STRESS DISTRIBUTION AS SHOWN BY POLARIZED LIGHT

Thesis for the Degree of B. S.

ARTHUR W. LYNCH

1929

Strains + stresses

THESIS

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Civil engineering - Strength of materials

STRESS DISTRIBUTION

as shown by

POLARIZED LIGHT

A Report Submitted to the Faculty

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Acknowledgement is hereby made for the help of Prof. C.M. Cade of the Civil Engineering Dept. in the preparation and study of this thesis.

Part I.

Introduction.

The distribution of stress and the determination of stresses set up in structures by the application of external and internal forces are questions of primary importance to the Engineer. We find an immense variety of problems arising from what seems a comparatively simple structure. Picture the stresses set up in the cylinder wall of an internal combustion engine. It is subjected to the pressure produced by explosion, variable heat strains, and internal stresses of unknown application. Our knowledge is deficient in the condition of loading of any number of problems. But to those who are familiar with the method of solution of fixed arches, and other non-statical problems, it is unnecessary to multiply instances where ample scope for investigation exists.

The experimental research of the physicist and the investigation of the mathematician are the authority upon which most of our structural design now hinges. Mechanical measurements can be made, based on the partially true theory that stress is proportional to strain, but this must, of its nature, take in length, breadth, and thickness. If the stress varies from point to point, as it does in a large marjority of cases, mechanical measurements can give no positive conception of conditions.

The mathematician comes nearer to a true analysis, but this is only true in simple structures. Consider the action of the supporting beam of an airplane wing. Reduction of weight necessitates a minimum cross-section at every point. There is a uniform thrust upward over the wing. (Fig.1) The beam is usually supported at the body end and by a brace attached a little past mid-way. It will be seen that as the stress increases the deflection increases, which again increases the stress, and by our present methods a series of approximations is our nearest approach to solution. Even in simple structures we find assumptions being made which warrant the advancement of a more accurate method.

In the study of measuring stress from point to point in a structure it is found that particles of a transparent medium have a rotational effect on polarized light proportional to their state of stress. Polarized light had been studied since 1669 but was not used in the determination of stress until Brewster and Prof. E.G. Coker recently experimented on its effectiveness.

The problem of stress analysis by the use of polarized light is gradually turning from experimental physics to an engineering method. The effect will be that the model of the structure, whether it is an arch bridge or the frame-work of a giant airplane, will be

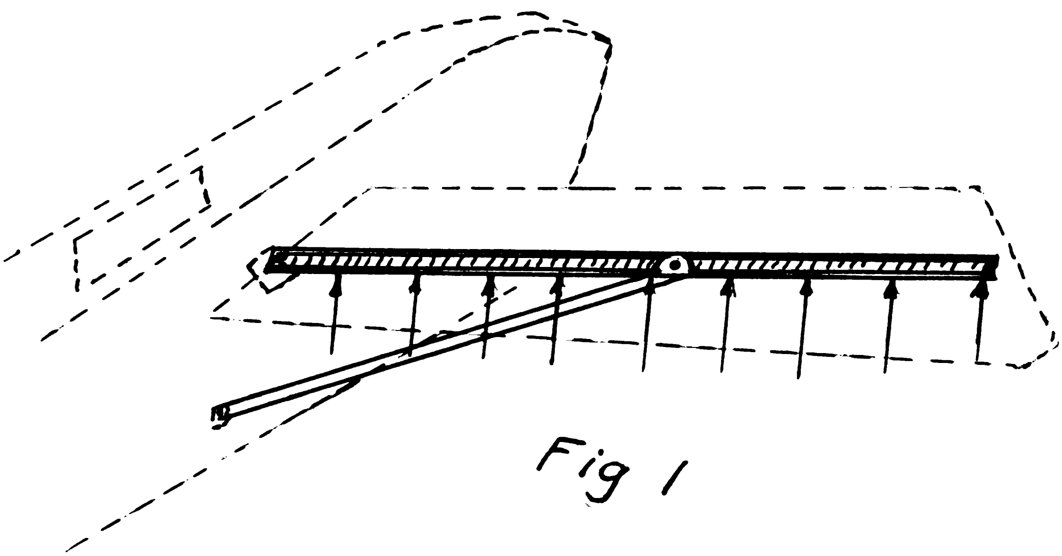


Fig 1

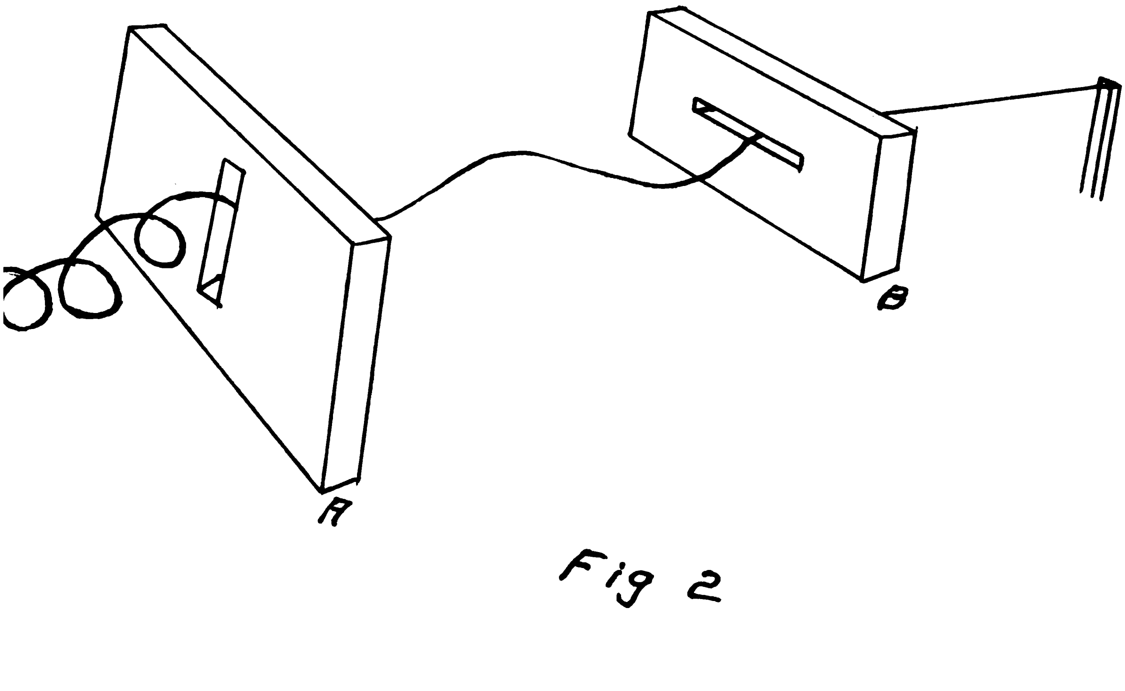


Fig 2

subjected as nearly as possible to the natural conditions which produce the maximum stresses and the resulting distribution of strain read directly under polarized light.

It is to be noted that studies have been made successfully of practical problems by the photo-elastic method by Prof. Coker of England, M. Mesnager of France, the General Electric Co., the Bureau of Aeronautics of the Navy Dept., and several others.

Prof. Coker, as a result of extended research and persistent study has perfected an arrangement of polarizer and analyser such that a quantitative study can be made of objects of such complex stress relations as a cement briquette. M. Mesnager constructed a glass model of a reinforced concrete bridge, since constructed over the Rhone at Balme, from which he determined the distribution of stress in the structure and the points of maximum danger.

A more complicated arrangement was made at M.I.T. when the Bureau of Aeronautics constructed to scale a miniature model of the Shenandoah entirely in celluloid. This model was subjected to conditions representative of those which the giant balloon would undergo during storms or other indefinite conditions of the air. Although a quantitative measurement in celluloid is as yet a difficult matter, the Navy Dept. hopes that these experiments will be of highly commensurable value in the design of airships, and will

aid in preventing the repetition of disasters similar to those occuring in recent years.

Prof. Heymans, in charge of the experiment, states: "By this photo-elastic method we can look into the vast and intricate net work of the dirigible and see exactly what is going on when she is laboring. We can see how she is carrying and distributing the load. We have made an analysis of the Shenandoah, showing exactly how the stresses are taken up by the members of the frame and wires. When we hear of new forces which the ship must meet in its ventures overhead we can try them out on the model here at Technology."

It is important today to bring to the general notice of the Engineer the possibilities of this system, and thus to insure its quick development and advance to practicability. Our effort in this thesis will be twofold; to substantiate by experiment a few ideas on the practical use of colorimetric effects; and to produce a cross-pollination of ideas on the subject, presented in such a manner as to be easily understandable to the average Engineer to whom it is of importance. No attempt will be made to derive a mathematical background, and little emphasis will be placed on the theoretical history of the subject.

Part II.

Theory of Light.

The average individual knows little of the nature of light. Whether he sees things or how he sees them interests him only in its ultimate completion, that he does see them. However, because of its relation to our subject, we will delve a little more into the theory of this great blessing. The Greeks elaborated several theories of vision. Democritus held that vision was caused by the projection of particles from the object into the pupil of the eye, while Euclid advanced the strange doctrine of ocular beams, according to which the eye itself sends out something which causes vision as soon as it meets something else emanated by the object.

During the time of Galileo most study was placed on the action of light until Huygens in 1678 advanced the wave theory. This theory gained ground until Newton threw his weight in favor of the corpuscular theory. He based his reasoning upon his experiments in spectra and straight line transmission of light, and so great was the respect of the scientific world for his opinions that for nearly a century this theory that we now consider false held complete sway. Thomas Young in 1804 revived the undulatory theory, after a century of neglect, in which he was given support by his contemporary, the brilliant Fresnel. Through a storm of contemptuous criticism from every scientific publication of the day these men finally succeeded in gaining recognition. The study of polarization added proof to their deductions, and from then until the present day the wave theory has given the most satisfactory explanation of the various phenominal of light.

For our purpose, then,"we may suppose that light is a vibratory motion set up in an elastic medium, called the luminiferous ether, which fills all space, and that the rays of ordinary light are caused by displacements which are transverse to the direction of the beam, and have any azimuth." (Note) Consider an infinite, all pervading mass of a highly sensitive, jelly like substance. Consider then a particle vibrating in this mass. It would cause condensations and rarefactions in the medium which would proceed in all directions as a wave transverse to the direction of projection, and which would eventually die out.

Consider now an object made up of an immense amount of these particles all vibrating and jostling about at once. It will be seen that they would send out waves of every imaginable length all vibrating in different planes but all transverse to the general beam in every direction. Thus we have built up a ray of light acting in all azimuths and consisting of waves of different length travelling at the same velocity. It is evident that the more violent the agitation, and

(Note) Coker Engr 1911

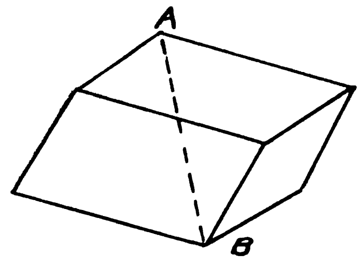
the greater the number of particles the farther the disturbance will travel.

We will next consider an explanation of some of the various light phenomena as sufficient substantiation of the theory for our article. It is commonly understood that light travels in a straight line. That is that while passing through a homogenous medium of constant density it does not vary or curve from the direction of ray. Therefore, if we allow light from a source at an infinite distance, which would show the rays of light to be parallel, to enter a darkened room through a small opening it will cast a spot of light of size equal to the hole on the opposite wall. Likewise the shadow of a sphere on a board perpendicular to the ray should be the same size as the sphere. It is noted, however, that the shadow of the sphere is smaller than the object and that the spot of light is slightly larger than the hole. This condition is called dispersion and it is explained that a ray of light bends lightly around a corner when passing an edge. The enumbra, or partially shaded part of the shadow, is caused by these rays which have been bent in slightly.

Conditions under which visual light differed from normal were noticed by the early Greeks. They first considered the problem of why the sun appeared to be larger at the horizon. It was noticed that color was produced similar to a rainbow when sun light was subjected to certain conditions. Before these conditions can be further considered it is necessary to define and illustrate two terms.

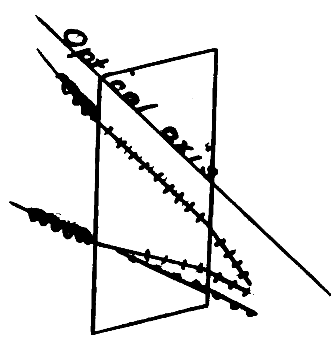
When a ray of white light strikes a glass grating there is reflected to the eye a complete rainbow of colors varying from red to violet. This is called the spectrum and the phenomena of its formation under these conditions is called diffusion. When light passes from one medium into another of different density the rays of light are bent in proportion to the difference in density of the media or the difference in rigidity, or both. This is known as refraction, and the angle thru which they are bent as the angle of refraction.

Newton stated that "Light is not similar or homogeneal, but consists of difform rays some of which are more refrangible than others." Thus, in the case of refraction we may have the formation of a spectrum. This phenomena, then, explains the apparent change in shape of an object projecting from air into water, the large appearance of the sun at the horizon, and also the spectrum formed by light transmitted through a glass prism. We will not consider the theory of this action, but will turn to subjects whose bearing on the subject is more immediate.

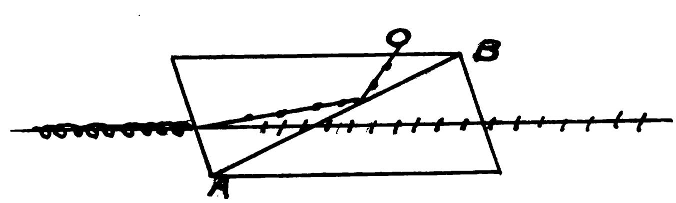


Calcite Crystal

Fig. 3



Significance of Optical Axis.



Nicol

Fig 4

Part III.

Polarization.

There is no need to go further into the history of the discovery and development of polarization. A peculiar property, known as double refraction, was found to be possessed by certain bodies of aelotropic structure, due to their molecular arrangement. If we take a Rhombohedral crystal of Iceland Spar, or Calcite, Fig. 3 we will find the refracted light broken into an ordinary and extraordinary ray. This natural crystal, which has the power of double refraction, is bounded by six parallelograms having angles of $101^{\circ}55'$ and $78^{\circ}5'$. If light passes through this crystal parallel to the axis of symmetry the ordinary and extraordinary rays coincide. This is called the optical axis.

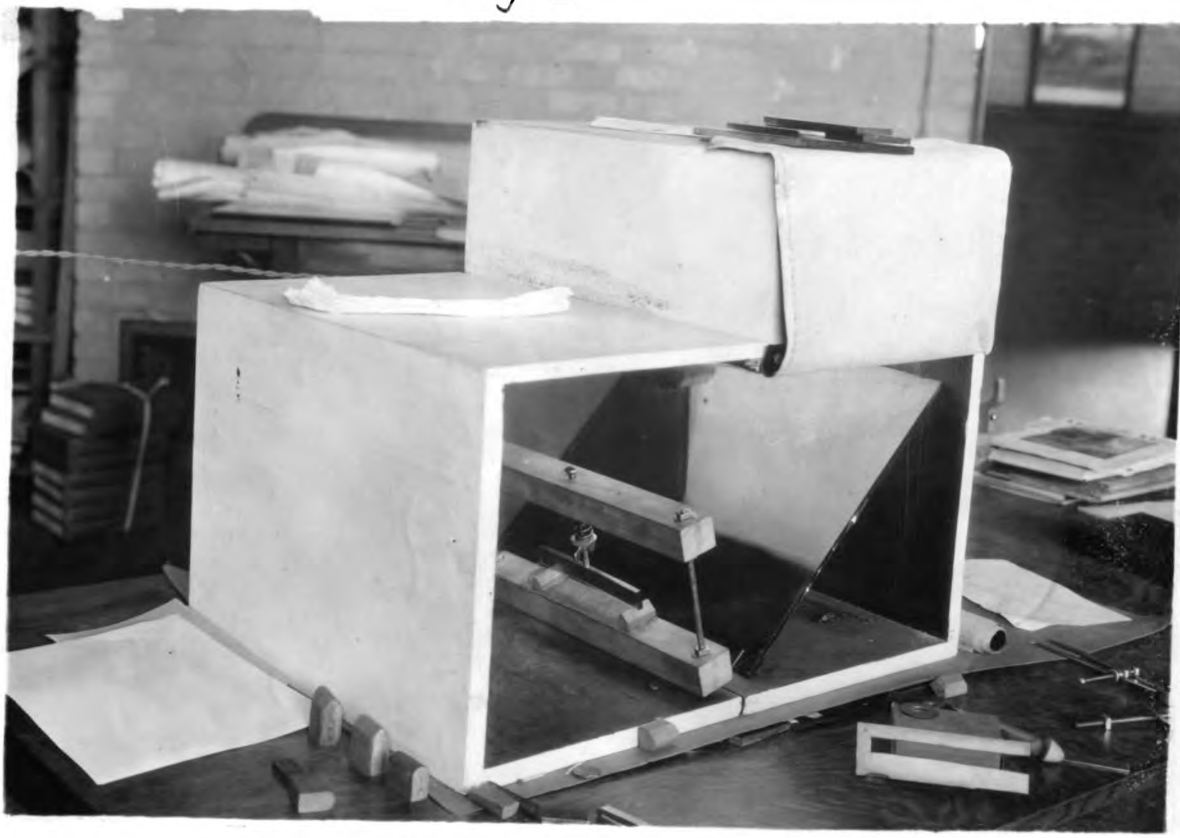
In passing a ray of light through a crystal of tourmaline the ordinary ray was found to be absorbed and the extraordinary ray, only, transmitted. This ray has the peculiar property of only vibrating in one plane. In other words it is Polarized.

Let us consider this phenomena, there is really no difficulty to its conception. We have said that a ray of ordinary light consisted of displacements transverse to the direction of the ray and having any azimuth. It is reasonable to suppose that a wave could be selected from this having only one azimuth if the others could be cut out. The situation is analogous to a rope (Fig. 2) which is being shaken in such a manner as to cause a nondescript series of waves to progress down its length. If a block(a)is placed over the rope with a vertical slot in it only those waves whose transverse motion were vertical would be allowed to pass through. We then have displacements in one known azimuth. If we place a second block (b) with the slot in a direction perpendicular to that in block(a)no vertical wave motion can pass through. We have now set up an apparatus for the production, detection, and the determination of direction of a polarized wave motion. If an optical setup block(a) would be called the polarizer and block (b) the analyser.

Certain substances have the property of rotating this plane of polarization. The amount of this rotation can be detected by the position of the analyser which allows no light to pass through, as compared to its position in a similar circumstance when the polarized light was not passed through the substance. This fact has been used largely in chemical analysis, and recently in the determination of stress.

A convenient arrangement of apparatus for obtaining plane polarized light is the modified form of a crystal of Iceland Spar devised by Nicol in which the rhomb is cut along plane A B (Fig. 4) and the cut surfaces, after being polished, are cemented together with Canada Balsam, which has a refraction index between that of the

Fig 6



Set up

Fig 5

A - Polariser B - Analyser C - Light Box ordinary and extraordinary ray. Thereby O is got rid of by internal reflection at the cut surfaces, and only the extraordinary ray is transmitted.

It is also noted that transparent substances polarize nearly all the light reflected from their surfaces when light is caused to enter them at a certain angle (known in optics as the polarizing angle of the medium in question.) Thus if we cover the back of two glass plates with a light absorbing substance, such as a mixture of lamp-black and turpentine, and cause light to strike the first at an angle of about 45° (the polarizing angle of glass) most of the light will be absorbed by the lamp-black on the back of the glass but a portion of it will be reflected and polarized at the surface. If we now cause this polarized light to strike the second plate at the same angle we have set up a polarizer and analyser between which quite large specimens can be examined. If the light is monochromatic light and dark lines will be obtained. If white light is used a succession of spectra will appear, varying with the strain produced in the specimen.

Of these methods, for our experiments, we have chosen an arrangement of glass reflectors shown and illustrated in Fig. 5 and 6. We considered these as offering the most convenience, with the least difficulty of construction. An explanation of the set up will be found with the illustrating figures. White light was used, propagated by twelve incandescent lamps, and was passed through a white asbestos paper before striking the polarizer.

It is not necessary to go deeply into the subject of polarization, nor to mathematically prove our statements. It is sufficient at present to describe certain effects which can be used to advantage in the solution of problems. For many experiments circularly polarized light is used. This is usually obtained by passing plane polarized light through quarter-wave plates. Their effect is to produce to beams of equal amplitude, vibrating at right angles, and differing in phase. This retards the wave motion in such a way as to cause the plane of polarization to move forward with a motion similar to a screw, each wave turning through a definite angle. If a Babinet's compensator is used either elliptically or circularly polarized light may be produced, depending on the part of the compensator passed through.

Our interest now turns to a study of stress, and we will leave polarized light until we take up its application to stress analysis, and its peculiar action when passed through a strained, transparent specimen.

Part IV

A Study of Stress.

The force exerted upon one body by another at a surface of contact is called the stress between the bodies. If then, we consider a single body as cut by an imaginary plane there would be forces set up at this plane known as the internal stresses of the body. If the forces are such that the particles of the body are pushed together at the imaginary surface this is known as a compressive stress. If the forces tend to pull the portions apart it is called tensile stress. Compressive stress at the surface of two bodies in contact is called bearing stress. Intensity of stress is spoken of in terms of the area concerned and the total stress. Thus we have lbs./sq. in. or dynes /sq.cm, or like units.

Since every body has a certain amount of elasticity it follows that wherever stress exist deformation must take place. The body is elongated or compressed depending on whether the stress is tensile or compressive. A deformation which remains after the stress is removed is called a set. It is of interest to note that only up to the last point where no set is obtained is the stress considered proportional to the strain. This point is called the elastic limit of the material and, after this condition of stress is reached, if more is applied, the strain increases out of proportion to the stress and some set remains. By mechanical or mathematical methods it has been impossible to determine this point accurately due to the influence of time and the gradualness with which the first set comes on.

In the study of a simple beam it will be seen that tension is effected along the bottom and compression along the top. This is easily seen by the way in which the particles tend to pull apart or push together. Between these stresses there exists a neutral plane. If a beam is twisted a stress called tortion is produced. All stresses in any structure whatsoever can be reduced to these types. The stress in any member consists of tension, compression, or tortion, or a combination of these. The identity of the type of stress is easily determined if the conditions of loading and the distribution from known points are understood. In photo-elastic work a strain in the material registers the same color regardless of type, showing only the amount and location of the strain. It is necessary therefore to identify the types at some easily determined point of the structure and by means of the neutral planes and points of no stress follow them through.

A number of assumptions are made in the mathematical determination of stress. Up to a certain point of excessive loading it is assumed that stress is proportional to strain, and up to this point it is held that any plane section perpendicular to the axis of moments before loading remains a plane section when stressed. The general theory of stress is limited by the elastic limit and the ordinary theory is further limited to symmetric bending. There is a ratio of definite relation between shear and moment, but the distortion caused by tension or compression is not in accordance, as far as our medium is concerned, with the distortion caused by the same ratio of lbs./sq. in. in shear.

The elements entering into the relations set up in the Bending Mom. equation of stress are the load and the beam, with the span considered as common to the load and the beam. In shear the section of the beam and the applied external forces are the controlling elements.

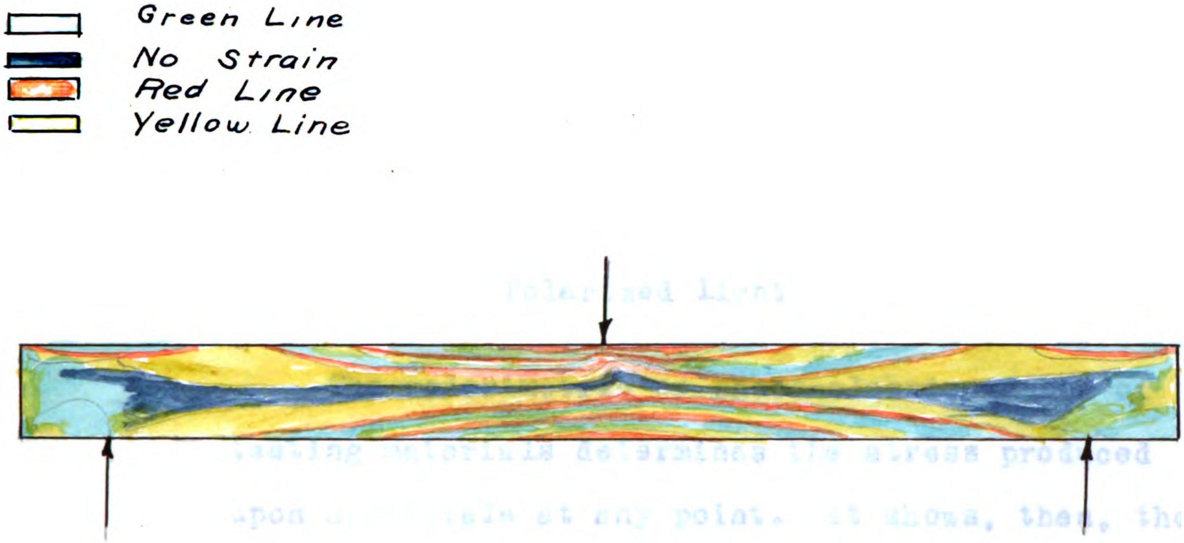


Fig 7

The interval between the same colors represents a strain equal to one wave length of light.

The strain produced by shear is not the same as that produced by compression or tension for a given intensity of lbs/sq. in.

Set was produced in the beam.

Part V.

Stress Analysis by the Use of Polarized Light

The colorimetric or photo-elastic method of testing materials determines the stress produced upon a particle at any point. It shows, then, the total or resultant strain upon that particle. For example, in Mechanics, we consider any section of a loaded simple beam as affected by two principle stresses; one normal to the section and the other acting along the section. In a vertical section we have the stress normal produced by the moment of the loading and the stress vertical produced by the shear of the support. In the optical determination of this stress the coloration representing its intensity would be seen in a line passing through the point at an angle in proportion to the ratio of shear to moment at the point.

An analogy may be made here between these lines of coloration and the contour lines of surface mapping. It is found that these lines are continuous and must either form a closed figure or run off the edge of the specimen. The reasoning of this fact needs no demonstration as it is in direct accordance with that of contour lines and is very simple.

It is also to be noted that the shape of the line of intensity through any point never changes although its color does as the load is applied or removed. This is fundamental. We reason that for a definite condition or state of loading the distribution of stress over a beam is proportional to the relative length, heighth, and breadth. Therefore, as the load is increased the distribution remains the same but the intensity increases in proportion to the load. We see, then, colors progressing one into anothers position as the load is changed. If the variation of strain is increased several bands may show where one was, but they will conform to the shape of that one, and if a minute study were made of shades the original band would have been subdivided along similar lines. Figure 7 shows a typical coloration of a simple beam seen under polarized light. Several spectra appear within the limits of stress and the furthest spectrum from the neutral plane indicates the greatest stress. As the load is increased a spectra of greater intensity moves toward the position of one of less intensity. This movement is carried out over the entire beam, but the general direction of the lines of stress intensity does not change.

The fact that polarized white light produces several spectra in accordance with the increase in intensity of strain in the specimen under observation suggests a simple and easily denoted denomination of the amount of strain in a particle. The number of the spectra could be used for the rougher measurement and the color for the fractional reading of the first.

For the purpose of our study in this treatise it will be sufficient to compute the stresses represented by the different spectra seen in a stressed beam of transparent bakelite.

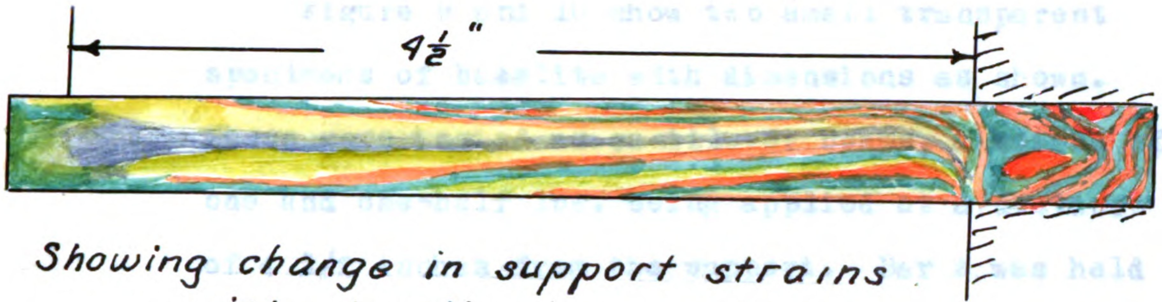
We have said that a line of definite coloration represents a certain intensity of strain, that a spectrum of coloration represents a certain range of intensity, and that these lines of color are in accordance with the resultant lines of stress. It is now in order that we should give some explanation of the presence of several spectra over a range of stress of one type (such as tension or compression.)

Light has a slower velocity in a denser medium. Therefore, an anology can be made between the impeded progress of light through a stressed specimen and a homogeneous wedge, it being commonly understood that stress varies the density of a substance.

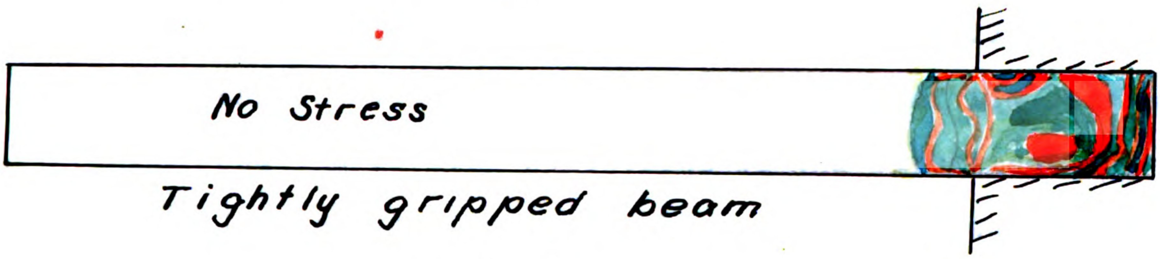
Consider the wedge ABC (Fig. 8) If a ray of light from the source D strikes the face AC part of it is reflected to the eye at E. The rest passes through the wedge and another part is reflected to the eye from the face AB. The second portion of the beam has travelled further before reaching the eye by an amount equal to twice the thickness of the wedge. Suppose now that this retaining amounts to one half the wave length of blue light. Then a crest and trough of blue light strikes the eye simultaneously, interfering with each other, and the eye sees white light minus the blue. As the ray strike farther down the wedge the longer rays reach interference points until white light minus the red is seen. At some point past this twice the wave length of blue is reached and blue again cancels itself. Thus as the ray is considered to progress down the wedge a succession of spectra is produced which might be used for an accurate measurement of the width at any point.

Applying this analogy to the phenomena of color bands obtained when a stressed specimen is observed under polarized light, we find a succession of spectra relative to the distribution and varying intensity of stress which afford an easy denomination of measurement. The change in color from red to red represents the interference of a definite color in each case and amounts to the impeding of light that wave length. For finer measurements it would be necessary to consider the varying colors in each spectra, but for our purpose it is sufficient to calibrate and measure by the number of spectra.

We may then consider a loaded simple beam at the center to be analogous in its interference phenomena to a wedge. The bottom would be in tension, would therefore have the least density, would impede the the progress of light the least, and would be comparable to the sharp end of the wedge. The top, or compression side, by the same reasoning would be represented by the wide part of the wedge.

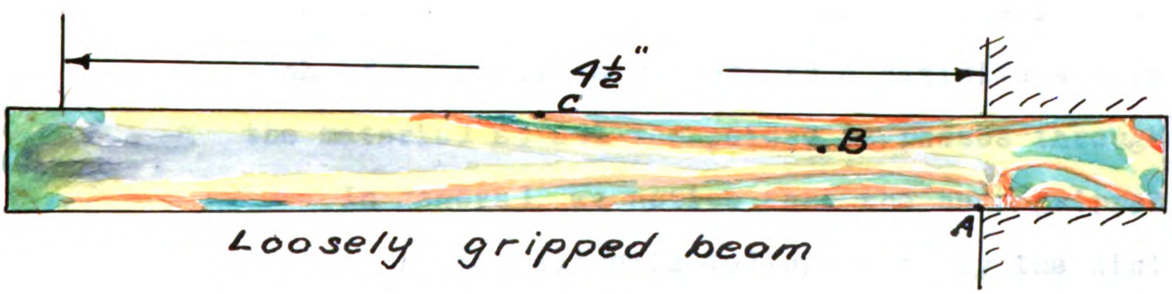


Showing change in support strains with loading



Tightly gripped beam

Fig.9



Loosely gripped beam

Fig 10

Figure 9 and 10 show two small transparent specimens of bakelite with dimensions as shown. These were tested as cantilever beams, a weight of one and one-half lbs. being applied at a distance of 4 1/2 inches from the support. Bar A was held tightly in the support, while Bar B was given enough compression only to keep it in place while the load was applied. Due to the coloration of the specimens and the working conditions of the experiment the line or location of each spectra is shown by the three colors of Red, Green, and Yellow. The location of the neutral axis is noted and the position of the red line of the first, second and third spectra at various points along the beam. The stress is then computed mathematically for these points. This sets up an elementary method of reading the stress directly. Each of these lines represents a definite strain in the material at all the points it passes through.

In general, we notice that the beam gripped loosely in the supports follows closely the distribution of stress assumed in the study of cantilever beams. The neutral axis is approximately in the center, the point of maximum compression is at the lower, outside corner of the support, compression is quite well distributed over the top of the support, and the width of the spectrum bands in compression and tension are approximately equal. In the tightly gripped bar, however, we notice that the tension spectrum bands are narrower than the compression bands, the neutral axis is nearer the top, and the change of compression in the support is distributed to the lower outside and upper inside corners. This condition was not universally true, although it was found to exist in several cases. It was probably due to an arrangement of material such that the shear components overthrew the general formation of stress lines.

The downward hook in the neutral plane at the support is common to all the cantilever beams tried in our experiment. This is due to the combining of tension.at the top of the beam with compression in the support. Because a mathematical solution assumes the location of the neutral plane, only bar B will be examined quantitatively. The computation and locations of points are below.

Cantilever Beam Fig 10
$4\frac{1}{2}'' \times .24'' \times .46''$

Point	Spectrum	Distance from Support	Distance from Neut. Pl.	M #/in.	Mom. Stress	Shear Stress
A	3rd	0"	0.23	6.75	370	13
B	1st	$\frac{3}{4}''$	0.08	5.63	107	10.8
C	2nd	$2\frac{1}{2}''$	0.23	3.0	164	5.8

$$S = \frac{Mc}{I}$$

$$I = \frac{1}{12} bd^2 = \frac{.24(.46)^2}{12} = .0092$$

Simple Beam Fig. 11
$5'' \times \frac{1}{4}'' \times \frac{3}{4}''$

Point	Spectrum	Distance from Support	Distance from Neut. Pl.	M #/in.	Mom. Stress	Shear Stress
A	3rd	$2\frac{1}{2}$	$\frac{3}{8}''$	5	168	0
B	2nd	$1\frac{1}{2}$	$\frac{1}{4}''$	3	67	10.7
C	2nd	$2\frac{1}{2}$	$\frac{1}{4}''$	5	111	0

$$I = \frac{0.25(.75)^2}{12} = 0.0112$$

It is evident that the difference in moduli of elasticity accounts for the lack of coordination between tension and compression, and shear strain. If sufficient time were available to measure the strain by mono-chromatic light in wave lengths it would be possible, by having specimens in pure tension and pure shear, to determine the relation of both and account for the position of every black line in a simple beam accurately. With our equipment at present, however, quantitative work must be largely a matter of guessing.

Fig. 11 shows this optical effects of a concentrated load of four lbs. at the center of a five inch beam of transparent bacolite three-quarter inches deep by 0.24 inches wide. There is nothing about these lines of stress contrary to our excepted distribution in like circumstance. We will, however, on a following page compute the stress represented by the red lines of the first and second spectra in tension and compression. This will act as a check on the results of the study of the cantilever.

It must here be noted that shear plays a very appreciable part in the location of stress lines and cannot be disregarded. It is also noticed that the strain caused by shear is not proportional to that caused by moment.

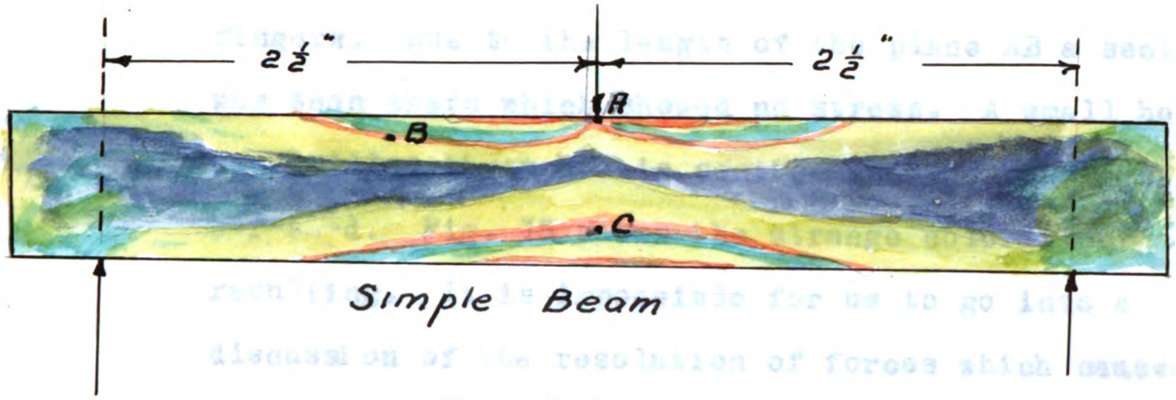
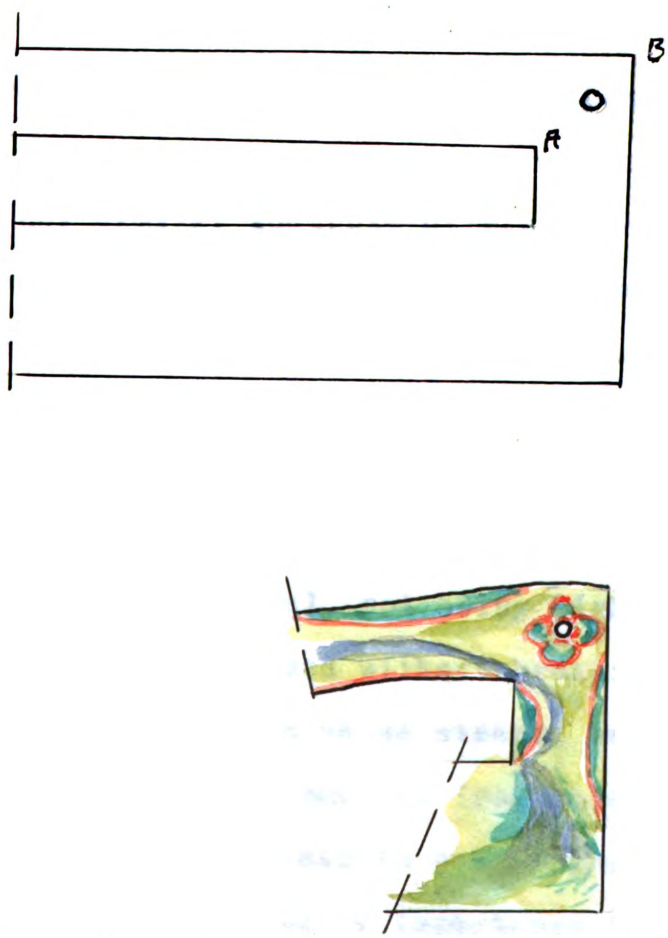


Fig 11



Strains around a hole

Fig 12

Fig. 12 shows a simple arrangement for producing cantilever moment by compression of the fingers. Due to the length of the plane AB a section was seen there which showed no stress. A small hole was drilled through this section and the beam again stressed. Fig. 13 shows the strange coloration resulting. It is impossible for us to go into a discussion of the resolution of forces which caused this action. Two many variables are concerned for a decision from one experiment. However, these figures do show the unlimited possibilities of photo-elastic work. Consider the new facts which might be discovered a colorimetric test of the Plate Girder. It may be urged as an objection that the reactions of these materials are not similar to those of steel and wood, but tests on such media as India Rubber and Jelly have yielded results which contradict this statement.

It is also known that after the last point of perfect resilience stress is no longer proportional to strain, and after the load is removed some strain, or set, remains. Our material, not being highly elastic, is easily overstressed without breaking. After the load is removed there can be no stress, yet some coloration remains. We can conclude from this that the coloration is due to the strain only. The detection of set in a structure might be of importance in many problems.

Part VI.

Conclusion.

In conclusion we wish to state that, due to limited time and crudeness of material, this study was not made with a physicists accuracy. It is the purpose of this paper, not to establish laws or units, but to convey some impression of the real importance and possibilities of the subject. Great companies such as the General Electric are interested in its study and their resources lend to their deductions the distinction of accuracy. We have accomplished our purpose if we succeed in producing a convincing argument for the general study of this phenomena as an Engineering Method.

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Houstoun - Treatise on Light

Cajorie - History of Physics

Preston - Theory of Light