SOME PROBLEMS IN THE DYNAMICS OF SMALL GROUP BALANCE

Thesis for the Degree of M. A.

MICHIGAN STATE UNIVERSITY

FRANCIS X. MULVIHILL

1967

[illegible] '104

ABSTRACT

SOME PROBLEMS IN THE DYNAMICS
OF SMALL GROUP BALANCE

by Francis X. Mulvihill

Heider's Balance Theory and the formalization of that theory by Harary and his associates have assumed that if a set of relations among three elements of a group are negative the set is in a state of imbalance. There are suggestions in the work of both, however, that this situation may be an exceptional case. In addition to the formalization of the theory, Harary and others were able to clarify the results of some previous research and to add some hypothesis not associated with Heider's theory.

The general purpose of this thesis is to relate these various propositions and generalizations to each other within a simple conceptual framework, viz. the cost and reward concepts of the Exchange Theory of George C. Homans. The theory is then specified to apply to the case of a three person discussion group and is formalized by means of a computer program.

The more specific purposes are:

1) to trace the dynamics of three groups under the assumption that

   a) a completely negative set of relations among all group members is unbalanced and

   b) the same type of relation is vacuously balanced, i.e. it is ignored as a factor in caculating degree of balance;

2) to determine whether there is any hidden implication in the definition of degree of balance which would tend to favor a valued relation rather than a zero relation.

The general computer model is described in general terms but many of its theoretical applications are omitted in order to more clearly see the effect of the items being investigated.

The choice of alternative situations for a subject is disproportionately dependent on relations between the other two subjects. Only where these relations have equal numbers of lines of each sign between them does choice depend largely on pairwise relations. Where most of the relations among members are negative, the simulation results assuming completely negative relations are vacuously balanced seem more reasonable. Conjectures in other cases are more difficult.

The degree of balance given by Cartwright and Harary permits but does not seem biased toward the preference of deletion of relations rather than negation. An alternate definition is suggested.

SOME PROBLEMS IN THE DYNAMICS OF SMALL GROUP BALANCE

By

Francis X. Mulvihill

A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

MASTER OF ARTS

Department of Sociology

1967

ACKNOWLEDGEMENTS

I would like to express my appreciation to the M.A. committee, viz. Thomas Conner and Herbert Karp and especially to the committee chairman, John T. Gullahorn, for their patience, aid and encouragement. I am also indebted to Sharon Baker and my wife, Beverly A. Mulvihill, for typing the preliminary and final drafts of the thesis. Finally, I thank the National Science Foundation which, through Grant GS-781 to John T. and Jeanne E. Gullahorn, supported the operation of the simulation.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF APPENDICES

BACKGROUND

One of the persistant concerns in Social Psychology has been the interdependence of affective and cognitive processes of interpersonal behavior. In the two person group composed of persons A and B, interest might center on A's liking for B and A's perception of B's liking for him as it affects A's perception of the logical soundness of B's opinion concerning some event. In other words, is A more likely to agree with B if he likes B than if he does not like B, other things being equal? The reverse relationship may also be considered: if B agrees with A, is A more likely to like B than if B disagrees with him? Finally, the question of the dynamics of the relationship might be considered. One aspect of the dynamics is the relative importance of each type of relationship, i.e. affective and cognitive, in determining the final relationship between the two. If A likes B, but B neither likes A nor agrees with A's opinion, is A likely to come to dislike B or is B likely to come to like A and agree with him? An obvious question here is the relative permanence or resistance to change of each of the two types of processes. This in turn leads to the question of the determinants of resistance to change which may be found in the situation itself. In the above example, high confidence of the individuals in their opinions would probably increase the likelihood that A would come to dislike B. But the affective relations themselves may help determine confidence. B's dislike of A may lead B to conclude that if he disagrees with A he must be correct in his opinion.

Although the addition of a third member increases the group elements by fifty percent, the complications accompanying this increase are disproportionately greater. The single set of relationships between A and B is increased threefold by introducing a set of relationships between A and C and between B and C. In addition to this, there is the question of the effect of the relationship between any two group members on the relationships of the third member with these two. If A likes B and B likes C, will the latter relationship affect A's relationship with C?

The purpose of this paper is to present a computer simulation model of a three person group discussion, such as that described above, employing the theoretical concepts discussed below in order to investigate the dynamics of the relationships which follow from the theory.

Fritz Heider (1) was directly concerned with the problem presented here, although his initial work was restricted to the case of two individuals in their relations to an impersonal object or event. He subsequently expanded his theory to include the case of three individuals (2). Heider conceived of relationships among elements of a group in terms of balance. In a two person group, a balanced state exists if the relations between them are all positive or all negative. In the case of three entities, a balanced state exists if all three relations are positive in all respects, or if two are negative and one is positive. He seemed somewhat unsure as to how to classify three negative relations among three group members. He initially considered this type as balanced, but then noted that it "does not seem to constitute a good psychological balance, since it is too indetermined." He then classified this as an unbalanced state. In regard to dynamics, Heider hypothesizes that if a balanced state does not exist, forces will arise to bring it into balance. If this is impossible, tension will result.

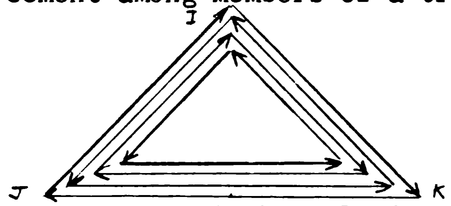
Cartwright and Harary (3) formalized the essential aspects of Heider's Balance Theory in terms of the mathematical theory of graphs. A finite graph is a finite collection of points and the finite set of all lines connecting all pairs of points. The individuals and objects of Balance Theory are coordinated to the points of the graph and relations are coordinated to lines. A path is a set of lines connecting two points of a graph. A cycle is a closed path and the length of the cycle is the number of lines constituting the cycle. A signed graph, or s-graph, is a graph whose lines may assume values of + or -. An s-graph of type T is a signed graph which includes T types of relations, e.g. a graph representing the variables of attraction and agreement would be an s-graph of type 2. The sign of a cycle is the product of the signs of the lines which constitute the cycle. A cycle whose sign is negative is unbalanced; a cycle whose sign is positive is balanced. If a set of points has one or more missing lines, i.e. if there are neutral or non-existent relations in the group, the set of points is said to be in vacuous balance. Degree of balance is the ratio of the number of positive cycles to the total number of cycles. Local balance is the balance of a graph at one point of the graph, i.e. the balance of those cycles which include that point. N-balance is the balance of those cycles having N lines or less. A finite digraph is a finite set of points and the finite set of all ordered lines between all pairs of these points. "Ordered" lines means that two lines connecting the same points but opposite in direction are considered to be different lines. A cycle of a digraph is a closed path whose lines are in the same direction. A semi-cycle is a closed path whose lines need not be in the same direction. An n-semi-cycle is a semi-cycle consisting of n lines. The concepts mentioned above in regard to graphs are extended here to apply to digraphs. An s-digraph of type T is a signed digraph representing T relations among the elements. Local balance is the balance of a digraph at one point. N-balance is the balance of those semi-cycles having N lines or less. The degree of balance of an s-digraph is the ratio of the number of positive semi-cycles to the total number of semi-cycles. A graph represents symmetric relations but a digraph is necessary to represent unsymmetric relations.

Besides facilitating the expansion of Heider's Balance Theory to collections of more than three elements, this formalization led to an important clarification. Heider had used the notation $\sim L$ to represent such consepts as dislike and $\sim U$ to represent such concepts as "has no connection with". The formalization called attention to the fact that $\sim L$ is an opposite relation and would be represented by a negative line, but $\sim U$ is a complementary relation and would be represented by the absence of any lines. It also made possible a reinterpretation of Jordan's experiments (4), in which some ambiguity in measures of psychological discomfort were clarified by differentiating between negative and missing relations. They found that the scores increased according to whether the situation was balanced, vacuously balanced or unbalanced.

In terms of graph theory, a group discussion among individuals

I, J and K could be represented as follows:

Figure 1 Graph Theory model of attraction and agreement among members of a triad.



where the two outer cycles represent the relation of agreement and the two inner cycles represent the relation of attraction. In graph theory convention, a solid line represents a positive relation, a dashed line represents a negative relation and the absence of a line represents a neutral or non-existent relation. The solid lines could be replaced by dashed lines or they could be removed, according to the type of relationship represented.

Harary (5) has hypothesized four tendencies in the dynamics of small groups, three of which are of interest here:[1]

1. Tendency toward balance: a group will endeavor to achieve a state of balance.
2. Tendency toward completeness: if two entities are not related, a bond will tend to appear between them.
3. Tendency toward positivity: there will be a marked preference for forming positive bonds rather than negative bonds.

The tendency toward completeness is of special interest here, since, as Berger, et. al. (6) have noted, Heider's theory does not postulate that a relation will occur. It merely postulates that if a relation does occur, it will tend to be in balance with other relations. The tendency

---

1. The fourth pertains to structures of more than three elements.

toward positivity is based on Jordan's (4) experiments in which he found that a measure of psychological discomfort was higher in a situation consisting of negative relations than in one of positive relations, beyond the discomfort attributable to imbalance.

Harary also introduces two concepts pertaining to the manner in which balance is achieved. A negation-minimal set of lines is the smallest number of lines in a graph which, when the signs are reversed, produce balance. A deletion-minimal set of lines is the smallest number of lines in a graph which, when deleted from the graph, produce balance. Although Harary does not attempt to relate all of these mathematical concepts to the substantive area of small group research he suggests that they may have implications for the understanding of empirical phenomena. Harary, Norman and Cartwright (7) also use these concepts but they suggest that the concepts may be applicable if there is cost connected with change. If this is the case, a negation-minimal or deletion-minimal set of lines would represent those relationships whose alteration would produce balance at the lowest cost. These concepts apply only in the case of complete balance, however, and will be used only analagously since this model does not treat the graph in its entirety in any one step.

Finally, Harary defines balance differently in the case of a completely negative graph with no missing lines than in the case of partially negative graphs. This definition, however, is an asymptotic expression, i.e. it pertains to the ultimate degree of balance toward which a completely negative graph will move. Since it pertains only to a complete graph and since it can not be used in a step by step simulation, it will not be considered here.

THEORY

The next step is to relate these various propositions, implications and findings to each other within a simgle theoretical framework. The conceptual framework of the Exchange Theory of George Homans (8) will be used to accomplish this. Specifically the theory will use the concepts of reward, cost and profit, i.e. reward minus cost, of social action, as determined by the values associated with the three types of balance among relations and the nature of the relations themselves.

These values are as follows:[2]

1. Balance among relations is positively valued (4, 20-25).
2. Imbalance among relations is negatively valued (4, 20-25).
3. Vacuous balance is less positively valued than balance but more positively valued than imbalance (4, 20-25; 5, 13-14).
4. Positive relations are positively valued (5, 15-16).
5. Negative relations are negatively valued (5, 15-16).
6. Neutral positions are less positively valued than positive relations but more positively valued than negative relations.[3]

---

2. (X,Y) following the value statements refer to the pages and lines in the preceding Background section in which the theoretical and empirical bases of these statements may be found.

3. This assumption is based on interpolation between statements 4 and 5 and by analogy with statement 3.

7. Change is negatively valued (6, 5-20).

8. Balance is more highly valued than positive relations (5, 15-16), e.g. a balanced cycle of one positive and two negative relations is more highly valued than an unbalanced cycle of one negative and two positive relations, even though the number of positive relations is greater in the latter situation.

In addition to these value statements, two assumptions are made concerning confidence:

1. An individual's confidence in his opinion is proportional to his degree of balance.

2. Where confidence is high, maintenance of opinion is more highly valued than the degree of balance.

The first assumption implies that group support of an individual's opinion through agreement with him is not necessary to maintain his confidence if the other relations with him are not positive. If I dislikes J and J disagrees with I, I will have high confidence in his own opinion, in spite of the disagreement, since he has another negative relation with the source of the disagreement.

The second assumption implies that if (1) I has high confidence in his opinion, (2) the degree of balance for I is low and (3) alteration of his other relations does not improve balance, then I will not change his opinion.

By the first assumption, however, his confidence may decrease, permitting him to change his opinion in the future.

Applying Homans' exchange concepts to the value statements, the theoretical basis of the model is as follows:

Proposition: Individuals act to maximize their rewards and/or minimize their costs.

Assertion 1: A negatively valued state is costly.

Combining the Proposition and Assertion 1,

Hypothesis 1: Individuals will act to avoid a negative state.

Assertion 2: A positively valued state is rewarding.

Combining the Proposition and Assertion 2,

Hypothesis 2: Individuals will act to maintain a positive state.

Assertion 3: Change of state is costly.

Combining the Proposition and the three Assertions,

Hypothesis 3: Individuals will change to a more positive state subject to the cost of changing from the original state.

In more empirically oriented terms, Hypotheses 1 and 2 state that individuals will not move to a more negative state under any circumstances. Hypothesis 3 states that individuals will move to a more positive state if the increase in the degree of balance or in the number of positive relations exceeds the cost of change.

The simulation is a specific statement of the interrelations among: the value statements, the assumptions regarding confidence and the three hypotheses.

The scope of the theory is limited by the following considerations:

1. Each group member holds a specific opinion.
2. The subject of the opinion is specific, i.e. the opinion is comprised of only one component. The members must agree

or disagree with each other; there is no partial agreement.

3. Opinions of each member are known to each other member and are correctly perceived by all members.

4. Each member correctly perceives each other member's attitudes.

5. There are at least three possible opinions.

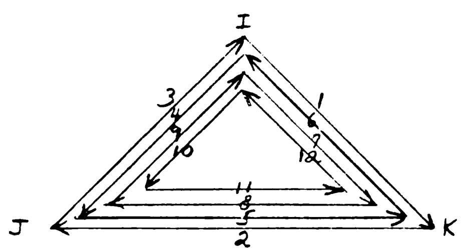
6. The members can not withdraw from the group.

THE SIMULATION

The next step in the development of the simulation is to translate the digraph model of Fig. 1 on page 5 into a form on which the computer can operate and to translate the theory into a set of logical rules which the computer can employ to determine the transition of the group from one state to another.

The lines of Figure 1 on page 5 are first assigned identification numbers:

Figure 2. Graph Theory model of a triad whose numbered lines represent types and directions of relations.



In matrix form, the rows of the matrix represent the originator of a relationship, the columns represent the receptor and the entries in the matrix cells represent the value of the relationship between the originator and the receptor. Figure 3 shows the matrix equivalent of the graph of Figure 2.

Figure 3. Matrix representation of the graph of Figure 2.

Matrix A

	I	J	K
I	0	10	7
J	9	0	11
K	12	8	0

Matrix B

	I	J	K
I	0	4	1
J	3	0	5
K	6	2	0

Matrix A represents the relations of attraction and Matrix B represents agreement. The possible values of the attraction matrix entries are +1, 0 and -1 corresponding to liking, neutrality, and disliking; the possible values of agreement are +1, and -1 representing agreement and disagreement.

The degree of balance can be calculated using these matrices. Although Cartwright and Harary do not define degree of balance for an s-digraph of type 2, they do define an s-graph of type 2 to be balanced if all of its cycles are positive but even this is atentative definition. Morrissette (9) differentiated two types of cycles: a mixed cycle is one which involves two types of relations; a pure cycle is one which involves only one type. A graph of type 2 is balanced if all cycles of length two and all pure cycles of length greater than two are balanced. This definition seems to contradict Heider's theory, however, since his central argument was that relations of different types will tend toward balance. It seems likely that, for any member of a triad, the relationship between the other two members will be of no consequence to him except as their relationship affects his relationship with each of them. This is represented in graph theory by the concept of local balance at point X, i.e. the balance of only those cycles which include point X. Using this concept and extending Cartwright and Harary's definitions of degree of balance of an s-digraph and balance of a type 2 graph, the degree of balance of a type 2 s-digraph at point X is defined as the ratio of the number of positive semi-cycles through point X to the total number of semi-cycles through point X. The total number of semi-cycles at any point in Figure 3 is $4^3 + 2C_2^4 = 76$.

An interesting consequence of this definition is that the degree of balance need not be the same for all individuals in a given situation. Although it is assumed here that every semi-cycle is equally important in determining the degree of balance, empirical tests are necessary to determine whether certain cycles are more important than others.

Appendix A is the flow chart which describes in detail the steps of the program. Appendix B lists the input data read into the computer as the program progresses. In regard to the random selection of groups determined by the group types specified by parameter card #3, if three members of a group have three possible positions available in regard to an issue in a group discussion, there are ten possible situations, disregarding the permutation of members among positions:

Table 1. Distributions of three group members among three opinion positions.

	Position Values 1	2	3	Total Difference Among Position Values	Average Difference
1	⋮			0	0
2		⋮		0	0
3			⋮	0	0
4	:	•		2	2/3
5		•	:	2	2/3
6		:	•	2	2/3
7	•	:		2	2/3
8	:		•	4	4/3
9	•		:	4	4/3
10	•	•	•	4	4/3

There has been some evidence that confidence is related to extremity of position. If this is the case it should be possible to indicate positions as well as agreement using binary values for agreement and confidence. If the first seven cases above are considered high or relatively high agreement groups on the basis that the average difference among members is less than one, this agreement can be reflected by unique configurations of agreement and confidence scores. Similarily, if the last three types are considered low or relatively low in agreement, on the basis that the average difference among members is greater than one, they also can be characterized by unique combinations of scores. Score combinations for each type are as follows:

Table 2. Agreement and confidence scores for one pattern of group types from Table 1.

Type	Agreement Scores			Confidence Scores		
	IJ	IK	JK	I	J	K
1	+	+	+	+	+	+
2	+	+	+	0	0	0
3	+	+	+	+	+	+
4	-	-	+	0	+	+
5	-	-	+	0	+	+
6	-	-	+	+	0	0
7	-	-	+	+	0	0
8	-	-	+	+	+	+
9	-	-	+	+	+	+
10	-	-	-	+	0	+

Duplicate situations, viz. 1,3; 4,5; 6,7; 8,9 are listed to indicate the probabilities of occurence of each type of situation, given that the group type is specified. It is on this basis that group types by agreement are selected. Attraction is specified by use of a 3:2:1 ratio for high and low attraction and a 1:2:1 ratio for neutral attraction, e.g. in high attraction groups (+) relations should appear 50% of the time, (0) relations 33% of the time and (-) relations 17% of the time.

Harary's concepts of negation-minimal and deletion-minimal sets of lines described above pertain to the attainment of balance for the entire group structure. In the dynamics of a group discussion, however, it is unlikely that all members would simultaneously change those relations necessary to achieve perfect balance. This model, therefore, considers the alternative relations available to each subject in turn and determines whether any of these alternatives produces a more rewarding state, i.e. a higher degree of balance and/ or a greater number of positive relations. No contention is made that this process approximates empirical reality. A simulation which does attempt to approximate reality is one developed by John T. and Jeanne E. Gullahorn (10). Their simulation, which gave rise to the simulation presented in this paper, uses essentially the same variables. But they also consider the type of activity involved, which uses a modification of Bales twelve categories to classify types. On the basis of this variable and the three variables common to both simulations, not only responses but the selection of the next actor is determined. Although the order in which subjects are selected does affect the outcome of a simulation, this factor is not relevant to the purpose of the simulation presented here, which is merely to investigate the dynamics of group interaction and determine what unforeseen implications or problems are involved in the theory.

When the focal subject is selected in the simulation process, the alternate states available to him are determined. This is done by entering all possible combinations of values of attraction and agreement in the row representing the focal subject. For example, if subject number two is the next subject, he may have any one of three values of liking for each of subjects number one and three. This produces nine different combinations of values in the second row of the attraction matrix. For each one of these combinations, there are also various combinations of the values of agreement between him and the other two subjects. The logical constraints on the number of combinations of agreement are described below. The degree of balance for the focal subject produced by each combination of values is compared with the degree of balance of the matrix representing his state after the prior subject had changed his relations. In the case of subject two, subject one would determine a more desireable state for himself and change to that state. The degree of balance for subject two produced by that state is the basis of comparison of alternate states. Any alternate state which has a degree of balance for subject two lower than the initial state is ignored as a possible future state. This causes the initial state to be considered as an alternate, i.e. it provides for the possibility that the subject may not change his current state. The alternate states with degrees of balance higher than the initial state are stored in matrix form for further operations. This step is the formalization of Hypothesis 1.

The constraints mentioned above are as follows. The first is that, of the eight mathematically possible agreement patterns, only five are logically possible, e.g. subject to the assumption that opinions are correctly perceived and have only one component it is logically impossible for I to agree with both J and K but for J and K to disagree with each other. The second constraint pertains to the possible alternate agreement patterns which a subject may choose. Taking I as the focal subject, if J and K disagree with each other, I may agree with either one or neither but not both. This limits I to three alternative states (including the original state). If J and K agree with each other, I may either agree with both or disagree with both. This limits I to two alternative states. The maximum number of alternatives therefore is 3 x 3 x 3 = 27 if J and K disagree or 3 x 3 x 2 = 18 if J and K agree, since I has liking relations with two others and each of these may assume values of +1, 0 or -1. The third constraint is that if confidence is positive, change of agreement is not possible.

After the selected alternate matrices have been stored, each one is tested for the number of changes between it and the initial matrix. Before preceeding with this step, however, the use of the parameters listed in Appendix A, parameter card # 6, must be clarified. As an example, suppose the initial degree of balance of a subject is .50, his liking for the other two subjects is +1 and 0 and his agreement with them is -1 and -1. Also suppose that his degree of balance in an alternate state is .75, his liking for the other two subjects is -1 and +1 and his agreement with them is -1 and +1. Then let the parameter values be as follows:

Table 3. Parameter Values

Parameter	Value	Variable
1	3.00	Cost of change
2	1.01	Upper balance limit
3	-0.01	Lower balance limit
4	1.00	Change of attraction in negative direction
5	0.00	Change of attraction in positive direction
6	2.00	Change of agreement in negative direction
7	1.00	Change of agreement in positive direction

In tabular form, the relevant aspects of the alternate state, initial state, differences between them and the appropriate parameters are as follows:

Table 4. Calculation of weights by Types of Change

	Alternate State	Initial State	Difference	Parameter Value	Absolute Value of Product
1. Balance for P	.75	.50	+.25	N.A.	
2. Liking for O_1	-1	+1	-2	1.00	2.00
3. Liking for O_2	+1	0	+1	0.00	0.00
4. Agreement with O_1	-1	-1	0	0.00	0.00
5. Agreement with O_2	+1	-1	+2	1.00	2.00
Sum of lines 2-5					4.00

Absolute values are used so that changes in opposite directions will not cancel each other when added together since the assumption is that any change regardless of direction is costly.

The sum of 4.00 is then multiplied by the first parameter divided by the maximum number of cycles, which in this case is seventy six. This division is used to bring the units of measurement of cost of change into agreement with the units of degree of balance.

The net degree of balance for the alternate state is therefore:

$$.75 - 4 \times 3/76 = .75 - .16 = .59.$$

Since this is higher than the degree of balance for the initial state, it is a possible state to which the individual would move.

This calculation is performed for all selected alternative states and the net degree of balance is stored. If there is one alternate state with the highest net degree of balance, that state is substituted for the initial state.

If two or more alternate states have equal net degrees of balance, which are also the highest, selection among them is made on the basis of the values of the relations in the matrix. This is expressed in terms of the difference between each alternate state and the original state. In the example of Table 4., if another state also had a net degree of balance of .59, but its values on lines 2-5 were 0, +1, -1 and +1, this state would be chosen since the liking score of neutrality toward O_1 is more highly valued than a negative score and all other scores are equal. If two or more scores with equal net degrees of balance also have equal values of relations, they are checked to determine if the initial state is one of the alternates. If it is, no changes are made. If it is not, selection among these equivalent states is made randomly, and the selected alternate state is substituted for the initial state.

If a substitution has been made, the degree of balance for each subject is recalculated on the basis of the new matrix. In any case, the next step is to check the confidence scores of the two non-focal subjects. The focal subject is excluded in this step on the assumption that he will wait for the reaction of other subjects before he changes his confidence. The second and third parameters of Table 3. are included in this calculation. If the subject's degree of balance is higher than or equal to the second parameter and his confidence is zero, his confidence is increased to one. If the subject's degree of balance is lower than or equal to the third parameter and his confidence is positive, his confidence is decreased to zero. Otherwise, there is no change in confidence. In the example of Table 3., it is impossible for confidence to change since the parameter values are outside the 0 to 1 range of the possible values of degrees of balance.

In the initial step of selection of alternate matrices, if there are no alternates the confidence of the focal subject is checked in the same manner discussed above for the non-focal subjects. This is a formalization of Heider's proposition that when balance is not attainable conflict will result. Conflict is formalized as loss of confidence.

If there have been three consecutive iterations with changes in neither degrees of balance nor confidence or if all degrees of balance are equal to one, the simulation of the group is terminated. Otherwise, the simulation is continued up to the number of iterations in columns 9-12 of parameter card #2. At termination of the group simulation, the final state of the group is printed.

If there are no more groups within a group type, the next group type is selected and the process is repeated. If there are no further group types, the program is terminated.

PURPOSE

As mentioned above, Heider was apparently uncertain of the proper classification for completely negative relations among three elements. His decision was to classify such relations as unbalanced which Cartwright and Harary formalized as negative cycles. Heider apparently did not consider the possibility of a situation which was neither balanced nor unbalanced, which is the type of situation which Cartwright and Harary labelled as vacuously balanced. Although the definition of vacuous balance pertains only to cycles which have zero valued lines, it seems reasonable, in view of Heider's uncertainty, to define a completely negative cycle of length greater than two as vacuously balanced. The first purpose of this paper, therefore, will be to compare the dynamics of the simulation under two conditions:

1) A completely negative cycle of length three is unbalanced, and

2) A completely negative cycle of length three is vacuously balanced.

Harary has noted that there is a tendency toward completion in groups, i.e. where there is no bond between members, a bond will tend to appear. The second purpose of the paper therefore is to determine, through the simulation, whether there is any implication in his definition of degree of balance which would favor the selection of a valued relation over a zero relation as a group strives to achieve balance.

Since these problems logically precede the simulation of theory and since the probability of detecting implications will be greater if extraneous restrictions on the dynamics are minimized, two aspects of the theory will be omitted here. First, the parameters for cost of change will be set equal to zero. This in effect, is a limitation of a restriction on the alternate states which are potential substitutes for the initial state. Second, the parameters for change of confidence will be set outside the range of values of the degree of balance, as in Table 3., so that confidence will not change.

The first part of the analysis therefore will be a two way comparison of the outcomes of the dynamics: the first dimension will be the group type, the second dimension will be the definition of completely negative cycles of length three. The second part of the analysis will be an investigation of the conditions under which valued and zero relations are selected as alternates.

ANALYSIS

Figures 4A-4C present the development of relations of a three person discussion group for each of three types of groups[4], classified by their degree of attraction and agreement. For any group member there are twelve semi-cycles of length two, i.e. six semi-cycles composed of each pair of lines with each of the other two members, and sixty four semi-cycles of length three, i.e. each line between any given pair of members taken in combination with each line connecting both of the other two pairs. The degree of balance is the number of these semi-cycles

---

4. The High Attraction, High Agreement case is not considered since it has no significant properties not included in the other three.

Figure 4A. Development of attraction and agreement among members of a Low Attraction, Low Agreement Group

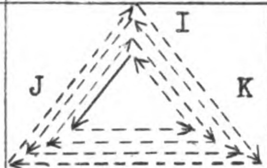
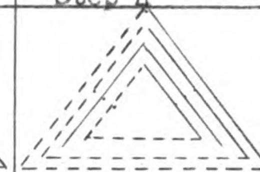
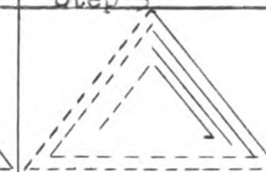
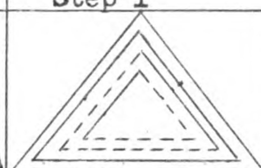
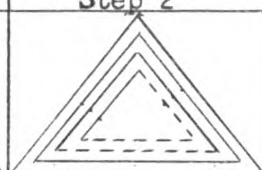
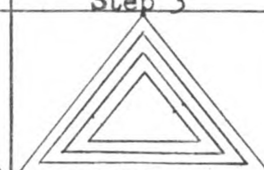
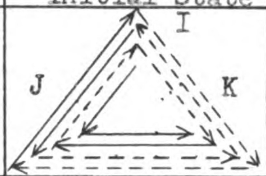
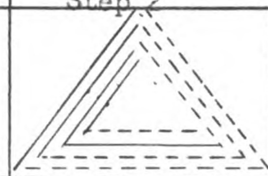
Method[a]	Initial State	Step 1	Step 2	Step 3	Step 4	Step 5
1.						
Balance	.33 .33 .37	.39 .43 .43	.49 .53 .54	.53 .51 .58	.75 .74 .79	1.00 1.00 1.00
2.	SAME					
Balance	.89 .89 1.00	1.00 1.00 1.00				

Figure 4B. Development of attraction and agreement among members of a Low Attraction, High Agreement Group

Method[a]	Initial State	Step 1	Step 2	Step 3	Step 4	Step 5
1.						
Balance	.47 .47 .47	.50 .49 .49	.64 .64 .61	1.00 1.00 1.00		
2.	SAME	NO ALTERNATIVES				
Balance	.53 .53 .53	.53 .53 .53				

[a] Method 1: Completely negative semi-cycles of length three are calaulated as negative semi-cycles.
Method 2: Completely negative semi-cycles of length three are calculated as zero semi-cycles.

Figure 4C. Development of attraction and agreement among members of a High Attraction, Low Agreement Group

Method[a]	Initial State	Step 1	Step 2	Step 3	Step 4	Step 5
1.		NO CHANGE				
Balance	.53 .48 .51	.53 .48 .51	.79 .75 .74	1.00 1.00 1.00		
2.	SAME					
Balance	.59 .54 .57	.64 .58 .64	.75 .71 .75	.92 .92 1.00	1.00 1.00 1.00	

[a]Method 1: Completely negative semi-cycles of length three are calculated as negative semi-cycles.
Method 2: Completely negative semi-cycles of length three are calculated as zero semi-cycles.

whose sign is positive divided by the total seventy six semi-cycles. Where zero relations exist, both numerator and denominator exclude the number of semi-cycles which include the zero relations. As noted in the table, the first method calculates the degree of balance assuming that completely negative semi-cycles of length three are unbalanced; the second method assumes they are vacuously balanced.

In Figure 4A, all relations are negative, except that I likes J. In the initial state, I has three positive and three negative semi-cycles of length two with J, six positive semi-cycles of length two with K and forty eight negative and sixteen positive semi-cycles of length three connecting all members. Since I has positive confidence, his agreement relations can not change. I, therefore, has nine alternate states; he can take any one of three attraction relations with the other two members. Of these nine possibilities, a positive relation with both J and K produces a degree of balance of .39, which is the single highest value possible. An alternative which seems reasonable in this case is for I to change to a negative or neutral relation with J, making all semi-cycles negative. Since a negative semi-cycle is considered unbalanced, however, this alternative is rejected. By changing his attraction toward K to positive he changes twelve of the forty eight negative 3-semi-cycles to positive, four of the sixteen positive 3-semi-cycles to negative and three of the six positive 2-semi-cycles with K to negative, for a net increase of five positive semi-cycles. The dominant factors in this change are the completely negative relations between J and K, the relatively large number of semi-cycles which include these relations and the predominantly negative relations between I and J. In more empirical terms,

this move is a consequence of I placing more emphasis on the fact that neither he nor K have may positive relations with J than on the fact that J neither likes him nor agrees with him. I therefore decides that K "can't be all bad". It seems reasonable to assert that these direct negative relations with J would become balanced before more complax third party relations with K are changed. This could be accomplished in the model by either reducing the weight of 3-semi-cycles relative to 2-semi-cycles or by calculating completely negative 3-semi-cycles as vacuously balanced. This problem will be discussed below. Another alternative not considered above is that of shifting to neutral relations. If I changed his attitude toward J to reutrality, three negative 2-semi-cycles and sixteen positive 3-semi-cycles would have been deleted. Since the proportion of positive semi-cycles deleted is greater than the original proportion, the degree of balance would decrease.
If I changed his attitude toward K to neutrality, three positive 2-semi-cycles would have been deleted. Although the number of negative semi-cycles deleted is greater than the number of positives, the proportion of positives deleted is again greater than the original proportion, and the degree of balance would decrease.

In the second step, J also has positive confidence in his opinion and therefore can not change his agreement relation. Of the nine possible alternatives, seven produce a higher degree of balance than the original state. Again, the fact that the two other members have predominantly negative relations is more important than the character of relations between J and the two other members taken individually. Since I has predominantly negative relations with K, and J has completely negative relations with K, J comes to like I. The fact that I likes J has little bearing on J's change.

Since K has zero confidence, and since I and J disagree, K has three possible agreement alternatives: he can maintain his disagreement with both I and J or he can change to agree with I or J. He can not agree with both since they do not agree with each other. The nine possible combinations of attraction combined with the three agreement alternatives produce a total of twenty seven alternative states. Of these, there are four which produce a higher degree of balance than the original. Of these four the three highest have equal degrees of balance and the fourth is only one percentage point lower than these. In all cases, K changes his opinion so as to agree with I. The attraction values with I and J are as follows:

Table 5. Attraction alternatives for K with equal degrees of balance.

Alternative	I	J	Balance
1.	+	0	.58
2.	0	-	.58
3.	+	-	.58
4.	0	0	.57

The first alternative is selected on the basis of the assumption that positive relations are more highly valued than neutral and neutral are more highly valued than negative.

In the original state, there are equal numbers of positive and negative 3-semi-cycles, since I and J have equal positive and negative relations between them. The predominant factors in K's changes, therefore, are the pairwise relations and not the third party relations. In other words, no matter what changes K makes, the number of positive and negative 3-semi-cycles will be equal, since the semi-cycles including one type of relation between I and J will be offset by the semi-cycles including the opposite type. Since K can not directly change I's liking for him, the only way to increase balance is to change his attraction and agreement relations with I to (1) both positive or (2) zero and positive respectively. In regard to J, the fact that J dislikes K prevents K from agreeing with or liking J.

The fact that K can achieve equal degrees of balance is of interest here. If K had maintained his negative relations with J, he would have the option of changing his negative attraction toward I to either zero or positive. In the initial state K has six positive semi-cycles with J, three positive and three negative semi-cycles with I and thirty two each of positive and negative 3-semi-cycles. If he changed to agreement with and neutrality toward I, he would eliminate the three negative semi-cycles with I and eight each of the positive and negative 3-semi-cycles. The total number of semi-cycles becomes fifty seven and the number of positives becomes thirty three which yields a degree of balance of .58. If K changed to agreement with and liking for I, the three negative semi-cycles with I would be changed to positive and this would be the only change. The total number of semi-cycles remains seventy six and the number of positives becomes forty four which also yields a degree of balance of .58.

In step 4, the positive relations between I and K and the negative relations between J and K cause I to come to dislike J in spite of the fact that J changed to like I. Again, the third party relations prevail.

In the final step, J can achieve perfect balance by changing his attraction relations with I and K to (1) dislike both, (2) dislike one and become neutral toward the other or (3) become neutral toward both. The last state is chosen since it has the least number of negative relations. If J had merely eliminated his positive relation with I, he would have reduced the number of 3-semi-cycles by twelve, all of which were negative and the number of 2-semi-cycles by three, which also are all negative. This eliminates all negative semi-cycles thus producing a balance of 1.00. This is an example of Harary's theorem (5) that a negation-minimal set is a deletion-minimal set and vice-versa, i.e. when the result is perfect balance, it makes no difference whether the lines causing imblance are changed in sign or deleted.

The same principle applies in the single step necessary to produce balance in Method 2 of Figure 4A. Since completely negative 3-semi-cycles are vacuously balanced in this method, there are only sixteen 3-semi-cycles and these are all positive. There are also six positive 2-semi-cycles between I and K and three positive and three negative 2-semi-cycles between I and J. By changing his positive relation with J to negative, I changes the three negative 2-semi-cycles to positive and the sixteen positive 3-semi-cycles to zero. Since all negative semi-cycles are eliminated, perfect balance is achieved and Harary's theorem applies again. Since zero relations are preferred to negative relations, deletion is chosen rather than negation.

In empirical terms, Method 2 says that I does not regard the relations between J and K as a factor when these relations are negative and his relations with J and K are also predominantly negative. The only criteria of change in this case are the pair relations.

Appendix C contains the verbal description of the dynamics of the groups of Figures 4B and 4C.

DISCUSSION

The most obvious problem regarding the definition of degree of balance used here is the disproportionate weight given to semi-cycles of length three in determining balance. With two non-symmetric variables and three group members, there are, for any single subject, twelve semi-cycles of length two and sixty four semi-cycles of length three. It seems unlikely that third party relationships are more than five times more important than direct relationships in determining behavior. But it also seems unlikely that any of these third party relationships is of no consequence whatever. There are at least five methods of weighting semi-cycles. The first is to give the same weight to all semi-cycles of the same length. The second is to weight cycles by the strength or importance of the type of relationship, e.g. to weight lines representing agreement differently than lines representing attraction. The third is to weight semi-cycles according to the direction and number of the component lines, e.g. lines originating from the focal subject may be more important than lines going to the focal subject. The fourth is to combine the first and second methods and the fifth is to combine the second and third methods. If the third method were used, the first would be unnecessary. It is doubtful whether such alternatives can be empirically tested, especially considering the complications involved in estimating parameters for costs associated with change and for the degree of balance associated with change of confidence. This is further complicated by the usual problems of instituting sufficiently tight controls in the experimental situation to determine such precise and complex details. The simulation would have to imcorporate provisions for the sequence of selection of subjects if it is to empirically verified, since the order makes a difference in the outcome.

In regard to the treatment of completely negative 3-semi-cycles, the following generalizations are noted:

When these semi-cycles are considered unbalanced and

1. where the relations between the non-focal subjects are predominantly negative, positive relations between the focal subject and one of the non-focal subjects tend to appear;
2. where the relations between the non-focal subjects are equally positive and negative, changes are determined by the direct relations between the focal and non-focal subjects;
3. where the relations between the non-focal subjects are predominantly positive, the focal subject tends to form relations of the same sign with both non-focal subjects. The sign of the relations depends on the predominant sign of the pairwise relations in the initial state.

When these semi-cycles are considered vacuously balanced, the consequences are somewhat different. Where relations between the non-focal subjects are predominantly negative, there is a stronger possibility that relations between the focal and non-focal subjects will both be negative, since completely negative semi-cycles are considered vacuously balanced. This causes more weight to be placed on the 2-semi-cycles than in the first method. Where relations between the non-focal subjects are equally positive and negative, changes are not completely determined by the direct relations between the focal and non-focal subjects. This is because the completely negative cycles do not cancel the cycles composed of the negative relations between the focal and non-focal subjects and the positive relations between non-focal subjects. Where relations between the non-focal subjects are predominantly positive, change will be determined similarily to the manner of the first method.

These differences give some suggestion on how verification of the treatment of completely negative 3-semi-cycles might be accomplished:

1. if completely negative 3-semi-cycles have no effect on choice of alternatives, changes in a predominantly negative situation should be made on the basis of pairwise relations only, or at least the negation of a positive relations should be as likely as the negation of a negative relation;
2. under the conditions of 1. above, changes should be made so that, if there are equal positive and negative relations between non-focal subjects, completely negative semi-cycles do not counter balance semi-cycles which have two negative relations between focal and non-focal subjects and positive

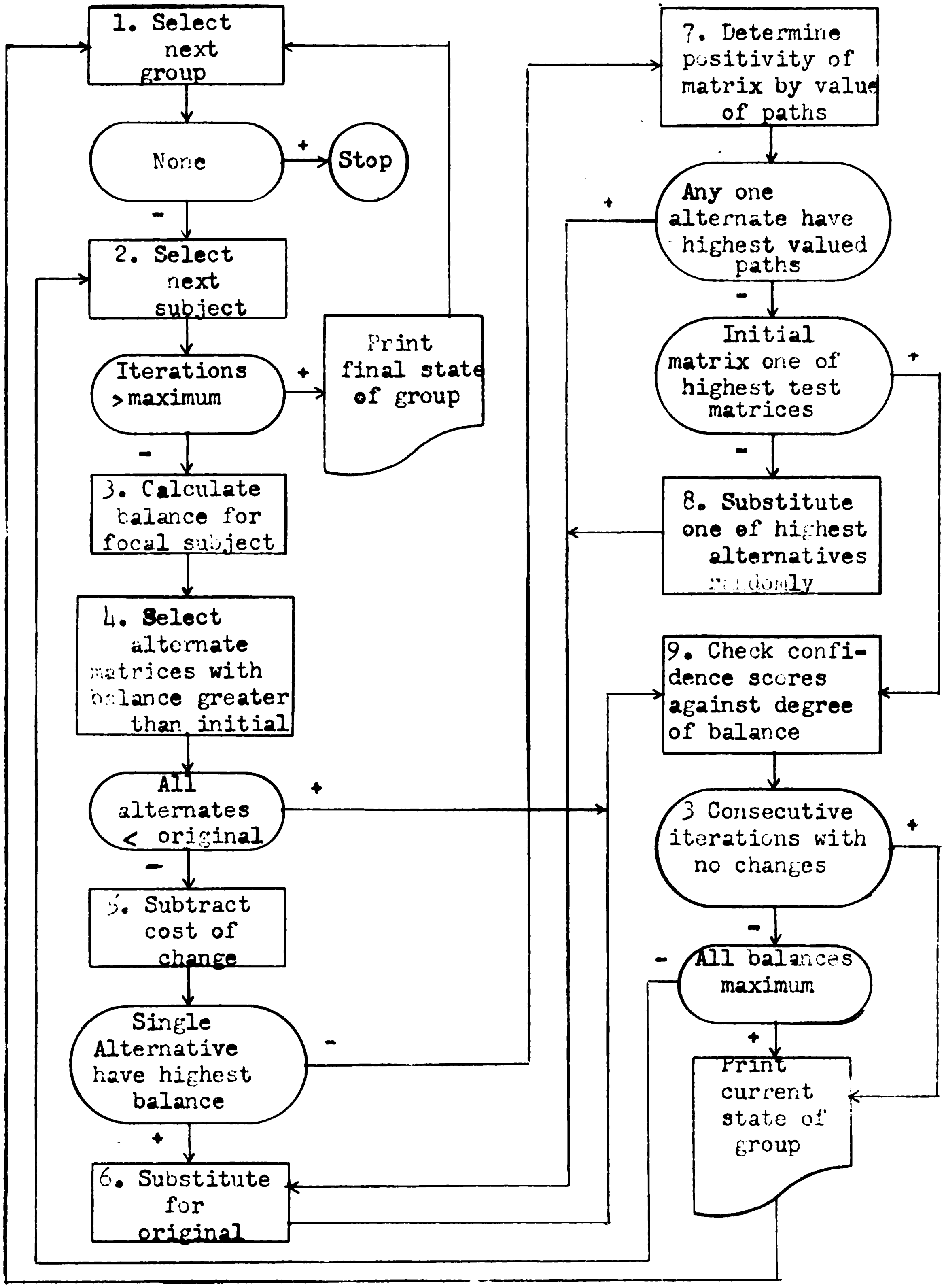
relations between non-focal subjects.

In view of the fact that estimation of parameters would probably be exceptionally difficult, if not impossible, and that this problem would obscure the way in which completely negative cycles are experienced by subjects, it probably would not be feasible to merely compare observed laboratory dynamics with the model to determine the proper definition. It would probably be necessary to determine during the course of an experiment the most relevant aspects of the situation for the subjects. The simulation described here should suggest some specific questions which could be asked of a subject in specific situations. These questions could be related to the costs of change and determinants of confidence as well as to perception of the imbalance.

In regard to the question of tendencies toward completion, it is obvious from the simulation and from Harary's own theory that there is no implication in his definition of degree of balance that valued relations will be preferred over zero relations when perfect balance can be achieved. In other words, if a subject is able to reach perfect balance by some particular move, it makes no difference whether the move is by negation or deletion. From the small sample of situations simulated in this paper, it appears that under other conditions completion is likely but not necessary. It is possible in the theoretical basis of the simulation and in the simulation itself to provide for the selection of valued rather than zero relations. It is also possible to define balance in such a way as to force completion. By defining the numerator of the degree of balance as the maximum number of semi-cycles in the digraph, i.e. the number of semi-cycles in the complete digraph, and the denominator as the number of positive semi-cycles, the tendency toward completion will be assured. In other words, a vacuously balanced cycle would appear as a zero in the numerator but as a one in the denominator. Perfect balance could be achieved only if all semi-cycles were positively balanced. A further refinement could be introduced by subtracting unbalanced from balanced semi-cycles in the numerator. This would provide for the assumption that vacuously balanced semi-cycles are more highly valued than balanced semi-cycles. The ultimate test of this definition is also reality.

APPENDIX A

FLOW DIAGRAM FOR PROGRAM



APPENDIX B

PARAMETER CARDS FOR "BALANCE"

CARD	COL.	ENTRY	FORMAT
1	1-4	Number of Group Types	1
	7-8	Logical Unit Number for External Storage	1
	12	0: If Matrices and Confidence Read from Cards	
		1: If Matrices and Confidence Chosen Randomly	1
2	1-4	Number of Times Each Group to be Run	1
	5-8	Number of Groups	1
	9-12	Number of Times Each Group to be Cycled	1
	13-16	Number of Subjects	1
	17-20	0: If Print only Initial and Final States	1
		1: If Print State After Each Cycle	1
	21-24	0: If No Punched Deck of Input Matrices and Confidence	1
3	4	1: If Low Cohesive	1
		2: If Neutral Cohesive	
		3: If High Cohesive	
	8	1: If Low Agreement	1
		2: If High Agreement	
4		Liking and Agreement Matrices	F2.0
5		Confidence	F2.0

OMIT CARDS 4,5 IF CARD #1, COL. 12 = 1.

IF NOT = 1, REPEAT CARDS 4,5 UP TO NUMBER IN COL.5-8, CARD #2

CARD	COL.	ENTRY	FORMAT
6	1-4	Number of Graph Cycles Equivalent to One Change of Position	F4.2
	5-8	Degree of Balance at Which Confidence Increases	F4.2

CARD	COL.	ENTRY	FORMAT
6	9-12	Degree of Balance at Which Confidence Decreases	F4.2
	13-16	Weight of Change of Liking in Negative Direction	F4.2
	17-20	Weight of Change of Liking in Positive Direction	F4.2
	21-24	Weight of Change of Agreement in Negative Direction	F4.2
	25-28	Weight of Change of Agreement in Positive Direction	F4.2

REPEAT CARD #6 UP TO NUMBER IN COL. 1-4, CARD #2

REPEAT CARDS #2-6 UP TO NUMBER IN COL. 1-4, CARD #1

NOTES on cards #4 and 5:

1. Card #4: Columns 2, 8, 16, 22, 30, 36 should be zero or blank, since these represent the relation of a subject with himself. The first 3 numbers on the card represent the first row of the Attraction matrix; the next three represent the first row of the Agreement matrix; the next three the second row of the Attraction matrix, etc. The possible values are 1 for liking, 0 for neutrality, -1 for disliking, 1 for agreement and -1 for disagreement.
2. Card #5: Columns 2,4 and 6 represent the confidence levels of subjects 1, 2 and 3 respectively. The possible values are 1 for confidence and 0 for no confidence.

APPENDIX C

VERBAL DESCRIPTION OF GROUP DYNAMICS FOR LOW ATTRACTION, HIGH AGREEMENT AND HIGH ATTRACTION, LOW AGREEMENT GROUPS

In Figure 4A., there are two positive agreement bonds and two negative attraction bonds connecting each pair of members. All nine attraction pattern alternatives have a degree of blance greater than or equal to the initial balance of .47, but the highest alternative is only .50. Since there are equal numbers of positive and negative relations with J and K, any move made by I will result in equal numbers of positive and negative 3-semi-cycles. The determination of the alternate state, therefore, depends entirely on the 2-semi-cycles. The equal numbers of positive and negative relations here produce two positive semi-cycles out of six total semi-cycles. By changing his liking for J and K to agree with the agreement relations, I increases the ratio of positive 2-semi-cycles to three out of six.

In step 2, J has three positive and three negative 2-semi-cycles with I, Two positive and four negative 2-semi-cycles with K and thirty two each of positive and negative 3-semi-cycles. In this case, however, the equal number of positive and negative relations do not cause the choice to be completely determined by the 2-semi-cycles. This is because J is able to change one or more of the relations, i.e. 2-semi-cycles are the sole determinant for J only when the relations between I and K are equally positive and negative. In this step, J's changes increase the number of positive 3-semi-cycles from thirty two to forty and decrease the negatives from thirty two to twenty four.

In step 3, K has equal numbers of positive and negative 2-semi-cycles with I and J and an excess of sixteen positive 3-semi-cycles over negatives. By changing his negative attraction relation with I and J to positive, all negative semi-cycles are eliminated, and perfect balance is achieved. Harary's equivalence theorem also applies here.

Under Method 2 the situation again is quite different. The equal number of positive and negative relations between J and K do not indicate equal numbers of positive and negative semi-cycles, since completely negative semi-cycles are now considered equal to zero and can not counter-effect the positive semi-cycles which include them. There are twenty four negative 3-semi-cycles, eight zero 3-semi-cycles and thirty two positive 3-semi-cycles in the digraph. There are also four negative and two positive 2-semi-cycles between I and each of the other two. The degree of balance of the 3-semi-cycles alone is 32/56 = .57; the degree of balance of the 2-semi-cycles is .33. If I changed his negative attraction relations with both J and K to positive relations, the degree of balance of the 2-semi-cycles would be increased to .50 but the degree of balance of the 3-semi-cycles would be reduced to .52. This is because the number of positive 3-semi-cycles remains the same while the total is increased to sixty two. The overall degree of balance for this situation is .51 and since this is lower than the original balance, no alternatives are available. Since this is the case with all members, the group has no way to increase its balance.

The most significant feature of this example is that, where completely negative semi-cycles are counted as zero and where the relations between non-focal subjects are equally positive and negative, the completely negative 3-semi-cycles do not counter effect the positive semi-cycles which include the common negative relations. For example, if I dislikes J, J dislikes K and and I agrees with K, that semi-cycle is balanced. If I also dislikes K, the semi-cycle consisting of the first two negative relations between I, J and K and the negative I →K relation would cancel the balanced semi-cycle only if completely negative semi-cycles were considered unbalanced. If completely negative semi-cycles are considered vacuously balanced, the positive semi-cycle is not cancelled. In the first example, I's dislike for J and J's dislike for K would probably lead I to side with K against J, other things being equal. Where this condition exists, it is probably a psychologically comfortable situation for I. If I also dislikes K, the question for empirical verification is whether the imbalance between the two relations between I and K is a sufficient reflection of I's psychological state, as reflected in his relation with K, or whether the imbalance of the completely negative relation is also necessary to sufficiently explain I's behavior.

In Figure 4A., there are two sets of pairwise relationships which were completely negative; in Figure 4B., there were no pairwise relationships which were completely negative. In Figure 4C., there is one pairwise relationship, between I and K, which is completely negative; the other two pairs have at least two positive relations.

In step 1, I is again faced with equal numbers of positive and negative relations between J and K, so that any changes must be made on the basis of semi-cycles of length two. Since any changes would decrease the degree of balance, I has no alternatives available to him.

In step 2, J originally has three negative and three positive semi-cycles with I, four negative and 2 positive semi-cycles with K and twenty four each of positive and negative 3-semi-cycles. The change shown in the table produces the single highest degree of balance available to him. The number of positive 2-semi-cycles increases from five to nine and the number of positive 3-semi-cycles increases from twenty four to thirty six. In this case, the completely negative relations between I and K contribute to J's moves but do not completely determine them.

In step 3, K's choice is also determined by semi-cycles of length two and three. K's selection is based on the pattern having the highest valued paths of four alternative patterns having the same degree of balance. This again is an example of Harary's theorem of the equivalence of negation and deletion when perfect balance is the end result of the change.

In step 1 of Method 2, there appears again the differential effect of the two methods of treating completely negative 3-semi-cycles where one pairwise relation consists of equal numbers of positive and negative relations. In contrast with step 1 of Figure 4B., the "No Change" situation occurs in Method 1 rather than Method 2. In Method 2, I has three negative and three positive semi-cycles with J, three positive semi-cycles with K and twenty four positive and eighteen negative 3-semi-cycles. There are also six completely negative semi-cycles which are considered to be zero. As in Figure 4B., the equal number of positive and negative relations no longer cancel each other, since the completely negative semi-cycles involved are counted as zero semi-cycles. By changing his attraction relation with J from positive to negative, I reduces the number of positive 2-semi-cycles from six to five but removes six of the eighteen negative 3-semi-cycles without decreasing the number of positive 3-semi-cycles. The semi-cycles composed of I's attraction toward J, the two positive relations between J and K and the three negative relations between I and K are brought into balance by changing the attraction relation to negative. The semi-cycles including the same relations between I and J and I and K but with the substitution of the negative relations between J and K for the positive are in balance when I likes J. When I reverses this relation, however, these semi-cycles become vacuously balanced rather than unbalanced. In more empirical terms, this says that the positive attraction relations between J and K are sufficient to bring I to dislike J but that the negative agreement relations are of no consequence in preserving I's liking for J. If completely negative semi-cycles were considered negative, I would not change his relationship with J because the fact that J had both positive and negative relations with K would place I in an ambivalent position. It is important to note here that attraction and agreement are assumed to be equally strong determinants of action.

In step 2, J initially has four negative and two positive semi-cycles with both I and K and twenty four positive, twelve zero and twelve negative 3-semi-cycles. Changing both attraction relations increases the number of positive 2-semi-cycles from four to six, increases the positive 3-semi-cycles from twenty four to thirty and decreases both the zero and negative 3-semi-cycles from twelve to nine. In this case, the differential treatment of completely negative

3-semi-cycles has no effect of J's choice of moves, i.e. he increases the degree of balance in either case by changing both attraction relations, but the treatment of these semi-cycles as zero produces a higher degree of balance.

In step 3, K is initially a virtual isolate. If completely negative 3-semi-cycles were considered unbalanced, K could not achieve perfect balance in this step because of the single negative relation between I and J, although changing his positive relation with J to negative would increase balance. By treating completely negative 3-semi-cycles as zero, however, perfect balance can be achieved and Harary's equivalence theorem together with the value assumptions determine K's choice of a neutral relation with J.

In the final step, the imbalance of I's pairwise relationships with J is the predominant factor in I's choice. The equivalence theorem and the value assumption again determine the particular choice.

REFERENCES

1. Fritz Heider, "Social Perception and Phenomenal Causality," Psychological Review, 51, 1944, pp. 358-374.

2. Fritz Heider, "Attitudes and Cognitive Organization," Journal of Psychology, 21, 1946, pp. 107-112.

3. Dorwin Cartwright and Frank Harary, "Structural Balance: A Generalization of Heider's Theory," Psychological Review, 63 1956, pp. 277-293.

4. Nehemiah Jordan, "Behavioral Forces that are a Function of Attitudes and Cognitive Organization," Human Relations, 6, 1953, pp. 273-287.

5. Frank Harary, "On the Measurement of Structural Balance," Behavioral Science, 4, 1959, pp. 316-323.

6. Joseph Berger, Bernard Cohen, J. Laurie Snell and Morris Zelditch, Jr., Types of Formalization in Small-Group Research. Boston, Mass., Houghton-Mifflin Company, 1962, pp. 28-29.

7. Frank Harary, Robert Z. Norman and Dorwin Cartwright, Structural Models: An Introduction to the Theory of Directed Graphs. New York, John Wiley and Sons, Inc., 1965, pp. 348-352.

8. George C. Homans, Social Behavior: Its Elementary Forms. New York, Harcourt, 1961.

9. Julian O. Morrissette, "An Experimental Study of the Theory of Structural Balance," Human Relations, 11, 1958, pp. 239-254.

10. John T. Gullahorn and Jeanne E. Gullahorn, "Empirical Validation of a Computer Model of Social Behavior," unpublished NSF application, 1967.