AN ANTENNA CALCULATOR

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

William Merton Nellis

1950

This is to certify that the

thesis entitled

AN ANTENNA CALCULATOR

presented by

William Merton Nellis

has been accepted towards fulfillment
of the requirements for

M.S. degree in E.E.

Major professor

Date May 25, 1950

AN ANTENNA CALCULATOR

by

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A THESIS

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Abstract

The basic theoretical considerations, the practical design, construction, and test of an electronic device for plotting the horizontal radiation pattern of a two tower vertical antenna array are presented in this paper.

The antenna calculator produces a voltage that changes in magnitude as does the magnitude of field strength about an antenna by adding a constant voltage whose envelope varies like the field strength about an antenna. This voltage is detected and displayed on a cathode ray oscilloscope with a circular sweep to give the polar radiation of an antenna.

The wide angle phase modulation is produced by pulse methods and frequency multiplication.

The system is suitable for demonstration of the principles of antenna radiation patterns.

References:

1. Reference Data for Radio Engineers, 3rd Edition, Federal Telephone and Radio Corporation.

2. Proceedings of the I.R.E., Dec. 1946, "The Antennalyzer", Brown and Morrison.

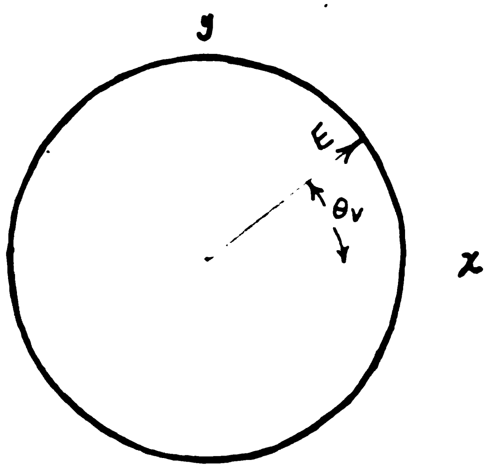
Table of Contents

An Antenna Calculator

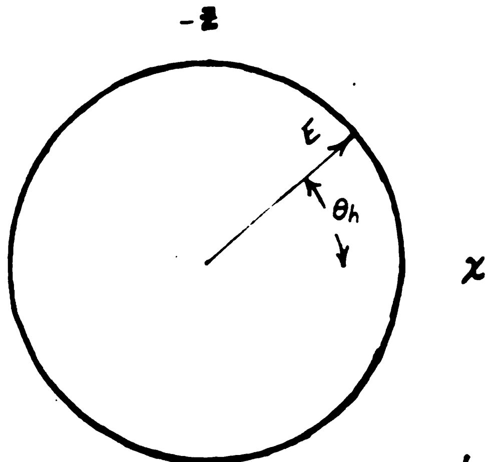
--for antenna directional characteristics--

The Problem

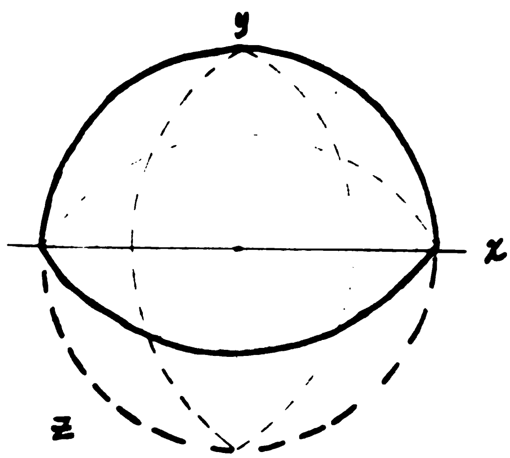
The intensity of radiation from an antenna or an array of antennas is generally investigated at some fixed distance from the antenna and in all directions about the antenna. A polar plot of this intensity about the antenna at some fixed distance is the radiation pattern of the antenna. Such a radiation pattern will give information regarding the ability of the antenna to radiate in any specified direction. Although the radiation pattern as defined above would necessitate a three dimensional system to completely describe it, a more easily obtained and equally useful radiation plot is that of the horizontal or vertical radiation pattern. These patterns are obtained by measuring strength of radiation at a fixed distance in any direction on a horizontal or vertical plane through the antenna. This type of pattern being a plane plot can be easily shown on paper and, to illustrate such patterns, Fig. 1 shows the horizontal and vertical pattern of a point source or isotropic antenna. The pattern in the horizontal plane is the same as that in the vertical plane and thus the complete pattern would be expected



a. Vertical



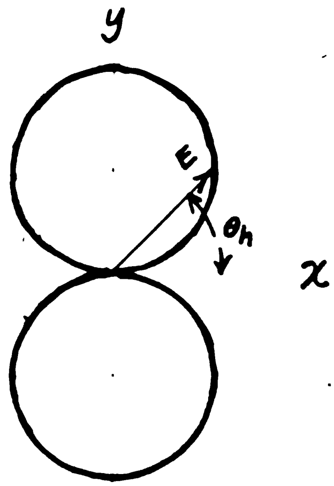
b. horizontal



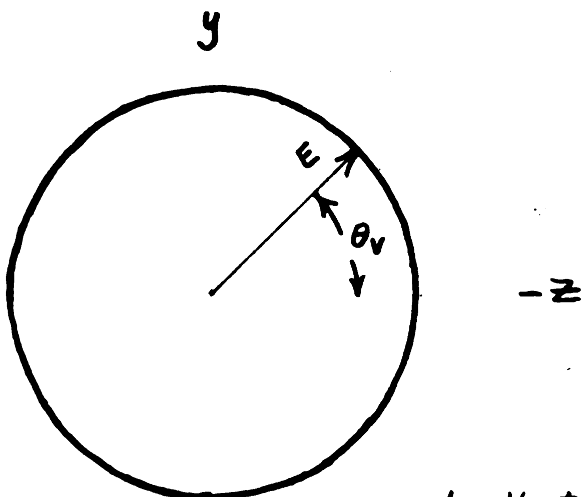
Total Pattern

E vs. θ

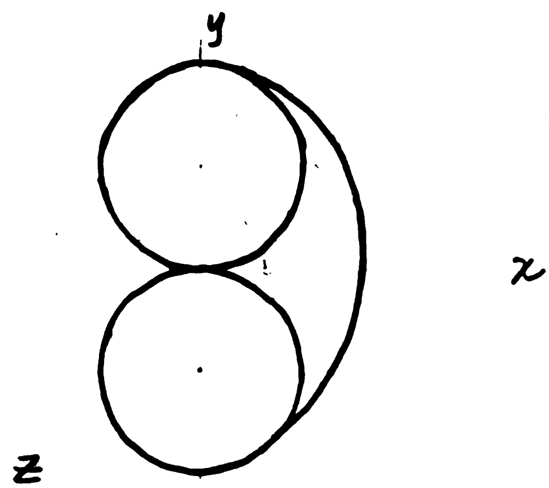
Fig. 1



a. horizontal



b. Vertical



Total Pattern
E vs. θ
Fig. 2

to be that of a sphere about the antenna. Fig. 2 shows the theoretical free-space patterns of a half wave vertical antenna. The total pattern is more complicated in this case because the horizontal and vertical patterns differ and the vertical pattern is not of the simple circular form. The total pattern would appear to be a doughnut shaped affair.

Antenna patterns can be calculated mathematically for individual antennas and arrays by use of proper assumptions and approximations. The problem undertaken in this paper is that of creating, designing, building, and testing of an electronic device for calculating antenna patterns equivalent to that which can be done by mathematics.

The equations for the pattern of an antenna are common and easily derived but the actual plotting of the function, $f(\theta)$, is a tedious job. The antenna calculator is designed to provide the antenna pattern plot on the screen of a cathode ray tube. The antenna calculator could incorporate flexibility such that the antenna parameters can be easily changed and the plot immediately presented. Functioning in this manner, the antenna calculator would show the result of a predetermined set of antenna parameters. Conversely, however, the calculator could also give the value of the parameters by adjusting the controls to give the desired pattern and then reading the parameters set on the controls.

The Theory

The equation for the horizontal radiation pattern of a two tower antenna array is developed by considering the problem as pictured in Fig. 3 which shows antenna 1 and antenna 2 spaced distance S. The radiation field is to be investigated at distance r and in direction θ from the antennas.

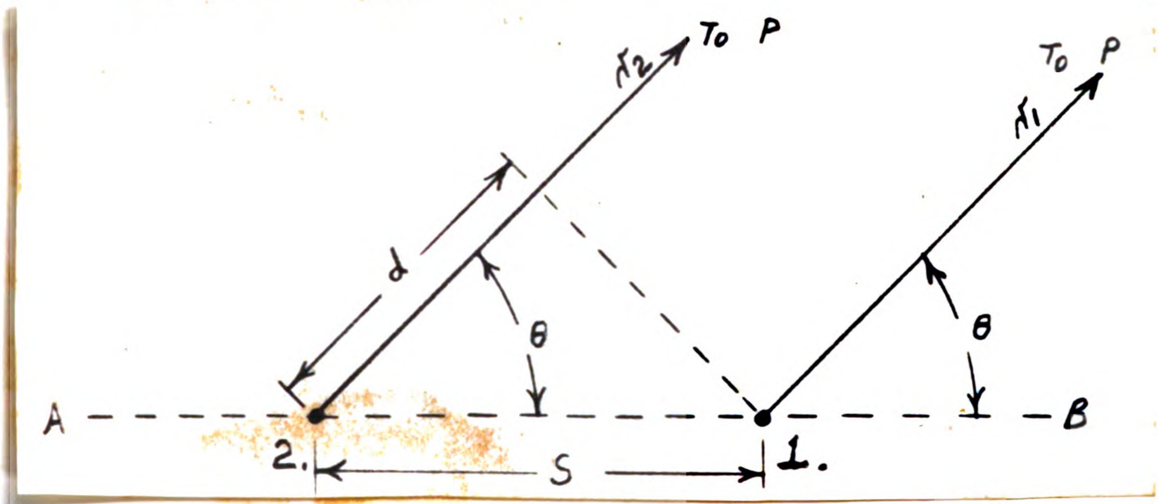


Fig. 3

The electric field intensity, E, produced at some point, P, direction θ and distance r from an antenna can be expressed in general as[1, 2]

$$E = k/r \; I \; F(\theta).$$

Where k is a proportionality factor containing such things as the intrinsic impedance of space, n, 2 , and

[1] r is taken great enough so that only radiation field need be considered and not the induction field.

[2] Reference Data for Radio Engineers, 3rd Edition, Federal Telephone and Radio Corporation.

factors derived from the length of the antenna. $F(\Theta)$ is the factor that modifies the strength of E depending on the direction Θ. For vertical antennas $F(\Theta)$ for the horizontal radiation pattern is unity[3], that is, the antenna radiates equally in all directions.

The problem shown in Fig. 3 will be solved under conditions of the following assumptions.

1. Each antenna radiates equally in all directions on the horizontal plane according to $E = KI$.
2. There is no mutual effect between antennas to alter the preceding assumption.
3. The point P at which the total E field is considered will be sufficiently far from the array so that the lines r_1 and r_2 can be considered parallel.
4. The electric fields of the two antennas will be considered to be equal in magnitude but differing in phase at point P. There is actually a difference in distance r_1 and r_2 to the point but this difference is negligible.

The following symbols will be used:

E_1 electric field at point P due to antenna 1.

E_2 electric field at point P due to antenna 2.

[3] Reference Data for Radio Engineers, 3rd Edition, Federal Telephone and Radio Corporation.

r_1 distance from antenna 1 to point P.

r_2 distance from antenna 2 to point P.

Θ direction angle from line A-B to point P.

I_1 current in antenna 1.

I_2 current in antenna 2.

M ratio of I_2 to I_1.

S spacing of antennas in electrical degrees.

d difference of r_1 and r_2.

ϕ phase angle difference of currents I_1 and I_2.

a phase angle between E_1 and E_2 incurred by difference in distance r_1 and r_2.

The field at point P caused by antenna I complete in magnitude and phase if antenna I is used as the reference is given as

$$E_1 = KI_1 \angle 0^\circ \quad (2)$$

and for antenna 2

$$E_2 = KI_2 \angle \phi + a \quad (3)$$

But the angle a can be expressed in terms of S and Θ as

$$a = S \cos \Theta \quad (4)$$

and

$$I_2 = M\, I_1$$

Therefore

$$E_2 = K\, M\, I_1 \angle \phi + S \cos \Theta \quad (5)$$

The total field is given by the vector addition of E_1 and E_2.

$$E_t = KI_1 + KMI_1 \cos x + jKMI_1 \sin x$$

Where $x = \phi + S \cos\theta$

or $$E_t = KI_1 \sqrt{(1+M\cos x)^2 + (M \sin x)^2}$$

and $$E_t = KI \sqrt{1+M^2+2M \cos(\phi + S \cos \theta)} \qquad (6)$$

The radical must be $F(\theta)$ by comparison of equation 6 with 1.

A plot of this magnitude vs. the angle θ would give the radiation pattern of the array. The equation shows that the pattern can be changed by changing any of the three parameters:

1. Antenna current phase, ϕ
2. Antenna current ratio, M
3. Antenna spacing, S

An electrical system may be set up to produce an output voltage having modulation on it according to equation 6 and, when this envelope of modulation is detected, the output of the detector will be a function exactly as equation 6.[4]

[4] George H. Brown and Wendell C. Morison, "The RCA Antennalyzer", Proc. I.R.E., Dec., 1946

The Antenna Calculator is to be a synthesis of circuits which will control Ø, M, and S and whose output will thus be a voltage having the form of equation 6.

The basic physical concepts upon which the foregoing equations were developed seem to give better insight to the problem than the preceding statement for these concepts say that the resultant field at any point is the sum of two fields which have various phase relations depending upon the direction Θ. This means that the resultant field, E, changes in magnitude as the two fields progress from in-phase to out-of-phase according to equation 6. An electrical circuit utilizing voltages analogous to fields can produce the same results as shown by block diagram Fig. 4.

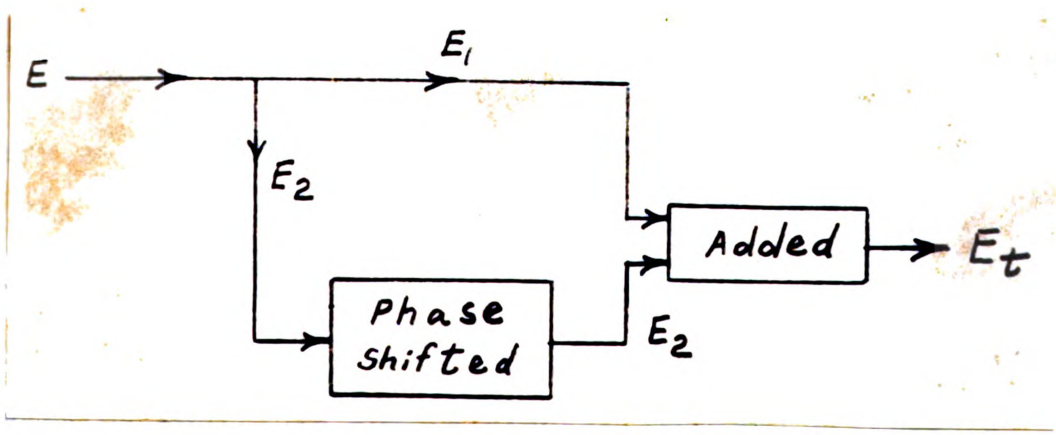


Fig. 4

The phase shift circuit will include a fixed shift Θ and the varying shift $S \cos \Theta$ which implies phase modulation at frequency $f = \Theta/2\pi t$. Proceeding to synthesize the calculator, we start with an alternating voltage source such as an oscillator which has output voltage

$$e = E \sin wt. \qquad (7)$$

From this voltage is obtained a similar voltage

$$e_1 = E_1 \sin wt$$

and another voltage which has been phase shifted amount $\emptyset$ and phase modulated according to $S \cos \Theta$.

$$e_2 = E_2 \sin (wt + \emptyset + S \cos \Theta) \qquad (8)$$

This expression is expanded to give

$$e_2 = E_2 \operatorname{Cos}(\emptyset + S \cos \Theta) \sin wt + E_2 \sin(\emptyset + S \cos \Theta) \cos wt.$$

By substituting $E_2 = E_1 M$ and adding 7 and 8 we obtain

$$e_t = E_1[1 + M \cos(\emptyset + S \cos \Theta)] \sin wt + E_1 M \sin(\emptyset\ S \cos \Theta) \cos wt.$$

Where $\Theta = 2\pi f_1 t$ and $w = 2\pi f_2 t$

f_1 is to be small compared to f_2 so that the relation

$$A \sin wt + B \cos wt = \sqrt{A^2 + B^2} \cdot \sin (wt + b)$$

may be used.

Equation 9 then becomes

$$e = E \sqrt{[1 + M \cos(\emptyset + S \cos \Theta)]^2 + M^2 \sin^2(\emptyset + S \cos \Theta)} \sin(wt + b)$$

The coefficient of $\sin(wt + b)$ is the envelope of the varying amplitude and is exactly the same radical as equation 6.

$$\sqrt{1 + M^2 + 2M \cos(\emptyset + S \cos \Theta)}$$

The envelope of the output voltage is then the same as equation 6 and if presented on a cathode ray oscilloscope display with a circular sweep would be the antenna pattern desired.

Specifications

Knowledge of the theory of the problem suggests certain limitations and simplifications to incorporate into the practical design of the antenna calculator. Such factors as the extent of problems to be solved, the type of solution, the accuracy, the precision, reliability, producibility, flexibility, utility, and cost are stated and discussed in the following material as specifications for the design of the antenna calculator.

General

1. The problem to be handled by this calculator will be limited to that of horizontal radiation patterns of an array of two vertical antennas having individual circular patterns. The vertical antenna is the most commonly encountered type of broadcast antenna making this calculator practical and usable for broadcast antennas. The limit of two antennas is imposed because of the primary factors of producibility and cost. However, with no modifications to the original calculator, but by the addition of new channels, the device will handle arrays having more elements.
2. The type of solution that will be presented is that of either rectangular or polar radiation plots (E vs. Θ) presented on a cathode ray display.

3. Accuracy on the rectangular presentation will be comparable to any mathematically obtained plot. Accuracy on the polar presentation will be slightly less accurate but will show distinctly and relatively all major and minor lobes and thus give the preferred concept of an antenna pattern adequate for demonstration of radiation pattern principles.
4. The precision of adjustment and components of the circuits will be within ordinary and easily obtained radio engineering standards.
5. The circuits will be reliable within the limits of ordinary component failure.
6. The unit will have flexibility incorporated within its three controls to give current magnitude ratios from 0 to 2:1, antenna current phase relations of at least 0 to 360 degrees, and antenna tower spacing from 0 to 360 degrees.
7. The usefulness is limited only by the number of antennas handled, current magnitudes, phases, and antenna spacing as specified above. Unit will be easily portable to various locations.
8. The theory must be adequately accomplished in practical circuits using standard components and use a regular cathode ray oscilloscope to present the patterns. Ordinary circuit layout

skill necessary for production.

9. The cost of the circuit is to be kept to the minimum and still meet the foregoing specifications. This will probably mean keeping the circuitry to a minimum necessary to satisfactorily perform the job. A conservative estimate of cost of components and hardware will be set at $50.00.

Electrical

1. The primary source of power is 110-120 volts, 60 cycle, single phase, a.c.
2. Transformer type of d.c. power and bias supplies to be used.
3. Use of standard 5 volt rectifier tubes and 6.3 volt tubes.
4. Standard chasis construction suitable for relay rack mounting.
5. Output available for either rectangular or polar coordinate display.
6. Output voltage level sufficient to give good deflection on standard oscilloscope.

The General Design

The foregoing theory predicts the possibility of an electrical circuit to produce antenna patterns on a cathode ray oscilloscope and certain specifications have been formulated to guide the design.

The main problem is that of obtaining phase shifts of adequate range to meet the specifications. After some study of the problem, a system devised by Kell[5] was decided upon to give the fixed and modulated phase shifts necessary. The sequence of operations on a sawtooth wave shown in Fig. 5 serve to explain how the phase shift is obtained. In row A is shown the sawtooth. At row B the sawtooth is clipped at various levels from the bottom. Row C shows that the result of row B is clipped at a constant level giving output pulses of various durations as shown in row D. Row E shows the result of differentiating the wave of row D. The positive pulses thus obtained change in position depending on how the original sawtooth was clipped. These pulses can be used to drive a tuned amplifier to change the pulses into sinusoidal voltages. The change in pulse position changes the phase of the sine voltage.

A block diagram of the proposed system is shown in Fig. 6. A 100 Kc. crystal oscillator was chosen as

[5] Ray D. Kell, RCA Laboratories. U.S. Patent No.2,061,734

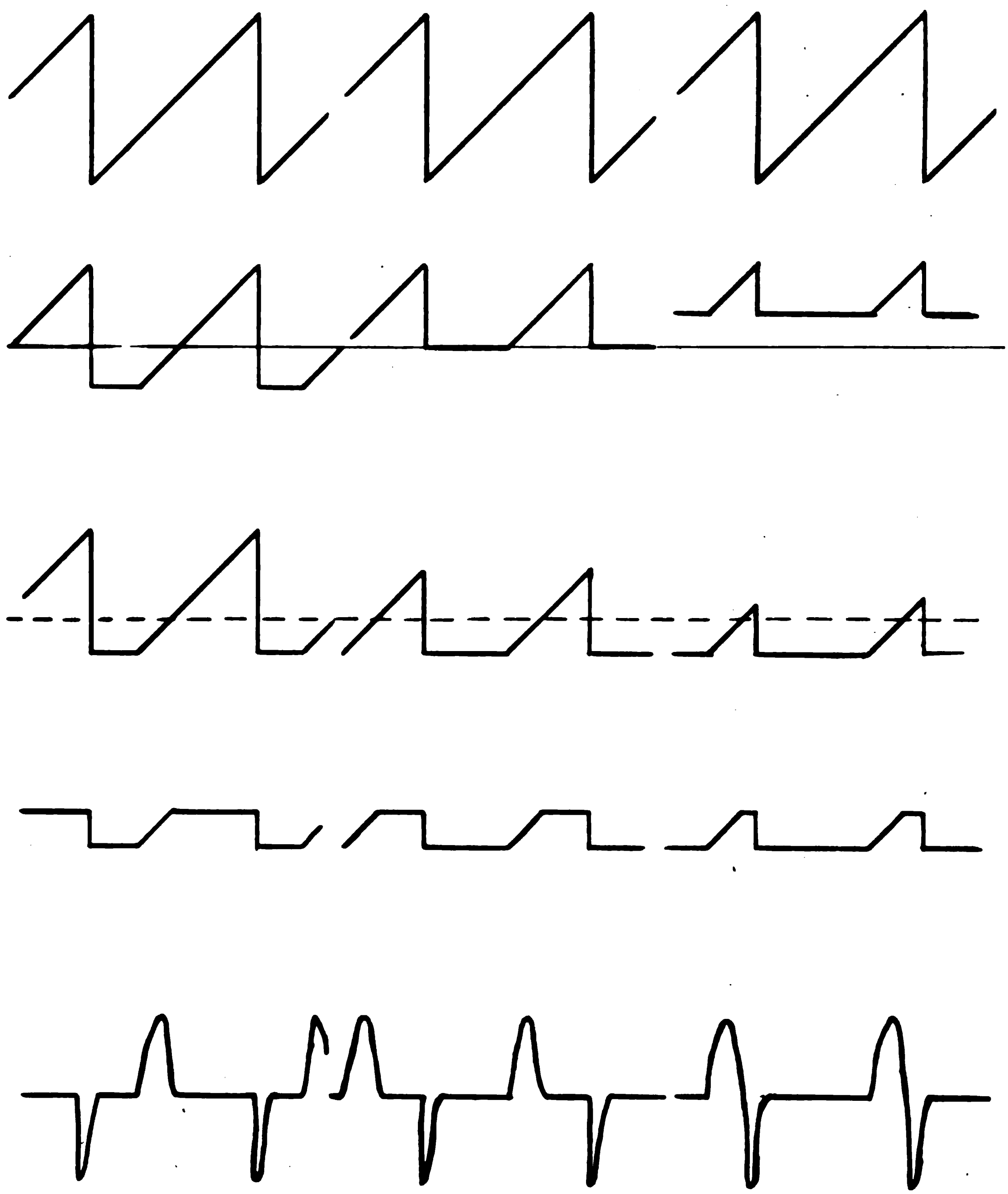


Fig. 5

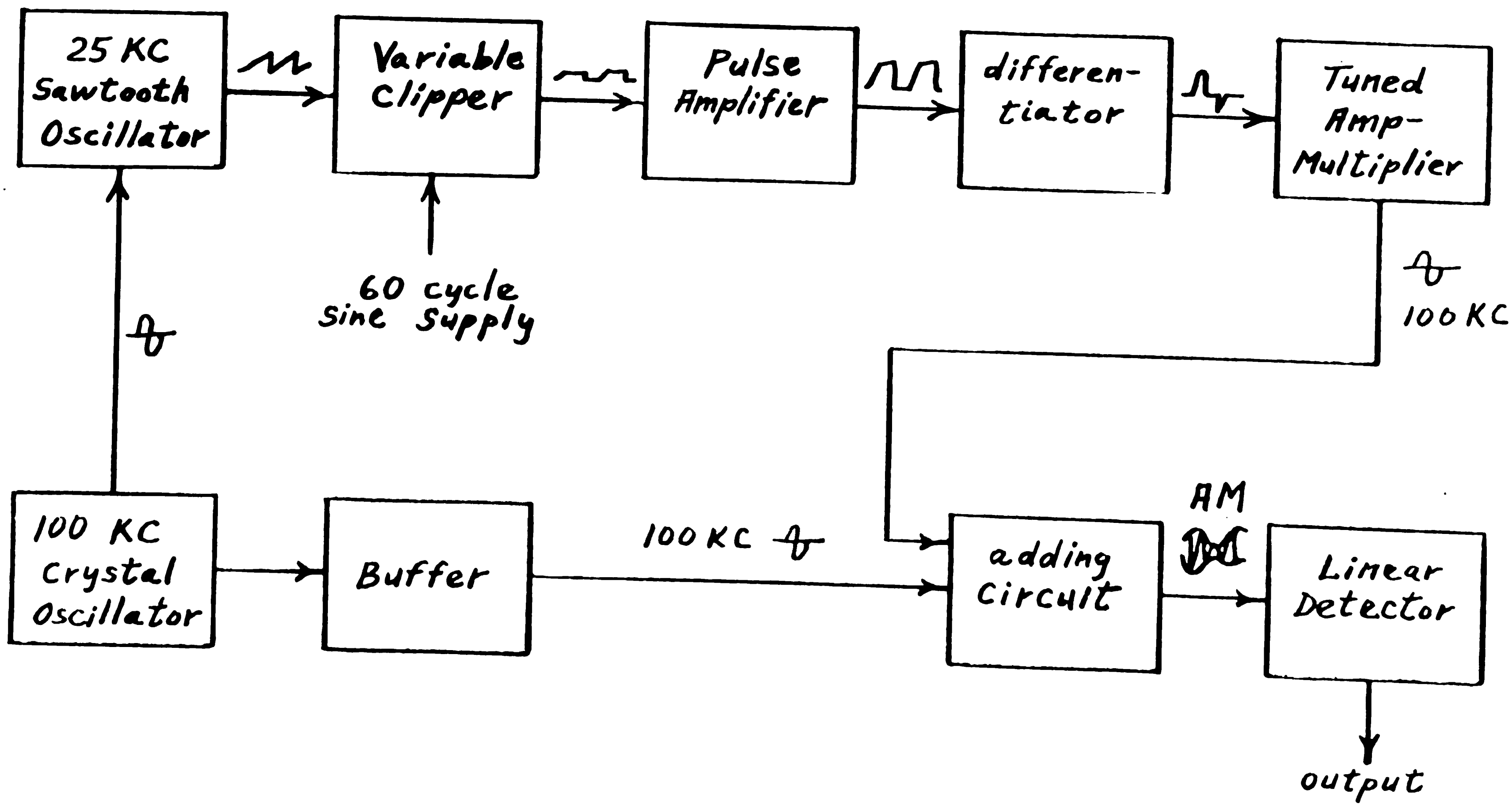
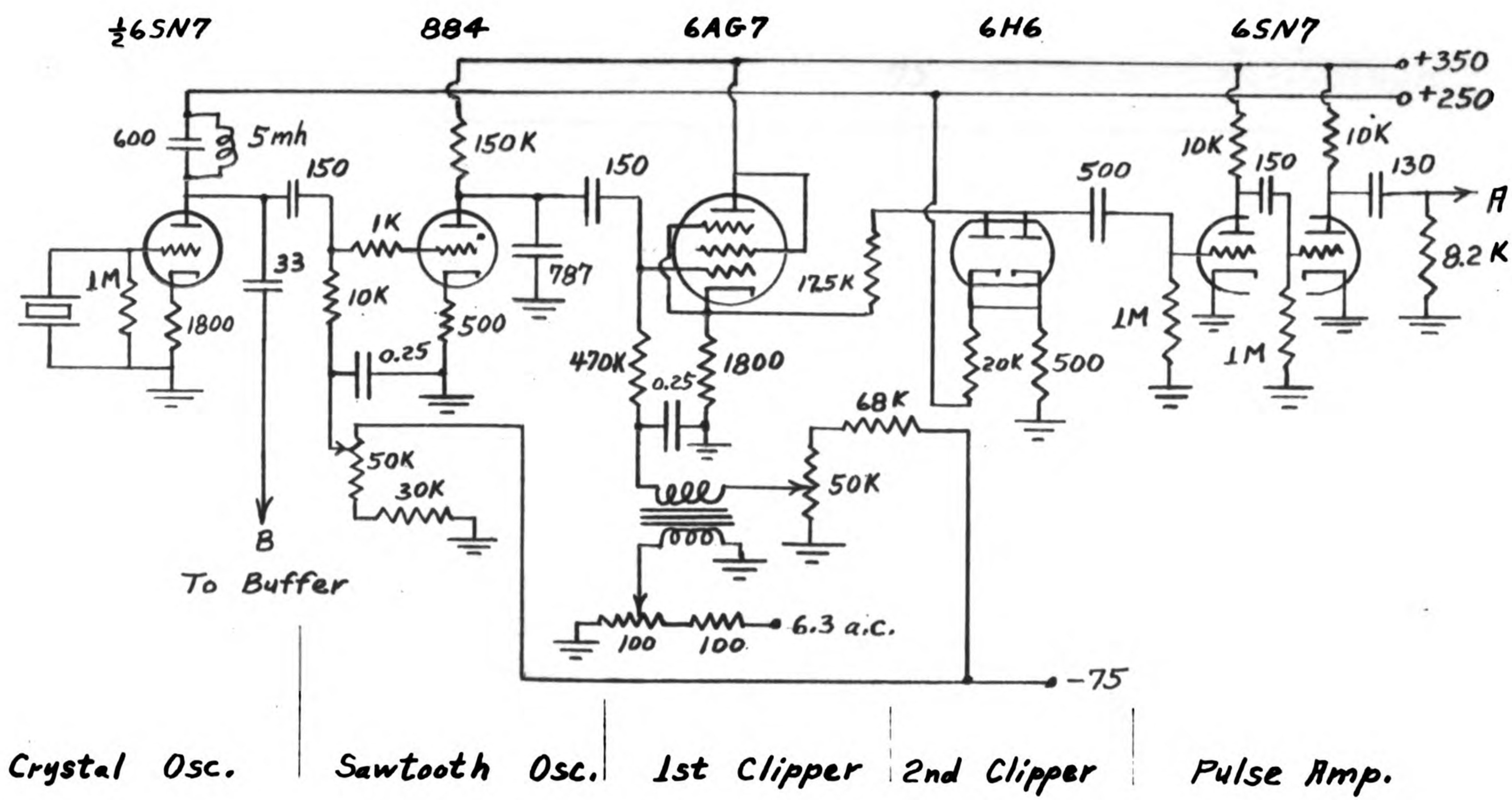


Fig. 6
the Analyzer System

the controlling stage because 100 kc is a low enough frequency to be easily utilized but high enough for crystal control. The 100 kc synchronizes a 25 kc sawtooth oscillator to obtain a lower frequency with which to produce the phase shifted pulses. The lower frequency allows the use of a gas tube sawtooth oscillator and requires multiplying back to 100 kc which will give still further phase shift. A maximum of 360 degrees phase shift is possible at 25 kc by the Kell system but this multiplied four times would give 1440 degrees. This is the maximum theoretical value, however, so if only 180 degrees could be obtained at 25 kc in the practical circuit, there would still be 720 degrees at the output, or a swing of 360 degrees, which would correspond to an antenna spacing of one wavelength, or 360 degrees, as formulated in the specifications.

The circuit diagram for the complete circuit is shown in Fig. 7 with all component values given.



½6SN7
884
6AG7
6H6
6SN7
+350
+250
600
5mh
150
150K
150
10K
10K
150
130
A
1M
1800
33
1K
10K
500
0.25
787
470K
0.25
1800
125K
500
1M
1M
8.2K
20K
500
68K
50K
30K
50K
B
To Buffer
100
100
6.3 a.c.
-75
Crystal Osc.
Sawtooth Osc.
1st Clipper
2nd Clipper
Pulse Amp.

Fig. 7-A

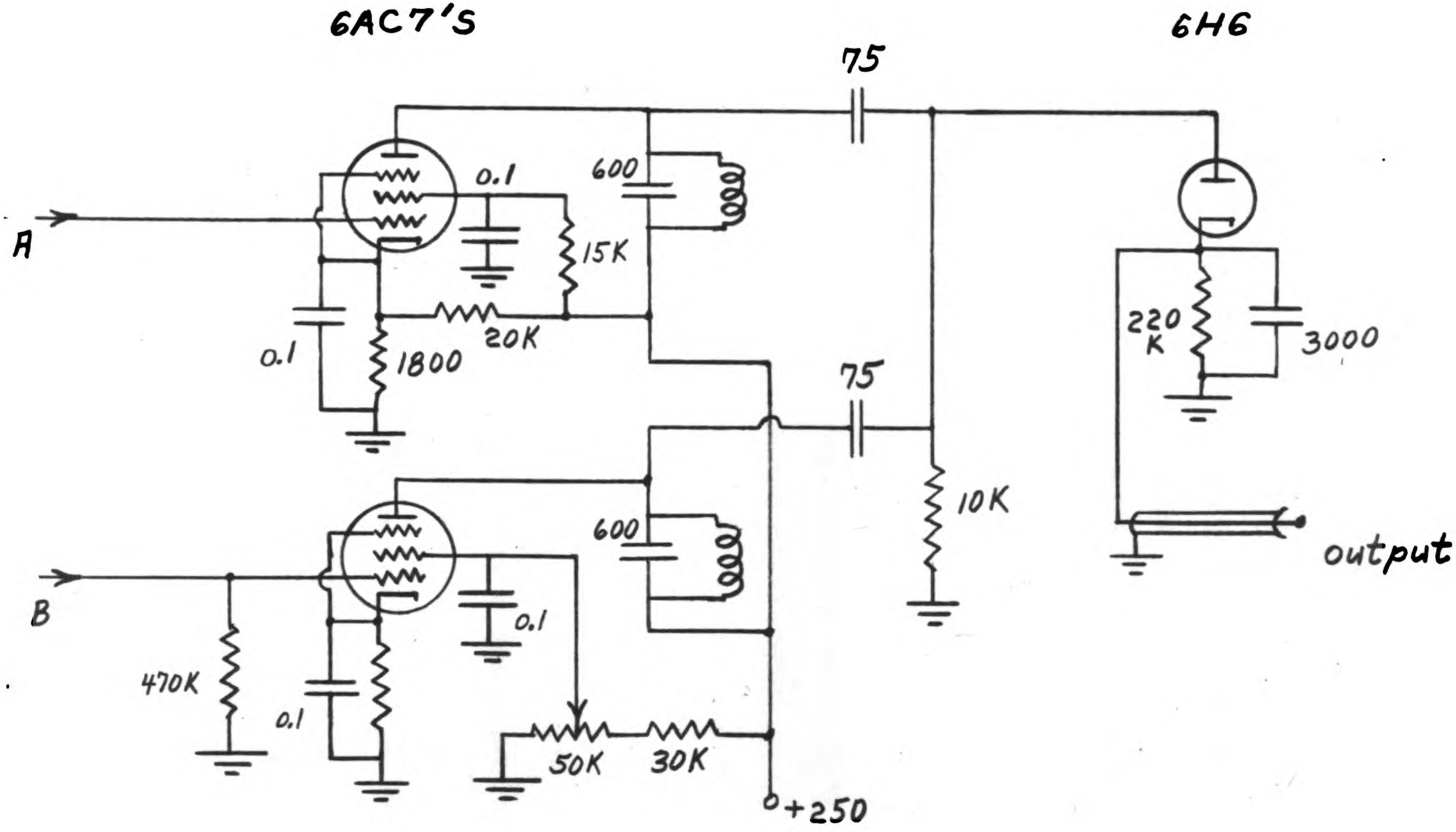

6AC7'S
6H6
75
75
A
B
0.1
600
15K
20K
0.1
1800
600
0.1
470K
0.1
50K
30K
+250
10K
220 K
3000
output

Fig. 7-B

The Calculator Circuit

The first circuit stage built was the triode crystal oscillator using a Bliley KV3 100 kilocycle crystal as the control. Output from the 100 kilocycle crystal oscillator is coupled to the grid of an 884 thyratron sawtooth oscillator to synchronize the sawtooth at 25 kilocycles. The sawtooth was made quite linear by using a charging voltage of 350 volts and allowing the condenser to charge to about 60 volts. This limited the condenser voltage rise to the nearly linear part of the exponential but still gave approximately 50 volts of sawtooth amplitude.

The sawtooth oscillator is followed by a 6AG7 cathode follower buffer stage which also clips the lower part of the sawtooth at various levels controlled by both d.c. and 60 cycle a.c. bias on this tube.

The cathode follower feeds a 6H6 shunt clipper biased at ten volts to remove the upper peak of the sawtooth ten volts above the previous negative peak clip. The output from this clipper is a series of sloped front pulses of various durations. These pulses are then amplified by two stages employing a 6SN7 and the resulting pulse is differentiated to give the required sharp positive pulse corresponding to the sloped front wave out of the clipper.

The positive pulse drives a class "C" tuned amplifier-multiplier which changes the pulse to a sinusoid of frequency 100 kilocycles. This 100 kilocycle sine wave can now be modulated in phase by the variable position driving pulse created by the clipping and differentiating circuits.

The phase modulated voltage is now mixed with a stable voltage direct from the crystal oscillator through a buffer amplifier and the resultant amplitude modulated voltage detected by a 6H6 linear detector whose output is either viewed on a cathode ray oscilloscope or fed to the polar plotter for polar coordinate display on an oscilloscope.

The Polar Plotter and Power Supply

The unit described in the foregoing material produces a time varying voltage that can be used to modulate a circular cathode ray tube sweep to obtain the polar coordinate antenna pattern. The signal voltage is only to modulate the circular sweep but not to actually appear in the output itself. In order to accomplish this, balanced modulation is used as shown in Fig. 8. The sweep is to be 60 cycles so the push-pull inputs to the modulators are 60 cycle sine waves and the plotting voltage is fed to parallel control grids. This keeps the plotting voltage balanced out of the output and only the 60 cycle amplitude modulated voltage appears. The balanced modulators are fed the 60 cycle voltages in quadrature so that the modulated output is also in quadrature and these two quadrature voltages, fed one to the cathode ray x input and the other to the y input, will produce the desired circular sweep modulated by the calculator voltage. The linearity and balance of the modulators is difficult to obtain so that a slight distortion of the pattern is present.

The power supply is of the conventional type as shown in Fig. 9. The polar plotter and the power supply were built on the same chasis. The complete calculator unit then occupies two 17 x 7 x 3 inch chasis.

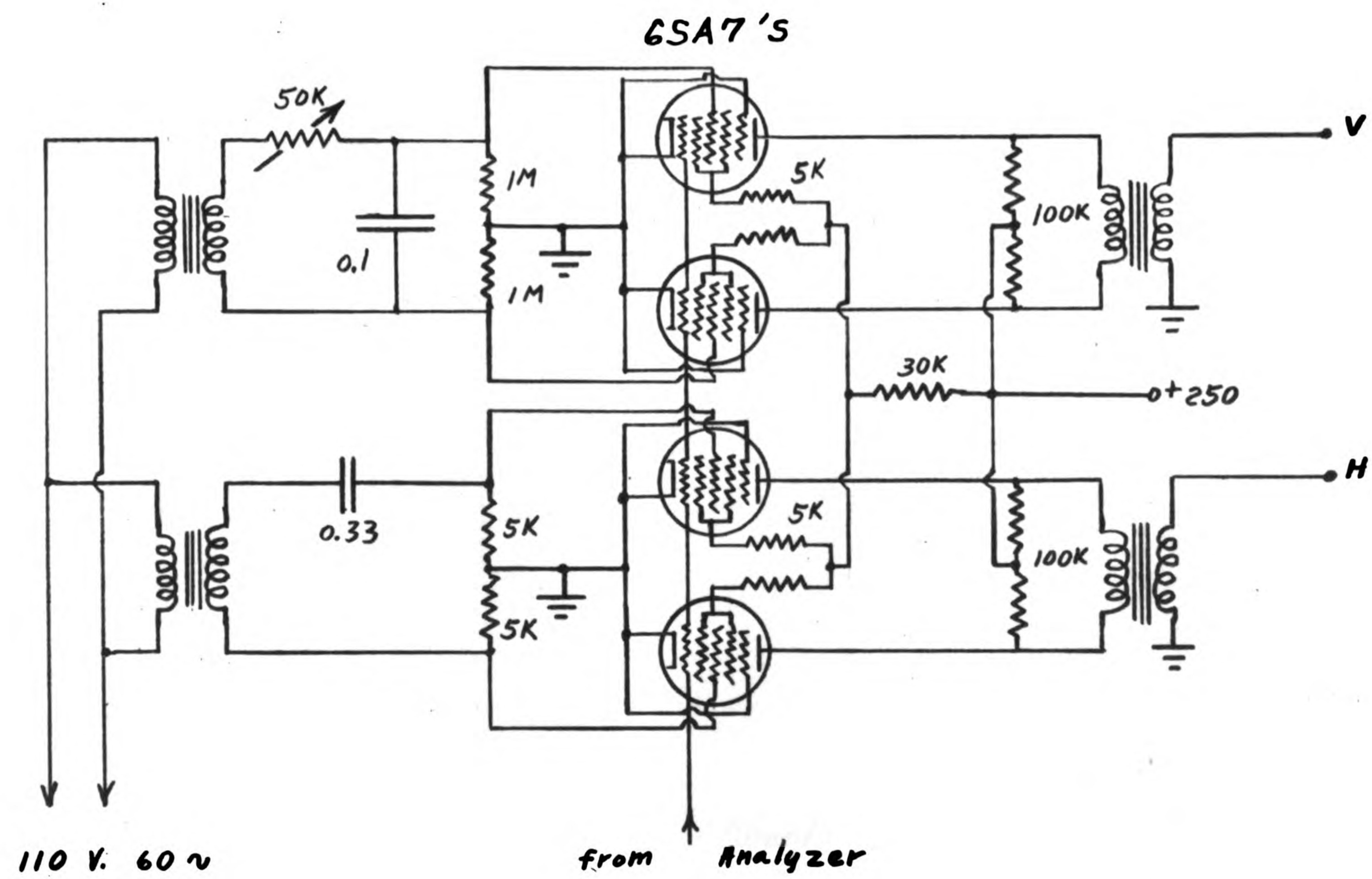
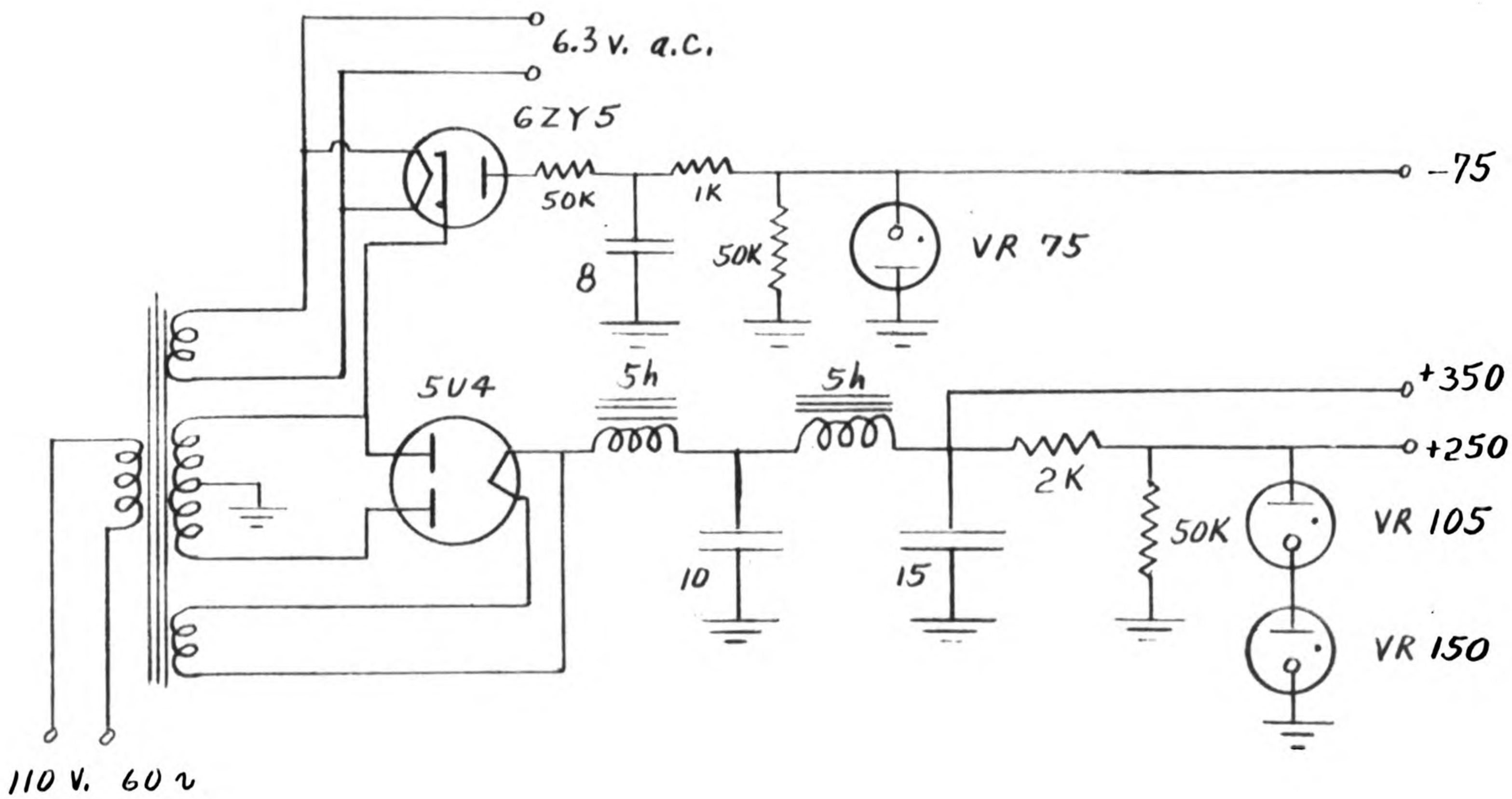

6SA7'S
50K
0.1
1M
1M
5K
5K
100K
100K
30K
+250
V
H
0.33
5K
5K
110 V. 60 ~
from Analyzer

Fig. -C



Power Supply

Fig. 9

The Results

The circuit operation gives results that follow the theoretical predictions very well. The placement of parts and general circuit layout can be seen in the photographs on the following pages. The four controls on the first chasis are for synchronizing the sawtooth, fixed phase shift, phase modulation control, and amplitude control for the fixed 100 kc channel. The second chasis contains the power supply and the balanced modulators of the polar plotter.

The calibration procedure is as follows:

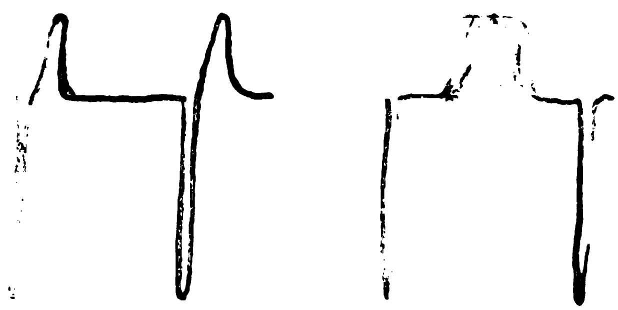
1. With antenna 1 shut off by the amplitude control, the polar and oscilloscope are adjusted for a circular pattern representing the pattern of antenna 2.
2. Antenna 1 is now turned on and with no phase modulation (antennas spaced zero degrees) the antenna current phase is adjusted until a minimum pattern is presented. This means that the currents are 180 degrees out of phase. A maximum pattern indicates in-phase currents and intermediate points can be calibrated by knowing the proper relative magnitude for addition at various phase angles.
3. The current ratio control can be calibrated by setting the current phase for 180 degrees, no modulation, and varying the magnitude of antenna 1 current. The

pattern going to zero will indicate equal currents and the ratio of total pattern magnitude to that of antenna 2 alone gives the ratio for other conditions.

4. The antenna spacing calibration can be obtained by noting the patterns of broadside and end fire arrays. The null point goes to zero when the spacing is 180 degrees and the control is linear from zero on up.

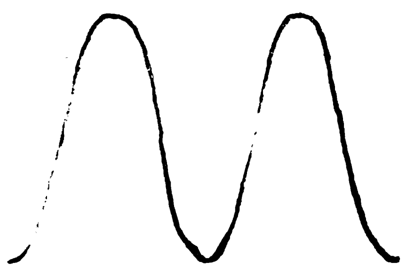
As predicted in the preliminary specifications, the polar coordinate representation of the antenna pattern is not as accurate as the rectangular because there is some non-linearity in the balanced modulators of the polar plotter. The major effects of antenna parameter changes can be seen with no difficulty. This unit would therefore make a very nice display unit.

output from first clipper



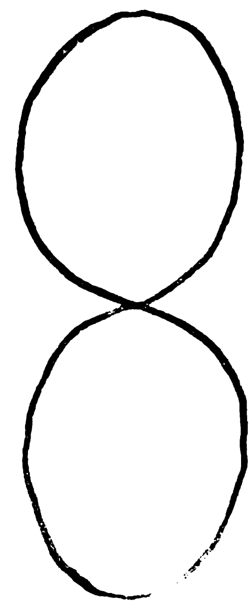
No Modulation Phase Modulated

output from Pulser



output from analyzer
$M=1 \quad \Phi=0 \quad S=180°$

Oscilloscope Traces at Various points of the circuit.



M=1 ϕ=0 S=180°

Oscilloscope Trace of Polar Pattern

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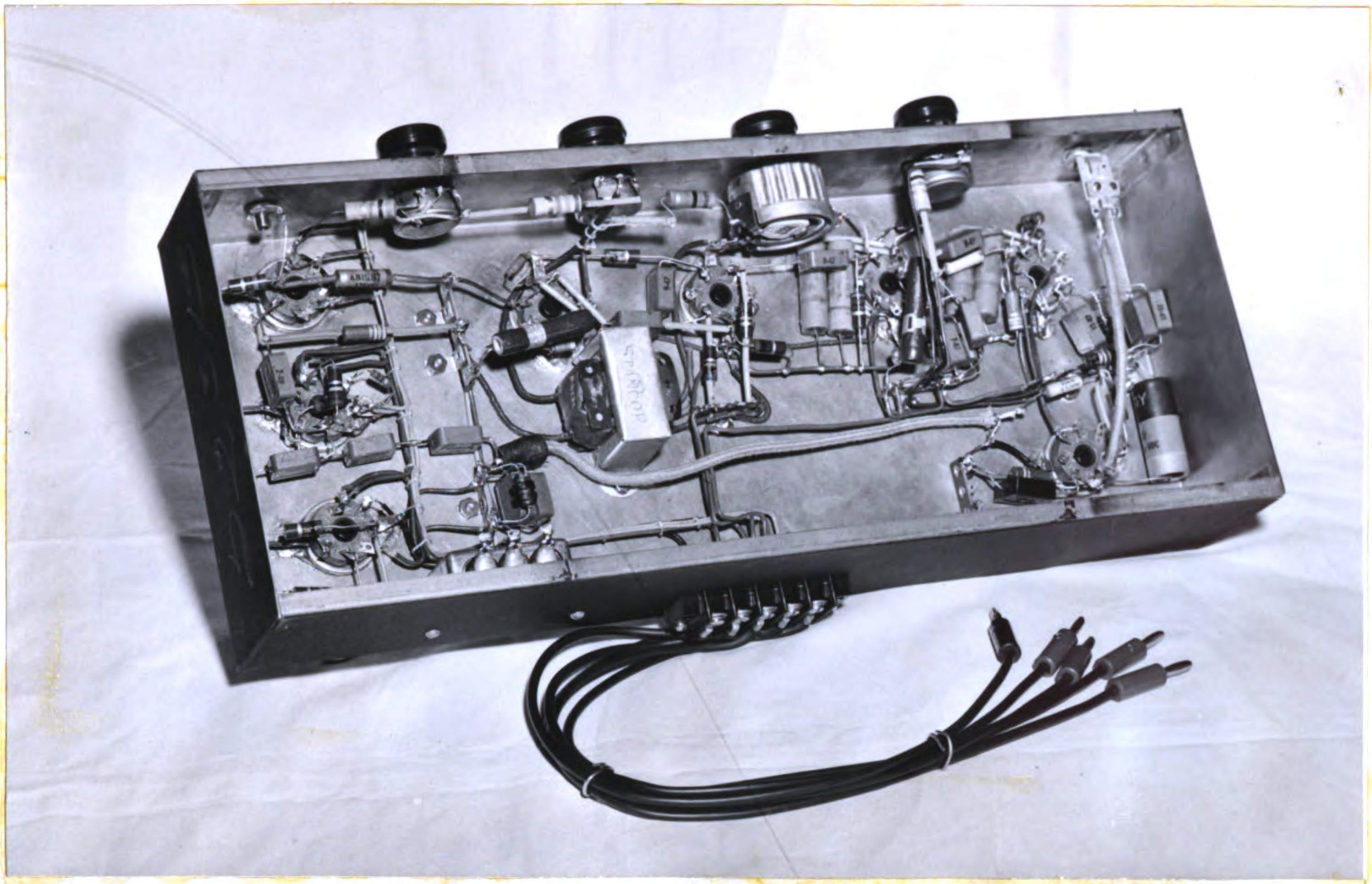
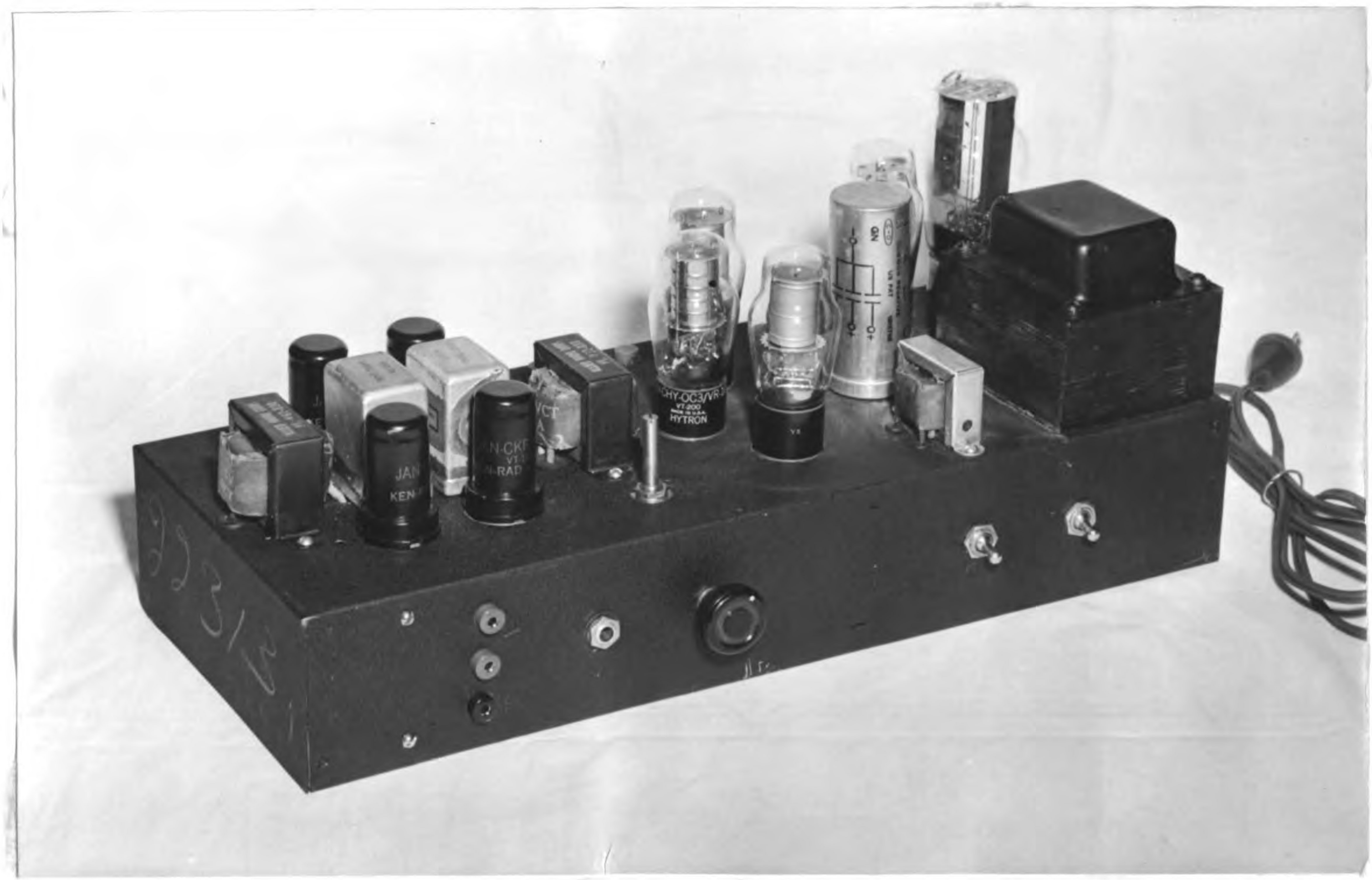
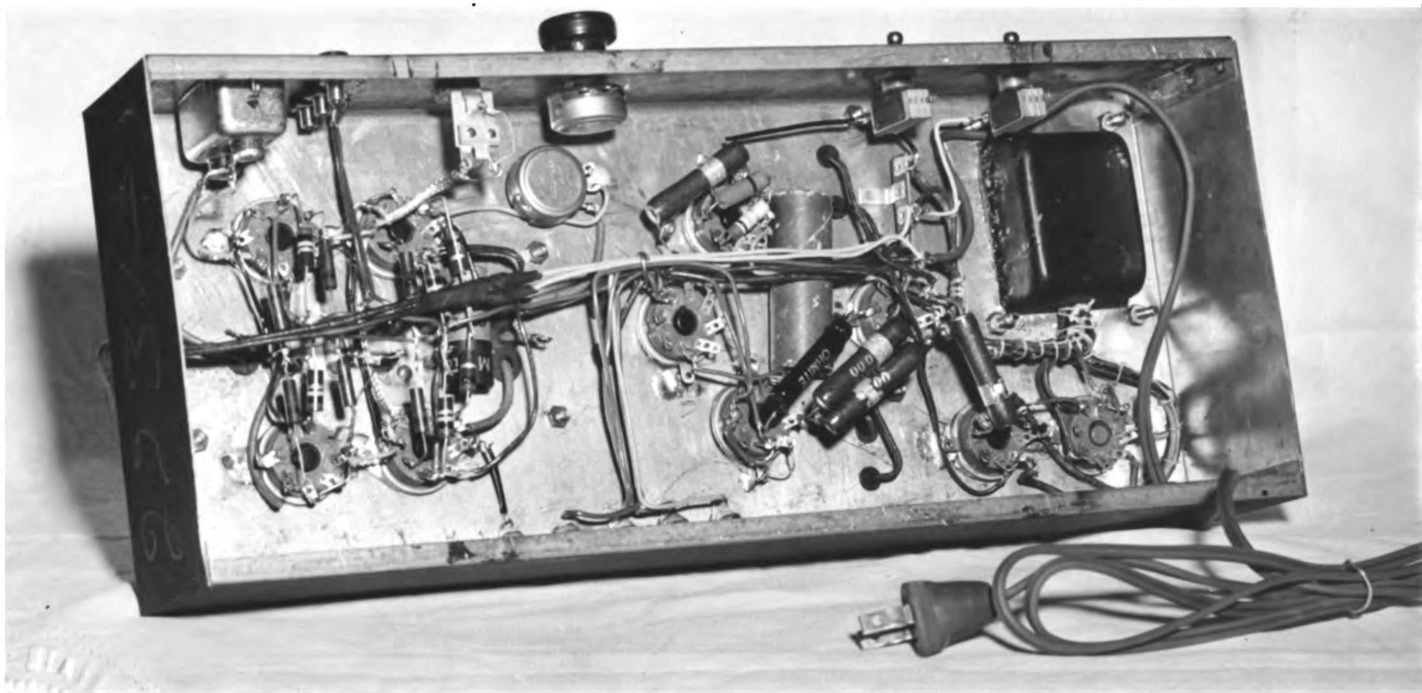
The calculator top



The calculator Bottom View



Power Supply and Polar Plotter



Power Supply and Polar Plotter