



ANALYSIS OF BOX CULVERTS

Thesis for the Degree of M. S.

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1953

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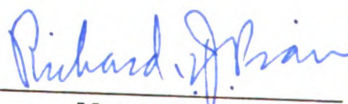
ANALYSIS OF BOX CULVERTS

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Major professor

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ANALYSIS OF BOX CULVERTS

By

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I. INTRODUCTION

A. Introductory Statement.

In the beginning of the twentieth century a new material of construction, reinforced concrete, came into existence. This material, being naturally continuous, demanded the study of Statically Indeterminate Structures. Hence, engineers were forced to learn more about the analysis and design of statically indeterminate structures. Accordingly various methods for the analysis of such structures have been developed. It is quite worthwhile to get well acquainted with these various methods. To achieve this purpose, box culvert which is one of the numerous statically indeterminate structures is taken as an example and various methods used to analyse it.

Out of numerous methods only six commonly used methods have been used to analyse this problem. These six methods are: 1. Moment Area Method. 2. Principle of Least Work Method (Castiglino's Theorem). 3. Virtual Work Method. 4. Slope-Deflection Method. 5. Moment Distribution Method. 6. Column Analogy Method.

B. Purpose and Scope.

The purpose of this study is two-fold. First is to analyse the box culverts by various methods used in the analysis of the Statically Indeterminate Structures thereby getting well acquainted with methods used in this problem. End moments of the box culverts have been determined by six widely used methods. Second is to introduce design tables showing moments, shear and normal thrust at critical sections of the box culverts. These tables are prepared for designer or checker and the use of the tables herein will eliminate calculations, thereby saving time.

Study has been restricted up to two cells a-symmetrical rectangular box culverts. But by going through this it can be seen that the box culverts having three or more cells can be analysed without much trouble.

C. Acknowledgement.

The author wishes to express his sincere thanks to Dr. R. H. J. Pian for his invaluable advise and suggestions without which this work would have been quite impossible.

II. GENERAL CONSIDERATIONS

A. Use of Culverts.

Culvert is a traverse drain or waterway under a road, railroad, canal, or channel. In ordinary engineering usage, however, culvert refers to only short structures through roadway or railroad embankments serving as passageways for water and normally not acting under hydrostatic pressure. In this discussion culverts shall be referred as defined in the latter manner. Bridges were used to span openings of considerable size and importance while culverts were relegated to minor openings. This distinction no longer applies arbitrarily as it is recognized that the efficiency and low cost of culverts make them desirable for a wide range of conditions. From a structural standpoint, culverts are advantageous because of their continuity. Unexpected loads or other unusual conditions are better resisted by culverts as all component parts contribute helpful restraint.

B. Culvert Loads.

Live Loads. These include moving concentrated super loads as truck wheel loads, with or without impact. For design purposes, the maximum live load on culvert is taken as that produced by heavy truck. In accordance with the common practice,

the standard truck train loading of the American Association of State Highway Officials can be adopted. The H-10 loading is used for heavily traveled secondary highways, and even for primary highways in most localities.

Pressures from wheel loads are assumed to be uniformly distributed on the culvert slabs as there is, in general, an intervening earth fill. Impact effect may be included in this uniformly distributed load.

Dead Loads. These include weight of embankment material on culvert, weight of the culvert and of contained water, lateral pressure on sides of the culvert. The total vertical load per lineal foot on a projecting culvert due to an embankment may be determined by the formula,

$$P = C_c \cdot w (B_c)^2$$

In which,

w = Weight of fill material per cubic foot.

B_c = Outside width of the culvert in feet.

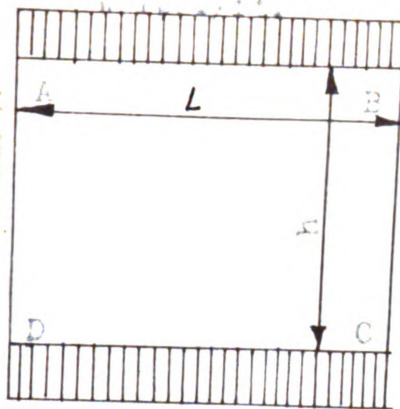
C_c = Coefficient depending on the depth of fill above culvert; width B_c ; projection of the culvert above natural surface, and the character of the fill material.

Weight of the empty culvert can be

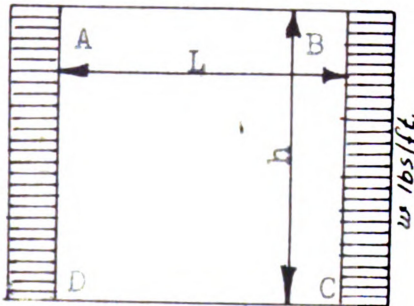
calculated very easily by assuming section.

Lateral pressure. Lateral pressure in the embankments are of two distinct types: active pressure and passive pressure. Active pressure on culverts is caused by the action of embankment in attempting to assume its natural condition of repose--that is its natural slope or angle of repose. Passive pressure is induced by the movement of structure against the supporting material. Its magnitude is a function of the amount of movement and also of the soil characteristics. Because of its indeterminate nature and changeable, uncertain magnitude, passive pressure is rarely depended upon to relieve stresses from more positive causes. Active pressure is direct function of vertical pressure, however, and may be estimated by fair accuracy (Rankine's Theory can be used). Active lateral pressures are assumed to be symmetrical on both sides of a culvert as care in back filling will permit large unbalanced pressures. Lateral pressure is computed in terms of equivalent fluid pressure. Its intensity depends on several factors, but is usually taken as one-third (angle of repose of the material 30 degrees) the vertical pressure at the point because it is more precise and safer usually. The load diagram is trapazoidal in shape and can be separated into uniform and triangular loading diagrams.

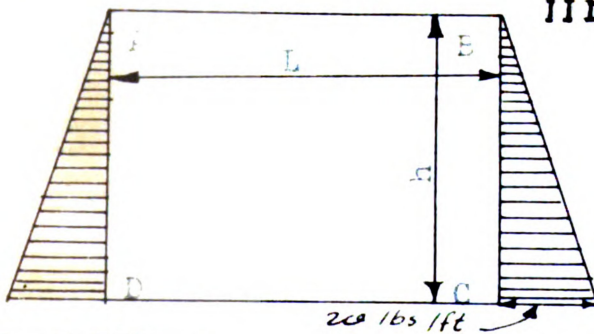
Above loading conditions can be classified as follows:



I. Uniform vertical load-total embankment and truck load in pounds per foot length of culvert.



II. Uniform lateral load (symmetrical on both sides) equal to lateral pressure at top of the culvert in pounds per square foot.



III. Triangular lateral load (symmetrical on both sides) equal to equivalent lateral unit pressure in pounds per square foot.

A combination of II and III will give any desired trapazoidal type of lateral loading.

III. ANALYSIS OF BOX CULVERT BY DIFFERENT METHODS

A. Virtual Work Method

(a) Rectangular Box Culvert with single cell:

Dimensions, moment of inertias of the component parts and loading are as shown in Fig. 1a.

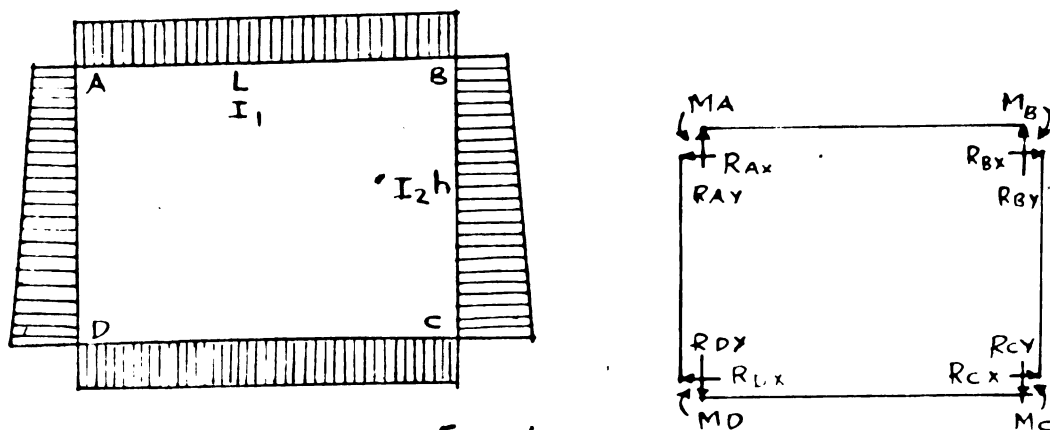


Fig. 1a.

Due to symmetrical loading

$$M_A = M_B \quad \text{and} \quad M_C = M_D$$

also from Fig. 1b.

$$R_A x h = \bar{W}h/3 + wh^2/2 + M_D - M_A \quad \text{and} \quad R_{Ay} = R_{By} = wL/2$$

Now relative change in slope at each corner will be zero,

$$\text{therefore,} \quad \theta_A = M.m.ds/EI = 0$$

which when simplified turns out to be as,

$$M_A + M_C = (\bar{W}h^2I_1 - wh^3I_1 - wL^3I_2)/6(LI_2 + hI_1) \dots (1)$$

Because horizontal deflection at any corner is zero,

$$\Delta h = M.m.ds/EI = 0$$

which can be easily simplified as,

$$M_C(2hI_1 - 3LI_2) + M_A hI_1 = 4\bar{W}h^2I_1/15 + wL^3I_2/4 + wh^3I_1/4 \dots (2)$$

Solving equations (1) and (2) simultaneously M_A and M_C can be determined.

If instead of rectangular box culvert there be a square one, then by putting $L = h$ and $I_1 = I_2$ two modified equations can be obtained which when solved will give the corner moments.

(b) Rectangular Box Culvert with two cells:

Dimensions, moment of inertias of the component parts of the culvert and loading are as shown in the Fig. 3a.

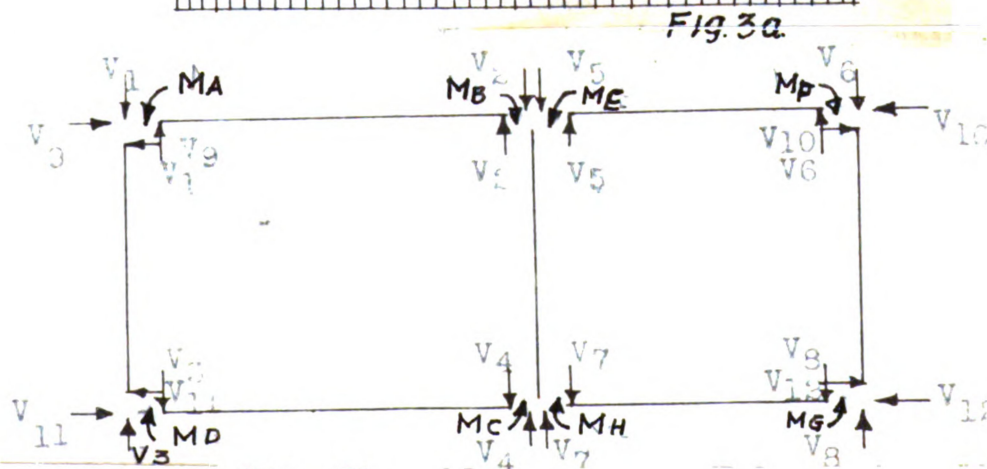
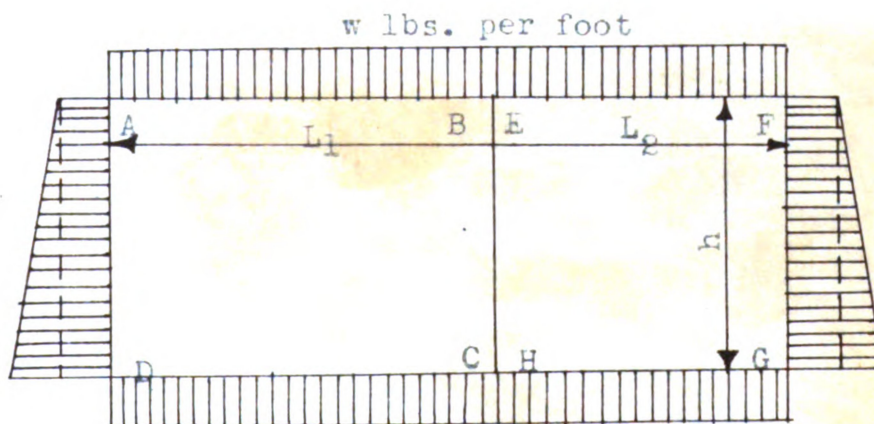


Fig. 3b

It can be seen from the Fig. 3b

that,

$$V_1 = V_3; \quad V_6 = V_8; \quad V_9 = V_{10}; \quad V_{11} = V_{12}$$

$$V_2 + V_5 = V_4 + V_7$$

Now,

$$V_1 L_1 = M_A + w \cdot L_1^2 / 2 - M_B$$

$$V_3 L_1 = M_D + w \cdot L_1^2 / 2 - M_C$$

Therefore,

$$M_A - M_D + M_C - M_B = 0 \dots \dots \dots (1)$$

Also,

$$V_5 L_2 = M_E + w.L_2^2/2 - M_F$$

$$V_7 L_2 = M_H + w.L_2^2/2 - M_G$$

And,

$$V_2 L_1 = M_B + w.L_1^2/2 - M_A$$

$$V_4 L_1 = M_C + w.L_1^2/2 - M_D$$

Therefore,

$$L_2 (M_B + M_C - M_A - M_D) = L_1 (M_E + M_H - M_F - M_G) \quad \dots \dots \dots (2)$$

Also,

$$V_9 h = M_A + w.h^2/2 + \bar{W}h/3 - M_D$$

$$V_{10} h = M_F + w.h^2/2 + \bar{W}h/3 - M_G$$

Now there are eight unknowns and the six additional equations may be expressed in terms of elastic energy. Since the relative horizontal displacement of the point A, at the left of the beam AB, and the point A, at the top of the member AD. is zero, Therefore, (see Fig. 4a & 4b)

$$\Delta_{A_x} = \int_A^A M.m.ds/EI = 0$$

Therefore,

$$\int_0^h M.y.dy/EI_2 + \int_0^{L_1} h.M.dy/EI_1 + \int_0^h M.y.dy/EI_2 = 0$$

Therefore,

$$- \int_0^h M_B y \cdot dy / EI_2 - (1/EI_1) \int_0^{L_1} (M_C h - wx^2 h/2 + V_4 x h) dx \\ - (1/EI_2) \int_0^h (M_A y - w \cdot h^2 y/2 - W \cdot y^4/3h + V_9 \cdot y^2) dy = 0$$

Which when simplified turns out to be,

$$(h^2/2EI_2) (M_A + M_B) + M_C h L_1 / I_1 + V_4 L_1^2 h / 2I_1 + V_9 h^3 / 3I_2 \\ - w \cdot L_1^3 h / 6I_1 - w \cdot h^4 / 4I_2 - W h^3 / 15I_2 = 0 \\ \dots \dots \dots (3)$$

Now the relative vertical displacement of the point A, will be zero,

Therefore from Fig. 5a and 5b.

$$\Delta_{A_y} = \int_A^A M \cdot m \cdot ds / EI = 0$$

Apply a unit vertical load at Point A,

Therefore,

$$\int_A^B M x \cdot ds / EI_1 + \int_B^C M \cdot L_1 \cdot ds / EI_2 + \int_C^D M \cdot x \cdot ds / EI_1 = 0$$

Therefore,

$$M_A L_1^2 / 2I_1 - w \cdot L_1^4 / 8I_1 + V_1 \cdot L_1^3 / 3I_1 + M_E L_1 h / I_2 \\ - w \cdot L_1^4 / 8I_1 + M_C L_1^2 / 2I_1 + V_4 L_1^3 / 3I_1 = 0$$

Therefore,

$$M_A L_1^2 / 2I_1 + M_C L_1^2 / 2I_1 + M_E L_1 h / I_2 + V_4 L_1^3 / 3I_1 \\ + V_1 L_1^3 / 3I_1 = w \cdot L_1^4 / 4I_1$$

$$\dots \dots \dots (4)$$

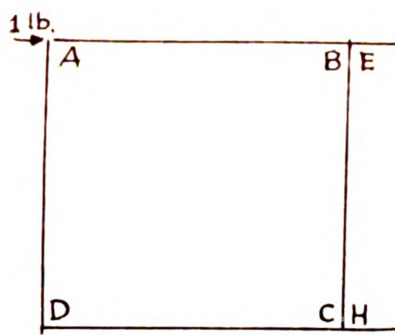


Fig. 4 a.

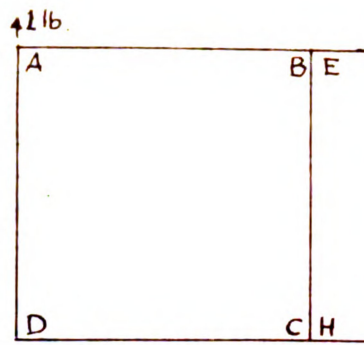


Fig. 5 a.

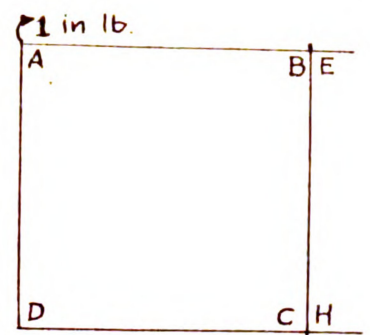


Fig. 6 a.

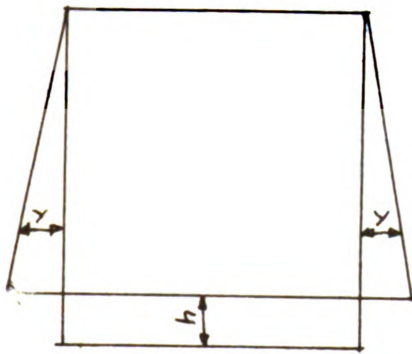


Fig. 4 b.

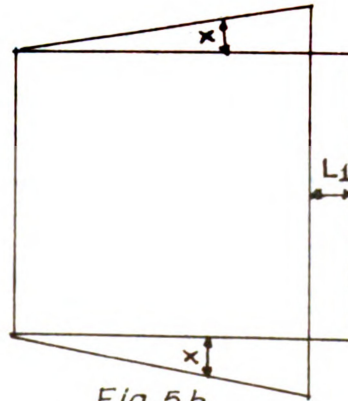


Fig. 5 b.

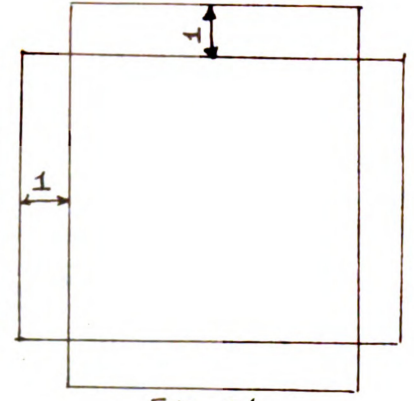


Fig. 6 b.

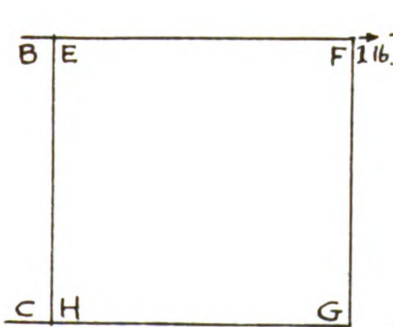


Fig. 7 a.

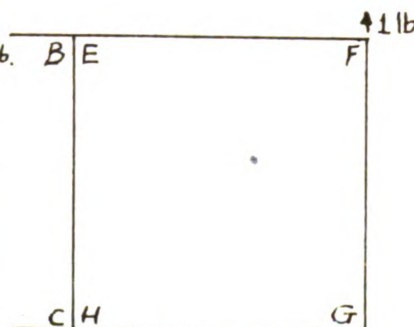


Fig. 8 a.

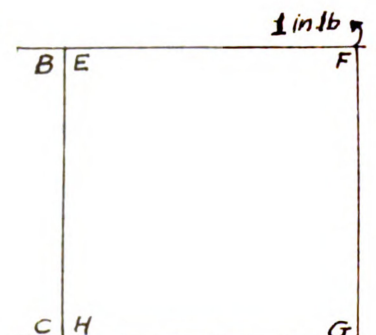


Fig. 9 a.

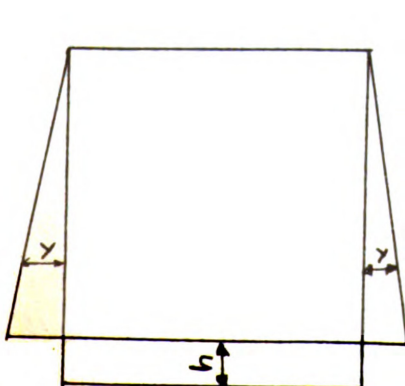


Fig. 7 b.

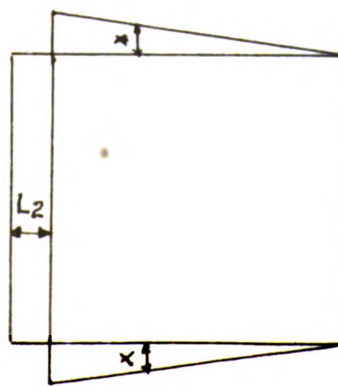


Fig. 8 b.

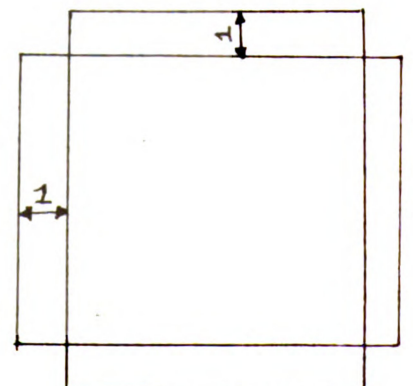


Fig. 9 b.

Now the relative change in the slope at point A, is zero.

Therefore,

$$\theta_A = \int_A^A M.m.ds/EI = 0$$

Apply a unit moment at point A, from Fig. 6a and 6b

Therefore,

$$M.ds/EI_1 + M.ds/EI_2 + M.ds/EI_1 + M.ds/EI_2 = 0$$

Therefore,

$$\begin{aligned} -\int M_A L_1^3/I_1 + w.L_1^2/6I_1 + v_1 L_1^2/2I_1 + M_B h/I_2 + M_C L_1/I_1 \\ + w.L_1^3/6I_1 - v_4 L_1^2/2I_2 = 0 \end{aligned}$$

Therefore,

$$\begin{aligned} (L_1/I_1)(M_A + M_C) + M_B h/I_2 + (L_1^2/2I_1)(v_1 + v_4) \\ = w.L_1^3/3I_1 \dots \dots \dots (5) \end{aligned}$$

Since the relative horizontal and vertical displacement and relative change in slope of the point F, at the right of the beam EF, and the point F, at the top of the member GF, are each separately equal to zero, therefore, following three equations can be obtained.

$$\begin{aligned} \Delta_{F_x} = \int_F^F M.m.ds/EI = 0 \\ \int_F^G M.y.ds/EI_2 + h \int_G^H M.ds/EI_3 + \int_H^E M.y.ds/EI_2 = 0 \end{aligned}$$

Therefore, from Fig. 7a and 7b.

$$\begin{aligned}
 & - M_F h^2 / 2I_2 + w.h^4 / 8I_2 + \bar{w}.h^3 / 15I_2 - v_{10} h^3 / 3I_2 \\
 & - M_G L_2^2 h / I_2 + w L_2^3 h / 6I_3 - v_8 L_2^2 h / 2I_3 - M_E h^2 / 2I_2 = 0
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 & (h^2 / 2I_2)(M_F + M_E) + M_G L_2^2 h / I_2 + v_{10} h^3 / 3I_2 + v_8 L_2^2 h / 2I_3 \\
 & = w.h^4 / 8I_2 + w.L_2^3 h / 6I_3 + \bar{w}h^3 / 15I_2 \\
 & \dots \dots \dots (6)
 \end{aligned}$$

Now,

$$\Delta_{F_y} = \int_F^F M.m.ds / EI = 0$$

Therefore, from Fig. 8a and 8b.

$$\int_G^H M.x.dx / EI_1 + L_2 \int_H^E M.ds / EI_2 + \int_E^F M.x.dx / EI_1 = 0$$

Therefore,

$$\begin{aligned}
 & - M_G L_2^2 / 2I_3 + w.L_2^4 / 8I_3 - v_8 L_2^3 / 3I_3 - M_E L_2^2 h / I_2 \\
 & + w.L_2^4 / 8I_3 - M_E L_2^2 / 2I_2 - v_5 L_2^2 / 3I_3 = 0
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 & (L_2^2 / 2I_3)(M_G + M_E) + M_E L_2^2 h / I_2 + (L_2^3 / 3I_3)(v_5 - v_8) \\
 & = w.L_2^4 / 4I_3 \dots \dots \dots (7)
 \end{aligned}$$

Now,

$$\Delta_F = \int_F^F M.m.ds / EI = 0$$

Therefore, from Fig. 9a and 9b.

$$\int_E^F M.ds/EI_1 + \int_F^G M.ds/EI_2 + \int_G^H M.ds/EI_1 + \int_H^E M.ds/EI_2 = 0$$

Therefore,

$$\begin{aligned} & - M_E L_2 / I_3 + w.L_2^3 / 6I_3 - V_5 L_2^2 / 2I_3 - M_F h / I_2 + w.h^3 / 6I_2 \\ & + \bar{w}h^2 / 12I_2 - V_{10} h^2 / 2I_2 - M_G L_2 / I_3 + w.L_2^3 / 6I_3 - V_8 L_2^2 / 2I_3 \\ & - M_E h / I_2 = 0 \end{aligned}$$

Therefore,

$$\begin{aligned} & (L_2 / I_3)(M_G + M_E) + (h / I_2)(M_F + M_E) + (L_2^2 / 2I_3)(V_5 + V_8) \\ & + V_{10} h^2 / 2I_2 = w.L_2^3 / 3I_3 + w.h^3 / 6I_2 + \bar{w}h^2 / 12I_2 \\ & \dots \dots \dots (8) \end{aligned}$$

By solving the above eight equations simultaneously, bending moment at each corner can be determined.

In this problem loading is general one and if any of these loading is missing then putting for that particular loading equal to zero new modified eight equations can be obtained from the above equations, and when these new equations will be solved simultaneously they will give moment at each corner of the culvert for that particular type of loading.

B. Moment Area Method

(a) Rectangular Box Culvert with single cell:

Dimensions, moment of inertias of component parts and loading are as shown in Fig. 1a.

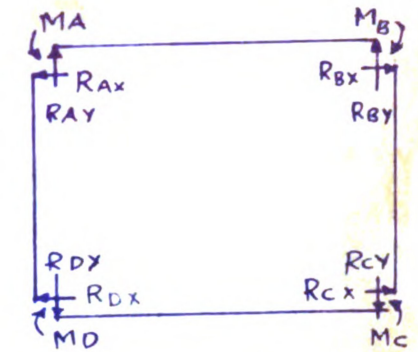
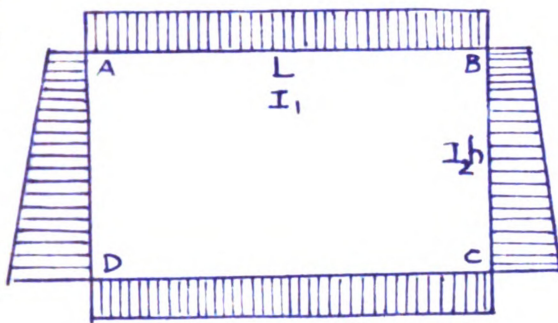


Fig. 1a.

Due to symmetrical loading, $M_A = M_B$ and $M_C = M_D$.

Now relative change in slope at any corner will be zero, therefore, the sum of the area under M/EI curve will be zero, which gives

$$M_A + M_C = (\bar{W}h^2I_1 + wL^3I_2 + wh^3I_1)/6(LI_2 + hI_1) \dots (1)$$

Also relative deflection at any corner is zero, therefore, statical moment of the area under M/EI curve about the corner will be zero.

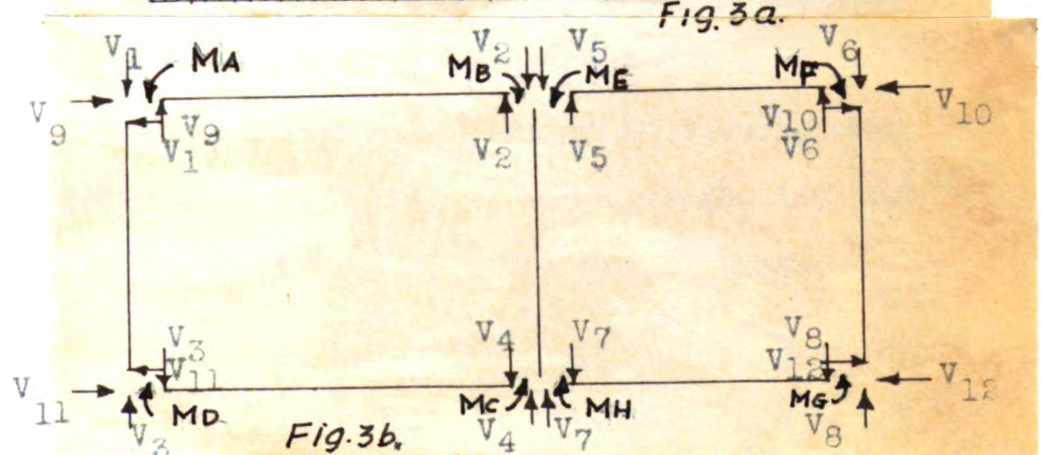
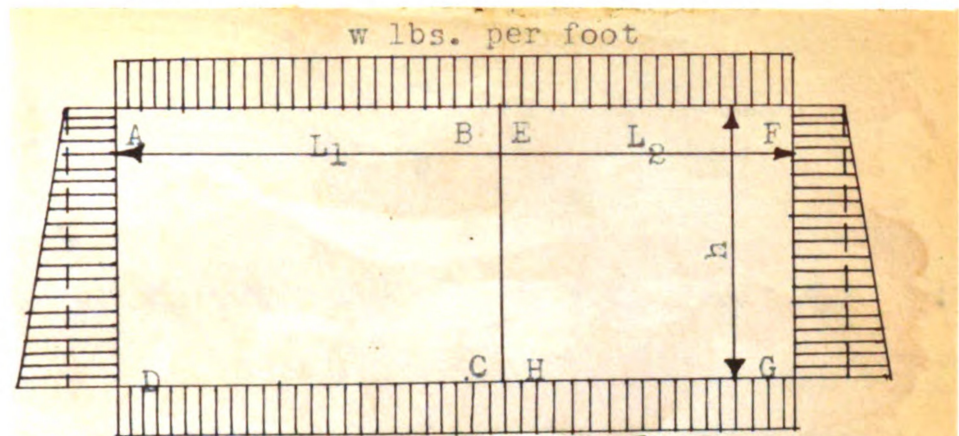
This will give

$$M_C(3LI_2 + 2hI_1) + M_AhI_1 = 4\bar{W}h^2I_1/15 + wL^3I_2/4 + wh^3I_1/4 \dots (2)$$

Solving equations (1) and (2) simultaneously M_A and M_C can be determined.

(b) Rectangular Box Culvert with Two Cells:

Dimensions, moment of inertias of the components parts of the culvert and loading are as shown in the Fig. 3a



It can be seen from the Fig. 3b

that,

$$V_9 = V_{10}; \quad V_{11} = V_{12}$$

$$V_1 = V_3; \quad V_6 = V_8; \quad V_2 + V_5 = V_4 + V_7.$$

Now,

$$V_1 L_1 = M_A + w \cdot L_1^2 / 2 - M_B$$

$$V_3 L_1 = M_D + w \cdot L_1^2 / 2 - M_C$$

Therefore,

$$M_A - M_D + M_C - M_B = 0 \quad \dots \dots \dots (1)$$

Also,

$$V_5 L_2 = M_E + w \cdot L_2^2 / 2 - M_F$$

$$V_7 L_2 = M_H + w \cdot L_2^2 / 2 - M_G$$

And,

$$V_2 L_1 = M_B + w \cdot L_1^2 / 2 - M_A$$

$$V_4 L_1 = M_C + w \cdot L_1^2 / 2 - M_D$$

Therefore,

$$L_2 (M_B + M_C - M_A - M_D) = L_1 (M_E + M_H - M_F - M_G) \quad \dots \dots \dots (2)$$

Now there are eight unknowns and the six additional equations may be expressed in terms of elastic energy. Since the relative horizontal displacement of the point A, at the left of the beam AB, and the point A, at the top of the member AD, is zero,

$$\text{therefore, } \Delta_{Ax} = \int_A^A m_1 \cdot M \cdot ds / F_1 \cdot EI = 0$$

(See Figures 4a and 4b)

Therefore,

$$\begin{aligned} & \int_B^C \frac{F_1 \cdot y \cdot M \cdot dy}{E \cdot I_2 \cdot F_1} + \int_C^D \frac{h \cdot F_1 \cdot M \cdot dy}{F_1 \cdot EI_1} + \int_D^A \frac{F_1 \cdot y \cdot M \cdot dy}{F_1 \cdot EI_2} = 0 \\ & \left[\frac{\text{Area} \cdot \bar{y}}{EI_2} \right]_B^C + h \left(\frac{\text{Area}}{EI_1} \right) \left[\right]_C^D + \left[\frac{\text{Area} \cdot \bar{y}}{EI_2} \right]_D^A = 0 \\ & \frac{1}{I_2} \left[M_H (h/2) (2h/3) + M_E (h/2) (h/3) + M_B (h/3) (h/3) + M_C (h/2) (2h/3) \right] \\ & \quad (h/I_1) \left[-(M_D + M_C) L_1 / 2 + w \cdot L_1^3 / 12 \right] \\ & \quad - (1/I_2) (M_D \cdot h^2 / 3 + M_A \cdot h^2 / 6) + w \cdot h^3 / 12 \cdot I_2 \cdot 2 + w h^2 \cdot 8h / 12 \cdot 15 I_2 \\ & \quad = 0 \end{aligned}$$

Therefore,

$$\begin{aligned}
 & M_H \cdot h^2 / 3I_2 + M_E \cdot h^2 / 6I_2 - M_B \cdot h^2 / 6I_2 - M_E \cdot h^2 / 3I_2 \\
 & - M_C \cdot hL_1 / 2I_1 - M_D \cdot hL_1 / 2I_1 - M_D \cdot h^2 / 3I_2 - M_A \cdot h^2 / 6I_2 \\
 & + w \cdot L_1^3 \cdot h / 12I_2 + w \cdot h^4 / 24I_2 + 2\bar{w} \cdot h^3 / 45I_2 = 0 \\
 & \dots \dots \dots (3)
 \end{aligned}$$

From Figures 5a and 5b.

$$\Delta_{Ay} = \int_A^A m_2 \cdot M \cdot ds / F_2 \cdot EI = 0$$

which is equal to,

$$\text{Area} \cdot \bar{x} / I_1 \Big]_A^B + (L_1 / I_2) \text{Area} \Big]_B^C + \text{Area} \cdot \bar{x} / I_2 \Big]_C^D = 0$$

which when simplified turns out into,

$$\begin{aligned}
 & M_E \cdot h \cdot L_1 / 2I_2 + M_H \cdot h \cdot L_1 / 2I_2 - M_B \cdot h \cdot L_1 / 2I_2 - M_C \cdot h \cdot L_1 / 2I_2 \\
 & - M_A \cdot L_1^2 / 6I_1 - M_B \cdot L_1^2 / 3I_1 - M_C \cdot L_1^2 / 6I_1 - M_D \cdot L_1^2 / 3I_1 \\
 & + w \cdot L_1^3 / 12I_1 = 0 \\
 & \dots \dots \dots (4)
 \end{aligned}$$

From figures 6a and 6b.

$$\Theta_A = \int_A^A m \cdot M \cdot ds / M \cdot EI = 0$$

That is sum of the area of moment diagram divided by I is zero.

Which will turn out as follows,

$$\begin{aligned}
 & M_A (hI_1 - L_1 I_2) / 2I_1 I_2 - M_B (L_1 I_2 + hI_1) / 2I_1 I_2 \\
 & M_D (hI_1 - L_1 I_2) / 2I_1 I_2 - M_C (L_1 I_2 + hI_1) / 2I_1 I_2 \\
 & (L_1 h / 2I_2) (M_E + M_H) + w \cdot L_1^3 / 6I_1 + w \cdot h^3 / 12I_2 + \bar{w} h^2 / 12I_2 = 0 \\
 & \dots \dots \dots (5)
 \end{aligned}$$

From the Figures 7a and 7b.

$$\Delta_{Fx} = \int_F^F m_3 \cdot M \cdot da / F_3 \cdot EI = 0$$

Therefore,

$$\text{Area} \cdot \bar{y} / I_2 \Big]_F^G + (h / I_3) \text{Area} \Big]_G^H + \text{Area} \cdot \bar{y} / I_2 \Big]_H^E = 0$$

Which when simplified turns out to be,

$$\begin{aligned} & (h^2 / 6I_2) (2M_B + M_C - 2M_E - M_F + 2M_G - M_H) \\ & - (L_2 h / 2I_3) (M_G + M_H) + w \cdot L_2^3 h / 12I_1 + w \cdot h^4 / 12I_2 \\ & + 2Wh^3 / 45I_2 = 0 \end{aligned}$$

. (6)

From the Figures 8a and 8b.

$$\Delta_{Fy} = \int_F^F m_4 \cdot M \cdot ds / F_4 \cdot EI = 0$$

Therefore,

$$\text{Area} \cdot \bar{x} / I_3 \Big]_G^H + (L_2 / I_2) \text{Area} \Big]_H^E + \text{Area} \cdot \bar{x} / I_3 \Big]_E^F = 0$$

Which when simplified turns out to be,

$$\begin{aligned} & (L_2 \cdot h / 2I_2) (M_B + M_C - M_E - M_H) + w \cdot L_2^4 / 12I_3 \\ & - (L_2^2 / 6I_3) (M_E + 2M_F + M_G + 2M_H) = 0 \end{aligned}$$

. (7)

From the Figures 9a and 9b.

$$\mathcal{O}_F = \int_F^F m_6 \cdot M \cdot ds / M \cdot EI = 0$$

That is area of moment diagram divided by I is equal to zero.

Which will turn out as follows,

$$\begin{aligned}
 & (h/2I_2)(M_B + M_C - M_E - M_F - M_G - M_H) + \bar{w}h^2/12I_2 \\
 & - (L_2/12I_3)(M_E + M_F + M_G - M_H) + w.h^3/12I_2 + w.L_2^3/6I_3 = 0 \\
 & \dots \dots \dots (8)
 \end{aligned}$$

By solving the above eight equations simultaneously bending moments at each corner can be determined.

In this problem general loading is taken but if any of these loadings is missing then putting for that particular loading equal to zero new modified eight equations can be obtained from these above eight equations, which when solved simultaneously will give bending moments at each corner of the culvert for that particular type of loading.

C. Principle of Least Work.

(a) Rectangular Box Culvert with Single Cell:

Dimensions, moment of inertias and load are as shown in the Fig. 1.

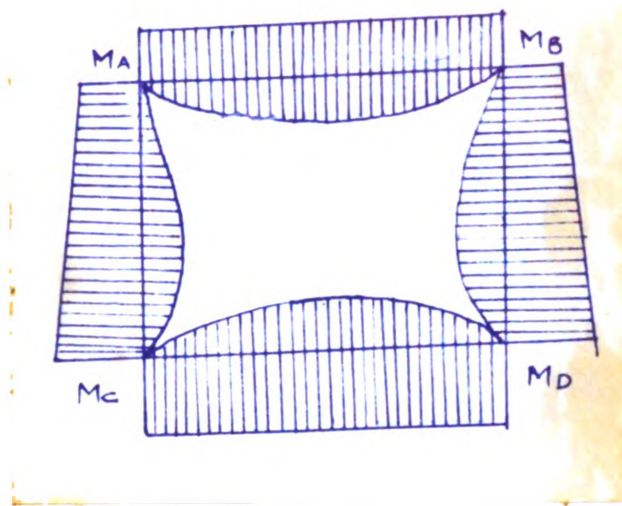


Fig. 1.

Total internal work in a structure are given by

$$U = (A_s \cdot \bar{Y}_s + A_1 \cdot \bar{Y}_1 + A_s \cdot \bar{Z})/EI$$

The constant term $A_s \cdot \bar{Y}_s/EI$ will drop out at the time of differentiation and, hence, can be dropped from the very beginning due to symmetrical loading $M_A = M_B$ and $M_C = M_D$.

Therefore, $U = A_1 Y_1/EI + A_s \bar{Z}/EI$

Now,

$$\frac{A\bar{Y}_1}{I} = \frac{MA^2L}{2I_1} + \frac{Mc^2L}{2I_2} + \frac{MA^2h}{I_2} + (MA Mc - 2MA + Mc^2) \frac{h}{3I_2}$$

$$As\bar{Z} = \frac{WL^3}{6I_1} MA + \frac{Wh^3}{12I_2} (MA + Mc) + \frac{Wh^2(7MA + 8Mc)}{90I_2}$$

$\frac{OU}{OMA} = 0$ which when solved will be,

$$\begin{aligned} MA(3LI_2 + 2hI_1) + MchI_1 - \frac{wh^3I_2}{4} - \frac{wh^3I_1}{4} \\ - 7 \frac{Wh^2I_2}{30} = 0 \dots \dots \dots (1) \end{aligned}$$

Also,

$\frac{dU}{dMc} = 0$, which when solved will be,

$$\begin{aligned} Mc(3LI_2 + 2hI_1) + MA hI_1 - \frac{wL^3I_2}{4} - \frac{wh^3I_1}{4} \\ - 8 \frac{Wh^2I_1}{30} = 0 \dots \dots \dots (2) \end{aligned}$$

By solving these two equations MA and Mc can be determined.

If the culvert is square instead of rectangular, then replacing h by L and I_2 by I_1 , moment at each corner can be very easily determined.

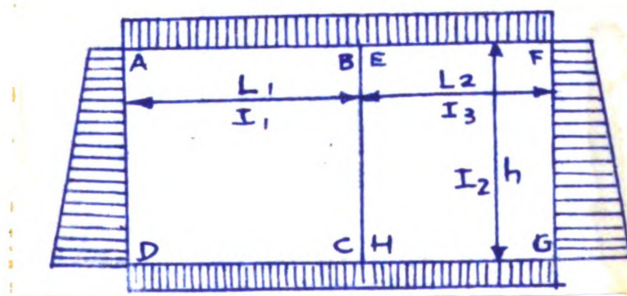


Fig. 2.

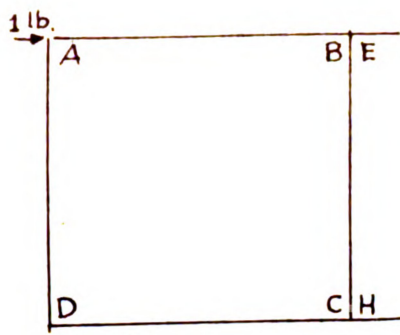


Fig. 4 a.

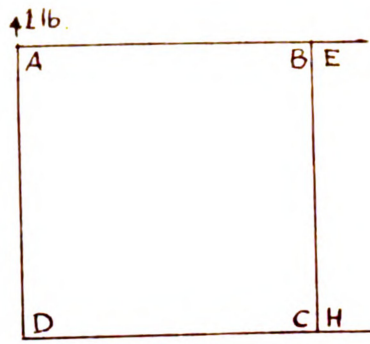


Fig. 5 a.

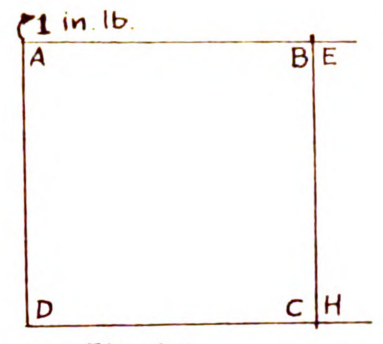


Fig. 6 a.

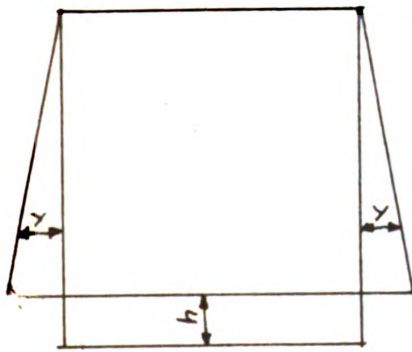


Fig 4 b

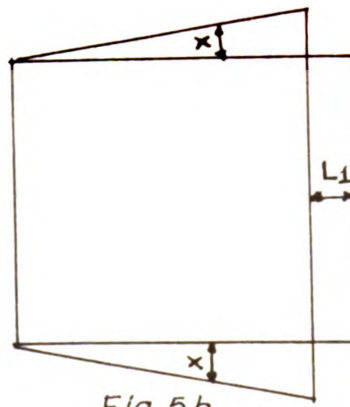


Fig. 5 b.

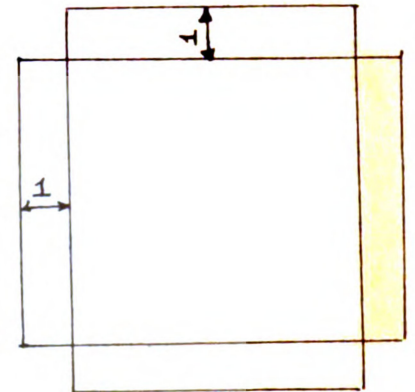


Fig. 6 b.

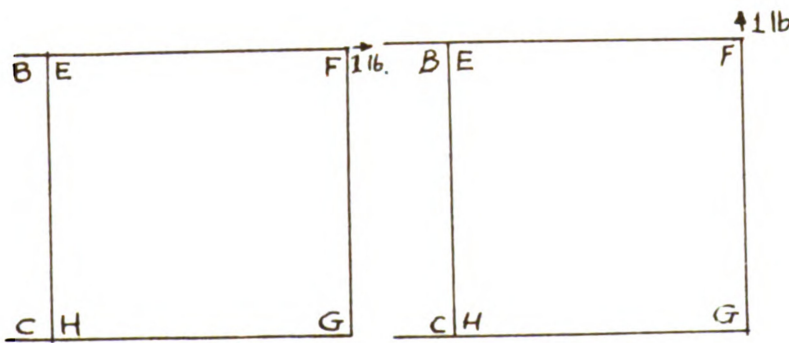


Fig. 7 a.

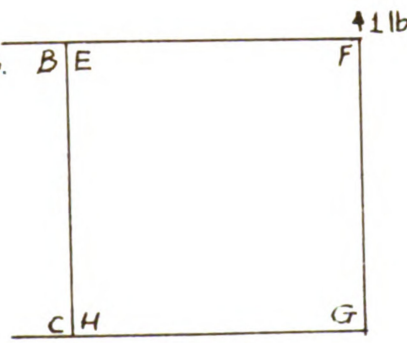


Fig. 8 a.

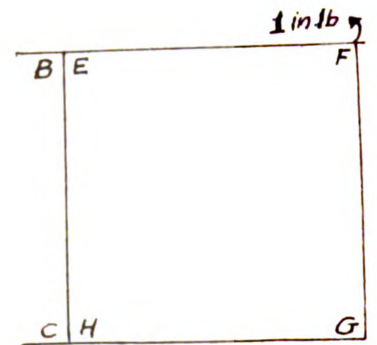


Fig. 9 a.

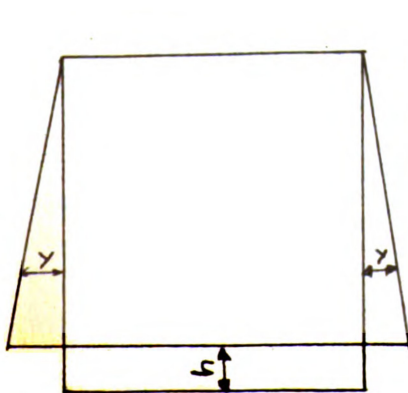


Fig 7b

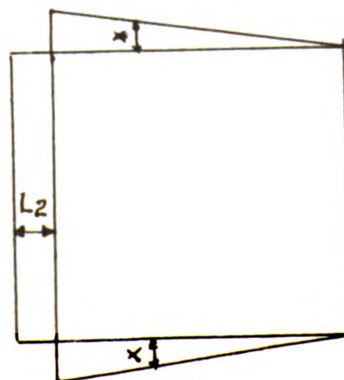


Fig. 8 b.

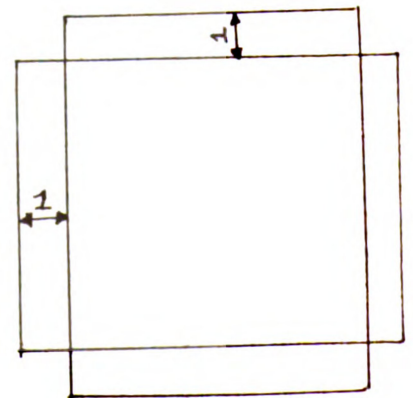


Fig. 9 b.

(b) Rectangular Box Culverts with Two Cells:

Case 1: Dimensions and sections and loading are as shown in the Fig. 2.

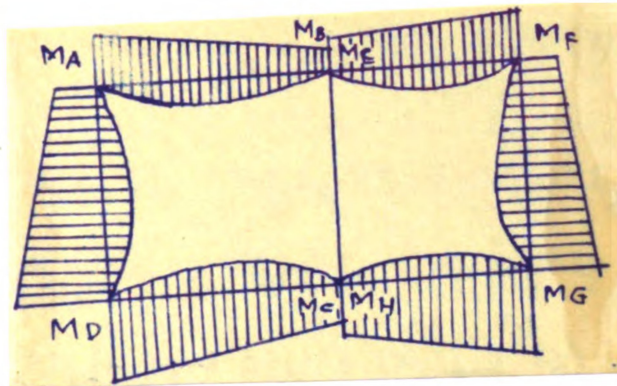


Fig. 2a.

Total internal work in structure is given by $U = (A_s \cdot \bar{y}_s + A_i \cdot \bar{y}_i + A_s \cdot \bar{z})EI$ (See Appendix)
Dropping the constant term $A_s \cdot \bar{y}_i/EI$ and E , the total internal work then will be,

$$U = (A_i \cdot \bar{y}_i/I - A_s \cdot \bar{z}/I)$$

Now,

$$\begin{aligned} A_i \cdot \bar{y}_i/I &= M_B^2 \cdot L_2/2I_2 + (L_2/6I_2)(M_A M_B + M_A^2 - 2M_B^2) \\ &+ M_E^2 L_1/2I_1 + (L_1/6I_1)(M_F M_E + M_F^2 - 2M_E^2) \\ &+ M_H^2 L_1/2I_1 + (L_1/6I_1)(M_H M_G + M_H^2 - 2M_G^2) \\ &+ M_C^2 L_2/2I_2 + (L_2/6I_2)(M_C M_D + M_D^2 - 2M_C^2) \end{aligned}$$

$$\begin{aligned}
& + M_A^2 h / 2I_3 + (h/6I_3) (M_A M_D + M_D^2 - 2M_A^2) \\
& + M_F^2 h / 2I_3 + (h/6I_3) (M_F M_G + M_G^2 - 2M_F^2)
\end{aligned}$$

And,

$$\begin{aligned}
As. \bar{z}/I &= (w.L_2^3/12)(M_A + M_B)/2I_2 + w.h^3(M_A + M_D)/24I_3 \\
& + (w.L_1^3/12)(M_F + M_E)/2I_1 + w.h^3(M_F + M_G)/24I_3 \\
& + (w.L_2^3/12)(M_D + M_C)/2I_2 + \bar{w}h^2(7M_A + 8M_D)/180I_3 \\
& + (w.L_1^3/12)(M_H + M_G)/2I_1 + \bar{w}h^2(7M_F + 8M_G)/180I_3
\end{aligned}$$

Therefore,

$$\begin{aligned}
dU/dM_A &= M_B L_2 / 6I_2 + 2M_A L_2 / 6I_2 + 2M_A h / 2I_3 \\
& + M_D h / 6I_3 + 4M_A h / 6I_3 + w.L_2^3 / 24I_2 = 0 \\
& = 2M_A (L_2 I_3 + hI_2) + (M_B L_2 + I_3) \\
& + M_D h I_2 - w.L_2^3 \cdot I_3 / 4 + \frac{w.h^3 I_2}{4} + \frac{7\bar{w}h^2 I_2}{30} = 0 \dots (1)
\end{aligned}$$

$$\begin{aligned}
dU/dM_B &= 2M_B L_2 / 2I_2 + M_A L_2 / 6I_2 - 4M_B L_2 / 6I_2 \\
& + w.L_2^3 / 24I_2 = 0 \\
& = 2M_B + M_A + w.L_2^2 / 4 = 0 \dots \dots \dots (2)
\end{aligned}$$

$$\begin{aligned}
dU/dM_C &= 2M_C L_2 / 2I_2 + M_D L_2 / 6I_2 - 4M_C L_2 / 6I_2 \\
& + w.L_2^3 / 24I_2 = 0 \\
& = 2M_C + M_D + w.L_2^2 / 4 = 0 \dots \dots \dots (3)
\end{aligned}$$

$$\begin{aligned}
dU/dM_D &= M_C L_2 / 6I_2 + 2M_D L_2 / 6I_2 + M_A h / 6I_3 \\
& + 2M_D h / 6I_3 + w.L_2^3 / 24I_2 = 0 \\
& = 2M_D (L_2 I_3 + hI_2) + M_C L_2 I_3 \\
& + M_A h I_2 + w.L_2^3 I_3 / 4 + \frac{w.h^3 I_2}{4} + \frac{8\bar{w}h^2 I_2}{30} = 0 \dots (4)
\end{aligned}$$

$$\begin{aligned}
dU/dM_E &= 2M_E L_1 / 2I_1 + M_F L_1 / 6I_1 - 4M_E L_1 / 6I_1 \\
& + w.L_1^3 / 24I_1 = 0 \\
& = 2M_E + M_F + w.L_1^2 / 4 = 0 \dots \dots \dots (5)
\end{aligned}$$

1. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

2. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

3. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

4. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

5. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

6. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

7. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

8. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

9. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

10. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

11.

12. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

13. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

14.

15. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

16. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

17. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

18. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

19. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

20. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

21. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

22. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

23. $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$$\begin{aligned}
dU/dM_F &= M_E L_1 / 6I_1 + 2M_F L_1 / 6I_1 + 2M_F h / 2I_3 \\
&+ M_G h / 6I_3 + 4M_F h / 6I_3 + w.L_1^3 / 24I_1 = 0 \\
&= 2M_F (L_1 I_3 + h I_1) + M_E L_1 I_3 \\
&+ M_G h I_2 + w.L_1^3 I_3 / 4 + \frac{w.h^3 I_2}{4} + \frac{7\bar{w}h^2 I_1}{30} \\
&= 0 \dots \dots (6)
\end{aligned}$$

$$\begin{aligned}
dU/dM_G &= M_H L_1 / 6I_1 + 2M_G L_1 / 6I_1 + M_F h / 6I_3 \\
&+ 2M_G h / 6I_3 + w.L_1^3 / 24I_1 = 0 \\
&= 2M_G (L_1 I_3 + h I_1) + M_H L_1 I_3 \\
&+ M_F h I_1 + w.L_1^3 I_3 / 4 + \frac{w.h^3 I_1}{4} + \frac{8\bar{w}h^2 I_1}{30} \\
&\dots \dots (7)
\end{aligned}$$

$$\begin{aligned}
dU/dH &= 2M_H L_1 / 2I_1 + M_G L_1 / 6I_1 - 4M_H L_1 / 6I_1 \\
&+ w.L_1^3 / 24I_1 = 0 \\
&= 2M_H + M_G + w.L_1^2 / 4 = 0 \dots \dots (8)
\end{aligned}$$

By solving above eight equations simultaneously required bending moments can be obtained.

If the culvert is rectangular box culvert with two cells having two rectangles of equal size, then bending moments at corners can be very easily calculated by making L_2 equal to L_1 and I_2 equal to I_1 and then substituting these values in the above equations.

If instead, there are two square cells, the moments can be determined very easily by making L_2 equal to L_1 equal to h , and I_1 equal I_2 equal I_3 , and substituting these values in the above equations and then solving them simultaneously.

D. Slope-Deflection Method.

Rectangular Box Culvert with single cell:

Dimensions, moment of inertias of the component parts of the culvert and loading are as shown in the Fig. 1.



Fixed end moments can be easily determined and they are as follows with their proper signs.

$$M_{AB} = + w.L^2/12 ; \quad M_{BA} = - w.L^2/12$$

$$M_{BC} = + \bar{W}.h/15 ; \quad M_{CB} = - \bar{W}.h/10$$

$$M_{CD} = + w.L^2/12 ; \quad M_{DC} = - w.L^2/12$$

$$M_{DA} = + \bar{W}.h/10 ; \quad M_{AD} = - \bar{W}.h/15$$

K - values will be:

$$K_1 = I_1/L \quad \text{and} \quad K_2 = I_2/h$$

Slope-deflection equations will be:

$$M_{AB} = -2.E.K_1(2\theta_A + \theta_B) + w.L^2/12.$$

$$M_{BA} = -2.E.K_1(2\theta_B + \theta_A) - w.L^2/12.$$

$$M_{BC} = -2.E.K_2(2\theta_B + \theta_C) + \bar{W}.h/15.$$

$$M_{CB} = -2.E.K_2(2\theta_C + \theta_B) - \bar{W}.h/10.$$

$$M_{CD} = -2.E.K_1(2\theta_C + \theta_D) + w.L^2/12.$$

$$M_{DC} = -2.E.K_1(2\theta_D + \theta_C) - w.L^2/12.$$

$$M_{DA} = -2.E.K_2(2\theta_D + \theta_A) + \bar{W}.h/10.$$

$$M_{AD} = -2.E.K_2(2\theta_A + \theta_D) - \bar{W}.h/15.$$

Now,

$$M_{AB} + M_{AD} = 0 \quad (1)$$

$$M_{BA} + M_{BC} = 0 \quad (2)$$

$$M_{CB} + M_{CD} = 0 \quad (3)$$

$$M_{DA} + M_{DC} = 0 \quad (4)$$

When the values from the slope-deflections equations will be substituted in the above four equations and when simplified, they will be as follows:

$$2E \left[2.\theta_A(K_1 + K_2) + K_1\theta_B + K_2\theta_D \right] = w.L^2/12 - \bar{W}.h/15. \quad \dots \dots \dots (1)$$

$$2E \left[2.\theta_B(K_1 + K_2) + K_1\theta_A + K_2\theta_C \right] = \bar{W}.h/15 - w.L^2/12. \quad \dots \dots \dots (2)$$

$$2E \left[2.\theta_C(K_1 + K_2) + K_1\theta_D + K_2\theta_B \right] = w.L^2/12 - \bar{W}.h/10. \quad \dots \dots \dots (3)$$

$$2E \left[2.\theta_D(K_1 + K_2) + K_1\theta_C + K_2\theta_A \right] = \bar{W}.h/10 - w.L^2/12 \quad \dots \dots \dots (4)$$

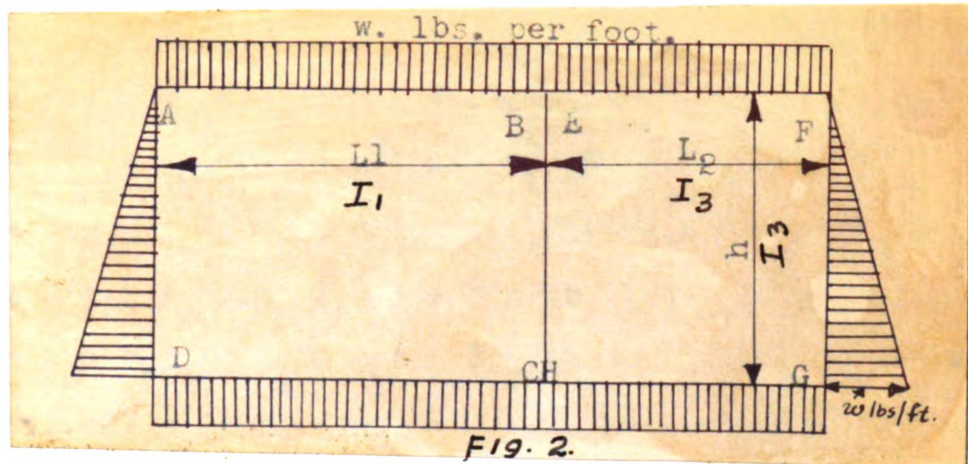
From these above four equations values of θ_A , θ_B , θ_C , θ_D , can be determined and from the values of the above quantities, moments at the corner of culvert can be calculated.

If there is uniform lateral load w pounds per foot height of the culvert instead of the above discussed loading, then replacing $\bar{W}.h/15$ and $\bar{W}.h/10$ each by $w.h^2/12$ with proper signs, and putting w , the uniform vertical load equal to zero, four revised equations can be obtained and from which end moments can be determined the same way in the above case.

If instead of a rectangular box culvert it is a square one, then by making $h = L$ and $I_1 = I_2$ in the above four equations, unknown "Thetas" can be obtained and, hence, corner moments of the culvert.

Rectangular Box Culvert with two cells:

Dimensions, moment of inertias of the component parts of culvert and loading are as shown in the Fig. 2.



Fixed end moments can be easily determined and they are as follows with proper signs:

$$\begin{aligned}
 M_{AB} &= w.L_1^2/12 & M_{BA} &= -w.L_1^2/12 \\
 M_{EF} &= w.L_2^2/12 & M_{FE} &= -w.L_2^2/12 \\
 M_{FG} &= \bar{W}.h/15 & M_{GF} &= -\bar{W}.h/10 \\
 M_{GH} &= w.L_2^2/12 & M_{HG} &= -w.L_2^2/12 \\
 M_{CD} &= w.L_1^2/12 & M_{DC} &= -w.L_1^2/12 \\
 M_{DA} &= \bar{W}.h/10 & M_{AD} &= -\bar{W}.h/15
 \end{aligned}$$

K - values will be:

$$K_1 = I_1/L_1 ; \quad K_2 = I_2/h ; \quad K_3 = I_3/L_2$$

Therefore,

the slope-deflection equations will be,

$$\begin{aligned}
 M_{AB} &= -2EK_1(2.\theta_A + \theta_B) + w.L_1^2/12 \\
 M_{BA} &= -2EK_1(2.\theta_B + \theta_A) - w.L_1^2/12
 \end{aligned}$$

$$M_{EF} = -2.E.K_3(2.\theta_E + \theta_F) + w.L_2^2/12$$

$$M_{FE} = -2.E.K_3(2.\theta_F + \theta_E) - w.L_2^2/12$$

$$M_{FG} = -2.E.K_2(2.\theta_F + \theta_G) - \bar{W}.h/15$$

$$M_{GF} = -2.E.K_2(2.\theta_G + \theta_F) + \bar{W}.h/10$$

$$M_{GH} = -2.E.K_3(2.\theta_G + \theta_H) - w.L_2^2/12$$

$$M_{HG} = -2.E.K_3(2.\theta_H + \theta_G) + w.L_2^2/12$$

$$M_{CD} = -2.E.K_1(2.\theta_C + \theta_D) + w.L_1^2/12$$

$$M_{DC} = -2.E.K_1(2.\theta_D + \theta_C) - w.L_1^2/12$$

$$M_{DA} = -2.E.K_2(2.\theta_D + \theta_A) + \bar{W}.h/10$$

$$M_{AD} = -2.E.K_2(2.\theta_A + \theta_D) - \bar{W}.h/15$$

$$M_{BC} = -2.E.K_2(2.\theta_B + \theta_C)$$

$$M_{CB} = -2.E.K_2(2.\theta_C + \theta_B)$$

Now,

$$M_{AB} + M_{AD} = 0 \quad \dots \dots \dots (1)$$

$$M_{DA} + M_{DC} = 0 \quad \dots \dots \dots (2)$$

$$M_{FE} + M_{FG} = 0 \quad \dots \dots \dots (2)$$

$$M_{FE} + M_{FG} = 0 \quad \dots \dots \dots (3)$$

$$M_{GF} + M_{GH} = 0 \quad \dots \dots \dots (4)$$

$$M_{BA} + M_{BC} + M_{EF} = 0 \quad \dots \dots \dots (5)$$

$$M_{HG} + M_{CD} + M_{CB} = 0 \quad \dots \dots \dots (6)$$

$$\theta_B = -\theta_E \quad \dots \dots \dots (7)$$

$$\theta_C = -\theta_H \quad \dots \dots \dots (8)$$

When the values from the slope-deflection equations will be substituted in these above eight equations, they will be:

$$2.E. [2.\theta_A(K_1 + K_2) + K_1\theta_B + K_2\theta_D] = w.L_1^2/12 - \bar{W}.h/15 \quad \dots \dots \dots (1)$$

$$2.E. [2.\theta_D(K_1 + K_2) + K_1\theta_C + K_2\theta_A] = \bar{W}.h/10 - w.L_1^2/12 \quad \dots \dots \dots (2)$$

$$2.E. [2.\theta_F(K_2 + K_3) + K_2\theta_E + K_3\theta_G] = \bar{W}.h/15 - w.L_2^2/12 \quad \dots \dots \dots (3)$$

$$2.E. [2.\theta_G(K_2 + K_3) + K_2\theta_F + K_3\theta_H] = w.L_2^2/12 - \bar{W}.h/10 \quad \dots \dots \dots (4)$$

$$2.E. [2.\theta_B(K_1 + K_2) + K_1\theta_A + K_2\theta_C + K_3(2\theta_E + \theta_F)] = w.L_2^2/12 - w.L_1^2/12 \quad \dots \dots \dots (5)$$

$$2.E. [2.\theta_C(K_1 + K_2) + K_1\theta_D + K_2\theta_B + K_3(2\theta_H + \theta_G)] = w.L_1^2/12 - w.L_2^2/12 \quad \dots \dots \dots (6)$$

And,

$$\theta_B = -\theta_E \quad \dots \dots \dots (7)$$

$$\theta_C = -\theta_H \quad \dots \dots \dots (8)$$

From the above eight equations when solved simultaneously, eight unknown Theta-values can be obtained. The values of slope thus obtained can be used in finding the moments at each corner of the culvert.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255	256	257	258	259	260	261	262	263	264	265	266	267	268	269	270	271	272	273	274	275	276	277	278	279	280	281	282	283	284	285	286	287	288	289	290	291	292	293	294	295	296	297	298	299	300	301	302	303	304	305	306	307	308	309	310	311	312	313	314	315	316	317	318	319	320	321	322	323	324	325	326	327	328	329	330	331	332	333	334	335	336	337	338	339	340	341	342	343	344	345	346	347	348	349	350	351	352	353	354	355	356	357	358	359	360	361	362	363	364	365	366	367	368	369	370	371	372	373	374	375	376	377	378	379	380	381	382	383	384	385	386	387	388	389	390	391	392	393	394	395	396	397	398	399	400	401	402	403	404	405	406	407	408	409	410	411	412	413	414	415	416	417	418	419	420	421	422	423	424	425	426	427	428	429	430	431	432	433	434	435	436	437	438	439	440	441	442	443	444	445	446	447	448	449	450	451	452	453	454	455	456	457	458	459	460	461	462	463	464	465	466	467	468	469	470	471	472	473	474	475	476	477	478	479	480	481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	607	608	609	610	611	612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	667	668	669	670	671	672	673	674	675	676	677	678	679	680	681	682	683	684	685	686	687	688	689	690	691	692	693	694	695	696	697	698	699	700	701	702	703	704	705	706	707	708	709	710	711	712	713	714	715	716	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	796	797	798	799	800	801	802	803	804	805	806	807	808	809	810	811	812	813	814	815	816	817	818	819	820	821	822	823	824	825	826	827	828	829	830	831	832	833	834	835	836	837	838	839	840	841	842	843	844	845	846	847	848	849	850	851	852	853	854	855	856	857	858	859	860	861	862	863	864	865	866	867	868	869	870	871	872	873	874	875	876	877	878	879	880	881	882	883	884	885	886	887	888	889	890	891	892	893	894	895	896	897	898	899	900	901	902	903	904	905	906	907	908	909	910	911	912	913	914	915	916	917	918	919	920	921	922	923	924	925	926	927	928	929	930	931	932	933	934	935	936	937	938	939	940	941	942	943	944	945	946	947	948	949	950	951	952	953	954	955	956	957	958	959	960	961	962	963	964	965	966	967	968	969	970	971	972	973	974	975	976	977	978	979	980	981	982	983	984	985	986	987	988	989	990	991	992	993	994	995	996	997	998	999	1000	1001	1002	1003	1004	1005	1006	1007	1008	1009	1010	1011	1012	1013	1014	1015	1016	1017	1018	1019	1020	1021	1022	1023	1024	1025	1026	1027	1028	1029	1030	1031	1032	1033	1034	1035	1036	1037	1038	1039	1040	1041	1042	1043	1044	1045	1046	1047	1048	1049	1050	1051	1052	1053	1054	1055	1056	1057	1058	1059	1060	1061	1062	1063	1064	1065	1066	1067	1068	1069	1070	1071	1072	1073	1074	1075	1076	1077	1078	1079	1080	1081	1082	1083	1084	1085	1086	1087	1088	1089	1090	1091	1092	1093	1094	1095	1096	1097	1098	1099	1100	1101	1102	1103	1104	1105	1106	1107	1108	1109	1110	1111	1112	1113	1114	1115	1116	1117	1118	1119	1120	1121	1122	1123	1124	1125	1126	1127	1128	1129	1130	1131	1132	1133	1134	1135	1136	1137	1138	1139	1140	1141	1142	1143	1144	1145	1146	1147	1148	1149	1150	1151	1152	1153	1154	1155	1156	1157	1158	1159	1160	1161	1162	1163	1164	1165	1166	1167	1168	1169	1170	1171	1172	1173	1174	1175	1176	1177	1178	1179	1180	1181	1182	1183	1184	1185	1186	1187	1188	1189	1190	1191	1192	1193	1194	1195	1196	1197	1198	1199	1200	1201	1202	1203	1204	1205	1206	1207	1208	1209	1210	1211	1212	1213	1214	1215	1216	1217	1218	1219	1220	1221	12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If the load is uniform lateral load w pounds per foot height of the culvert, the moment at each corner can be obtained by replacing $\bar{W}.h/10$ and $\bar{W}.h/15$ by $w.h^2/12$ with proper signs and putting w -the uniform vertical load equal to zero, and then solving new equations thus obtained, to obtain slopes which can be used to find corner moments.

If instead of two cell rectangular box culvert, there be two cell square one, then putting L_1 , L_2 and h each equal to L and also I_1 , I_2 , and I_3 each equal to I , above eight equations can be modified and these new modified equations can be used in finding moments at the corners of this type of culvert as usual.

E. Moment Distribution Method.

Sign Conventions Used in Solving This Problem:

Moments which indicate tension on the inside face are taken as positive or negative otherwise. Shears that indicate summation of the forces at the left of the section acting outward when viewed from within the culvert are considered as positive. Square Box Culvert having symmetrical uniform vertical load w per foot length of the culvert and the same moment of inertia for all component parts.

Distributing factors for this particular case shall be $1/2$. Assuming 100 units of fixed end moments and distributing these moments according to above distributing factors, which is not very difficult, it was found that bending moment at each corner comes to 50% of the assumed fixed end moment.

Similarly the bending moments at each corner due to uniform lateral load shall be 50% of the fixed end moment.

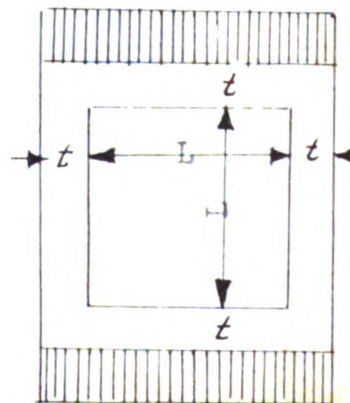


Fig. 1.

Same culvert as in the previous case with triangular load varying from zero at one end to w pounds per foot of culvert length at the other end as shown in the following Fig. Fixed end moment due to this type of loading will be at A and B equal to $\bar{W} \cdot L / 15$ and at C and D, to $\bar{W} \cdot L / 10$, where $\bar{W}$ is total load on one side of the culvert due to above type of loading and it comes to $w \cdot L / 2$. Assuming fixed end moment at A equal to 100 units, consequently fixed end moment at C will be equal to 150, and distributing these moments according to the distributing factors equal to $1/2$, the final bending moments at the corners A and B were found as 56% and the corner C and D, were 43% of the assumed fixed end moment at respective corner.

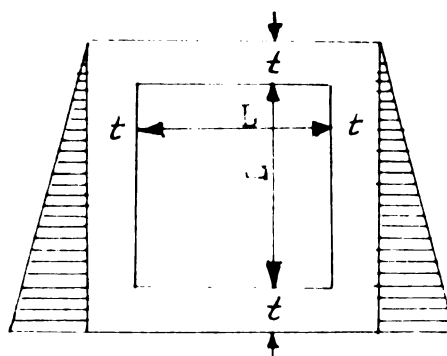


Fig. 2.

Rectangular Cell Box Culvert:

Dimensions and sections of the component parts are as shown in the Fig. 3. Load is uniform vertical load w pounds per foot length of the culvert.

Distributing factors:

Taking one foot width of culvert,

$$I_1 = 1(1.5t)^3/12 \quad \text{and,}$$

$$I_2 = 1(1.t)^3/12$$

$$K_1 = I_1/L(1.5) \\ = 2.25t^3/12L \quad \text{and,}$$

$$K_2 = t^3/12L$$

$$K_1 = 2.25K_2$$

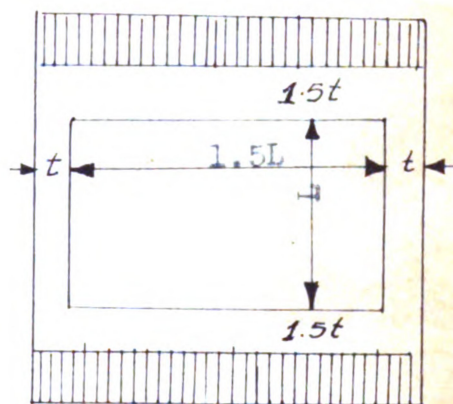


Fig. 3.

Therefore,

Distributing factors will be,

$$1/1 + 2.25 \quad \text{and} \quad 2.25/1 + 2.25$$

$$= 0.31 \quad \text{and} \quad 0.69.$$

Due to symmetry the final bending moments at all the four corners shall be the same. Fixed end moment of 100 units was assumed and when distributed by the above distributing factors final moment at each corner came equal to 31% of the assumed fixed end moment.

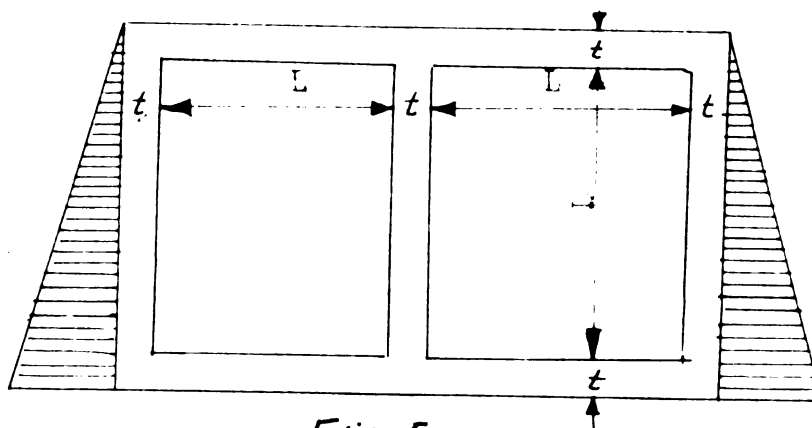
Box Culverts with Two Square Cells:

Case 2. Dimensions and sections of the component parts are the same as in the previous case. Loading is uniform lateral load w pounds per foot length of the culvert height. Distribution factor will be the same as in the previous case. Fixed end moments are assumed as 100 units and these moments then were distributed according to proper distributing factors. Moments at the four end corners turned out to be 66% and at central diaphragm to be 33% of the assumed fixed end moments.

Box Culverts with Two Square Cells:

Case 3. Dimensions, sections and distributing factors are the same as in the previous cases. Loading is triangular lateral load. Fixed end moment at A and B were assumed to be 100 units and consequently fixed end moments D and G comes equal to 150 units, because fixed end moments at A and F due to triangular lateral load varying from zero at one end to w per foot height of the culvert at the other end, will be $W.h/15$, and D and G will be $W.h/10$, where W is the total lateral load. When these fixed end moments were distributed according to proper distributing factors, the final moments turned out to be:

At A and F 71%; at D and G 62%; at B-E 36% and at C-H 31% of the assumed fixed end moments.



Rectangular Box Culverts with Two Cells:

Case 1. Dimensions, moment of inertias of component parts of the culvert and loading are as shown in the Fig. 6.

$$I_1 = (1.5t)^3/12 \quad \text{and} \quad K_1 = 2.25t^3/12.L$$

$$I_2 = t^3/12 \quad \text{and} \quad K_2 = t^3/12.L$$

Therefore, distribution factors at all the outer corners will be .31 and .69 as shown in the sketch, and the distribution factors at the diaphragm will be .41, .41, and .18. Assumed fixed end moments were 100 units and when these moments were distributed according to above factors, the final moments at corners A, F, G and D turned out to be 20% of the assumed fixed end moments. The moment at B-E and C-H comes to 140%. Procedure is shown on the next page.

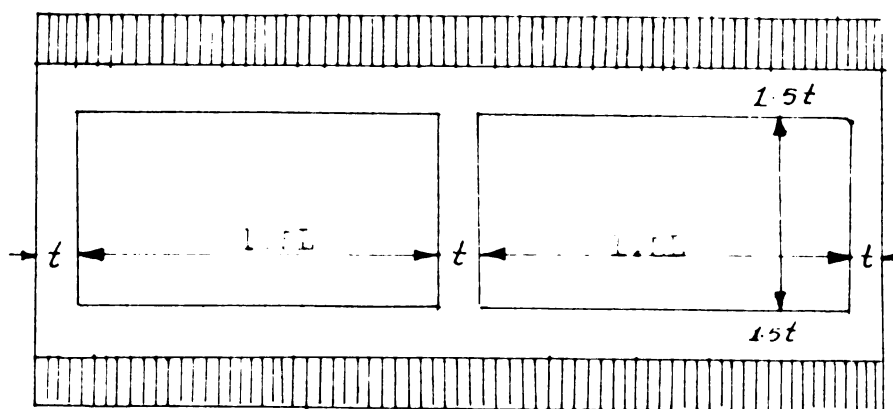
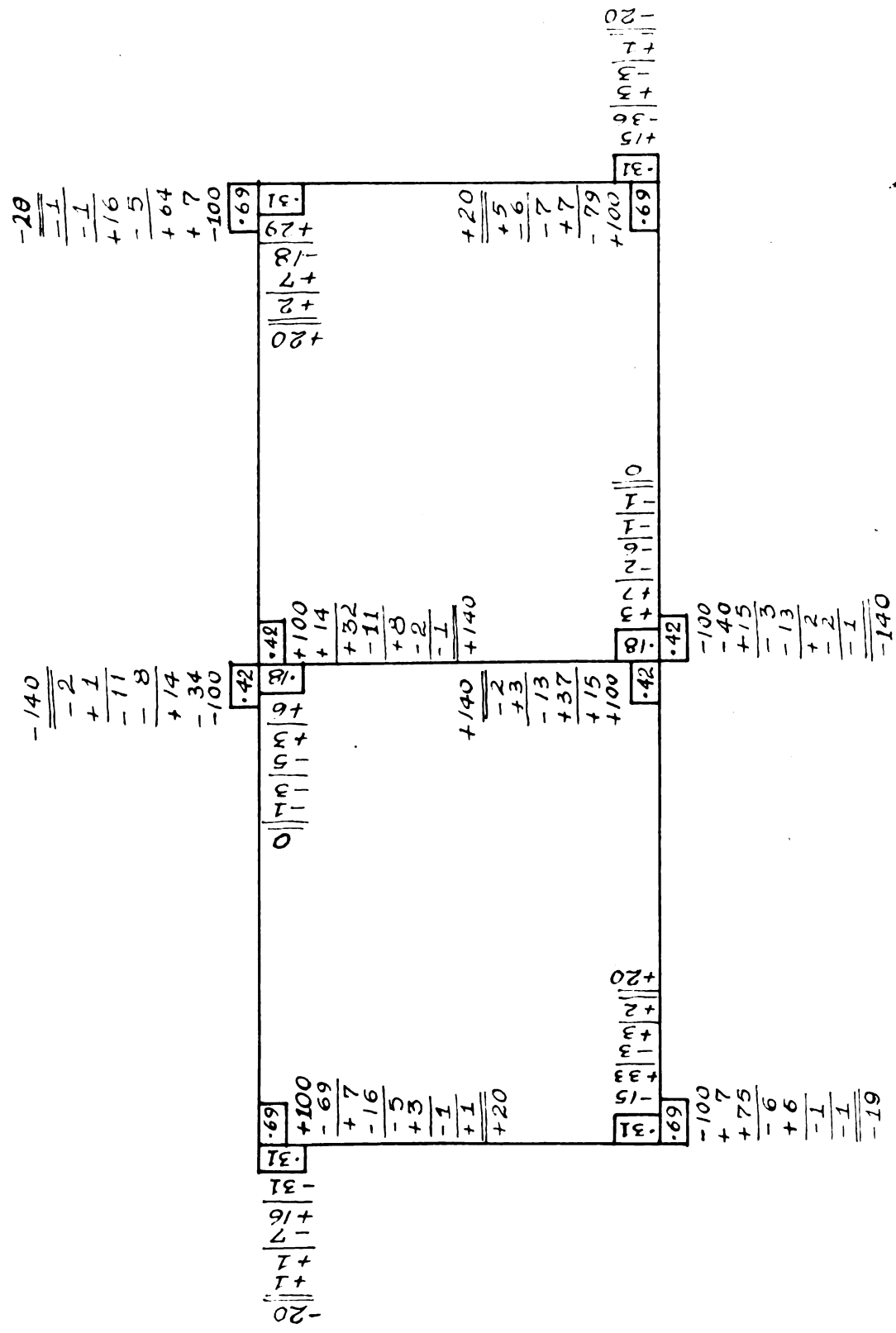
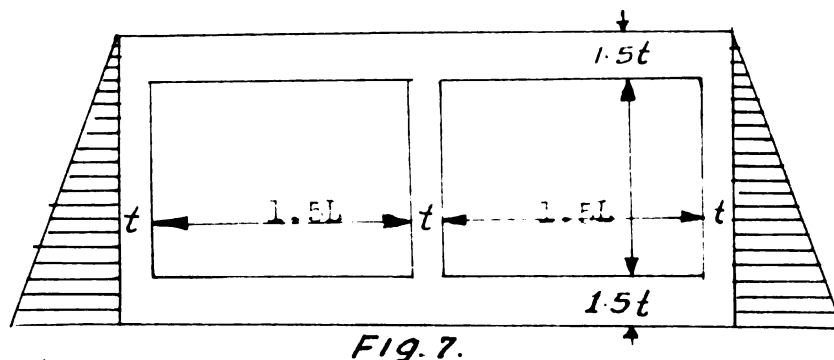


Fig. 6



Case 2. Everything the same as in the previous case but loading is uniform lateral load. Assumed fixed end moments were 100 units, when distributed according to proper distributing factors, the final moment at A, F, G and D turned out to be 81% and at B-E and C-H 41%. Procedure is shown on the next page.



Case 3. Everything same as in the first case except loading is triangular lateral loading. Assumed fixed end moments at A and F were 100 units and consequently at G and D their value will be 150 units. When these moments were distributed according to proper distributing factors, the final moment at A and F turned out to be 87% and at G and D 78%; at B-E 43% and at C-H 39% of the fixed end assumed moments. Procedure is shown on the next page.

Box Culverts with Two Rectangular Cells:

Case 1. Dimensions and sections of the component parts are as shown in the Fig. 8.

Distributing Factors:

$$I_1 = t^3/12.$$

$$I_2 = (1.25t)^3/12 \text{ and } I_3 = (1.25t)^3/12$$

$$K_1 = 2.25 \cdot t^3/12L$$

$$K_2 = t^3/12L$$

$$K_3 = 1.25^2 t^3/12L.$$

Distributing factor for member BA = K_{BA} / K

$$= 1.56/1-1.56-2.25$$

$$= 0.32.$$

Similarly other distributing factors can be easily determined and their values are as shown in the Fig. 13.

For uniform vertical load w per foot length of the culvert length, final moments after distributing assumed fixed end moment equal to 100 units for the span EF turned out to be as follows:

At A and D equal to 14%

At F and G equal to 21%

At B-E and C-H equal to 120%.

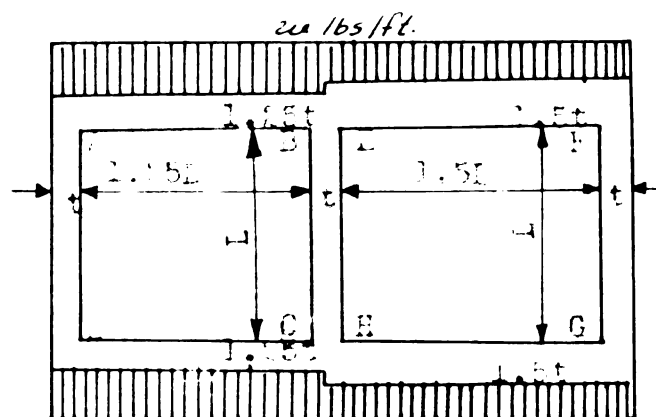


Fig. 8

[illegible]

100

[illegible]

Box Culverts with Two Rectangular Cells:

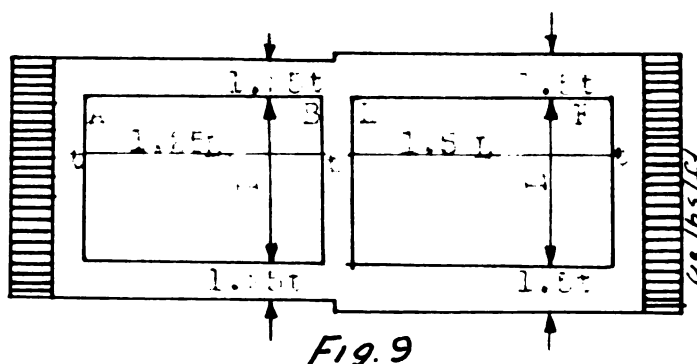
Case 2. Dimensions, sections and distributing factors are same as in the previous case. Loading is uniform lateral load w per foot height of the culvert. Assumed fixed end moments at A and F were 100 units and at D and G were 100 units. These moments were distributed according to the proper distributing factors and the final moments turned out to be as follows:

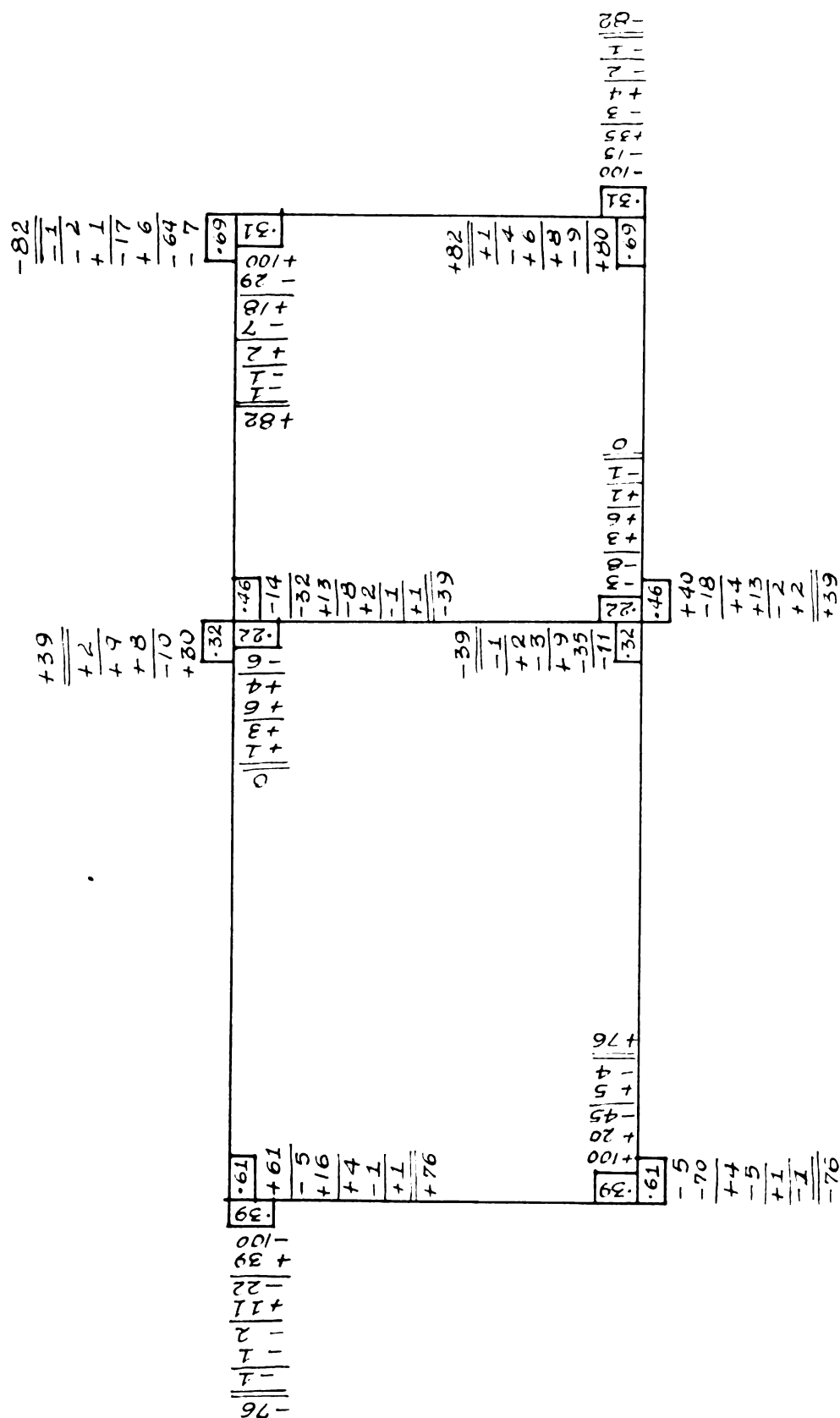
At A and D equal to 75%

At F and G equal to 81%

At B-E and C-H equal to 39%

of the assumed fixed end moments.





Box Culverts with Two Rectangular Cells:

Case 3. Dimensions, sections and distributing factors are same as previous cases. Loading is triangular lateral load varying from zero at one end to w per foot height of culvert at the other end. Fixed end moments at A and F were assumed to be 100 units and at D and G to be 150. These moments were distributed according to proper distributing factors and final moments turned out to be as follows:

At A equal to 79%

At B-E equal to 37%

At F equal to 55%

At G equal to 85%

At C-H equal to 38%

At D equal to 74%

of the assumed fixed end moments.

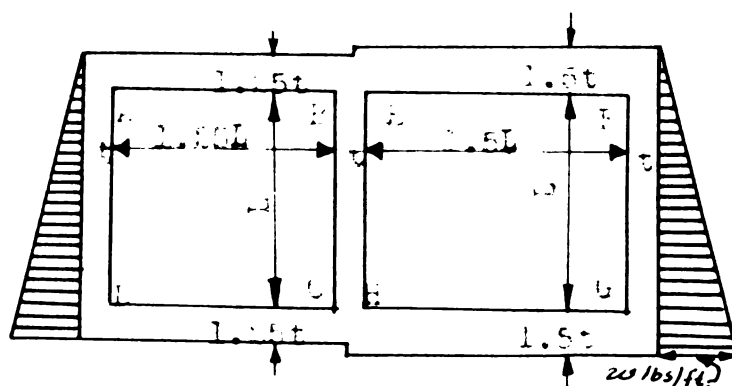


Fig. 10

F. Column Analogy.

Sign Conventions Used in Solving This Problem:

Bending moments in the beams will be considered positive if they produce tension on the inside of the elastic ring. Shears are positive if they accompany positive rate of change of bending moment, increase up or to the right. Square box culvert symmetrical loading and same moment of inertia for all component parts. Loading is uniform vertical load w per foot length of the culvert.

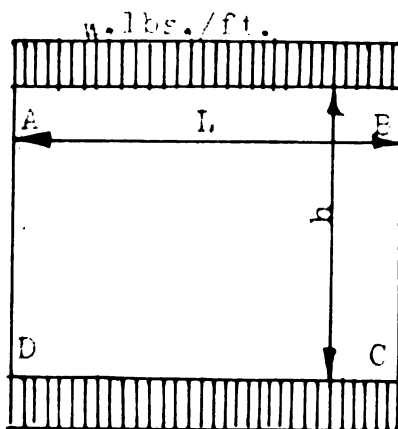


Fig. 1.

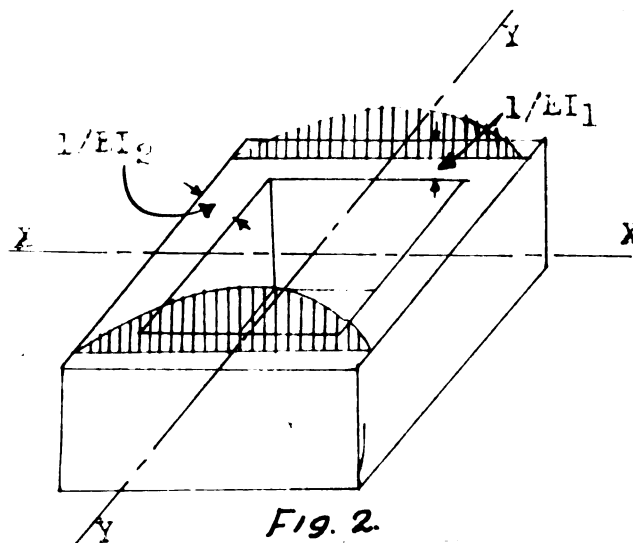


Fig. 2.

EI is same for all members and is taken as unity.

Area of Analogous Column,

$$A = 4L.$$

$$\bar{y} = \frac{L \cdot L + 2L \cdot L/2}{4L} = L/2.$$

$$\begin{aligned} I_{xx} = I_{yy} &= 2L \cdot (L/2)^2 + 2 \cdot L^3/12. \\ &= 5 \cdot L^3/12. \end{aligned}$$

Load on Analogus Column:

$$W = 2.w.L^2L/3.8.$$

$$= w.L^3/12,$$

$$\text{Total load} = 2.w.L^3/12.$$

$$= w.L^3/6.$$

$$\text{Therefore, } f_A = M_A = w.L^3/6.4.L.$$

$$= w.L^2/24.$$

($M_{xx} = M_{yy} = 0$; because loading on the analogous column is symmetrical)

Due to symmetry the moments at all corners will be the same.

Rectangular Cell Box Culvert:

Case 1.

Dimensions and sections of the component parts are as shown in the fig. (a) below. Load is uniform vertical load w per foot length of the culvert.

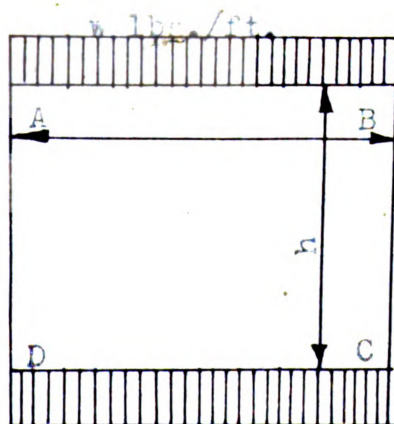


Fig. 3

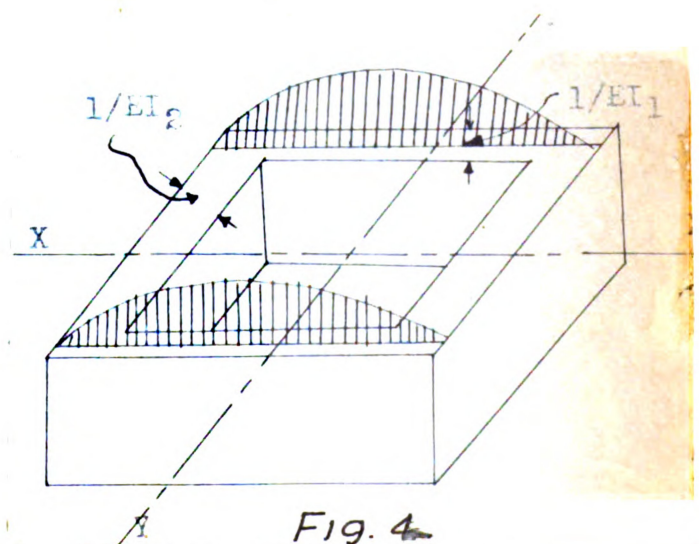


Fig. 4

Rectangular Cell Box Culvert:

Case 2.

Dimensions and sections of the component parts are the same as in the previous case. Load is triangular loading, uniformly increasing from one end to the other end according to the law $w.y/h$. Therefore,

$$\begin{aligned}\text{Total load } \bar{W} &= (w.h/h)(h/2) \\ &= w.h/2.\end{aligned}$$

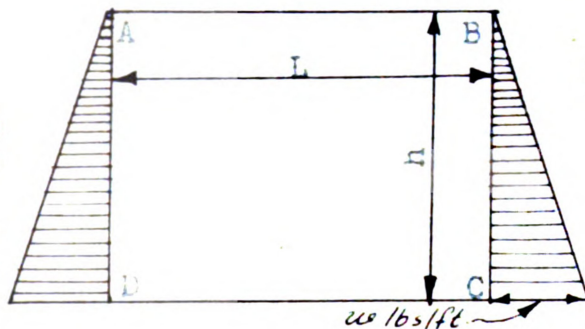


Fig. 5

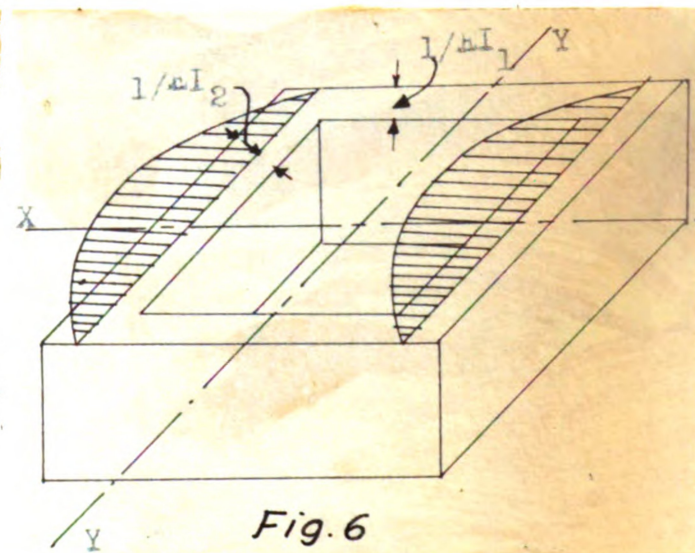


Fig. 6

Area of the section of the analogous column,

$$A = 2(L.I_2 + h.I_1)/I_1.I_2$$

Load on the column will be area of moment diagram due to load alone,

$$\begin{aligned}W &= 2.\bar{W}.h^2/12.I_2 \\ &= \bar{W}.h^2/6I_2\end{aligned}$$

$$M_{xx} = (Wh^2/6I_2)(8/15 - 1/2)h = \bar{W}.h^3./180I_2$$

$$M_{yy} = 0$$

$$\begin{aligned} I_{xx} &= 2.(L/I_1)(h/2)^2 + 2(1/12)(1/I_2)(h)^3 \\ &= Lh^2/2I_1 + h^3/6I_2 \\ &= (h^2/6) \left[(3LI_2 + hI_1)/I_1I_2 \right] \end{aligned}$$

Due to symmetry of load and section of the analogous column,

$$\begin{aligned} M_A = M_B = f_A &= W/A - M_{xx}.y/I_{xx}. \\ &= \frac{\bar{W}h^2.I_1I_2}{6.I_2.2(LI_2 + hI_1)} - \frac{\bar{W}h^3.I_1I_2.6.h}{180.I_2.h^2(3LI_2 + hI_1).2.} \\ &= \frac{\bar{W}.h^2.I_1}{12(L.I_2 + h.I_1)} - \frac{\bar{W}.h^2.I_1}{60(3L.I_2 - h.I_1)} \\ &= -(\bar{W}.h^2.I_1/12) \left[1/(L.I_2 - h.I_1) - 1/5(3LI_2 - hI_1) \right] \end{aligned}$$

Similarly,

$$\begin{aligned} M_C = M_D = f_C &= W/A + M_{xx}.y/I_{xx}. \\ &= \bar{W}.h^2.I_1/12 \left[(1/L.I_2 - h.I_1) \right. \\ &\quad \left. + (1/5(3LI_2 - hI_1)) \right] \end{aligned}$$

IV. DESIGN TABLES FOR BOX CULVERTS.

The following tables are prepared by m.d. method:

TABLE I. Square Culverts

Coefficients for Moment, M, Thrust, N, and Shear, V,
in Traverse Section 1 ft. wide.

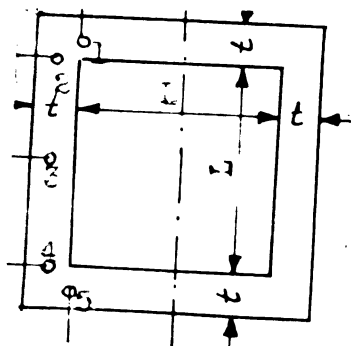


Fig. 1

Traverse Section

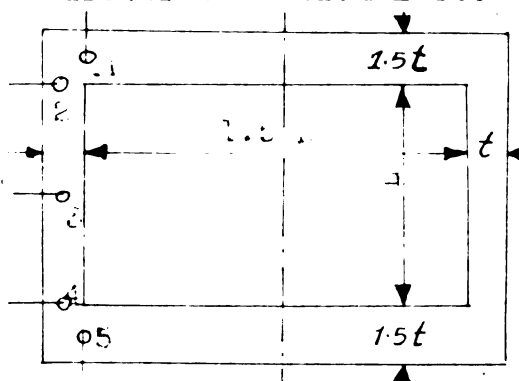
Signs

- Moment, M, indicates tension on inside face.
- Thrust, N, indicates compression on section.
- Shear, V, indicates that the summation of the forces at the left of the section acts outward when viewed from within.

	Uniform vertical load			Uniform lateral Load			Triangular lateral load		
	$.0417.w.L^2$	wL		$.0417.w.L^2$	wL		$.0188.w.L^2$	$wh/2$	
	M	N	V	M	N	V	M	N	V
1	+ 2			- 1	+ .50		- 1	+ .16	
2	- 1		+ .50	- 1	+ .50		- 1	+ .16	
3	- 1	+ .50		- 1		- .50	- 1		- .1
4	- 1	+ .50		+ 2			+ 2.22		
5	- 1	+ .50		- 1	+ .50		- 1.22		+ .3
6	- 1	- .50		- 1	+ .50		- 1.22	+ .34	
7	+ 2			- 1	+ .50		- 1.22	+ .34	

TABLE II. Rectangular Culverts

Coefficients for Moment, M, Thrust, N, and Shear, V, in
Traverse Section 1 ft. wide.



Signs

- Shear, V, indicates that the summation of the forces at the left of the section acts outward when viewed from within.

Fig. 2 Traverse Section.

- Moment, M, indicates tension on inside face.
- Thrust, N, indicates compression on section.

	Uniform vertical load			Uniform lateral load			Triangular lateral load		
	M	N	V	M	N	V	M	N	V
	$.0833wL^2$			$.0833wL^2$			$w.h^3$		
	$w(1.5L)^2$			$w.L$			$wh/2$		
1	†1.2			-.126	†.5	-.033	†.16		
2	-.15		-.5	-.278	†.5	-.033	†.16		
3	-.30	-.5		-.278		-.5	-.016		-.16
4	-.30	-.5		†.125			†.34		
5	-.30	-.5		-.278		-.5	-.016		-.33
6	-.15		-.5	-.278	†.5	-.033	†.33		
7	†1.20			-.126	†.5	-.033	†.33		

TABLE III. Box Culverts of
Two Square Cells.

Coefficients for Moment, M, Thrust, N, and Shear, V, in
Traverse Section 1 ft. wide.

Signs: They are same as in the previous cases.

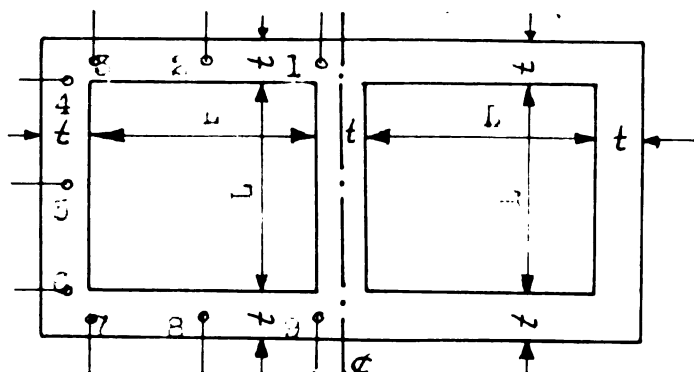


Fig. 3. Traverse Section

	Uniform vertical load			Uniform lateral load			Triangular lateral load		
	M	N	V	M	N	V	M	N	V
	$.0139wL^2$	$w.L$		$.0556wL^2$	$w.L$		$.0123wL^3$	$w.L^2/2$	
1	-4		-.292	+.50	+.5		+.1	+.16	
2	+.2			-0.25	+.5		-0.5	+.16	
3	-1		+.208	-1	+.5		-2	+.16	
4	-1	+.208		-1		-.5	-2		-.16
5	-1	+.208		+.1.25			+.2.82		
6	-1	+.208		-1	-.5	+.5	-2.52		+.34
7	-1		-.208	-1	+.5		-2.52	+.34	
8	+.2			-0.25	+.5		-0.63	+.34	
9	-4		+.292	+.50	+.5		+.1.26	+.34	

4. The fourth part of the document discusses the implications of the study for practice and policy. It suggests that the findings can be used to inform decision-making and to develop strategies to improve the performance of the organization.

5. The fifth part of the document concludes the study by summarizing the key findings and highlighting the limitations of the research. It also suggests areas for future research and provides a list of references.

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1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	8											

TABLE IV. Box Culverts of Two
Rectangular Cells.

Coefficients for Moment, M, Thrust, N, and Shear, V,
in Traverse Section 1 ft. wide.

Signs: Same as in the previous cases.

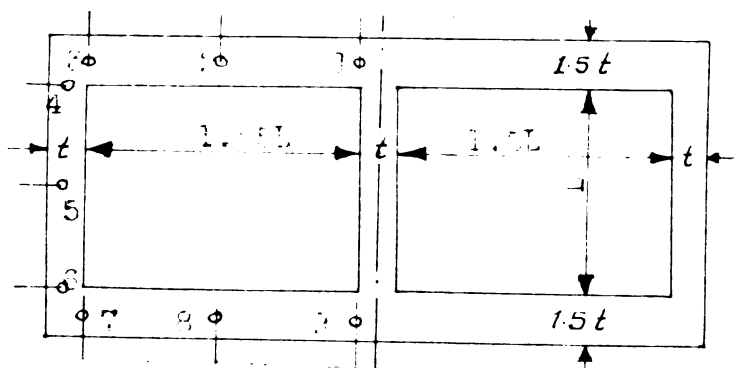


Fig. 4. Traverse Section

	Uniform vertical load			Uniform lateral load			Triangular lateral load		
	M	N	V	M	N	V	M	N	V
	$.0079wL^2$	wL		$.042.wL^2$	$w.L$		$.0142wh^2$	$wh/2$	
1	-3.7		-.272	+1	+1.5		+1	+1.16	
2	+5.7			-.50	+1.5		-1	+1.16	
3	-1		+1.228	-2	+1.5		-2	+1.16	
4	-1	+1.228		-2		-.5	-2		-.16
5	-1	+1.228		+1			+2.1		
6	-1	+1.228		-2		+1.5	-2.7		+1.34
7	-1		-.228	-2	+1.5		-2.7	+1.34	
8	+5.7			-.50	+1.5		-.70	+1.34	
9	-3.7		+1.272	+1	+1.5		+1.4	+1.34	

TABLE V. Box Culverts of Two A-symmetrical
Rectangular Cells.

Coefficients for Moment, M, Thrust, N, and Shear, V, in
Traverse section 1 ft. wide.

Signs: Same as in the previous tables.

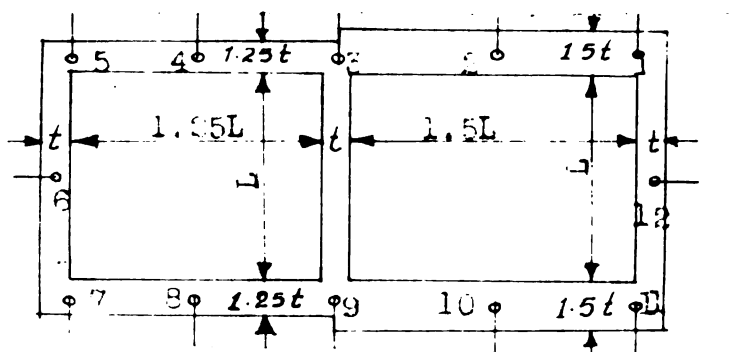


Fig. 5. Traverse Section

Uniform vertical load			Uniform lateral load			Triangular lateral load		
M	M	V	M	N	V	M	N	V
$w.L^2$	wL		$w.L^2$	wL		$w.L^2$	wL	
1	-0.018	†.495	†.066	†.5		-0.018	†.42	-.42
2	†0.058		†.05	†.5		-0.015	†.42	
3	-0.133	-1.68	†0.033	†.5		-0.011	†.42	
4	†0.075		-0.042	†.5		-0.02	†.42	
5	-0.018	†.493	†.493	-.053	-.5	-0.026	†.42	-.42
6	-0.018	†.493	†.030			†0.03		
7	-0.018	†.493	-.493	-.053	†.5	-0.037	†.58	†.58
8	†0.075		-.042	†.5		-0.034	†.58	
9	-0.133	†.913	†1.68	†.033	†.5	-0.03	†.58	
10	†0.058		†.05	†.5		-0.036	†.58	
11	-0.018	-.495	-.066	†.5		-0.-52	†.58	†.58
12	-0.018	-.495	†.017			†0.033		

V. NUMERICAL EXAMPLES

Example No. 1.

Analysis of Rectangular Box Culvert:

Assumed data:

Waterway opening: 52 sq. ft.

Truck loading: 10-ton truck on
unsurfaced secondary highway.

Fill on top of culvert: 5 ft.

For an opening of 52 sq. ft., a rectangular culvert having a span-height ratio of 1.5 to 1 would require approximately a clear span of 9 ft. and a clear height of 6 ft. ($9 \times 6 = 54.0$ sq. ft.) Estimate t equal to 7 in. (Slab thickness equals $1.5 \times 7 = 10.5$ in.)

Load factors for the use of Table II:

$$\begin{aligned}\text{Outside width of the culvert} &= 9 - 2 \times 7/12 \\ &= 10.17 \text{ ft.}\end{aligned}$$

$$\text{Live load from the 10-ton truck} = 1,710$$

$$\text{Dead load from the fill} = 5 \times 100 \times 10.17 = \underline{\underline{5090}}$$

$$\text{Uniform vertical load w lb. per foot} = 6,800$$

$$\begin{aligned}\text{Uniform lateral load w lbs. per sq. ft.} &= \\ &5 \times 100/3 = 167\end{aligned}$$

(Assumed angle of repose is 30 degrees)

$$\text{Triangular lateral load per ft. height} = 100/3 = 33$$

Computations based on the constants are given in the schedule on the next page.

Computation Schedule for 9x6 ft. Culvert

Load conditions				Sections			
Moment, M,	4		5		6		7
Thrust, N, or	M	N	M	N	M	V	M
Shear, V	ft.lb.	lb.	ft.lb.	lb.	ft.lb.	lb.	ft.lb.

I. Uniform

vertical load

M: -1630 -1630 -780 ‡6510

N or V: ‡3400 ‡3400 -3400

II. Uniform

lateral load

M: ‡530 -240 -460 -460

III. Triangular

lateral load

M: ‡360 -170 -350 -350

Total M: -740 -2040 -1590 ‡5700

Total N or V: ‡3400 ‡3400 ‡3400

Example No. 2.

Analysis of Box Culvert of Two Square Cells:

Assumed data:

Waterway opening: 160 sq. ft.

Truck loading: 10-ton truck on
unsurfaced secondary highway.

Fill on the top of culvert: 5 ft.

For an opening of 160 sq. ft., a culvert having two square openings would require a clear span of approximately 9 ft. (Waterway = $2 \times 9 \times 9 = 162$ sq.ft.) Estimate t equal to 9 in. (Thickness of slab and walls)

Load factors for use of Table III.

Outside width of culvert = $2 \times 9 + 3 \times 0.75 = 20.25$ ft.

Live load from 10-ton truck = 1,710

Dead load from fill,

$$5 \times 100 \times 20.25 = \underline{10,130}$$

Uniform vertical load,

$$w \text{ lb. per foot} = 11,840$$

Uniform lateral load,

$$w \text{ lb. per sq. ft.} = 5 \times 100/3 = 167$$

Triangular lateral load

$$w \text{ lb. per foot of height} = 100/3 = 33$$

Computations based on the constants are given in the schedule on the next page.

Computation Schedule for Culvert Having

Two 9 x 9 ft. Openings.

Load conditions	Sections							
	5		6		7	8	9	
Moment, M,								
Thrust, N, or	M	N	M	N	M	M	M	V
Shear, V	ft. lb.	lb.	ft.lb.	lb.	ft.lb.	ft.lb.	ft.lb.	lb.
I. Uniform								
vertical load								
M:	-1620		-1620		-1620	-3230	-6460	
N or V:		‡2460		‡2460				‡3450
II. Uniform								
lateral load								
M:	‡1100		-570		-570	-220	‡440	
III. Triangular								
lateral load								
M:	‡1070		-550		-950	-240	‡490	
Total M:	-550		-2740		-3140	‡2770	-5530	
Total N or V:		‡2460		‡2460				‡3450

VI. CONCLUSION

After studying the methods applied in this thesis it can be seen that Virtual Work Method, Principle of Least Work Method, and Slope-deflection Method, all involve solving numerous simultaneous equations, which is rather tedious and takes a great deal of time. So far as Column Analogy Method is concerned, its application has been limited to one cell box culverts. The only method which can be considered as best suited for this type of problem is Moment Distribution Method.

Tables given herein are prepared by Moment Distribution Method. These tables cover a wide range of design conditions and their use entails a minimum of calculation.

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