A REVIEW OF PHOTOELASTIC THEORY AND METHODS, WITH AN ACCOUNT OF THE RESULTS FROM AN APPLICATION TO SCREW THREADS

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THESIS

A REVIEW OF PHOTOELASTIC THEORY AND METHODS, WITH AN ACCOUNT OF THE RESULTS FROM AN APPLICATION TO SCREW THREADS

by

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PREFACE

The first discoveries and subsequent developments in photoelasticity were regarded as a branch of physics. However, with the recent improvements in photoelastic equipment and technique, it has become the ideal method for the commercial engineer in the experimental determination of stresses. Hence, the large majority of papers dealing with photoelastic theory were written in this period by physicists who did not always appreciate the desire of the engineer for practical information in a concise form. Little was said regarding actual laboratory procedure and technique, and treatment of the theory was largely mathematical, being unduly long and abstract.

However, in the last few years there have appeared several papers written by investigators who were commercial engineers, and these papers have well covered actual laboratory procedures. The authors have largely confined themselves in these papers to the results of the particular investigations undertaken, assuming that the reader has studied previous accounts of the theory. It is therefore the purpose of Part I of this thesis to review, in as brief and simple a manner as possible, the fundamentals of photoelastic theory as well as to give a unified account, in review form, of the laboratory procedure and technique as given in the current literature.

As stated above, it was decided to plan this thesis in two parts. The first part is an account of the theory involved, while the second part deals with a particular investigation performed by the writer.

In giving an account of photoelastic theory and practice, an effort was made to keep this as brief and simple as possible, yet giving a comprehensive review of the field to the present time. It is realized by the writer that this cannot be too easy or too short, but maximum use was made of pictorial methods of presentation, thereby clarifying and reducing the amount of text material needed. Given in resume form using a large number of sources, it is believed that this method will be of great value in assisting those to whom this thesis is submitted in evaluating the second part, as well as being of help to those who may have occasion to use this paper as a reference in the future.

It is assumed that the reader will be familiar with the subject matter of strength of materials, but Chapter III is included only as a review of those elements which apply to photoelastic work. It is also imperative that a clear conception of principal stresses be had before proceeding with the work.

Chapter V includes a discussion of the new polarizing material Polaroid and its action upon ordinary white and monochromatic light. Polaroid has found wide favor in photoelastic work and has many advantages over the formerly used Nicol prisms, hence this polarizing medium is given a complete discussion.

An explanation of the polariscope and its essential elements is also given in Chapter V, as well as drawings and a discussion of the construction of the polariscope built by the writer.

Ordinarily, sufficient information may be obtained from the stress pattern without separating the principal stresses p and q. Occasionally this is required, and methods for this procedure are reviewed in Chapter VII, these including the graphical integration method, the extensometer method, the interferometer method, the membrane method and others.

The results of the writer's work on stresses in screw threads with undercut shanks is given in regular engineering report form in Part II.

The author wishes to express his appreciation for the many helpful suggestions received from faculty members of the Engineering Department. Especially does he wish to acknowledge his gratitude to Mr. Seble and Mr. Pearson, laboratory technicians, for the valuable assistance which they have rendered in the construction of apparatus.

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PART I

PHOTOELASTIC THEORY AND PRINCIPLES

CHAPTER I

INTRODUCTION

1. Definition. Photoelasticity is an experimental optical method of finding stresses and stress distributions in machine parts and structural members, using polarized light and transparent models made of plastics, such as Bakelite. Use is made of the effect of a stressed transparent model on the passage of polarized light through the model. Double refraction and interference phenomena combine to produce a stress pattern which is thrown upon a screen or recorded by means of a camera, from which the various stresses may be evaluated. The optical equipment used in this method is termed a polariscope, which consists of a light source, polarizing equipment, a suitable lens system, a means of supporting and loading the model, and a screen or camera for viewing or recording the image produced.

The application of the photoelastic method in engineering work consists of finding the stresses present in the model, using the information given by the stress pattern. The model results may then be easily transferred to the prototype if there exists a difference in scale of size or loading.

2. History and Advantages of Photoelasticity. In 1816, Sir David Brewster discovered that glass became

doubly refracting in polarized light when placed under load. All photoelastic stress investigations are based on this fundamental phenomena of double refraction of transparent materials when stressed. This property of double refraction not only accounts for what is happening within the stressed model, but is the basis for the explanation of how polarized light is produced.

The first part of Sir David's paper, read before the Royal Society on Feb. 19, 1861, may be summarized in the following statement: When light passes through a plate of glass which is stressed transversely to the direction of propagation, the axes of polarization in the glass are along, and perpendicular to, the direction of stress. Thus we see that stressed glass becomes double refracting and splits the entering polarized ray into two components, the speed of each component being influenced differently while traversing the glass due to the stressed condition at that point. In this way the relative retardation of each ray was seen to have a definite relationship with the stress. This phenomena is basic in photoelastic work, and will be discussed at greater length in a later chapter.

Other 19th century physicists contributed greatly to the fundamental ideas of photoelasticity, among these being Neumann, Maxwell, and many others.

Among the first to make an engineering application of the method was Mesnager, a French engineer, who constructed a glass model of a proposed bridge over the Rhone river in order to check results of the designers. Coker and Filon did excellent work in applying the method to engineering problems using celluloid, which was a great improvement over glass because of its greater optical sensitivity and ease of cutting to shape.

However, it has been only in the last five to ten years that the greatest advances have been made in the subject, these recent improvements being largely in procedure, equipment and materials. Among these are the use of a monochromatic light source and the use of Bakelite as a material. These recent refinements have aided greatly in making the photoelastic method a practical one for the engineer.

In designing a structural member or machine part, one may find stresses and stress distributions in two ways. The first method is to compute mathematically the stresses at various points or sections, using the conventional formulas of stress analysis. These formulas, however, are based on a number of assumptions, these holding only in special cases. Some of these assumptions are: that longitudinal stresses follow a linear law in bending; that axial loads produce a uniform stress distribution; that horizontal and vertical shear stresses follow a parabolic law of distribution in beams in bending. This latter assumption has been shown to be greatly in error in most cases by M. M. Frocht.[1] From his investigations it was found that the maximum vertical shears are much higher than the maximum computed by the parabolic formula, and furthermore this maximum does not occur at the neutral axis but close to the point of load

[1] "The Place of Photoelasticity in Engineering Instruction", M.M.Frocht. Carnegie Institute of Technology. 1937.

application.

The fact that these formulas generally yield incorrect results in the case of the second assumption may be illustrated by a simple prismatic bar of rectangular cross section subjected to pure tension as shown in the figure below. The stress will closely approximate that

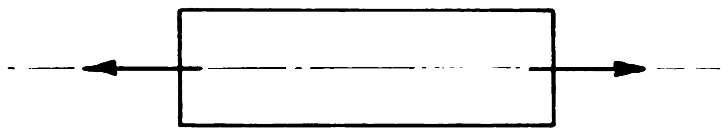


Figure 1

given by the formula $s = \frac{P}{A}$. However, if a small hole be put in the bar as shown in the following figure, the stress

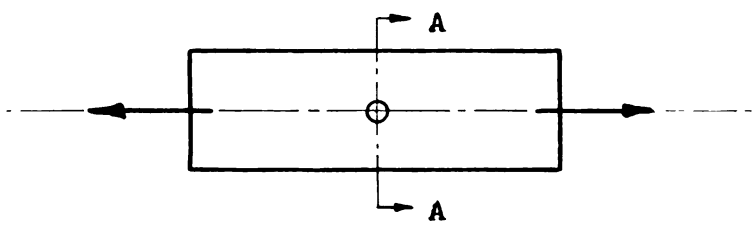
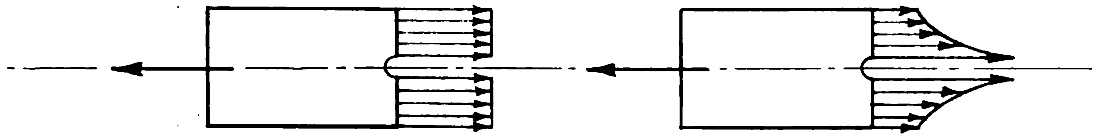


Figure 2

found from the formula $s = \frac{P}{A'}$, where A' is the area of the cross section at A-A after the hole has been placed in the bar, the stress will be approximately the same as in the previous case. But experiments have shown that the stress obtained by formula in the second instance is greatly in error, since the effect of a small circular hole is known to increase the stress to about <u>three</u> times this value. Instead of being uniform as assumed, the actual stress distribution contains definite stress concentrations, these stresses increasing greatly at the edges of the hole as shown in the figure on the following page.



By formula Actual

Figure 3

Thus it may be seen that the elementary formulas may be used only with caution in simple cases, and that they are not applicable in the case of members of complicated or irregular shape containing holes, notches, grooves, reentrant corners etc.

Results obtained from the photoe;astic method have had their accuracy well established by precise agreement with results from the theory of elasticity for the more simple cases. Photoelastic results are not only exact and reliable, but the scope of the method goes far beyond available mathematical solutions. It includes statically indeterminate cases, all problems having equally simple photoelastic determinations. The photoelastic method precludes the need of questionable assumptions as in the case of theoretical solutions, which are generally long and difficult.

In those cases where analytical methods fail to give information on stress distributions, recourse may be had to the experimental method, using models of similar shape and subjected to the same type of loading as the prototype. The most valuable of these experimental methods is that of photoelasticity. This has been of inestimable value to designers in supplying data on stress concentrations, not only making it possible to increase strength by adding material at these points but also to conserve material and weight at points of low stress. The commercial applications of photoelastic work in design are innumerable.

In recent years it has also proven an effective means in the educational field of bringing to the student a way to actually see stress distributions in various structural members and machine shapes. It has also served as a means of checking the accuracy of test specimens used in determining the strength of various materials. For example, in the standard cement testing briquet, the usual tensile strength given for the section of minimum area is around 40% less than the actual stress existing over the outside part of this section where failure begins.

Most photoelastic investigations have been performed using two-dimensional stress problems. Although the two-dimensional case covers a wide variety of problems met in practice, it remained until recently to develop a satisfactory method of applying photoelasticity to three-dimensional stress problems. However, there have been developed two methods suitable for three-dimensional work. It is interesting to note that at the present time, photoelasticity is the only experimental method of finding the true stress conditions within a solid body. The first of these is the fixation method[1], wherein a three-dimensional model

1 M. Hetenyi, "Fundamentals of Three-dimensional Photoelasticity", Jour. App. Mech., 5, A149 (1938).

is loaded in the manner of its prototype and heated while under load. Upon cooling, "slices" are cut from the model along suitable cross sections and examined in the polariscope. It has been shown that the patterns are correctly preserved by this "freezing" method, and that sawing of the slices does not disturb the pattern. The second method is the scattering method[1]. This is a very recent method, and the reader is referred to the Bibliography for references to these methods. In this thesis, two-dimensional photoelasticity only will be discussed.

[1] Fried & Weller, "Photoelastic Analysis of Two- and Three-dimensional Stress Systems", Eng. Exp. Station Bulletin No. 106, Ohio State University.

CHAPTER II

GENERAL METHODS OF INVESTIGATION

1. General procedure. So that the reader may more easily follow the treatment of each individual topic, a brief indication of the stress determinations made from the photoelastic pattern will be given here.

The first step is to make a scale model out of Bakelite or other suitable transparent material, and record the pattern produced by the stressed model when placed in the polariscope. By methods to be described in Chapter VII, the following more important determinations may be made:

(a) Shear stresses.

A stressed model projects an image on the screen, this image consisting of a number of brightly colored bands. (If a monochromatic source of light is used instead of white light, these colored bands appear as plain black and white fringes.) These colored bands, called isochromatics, are actual representations of the maximum shear distribution in the model. Thus by a glance at the image on the screen, the lines of maximum shear stress may be clearly seen. In a straight beam loaded by couples at the extremities, these shear lines will be straight, parallel, and horizontal.

By a simple calibration experiment the actual value in p.s.i. of these stresses may be found at any point in the model.

(b) Localized points of high stress.

The spacing or degree of concentration of lines in any region is indicative of the amount of stress present at that point. That is, in a region where the lines are far apart indicates the presence of little stress. A region having a large number of closely packed lines will be one of high stress. This can be used in an analytical manner, as will be shown later.

(c) Principal stresses.

Superimposed upon the isochromatic, or shear line network is a second set of black lines, called isoclinics. These lines have the property of shifting their position while the model is under a constant load when the polarizer and analyzer are rotated together, the axis of polarization of one being kept at 90° to the axis of the other. The explanation of this will be given in a later chapter.

By sketching the positions of these isoclinics for various angular positions of the polarizer and analyzer, a network of lines is obtained from which the directions of the principal stresses (tension and compression) may be completely determined.

By subsequent methods, these principal stresses may be evaluated for any point in the model. This procedure is more involved than those previously described, although this determination is not necessary for many problems.

(d) Boundary stresses.

The fundamental photoelastic equation is

$$p - q = 2nF$$

where p and q are the principal stresses, n is the fringe order, and F is a constant which may be determined by calibration. Since the boundary is free from shear, the stresses acting upon it are either purely tensile or compressive. Hence in the formula given, one principal stress reduces to zero, making it extremely easy to plot the boundary stresses. When q is zero, the equation reduces to $p = 2nF$. Methods of determining F and n will be given later.

The results obtainable from a photoelastic investigation have been indicated in only a very brief manner here. The methods used to obtain these results, and their practical application will receive full attention in Chapter VII.

CHAPTER III

ELEMENTARY THEORY OF STRESS AS APPLIED TO PHOTOELASTICITY

1. Definitions. Before proceeding, it will be well to define the symbols ordinarily used in discussions of elastic theory:

ϕ........Angle from the X-axis to the direction of P measured counterclockwise.

P,Q......Principal stresses.

(P-Q)....Principal stress difference.

S_x, S_y....Normal components of stress in X and Y directions respectively.

v_{xy}, v_{yx}..Shearing components of stress.

Stresses normal to a plane will be denoted by S, the subscript showing the direction in which the stress is acting; e.g., S_x would be a normal stress acting in the X-direction.

Stresses acting normal to an inclined plane will be denoted by S_n.

Stresses acting parallel to a plane (shear stresses) will be denoted by v, the first letter in the double subscript showing which plane the shearing stress is acting upon, and the second showing its direction (parallel to the X or Y axis.)

2. Two cases of tension.

(a) Simple, or pure tension.

To obtain a clear understanding of how the various stresses on an element change in magnitude when

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2. Two cases of tension.

(a) Simple, or pure tension.

To obtain a clear understanding of how the various stresses on an element change in magnitude when the element is rotated, a case of simple tension in a prismatic bar may be considered, Figure 4.

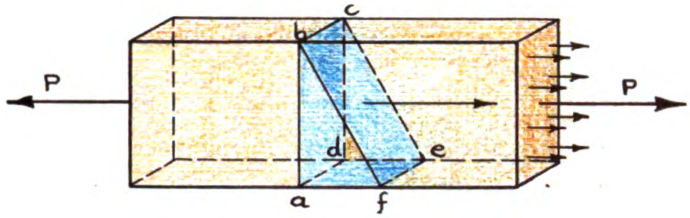


Figure 4

The bar is loaded axially in the one direction only, so that it is subjected to pure tension. It will be evedent that the normal stress on a perpendicular section abcd (Area A), is $S = \frac{P}{A}$. Now examine a section bcef (Area A') which is inclined at an angle ϕ to the normal section. Taking half the bar as a free body in Figure 5, it will be

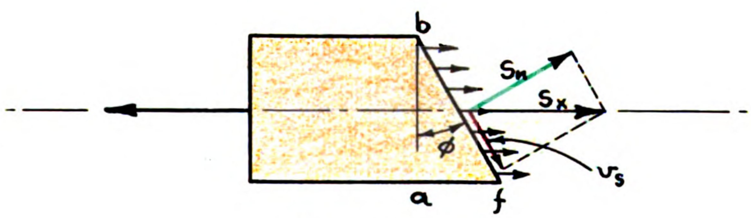


Figure 5

seen that the stress S_x may now be resolved into two components. One, S_n, is normal to the particular inclined section that is being considered, the other, v_s, is a shearing stress parallel to this plane. Hence, on planes other than a perpendicular section, even in pure tension, there appears a <u>shear</u> stress on those planes taken at various angles. It may next be asked when this shearing stress is a maximum, and at what angle.

First, an expression may be written for the stress S_x over the section ab as it is rotated to any angle ϕ:

$$S_x = \frac{P}{\text{Area, section abcd} = A} \qquad (\text{For } \phi = 0^\circ)$$

$$= \frac{P}{\dfrac{A}{\cos \phi}} = \frac{P}{A} \cos \phi \qquad (\text{For any } \phi)$$

A series of sketches is shown for variations of S_x with the angle ϕ in Figure 6.

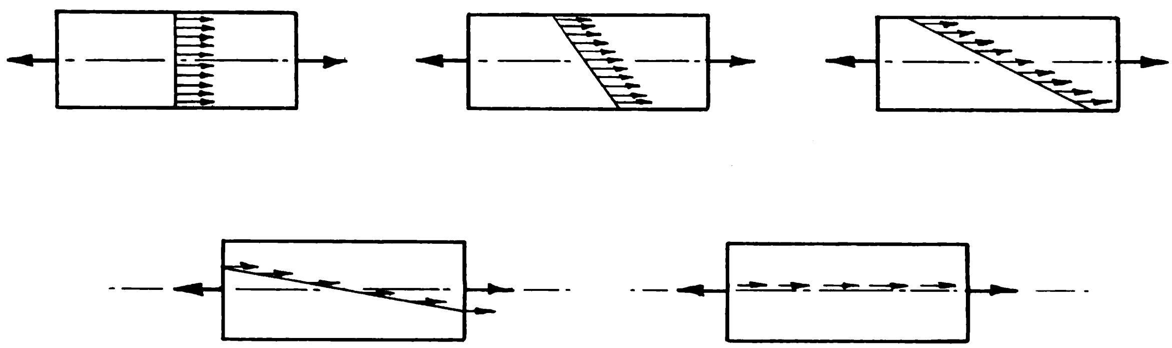


Figure 6

It will be remembered that this shows the stress S_x only, it being the stress over the inclined section in the direction of load.

The components of S_x may now be considered, these being the normal stress S_n and the shear stress v_s. A study of the shear stress is important for there are materials which are much weaker in shear than in tension.

Writing the expressions for the normal and tangential (shear) stresses:

Normal:

$$S_n = \frac{P \cos \phi}{\dfrac{A}{\cos \phi}} = \frac{P}{A} \cos^2 \phi = S_x \cos^2 \phi \quad \text{..(2-1)}$$

Shear:

$$v_s = \frac{P}{\frac{A}{\cos \phi}} = S_x \cos \phi \sin \phi$$

$$= \frac{S_x \sin 2\phi}{2} \quad \text{.....................(2-2)}$$

From this it is evident that

Max. value of $S = S_x$ when $\phi = 0^\circ$

Min. value of $S = 0$ when $\phi = 90^\circ$

Max. value of $v = \frac{S_x}{2}$ when $\phi = 45^\circ$

Min. value of $v = 0$ when $\phi = 0^\circ$ or 90°

Letting $\phi_1 = 90^\circ - \phi$ and referring to the formula for shear stress, it will be evident that values of v for $\phi = \phi_1$ and $\phi = 90^\circ - \phi_1$ are equal and that the values of v for $\phi = \phi_1$ and $\phi = 90^\circ + \phi_1$ are equal but opposite in sign.

Also from equation (2-1),

$$S_1 = S_x \cos^2 \phi, \quad \text{(See Fig. 7 below)}$$

and

$$S_2 = S_x \cos^2 (90^\circ + \phi_1)$$

$$= S_x \sin^2 \phi$$

from which

$$S_1 + S_2 = S_x(\cos^2 \phi_1 + \sin^2 \phi_1)$$

$$= S_x.$$

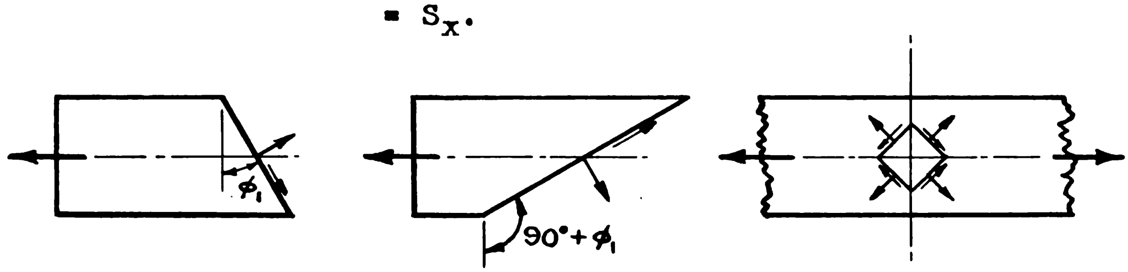
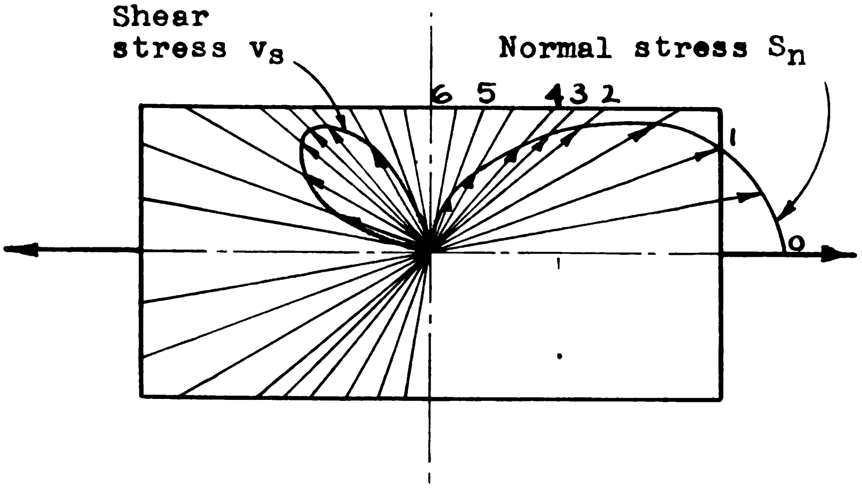


Figure 7

From the preceding statements two important conclusions may be stated:

1. The sum of the normal stresses acting on two perpendicular sides of an element within a bar under pure tension is constant and is equal to S_x max.
2. The tangential (shearing) stresses on two perpendicular sides of an element within a bar under pure tension are of equal magnitude.

A graphic representation of how the two stresses S_n and v_s vary as the plane is turned through various angles in shown in Figure 8. In this example the area was taken as 1 sq. in. and the load as 2000 lbs. This appears on the following page.



Plane	Angle	S_n, p.s.i.	v_s, p.s.i.
0	0°	2000	0
1	20°	1766	643
2	40°	1174	985
3	45°	1000	1000
4	50°	826	985
5	70°	234	643
6	90°	0	0

Figure 8

2. Two cases of tension (cont).

(b) Combined tensions.

Consider now the stress condition in a thin prismatic bar subjected to two tensions at right angles to each other and acting along the X and Y axes, Figure 9.

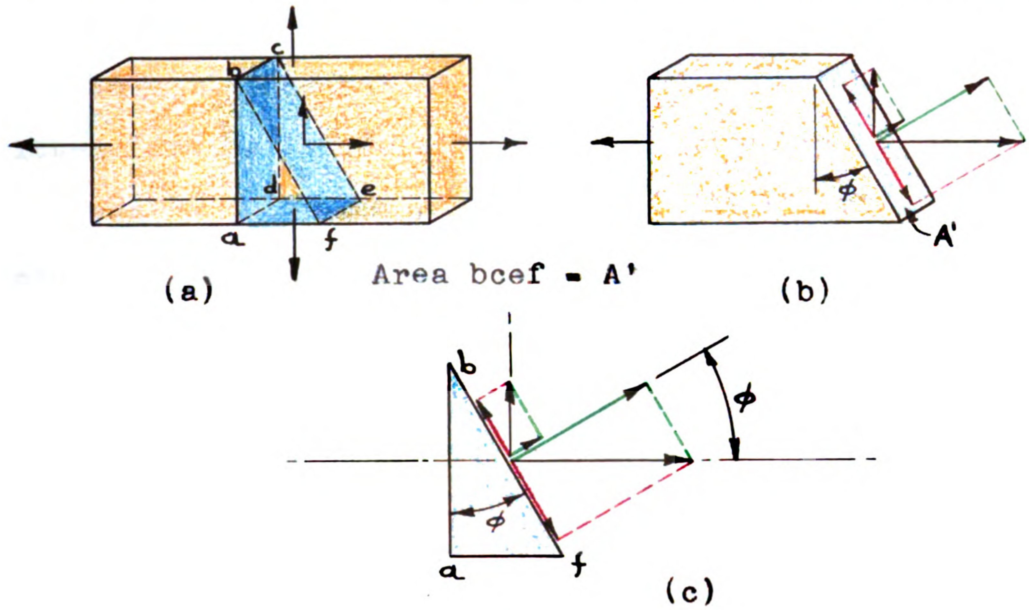


Figure 9

The conditions of static equilibrium may be applied to the above sketch. It must be remembered that the vectors shown represent unit stresses and not total forces. Hence these unit stresses must be multiplied by the areas over which they act to obtain the total forces used in the equations of equilibrium. Let A' be the area of face bf; then the area of face ab is A' cos ϕ and the area of face af is A' sin ϕ.

Normal stress:

Considering all forces acting on face bf as projected in a direction normal to bf, the equation of equilibrium may be written for S_n:

$$S_n A' = (S_x A' \cos \phi)(\cos \phi) + (S_y A' \sin \phi)(\sin \phi)$$

The areas A' cancel, leaving:

$$S_n = S_x \cos^2 \phi - S_y \sin^2 \phi$$

$$= \frac{S_x - S_y}{2} - \frac{S_x - S_y}{2} \cos 2\phi \ldots\ldots\ldots\ldots (2\text{-}3)$$

Shear stress:

In a similar way an equation of equilibrium may be written for the forces acting parallel to the plane bf:

$$v_s A' = (S_x A' \cos \phi)(\sin \phi) - (S_y A' \sin \phi)(\cos \phi)$$

Again the areas A' cancel, leaving:

$$v_s = (S_x - S_y) \sin \phi \cos \phi$$

$$= \frac{S_x - S_y}{2} \sin 2\phi \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}4)$$

Maximum and minimum stresses.

As in the case of pure tension, it is evident upon examining the variation of the stresses on a section as it is rotated through 90° that the normal stresses acting upon the section reach a maximum or minimum value when $\phi = 0°$ or 90°. Under these circumstances the shear stress disappears. These particular directions are "Principal Stress Directions", and the normal stresses in this case become the "Principal Stresses".

In this particular case of two axial tensions it should be noted that the directions of principal stress happen to coincide with the directions of the external forces applied along the axes of the bar. This is not the case for more complicated shapes and systems of loading. The case just discussed helps to introduce the reader to the idea of principal stresses; the general case will be discussed in Article 4. The general case considers the effect of shear stresses applied.

3. Equality of shear stresses.

Before studying the general case of plane stress, the relation between shearing stresses on planes at right angles may be briefly shown. This relation has been stated previously in Article 2a, but is usually proved as given here.

Let Figure 10 represent a small prismatical element of unit thickness cut out of a stressed member. If a shearing stress v_s acts on the right hand face, the shearing force on this face will be $v_s dy$. There must be a shearing force equal but opposite in direction acting on the left hand face in order that the sum of the vertical forces be zero to satisfy the conditions of equilibrium. However, these two forces form a couple, and so to prevent rotation, there must be a second couple made up of shearing forces $v_s' dx$ which act on the top and bottom faces. These two couples must be numerically equal and must act in opposite directions. The first couple may be seen to have a moment arm dx, and the second couple a moment arm of dy. Then

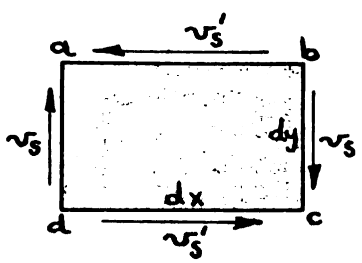


Figure 10

$$(v_s dy)dx = (v_s' dx)dy$$

from which

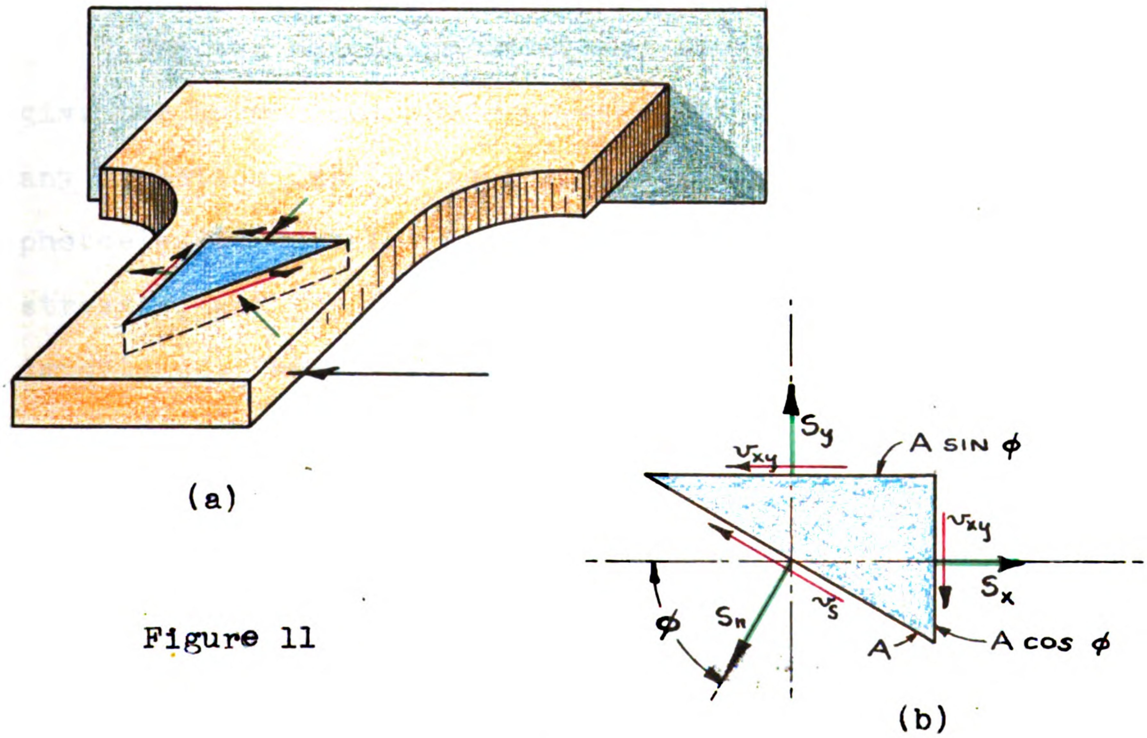
$$v_s = v_s' \quad \text{..................(2-5)}$$

Thus it may be seen that unit shearing stresses acting on planes at right angles to each other are numerically equal. Hence in the following discussion $v_{xy} = v_{yx}$, and either may be written in any case.

4. General case of plane stress.

The general case of an element subjected to plane stress may now be considered, such as when an irregularly shaped member is under any type of external loading or combination of loads.

Referring to Figure 11, a small elementary prism may be imagined to be taken from a stressed member and isolated



(a)

(b)

Figure 11

as a free body in Figure 11b. Representing the face ab as area A, then the area of face ac is A sin ϕ and the area of face cb is A cos ϕ.

Using the method of Article 2b of applying the conditions of static equilibrium to the prism:

$$S_nA = S_xA \cos \phi \cos \phi + S_yA \sin \phi \sin \phi$$
$$- v_{xy}A \cos \phi \sin \phi - v_{xy}A \sin \phi \cos \phi$$

The areas cancel, and this reduces to:

$$S_n = S_x \cos^2 \phi + S_y \sin^2 \phi - 2v_{xy} \sin \phi \cos \phi$$
$$= \frac{S_x - S_y}{2} - \frac{S_x - S_y}{2} \cos 2\phi - v_{xy} \sin 2\phi \text{..(2-6)}$$

In a similar manner expressions for v_s may be obtained. These are:

$$v_s = (S_x - S_y) \sin \phi \cos \phi + v_{xy}(\cos^2 \phi - \sin^2 \phi)$$
$$= \frac{S_x - S_y}{2} \sin 2\phi + v_{xy} \cos 2\phi \text{..............(2-7)}$$

The two equations developed above, (2-6 and 2-7), give the normal and shear stresses at any point and for any direction for the general case of plane stress. In photoelastic work the principal stresses, maximum shear stresses, and the directions of these planes are of prime importance. These will now be discussed for the general case.

Taking the equation of normal stress (2-6) and differentiating it with respect to ϕ, one obtains:

$$\tan 2\phi = - \frac{2v_{xy}}{S_x - S_y} \text{ ....(2-8)}$$

Differentiating the equation for v_s, gives:

$$\tan 2\phi = \frac{S_x - S_y}{2v_{xy}} \text{ ....(2-9)}$$

Considering equation (2-8), it is evident that there are two values of 2ϕ between 0° and 360°, these differing by

180° because they have the same value for the tangent. From this we infer a statement important in photoelastic work: Since the values of ϕ differ by 90°, the planes on which principal stresses occur are 90° apart, or, more simply, the principal stresses are at right angles to each other.

Considering equation (2-9), a similar reasoning shows that the planes on which the maximum shearing stresses occur are at right angles to each other.

It is now desired to find the planes on which the shear stress is zero. To do this, equation (2-7) may be rewritten after setting $v_s = 0$. Solving for tan 2ϕ gives:

$$\tan 2\phi = - \frac{2v_{xy}}{S_x - S_y}$$

Since by this procedure an expression is obtained which is the same as equation (2-8), a further important conclusion may be made: The shear stress is zero on planes of maximum and minimum normal stress (principal stresses).

If it be noted that the right hand member of equation (2-8) is the negative reciprocal of the right hand member of equation (2-9), and that since the values of 2ϕ differ by 90° then the values of ϕ differ by 45°, a last conclusion may be drawn that will be of interest photoelastically: The planes of maximum shear stress are at 45° to the planes of principal stress.

5. Summary.

It has been shown that the normal and shear stresses on a section of a stressed member vary with the rotation of the section. The maxima and minima of these stresses were investigated mathematically as well as the directions of the planes on which they occurred.

Quantitative statements of the amounts of the stresses were given by the numbered equations of the preceding article. Directions of the planes and relations between planes of principal stress and planes of maximum shear will be given again in this summary.

The following conclusions developed in this chapter are important in evaluating stress patterns and understanding photoelastic theory. They are given here in summary form:

1. When the normal stresses reach a maximum or a minimum they become the principal stresses, and the shear stress then is zero.
2. The principal stresses are on planes which are at right angles to each other.
3. The planes on which the maximum shearing stresses occur are at right angles to each other.
4. The shear stress is zero on planes of maximum and minimum normal stress (principal stresses).
5. The planes of maximum shear stress are at 45° to the planes of principal stress.
6. The maximum shear stress is given by the formula

$$v_{max} = \frac{P - Q}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2\text{-}10)$$

where P and Q are principal stresses.

Hereafter in the text, the two maximum and minimum normal stresses (principal stresses) denoted formerly by maximum or minimum S_x and S_y, will be denoted by P and Q.

The formula (2-10) is basic in interpreting stress patterns, since the isochromatic lines of a pattern are interpreted as loci of points of maximum shear stress, or or loci of points of equal (P - Q) stress. The formula for maximum shear (2-10) is a special case of the plain shear formula, this being

$$v_s = \frac{P - Q}{2} \sin 2\phi$$

The fraction is a maximum when sin $2\phi = 1$. This occurs when 2ϕ is 90°, ϕ then being 45°.

CHAPTER IV

GENERAL OPTICAL THEORY

1. Theories of light.

The nature of light is a difficult subject, and no theory exists yet which is completely satisfactory. A brief statement of the theories which have been proposed are listed here.

(a). Corpuscular theory of Newton. About 1866 Newton proposed that light consisted of a stream of very small particles or "corpuscles", radiating from a source in straight lines. This explained reflection, but could not explain later phenomena and was rejected. Physicists recently returned to this theory but only for certain unusual phenomena in conjunction with modern theory.

(b). Wave theory of Huygens. Huygens suggested, about 1678, that light consisted of a longitudinal motion, such as that of sound. This explained reflection, refraction, and double refraction.

(c). Modern wave theory of Fresnel. This theory stated that light was of a wave form, but the waves were of a transverse nature such as the wave motion of a vibrating wire. The motion was considered to be oscillatory in a direction perpendicular to the direction of propagation.

(d). Electro-magnetic theory of Maxwell. This retains the transverse wave motion theory but in addition states that there are electrical oscillations which give rise to magnetic fields, each being perpendicular to the other and to the path of the ray.

The exact nature of light is still an unsettled one, and one theory is used to explain certain phenomena, while another theory is used to explain other phenomena. As yet, no one theory is entirely adequate. However, it is definitely established that for ordinary phenomena such as polarization and double refraction which are dealt with in photoelasticity, the light may be regarded simply as a transverse wave motion. Hence in the following discussion this will be the theory assumed as basis for the explanations. This is theory (c) given above.

2. Nature of light.

(a). White light. Ordinary white light is composed of the different monochromatic colors. Blended together, their combined effect is the appearance of white light. When white light is passed through a prism, it is bent by refraction and separated into its colored components, thus giving the spectrum of colors.

White light may be thought of as being composed of vibrations lying in planes passed at <u>all</u> angles through a center line whose direction is in the direction of the ray. Each vibration, or wave, is a sine curve, and may be represented as shown in Figure 12 by a rotating vector. The length of the vector is equal to the amplitude, and the position of a point (at the end of the vector) from the X-axis at any time is plotted against time. This figure also gives the notation which will be used. It should be remembered

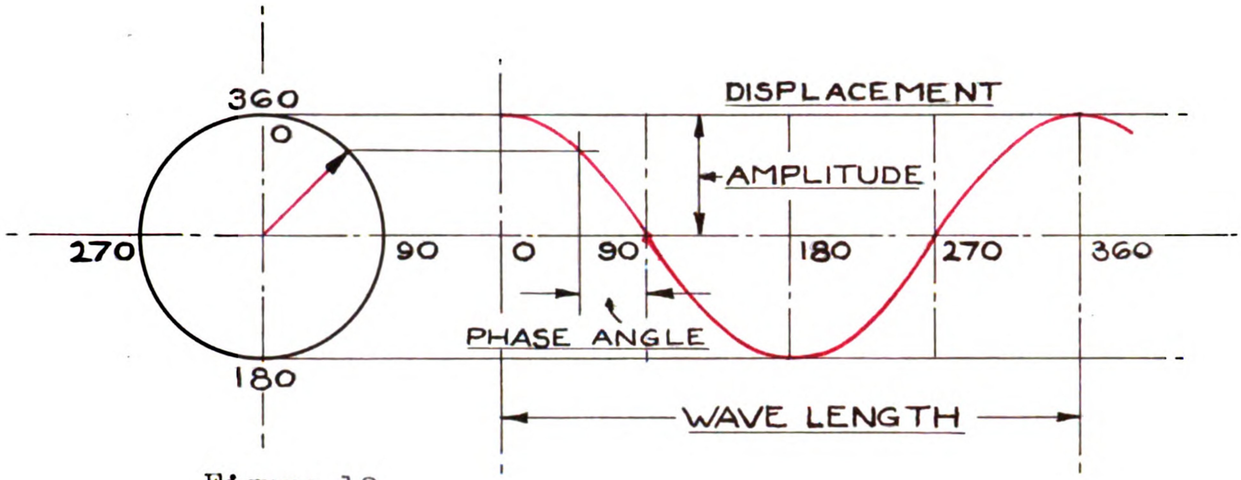


Figure 12

that white light is composed of several different colors, and hence will have vibrations of several different wave-lengths. Figure 13a shows the nature of ordinary light for

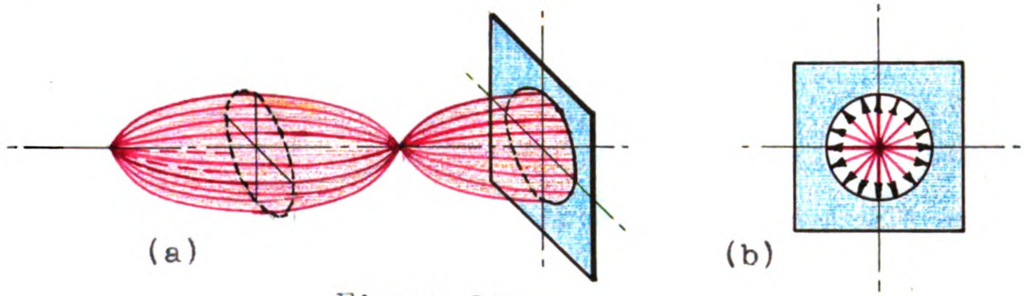


Figure 13

one wave length only, while Figure 13b gives a vector representation of the vibrations in the different planes at maximum amplitude. Figure 14 shows, on one plane only, the

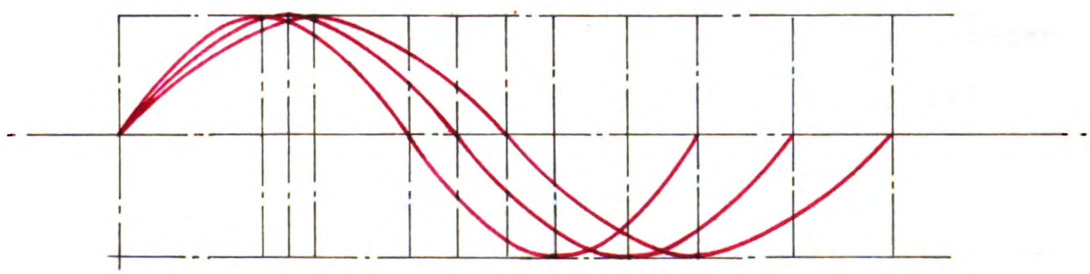


Figure 14

different wave-lengths which would exist in white light.

Hence it is evident that ordinary white light consists of random, chaotic vibrations which are not ordered with respect to any one direction or plane. Polarized light, it will be shown, has definite directional properties.

The intensity or brightness of light is proportional to the square of the amplitude. Color is a function of the frequency. This does not change when the light is reflected, refracted, or transmitted through various media. Although the frequency does not change, the velocity varies when light is transmitted and depends on the material. The velocity is equal to the product of the wave-length and the frequency; hence the wave-length also changes. The velocity is given by the familiar equation:

$$V = n\lambda \qquad (3\text{-}1)$$

In this discussion the velocity of light through air, then Bakelite, then emergence into air again, will be of greatest interest. It will be evident that the light will regain its original speed upon emergence from the Bakelite to the air again. In this case, no effect would be apparent but as will be seen later, the Bakelite will have a different effect on two certain ray components, which gives a permanent retardation of one behind the other upon emergence.

(b). Monochromatic light. Monochromatic light is light of a single color and hence of a certain frequency. In other words, it is simply a special case of white light. Noting this limitation, the discussion of the preceding article applies to monochromatic light as a special case of white light.

(c). Polarized light. There are several methods of polarizing light, one of which is by means of discs of Polaroid. These various methods will be discussed in Chapter V, but Polaroid is mentioned here since it serves admirably as a means of visualizing how polarized light is produced.

Polarized light may be obtained then, by passing ordinary light through a disc of Polaroid. This material has optical properties that are directional, which may be described by saying that the Polaroid has an optical axis. In the direction of this axis, the Polaroid may be thought of as having a number of very small slots which stop all of the vibrations except those in a single plane. Thus, the light emerging from the Polaroid will be polarized, the vibrations all being in a single plane. The total effect over the surface of the Polaroid will be to transmit polarized light, the vibrations of which will be in parallel planes.

This polarized condition of the light may be detected by the use of a second disc of Polaroid. This too may be thought of as having a large number of tiny slots oriented in the same direction. Thus, if the axis of the second Polaroid is set parallel to that of the first, the polarized light will be transmitted, while if the axis of the second disc is set at 90° to that of the first, the vibrations will be stopped and no light will be transmitted.

The first of a pair of Polaroid discs used in the manner described above is termed the polarizer, while the second disc is termed the analyzer, since it detects or analyses the polarized condition of the light. A pair of Polaroids used in this manner comprises the fundamental parts of a polariscope. The beam of parallel, polarized light between the polarizer and analyzer is termed the field of the polariscope, and it is in this field that the stressed transparent model is placed.

The action of Polaroids upon light is shown in Figure 15, which appears on the following page.

(d). Circularly polarized light. For reasons to be discussed later, it is sometimes desirable to examine the model in circularly polarized light. This condition is obtained by the use of quarter-wave plates which are made of mica or quartz. These materials may be cut so that they possess a principal crystalline axis called the optical axis, and will have different optical properties in two perpendicular directions.

Let it be assumed that a disc of mica or quartz be placed in the path of a ray of polarized light so that the optical axis of the crystal is at 45° to the plane of polarized light. The polarized light, it will be remembered, is vibrating in one direction only, and may be represented by a single vector lying in the plane of polarization. When the polarized ray strikes the mica disc, the single component splits into two components which then proceed through the mica at 90° to each other and at 45° to the original plane of polarization. Due to the directional properties of the mica, one ray will travel <u>faster</u> than the other. This is due to the fact that the molecules are closer together in one direction; thus the vibrating electrons have more difficulty in traversing the mica in this direction than do the electrons penetrating the mica at 90°, on which plane the molecules are farther apart. Thus one component of the original wave vector travels through the mica at a greater speed than does the other component. The mica is set at 45° to the plane of polarization of the incident light in order that the two components formed will be equal.

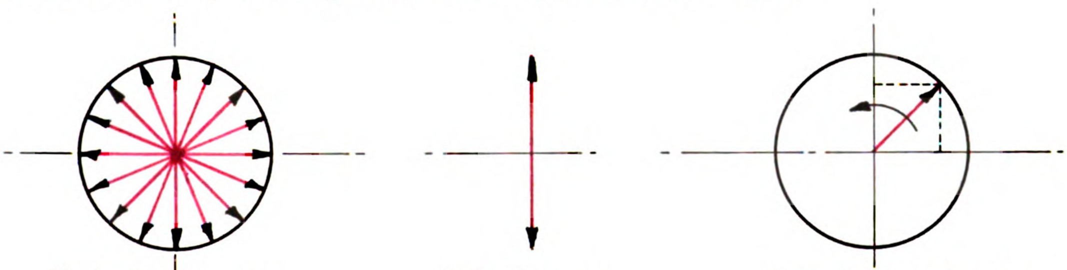
Now if the thickness of the mica be such that the two components reach the far side when they are $\frac{1}{4}$ of a wave out of phase, they will retain this retardation upon entry into the air again where the speed of each component will resume a constant velocity. In other words, upon leaving the mica they will have a permanent relative retardation of $\frac{1}{4}$ of a wave length. A crystal whose thickness is prepared to produce this effect is termed a $\frac{1}{4}$-wave plate.

Figure 16, on the following page, shows pictorially the effect of a $\frac{1}{4}$-wave plate on polarized light.

After leaving the $\frac{1}{4}$-wave plate, the two components will be vibrating at right angles to each other and $\frac{1}{4}$ wave out of phase. Each component will have a simple harmonic vibration in its plane, and will be in the form of a sine curve. If these two components be combined at each instant to form a resultant vector, it will be apparent that the terminus of this resultant vector will trace out a helix. The projection of this motion on a plane perpendicular to the ray would be a circle. Hence the name circularly polarized light.

Circularly polarized light may be converted back to plane polarized light by means of a second $\frac{1}{4}$-wave plate with its principal optic axis at 90° to that of the first. This second $\frac{1}{4}$-wave plate will retard the ray component advanced by the first and restore the light to the plane polarized condition. The use of circularly polarized light is to remove the isoclinic lines from the pattern. This will be discussed in a later chapter.

(e). Vector representation and summary. A vector representation of each type of light will help to fix them in mind. They are given in Figure 17, below.



(a) Ordinary Light (b) Plane Polarized (c) Circularly Polarized

Figure 17

White light is composed of random vibrations in all planes, and is composed of the different colors having different wave lengths.

Plane-polarized white light consists of vibrations in one direction only, but still contains vibrations of different wave-lengths.

Ordinary monochromatic light consists of vibrations in all planes, but has only one wave length.

Plane-polarized monochromatic light has vibrations in one direction only, and has but a single wave length.

3. Double refraction and interference.

When a transparent isotropic material such as Bakelite is stressed and placed in the path of plane polarized light, it behaves as a temporary doubly-refracting crystal much as did the $\frac{1}{4}$-wave plate discussed in the preceding article. However, the difference is that the $\frac{1}{4}$-wave plate possessed directional properties which were parallel and perpendicular at all points of the material. In the case of the Bakelite model, these directional properties vary in direction and amount at different points throughout the model. The distribution, or amount and direction of these properties, are dependent at any particular point upon the amount and direction of the internal stresses at that point.

Let a particular point be examined in a stressed transparent model, as in Figure 18 below. As the plane-polarized ray enters the model, the single component splits into two components at right angles to each other, these two components taking the directions of the principal stresses at that point. The principal stresses, it will be remembered, are always at right angles to each other. The model may be thought of as having two tiny slots at the point in question,

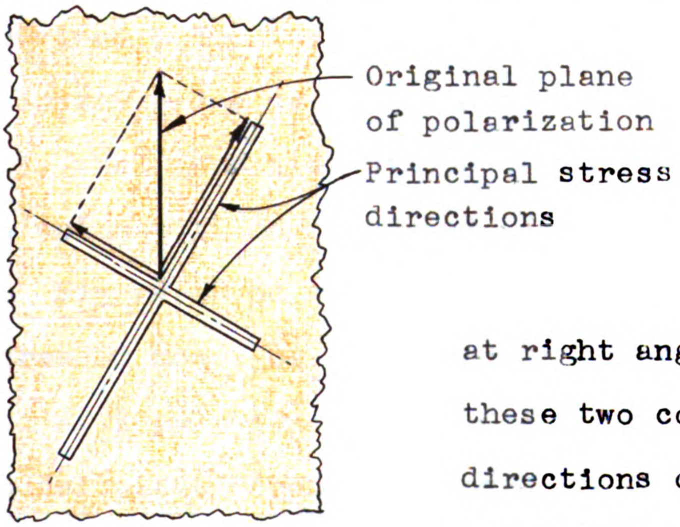


Figure 18

each slot being along a principal stress direction. The two ray components entering these slots travel at different velocities just as they did in the case of the $\frac{1}{4}$-wave plate, and for the same reason. However, in this case the orientation and length of these slots is not the same for all points in the model, but will vary according to the directions and amounts of the principal stresses at each point.

As stated previously, the action of the stressed Bakelite on plane polarized light is very similar to that of the $\frac{1}{4}$-wave plate. In the case of the $\frac{1}{4}$-wave plate, its axis is always set at 45° to the plane of polarization to give equal length to each vector produced. In other words, the $\frac{1}{4}$-wave plate is simply a special condition of what is now being considered. The length of each imaginary slot in the stressed model is proportional to the stress in that direction, and it is only in special cases where the P and Q stresses are equal (or the difference between them is zero), that each slot will be equal. This is a special case which will be discussed later.

Figure 19 on the following page shows the action of a stressed model on polarized light when placed in the field of a polariscope. The plane polarized light is in a vertical plane. Upon reaching the stressed model (considering a single point as shown on the drawing) the single vertical vector splits into two components at right angles to each other. These two components take the directions of the principal stresses at that particular point, and start through the model. Each component will travel with a different velocity while moving through the loaded model. Thus they will be out of phase with each other when they leave the model. The difference in speed existing while the rays are within the model is caused by a stretching of the molecules in the direction of tension, leaving more space for the electrons vibrating in the direction of a tensile stress. Similarly compression (at right angles to tension) will crowd the molecules closer together, and the speed of the two components in the direction of the tensile and compressive stresses will obviously be different. In other words, the change in velocity of each component will be proportional to the stress in that direction.

Upon leaving the model, the two component rays are vibrating at right angles to each other and are out of phase with each other an amount which is proportional to the difference between the principal stresses. Since the two components regain a constant velocity upon reentering the air, this difference in phase, or retardation, will be maintained. All that remains is to resolve the two rays into two components in the same plane so that interference will be obtained. This is done by passing the two rays through a second Polaroid disc, the analyzer.

Figure 19a, on the following page, gives a vector representation of plane-polarized light. Figure 19b indicates how the single vertical vector is split into two vectors at right angles and in the direction of the principal stresses. Figure 19c shows how the two rays are re-

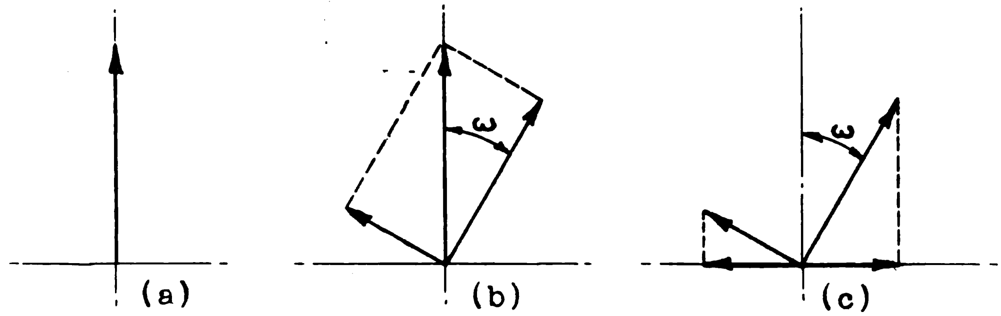


Figure 19

solved into coplanar horizontal components in order to secure interference. A succession of these rays coming from related points in the model whose principal stress differences are the same will form a band, or fringe, on the screen. This band will be a locus of points whose principal stress differences are the same.

A word may be said here concerning the thickness of the model. If the model is of constant thickness throughout, all rays will have traversed the same distance in going through the model, and hence all relative retardations will be proportional. Hence the thickness of the model does not matter. However, the thickness should be small enough compared with the dimensions of the model so that conditions of plane stress obtain. A good thickness for models of average size is usually taken as $\frac{1}{4}$".

Considering the total effect of several points along a section of the model having various stress-differences, it will be seen that with monochromatic light a series of dark and light bands will appear on the screen. This is the result of interference of the light waves which either reinforce or cancel each other, thus producing bands of varying intensity on the screen.

It has now been shown that the difference in the principal stresses produced a retardation which was proportional to this difference, and that the analyzer reduced these out-of-phase vibrations to components which were coplanar. It has also been shown that by interference of the final vibrations, bands are produced on the screen due to varying intensities. These bands form the image on the screen which will give the desired information pertaining to the stresses producing them. Interpretation and evaluation of these bands is discussed in Chapter VII.

4. Appearance of pattern with white light and with monochromatic light.

The explanation of the preceding articles was based upon monochromatic light as a source. This forms the more simple case for discussion, since it is a special case of white light.

Monochromatic light. When a monochromatic source of light is used, the image thrown upon the screen will consist of black and white fringes.

White light. When a white source of light is used, the image thrown upon the screen will consist of colored bands, these bands being termed isochromatics, meaning lines of equal color. In the colored image, the pattern (distribution, shape and position of the bands) is the same as the monochromatic pattern. However, in place of each black fringe, there will be a complete spectrum of colors. This spectrum of colors from yellow through orange, red, violet, green etc. and back to yellow, is repeated once on the white light source pattern for every corresponding black fringe appearing on the monochromatic pattern. Thus the two are actually the same, but each has different advantages in evaluating stresses. These will be discussed later.

Monochromatic light is a special case of white light, and interference gives only the black and white fringes. White light, however, is composed of the spectrum of colors and forms a more complex case to explain. The discussion of the preceding article based on monochromatic light applies the same in this case, the process simply being repeated for each color at a point.

CHAPTER V

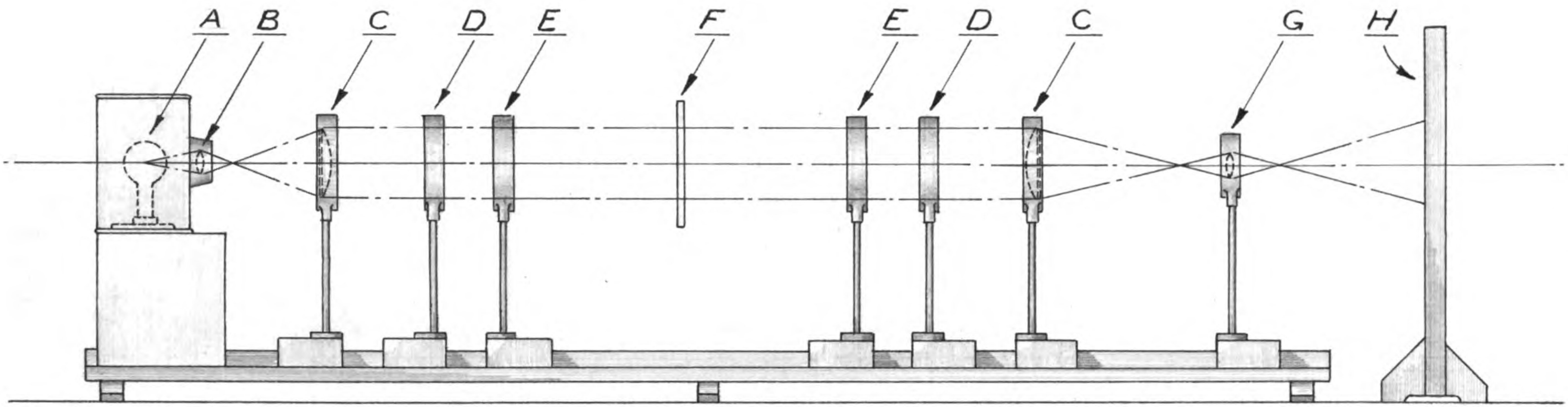
THE POLARISCOPE

1. Theory and elements of the polariscope.

The essential elements of a polariscope would include a light source, a suitable lens system, a means for supporting and loading the model, and a screen or camera for viewing or recording the image produced. Two sketches of polariscopes are given on the following pages, the first with a white light source and a screen, the second with a monochromatic light source and a camera.

The lens system shown is designed to give a beam of parallel rays for passage through the polarizer, model, and analyzer. If the light is a point source, two collimating lenses may be used. The first will give a parallel beam of rays for passage through the model, while the second will converge these parallel rays. The projection lens is used for focusing the image on the screen. The various lenses and other parts are usually mounted in suitable holders, these having graduations in 5 or 10 degree increments for the Polaroids and $\frac{1}{4}$-wave plates. These holders are supported by rods mounted on some type of saddle, these in turn being supported on an optical bench of the proper rigidity.

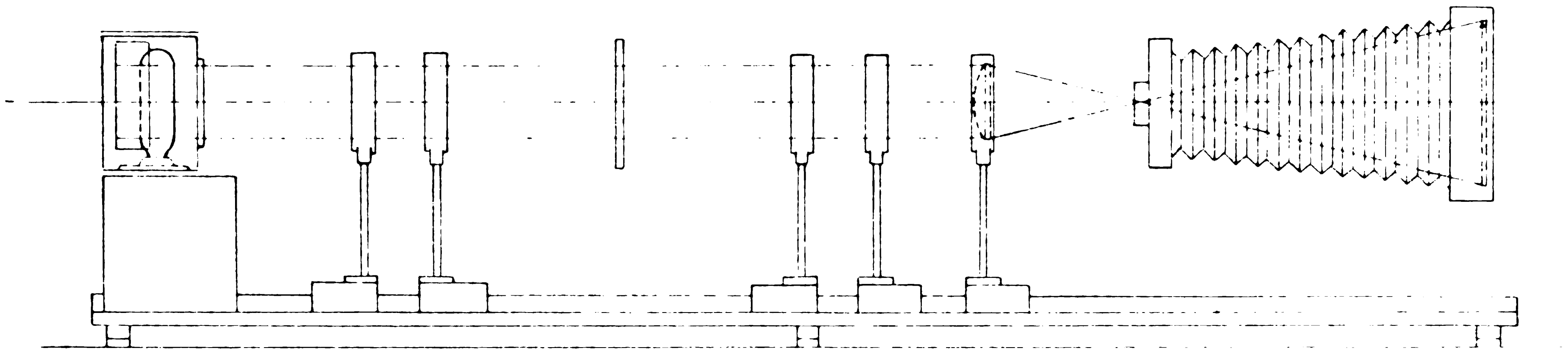
The relative positions of these elements may now be discussed. Since the use of Polaroid has become such a popular means of obtaining polarized light, it will be assumed that the polariscope being discussed is equipped with it. The first collimating lens after the light source will



OPTICAL SYSTEM OF THE POLARISCOPE

A - SOURCE OF LIGHT
B - COLLECTING LENS
C - COLLIMATING LENS
D - POLAROID DISCS

E - QUARTER-WAVE PLATES
F - STRESSED MODEL
G - CONDENSING LENS
H - SCREEN



POLARISCOPE WITH MONOCHROMATIC LIGHT SOURCE
AND CAMERA

give a beam of parallel rays, which will strike the polarizer perpendicularly. The polarizer, both $\frac{1}{4}$-wave plates, and analyzer are all mounted in rings calibrated in degrees, and may all be rotated. For a circular polariscope, the first $\frac{1}{4}$-wave plate would be placed directly after the polarizer, and would have its axis inclined at 45° to that of the polarizer. The second $\frac{1}{4}$-wave plate must be placed directly in front of the analyzer. This second $\frac{1}{4}$-wave plate must have its axis set at 90° to that of the first $\frac{1}{4}$-wave plate. Summarizing, we note that the axis of the first $\frac{1}{4}$-wave plate is set at 45° to that of the polarizer, and is placed just after it; that the axis of the second $\frac{1}{4}$-wave plate is at 45° to that of the polarizer and analyzer and at 90° to that of the first $\frac{1}{4}$-wave plate, the second $\frac{1}{4}$-wave plate being placed just before the analyzer; that the axis of the analyzer is always set at 90° to that of the polarizer.

2. Plane and circular polariscopes.

When two $\frac{1}{4}$-wave plates are used, their function is to remove the isoclinics in order that the isochromatics may be studied. For study of the isoclinics, then, the $\frac{1}{4}$-wave plates are not needed. A polariscope with just polarizer and analyzer is termed a <u>plane</u> polariscope, since the model is being examined with plane polarized light. A polariscope with $\frac{1}{4}$-wave plates added is termed a <u>circular</u> polariscope.

3. Light sources.

It was stated that a monochromatic source of light will give a pattern consisting of black and white fringes, while a white light source will give a pattern consisting of colored bands, or isochromatics, where each repetition of the spectrum is equivalent to a black and white pair of bands on the monochromatic pattern. Both of these light sources may be used in a photoelastic determination, each having advantages.

The photoelastic stress pattern, using a plane polariscope, is composed of two sets of lines, these being the isoclinic lines and the isochromatic lines. The isoclinic lines are always black, regardless of the light source, while the isochromatic lines are also black with a monochromatic source, but are colored with a white light source. Since the black isoclinics must be plotted with the isochromatics present also, it is of advantage to use a white light source for this particular investigation so that the black isoclinics are easily discernible against the colored background. With a monochromatic source, however, it is sometimes difficult to distinguish the isoclinics from the isochromatics, both of which are then black.

A white light source is also useful for identifying points in the model for which the principal stresses are equal. These points are termed isotropic points and will be discussed later. A third use for a white light source is for classroom and lecture purposes, since the image formed is much brighter than that obtained with any monochromatic source.

However, for quantitative dterminations of stresses, the white light source requires a method which is lacking in precision, and is now obsolete. For taking photographs and obtaining data, the monochromatic source is the better one to use.

4. Polarizing methods.

There are several means for obtaining polarized light. The principles and advantages of the more important methods will be discussed briefly.

(a) Polarization by reflection and glass plates. If a beam of light strikes a glass plate held at an angle to the beam, part of the light is reflected and part refracted. The most advantageous angle is about 57°, this giving the greatest amount of polarized light by reflection.

Better results may be obtained by using a pile of glass plates, this being termed a transmission polarizer. However, these methods are inefficient and are difficult to arrange in a polariscope. A stack of twelve plates will polarize only about 50% of the light, which is the maximum possible.

Polariscopes have been built using these methods in order to avoid the high cost of Nicol prisms. However, since the advent of Polaroid, these means are rather obsolete.

(b) The Nicol prism. The Nicol prism has been a common means of obtaining polarized light of good quality. These are made by first cutting in halves a rhomb of Iceland spar, which is a pure, transparent form of calcite. The cut faces are then polished and cemented together again with Canada balsam. Light entering one end of the prism is broken up into two beams by double refraction, one vibrating vertically and the other at right angles to it. The latter beam is totally reflected out and absorbed, while the other plane-polarized beam is transmitted.

The Nicol prism gives polarized light of good quality but is subject to many disadvantages, some of which are almost prohibitive at present. These are:

(1) Small aperture. Photoelastic work calls for a wide field of polarized light in which to view the model. The polarized beam from the Nicol must be diverged before being made parallel, which causes a great loss of intensity. This also calls for extra lenses, which impairs the efficiency of the polariscope.

(2) Nicol prisms are delicate and must be handled with care, since even small ones are expensive. They must also be protected from the heat of the light source by a glass water-jar, used to act as a cooler.

(3) Nicol prisms are difficult to obtain at present, since the supply of large pieces of Iceland spar has practically ceased.

(4) High cost. Nicol prisms rarely come into the market. When they do, they command a high price, since they cannot be replaced.

(c). Polaroid.

Recent advances and wide industrial application of photoelasticity have undoubtedly been greatly stimulated by the fairly recent development of Polaroid. This is a low cost material giving a wide field of polarized light of excellent quality. Polaroid discs are made by the Polaroid Corporation, and are composed of a large number of crystals of iodoquinine sulphate ($4C_{20}H_{24}N_2O_2 \cdot 3SO_4H_2 \cdot 6I$ $3H_2O$). Each of these crystals have the property of being able to polarize light. but the mass of crystals must have their polarizing axes all turned the same way in uniform orientation, thus producing the same effect as would a single large crystal. This is accomplished by spraying the solution onto a stretched rubber membrane. When the solution has attained a certain viscosity during drying, the membrane is released and stretched again at right angles to the original direction of stretch, thus orienting the polarizing axes of all the crystals in a common direction.

The action of each of these crystals is similar to that of the Nicol prism. The ray which is reflected out in the Nicol is absorbed by the Polaroid itself.

The structure of Polaroid is very minute, even at 1000 magnifications. Its polarizing effectiveness is over $99\frac{1}{2}\%$ perfect, and gives uniform and homogeneous polarization over the entire area of the Polaroid disc. It is not affected by age, shock, or exposure to temperatures up to 250° F. It has a further advantage over the Nicol prism in that fewer lenses are required in a polariscope equipped with Polaroid.

5. Construction of polariscope used for this investigation.

The essential elements of a polariscope are: a

Light source, a suitable lens system, a means for supporting and loading the model, a means of obtaining circularly polarized light when desired, and a screen or camera for viewing or recording the image produced. A sketch of the polariscope is again given on the next page for reference. The entire optical system must be readily adjustable to make possible a common optical axis vertically, and for ease in procuring the correct focal adjustment of the lenses along the optical axis.

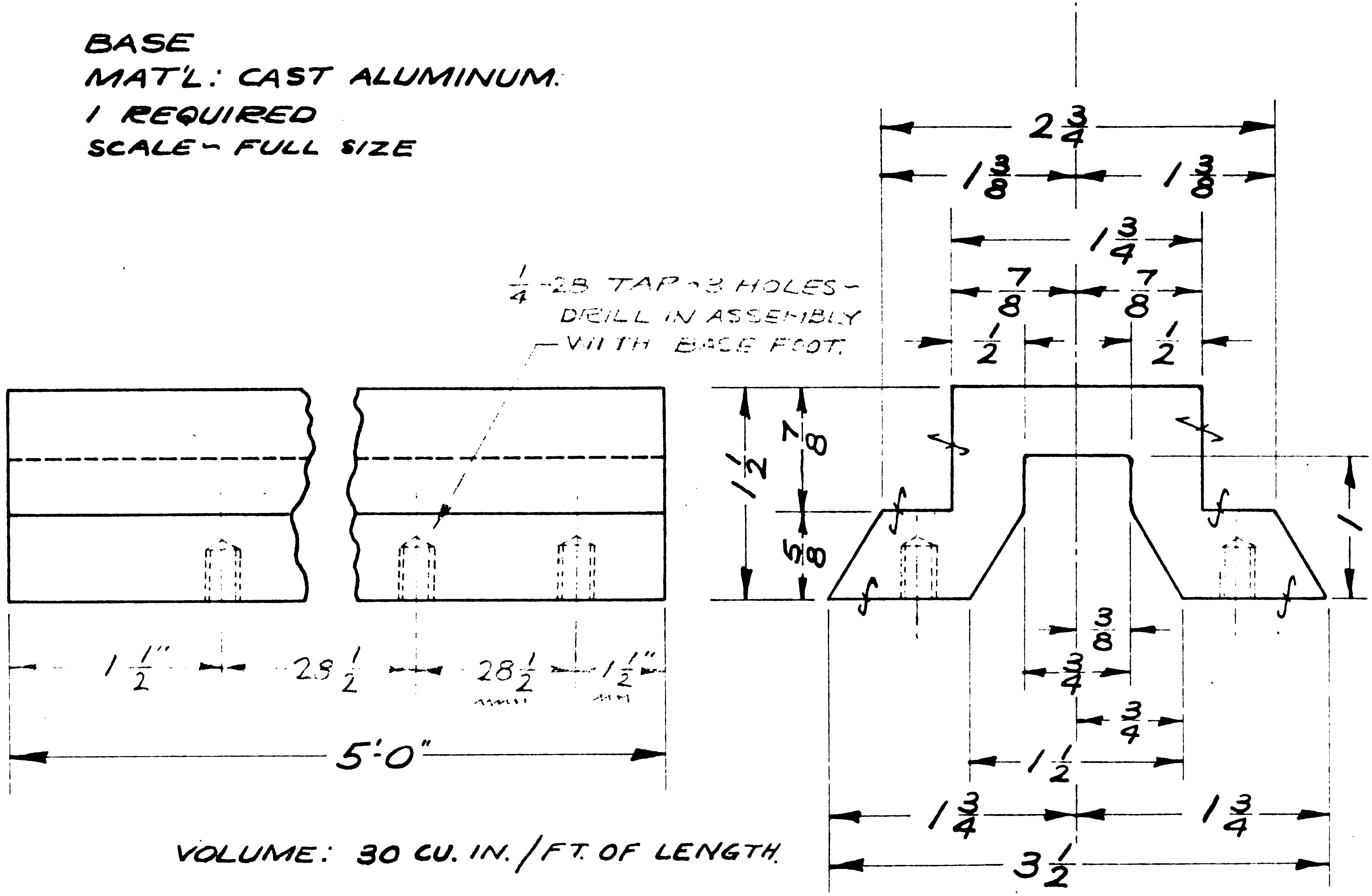
The optical rail.

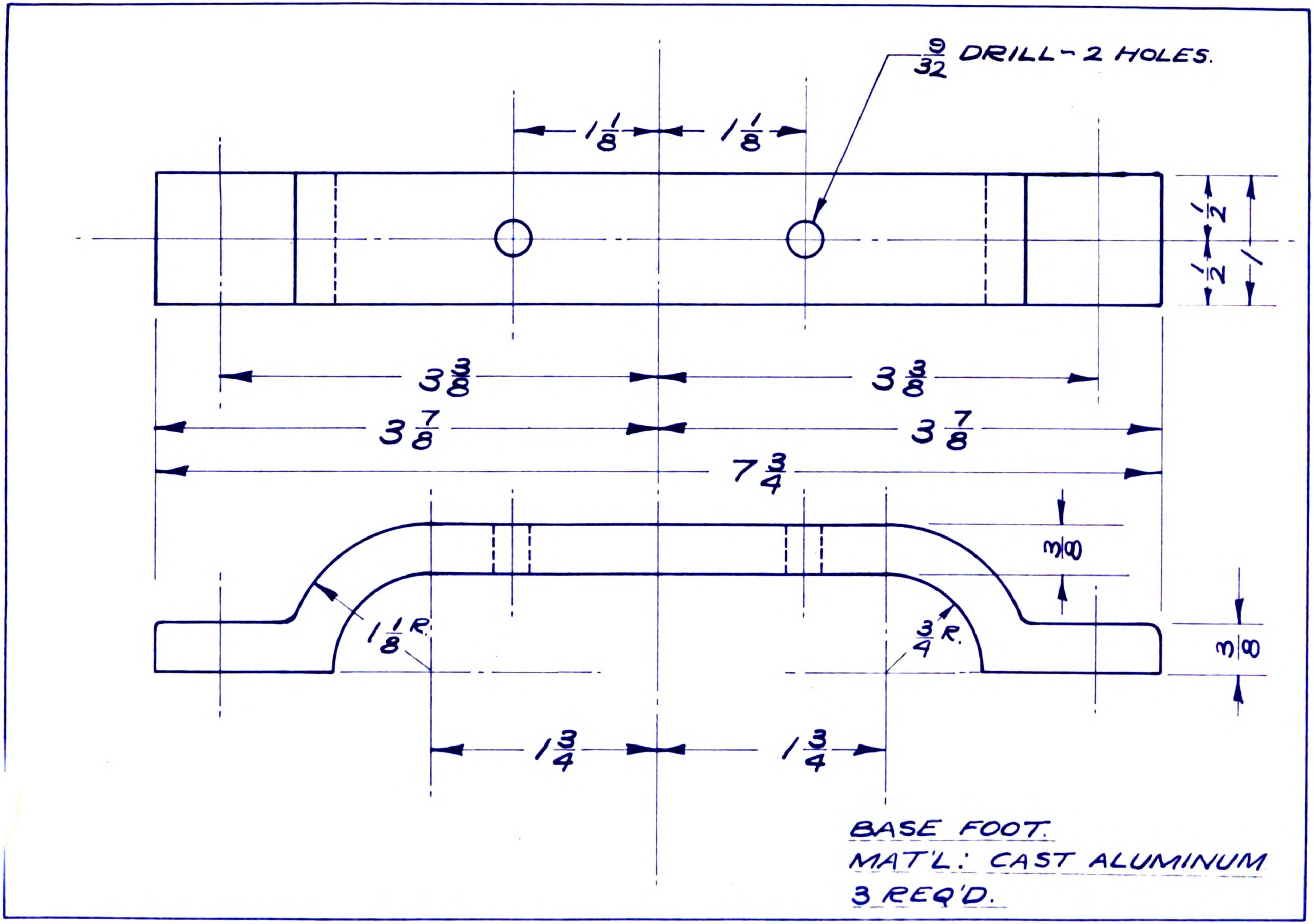
It is common in optical apparatus to secure the supports to the rail by means of a T-slot. However, in a polariscope it is continually necessary to remove and replace the $\frac{1}{4}$-wave plates and to make other adjustments. A T-slot would necessitate the removal of all the pieces when only one item is to be removed. Hence the rail was designed as shown in the print so that any part might be quickly and easily removed without disturbing any other part.

In order that the rail might not be of excessive weight, it was decided to cast it of aluminum. As the rail must be rigid and straight to insure optical alignment, it was made of a fairly heavy section.

The casting of the rail presented several difficulties. First, the core would not cope out in green sand, and it was found necessary to ram up a false cope, constructing the core on the drag section of the mold. A casting of this length in aluminum is liable to excessive warp-

BASE
MAT'L: CAST ALUMINUM.
1 REQUIRED
SCALE – FULL SIZE





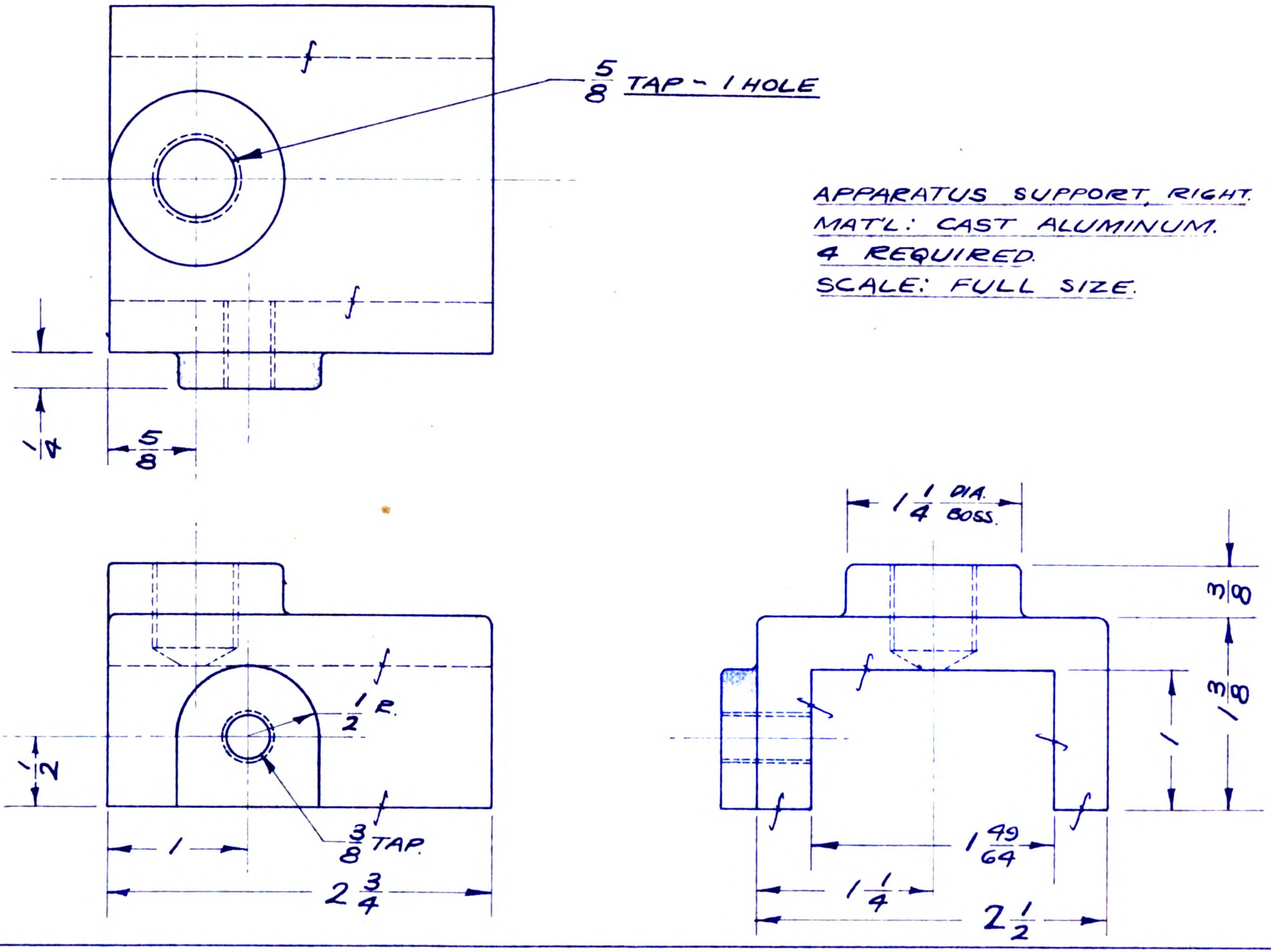
9/32 DRILL - 2 HOLES.
1 1/8
1 1/8
1/2
1/2
1
3 3/8
3 3/8
3 7/8
3 7/8
7 3/4
3/8
1 1/8 R.
3/4 R.
3/8
1 3/4
1 3/4
BASE FOOT.
MAT'L.: CAST ALUMINUM
3 REQ'D.

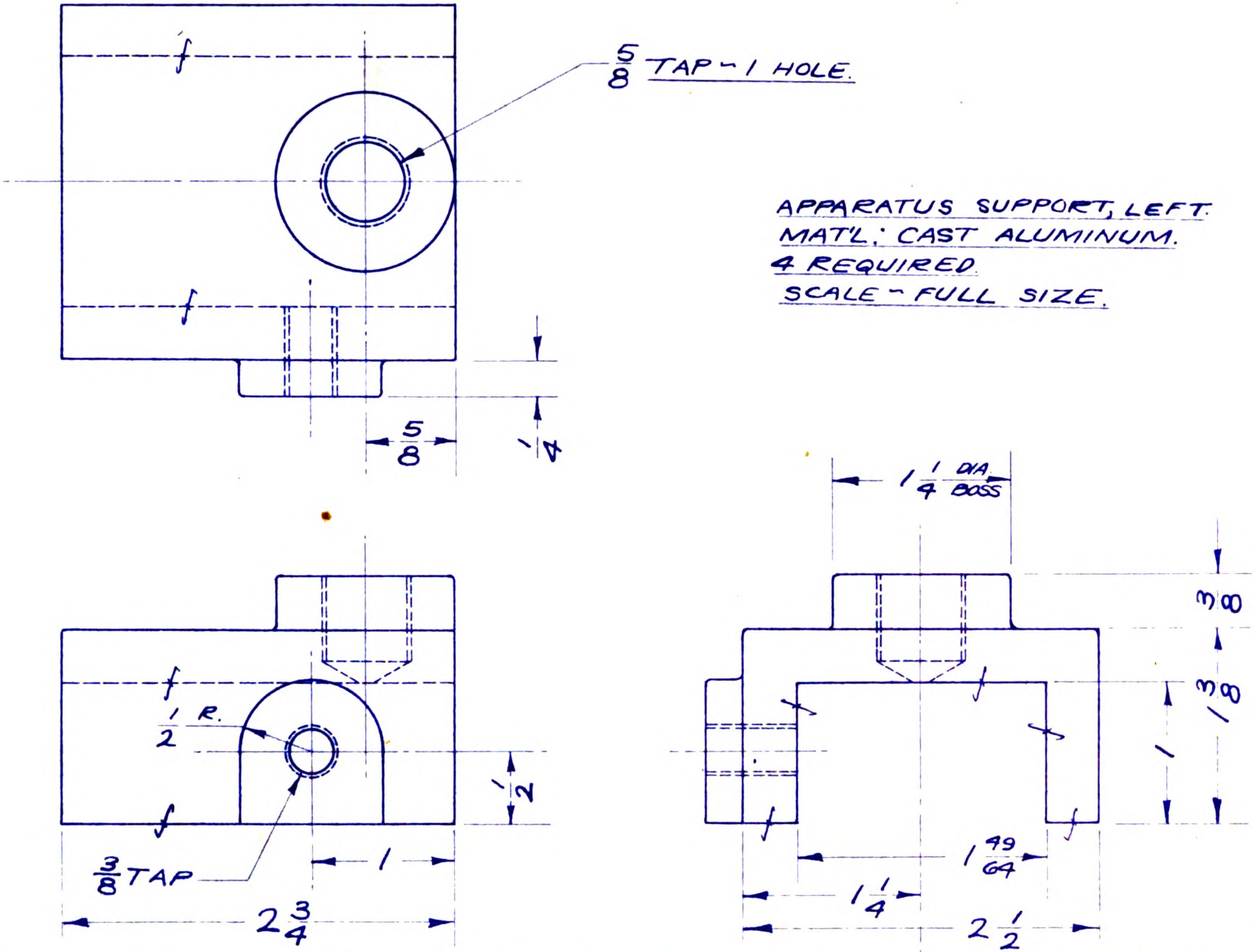
age, and since this could not be allowed in this piece to any great extent, precautionary mearsures were taken. It was found after a number of trials, that satisfactory results could be obtained by pouring through two sprues each gated to the bottom of the mold in three different places. In other words, the casting was fed from six gates equally spaced along the length of the mold. Five equally spaced heavy risers were used, and the metal was poured at as low a temperature as possible.

Supports.

The various parts of the optical system are supported by saddles fitting over the ways of the optical rail. Each saddle is tapped to hold a telescoping rod, this being for height adjustment.

The saddle is an aluminum casting machined for a sliding fit over the rail ways and held in any desired position by a thumb screw tightening against the rail. The boss, which is drilled and tapped for the support rods, is located at the end, and half the saddles are made reversed from the other half in order that parts of the optical system may be placed close together. The telescoping rod is held at the desired height by slitting and taper threading the tube half, and screwing a nut containing a pipe tapped center down over it, clamping it against the rod. This makes both an effective and a neat looking clamp. The end of the rod is threaded for attachment to any of the parts of the optical system. Details may be found in the prints.


5/8 TAP ~ 1 HOLE
APPARATUS SUPPORT, RIGHT.
MAT'L: CAST ALUMINUM.
4 REQUIRED.
SCALE: FULL SIZE.
1/4
5/8
1 1/4 DIA. BOSS.
3/8
1 3/8
1
1/2 R.
1/2
3/8 TAP.
1
2 3/4
1 49/64
1 1/4
2 1/2



APPARATUS SUPPORT, LEFT.
MAT'L.: CAST ALUMINUM.
4 REQUIRED.
SCALE - FULL SIZE.

Collimating lens.

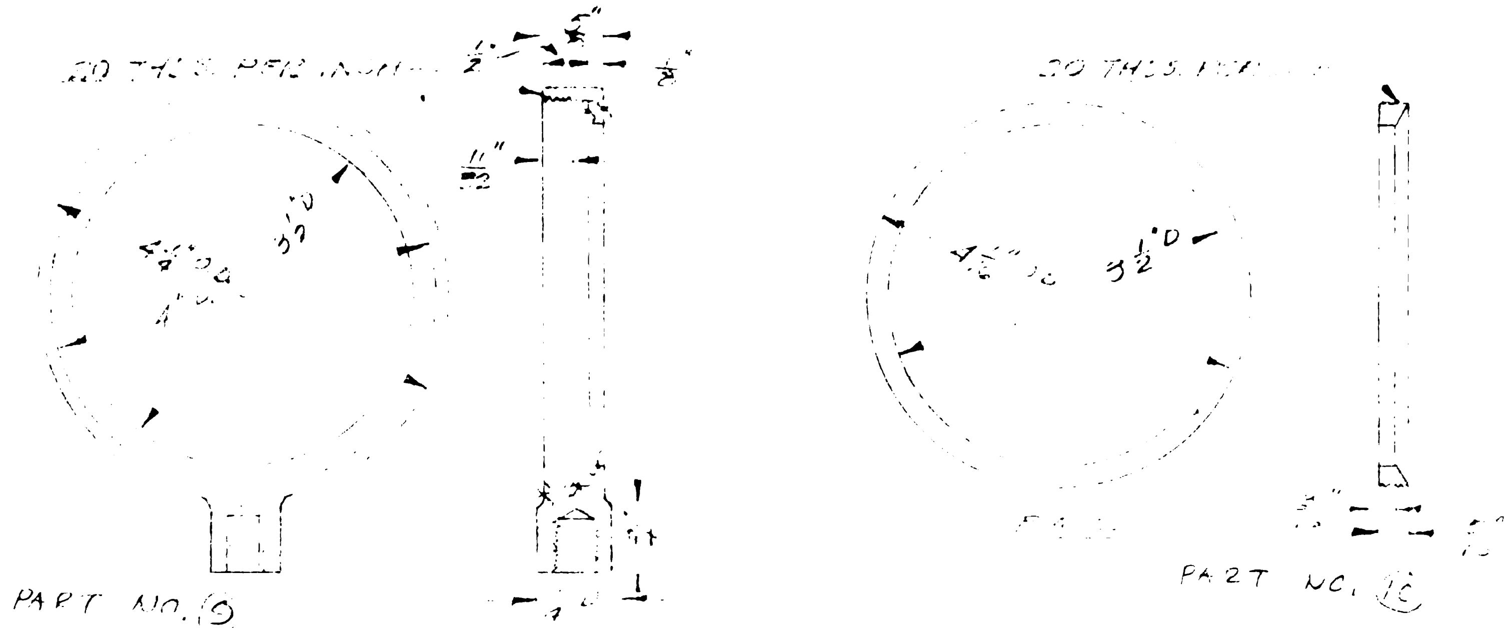
In order to bring the parallel beam of light to a focus, a 4" plano-convex collimating lens of 7" focal length was used. The lens is mounted in a pair of rings turned from aluminum castings. The outer ring is of L-section and forms the case. This outer ring is cast with a boss, this being drilled and tapped to receive the support rod. The inner ring screws into the outer one, holding the lens between them. Rubber gaskets were placed on either side of the lens.

Polaroid and $\frac{1}{4}$-wave plate mounts.

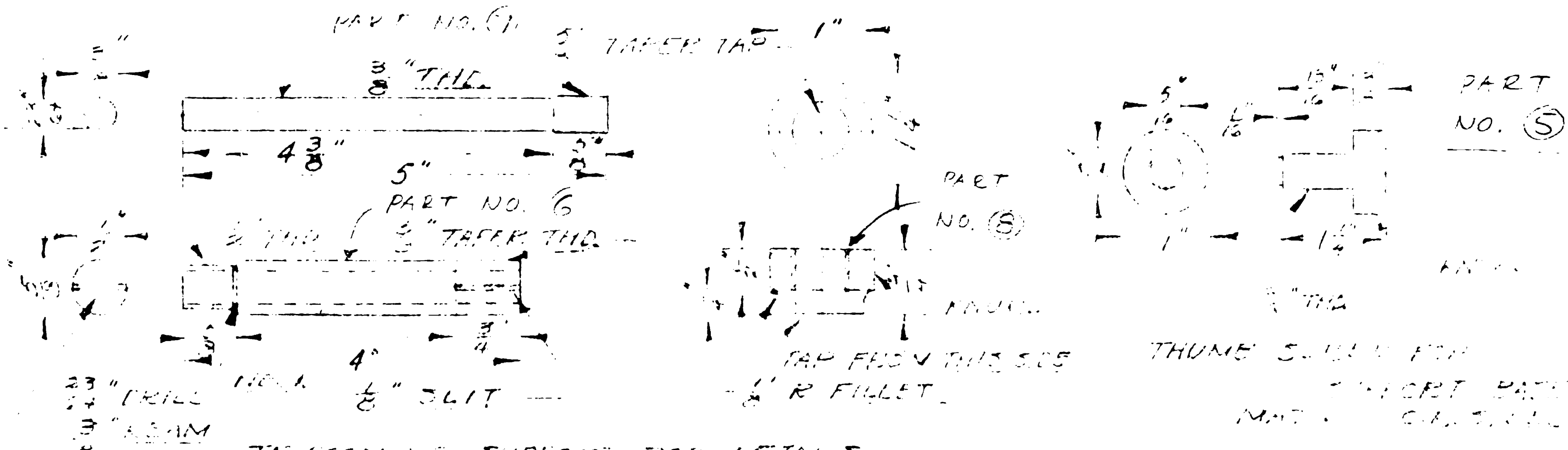
The mounting of the Polaroids and $\frac{1}{4}$-wave plates must be such that they may be rotated to any desired position of the polarizing axes. This was accomplished by mounting the Polaroid (or $\frac{1}{4}$-wave plates) in a pair of rings, then mounting this assembly inside a second pair of rings. The outer ring was graduated in 10° intervals in a milling machine, mounting the ring in a dividing head. Numerals were then stamped on. Polaroids and $\frac{1}{4}$-wave plates were of 4" diameter.

Projection lens.

When it was desired to throw the image on a screen, it was necessary to provide a projection lens in place of the camera lens. A 1 5/8" lens salvaged from a motion picture projector was used for this purpose, placed so that its focal point coincided with that of the collimating lens. The mounting for this lens is similar to that for the collimating lens.



COLLIMATING LENS MOUNTING DETAILS
MAT'L — CAST ALUMINUM



TELESCOPING SUPPORT ROD DETAILS
MAT'L C.R. STEEL

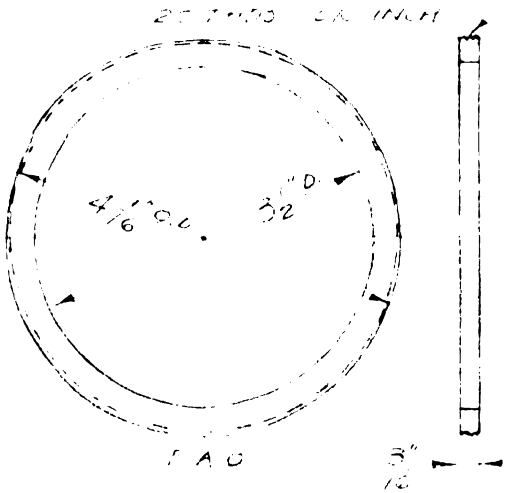
THUMB SCREW FOR SUPPORT ROD

SCALE — HALF SIZE

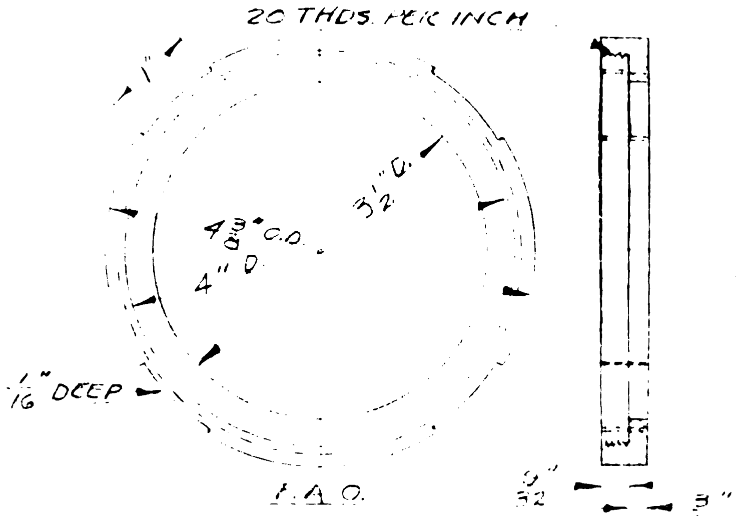
DETAILS OF POLAROID MOUNTINGS

SCALE — HALF SIZE

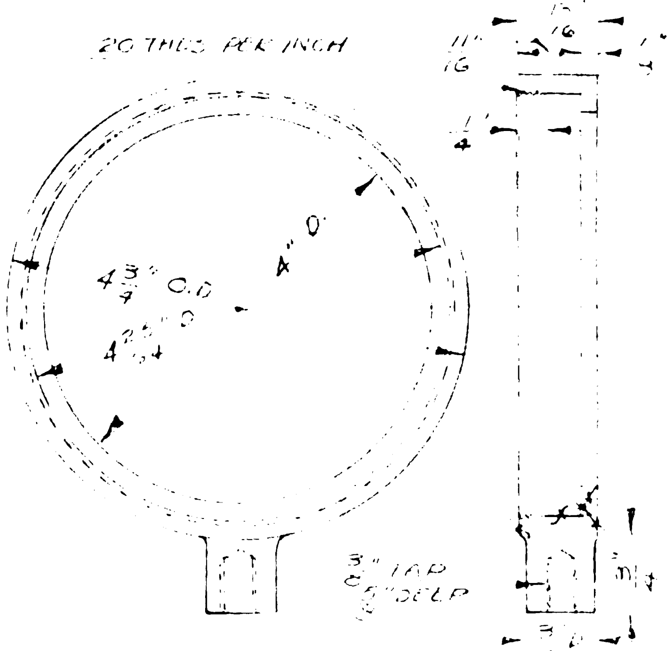
MATERIAL :
CAST ALUMINUM



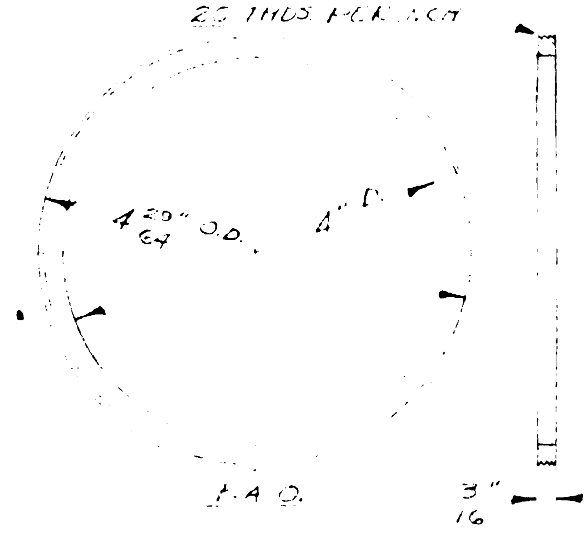
PART NO. 19



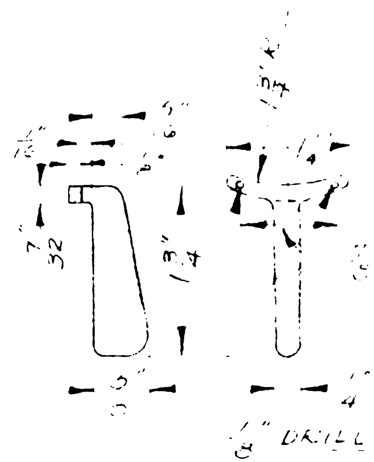
PART NO. 13



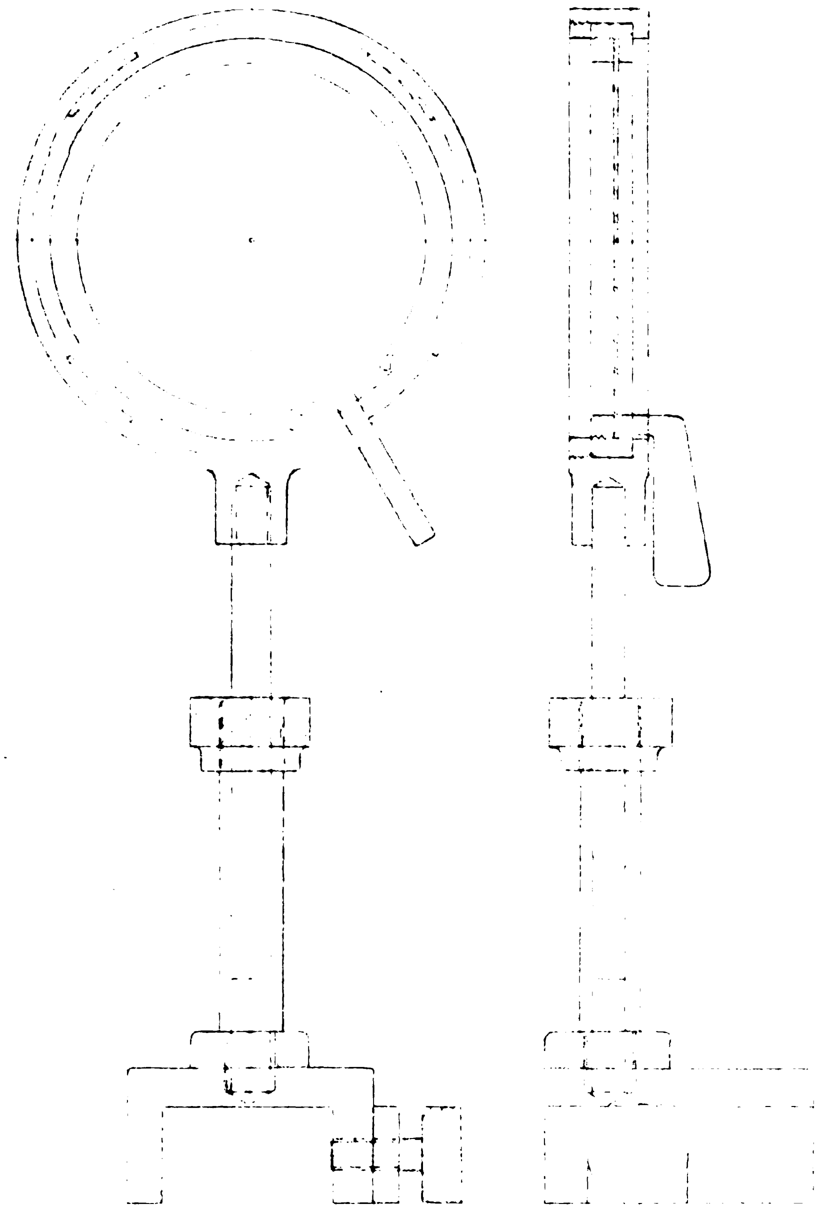
PART NO. 11



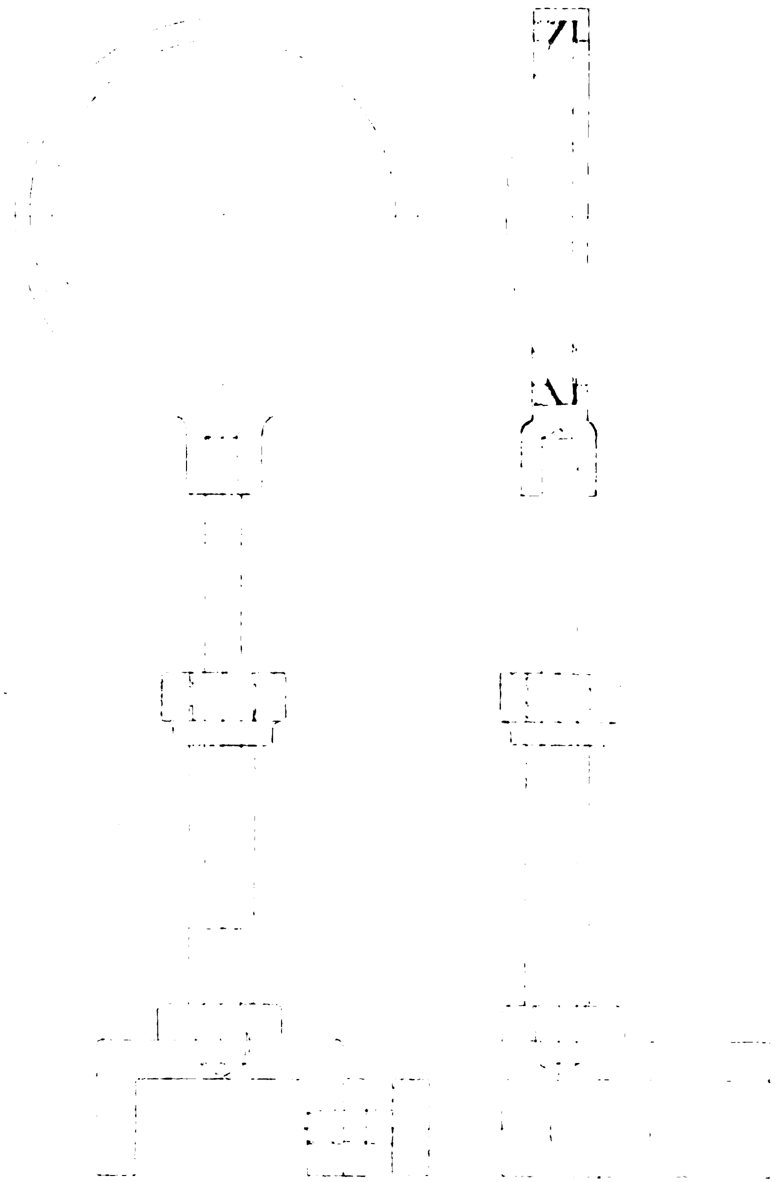
PART NO. 12

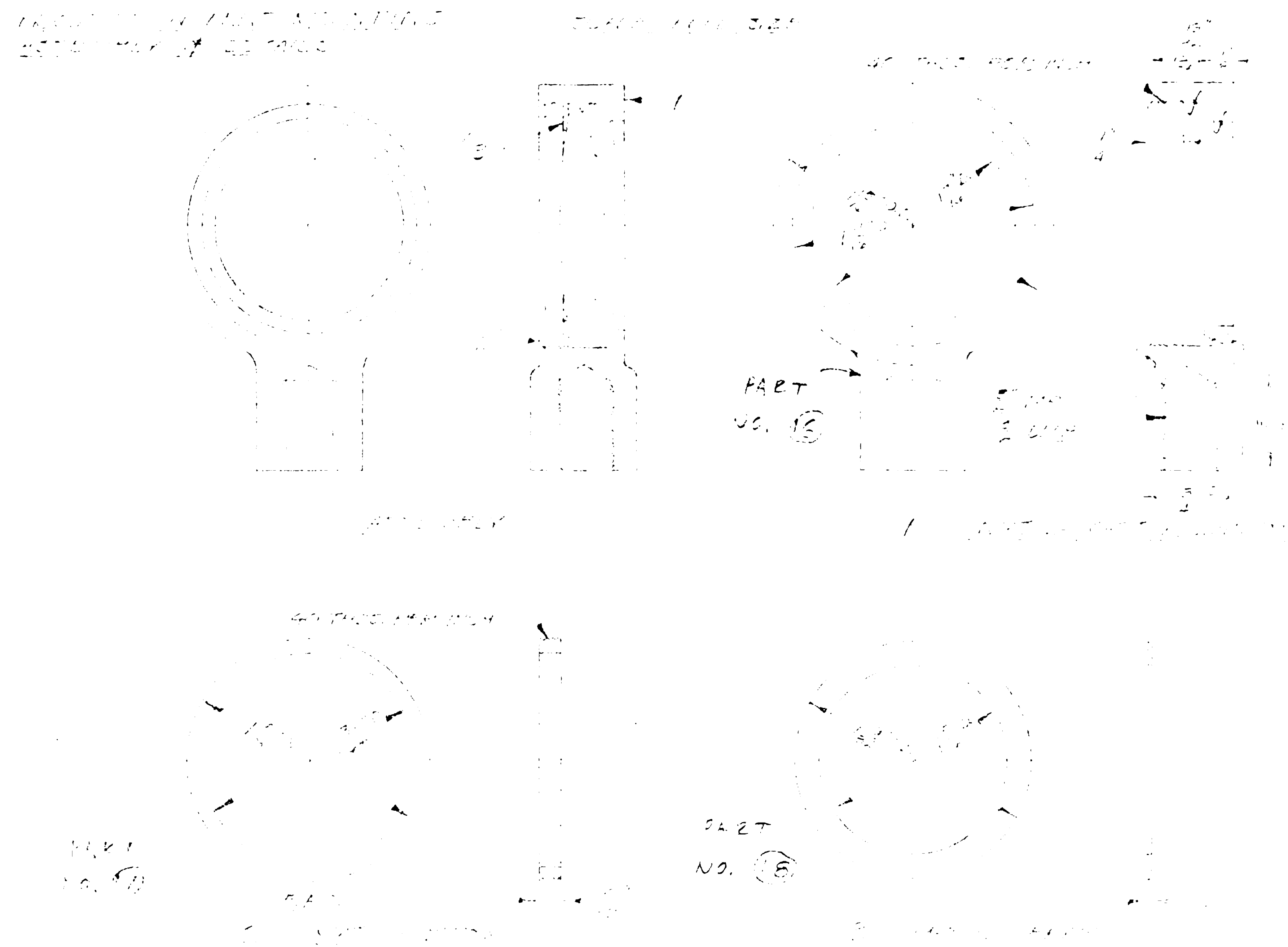


PART NO. 15



POLAROID & QUARTER WAVE PLATE
MOUNTING ASSEMBLY




PART
NO. 16

Light source.

Both white and monochromatic light sources were used. For the white light source, a 500-watt projection bulb with a pre-focus socket was used. The lamp housing is shown in the accompanying print.

The monochromatic light source used for taking pictures was kindly loaned by the Physics Department. The single unit consists of a mercury light, parabolic reflector, and other necessary parts. The unit produces a field of parallel rays. Hence no collimating lens is necessary, the unit being placed just before the first Polaroid disc. Suitable Wratten and Wainwright filters were used to obtain green light of 5461 A^{o}.

Details of the parts described are shown in the prints and photographs included in this section.

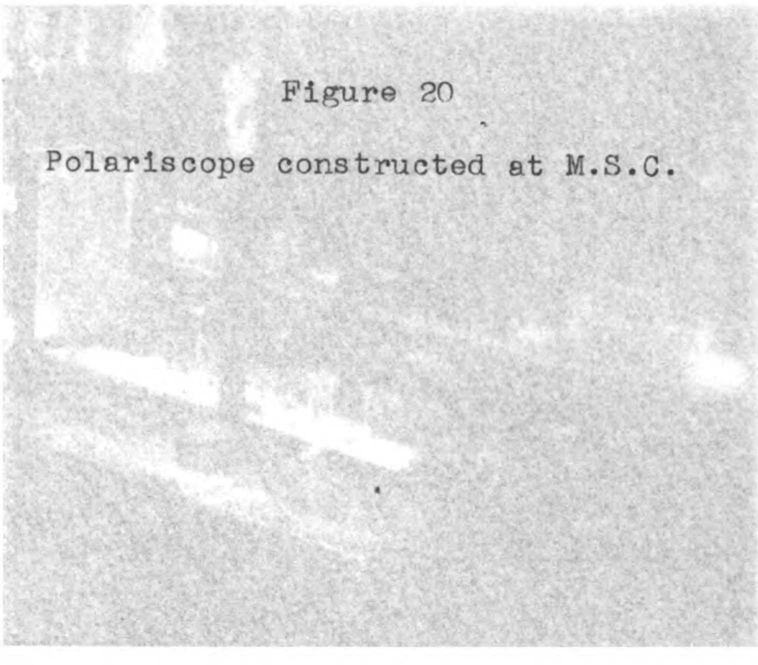


Figure 20

Polariscope constructed at M.S.C.

CHAPTER VI

THE MODEL;

MATERIALS, PREPARATION, LOADING

1. Materials.

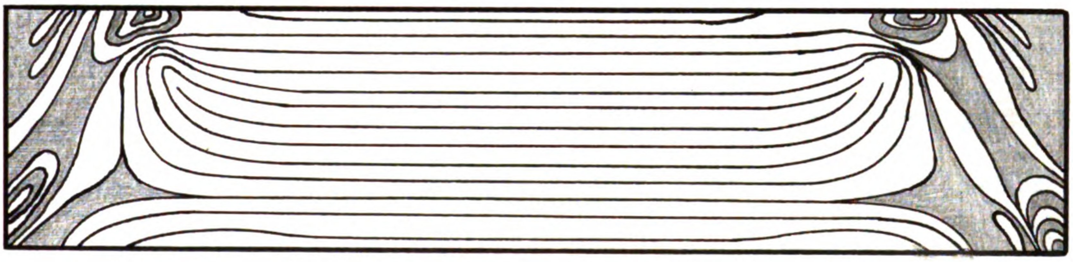
Several materials have been used for photoelastic analysis work, each having its advantages and disadvantanges. Materials used by early investigators included glass and celluloid, but these have very poor optical sensitivity. Recent materials include pyralin, marblette, and Bakelite. Bakelite has been used most widely in recent years, and the type BT-61-893 has been accepted as having the best general properties.

A list of desirable properties to consider has been listed by Filon (Ref. 3), Solakian (Ref. 44), and other investigators. Some of these properties may be listed here: (1) High optical sensitivity, (2) Good machinability, (3) Linear stress-strain relation, (4) Absence or easy removal of initial double refraction, (5) Little or no creep, (6) Little or no "edge effect", (7) High transparency. Mindlin (Ref. 33) has given a list of materials arranged in order according to their desirability from the viewpoint of optical sensitivity only:

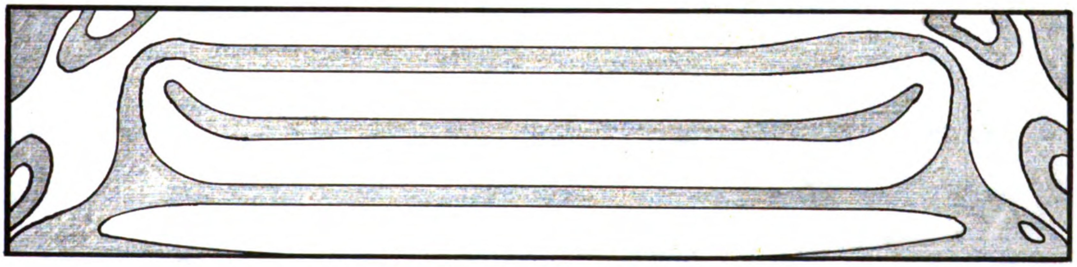
Phenolite
Bakelite
Celluloid
Pollopas
Charmoid
Vinylite
Glass

A newly developed plastic, Lucite, also ranges near the top of this list according to the producers. Marblette also has a high optical sensitivity, but has a lower elastic limit.

A material having a higher coefficient of optical sensitivity will give a pattern having more isochromatics, and hence gives more accurate data. Figure 21 shows a comparison between the optical sensitivity of Bakelite and celluloid, using two beams loaded under identical conditions.



BAKELITE



CELLULOID

Figure 21

2. Methods of preparation.

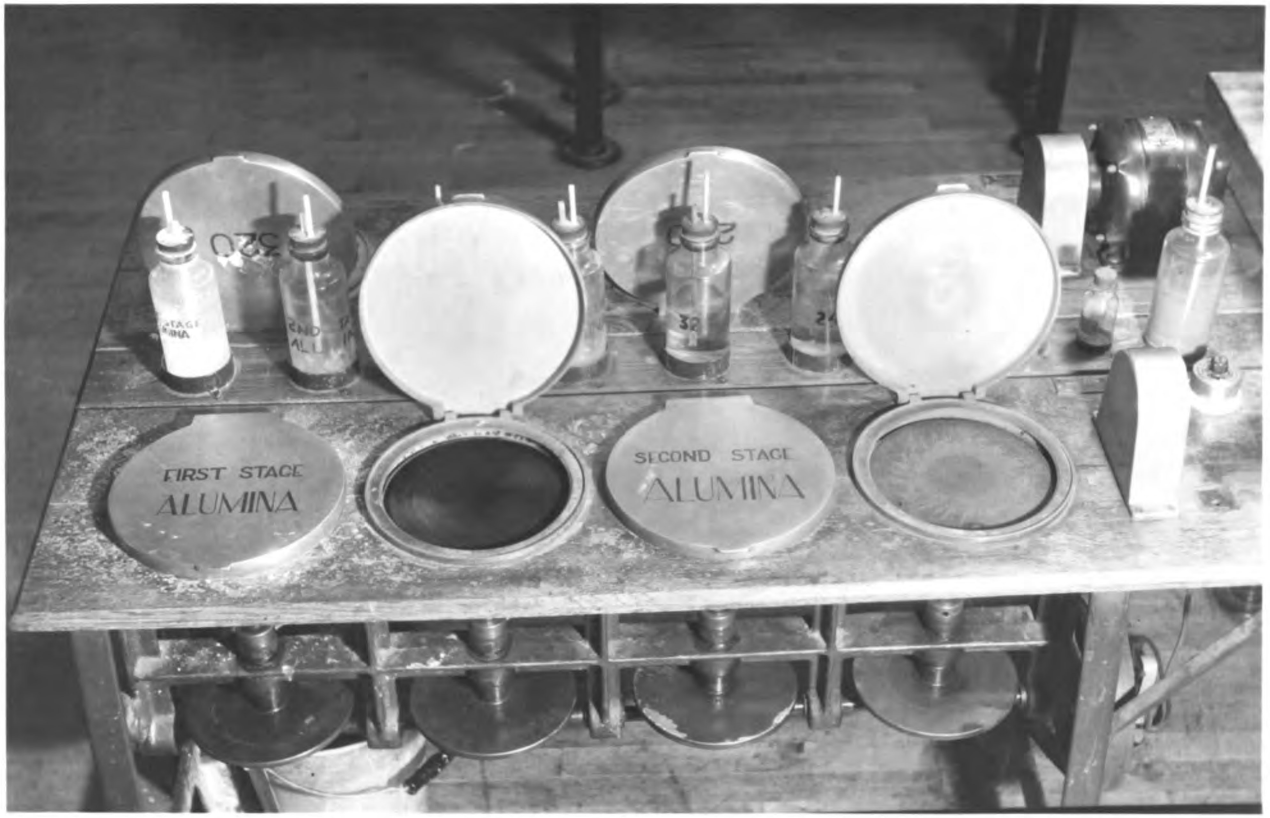
Bakelite may be obtained in sheets about 10" x 18" in unpolished form, $\frac{1}{4}$" being the usual thickness used. It may also be obtained, at a higher price, ready polished and annealed. In the latter case it is only necessary to mark out the model desired and carefully form it to shape.

The general procedure for this will now be described.

In the event that unpolished pieces are obtained, a piece should be cut from the sheet so that there is about 1/8" of material in excess of the finished dimensions. This is to obtain a perpendicular edge on the finished model, since polishing wears down the edge surfaces slightly. If the model blank is warped or has uneven surfaces, these should be made plane and parallel by surfacing on a milling machine, turning in a lathe, or if the condition is not excessive, by hand surfacing with emery cloth. Usually the unpolished sheets are sufficiently plane and parallel so that they may be polished directly.

The author has obtained good results by beginning the polishing process with a No. 240 metallographic polishing wheel, then working through No. 320, levigated alumina of two grades, and finishing with a rouge wheel. The 240 and 320 wheels were canvas covered, the remainder being velvet covered. Scratches may be eliminated by holding the model each time at right angles to the position on the preceding wheel. The model should be washed in water at room temperature when changing wheels. A wheel speed of about 250 R.P.M. is recommended. The models should always be polished wet, using the abrasive in suspension. In Figure 22 is shown a series of metallographic polishing wheels used by the author, which serve very well in polishing models.

Figure 22


FIRST STAGE
ALUMINA
SECOND STAGE
ALUMINA
320

A hand or power jig saw may be used for cutting the blank roughly to shape before polishing, but the final shaping must be done much more carefully. The excess material may be removed slowly on a jig saw fairly close to the finish line, the final cut being made on a milling machine using sharp cutters and lard oil lubrication. Holes should not be drilled, but either reamed, bored, or turned on a lathe. On models of irregular shape, good results may be obtained by careful filing. Mindlin (Ref. 33) gives three points to observe in finishing models: (1) Avoid heating the material, (2) Form sharp edges, (3) Have the boundary surfaces accurately normal to the faces of the model. The model will be useless if fast or heavy cuts are taken.

In some materials, notably Bakelite, there occurs a phenomenon known as "edge effect". This is due to a "drying out" of the edges which have just been cut. This effect causes a change in the optical properties of the material near the edges, thus causing an error. Figure 23 shows a portion of a model exhibiting edge effect. It is characterized by a hooking back of the fringes near the boundary. In some materials this effect appears within a few hours; in others not until several hours have elapsed. Hence it is best to use the model directly after it has been finished.

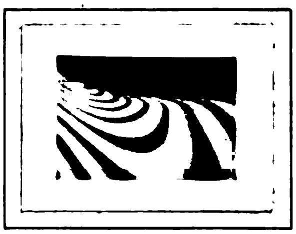


Figure 23

3. Annealing.

When the rough, unannealed sheets are used, the piece should be inserted between the crossed Polaroids and examined for initial stresses. There is usually some initial stress evident at the boundaries of the pieces. Ordinarily, the clear areas may be marked and the models carefully cut from these areas. If initial stresses are present in a piece to be used as a model, it is necessary to anneal the piece. The writer has found that a small electric oven of the type used for household cooking serves admirably for this purpose. However, to insure a uniform temperature and slow cooling rate, several fairly large pieces of metal were placed inside, and the oven covered with blankets. The oven may be brought up to heat fairly rapidly, using about three hours for this. A soaking temperature of about 200° F. may be used, the model being kept at this temperature for about three hours. The heat may then be turned off, the model being left to cool overnight. Before removing the model, it is well to leave the oven door open for about an hour before bringing the model out into the room. While in the oven, the model should be fully supported on a piece of asbestos board to prevent warping.

4. The loading frame constructed at M.S.C.

A desirable loading device should possess the following characteristics: (1) Capability of loading models with a wide variety of loading systems, (2) Accurate means of measuring the applied load, (3) Large, clear opening for passage of the optical field, (4) Provision for moving the loading frame so that different parts of a large model may be brought into the field.

The design of the loading frame constructed by the author is very similar to that of H. E. Wessman (Ref. 50), with the exception of being somewhat smaller. It consists of a frame sliding within a larger frame, thus making provision for bringing different parts of a large model into view. The inner frame is adjusted vertically by means of a handwheel and screw. Loading is by means of a lever supporting weights at its extremity. Models may be loaded in tension, compression, and under various types of bending. Materials used in constructing the frame were largely angle iron and strap iron. Details are given in the accompanying print. A photograph of the frame is shown in Figure 24.

5. Methods of loading.

From an inspection of the drawing and photograph, it may be seen that models may be tested in tension, compression, or under various types of bending. For models to be tested in tension, holes should be reamed to take a bushing and pin. Great care must be taken to see that a truly axial load is applied with no eccentricity. Various types of compression loading may be easily arranged. Precaution should be taken to see that the applied load or loads are exactly vertical and in good alignment. When testing beams and other models in bending, distances should be carefully and accurately determined. For com-

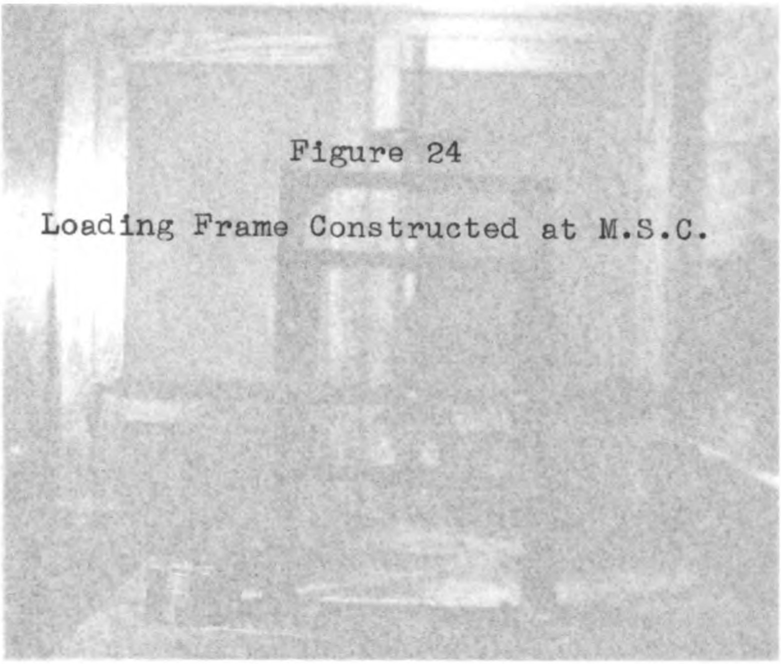


Figure 24

Loading Frame Constructed at M.S.C.

pression and bending tests, the model should be checked to see that it remains vertical when under a heavy load.

In many cases where it is desired to load a model at several points, loading may be accomplished by looping music wire over the model at the point or points desired and loading directly with known weights. For side loading, music wire may be used with turnbuckles and small spring balances.

CHAPTER VII

PHOTOELASTIC DETERMINATION OF STRESSES

1. General.

In making photoelastic stress determinations, the first step is to make a suitable model of the structure or part to be studied, the model being made of Bakelite or other similar material. This is then placed in the loading frame and loaded in the same manner as is the prototype. Examination with white light in a circular polariscope will give a brightly colored pattern consisting of colored bands, called isochromatics. If a monochromatic light source is used, the pattern will consist of black and white fringes, as explained in Chapter IV. Evaluation of this pattern will give the shear and boundary stresses.

If the $\frac{1}{4}$-wave plates be removed from the circular polariscope, a new set of lines will appear, these being the isoclinic lines. These lines are always black, regardless of the light source, and will be superimposed on the isochromatic pattern. However, the isoclinics may be unmistakably recognized, and are easily discerned from the isochromatics. The direction of the isoclinics are independent of the load, and will have the same direction at all loads for any particular setting of the Polaroids. The directions of the isoclinics will vary as the Polaroids are rotated angularly, this being for a constant load. Directions of the principal stresses are determined from the isoclinic lines.

The above material has been given in Chapter IV, but is included here briefly as a review before discussing evaluation of the patterns.

2. Interpretation of fringe value and stress pattern.

The general shear formula is

$$v_s = \frac{P - Q}{2} \sin 2\phi$$

It was explained in Chapter III that this shear stress becomes a maximum when sin 2ϕ is 1. This occurs when $2\phi = 90^\circ$, or $\phi = 45^\circ$, when the above formula becomes

$$v_{max} = \frac{P - Q}{2}$$

Photoelastic analysis is based on the fundamental stress-optic law, which states that in a model of constant thickness, the fringe order is constant at all points where the principal stress difference is constant.

This is expressed mathematically as

$$n = ct(P - Q) \qquad \text{(7-1)}$$

where n is the fringe order, t is the thickness of the model, P and Q are the principal stresses, and c is a constant to be determined.

Application of this law to the interpretation of stress patterns may now be discussed.

Equation (7-1) may be rewritten:

$$P - Q = \frac{n}{ct}$$

Along a particular fringe n is constant, and hence along a locus of such points,

$$P - Q = K$$

From this, it may now be said that a fringe represents the locus of points at which there is a constant difference between the principal stresses.

It will be remembered that the maximum shear stress is given by

$$v_{max} = \frac{P - Q}{2}$$

or, rewriting this:

$$P - Q = 2v_{max}$$

If this last expression for P - Q is substituted in the stress-optic equation (7-1), there results:

$$v_{max} = \frac{n}{2ct}$$

or

$$v_{max} = Fn$$

where F is given by

$$F = \frac{1}{2ct}$$

F is known as the fringe value of the model, and is a constant for any one material or sheet. Methods of determining F will be given later.

It may now be seen that where n is constant, as along a fringe, v_{max} is also constant; hence a fringe may now be defined as the locus of points of equal maximum shear stress.

It may also be seen from this equation that the maximum shear stress is directly proportional to the fringe order. Hence for a point in a model at which several fringes have passed by during loading, the stress will be greater than for a point at which only a few fringes have passed, the stresses at the two points being proportional to the number of fringes passing each point. For example, if 12 fringes have passed a certain point, the stress for that point will be 3 times as great as that for a point where only 4 fringes have passed.

3. Points of concentrated stress.

In some cases sufficient information regarding points of high stress concentration may be obtained by merely examining the pattern and making no quantitative analysis. From the preceding article it is evident that areas of high stress concentration may be observed as areas containing fringes which are closely spaced, as at sharp corners, fillets, etc. Likewise areas of low stress are those containing only a few widely spaced lines. Weight may be saved by merely cutting these areas down.

4. Calibration and adjustment of the polariscope.

It is recommended that a test model be made similar to that suggested by Orton (Ref. 39) and given in Figure 25 on the following page.

The holes should be fitted with steel bushings, which are smooth and a snug fit. The model is loaded vertically in tension in the center of the field. Care should be taken to see that there is little or no eccentricity of loading.

As load is applied slowly on <u>any</u> model, each point will alternate in brightness and darkness as the black

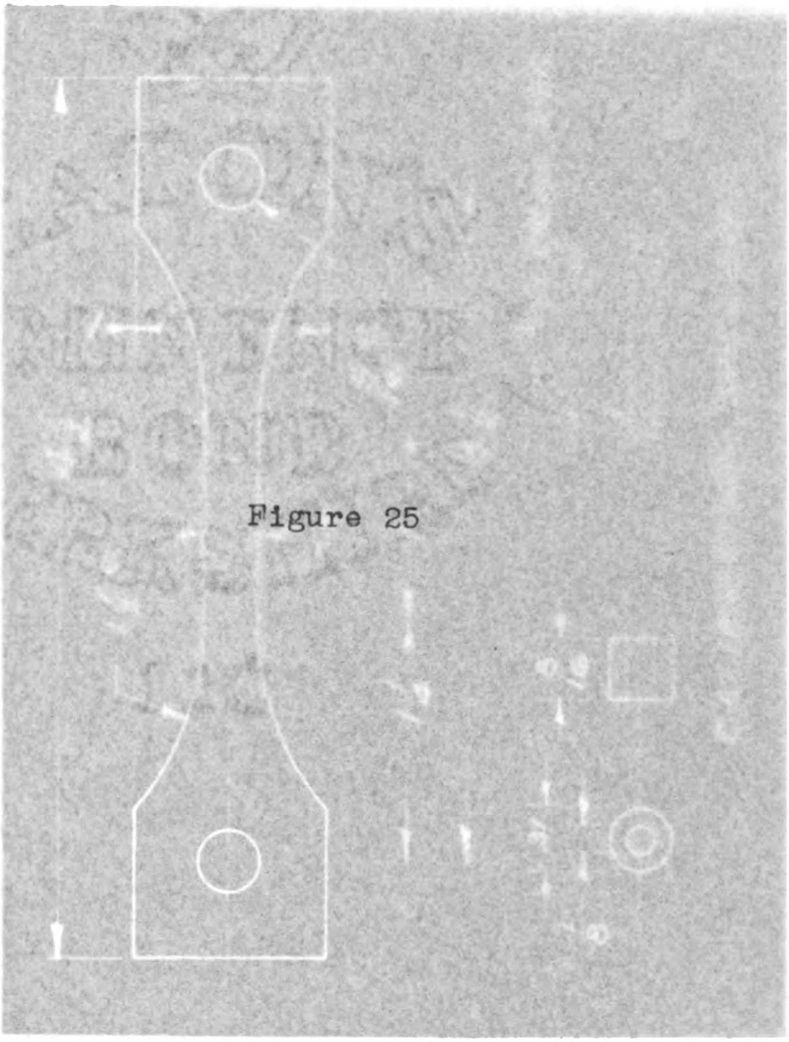
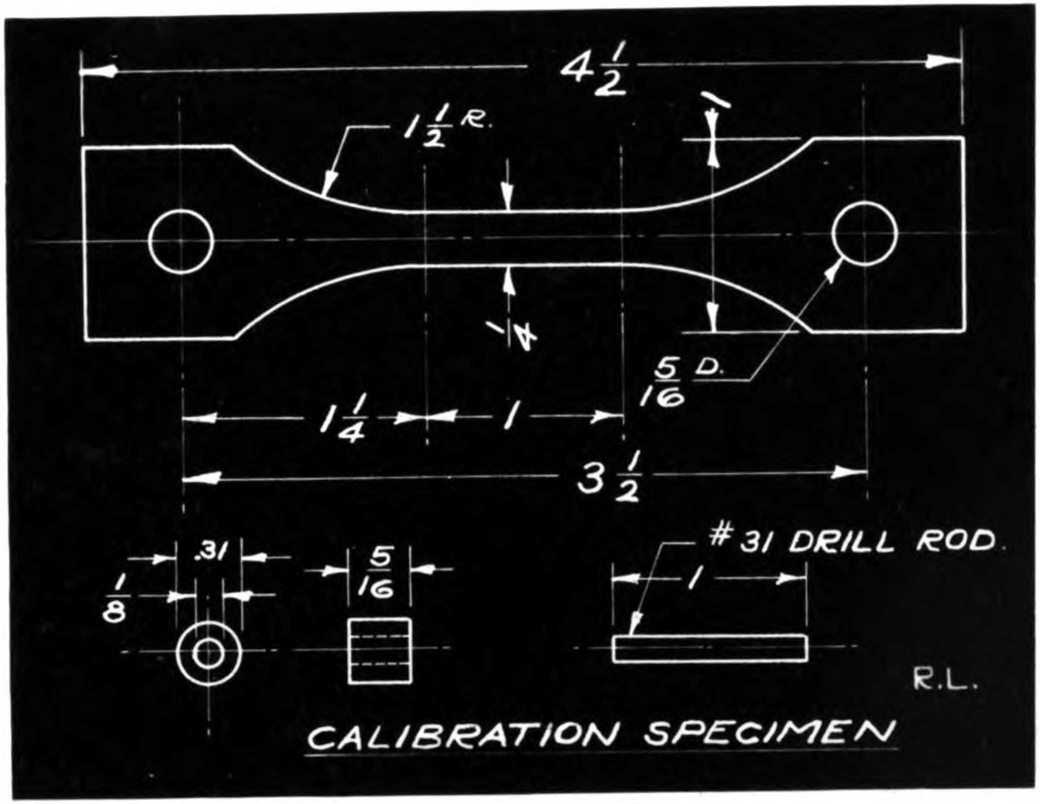
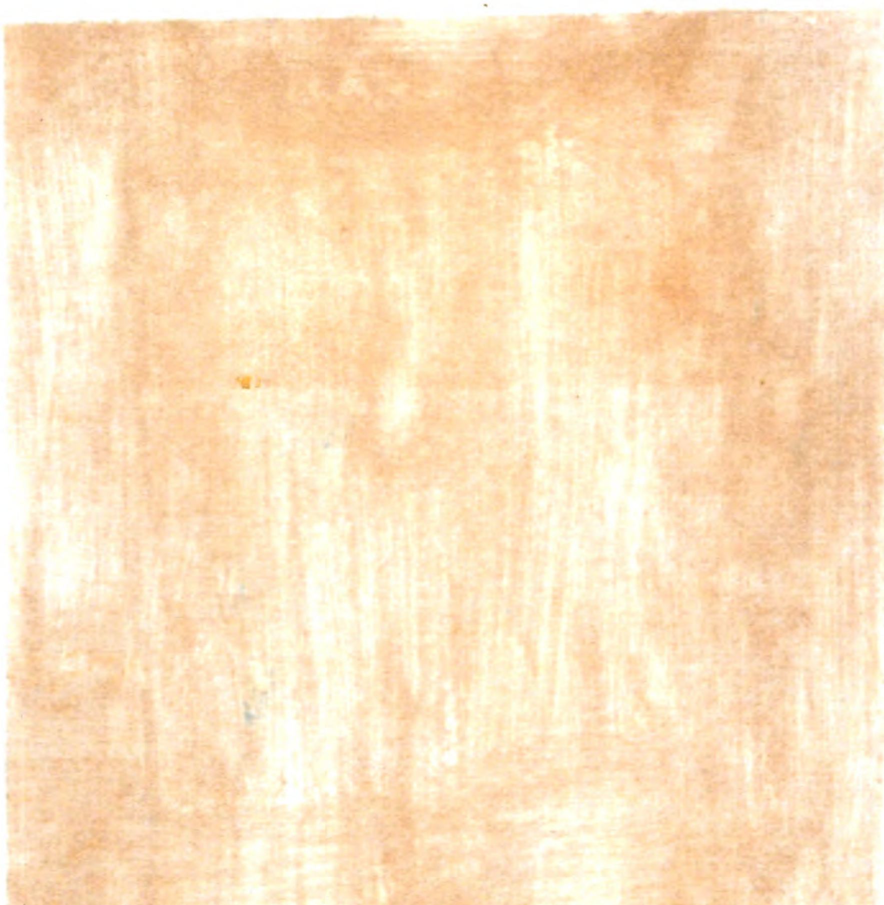


Figure 25



4 1/2
1 1/2 R.
1
1/4
5/16 D.
1 1/4
1
3 1/2
31 DRILL ROD.
.31
1/8
5/16
1
R.L.
CALIBRATION SPECIMEN



and white fringes pass the point. A diagramatic repre-

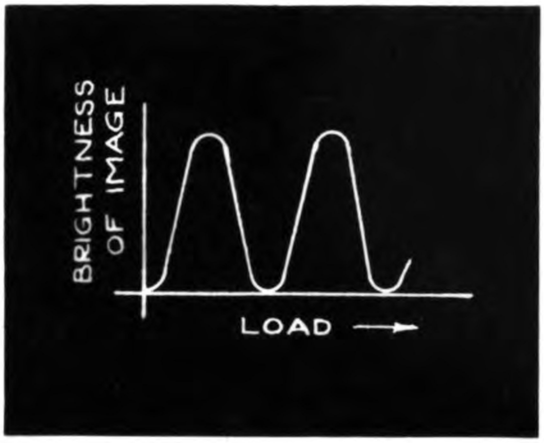


Figure 26

sentation of this variation in intensity is given in Figure 26 above.

The same will now occur when the specimen used for adjusting is loaded. As load is applied slowly, the model will brighten at its midpoint, then darken again. When maximum darkness occurs again, the model will at this time be stressed to the first order. By dividing the load by the area of the mid-section (width x thickness), the material will be calibrated and the fringe value determined. This first value obtained should not be used, but the model should be loaded until several fringes have passed, and an average value computed. This completes the calibration of the material. In the next article, a second method of calibrating will be described.

To adjust the polariscope, the same specimen may be used. If Polaroids are used, the directions of the polarizing axes will be known when purchased. The polarizer and analyzer should be set at the 45° position (polarizer and analyzer are always set at right angles to each other), which will give maximum light at the midpoint of the specimen. If the specimen now be loaded so that the second order fringe is centered about the centerline of the model and then backed off to one-fourth the distance from the first to the second order, the effect of a $\frac{1}{4}$-wave plate will be produced, since the vertical component emerges from the specimen a $\frac{1}{4}$-wave length ahead of the horizontal component.

This temporary $\frac{1}{4}$-wave plate may now be used in checking the plates of the polariscope. One $\frac{1}{4}$-wave plate may be placed in position and rotated until maximum darkness is obtained at the center of the specimen. In order to cancel the $\frac{1}{4}$-wave advance occurring in the specimen, the retarding axis of this $\frac{1}{4}$-wave plate must therefore be vertical. By loosening the clamping ring, the graduated ring should now be set so that the zero is horizontal. This will be the advancing axis of the plate.

The zero of the adjusted $\frac{1}{4}$-wave plate may now be moved to the vertical position, the second plate placed in the polariscope and rotated until the field on the screen is at maximum darkness. (intensity of the specimen is disregarded for this). The retarding axis of this second plate will then be vertical, and the graduated ring should now be set with the zero horizontal, as before. Adjustment is now complete.

To prepare the polariscope for use, the $\frac{1}{4}$-wave plates should be removed, the polarizer set vertical and the analyzer horizontal, thus giving maximum field darkness. The first $\frac{1}{4}$-wave plate may then be placed in position with the axis set at 45°. The second $\frac{1}{4}$-wave plate, when placed in position, should be rotated until maximum darkness is again obtained. The polariscope is then ready for use.

In using the tension strip, bushings are used as shown in Figure 27, these bushings having slots for accurately centering the load.

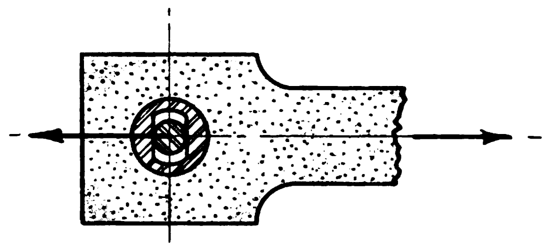


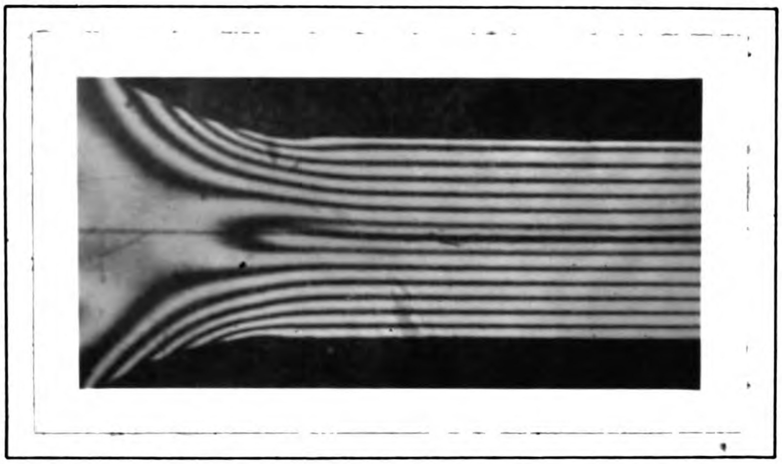
Figure 27

5. Calibration by bending.

It is usually difficult to accurately center the tension strip just described, and a small eccentricity will cause a large calibration error. In the beam method to be described, all that is needed is a reasonably accurate determination of the load positions with assurance of their symmetrical disposition about the center of the beam.

In calibrating by the beam method, a piece of the shape shown in Figure 28 on the following page is first made, with four bushings placed tangent to the center line. When loaded as shown, the middle portion of the beam will be subjected to a constant bending moment and

Calibration Beam in Bending



zero vertical shear. The isochromatics, or fringes, will be straight, horizontal, and parallel. When loading begins, the fringes start at the beam boundaries

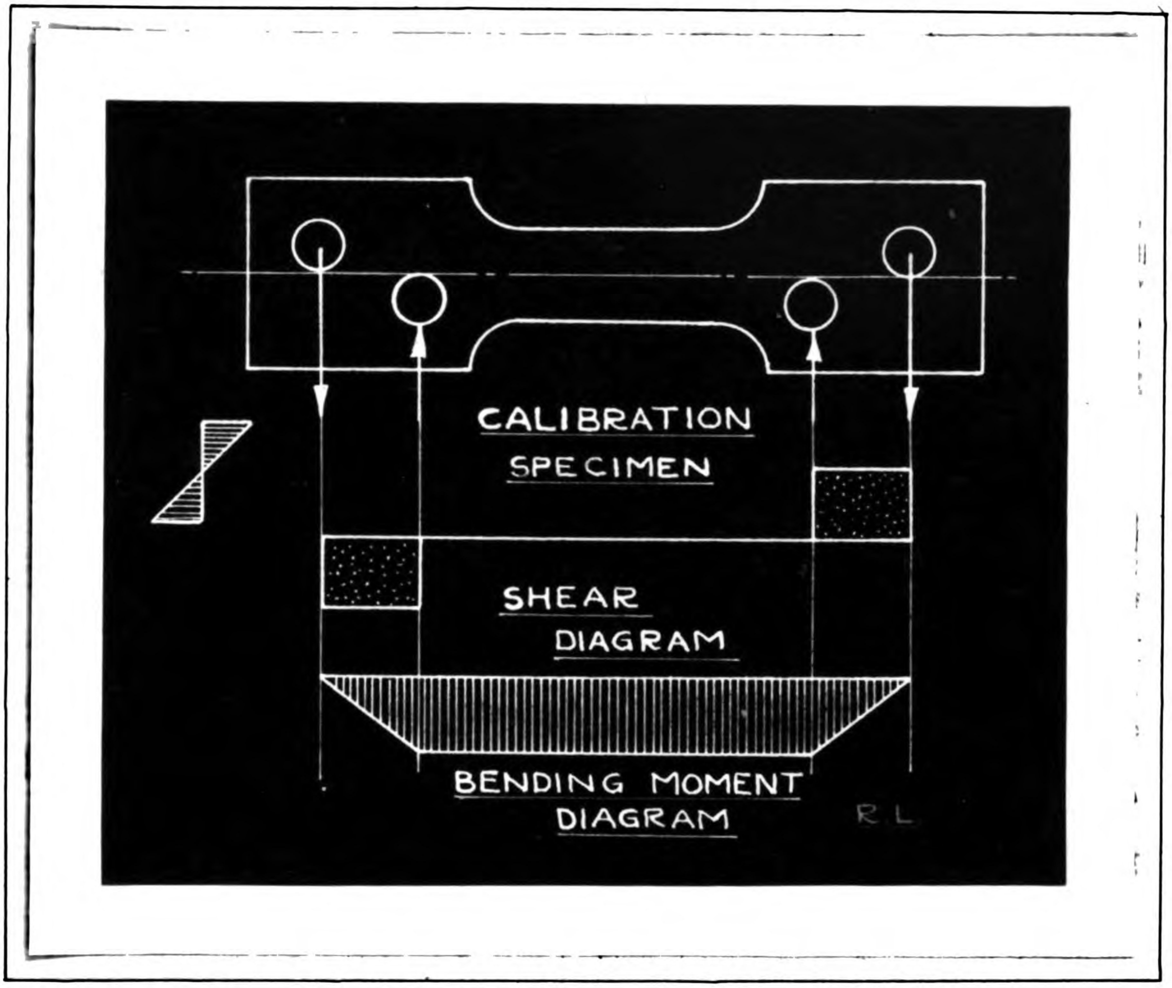


Figure 28

and progress toward the neutral axis of the beam. By counting the number of fringes which pass the boundary and relating this to the stress obtained from the formula $s = \frac{MC}{I}$, the fringe value, or stress which each fringe represents, is obtained.

6. Isochromatics.

Isochromatics are lines of constant color and are generally referred to as fringes when monochromatic light is used.

In any stressed body, there exist principal stresses at every point. In photoelastic analysis using transparent materials, interference fringes are produced by the polarized light, these fringes being proportional to the difference between the principal stresses, this being denoted by (P-Q).

To determine the maximum shear stress at any point, the fringe order is first obtained. This is simply the number of fringes which have passed that point during loading. This fringe order is then multiplied by the fringe value, or stress per fringe as determined from the calibration beam. The value obtained is the difference between the principal stresses, (P-Q). The maximum shear stress is $\frac{P - Q}{2}$ and hence can be obtained immediately by dividing the value found above by 2.

Thus the maximum shear stress at any point or along any section may be rapidly and easily determined.

7. Boundary stresses.

Consider a small element in the boundary of a body, the boundary considered being free from external loads at that point. Since there can be no stress normal to the boundary and the shear stress on one face of the element is zero, it follows that one of the principal stresses is zero. Hence at a free boundary there is only one principal stress, its direction being tangent to the boundary.

This may be shown as follows: Writing the fundamental equation:

$$P - Q = \frac{n}{ct}$$

and remembering that

$$F = \frac{1}{2ct},$$

it follows that

$$P - Q = 2nF.$$

For a free boundary condition, Q may be set equal to zero. This gives:

$$P = 2nF.$$

Hence tensile or compressive boundary stresses may be easily and directly evaluated.

8. Isoclinics; principal stress directions.

If the $\frac{1}{4}$-wave plates are removed from the polariscope, a few black bands will appear on the screen. These lines are termed isoclinics. Although the isochromatics change position with a change in load, the isoclinics are independent of the load (excepting zero load) for any particular setting of the polarizer and analyzer. If the polarizer and analyzer be rotated, the directions of these lines will be seen to change. As explained in Chapter IV, these lines are loci of points whose principal stress directions coincide with the axes of polarizer and analyzer for each setting.

The first step in obtaining the directions of the principal stresses is to set polarizer and analyzer to their zero positions; i.e., polarizer axis vertical, analyzer axis horizontal. The center lines of the isoclinics for this setting are then traced on a sheet of paper which may be attached to the screen. Rotating polarizer and analyzer 10° will give rise to new positions of the isoclinics. These may then be traced on the paper. Each set of lines obtained should be identified by marking the setting for which they were obtained.

After the isoclinic pattern is obtained, the directions of the principal stresses are drawn in. The isoclinics themselves are <u>not</u> principal stress directions, but each isoclinic gives the angle which a principal stress direction forms with the X-axis for a particular point on the isoclinic. An easy manner in which to perform the work is to place a series of small crosses inclined at the proper angle along each iso-

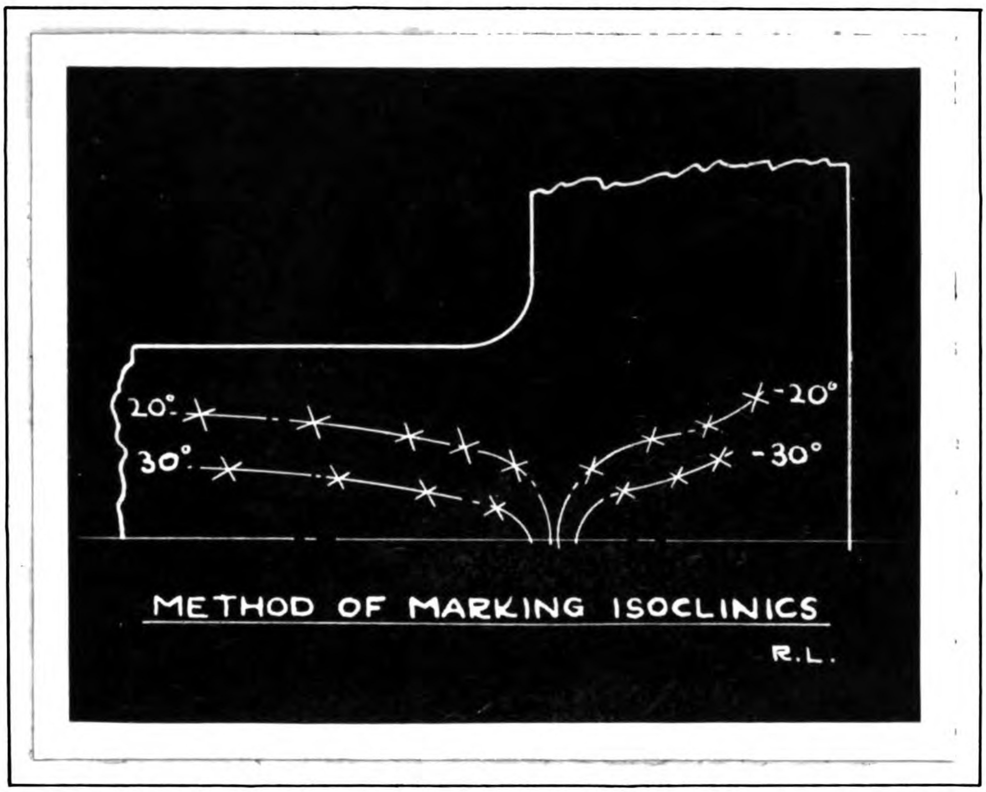


Figure 29

clinic as shown in Figure 29, on the preceding page. For example, all the crosses along a 10° isoclinic would be made inclined at 10° to the axes. A protractor-triangle has been found to be extremely helpful in this work.

Lines may now be drawn through the inclined crosses, forming a series of lines crossing each isoclinic at the proper angle, as shown in Figure 30.

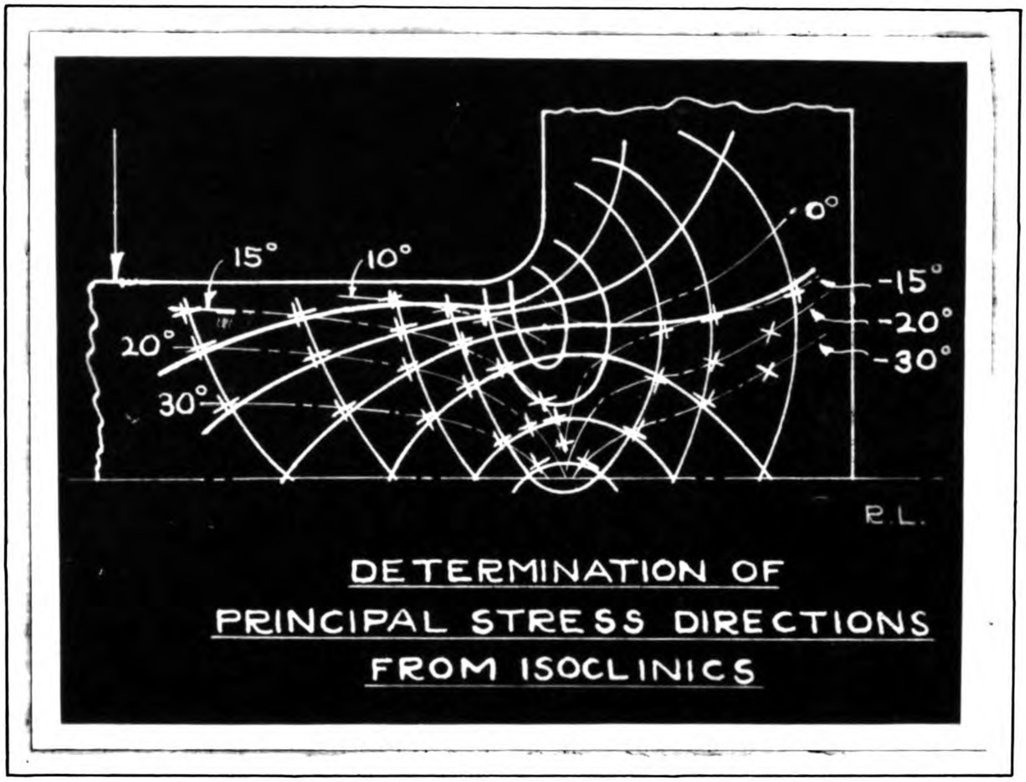


Figure 30

The resulting network of lines will be the directions of the principal stresses. It should be remembered that the complete network consists of <u>two sets</u> of lines, each set always intersecting the other set at right angles at all points. These lines will also be always parallel and perpendicular to the boundaries.

The above illustrations are for a cantilever beam. Figure 31 shows a tension strip, the lower half showing

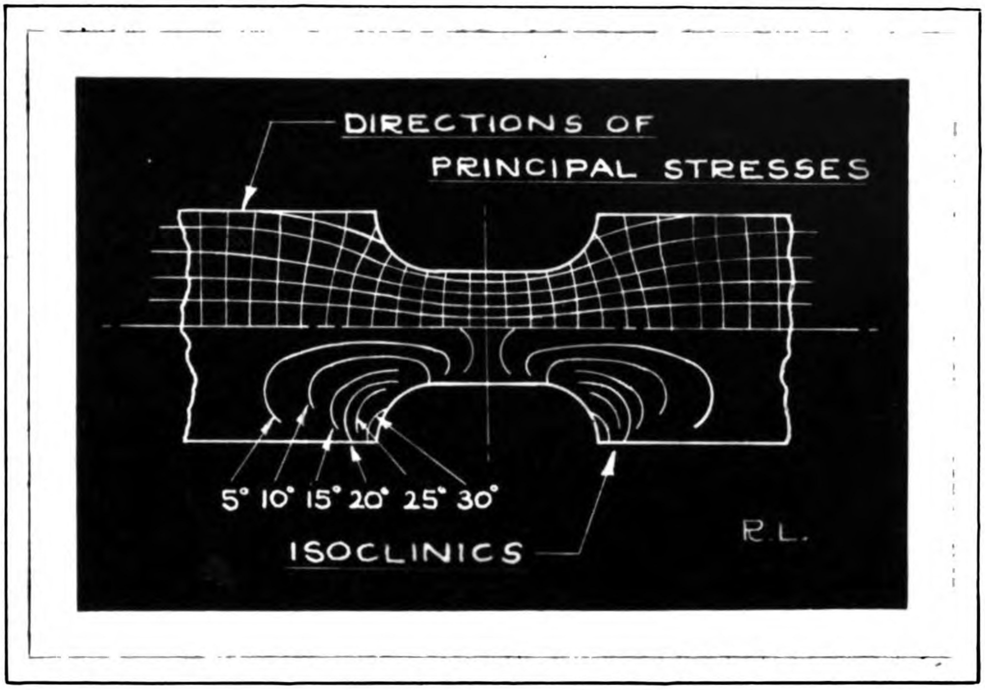


Figure 31

the isoclinics, the upper half the principal stress directions.

9. Shear lines.

Since the maximum shear stresses are at 45° to the principal stresses, the directions of the shear stresses may be obtained from the network of principal stress directions by simply drawing a set of lines which intersect the principal stress directions at 45° at all points.

10. Principal stresses.

For a great majority of investigations, the information obtained from the procedures already described are sufficient. These include shear stresses and tensile and compressive boundary stresses. Principal stress directions are also easily obtained and are sometimes of interest.

However, there are occasional problems in which the amounts of the principal stresses at interior points are desired. This involves a more extensive procedure, and calls for additional equipment. The process is also time-consuming.

A brief summary of the more important methods will be given here. These methods have been well covered in recent literature.

(a). Lateral extensometer method.

To find the individual principal stresses P and Q, it is first necessary to determine the principal stress difference P-Q for those regions in which investigation is desired. To separate P and Q, it is necessary to obtain a quantity (P + Q), so that this may be combined with the (P - Q) values found for a point. It was first discovered by Mesnager that the strain normal to the model is proportional to the sum of the principal stresses. Hence (P + Q) is constant along lines of constant thickness, these lines being determined by use of a lateral extensometer. For a Bakelite sheet $\frac{1}{4}$" in thickness, an increase of one fringe produces roughly a displacement of about 0.00001 in. It will be seen that this is a delicate and tedious method.

(b). Interferometer method.

These methods are based on interference phenomena of light reflected from the surface of a steel model which is shaped and loaded in the same manner as the Bakelite model.

Figure 32 shows the equipment used by Max M. Frocht, Ref. 6.

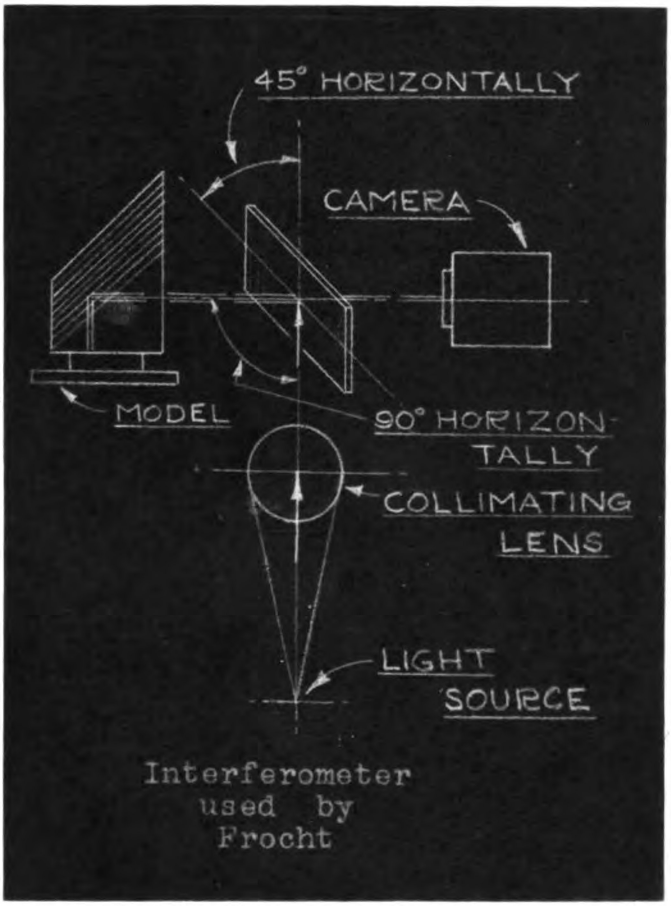


Figure 32

In Figure 33 on the following page is shown the interferometer as used by Brahtz and Soehrens, (Ref. 2).

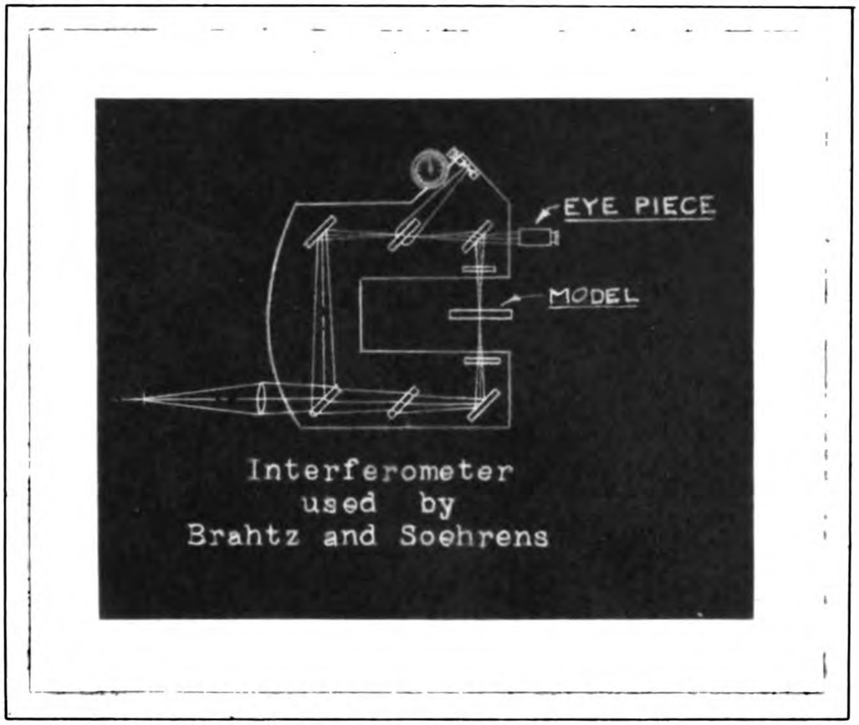


Figure 33

These methods are based on the fact that each interference fringe appearing in the interferometer will represent a path of constant thickness, and hence a path of constant (P + Q). Thus the (P + Q) values of points on these lines may be combined directly with the corresponding points whose (P - Q) values are known. This will yield the amount of each individual principal stress at the point desired.

The disadvantage of this method is the cost of interferometer equipment.

(c). Membrane method.

In the application of this method, a plate or box is constructed so that an opening is formed in a flat surface, the opening having the same shape as the model being investigated. A membrane is then stretched over the opening and fastened along the boundaries such that the ordinates of the membrane are proportional to the (P + Q) values at those boundary points. It may then be shown that the ordinate of any point on the interior region of the membrane is proportional to the (P + Q) value at that point. Combining this with the (P - Q) value of the point will separate the principal stresses.

Weibel (Ref. 40) has obtained good results from this method by using a soap film as a membrane. McGivern and Supper (Ref. 23) used the same method making use of a thin rubber sheet as a membrane.

This method is an easy one to apply, but is limited to regions near those boundaries along which (P + Q) can be previously determined.

(d) Numerical and Graphical methods.

These methods of separating the principal stresses for interior points require no additional photo-elastic equipment or laboratory procedure after the (P - Q) stresses have been determined and the directions of the principal stresses obtained.

Numerical method. Liebmann has developed a process in which Laplace's equation is solved for certain boundary values by successive numerical approximations. Shortley, Fried, and Weller (Ref. 9, Bulletin) have improved on the process by devising means for obtaining a more rapid convergence of the successive approximations.

Among others developing similar lethods are Poritsky, Snively, and Wylie, (Ref. 29), Neuber, (Ref. 26), and Frocht, (Ref. 8).

11. Stress concentration factors.

It is well known that sharp corners, fillets, small holes etc. have a marked tendency to greatly increase the stress concentration wherever they occur. The factor of stress concentration is defined as the ratio of the maximum stress to the average stress for a particular region, and is denoted by K:

$$K = \frac{S_{max}}{S_{ave.}}$$

Photoelastically, these may be determined by a comparison of fringe orders, these being directly proportional to the stresses:

$$S_{max} = 2Fn_{max}\,, \qquad S_{ave} = 2Fn_{ave}.$$

Substituting:

$$K = \frac{n_{max}}{n_{ave}}$$

Thus the stress concentration factor may be obtained directly from the stress pattern, no knowledge of loads, dimensions, or material being necessary.

Stress concentration factors are usually plotted against the ratio r/d, r being the radius of the fillet, groove, or hole, and d being the depth of the piece at that point. Frocht has plotted these curves for investigations of several types (Ref. 10 & 11).

It is interesting to note that as the ratio r/d decreases, the stress concentration rises to extremely high values.

12. Recent developments.

The photoelastic method is now being applied with success to three-dimensional problems (Ref. 8, 13, 15, 18, 28). Dynamic problems, such as impact stresses, stress conditions in rotating discs etc. are being studied by motion picture and stroboscopic means.

13. Stress pattern photographs.

On the following sheets are shown photographs taken by the author of typical models loaded as described. All models are of Bakelite. Photographs were taken with monochromatic light (5461 A^{o}) on Wratten and Wainwright Metallographic plates. Azo No. 4 printing paper was used in making prints.

Figure 34

Tension Strip with Circular Hole

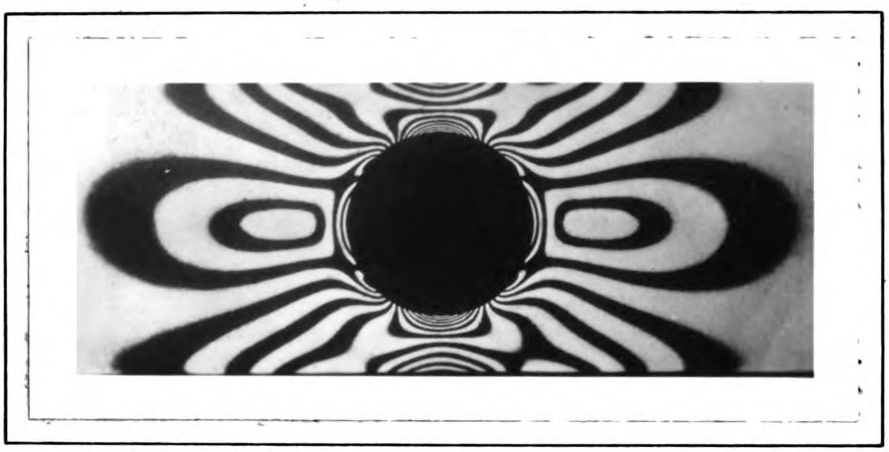


Figure 35

Calibration Beam in Bending

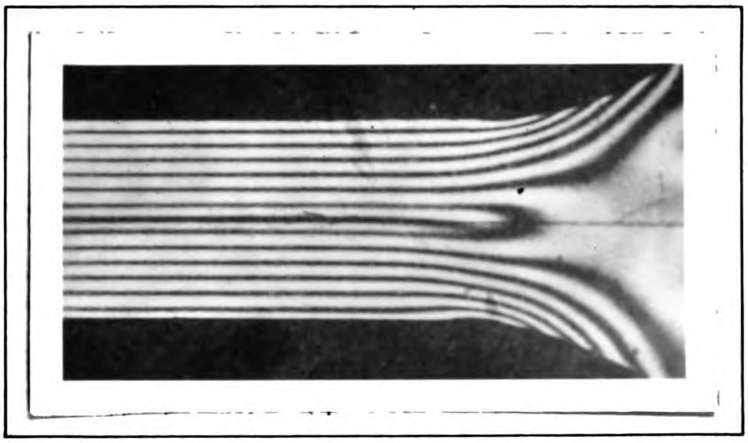
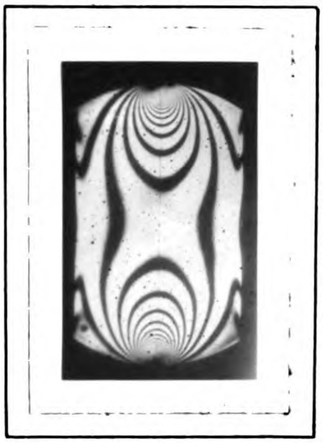


Figure 36

Rocker Loaded Vertically



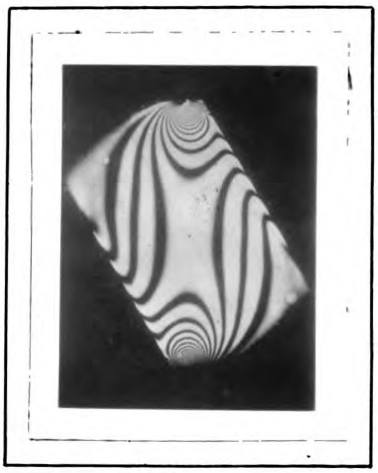
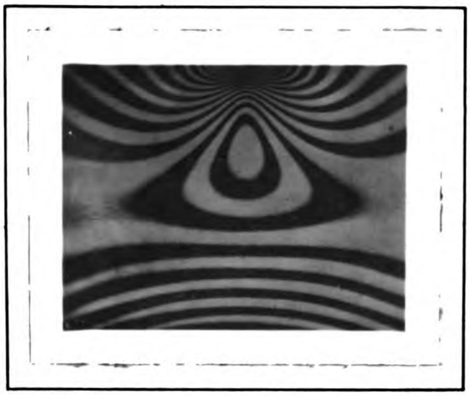
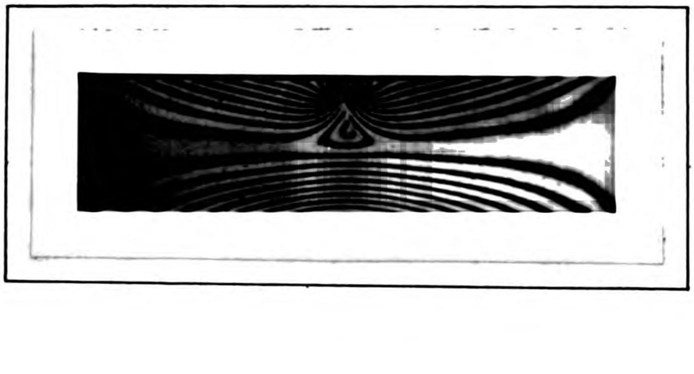


Figure 37

Centrally Loaded Beam





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