ANALYSIS OF BOX CULVERTS

Thesis for the Degree of M. S.

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This is to certify that the

thesis entitled

ANALYSIS OF BOX CULVERTS

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ANALYSIS OF BOX CULVERTS

By

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CONTENTS

I. INTRODUCTION

A. Introductory Statement.

In the beginning of the twentieth century a new material of construction, reinforced concrete, came into existance. This material, being naturally continuous, demanded the study of Statically Indeterminate Structures. Hence, engineers were forced to learn more about the analysis and design of statically indeterminate structures. Accordingly various methods for the analysis of such structures have been developed. It is quite worthwhile to get well acquainted with these various methods. To achieve this purpose, box culvert which is one of the numerous statically indeterminate structures is taken as an example and various methods used to analyse it.

Out of numerous methods only six commonly used methods have been used to analyse this problem. These six methods are: 1. Moment Area Method. 2. Principle of Least Work Method (Castiglino's Theorem). 3. Virtual Work Method. 4. Slope-Deflection Method. 5. Moment Distribution Method. 6. Column Anology Method.

B. Purpose and Scope.

The purpose of this study is two-fold. First is to analyse the box culverts by various methods used in the analysis of the Statically Indeterminate Structures thereby getting well acquainted with methods used in this problem. End moments of the box culverts have been determined by six widely used methods. Second is to introduce design tables showing moments, shear and normal thrust at critical sections of the box culverts. These tables are prepared for designer or checker and the use of the tables herein will eliminate calculations, thereby saving time.

Study has been restricted up to two cells a-symmetrical rectangular box culverts. But by going through this it can be seen that the box culverts having three or more cells can be analysed without much trouble.

C. Acknowledgement.

The author wishes to express his sincere thanks to Dr. R. H. J. Pian for his invaluable advise and suggestions without which this work would have been quite impossible.

II. GENERAL CONSIDERATIONS

A. Use of Culverts.

Culvert is a traverse drain or waterway under a road, railroad, canal, or channel. In ordinary engineering usage, however, culvert refers to only short structures through roadway or railroad embankments serving as passageways for water and normally not acting under hydrostatic pressure. In this discussion culverts shall be refered as defined in the latter manner. Bridges were used to span openings of considerable size and importance while culverts were relegated to minor openings. This distinction no longer applies arbitrarily as it is recognized that the efficiency and low cost of culverts make them desirable for a wide range of conditions. From a structural standpoint, culverts are advantageous because of their continuity. Unexpected loads or other unusual conditions are better resisted by culverts as all component parts contribute helpful restrain.

B. Culvert Loads.

Live Loads. These include moving concentrated super loads as truck wheel loads, with or without impact. For design purposes, the maximum live load on culvert is taken as that produced by heavy truck. In accordance with the common practice, the standard truck train loading of the American Association of State Highway Officials can be adopted. The H-10 loading is used for heavily traveled secondary highways, and even for primary highways in most localities.

Pressures from wheel loads are assumed to be uniformly distributed on the culvert slabs as there is, in general, an intervening earth fill. Impact effect may be included in this uniformly distributed load.

Dead Loads. These include weight of embankment material on culvert, weight of the culvert and of contained water, lateral pressure on sides of the culvert. The total vertical load per lineal foot on a projecting culvert due to an embankment may be determined by the formula,

$$P = Cc.w(Bc)^2$$

In which,

w = Weight of fill material per cubic foot.

Bc = Outside width of the culvert in feet.

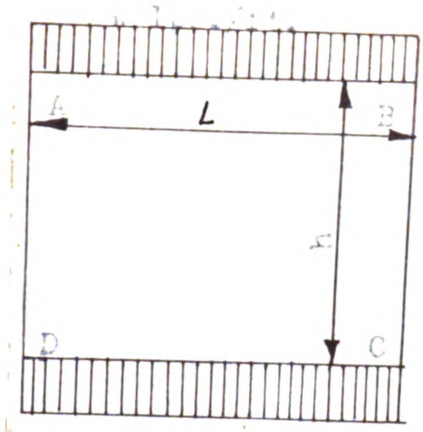
Cc = Coefficient depending on the depth of fill above culvert; width Bc; projection of the culvert above natural surface, and the character of the fill material.

Weight of the empty culvert can be

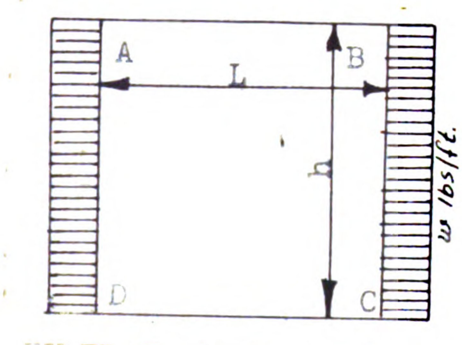
calculated very easily by assuming section.

Lateral pressure. Lateral pressure in the embankments are of two distinct types: active pressure and passive pressure. Active pressure on culverts is caused by the action of embankment in attempting to assume its natural condition of repose--that is its natural slope or angle of repose. Passive pressure is induced by the movement of structure against the supporting material. Its magnitude is a function of the amount of movement and also of the soil characteristics. Because of its indeterminate nature and changeable, uncertain magnitude, passive pressure is rarely depended upon to relieve stresses from more positive causes. Active pressure is direct function of vertical pressure, however, and may be estimated by fair accuracy (Rankine's Theory can be used). Active lateral pressures are assumed to be symmetrical on both sides of a culvert as care in back filling will permit large unbalanced pressures. Lateral pressure is computed in terms of equivalent fluid pressure. Its intensity depends on several factors, but is usually taken as one-third (angle of repose of the material 30 degrees) the vertical pressure at the point because it is more precise and safer usually. The load diagram is trapazoidal in shape and can be separated into uniform and triangular loading diagrams.

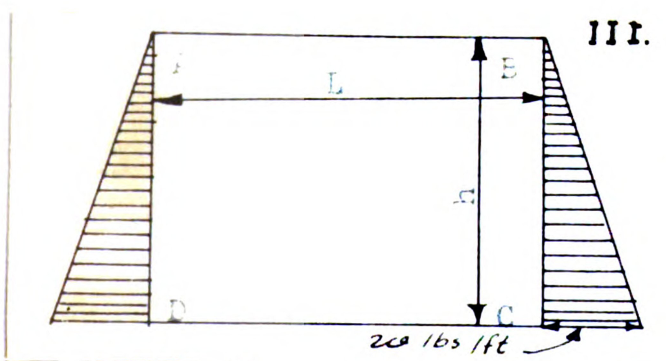
Above loading conditions can be classified as follows:



I. Uniform vertical load-total embankment and truck load in pounds per foot length of culvert.



II. Uniform lateral load (symmetrical on both sides) equal to lateral pressure at top of the culvert in pounds per square foot.



III. Triangular lateral load (symmetrical on both sides) equal to equivalent lateral unit pressure in pounds per square foot.

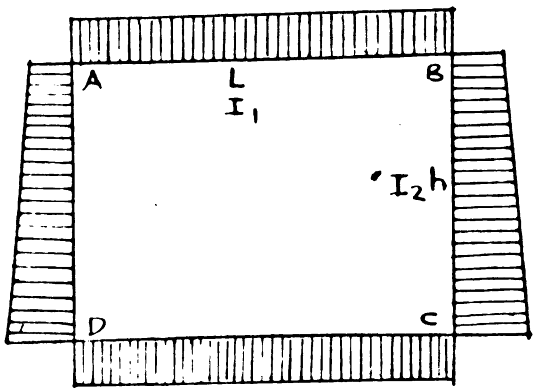
A combination of II and III will give any desired trapazoidal type of lateral loading.

III. ANALYSIS OF BOX CULVERT BY DIFFERENT METHODS

A. Virtual Work Method

(a) Rectangular Box Culvert with single cell:

Dimensions, moment of inertias of the component parts and loading are as shown in Fig. 1a.



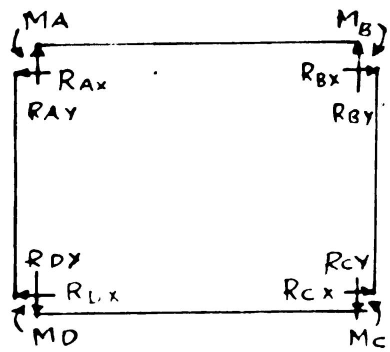


Fig. 1a.

Due to symmetrical loading

$M_A = M_B$ and $M_C = M_D$

also from Fig. 1b.

$R_{Ax}h = \overline{W}h/3 + wh^2/2 + M_D - M_A$ and $R_{Ay} = R_{By} = wL/2$

Now relative change in slope at each corner will be zero,

therefore, $O_A = M.m.ds/EI = 0$

which when simplified turns out to be as,

$M_A + M_C = (\overline{W}h^2I_1 - wh^3I_1 - wL^3I_2)/6(LI_2 + hI_1)$ . . . . (1)

Because horizontal deflection at any corner is zero,

$$Ah = M.m.ds/EI = 0$$

which can be easily simplified as,

$M_c(2hI_1 - 3LI_2) + M_AhI_1 = 4\overline{W}h^2I_1/15 + wL^3I_2/4 + wh^3I_1/4$

. . . . (2)

Solving equations (1) and (2) simultaneously M_A and M_C can be determined.

If instead of rectangular box culvert there be a square one, then by putting $L = h$ and $I_1 = I_2$ two modified equations can be obtained which when solved will give the corner moments.

(b) Rectangular Box Culvert with two cells:

Dimensions, moment of inertias of the component parts of the culvert and loading are as shown in the Fig. 3a.

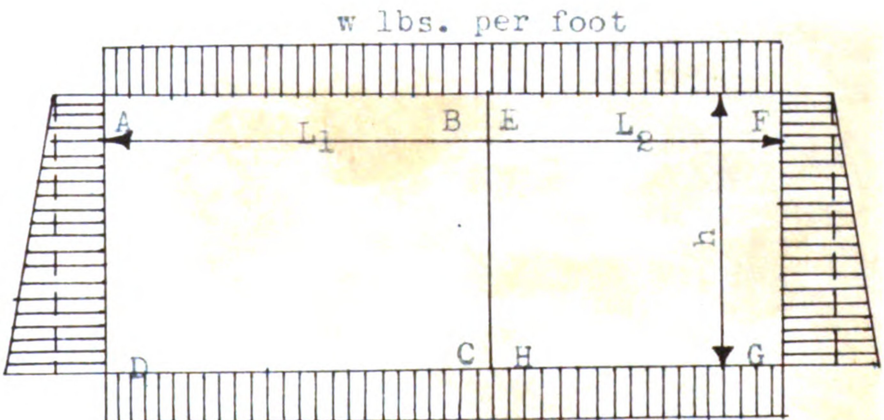


Fig. 3a.

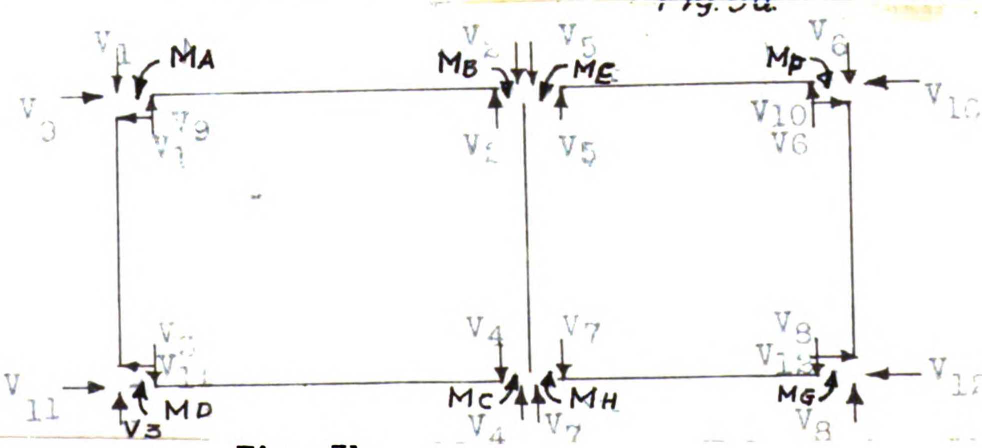


Fig. 3b

It can be seen from the Fig. 3b that,

$$V_1 = V_3; \quad V_6 = V_8; \quad V_9 = V_{10}; \quad V_{11} = V_{12}$$

$$V_2 + V_5 = V_4 + V_7$$

Now,

$$V_1 L_1 = M_A + w.L_1^2/2 - M_B$$

$$V_3 L_1 = M_D + w.L_1^2/2 - M_C$$

Therefore,

$$M_A - M_D + M_C - M_B = 0 \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (1)$$

Also,

$$V_5L_2 = M_E + w.L_2^2/2 - M_F$$

$$V_7L_2 = M_H + w.L_2^2/2 - M_G$$

And,

$$V_2L_1 = M_B + w.L_1^2/2 - M_A$$

$$V_4L_1 = M_C + w.L_1^2/2 - M_D$$

Therefore,

$$L_2(M_B + M_C - M_A - M_D) = L_1(M_E + M_H - M_F - M_G)$$

. . . . (2)

Also,

$$V_9h = M_A + w.h^2/2 + \overline{W}h/3 - M_D$$

$$V_{10}h = M_F + w.h^2/2 + \overline{W}h/3 - M_G$$

Now there are eight unknowns and the six additional equations may be expressed in terms of elastic energy. Since the relative horizontal displacement of the point A, at the left of the beam AB, and the point A, at the top of the member AD. is zero, Therefore, (see Fig. 4a & 4b)

$$\Delta_{A_x} = \int_A^A M.m.ds/EI = 0$$

Therefore,

$$\int_0^h M.y.dy/EI_2 + \int_0^{L_1} h.M.dy/EI_1 + \int_0^h M.y.dy/EI_2 = 0$$

Therefore,

$$- \int_0^h M_B y.dy/EI_2 - (1/EI_1)\int_0^{L_1}(M_C h - wx^2h/2 + V_4 xh)dx$$
$$- (1/EI_2)\int_0^h (M_A y - w.h^2y/2 - W.y^4/3h + V_9.y^2)dy = 0$$

Which when simplified turns out to be,

$$(h^2/2I_2)(M_A + M_B) + M_C hL_1/I_1 + V_4 L_1^2 h/2I_1 + V_9 h^3/3I_2$$
$$- w.L_1^3 h/6I_1 - w.h^4/4I_2 - Wh^3/15I_2 = 0$$

. . . . (3)

Now the relative vertical displacement of the point A, will be zero,

Therefore from Fig. 5a and 5b.

$$\Delta_{A_y} = \int_A^A M.m.ds/EI = 0$$

Apply a unit vertical load at Point A,

Therefore,

$$\int_A^B Mx.ds/EI_1 + \int_B^C M.L_1.ds/EI_2 + \int_C^D M.x.ds/EI_1 = 0$$

Therefore,

$$M_A L_1^2/2I_1 - w.L_1^4/8I_1 + V_1.L_1^3/3I_1 + M_E L_1 h/I_2$$
$$- w.L_1^4/8I_1 + M_C L_1^2/2I_1 + V_4 L_1^3/3I_1 = 0$$

Therefore,

$$M_A L_1^2/2I_1 + M_C L_1^2/2I_1 + M_E L_1 h/I_2 + V_4 L_1^3/3I_1$$
$$+ V_1 L_1^3/3I_1 = w.L_1^4/4I_1$$

. . . . (4)

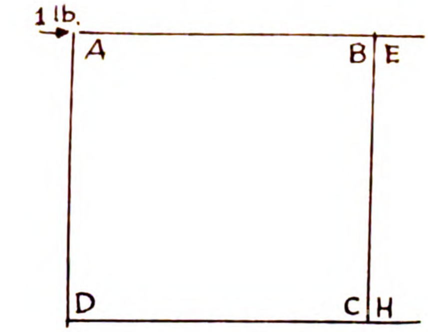


Fig. 4 a.

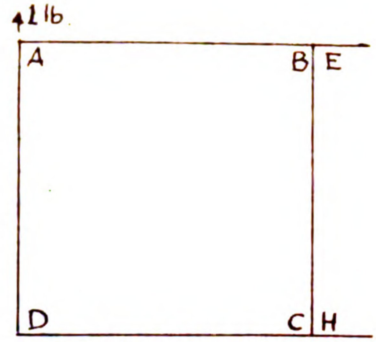


Fig. 5 a.

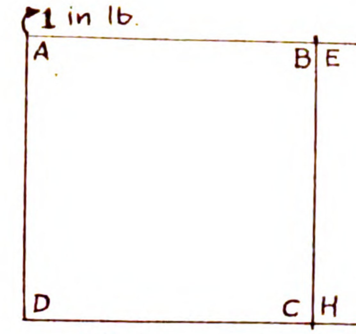


Fig. 6 a.

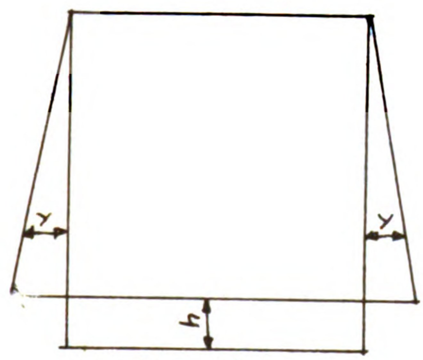


Fig. 4 b.

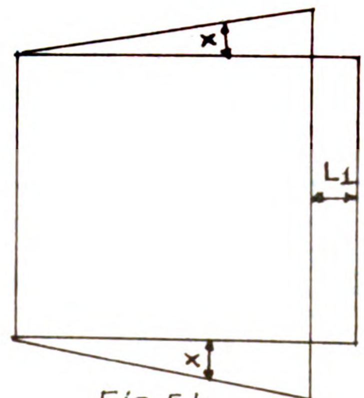


Fig. 5 b.

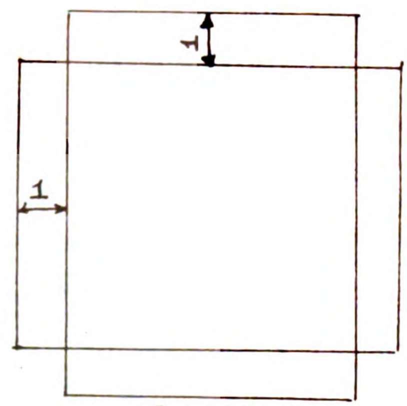


Fig. 6 b.

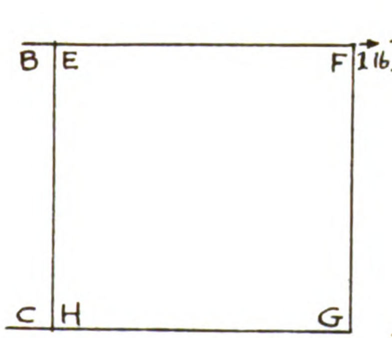


Fig. 7 a.

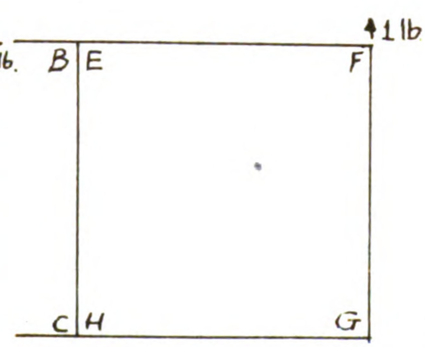


Fig 8 a.

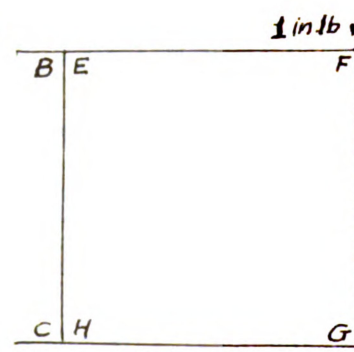


Fig. 9 a.

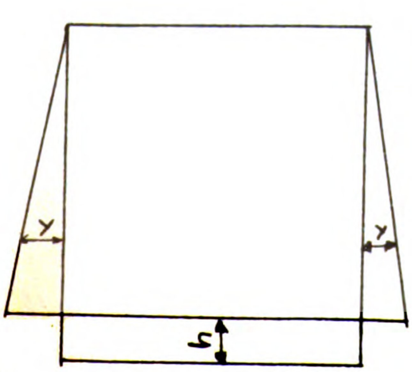


Fig 7b.

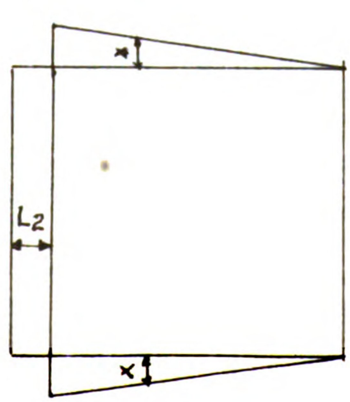


Fig. 8 b.

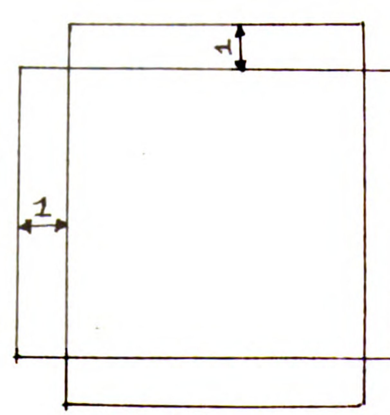


Fig. 9 b.

Now the relative change in the slope at point A, is zero.

Therefore,

$$\theta_A = \int_A^A M.m.ds/EI = 0$$

Apply a unit moment at point A, from Fig. 6a and 6b

Therefore,

$$M.ds/EI_1 + M.ds/EI_2 + M.ds/EI_1 + M.ds/EI_2 = 0$$

Therefore,

$$-\int M_A L_1^3/I_1 + w.L_1^2/6I_1 + V_1 L_1^2/2I_1 + M_B h/I_2 + M_C L_1/I_1$$
$$+ w.L_1^3/6I_1 - V_4 L_1^2/2I_2 = 0$$

Therefore,

$$(L_1/I_1)(M_A + M_C) + M_B h/I_2 + (L_1^2/2I_1)(V_1 + V_4)$$
$$= w.L_1^3/3I_1 \quad \ldots\ldots\ldots\ldots \quad (5)$$

Since the relative horizontal and vertical displacement and relative change in slope of the point F, at the right of the beam EF, and the point F, at the top of the member GF, are each separately equal to zero, therefore, following three equations can be obtained.

$$\Delta_{F_x} = \int_F^F M.m.ds/EI = 0$$

$$\int_F^G M.y.ds/EI_2 + h \int_G^H M.ds/EI_3 + \int_H^E M.y.ds/EI_2 = 0$$

Therefore, from Fig. 7a and 7b.

$$- M_F h^2/2I_2 + w.h^4/8I_2 + W.h^3/15I_2 - V_{10}h^3/3I_2$$

$$- M_G L_2 h/I_2 + wL_2^3 h/6I_3 - V_8 L_2^2 h/2I_3 - M_E h^2/2I_2 = 0$$

Therefore,

$$(h^2/2I_2)(M_F + M_E) + M_G L_2 h/I_3 + V_{10}h^3/3I_2 + V_8 L_2^2 h/2I_3$$

$$= w.h^4/8I_2 + w.L_2^3 h/6I_3 + \overline{W}h^3/15I_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

Now,

$$\Delta_{F_y} = \int_F^F M.m.ds/EI = 0$$

Therefore, from Fig. 8a and 8b.

$$\int_G^H M.x.dx/EI_1 + L_2 \int_H^E M.ds/EI_2 + \int_E^F M.x.dx/EI_1 = 0$$

Therefore,

$$- M_G L_2^2/2I_3 + w.L_2^4/8I_3 - V_8 L_2^3/3I_3 - M_E L_2 h/I_2$$

$$+ w.L_2^4/8I_3 - M_E L_2^2/2I_2 - V_5 L_2^2/3I_3 = 0$$

Therefore,

$$(L_2^2/2I_3)(M_G + M_E) + M_E L_2 h/I_2 + (L_2^3/3I_3((V_5 - V_8)$$

$$= w.L_2^4/4I_3 \quad \ldots\ldots\ldots\ldots\ldots (7)$$

Now,

$$\theta_F = \int_F^F M.m.ds/EI - 0$$

Therefore, from Fig. 9a and 9b.

$$\int_E^F M.ds/EI_1 + \int_F^G M.ds/EI_2 + \int_G^H M.ds/EI_1 + \int_H^E M.ds/EI_2 = 0$$

Therefore,

$$- M_E L_2/I_3 + w.L_2^3/6I_3 - V_5 L_2^2/2I_3 - M_F h/I_2 + w.h^3/6I_2$$
$$+ \bar{W}h^2/12I_2 - V_{10}h^2/2I_2 - M_G L_2/I_3 + w.L_2^3/6I_3 - V_8 L_2^2/2I_3$$
$$- M_E h/I_2 = 0$$

Therefore,

$$(L_2/I_3)(M_G + M_E) + (h/I_2)(M_F + M_E) + (L_2^2/2I_3)(V_5 + V_8)$$
$$+ V_{10}h^2/2I_2 = w.L_2^3/3I_3 + w.h^3/6I_2 + \bar{W}h^2/12I_2$$

. . . . (8)

By solving the above eight equations simultaneously, bending moment at each corner can be determined.

In this problem loading is general one and if any of these loading is missing then putting for that particular loading equal to zero new modified eight equations can be obtained from the above equations, and when these new equations will be solved simultaneously they will give moment at each corner of the culvert for that particular type of loading.

B. Moment Area Method

(a) Rectangular Box Culvert with single cell:

Dimensions, moment of inertias of component parts and loading are as shown in Fig. 1a.

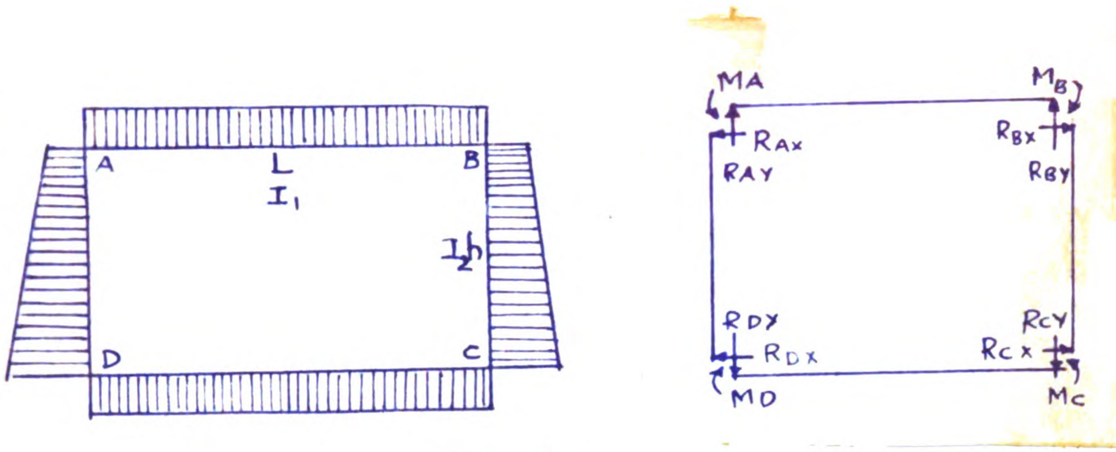


Fig. 1a.

Due to symmetrical loading, $M_A = M_B$ and $M_C = M_D$.

Now relative change in slope at any corner will be zero, therefore, the sum of the area under M/EI curve will be zero, which gives

$$M_A + M_C = (\overline{W}h^2 I_1 + wL^3 I_2 + wh^3 I_1)/6(LI_2 + hI_1) \quad . . . (1)$$

Also relative deflection at any corner is zero, therefore, statical moment of the area under M/EI curve about the corner will be zero.

This will give

$$M_C(3LI_2 + 2hI_1) + M_A hI_1 = 4\overline{W}h^2 I_1/15 + wL^3 I_2/4 + wh^3 I_1/4 \quad . . . (2)$$

Solving equations (1) and (2) simultaneously M_A and M_C can be determined.

(b) Rectangular Box Culvert with Two Cells:

Dimen$ions, moment of inertias of the components parts of the culvert and loading are as shown in the Fig. 3a

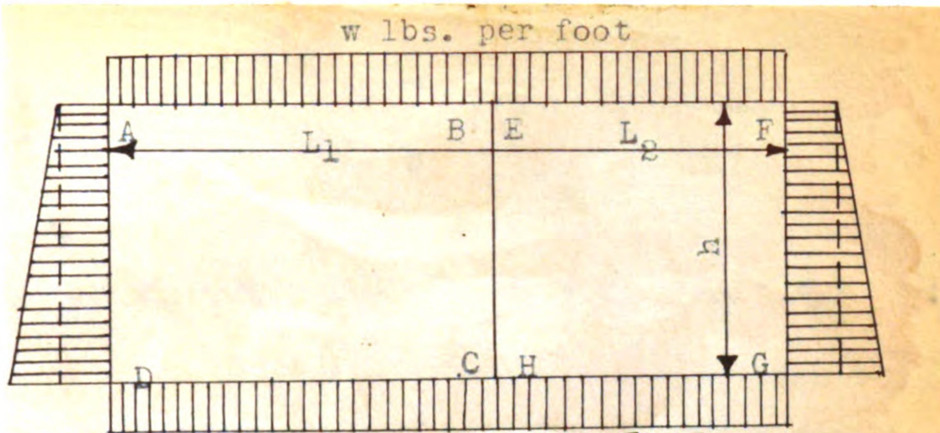


Fig. 3a.

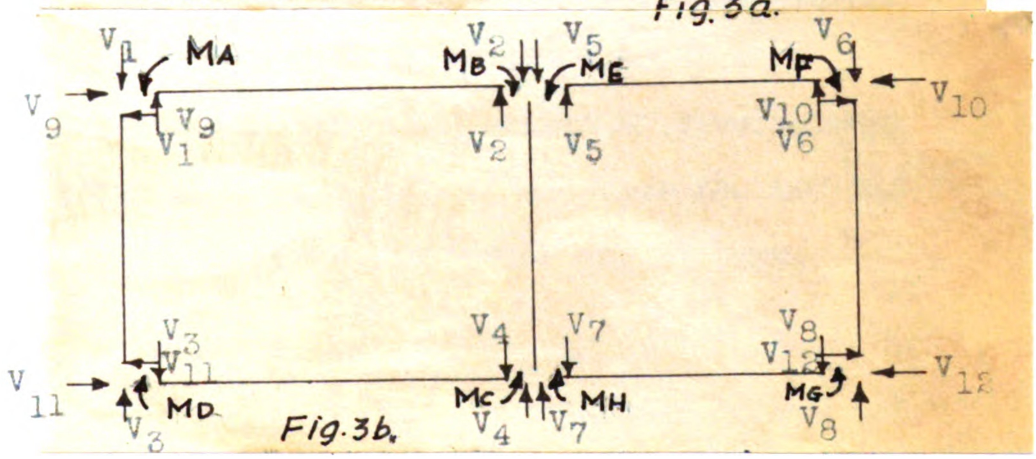


Fig. 3b.

It can be seen from the Fig. 3b

that,

$$V_9 = V_{10}; \quad V_{11} = V_{12}$$

$$V_1 = V_3; \quad V_6 = V_8; \quad V_2 + V_5 = V_4 + V_7 .$$

Now,

$$V_1 L_1 = M_A + w.L_1^2/2 - M_B$$

$$V_3 L_1 = M_D + w.L_1^2/2 - M_C$$

Therefore,

$$M_A - M_D + M_C - M_B = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

Also,

$$V_5L_2 = M_E + w.L_2^2/2 - M_F$$

$$V_7L_2 = M_H + w.L_2^2/2 - M_G$$

And,

$$V_2L_1 = M_B + w.L_1^2/2 - M_A$$

$$V_4L_1 = M_C + w.L_1^2/2 - M_D$$

Therefore,

$$L_2(M_B + M_C - M_A - M_D) = L_1(M_E + M_H - M_F - M_G)$$

. . . . (2)

Now there are eight unknowns and the six additional equations may be expressed in terms of elastic energy. Since the relative horizontal displacement of the point A, at the left of the beam AB, and the point A, at the top of the member AD, is zero,

therefore, $\Delta_{Ax} = \int_A^A m_1.M.ds/F_1.EI = 0$

(See Figures 4a and 4b)

Therefore,

$$\int_B^C F_1.y.M.dy/E.I_2.F_1 + \int_C^D h.F_1.M.dy/F_1.EI_1 + \int_D^A F_1.yM.dy/F_1EI_2 = 0$$

$$\left.\text{Area}.\bar{y}/EI_2\right]_B^C + \left.h(\text{Area}/EI_1)\right]_C^D + \left.\text{Area}.\bar{y}/EI_2\right]_D^A = 0$$

$$\frac{1}{I_2}\left[M_H(h/2)(2h/3) + M_E(h/2)(h/3) + M_B(h/3)(h/3) + M_C(h/2)(2h/3)\right]$$

$$(h/I_1)\left[-(M_D + M_C)L_1/2 + w.L_1^3/12\right]$$

$$- (1/I_2)(M_D.h^2/3 + M_A.h^2/6) + w.h_3^3.h/12.I_2.2 + Wh^2.8h/12.15I_2$$

$$= 0$$

Therefore,

$$M_H.h^2/3I_2 + M_E.h^2/6I_2 - M_B.h^2/6I_2 - M_E.h^2/3I_2$$
$$- M_C.hL_1/2I_1 - M_D.hL_1/2I_1 - M_D.h^2/3I_2 - M_A.h^2/6I_2$$
$$+ w.L_1^3.h/12I_2 + w.h^4/24I_2 + 2\bar{W}.h^3/45I_2 = 0$$
$$\ldots\ldots\ldots\ldots (3)$$

From Figures 5a and 5b.

$$\Delta_{Ay} = \int_A^A m_2.M.ds/F_2.EI = 0$$

which is equal to,

$$\text{Area}.\ \bar{x}/I_1\Big]_A^B + (L_1/I_2)\ \text{Area}\Big]_B^C + \text{Area}.\bar{x}/I_2\Big]_C^D = 0$$

which when simplified turns out into,

$$M_E.h.L_1/2I_2 + M_H.h.L_1/2I_2 - M_B.h.L_1/2I_2 - M_C.h.L_1/2I_2$$
$$- M_A.L_1^2/6I_1 - M_B.L_1^2/3I_1 - M_C.L_1^2/6I_1 - M_D.L_1^2/3I_1$$
$$+ w.L_1^3/12I_1 = 0$$
$$\ldots\ldots\ldots\ldots (4)$$

From figures 6a and 6b.

$$\theta_A = \int_A^A m.M.ds/M.EI = 0$$

That is sum of the area of moment diagram divided by I is zero.

Which will turn out as follows,

$$M_A(hI_1 - L_1I_2)/2I_1I_2 - M_B(L_1I_2 + hI_1)/2I_1I_2$$
$$M_D(hI_1 - L_1I_2)/2I_1I_2 - M_C(L_1I_2 + hI_1)/2I_1I_2$$
$$(L_1h/2I_2)(M_E + M_H) + w.L_1^3/6I_1 + w.h^3/12I_2 + \bar{W}h^2/12I_2 = 0$$
$$\ldots\ldots\ldots\ldots (5)$$

From the Figures 7a and 7b.

$$\Delta_{Fx} = \int_F^F m_3.M.da/F_3.EI = 0$$

Therefore,

$$Area.\bar{y}/I_2\Big]_F^G + (h/I_3)\ Area\Big]_G^H + Area.\bar{y}/I_2\Big]_H^E = 0$$

Which when simplified turns out to be,

$$(h^2/6I_2)(2M_B + M_C - 2M_E - M_F + 2M_G - M_H)$$

$$- (L_2h/2I_3)(M_G + M_H) + w.L_2^3h/12I_1 + w.h^4/12I_2$$

$$+ 2\bar{W}h^3/45I_2 = 0$$

$$\ldots\ldots\ldots\ldots (6)$$

From the Figures 8a and 8b.

$$\Delta_{Fy} = \int_F^F m_4.M.ds/F_4.EI = 0$$

Therefore,

$$Area.\bar{x}/I_3\Big]_G^H + (L_2/I_2)Area\Big]_H^E + Area.\bar{x}/I_3\Big]_E^F = 0$$

Which when simplified turns out to be,

$$(L_2.h/2I_2)(M_B + M_C - M_E - M_H) + w.L_2^4/12I_3$$

$$- (L_2^2/6I_3)(M_E + 2M_F + M_G + 2M_H) = 0 \ldots\ldots\ldots (7)$$

From the Figures 9a and 9b.

$$\theta_F = \int_F^F m_6.M.ds/M.EI = 0$$

That is area of moment diagram divided by I is equal to zero.

Which will turn out as follows,

$$(h/2I_2)(M_B + M_C - M_E - M_F - M_G - M_H) + \bar{W}h^2/12I_2$$
$$- (L_2 12I_3)(M_E + M_F + M_G - M_H) + w.h^3/12I_2 + w.L_2^3/6I_3 = 0$$
$$\quad . . . . (8)$$

By solving the above eight equations simultaneously bending moments at each corner can be determined.

In this problem general loading is taken but if any of these loadings is missing then putting for that particular loading equal to zero new modified eight equations can be obtained from these above eight equations, which when solved simultaneously will give bending moments at each corner of the culvert for that particular type of loading.

C. Principle of Least Work.

(a) Rectangular Box Culvert with Single Cell:

Dimensions, moment of inertias and load are as shown in the Fig. 1.

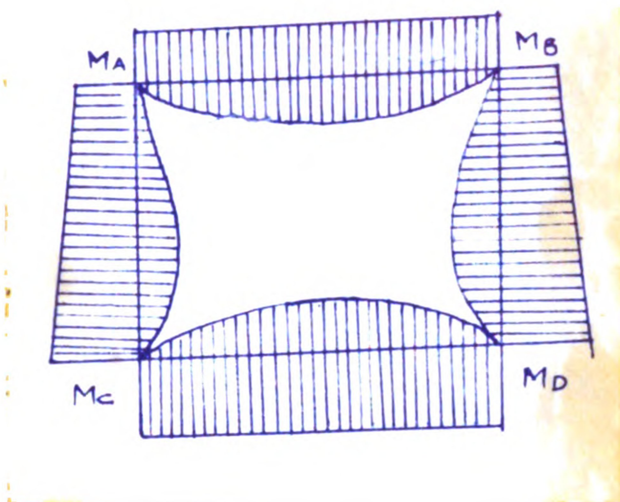


Fig. 1.

Total internal work in a structure are given by

$U = (As.\ \overline{Y}s + Ai.\ \overline{Y}i + As.\overline{Z})/EI$

The constant term $As.\ \overline{Y}s./EI$ will drop out at the time of differentiation and, hence, can be dropped from the very beginning due to symmetrical loading $M_A = M_B$ and $Mc = MD$.

Therefore, $U = Ai\ Yi/EI + As\overline{Z}/EI$

Now,

$$\frac{A_i\bar{Y}_i}{I} = \frac{M_A^2 L}{2I_1} + \frac{M_c^2 L}{2I_2} + \frac{M_A^2 h}{I_2} + (M_A M_c - 2M_A + M_c^2)\frac{h}{3I_2}$$

$$A_s\bar{Z} = \frac{wL^3}{6I_1} M_A + \frac{Wh^3}{12I_2} (M_A + M_c) + \frac{\bar{W}h^2(7M_A + 8M_c)}{90I_2}$$

$\frac{OU}{OM_A} = 0$ which when solved will be,

$$M_A(3LI_2 + 2hI_1) + M_c hI_1 - \frac{wh^3}{4}I_2 - \frac{wh^3}{4}I_1$$

$$- 7\frac{\bar{W}h^2}{30}I_2 = 0 \ldots\ldots\ldots\ldots (1)$$

Also,

$\frac{dU}{dM_c} = 0$, which when solved will be,

$$M_c(3LI_2 + 2hI_1) + M_A hI_1 - \frac{wL^3}{4}I_2 - \frac{wh^3}{4}I_1$$

$$- 8\frac{\bar{W}h^2}{30}I_1 = 0 \ldots\ldots\ldots\ldots (2)$$

By solving these two equations MA and Mc can be determined.

If the culvert is square instead of rectangular, then replacing h by L and I_2 by I_1 , moment at each corner can be very easily determined.

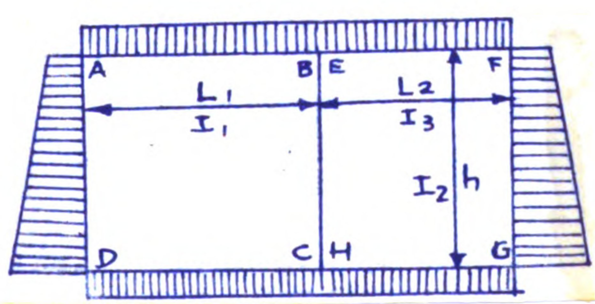


Fig. 2.

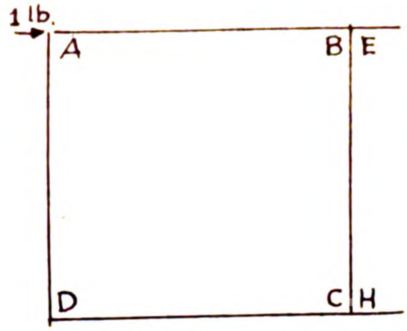


Fig. 4 a.

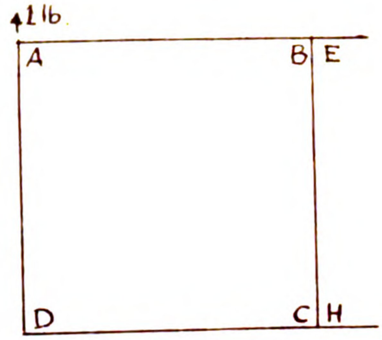


Fig. 5 a.

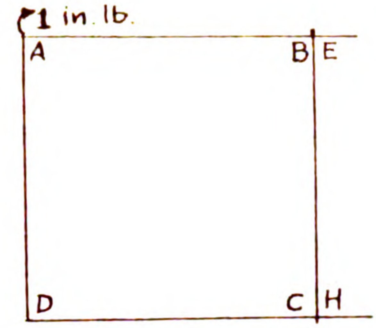


Fig. 6 a.

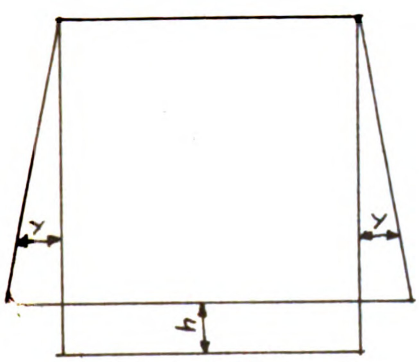


Fig 4 b.

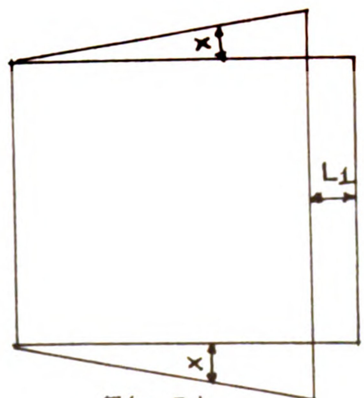


Fig. 5 b.

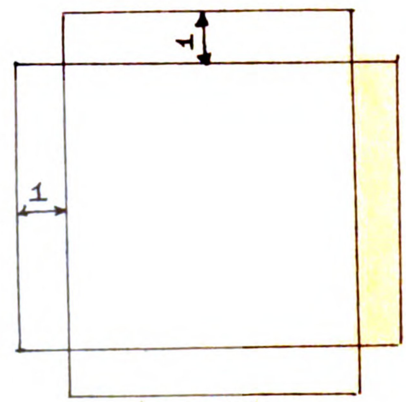


Fig. 6 b.

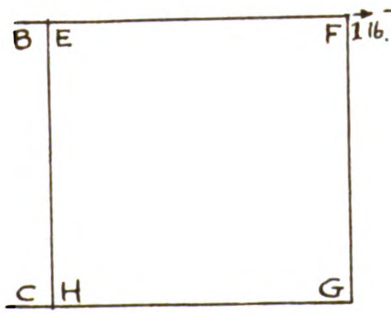


Fig. 7 a.

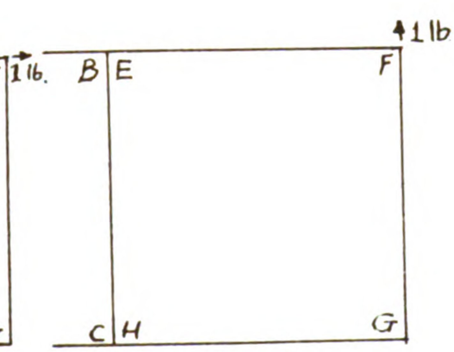


Fig. 8 a.

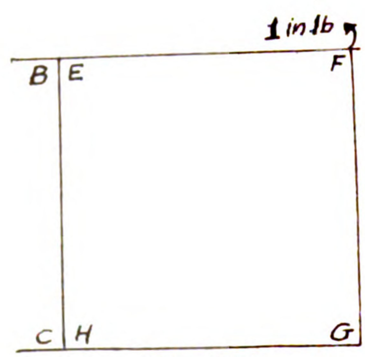


Fig. 9 a.

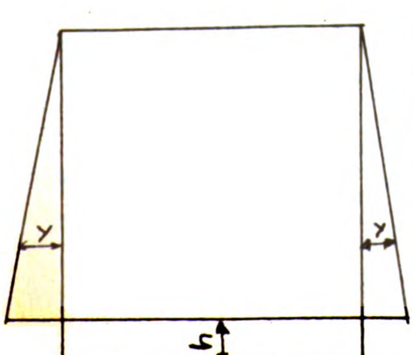


Fig. 7b.

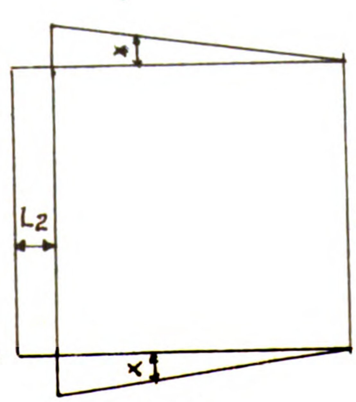


Fig. 8b.

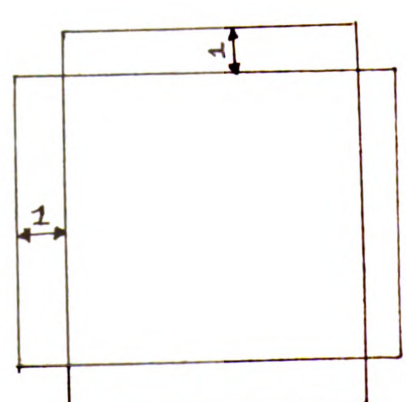


Fig. 9b.

(b) Rectangular Box Culverts with Two Cells:

Case 1: Dimensions and sections and loading are as shown in the Fig. 2.

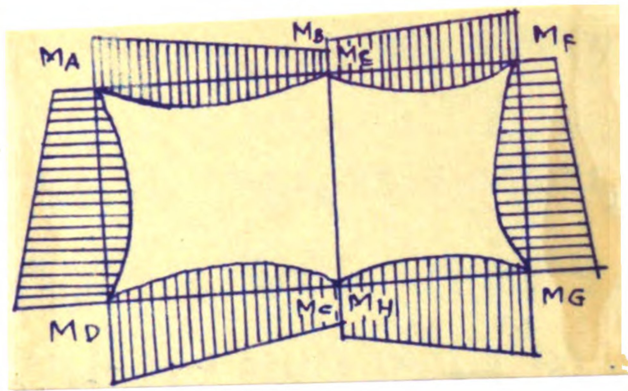


Fig. 2a.

Total internal work in structure is given by $U = (As.\bar{y}_s + Ai.\bar{y}_1 + As.\bar{z})EI$ (See Appendix) Dropping the constant term $As.\bar{y}_1/EI$ and E, the total internal work then will be,

$$U = (Ai.\bar{y}_1/I - As.\bar{z}/I)$$

Now,

$$\begin{aligned} Ai.\bar{y}i/I = {} & M_B^2.L_2/2I_2 + (L_2/6I_2)(M_A M_B + M_A^2 - 2M_B^2) \\ & + M_E^2 L_1/2I_1 + (L_1/6I_1)(M_F M_E + M_F^2 - 2M_E^2) \\ & + M_H^2 L_1/2I_1 + (L_1/6I_1)(M_H M_G + M_G^2 - 2M_H^2 \\ & + M_C^2 L_2/2I_2 + (L_2/6I_2)(M_C M_D + M_D - 2M_C) \end{aligned}$$

$$+ M_A^2 h/2I_3 + (h/6I_3)(M_A M_D + M_D^2 - 2M_A^2)$$

$$+ M_F^2 h/2I_3 + (h/6I_3)(M_F M_G + M_G^2 - 2M_F^2)$$

And,

$$As.\bar{z}/I = (w.L_2^3/12)(M_A + M_B)/2I_2 + w.h^3(M_A + M_D)/24I_3$$

$$+ (w.L_1^3/12)(M_F + M_E)/2I_1 + w.h^3(M_F + M_G)/24I_3$$

$$+ (w.L_2^3/12)(M_D + M_C)/2I_2 + \bar{W}h^2(7M_A + 8M_D)/180I_3$$

$$+ (w.L_1^3/12)(M_H + M_G)/2I_1 + \bar{W}h^2(7M_F + 8M_G)/180I_3$$

Therefore,

$$dU/dM_A = M_B L_2/6I_2 + 2M_A L_2/6I_2 + 2M_A h/2I_3$$

$$+ M_D h/6I_3 + 4M_A h/6I_3 + w.L_2^3/24I_2 = 0$$

$$= 2M_A(L_2 I_3 + hI_2) + (M_B L_2 + I_3)$$

$$+ M_D h I_2 - w.L_2^3.I_3/4 + \frac{w.h^3}{4}I_2 + \frac{7\bar{W}h^2}{30}I_2 = 0 \quad . \quad . \quad (1)$$

$$dU/dM_B = 2M_B L_2/2I_2 + M_A L_2/6I_2 - 4M_B L_2/6I_2$$

$$+ w.L_2^3/24I_2 = 0$$

$$= 2M_B + M_A + w.L_2^2/4 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

$$dU/dM_C = 2M_C L_2/2I_2 + M_D L_2/6I_2 - 4M_C L_2/6I_2$$

$$+ w.L_2^3/24I_2 = 0$$

$$= 2M_C + M_D + w.L_2^2/4 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

$$dU/dM_D = M_C L_2/6I_2 + 2M_D L_2/6I_2 + M_A h/6I_3$$

$$+ 2M_D h/6I_3 + w.L_2^3/24I_2 = 0$$

$$= 2M_D(L_2 I_3 + hI_2) + M_C L_2 I_3$$

$$+ M_A h I_2 + w.L_2^3 I_3/4 + \frac{w.h^3}{4}I_2 + \frac{8\bar{W}h^2}{30}I_2 = 0 \quad . \quad . \quad . (4)$$

$$dU/dM_E = 2M_E L_1/2I_1 + M_F L_1/6I_1 - 4M_E L_1/6I_1$$

$$+ w.L_1^3/24I_1 = 0$$

$$= 2M_E + M_F + w.L_1^2/4 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

$$dU/dM_F = M_E L_1/6I_1 + 2M_F L_1/6I_1 + 2M_F h/2I_3$$

$$+ M_G h/6I_3 + 4M_F h/6I_3 + w.L_1^3/24I_1 = 0$$

$$= 2M_F(L_1 I_3 + hI_1) + M_E L_1 I_3$$

$$+ M_G hI_2 + w.L_1^3 I_3/4 + \frac{w.h^3}{4} I_2 + \frac{7\overline{W}h^2}{30} I_1$$

$$= 0 \; . \; . \; . \; . \; . \; (6)$$

$$dU/dM_G = M_H L_1/6I_1 + 2M_G L_1/6I_1 + M_F h/6I_3$$

$$+ 2M_G h/6I_3 + w.L_1^3/24I_1 = 0$$

$$= 2M_G(L_1 I_3 + hI_1) + M_H L_1 I_3$$

$$+ M_F hI_1 + w.L_1^3 I_3/4 + \frac{w.h^3}{4} I_1 + \frac{8\overline{W}h^2}{30} I_1$$

$$. \; . \; . \; . \; . \; (7)$$

$$dU/dH = 2M_H L_1/2I_1 + M_G L_1/6I_1 - 4M_H L_1/6I_1$$

$$+ w.L_1^3/24I_1 = 0$$

$$= 2M_H + M_G + w.L_1^2/4 = 0 \; . \; . \; . \; . \; . \; . \; (8)$$

By solving above eight equations simultaneously required bending moments can be obtained.

If the culvert is rectangular box culvert with two cells having two rectangles of equal size, then bending moments at corners can be very easily calculated by making L_2 equal to L_1 and I_2 equal to I_1 and then substituting these values in the above equations.

If instead, there are two square cells, the moments can be determined very easily by making L_2 equal to L_1 equal to h, and I_1 equal I_2 equal I_3, and substituting these values in the above equations and then solving them simultaneously.

D. Slope-Deflection Method.

Rectangular Box Culvert with single cell:

Dimensions, moment of inertias of the component parts of the culvert and loading are as shown in the Fig. 1.

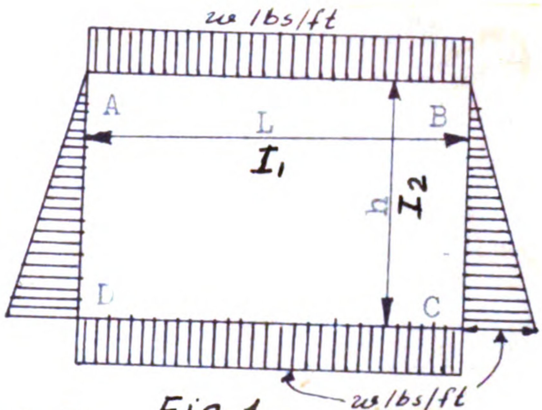


Fig. 1.

Fixed end moments can be easily determined and they are as follows with their proper signs.

$$M_{AB} = + w.L^2/12 \; ; \qquad M_{BA} = - w.L^2/12$$
$$M_{BC} = + \overline{W}.h/15 \; ; \qquad M_{CB} = - \overline{W}.h/10$$
$$M_{CD} = + w.L^2/12 \; ; \qquad M_{DC} = - w.L^2/12$$
$$M_{DA} = + \overline{W}.h/10 \; ; \qquad M_{AD} = - \overline{W}.h/15$$

K - values will be:

$$K_1 = I_1/L \text{ and } K_2 = I_2/h$$

Slope-deflection equations will be:

$$M_{AB} = -\ 2.E.K_1(2\theta_A + \theta_B) + w.L^2/12.$$
$$M_{BA} = -\ 2.E.K_1(2\theta_B + \theta_A) - w.L^2/12.$$
$$M_{BC} = -\ 2.E.K_2(2\theta_B + \theta_C) + \overline{W}.h/15.$$
$$M_{CB} = -\ 2.E.K_2(2\theta_C + \theta_B) - \overline{W}.h/10.$$
$$M_{CD} = -\ 2.E.K_1(2\theta_C + \theta_D) + w.L^2/12.$$
$$M_{DC} = -\ 2.E.K_1(2\theta_D + \theta_C) - w.L^2/12.$$
$$M_{DA} = -\ 2.E.K_2(2\theta_D + \theta_A) + \overline{W}.h/10.$$
$$M_{AD} = -\ 2.E.K_2(2\theta_A + \theta_D) - \overline{W}.h/15.$$

Now,

$$M_{AB} + M_{AD} = 0 \quad \ldots\ldots\ldots\ldots \quad (1)$$
$$M_{BA} + M_{BC} = 0 \quad \ldots\ldots\ldots\ldots \quad (2)$$
$$M_{CB} + M_{CD} = 0 \quad \ldots\ldots\ldots\ldots \quad (3)$$
$$M_{DA} + M_{DC} = 0 \quad \ldots\ldots\ldots\ldots \quad (4)$$

When the values from the slope-deflections equations will be substituted in the above four equations and when simplified, they will be as follows:

$$2E\left[2.\theta_A(K_1 + K_2) + K_1\theta_B + K_2\theta_D\right] = w.L^2/12 - \overline{W}.h/15. \qquad \ldots\ldots\ldots\ (1)$$

$$2E\left[2.\theta_B(K_1 + K_2) + K_1\theta_A + K_2\theta_C\right] = \overline{W}.h/15 - w.L^2/12. \qquad \ldots\ldots\ldots\ (2)$$

$$2E\left[2.\theta_C(K_1 + K_2) + K_1\theta_D + K_2\theta_B\right] = w.L^2/12 - \overline{W}.h/10. \qquad \ldots\ldots\ldots\ (3)$$

$$2E\left[2.\theta_D(K_1 + K_2) + K_1\theta_C + K_2\theta_A\right] = \overline{W}.h/10 - w.L^2/12 \qquad \ldots\ldots\ldots\ (4)$$

From these above four equations values of θ_A, θ_B, θ_C, θ_D, can be determined and from the values of the above quantities, moments at the corner of culvert can be calculated.

If there is uniform lateral load w pounds per foot height of the culvert instead of the above discussed loading, then replacing $\overline{W}.h/15$ and $\overline{W}.h/10$ each by $w.h^2/12$ with proper signs, and putting w, the uniform vertical load equal to zero, four revised equations can be obtained and from which end moments can be determined the same way in the above case.

If instead of a rectangular box culvert it is a square one, then by making h = L and $I_1 = I_2$ in the above four equations, unknown "Thetas" can be obtained and,hence, corner moments of the culvert.

Rectangular Box Culvert with two cells:

Dimensions, moment of inertias of the component parts of culvert and loading are as shown in the Fig. 2.

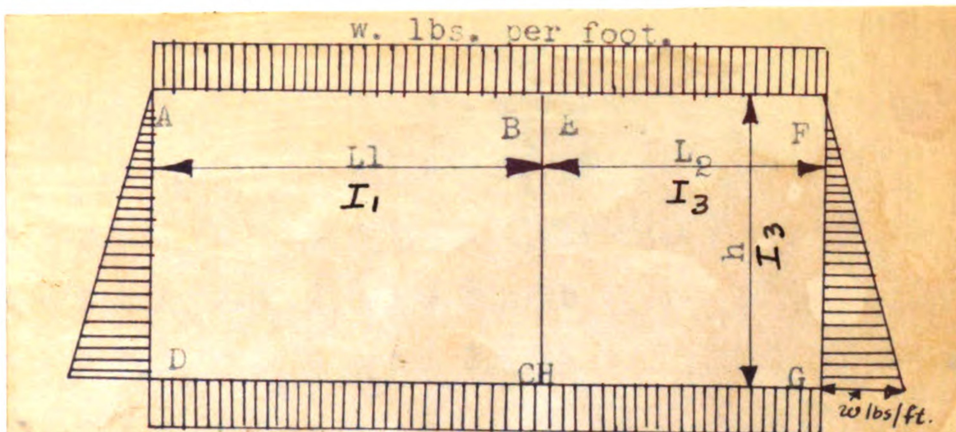


Fig. 2.

Fixed end moments can be easily determined and they are as follows with proper signs:

$$M_{AB} = w.L_1^2/12 \qquad M_{BA} = -\ w.L_1^2/12$$

$$M_{EF} = w.L_2^2/12 \qquad M_{FE} = -\ w.L_2^2/12$$

$$M_{FG} = \overline{W}.h/15 \qquad M_{GF} = -\ \overline{W}.h/10$$

$$M_{GH} = w.L_2^2/12 \qquad M_{HG} = -\ w.L_2^2/12$$

$$M_{CD} = w.L_1^2/12 \qquad M_{DC} = -\ w.L_1^2/12$$

$$M_{DA} = \overline{W}.h/10 \qquad M_{AD} = -\ \overline{W}.h/15$$

K - values will be:

$$K_1 = I_1/L_1 \ ; \quad K_2 = I_2/h \ ; \quad K_3 = I_3/L_2$$

Therefore, the slope-deflection equations will be,

$$M_{AB} = -2EK_1(2.\theta_A + \theta_B) + w.L_1^2/12$$

$$M_{BA} = -2EK_1(2.\theta_B + \theta_A) - w.L_1^2/12$$

$$M_{EF} = -\ 2.E.K_3(2.\theta_E + \theta_F) + w.L_2^2/12$$
$$M_{FE} = -\ 2.E.K_3(2.\theta_F + \theta_E) - w.L_2^2/12$$
$$M_{FG} = -\ 2.E.K_2(2.\theta_F + \theta_G) - \overline{W}.h/15$$
$$M_{GF} = -\ 2.E.K_2(2.\theta_G + \theta_F) + \overline{W}.h/10$$
$$M_{GH} = -\ 2.E.K_3(2.\theta_G + \theta_H) - w.L_2^2/12$$
$$M_{HG} = -\ 2.E.K_3(2.\theta_H + \theta_G) + w.L_2^2/12$$
$$M_{CD} = -\ 2.E.K_1(2.\theta_C + \theta_D) + w.L_1^2/12$$
$$M_{DC} = -\ 2.E.K_1(2.\theta_D + \theta_C) - w.L_1^2/12$$
$$M_{DA} = -\ 2.E.K_2(2.\theta_D + \theta_A) + \overline{W}.h/10$$
$$M_{AD} = -\ 2.E.K_2(2.\theta_A + \theta_D) - \overline{W}.h/15$$
$$M_{BC} = -\ 2.E.K_2(2.\theta_B + \theta_C)$$
$$M_{CB} = -\ 2.E.K_2(2.\theta_C + \theta_B)$$

Now,

$$M_{AB} + M_{AD} = 0 \qquad \ldots\ldots\ldots (1)$$
$$M_{DA} + M_{DC} = 0 \qquad \ldots\ldots\ldots (2)$$
$$M_{FE} + M_{FG} = 0 \qquad \ldots\ldots\ldots (2)$$
$$M_{FE} + M_{FG} = 0 \qquad \ldots\ldots\ldots (3)$$
$$M_{GF} + M_{GH} = 0 \qquad \ldots\ldots\ldots (4)$$
$$M_{BA} + M_{BC} + M_{EF} = 0 \qquad \ldots\ldots\ldots (5)$$
$$M_{HG} + M_{CD} + M_{CB} = 0 \qquad \ldots\ldots\ldots (6)$$
$$\theta_B = -\ \theta_E \qquad \ldots\ldots\ldots (7)$$
$$\theta_C = -\ \theta_H \qquad \ldots\ldots\ldots (8)$$

When the values from the slope-deflection equations will be substituted in these above eight equations, they will be:

$$2.E.\left[2.\theta_A(K_1 + K_2) + K_1\theta_B + K_2\theta_D\right] = w.L_1^2/12 - \overline{W}.h/15 \qquad \ldots\ldots\ldots\ldots (1)$$

$$2.E.\left[2.\theta_D(K_1 + K_2) + K_1\theta_C + K_2\theta_A\right] = \overline{W}.h/10 - w.L_1^2/12 \qquad \ldots\ldots\ldots\ldots (2)$$

$$2.E.\left[2.\theta_F(K_2 + K_3) + K_2\theta_E + K_3\theta_G\right] = \overline{W}.h/15 - w.L_2^2/12 \qquad \ldots\ldots\ldots\ldots (3)$$

$$2.E.\left[2.\theta_G(K_2 + K_3) + K_2\theta_F + K_3\theta_H\right] = w.L_2^2/12 - \overline{W}.h/10 \qquad \ldots\ldots\ldots\ldots (4)$$

$$2.E.\left[2.\theta_B(K_1 + K_2) + K_1\theta_A + K_2\theta_C + K_3(2\theta_E + \theta_F)\right] = w.L_2^2/12 - w.L_1^2/12 \qquad \ldots\ldots\ldots\ldots (5)$$

$$2.E.\left[2.\theta_C(K_1 + K_2) + K_1\theta_D + K_2\theta_B + K_3(2\theta_H + \theta_G)\right] = w.L_1^2/12 - w.L_2^2/12 \qquad \ldots\ldots\ldots\ldots (6)$$

And,

$$\theta_B = -\theta_E \qquad \ldots\ldots\ldots\ldots (7)$$

$$\theta_C = -\theta_H \qquad \ldots\ldots\ldots\ldots (8)$$

From the above eight equations when solved simultaneously, eight unknown Theta-values can be obtained. The values of slope thus obtained can be used in finding the moments at each corner of the culvert.

If the load is uniform lateral load w pounds per foot height of the culvert, the moment at each corner can be obtained by replacing $\overline{W}.h/10$ and $\overline{W}.h/15$ by $w.h^2/12$ with proper signs and putting w-the uniform vertical load equal to zero, and then solving new equations thus obtained, to obtain slopes which can be used to find corner moments.

If instead of two cell rectangular box culvert, there be two cell square one, then putting L_1, L_2 and h each equal to L and also I_1, I_2, and I_3 each equal to I, above eight equations can be modified and these new modified equations can be used in finding moments at the corners of this type of culvert as usual.

E. Moment Distribution Method.

Sign Coventions Used in Solving This Problem:

Moments which indicate tension on the inside face are taken as positive or negative otherwise. Shears that indicate summation of the forces at the left of the section acting outward when viewed from within the culvert are considered as positive. Square Box Culvert having symmetrical uniform vertical load w per foot length of the culvert and the same moment of inertia for all component parts.

Distributing factors for this particular case shall be 1/2. Assuming 100 units of fixed end moments and distributing these moments according to above distributing factors, which is not very difficult, it was found that bending moment at each corner comes to 50% of the assumed fixed end moment.

Similarly the bending moments at each corner due to uniform lateral load shall be 50% of the fixed end moment.

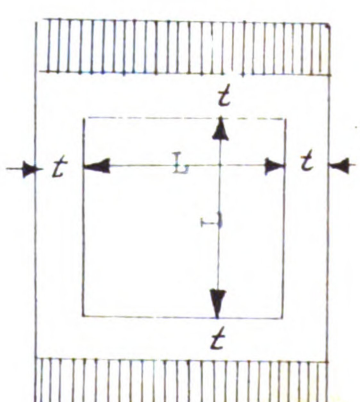


Fig. 1.

Same culvert as in the previous case with triangular load varying from zero at one end to w pounds per foot of culvert length at the other end as shown in the following Fig. Fixed end moment due to this type of loading will be at A and B equal to $\overline{W}.^{L}/15$ and at C and D, to $\overline{W}^{L}/10$, where $\overline{W}$ is total load on one side of the culvert due to above type of loading and it comes to $w.^{L}/2$. Assuming fixed end moment at A equal to 100 units, consequently fixed end moment at C will be equal to 150, and distributing these moments according to the distributing factors equal to 1/2, the final bending moments at the corners A and B were found as 56% and the corner C and D, were 43% of the assumed fixed end moment at respective corner.

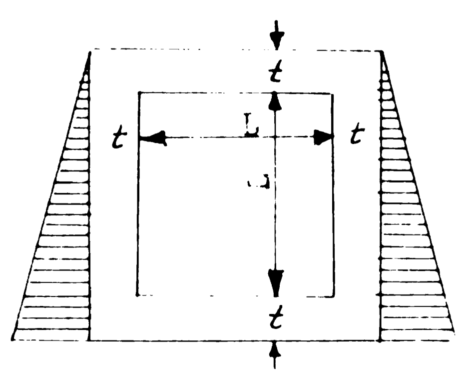


Fig. 2.

Rectangular Cell Box Culvert:

Dimensions and sections of the component parts are as shown in the Fig. 3. Load is uniform vertical load w pounds per foot length of the culvert.

Distributing factors:

Taking one foot width of culvert,

$I_1 = 1(1.5t)^3/12$ and,

$I_2 = 1(1.t)^3/12$

$K_1 = I_1/L(1.5)$

$= 2.25t^3/12L$ and,

$K_2 = t^3/12L$

$K_1 = 2.25K_2$

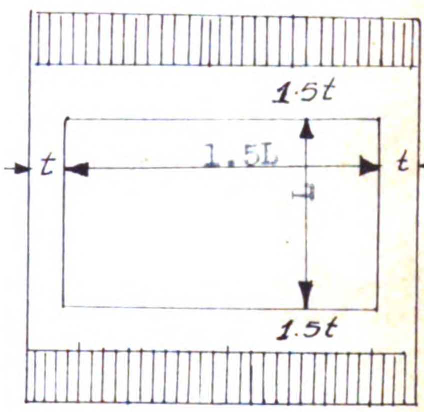


Fig. 3.

Therefore,

Distributing factors will be,

$1/1 + 2.25$ and $2.25/1 + 2.25$

$= 0.31$ and 0.69.

Due to symmetry the final bending moments at all the four corners shall be the same. Fixed end moment of 100 units was assumed and when distributed by the above distributing factors final moment at each corner came equal to 31% of the assumed fixed end moment.

Box Culverts with Two Square Cells:

Case 2. Dimensions and sections of the component parts are the same as in the previous case. Loading is uniform lateral load w pounds per foot length of the culvert height. Distribution factor will be the same as in the previous case. Fixed end moments are assumed as 100 units and these moments then were distributed according to proper distributing factors. Moments at the four end corners turned out to be 66% and at central diaphram to be 33% of the assumed fixed end moments.

Box Culverts with Two Square Cells:

Case 3. Dimensions, sections and distributing factors are the same as in the previous cases. Loading is triangular lateral load. Fixed end moment at A and B were assumed to be 100 units and consequently fixed end moments D and G comes equal to 150 units, because fixed end moments at A and F due to triangular lateral load varying from zero at one end to w per foot height of the culvert at the other end, will be W.h/15, and D and G will be W.h/10, where W is the total lateral load. When these fixed end moments were distributed according to proper distributing factors, the final moments turned out to be:

At A and F 71%; at D and G 62%; at B-E 36% and at C-H 31% of the assumed fixed end moments.

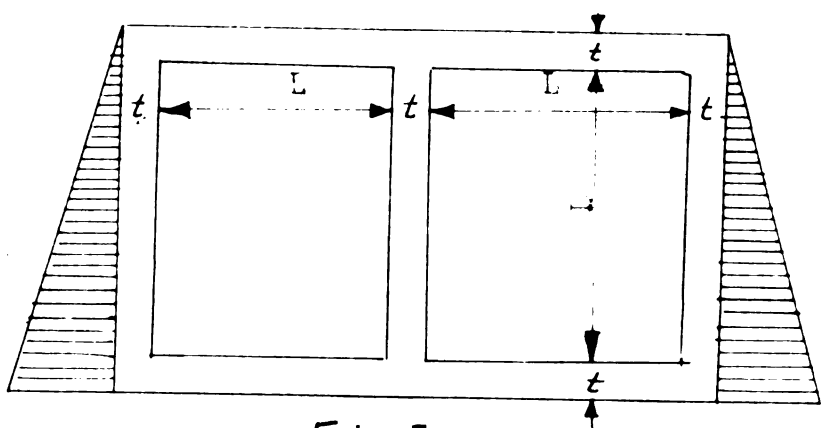


Fig.5.

Rectangular Box Culverts with Two Cells:

Case 1. Dimensions, moment of inertias of component parts of the culvert and loading are as shown in the Fig. 6.

$$I_1 = (1.5t)^3/12 \quad \text{and} \quad K_1 = 2.25t^3/12.L$$
$$I_2 = t^3/12 \quad \text{and} \quad K_2 = t^3/12.L$$

Therefore, distribution factors at all the outer corners will be .31 and .69 as shown in the sketch, and the distribution factors at the diaphram will be .41, .41, and .18. Assumed fixed end moments were 100 units and when these moments were distributed according to above factors, the final moments at corners A, F, G and D turned out to be 20% of the assumed fixed end moments. The moment at B-E and C-H comes to 140%. Procedure is shown on the next page.

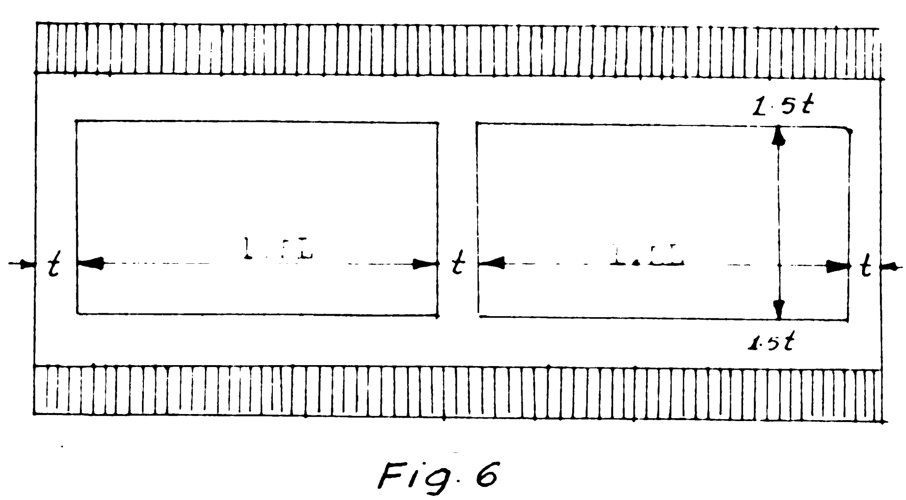
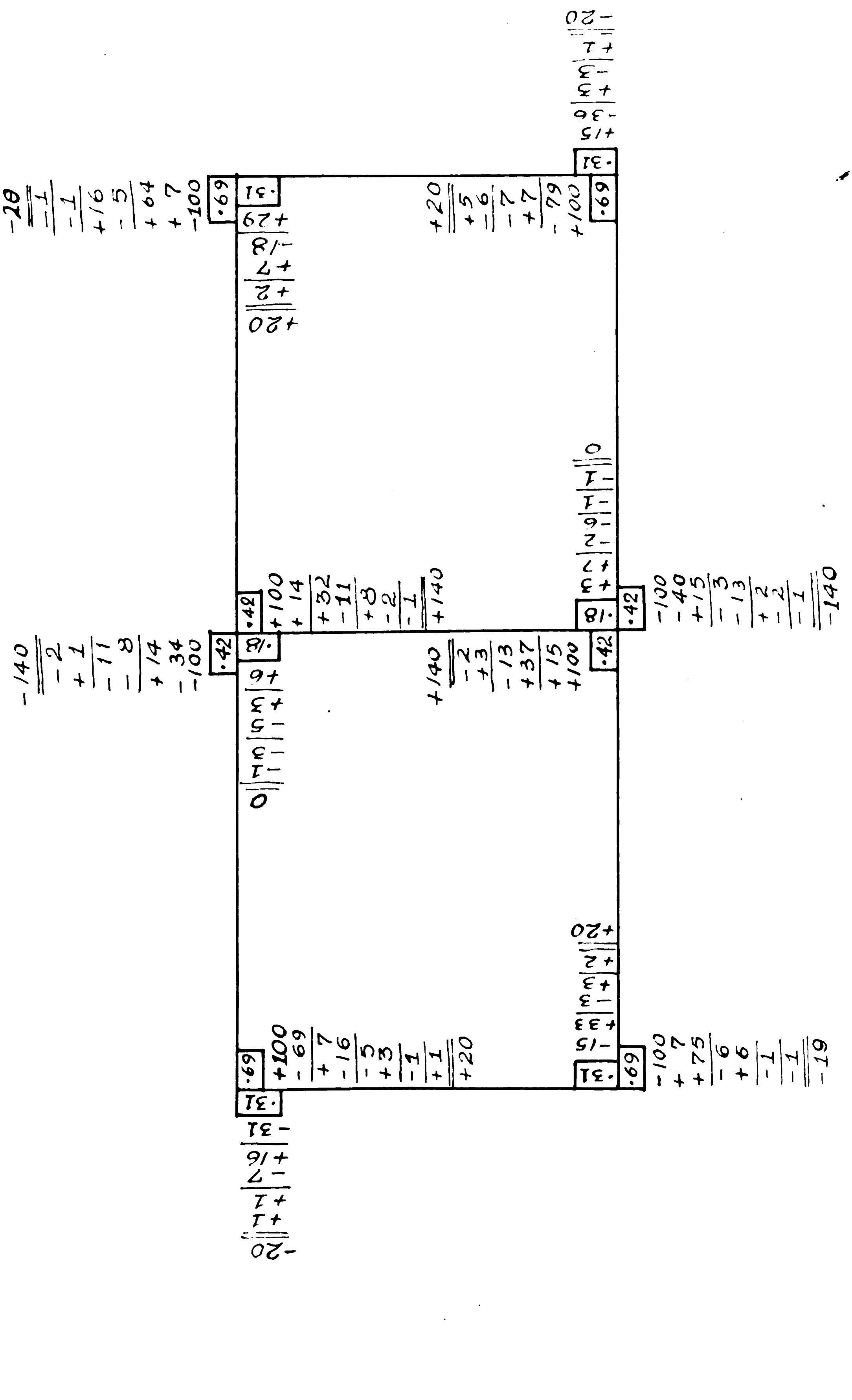


Fig. 6



Case 2. Everything the same as in the previous case but loading is uniform lateral load. Assumed fixed end moments were 100 units, when distributed according to proper distributing factors, the final moment at A, F, G and D turned out to be 81% and at B-E and C-H 41%. Procedure is shown on the next page.

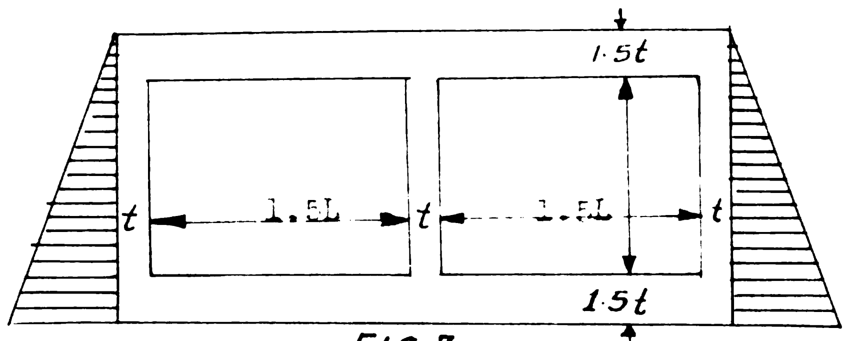
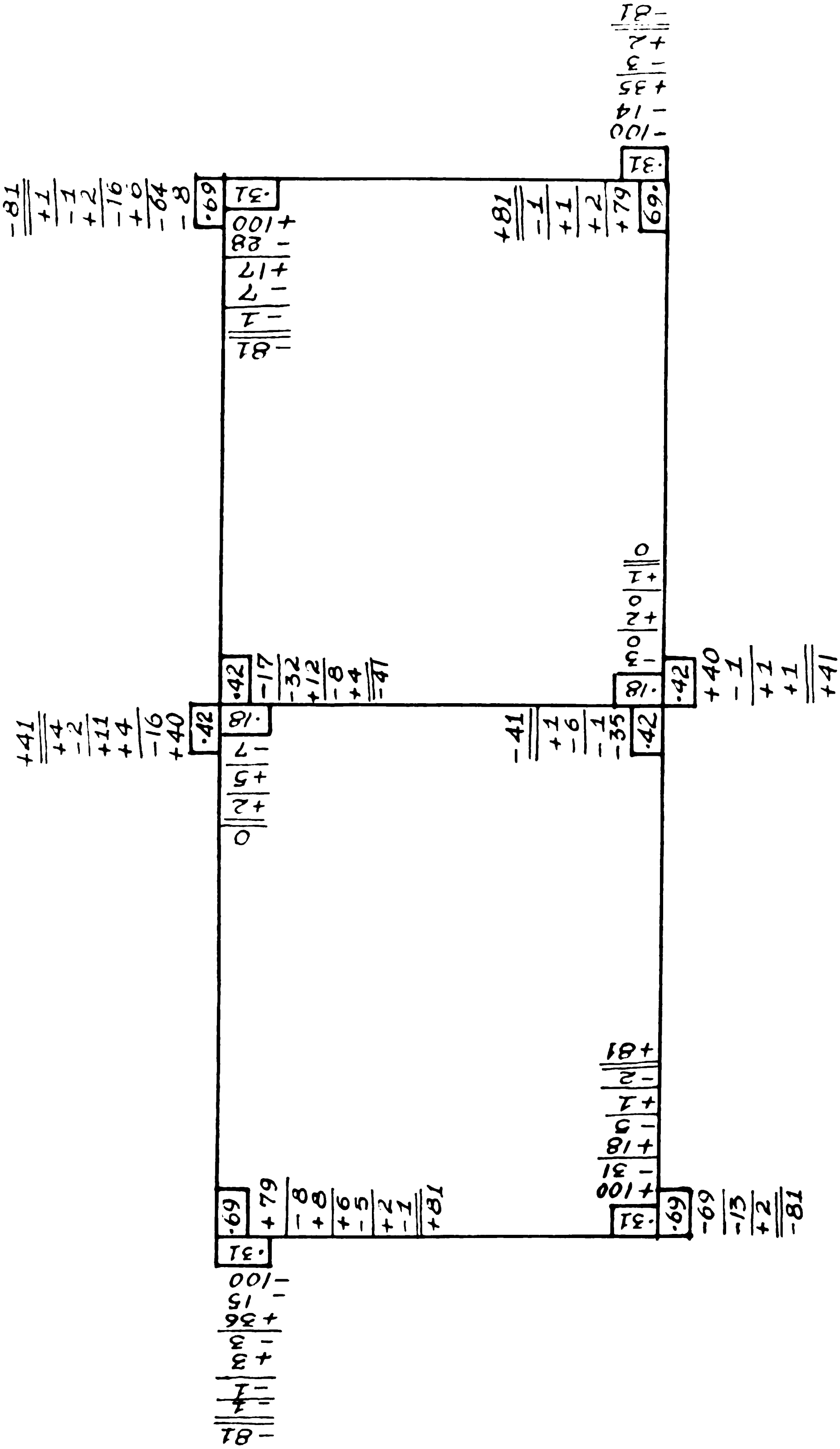
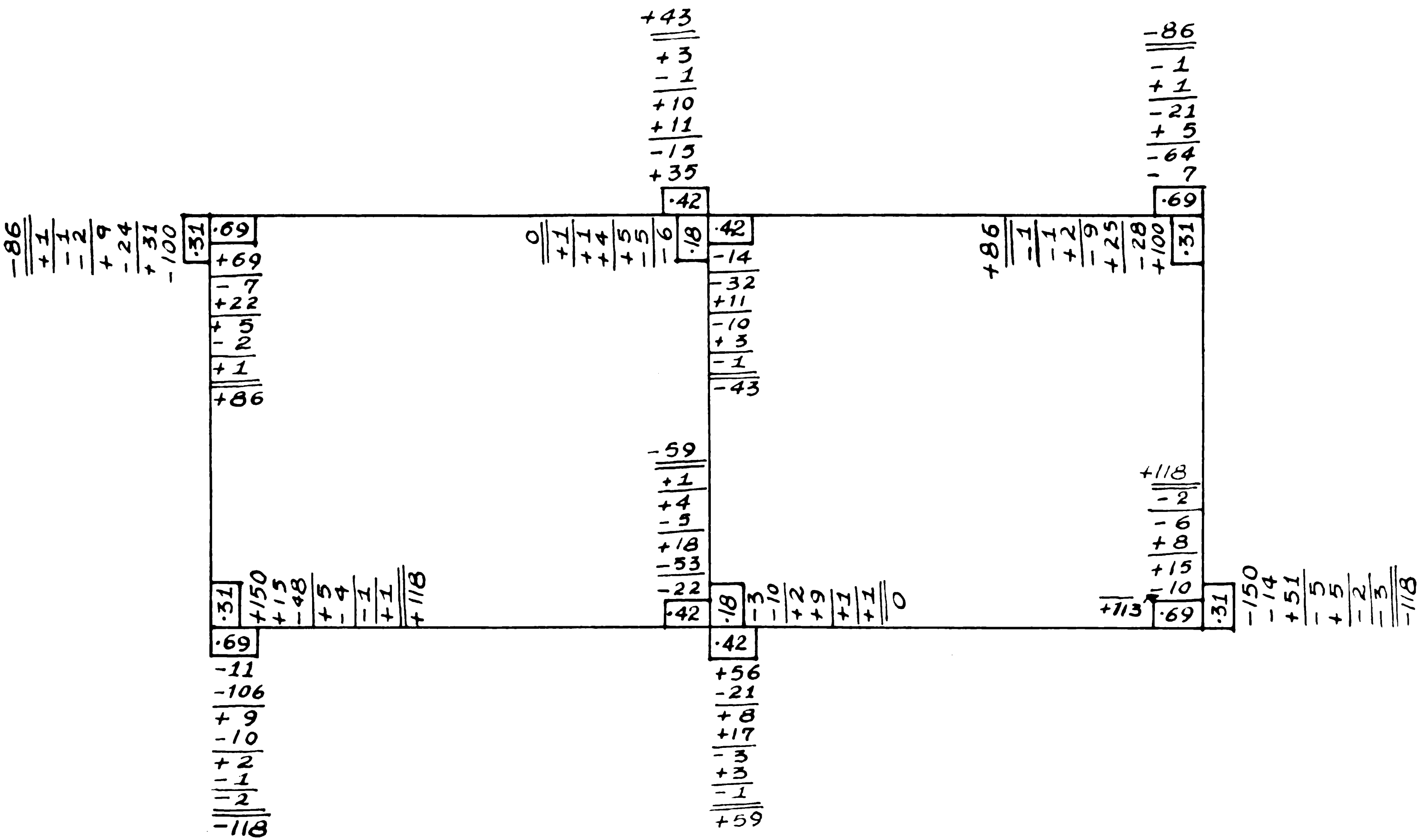


Fig. 7.

Case 3. Everything same as in the first case except loading is triangular lateral loading. Assumed fixed end moments at A and F were 100 units and consequently at G and D their value will be 150 units. When these moments were distributed according to proper distributing factors, the final moment at A and F turned out to be 87% and at G and D 78%; at B-E 43% and at C-H 39% of the fixed end assumed moments. Procedure is shown on the next page.





Rectangular Box Culvert with Two Cells (Symmetrical)
Case 7, Triangular Lateral Load.

Box Culverts with Two Rectangular Cells:

Case 1. Dimensions and sections of the component parts are as shown in the Fig. 8.

Distributing Factors:

$$I_1 = t^3/12.$$

$$I_2 = (1.25t)^3/12 \text{ and } I_3 = (1.25t)^3/12$$

$$K_1 = 2.25.t^3/12L$$

$$K_2 = t^3/12L$$

$$K_3 = 1.25^2 t^3/12L.$$

Distributing factor for member BA $= K_{BA}/\ K$

$= 1.56/1\text{-}1.56\text{-}2.25$

$= 0.32.$

Similarly other distributing factors can be easily determined and their values are as shown in the Fig. 13.

For uniform vertical load w per foot length of the culvert length, final moments after distributing assumed fixed end moment equal to 100 units for the span EF turned out to be as follows:

At A and D equal to 14%

At F and G equal to 21%

At B-E and C-H equal to 120%.

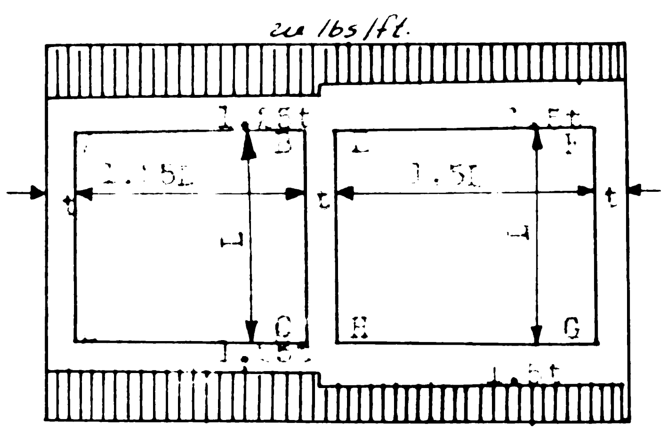
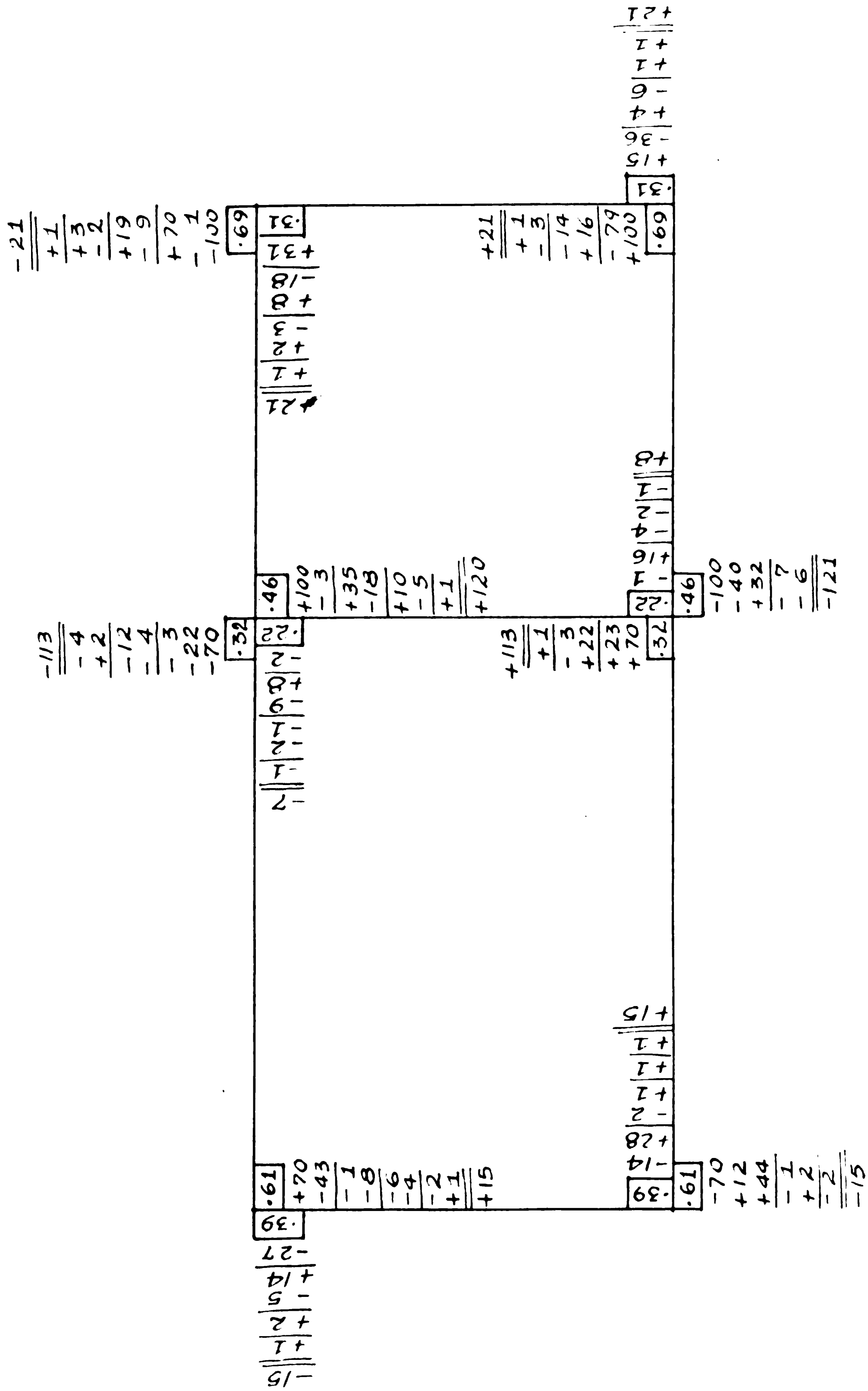


Fig. 8



Box Culverts with Two Rectangular Cells:

<u>Case 2</u>. Dimensions, sections and distributing factors are same as in the previous case. Loading is uniform lateral load w per foot height of the culvert. Assumed fixed end moments at A and F were 100 units and at D and G were 100 units. These moments were distributed according to the proper distributing factors and the final moments turned out to be as follows:

At A and D equal to 75%

At F and G equal to 81%

At B-E and C-H equal to 39%

of the assumed fixed end moments.

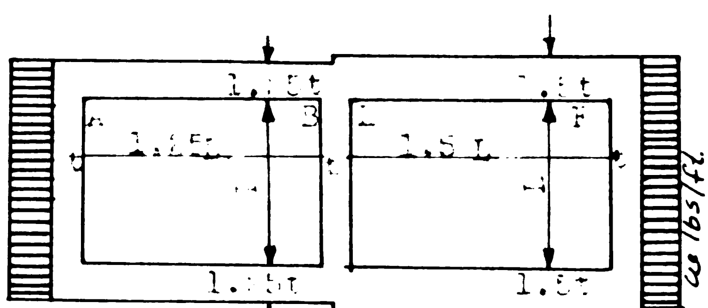
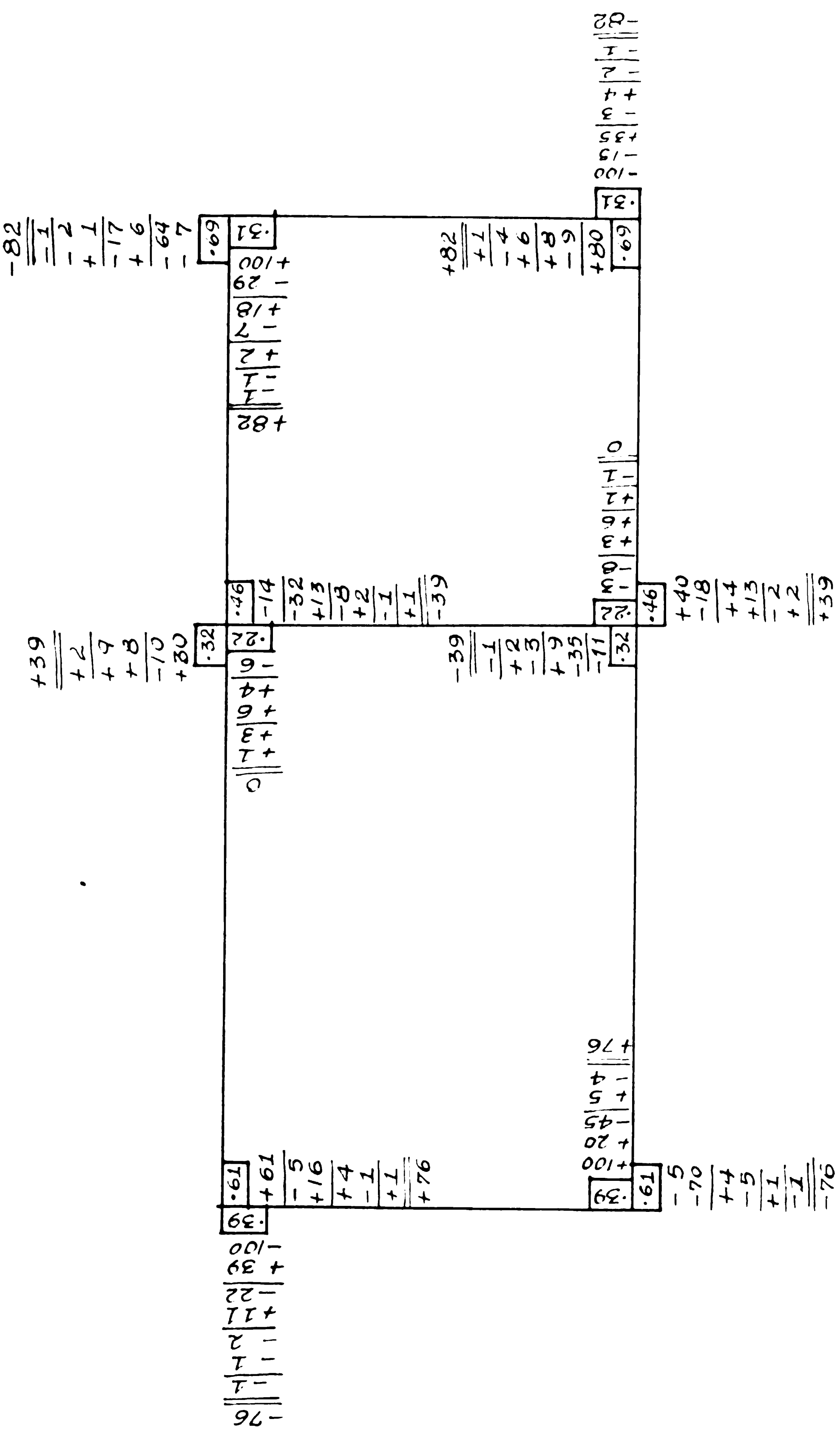


Fig. 9



Box Culverts with Two Rectangular Cells:

Case 3. Dimensions, sections and distributing factors are same as previous cases. Loading is triangular lateral load varying from zero at one end to w per foot height of culvert at the other end. Fixed end moments at A and F were assumed to be 100 units and at D and G to be 150. These moments were distributed according to proper distributing factors and final moments turned out to be as follows:

At A equal to 79%

At B-E equal to 37%

At F equal to 55%

At G equal to 85%

At C-H equal to 38%

At D equal to 74%

of the assumed fixed end moments.

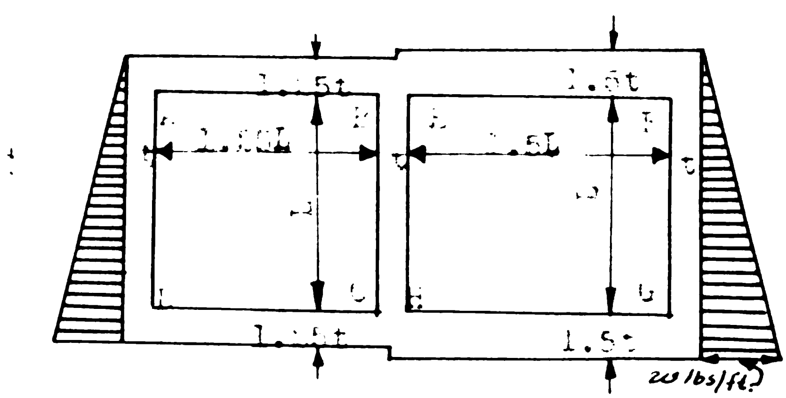
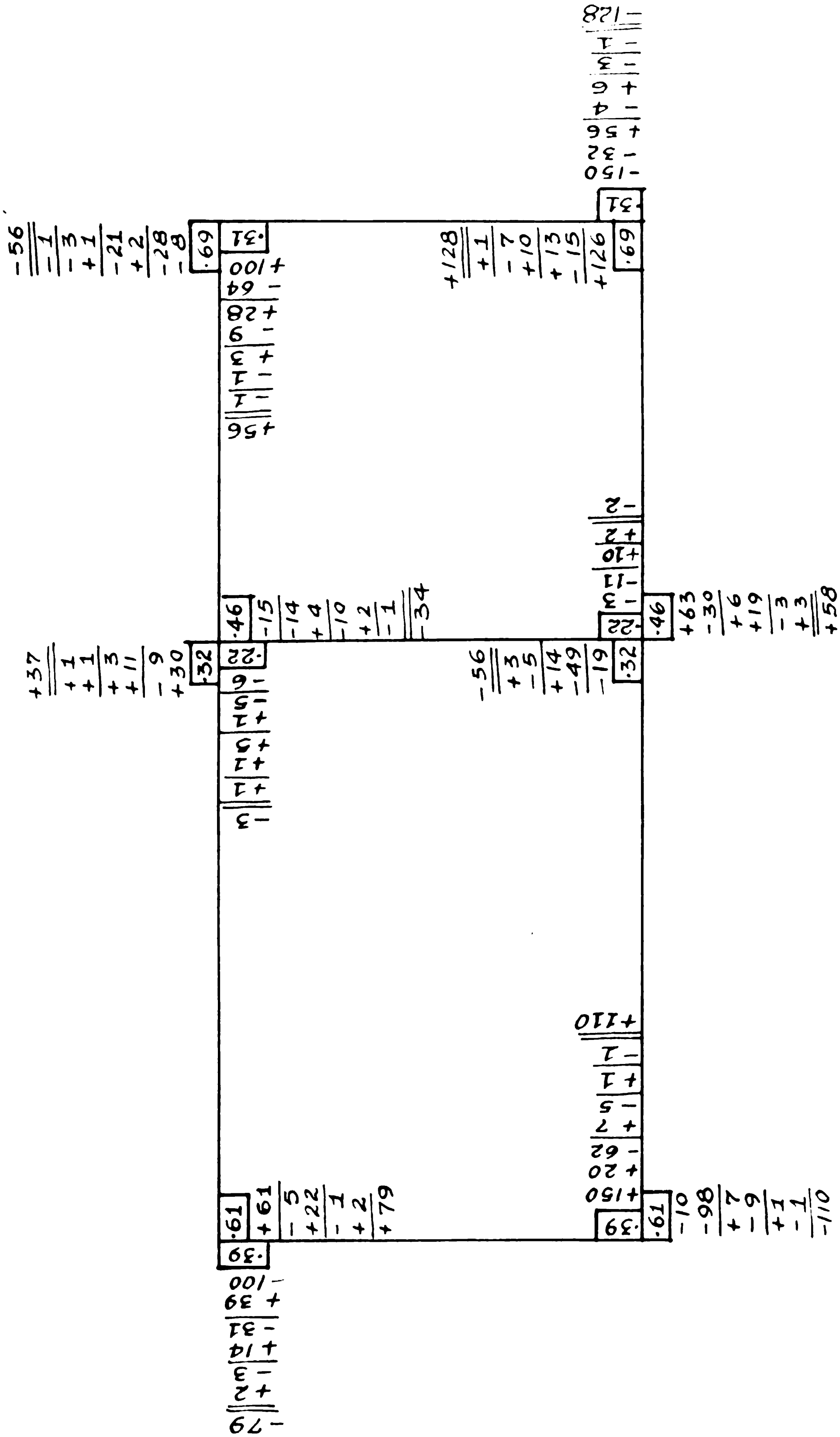


Fig.10



F. Column Anology.

Sign Conventions Used in Solving This Problem:

Bending moments in the beams will be considered positive if they produce tension on the inside of the elastic ring. Shears are positive if they accompany positive rate of change of bending moment, increase up or to the right. Square box culvert symmetrical loading and same moment of inertia for all component parts. Loading is uniform vertical load w per foot length of the culvert.

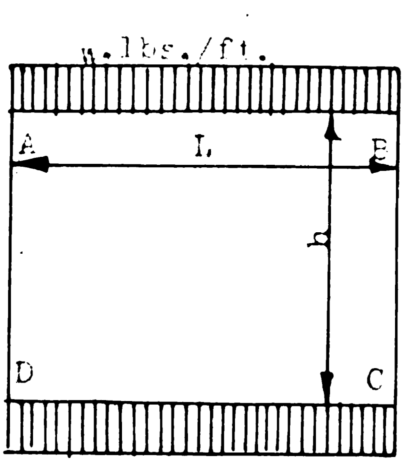


Fig. 1.

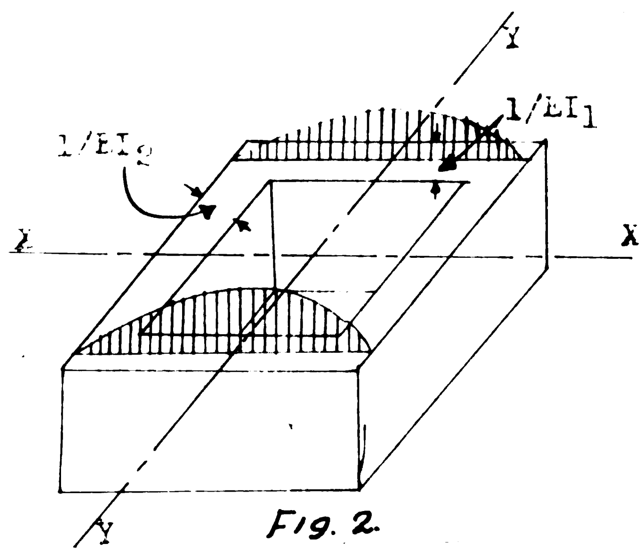


Fig. 2.

EI is same for all members and is taken as unity.

Area of Anologus Column,

$$A = 4L.$$

$$\bar{y} = \frac{L.L + 2L.L/2}{4L} = L/2.$$

$$Ixx = Iyy = 2L.(L/2)^2 + 2.L^3/12.$$

$$= 5.L^3/12.$$

Load on Anologus Column:

$$W = 2.w.L^2.L/3.8.$$
$$= w.L^3/12,$$

Total load = $2.w.L^3/12.$

$$= w.L^3/6.$$

Therefore, $f_A = M_A = w.L^3/6.4.L.$

$$= w.L^2/24.$$

($M_{xx} = M_{yy} = 0$; because loading on the anologus column is symmetrical)

Due to symmetry the moments at all corners will be the same.

Rectangular Cell Box Culvert:

<u>Case 1.</u> Dimensions and sections of the component parts are as shown in the fig. (a) below. Load is uniform vertical load w per foot length of the culvert.

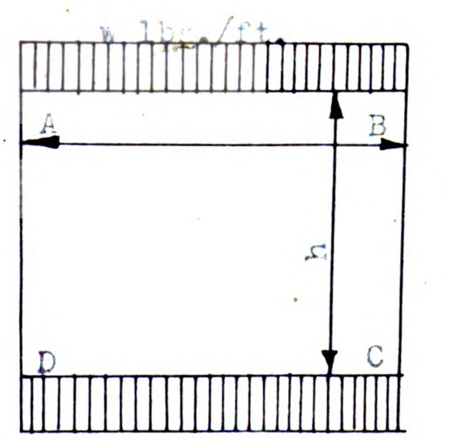


Fig. 3

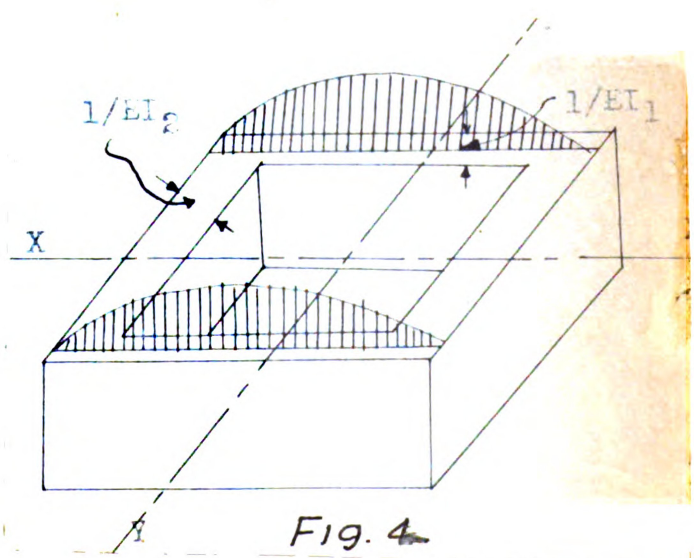


Fig. 4

Rectangular Cell Box Culvert:

Case 2. Dimensions and sections of the component parts are the same as in the previous case. Load is triangular loading, uniformly increasing from one end to the other end according to the law w.y/h. Therefore,

$$\text{Total load } \overline{W} = (w.h/h)(h/2)$$
$$= w.h/2.$$

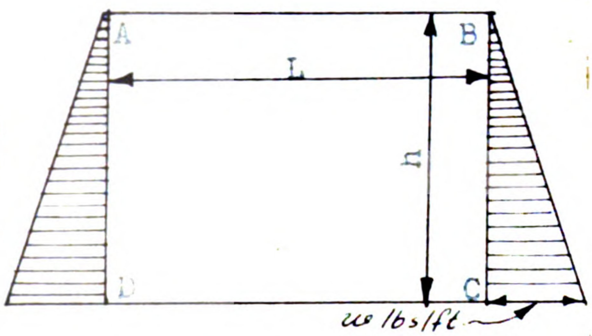


Fig. 5

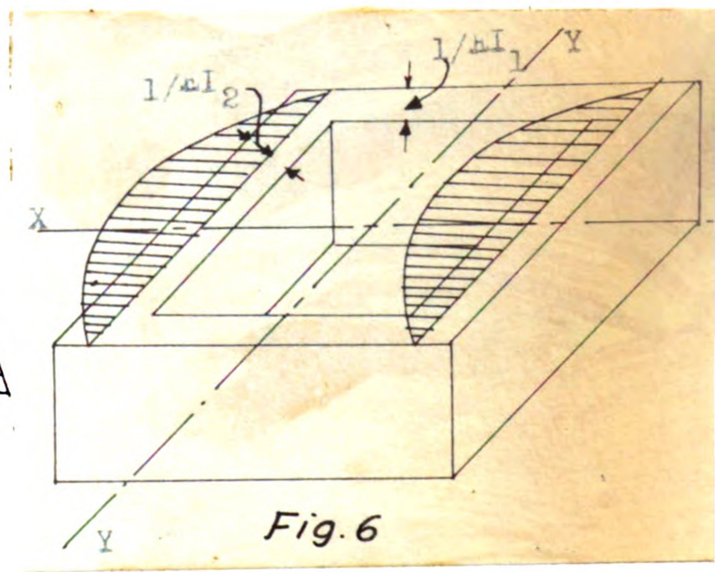


Fig. 6

Area of the section of the anologus column,

$$A = 2(L.I_2 + h.I_1)/I_1.I_2$$

Load on the column will be area of moment diagram due to load alone,

$$W = 2.\overline{W}.h^2/12.I_2$$
$$= \overline{W}.h^2/6I_2$$

$$Mxx = (Wh^2/6I_2)(8/15 - 1/2)h = \overline{W}.h^3./180I_2$$

$$Myy = 0$$

$$Ixx = 2.(L/I_1)(h/2)^2 + 2(1/12)(1/I_2)(h)^3$$

$$= Lh^2/2I_1 + h^3/6I_2$$

$$= (h^2/6)\left[(3LI_2 + hI_1)/I_1I_2\right]$$

Due to symmetry of load and section of the anologus column,

$$M_A = M_B = f_A = W/A - Mxx.y/Ixx.$$

$$- \frac{\overline{W}h^2.I_1I_2}{6.I_2.2(LI_2 + hI_1)} - \frac{\overline{W}h^3I_1I_2.6.h}{180.I_2.h^2(3LI_2 + hI_1).2.}$$

$$= \frac{\overline{W}.h^2.I_1}{12(L.I_2 + h.I_1)} - \frac{\overline{W}.h^2.I_1}{60(3L.I_2 - h.I_1)}$$

$$= -(\overline{W}.h^2.I_1/12)\left[1/(L.I_2-h.I_1) - 1/5(3LI_2-hI_1)\right]$$

Similarly,

$$M_C = M_D = f_C = W/A + Mxx.y/Ixx.$$

$$= \overline{W}.h^2.I_1/12)\left[(1/L.I_2 - h.I_1) + (1/5(3LI_2 - hI_1)\right]$$

IV. DESIGN TABLES FOR BOX CULVERTS.

The following tables are prepared by m.d. method:

TABLE I. Square Culverts

Coefficients for Moment, M, Thrust, N, and Shear, V, in Traverse Section 1 ft. wide.

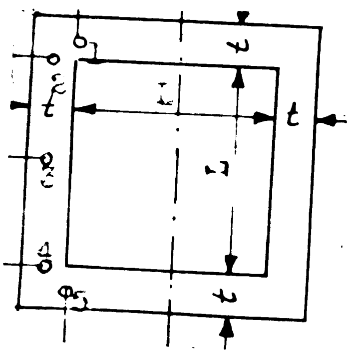


Fig. 1

Traverse Section

Signs

- Moment, M, indicates tension on inside face.
- Thrust, N, indicates compression on section.
- Shear, V, indicates that the summation of the forces at the left of the section acts outward when viewed from within.

	Uniform vertical load			Uniform lateral Load			Triangular lateral load		
	$.0417.w.L^2$	wL		$.0417.w.L^2$	wL		$.0188.w.L^2$	wh/2	
	M	N	V	M	N	V	M	N	V
1	+ 2			- 1	+.50		- 1	+.16	
2	- 1		+.50	- 1	+.50		- 1	+.16	
3	- 1	+.50		- 1		-.50	- 1		-.1
4	- 1	+.50		+ 2			+2.22		
5	- 1	+.50		- 1	+.50		-1.22		+.3
6	- 1	-.50		- 1	+.50		-1.22	+.34	
7	+ 2			- 1	+.50		-1.22	+.34	

TABLE II. Rectangular Culverts

Coefficients for Moment, M, Thrust, N, and Shear, V, in Traverse Section 1 ft. wide.

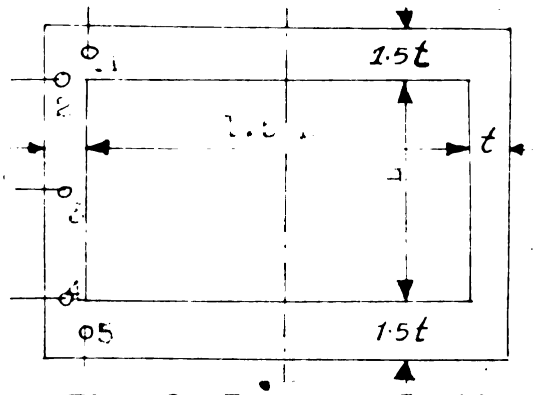


Fig. 2 Traverse Section.

Signs

- Shear, V, indicates that the summation of the forces at the left of the section acts outward when viewed from within.
- Moment, M, indicates tension on inside face.
- Thrust, N, indicates compression on section.

	Uniform vertical load			Uniform lateral load			Triangular lateral load		
	M	N	V	M	N	V	M	N	V
	$.0833wL^2$	$w(1.5L)^2$		$.0833wL^2$	$w.L$		$w.h^3$	$wh/2$	
1	+1.2			-.126	+.5		-.033	+.16	
2	-.15		-.5	-.278	+.5		-.033	+.16	
3	-.30	-.5		-.278		-.5	-.016		-.16
4	-.30	-.5		+.125			+.34		
5	-.30	-.5		-.278		-.5	-.016		-.33
6	-.15		-.5	-.278	+.5		-.033	+.33	
7	+1.20			-.126	+.5		-.033	+.33	

TABLE III. Box Culverts of Two Square Cells.

Coefficients for Moment, M, Thrust, N, and Shear, V, in Traverse Section 1 ft. wide.

Signs: They are same as in the previous cases.

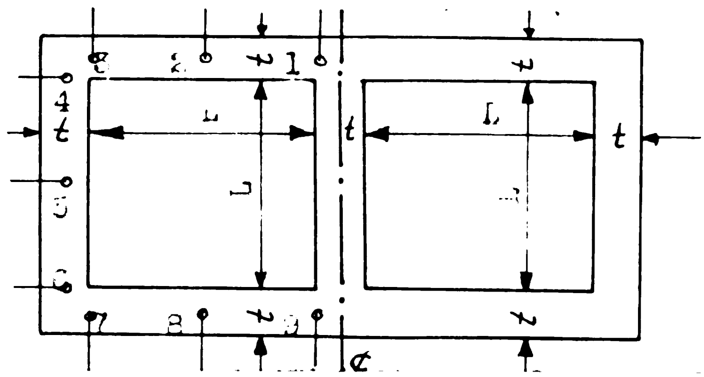


Fig. 3. Traverse Section

	Uniform vertical load			Uniform lateral load			Triangular lateral load		
	M	N	V	M	N	V	M	N	V
	$.0139wL^2$	w.L		$.0556wL^2$	w.L		$.0123wL^3$	$w.L^2/2$	
1	-4		-.292	+0.50	+.5		+1	+.16	
2	+2			-0.25	+.5		-0.5	+.16	
3	-1		+.208	-1	+.5		-2	+.16	
4	-1	+.208		-1		-.5	-2		-.16
5	-1	+.208		+1.25			+2.82		
6	-1	+.208		-1	-.5	+.5	-2.52		+.34
7	-1		-.208	-1	+.5		-2.52	+.34	
8	+2			-0.25	+.5		-0.63	+.34	
9	-4		+.292	+0.50	+.5		+1.26	+.34	

TABLE IV. Box Culverts of Two Rectangular Cells.

Coefficients for Moment, M, Thrust, N, and Shear, V, in Traverse Section 1 ft. wide.

Signs: Same as in the previous cases.

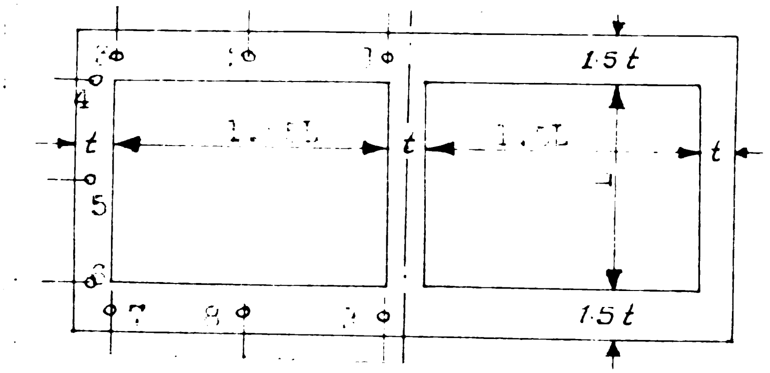


Fig. 4. Traverse Section

	Uniform vertical load			Uniform lateral load			Triangular lateral load		
	M	N	V	M	N	V	M	N	V
	$.0079wL^2$	wL		$.042.wL^2$	w.L		$.0142wh^2$	wh/2	
1	-3.7		-.272	+1	+.5		+1	+.16	
2	+5.7			-.50	+.5		-1	+.16	
3	-1		+.228	-2	+.5		-2	+.16	
4	-1	+.228		-2		-.5	-2		-.16
5	-1	+.228		+1			+2.1		
6	-1	+.228		-2		+.5	-2.7		+.34
7	-1		-.228	-2	+.5		-2.7	+.34	
8	+5.7			-.50	+.5		-.70	+.34	
9	-3.7		+.272	+1	+.5		+1.4	+.34	

TABLE V. Box Culverts of Two A-symmetrical Reotangular Cells.

Coefficients for Moment, M, Thrust, N, and Shear, V, in Traverse section 1 ft. wide.

Signs: Same as in the previous tables.

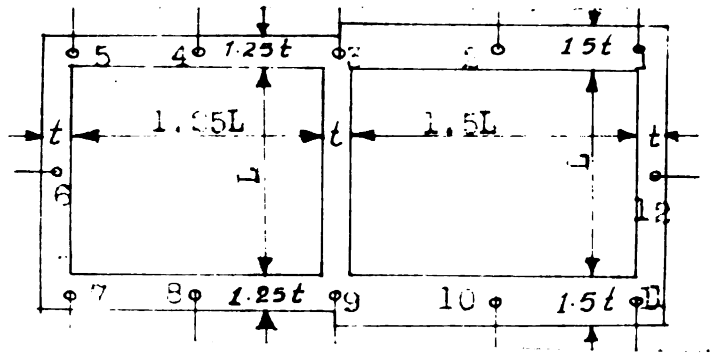


Fig. 5. Traverse Section

	Uniform vertical load			Uniform lateral load			Triangular lateral load		
	M / $w.L^2$	N / wL	V	M / $w.L^2$	N / wL	V	M / $w.L^2$	N / wL	V
1	-0.018		+.495	+.066	+.5		-0.018	+.42	-.42
2	+0.058			+.05	+.5		-0.015	+.42	
3	-0.133	-1.68	-.913	+0.033	+.5		-0.011	+.42	
4	+0.075			-0.042	+.5		-0.02	+.42	
5	-0.018	+.493	+.493	-.053		-.5	-0.026	+.42	-.42
6	-0.018	+.493		+.030			+0.03		
7	-0.018	+.493	-.493	-.053		+.5	-0.037	+.58	+.58
8	+0.075			-.042	+.5		-0.034	+.58	
9	-0.133	+.913	+1.68	+.033	+.5		-0.03	+.58	
10	+0.058			+.05	+.5		-0.036	+.58	
11	-0.018		-.495	-.066	+.5		-0.-52	+.58	+.58
12	-0.018	-.495		+.017			+0.033		

V. NUMERICAL EXAMPLES

Example No. 1.

Analysis of Rectangular Box Culvert:

Assumed data:

Waterway opening: 52 sq. ft.

Truck loading: 10-ton truck on unsurfaced secondary highway.

Fill on top of culvert: 5 ft.

For an opening of 52 sq. ft., a rectangular culvert having a span-height ratio of 1.5 to 1 would require approximately a clear span of 9 ft. and a clear height of 6 ft. (9 x 6 = 54.0 sq. ft.)

Estimate t equal to 7 in. (Slab thickness equals 1.5 x 7 = 10.5 in.)

Load factors for the use of Table II:

Outside width of the culvert = 9 - 2 x 7/12
= 10.17 ft.

Live load from the 10-ton truck = 1,710

Dead load from the fill = 5 x 100 x 10.17 = 5090

Uniform vertical load w lb. per foot = 6,800

Uniform lateral load w lbs. per sq. ft. =
5 x 100/3 = 167

(Assumed angle of repose is 30 degrees)

Triangular lateral load per ft. height = 100/3 = 33

Computations based on the constants are given in the schedule on the next page.

Computation Schedule for 9x6 ft. Culvert

Load conditions	Sections						
Moment, M,	4		5		6		7
Thrust, N, or	M	N	M	N	M	V	M
Shear, V	ft.lb.	lb.	ft.lb.	lb.	ft.lb.	lb.	ft.lb.
I. Uniform vertical load							
M:	-1630		-1630		-780		+6510
N or V:		+3400		+3400		-3400	
II. Uniform lateral load							
M:	+530		-240		-460		-460
III. Triangular lateral load							
M:	+360		-170		-350		-350
Total M:	-740		-2040		-1590		+5700
Total N or V:		+3400		+3400		+3400	

Example No. 2.

Analysis of Box Culvert of Two Square Cells:

Assumed data:

Waterway opening: 160 sq. ft.

Truck loading: 10-ton truck on unsurfaced secondary highway.

Fill on the top of culvert: 5 ft.

For an opening of 160 sq. ft., a culvert having two square openings would require a clear span of approximately 9 ft. (Waterway = 2 x 9 x 9 = 162 sq.ft.) Estimate t equal to 9 in. (Thickness of slab and walls)

Load factors for use of Table III.

Outside width of culvert = 2 z 9 - 3 x 0.75 = 20.25 ft.

Live load from 10-ton truck = 1,710

Dead load from fill,

5 x 100 x 20.25 = 10,130

Uniform vertical load,

w lb. per foot = 11,840

Uniform lateral load,

w lb. per sq. ft. = 5 x 100/3 = 167

Triangular lateral load

w lb. per foot of height = 100/3 = 33

Computations based on the constants are given in the schedule on the next page.

Computation Schedule for Culvert Having Two 9 x 9 ft. Openings.

Load conditions	Sections							
Moment, M,	5		6		7	8	9	
Thrust, N, or	M	N	M	N	M	M	M	V
Shear, V	ft. lb.	lb.	ft.lb.	lb.	ft.lb.	ft.lb.	ft.lb.	lb.
I. Uniform vertical load								
M:	-1620		-1620		-1620	-3230	-6460	
N or V:		+2460		+2460				+3450
II. Uniform lateral load								
M:	+1100		-570		-570	-220	+440	
III. Triangular lateral load								
M:	+1070		-550		-950	-240	+490	
Total M:	-550		-2740		-3140	+2770	-5530	
Total N or V:		+2460		+2460				+3450

VI. CONCLUSION

After studying the methods applied in this thesis it can be seen that Virtual Work Method, Principle of Least Work Method, and Slope-deflection Method, all involve solving numerous simultaneous equations, which is rather tedious and takes a great deal of time. So far as Column Anology Method is concerned, its application has been limited to one cell box culverts. The only method which can be considered as best suited for this type of problem is Moment Distribution Method.

Tables given herein are prepared by Moment Distribution Method. These tables cover a wide range of design conditions and their use entails a minimum of calculation.

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