THESIS

THE USE OF POLARIZED LIGHT IN STRESS ANALYSIS

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1924

THESIS

Photoelasticity

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THE USE OF POLARIZED LIGHT IN STRESS ANALYSIS.

A Report Submitted to the Faculty
of the
Michigan Agricultural College

By
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Candidate for the Degree
of
Bachelor of Science
June 16, 1924.

Acknowledgment is hereby made for the help of my father, Professor Edwin Morrison of the Physics Department, without whose suggestions this theses could not have been written.

Table of Contents.

Part I.

Introduction.

The purely mechanical methods by which experimental solutions of stress problems are attacked are both difficult and often questionable in their results. One has to assume at the start that the stress is proportional to the magnitude of the strain. This principle can hold only within the elastic limits of the material and beyond the elastic limit these methods fail entirely. Furthermore stresses within the body cannot be measured by mechanical devices. Whatever the arrangement may be, it is not possible to measure the stress at a point is the body is subjected to stress varying from point to point.

In a majority of the problems which arise in actual practice the stresses change rapidly from point to point and our information has been obtained either from using mechanical equipment or from mathematical investigations. By mathematical research it may be possible to obtain exact solutions of a variety of complicated problems; but even the simplest of problems offer great difficulty in solution. For example, the determination of the stresses in hooks, chain links, riveted plates, the effect of notches and holes of various forms in tension and compression members, beams, pillars and shafts, the distribution of stress in built up structures such as plate girders, riveted frames, masonry dams, etc., etc.

The optical method of determining stress distribution and intensity has proved to be very acceptable. During the last few years this subject has been attacked with a very satisfactory degree of success by a number of investigators, among which are Professors Filton and Coker and Major Low of England and M. Mesnager of France.

Perhaps the most notable application of optical stress analysis has been made by Mesnager who, from a glass model, made a determination of the stress distribution in a bridge of reinforced concrete to be placed ofer the river Rhone at Balme, France.

The General Electric Company has done considerable research work along this line, in determining the stress distributions in various types of steam turbine bucket dovetains and tenons with different types of load.

The Bureau of Aeronautics of the Navy Department has recently made a study of a celluloid model of the airship "Shenandoah" by means of photo-elastic methods. The work was done in the laboratories of the Massachusetts Institute of Technology. The Navy Department will give no definite statement of the results of the tests, but they have expresses themselves as being well pleased with the data obtained. They state that the experiments will be of "distinct value and of material aid in the design of airships to prevent a repetition of the 'ZR - 2' and 'Roma' disasters".

Prof Heynans who conducted the tests states: "By this photo-elastic method we can look into the vast and intricate net work of the dirigible and see exactly what is going on when it is laboring. We can see how she is carrying and distributing the load. We have made an analysis of the 'Shenandoah', showing exactly how the stresses are taken up by the members of the frame and the wires. When we

hear of new forces which the ship must meet in its ventures overhead we can try them out on the model here at Technology."

The model consists of several thousand pieces of celluloid machined exactly to scale and fitted together in a miniature duplicate of the air ship.

Part II.

The Theory of Polarized Light.

In order to understand the photo-elastic method some knowledge must be had of light waves and of plane polarized light. It is of interest to note that the phenomenom of polarization has afforded conclusive proof that light waves are transverse waves, - that is the ether particles which transmit light vibrate at right angles to the direction in which the disturbance is being propagated.

Light is refracted in a definate way when it passes from air into glass or other transparent substances. There are certain crystals, however, which behave in a different way toward light. Bartholinus in 1669 first noticed this as a doubling of an object seen through a parallel sided crystal of iceland spar or calcite. He found that when a beam of light strikes the surface of such a calcite plate perpendicularly, one portion of the light goes straight through in the same manner it would in a plate of glass, while another portion is bent slightly on entering the first surface, goes through the plate in this oblique direction and then bends back into a direction parallel with the original one as it leaves the second surface. This crystal which divided the light falling upon it into two parts, each part being refracted according to a different law is called a "doubly refracting crystal. The first of the two parts described is called the ordinary ray, because it follows the ordinary laws of refraction, and the second part is called the extraordinary ray.

A natural crystal of calcite is in the form of a rhombohedron two opposite solid angles of which are bounded by three obtuse angles. The optical axis of this crystal is parallel to a line drawn through one of these solid angles equally inclined to all three faces.

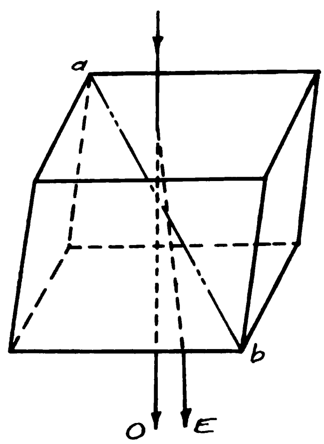


Fig. 1

In Figure 1, a b represents the optical axis; O the ordinary ray; and E the extraordinary ray.

When a plate is cut out of a calcite crystal so that its faces are perpendicular to the optical axis, then a ray of light striking one face perpendicularly is not doubly refracted, and it passes through the plate in a direction parallel to the optical axis, which is the axis about which the crystal is symetrical.

A crystal called tourmaline acts in a similar manner to calcite but in addition it has the property of absorbing the ordinary ray completely within a very short distance, while it is fairly transparent to the extraordinary ray.. If two plates of tourmaline are placed upon each other, only the extraordinary ray will get through the first plate. This will go on through the second plate if the optical axes of the two plates are parallel. If the second plate is turned so that the axes are not parallel when the extraordinary ray enters the second plate it is divided into two parts, of which the ordinary part is absorbed and the extraordinary gets through. This part that gets through becomes gradually fainter as the turning continues, and disappears entirely when the plate has been turned through 90° from the first position. On passing the 90° position the light reappears, and regains full brightness at 180°.

A mechanical analogy will explain the phenomen shown by the crossed tourmaline crystals. Imagine a long flexible rubber tube A B, Figure 2, with one end fastened to the wall and the other end held in the hand. By moving the end of the tube held in the hand, A, to and fro it is possible to cause transverse waves to travel

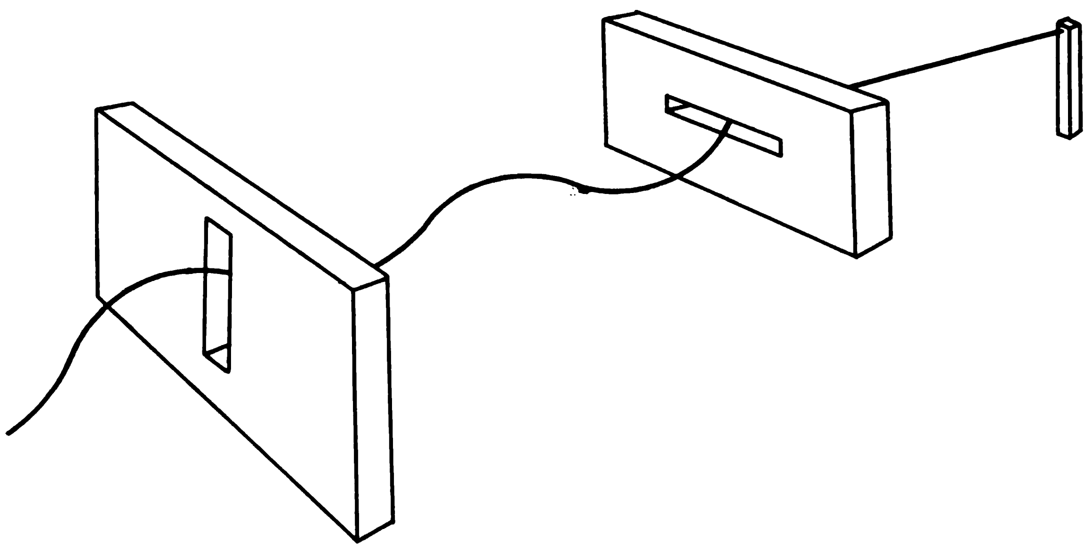


Fig. 2

down the length of the tube. If a block of wood with a slot cut in it is placed over the tube, it is evident that the motion of the tube will not be interfered with so long as the slot is parallel to the direction of motion. If the tube is vibrated at right angles to the slot the vibrating motion will not be able to pass the block of wood. Imagine that the end of the tube which is held in the hand is caused to vibrate in a number of different directions. If the slot is in a vertical position, of all of these vibratory motions imparted to the tube only those which are in a vertical direction will be transmitted or passed through the block. If a second slotted block is placed over the tube, those vibrations which pass the first slot will pass the second providing the second slot is parallel to the first. If the second slot is placed at right angles to the first, no vibration will pass.

This leads us to assume that ordinary light consists of a transverse wave motion, the vibrations taking plase in many directions. When such a beam passes through a tourmaline crystal, only vibrations in a certain direction are allowed to pass, so that the transmitted team differs from ordinary light in that the vibratory motions of the ether particles are all in the same plane. This being the case, it is evident that this beam of light can pass a second tourmaline crystal only when it is parallel to the first.

The beam which passes the first tourmaline plate is said to be "plane polarized" and the plate is called a "polarizer". The second plate acts as a detector of the polarized condition and is called an "analyzer".

In order to use calcite as a polariser or analyser it is necessary to devise some means of getting rid of one of the beams within the crystal, which tourmaline does naturally. The Nicol prism was devised for this purpose and makes use of the principal of total reflection. Huyghens showed that the double refraction is accounted for by the fact that the ordinary and extraordinary rays travel in the crystal with different speeds. That is, calcite has two different indices of refraction for the two beams. A rhomb of calcite may be cut as shown in Figure 3, and the two halves are cemented together again with Canada balsam, after the two oblique surfaces have been polished. The refractive index of the balsam is intermediate between that of calcite for the ordinary and extra-

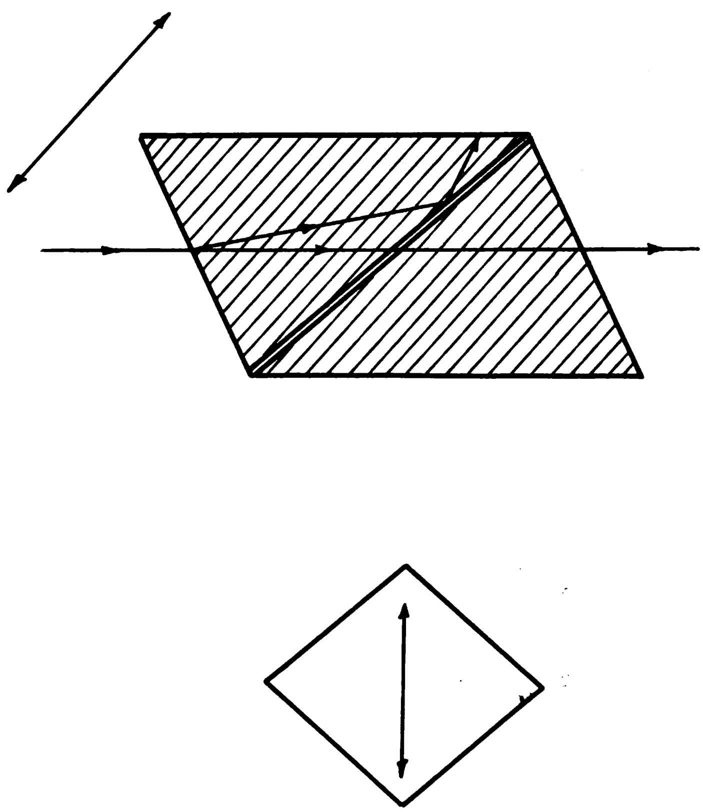


Fig. 3.

ordinary rays. The former meets the balsam film at an angle greater than the critical angle and is totally reflected to one side and may be absorbed by some dark surface. We, therefore, have a plane polarised beam of polarised light emerging from the prism, whose direction of vibration may be changed by rotating the prism.

Figure 4 gives an idea of the action of the Nicol prism. It shows a ray of light with its random vibrations before entering the prism and emerging on the farther side as a ray of plane polarised light.

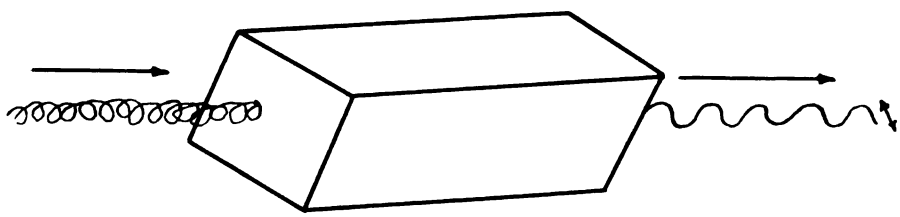


Fig. 4.

If a plane polarised beam of light is passed through a plate of quartz or mica of the proper thickness it may be changed to a circular polarized beam. That is, the ether particles move in a circular orbit.

A thin plate of mica or quartz is cut with its optical axis parallel to the plate. As a beam of polarised light passes through this plate, ordinary and extraordinary rays are again produced, executing vibrations in directions 90° to each other. Figure 5

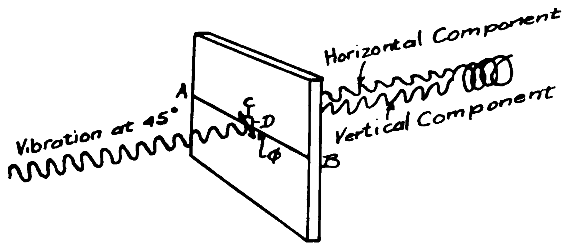


Fig. 5

shows a beam of polarised light which strikes the plate perpendicular to the surface of the plate and to the optical axis, A. B. In this case no bending of either ray occurs, but one is retarded more than the other, the difference in retardation depending on the thickness of the plate. We have, therefore, an emergent ray of light made up of two components, the extraordinary ray vibrating in the horizontal plane and the ordinary in the vertical plane. If the plate is cut to such a thickness that one of these rays is retarded a quarter of a wave length with respect to the other, any other particle, say at P, is made to vibrate up and down and side ways at the same time. If one train is a quarter of a period behind the other, the resultant vibration is a circular one and circularly polarized light is said to be produced.

In order to produce a separation into two perpendicularly polarised rays, as described above, the direction of vibration of the incident light must make such an angle with the optic axis A B that neither of the components C or D vanish. These component vibrations must be equal to produce circularly polarized light. Therefore the direction of vibration must make an angle of 45° with the optical axis in order to make the component vibrations equal. The plate may then be placed four different positions 90° apart to produce circularly polarised light. If the axis A B shown in Figure 5 was changed to the vertical, the incident ray would still make an angle of 45° with this axis, but the relative retardation of the horizontal and vertical component emergent rays would be reversed in sign. The result is that the direction of vibration of the circularly polarized beam takes place in the opposite sense. Therefore in order to reverse the direction of circular vibration in a circularly polarised beam it is merely necessary to rotate the quarter wave plate through 90° in its plane. If polarized light is passed through two quarter wave plates with axes at 90° with each other the effect of one plate is neutralized by the other, and the ray remains unaltered.

Mica and quartz are commonly used to make quarter wave plates, mica being usually used in large plates.

Part III.

The Action of Polarized Light on a Stressed Transparent Model.

Before considering the action of a stressed member on a ray of circularly polarized light it is necessary to know something of stress in general in a homogeneous medium.

A state of stress can always be represented by two stresses at right angles, and if their directions and magnitudes are known over the whole specimen the state of stress is completely determined. These principal stresses, called the p and q systems always intersect at right angles and are normal and tangent at boundaries where there are no applied stresses. Figure 6 shows the principal

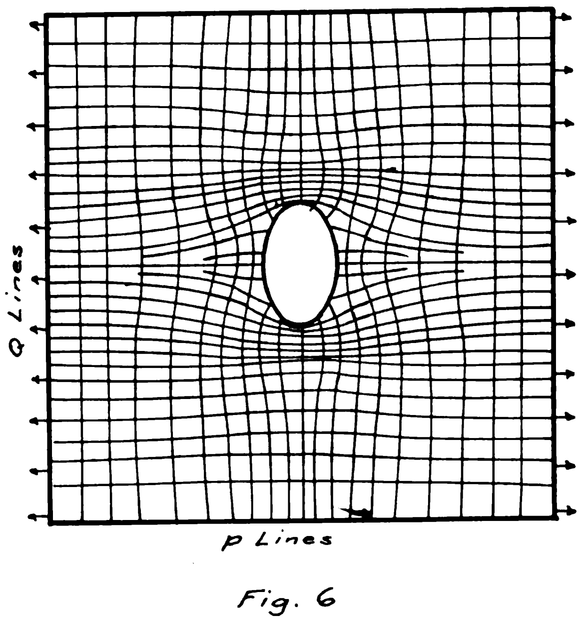


Fig. 6

stress lines for an elliptical hole in a member under tension. At all free boundaries there can be no normal stress so that either p or q must vanish.

These principal stresses always coincide with the direction of zero shear, so that the shear at any point has a maximum value in planes at 45° to the principal stress directions. This shear intensity varies according to the sine law from a maximum of 1/2 (p - q) at 45° to zero in the principal stress directions.

These facts hold true for any system of plane stress and are of great aid in estimating stress distribution.

For three dimensional stress the same laws hold true, with an extension to three dimensions, namely:-

(1) The principal stress directions are represented by three systems of lines at 90° to each other.

(2) At any bounding surface, where no applied forces exist one of these directions is normal and the other two coincident with that surface.

(3) Any plane coinciding with two of these stress directions is a plane of zero shear.

(4) All planes at 45° to these planes are planes of maximum shear.

(5) The shear varies from a maximum in these 45° planes to zero in the principal stress planes according to the sine law.

When a plane stressed specimen of celluloid is placed in a beam of circularly polarized light, an action occurs similar to that occuring in the quarter wave plate. That is, the wave of light may be thought of as being separated in two polarized components with vibrations at right angles, and in this case with their directions of vibrations coinciding with the principal stresses mentioned above. If the principal stresses are unequal one of the vibrations is retarded with respect to the other and this retardation is proportional to the principal stress differences at the point considered. This in the law which connects the light effect with stress. Mathematically it may be expressed as follows:-

Relative retardation = e (p - q) t (1)

where p = one principal stress

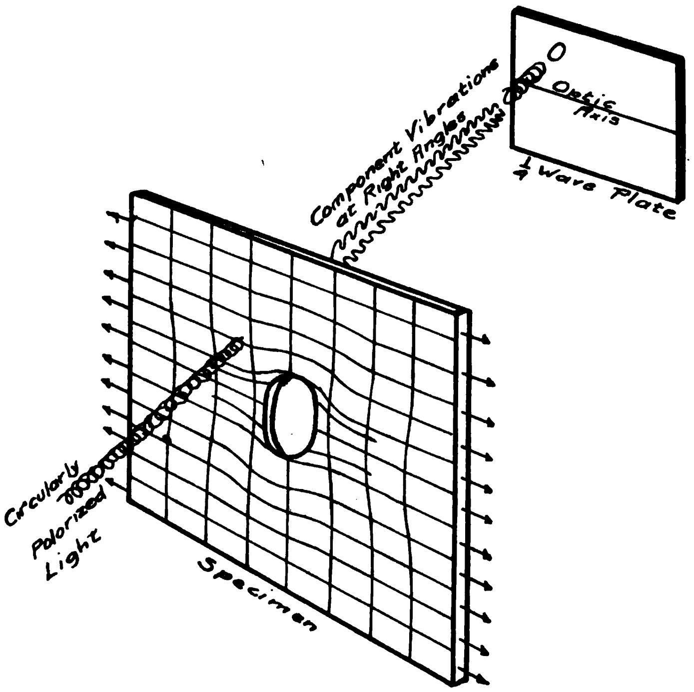


Fig. 7

q = the other principal stress

t = the thickness of the specimen

c = the optical constant for the material used.

Figure 7 shows a beam of circularly polarized light passing through a stressed specimen. When the light emerges the polarised components at right angles are retarded different amounts with respect to one another according to the law stated above, and therefore the emergent vibration is plane circularly or elliptically polarised depending on the amount of the relative retardations of the components.

Figure 8 showes the various changes this vibration goes through for every eighth wave length relative retardation. For

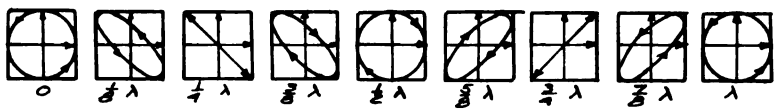


Fig. 8

whole wave length retardations the light vibrates just as the incident; for odd half wave lengths, 1/2 , 3/2 , 5/2 etc. the vibration is circular but in the opposite direction to the incident. for odd quarter wave lengths the vibrations are plane polarised, and for odd eighth wave lengths elliptically polarized. The reason for using circularly polarised light on the stressed sample is that no matter what the principal stress directions are, the plane polar ised components parallel to these directions, into which it is separated, must always be of equal amplitude. This is true because linear components of a circular motion at right angles to each other are always of equal amplitude. Therefore it makes no difference at what angle the specimen be turned in a plane perpendicular to the light direction, or what direction the principal stresses p and q take, the vibrations of the light emerging from the specimen are always of the same character and therefore like the forms shown in Figure 8. The angle of the emerging plane and elliptically polarised light, however, does depend on the stress directions, and changes as the sample is rotated in its plane. On referring to Figure 7 the two ellipses shown will turn as the sample is turned.

After transmission through the sample as explained above the light is of the character shown in Figure 7. The direction of the component vibrations depending on the principal stress directions, but the relative retardations depending only on the difference in magnitude of the principal stresses for a specimen of given thickness, according to the law stated in Equation (1). If this light is transmitted through another quarter wave plate and another Nicol prism polariser, the result is that for all points of the specimen where relative retardations of integral wave lengths (see Figure 8) are produced the light is all cut off; for odd half wave lengths it is all transmitted, and the amplitude of vibration varies according to the sine law for intermediate points. This law is expressed mathematically as follows:-

$$A = A_m \sin \pi \lambda$$

where A = amplitude

A_m = maximum amplitude

λ = relative retardation of vibration components in wave lengths.

The intensity of light varies as the square of the amplitude and we have:-

$$I = I_m \sin^2 \pi \lambda$$

where I = intensity of illumination

I_m = maximum intensity.

All the light transmitted through this second Nicol prism is of course polarized in one plane.

The above discussion is true only for a single wave length, or monochromatic light only. If red light were used, the light projected on a screen would consist of a system of black and red bands. Second and higher orders of red would appear when the difference between the principal stresses p and q was great enough. Where p and q were equal or where the stress was zero the red would all be cut out and black would result. Black would also result where p and q differed by an amount which produced an even wave length relative retardation. For odd half wave length relative retardations the maximum red would appear.

Similar results are produced by any single wave length or color, but different colors are retarded different amounts by a given stress. the result is that the system of bands of one color do not exactly overlap those of another color. they are shifted with respect to each other and for several colors the shifting results in repeated series of colors in place of what was a single bright and dark band in the case of a single color. This is illustrated in Figure 9, which shows a series of colors obtained from actual observation, at the General Electric Laboratories, on a sample of celluloid 0.18 inches thick and 3/8 inches wide. The

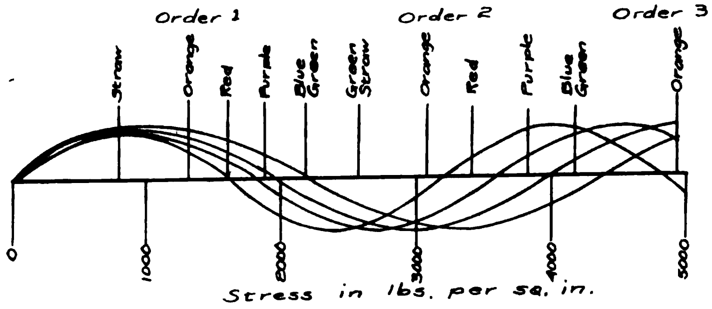


Fig. 9.

stress was carried up to 5000 pounds per square inch. Four different colors are shown with their characteristic variations of intensities superimposed, which gives a series approximating the actual. They indicate the way the color series is produced. As p and q differ more and more this series of color may pass through more than one order. As (p - q) increases (Equation 1) the color passes through drfinite series. For celluloid this sequence is about as follows: beginning at (p = q), black, straw, orange, red, blue green, and again straw, orange, red, blue green, etc.
The color effect gives a measure of (p - q) only, and not the values of p or q.

In order to have a complete determination of the stress it is necesstay to have $(p + q)$ also. This is made by means of an extensometer measurement. only color observations are necessary, however, where p or q vanish. The principal stresses p and q are always normal and tangent to free edges and also there can be no normal stress at a free edge. Therefore, p or q must vanish and the remaining stress is tangent to the edge. At all free boundaries, therefore, in a plane stresses specimen the order of color is a direct measurement of the stress magnitude. Since edges are usually regions of maximum stress this is a fact of importance. For a simple rectangular tension membber or for parts of a beam of uniform cross section, there exists only one set of principal stresses and here also the color gives the stress directly. The neutral axis of the beam may show as a dark band. For many cases, however dark areas will represent areas where p and q have finite values but are equal. The stress magnitudes corresponding to particular colors anc be read directly from the color. A sample of the same material as the specimen under investigation is taken and the various colors are calibrated in terms of stress. Another method is to balance out the color until a dark field is produced, using a piece of the same material, on which the intensity is measured by a spring balance.

Part IV.

Description of Apparatus used in the Coker Photo-Elastic Method.

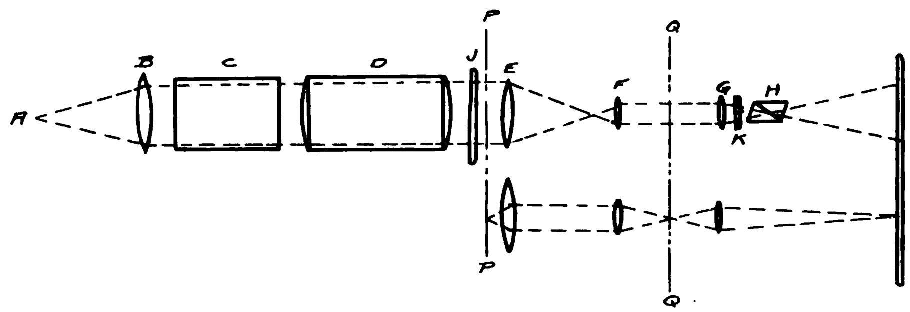


Fig. 9.

Figure 9 is a diagramatic representation of the optical apparatus used and shows the path of the light rays. The apparatus described is in use at the General Electric Company's Research Laboratories.

Light from a source A passes through a condensing lens B and a water screen C to reduce the heat rays. It is then passed through a polarizer D. This unit consists of two 4 1/2 inch condensing lenses and two small concave lenses arranged as shown in Figure 10, in combination with a Nicol polarizing prism of calcite. The later is less than one inch in diameter, but with the combination of lenses gives a 4 1/2 beam of polarized light.

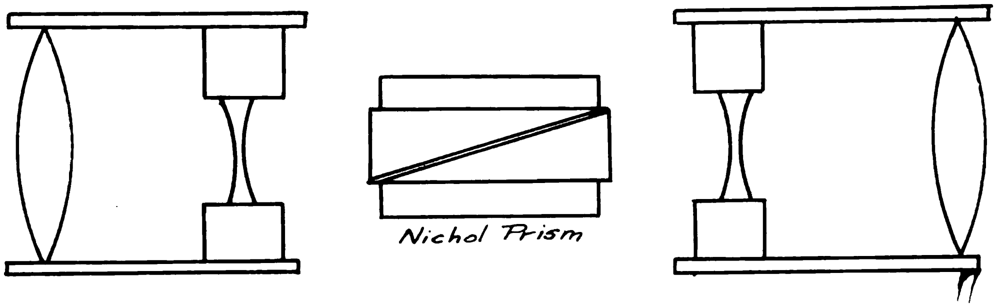


Fig. 10

The plane polarized light is then passed through a quarter wave plate of mica J, which changed it to circularly polarized light. This is passed through the transparent specimen under investigation. This specimen is in the plane marked P. The light then passes through lenses E, F and G and through the quarter wave plate K which is similar to J except that its axis is at 90° to that of J and therefore counteracts the effect of J. The resulting light is analysed by the polariser H with its plane of polarization 90° to that of the polarizer D. The light is finally projected upon the screen I so that points in either the plane P or Q in which ever the specimen is placed, are brought to a focus on this screen. The colors produced depend directly on the stress distribution in the specimen.

The models are usually made of celluloid on account of its great flexibility and toughness and the ease with which it can be drilled, turned or machined. This material has a value of E = 355,000. There is no very pronounced elastic limit and the stress deformation curve is very nearly straight up to a load of 1900 pounds per square inch, which may be taken as the elastic limit of the material.

Part V gives the results of one of a number of experiments conducted by Prof. Coker by this method.

Part V.

Tests on the Stress Distribution in Cement Briquettes by Photo-Elastic Methods.

The stress distribution in cement briquettes is of a very complex nature. The loads are applied very obliquely to the contour at four points and the shape of the briquette invites much complexity. This is at once evident from the lines of principal stress in the British standard form in which the contact loads applied by the grips produce a complicated stress system which would be practically impossible to unravel except by experiment.

In experiments conducted in 1913 Prof. Coker found it impossible to measure the stress distribution completely, but an attempt was made to find the stress difference (p - q) across the minimum and principal section and thereby fix the relation between the maximum stress there and the mean applied load, since at the contour q must be zero at every place untouched by the grips. Some earlier measurements on the British standard briquette indicate that this maximum stress is about 1.75 times the value of the mean average stress across the section, and in the standard American form of that date the corresponding value was found to be 1.70, while in the continental form it appeared to rise to about 1.95 times the mean stress.

These measurements have been repeated and amplified recently, so that it is now possible to give a better idea of the actual stress systems at the central sections of each form, and Figure 11

shows the distribution obtained at the minimum sections of each briquette.

As will be observed the distinctive feature of the distribution in each case is an extremely variable tension p across

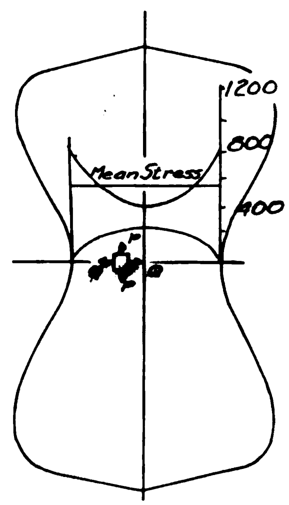


Fig. 11.

the minimum cross-section accompanied by a variable cross-stress q at right angles of considerable magnitude. In the British form, for example, with a load giving a mean average stress of 500 pounds per square inch, the highest value of p at the outer contour is 870 pounds per square inch, or 1.74 times the value of the mean stress, which sinks to 405 pounds per square inch at the center, or slightly more than 80 percent of the mean average value. In addition to this stress there is a cross-stress which rises rapidly from a zero value at the contour to a value of about 255 pounds per square inch for the central six-tenths of the cross-section, so that the manner of loading and the form of the section calls into play a cross-stress of 47 per cent of the mean average stress due to pull.

Therefore, a member of this form is certainly not in pure tension, and the color bands which are observed on the model will indicate this quite clearly.

It is important to show that the stress distribution in a cement briquette is similar to that in a transparent model and recent experiments show that the sum of the principal stresses (p + q) at the waist are almost exactly the same as those found in a transparent model when the briquette has been made for some time. This is what may be expected, for it is very probable that the stress in a cement briquette of considerable age is of exactly the same kind as experiments on transparent models show since old cement has very perfect elastic properties. The fact that sound vibrations are transmitted very readily through cement partition walls and floors shows this, and in another direction it has been proved that such cement possesses very perfect thermo-elastic properties. This latter property is not, however, possessed, or only imperfectly so, by cement which has been recently moulded, so that so far as this evidence can be taken into account it points to the conclusion that briquettes tested in seven days and in twenty eight days, as is provided for in standard specifications, will not be under exactly the same type of stress as transparent models show. They are in fact most probably in a semi-elastic condition in which the stress distribution is less variable than in a purely elastic state, so that any provisions found necessary from experiments on elastic bodies will more than cover the requirements sought. The older the briquette the more nearly the stress distribution approximates that of the elastic condition of the transparent model. The chief bar to uniformity appears to be in the varying shapes of the briquettes themselves, but it does not appear impossible to devise a simple means whereby this difficulty can be overcome without the necessity of a wholesale scrapping of existing testing machinery. Briefly, Prof. Coker's proposal is to examine how far uniformity of testing can be obtained by maintaining in every case the existing forms of the ends and lengthening the central part by addition of a parallel part of the least possible length which will satisfy the condition that stress in that part is a uniform tension. Thus if we take the British standard briquette, Figure 12 and introduce into it a defin te

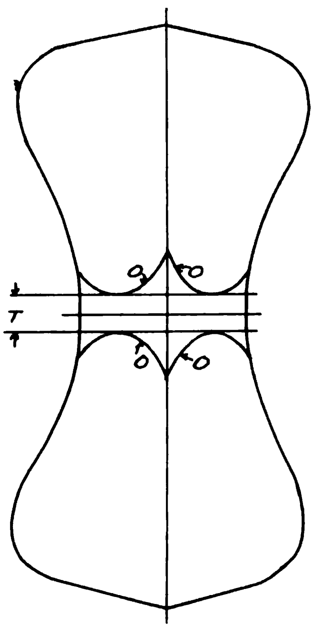


Fig. 12

parallel length, the part in pure tension is at once defined by the zero isoclinics as shown in Figure

Having established this length ? by experiment, and this has already been done, it is a simple matter to make the length in pure tension any quantity considered desirable, and this need only be a small fraction of an inch. Further experiment will be necessary to find in the cement briquette the stress distribution at the waist at various ages. It has been established by recent measurements that there is uniformity of lateral contraction under load in each type of briquette lengthened in this way, thereby indicating the probability that uniform tension exists at these central cross-sections.

Should there be no further difficulties it would only be necessary to alter the moulds now in use to obtain not only uniformity in the comparison of results, but also a real tension test of cement which there is some reason to believe has never yet been attained under existing conditions, owing to the variable tension at the waist, and the want of correspondence of this variability in briquettes of different patterns.

Part VI.

Studies Conducted in the Physics Laboratory at M. A. C.

Owing to difficulties encountered in obtaining apparatus for use in the Coker Photo-Elastic method, my father, Prof. Morrison, has devised another method. Instead of evaluating the stress in terms of color, which is hard to estimate, this method will make use of the varying wave lengths of light in determining stress intensity.

The apparatus, Figure 13, consists of an adapted Norrenburg Polariscope. Light from a 100 watt lamp A passes through a standard ray filter B which will give a beam of light of

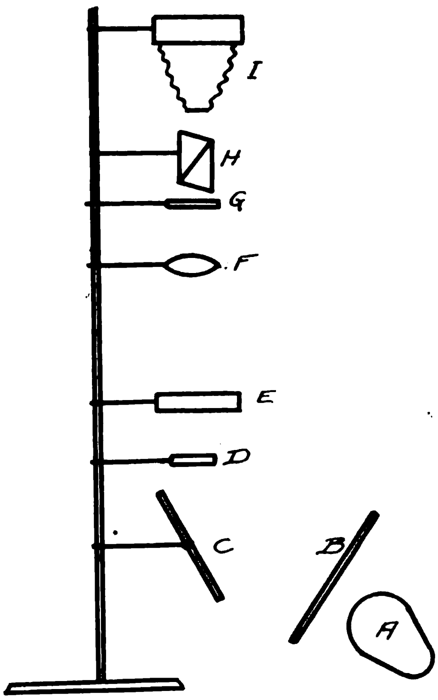


Fig. 13.

wave length. This beam is polarized by means of a glass plate C and is reflected vertically and passes through the quarter wave plate D, where it is circularly polarized. After passing through the model E being studied, the beam is focused into the Nicol analyzing prism H by means of the lens F of 15 centimeters focal length. A quarter wave plate is placed between the lens and analyzer. A picture of the effects produced is taken by means of a camera I.

The method of procedure will be as follows. A standardizing beam of glass or celluloid will be placed in a clamp as shown in Figure 14. By placing weights on the lever arm a known load may be concentrated at the center of the beam. The loaded beam will then be placed in the apparatus at point E and a picture will be taken of the stress distribution, using a light of μ_1 wave length.

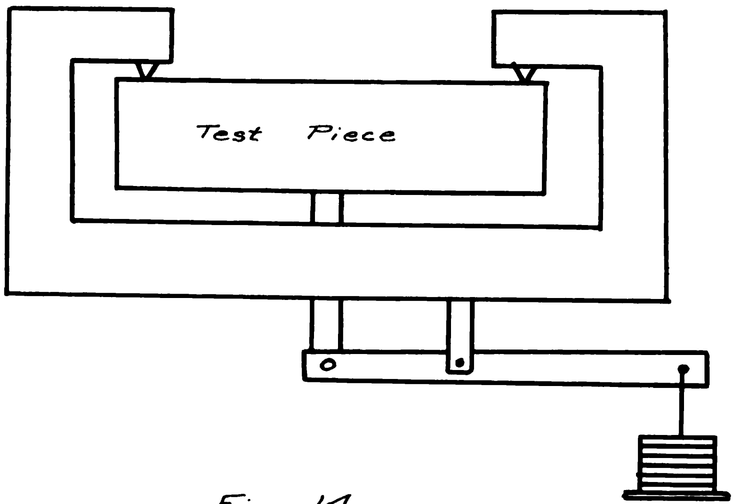


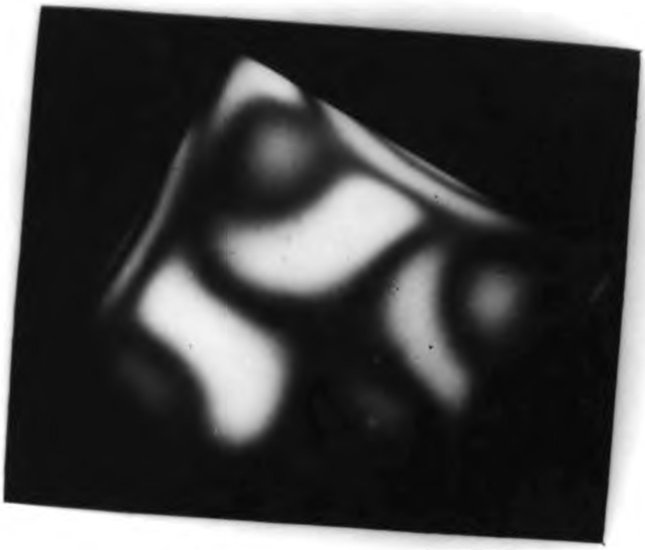
Fig. 14.

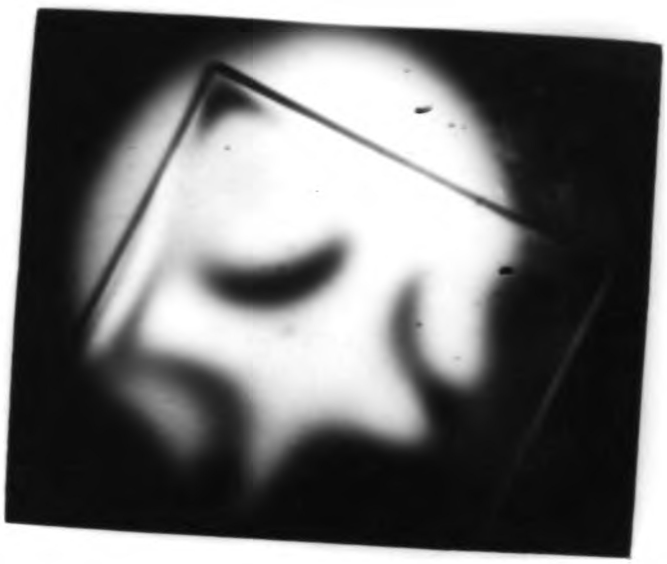
The load on tho beam will then be changed and another picture taken. The standard ray filter will then be changed to one giving light of a wave length μ_2 and two pictures taken using the same loads as before. Therefore, by measuring the width of the stress lines produced and knowing the wave lengths of the light used and the loads used it will be possible to compute the amount of the stress at any point in terms of wave lengths of light.

By placing a model made of the same material as the the standard beam in the apparatus, and loading the model to produce an unknown stress distribution, and by taking pictures using the standard ray filters it will be possible to evaluate the stresses.

The two pictures show the stress in a small piece of unannealed glass.

The apparatus has just been set up and further study will be made.





Part VII.

Conclusions.

The photo-elastic method has been used to show stress distribution and to measure the stresses themselves by the Americal Navy.

A prominent engineer in France has analyzed the stresses in a reinforced concrete bridge which has been actually constructed over the Rhone by this method.

A great industrial concern like the General Electric Company has used this method in designing steam turbine buckets.

Experiments conducted in the Physics Laboratory at M. A. C. have shown the possibility of using this method.

From the above statements it is to be seen that the photo-elastic method of stress determination is being used more extensively and is deserving of further study.

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