TRANSIENT BEHAVIOR OF AN ENGINE-TORQUE CONVERTER-LOAD SYSTEM

Thests for the Degree of M. S. MICHIGAN STATE UNIVERSITY John Allen McMennamy 1966 THESIS



ABSTRACT

TRANSIENT BEHAVIOR OF AN ENGINE-TORQUE CONVERTER - LOAD SYSTEM

by John Allen McMennamy

The problem of controlling a system is basically that of understanding what type of response can be expected from a given input to the system. With the underlying aim to design a controller for a particular system, this study attempts to characterize the performance of an internal combustion engine-hydrodynamic transmission-inertia load system.

To accomplish this a general system is analyzed mathematically and methods to find the appropriate constants and functions for a particular case are presented. The equations for this particular system were then programmed for an electrical analog computer to simulate the physical system. Tests were conducted to check the accuracy of the analog system, and a reasonable agreement was obtained.

The influence of engine inertia, load inertia, a constant load, gear ratio and torque converter size was studied by using only the analog computer. In order to present the results of this study in as useful a form as possible nondimensional terms were used.

Approved

TRANSIENT BEHAVIOR OF AN ENGINE-TORQUE

CONVERTER - LOAD SYSTEM

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John Allen McMennamy

A THESIS

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Chapter 1

INTRODUCTION

This project is a part of a group of projects that will eventually comprise a complete laboratory test track. The completed test track apparatus will consist of a car that rides on rails, a prime mover, and a control unit to govern the motion of the car.

The car might carry a soil bin and will be constructed such that the function of a tool or traction device can be studied as the car moves on the rails. The engine and transmission used in this project will eventually be used as the prime mover for the car.

The control unit of the test track must automatically control the initial acceleration of the car, and then control the speed of the car while tests are being made on the soil in the car. It should do this independent of loading conditions. A minimum stress level on the components of the test track and the soil in the soil bin is desired during acceleration; therefore, the optimum control unit should give a constant acceleration from the start to full speed. On the test length of track a constant velocity should be maintained.

A knowledge of the parameters affecting transient operation of the engine-transmission-load system is essential to designing a control unit to govern the motion of the car.

Little was reported in the literature examined about the transient performance of internal combustion engines, but much has been written about the analysis and performance of "systems". The effect of engine inertia seems to be recognized in literature and practice

but any other transient characteristics of the engine do not seem to have been discussed. No systematic investigation of the transient behavior of an engine-hydrodynamic transmission system was found in the literature.

This study employs the electrical analog computer to study performance parameters and predict performance, but according to C. F. Sandford, Advanced Design Engineer at Oldsmobile Division of General Motors Corporation, the digital computer can be used to predict the performance of an automobile with some accuracy.

Since vehicles in general-tractors, trucks, automobiles, etc. -can be characterized as an engine-transmission-load system, with considerable inertia involved, the knowledge of what parameters affect transient performance should be of some general interest.

STATEMENT OF THE PROBLEM

The general objectives were outlined in the introduction, but the specific objectives are to find what parameters govern the transient performance of an internal combustion engine-hydrodynamic transmission-load system and determine the extent to which each parameter influences performance under various loading conditions, with main consideration given to the inertias involved.

Some of the load conditions of special interest are:

- 1. The rapid acceleration with constant rate of acceleration of a large inertia load.
- 2. The instantaneous superposition of a relatively constant load on the load described above.

PROCEDURE

A mathematical analysis was made in order to determine what parameters might effect the engine-transmission-load system, and from the resulting set of equations an electrical analog was constructed on an analog computer.

The response of the electrical analog and of the physical transmission-load system were compared in order to determine if the parameters chosen for the mathematical model were sufficient to actually describe transient performance.

Once it was established that the mathematical model could accurately predict transient performance the investigation was continued to determine the extent to which each performance parameter influenced the system under various loading conditions.

Chapter 2

APPARATUS

The principal components used in this project were an engine, transmission, dynamometer and an automotive rear axle as a "physical system" and an electrical analog computer with an X-Y recorder. The instrumentation is described separately in the next section.

Figure 2-1 and 2-2 shows photographs and a schematic sketch of the engine and transmission coupled to the dynamometer. Figure 5-2 is a photograph of the analog computer used.

The following specification for the components are published by the manufacturer of each respective piece of equipment.

Engine:

Manufacturer	Oldsmobile Division of General
	Motors Corporation
Туре	F-85 Cutlass 90° V-Type
Number of Cylinders	8
Bore and Stroke	3.9385 in.X 3.3850 in.
Piston Displacement	330 cubic in.
Compression Ratio	10.25 to 1
Firing Order	1 -8 -4 -3 -6 -5 -7 -2
Brake Horse Power	320 at 5200 RPM
Maximum Torque	335 ft.lb. at 3600 RPM





Figure 2-1. Photograph and schematic diagram of the complete "physical" system.



Figure 2-2. Photographs of the engine - transmission unit and the automotive differential used for braking connected to the dynamometer.

Transmission:

Manufacturer	Oldsmobile Division of General
	Motors Corporation
Manufacturer Name	Jetaway
Туре	Torque converter plus 2-speed
	gear transmission
Torque Converter	Variable pitch stator type
Low stator angle	40 [°]
High stator angle	57 [°]
Gear Transmission	Planetary type
Low ratio	1.76
Drive ratio	1.00
Reverse ratio	1.76

Rear Axle:

Manufacturer	Oldsmobile Division of General
	Motors Corporation
Туре	1966 F-85 Cutlass
Gear Ratio	3.55 to 1
Brakes	Self-energizing drum brakes
Drum diameter	9.5 inches I.D.
Total lining area	62.5 sq. in.

Dynamometer:

Manufacturer	General Electric
Туре	Cradled D.C. Generator

Capacity	200 h.p.
Speed range	2500 -6 000 RPM
Moment of inertia	
of rotor	2.73 ft.lb. sec^2 .

Electrical Analog Computer:

Manufacturer	Electronic Associates Inc.
Model	TR-48
Туре	D.C. Analog
Number of Amplifiers	48
Output voltage range	
of amplifier	± 10 volts min.
Offset (at summing	
junction)	
As unity gain inverter with 10K resistor	\pm 30 μ volts max.
As unity gain inverter with 100K resistor	\pm 50 μ volts max.
X-Y Recorder:	
Manufacturer	Varian Associates
Model	F-80
Accuracy	
Static	0.2% of full scale
Dynamic	0.25% of full scale
Time base	3% all ranges (adjustable to 1% on any range)
Writing speed	17 inches/sec. each axis

The throttle position of the engine was controlled manually by a lever on the front of the engine stand. An attempt was made to arrange the throttle linkage so that relative motion between the engine and test stand would not affect the throttle setting.

The transmission gear ratio was controlled by a selection lever on the transmission, and the stator angle of the torque converter was controlled by a manual switch connected to a solenoid in the transmission.

Instrumentation

The following values were measured: Engine speed versus time Throttle position of the engine Speed out of transmission versus time Gear ratio and stator angle of the transmission Output torque

The following discussion describes the device used to measure each of the values under various conditions and what means of calibration was employed.

Engine Speed

Steady state speed of the engine was measured with a Strobotac. The Strobotac was calibrated at 900 and 3600 RPM internally with line frequency (60 cps). Intermediate values were checked with a counter and timer built into a Standard Electric Time Company electric tachometer unit on the dynamometer. Discrepancies were noted so that readings taken with the Strobotac could be corrected.

Transient engine speeds were measured with a Barber Colman Company D.C. generator with a 5 volt per 1000 RPM output at an infinite ohm load. The generator was connected to the vertical input of a Type 532 Dual Trace, Tektronix oscilloscope. The trace on the oscilloscope was photographed with a Polaroid Land Camera mounted on the oscilloscope. The vertical gain of the oscilloscope was calibrated with the Strobotac to give a vertical scale of 100 radians/ second/cm. Different horizontal sweep velocities were used to give the desired horizontal time scale.

Throttle Position of the Engine

Only fully open and fully closed throttle positions were used; therefore, the position could be judged by the operator of the throttle. Tests, which are mentioned in Chapter 4, indicated that engine speed change and throttle movement can be correlated.

Speed Out of the Transmission

Steady state speed out of the transmission is measured with the tachometer on the dynamometer. The dynamometer, which is directly connected to the transmission output shaft, has a Standard Electric Time Company tachometer which consists of an A.C. generator connected to a synchronous motor in the tachometer. The tachometer was checked against a counter and timer which is built into the tachometer unit.

Transient output speeds were measured with a Barber Colman D. C. generator, with a 24 volt per 1000 RPM output at an infinite load. The generator was connected to a vertical input of a Type 532 Dual Trace Tektronix oscilloscope. The same calibration procedure was used for the transmission output speed as was used for the engine speed measurement. Since the two channels of the oscilloscope are independently adjustable, simultanenous readings of engine and output speed can be made with identical time scales.

Gear Ratio and Stator Angle of the Transmission

Only the low gear ratio of the transmission was used in the tests; therefore, it was only necessary to place the gear selector in low gear to hold it there.

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Only the high angle of the variable pitch stator was used. To position the stator at the high angle the solenoid which controls the stator angle was energized.

Output Torque

Engine torque was calculated from torque measured at the output shaft of the transmission. The torque out of the transmission was measured with a General Electric 200 h. p. cradled electric dynamometer. The engine speed, output speed and output torque were measured simultaneously so that the engine torque could be calculated using the transmission characteristics curve, Figure 4-4. Friction loss in the gear transmission between the torque converter and the output shaft of the transmission unit was neglected. The rear axle assembly was not connected during this test.

Chapter 3

MATHEMATICAL ANALYSIS

Each major component of the system, i.e., engine, torque converter, gear transmission and load, is analyzed in this chapter in order that an electrical analog of the engine-load system can be constructed. An effort was made to consider the effects of surrounding components in the analysis of each component so that the resulting equations could be linked together to form a complete system.

The Engine

The engine, which is the prime mover, is ordinarily controlled by adjusting the throttle position; therefore, throttle position is referred to as "input" to the engine.

Under steady state operation of the engine the torque developed by the engine is usually expressed as a function of engine speed and throttle opening. Figure 3-1 is a typical example of a family of torque curves for various throttle positions.

External effects, such as, atmospheric conditions, lubricant temperature, coolant temperature and history of operation, may affect the output, i.e., torque at some output speed; however, since the time of operation of the engine is for each run expected to be relative short, less than 10 seconds, these effects are considered to be constant for this case.

The steady state torque curve of the engine can then be expressed mathematically as:

$$T_{E} = f_{1}(x, \omega_{E})$$
(3-1)

 T_E = output or torque developed by the engine f_1 = steady state engine torque function x = input to engine (throttle position) ω_E = speed of engine

The function, f_l , relating the steady state torque to input and engine speed depends on the design of the engine. It is determined empirically for this study.

When the throttle is fully closed the engine will run at idle speed if $T_E = 0$. At speeds greater than idle speed the "drag" on the engine gives negative values for T_E . The drag in the engine is normally associated with friction and pumping losses in the engine and power requires of the engine accessories.

Since the drag in the engine is also some function of engine speed, equation (3-1) is adequate to express the output torque for any throttle position. Figure 3-2 shows typical relationships between friction torque and engine speed.

Whenever engine speed changes the inertia of the engine affects equation (3-1). From kinetics it is known that the acceleration of a rotating mass is proportional to the torque applied to it. For transient operation the engine torque function should then include acceleration values. If this is done the torque of the engine can be expressed:



Figure 3-1. Family of torque curves for various throttle positions. Persson (1965).



Figure 3-2. Friction torque versus engine speed for two engines. Barger (1963).

$$\Gamma_{E} = f_{2}(x, \dot{x}, \omega_{E}, \dot{\omega}_{E}) \qquad (3-2)$$

 $\dot{\mathbf{x}}$ = time rate of change of input to engine

f₂ = transient engine torque function

 $\dot{\omega}_{\rm F}$ = angular acceleration of engine

Except for the influence of engine inertia on f_2 , little is reported in the literature about the nature of f_2 .

The Torque Converter

The transmission, previously described as a torque converter with a gear transmission attached, is treated as two separate units for the purpose of the analysis.

In considering the hydrodynamic torque converter it should be pointed out that there must be relative motion between the input member, called the impeller, and the output member, called the turbine or runner, for torque to be transmitted by the torque converter. The stator or reactor which distinguishes a torque converter from a fluid coupling is usually stationary as its name implies.

According to Heldt (1955) the general properties of a hydrodynamic torque converter can be expressed in terms of this relative motion between members. The three characteristic terms he discusses are the torque ratio, speed ratio and power efficiency of the torque converter. The torque ratio is the ratio of output torque to the input torque applied to the impeller; the speed ratio is the ratio of runner speed to impeller speed. Power efficiency can be expressed as the product of torque ratio and speed ratio and is therefore not an independent variable. Power ratio is by definition the ratio of output power to input power.

The performance properties of the torque converter which are described by these three characteristic terms have a fixed relationship determined by the design of the torque converter. The relationships are determined empirically. The relationship is often presented graphically as in Figure 4-4, but for the purpose of this mathematical analysis it is expressed:

$$T_{R} = f_{3}(\omega_{R})$$
(3-3)

 T_R = torque ratio of torque converter f_3 = torque ratio-speed ratio function ω_R = speed ratio of torque converter Torque ratio and speed ratio are defined:

$$T_{R} = \frac{T_{o}}{T_{i}}$$
(3-4)

T_o = torque out of torque converter T_i = torque into torque converter

$$\omega_{\rm R} = \frac{\omega_{\rm o}}{\omega_{\rm i}} \tag{3-5}$$

 ω_0 = speed out of torque converter ω_i = speed into torque converter

The relationship of efficiency to speed ratio and torque ratio does not contribute to the solution of the set of equations comprising the mathematical model; therefore, it is mentioned here only because it is often used and furnishes easily interpreted information about the system.

The torque and speed into the impeller must be the same as the torque and speed out of the engine since the engine crank shaft is connected directly to the impeller of the torque converter. This imposes a condition on equation (3-2) permitting only certain combinations of $T_{\rm F}$ and $\omega_{\rm F}$.

Heldt (1955) proposes that there exist a fundamental relationship between torque, speed and linear dimensions of the torque converter unit. He expressed this relationship:

$$T_i = C n^2 D^5$$
 (3-6)

- $T_i = torque at impeller$
- n = speed of the input shaft
- D = any characteristic linear dimension such as the maximum diameter of the impeller

Slip can be defined:

$$slip = 1 - \omega_R \tag{3-7}$$

In the consideration of a given transmission D is constant; therefore, C will be a function of only slip, and equation (3-6) can be written:

$$\Gamma_{\rm E} = \omega_{\rm E}^2 [f_4 (1 - \omega_{\rm R})]$$
 (3-8)

 f_4 = converter torque function

Figure 4-5 illustrates the relationship involved in equation (3-8) for the torque converter being studied. The torque converter as a unit is subject to the effects of inertia and drag on a rotating mass; however, these effects will be associated with the adjacent units, namely the engine and gear transmission. How and to what extent the association is made is discussed in Chapter 4.

The Gear Transmission

The output shaft of the torque converter is directly connected to the input shaft of the gear transmission; therefore, ω_0 and T_0 are the input values for the gear transmission.

If power loss in the gear transmission is not considered the power relationship for this element is:

$$T_{o}\omega_{o} = T_{L}\omega_{L} \tag{3-9}$$

 T_L = torque out of transmission applied to load ω_L = speed of output shaft of gear transmission

Since the speed relationships of the gear transmission remain constant for a given gear arrangement, the "ratio" of the transmission is by convention:

$$R_{T} = \frac{\omega_{o}}{\omega_{L}}$$
(3-10)

 R_{T} = ratio of gear transmission

It follows from equations (3-9) and (3-10) that:

$$T_{L} = R_{T} T_{o}$$
(3-11)

If the inertia of the transmission is considered equation (3-11) becomes:

$$T_{L} = R_{T} T_{o} - I_{T} \dot{\omega}_{L} \qquad (3-12)$$

 I_T = moment of inertia of rotating transmission parts $\dot{\omega}_L$ = angular acceleration of output shaft

It should be noted that I_T must be expressed relative to the output shaft, and I_T may change when R_T is changed due to the difference in gear arrangements.

The Load

The load can be characterized in general terms by the differential equation:

$$T_{L} = I_{L}\dot{\omega}_{L} + f_{5}(\omega_{L}) + T(t)$$
 (3-13)

 I_L = moment of inertia of the load $\dot{\omega}_L$ = angular acceleration of the load f_5 = load-speed function T(t) = time dependent load

By changing I_L , f_5 , and T(t) equation (3-12) can be altered to simulate many loading conditions.

The Mathematical Model

A set of <u>n</u> independent equations containing <u>n</u> unknown is needed to form a workable mathematical model. The value of one unknown variable, the independent variable, is usually established so that <u>n</u>-1 independent equations are needed to solve for the remaining <u>n</u>-1 unknowns, dependent variables.

Listing the known and unknown terms in the previously developed equations:

Unknown Variables

f₁ = steady state engine = throttle position х T_{F} = torque out of engine torque function $f_2 = transient engine$ T_o = torque out of torque torque function converter = T_R to ω_R function $T_{I} = load torque$ f3 T_{R} = torque ratio of converter f_{4} = converter torque $\omega_{\rm F}$ = speed of engine function ω_{o} = speed out of converter $f_5 = load-speed$ function $\omega_{\rm L}$ = speed of load I_T = moment of inertia of $\omega_{\rm R}$ = speed ratio of converter the gear transmission $\dot{\mathbf{x}} = d\mathbf{x}/dt$ = moment of inertia of I. $\dot{\omega}_{\rm E} = d\omega_{\rm F}/dt$ the load $\dot{\omega}_{0} = d\omega_{0}/dt$ R_{T} = speed ratio of the gear $\dot{\omega}_{\rm T} = d\omega_{\rm T}/dt$ transmission T(t) = time dependent load I_F = moment of inertia of the engine

Knowns

The definitions of the last four unknown variables are actually equations; therefore, since these four variables contribute four equations they need not be considered independent variables. There are, therefore, nine unknown variables. Some of the functions and constants listed as "known" have to be determined in preliminary tests. These preliminary tests are described in Chapter 4. The insertion of these evaluated constants and functions in earlier stated equations gives the following set of equations:

$$T_{E} = \begin{cases} -0.32 \omega_{E} , \text{ for } x = 0 \\ 240 - 0.35 \dot{\omega}_{E}, \text{ for } x = 1 \end{cases}$$

$$T_{R} = \begin{cases} 2.02 - 1.165 \omega_{R}, \text{ for } \omega_{R} < 0.875 \\ 1.0, \text{ for } \omega_{R} \ge 0.875 \end{cases}$$

$$(4-6)$$

$$T_{R} = \frac{T_{E}}{T_{E}}$$
(3-4)

$$\omega_{\rm R} = \frac{\omega_{\rm o}}{\omega_{\rm E}} \tag{3-5}$$

$$\omega_{0} = \begin{cases} \omega_{L}, & \text{for } R_{T} = 1.00 \\ 1.76 \omega_{L}, & \text{for } R_{T} = 1.76 \end{cases}$$
(4-10)

$$T_{L} = 2.81 \dot{\omega}_{L} + T(t)$$
 (4-11)

$$T_{L} = R_{T} T_{o} - 0.06 \dot{\omega}_{L}$$
 (4-12)

$$\frac{T_E}{\omega_E^2} = 3.04 \times 10^{-3} (1 - \omega_R)^{1/4}$$
 (4-9)

with the dimensions of foot pounds for torque, radians/second for angular speed, radians/second² for angular acceleration, and foot pound second² for inertia.

There are, therefore, eight equations for the mathematical model. As was previously stated it is only necessary to establish the value of the independent variable and then solve the $\underline{n} - 1 = 8$ equations for the remaining $\underline{n} - 1 = 8$ dependent variables.

Chapter 4

EVALUATION OF CONSTANTS AND EMPIRICAL FUNCTIONS

The evaluation of constants and empirical functions for the equations developed in Chapter 3 is important for the outcome of this study.

The constants and functions are developed in the same order that they were introduced in Chapter 3.

Steady State Engine Torque Function

The steady state torque function, f_1 , relates throttle position, x, and engine speed, ω_E , to the output torque of the engine at steady state conditions. The notation relating x to throttle position is arbitrarily given as: x = 1.0 for the throttle fully opened, and x = 0for the throttle fully closed.

When equation (3-1) is evaluated $f_1(1, \omega_E)$ is found to be fairly constant at 240 ft. lb. in the speed range of expected operation. The evaluation of $f_1(0, \omega_E)$ is discussed under the evaluation of f_2 .

Transient Engine Torque Function

The transient engine forque function, f_2 , was presented as being dependent on x, \dot{x} , ω_E and $\dot{\omega}_E$; however, it will be assumed that the effect of throttle acceleration, \dot{x} , on f_2 can be neglected. This assumption is based on the results of a test in which the engine was given an input of x = 1 from idle speed (400 R. P. M.). The time from the initiation of the input to the time the engine responded was measured from Figure 4-1 as 0.14 second. A lag in response



HORIZONTAL SCALE: 0.10 SECOND / GRID LINE VERTICAL SCALE: APPROX. 100 RAD. / SEC. / GRID LINE



Figure 4-1. Measurement of the lag in response of the engine at idle speed. The discontinuity in the straight line indicates the time at which throttle movement was initiated.

of this order is not critical in the intended use of the engine, and this lag is expected to decrease with increased engine speed due to increased manifold velocities. The test also indicated an almost constant acceleration corresponding to almost full torque from the moment the engine responded. There seemed to be no gradual increase in torque developed by the engine.

It was mentioned in Chapter 3 that engine inertia affects the torque of the engine under changing speed conditions. It is a basic assumption of this study that the only difference in the torque of the engine under steady state and transient conditions is that torque associated with engine inertia. That is to say, when the engine is not accelerating, $f_1 = f_2$, and when the engine is accelerating:

$$T_{E} = f_{2}(x, \omega_{E}, \dot{\omega}_{E}) = f_{1}(x, \omega_{E}) - I_{E} \dot{\omega}_{E}$$

$$(4-1)$$

 I_{F} = moment of inertia of engine

 I_E is considered to include the moment of inertia of transmission and torque converter parts which rotate when the transmission is in "neutral" or "park".

If x = 1 and the transmission is in "neutral" so that the drag of the torque converter is included but there is no external load equation (4-1) becomes:

$$0 = f_{1}(1, \omega_{E}) - I \dot{\omega}_{E}$$
 (4-2)

Since $f_1(1, \omega_E) = 240$ ft. lb. from the constant speed test, it follows that $I_E \dot{\omega}_E = 240$ ft. lb.


PHOTOGRAPH



HORIZONTAL SCALE : 0.5 SECOND / GRID LINE VERTICAL SCALE : 100 RAD. / SEC. / GRID LINE



Figure 4-2. The average values of photographs 1 and 2 are plotted above. Curve a - b is for x = 1 and no external load applied. Curve b - c is for x = 0 and no external load applied.

Figure 4-2 shows plots of engine speed against time under no output load conditions. At point <u>a</u> on Figure 4-2 the engine first responds to the fully opened throttle and the throttle is left fully open almost to point <u>b</u> where it is closed. The slope of curve <u>a-b</u> is $\dot{\omega}_E$ under full torque and is relatively linear at 680 rad./sec. The evaluation of I_E in equation (4-2) gives a value of 0.35 ft. lb. sec., which is a reasonable value for this size engine. A 1961 Oldsmobile V-8, which has a displacement of 394 cubic inches has a moment of inertia of 0.2937 for its crankshaft, flywheel, damper, damper pulley and driving torus of its fluid coupling. If the inertia of the connecting rods, accessories and additional transmission parts were added, the value would be a little higher.

Now that a value for engine inertia has been established the value of $f_1(0, \omega_E)$ can be evaluated. If the transmission is in "neutral" and x=0, i.e., the throttle is closed, equation (4-1) becomes:

$$0 = f_1 (0, \omega_F) - 0.35 \dot{\omega}_F$$
 (4-3)

 $\dot{\omega}_{\rm F}$ = radian/sec.

The curve <u>b-c</u> in Figure 4-2 shows a plot of engine speed against time under no load and x = 0 conditions; therefore, the curve <u>b-c</u> has the equation (4-3). Evaluation of the slope, $\dot{\omega}_E$, at different engine speeds gives the points plotted in Figure 4-3.

The dotted line in Figure 4-3 is for friction torque as determined by the manufacturer's motoring test. The motoring test is made on an engine with no accessories, and the ignition is off. According to Gish, McCullough, Retzloff and Mueller (1957) the friction torque obtained



Figure 4-3. Engine drag and friction torque versus engine speed.



Figure 4-4. Torque converter characteristics at the high angle

stator setting.

from the motoring test and a firing test, i.e., a test with the engine running, are different; therefore, it is hoped that the method used here gives more meaningful results.

The equation of the straight line drawn through the points in Figure 4-3 is:

$$T_E = f_1(0, \omega_E) = -0.32 \omega_E$$
 (4-4)

This corresponds to a mean pressure of 16 psi per 100 rad./sec.

With the functions previously developed, equation (3-2) can be expressed:

$$T_{E} = \begin{cases} 0.32 \,\omega_{E}, & \text{for } x = 0 \\ 240 - 0.35 \,\dot{\omega}_{E}, & \text{for } x = 1 \end{cases}$$
(4-5)

Torque Ratio to Speed Ratio Function

To evaluate f_3 in equation (3-4) T_R is plotted against ω_R as in Figure 4-4. The plot of these ratios was obtained from dynamometer data supplied by the transmission manufacturer. The dynamometer data consisted of plots of input speed, speed ratio, torque ratio, and efficiency against output speed. The input torque was a constant 280 ft. lb. throughout the test, and the stator was set in the high-angle positions throughout the test.

As can be seen from Figure 4-4 the torque ratio decreases at almost a linear rate from 2.02 down to 1.00 and then remains at 1.00 for speed ratios beyond 0.875. The equation of the straight line in Figure 4-4 is:

$$T_{R} = \begin{cases} 2.02 - 1.165 \,\omega_{R}, & \text{for } \omega_{R} \leq 0.875 \\ 1.0, & \text{for } \omega_{R} \geq 0.875 \end{cases}$$
(4-6)

Converter Torque Function

If equation (3-8) is rewritten:

$$\frac{T_E}{\omega_E^2} = f_4(1 - \omega_R)$$
(4-7)

it is seen that f_4 relates the slip of the torque converter, $1 - \omega_R$, to the ratio T_E / ω_E^2 . These terms are plotted in Figure 4-5 from the dynamometer data described in the preceeding section. The equation for the line in Figure 4-5 is:

$$\frac{T_E}{RPM_E^2} = 3.34 \times 10^{-5} (1 - \omega_R)^{1/4}$$
 (4-8)

If RPM_E is converted to radians per second to be compatible with the units in the previous equations, equation (4-8) becomes:

$$\frac{T_E}{\omega_E^2} = 3.04 \times 10^{-3} (1 - \omega_R)^{1/4}$$
 (4-9)

Speed Ratio of Gear Transmission

The gear ratio of the transmission, R_T , is given by the manufacturer as:



$$R_T = 1.00$$
, for top gear
 $R_T = 1.76$, for low gear
 $R_T = 1.76$, for reverse gear

Reverse will not be used; therefore, equation (3-10) can be written:

$$\omega_{0} = \begin{cases} \omega_{L}, & \text{for } R_{T} = 1.00 \\ 1.76 \omega_{L}, & \text{for } R_{T} = 1.76 \end{cases}$$
(4-10)

Moment of Inertia of the Transmission

According to Kosier and McConnell (1957) the polar-moment of inertia of automobile engine-transmission combinations fall within the range of 0.40 to 0.50 ft. lb. sec. This would imply that a common value of I_T would be in the range of 0.15 to 0.05 ft. lb. sec.², since I_E has been given the value of 0.35 ft. lb. sec.².

The manufacturer of the 1959 Oldsmobile Hydromatic transmission gives the following values for its moment of inertia at the drive shaft:

These values include the inertia of the torus cover and the final drive unit. The Jet-a-way used is expected to have inertial values similar to the Hydromatic due to construction similarities. The similarities are more pronounced in the first and third speeds of the Hydromatic than in the other two speeds. Since many of the transmission parts have been included in the value of engine inertia, the value of 0.06 ft. lb. sec.² is chosen for I_T . This value falls in the range suggested previously.

Moment of Inertia of the Load

The inertial load on the prime mover for the laboratory test track outlined in the introduction is estimated at 1.00 ft. lb. sec.² on the basis of a mass of 5000 lbs. and a speed ratio of the drive system of 14 rad./sec. = 1 ft./sec. The armature of the electric dynamometer used in this study has a moment of inertia of 2.73 ft. lb. sec.², but because of the additional inertia of drive shafts, couplings and the differential used for braking, the inertia is estimated to be 2.75 ft. lb. sec.². By keeping the gear transmission in low gear, $R_T = 1.76$, the effective inertia as seen at the output shaft from the converter can be lowered to 0.89 ft. lb. sec.².

Therefore, the armature of the electric dynamometer described in Chapter 2 and its attached parts corresponds sufficiently well to the intended use and can be used for an inertial load, with the transmission in low gear.

Load Speed Function

The load speed function, f_5 , would normally be associated with some frictional drag for this system. Since the inertia of the load is relatively large, it is assumed that the torque $f_5(\omega_L)$ can be neglected in this study.

Time Dependent Load

The time dependent part of T_L , T(t), can have any value from 0 to 100 ft. lb. and it may be superimposed upon the inertial load at any rate and time. Since T(t) is so independent, it will be left in the form T(t) except when particular cases are referred to. (See Chapter 6).

If I_T is added to I_L , equation (3-12) and (3-13) can be combined to give:

$$T_{I} = 2.81 \dot{\omega}_{I} + T(t)$$
 (4-11)

or

$$T_{L} = \dot{\omega}_{0} \quad \frac{2.81}{R_{T}^{2}} + \frac{T(t)}{R_{T}}$$

Equation (4-11) imposes the condition that:

$$T_{L} = R_{T} T_{0} - 0.06 \dot{\omega}_{L}$$
 (4-12)

Chapter 5

THE ELECTRICAL ANALOG

The electrical analog of the engine-transmission-load system is the equations developed in Chapters 3 and 4 programmed on the EAI TR-48 analog computer.

Some of the equations listed at the end of Chapter 3 were rewritten for easier use on the analog computer. The equations used became:

$$T_E = 240 - 0.35 \dot{\omega}_E$$
 (4-5)

$$T_{R} = \begin{cases} 2.02 - 1.165 \,\omega_{R}, & \text{for } \omega_{R} \leq 0.875 \\ 1.00, & \text{for } \omega_{R} \geq 0.875 \end{cases}$$
(4-6)

$$T_{o} = T_{R} T_{E}$$
 (3-4)

$$\omega_{\rm R} = \frac{\omega_{\rm o}}{\omega_{\rm E}} \tag{3-5}$$

$$\omega_{\rm E} = \left(\frac{{\rm T}_{\rm E}}{3.04 \times 10^{-3} (1 - \omega_{\rm R})^{1/4}}\right)^{1/2}$$
(4-9)

$$\omega_{\rm L} = \frac{\omega_{\rm o}}{R_{\rm T}} \tag{4-10}$$

$$\dot{\omega}_{0} = \frac{R_{T}^{2}}{I_{L} + I_{T}} (T_{0} - \frac{T(t)}{R_{T}})$$
 (4-11)

There are only seven equations listed because load torque, T_L , has been eliminated. Additional equations needed for programming were:

$$\dot{\omega}_{\rm E} = \frac{\mathrm{d}\omega_{\rm E}}{\mathrm{d}t} \tag{5-1}$$

$$\omega_{o} = \omega_{oi} + \int \dot{\omega}_{o} dt \qquad (5-2)$$

 ω_{oi} = initial output speed.

$$a_{\rm L} = \int \omega_{\rm L} \, dt \tag{5-2}$$

 $a_{L} = distance in radians.$

These "physical" equations were adapted for use on the computer, i.e., they were transformed into "scaled" equations, by the method suggested by Ashley (1963). One second on the computer was scaled to equal 10 second in real time in order to obtain accurate plots with the Varian X-Y recorder. The scaled equations are listed below with the components used to form the equation noted at the end of each equation. The bar over the variables name indicates that it is a scaled value. Potentiometers are prefixed by "p", and amplifiers are prefixed by "a".

$$\overline{T}_{E} = 7.20 - 0.522 \,\overline{\omega}_{E}$$
; a 46, p 16, p 40 (4-5)

$$\overline{T}_{R} = 10.10 - 0.583 \,\overline{\omega}_{R}$$
; a 32, a 44, p 35, p 36
p 37 for $\overline{T}_{R} \ge 5.00$ (4-6)

$$\overline{T}_{0} = \frac{\overline{T}_{R} \overline{T}_{E}}{10.00}$$
; a 23, a 41 (3-4)

$$\overline{\omega}_{\rm R} = 10.00 \frac{\overline{\omega}}{2} / \overline{\omega}_{\rm E}$$
; all (3-5)

$$\overline{\omega}_{E} = \left(\frac{0.128 \overline{T}_{E}}{(10 - \overline{\omega}_{R})^{1/4}}\right)^{1/2}; a 06, a 07, a 17, a 18,a 19, a 20, a 42, a 47,a 47, p 05, p 20, p 23,p 41 (4-9)$$

$$\overline{\omega}_{L} = \frac{\overline{\omega}_{O}}{R_{T}} ; a 00, a 01, p 01 \qquad (4-10)$$

$$\overline{\omega}_{0} = 1.33 \frac{R_{T}^{2}}{I_{L} + I_{T}} (\overline{T}_{0} - \frac{\overline{T(t)}}{R_{T}}); a 40, p 28, p 30$$
(4-11)

$$\overline{\dot{\omega}}_{\rm E} = 10.00 \frac{d\omega_{\rm E}}{dt} = \frac{d\overline{\omega}_{\rm E}}{d\tau}$$
; a 03, a 08, a 10 (5-1)

 τ = scaled time.

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$$\overline{\omega}_{o} = \omega_{oi} + \int \overline{\dot{\omega}}_{o} d\tau$$
; a 22, p 28, p 27 (5-2)

$$\overline{a}_{L} = \int \overline{\omega}_{L} d\tau \qquad ; a 02, a 14, p 00 \qquad (5-3)$$

Figure 5-1 is a schematic diagram of the analog computer program of the above equations.

The following list shows where each variable is available in the circuit and the scale factor used to convert the voltage to the appropriate units:



Variable	Where Available	Voltage Reading
${}^{\mathrm{T}}{}_{\mathrm{E}}$	a 46	0.030 T _E
Т _о	a 40	0.015 T _o
T _R	a 44	5.00 T _R
$^{\omega}$ E	a 21	0.020 ω _E
ω	a 22	0.020 ω _ο
$^{\omega}$ L	a 01	0.020 ω _L
ω _R	a ll	10.00 ω _R
ώ _E	a 03	0.002
ů	p 28	0.02 ů ₀
^a L	a 14	0.02 a _L

Figure 5-2 is a photograph of the analog computer programmed with the above equations.



The analog computer and the X-Y recorder used in this study. Figure 5-2. Comparison of the Real and Analog Systems

Two tests were conducted to show that the mathematical model could predict the transient performance of the engine-transmissionload system. Both tests consisted of a comparison between speed versus time plots obtained from the analog computer and the real system.

Photograph 2 in Figure 5-3 shows the speed versus time plot obtained from the real system when the output was braked and then released at time, t = 0. In this test the throttle was fully opened before the test started. The upper trace is engine speed and the lower trace is speed out of the transmission. Recall that in Chapter 2 it was stated that only low gear, $R_T = 1.76$, was used in the tests.

Photograph 1 in Figure 5-3 shows the speed versus time plot obtained from the real system when the output was almost stopped when the throttle position was changed from almost closed to fully open. The brakes were released 0.2 seconds when the engine responded. This gave an initial output speed of about 20 rad./sec. when the throttle was opened. As in photograph 2 the upper trace is engine speed and the lower trace is output speed.

The speed versus time curves plotted below the photographs were ploted by the x-y recorder connected to the analog computer. As can be seen the general shapes of the curves obtained from the real and analog system are similar, and in fact the absolute values are relatively close.



Photograph 1

Photograph 2



Figure 5-3. Comparison of the physical and the analog system. See page 43 for explanation.

On the basis of the results of these two tests it will be assumed that the analog computer program is accurate enough to proceed to investigate the system using only the analog computer, i.e., any following results have not been checked for accuracy with the real system.

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Chapter 6

STUDY OF SYSTEM RESPONSE USING THE ELECTRICAL ANALOG

The results of the study of the parameters governing a particular system, such as the laboratory test track described in the introduction are of value only for this case with the indicated size of components; however, by using dimensionless parameters where possible and by expanding the set of values on the analog computer it is possible to obtain results which can be applied to many different systems.

The dimensionless parameters that were developed are based on the assumption that the factors that are most significant in an engine-transmission-load system are the maximum torque that the engine can develop and the load inertia. These quantities have therefore been selected as the standard values with which the other quantities in the system are compared when formulating the general nondimensional parameters. In many cases the load of a system is given and the engineer is given the task of selecting a torque converter and a gear ratio to most effectively move the load.

To generalize the problem, the previously developed equations are rewritten:

$$T_{F} = f_{1}(1) - I_{F}\dot{\omega}_{F} \qquad (6-1)$$

$$T_{R} = T_{RST} - K_{1} \omega_{R}$$
 (6-2)

 T_{RST} = stall torque ratio of the torque converter K_1 = constant

 T_{RST} and K_1 are dependent on the design of the torque converter, but not on its size.

$$T_{R} = \frac{T_{o}}{T_{E}}$$
(6-3)

$$\omega_{\rm R} = \frac{\omega_{\rm o}}{\omega_{\rm E}} \tag{6-4}$$

$$\omega_{\rm E}^2 = \frac{T_{\rm E}}{K_2 (1 - \omega_{\rm R})^n}$$
(6-5)

n = exponent dependent on the design of the torque converter

K₂ = constant dependent on the size of the torque converter.

$$\dot{\omega}_{0} = \frac{R_{T}^{2}}{I_{L} + I_{T}} (T_{0} - \frac{T(t)}{R_{T}})$$
 (6-6)

 T_{RST} , K_1 and n will be assigned the same values that were found for the physical system used in this project. They are also appropriate for similar devices of different size.

Since the constant maximum engine torque, $f_1(1)$, is considered to be most important the other variables were expressed relative to $f_1(1)$ as follows:

$$\tau_{\rm E} = \frac{{\rm T}_{\rm E}}{{\rm f}_1(1)} \tag{6-7}$$

$$\tau_{0} = \frac{T_{0}}{f_{1}(1)}$$
 (6-8)

$$\tau' = \frac{T(t)}{f_1(1)}$$
(6-9)

$$i_{\rm E}^2 = \frac{I_{\rm E}}{f_1(1)}$$
 (6-10)

$$i_{\rm L}^2 = \frac{I_{\rm L} + I_{\rm T}}{f_1(1)}$$
 (6-11)

$$\kappa^{2} = \frac{K_{2}}{f_{1}(1)}$$
(6-12)

The first three variables are nondimensional and the last three variables have the dimension time. In a simple system governed by the relation $T = I\dot{\omega}$ it is observed that the time for a given rotation is proportional to $\sqrt{I/T}$ or i; therefore, a nondimensional time is chosen as:

$$\theta = \frac{t}{i_L} \tag{6-13}$$

For the simple system, governed by $T = I\dot{\omega}$, the plots of some distance a versus θ would be identical for various values of T and I as long as i is constant; therefore, plots against θ will accent relative values rather than magnitudes.

Inserting the new variables defined in equations (6-7) through (6-13) in the first set of general equations gives:

$$1 - \tau_{E} = \left(\frac{i_{E}}{i_{2}}\right)^{2} - \frac{d^{2} \alpha_{E}}{d\theta^{2}}$$
(6-14)

 a_E = rotation of the engine crankshaft in radians.

$$\tau_{o} = \tau_{E} (T_{RST} - K_{1} \omega_{R})$$
(6-15)

$$\tau_{\rm E} = \left(\frac{\kappa}{{\rm i}_{\rm L}}\right)^2 \left(\frac{{\rm d}_{\rm E}}{{\rm d}_{\rm \theta}}\right)^2 \left(1 - \omega_{\rm R}\right)^{\rm n} \tag{6-16}$$

$$\frac{d^2 a_o}{d\theta^2} = R_T^2 \tau_o - R_T \tau'$$
(6-17)

 a_{o} = rotation of the output shaft of the torque converter

This set of nondimensional equations is used to evaluate the significance of the system parameters.

The parameters chosen for consideration are the relative engine inertia i_E , the relative load inertia i_L , the torque converter size parameter κ , the superimposed load factor $\dot{\tau}$, and the gear ratio R_T . Table 6-1 indicates the values of the variables used in the analog computer solution of equations (6-14) through (6-17). The values in column 2 were maintained for all the variables except the variable being studied, i.e., the variable being studied was assigned the three values listed for it in Table 6-1, but all other variables remained at their value assigned in column 2. All tests were started from a stalled condition.

Real System Magnitude		
3		
240		
0.70		
3.00		
0^{-3} 7.78 x 10 ⁻³		
390		
0.50		

Table 6-1

The real system variable magnitudes were transformed into the nondimensional system by equations (6-7) to (6-13) to give the following table of magnitudes.

Ta	Ъl	е	6	-2
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Variable	Dimension	n Nondimensional System Magnitudes			
		1	2	3	
Engine torque, f ₁ (1)	1	1	1.	1.	
Engine inertia, i _E	sec.	0	0.038	0.054	
Load inertia, i _L	sec.	0.03.7	0.065	0.112	
Torque converter size coefficient, ĸ	sec.	3.15×10^{-3}	3.55×10^{-3}	5.70 $\times 10^{-3}$	
Superimposed load, π '	1	0	0.71	1.54	
Gear ratio, R _T	1	4.00	1.76	0.50	

Dependent variables were chosen to be the relative output torque $\tau_0^R T$, load speed $\omega_L^i L$, engine speed $\omega_E^i L$, and rotation a_L . These variables were plotted against relative time, θ .

The Effect of Engine Inertia

Figure 6-1 illustrates that engine inertia can lower the torque into the torque converter and causes output torque to drop off a corresponding amount. $\dot{\omega}_E$ for curves 2 and 3 are not very different; therefore, $\tau_0 R_T$, a_L and $\omega_L i_L$ are all about the same for these two curves. The curves indicate some "safety factor" in estimating a value of engine inertia since the boundary given by $I_E = 0$ is not very different from the other two curves. Also, doubling the intermediate value did not drastically effect the curves for this size of load; however, the effects of engine inertia are greater at lighter loads as is pointed out in the following discussion.

The Effect of a Superimposed Load

The fact that the $\tau_0 R_T$ curves 1 and 2 in Figure 6-2 drop below 1.76 indicates that there is a reduction in input torque due to the effects of engine inertia. Values of τ' above approximately 0.9 superimposed on the inertia load should make more effective use of the torque converter since τ_0 would not become much less than one.



Figure 6-1. The effect of i_E . The values of i_E in the above curves are: $i_E = 0$ for curve 1, $i_E = 0.038$ for curve 2, and $i_E = 0.054$ for curve 3.



Figure 6-2. The effect of τ' . The values of T(t) in the above curves are: $\tau' = 0$ for curve 1, $\tau' = 0.71$ for curve 2, $\tau' = 1.54$ for curve 3.

The Effect of Transmission Gear Ratio

Transmission gear ratios have a large effect on the transient performance of this system. The extreme affects of gear ratio are illustrated in Figure 6-3. When $R_T = 0.5$ the output speed and torque are about the same as obtained with a straight gear drive in place of the torque converter and gear drive; however, the engine speed remains relatively constant. The maintenance of a constant engine speed is not too important if the engine torque curve is flat, but if the torque curve is sharply peaked there is a definite advantage of operating at the peak torque speed.

The opposite extreme is observed in curve 1 for $R_T = 4.0$. The engine acceleration is large enough to cause $\tau_0 R_T$ to fall below the value for a higher gear, and this accounts for the decreased output values.

Note that at $\theta = 5$ the output speed for curve 1 is below that of curve 2 but the distance traveled for 1 is still greater than that for 2.

Situations similar to this are often observed in automobile racing. When gear ratios are changed the elapse time between points are changed with an opposite affect on the speeds obtained. This phenomenon is attributed to many factors, but the influence of engine inertia is often overlooked.



Figure 6-3. The effect of R_T . The values of R_T in the above curves are: $R_T = 4.0$ for curve 1, $R_T = 1.76$ for curve 2, and $R_T = 0.5$ for curve 3.

The Effect of Load Inertia

The affect of changing load inertia and changing gear ratios would not be distinguishable if it were not for the presence of the output parameters $\omega_L i_L$ and α_L . When i_L is decreased as in curve 1 of Figure 6-4, τ' becomes a substantial part of the load on the torque converter. Note that the speed and time parameter scales are influenced by i_L .

Although the change of engine speed is about the same with respect to θ for the three curves the difference in $\dot{\omega}_E$ shows up in the $\tau_0 R_T$ versus θ plot. The sharp drop in the torque of curves 1 and 2 accounts for the decrease in relative output speed of these lighter loads. The absolute magnitudes are plotted against time, t, as broken lines.

The Effect of Torque Converter Size

Torque converter size is associated with its torque capacity at various speeds, and this is illustrated clearly in the $\omega_{\rm E} i_{\rm L}$ plot of Figure 6-5. The stall speeds are different, but $\dot{\omega}_{\rm E}$ is similar for all three curves; therefore, loss of torque due to engine inertia should be about the same. The increased performance with the smaller torque converter is explained by the fact that the slip and hence the torque ratio is greater with the smaller unit.

Changing the size of the torque converter would be expected to change the effective inertia of the engine and load, but this affect was not considered.



Figure 6-4. The effect of i_L . The values of i_L in the above curves are: $i_L = 0.037$ for curve 1, $i_L = 0.065$ for curve 2, and $i_L = 0.112$ for curve 3.



Figure 6-5. The effect of κ . The values of κ in the above curves are: $\kappa = 3.15 \times 10^{-3}$ for curve 1, $\kappa = 3.55 \times 10^{-3}$ for curve 2, and $\kappa = 5.7 \times 10^{-3}$ for curve 3.

There is a practical limit to how small a torque converter can be used without excessive engine speeds. The shape of the engine torque curve would also be a prime consideration in selecting a torque converter.

USE OF RESULTS AND SUGGESTIONS FOR FURTHER STUDY

It is evident from the results that engine inertia is a prime factor governing system performance whenever "relatively" high accelerations are involved. What is considered fast accelerations for a tractor engine may be insignificant for a motor cycle engine; therefore, dimensionless parameters were used so that the results could be applied to various systems.

It is the author's opinion that transmissions used in many vehicles are selected on a compromise of maximum performance capability, efficiency of operation and cost, and this selection is based on experience rather than the scientific approach. However, in special vehicles where maximum performance or maximum utilization of the components is the only consideration a mathematical study similar to the one used in this study could be useful.

The analog computer program developed can be used to simulate other systems than the one in this project, and it should be a valuable tool in determining the characteristics of hypothetical systems. Caution should be used in applying cyclic inputs because of lag in engine response which was neglected in this study. Further investigation of the cause and effects of this lag would expand the usefulness of this analog.

Preliminary tests that have not been mentioned showed that the first 0.2 seconds of the engine speed and output speed curves are fairly independent of the load when the test is started from an idling condition.

In fact the output curve appears as if there was a lag in response during the first 0.2 sec.; however, this lag is due to the engine inertia and torque converter characteristics. Because of this all the curves in the results were started from stall speeds. The study of the effect of starting tests below and above equilibrium speeds would be interesting because these conditions often exist, particularly when gear ratios are changed to change the output speed range.

It would be interesting to study the application of the hydrodynamic torque converter to systems with widely varing load conditions. Farm tractors, stationary machinery and off-road machines may use the torque converter type transmission to some advantage; however, governing the output speed is a problem. As was mentioned in the introduction, something must be known of the operating characteristics of a system before it can be effectively controlled. It is hoped that the theoretical background presented in this study will be helpful in this aspect.

SUMMAR Y

A mathematical model was developed to express the transient behavior of the complete engine-hydrodynamic transmission-load system. These equations were programmed on an electrical analog computer, which greatly contributed to the amount of information obtained.

Laboratory methods were presented to find engine inertia by acceleration test and drag torque of the engine by deceleration test. These tests are performed while the engine is firing.

It was established for two cases that the electrical analog behaved similarly to the physical system. The influence of various system parameters were then investigated using only the electrical analog. The effect of engine inertia, load size, gear ratio and torque converter size on the engine speed, output speed, output torque and distance traveled was presented.

From the test conducted it appears that engine inertia has a large influence on system performance under light loads and that torque converter characteristics predominate under heavier loads.

Transient engine performance appears to be the same as steady state performance except for the influence of engine inertia and a small lag in response. There is an additional lag in response at the output of the transmission when the engine is given an input from an idling condition. This lag is due to the time involved in engine acceleration from idle to a stall condition at full torque.

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