AN AUTOMATIC IMPEDANCE MATCHER FOR A HALF-WAVE DIPOLE ANTENNA

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Harmon D. Strieter

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AN AUTOMATIC IMPEDANCE MATCHER FOR A HALF-WAVE DIPOLE ANTENNA

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PREFACE

The purpose of this thesis is to present the development of a method of automatic impedance matching for a specific type of antenna.

The author wishes to express his thanks to the members of the Electrical Engineering Department at Michigan State College for their kindness and assistance in meeting the requirements for the Master of Science degree. Special thanks are due Professor I. B. Baccus, Professor R. J. Jeffries and Professor I. O. Ebert for their kindness and tact during a difficult period; and to Dr. J. A. Strelzoff for his help and advice during the development of this thesis.

Harmon D. Strieter

Outline

TRANSMISSION LINES

As a means of transporting power from one point to another, transmission lines are resorted to. There are several factors that affect the proper choice of a particular transmission line for a given purpose.

A transmission line may be made up of parallel wires, of parallel plates, of coaxial conductors, or, in general, of any two conductors separated by a dielectric material.

We can diagrammatically indicate a transmission line as follows:

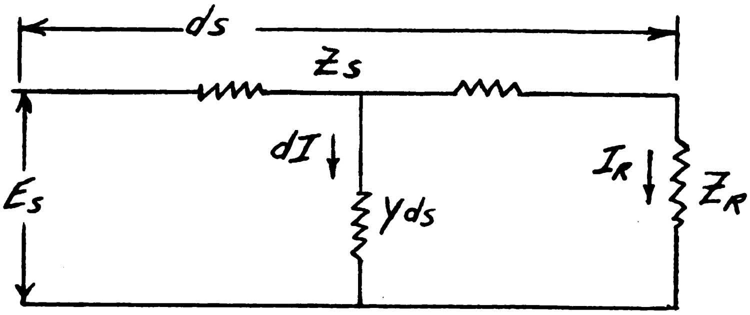


Figure 1

We can think of the transmission line being made up of cascaded infinitesimal "T" sections, of which one is shown in the above diagram. The elemental section is of length ds and carries a current I. The series impedance of the element is Zds ohms and the voltage drop in the length ds is:

$$dE = IZds$$

or

$$\frac{dE}{ds} = IZ \qquad (1)$$

The shunt admittance per unit length of line is Y mhos and thus the admittance of the element of line is Yds mhos. The current dI that flows across the line or from one conductor to the other is:

$$dI = EYds$$

or (2)

$$\frac{dI}{ds} = EY$$

Differentiating equations 1 and 2 with respect to s:

$$\frac{d^2E}{ds^2} = Z\frac{dI}{ds} = ZyE$$

$$\frac{d^2I}{ds^2} = Y\frac{dE}{ds} = ZyI$$

Solving these equations by the rules of differentiation:

$$(D^2 - Zy)E = 0$$
$$(D^2 - Zy)I = 0$$
$$D = Zy$$

from which we can write:

$$E = Ae^{\sqrt{Zy}s} + Be^{-\sqrt{Zy}s} \qquad (3)$$

$$I = Ce^{\sqrt{Zy}s} + De^{-\sqrt{Zy}s} \qquad (4)$$

To solve for the constants A, B, C, and D, the constants of intergration, we must evaluate the boundary conditions.

At $s = o$
$I = I_r$
$E = E_r$

Then equations 3 and 4 become:

$$E_r = A + B \qquad (5)$$
$$I_r = C + D \qquad (6)$$

Differentiating equations 3 and 4 so that we may use the same set of boundary conditions, we get:

$$\frac{dE}{ds} = A\sqrt{Zy}e^{\sqrt{Zy}s} - B\sqrt{Zy}e^{-\sqrt{Zy}s}$$

$$\frac{dI}{ds} = C\sqrt{Zy}e^{\sqrt{Zy}s} - D\sqrt{Zy}e^{-\sqrt{Zy}s}$$

Using equations 1 and 2, we get that:

$$I = A\sqrt{\frac{Y}{Z}}\,e^{\sqrt{Zy}\,s} - B\sqrt{\frac{Y}{Z}}\,e^{-\sqrt{Zy}\,s}$$

$$E = C\sqrt{\frac{Z}{y}}\,e^{\sqrt{Zy}\,s} - D\sqrt{\frac{Z}{y}}\,e^{-\sqrt{Zy}\,s}$$

at s = o

$$I_r = A\sqrt{\frac{Y}{Z}} - B\sqrt{\frac{Y}{Z}} \qquad (7)$$

$$E_r = C\sqrt{\frac{Z}{y}} - D\sqrt{\frac{Z}{y}} \qquad (8)$$

Solving equations 5, 6, 7, and 8 simultaneously, we obtain the values for the constants of intergration as follows:

$$A = \frac{Er}{2} + \frac{Ir}{2}\sqrt{\frac{Z}{y}}$$

$$B = \frac{Er}{2} - \frac{Ir}{2}\sqrt{\frac{Z}{y}}$$

$$C = \frac{Ir}{2} + \frac{Er}{2}\sqrt{\frac{Y}{Z}}$$

$$D = \frac{Ir}{2} - \frac{Er}{2}\sqrt{\frac{Y}{Z}}$$

We may then rewrite equations 3 and 4:

$$E = \frac{Er + Ir\sqrt{\frac{Z}{Y}}}{2}e^{\sqrt{Zy}\,s} + \frac{Er - Ir\sqrt{\frac{Z}{Y}}}{2}e^{-\sqrt{Zy}\,s} \qquad (9)$$

$$I = \frac{(Ir + Er\sqrt{\frac{Y}{Z}})}{2}e^{\sqrt{Zy}\,s} + \frac{Ir - Er\sqrt{\frac{Y}{Z}}}{2}e^{-\sqrt{Zy}\,s} \qquad (10)$$

Since Er = IrZr and by definition $Zo = \sqrt{\frac{Z}{y}}$, equations 9 and 10 can be rewritten as follows:

$$E = \frac{Er(Zb + Zo)}{2\,Zs}\,e^{\sqrt{Zy}\,s} + \frac{Zr - Zo}{Zr + Zo}\,e^{-\sqrt{Zy}\,s} \qquad (11)$$

$$I = \frac{Ir(Zb + Zo)}{2\,Zo}\,e^{\sqrt{Zy}\,s} - \frac{Zr - Zo}{Zr + Zo}\,e^{-\sqrt{Zy}\,s} \qquad (12)$$

By further manipulation, equations 11 and 12 may be rearranged in terms of hyperbolic functions:

$$E = Er\cosh\sqrt{Zy}\,s + IrZo\sinh\sqrt{Zy}\,s \qquad (13)$$

$$I = Ir\cosh\sqrt{Zy}\,s + \frac{Er}{Zo}\sinh\sqrt{Zy}\,s \qquad (14)$$

These equations fully define the voltage and current at any point on the general transmission line.

Of primary importance to the subject being developed is the input impedance of the transmission line. Define $\gamma = \sqrt{zy}$

then $$Zs = \frac{Es}{Is} = \frac{Er \cosh \gamma l + IrZo \sinh \gamma l}{Ir \cosh \gamma l + \frac{Er}{Zo} \sinh \gamma l}$$

$$= Zo \frac{Zr \cosh \gamma l + Zo \sinh \gamma l}{Zo \cosh \gamma l + Zr \sinh \gamma l}$$

A particular case that is of great interest is when the load impedance Zr is equal to the characteristic impedance of the transmission line Zo. Then,

$$Zs = Zo \frac{Zo \cosh \gamma l + Zo \sinh \gamma l}{Zo \cosh \gamma l + Zo \sinh \gamma l} = Zo$$

From the practical standpoint, the proper termination of the transmission line is most easily met when the characteristic impedance is a real constant. This will make the transmitted energy a maximum.

Consider a source with an internal impedance Zi feeding a load impedance Zr. Assume the voltage of the source unity. The impedances will be of the form

$$Zi = Ri + jXi$$
$$Zr = Rr + jXr$$

The power delivered to the load impedance may be formulated on the basis of the simple circuit shown in Figure 2:

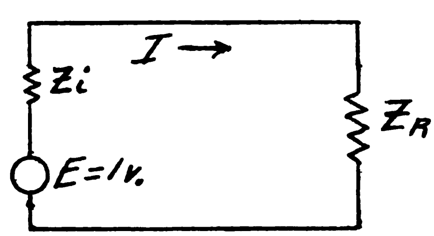


Figure 2 $$Pr = I^2 R_r = \left(\frac{E}{Zi + Zr}\right)^2 R_r = \frac{Rr}{(R + Rr)^2 + (X + Xr)^2}$$

The maximum value of this expression is to be determined. Differentiating this expression with respect to Xr and Rr:

$$\frac{\partial P_R}{\partial X_R} = -\frac{R_R 2 (X_i + X_R)}{[(R_i + R_R)^2 + (X_i + X_R)^2]^2}$$

$$\frac{\partial P_R}{\partial R_R} = \frac{(R_i + R_R)^2 + (X_i + X_R)^2 - 2R_R(R_i + R_R)}{[(R_i + R_R)^2 + (X_i + X_R)^2]^2}$$

Setting these partial differentiations to zero, we obtain:

$$\frac{\partial P_R}{\partial X_R} = 0 \quad \text{if } X_i = -X_R$$

$$\frac{\partial P_r}{\partial R_r} = 0 \quad \text{if } R_i = R_R$$

This gives us the expression for the conditions necessary for maximum power transfer. Thus, if the two impedances are pure resistances, the condition for maximum power transfer is simply that they shall be equal. When they have reactance components however, the latter should be equal in magnitude and of opposite sign. This is equivalent to a resonance condition.

In the transmission line, the characteristic impedance Zo is the one impedance and the termination of the other. Since in communication systems, the power available is relatively low, it is essential that the system be designed to deliver the largest part possible of the input power.

Later on we shall see that the condition for maximum power transfer to be realized simultaneously with no reflection occurs when the characteristic impedance Zo reduces to a constant.

Reflection

Referring to equations 11 and 12 for the voltages and currents on the transmission line,

$$E = \frac{Er(Zr + Zo)}{2Zr} e^{\sqrt{zy}s} + \frac{Zr - Zo}{Zr + Zo} e^{-\sqrt{zy}s} \qquad (11)$$

$$I = \frac{Ir(Zr + Zo)}{2Zo} e^{\sqrt{zy}s} - \frac{Zr - Zo}{Zr + Zo} e^{-\sqrt{zy}s} \qquad (12)$$

we observe that in the general case in which the load impedance Zr is not equal to the characteristic impedance of the line Zo, each equation consists of two terms, one of which varies exponentially with plus s, the other with minus s. Taking s to be positive as measured from the receiving end, the wave that travels from the sending end to the receiving end can be identified as the component varying with $e^{\gamma s}$, represents a wave of voltage or current progressing from the receiving end toward the sending end. This wave is called the reflected wave. A reflected wave will always be present unless the load impedance is equal to the characteristic impedance. Reflected waves will be present in varying degrees depending on the amount of mismatch present, with the reflected wave being a maximum under the two extremes conditions of open or short circuited terminations.

The ratio of amplitudes of the reflected and incident voltage waves at the receiving end of the line is called the "reflection coefficient."

From equation 11, with s = 0, we get that the reflection coefficient $K = \frac{\text{Reflected voltage at load}}{\text{Incident voltage at load}}$

$$K = \frac{Zr - Zo}{Zr + Zo}$$

Then equations 11 and 12 may be rewritten using the reflection coefficient as follows:

$$E = \frac{Er(Zr + Zo)}{2Zr}\left[e^{\gamma s} + Ke^{-\gamma s}\right] \qquad (13)$$

$$I = \frac{Ir(Zr + Zo)}{2Zo}\left[e^{\gamma s} - Ke^{-\gamma s}\right] \qquad (14)$$

Again we see that if Zr = Zo, the expressions for the voltage and current at any point on the line simplify to

$$E = Epe^{\gamma s} \qquad (15)$$

$$I = Ipe^{\gamma s} \qquad (16)$$

Since we are concerned primarily with high frequency lines, and since the transmission lines that we are concerned with will be relatively short, the attenuation terms may be neglected. In general $\gamma = \alpha + j\beta$ where α is the attenuation constant and β is the phase constant. Thus, for our lines of negligible attenuation $\alpha = 0$, equations 13 and 14 may be rewritten.

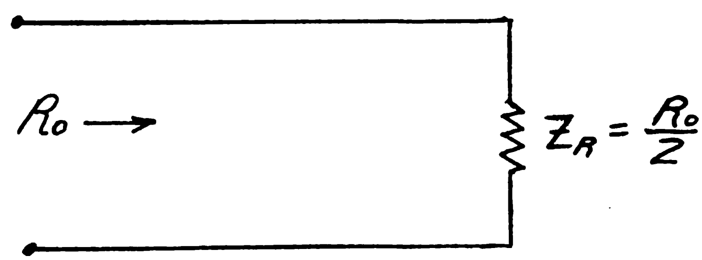
$$E = \frac{Er(Zr + Zo)}{2Zr}\left[e^{j\beta s} + Ke^{-j\beta s}\right] \qquad (17)$$

$$I = \frac{Ir(Zr + Zo)}{2Zo}\left[e^{j\beta s} - Ke^{-j\beta s}\right] \qquad (18)$$

The term varying with $e^{j\beta s}$ has previously been identified as a wave progressing from the source toward the load, and the term involving $e^{-j\beta s}$ as the reflected wave moving from the load back to the source. The magnitude of the reflected wave is dependent on the value K, the reflection coefficient.

The actual voltage at any point on the transmission line is the sum of the incident and reflected wave voltages at that point. The resultant total voltage wave appears to stand still on the line oscillating in magnitude with respect to time but having fixed positions of maxima and minima. This is due to the phase of the reflected and incident waves canceling or adding to the amplitude of each other.

For an example which will help to clarify what has been said previously, let us take a typical case of a transmission line 290° electrical length and terminated in a resistance of one-half the characteristic resistance of the line.



A better understanding of the conditions present may be had if we transform equations 17 and 18 to another form.

$$E = \frac{Er}{Zr}\left[Zr\frac{e^{jBs}+e^{-jBs}}{2} + jRo\frac{e^{jBs}-e^{-jBs}}{2j} \right] \qquad (19)$$

$$I = \frac{Ir}{Zo}\left[Zo\frac{e^{jBs}+e^{-jBs}}{2} + jZr\frac{e^{jBs}-e^{-jBs}}{2j} \right] \qquad (20)$$

recognizing that

$$\cos Bs = \frac{e^{jBs}+e^{-jBs}}{2}$$

and $\sin Bs = \dfrac{e^{jBs}-e^{-jBs}}{2j}$ $\qquad$ B is defined as $\dfrac{2\pi}{\lambda}$

equations 19 and 20 degenerate into

$$E = Er(\cos\frac{2\pi}{\lambda} + j\frac{Ro}{Zr}\sin\frac{2\pi}{\lambda}s) \qquad (21)$$

$$I = Ir(\cos\frac{2\pi}{\lambda}s + j\frac{Zr}{Ro}\sin\frac{2\pi}{\lambda}s) \qquad (22)$$

Solving these equations with our typical example in mind and plotting the amplitudes of the resultant current and voltage waves versus distance, we arrive at the following graph:

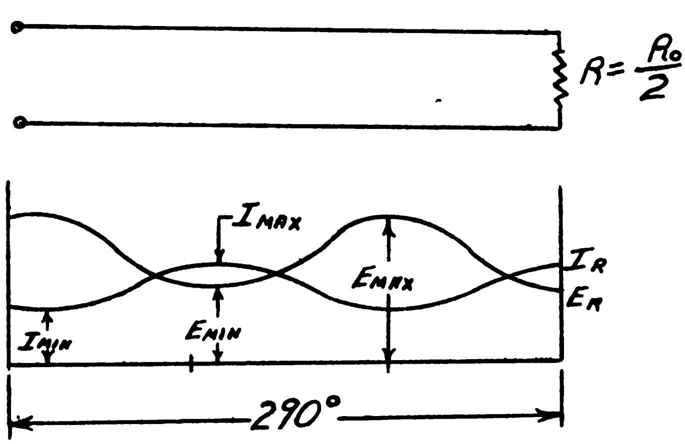


Figure 3

The plotting of the resultant voltage and current for this simple case clearly shows the maxima and minima present. These waves are known as standing waves and the ratio of the maximum voltage or current (anti-node) to the minimum voltage or current (node) is called the standing wave ratio.

Thus: $\frac{E_{max}}{E_{min}} = SWR$

$\frac{I_{max}}{I_{min}} = SWR$

Standing wave ratio measurements can be readily made on open wire transmission lines though for coaxial lines, a slotted line must be used. The greater the magnitude of the SWR, the greater the reflection factor and mismatch present.

The standing wave ratio may be defined in terms of the reflection coefficient as

$$\rho = \frac{1 + |K|}{1 - |K|}$$

This may be rearranged as

$$|K| = \frac{\rho - 1}{\rho + 1} = \frac{|E_{max}| - |E_{min}|}{|E_{max}| + |E_{min}|}$$

Figure 3 is a plot of equation 23 that permits obtaining the magnitude of K from a knowledge of the standing wave ratio.

Practically speaking, it is almost impossible to obtain a standing wave ratio of unity. However, this ideal can be closely approximated and by careful engineering design and use of impedance matchers, the power lost due to reflection can be kept very small. A discussion of matching the transmission line to the load or radiator will be presented illustration some of the methods and circuits whereby the standing wave ratio may be kept a minimum.

Antenna Impedance Matching Networks

Every antenna system, no matter what its physical form, will have a definite value of impedance at the point where the line is to be connected. The problem is to transform this antenna input-impedance to the proper value for matching the line. The conditions existing at the load determine the standing wave ratio on the line.

There are various types of matching networks that are available to match the antenna input impedance to the transmission line. In the following section a few methods of impedance matching will be given, though for a more thorough and rigorous treatment of this phase, the reader is referred to the references as given in the appendix.

The Quarter-Wave Transformer

The expression for the input impedance of a lossless line may be obtained for equations 21 and 22. Dividing equation 21 by 22, we obtain

$$Z_s = \frac{E_r\left(\cos\frac{2\pi}{\lambda}s + j\frac{R_o}{Z_r}\sin\frac{2\pi}{\lambda}s\right)}{I_r\left(\cos\frac{2\pi}{\lambda}s + j\frac{Z_r}{R_o}\sin\frac{2\pi}{\lambda}s\right)}$$

$$= R_o\,\frac{Z_r\cos\frac{2\pi}{\lambda}s + j\,R_o\sin\frac{2\pi}{\lambda}s}{R_o\cos\frac{2\pi}{\lambda}s + j\,Z_r\sin\frac{2\pi}{\lambda}s}$$

$$= R_o\,\frac{Z_r + j\,R_o\tan\frac{2\pi}{\lambda}s}{R_o + jZ_r\tan\frac{2\pi}{\lambda}s} \qquad (23)$$

This may be further rearranged as

$$Z_s = R_o\,\frac{\dfrac{Z_r}{\tan\frac{2\pi s}{\lambda}} + jR_o}{\dfrac{R_o}{\tan\frac{2\pi s}{\lambda}} + jZ_r} \qquad (24)$$

If the line is made a quarter-wave long, that is, if $s = \lambda/4$ then equation 24 degenerates to

$$Zs = \frac{Ro^2}{Zr}$$

This states that the input impedance of the line is equal to the square of the characteristic resistance of the line divided by the load impedance. Thus we can think of the quarter-wave line as a transformer to match a load of Zr to a source of Zs ohms. This match can be obtained if the characteristic impedance Ro' of the matching quarter-wave section of line is properly chosen so that

$$Ro' = \left|\sqrt{ZsZr}\right| \qquad (25)$$

To couple a transmission line to an antenna of impedance Zr, equation 24 may be used. The antenna impedance is then transformed to a value equal to the characteristic impedance of the transmission line. This value of Ro', the characteristic impedance of the matching section, is just the value required to achieve critical coupling and maximum power transfer and consequently, the standing wave ratio will be essentially unity.

The Half Wave Line as a Transformer

From equation 23, when a length of line having $s = \lambda/2$ is used

$$Zs = Ro\,\frac{Zr + jRo\tan}{Ro + jZr\tan} = Zr$$

A half-wave length of line may then be considered as a one to one transformer. This has a very useful application if the antenna impedance equals the characteristic impedance of the transmission line and the line itself matches the impedance of the transmitter. However, these conditions are the exception rather than the usual case.

The exponential line for impedance transformation (commonly called the "Delta" match); the "T" matching section, the "Gamma" match, are all methods which may be used to match the antenna input impedance to the transmission line. These methods are infrequently used and are more difficult to construct and adjust then the quarter-wave matching transformer.

There are many other variations of circuitry that enable an effective match between the antenna and transmission line, but the subject itself would be a volumnetric topic and be much too lengthy for the purposes of this paper. Reference may be made to several periodicals as listed in the Appendix for further information.

Coupling the Transmitter to the Line

As has been mentioned previously, the transmitter, or radio frequency amplifier, requires a definite value of load resistance if the desired power output is to be obtained. Since the input impedance Zs will seldom be the same as the load impedance required by the transmitting tube or tubes, the impedance must be transformed.

The principle of impedance transformation is illustrated in the following figure:

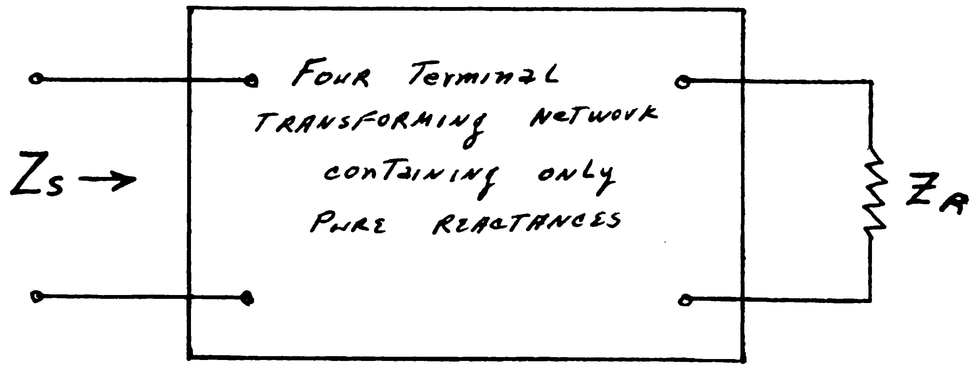


Figure 4

Figure 4 shows a network connecting a load impedance Zr which may be thought of as the input impedance of the transmission line and radiator, to the two input terminals 1-2. If this connecting network is made of pure reactances, any power delivered to the input terminals 1-2 must in turn be transferred to the load Zr. However, the resistive and reactive components of the input or driving point impedance at 1-2 will, in general, be different from the impedance Zr connected to the output terminals 3-4. Hence, the reactance network may be considered to be and "impedance-transforming" network changing the impedance Zr into an impedance Zs.

If the network contains resistive components, then the output power would be less than the input power and this power loss would be undesirable. Therefore, the networks are made of reactances with the lowest possible resistance.

Two reactance of opposite sign may be arranged as in Figure 5 to transform at one frequency a load resistance Rr to provide a desired load Rin for the generator, where Rr $<$ Rin.

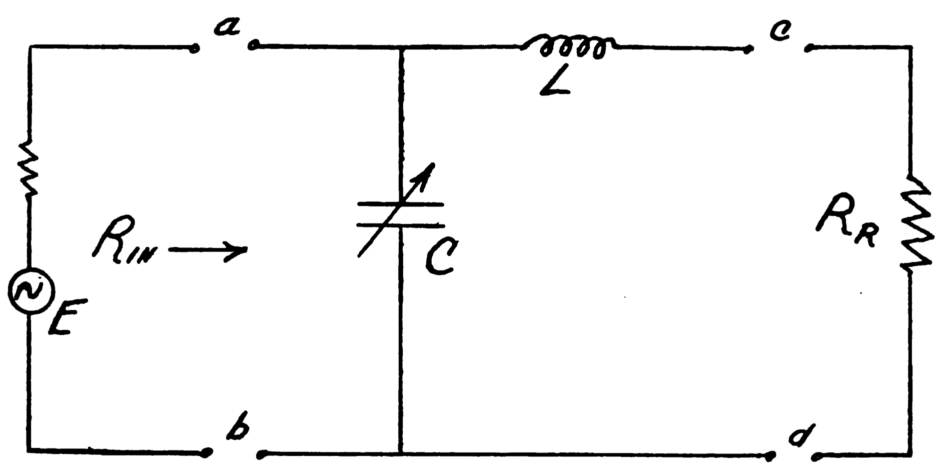


Figure 5

Operation of this circuit may be readily understood if it is noted that the circuit to the right of the terminals a,b constitutes a parallel circuit that at anti-resonance appears as a resistance load on the generator. The value of this resistance load is a function of the L to C ratio chosen for the reactance matching section. Thus one can select the values resulting in an antiresonant load Rin for matching to Rg. Simultaneous conditions to be realized are that the circuit to the right of a,b be in anti-resonance and have an anti-resonant impedance equal to Rin.

$$w = \sqrt{\frac{1}{LC} - \frac{Rr^2}{L^2}} \qquad (26)$$

$$R_{antiresonance} = Rin = \frac{L}{CRr} \qquad (27)$$

from which L = RinRrC

Inserting this expression for L in equation 26 and solving for C, we obtain

$$C = \frac{1}{wRin}\sqrt{\frac{Rin}{Rr} - 1} \qquad (28)$$

Likewise, from equation 27,

$$C = \frac{L}{RinRr}$$

Inserting into equation 26, leads to

$$L = \frac{R}{w}\sqrt{\frac{Rin}{Rr} - 1} \qquad (29)$$

We note that L will be imaginary if Rr > Rin. Therefore, if Rr > Rin, the matching circuits components of C and L should be interchanged. We will then obtain through a similar process

$$C = \frac{1}{WRr}\sqrt{\frac{Rr}{Rin} - 1} \qquad (30)$$

$$L = \frac{Rin}{w}\sqrt{\frac{Rr}{Rin} - 1} \qquad (31)$$

For matched conditions Rin will be equal to Rg and thus maximum power will be transferred.

The above circuitry is often used for matching a radio frequency amplifier to the transmission line. These are quite useful and their simplicity is such that they are easily constructed and adjusted.

Generally, inductive coupling, or coupling by means of the magnetic field, is used to transfer the radio frequency power to the antenna coupler, transmission line and thus to the radiator. A modification of inductive coupling, called link coupling particularly adapts itself to good constructional procedure and has found a wide acceptance in the design of antenna coupling impedance matching. Link coupling gives the effect of inductive coupling between two coils that have no mutual inductance. The link coils usually have a small number of turns compared with the tank coil of the final radio frequency amplifier. The number of turns is not greatly important because the coefficient of coupling is relatively independent of the number of turns on either coil. It is more important that both link coils should have about the same number of turns. Usually the length of the coil is small compared with the wavelength and therefore, the length is not critical if this is so. However, if the construction of the link necessitates that the length becomes an appreciable fraction of a wavelength, transmission line methods must be used so that the input impedance to the radio frequency amplifier is of the proper value for maximum power transfer.

The following diagram, showing a radio frequency amplifier coupled by means of line coupling to an impedance matching network clarifies what is meant by link coupling.

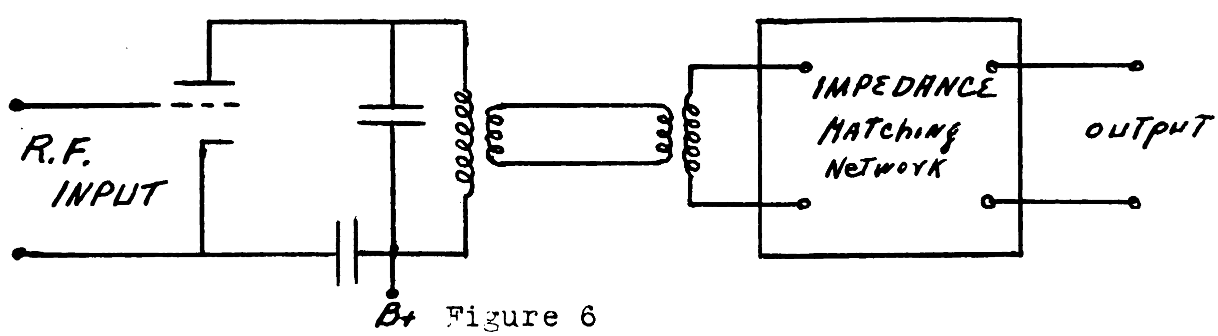


Figure 6

The pi impedance matching network, illustrated in Figure 7 is much more general in its application than the network given above. The pi network has the advantage of being able to match a wider range of impedances.

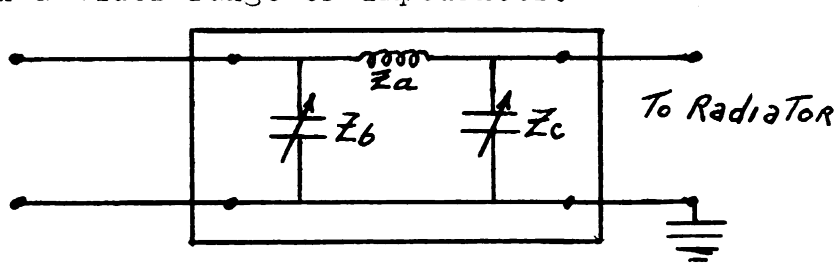


Figure 7

For the design consideration, let us take the general "T" section network of pure reactances. Figure 8 shows a generator of internal resistance Ri connected to a load R_2 through the T section network.

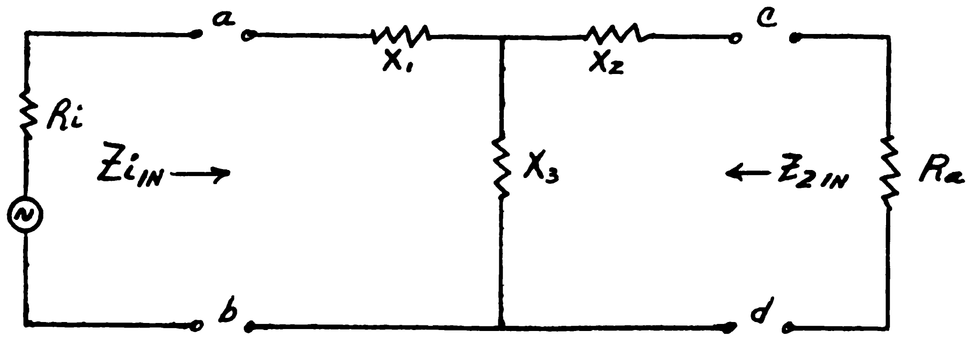


Figure 8

For the generator to be matches to the load, it is necessary that the impedance Z_{1in} at the terminals a,b be equal to R_1. With the load R_2 connected, the impedance Z_{1in} must be

$$Z_1in = Ri = jXi + \frac{(jX3)(R_2 + jX2)}{R_2 + jX2 + jX_3}$$

$$= \frac{-X1X2 - X1X3 - X2X3 + jX1R2 + jX3R2}{R_2 + j(X_2 + X_3)}$$

This can be simplified to

$$R_1R_2 + jR_3R_1 + jX_3R_1 = -X_1X_2 - X_1X_3 - X_2X_3 + jX_1R_3 + jX_3R_2 \quad (32)$$

Equating the real terms,

$$R_1R_2 = -X_1X_2 - X_1X_3 - X_2X_3 \quad (33)$$

Since R_1 and R_2 are real and positive, one or more of the terms on the right side of this equation must be positive. This can be accomplished only by having one of the reactances to be opposite in sign to the other. Thus one reactive arm of the T section must be opposite in sign to the other two.

That is, the T section must be composed of one capacity and two inductances or two capacities and one inductance.

By equating the reactive terms of Equation 32:

$$R_1(X_2 + X_3) = R_2(X_1 + X_3) \qquad (34)$$

$$X_1 + X_3 = \frac{R_1}{R_2}(X_2 + X_3) \qquad (35)$$

Equation 33 may be rewritten as

$$R_1 R_2 = -(X_1 + X_3)(X_2 + X_3) - X_3^2 \qquad (36)$$

Equations 35 and 36 may be combined in one of two ways, giving:

$$X_2 + X_3 = \pm\sqrt{\frac{R_2}{R_1}(X_3^2 - R_1R_2)}$$

$$X_2 = \pm\sqrt{\frac{R_2}{R_1}(X_3^2 - R_1R_2)} - X_3 \qquad (37)$$

The second combination yields:

$$X_1 + X_3 = \pm\sqrt{\frac{R_1}{R_2}(X_3^2 - R_1R_2)}$$

$$X_1 = \pm\sqrt{\frac{R_1}{R_2}(X_3^2 - R_1R_2)} - X_3 \qquad (38)$$

Equations 37 and 38 supply the values for X_1 and X_2 arms of the T section in terms of the third arm X_3. Two values of each are possible depending on the choice of the plus signs or the minus signs of the radicals. Since there are three unknowns and only two equations, it is necessary to assume a value for one of the reactance arms; after which the other two are readily determined.

By using the T to π transformations, the T sections developed by the equations may be readily transferred to π sections. The transformation formulas are readily computed from the equivalent input impedance with the terminals open circuited or shot circuited.

The transformation equations for the T to network section are given as follows:

$$Z_a = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_2}$$

$$Z_b = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_3}$$

$$Z_c = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_1}$$

A further advantage of section networks, in addition to its impedance matching capabilities, is its action as a low pass filter and the filtering action reduces unwanted harmonics materially. This type of network is a convenient method of matching an end fed Hertz (or random length) antenna.

<u>Automatic Impedance-Matching</u>

Automatic impedance matching devices are desirable in matching the input impedance of a given antenna to the characteristic impedance of a given feeder line, such that the power to the antenna is at all times, within a given frequency range, the optimum available in the feeder.

The remainder of this paper presents a solution to the specific problem of automatically matching the input impedance of a "doublet " antenna to a 75 ohm transmission line at frequencies from 6.8 to 7.5 megacycles. The limits of acceptable impedance matching must be such that the standing wave ratio will not exceed 1-1.25 at any frequency within this range.

The general block diagram of the circuit is as follows:

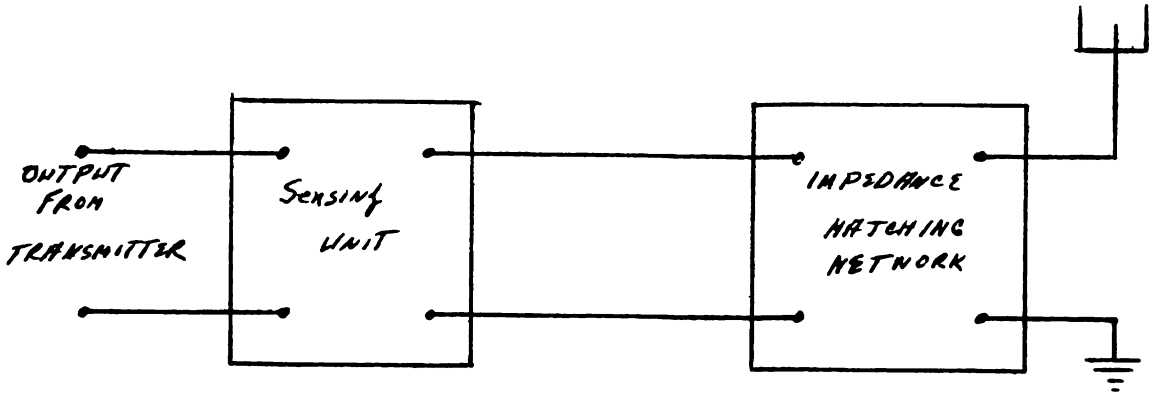


Figure 9

Sensing Unit

For optimum power transfer, the power factor should be essentially unity. This factor lends itself admirably for use as a "sensing " device for, with unity power factor, the input impedance of the antenna is purely resistive with the reactive components balanced out. By having a device that will measure the relative differences of the phase angle between voltage and current, this will enable the designer to use the phase angle differences as the "error-sensing" network.

The Foster- Seely discriminator circuit, developed for frequency modulation detection, with a few minor variations adapts itself admirably for use as a phase angle detector.

The basic circuit of the detector is shown in Figure 10.

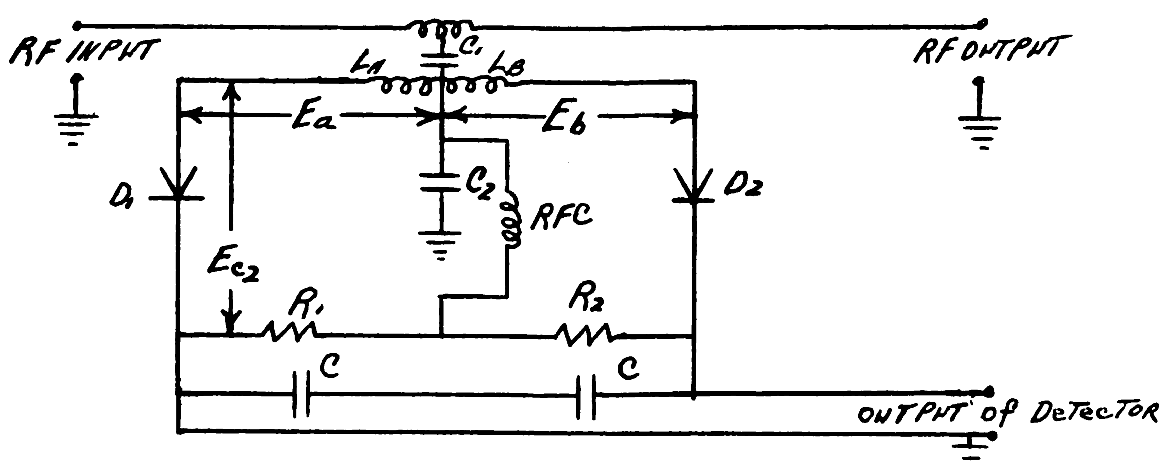
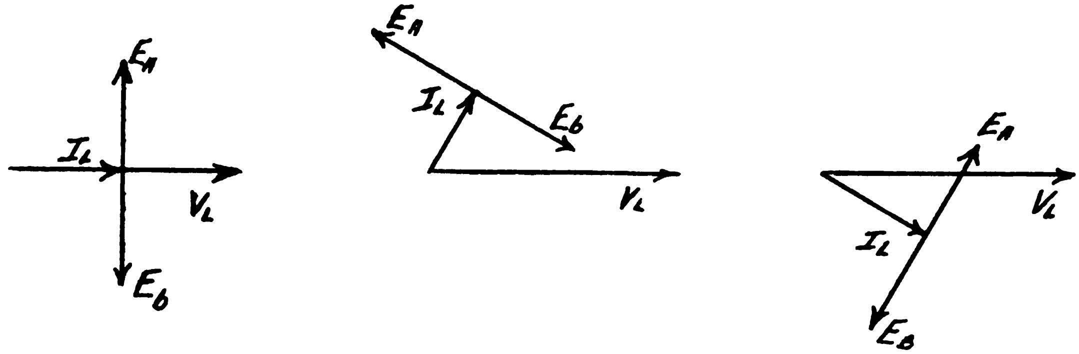


Figure 10

The significant distinction is that the circuit of the phase-angle detector is untuned, so that it is phase-conscious, rather than frequency-conscious as is the more familiar circuit of the Foster-Seeley frequency discriminator.

Qualitatively, the operation of the phase-angle detector can best be described by reference to the vector diagrams of Figure 11 and 12.



Unity P.F. (A) Leading P.F. (B) Lagging P.F. (C)

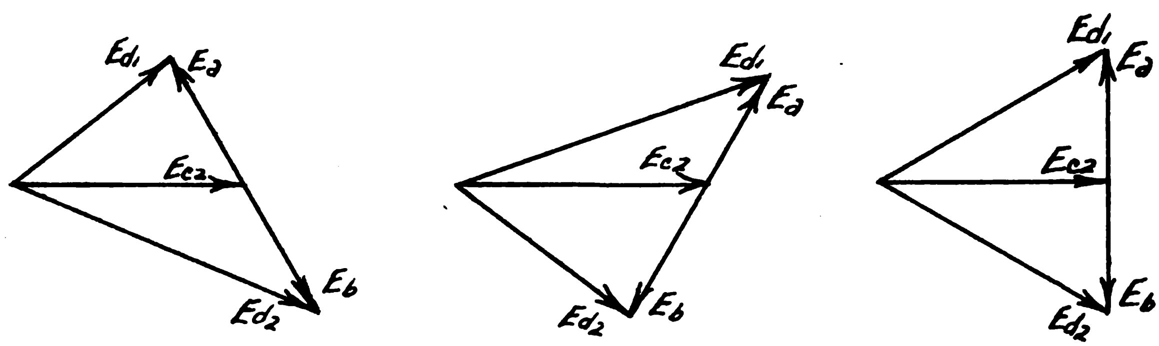
Figure 11

The voltage across capacitor C_2 is always in phase with the voltage, V_1, across the transmission line. Similarly, the voltage developed across the inductor L_2 is always different in phase from the line current, I_L, by 90 degrees.

Using the base of the vector diagram as the voltage across C_2 (E_{c2}, and with the coil center tapped, then the voltage E_a, across L_a, leads the line current by 90 degrees. The voltage E_b, across L_b lags the line current by 90 degrees. Thus, with the coil center-tapped, E_a equals E_b and with respect to the voltage and current in the transmission line, the vector diagrams of Figure 11 correctly describe the conditions present.

Referring to the secondary side of the phase-angle detector of Figure 12, the operation may be described as follows:

The voltage across either D_1 or D_2, is the sum of the voltage across C_2 and the voltage across the center-tapped coil, L_a or L_b. With a change in phase angle between the voltage across C_2 (which voltage is always in phase with the transmission line voltage V_L and the voltage across the coil), causes the vector sum of these voltages to change and the rectified voltages developed across the diode load resistors, R_1 and R_2 will change.



Leading P. F. (A) Lagging P.F. (B) Unity P.F. (C)

Figure 12

Referring to Figure 12, it is seen that when the line current leads the line voltage, the voltage E_{d1} will be less than E_{d2} and the output voltage will be positive. Likewise, when the line current lags the line voltage, Figure 12B, E_{d1} will be greater than E_{d2} and the output voltage will be negative. At unity power factor, the rectified voltages are equal and the output voltage is ~~less~~ than zero.

Thus, the two prime requisites, zero output at unity power-factor (resistive load), and the sign of the error voltage being dependent on the sign of the phase angle, are met so that this device is suitable to control a servo-system to correct the phase angle to zero.

With the sensing unit now available, the choice of the coupling network must be decided. With the design criterion in mind, the small range of frequencies envolved makes it logical that the following network would be well suited.

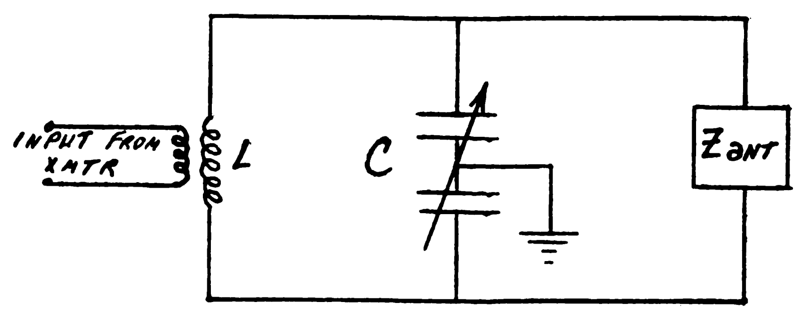


Figure 13

Again, from practical experience, the reactance of the double antenna that is "cut" for the mid-frequency of this range of frequencies, will be fairly small such that the component of L and C will not be too critical. It was decided that the coupling network in use by the author would be adapted for automatic control of the impedance matching of the transmission line to the antenna. This was built up several years ago from surplus components then available.

An ordinary D.C. amplifier amplifies the output from the "Sensing" circuit and controls the reversible motor which in turn, controls the variable condenser such that the optimum output is fed into the radiating system at all times.

The following schematic was subsequently built up for obtaining the experimental results.

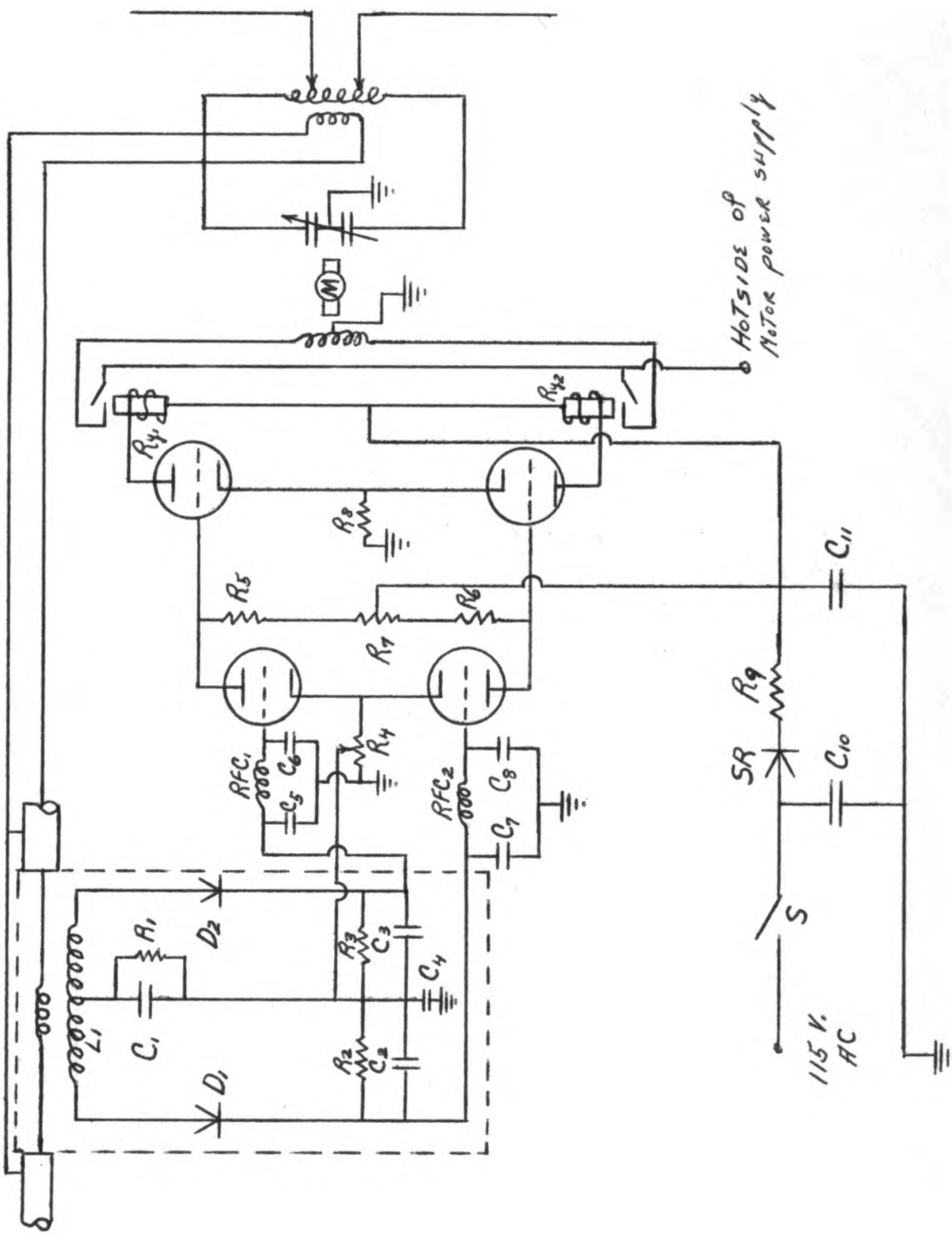


FIGURE 14

Parts List for Tuner:

C_1 200 mmf

C_2, C_3, C_9 100 mmf

C_4 .01 mfd

C_5, C_6, C_7, C_8 .015 mfd

C_{10} .05 mfd, 600 volts

C_{11} 40 mfd, 200 volts

D_1, D_2, 1N34 Rectifier

R_1 30K, ½ watt

R_2, R_3 0.1 megohm, ½ watt

R_4 50K potentiometer

R_5, R_6 47K, ½ watt

R_7 0.1 megohm potentiometer

R_8 20K, ½ watt

R_9 50 ohms, ½ watt

L_1 1 turn coil, center tapped

R_{y1}, R_{y2} S.p.s.t. relay, 5000 ohm

RFC, RFC_2 2.5 mho, R.F. choke

SR Selenium rectifier

As can be seen from the schematic, the crucial point is the development of this diagram is the "sensing" unit. After the proper determination of a suitable circuit for the measurement of error, straightforward circuitry results.

For details on the operation of the direct current amplifiers, various references are given in the appendix that give a concise explanation of this simple circuit.

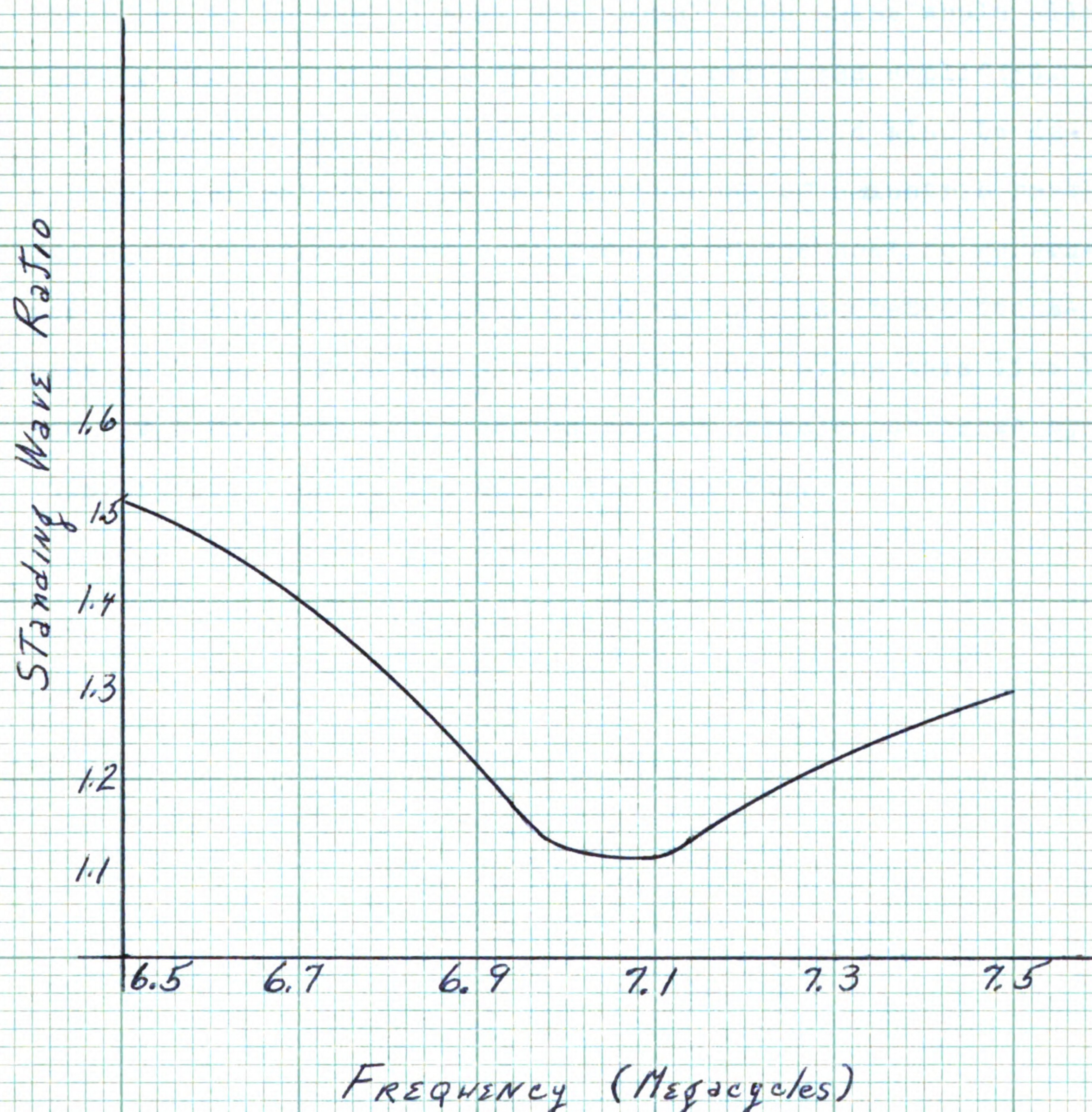
Experimental Results:

The following data was obtained from the prototype model as set up in the laboratory.

Frequency in Mcs.	Standing Wave Ratio
6.5	1.5
6.6	1.45
6.7	1.4
6.8	1.3
6.9	1.2
7.0	1.1
7.1	1.1
7.2	1.2
7.3	1.2
7.4	1.25
7.5	1.3

This information can be conveniently graphed (refer to Figure 15) and shows that our final results are quite acceptable. Within the range of frequencies desired, the standing wave ratio is well within tolerable limits.

Experimental Results
Frequency in Megacylces
Vs
Standing Wave Ratio



Conclusion:

No particular difficulty was experienced in obtaining the components for the antenna turner. Surplus military parts were available and adapted for use. The most critical component was found to be the plate relays that control the reverseable motor. These had to be set so that the normal tube noise would allow them to remain open until the sensing error was amplified.

Automatic impedance-matching devices would also be desireable for matching the load impedances of dielectric heating equipment to the output impedance of the heating generator or to the characteristic impedance of associated transmission lines. Work has already been done along this line by the Naval Research Laboratories and an article published by this organization is listed in the Appendix.

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