 \text{ or multiple of } 180^\circ \quad \text{-- (B)}$$

and

$$\beta_x = \sin\left\{\theta_1 + (x - 1)\,\phi\right\}$$

β_x = relative amplitude of disc number x

Formulas defining ϕ, θ_1 and θ_{n+1} involve the inpedances of the system (either mechanical or electrical) and their form can be greatly simplified by introducing the parameter, F., which is defined as the ratio of the driving frequency to the frequency of one of the equal discs on one of the springs i.e.

$$F = \frac{f}{f_o}$$

F = frequency ratio

f = driving frequency

$$f_o = \frac{1}{2\pi}\sqrt{\frac{K}{I}}$$

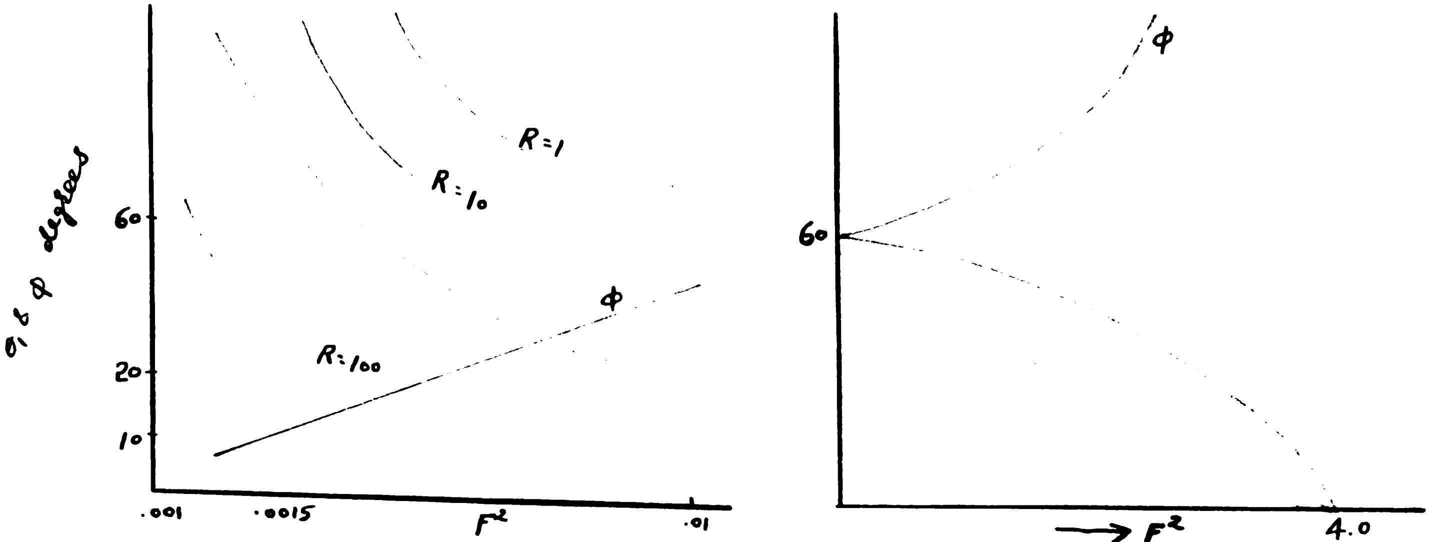
It can be shown that

$$\tan\theta_1 = \frac{-\sqrt{\frac{4}{F^2} - 1}}{2R_1 - 1}$$

$$\tan\theta_{n+1} = \frac{-\sqrt{\frac{4}{F^2} - 1}}{2\,R_{n+1} - 1}$$

$$\cos\phi = 1 - \frac{F^2}{2} \qquad R_1 = \frac{I_1}{I}$$

$$R_{n+1} = \frac{I_{n+1}}{I}$$



Figures show various values of R.

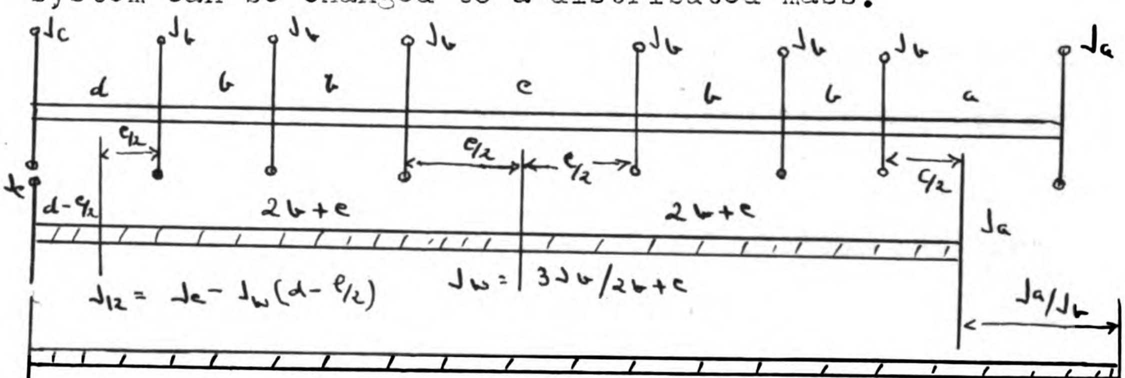
It may be seen that for values of R greater than 1/2 θ is negative. Only absolute values are given in the chart. Curves for ϕ all have + ive. slopes while the curve for θ, has a -ive. slope.

The problem of finding the natural frequencies for a given system involves guessing at a value of F^2, reading ϕ, θ_1, θ_{n+1}, from curves and checking these values in the equation (1). At the first natural frequency the right hand member of the equation (13) is zero; at the second frequency the right hand member is 180°; and at the third, 360° and so on. It is interesting to note that F^2 cannot be greater than 4, i.e. the highest natural frequency of the system is not more than $2f_0$.

CONCLUSION

The real difficulty in making accurate calculations for the amplitude of vibration at a natural frequency is due to the lack of knowledge and uncertainty of the damping forces. Various empirical formulas have been attacked in another way by the use of a term which K Wilson has called the "equilibrium amplitude".

Consider reduction in the arithmetical work of torsional vibration. Calculation for multi-cylinder engines results from the assumption that the crankshaft and masses attacked along its length are a uniform shaft with mass and elasticity. In a six cylinder engine, a concentrated mass system can be changed to a distributed mass.



$$L_1 = 4b + \frac{3c}{2} + a1 + \frac{J_a}{J_\omega}$$

$$J_1 = J_a + 6J_b + J_\omega(d - c/2)$$

Calculations for torsional vibration frequency of an engine and flywheel, arranged as a uniform shaft with mass and elasticity with a concentrated mass at its end are simple table for

$$T(\phi n) = \frac{180}{\pi \phi n} \tan \phi n$$ is used.

The writer prefers using the inertia factors in #. in^2., but this is not necessary. The value of ϕ is defined as

$$360\sqrt{\frac{I}{C}}\;g \qquad \text{with } J \ \#.\ \text{in.}^2 \text{ and}$$
$$C \ \#.\ \text{in./radian}$$

In solving more complicated systems, it often happens that some part of the system has a larger influence on one of the natural frequencies than any other part. But it is not difficult to recognize. For an example, in a marine installation, consisting of an engine, flywheel, propeller shafting, and propeller, it is easy to see that the first natural frequency is determined for the most part by the flywheel, propeller, and proper shafting as a simple two mass system. It can also be seen that the second natural frequency is determined for the most part by the flywheel and engine as a uniform heavy shaft with a concentrated mass at the end.

The reduction method of calculation has been developed to take advantage of this characteristic. The reduction method used on the same equivalent system requires no more work than the Holzer method. But when the reduction method is used on an equivalent system with a distributed mass for the moving parts of multi-cylinder engine, considerable saving of calculations results. The reduced inertia factor from the end to the first node is + ive. and is greater than the original value. The reduced inertia factor at a node is infinite. Just after passing a node, it is minus infinity and increases through zero to positive infinity when the next node is reached.

The variety of complicated techniques outlined above for calculating and testing torsional vibration problems indicates a fundamental reason for the many uncertainties and disagreements which have clouded the subject. Considerable strides have been made in test instrumentation, and there are a few well established measuring techniques. Calculation, however, still depends largely on the experience and discretion of the calculator. This is so because the more precise classical methods, by which it would be possible to collect and correlate the masses of data available, are extremely laborious.

Analogies are useful for analysis in unexplored fields. By means of analogies, an unfamiliar system may be compared with one that is better known. Although not generally so considered, the electrical circuit is the most common and widely exploited vibrating system. By means of analogies, the knowledge in electrical circuits may be applied to the solution of problems in mechanical and acoustical systems. In this procedure, the mechanical or acoustical vibrating system is converted into the analogous electric circuit. Any mechanical system can be reduced to an electrical network, and the problem may be solved by electrical circuit theory.

$$r_R = \frac{T}{\omega} = \left(\frac{F_R}{\theta} \right)$$

T = applied torque.

ω = angular velocity, radian/sec.

r_R = mechanical rotational resi.

Mechanical rectilineal internal energy:

$$f_m = m \frac{du}{dt}$$

$\frac{du}{dt}$ = acc. in cm./sec.^{2}

f_m = driving forcein dynes.

m = mass in grams.

Mechanical rotational internal energy associated with moment of inertia.

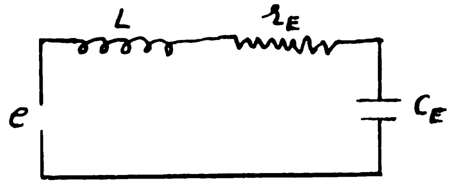
M. of I. given by $f_R = I \frac{d\theta}{dt}$

I = gm. (cm.)^2

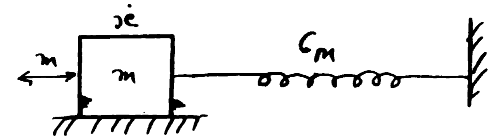
$\frac{d\theta}{dt}$ = angular acc. rad./sec.^{2}

f = torque in dynes. cm.

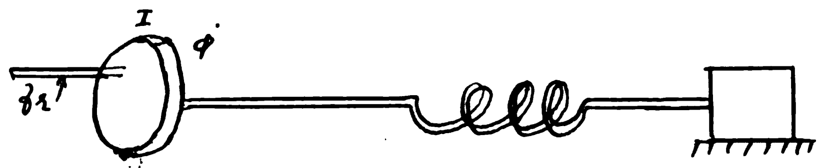
Friction, mass, and compliance govern the movements of physical bodies in the same manner that resistance, inductance, and capacitance govern the movement of electricity.



electrical



rectilineal



rotational mechanical

One degree of freedom.

Principal of the conservation of energy forms one of the basic theorems in most sciences.

K.E. in magnetic field

electrical

$$T_{K.E.} = \frac{1}{2} Li^2$$

K.E. stored in mass mechanical rectilineal

$$T_{K.M.} = \frac{1}{2} m \dot{x}^2$$

m = gms.

$\dot{x}$ = cm./sec.

K.E. stored in moment of inertia

$$T_{K.R.} = \frac{1}{2} I \phi^2$$

I = gm. (CM)^2

ϕ = radi./sec.

D'Alemberrt's Principal

Equation for rotational system

$$I \frac{d^2\phi}{dt^2} + r_R \frac{d\phi}{dt} + \frac{\phi}{C_R} = F_R e^{j\omega t}$$

$f_r = F_R e^{j\omega t}$ external applied force

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