STABILITY OF TRANSMISSION CIRCUITS

Thesis for the Degree of M. S.

H. J. Kurtz

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THESIS

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STABILITY

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INTRODUCTION

The purpose of this thesis is to develop the theory for the condition of maximum stability in a transmission circuit under varying conditions of load, power factor, line constants, etc. and to represent by means of charts the synchronizing power corresponding to any rational combination of generator terminal voltage, generator resistance and reactance, transmission circuit resistance and reactance, load power factor, and effective load resistance and reactance.

The scope of this work is necessarily limited, hence the stability of circuits during transient periods is not discussed and steady-state conditions are assumed in the following discussion.

Several oscillograms were taken of the voltages and currents of two generators operating in parallel, under varying conditions of excitation, phase displacement and line constants, and operating just within the stability limit.

GENERAL CONSIDERATIONS

Until a very few years ago the loads on power systems had a comparatively low power factor causing poor generator and transmission line regulations. However, the size of the loads and the length of the lines were small, so that the generator field rheostats were entirely adequate to control the voltage without loss of synchronism.

Recent demand for customer power factor improvement and increase in length and voltage of transmission lines have combined to create a general tendency toward better power factor. Hence during the light load period on a long transmission line the generator load becomes decidedly leading. As a result of this, unstable voltages are produced and field rheostat control alone is no longer sufficient. If the transmission line is of considerable length the generators may even become self-exciting and reverse field excitation necessary to control the voltage. This is, in effect, a resonant circuit between the capacity of the transmission line and the inductance of the generator. When the field is reversed the generator is liable to slip a pole which will cause abnormal voltages.

From the above considerations it is evident that it is desirable to make a study of transmission stability especially when systems of high voltages are interconnected.

In general, there are three main types of instability in electric systems:

(1) The transients of readjustment to changed circuit conditions.

(2) Unstable electrical equilibrium, the condition in which the effect of a cause increases the cause.

(3) Permanent instability resulting from a combination of circuit constants which cannot co-exist.

Transients are the phenomena by which, at the change of circuit conditions, current, voltage, load, etc. readjust themselves from the values corresponding to the initial conditions to the values corresponding to the new conditions of the circuit. For example, when a switch is closed and a load put on a circuit, the current cannot increase instantly due to inductance of the circuit and some time must elapse so that electro-magnetic energy can be stored. Also when a switch is closed in a motor circuit the transient period corresponds to the period of acceleration of the motor.

The characteristic of transients is that they are of very short duration and exist between two periods of steady state conditions.

It should be kept in mind when dealing with transients that resistance, inductance, capacitance, and leakance, the ordinary constants of a circuit, are no longer constants, and that the results obtained when they are used as constants are only approximations.

If the effect brought about by a cause is such as to reduce the cause, the effect limits itself and stability results. If, however, the reverse is true, the effect continues with increasing intensity and instability results. This applies equally well to all phenomena.

Instability may manifest itself in three (different) forms: (1) Instability leading up to stable conditions, (2) Instability causing permanent interruption to service, (3) Instability leading again to stability, and thus periodically repeating itself. Whether instability results, and what form it assumes depends, not only on the cause, but on the circuit taken as a whole.

In connection with transmission line stability, the effect of the armature of the generator on the field excitation must be considered. For a unity power factor load the field set up by the ampere-turns of the armature is at right angles to the field set up by the ampere-turns of the field and causes cross-magnetization. This results in a shifting of the field flux and under heavy load conditions actually demagnetizes the field which, in turn, reduces the induced voltage of the generator. If the load is zero power factor lagging, the field caused by the ampere-turns of the armature is directly opposed to the main field of the generator, and hence, causes a considerable reduction in the effective field and induced voltage of the generator. If the load is zero power factor leading, the field set up by the ampere-turns of the atmature is in phase with the main field and results in a field and induced voltage much larger than normal.

From these considerations it is evident that, for all possible load conditions on a generator, it is necessary to have a wide range of field excitation if constant terminal voltage is to be maintained.

THEORETICAL CONSIDERATIONS

When two or more alternators are operating in parallel, the natural reactions which result from a departure from synchronism are such as to reestablish it. This is the principle which makes possible the parallel operation of generators.

If two equal generators are considered operating in parallel and their induced voltages not quite equal, there will be a circulating current between the machines caused by the difference between the two voltages and flowing thru the series circuit consisting of the synchronous impedances of the two machines and the impedance of the circuit between the machines. This circulating current may be caused by the two voltages being either out of phase or differing in magnitude. When it is produced by a difference in phase it produces synchronizing acting. When resulting from an inequality of voltages, it equalizes the terminal voltages, mainly by the effect of armature ampere-turns on the main field flux. For two equal generators the circulating current can be expressed by the formula

$$I_c = \frac{E_1 - E_2}{Z} \tag{1}$$

where Z represents the synchronous impedances of the two machines and the line between them in series.

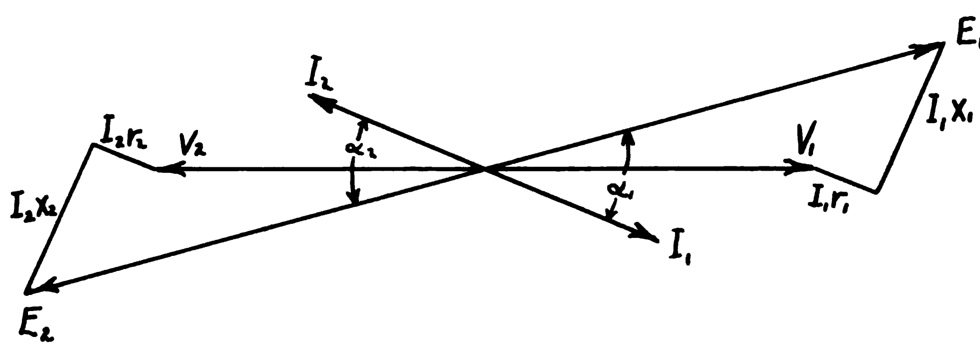


Fig. 1

Fig. 1 represents two equal generators with their armatures taken as a series circuit and with equal excitations and equal loads. V represents the terminal voltage and E represents the induced voltage. The terminal voltages are equal and in phase when considered with respect to the parallel circuit but are opposite in phase when considered with respect to the series circuit.

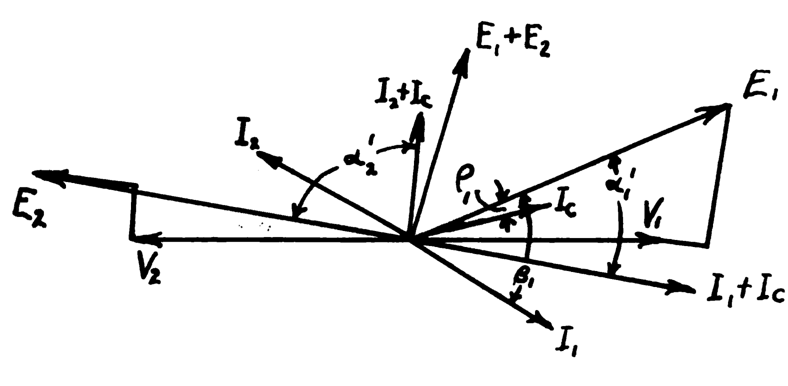


Fig. 2

Fig. 2 represents two equal generators with equal terminal voltages but with the induced voltages out of phase. Since E_1 and E_2 are not in opposition it follows that their sum is not zero and hence there must be a circulating current flowing thru the two armatures represented by the formula $I_c = \frac{E_1 + E_2}{2Z}$ where Z ($Z = r + jX$) represents the synchronous impedance of one machine. This circulating current lags behind the voltage $(E_1 + E_2)$ by the angle $\tan^{-1} \frac{X}{r}$, where X is the synchronous reactance and r the effective resistance. Since the ratio of the reactance to resistance is usually large, this angle is nearly 90^o.

The circulating current has a component in phase with E_1 and a component in opposition to E_2. Hence it produces generator action with respect to machine No. 1 and motor action with respect to machine No. 2, so that the result is to slow down machine No. 1 and to accelerate machine No. 2, thereby tending to bring the voltages E_1 and E_2 in phase.

The current carried by the armatures is the vector sum of the circulating current and the load current. The current delivered by the two generators is the vector difference between the two armature currents.

The change in the output of the generators when the voltages are out of phase is partly due to the power developed by the circulating current considered with respect to the induced voltages in the generators, and partly due to the change in phase and magnitude of the generated voltages with respect to the load current supplied by each generator. Referring to Figs. 1 and 2, the change in the power developed by generator No. 1 due to a phase displacement is

$E_1(I_1 + I_c) \cos a_1^1 - E_1I_1 \cos a_1$. Assuming constant terminal voltage, I_1 will not change and

$$E_1(I_1+I_c) \cos a_1^1 - E_1I \cos a_1 = E_1I_1 \cos \beta_1 - E_1I_1 \cos a_1 + E_1I_c \cos p_1 \quad (2)$$

I_c is sometimes called synchronizing current since a large part of the synchronizing power is caused by this current directly.

I_c is the only current which tends to restore synchronism when there is no load on the system, and this occurs only when I_c lags behind $(E_1 + E_2)$. If E_1 and E_2 are equal and I_c is in phase with their sum, there would be equal positive projections on E_1 and E_2 and an equal generator action would be produced on each machine, and consequently no synchronizing power developed. The synchronizing power developed by I_c is dependent upon its lag behind $(E_1 + E_2)$, hence inductance is necessary for the parallel operation of generators. By inserting capacity between the generators in parallel, the circulating current can be made to lead $(E_1 + E_2)$. The action of I_c under this condition is to change the IZ drop thru the machines to an IZ rise, thereby reducing the induced voltages. Generators inherently have inductance in their armatures, so that the natural tendency is for proper parallel operation and their stability depends upon the amount of resistance and inductance in their armatures and to some extent upon the constants of the circuit and load.

If the constants of two generators are not in the inverse ratio of their ratings they may be paralleled by properly adjusting the inputs and field excitations so they will assume any desired load. There will be a circulating current between the armatures, however, which is desirable and necessary since it is in this way that the terminal voltages are equalized.

Changing the input to a generator operating in parallel with others changes its load and phase position, but does not change the value of its generated voltage appreciably. Changing the excitation changes the induced voltage and its phase position, but does not change the load appreciably.

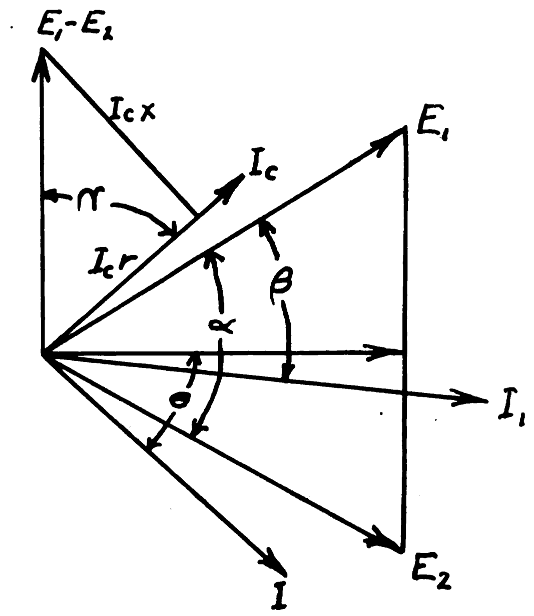


Fig. 3

Fig. 3 shows the vector diagram for two equal generators drawn with respect to their parallel circuit. I represents the value of current in each generator which is in phase with the load current I_o. I_c the circulating current equals $\frac{E_1 - E_2}{2Z}$.

If P_1 represents the power developed by generator No. 1 when the induced voltages are displaced by an angle a, and I_1 the armature current, then

$$P_1 = I_1 E_1 \cos\beta = \left(\frac{I_o}{2} + \frac{E_1 - E_2}{2Z}\right) E_1 \cos\beta = (I + I_c)\, E_1 \cos\beta$$

$$P_1 = I_1E_1 \cos\theta \cos\frac{a}{2} - IE_1 \sin\theta \sin\frac{a}{2} + I_c^2 r + \frac{E_1^2 x}{Z} \sin\frac{a}{2} \cos\frac{a}{2} \quad (3)$$

Similarly

$$P_2 = IE_2 \cos\theta \cos\frac{a}{2} + IE_2 \sin\theta \sin\frac{a}{2} + I_c^2 r - \frac{E_2^2 x}{Z} \sin\frac{a}{2} \cos\frac{a}{2} \quad (4)$$

The change in power developed by each generator due to phase displacement is found by subtracting $E_1I \cos\theta$ and $E_2I \cos\theta$ from equations (3) and (4) respectively. Making this subtraction gives the change in power developed by generator No. 1 equal to

$$EI \cos\theta \left(\cos\frac{a}{2} - 1\right) - IE \sin\theta \sin\frac{a}{2} + I_c^2 r + \frac{E^2 x}{Z^2} \sin\frac{a}{2} \cos\frac{a}{2} \quad (5)$$

Similarly the change in power developed by generator No. 2 is equal to

$$EI \cos\theta \left(\cos\frac{a}{2} - 1\right) + EI \sin\theta \sin\frac{a}{2} + I_c^2 r - \frac{E^2 x}{Z^2} \sin\frac{a}{2} \cos\frac{a}{2} \quad (6)$$

The first and third terms of equations (5) and (6) are equal and of the same sign so that they must represent generating action in each machine and hence do not cause synchronizing action but tend to slow down the system frequency. The second and fourth terms are equal but of opposite signs and hence must represent the cynchronizing power acting between the two machines and is equal to

$$P_s = E\left[\frac{Ex}{Z^2} \cos\frac{a}{2} - I \sin\theta\right] \sin\frac{a}{2}\,. \quad (7)$$

This value of power is one half the difference between the powers developed by the two generators. From equation (7) it can readily be seen that for everything constant except the generator power factor, the synchronizing power is a maximum when θ is equal to 90° and negative. This means that maximum stability is obtained for a zero power factor leading load.

Equating $\cos\frac{a}{2}\sin\frac{a}{2}$ to $\frac{\sin a}{2}$ and rewriting (7) P_s becomes

$$P_s = E\left[\frac{Ex}{2Z^2}\sin a - I\sin\theta\sin\frac{a}{2}\right] \tag{8}$$

By differentiating (8) with respect to a, equating to zero and solving for a, the value of a for maximum synchronizing power is obtained

$$\frac{dP_s}{da} = E\left[\frac{Ex}{2Z^2}\cos a - \frac{I\sin\theta}{2}\cos\frac{a}{2}\right] = 0$$

$$a = \cos^{-1}\left[\frac{I^2\sin^2\theta}{4E^2x^2} \pm \frac{I\sin\theta}{4E^2x^2}\sqrt{I^2\sin^2\theta + 8E^2x^2}\right] \tag{9}$$

Similarly,

$$\frac{dP_s}{dE} = \frac{Ex}{Z^2}\sin a - I\sin\theta\sin\frac{a}{2} = 0$$

$$E = \frac{IZ^2\sin\theta}{x\cos\frac{a}{2}} \tag{10}$$

This gives the value of E for maximum synchronizing power.

The above statements regarding maximum synchronizing power and hence maximum stability, were made assuming the induced voltage constant. However, in practice the excitation is usually varied to maintain constant terminal voltage and under these conditions the synchronizing power becomes greatest for inductive loads for a given phase displacement between the induced voltages.

If R_0 and X_0 are the resistance and reactance respectively of the load, then

$$I = \frac{E\cos\frac{a}{2}}{\text{total } Z} = \frac{E\cos\frac{a}{2}}{\sqrt{(2R_0+r)^2+(2X_0+X)^2}} = \frac{I_0}{2}$$

and

$$\sin\theta = \frac{\text{total } X}{\text{total } Z} = \frac{2X_0+X}{\sqrt{(2R_0+r)^2+(2X_0+X)^2}}$$

Substituting these values in equation (7) gives for the value of synchronizing power

$$P_s = \frac{E^2}{2}\left[\frac{X}{Z^2} - \frac{2X_0 + X}{(2R_0+r)^2+(2X_0+X)^2}\right] \sin a \qquad (11)$$

If a short circuit should occur with no impedance between it and the generators the equation for synchronizing power becomes equal to zero. This means that instability would result and the generators would fall out of synchronism.

By expressing the induced voltage in terms of the terminal voltage, an equation for synchronizing power can be written in terms of the phase displacement, the various resistances and reactances, and the terminal voltage.

$$E = V + I(r + jx)$$

$$V = 2IZ_0 = 2I\sqrt{R_0^2 + X_0^2}$$

since both generators are supplying equal loads.

$$E = I(\text{total } Z) = I\sqrt{(2R_0+r)^2+(2X_0+X)^2}$$

$$E = \frac{V}{2}\sqrt{\frac{(2R_0+r)^2+(2X_0+X)^2}{R_0^2 + X_0^2}}$$

Substituting this value for E in equation (11) and simplifying gives

$$P_s = \frac{V^2}{2}\left[\frac{X}{Z^2} + \left(\frac{X^2 - r^2}{2Z^2}\right)\left(\frac{X_0}{R_0^2 + X_0^2}\right) + \frac{R_0 r X}{Z^2(R_0^2 + X_0^2)}\right] \sin a \qquad (12)$$

When the load is zero the equation reduces to

$$P_s = \frac{V^2 X \sin a}{2Z^2} = \frac{V^2 X \sin a}{2(r^2 + X^2)}$$

The maximum synchronizing power that can be obtained for the zero load condition by separately varying the resistance and reactance is shown by the following, by setting the first derivative equal to zero and solving for the variable quantity.

$$\frac{dP_s}{dX} = \frac{2(r^2 + X^2)\,V^2 \sin a - 4V^2X^2 \sin a}{4(r^2 + X^2)^2} = 0$$

Solving for X gives

$$X = r \tag{14}$$

The other values for maximum synchronizing can be obtained by inspection of equation (13) as well as by differentiation. The synchronizing power varies directly as the square of the terminal voltage; it is maximum when $a = 90^\circ$; and it is maximum when $r = 0$.

The maximum synchronizing power when the system is carrying a load is shown by the following. Rewriting equation (12) in four terms gives

$$P_s = \frac{V^2X \sin a}{2(r^2 + X^2)} + \frac{V^2X^2 \sin a\, X_0}{4(R_0^2+X_0^2)(r^2+X^2)} - \frac{V^2r^2 \sin a\, X_0}{4(R_0^2+X_0^2)(r^2+X^2)} + \frac{V^2rX\,R_0 \sin a}{2(r^2+X^2)(R_0^2+X_0^2)} \tag{15}$$

Differentiating equation (15) with respect to r, equating to zero gives

$$\frac{dP_s}{dr} = \frac{V^2X \sin a}{2}\left(-\frac{2r}{(r^2+X^2)^2}\right) + \frac{V^2r^2\,X_0 \sin a}{4(R_0^2 + X_0^2)}\left(-\frac{2r}{(r^2+X^2)^2}\right)$$

$$-\frac{V^2X_0 \sin a}{4(R_0^2 + X_0^2)}\left(\frac{2r(r^2+X^2) - r^2(2r)}{(r^2 + X^2)}\right) + \frac{V^2X\,R_0 \sin a}{2(R_0^2 + X_0^2)}\left(\frac{r^2+X^2-r(2r)}{(r^2 + X^2)^2}\right) = 0$$

Solving for r gives

$$r = \frac{-R_0^2 + X_0^2 + xX_0 \pm \sqrt{(-R_0^2 + X_0^2 + xX_0)^2 + R_0^2X^2}}{R_0} \tag{16}$$

This represents not only the effective resistance of the generator armature, but also the resistance of the circuit up to the point of synchronizing.

Differentiating equation (15) with respect to X gives

$$\frac{dP_s}{dx} = \frac{V^2 \sin a}{2}\left(\left[\frac{r^2+x^2-x(2x)}{(r^2 + x^2)^2}\right] - \frac{X_0}{2(R_0^2+X_0^2)}\left[\frac{2x(r^2+x^2)-x^2(2x)}{(r^2 + x^2)^2}\right]\right.$$

$$\left.-\frac{r^2X_0}{2(R_0^2+X_0^2)}\left[-\frac{2x}{(r^2+X_{\cancel{0}}^2)}\right] + \frac{rR_0}{R_0^2+X_0^2}\left[\frac{r^2+x^2-x(2x)}{(r^2 + x^2)^2}\right]\right) = 0$$

Solving for x gives $\quad x = r \quad$ (17)

This value of x also includes the reactance of the circuit up to the point of synchronizing, as well as the synchronous reactance of the generator.

Differentiating equation (15) with respect to R_0 gives

$$\frac{dP_s}{dR_0} = \frac{V^2 \sin a}{2(r^2+x^2)} \left(\frac{X_0 x^2}{2} \left[- \frac{2R_0}{(R_0^2 + X_0^2)^2} \right] - \left[\frac{-X_0 r}{2} \; \frac{2R_0}{(R_0^2 + X_0^2)^2} \right] + rx \left[\frac{R_0^2 + X_0^2 - 2R_0^2}{(R_0^2 + X_0^2)^2} \right] \right) = 0$$

Solving for R_0 gives $\quad R_0 = \dfrac{X_0 r}{x}$ or $-\dfrac{X_0 r}{r} \quad$ (18)

This gives the value of the effective resistance of the load for the maximum synchronizing power.

Differentiating equation (15) with respect to X_0 gives

$$\frac{dP_s}{dX_0} = \frac{V^2 \sin a}{2(r^2+x^2)} \left(\frac{x^2}{2} \left[\frac{R_0^2 + X_0^2 - 2X_0^2}{(R_0^2 + X_0^2)^2} \right] - \frac{r^2}{2} \left[\frac{R_0^2 + X_0^2 - 2X_0^2}{(R_0^2 + X_0^2)^2} \right] + \left[\frac{-2r\,x\,R_0 X_0}{(R_0^2 + X_0^2)^2} \right] \right) = 0$$

Solving for X_0 gives $\quad X_0 = \pm R_0 \dfrac{r - x}{r + x}$ or $R_0 \dfrac{x - r}{x + r} \quad$ (19)

Differentiating (15) with respect to a gives

$$\frac{dP_s}{da} = \frac{V^2}{2(r^2+x^2)} \left([x \cos a] + \frac{1}{2} \left[\frac{x^2 X_0 \cos a}{R_0^2 + X_0^2} \right] - \frac{1}{2} \left[\frac{r^2 X_0 \cos a}{R_0^2 + X_0^2} \right] + \left[\frac{r\,x\,R_0 \cos a}{R_0^2 + X_0^2} \right] \right) = 0$$

Solving for a gives $\quad a = 90^\circ$ for maximum synchronizing power. (20)

Substituting the values $\dfrac{R_0}{Z_0}$ for $\cos \theta$ and $\dfrac{X_0}{Z_0}$ for $\sin \theta$ in equation (15), the value of synchronizing power becomes

$$P_s = \frac{V^2 \sin a}{2(r^2+x^2)} \left(x + \frac{x^2 \sin \theta \cos \theta}{2R_0} - \frac{r^2 \sin \theta \cos \theta}{2R_0} + \frac{rx \cos^2 \theta}{R_0} \right) \quad (21)$$

Assuming the effective resistance of the load constant, differentiating with respect to θ gives

$$\frac{dP_s}{d\theta} = \frac{x^2}{2R_o}[\cos^2\theta - \sin^2\theta] - \frac{r^2}{2R_o}[\cos^2\theta - \sin^2\theta] + \frac{rx}{R_o}$$

$$[-2\sin\theta\cos\theta] = 0$$

Solving for θ gives

$$\theta = \cos^{-1}\frac{r + x}{\sqrt{r^2+x^2}} \text{ or } \cos^{-1}\frac{r - x}{\sqrt{r^2+x^2}} \tag{22}$$

Rewriting equation (15) again, keeping in mind that the power factor is to vary and that the effective reactance of the load is to remain constant, gives

$$P_s = \frac{V^2 \sin a}{2(r^2+x^2)}\left[x + \frac{(x^2 - r^2)\sin^2\theta}{2X_o} + \frac{r\,x\cos\theta\sin\theta}{X_o}\right] \tag{23}$$

Differentiating with respect to θ gives

$$\frac{dP_s}{d\theta} = \frac{(x^2 - r^2)\,2\cos\theta\sin\theta}{2X_o} + \frac{r\,x(\cos^2\theta - \sin^2\theta)}{X_o} = 0$$

Solving for θ gives

$$\theta = \cos^{-1} \pm\frac{r}{\sqrt{r^2+x^2}} \text{ or } \pm\frac{x}{\sqrt{r^2+x^2}} \tag{24}$$

The above discussion has been devoted to the consideration of two equal generators operating in parallel. In the following discussion there will be considered the general case, that of a generator operating in parallel with a large inter-connected system, or, what amounts to the same thing, two generators of unlike characteristics operating in parallel. Fig. 4 shows the relations existing between the various voltages and currents for this condition, drawn with respect to the parallel circuit. V is the voltage at the point where the effective load is taken off and is used as the reference vector. E_1 and E_2 are the induced voltages respectively in the two machines, displaced by the angle a, and unequal in magnitude. I_1 and I_2 are the armature

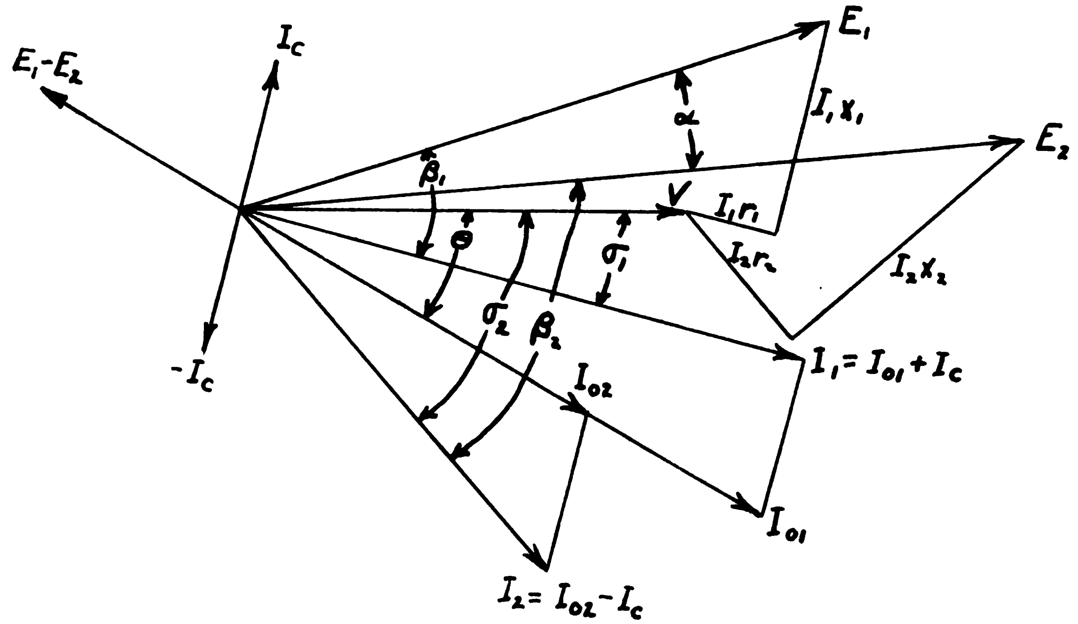


Fig. 4

currents respectively of the two machines. I_{01} and I_{02} are the components of the two armature currents which are in phase with the load current I_0. I_c is the circulating current caused by the difference between the induced voltages of the two machines ($E_1 - E_2$), the armature currents being the vector sum of the circulating current and the components in phase with the load current. The difference between V and the induced voltage of the machines is the impedance drop in the circuits from the point of paralleling back to the machines plus the impedance drop in the machines themselves.

The values for the induced voltages can be expressed as follows:

$$E_1 = V + I_1Z_1 = V + I_1r_1 \cos \sigma_1 + I_1X_1 \sin \sigma_1 + j(I_1X_1 \cos \sigma_1 - I_1r_1 \sin \sigma_1) = e_1 + je_1^1 \quad (25)$$

$$E_2 = V + I_2Z_2 = V + I_2r_2 \cos \sigma_2 + I_2X_2 \sin \sigma_2 + j(I_2X_2 \cos \sigma_2 - I_2r_2 \sin \sigma_2) = e_2 + je_2^1 \quad (26)$$

The voltage which causes the circulating current can be expressed similarly,

$$E_1 - E_2 = I_1r_1 \cos \sigma_1 + I_1X_1 \sin \sigma_1 - I_2r_2 \cos \sigma_2 - I_2X_2 \sin \sigma_2 + j(I_1X_1 \cos \sigma_1 - I_1r_1 \sin \sigma_1 - I_2X_2 \cos \sigma_2 + I_2r_2 \sin \sigma_2) \quad (27)$$

The angle α is equal to $\tan^{-1} \frac{x_1 + x_2}{r_1 + r_2}$ and , in most cases will be nearly ninety degrees. When the circuit between the generators is of comparatively high resistance, the angle α will be somewhat less than ninety degrees.

By representing $(E_1 - E_2)$ as $e_c + je_c^1$, the value of the circulating current can be expressed as

$$I_c = \frac{[r_1 + r_2 - j(x_1 + x_2)][e_c + je_c^1]}{(r_1 + r_2)^2 + (x_1 + x_2)^2} \quad (28)$$

The total power developed by generator No. 1 is $P_1 = E_1I_1 \cos \beta_1$. The power developed by generator No. 1 which supplies the load, circuit losses and machine losses, is $VI_{01} \cos \theta + I_1^2r_1$. Hence the synchronizing power developed by generator No. 1 must be

$$P_s = E_1I_1 \cos \beta_1 - (VI_{01} \cos \theta + I_1^2r_1) \quad (29)$$

Expressing E_1 in terms of V, equation (29) becomes

$$P_s = \sqrt{(V+I_1r_1 \cos \sigma_1+I_1X_1 \sin \sigma_1)^2+(I_1X_1 \cos \sigma_1-I_1r_1 \sin \sigma_1)^2}\; I_1 \cos \beta_1 - (VI_{01} \cos \theta + I_1^2r_1) \quad (30)$$

$\beta_1 = \tan^{-1} \frac{e_1^1}{e_1} + \tan^{-1} \frac{i_1^1}{i_1}$ & $\sigma_1 = \tan^{-1} \frac{i_1^1}{i_1}$, where $E_1 = e_1 + je_1^1$ and $I_1 = i_1 + ji_1^1$.

If the values of β_1 and σ_1 are substituted in equation (30) the expression becomes quite cumbersome and, unless the two generators have constants which deviate considerably from the inverse ratio of their ratings, it would be advisable to use equation (21) instead of equation (30) for an approximate result.

Rearranging equation (21) in the form $P_s = P_1 + P_2 - P_3 + P_4$ where

$$P_1 = \frac{V^2X \sin a}{2(r^2+x^2)}, \quad P_2 = \frac{V^2X^2 \sin \theta \cos \theta \sin a}{4R_0\ (r^2 + x^2)},$$

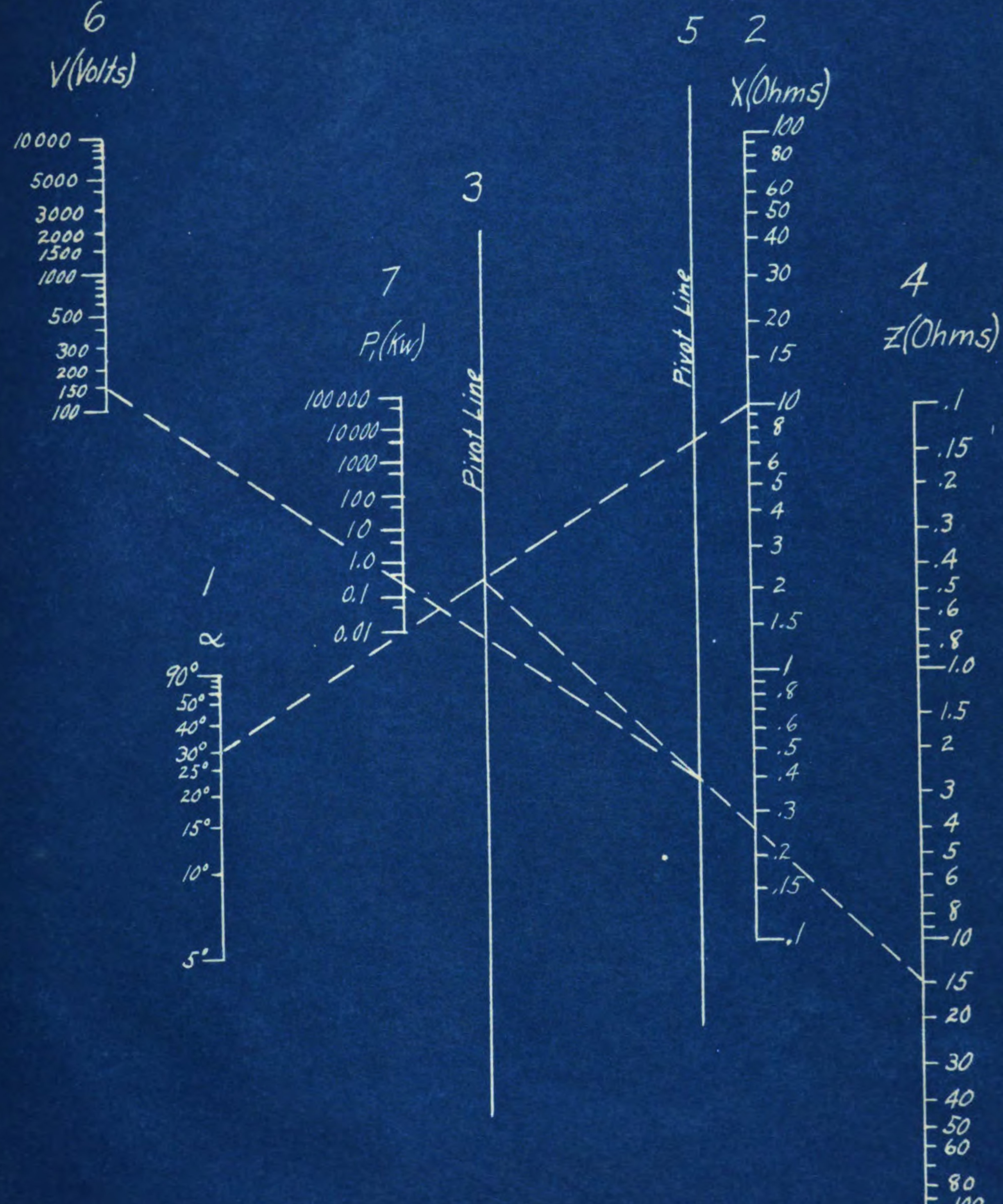
$$-P_3 = \frac{r^2V^2 \sin \theta \cos \theta \sin a}{4R_0\ (r^2 + x^2)}, \quad P_4 = \frac{V^2r\ x \cos^2\theta \sin a}{2R_0\ (r^2 + x^2)},$$

and choosing proper values for the variables, the value of synchronizing power can be put in chart form.

From Chart I the value of P_1 can be obtained by placing a straight edge from the value of a to the value of x, rotate the straight edge about the intersection of pivot line (3) to the value of Z, rotate the straight edge about the intersection of pivot line (5) to the value of V and read the value of kilowatts on the P_1 scale.

In a similar manner the values of P_2, P_3 and P_4 can be put in chart form. However, P_1 represents the synchronizing power due to the circulating current and is most important. If the load is zero P_1 represents the total synchronizing power. If the load is not zero, the values of P_2, P_3 and P_4 depend upon the effective values of the resistance and reactance of the load. If the reactance of the load is zero, P_2 and P_3 become zero.

CHART I


6
V(Volts)
10000
5000
3000
2000
1500
1000
500
300
200
150
100
7
P_1(Kw)
100 000
10000
1000
100
10
1.0
0.1
0.01
1
α
90°
50°
40°
30°
25°
20°
15°
10°
5°
3
Pivot Line
5
Pivot Line
2
X(Ohms)
100
80
60
50
40
30
20
15
10
8
6
5
4
3
2
1.5
1
.8
.6
.5
.4
.3
.2
.15
.1
4
Z(Ohms)
.1
.15
.2
.3
.4
.5
.6
.8
1.0
1.5
2
3
4
5
6
8
10
15
20
30
40
50
60
80
100

TESTS AND OSCILLOGRAMS

In connection with the study of stability of transmission circuits several oscillograms were taken of the voltages and currents of the two laboratory generators G2 and G3 operating in parallel near the point of instability.

Fig. 5 is a connection diagram showing a typical set of conditions under which the oscillograms were taken.

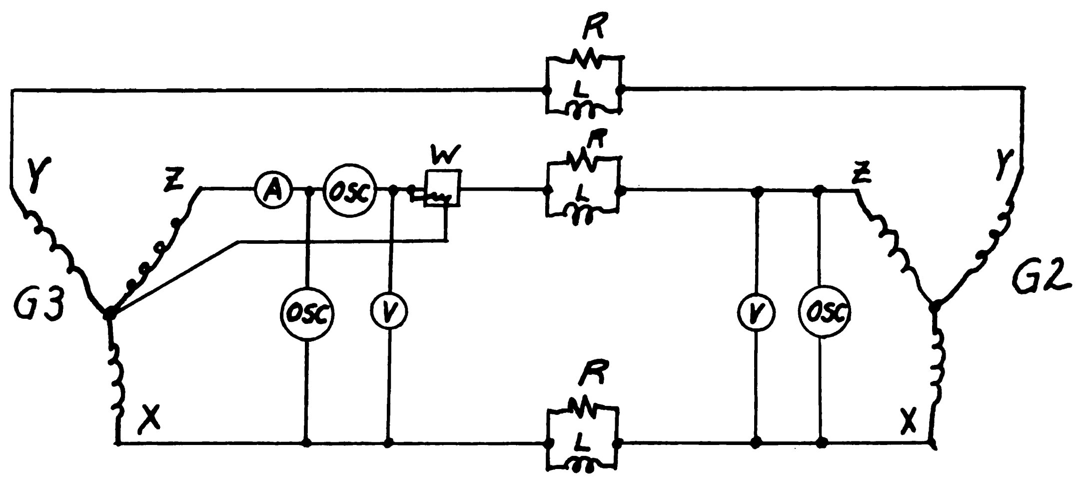


Fig. 5

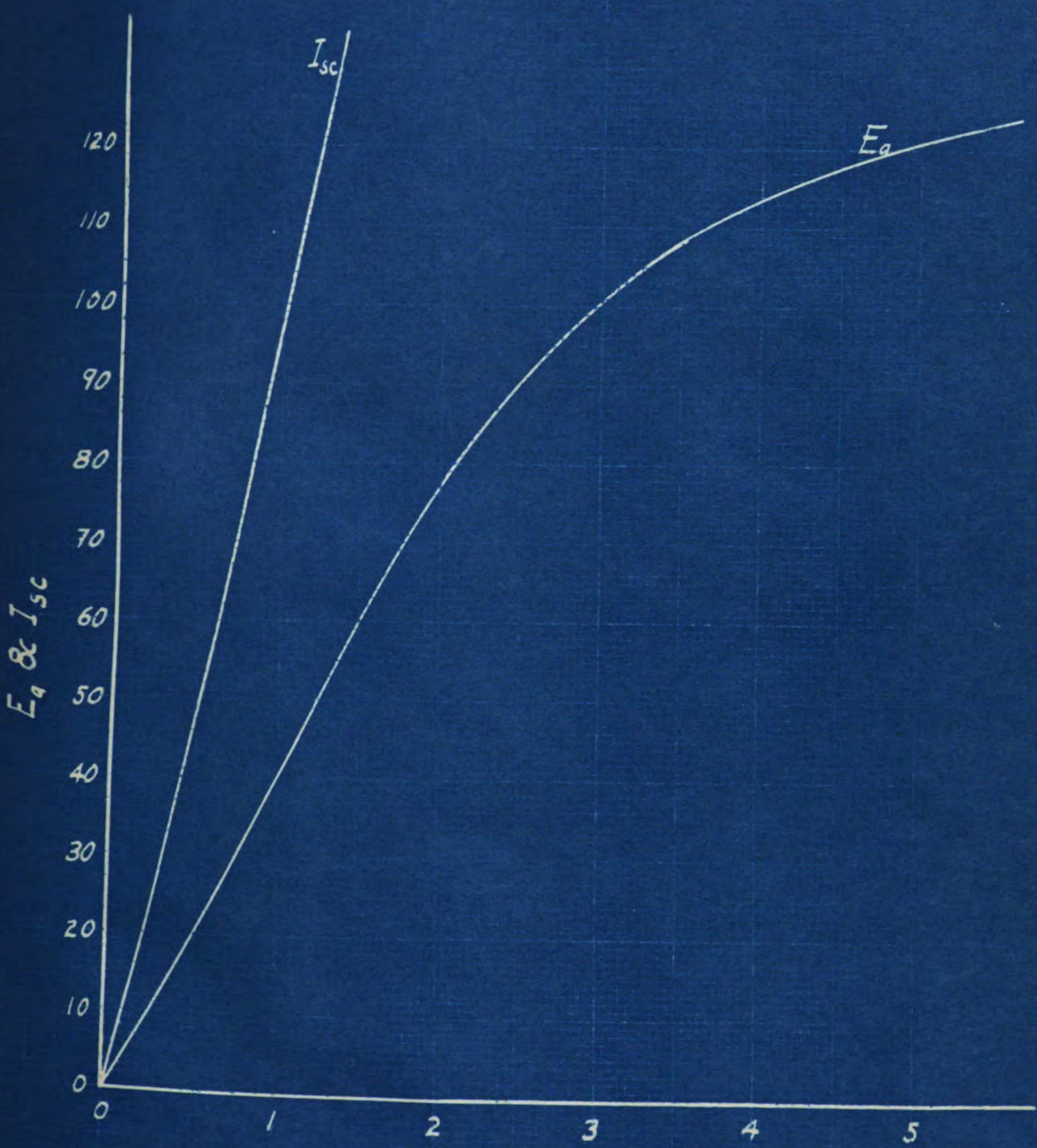
The following two pages show the short circuit and saturation curves of G2 and G3 respectively.

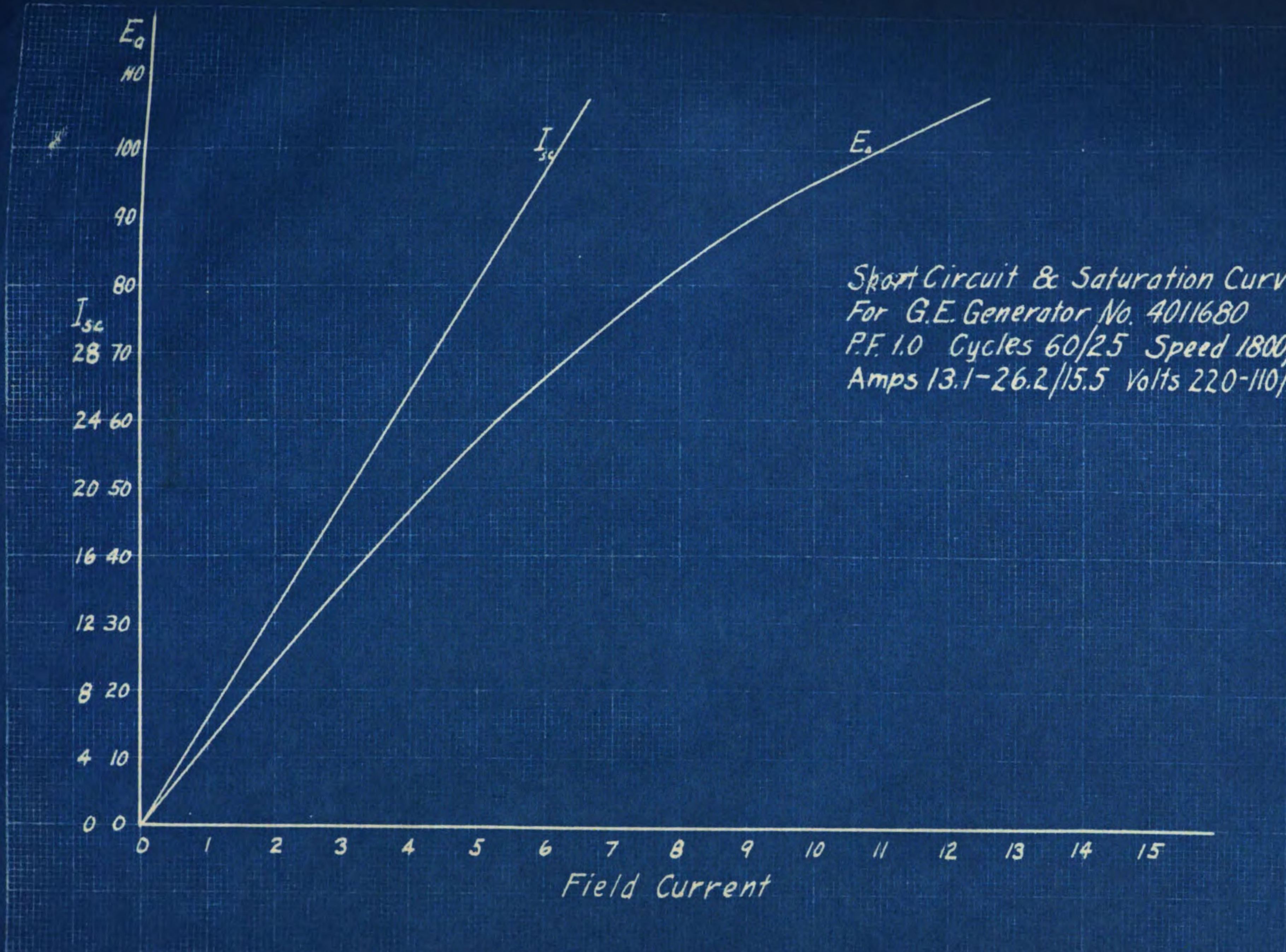
The synchronous impedance to neutral as calculated from these curves is 0.238 ohm for G2 and 1.045 ohms for G3.

The average D.C. resistance to neutral of G2 is 0.10 ohm. The average D.C. resistance to neutral of G3 is 0.113 ohm.

As can be seen from Fig. 5, all the readings and oscillograms were taken of values of current and voltage in the X and Z phases.

Short Circuit & Saturation Curves
For Fort Wayne Generator No. 10525
Kw. 25 Amps 120 Volts F.L. 120
Speed 1800 Phase 3 Cycles 60




Short Circuit & Saturation Curv
For G.E. Generator No. 4011680
P.F. 1.0 Cycles 60/25 Speed 1800
Amps 13.1-26.2/15.5 Volts 220-110/
E_a
I_{sc}
E_a
I_{sc}
110
100
90
80
28 70
24 60
20 50
16 40
12 30
8 20
4 10
0 0
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Field Current
MICHIGAN STATE COLLEGE

Oscillograms 1A, 1B, 1C, 2A, 2B, and 2C were taken with constant generator excitations on both machines, the driving D.C. motor of G3 overexcited and 2.63 ohms resistance in each of the lines between the generators. For these conditions the hunting was very bad and with a prolonged period so that it was necessary to take a slow oscillogram to include the part showing the maximum circulating current. This is shown by 1A. The central portion of 1B shows a reduced circulating current corresponding to the period when G3 was supplying synchronizing power to G2. The approximate meter readings for 1A, 1B and 1C are as follows:

V_2	V_3	I_z	W_z	f
75-110	65-120	0-40	0-2000	57.2-58.2

Oscillograms 3A and 3B represent the same conditions that oscillograms 1A, 1B and 1C do except that there is 4.15 ohms resistance in the lines. The approximate meter readings are:

E_2	E_3	I_z	W_z	f
55-115	82-113	0-25	0-1370	57.9-58.9

Oscillogram 3C represents the same conditions as the previous ones except that the resistances in the lines were 5.84 ohms. The approximate meter readings are:

E_2	E_3	I_z	W_z	f
106-108	99	10-11	200-240	58.5

This condition has considerably less hunting than the previous cases, however, the resistance in the circuit is too large to be practical.

Oscillogram 4A represents the condition of constant driving speeds for both machines, G3 underexcited and 2.63 ohms resistance in each line. The meter readings are:

E_2	E_3	I_z	W_z	f
92.5	56.5	20.5-21.5	90-120	59.1

Oscillogram 4B represents the same conditions as 4A except G3 is overexcited. The meter readings are:

E_2	E_3	I_z	W_z	f
90	95	7-7.5	50-100	58.5

Oscillogram 4C represents the same conditions as 4A except that the line resistance is 5.84 ohms. The meter readings are:

E_2	E_3	I_z	W_z	f
104-105	20-70	8-12	200-1500	59

Oscillogram 5A represents the condition of constant driving speeds, G3 under excited and reactance coils in the lines. The value of reactance and resistance of the coils is approximately 14.25 ohms and 0.61 ohm respectively. There was no hunting and the meter readings are:

E_2	E_3	I_z	W_z	f
122	80	4.1	30	59.9

Oscillogram 5B represents the same condition as 5A except that G3 is overexcited. The meter readings are:

E_2	E_3	I_z	W_z	f
82	120	5.4	250	61.5

Oscillogram 5C represents the same condition as 5A except that the driving motor of G3 was accelerated. The meter readings are:

E_2	E_3	I_z	W_z	f
105.5	60	4.3	210	62.0

Oscillogram 6A represents the same conditions as 5C except that the driving motor of G3 was retarded. The meter readings are:

E_2	E_3	I_z	W_z	f
107	123	4.7	275	60.9

Oscillogram 6B represents the same conditions as 5A except that G3 is over excited and 15.1 ohms resistance was connected in parallel with the reactance coils. The meter readings are:

E_2	E_3	I_z	W_z	f
73	113	4.0	150	63.5

Oscillogram 6C represents the same conditions as 6B except that G3 was underexcited. The meter readings are:

E_2	E_3	I_z	W_z	f
100.5	(50-55)	2.7	20	62.4

Oscillogram 7A represents G2 and G3 paralleled as single phase generators, their Y-phases being open. A resistance of 2.8 ohms was connected in the line, a load of 7.7 ohms resistance and 14.2 ohms reactance was connected across the line and the driving motor of G3 accelerated. The meter readings are:

E_2	E_3	I	W(line)	W(load)	f
86	?	16	80	640	60.2

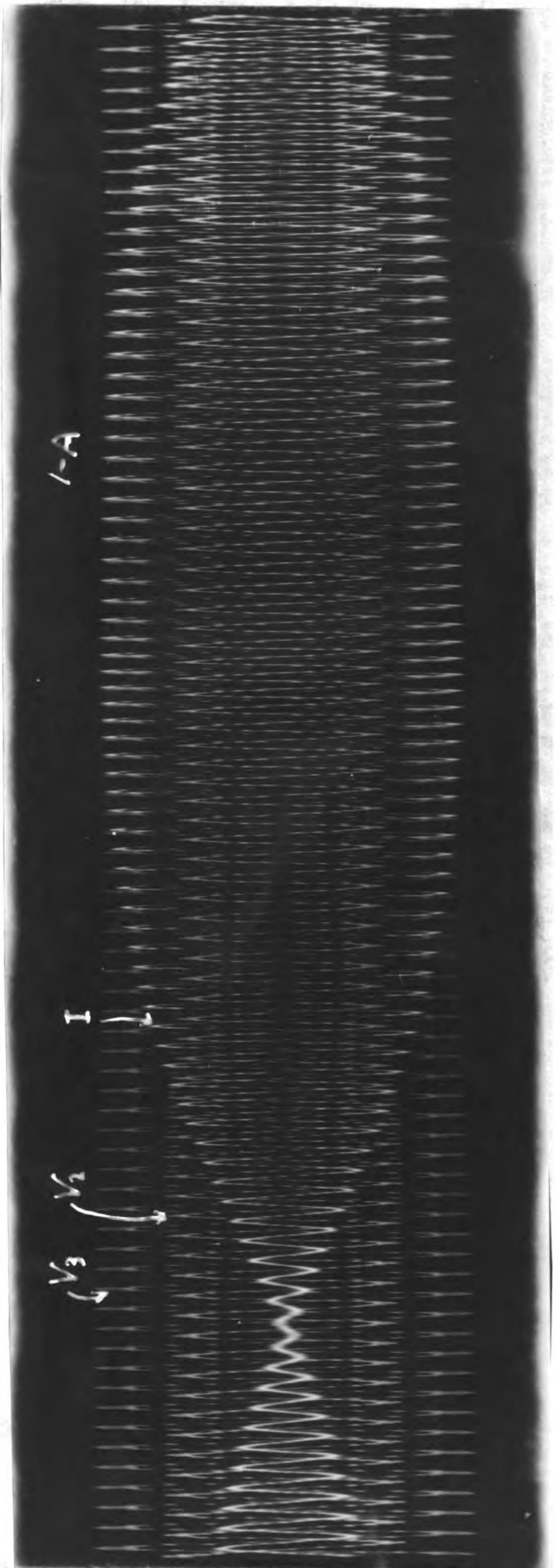
Oscillogram 7B represents the same conditions as 7A except that G3 was underexcited. The meter readings are:

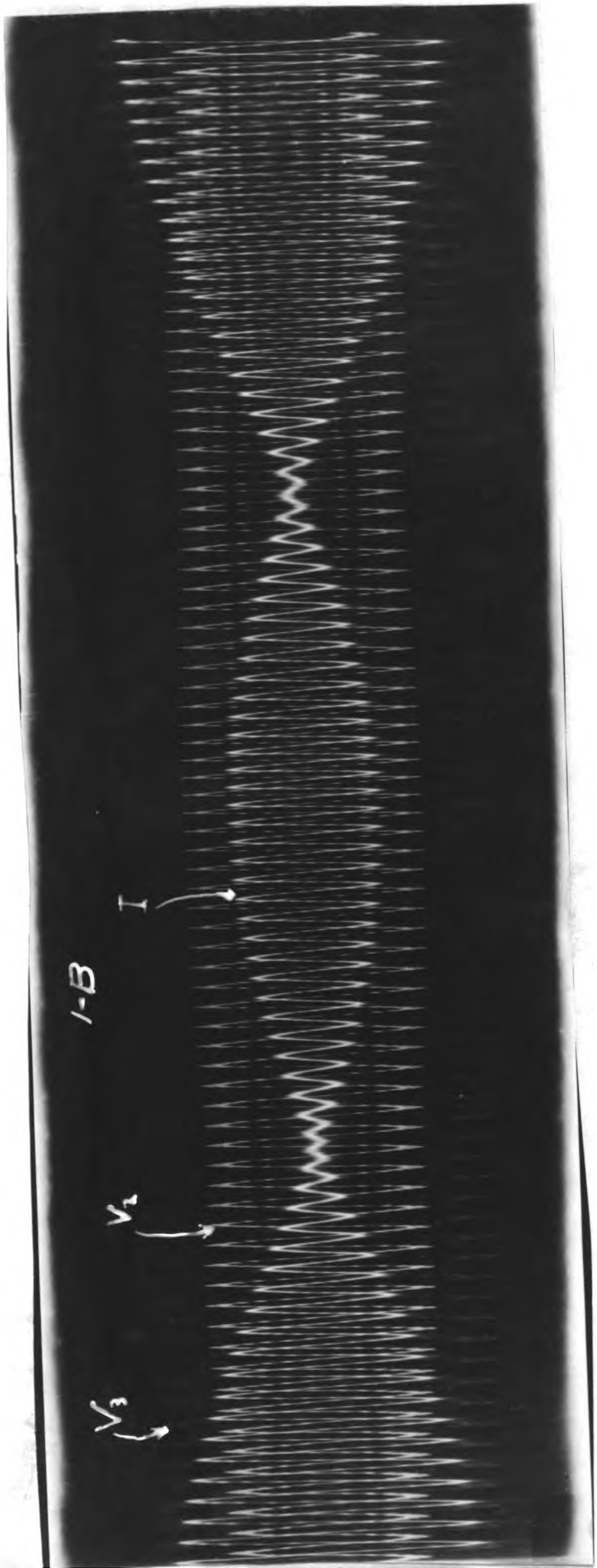
E_2	E_3	I	W(line)	W(load)	f
106	83	15	20	600	60.2

Oscillogram 7C represents the same conditions as 7A except that the reactance was used alone as the load and the driving motor of G3 was retarded. The meter readings are:

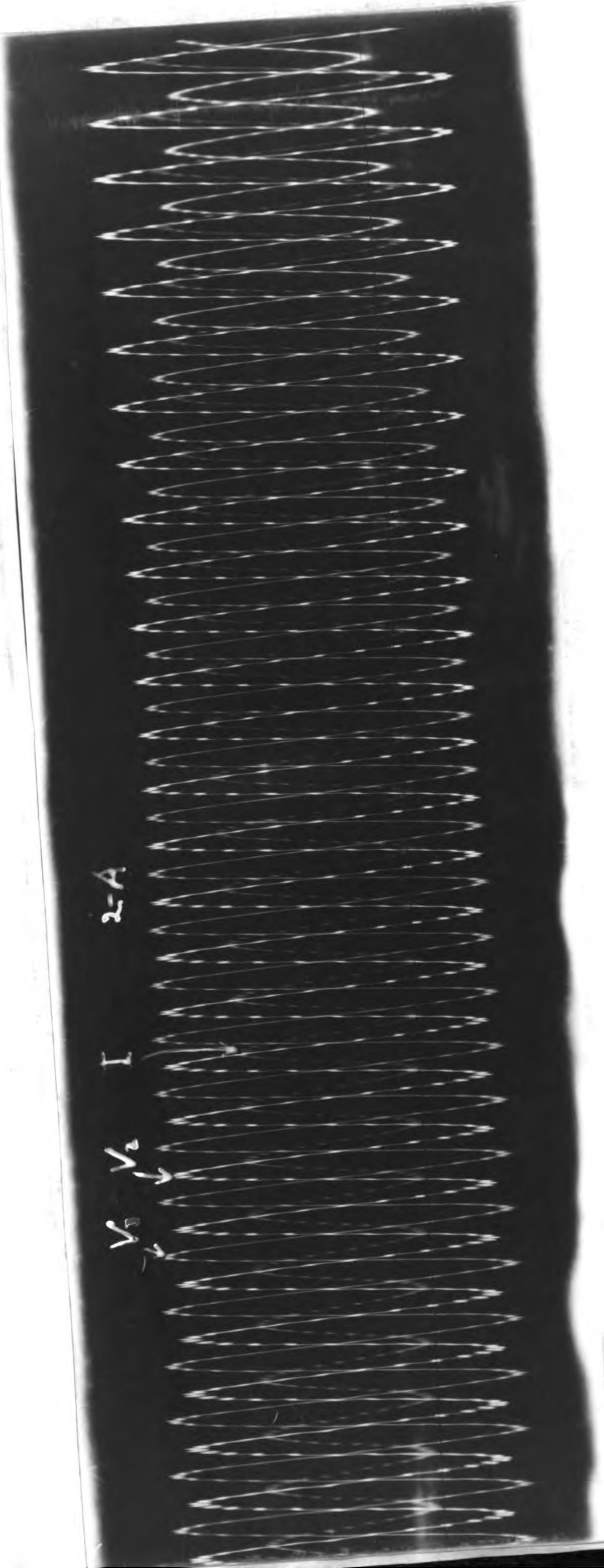
E_2	E_3	I	W(line)	W(load)
103	103	15	800	40

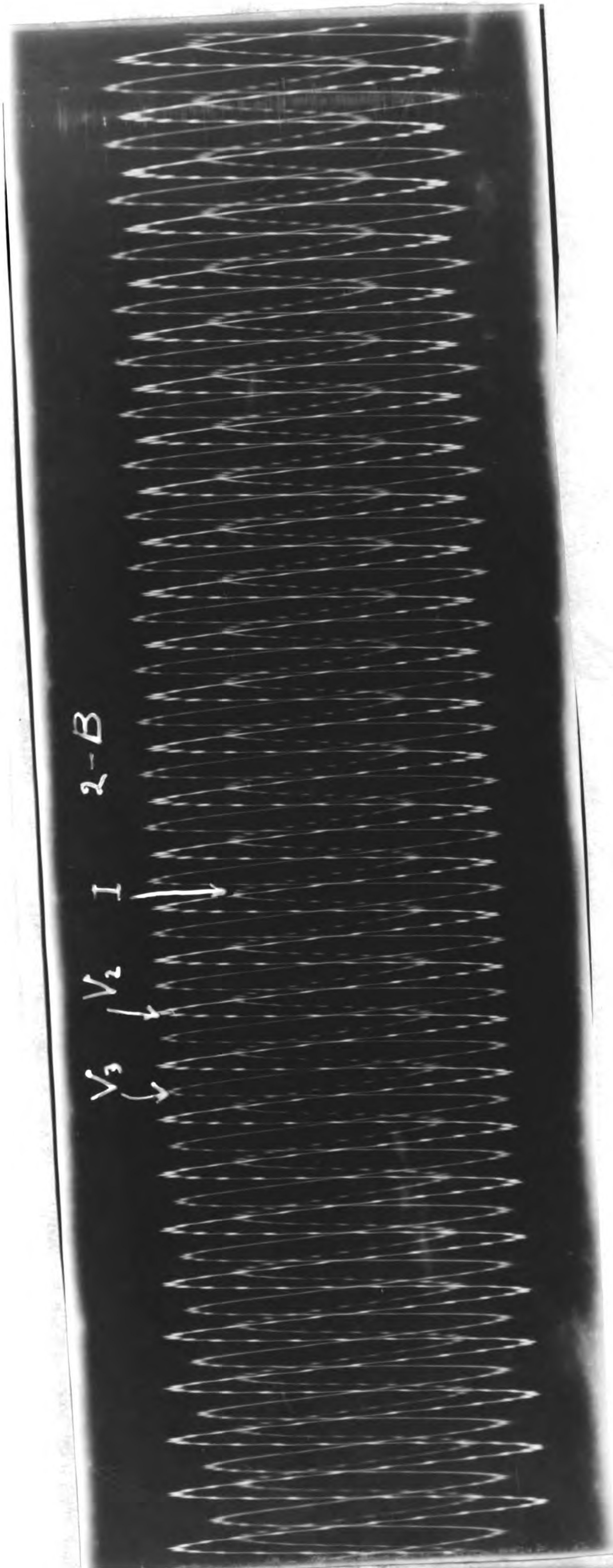
In order to check the experimental results with the value of synchronizing power which can be obtained from Chart I, suppose the conditions represented by oscillogram 5A are tested. The angle a or the displacement between the induced voltages of the two generators can be assumed to be approximately the same as the phase difference between the voltages shown on the oscillogram since the current is comparatively small and the reactance is practically all in the line. This angle by measurement is approximately 120°. The total reactance in series to neutral is the sum of the two synchronous impedances $(14.25 + 1.04 + .24 = 15.53)$ plus the reactance in the line. The total resistance $(.10 + .113 + .61 = .823)$ is negligible in comparison with the reactance. Using these values together with the voltage to neutral for this case, the value of synchronizing power is 40 watts as compared with 30 watts read on the wattmeter.

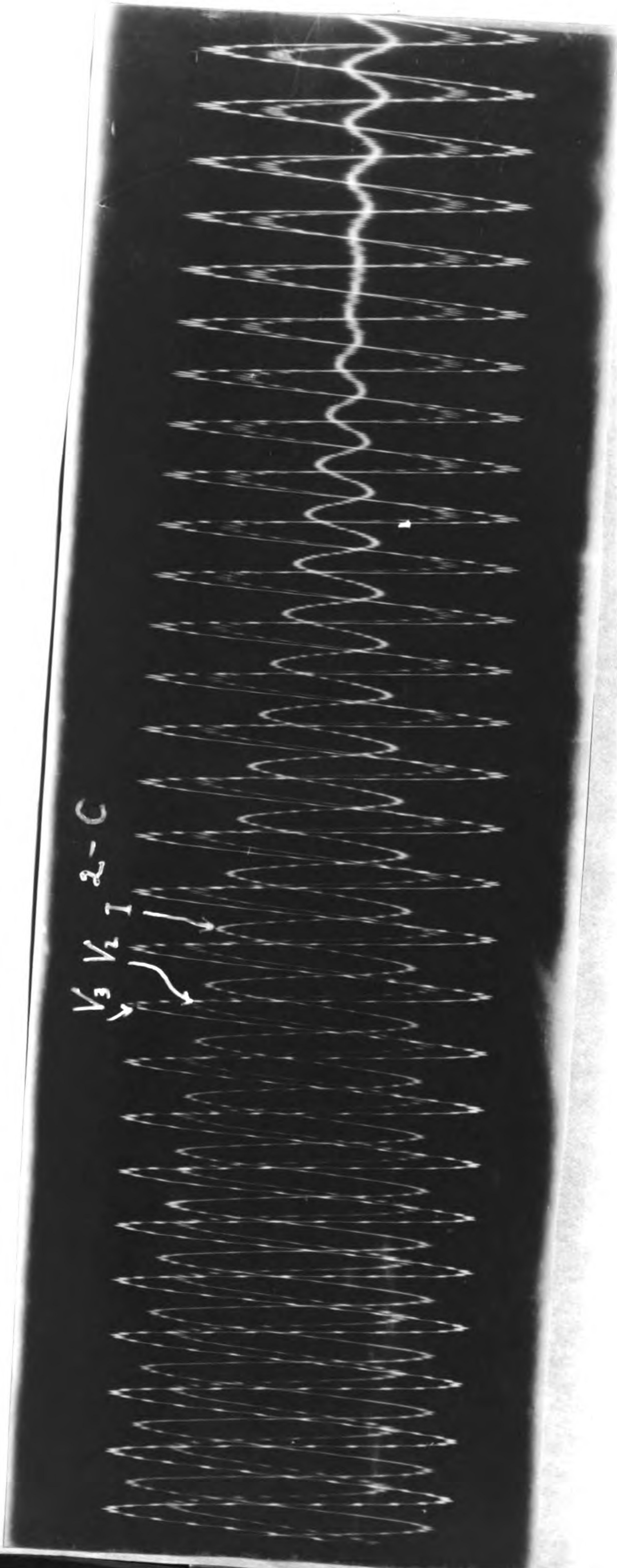

I-A
I
V_2
V_3

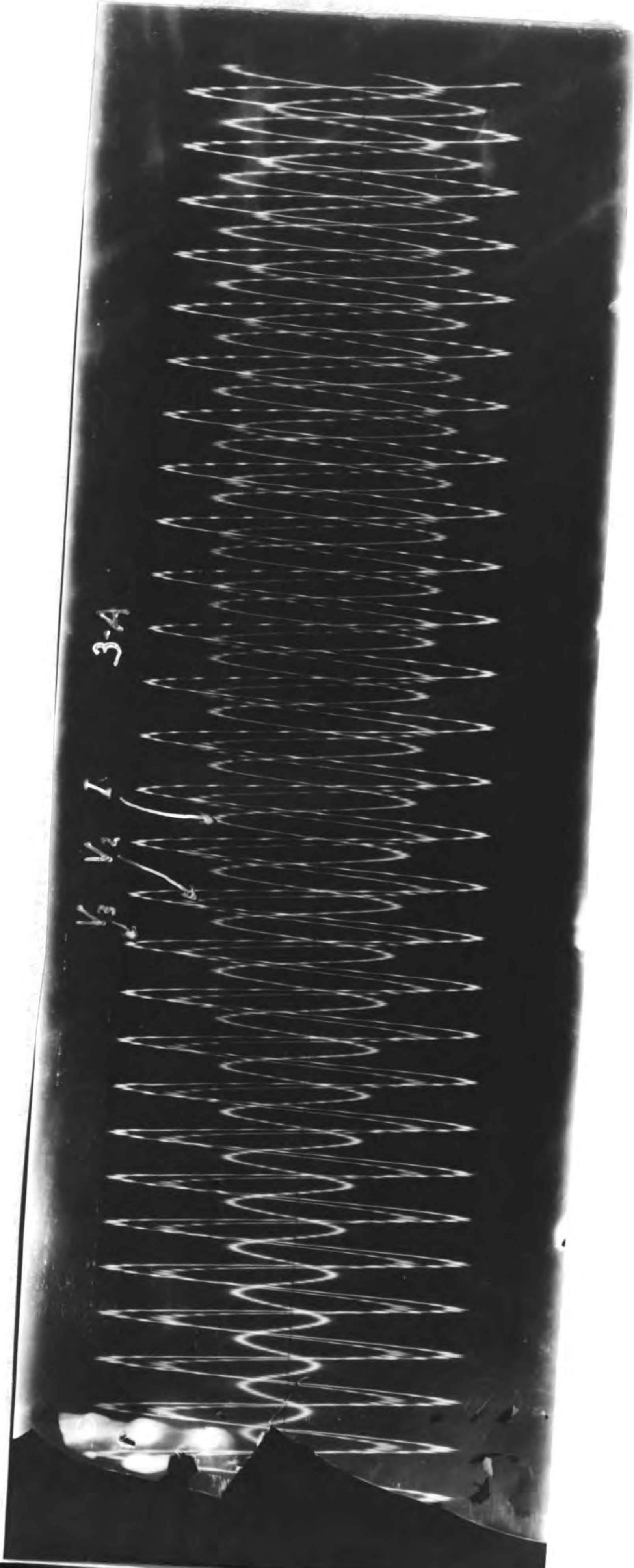

1-B
I
V_2
V_3

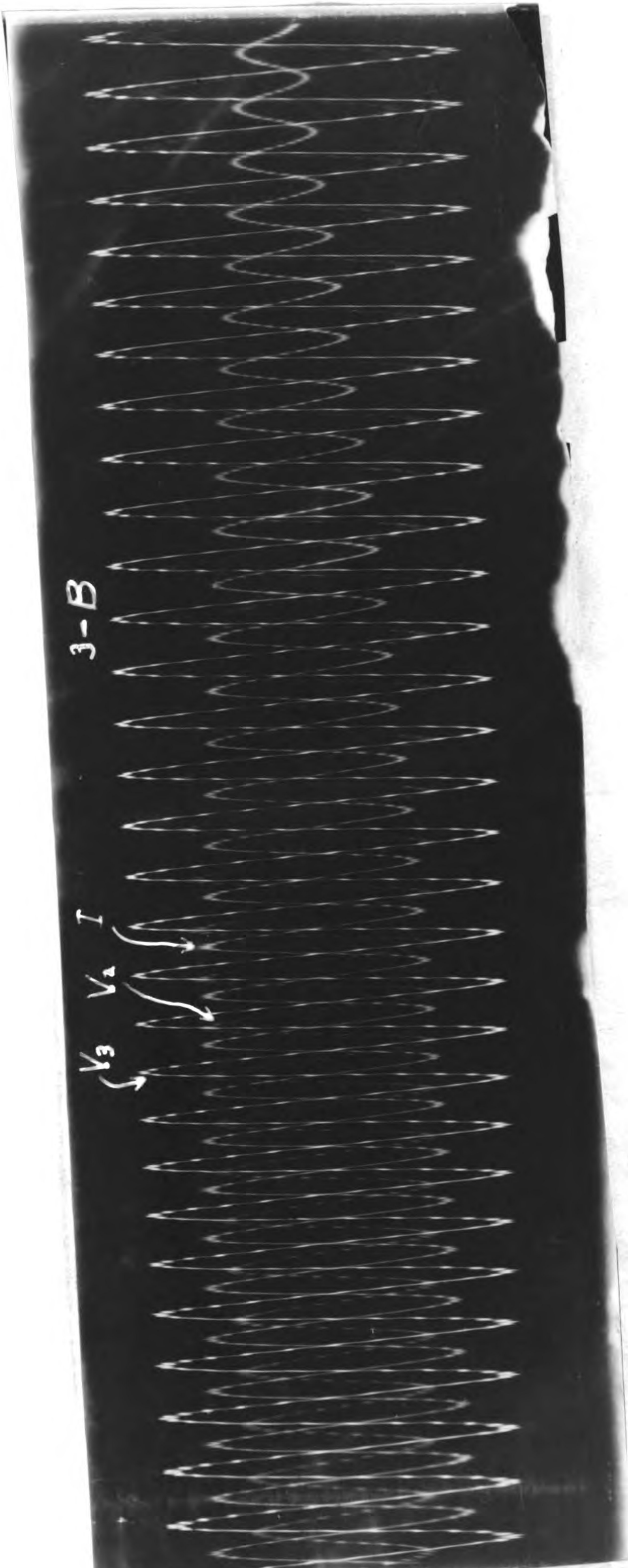

1-C
I
V_2
V_3

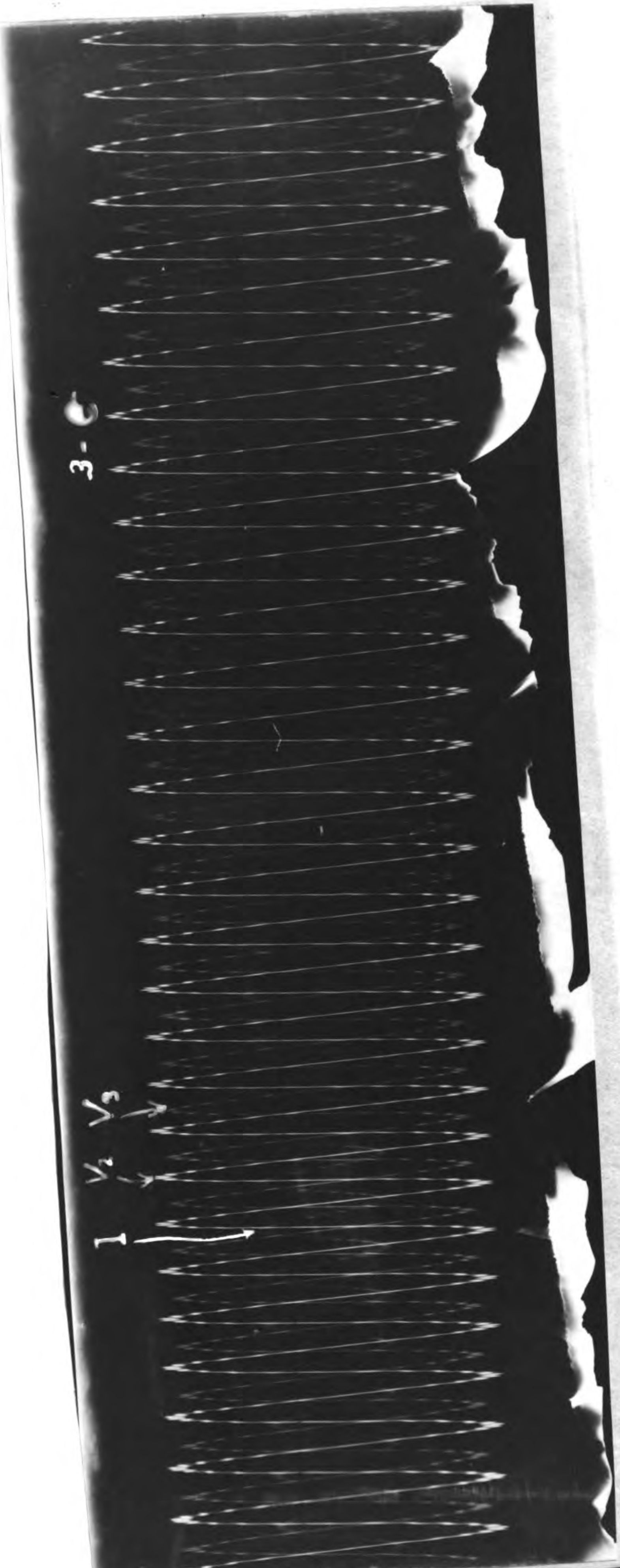

2-A
I

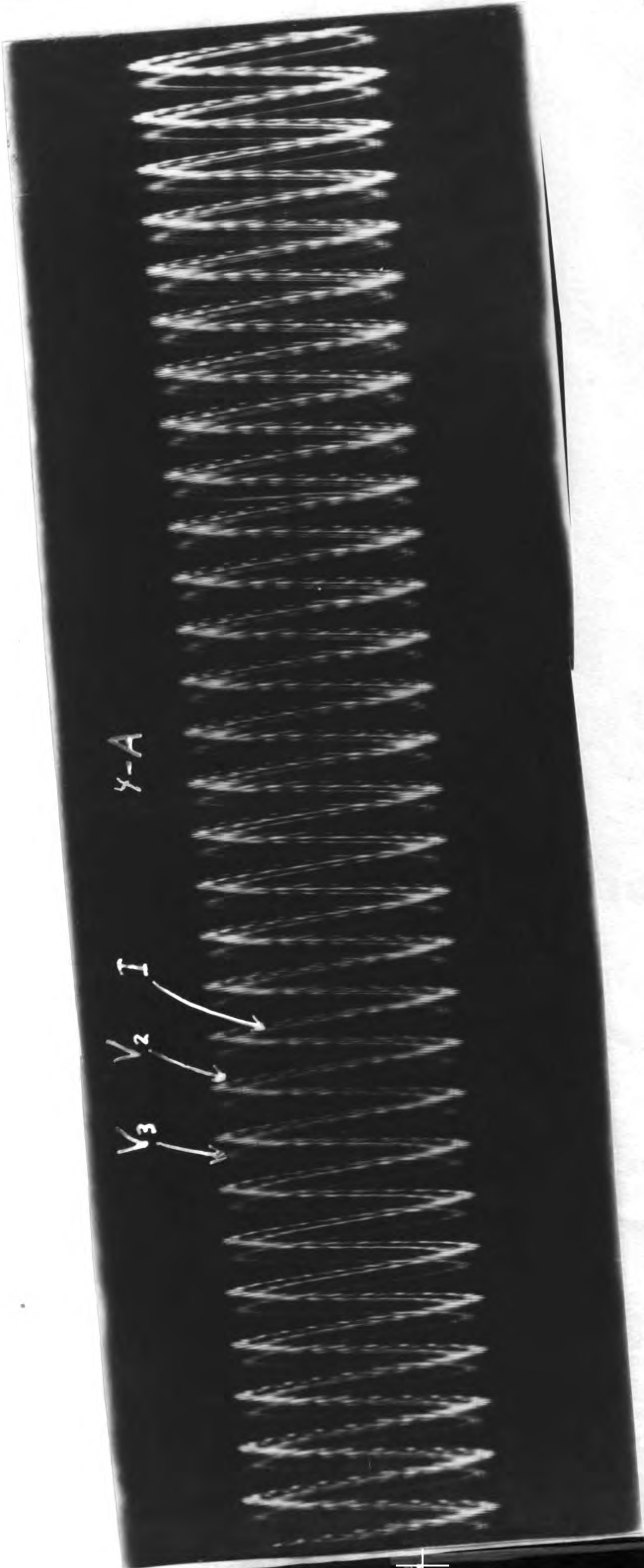

2-B
I
V_2
V_3


V3 V2 I 2-C

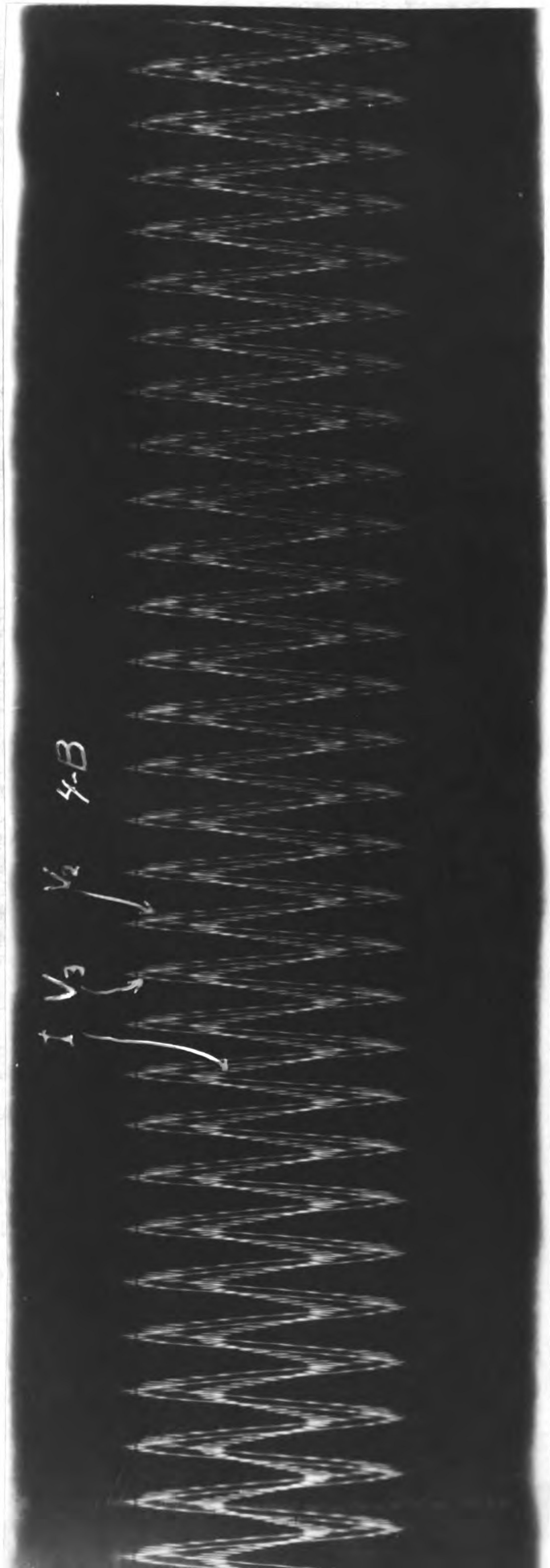

3-A
V_3
V_2
I


3-B
I
V2
V3

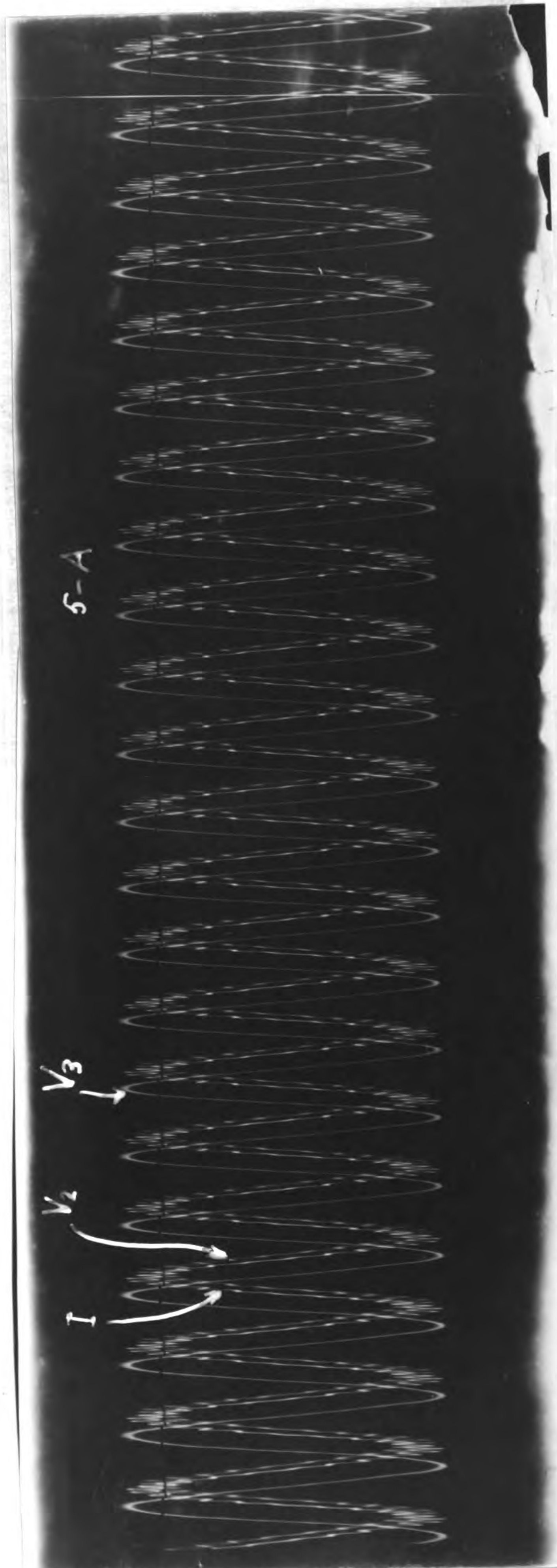

3-C
I
V_2
V_3

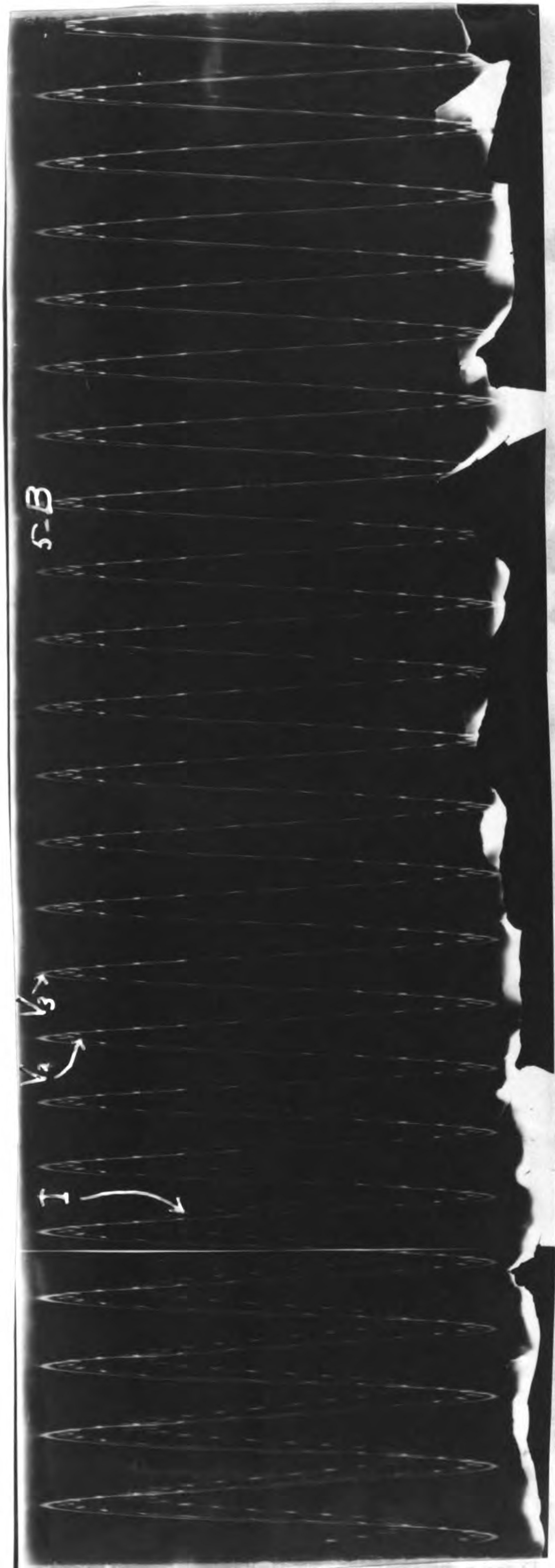


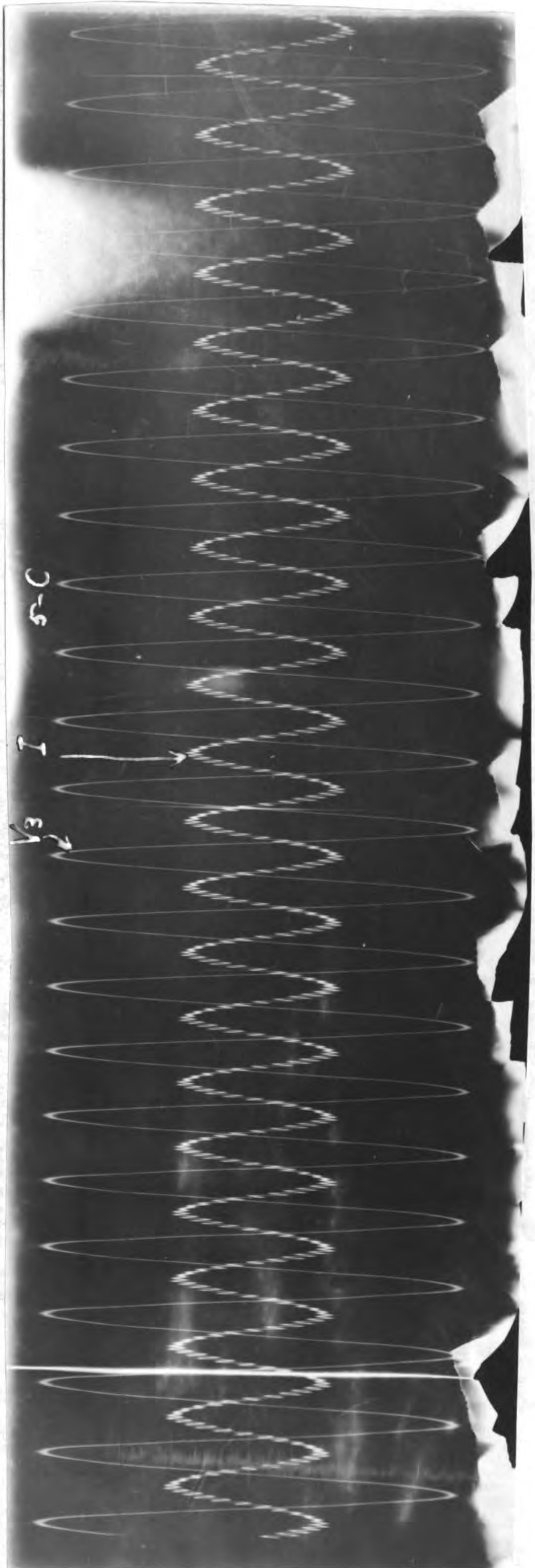
Y-A
I
V2
V3

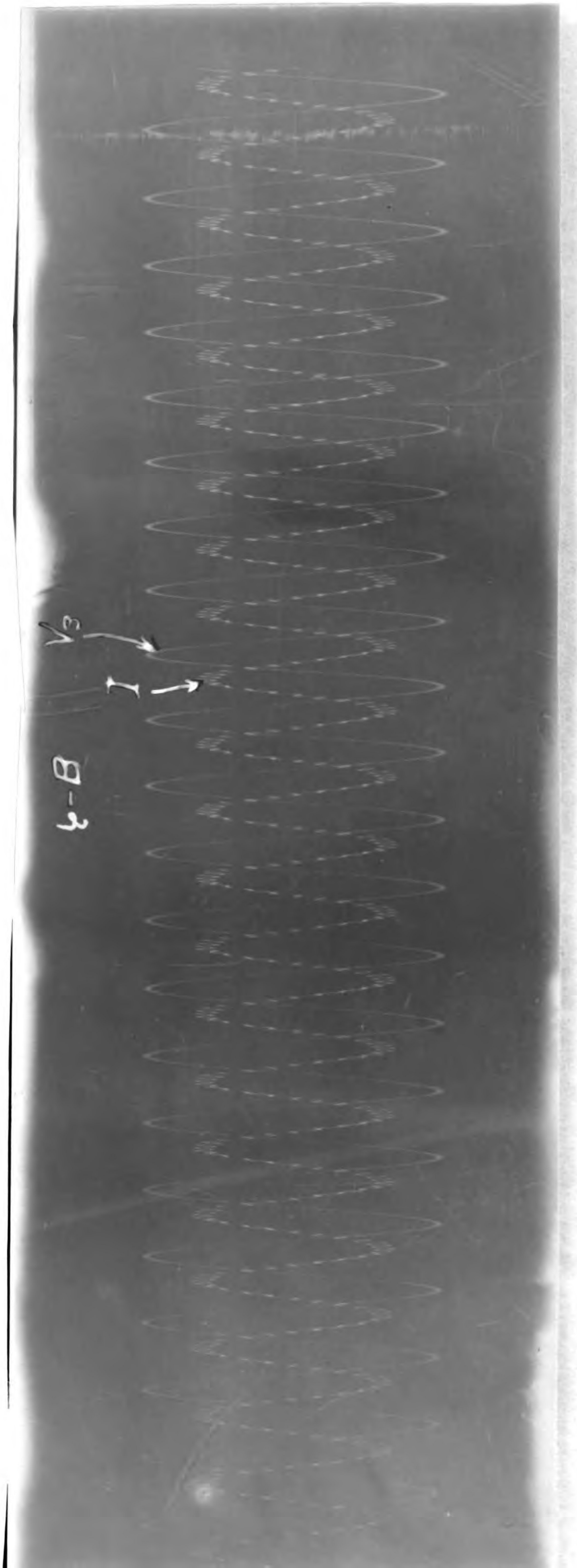

I
V3
V2
4-B

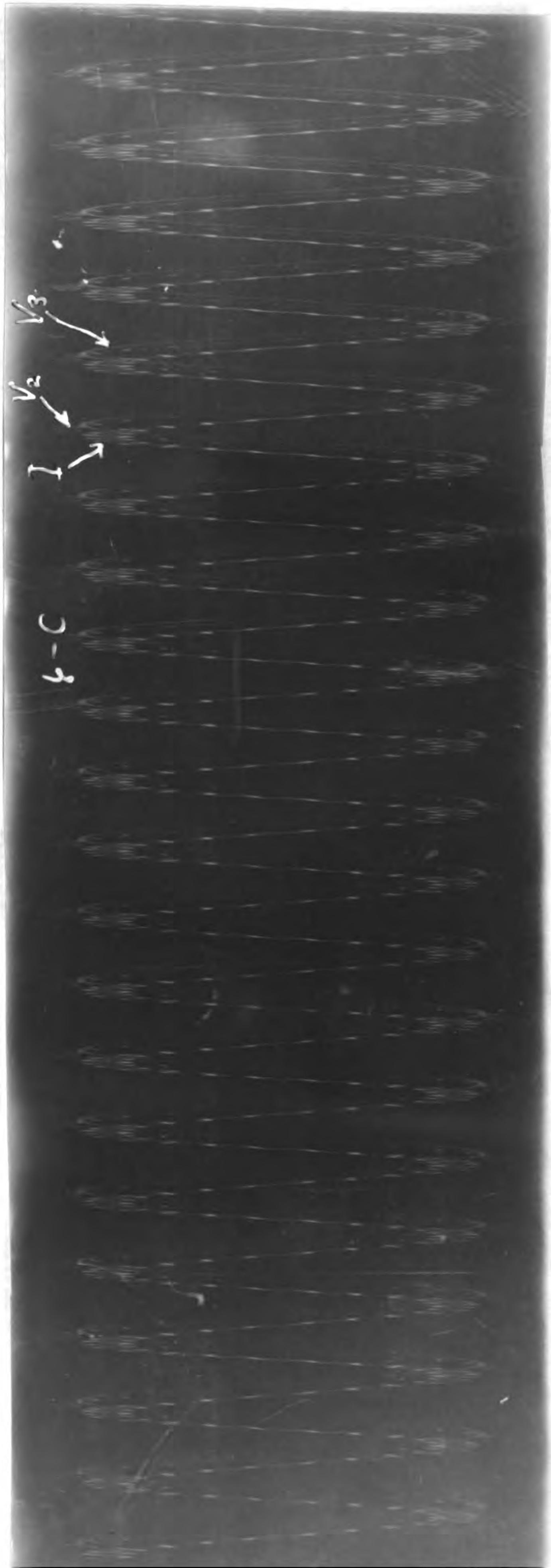

4-C
V_2
V_3
I


5-A
V3
V2
I


5-B
V_2
V_3
I

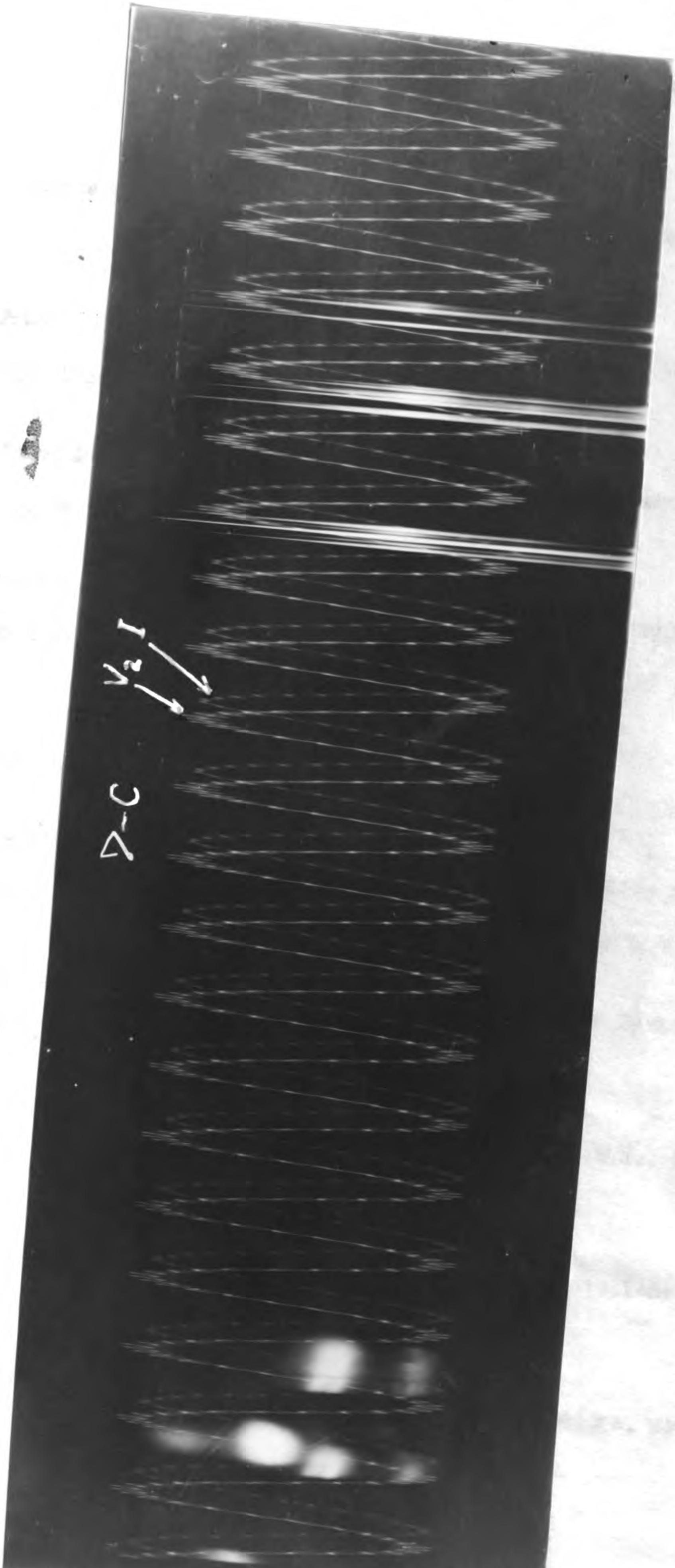

5-C
I
V3


V3
I


V3
V2
I


I
7-A


V3
I
V2
7-B


D-C
V2
I

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