115
714
THS

THE EFFECT OF VARIATION OF THE PHYSICAL CONSTANTS OF A THREE MASS SYSTEM UPON THE RESONANT FREQUENCIES

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

Zigurds Juris Levensteins

1954

THESIS

This is to certify that the

thesis entitled

THE EFFECT OF VARIATION OF THE PHYSICAL CONSTANTS OF A THREE MASS SYSTEM UPON THE RESONANT FREQUENCIES

presented by

ZIGURDS J. LEVENSTEINS

has been accepted towards fulfillment of the requirements for

MASTER OF SCIENCE **degree in** APPLIED MECHANICS

Charles O. Harris
Major professor

Date December 6, 1954

O-169

THE EFFECT OF VARIATION OF THE PHYSICAL CONSTANTS OF A THREE MASS SYSTEM UPON THE RESONANT FREQUENCIES

by

Zigurds Juris Levensteins

A THESIS

Submitted to the School of Graduate Studies of
Michigan State College of Agriculture and Applied Science
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

Department of Applied Mechanics

1954

ACKNOWLEDGEMENT

The author wishes to express his sincere thanks to Dr. Charles O. Harris, under whose lofty inspiration, constant supervision, and unfailing interest this investigation was undertaken and to whom the results are herewith dedicated.

THE EFFECT OF VARIATION OF THE PHYSICAL CONSTANTS OF A THREE MASS SYSTEM UPON THE RESONANT FREQUENCIES

by

Zigurds Juris Levensteins

AN ABSTRACT

Submitted to the School of Graduate Studies of
Michigan State College of Agriculture and Applied Science
in partial fulfillment of the requirements
for the degree of

MASTER OF SCIENCE

Department of Applied Mechanics

Year 1954

Approved ______________________

Zigurds Juris Levensteins

The object of this study was to investigate the relationship between the resonant frequencies and the physical constants in a three mass system. In particular, it was desired to see the effect that variation of the physical constants of the system has upon the distribution of the resonant frequencies.

Two methods of analysis were used. First the sixth degree algebraic resonance equation was solved by a direct method, obtaining expressions for the three resonant frequencies in a trigonometric form as functions of the physical constants of the system. Second, by an inverse method of approach, relationships between the resonant frequencies and the physical constants of the system were derived in an algebraic form.

It was concluded from the direct method of solution that the expressions were well suited for the computation of the resonant frequencies in any given three mass system. Also, the results obtained by the direct method of analysis showed that it is possible to have only two distinct resonant frequencies in a three mass system, provided that the physical constants of the system have a particular relationship. However, this relationship is so complicated that it appears to be difficult to devise a physical system in which the masses and springs fulfill the required relationship.

For the purpose of determining the effect that variation of the physical constants of the system has upon the distribution of the resonant frequencies, it was found that the inverse method of solution was simpler than the trigonometric form as found by the direct method.

Some numerical calculations were made with the inverse method solution. By varying one physical constant of the system at a time and keeping all others unchanged, the effect that each constant has on the distribution of the resonant frequencies was determined.

TABLE OF CONTENTS

Object

The object of this study was to investigate the resonance conditions of a vibrating three mass system and determine the relationship between the resonant frequencies and the physical constants of the system. In particular, it was desired to see the effect that variation of the physical constants of the system have upon the distribution of the resonant frequencies.

Introduction

Vibration is a repetitive motion. It can develop on every occasion where there is an elastic body and a repetitive force acting on it. Therefore, it is a very important problem in engineering.

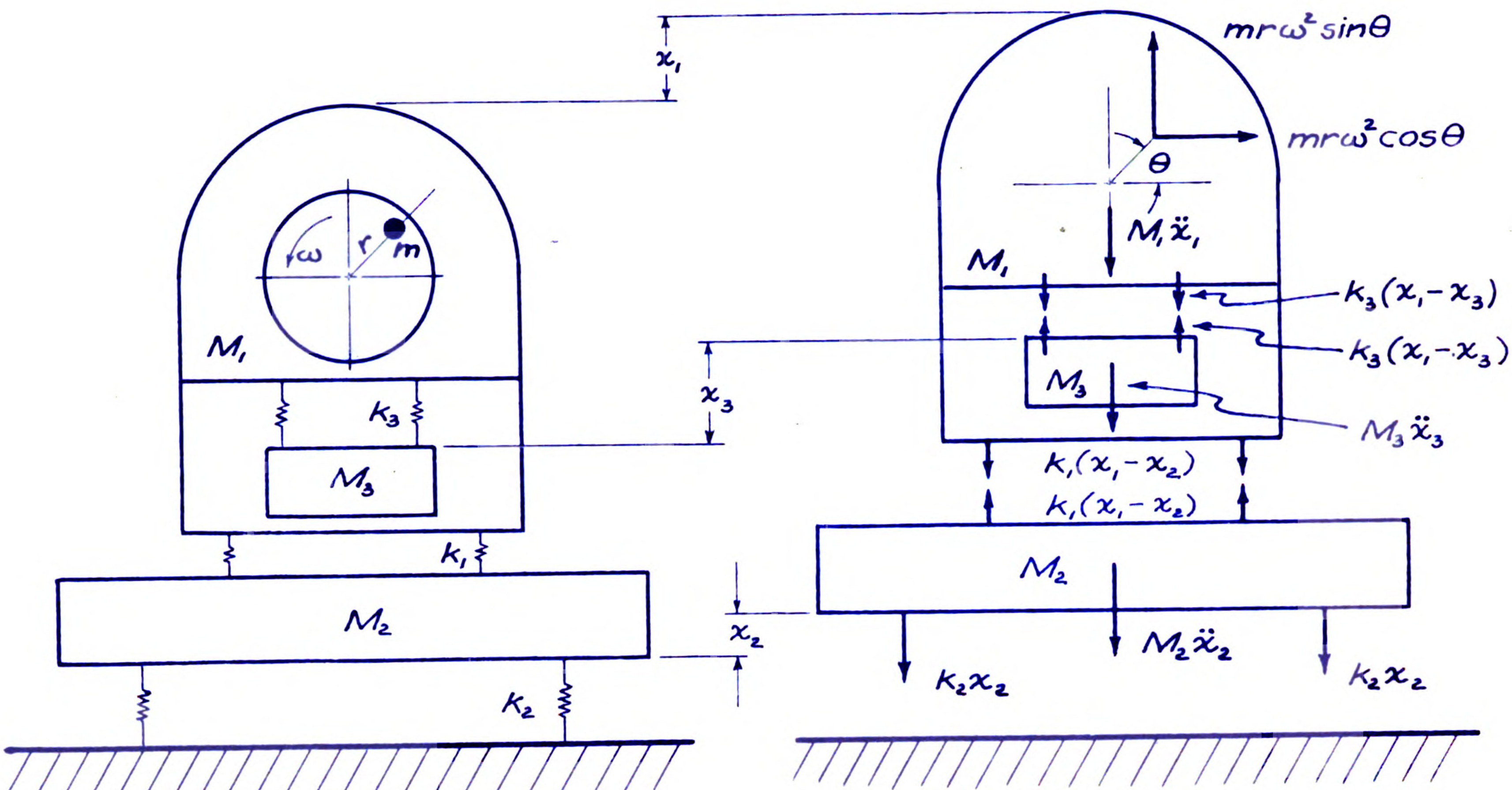
To describe completely the vibrating system at any instant, a certain number of independent coordinates is necessary. This number of independent coordinates is called the number of degrees of freedom of the system. There are systems with a single degree of freedom as well as systems with infinitely many degrees of freedom. The simplest vibrating system is with one degree of freedom and is comparatively easy to analyze, but as the number of degrees of freedom increases the problem becomes exceedingly difficult. A system with three degrees of freedom was chosen for this analysis, so let us turn to the investigation of that particular type.

The Steady State Motion of a Three-Mass System

Figure one shows a system with three degrees of freedom. It consists of three masses, M_1, M_2, and M_3 and three springs with spring constants k_1, k_2, and k_3, respectively. It is assumed that the springs have negligible mass. Furthermore, all friction in the system (causing damping of the vibrations) is neglected. A harmonic driving force $mr\omega^2\sin\theta$ caused by a rotating unbalance is applied to the mass M_1. Thus vibration of the system will occur. It seems reasonable to assume the frequency of the vibration to be the same as the frequency of the driving force. This assumption is supported by experimental results and theory.

Figure 1a shows the configuration of the system in static equilibrium, and it is assumed that the vibration will be about this configuration. Only the vertical vibrations of the system will be considered in this analysis; thus in this case the independent coordinates necessary to describe the system are the three vertical displacements x_1, x_2, and x_3 of the masses M_1, M_2, and M_3, respectively. Only steady state vibrations of the system will be considered, neglecting the transient vibrations.

Figure 1b shows the configuration of the system when it is vibrating in the steady state. Also Figure 1b shows all forces acting on the masses M_1, M_2, and M_3 at any time t. By applying Newton's Second Law of Motion to the system three



(a) CONFIGURATION OF STATIC EQUILIBRIUM

(b) CONFIGURATION IN STEADY STATE

FIG. 1. SYSTEM

simultaneous second order differential equations are obtained. They are:

$$M_1\ddot{x}_1 + k_1(x_1 - x_2) + k_3(x_1 - x_3) = mr\omega^2 \sin\omega t, \quad (1)$$

$$M_2\ddot{x}_2 - k_1(x_1 - x_2) + k_2 x_2 = 0, \quad (2)$$

$$M_3\ddot{x}_3 - k_3(x_1 - x_3) = 0 \; . \quad (3)$$

Where it is assumed that the frequency of the driving force $mr\omega^2 \sin\theta$ is constant, so that $\theta = \omega t$ and θ can be replaced by ωt .

For steady state vibrations, the solutions of equations (1), (2), and (3) can be assumed in the forms:

$$x_1 = X_1 \sin\omega t, \quad (4)$$

$$x_2 = X_2 \sin\omega t, \quad (5)$$

$$x_3 = X_3 \sin\omega t. \quad (6)$$

Where X_1, X_2, and X_3 are the amplitudes of motion of M_1, M_2, and M_3, respectively. Substition of equations (4), (5), and (6) in equations (1), (2), and (3) yields,

$$
\begin{aligned}
&(k_1+k_3-\omega^2M_1)X_1-k_1X_2-k_3X_3=mr\omega^2\,,\\
&-k_1X_1+(k_1+k_3-\omega^2M_2)X_2=0\,,\\
&-k_3X_1+(k_3-\omega^2M_3)X_3=0\,,
\end{aligned}
\tag{7}
$$

which is a system of three simultaneous algebraic equations that can be solved for X_1, X_2, and X_3.

Solving the system by determinants and introducing the notations,

$$\omega_1^2=\frac{k_1}{M_1}\,;\quad \omega_2^2=\frac{k_2}{M_2}\,;\quad \omega_3^2=\frac{k_3}{M_3}\,;$$

$$\mu=\frac{M_2}{M_1}\,;\quad \nu=\frac{M_3}{M_1}$$

the amplitudes of motion are,

$$X_1=\frac{\frac{mr}{M_1}\left(\frac{\omega}{\omega_3}\right)^2\left[1-\left(\frac{\omega}{\omega_3}\right)^2\right]\left[\frac{1}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^2+\left(\frac{\omega_2}{\omega_3}\right)^2-\left(\frac{\omega}{\omega_3}\right)^2\right]}{D}\,, \tag{8}$$

$$X_2=\frac{\frac{mr}{M_1}\left(\frac{\omega}{\omega_3}\right)^2\left[1-\left(\frac{\omega}{\omega_3}\right)^2\right]\frac{1}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^2}{D}\,, \tag{9}$$

$$X_3 = \frac{\frac{mr}{M_1}\left(\frac{\omega}{\omega_3}\right)^2\left[\frac{1}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^2+\left(\frac{\omega_2}{\omega_3}\right)^2-\left(\frac{\omega}{\omega_3}\right)^2\right]}{D} . \tag{10}$$

D is the coefficient determinant of the system of equations (7), and is,

$$D = \left[1-\left(\frac{\omega}{\omega_3}\right)^2\right]\left\{\left[\frac{1}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^2+\left(\frac{\omega_2}{\omega_3}\right)^2-\left(\frac{\omega}{\omega_3}\right)^2\right]\left[\left(\frac{\omega_1}{\omega_3}\right)^2+\nu-\left(\frac{\omega}{\omega_3}\right)^2\right] - \frac{1}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^4\right\} - \nu\left[\frac{1}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^2+\left(\frac{\omega_2}{\omega_3}\right)^2-\left(\frac{\omega}{\omega_3}\right)^2\right] . \tag{11}$$

Resonant Frequencies

Whenever the denomination D is equal to zero the amplitudes of motion X_1, X_2, and X_3 will become infinite. The condition under which an amplitude tends to increase without limit is called a resonant condition. It is very important to know for what values of the driving frequency ω the amplitudes become infinite in any given vibrating system, because at those frequencies an extremely violent shaking

of the system will result. In some problems this condition is desired, in others it is to be avoided. To determine the values of ω for which resonance occurs, the coefficient determinant D must be equated to zero. Thus an algebraic equation of sixth degree in $\frac{\omega}{\omega_3}$ is obtained,

$$\left(\frac{\omega}{\omega_3}\right)^6 - A\left(\frac{\omega}{\omega_3}\right)^4 + B\left(\frac{\omega}{\omega_3}\right)^2 - C = 0 . \tag{12}$$

The coefficients A, B, and C are functions of the physical constants of the vibrating system, namely, ω_1, ω_2, ω_3, μ, and ν. They are:

$$A = \left(1+\frac{1}{\mu}\right)\left(\frac{\omega_1}{\omega_3}\right)^2 + \left(\frac{\omega_2}{\omega_3}\right)^2 + \nu + 1 , \tag{13}$$

$$B = \left(1+\frac{1}{\mu}\right)\left(\frac{\omega_1}{\omega_3}\right)^2 + \left(\frac{\omega_2}{\omega_3}\right)^2 + \left(\frac{\omega_1}{\omega_3}\right)^2\left(\frac{\omega_2}{\omega_3}\right)^2 + \frac{\nu}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^2 + \nu\left(\frac{\omega_2}{\omega_3}\right)^2 , \tag{14}$$

$$C = \left(\frac{\omega_1}{\omega_3}\right)^2\left(\frac{\omega_2}{\omega_3}\right)^2 . \tag{15}$$

An equation like (12) must have six roots, but only three of them are different numerically; that is, in absolute value there are only three roots. This is true because equation (12) can be written,

$$\lambda^3 - A\lambda^2 + B\lambda - C = 0, \tag{16}$$

by letting,

$$\left(\frac{\omega}{\omega_3}\right)^2 = \lambda .$$

Thus it can be seen that a system with three degrees of freedom has three distinct values of the driving frequency for which resonance occurs. Since the coefficients of the resonance equation (12) are functions of the physical constants of the system, the location of the resonant points can be varied by changing the masses and spring constants in the system.

Now assume that the roots of equation (16) are λ_1, λ_2, and λ_3. Then the following relationship between the coefficients A, B, and C and the roots λ_1, λ_2, and λ_3 must be true,

$$A = \lambda_1 + \lambda_2 + \lambda_3 , \tag{17}$$

$$B = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1 , \tag{18}$$

$$C = \lambda_1\lambda_2\lambda_3 . \tag{19}$$

From equation (19) it can be seen that $\frac{\omega}{\omega_3}=0$ can never be a resonant point, since that would require constant C to be zero, but for the system to be of three degrees of freedom C must be different from zero as can be seen from equation (15).

Trigonometric Form of Solution

A trigonometric form of solution of equation (16) will now be attempted. First, to eliminate the quadratic term let,

$$\lambda = z + \frac{A}{3} \, .$$

Then,

$$z^3 + 3Hz + G = 0, \tag{20}$$

where,

$$H = \frac{3B - A^2}{9}, \tag{21}$$

and,

$$G = \frac{9AB - 2A^3 - 27C}{27}, \tag{22}$$

and A, B, and C are as previously given by equations (13), (14), and (15). From physical considerations it can be assumed that the roots λ_1, λ_2, and λ_3 of equation (16) and therefore also z_1, z_2, and z_3 of equation (20) are distinct positive real quantities. Hence the discriminant Δ of equation (20) must be positive.

The discriminant of a reduced cubic equation is,

$$\Delta = -27(G^2+4H^3)\,.$$

Therefore, $G^2+4H^3<0$ and since G^2 cannot be negative, H must be, from which it follows that

$$A^2>3B\,.$$

This is one condition that must be fulfilled in order for a system with three degrees of freedom to have three resonant points. It can be proven that the solution of equation (20) can be given in a trigonometric form* as,

$$z=2\sqrt{-H}\cos\left[\frac{\arccos\left(-\frac{G}{2\sqrt{-H^3}}\right)+2k\pi}{3}\right], \tag{23}$$

where $k=0,1,2$. Substitution of equations (21) and (22) into equation (23) yields,

$$z=\frac{2}{3}\sqrt{A^2-3B}\cos\left\{\frac{\arccos\left[-\frac{(9AB-2A^3-27C)\sqrt{A^2-3B}}{2(A^2-3B)^2}\right]+2k\pi}{3}\right\}.$$

$$k=0,1,2\,.$$

---

*Conkwright, N. B. Introduction to the Theory of Equations. New York: Ginn and Company, 1941. p. 76.

Noting that $\lambda = Z + \frac{A}{3}$, this expression gives the three desired roots of equation (16),

$$\lambda_1 = \frac{2}{3}\sqrt{A^2-3B}\cos\left\{\frac{\arccos\left[-\frac{(9AB-2A^3-27C)\sqrt{A^2-3B}}{2(A^2-3B)^2}\right]}{3}\right\} + \frac{A}{3}, \tag{24}$$

$$\lambda_2 = \frac{2}{3}\sqrt{A^2-3B}\cos\left\{\frac{\arccos\left[-\frac{(9AB-2A^3-27C)\sqrt{A^2-3B}}{2(A^2-3B)^2}\right]+2\pi}{3}\right\} + \frac{A}{3}, \tag{25}$$

$$\lambda_3 = \frac{2}{3}\sqrt{A^2-3B}\cos\left\{\frac{\arccos\left[-\frac{(9AB-2A^3-27C)\sqrt{A^2-3B}}{2(A^2-3B)^2}\right]+4\pi}{3}\right\} + \frac{A}{3}. \tag{26}$$

Thus it can be seen that the location of the resonant points represented above by equations (24), (25), and (26) are dependent on all the physical constants of the system ω_1, ω_2, and ω_3, μ , and ν , but the functional relationship between them is extremely complicated. Since the cosine funtion cannot be larger than plus one or smaller than minus one, for any given system the maximum and minimum values of resonant frequencies are,

$$\max.\left(\frac{\omega}{\omega_3}\right)_{res.}=\sqrt{\frac{A}{3}+\frac{2}{3}\sqrt{A^2-3B}}\ , \qquad (27)$$

$$\min.\left(\frac{\omega}{\omega_3}\right)_{res.}=\sqrt{\frac{A}{3}-\frac{2}{3}\sqrt{A^2-3B}}\ . \qquad (28)$$

Conditions for Two Resonant Frequencies to be Equal

Now let us look at the possibility of making the roots of the resonance equation equal. Assume that the roots of equation (20) are Z_1 and $Z_2 = Z_3$. The latter can be true if and only if the discriminant Δ is equal to zero, that is

$$G^2+4H^3=0\ . \qquad (29)$$

In equation (23) let

$$\phi=\arccos-\frac{G}{2\sqrt{-H^3}}\ ,$$

but $H^3 = -\frac{G^2}{4}$; if there are repeated roots, therefore,

$$\phi=\arccos-\frac{G}{2\sqrt{\frac{G^2}{4}}}=\arccos\pm 1\ ,$$

depending on whether G is positive or negative. If $G>0$, $\phi = \arccos -1$; and $\phi = \pi$. If $G<0$; $\phi = \arccos +1$; and $\phi = 0$. Therefore, if there are repeated roots of the resonance equation (16), they must be,

$$\lambda_1 = -\frac{2}{3}\sqrt{A^2-3B} + \frac{A}{3},$$

$$\lambda_2 = \frac{1}{3}\sqrt{A^2-3B} + \frac{A}{3}, \tag{30}$$

$$\lambda_3 = \frac{1}{3}\sqrt{A^2-3B} + \frac{A}{3},$$

if $G>0$; and,

$$\lambda_1 = \frac{2}{3}\sqrt{A^2-3B} + \frac{A}{3},$$

$$\lambda_2 = -\frac{1}{3}\sqrt{A^2-3B} + \frac{A}{3}, \tag{31}$$

$$\lambda_3 = -\frac{1}{3}\sqrt{A^2-3B} + \frac{A}{3}.$$

If $G<0$. For equations (30) to be possible in a physical system,

$$-\frac{2}{3}\sqrt{A^2-3B} + \frac{A}{3} > 0,$$

or $B > \left(\frac{A}{2}\right)^2$. For equations (31) to be possible in a physical system,

$$-\frac{1}{3}\sqrt{A^2-3B}+\frac{A}{3}>0,$$

or $B>0$, which is always true, since all quantities in equation (14) are positive.

Distribution of Resonant Frequencies

To see the effect that variation of the physical constants of the system have upon the distribution of the resonant frequencies, as given by equations (24), (25), and (26), different values of ω_1, ω_2, ω_3, μ , and ν must be substituted in the equations (13), (14), and (15) to obtain the quantities A, B, and C. Then in turn the known A, B, and C must be substituted in equations (24), (25), and (26); and the variation of λ_1, λ_2, and λ_3 noted. However, due to the complexity of the expressions for the resonant frequencies, and also quantities A, B, and C, the substitution and computation would be a very tedious and lengthy process. Therefore, let us try another method of approach.

Equations (13), (14), and (15) give the quantities A, B, and C (they are coefficients of the resonance equation (16)) in terms of the physical constants of the system. Equations (17), (18), and (19) give the relationship that must exist between the roots of the resonance equation λ_1, λ_2, and

λ_3 and A, B, and C. Therefore, the right hand sides of equations (13), (14), and (15) may be equated to the right hand sides of equations (17), (18), and (19), respectively, because the left hand sides of the above equations are also equal, respectively. This gives:

$$\lambda_1+\lambda_2+\lambda_3=\left(1+\frac{1}{\mu}\right)\left(\frac{\omega_1}{\omega_3}\right)^2+\left(\frac{\omega_2}{\omega_3}\right)^2+\nu+1\,, \tag{32}$$

$$\lambda_1\lambda_2+\lambda_2\lambda_3+\lambda_3\lambda_1=\left(1+\frac{1}{\mu}\right)\left(\frac{\omega_1}{\omega_3}\right)^2+\left(\frac{\omega_2}{\omega_3}\right)^2+\left(\frac{\omega_1}{\omega_3}\right)^2\left(\frac{\omega_2}{\omega_3}\right)^2 + \frac{\nu}{\mu}\left(\frac{\omega_1}{\omega_3}\right)^2+\nu\left(\frac{\omega_2}{\omega_3}\right)^2\,, \tag{33}$$

$$\lambda_1\lambda_2\lambda_3=\left(\frac{\omega_1}{\omega_3}\right)^2\left(\frac{\omega_2}{\omega_3}\right)^2\,. \tag{34}$$

In this system of three simultaneous algebraic equations, eight quantities are involved, namely, λ_1, λ_2, λ_3, ω_1, ω_2, ω_3, μ, and ν. By assuming five of them, the other three can be uniquely determined. Suppose we assume values for λ_1, ω_2, ω_3, μ, and ν and solve for ω_1, λ_2, and λ_3. From equation (34),

$$\left(\frac{\omega_1}{\omega_3}\right)^2=\frac{\lambda_1\lambda_2\lambda_3}{\left(\frac{\omega_2}{\omega_3}\right)^2}\,. \tag{35}$$

Substituting this expression in equations (32) and (33) and solving for ${}_2$ ${}_3$ yields,

$$\lambda_2\lambda_3 = \frac{(\lambda_1-\nu-1)\left[\left(\frac{\omega_2}{\omega_3}\right)^2-\lambda_1\right]}{(1-\lambda_1)\left[\left(1+\frac{1}{\mu}\right)\frac{\lambda_1}{\left(\frac{\omega_2}{\omega_3}\right)^2}-1\right]+\frac{\nu\mu}{\mu\left(\frac{\omega_2}{\omega_3}\right)^2}} \,. \tag{36}$$

Substitution of equation (36) into equation (35) gives the unknown $\left(\frac{\omega_1}{\omega_3}\right)^2$,

$$\left(\frac{\omega_1}{\omega_3}\right)^2 = \frac{(\lambda_1-\nu-1)\left[\left(\frac{\omega_2}{\omega_3}\right)^2-\lambda_1\right]}{(1-\lambda_1)\left[\left(1+\frac{1}{\mu}\right)-\frac{\left(\frac{\omega_2}{\omega_3}\right)^2}{\lambda_1}\right]+\frac{\nu}{\mu}} \,. \tag{37}$$

It can be seen from equation (37) that an interesting relationship is obtained if λ_1 is chosen to be unity; then,

$$\left(\frac{\omega_1}{\omega_3}\right)^2 = \mu\left[1-\left(\frac{\omega_2}{\omega_3}\right)^2\right] \,. \tag{38}$$

Recalling that $\lambda = \left(\frac{\omega}{\omega_3}\right)^2$, $\lambda_1 = 1$ means that $\omega = \omega_3$. Hence, if the frequency of the driving force is equal to the natural frequency of the mass M_3 and spring k_3 considered alone, then the equation (38) holds true. Substituting in this equation the previously defined quantities of ω_1, ω_2, and μ we get,

$$k_1 = -k_2 + M_2\omega_3^2 \ . \qquad (39)$$

Knowing $\left(\frac{\omega_1}{\omega_3}\right)^2$ from equation (37) the quantities A, B, and C may be computed. Then by knowing A, B, and C the other two resonant frequencies may be found from the equation,

$$\lambda = \frac{-(\lambda_1 - A) \pm \sqrt{(\lambda_1 - A)^2 - \frac{4C}{\lambda_1}}}{2} \ , \qquad (40)$$

where using the plus sign before the radical gives one root and the minus sign the other.

Thus this inverse method of approach gives expressions that are simpler for variation and computation of the parameters.

Numerical Examples

Now let us apply the inverse method developed in the latter part of the previous section to obtain results in graphical form.

First, parameters ν and $\left(\frac{\omega_2}{\omega_3}\right)^2$ were chosen as variables and the resonant frequency ratio $\sqrt{\lambda_1}$ was held constant at 1.1. The parameter ν was chosen to vary from 0.01 to 0.1 and $\left(\frac{\omega_2}{\omega_3}\right)^2$ was given the values 0.01, 0.02, 0.03, and 0.04, successively. Figure 2 shows the results obtained for the variation of the resonant frequency ratios; and Figure 3 the variation of

frequency, and spring characteristics of the mass M_1. Figure 2 shows that the middle resonant frequency ratio is quite sensitive to variations in ν, but its variation is negligible with respect to $\left(\frac{\omega_2}{\omega_3}\right)^2$, at least up to $\left(\frac{\omega_2}{\omega_3}\right)^2 = 0.04$. On the other hand, the lowest resonant frequency ratio seems to have no variation with respect to ν, but it increases with increasing values of $\left(\frac{\omega_2}{\omega_3}\right)^2$. Figure 3 shows that the parameter $\left(\frac{\omega_1}{\omega_3}\right)^2$, along with the spring constant k_1, is greatly influenced by variations in ν, but the variation of $\left(\frac{\omega_2}{\omega_3}\right)^2$ has negligible effect on both of them, at least in the range of values used.

Second, for two different values of the parameter ν ($\nu = 0.1$ and $\nu = 0.06$), the parameter μ was made to vary from 1.5 to 2.5. The effect of this variation on the resonant frequency ratios is shown in Figure 4. For the lowest resonant frequency ratio, the magnitudes for it, obtained by the two different values of the parameter ν were so nearly equal that it was impossible to draw two separate curves. Figure 4 shows that the variation of the resonant frequency ratios with parameter μ is very slight. This is particularly so for the lowest and highest resonant frequency ratios. If variation of the resonant frequency ratios with the parameter μ is to be avoided, it is evident from Figure 4 that a choice of higher values of the parameter ν is advantageous.

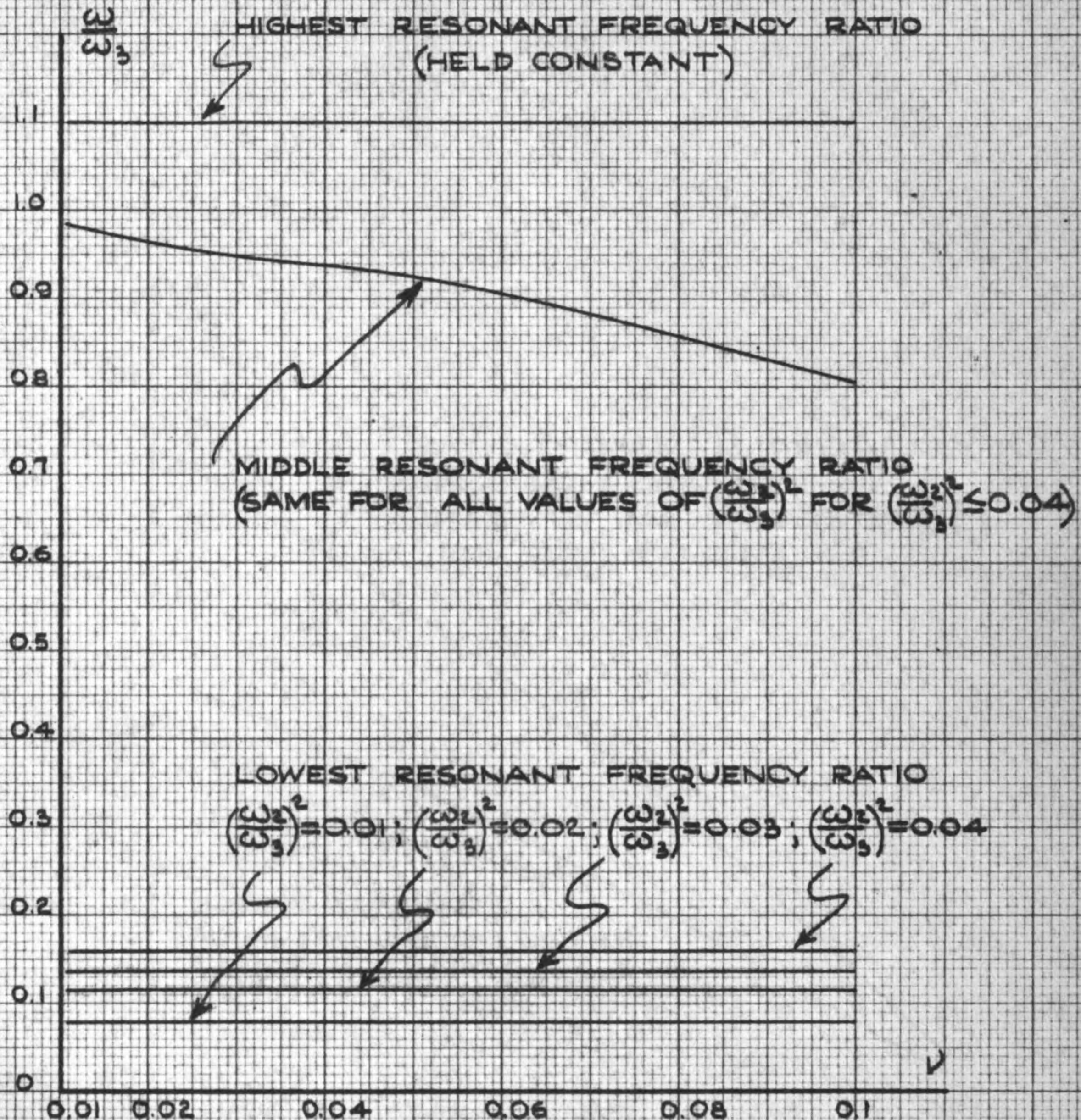


FIG. 2
VARIATION OF RESONANT FREQUENCY RATIOS WITH MASS RATIO ν.

MASS RATIO $\mu = 2$.

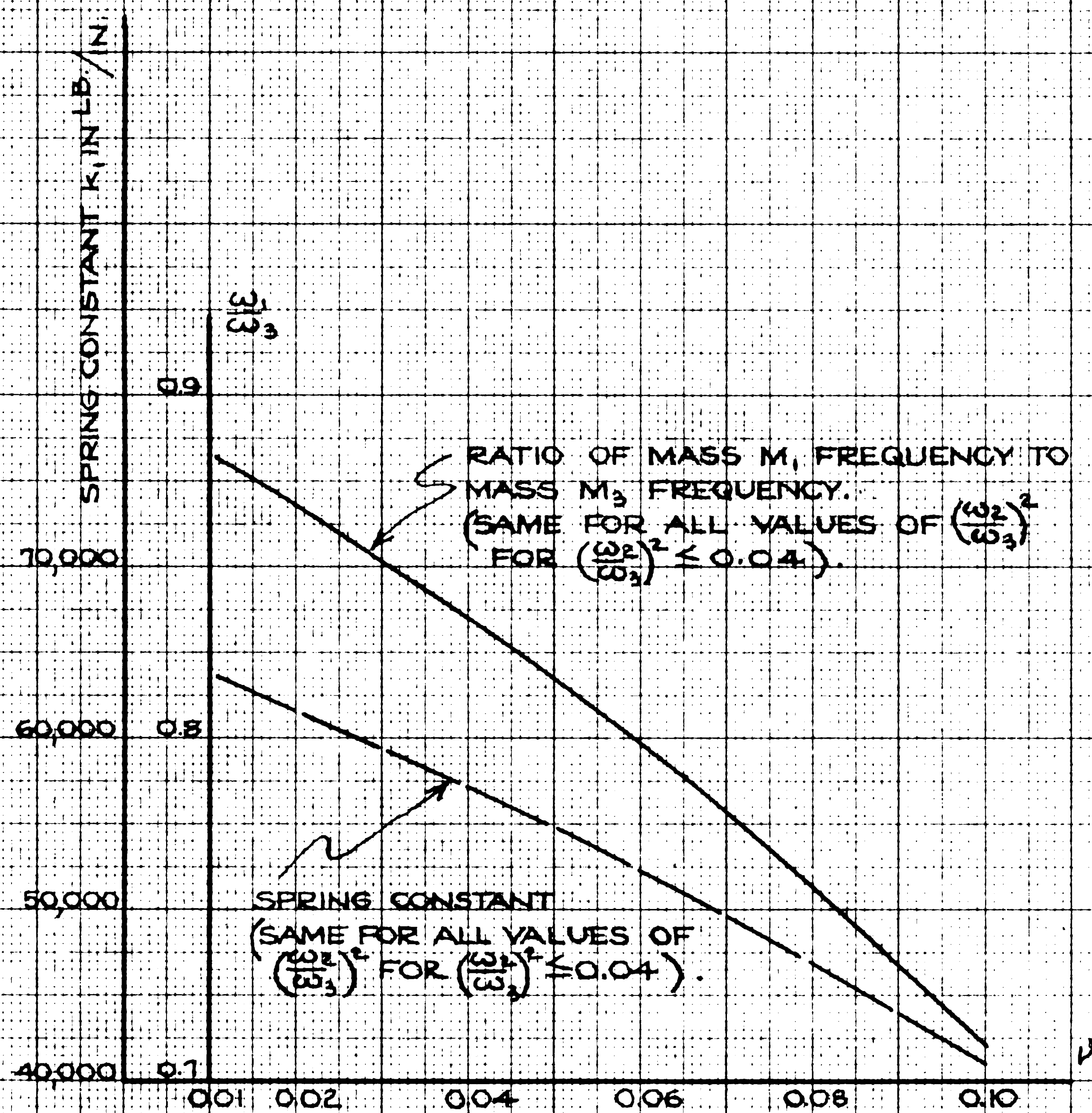


FIG. 3.

VARIATION OF FREQUENCY AND SPRING CHARACTERISTICS OF MASS M_1 WITH MASS RATIO ν.

HIGHEST RESONANT FREQUENCY RATIO HELD CONSTANT AT 1.10;
MASS RATIO $\mu = 2$;
NATURAL FREQUENCY OF MASS M_3 AND SPRING $(\omega_3) = 1200$ R.P.M.

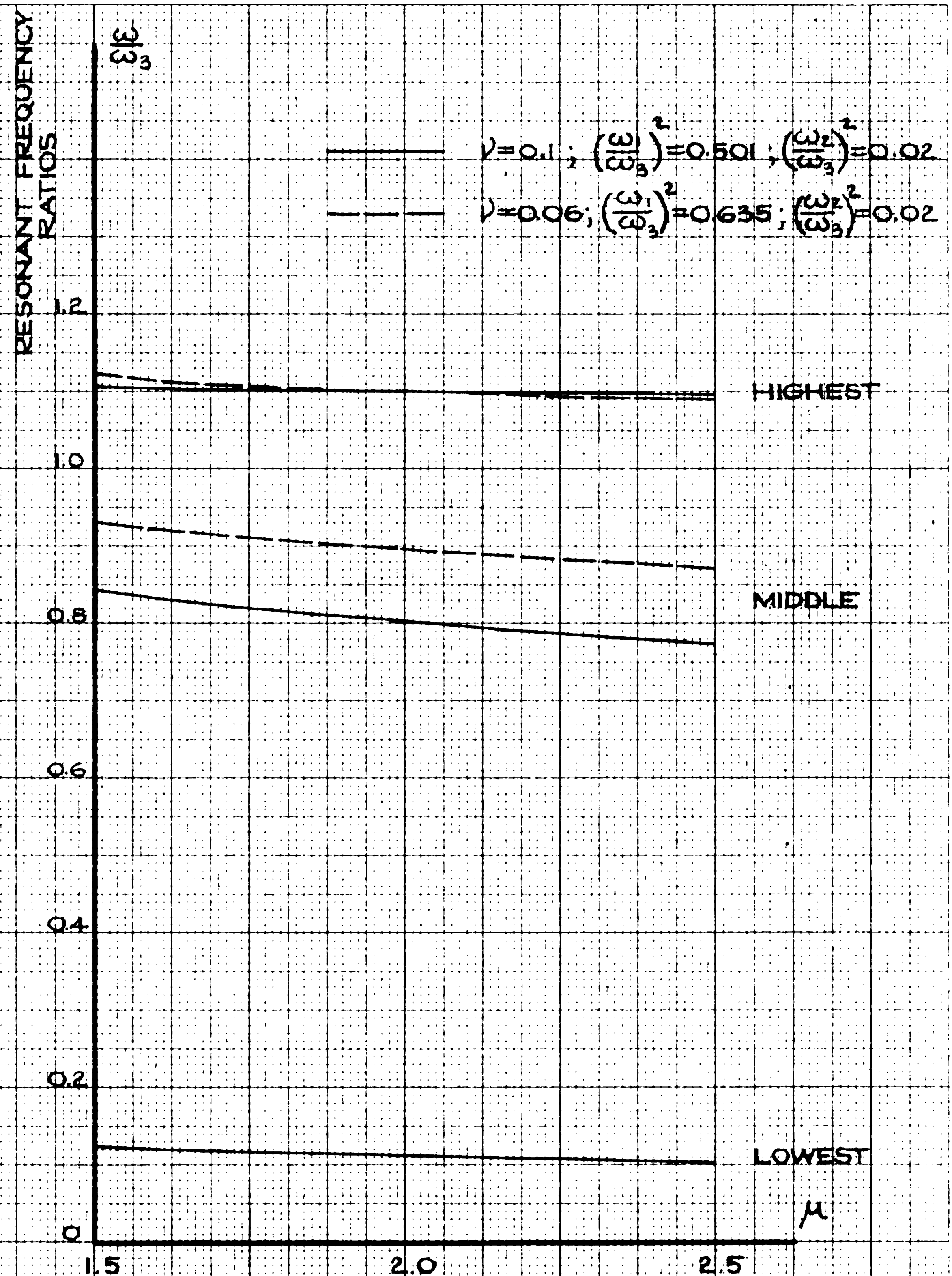


FIG. 4.
VARIATION OF RESONANT FREQUENCY RATIOS WITH MASS RATIO μ.

Conclusions

From this study of a vibrating three mass system it may be concluded:

1. A direct method of solution of the sixth degree algebraic resonance equation, where the solution is expressed in a trigonometric form, offers good possibilities for computing the resonant frequencies of any given vibrating system with three degrees of freedom.

2. It appears to be possible to have a vibrating system with three degrees of freedom that has only two distinct resonant frequencies; that is, the resonance equation may have repeated roots. However, it seems difficult to devise such a system, because of the extremely complicated relationships that the physical constants of the system have to satisfy.

3. An inverse method of solution gives simpler relationships between the resonant frequencies and the physical constants of the system than the trigonometric form. The expressions obtained by the inverse method are well suited for determining the effect that variations of the physical constants have upon the distribution of the resonant frequencies.

4. A notable simplification in the relationships between the resonant frequencies and the physical constants of the system is obtained by making the frequency of the driving force equal to the natural frequency of mass M_3 and spring k_3 alone.

BIBLIOGRAPHY

Conkwright, N. B. Introduction to the Theory of Equations. New York: Ginn and Company, 1941.

Thomson, W. T. Mechanical Vibrations. New York: Prentice-Hall, Incorporated, 1948.

Timoshenko, S. Vibration Problems in Engineering. New York: D. Van Nostrand Company, Incorporated, 1937.

Timoshenko, S., and D. H. Young. Advanced Dynamics. New York: Mc Graw-Hill Book Company, Incorporated, 1948.

ROOM USE ONLY