MULTIVARIATE FRACTIONAL RESPONSE MODELS IN A PANEL SETTING WITH AN APPLICATION TO PORTFOLIO ALLOCATION

By

Michael Anthony Carlton

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Economics – Doctor of Philosophy

2013

ABSTRACT

MULTIVARIATE FRACTIONAL RESPONSE MODELS IN A PANEL SETTING WITH AN APPLICATION TO PORTFOLIO ALLOCATION

By

Michael Anthony Carlton

Several papers use subjective survival probabilities as a measure of mortality risk in studying economic behavior. The first chapter “Wealth Holdings, Asset Allocation and Mortality: A Test of the Information Content of Subjective Survival Probabilities” studies whether subjective survival probability measures contain any additional information that can explain differential wealth holdings and asset allocation among households. We find some evidence that survival probabilities can explain differences in household wealth holding and allocation once we control for other factors that affect decision-making. We also find that the estimated impact of subjective survival is sensitive to the inclusion of reported survival probabilities of one.

Some fractional response variables, like the proportion of financial wealth allocated across multiple assets, must satisfy an adding up restriction. In the second chapter “A Model for Multivariate Fractional Responses with an Application to Asset Allocation”, we develop a two-step procedure where we estimate a model with multiple fractional response variables exploiting the fact that these variables sum to one in each period and are correlated over time. The first step entails estimation of the multivariate fractional responses using the multinomial quasi-likelihood function which explicitly imposes the adding-up restriction and the second step uses the Classical Minimum Distance estimator to account for serial correlation.

Many panel data estimators implicitly assume that we have a balanced panel at our disposal. Unfortunately this is rarely the case and dropping observations is an unsatisfactory solution to the problem. Estimation of fractional responses in a panel requires assumptions about

the distribution of the unobserved effect and its relationship with observables, which requires special treatment in an unbalanced panel. In the third chapter, “Estimation of a Multivariate Fractional Response Model with Unbalanced Panel Data”, we extend the approach in Wooldridge (2010) to the case of multiple fractional responses and apply this to unbalanced panel data on the allocation of financial wealth across several assets.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS

AHEAD..Asset and Health Dynamics among the Oldest Old

ANOVA ....Analysis of Variance

APE..Average Partial Effect

ASF ....Average Structural Function

CAMS ....Consumption and Activities Mail Survey

CDF..Cumulative Distribution Function

CMD ....Classical Minimum Distance

HRS ....Health and Retirement Study

IRA..Individual Retirement Account

OLS ....Ordinary Least Squares

P75 ....Subjective Probability of Living to Age 75

Pr(Live to 75) ....Subjective Probability of Living to Age 75

QMLE ....Quasi-Maximum Likelihood Estimates

CHAPTER 1

WEALTH HOLDINGS, ASSET ALLOCATION, AND MORTALITY: A TEST OF THE INFORMATION CONTENT OF SUBJECTIVE SURVIVAL PROBABILITIES

1.1 Introduction

Economic theory predicts that individuals save to finance their consumption in retirement. An important factor in deciding the level and allocation of wealth to different assets is the length of time the individual expects to live after retirement. The risk that an individual faces if they do not properly account for their survival expectations is that they will outlive their assets. Social Security and pensions can help to alleviate this risk by providing a fixed stream of income in retirement, but if individuals are attempting to smooth consumption there is a potential for a significant decrease in utility later in life from consuming too much early in life. Therefore, we should see that forward-looking individuals adjust their current behavior to their individual life expectancy.

It is straightforward to determine whether mortality has an impact on wealth holding and asset allocation. With repeated observations on individuals, we can simply see if wealth holding and asset allocation is different for individuals with longer actual lifetimes. The drawback to this approach is that it is looking at actual lifetimes, not expected lifetimes. In any period, an individual does not know their actual date of death but only has some idea of their probability to reach a target age. One can use life-table probabilities as a proxy for individual survival, but since these are averages over the entire population, they only vary by age, race, and gender. The ideal measure that we would like to use is individual's expected survival probabilities at the point in time that they are making their decision to save and allocate wealth.

Our goal in this paper is to determine whether we can explain any of the heterogeneity in wealth holdings and asset allocation using subjective survival probabilities elicited in the Health and Retirement Study (HRS). The HRS collects detailed information on household asset holdings as well as measures of subjective survival probabilities. Therefore, we have a source of data that contains the information that we would need to determine the impact of subjective survival probabilities on wealth holding and allocation behavior.

Several studies have analyzed the validity of these subjective survival probabilities in terms of actual mortality risk. Hurd and McGarry (1995) find that these measures are internally consistent, match closely to life table averages and covary with known correlates of actual mortality[1]. In a follow-up study, Hurd and McGarry (2002) find that these probabilities are also good predictors of actual mortality experiences of the HRS respondents. Smith, Taylor and Sloan (2001) support this finding but also point out that there is a large portion of the sample that report very small changes in their survival probabilities over time.

Elder (2012) looks closely at the subjective survival probabilities in the HRS and finds systematic differences as compared to life tables probabilities. He finds that the subjective survival probabilities do not account for yearly increases in mortality rates and that individuals do not update their survival probabilities as expected. He also finds that the life-table survival probabilities are considerably better predictors of actual survival than the subjective survival probabilities. Perhaps most concerning is that Elder (2012) provides compelling evidence that subjective survival probabilities in the HRS contain significant noise; in fact so much random noise that it may overwhelm any individual information that reflects actual heterogeneity.

While the above studies provide evidence that subjective survival probabilities match aggregate life-table probabilities but systematically differ from how mortality rates actually vary

[1] Hurd and McGarry (1995) do note that blacks report higher subjective survival probabilities than their white counterparts. This is inconsistent with the life-table probabilities.

over the lifetime it is still important to determine whether they are useful in explaining heterogeneity in economic behavior. What is important to note is that the findings in Elder (2012) suggest that we are likely to see very little impact from subjective survival probabilities simply because there is little signal in these measures.

Our empirical work focuses on estimating reduced-form relationships of wealth holdings and asset allocation with subjective survival probabilities elicited from individuals. We attempt to control for the heterogeneity in factors that economic theory tells us should affect the wealth holding and allocation decision including basic demographics (age, education, gender, etc.), income (both current and permanent), as well as proxies for an individual's time preference, cognition, and risk aversion. Since the HRS collects financial data at the household level, we model the household's wealth holding and the proportion of financial wealth allocated to stocks, bonds, CDs, and checking/savings/money market accounts. We estimate different models for single and married households since multi-person households have to take into account the survival expectations of both members. To estimate our allocation equations we use the Quasi-Maximum Likelihood approach proposed in Papke and Wooldridge (1996, 2008) which is appropriate for estimating fractional response variables.

We find mixed evidence for the impact of subjective survival on wealth holdings[2]. For single households, we find that a one-percentage point difference in reported subjective survival probability (a difference of 0.01) leads to about $266 more Net Worth and about $124 more Financial Wealth holdings, both significant at the 5% level. For married households we find that a one-percentage point difference in the husband's subjective survival probability leads to

[2] We find mixed evidence in terms of statistical significance of our estimates. Regardless of statistical significance, all of our estimated impacts for subjective survival probabilities are very small in magnitude.

increased Net Worth of about $241 and about $29 more Financial Wealth but both estimates are statistically insignificant at standard levels. A one-percentage point difference for the wife's subjective survival probability is associated with about $425 more in Net Worth (statistically significant at the 5% level) but, contrary to theory, leads households to hold about $22 less in Financial Wealth, though this estimate is not statistically significant.

There appears to be no impact of survival probability on the proportion of financial wealth allocated across assets for single households though we note that we see the correct sign of the impact of survival probabilities on the proportion of wealth allocated to stocks and checking. For married households, we find that a one-percentage point difference in the husband's subjective survival probability leads the household to allocate about 0.026% more to stocks and a one-percentage point difference in the wife's subjective survival probability leads the household to allocate around 0.022% more financial wealth to stocks; these are significant at the 5% and 10% levels. The average partial effects for the subjective survival probabilities in our checking equation are negative for both husband and wife but are very small and statistically insignificant at standard levels.

We find no evidence that our estimated impacts of subjective survival probabilities are sensitive to the inclusion of a measure of permanent income. If we drop our measure of permanent income, we find that the estimated impact of subjective survival increases slightly for wealth holdings and asset allocation equations. We also show that our estimated impacts for subjective survival on wealth holding and asset allocation look very different when we treat reported survival probabilities of one differently. This is consistent with the findings from Elder (2012) which suggest that there is significant measurement error in these reported survival probabilities.

The outline of this paper is as follows. Section 2 briefly discusses the underlying theory and reviews studies that use subjective survival probabilities to explain economic behavior. Section 3 briefly explains the HRS, provides descriptive statistics and patterns in the data, and details the analysis of the informational content of survival probability measures in terms of wealth holding and allocation. Section 4 presents the results of our analysis and Section 5 concludes.

1.2 Theory and Literature Review

Economic theory indicates that forward-looking individuals with higher survival probabilities will hold more wealth and allocate a larger portion of their wealth to the risky asset than individuals with lower survival probabilities. This is because with time-separable utility the introduction of non-zero survival probabilities effectively multiplies a time varying factor to the individual's constant discount rate. Bernheim, Skinner, and Weinberg (2001) perform a simple simulation and demonstrate that individuals with lower discount rates hold more wealth than individuals with higher discount rates. Cocco, Gomes, and Maenhout (2005) and Sahm (2007) show that the underlying policy function that defines the optimal portfolio choice of the household is a function of total wealth: the sum of discounted future income and current financial wealth holdings. With uncertain lifetimes, the individual will discount future income by incorporating the probability that they will earn that future income stream. They show that the proportion of financial wealth allocated to the risky asset is positively related to how "certain" their future income stream is. DeNardi et al. (2009) also document that higher survival probabilities (even if they are small) will lead even the oldest and sickest individuals to spend down their retirement wealth very slowly. DeNardi et al. (2009) also show that the impact of lower survival probabilities leads individuals to decrease their wealth holdings.

The basic premise here is that individuals will smooth the marginal utility of consumption over time by setting the marginal utility of current consumption to the marginal utility of the discounted future consumption stream. As long as individuals discount the future by including the probability of survival we will see that a person with higher survival probabilities will hold more wealth and allocate a larger proportion of wealth to the risky asset.

Several studies attempt to link subjective survival probabilities to economic behavior and these tend to focus on the areas of retirement, bequests, wealth holding, and consumption. Hurd et al. (2002) analyze the decisions to retire and claim Social Security benefits early. Using responses to subjective survival to age 85 and Social Security earnings data matched to the first four waves of the HRS, they estimate their model on two different samples: those who are retired prior to age 62 and those not retired by 62. They find that there is a small, statistically significant increase in retirement and claiming of benefits but only for those individuals that report that their probability of living until age 85 is zero.

Delevande et al. (2006) revisit the retirement and claiming issue with the hope of obtaining estimates that are more accurate by instrumenting for measurement error in the reported survival probabilities. They use the response to the question of survival to the age of 75 instead of 85 used by Hurd et al. (2002). As instruments for the subjective survival probability, they use demographic information, mortality experience of parents, and an optimism index, which is a predicted value generated from a factor analysis of the remaining subjective probability questions. This optimism index reflects the correlation of the responses to all probability expectation questions in the HRS and potentially represents the unobserved heterogeneity in individual expectations. Notably this constructed optimism index has a large and statistically significant impact on their instrumental variable for the subjective survival probability. Similar to Hurd et al. (2002) they estimate the impact of survival probabilities on claiming behavior for the group of respondents that are retired by age 62 and the impact of survival probability on early retirement and early claiming for the sample of respondents that are still working at age 62. Using the raw subjective survival probability measures they find no statistically significant impact of survival on either claiming or retirement. When they instrument for subjective survival they find that there is a statistically significant impact on early claiming but not on retirement. Their results suggest that a five-percentage point increase in the predicted survival probability will lead to a 1.9 percentage point decrease in the number of people that will claim Social Security benefits early. Delevande et al. (2006) claim that using instrumental variables to rid the subjective survival probability of measurement error is the reason that they are able to find a statically and economically significant effect of subjective survival on claiming behavior.

Gan, Gong, Hurd, and McFadden (2004) study the impact of subjective survival probability measures on the bequest behavior of elderly households. They derive estimable equations from a life-cycle model so that they can estimate structural parameters that can describe the individual's preferences for bequests. To account for focal points and to calculate a survival curve they construct a measure of yearly mortality rates based on responses to subjective survival questions in the Asset and Health Dynamics among the Oldest Old (AHEAD) study. In addition, they estimate their equations using the life-table survival curves to compare the predictive power of the subjective survival curves. Since they are estimating a life-cycle model, they are able to simulate consumption and wealth trajectories and compare model predictions to actual decisions within the AHEAD panel. They find that their predicted consumption and wealth trajectories using the survival curves derived from the subjective survival probabilities outperform the predicted values using life-table survival. They also note that their estimates suggest that bequest motives of the older population represented in the AHEAD are very small and that most bequests are involuntary or accidental.

Salm (2006) attempts to estimate the structural parameters from the Euler equation for consumption derived from a simple life-cycle model. He uses data from single households that completed the HRS interview in 2000 and 2002 and completed the Consumption and Activities Mail Survey (CAMS) in 2001 and 2003. Using the subjective survival probabilities, he constructs yearly survival rates following Gan et al. (2004). He interprets the inverse of his estimated coefficient on the subjective survival probability as his estimated value for the risk aversion parameter. To allow for precautionary savings he estimates the model including an estimated variance of out-of-pocket medical expenses to proxy for consumption risk. Salm (2006) finds that higher subjective survival probabilities lead to decreases in the growth rate of consumption.

Perry (2005) estimates an empirical model derived from the Euler equation for consumption. He constructs yearly survival rates from the subjective survival probability responses in the HRS. Perry (2005) constructs his measure of consumption by looking at differences between wealth holdings across periods. He finds that there is no statistically significant relationship between the constructed subjective survival probabilities and his measure of consumption. Perry (2005) points to substantial measurement error in his measure of consumption as the culprit for the lack of statistical significance in his estimates.

Bloom et al. (2006) studies the impact of subjective survival probabilities on retirement and wealth holdings of both single and married households. Bloom et al. (2006) take a sample of individuals that were aged 50-70 in 1992 and estimate the relationship between their retirement decisions and wealth holding and subjective survival probability to age 75. To correct for potential measurement error they instrument subjective survival probabilities using mortality risk factors and parental mortality experience. They find no statistically significant impact of subjective survival on either retirement or wealth holding for single households. Looking at married households, they find no statistically significant relationship for the retirement decision of either the husband or wife. For married households they find that once they instrument for the subjective survival probability they estimate a statistically significant impact of survival probabilities of both spouses. Their estimates suggest that a ten percentage point change in the husband's survival probability leads to a $27,600 increase in wealth (significant at the 10% level) and a ten percentage point increase in the wife's survival probability leads to a $32,600 increase in wealth. Bloom et al. (2006) interpret these findings as evidence that households save more in the face of higher expected lifetimes because there is no incentive to postponing retirement.

DeNardi, French, and Jones (2009) look at the impact of survival uncertainty, medical expenses, and health uncertainty on the wealth holdings of elderly individuals in the AHEAD. They estimate a structural model using the Method of Simulated Moments and matching the medians that were in the data to the medians estimated by the structural model. They did not use the subjective survival probabilities that are collected in the AHEAD survey; instead, they estimate future survival relying on the actual mortality experience in the panel[3]. They find that the structural model fits the data very well. This lends much more credibility to the simulations that they perform to isolate the impact of differential mortality on wealth holdings. Since they have modeled their survival probabilities as a function of gender, health status, and permanent

[3] The model for survival probabilities includes the prior period's health status, permanent income, and gender.

income, they perform simulations that show the impact of changes within the components of survival. It appears that the impact of health, gender, and permanent income all have similar impacts on the wealth holdings of elderly households. The simulations clearly point to the fact that the slow spend down of wealth in old age is due to uncertain lifetime. As long as there is a possibility of outliving one's assets there will be a significant precautionary savings motive.

With the exception of Hurd et al. (2002), Bloom et al. (2006), and Delevande et al. (2006), all of the cited works estimate either an Euler equation or a structural model using either the HRS or the AHEAD. Most of the studies that use the subjective survival probabilities either transform them into yearly survival curves or use instrumental variable techniques to correct for measurement error. Our work is probably closest to Bloom et al. (2006), although we take serious the findings of the other studies that we have reviewed. We estimate models for wealth holdings of both single and married households and find a statistically significant but small impact on wealth holdings for single and married households. While there are a few empirical studies of asset allocation in the HRS they look at health status and not survival probabilities (Rosen and Wu (2004) and Berkowitz and Qiu (2006)). Lillard and Willis (2001) estimate a model of asset allocation including a measure of the number of focal point responses across all subjective probability questions, which they interpreted as a measure of cognition. They found that fewer focal point responses correlate with increased allocation to the risky asset.

1.3 Model Specification, Descriptive Statistics, and Estimation Strategy

The goal of our empirical exercise is to determine whether subjective survival probabilities help to explain the differences in wealth holding and allocation behavior between households in the Health and Retirement Study. We estimate reduced form equations to determine the relationship between household wealth holdings, allocation of wealth and survival probabilities. In our model specification we include basic demographic characteristics: age, education, working status, household size, gender, marital status, and current income. Theory dictates that the household's wealth holding and allocation decisions are a function of not only their current resources but also what they expect their future resources will be. To this end, we include a measure of permanent income in our model specifications. Differences in risk aversion among households can affect the amount of wealth and the allocation of wealth across assets. To proxy for risk aversion we include responses to questions regarding a household's willingness to accept different income gambles. Households can also differ in their discount rates. We include responses to questions about the length of time that the individual considers for financial planning as a proxy for discount rates. Previous research (Elder (2012), McArdle et al. (2009) and Lillard and Willis (2001)) suggest that an individual's cognition can also affect the outcome of the wealth holding and asset allocation decisions; we include measures of word recall and simple numerical calculations captured in the HRS as proxies for cognition.

We will estimate separate equations for single and married households. We present equations for single households; married households will include the same set of regressors for both the husband and wife. Let i represent the household and t represent the year of the survey. To estimate the impact of survival probabilities on wealth holding we specify the following equation:

$$W_{it} = \beta_1 \cdot \Pr(\text{Live to 75}) + \boldsymbol{\beta}\mathbf{X}_{it} + u_{it} \qquad (1.3.1)$$

where W_{it} is a measure of wealth, either Net Worth or Financial Wealth, $\Pr(\text{Live to 75})$ represents our measure of subjective survival probability from the HRS (see below for the specific question used to solicit the survival probability), and $\mathbf{X}_{it}$ are control variables that we detail below. We also estimate the relationship between survival probabilities and the allocation of Financial Wealth across four types of assets; stocks, bonds, CDs, and checking/savings/money market accounts:

$$y_{itg} = \Phi(\theta_1 \cdot \Pr(\text{Live to 75}) + \boldsymbol{\theta}\mathbf{Z}_{it}) + \varepsilon_{it} \qquad (1.3.2)$$

where y_{itg} represents the proportion of financial wealth allocated to asset g in year t by household i. Here $\mathbf{Z}_{it}$ represents control variables that will include the same variables we use in the wealth holding equations with the addition of a measure of total wealth available to the household at time t. Due to the fractional nature of the asset share equation in (1.3.2) we have followed Papke and Wooldridge (1996, 2008) and specified that the conditional mean function is nonlinear; Φ represents the standard normal cumulative distribution function.

We include controls for standard demographic variables that can affect behavior such as age, age^2, age^3, household size, education, as well as controls for year effects. Theory tells us that it is necessary to control for current income as well as the value of lifetime income. We measure current income as the sum of all non-capital income of the household as reported in all waves. As a measure of lifetime resources we follow Altonji and Doraszelski (2005) and calculate a household's permanent income. To do this we regress current household income on age, age^2, age^3, age^4, a set of year dummies, household size, and indicators for marital status, gender and education level.[4] For each household we calculate the average of the residuals from this regression and then compute the permanent income of the household as the sum of the average residual and the predicted value of income for the education level of the individual and

[4] For married households, we do not include indicators for gender or marital status.

assuming that the individual is at the average age in our sample.[5] Since there is likely a nonlinear impact of these income measures we also include squares of current and permanent income as well as their cross product in our wealth holding equations[6].

Theory dictates that total wealth has an impact on the allocation of wealth across available assets. Ideally, the measure of wealth that we would include in our model would include pension wealth, Social Security wealth and the present value of all future income. For our measure of wealth we use the log of Net Worth which is the sum of Financial Wealth (the value of holdings in stocks, bonds, CDs and checking/savings/money market accounts) and Non-Financial Wealth (the value of holdings in IRA/Keogh accounts, housing, vehicles, other real estate, and trusts) minus any debt associated with these asset holdings.[7] Our hope is that the combination of Net Worth and the estimated permanent income measure will act as a good proxy for the total wealth measure that theory dictates will affect decision-making.

In addition to controlling for differences in household demographics, income, and wealth there is also a need to control for risk aversion and discount rates of households. To proxy for discount rates we will use responses to HRS question regarding the financial planning horizon of the household[8]. To control for risk aversion we will use questions meant to solicit aversion to income risk and we will categorize individuals based on their responses to a series of unfolding

---

[5] For single households we use calculate permanent income at age 58. For married households we use 58 for the husband and 54 for the wife, both of these are the average ages in our sample.

[6] In our asset allocation equations we actually use permanent log income, which we estimate in a similar manner to permanent income with the exception that we regress the log of current income on the demographic variables.

[7] We use RAND imputations for missing values of income and asset holdings when the respondent was unable to give exact values.

[8] The choices available from the planning horizon question are: *Next Few Months*, *Next Year*, *Next Few Years*, *5-10 Years* and *10+ Years*. We use *Next Few Months* as our base category.

questions regarding different income scenarios following Barsky, et al. (1997). This essentially categorizes an individual into one of four groups: High Risk Aversion, Medium-High Risk Aversion, Medium-Low Risk Aversion and Low Risk Aversion. We use High Risk Aversion as our base group.

We include two additional controls to proxy for the cognitive ability of the individual. The HRS collects several measures of individual cognition, but not all measures are available in all waves of the survey. In every wave of the HRS, respondents are given a list of nouns and then asked to repeat this list immediately and at the end of the cognition section. Our first measure of cognitive ability is the proportion of words recalled at the end of the cognition section. In addition, each individual performs a series of five simple numerical calculations. First, the individual subtracts 7 from 100. The next question in the series asks the individual to subtract 7 from the answer to the previous calculation. The individual performs this calculation a total of five times. As a second measure of cognition, we use the number of correct calculations, ranging from 0 to 5. By no means do we think that these two measures will completely capture the cognitive ability of the individual. Our hope is that accounting for education level and some time-varying measure of cognitive ability that we can accurately proxy for the cognitive ability of an individual.[9]

We use data from the Health and Retirement Study (HRS). The HRS began in 1992 and was nationally representative of all non-institutionalized individuals aged 51-61 in that year. In 1998 the HRS combined with the Asset and Health Dynamics of the Oldest Old (AHEAD) and added several new cohorts to be nationally representative of the population of non-

[9] McArdle, Smith, and Willis (2009) study the ability of cognitive measures in the HRS to explain wealth holdings and allocation. They find that word recall is positively correlated with allocation to the risky asset (stocks) and the amount of wealth held. Their analysis does not account for survival probabilities.

institutionalized individuals aged 51 and older. In addition to initial respondents, spouses are interviewed and followed in subsequent interviews. We use data for all cohorts from Waves 1-7 (1992-2004) of the HRS. In addition to basic demographic variables, the HRS collects detailed information on wealth, allocation, income and its sources, health and measures of the probability that future events occur. The main variable of interest in this analysis is the response to the following question:

> *On a scale from 0 to 100, where 0 is no chance and 100 is absolutely certain, what are the chances that you will live to age 75 or older?*

We use responses to this question as a measure of subjective survival probabilities.

We construct our data set by household. For married households we combine spouses in each wave so that our panel consists of household observations by year. To select individuals for our sample we include observations that satisfy the following criteria:

(i) respondent is younger than 65 at the time of the interview,

(ii) respondent does not have missing values for subjective survival probability,

(iii) responses are not from a proxy interview and

(iv) the respondent is still living at the time of the interview.

These criteria are applied at the individual level so married households that do not have both spouses are dropped. The decision to allocate wealth across assets is conditional on holding positive financial wealth; therefore, for estimating allocation relationships we drop observations where financial wealth is zero. Imposing the above criteria for married households leaves us with 6,608 unique households (18,603 observations) to estimate wealth holding equations (we call this our Wealth Sample). If we drop those observations where households are holding zero financial wealth we are left with 6,048 households (16,690 total observations) which we will call our

Allocation Sample. Applying the criteria to single households we are left with 5,022 households (14,275 total observations) for our Wealth Sample and dropping observations with zero financial wealth we are left with 4,053 households (10,573 total observations) in our Allocation Sample.

We calculate means and medians for the Wealth and Allocation Samples for both single and married households. Table 1 displays these descriptive statistics for single households. We can see that single households are predominately white females. A majority of individuals have not completed college (a little over 80%). Most single households hold most of their net worth in non-financial wealth. Between the Wealth and Allocation sample, we can see that individuals that hold positive financial wealth tend to report higher survival probabilities, have slightly more education, income, and wealth. On average, it appears that single households tend to hold higher amounts of their financial wealth in checking, savings, and money market accounts.

Table 2 displays descriptive statistics for the Wealth and Allocation Samples for married households. Compared to single households, married household members tend to have slightly higher education. Married households also report holding a large portion of their net worth in non-financial wealth. Married households earn more income on average and they have higher wealth holdings relative to single households. Comparing the Wealth Sample to the Allocation Sample, it appears that households with positive financial wealth are better educated, earn slightly more income, and hold more wealth. In comparison to single households it appears that married households allocate about 10% more of their financial wealth to stocks and ten percent less to checking, savings, and money market accounts.

Table 3 displays the raw relationship between average wealth holdings and average asset allocation and survival probabilities for single households. Tables 4 and 5 show the same relationship for married households; Table 4 uses the husband's reported survival probability and

Table 5 uses the wife's reported survival probability. We can see that average wealth holdings are higher for higher values of survival with the exception of those individuals that report a survival probability of one. We can also see that the average proportion of financial wealth to stocks is increasing with higher survival probabilities and the allocation to checking is decreasing with higher survival probabilities. Since stocks and checking are the most risky and least risky of the four financial assets this finding seems to fit well with what theory predicts. The objective of this study is to determine whether this correlation still exists once we have removed the impact of other factors that affect wealth holding and allocation behavior.

1.4 Results and Discussion

Using equation (1.3.1) we estimate models of wealth holding for single households; results are presented in Table 6.[10] We see a positive impact of survival probabilities on both Net Worth and Financial Wealth; a one-percentage point difference in survival probability is associated with about $256 more Net Worth and $124 more Financial Wealth, both estimates are significant at the 5% level. Education plays a key role in explaining differences in wealth holdings among single households. Households that have a college degree have nearly $111,000 more in Net Worth (and nearly $51,000 more Financial Wealth) than individuals that have not completed High School. There also appears to be a significant positive impact of a longer financial planning horizon. Those households that plan for more than ten years into the future have about $95,000 more in Net Worth and $39,000 more Financial Wealth than households that only plan for the next few months.

We now turn to the results of estimating equation (1.3.2) using the Quasi-Maximum

[10] Throughout the paper (including the tables) we refer to a one-percentage point change in the subjective survival probability. This is equivalent to a 0.01 change in our measured variable. The reported coefficients and average partial effects in the tables are already calculated for a 0.01 change in subjective survival probability.

Likelihood Estimation proposed in Papke and Wooldridge (1996, 2008) for fractional response variables. Table 7 presents the estimated average partial effects from estimation of this fractional probit on the proportion of Financial Wealth allocated to the four different assets: stocks, bonds, CDs, and checking/savings/money market accounts.

There appears to be no impact of survival probabilities on the allocation of wealth for single households. While the signs on the average partial effects of survival probability on stocks and checking are what theory predicts (positive and negative, respectively) they are estimated to be nearly zero and are all statistically insignificant at any standard level. We see that households with a college degree allocate 11% more of their financial wealth to stocks and 10% less to checking accounts than households that have not completed High School, both significant at the 5% level. Financial Planning Horizon also plays a role in asset allocation, we estimate that households with a horizon of ten years or more will allocate 5% more financial wealth to stocks, 1% more to bonds, and 6% less to checking accounts than those households that have a horizon of only a few months.

Table 8 contains the results from estimation of the wealth holding equation (1.3.1) for married households. We see a positive impact of a wife's subjective survival on Net Worth holdings; a one-percentage point difference in a wife's survival probability is associated with about $425 more in Net Worth, this is significant at the 5% level. A one-percentage point difference in the husband's subjective survival is associated with about $241 more Net Worth, but this estimate is not statistically significant. Looking at the impact of subjective survival on Financial Wealth, we see that the estimated impact for a husband's survival is very small and insignificant, and that the estimate of the impact of a wife's survival probability is actually negative, though both estimates have very large standard errors. The estimated impact of education on wealth holdings is not as clear-cut for married households as for single households. We see that the husband's education has no statistically significant relationship with Net Worth; it appears that having less than high school education leads to more wealth holdings than if the husband completed high school or some college. The wife's education level does have the expected impact on Net Worth. If the wife has a college degree, we estimate that household to hold about $84,000 more in Net Worth than a similar household with a wife that did not complete high school. Our estimates for education in the financial wealth regressions make a little more sense. We find that a college degree for the husband leads to about $37,000 more in financial wealth. If the wife also completed college, we estimate that the household's financial wealth will increase by $38,000. We also see that having the longest financial planning horizon (10 + years) has a positive impact on wealth holdings. If both spouses have horizons of ten years or more we see them holding about $110,000 more in Net Worth and about $52,000 more in Financial Wealth.

Table 9 displays the results of the fractional probit estimation of equation (1.3.2) on the allocation of Financial Wealth across the four asset types. We see that there is a statistically significant impact of both husband and wife survival probabilities on the allocation to stocks and CDs. A one-percentage point difference in the husband's subjective survival leads households to hold about 0.026% more of their financial wealth in stocks and about 0.013% less in CDs, these estimates are significant at the 5% and 10% level respectively. A one-percentage point difference in the wife's survival probability leads the household to hold about 0.022% more financial wealth in stocks and 0.019% less financial wealth in CDs, both these estimates are significant at the 10% level. While the estimated average partial effects of husband and wife survival probabilities are statistically insignificant for the allocation of wealth to checking we can see that we estimate negative, though small impacts of higher survival.

Similar to our findings for single households we see that higher education levels of both spouses lead to increased allocation to stocks and decreased allocation to checking. Not only is the estimated impact of a college degree statistically significant for both spouses for all assets but the magnitude is also large compared to the estimated impact of subjective survival to age 75. We see that the financial planning horizon of the husband only is positively associated with the allocation of wealth to stocks and negatively associated with the allocation to checking. This is not the case for the wife's planning horizon. For stock allocation, we see that longer planning horizons have a negative impact relative to the shortest horizon of only a few months. Looking at the results for the allocation of wealth to checking we can see that only the longest planning horizon (10+ years) has a negative impact, though none of the estimated average partial effects for the wife's planning horizon are statistically significant.

One main reason that our results for single and married households differ is that married households need to consider the impact of the characteristics of both household members. Women live longer than men and therefore would be the more likely of the two spouses to have to face the risk of outliving assets. While we find an impact of the wife's increased survival probabilities on Net Worth, it is very small and does not appear to carry over to the financial wealth of the households. Looking at the allocation decision, we see that the impact of the wife's survival probabilities is smaller than the estimated impact of the husband's. In addition, it appears that only the financial planning horizon of the husband has any impact on allocation. We see that the longest planning horizon (relative to the shortest horizon) for the husband increases the proportion allocated to stocks by about 3% and decreases the proportion allocated to checking by 4%. It appears that the characteristics of the wife play a small role in the decision of how much to save and how to allocate wealth across assets. This could be consistent with the idea that the household maximizes their joint utility, i.e. they only consider the lifetime of the household when both spouses are alive. If this is the case, we would see that divorced or widowed women would have less wealth at their disposal. From our analysis of single households, we see that divorced women have less wealth but widowed individuals have greater wealth. The asset allocation of divorced and widowed households in the analysis of singles indicates that they allocate less to stocks and more to checking, but none of these estimates are statistically significant. A more detailed analysis into the decision making of married households and the way that they pool their characteristics to make wealth and allocation decisions is definitely an avenue for future research.

We test the sensitivity of our results for subjective survival responses for wealth holding and asset allocation for single and married households to dropping variables that include our constructed measure of permanent income. We remove permanent income, its square, and the cross product of permanent income and current income from the wealth holding equations and we remove permanent log income from the asset equations.

For single households, we can see that removing permanent income from our wealth holding equations leads to an increased impact of subjective survival probabilities. Without controlling for permanent income, we see that a one-percentage point difference in subjective survival leads single households to hold \$339 more Net Worth and \$156 more Financial Wealth. Removing permanent log income from our asset allocation equations we see that the impact of subjective survival does not change much. There is no statistically significant impact of difference in subjective survival, though we do see that the estimated impacts of higher subjective survival increased slightly for stocks and bonds and decreased slightly for CDs and checking.

When we remove permanent income variables from the wealth holding equations for married households, we estimate that a one-percentage point difference in husband's subjective survival probability leads the household to hold \$356 more in Net Worth and \$66 more in Financial Wealth. A one-percentage point difference in the wife's subjective survival probability increases Net Worth by \$522 and Financial Wealth by \$12. While the estimated impact for Financial Wealth remains statistically insignificant and economically small for both spouses we see that the estimated impact of a one-percentage point difference in subjective survival probability for either spouse is generating about \$100 more Net Worth and that the estimate for a husband's subjective survival probability is now significant at the 10% level. Removing permanent log income from the asset allocation equations has a similar effect as in single households. The estimated impact of subjective survival probabilities slightly increases for stocks and bonds and slightly decreases for CDs and checking. We see that our estimated impact for a one-percentage point difference in the husband's subjective survival probability leads to 0.027% more of the households wealth allocated to stocks and only 0.015% less wealth allocated to checking. We see that the one-percentage point difference in the wife's subjective survival probability is now driving a larger impact on the allocation of wealth to stocks. Including permanent log income we see that the impact was around 0.022% more in stocks (significant at the 10% level), but dropping our permanent log income measure we see that this estimate increases slightly to 0.023% and remains statistically significant at the 10% level. In addition, including permanent log income we estimated a negative (statistically insignificant) impact of increased subjective survival probabilities of both spouses on the proportion of wealth allocated to checking. When we drop permanent log income, the estimated impacts become slightly more negative.

Overall, excluding our measure of permanent income from our wealth holding and allocation equations leads us to find nearly identical coefficients and average partial effects as estimates from models that include permanent income. Removing permanent income, we see the estimated impacts of subjective survival on wealth holdings and asset allocation move away from zero. This suggests that the permanent income is likely weakly positively correlated to the reported subjective survival probabilities. Controlling for a measure of expected lifetime resources appears to have a negligible effect on the estimated impact of reported survival probabilities. It is interesting though to note that the inclusion of a measure of permanent income shrinks the estimated impacts for survival probabilities to zero.

Bunching at focal points has been a concern of the subjective survival probabilities in the HRS since Hurd and McGarry (1995). The worry is that focal point responses may reflect either difference in cognition (Lillard and Willis (2001)) or measurement error (Bloom et al. (2006)). We provide evidence that the relationship between wealth holdings and asset allocation in the raw data appears to support theoretical predictions; average wealth holding and allocation to the risky asset increase as subjective survival probabilities increase, with the exception of households that report survival probabilities of one. Bloom et al. (2006) points out that households that report survival probabilities of one appear to have mortality rates similar to those people responding with survival probabilities closer to 0.7 or 0.8.

To assess the impact of these focal points on our estimates we treat focal point responses of one differently. First, we estimate our wealth holding and asset allocation equations dropping observations where the respondent reports a survival probability of one.[11] Second, recognizing that a focal point response may reflect some underlying time-constant heterogeneity of the household, we estimate our models dropping the all observations for the household if any respondent has ever reported a survival probability of one.

For single households, excluding observations where the respondent reported a survival probability of one leads us to estimate a larger impact of subjective survival probabilities on wealth holdings. Despite increases in the standard errors from reducing the number of observations we can see that we are estimating that a one-percentage point difference in subjective survival probability translates into $394 more Net Worth and $182 more Financial Wealth[12]. If we drop households that ever reported a survival probability of one our estimate of the impact of a one-percentage point difference in subjective survival on Net Worth is $466 and $143 on Financial Wealth.

More interesting is the impact that dropping focal point responses has on the estimated impact of subjective survival probabilities for asset allocation. When we drop observations where the respondent gives a survival probability of one we estimate that a one-percentage point difference in subjective survival increases the allocation of financial wealth to stocks by 0.032 % and decreases allocation to checking by 0.035%; both estimates are statistically significant at the 5% level despite the increases in standard error due to fewer observations. We see an even larger impact of removing households that ever reported a survival probability of one: we estimate that a one-percentage point difference in survival probabilities leads to 0.05% more financial wealth

[11] For married households, we drop the observation if either member of the household reports a survival probability of one.

[12] Though not statistically significant, this is $13,837 more Net Worth and $5,877 more Financial Wealth than estimates including observations with reported survival probabilities of one.

allocated to stocks and 0.047% less allocated to checking; both estimates are significant at the 5% level.

Treating reported survival probabilities of one differently leads to significantly different estimates of the impact of subjective survival on wealth holdings and asset allocation for married households. Removing observations where either the husband or the wife reported a survival probability of one leads us to estimate that a one-percentage point difference in the husband's survival probability results in only $57 more Net Worth and $5 less Financial Wealth (both estimates are statistically insignificant). A one-percentage point difference in the subjective survival probability of the wife leads to an estimated $698 (significant at the 5% level) difference in Net Worth and $129 difference in Financial Wealth. Removing focal point responses causes the estimate for the wife's subjective survival to increase for wealth holding equations and the estimate for the husband's subjective survival to decrease for both wealth holding equations.

Removing households where either spouse ever reported a survival probability of one leads us to estimate that a one-percentage point difference in the husband's subjective survival probability results in $171 less Net Worth and $14 more Financial Wealth. A one-percentage point difference in the wife's subjective survival probability leads to $497 more Net Worth and $215 more Financial Wealth. Interestingly the estimated impacts of subjective survival probabilities are statistically insignificant once we remove focal point households.

When we remove observations where either spouse reported a survival probability of one we see that a one-percentage point difference in the husband's survival probability results in 0.035% more financial wealth allocated to stocks (significant at the 5% level), 0.021% less allocated to CDs (significant at the 5% level) and 0.017% less in checking (statistically insignificant). A one-percentage point difference in the wife's survival probability results in

0.032% more financial wealth allocated to stocks (significant at the 5% level), and 0.026% less allocated to CDs (significant at the 10% level). When we drop households that ever report a survival probability of one we estimate that a one-percentage point difference in the husband's subjective survival leads us to find a statistically significant (5% level) increase of 0.012% of financial wealth to bonds. Perhaps more interesting is the fact that we now estimate that a one-percentage point difference in subjective survival for the wife leads the household to hold 0.06% more financial wealth in stocks and 0.036% less in CDs, while still small in magnitude they are considerably larger than our estimates that include all households.

It is apparent that the estimated impact of subjective survival probabilities on wealth holding and asset allocation are sensitive to the inclusion of reported survival probabilities of one. Our sensitivity analysis suggests that households that ever report a survival probability of one act very differently than the rest of our sample. Removing all observations for households that ever report a survival probability of one leads us to find that single households tend to increase their Net Worth more than their Financial Wealth in response to an increase in longevity risk. We also find that single households increase their allocation of financial wealth to stocks and decrease their allocation to checking. For married households we find that once we remove the focal point households the wife's survival probabilities becomes more important in the wealth holding and asset allocation decisions. While the estimates of the impact on wealth holding of wife's survival probabilities are not statistically significant at standard levels[13], they do coincide well with the allocation equation findings that the wife's survival leads to a significant positive impact on stock allocation. In fact, despite the lack of statistical significance the estimated impact of the wife's subjective survival probability is not much different than the

[13] The estimated impact for Net Worth is significant at the 14% level and for Financial Wealth at the 12% level.

estimated impacts for the single household's survival probability.

It appears that including households that ever report survival probabilities of one has a significant impact on the estimated impacts of subjective survival on wealth holding and asset allocation. Excluding these "focal point" households leads us to find estimated impacts that seem reasonable from a theoretical perspective and are generally statistically significant, despite increased standard errors due to smaller sample sizes. While there is still likely significant measurement error present in the subjective probabilities, we have identified that at a minimum the inclusion of households that report survival probabilities of one greatly affects estimates of wealth holding and allocation equations.

1.5 Conclusion

Theory points to the need to include estimates of an individual's subjective survival probability in estimating models of economic decision-making. Several studies use the subjective survival probabilities in the HRS to explain economic behavior with mixed results. We present here a very simple analysis that attempts to explain differences in wealth holding and asset allocation behavior using differences in subjective survival probabilities.

We find that there does appear to be a small, statistically significant impact of subjective survival probabilities on wealth holding (Net Worth, particularly) and asset allocation (stocks, in general). We find that our results for subjective survival probabilities are insensitive to the inclusion of a measure of permanent income, though we do see that including a measure of permanent income causes the estimated impact of subjective survival probabilities to move towards zero. If we exclude our measure of permanent income, we find that our estimated impact of subjective survival increases (but not by the total impact of permanent income) and the estimated impact on asset allocation moves away from zero. We also find that inclusion of households that ever report a survival probability of one has a significant detrimental impact on the estimated impact of subjective survival for asset allocation equations for single households and for wife's in married households. It appears that longevity risk plays a statistically significant role for single households, which contradicts the findings of Bloom et al. (2006).

By no means does our study attempt to contradict the hypothesis that there is measurement error in these subjective survival probabilities. In fact, it appears that bunching does represent a form of measurement error but it may be distinctly different (or additive) to the measurement error that is most likely present in all the subjective survival responses. Elder (2012) finds that life table probabilities are better predictors of in-sample mortality and that subjective survival probabilities probably reflect more measurement error than they reflect heterogeneity in mortality expectations of HRS respondents. These findings raise questions about both the validity of these subjective survival probabilities as proxies for mortality risk and how we can interpret the estimated relationships between these measures and decision-making. Our results suggest that controlling for other factors that can affect a household's wealth holding and allocation decisions (such as permanent income) almost completely removes the correlation with subjective survival that we see in the raw data. In addition, we find that excluding households that ever report a survival probability of one significantly affects our findings. We are not sure what these focal point responses mean in terms of mortality or if they represent heterogeneity in cognition but it does suggest that we are probably not able to interpret these probabilities as reflecting actual mortality risk.

Future avenues for research would be to try different specifications for married households, particularly around different types of objective functions that combine attributes of the spouses differently. In addition, including the cohort survival probability module as a standard part of the HRS expectations survey would help to tease out measurement error and the learning process that respondents go through when estimating their expectations. It might also be beneficial to look into the impact of bequest intentions as they may play a large role for some households compared to others.

CHAPTER 2

A MODEL FOR MULTIVARIATE FRACTIONAL RESPONSES WITH AN APPLICATION TO ASSET ALLOCATION

2.1 Introduction

Many interesting economic variables are fractional in nature. By definition, some fractional response variables are related, such as the budget share of goods in a demand system or the allocation of wealth across available assets. Since these fractional responses are shares of a total, they must satisfy an adding up restriction. Due to the limited nature of the fractional responses we are often interested in estimating relationships using nonlinear conditional mean functions. Analogous to the case where we estimate linear conditional means, the adding up restriction imposes constraints on the marginal effects of the covariates in our model. Since the marginal effects of covariates in a nonlinear conditional mean are themselves nonlinear, this causes some difficulty in estimation as it amounts to imposing nonlinear constraints on our coefficients. We get around this issue by imposing the adding up restriction using the multinomial quasi-likelihood.

Sivakumar and Bhat (2002) and Mullahy (2010) show how this approach works when using cross-sectional data. In fact, this approach extends very naturally to the panel data case, even if we specify a time-invariant unobserved effect for each cross-sectional unit. We can appeal to the Correlated Random Effects approach to estimation (Chamberlain (1980) and Mundlak (1978)) and specify that the conditional mean of the unobserved effect is a parametric function of the time-averages of the independent variables. As pointed out in Wooldridge (2010), this approach is only appropriate when we have a balanced panel. With an unbalanced panel, at a minimum, we violate the assumption that the unobserved effect has a constant variance across

cross-section units, since we have a different number of observations for each unit. We address this concern by exploiting the independence of our cross-section units. To do this we estimate the multinomial quasi-likelihood on balanced panel subsets of the full, unbalanced panel. We create the balanced panels by combining households that are observed in the sample for the same number of time periods. Since the households in different balanced panel subsets are independent of each other we are able to take the weighted average of the average partial effects (and their variances) estimated in each balanced panel subset to get the average partial effect for the entire unbalanced panel.

This approach ignores the potential serial correlation present within the cross-sectional unit. A natural next step is to correct for this serial correlation by using a weighting matrix that accounts for serial correlation. To do this we use Classical Minimum Distance estimation. Simply put, this method takes the estimated coefficients from the multinomial quasi-likelihood estimation on each balanced panel subset of data and estimates a single coefficient for each covariate that is a weighted average of all the balanced panel subset coefficients. The weighting matrix that we construct contains the estimated variance-covariance matrices from the multinomial quasi-likelihood estimated on each balanced panel subset and estimates of the serial correlation within and across equations over time.

This paper takes the single equation fractional probit technique laid out in Papke and Wooldridge (2008) and extends it to cover multiple fractional responses that must meet an adding up restriction. In addition, we estimate an unbalanced panel using the Correlated Random Effects framework by exploiting the independence of our cross-sectional units. Lastly, we also show how we can apply Classical Minimum Distance to this procedure to potentially derive more efficient estimates. Section 2 briefly covers some of the work done on fractional response variables. Section 3 presents the single equation specification of our multiple fractional responses. Section 4 combines the single equations together using the multinomial quasi-likelihood. Section 5 lays out the Classical Minimum Distance estimator. Section 6 describes an alternative estimator that may be more parsimonious. Section 7 describes the data that we use in our analysis and presents results of estimation and section 8 concludes.

2.2 Fractional Response Variables

Gourieroux, Monfort, and Trognon (1984) summarizes the theory underlying the quasi-maximum likelihood estimator. They show that if a random variable comes from the linear exponential family and we correctly specify the conditional mean, then estimation by quasi-maximum likelihood generates consistent, though potentially inefficient, estimates for the impacts of our covariates. This finding holds under the general conditions that we require for M-Estimation. Under the standard assumptions, the quasi-maximum likelihood estimators are asymptotically normal and converge at the rate of $N^{-1/2}$.

Papke and Wooldridge (1996) estimate a single fractional response variable in the cross-section using the logit functional form for the conditional mean function. Papke and Wooldridge (2008) estimate a single equation fractional response variable in the panel data context using the probit functional form for the conditional mean. They insert the time-averages of their coefficients in their conditional mean function, thus parametrically “removing” the unobserved effect. They also explore the possibility of improving efficiency by constructing a “working” correlation matrix that allows for some correction of the serial correlation within cross-sectional units. This technique essentially follows the logic that even if we incorrectly specify the correlation structure that we can gain some efficiency by not ignoring its presence.

Sivakumar and Bhat (2002), Mullahy (2010) and Koch (2010) study the multivariate fractional response case in cross-sectional data. They follow a similar approach that we take in this paper and exploit the robustness properties of the multinomial quasi-likelihood. They specify their conditional means using the multinomial logit specification that imposes the adding up restriction across equations. In this paper, we use the probit specification of the conditional mean functions since it more easily accommodates the use of the Chamberlain-Mundlak device to control for the presence of an unobserved effect.

2.3 Single Equation Fractional Probit

A natural starting point for our analysis is to ignore the relationship between our fractional response variables and proceed as though we will estimate each equation separately. We do this for two reasons; first our procedure essentially takes our single equation framework and extends it to the multiple fractional response case. Second, the single equation framework gives us a baseline set of estimates to compare to the results of the procedure that we present here.

Assume that at each time period t, we observe household i allocate a proportion of their financial wealth to each asset g. We assume that $g = 1, 2, ..., G$ exhausts all possible investment options so that in each period the household invests all their financial wealth. This assumption implies that $\sum_g^G y_{itg} = 1$ for each $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$. In addition to our response vector, we observe a set of covariates $\mathbf{x}_{it}$ and an unobserved effect $\mathbf{c}_{ig}$. Our goal is to estimate a model of the conditional mean of y given $\mathbf{x}$ to determine the impact of each covariate. By construction, y_{itg} is bounded between zero and one. Assuming that we use the same set of covariates for each equation we can specify the conditional mean as:

$$E(y_{itg} \mid \mathbf{x}_{it}, c_{ig}) = \Phi(\mathbf{x}_{it}\boldsymbol{\beta}_g + c_{ig}); \quad i = 1,\ldots,N; \; t = 1,\ldots,T; \text{ and } g = 1,\ldots,G \quad (2.3.1)$$

We point out that the above specification completely ignores the fact that these fractional responses sum to one. It is simply acting as though we are treating each asset equation by itself. Nevertheless, it is useful to study what we can derive from these single equation models since our approach here essentially takes the specification in (2.3.1) for each equation and uses the multinomial quasi-likelihood to impose the adding up constraint.

Our interest lies in estimating the average partial effects of the covariates, $\mathbf{x}_{it}$ on the proportion of financial wealth allocated to each asset. Following the results for a probit model the direction of the partial effect is determined by the sign of $\boldsymbol{\beta}_g$. For a continuous variable x_{tk} (dropping the i subscript);

$$\frac{\partial E(y_{tg} \mid \mathbf{x}_t, c_g)}{\partial x_{tk}} = \beta_{gk}\phi(\mathbf{x}_t\boldsymbol{\beta}_g + c_g) \quad (2.3.2)$$

To obtain the average partial effect we can average this over the distribution of c_g and $\mathbf{x}_t$.

To consistently estimate the coefficients in this single equation framework we require that the covariates are not correlated with the error term. We assume that $\mathbf{x}_{it}$ is strictly exogenous conditional on the unobserved effect,

$$E(y_{itg} \mid \mathbf{x}_i, c_{ig}) = E(y_{itg} \mid \mathbf{x}_{it}, c_{ig}). \quad (2.3.3)$$

Following Chamberlain (1980) and Mundlak (1978) we assume that

$$c_{ig} = \psi_g + \overline{\mathbf{x}}_i\boldsymbol{\xi}_g + a_{ig}; \text{ where } a_{ig} \mid \mathbf{x}_i \sim Normal(0, \sigma_a^2). \quad (2.3.4)$$

Using (2.3.4) we can rewrite (2.3.1) as:

$$E(y_{itg} \mid \mathbf{x}_i, a_{ig}) = \Phi(\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i\boldsymbol{\xi}_g + a_{ig}). \quad (2.3.5)$$

So the mean of y_{itg} conditional on $\mathbf{x}_i$ is:

$$E(y_{itg} \mid \mathbf{x}_i) = E(\Phi(\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i\boldsymbol{\xi}_g + a_{ig} \mid \mathbf{x}_i) = \Phi\left(\frac{\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i\boldsymbol{\xi}_g}{(1+\sigma_{ag}^2)^{-1/2}}\right) \quad (2.3.6)$$

or

$$E(y_{itg} \mid \mathbf{x}_i) = E(\Phi(\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i\boldsymbol{\xi}_g + a_{ig} \mid \mathbf{x}_i)) = \Phi\left(\psi_{ga} + \mathbf{x}_{it}\boldsymbol{\beta}_{ga} + \overline{\mathbf{x}}_i\boldsymbol{\xi}_{ga}\right) \quad (2.3.7)$$

where we use the a subscript to denote that our estimated coefficients are scaled by $(1+\sigma_{ag}^2)^{-1/2}$. Woodridge (2002, Section 15.8.2) shows that the mixing properties of the normal distribution lead to equation (2.3.7).

For identification of the scaled coefficients in (2.3.7) we require that there is no perfect collinearity between the elements of $\mathbf{x}_{it}$ and that there is enough variation in $\mathbf{x}_{it}$ over time. With the above parametric specification of the unobserved effect we can now write the Average Structural Function (ASF) following Blundell and Powell (2003):

$$ASF_g(\mathbf{x}_t) = E_{\overline{\mathbf{x}}_i}\left[\Phi\left(\psi_{ga} + \mathbf{x}_t\boldsymbol{\beta}_{ga} + \overline{\mathbf{x}}_i\boldsymbol{\xi}_{ga}\right)\right] \quad (2.3.8)$$

A consistent estimator of $ASF_g(\mathbf{x}_t)$ is:

$$\widehat{ASF}_g(\mathbf{x}_t) = N^{-1}\sum_{i=1}^{N}\Phi\left(\hat{\psi}_{ga} + \mathbf{x}_t\hat{\boldsymbol{\beta}}_{ga} + \mathbf{x}_i\hat{\boldsymbol{\xi}}_{ga}\right). \quad (2.3.9)$$

where $\hat{\psi}_{ga}$, $\hat{\boldsymbol{\beta}}_{ga}$, and $\hat{\boldsymbol{\xi}}_{ga}$ are consistent estimates of the scaled coefficients in equation (2.3.7). For a continuous variable x_j we can calculate the average partial effect by taking the derivative of (2.3.9) and average over both i and t:

$$\widehat{APE}_g(x_j) = \frac{\partial\Phi(\hat{\psi}_{ga} + \mathbf{x}_{it}\hat{\boldsymbol{\beta}}_{ga} + \overline{\mathbf{x}}_i\hat{\boldsymbol{\xi}}_{ga})}{\partial x_j} = NT^{-1}\sum_{t=1}^{T}\sum_{i=1}^{N}\hat{\beta}_{ga,j}\phi(\hat{\psi}_{ga} + \mathbf{x}_{it}\hat{\boldsymbol{\beta}}_{ga} + \overline{\mathbf{x}}_i\hat{\boldsymbol{\xi}}_{ga}) \quad (2.3.10)$$

For a binary x_j we can calculate the average partial effect by calculating the difference in the Average Structural Function evaluated at zero and one.

The above presentation closely mirrors that of Papke and Wooldridge (2008) for the single equation fractional probit model. We laid out the basic specification of the conditional mean functions for each of our asset equations. We can use the individual conditional mean functions and the previous results to estimate each equation assuming that there is no correlation between equations. Estimation of the individual equation conditional means would entail maximizing the Bernoulli quasi-likelihood function. While this works, it ignores the fact that there is a definite relationship between the fractional responses, i.e. that they sum to one for each household in each time period.

Analogous to the linear case, imposing the adding up restriction places a constraint on the marginal effects of the covariates. The adding up restriction states that $\sum_{g=1}^{G} y_{itg} = 1$. Taking expectations of this conditional on $\mathbf{x}_{it}$ and c_{ig} gives us the following restriction:

$$E(y_{itg} \mid \mathbf{x}_{it}, c_{ig}) = 1. \tag{2.3.11}$$

Taking the derivative of (2.3.11) with respect to x_k:

$$\begin{aligned}\sum_{g=1}^{G} \frac{\partial E(y_{itg} \mid \mathbf{x}_{it}, c_{ig})}{\partial x_k} &= \sum_{g=1}^{G} \beta_{gk} \phi(\mathbf{x}_{it}\boldsymbol{\beta}_g + c_{ig}) \\ &= \sum_{g=1}^{G} \beta_{gk} \phi(\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i \boldsymbol{\xi}_g) = 0.\end{aligned} \tag{2.3.12}$$

Equation (2.3.12) shows that the adding up restriction implies that the sum of the marginal effects of a specific covariate across equations is zero. Therefore, the effect of a change in a covariate will lead to a reallocation of financial wealth across the available assets.

A very simple way to impose the adding up restriction in estimation is to exploit the multinomial quasi-likelihood function.

2.4 The Multinomial Quasi-Likelihood

We use the multinomial quasi-likelihood function to allow for correlation across equations and impose the restriction that our dependent variables must sum to one in each period for each household. We assume our fractional responses $(y_{it1}, y_{it2}, ..., y_{itG})$ are exhaustive and mutually exclusive categories representing the population and that $y_{itg} \in [0,1]$ and $\sum_{g=1}^{G} y_{itg} = 1$. Let $\mathbf{x}_{it}$ be a set of covariates that affect our quantity of interest, $E(y_{itg} \mid \mathbf{x}_i)$.

Assume that

$$E(y_{itg} \mid \mathbf{x}_i) = \Phi(\psi_{o,t,g,a} + \mathbf{x}_{it}\boldsymbol{\beta}_{o,t,g,a} + \overline{\mathbf{x}}_i\boldsymbol{\xi}_{o,t,g,a}) = \Phi(\mathbf{x}_{it}, \boldsymbol{\theta}_{o,t,g,a}) \quad (2.4.1)$$

For identification of the parameters the multinomial quasi-likelihood function requires that the following constraint holds:

$$\begin{aligned}\Phi(\mathbf{x}_{it}, \boldsymbol{\theta}_{o,t,G,a}) &= E(y_{itG} \mid \mathbf{x}_i) \\ &= 1 - \Phi(\mathbf{x}_{it}, \boldsymbol{\theta}_{o,t,1,a}) - \Phi(\mathbf{x}_{it}, \boldsymbol{\theta}_{o,t,2,a}) - \cdots - \Phi(\mathbf{x}_{it}, \boldsymbol{\theta}_{o,t,G-1,a}).\end{aligned} \quad (2.4.2)$$

So that for any $\boldsymbol{\theta}_t = \{\boldsymbol{\theta}_{t,1,a}, \boldsymbol{\theta}_{t,2,a}, ..., \boldsymbol{\theta}_{t,G-1,a}\}$,

$$\sum_{g=1}^{G} \Phi_g(\mathbf{x}_{it}, \boldsymbol{\theta}_t) = 1.$$

Assuming that the conditional mean functions are continuously differentiable in $\boldsymbol{\theta}_t$ the adding up restriction implies

$$\nabla_{\boldsymbol{\theta}_t}\Phi_G(\mathbf{x}_{it}, \boldsymbol{\theta}_t) = -\nabla_{\boldsymbol{\theta}_t}\Phi_1(\mathbf{x}_{it}, \boldsymbol{\theta}_t) - \nabla_{\boldsymbol{\theta}_t}\Phi_2(\mathbf{x}_{it}, \boldsymbol{\theta}_t) - \cdots - \nabla_{\boldsymbol{\theta}_t}\Phi_{G-1}(\mathbf{x}_{it}, \boldsymbol{\theta}_t).$$

Assuming that we have a random draw i the multinomial quasi-likelihood equation for each observation is:

$$\ell_{it}(\boldsymbol{\theta}_t) = y_{it1}\log[\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_t)] + y_{it2}\log[\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_t)] + \cdots + y_{itG}\log[\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_t)]. \quad (2.4.3)$$[14]

The score function is

$$\begin{aligned}\mathbf{s}_{it}(\boldsymbol{\theta}_t) &= \frac{y_{it1}}{\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_t)}\nabla_{\boldsymbol{\theta}_t}\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_t)' + \frac{y_{it2}}{\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_t)}\nabla_{\boldsymbol{\theta}_t}\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_t)' + \cdots \\ &\quad + \frac{y_{itG}}{\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_t)}\nabla_{\boldsymbol{\theta}_t}\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_t)'\end{aligned} \quad (2.4.4)$$

Assuming the correct specification of the conditional mean

$$\begin{aligned}E(\mathbf{s}_{it}(\boldsymbol{\theta}_t)\,|\,\mathbf{x}_i) &= \frac{E(y_{it1}\,|\,\mathbf{x}_i)}{\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_t)}\nabla_{\boldsymbol{\theta}_t}\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_t)' + \frac{E(y_{it2}\,|\,\mathbf{x}_i)}{\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_t)}\nabla_{\boldsymbol{\theta}_t}\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_t)' + \cdots \\ &\quad + \frac{E(y_{itG}\,|\,\mathbf{x}_i)}{\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_t)}\nabla_{\boldsymbol{\theta}_t}\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_t)' \\ &= \nabla_{\boldsymbol{\theta}_t}\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_t)' + \nabla_{\boldsymbol{\theta}_t}\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_t)' + \cdots + \nabla_{\boldsymbol{\theta}_t}\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_t)' \\ &= \mathbf{0}.\end{aligned}$$

Let $\mathbf{H}_t(\mathbf{x}_i,\mathbf{y}_i,\boldsymbol{\theta}_t)$ be the Hessian, it can be shown that

$$\begin{aligned}-E[\mathbf{H}_t(\mathbf{x}_{it},\mathbf{y}_{it},\boldsymbol{\theta}_{o,t})\,|\,\mathbf{x}_i] &= \frac{\nabla_{\boldsymbol{\theta}_t}\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})'\nabla_{\boldsymbol{\theta}_t}\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})}{\Phi_1(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})} + \frac{\nabla_{\boldsymbol{\theta}_t}\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})'\nabla_{\boldsymbol{\theta}_t}\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})}{\Phi_2(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})} + \cdots \\ &\quad + \frac{\nabla_{\boldsymbol{\theta}_t}\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})'\nabla_{\boldsymbol{\theta}_t}\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})}{\Phi_G(\mathbf{x}_{it},\boldsymbol{\theta}_{o,t})}\end{aligned}$$

and is consistently estimated by

[14] We point out that using the Normal cumulative distribution function does not explicitly restrict the predicted values of the “omitted” equation from falling outside of [0,1]. Using the logarithm function in the quasi log-likelihood implicitly imposes this restriction. In our application, we do not see this as a cause for concern as all predicted values of the “omitted” equation fall within [0,1], though this may be a concern where there is a greater frequency of zeros and ones in the response variable.

$$\hat{\mathbf{A}}_t = N^{-1}\sum_{i=1}^{N}\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)'\mathbf{W}_t(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)$$

where

$$\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t) = \begin{pmatrix} \nabla_{\boldsymbol{\theta}_t}\Phi_1(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t) \\ \nabla_{\boldsymbol{\theta}_t}\Phi_2(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t) \\ \vdots \\ \nabla_{\boldsymbol{\theta}_t}\Phi_G(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t) \end{pmatrix}$$

and

$$\mathbf{W}_t(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t) = \begin{pmatrix} \frac{1}{\Phi_1(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)} & 0 & \cdots & 0 \\ 0 & \frac{1}{\Phi_2(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & \frac{1}{\Phi_G(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)} \end{pmatrix}.$$

Define

$$\hat{\mathbf{B}}_t = N^{-1}\sum_{i=1}^{N}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}}_t)\mathbf{s}_{it}(\hat{\boldsymbol{\theta}}_t)'.$$

Then an estimator of the variance matrix that only relies on the correct specification of the conditional means is given by

$$\begin{aligned}\hat{\mathbf{A}}_t^{-1}\hat{\mathbf{B}}_t\hat{\mathbf{A}}_t^{-1}/N &= \left(\sum_{i=1}^{N}\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)'\mathbf{W}_t(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\right)^{-1}\left(\sum_{i=1}^{N}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}}_t)\mathbf{s}_{it}(\hat{\boldsymbol{\theta}}_t)'\right) \\ &\cdot\left(\sum_{i=1}^{N}\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)'\mathbf{W}_t(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\right)^{-1}.\end{aligned} \tag{2.4.5}$$

In the above presentation we laid out the basic approach to estimating the multinomial quasi-likelihood for each time period in a balanced panel. With a few minor changes we can rewrite the above equations to estimate a pooled version of the multinomial quasi-likelihood where we do not allow the estimated parameter vector to vary by time period. We can write the partial multinomial quasi-likelihood function as

$$\ell_i(\boldsymbol{\theta}) = \sum_{t=1}^{T} y_{it1} \log[\Phi_1(\mathbf{x}_{it}, \boldsymbol{\theta})] + y_{it2} \log[\Phi_2(\mathbf{x}_{it}, \boldsymbol{\theta})] + \cdots + y_{itG} \log[\Phi_G(\mathbf{x}_{it}, \boldsymbol{\theta})].$$

Then the score function is

$$\mathbf{s}_i(\boldsymbol{\theta}) = \sum_{t=1}^{T} \mathbf{s}_{it}(\boldsymbol{\theta}) = \sum_{t=1}^{T} \left(\begin{array}{c} \dfrac{y_{it1}}{\Phi_1(\mathbf{x}_{it}, \boldsymbol{\theta})} \nabla_{\boldsymbol{\theta}} \Phi_1(\mathbf{x}_{it}, \boldsymbol{\theta})' + \dfrac{y_{it2}}{\Phi_2(\mathbf{x}_{it}, \boldsymbol{\theta})} \nabla_{\boldsymbol{\theta}} \Phi_2(\mathbf{x}_{it}, \boldsymbol{\theta})' + \cdots \\ \cdots + \dfrac{y_{itG}}{\Phi_G(\mathbf{x}_{it}, \boldsymbol{\theta})} \nabla_{\boldsymbol{\theta}} \Phi_G(\mathbf{x}_{it}, \boldsymbol{\theta})' \end{array} \right)$$

and the negative of the expected Hessian is

$$-E[\mathbf{H}(\mathbf{x}_i, \mathbf{y}_i, \boldsymbol{\theta}_o) \mid \mathbf{x}_i] = \sum_{t=1}^{T} \left(\begin{array}{c} \dfrac{\nabla_{\boldsymbol{\theta}} \Phi_1(\mathbf{x}_{it}, \boldsymbol{\theta}_o)' \nabla_{\boldsymbol{\theta}} \Phi_1(\mathbf{x}_{it}, \boldsymbol{\theta}_o)}{\Phi_1(\mathbf{x}_{it}, \boldsymbol{\theta}_o)} + \dfrac{\nabla_{\boldsymbol{\theta}} \Phi_2(\mathbf{x}_{it}, \boldsymbol{\theta}_o)' \nabla_{\boldsymbol{\theta}} \Phi_2(\mathbf{x}_{it}, \boldsymbol{\theta}_o)}{\Phi_2(\mathbf{x}_{it}, \boldsymbol{\theta}_o)} + \cdots \\ \cdots + \dfrac{\nabla_{\boldsymbol{\theta}} \Phi_G(\mathbf{x}_{it}, \boldsymbol{\theta}_o)' \nabla_{\boldsymbol{\theta}} \Phi_G(\mathbf{x}_{it}, \boldsymbol{\theta}_o)}{\Phi_G(\mathbf{x}_{it}, \boldsymbol{\theta}_o)} \end{array} \right).$$

The consistent estimator of the Hessian is

$$\hat{\mathbf{A}} = NT^{-1} \sum_{i=1}^{N} \sum_{t=1}^{T} \nabla_{\boldsymbol{\theta}} \boldsymbol{\Phi}(\mathbf{x}_{it}, \hat{\boldsymbol{\theta}})' \mathbf{W}(\mathbf{x}_{it}, \hat{\boldsymbol{\theta}}) \nabla_{\boldsymbol{\theta}} \boldsymbol{\Phi}(\mathbf{x}_{it}, \hat{\boldsymbol{\theta}})$$

where

$$\nabla_{\boldsymbol{\theta}}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}) = \begin{pmatrix} \nabla_{\boldsymbol{\theta}}\Phi_1(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}) \\ \nabla_{\boldsymbol{\theta}}\Phi_2(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}) \\ \vdots \\ \nabla_{\boldsymbol{\theta}}\Phi_G(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}) \end{pmatrix}$$

and

$$\mathbf{W}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}) = \begin{pmatrix} \frac{1}{\Phi_1(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})} & 0 & \cdots & 0 \\ 0 & \frac{1}{\Phi_2(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})} & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\Phi_G(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})} \end{pmatrix}.$$

Define

$$\hat{\mathbf{B}} = NT^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})'.$$

Then a consistent estimator of the variance matrix that only relies on the correct specification of the conditional means is

$$\hat{\mathbf{A}}^{-1}\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1}/N = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_{\boldsymbol{\theta}}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})'\mathbf{W}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})\nabla_{\boldsymbol{\theta}}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})\right)^{-1} \cdot \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})'\right) \cdot \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\nabla_{\boldsymbol{\theta}}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})'\mathbf{W}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})\nabla_{\boldsymbol{\theta}}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}})\right)^{-1}.$$

2.5 Classical Minimum Distance Estimation

Classical Minimum Distance estimation (CMD) is similar to Generalized Method of Moments and involves minimizing the Euclidean distance between a set of "reduced-form" parameters and their structural counterparts. The basic exposition is as follows and can be found in Wooldridge (2002, Section 14.6).

Suppose that we have a $P \times 1$ vector of structural parameters $\boldsymbol{\theta}_o$ related to $\boldsymbol{\pi}_o$ a $S \times 1$ (where $S > P$) vector of reduced-form parameters such that:

$$\boldsymbol{\pi}_o = \mathbf{h}(\boldsymbol{\theta}_o) \tag{2.5.1}$$

where $\mathbf{h}$ is a known, continuously differentiable function that maps the structural parameters to the reduced-form parameters. To perform CMD estimation we first obtain estimates of the reduced-form parameters, say $\hat{\boldsymbol{\pi}}$, and then find an estimator $\hat{\boldsymbol{\theta}}$ that minimizes the weighted Euclidean distance between $\hat{\boldsymbol{\pi}}$ and $\mathbf{h}(\hat{\boldsymbol{\theta}})$. Analogous to GMM, we can use any weighting matrix so long as it is positive semi-definite, such as an $S \times S$ identity matrix, but the optimal weighting matrix is the one that makes the CMD estimator the minimum chi-square estimator.

The CMD estimator solves the following problem:

$$\min_{\boldsymbol{\theta} \in \boldsymbol{\Theta}} \{\hat{\boldsymbol{\pi}} - \mathbf{h}(\boldsymbol{\theta})\}' \hat{\boldsymbol{\Xi}}^{-1} \{\hat{\boldsymbol{\pi}} - \mathbf{h}(\boldsymbol{\theta})\} \tag{2.5.2}$$

where $\operatorname{plim}_{N \to \infty} \hat{\boldsymbol{\Xi}} = \boldsymbol{\Xi}_o$. Assuming that $\hat{\boldsymbol{\pi}}$ is a consistent estimator of $\boldsymbol{\pi}_o$ and that $\sqrt{N}(\hat{\boldsymbol{\pi}} - \boldsymbol{\pi}_o) \overset{a}{\sim} \text{Normal}(\mathbf{0}, \boldsymbol{\Xi}_o)$ then $\hat{\boldsymbol{\Xi}}^{-1}$ is the inverse of any consistent estimate of the asymptotic variance of $\sqrt{N}(\hat{\boldsymbol{\pi}} - \boldsymbol{\pi}_o)$. The solution to the optimization problem in (2.5.2) is $\hat{\boldsymbol{\theta}}$ which minimizes the weighted Euclidean distance between $\hat{\boldsymbol{\pi}}$ and $\mathbf{h}(\hat{\boldsymbol{\theta}})$ it can be shown that $\sqrt{N}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_o) \overset{a}{\sim} \text{Normal}[0, \mathbf{H}_o' \boldsymbol{\Xi}_o^{-1} \mathbf{H}_o]$ where $\mathbf{H}_o = \mathbf{H}(\boldsymbol{\theta}_o)$ and $\mathbf{H}(\boldsymbol{\theta}) \equiv \nabla_{\boldsymbol{\theta}} \mathbf{h}(\boldsymbol{\theta})$ the $S \times P$ Jacobian of $\mathbf{h}(\boldsymbol{\theta})$. The appropriate estimator of $\widehat{\text{Avar}}(\hat{\boldsymbol{\theta}})$

$$\widehat{\text{Avar}}(\hat{\boldsymbol{\theta}}) \equiv (\mathbf{H}' \boldsymbol{\Xi}^{-1} \mathbf{H})^{-1} / N = (\mathbf{H}' [\widehat{\text{Avar}}(\hat{\boldsymbol{\pi}})]^{-1} \hat{\mathbf{H}})^{-1} \tag{2.5.3}$$

If the mapping function $\mathbf{h}$ is linear then we can state the general form of the CMD estimator as follows (Wooldridge, 2002, Problem 14.7):

$$\hat{\boldsymbol{\theta}} = (\mathbf{H}'\hat{\boldsymbol{\Xi}}^{-1}\mathbf{H})^{-1}\mathbf{H}'\hat{\boldsymbol{\Xi}}^{-1}\hat{\boldsymbol{\pi}}. \tag{2.5.4}$$

This implies that the optimal CMD estimator of the structural parameter vector is a weighted average of the reduced form parameter vector where the weighting matrix is the estimated asymptotic variance matrix of the reduced-form parameters.

The mapping function that we use in the CMD estimation step is indeed linear and therefore we can use (2.5.4) to construct the estimates of the structural parameter vector. To obtain our reduced-form parameter vector we maximize the multinomial quasi-likelihood for each time period within each balanced panel subset. Using these estimates we can then construct the weighting matrix that we will use in the CMD estimation. The weighting matrix will be a block matrix where the estimated variance matrices from each time period are on the diagonal and the off-diagonal block matrices contain estimates of the covariance within and across equations over time.

Given the results in section 3 and the specifications of the conditional mean functions in section 2 we define the diagonal elements of the weighting matrix as

$$\begin{aligned}\hat{\mathbf{A}}_t^{-1}\hat{\mathbf{B}}_t\hat{\mathbf{A}}_t^{-1}/N = {} & \left(\sum_{i=1}^{N}\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)'\mathbf{W}_t(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\right)^{-1}\left(\sum_{i=1}^{N}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}}_t)\mathbf{s}_{it}(\hat{\boldsymbol{\theta}}_t)'\right) \\ & \cdot\left(\sum_{i=1}^{N}\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)'\mathbf{W}_t(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\right)^{-1}.\end{aligned} \tag{2.5.5}$$

We construct the off-diagonal elements of the weighting matrix in a similar manner

$$\hat{\mathbf{A}}_t^{-1}\hat{\mathbf{B}}_{ts}\hat{\mathbf{A}}_s^{-1}/N = \left(\sum_{i=1}^{N} \nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)'\mathbf{W}_t(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\nabla_{\boldsymbol{\theta}_t}\boldsymbol{\Phi}(\mathbf{x}_{it},\hat{\boldsymbol{\theta}}_t)\right)^{-1}\left(\sum_{i=1}^{N}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}}_t)\mathbf{s}_{is}(\hat{\boldsymbol{\theta}}_s)'\right)$$

$$\cdot\left(\sum_{i=1}^{N} \nabla_{\boldsymbol{\theta}_s}\boldsymbol{\Phi}(\mathbf{x}_{is},\hat{\boldsymbol{\theta}}_s)'\mathbf{W}_s(\mathbf{x}_{is},\hat{\boldsymbol{\theta}}_s)\nabla_{\boldsymbol{\theta}_s}\boldsymbol{\Phi}(\mathbf{x}_{is},\hat{\boldsymbol{\theta}}_s)\right)^{-1} \qquad (2.5.6)$$

for $s \neq t$.

The weighting matrix that we use in CMD estimation is

$$\hat{\boldsymbol{\Omega}} = \begin{pmatrix} \hat{\mathbf{A}}_1^{-1}\hat{\mathbf{B}}_{11}\hat{\mathbf{A}}_1^{-1} & \hat{\mathbf{A}}_1^{-1}\hat{\mathbf{B}}_{12}\hat{\mathbf{A}}_2^{-1} & \cdots & \hat{\mathbf{A}}_1^{-1}\hat{\mathbf{B}}_{1T}\hat{\mathbf{A}}_T^{-1} \\ \hat{\mathbf{A}}_2^{-1}\hat{\mathbf{B}}_{21}\hat{\mathbf{A}}_1^{-1} & \hat{\mathbf{A}}_2^{-1}\hat{\mathbf{B}}_{22}\hat{\mathbf{A}}_2^{-1} & \cdots & \hat{\mathbf{A}}_2^{-1}\hat{\mathbf{B}}_{2T}\hat{\mathbf{A}}_T^{-1} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{A}}_T^{-1}\hat{\mathbf{B}}_{T1}\hat{\mathbf{A}}_1^{-1} & \hat{\mathbf{A}}_T^{-1}\hat{\mathbf{B}}_{T2}\hat{\mathbf{A}}_2^{-1} & \cdots & \hat{\mathbf{A}}_T^{-1}\hat{\mathbf{B}}_{TT}\hat{\mathbf{A}}_T^{-1} \end{pmatrix}. \qquad (2.5.7)$$

We calculate this weighting matrix for each balanced panel subset in our data. The mapping function that we use is linear, it maps the T reduced-form coefficient vectors to the single structural parameter vector for each balanced panel subset. Therefore, we can state that the CMD estimator $\tilde{\boldsymbol{\beta}}$ takes the following form

$$\tilde{\boldsymbol{\beta}} = (\mathbf{H}'\hat{\boldsymbol{\Omega}}^{-1}\mathbf{H})^{-1}\mathbf{H}'\boldsymbol{\Omega}^{-1}\hat{\boldsymbol{\beta}} \qquad (2.5.8)$$

where $\hat{\boldsymbol{\Omega}}$ is defined above, $\hat{\boldsymbol{\beta}}$ represents the $T(G-1)K\times 1$ (K covariates, $G-1$ equations, T time periods in each balanced panel subset) vector of parameter estimates from the multinomial quasi-likelihood maximization and $\mathbf{H}$ is a $T(G-1)K\times(G-1)K$ matrix that maps the reduced-form parameter estimates to the structural parameter estimates:

$$\mathbf{H} = \begin{pmatrix} \mathbf{I}_{(G-1)K}^{t=1} \\ \mathbf{I}_{(G-1)K}^{t=2} \\ \vdots \\ \mathbf{I}_{(G-1)K}^{t=T} \end{pmatrix}$$

where $\mathbf{I}_{(G-1)K}^{t=j}$ is a $(G-1)K \times (G-1)K$ identity matrix for each time period j.

After deriving the CMD estimates of the structural parameters in (2.5.8), we can calculate the average partial effects of our covariates as in Section 2 using $\tilde{\boldsymbol{\beta}}$. Therefore, our proposed procedure is

Procedure 1:

1. For each time period obtain the multinomial quasi-likelihood estimates of the coefficients, the gradient vector, and the robust variance matrix.

2. Create the weighting matrix as in (2.5.7), and the matrix $\mathbf{H}$

3. Obtain the MD coefficient estimates using (2.5.8) and the asymptotic variance matrix of these estimated coefficients as in (2.5.3).

The above presentation necessarily assumes that we have a balanced panel; this is due to the use of the Chamberlain-Mundlak device employed to remove the unobserved effect. If we have an unbalanced panel then, at a minimum, our assumption of a common variance across households, conditional on the number of time periods, is incorrect. Households in the panel for fewer time periods will necessarily have a larger estimated variance for their time-averaged observables. One solution to this problem is to drop observations so that we have a balanced panel. While this works technically, it has the potential to cause problems that are even more worrisome. Fortunately, there is a very simple way to allow us to use this procedure in the context of an unbalanced panel.

Since we have independence within the cross-section, i.e., households are randomly sampled and independent within each time period, we can simply subset our data based on the number of times that we observe households and then average the estimated average partial effects across these subsets. This entails creating balanced panels that contain households that are observed for the same number of time periods. We estimate the multinomial quasi-likelihood on each of these balanced panels by time period, followed by the minimum distance estimation to account for serial correlation. We then calculate the average partial effects for each balanced panel and average these across all the balanced panels.

We adapt Procedure 1 in the following way to allow for estimation on an unbalanced panel:

Procedure 2:

1. Subset the panel into balanced panels which contain all households that have exactly j observations, where $j = 2, ..., T$.

2. For each time period in each balanced panel subset maximize the multinomial quasi-likelihood obtaining the estimates of the coefficients, the gradient vector, and the robust variance matrix.

3. Apply CMD estimation as detailed above to each subset of the data.

4. Calculate the average partial effects and their variances for each balanced panel subset and then average these across all balanced panel subsets.

2.6 An Alternative Specification

The procedure presented above essentially takes single equation fractional probits and combines them into a system of equations. While this is the logical first step in estimating multiple fractional response variables, it assumes that there are G sources of heterogeneity. This is a very general assumption and acts as though we have a different distribution for the unobserved effect for each equation. We know that heterogeneity is at the household level, and we want to control for these time-invariant unobserved characteristics. Suppose that what we cannot measure is household risk aversion and that risk aversion is time-invariant but correlated with our covariates. If we do not control for risk aversion then our estimated coefficients will be inconsistent. Although risk aversion is constant over time and across assets, we anticipate that the *expected value* of the impact of risk aversion will vary depending on the asset under consideration. A very risk-averse household will allocate a smaller proportion of their wealth to the risky asset and a larger proportion to less risky assets. Therefore, the expected value of the unobserved effect should vary by equation. Our model allows for this in a very general way. One concern is that the model that we propose is too general and leads to estimation of too many parameters.

A potential solution is to specify a slightly different model where instead of estimating a different parameter for each time average and intercept by equation we estimate a single parameter vector for the time averages and then estimate parameters to allow the effect of the time-averaged covariates to vary by equation. We can adapt the above model as follows:

$$E(y_{itg} \mid \mathbf{x}_{it}, c_i) = \Phi(\mathbf{x}_{it}\boldsymbol{\beta}_g + \delta_g c_i).$$

Here we introduce an equation specific parameter δ_g that allows the impact of the unobserved effect to vary across equations. In addition, we make the assumption that there is a single source of heterogeneity.

$$c_i = \Psi + \xi\bar{\mathbf{x}}_i + a_i$$

$$a_i \mid \mathbf{x}_i \sim N(0, \sigma_a^2)$$

Again, we assume strict exogeneity conditional on the unobserved effect:

$$E(y_{itg} \mid \mathbf{x}_i, c_i) = E(y_{itg} \mid \mathbf{x}_{it}, c_i)$$

Plugging c_i into $E(y_{itg} \mid \mathbf{x}_{it}, c_i)$ gives us the mean conditional on observed and unobserved factors:

$$E(y_{itg} \mid \mathbf{x}_{it}, a_i) = \Phi(\delta_g \Psi + \mathbf{x}_{it}\boldsymbol{\beta}_g + \delta_g \xi\bar{\mathbf{x}}_i + \delta_g a_i)$$

where $\delta_g a_i \mid \mathbf{x}_i \sim Normal(0, \delta_g^2 \sigma_a^2)$

Therefore the conditional mean of y_{itg} given $\mathbf{x}_i$ is

$$E(y_{itg} \mid \mathbf{x}_i) = \Phi\left(\frac{\delta_g \Psi + \mathbf{x}_{it}\beta_g + \delta_g \xi\bar{\mathbf{x}}_i}{(1+\delta_g^2 \sigma_a^2)^{1/2}}\right)$$

This is nearly identical to equation (2.3.6) above, except that we have a different scale factor. Regardless, we will still be able to derive the consistent estimates of the average partial effects.

We can estimate the above conditional means using the multinomial quasi-likelihood. Since we cannot identify a parameter for each equation, we would need to normalize one say, δ_G to one. We do not estimate this specification here, but we point out that in other applications this can be considered since it is more parsimonious and may be more efficient.

2.7 Results

We use data from the Health and Retirement Study (HRS). The HRS began in 1992 and was nationally representative of all non-institutionalized individuals aged 51-61 in that year. In 1998, the HRS combined with the Asset and Health Dynamics of the Oldest Old (AHEAD) and added several new cohorts to be nationally representative of the population of non-institutionalized individuals aged 51 and older. Both initial respondents and their spouses are interviewed and followed in subsequent waves. We use data for all cohorts from Waves 1-7 (1992-2004) of the HRS. In addition to basic demographic variables, the HRS collects detailed information on wealth holdings, allocation of wealth, income and its sources, health, and measures of the probability that future events occur. We use subjective survival probability, age, word recall, household income, and non-financial wealth as regressors in our model.

Our measure of subjective survival probabilities comes from responses to the following question:

> *On a scale from 0 to 100, where 0 is no chance and 100 is absolutely certain, what are the chances that you will live to age 75 or older?*

Our measure of income is calculated as the sum of all non-capital income received by the respondent and spouse during the year. To calculate wealth variables we use responses to questions about the value of holdings within various financial assets. The assets that are measured are stocks and/or mutual funds, bonds (corporate and government), Certificates of Deposit (CDs) and checking, saving and/or money market accounts. We also have information about other assets owned by the household including housing, vehicles, other real estate, IRA/Keogh accounts, and trusts. We distinguish between two measures of wealth holdings. Non-financial wealth consists of the value of housing, real estate, IRA/Keogh accounts, vehicles, and trust holdings, less any associated debt. Financial Wealth is the value of all holdings in stocks/mutual funds, bonds, Certificates of Deposit and savings, checking and money market accounts. HRS respondents are given a list of nouns and then asked to repeat this list immediately and then again at the end of the cognition section. We use the proportion of words recalled at the end of the section as a proxy for cognition.

We construct our data set by household. We combine spouses in each wave so that our panel consists of household observations by year. To select individuals for our sample we drop observations that satisfy the following criteria:

(i) older than 65,

(ii) missing values for subjective survival probability,

(iii) is a proxy interview or

(iv) is deceased.

We apply these criteria at the individual level so households that do not have both spouses are dropped. The decision to allocate wealth across assets is conditional on holding positive financial wealth; thus, we drop observations where financial wealth is zero. In addition, we drop households that are only observed for one time period. Imposing the above criteria for married households leaves us with 3,872 households (14,514 total observations).

In Table 13, we display means and medians of the relevant variables in our analysis. We can see from Table 13 that the average allocation of wealth generally falls into two assets: stocks and checking. There is some investment in CDs but very little in Bonds. There is some concern regarding the time variation in the HRS in particular with subjective survival probability measures.[15] Table 14 presents the proportion of the variation that is between households in our dataset. We can see that the bulk of the variation in our variables is between households, but there is some variation over time that we may be able to exploit.

As a first step, we estimate our allocation equations by linear fixed effects. In general linear fixed effects coefficients should provide fairly good estimates of the average partial effects and provide a reasonable baseline to compare other estimation techniques. The results of the

[15] Smith et al. (2001) and Elder (2010) point to the lack of appropriate variation in the subjective survival probabilities collected in the HRS.

linear fixed effects estimation are presented in Table 15.

We now turn to different methods of estimating the average partial effects of our covariates using a nonlinear conditional mean specification. As a first pass, we know that we can estimate each equation using a single equation nonlinear method. We follow Papke and Wooldridge (2008) and use the procedure that they laid out for the estimation of a pooled fractional probit. Since we have an unbalanced panel we cannot simply estimate the single equation fractional probit on the entire panel. Instead we subset the unbalanced panel as described above, maximize the Bernoulli quasi-likelihood for each balanced panel subset, calculate the average partial effects and then average these across the balanced panel subsets. The results from this estimation approach are presented in Table 16. With the exception of the estimated average partial effects on Husband and Wife Word Recall, the estimated average partial effects are similar to those estimated by linear fixed effects but we can see that estimating using balanced panel subsets significantly increased our estimated standard errors so that none of our covariates are statistically significant. The difference in the estimated average partial effects for both Husband and Wife Word Recall may suggest that there is some nonlinearity in the impact of this variable on the allocation of wealth

Next, we maximize the multinomial quasi-likelihood on each balanced panel subset, calculate the average partial effects for each balanced panel subset, and then average these over the balanced panel subsets. Table 17 contains the estimated average partial effects and the standard errors from this approach. Overall, we can see that the estimated average partial effects for all equations are very similar to those estimated by the single equation fractional probit technique. Looking at the standard errors, we can see that they are nearly identical to the fractional probit standard errors. It appears that imposing the adding up restriction does not affect the estimates of either the average partial effects or the standard errors.

Table 18 presents the estimated average partial effects and standard errors from Procedure 2 outlined above. Our main points of comparison here are Tables 15, 16 and 17, the average partial effects estimated by linear fixed effects, maximizing the Bernoulli quasi-likelihood and maximizing the multinomial quasi-likelihood on the balanced panel subsets. Looking first at the estimated standard errors, we can see that they are smaller than those in both Tables 16 and 17 but larger than those estimated by linear fixed effects. This suggests that accounting for serial correlation within and between equations has led to more precise estimates relative to the estimation that uses balanced panel subsets. In particular, the estimated standard errors for the allocation of wealth to Bonds and CDs are almost half the size of the standard errors estimated by maximizing the pooled multinomial quasi-likelihood.

Turning to the estimated average partial effects we can see that there are some differences between the results in Table 18 compared to Tables 15, 16 and 17. Tables 19, 20 and 21 show the p-values of testing the difference in estimated average partial effects across the different estimation approaches. Tables 19 and 20 compare the estimated average partial effects of Procedure 2 to the estimated partial effects from maximizing the Bernoulli quasi-likelihood for each asset equation on balanced panel subsets and from maximizing the multinomial quasi-likelihood on balanced panel subsets. Given the large standard errors from estimation on balanced panel subsets with either the single equation or multiple equation approaches we only see a statistically significant difference (at the 5% level) for the estimated average partial effect of Log(Income) on the allocation of wealth to Bonds. Looking at Table 21, which compares estimated average partial effects between Procedure 2 and linear fixed effects we can see that the estimated average partial effect of Log(Income) on bonds is also statistically significantly different than the linear fixed effect estimate. It is unclear whether we should expect a negative correlation between income and bond holdings, since it is not certain what increasing income levels mean for the riskiness of that income. Theoretically a more risky income stream would lead to an increased incentive to allocate wealth to less risky investments, but this will depend upon the correlation between the riskiness of the return from each asset and income. Since the estimated average partial effects from linear fixed effects, maximizing the Bernoulli quasi-likelihood for each equation and maximizing the multinomial quasi-likelihood are relatively close to zero with large standard errors it seems plausible that the effect is negative but not controlling for serial correlation leads us to estimate a very small impact from income.

Simply looking at the magnitude of the estimated average partial effects it appears that the largest differences occur within the stock and checking equations. The estimated average partial effects that are the most different are for Husband and Wife Age, Husband and Wife Pr(Live to 75), and Husband and Wife Word Recall. While the impacts of these covariates have changed for both Husband and Wife there are distinct differences between the two members of the household. The average partial effect of a ten percentage point increase in survival probability on the allocation of wealth to stocks increased nearly tenfold for the Husband, but decreased twofold for the Wife. We can also see that the estimated average partial effect on stocks from Husband Word Recall is about 0.003 percentage points greater in Table 18 compared to the linear fixed effects estimate in Table 15, but this is not the case when comparing Wife Word Recall. Nevertheless, though the magnitudes of the estimated average partial effects are larger from Procedure 2 we cannot reject the null hypothesis that they are statistically equivalent to the estimates from maximizing the single equation Bernoulli quasi-likelihoods or the multinomial quasi-likelihood.

Turning to Table 21, which compares the estimated average partial effects between linear fixed effects and Procedure 2, the covariates with statistically significant different estimates (at the 5% level) are Husband Age for stocks, Wife Pr(Live to 75) and Log(Income) for bonds and Husband Age and Wife Age for checking. The differences in the estimated average partial effects for Husband and Wife Age are not surprising given the fact that Procedure 2 is using the between household variation from time averages within years of the balanced panel subsets to estimate the impact of age. Perhaps more surprising is the fact that we now estimate a negative average partial effect for Wife Pr(Live to 75) for bonds. Though this estimated average partial effect from Procedure 2 is not statistically different from the linear fixed effect estimate it suggests that the Wife may prefer less risky assets (and potentially more liquid in the case of CDs and Checking) when facing an increased lifespan. While this is purely conjecture it may suggest that there is an avenue for research into the different preferences of household members against both the expected risk and return from assets and the liquidity of these assets.

There are several reasons that we would expect to see differences in the estimated average partial effects from our Procedure 2. First, the two-step procedure maximizes the multinomial quasi-likelihood for each time period within each balanced panel subset; this technique exploits the variation in the differences from the time averages between households. Second, Procedure 2 is weights the coefficients based on the serial correlation in the allocation of wealth to each asset, the serial correlation in the covariates and the correlation of the covariates across equations within each time period and also across time. Our procedure combines all of these correlations into weights that are then applied to the coefficients from each time period within the balanced panel subset. What is promising is that while we do see some statistically different estimated average partial effects from Procedure 2 compared to maximizing single equation Bernoulli quasi-likelihoods, maximizing multiple equation multinomial quasi-likelihoods and single equation linear fixed effects the actual impact from these covariates all point to relatively small effects on the allocation of wealth across these assets.

Overall it seems that our estimated impacts from the procedure we laid out here are reasonably close to estimates from models that we know will generate consistent but inefficient results. In addition, despite slightly different estimated average partial effects our proposed procedure still estimates small impacts of these covariates on the allocation decision which is consistent with the other approaches that we used.

One of the advantages of the method that we proposed here is that it allows the effect of the covariates to affect our response variables in a non-linear way. Even though we have estimated our multinomial QMLE on time periods within balanced panel subsets we can still construct the Average Structural Function (ASF) for the entire sample:

$$\widehat{ASF}(\mathbf{x}_t) = N^{-1}\sum_{i=1}^{N}\sum_{r=2}^{T} 1[T_i = r]\Phi\left(\hat{\Psi}_r + \mathbf{x}_t\hat{\boldsymbol{\beta}}_{gr} + \bar{\mathbf{x}}_i\hat{\boldsymbol{\xi}}_{gr}\right) \text{ for } r = 2,\ldots,T.$$

We plot $\widehat{ASF}(\mathbf{x}_t)$ for Husband Word Recall for the allocation of wealth to Stocks, Bonds, CDs, and Checking in Figures 1 – 4 and for Wife Word Recall in Figures 5 – 8. As a point of reference, we also plot the corresponding ASF from the linear fixed effects estimation, which are straight lines because the coefficient estimates are constant over the entire range of the covariate. From figure 1 we can see that there is some nonlinearity in the impact of Husband Word Recall on stocks. It seems that the impact of remembering additional words increases slightly as more words are recalled. Figure 2 displays the average structural function for bonds. There appears to be a steep decline in the predicted proportion of wealth allocated to bonds as the number of words recalled increases, but this impact dampens as more and more words are recalled. For the allocation of wealth to CDs we see a relatively stable small impact of Husband Word Recall that gets slightly larger as the number of words recalled increases. In figure 4 we can see that the impact of Husband Word Recall on the proportion of wealth allocated to checking is initially small, but increases as the number of words recalled increases.

Turning to Wife Word Recall we can see in Figure 5 that the average structural function is fairly linear across the range of word recall. Figure 6 shows that there is some significant nonlinearity in the impact of word recall on the allocation of wealth to bonds. It appears that impact of recalling additional words increases once the wife has recalled greater than 50% of the words given in the survey. Figure 7 shows that there is a slight dampening effect of recalling additional words on the allocation of wealth to CDs, but that it sets in near the end of the range of possible values. As we can see from Figure 8, there appears to be “diminishing returns” to the impact of additional words recalled by the wife and it seems that this begins to set in slightly past 60% of the words recalled.

2.8 Conclusion

In this paper, we have presented a two-step estimation procedure for cases where we have multiple fractional responses that must satisfy an adding up restriction. We leveraged the properties of the multinomial quasi-likelihood and the normal cumulative distribution function to account for the adding-up restriction and the restricted range of our multiple response variables. In our application, we were able to exploit the random sampling of cross-sectional units to derive consistent and more slightly more efficient estimates of the average partial effects of our covariates on the allocation of financial wealth to various assets in an unbalanced panel while still controlling for a time-invariant unobserved effect. Since the multinomial quasi-likelihood function is a member of the linear exponential family we are able to achieve consistent results under the assumption that we have correctly specified the conditional mean. Our second step allows us to increase the efficiency of our estimates from using nonlinear methods on balanced panel subsets of our data while still estimating average partial effects that are (for the most part) not statistically different from these other approaches.

We also laid out an alternative estimation approach that makes a further assumption on the impact of our time averages. This refinement is a clear next step along this line of research. There is also much to be gained in applying the two-step procedure to other problems where we have multivariate fractional responses that must satisfy an adding-up restriction. In particular, it would be useful to see how well this estimation technique performs in balanced panels and also in panels where there is more within variation in the covariates. It would also be fruitful to see the impact that different assumptions about the serial correlation have on the estimates from the CMD step. One assumption that is obvious and that it used in the Generalized Estimating Equation literature is to assume that the correlation within units over time is exchangeable. Papke and Wooldridge (2008) find that there is almost no difference between their estimates when assuming an exchangeable correlation structure compared to a completely unstructured correlation.

CHAPTER 3

ESTIMATION OF A MULTIVARIATE FRACTIONAL RESPONSE MODEL WITH UNBALANCED PANEL DATA

3.1 Introduction

Estimation using panel datasets is more prevalent with the increase of available data on repeated observations on micro-level units. With panel data we are careful to remove unobserved effects by differencing the data or using fixed-effects. With a linear model this is straightforward and allows for the unobserved effect to be correlated with observables and having a balanced panel is not necessary. Once we begin to look at estimation of nonlinear models on panel data this issue becomes a little more complicated.

Treating the unobserved effect as a parameter to estimate leads to inconsistent estimates, referred to as the “incidental parameters problem”. In general it is assumed that the unobserved effect is part of the conditional mean that we specify for our dependent variable. As this unobserved effect is folded into a nonlinear function we are not able to remove it via differencing or time-demeaning. We can avoid this problem either by ignoring it or using a random-effect technique that assumes that the unobserved effect is not correlated with any observables. While the random effects procedure provides a solution, the assumption that the unobserved effect is uncorrelated with the independent variables can lead to inconsistent estimates if violated.

Another approach to controlling for the unobserved effect in panel data is through Correlated Random Effects models. The approach makes parametric assumptions about the way that the time-invariant effect is related to other independent variables in the conditional mean model. The usual set up assumes that there is a linear relationship between the unobserved effect and the time-averages of the regressors. This Chamberlain-Mundlak device is relatively simple to implement assuming that we have a balanced panel.

In the context of an unbalanced panel care needs to be taken when employing the Chamberlain-Mundlak device to control for unobserved effects. The standard assumption is that the unobserved effect c_i is distributed $Normal(\psi + \xi \bar{x}_i, \sigma_c)$. This assumption is not accurate if some cross-section units have fewer observations than others since the estimated variance of the unobserved effect will be larger for units that are observed fewer times. The most obvious approach to remedy this problem is to simply choose the largest balanced panel, essentially dropping cross-section units that we do not observe for a set number of periods. This approach is dangerous for two reasons. First, reducing the number of observations decreases the accuracy of our estimates. Second, even if we assume that selection is uncorrelated with the idiosyncratic errors there is a potential for selection to be correlated with the observables and unobserved heterogeneity which will lead to inconsistent estimates of their effects on our dependent variable. Wooldridge (2010) presents a modification of the Correlated Random Effects approach that allows selection to be correlated with the covariates and the unobserved effect. This modification allows us to apply the Correlated Random Effects approach to be applied to unbalanced panel data.

Wooldridge (2010) presents an example using a standard probit (or fractional probit) model and suggests that we can extend this modification to multivariate fractional responses. Sivakumar and Bhat (2002) and Mullahy (2010) use the multinomial quasi-likelihood to estimate models of multivariate fractional responses that have an adding up restriction and estimate their models using cross-sectional data. Extending the models to balanced panel data is relatively straightforward as we can simply estimate pooled models on repeated cross-section observations.

Of course, we would want to make sure that we use clustered standard errors to make inference robust. In a balanced panel we could use the Correlated Random Effects approach and control for the unobserved heterogeneity by making parametric assumptions of the moments of its distribution.

In this paper we combine the approach for multivariate fractional responses and the modifications to the Correlated Random Effects approach to estimate models on multiple fractional responses on cross-section units that are observed over time, but not for the same number of time periods. We do this by combining the conditional mean specifications of each of our fractional responses and estimate using the multinomial quasi-likelihood function which explicitly imposes the adding up restriction of the multivariate fractional responses. We assume that selection is uncorrelated with the idiosyncratic error but allow it to be correlated with the unobserved effect and the covariates in our conditional mean.

The outline of the paper is as follows. Section 2 presents the single equation fractional probit model discussed in Papke and Wooldridge (2008), section 3 extends the single equation fractional probit to allow for selection following Wooldridge (2010), section 4 presents the multinomial quasi-likelihood and details how we will estimate the multivariate fractional response models on an unbalanced panel simultaneously, section 5 presents results from estimation of this model comparing it to results from other estimation methods and section 6 concludes.

3.2. Single Equation Fractional Probit Models in a Balanced Panel

We begin by laying down the framework for estimating a model of portfolio allocation on a balanced panel. We proceed in this manner since it makes obvious the modifications that we need to make for the unbalanced panel case and demonstrates the impact that these modifications will have on the estimation of the average partial effects.

Assume that at each time period t, we observe household i allocate a proportion of their financial wealth to each asset g. We assume that $g = 1, 2, \ldots, G$ exhausts all possible investment options so that in each period the household invests all their financial wealth. This assumption implies that $\sum_g^G y_{itg} = 1$ for each $i = 1, 2, \ldots, N$ and $t = 1, 2, \ldots, T$. In addition to our response vector, we observe a set of covariates $\mathbf{x}_{it}$ and an unobserved effect $\mathbf{c}_{ig}$. Our goal is to estimate a model of the conditional mean of y given $\mathbf{x}$ to determine the impact of each covariate. By construction, y_{itg} is bounded between zero and one. Assuming that we use the same set of covariates for each equation we can specify the conditional mean as:

$$E(y_{itg} \mid \mathbf{x}_{it}, c_{ig}) = \Phi(\mathbf{x}_{it}\boldsymbol{\beta}_g + c_{ig}); \quad i = 1, \ldots, N; \; t = 1, \ldots, T; \text{ and } g = 1, \ldots, G \quad (3.2.1)$$

We point out that the above specification completely ignores the fact that these fractional responses sum to one. It is simply acting as though we are treating each asset equation by itself. Nevertheless, it is useful to study what we can derive from these single equation models since our approach here essentially takes the specification in (3.2.1) for each equation and uses the multinomial quasi-likelihood to impose the adding up constraint.

Our interest lies in estimating the average partial effects of the covariates, $\mathbf{x}_{it}$ on the proportion of financial wealth allocated to each asset. Following the results for a probit model the direction of the partial effect is determined by the sign of $\boldsymbol{\beta}_g$. For a continuous variable x_{tk} (dropping the i subscript);

$$\frac{\partial E(y_{tg} \mid \mathbf{x}_t, c_g)}{\partial x_{tk}} = \beta_{gk}\phi(\mathbf{x}_t\boldsymbol{\beta}_g + c_g) \quad (3.2.2)$$

To obtain the average partial effect we can average this over the distribution of c_g and and $\mathbf{x}_t$.

To consistently estimate the coefficients in this single equation framework we require that the covariates are not correlated with the error term. We assume that $\mathbf{x}_{it}$ is strictly exogenous conditional on the unobserved effect,

$$E(y_{itg} \mid \mathbf{x}_i, c_{ig}) = E(y_{itg} \mid \mathbf{x}_{it}, c_{ig}). \tag{3.2.3}$$

Following Chamberlain (1980) and Mundlak (1978) we assume that

$$c_{ig} = \psi_g + \overline{\mathbf{x}}_i \boldsymbol{\xi}_g + a_{ig}\text{; where } a_{ig} \mid \mathbf{x}_i \sim Normal(0, \sigma_a^2). \tag{3.2.4}$$

Using (3.2.4) we can rewrite (3.2.1) as:

$$E(y_{itg} \mid \mathbf{x}_i, a_{ig}) = \Phi(\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i \boldsymbol{\xi}_g + a_{ig}). \tag{3.2.5}$$

So the mean of y_{itg} conditional on $\mathbf{x}_i$ is:

$$E(y_{itg} \mid \mathbf{x}_i) = E(\Phi(\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i \boldsymbol{\xi}_g + a_{ig} \mid \mathbf{x}_i) = \Phi\left(\frac{\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i \boldsymbol{\xi}_g}{(1+\sigma_{ag}^2)^{-1/2}}\right) \tag{3.2.6}$$

or

$$E(y_{itg} \mid \mathbf{x}_i) = E(\Phi(\psi_g + \mathbf{x}_{it}\boldsymbol{\beta}_g + \overline{\mathbf{x}}_i \boldsymbol{\xi}_g + a_{ig} \mid \mathbf{x}_i)) = \Phi\left(\psi_{ga} + \mathbf{x}_{it}\boldsymbol{\beta}_{ga} + \overline{\mathbf{x}}_i \boldsymbol{\xi}_{ga}\right) \tag{3.2.7}$$

where we use the a subscript to denote that our estimated coefficients are scaled by $(1+\sigma_{ag}^2)^{-1/2}$. Woodridge (2002, Section 15.8.2) shows that the mixing properties of the normal distribution lead to equation (3.2.7).

For identification of the scaled coefficients in (3.2.7) we require that there is no perfect collinearity between the elements of $\mathbf{x}_{it}$ and that there is enough variation in $\mathbf{x}_{it}$ over time. With the above parametric specification of the unobserved effect we can now write the Average Structural Function (ASF) following Blundell and Powell (2003):

$$ASF_g(\mathbf{x}_t) = E_{\overline{\mathbf{x}}_i}\left[\Phi\left(\psi_{ga} + \mathbf{x}_t\boldsymbol{\beta}_{ga} + \overline{\mathbf{x}}_i\boldsymbol{\xi}_{ga}\right)\right] \qquad (3.2.8)$$

A consistent estimator of $ASF_g(\mathbf{x}_t)$ is:

$$\widehat{ASF}_g(\mathbf{x}_t) = N^{-1}\sum_{i=1}^{N}\Phi\left(\hat{\psi}_{ga} + \mathbf{x}_t\hat{\boldsymbol{\beta}}_{ga} + \mathbf{x}_i\hat{\boldsymbol{\xi}}_{ga}\right). \qquad (3.2.9)$$

where $\hat{\psi}_{ga}$, $\hat{\boldsymbol{\beta}}_{ga}$, and $\hat{\boldsymbol{\xi}}_{ga}$ are consistent estimates of the scaled coefficients in equation (3.2.7). For a continuous variable, x_j we can calculate the average partial effect by taking the derivative of (3.2.9) and average over both i and t:

$$\widehat{APE}_g(x_j) = \frac{\partial\Phi(\hat{\psi}_{ga} + \mathbf{x}_{it}\hat{\boldsymbol{\beta}}_{ga} + \overline{\mathbf{x}}_i\hat{\boldsymbol{\xi}}_{ga})}{\partial x_j} = NT^{-1}\sum_{t=1}^{T}\sum_{i=1}^{N}\hat{\beta}_{ga,j}\phi(\hat{\psi}_{ga} + \mathbf{x}_{it}\hat{\boldsymbol{\beta}}_{ga} + \overline{\mathbf{x}}_i\hat{\boldsymbol{\xi}}_{ga}) \qquad (3.2.10)$$

For a binary x_j we can calculate the average partial effect by calculating the difference in the Average Structural Function evaluated at zero and one.

3.3 Single Equation Fractional Probit Models in an Unbalanced Panel

Once we are dealing with an unbalanced panel we need to make explicit the fact that each cross section unit in our sample is not observed for the full span of time. Let T_i represent the number of periods that we observe each cross sectional unit. We also introduce a selection indicator to index whether an observation is used in estimation. Let s_{it} be an indicator that takes on a value of one if the observation is used in estimation and zero otherwise. To get consistent estimates of the average partial effects we need to make explicit the assumption that selection is not correlated with the idiosyncratic error term. Therefore, in addition to the usual strict exogeneity assumption we assume that selection is conditionally ignorable. We write this assumption as follows:

$$E(y_{itg} \mid \mathbf{x}_i, c_{ig}, \mathbf{s}_i) = E(y_{itg} \mid \mathbf{x}_{it}, c_{ig}) = \Phi(\mathbf{x}_{it}\boldsymbol{\beta}_g + c_{ig}) \qquad (3.3.1)$$

Now we need to specify a model for the distribution of the unobserved effect. In the typical case this amounts to specifying that the expected value of the unobserved effect varies linearly with the time averages of the observed covariates.

In an unbalanced panel, we need to specify that the mean and the variance of the unobserved effect vary by T_i since intuitively the estimated moments will vary simply because we are using a different number of observations to estimate the parameters of the distribution. Following Wooldridge (2010), we define $\mathbf{w}_i$ to represent functions of the covariates and selection $\{(s_{it}, s_{it}\mathbf{x}_{it} : t = 1,...,T\}$. A straightforward extension of the standard Chamberlain-Mundlak device would be to allow $E(c_{ig} \mid \mathbf{w}_i)$ to vary by the number of time periods that we observe our cross-section units[16]:

$$E(c_{ig} \mid \mathbf{w}_i) = \sum_{r=1}^{T} \psi_{gr} \cdot 1[T_i = r] + \bar{\mathbf{x}}_i \boldsymbol{\xi}_g \qquad (3.3.2)$$

We allow the variance of c_{ig} to vary across the unbalanced panel through the following specification:

$$Var(c_{ig} \mid \mathbf{w}_i) = \exp\left(\tau + \sum_{r=1}^{T-1} 1[T_i = r] \cdot \boldsymbol{\omega}_{gr} \right) \qquad (3.3.3)$$

The specification in (3.3.3) allows the variance of the unobserved effect to vary with the number of time periods that we observe a cross-section unit in our panel.

[16] Wooldridge (2010) actually specifies that the conditional mean of the unobserved effect includes the interaction between the indicator for number of time periods in sample and the time averages of the covariates. We only include the indicator variables here due to non-convergence of our likelihood functions in estimation.

Given equations (3.3.2) and (3.3.3) and assuming c_{ig} conditional on $\mathbf{w}_i$ is distributed normal we can rewrite the conditional mean for a given asset as

$$E(y_{itg} \mid \mathbf{x}_{it}, \mathbf{w}_i, s_{it} = 1) = \Phi\left[\frac{\mathbf{x}_{it}\boldsymbol{\beta}_g + \sum_{r=2}^{T}\psi_{gr} \cdot 1[T_i = r] + \bar{\mathbf{x}}_i\boldsymbol{\xi}_g}{\left\{1 + \exp\left(\tau + \sum_{r=1}^{T-1} 1[T_i = r] \cdot \boldsymbol{\omega}_{gr}\right)\right\}^{1/2}}\right] \tag{3.3.4}$$

We can reparameterize equation (3.3.4) so that the denominator is unity if we have a balanced panel:

$$E(y_{itg} \mid \mathbf{x}_{it}, \mathbf{w}_i, s_{it} = 1) = \Phi\left[\frac{\mathbf{x}_{it}\boldsymbol{\beta}_g + \sum_{r=1}^{T}\psi_{gr} \cdot 1[T_i = r] + \bar{\mathbf{x}}_i\boldsymbol{\xi}_g}{\exp\left(\sum_{r=2}^{T} 1[T_i = r] \cdot \boldsymbol{\omega}_{gr}\right)^{1/2}}\right] \tag{3.3.5}$$

Given the specification in (3.3.5) we follow Blundell and Powell (2003) and construct the Average Structural Function as:

$$ASF_g(\mathbf{x}_t) = E_{\mathbf{x}_i}\left[\Phi\left(\frac{\mathbf{x}_t\boldsymbol{\beta}_g + \sum_{r=1}^{T}\psi_{gr} \cdot 1[T_i = r] + \bar{\mathbf{x}}_i\boldsymbol{\xi}_g}{\exp\left(\sum_{r=2}^{T} 1[T_i = r] \cdot \boldsymbol{\omega}_{gr}\right)^{1/2}}\right)\right] \tag{3.3.6}$$

A consistent estimate of this is

$$\widehat{ASF}_g(\mathbf{x}_t) = N^{-1}\sum_{i=1}^{N}\Phi\left(\frac{\mathbf{x}_t\hat{\boldsymbol{\beta}}_g + \sum_{r=1}^{T}\hat{\psi}_{gr} \cdot 1[T_i = r] + \bar{\mathbf{x}}_i\hat{\boldsymbol{\xi}}_g}{\exp\left(\sum_{r=2}^{T} 1[T_i = r] \cdot \hat{\boldsymbol{\omega}}_{gr}\right)^{1/2}}\right) \tag{3.3.7}$$

where $\hat{\boldsymbol{\beta}}_g$, $\hat{\psi}_{gr}$, $\hat{\boldsymbol{\xi}}_g$ and $\hat{\boldsymbol{\omega}}_{gr}$ are consistent estimates of the coefficients in (3.3.5). For a continuous variable x_k we can calculate the average partial effect by taking the derivative of (3.3.7):

$$\hat{\beta}_{gk}\left\{N^{-1}\sum_{i=1}^{N}\phi\left[\frac{\mathbf{x}_t\hat{\boldsymbol{\beta}}_g+\sum_{r=1}^{T}\hat{\psi}_{gr}\cdot 1[T_i=r]+\overline{\mathbf{x}}_i\hat{\boldsymbol{\xi}}_g}{\exp\left(\sum_{r=2}^{T}1[T_i=r]\cdot\hat{\boldsymbol{\omega}}_{gr}\right)^{1/2}}\right]\right\} \tag{3.3.8}$$

where $\phi[\bullet]$ represents the standard normal probability distribution function. Once again we can see that $\hat{\beta}_{gk}$ determines the direction of the partial effect.

3.4 The Multinomial Quasi-Likelihood: Multivariate Fractional Responses

We now describe the multinomial quasi-likelihood function and detail the how estimation proceeds given the specifications for the conditional mean and variance. Since an observation can only have a log-likelihood value if it is included in estimation the following exposition necessarily assumes that the likelihood is only defined for observations where $s_{it}=1$.

The multinomial quasi-likelihood function for a random draw i in a specific time period t is given by:

$$\begin{aligned}\ell_{it}(\boldsymbol{\theta}) = {} & y_{i1t}\log\left[m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)\right]+y_{i2t}\log\left[m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)\right]+\cdots \\ & \cdots+y_{iGt}\log\left[m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)\right]\end{aligned} \tag{3.4.1}$$

Where

$$m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_g)=\Phi\left(\frac{\mathbf{x}_{it}\boldsymbol{\beta}_g+\sum_{r=1}^{T}\psi_{gr}\cdot 1[T_i=r]+\bar{\mathbf{x}}_i\boldsymbol{\xi}_g}{\exp\left(\sum_{r=2}^{T}1[T_i=r]\cdot\boldsymbol{\omega}_{gr}\right)^{1/2}}\right).$$ [17]

For identification of the parameters the multinomial quasi-likelihood requires that the following constraint hold:

$$m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)=1-m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)-m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)-\cdots-m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_{G-1}) \quad (3.4.2)$$

Since we will be pooling observations the partial quasi-likelihood function is

$$\ell_i(\boldsymbol{\theta})=\sum_{t=1}^{T} y_{i1t}\log\left[m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)\right]+y_{i2t}\log\left[m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)\right]+\cdots+y_{iGt}\log\left[m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)\right]$$

(3.4.3)

The score function is then

$$\mathbf{s}_i(\boldsymbol{\theta})=\sum_{t=1}^{T}\mathbf{s}_{it}(\boldsymbol{\theta})=\sum_{t=1}^{T}\left(\begin{array}{l} y_{it1}\cdot\dfrac{\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)'}{m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)}+y_{it2}\cdot\dfrac{\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)'}{m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)}+\cdots \\ \cdots+y_{itG}\cdot\dfrac{\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)'}{m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)}\end{array}\right) \quad (3.4.4)$$

Define $\mathbf{H}_i(\mathbf{x}_i,\mathbf{y}_i,\boldsymbol{\theta})$ as the Hessian then

[17] We note that using the Normal cumulative distribution function does not explicitly restrict the predicted value of the “omitted” equation to fall in the unit interval, though it is implicitly imposed by the logarithm function. In our application, the predicted values of the omitted equation remain in the unit interval. We also estimate an approximate multinomial logit form of the system which does explicitly impose the adding-up restriction of the equations, the results are nearly identical.

$$-E[\mathbf{H}_i(\mathbf{x}_i,\mathbf{y}_i,\boldsymbol{\theta}\mid\mathbf{x}_i)]=\sum_{t=1}^{T}\left(\begin{array}{c}\dfrac{\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)'\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)}{m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_1)}+\cdots\\ \dfrac{\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)'\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)}{m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_2)}+\cdots\\ \cdots+\dfrac{\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)'\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)}{m(\mathbf{x}_{it},\mathbf{w}_i,\boldsymbol{\theta}_G)}\end{array}\right) \quad (3.4.5)$$

this is consistently estimated by:

$$\widehat{\mathbf{A}}=(NT)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\boldsymbol{\mu}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})'\mathbf{W}_{it}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})\boldsymbol{\mu}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}}) \quad (3.4.6)$$

where

$$\boldsymbol{\mu}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})=\begin{pmatrix}\nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}}_1)\\ \nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}}_2)\\ \vdots\\ \nabla_{\boldsymbol{\theta}}m(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}}_G)\end{pmatrix} \quad (3.4.7)$$

and

$$\mathbf{W}_{it}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})=\begin{pmatrix}\dfrac{1}{m(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}}_1)} & 0 & \cdots & 0\\ 0 & \dfrac{1}{m(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}}_2)} & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \dfrac{1}{m(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}}_G)}\end{pmatrix}. \quad (3.4.8)$$

Define $\hat{\mathbf{B}}=\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})'$, then we can construct the robust sandwich form of the asymptotic variance-covariance matrix as

$$\hat{\mathbf{A}}^{-1}\hat{\mathbf{B}}\hat{\mathbf{A}}^{-1}/N = \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\boldsymbol{\mu}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})'\mathbf{W}_{it}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})\boldsymbol{\mu}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})\right)\bullet$$

$$\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})\mathbf{s}_{it}(\hat{\boldsymbol{\theta}})'\bullet \tag{3.4.9}$$

$$\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\boldsymbol{\mu}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})'\mathbf{W}_{it}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})\boldsymbol{\mu}(\mathbf{x}_{it},\mathbf{w}_i,\hat{\boldsymbol{\theta}})\right)^{-1}$$

3.5 Results

We use data from the Health and Retirement Study (HRS). The HRS began in 1992 and was nationally representative of all non-institutionalized individuals aged 51-61 in that year. In 1998, the HRS combined with the Asset and Health Dynamics of the Oldest Old (AHEAD) and added several new cohorts to be nationally representative of the population of non-institutionalized individuals aged 51 and older. Both initial respondents and their spouses are interviewed and followed in subsequent waves. We use data for all cohorts from Waves 1-7 (1992-2004) of the HRS. In addition to basic demographic variables, the HRS collects detailed information on wealth holdings, allocation of wealth, income and its sources, health, and measures of the probability that future events occur. We use subjective survival probability, age, word recall, household income, and non-financial wealth as regressors in our model.

Our measure of subjective survival probabilities comes from responses to the following question:

> *On a scale from 0 to 100, where 0 is no chance and 100 is absolutely certain, what are the chances that you will live to age 75 or older?*

Our measure of income is calculated as the sum of all non-capital income received by the respondent and spouse during the year. To calculate wealth variables we use responses to questions about the value of holdings within various financial assets. The assets that are measured are stocks and/or mutual funds, bonds (corporate and government), CDs, and checking, saving and/or money market accounts. We also have information about other assets owned by the household including housing, vehicles, other real estate, IRA/Keogh accounts, and trusts. We distinguish between two measures of wealth holdings. Non-financial wealth consists of the value of housing, real estate, IRA/Keogh accounts, vehicles, and trust holdings, less any associated debt. Financial wealth is the value of all holdings in stocks/mutual funds, bonds, CDs, and checking, savings, and money market accounts. HRS respondents are given a list of nouns and then asked to repeat this list immediately and then again at the end of the cognition section. We use the proportion of words recalled at the end of the section as a proxy for cognition.

We construct our data set by household. We combine spouses in each wave so that our panel consists of household observations by year. To select individuals for our sample we drop observations that satisfy the following criteria:

(i) older than 65,

(ii) missing values for subjective survival probability,

(iii) is a proxy interview or

(iv) is deceased.

We apply these criteria at the individual level so households that do not have both spouses are dropped. The decision to allocate wealth across assets is conditional on holding positive financial wealth; thus, we drop observations where financial wealth is zero. In addition, we drop households that are only observed for one time period. Imposing the above criteria for married households leaves us with 3,872 households (14,514 total observations).

In Table 22, we display means and medians of the relevant variables in our analysis. We can see from Table 22 that the average allocation of wealth generally falls into two assets: stocks and checking. There is some investment in CDs but very little in Bonds. There is some concern regarding the time variation in the HRS in particular with subjective survival probability measures.[18] Table 23 presents the proportion of the variation that is between households in our dataset. We can see that the bulk of the variation in our variables is between households, but there is some variation over time that we may be able to exploit.

As a first step we estimate our allocation equations by linear fixed effects. In general linear fixed effects coefficients should provide fairly good estimates of the average partial effects therefore they should provide us with a reasonable baseline to compare other estimation techniques. The results of the linear fixed effects estimation are presented in Table 24.

We now turn to different methods of estimating the average partial effects of our covariates using nonlinear estimation. As a first pass we know that we can estimate each equation using a single equation nonlinear method. We follow Papke and Wooldridge (2008) and use the procedure that they laid out for the estimation of a pooled fractional probit. In this approach we are ignoring the unbalanced nature of the panel and simply maximizing the Bernoulli quasi-likelihood function for each asset equation over the full dataset. The results from this approach are presented in Table 25. While we have controlled for an unobserved effect in this estimation we have not accounted for the fact that we have an unbalanced panel, in fact we have simply assumed that each household is observed the same number of times.

Table 26 presents the results of maximizing the Bernoulli quasi-likelihood for each balanced panel subset of the full, unbalanced panel and then averaging the estimated average partial effects (and their variances) across the balanced panel subsets. We can see that the

[18] Smith et al. (2001) and Elder (2010) point to the lack of appropriate variation in the subjective survival probabilities collected in the HRS.

estimated average partial effects are nearly identical to the estimates provided by completely ignoring the unbalanced nature of the data. The difference in standard errors between these two approaches is significant though. The increase in the estimated standard errors arises from the fact that there are considerably fewer observations within each balanced panel subset and this naturally leads to larger variances. While this approach assumes that there is a difference in the estimated moments of the unobserved effect depending on the number of times that a household is viewed in the panel it does not provide a way to test whether the number of times that we observe a household has a statistically significant impact on the estimates.

Our next step is to estimate each asset equation on the full, unbalanced panel correcting for the unbalanced nature of the panel by allowing the mean and the variance of the unobserved effect to vary with the number of time periods that we observe a household. We maximize the Bernoulli quasi-likelihood and include indicator variables for the number of time periods that a household is in the sample in the specification of the conditional mean and the conditional variance of the unobserved effect. This is what Wooldridge (2010) proposes; except that we have made the assumption that there is a constant slope on the time averages of the covariates in the model. The results of this estimation approach are displayed in Table 27.

There are some differences in the estimated average partial effects in Table 27 for some covariates when we compare them to those estimated by the other methods in Tables 24, 25 and 26. This appears to be most prevalent in the estimated average partial effects for the covariates in the bond equation. For the most part, the larger average partial effects are not that worrisome since the estimated standard errors are large enough so that they are statistically insignificant at standard levels. The only estimated average partial effect which stands out as statistically different is Wife Pr(Live to 75). While the overall impact on allocation is still small – a ten percentage point increase in Wife Pr(Live to 75) leads to a 0.005 percentage point increase in the allocation of financial wealth to bonds - it is still five times larger than the estimated average partial effect from the other single equation estimates. Perhaps most striking is that in the other single equation estimation approaches we very nearly have average partial effects that sum to zero across the four equations, but this does not appear to be the case in Table 27. In fact it appears that none of the marginal effects of any covariate sum to zero across equations.

We also test the joint significance of the indicator variables for the total number of time periods that a household is observed in both the mean and the variance equation. From our test of joint significance we can see that the allocation of wealth to CDs is the only equation where we see that we obtain a test statistic of 52.16 which is significant at the 5% level, but this is only for the mean equation. This implies that there is little evidence of variation in the moments of the unobserved effects at the single equation level, with the exception of the mean of c_i for the allocation of wealth to CDs.

Our next step is to estimate the allocation equations by maximizing the multinomial quasi-likelihood function. We first estimate the multinomial quasi-likelihood function on the entire unbalanced panel ignoring the fact that we observe households for a different number of time periods. The results of this approach are shown in Table 28. We can see that our estimated average partial effects are very similar to those that we estimated using linear fixed effects and maximizing the Bernoulli quasi-likelihood for each equation. It appears that the biggest difference in estimated average partial effects is for Log (Income) which is larger in the single equation approaches than it is for the multinomial approach. This is not surprising as the multinomial approach leads us to estimated coefficients that maximize a likelihood that is restricted to values that satisfy the adding up constraint.

As a next step we acknowledge the unbalanced nature of our panel by maximizing the pooled multinomial quasi-likelihood for each balanced panel subset and then average the estimated average partial effects and their variances across the balanced panel subsets. Table 29 displays the results of this estimation approach. Once again it is clear that our standard errors have increased significantly since the sample sizes of the balanced panel subsets are smaller than the full panel. Nevertheless, there appears to be little difference in the estimated average partial effects compared to the estimates in Tables 24, 25 and 26. In fact, see that the estimated impact of Log (Income) are closer to those estimated using the single equation approaches.

Table 30 presents the results of maximizing the multinomial quasi-likelihood on the full, unbalanced panel and specifying that the moments of the unobserved effect to vary based on the number of times that we observe households in the panel. The estimated average partial effects are nearly identical to those estimated by linear fixed effects. Unlike the estimates from the maximization of the Bernoulli quasi-likelihood in table 27 the estimated average partial effects for each covariate sum to zero (with some rounding error) in table 30. This is due to the fact that the adding up restriction is naturally imposed by the specification of the conditional mean function and therefore restricts the maximization routine to focus on coefficients that satisfy the restriction. Looking at the joint significance test for the indicator variables for the total number of times that a household is observed we can see that there are some slight differences from the similar test presented in Table 27. First, since we are "excluding" the checking equation in the multinomial approach we do not have coefficients to test so these are missing. Second, it appears that both the conditional mean and conditional variance of the unobserved effect for the allocation of wealth to bonds differs across the number of time periods that we observe households. Similar to the estimates in table 27 we see that the conditional mean of the unobserved effect in the allocation of wealth to CDs does differ depending on the total number of time periods that a household is in sample. The difference in the bond allocation equation is most likely due to the imposition of the adding up restriction. Once we force the impact of these covariates to cancel out across the four assets it restricts the search for coefficients to those that satisfy the adding up constraint. Since the allocation of wealth to bonds is so small (in fact for a majority of households it is zero more often than not) and the quasi-likelihood function is flatter than for other assets, a single equation approach is likely to find many coefficients that maximize the flat likelihood but that do not maximize the multinomial quasi-likelihood.

As another point of comparison, we include Table 31 which shows the estimated average partial effects from estimating equation (3.3.5) but replacing the Normal Cumulative Distribution Function with the multinomial logit functional form. We should note that the below estimation equation is an approximation, the actual derivation of the appropriate specification of the conditional mean and variance function when using the multinomial logit functional form is more complicated. We perform this additional estimation since using the Normal Cumulative Distribution Function does not explicitly impose the restriction that the predicted value of the omitted equation must be in the unit interval; instead we are relying on the logarithm to impose this constraint. Specifically we are estimate the following equation:

$$E(y_{itg} \mid \mathbf{x}_{it}, \mathbf{w}_i, s_{it} = 1) = \frac{\exp\left[\dfrac{\mathbf{x}_{it}\boldsymbol{\beta}_g + \sum_{r=1}^{T}\psi_{gr}\cdot 1[T_i = r] + \bar{\mathbf{x}}_i\boldsymbol{\xi}_g}{\exp\left(\sum_{r=2}^{T} 1[T_i = r]\cdot\boldsymbol{\omega}_{gr}\right)^{1/2}}\right]}{1 + \sum_{g=1}^{G-1}\exp\left[\dfrac{\mathbf{x}_{it}\boldsymbol{\beta}_g + \sum_{r=1}^{T}\psi_{gr}\cdot 1[T_i = r] + \bar{\mathbf{x}}_i\boldsymbol{\xi}_g}{\exp\left(\sum_{r=2}^{T} 1[T_i = r]\cdot\boldsymbol{\omega}_{gr}\right)^{1/2}}\right]}. \quad (3.5.1)$$

We can see from Table 31 that the estimated average partial effects are nearly identical to those reported in Table 30, though the estimated standard errors for all equations are smaller than those estimated using the multinomial fractional probit approach. An interesting difference between Table 30 and 31 arises in the χ^2 test for ψ_{gr} and ω_{gr} for the Bond equation. In table 30 we can see that there the covariates are in general not statistically significant, but the indicator variables for the length of time in sample are significant in both the conditional mean and variance equation. Using the multinomial logit functional form we see that the these indicator variables are no longer statistically significant in either the conditional mean or variance equation, but that now the estimated partial effects are more precisely estimated. The fact that the estimated average partial effects are nearly identical across all estimation approaches suggests that we can leverage the flexibility (and potential efficiency) of non-linear estimation techniques on fractional response variables in an unbalanced panel.

Overall our estimated average partial effects from the procedure laid out in this paper are nearly identical to those estimated by methods that we know will give us consistent estimates, but it appears that estimating a nonlinear model that appropriately imposes the adding up restriction and also accounts for the unbalanced nature of our panel has led to a happy compromise between ignoring this information and over controlling for it.

3.6 Conclusion

One of the major criticisms of estimating nonlinear models on panel data with a correlated random effects approach is that it cannot handle unbalanced panels. Wooldridge (2010) offers a simple solution to this problem that is parsimonious and easily estimable by standard software. We have extended this approach to multiple fractional response variables and have exploited the fact that we have an adding up restriction that allows us to use the multinomial quasi-likelihood function. Since the multinomial quasi-likelihood is in the linear exponential family we can be confident that we have consistent estimates of the average partial effects of our covariates.

We have also compared our results from our procedure to other estimation methods that will give us consistent estimates of the average partial effects. We have found that the estimation procedure we laid out here lies somewhere in between ignoring the unbalanced nature of the panel and more drastic corrections for it such as estimation of standard correlated random effect models on balanced panel subsets. Since we have explicitly modeled that the conditional mean and variance of our unobserved effect varies with the number of time periods we observe a cross-sectional unit we are able to use the full data set to estimate our average partial effects. This has allowed us to maintain consistency while increasing the precision of our estimated average partial effects.

Possible avenues for future research would be to attempt to more parsimoniously specify that the unobserved effect will differ across equations. Here we have allowed for each covariate in the conditional mean and variance of the unobserved effect to be different for each equation. It might be fruitful to instead specify that the unobserved effect is the same across each equation but instead there is an overall scale factor for each equation's conditional mean and variance function. With the appropriate normalization we could then estimate fewer coefficients leading to possible more efficiently estimated average partial effects.

APPENDIX

Table 1. Summary Statistics for Single Households

	Wealth Sample		**Allocation Sample**	
	Mean	**Median**	**Mean**	**Median**
Age	58	58	58	59
White	0.62		0.71	
Black	0.27		0.21	
Hispanic	0.09		0.06	
Other	0.03		0.03	
Less than High School	0.28		0.19	
High School	0.34		0.35	
Some College	0.20		0.23	
College	0.08		0.10	
Post Grad	0.10		0.13	
Male	0.29		0.29	
Divorced/Separated	0.56		0.56	
Widowed	0.30		0.30	
Never Married	0.14		0.14	
Working	0.57		0.63	
Retired	0.18		0.20	
Unemployed	0.04		0.03	
Disabled	0.18		0.12	
Household Size	1.81	1	1.72	1
Word Recall	0.63	0.71	0.67	0.71
Series 7 Correct	2.34	2	2.66	3
High Risk Aversion	0.62		0.63	
Medium-High Risk Aversion	0.12		0.13	
Medium-Low Risk Aversion	0.10		0.10	
Low Risk Aversion	0.14		0.13	
Plan for Next Few Months	0.25		0.21	
Plan for Next Year	0.11		0.11	
Plan for Next Few Years	0.27		0.28	
Plan for 5-10 years	0.26		0.29	
Plan for 10+ years	0.10		0.10	
Pr(Live to 75)	0.63	0.70	0.65	0.75
Net Worth	$131,430	$38,530	$168,903	$64,186
Financial Wealth	$35,029	$1,485	$47,295	$4,733
Current Income	$22,764	$15,300	$27,155	$19,407
Log(Current Income)	9.55	9.64	9.77	9.87
Permanent Income	$21,405	$15,303	$25,585	$18,983
Permanent Log Income	9.44	9.50	9.65	9.71
Stocks			0.17	
Bonds			0.02	
CDs			0.08	
Checking			0.74	

Table 1 (cont'd).

Observations	**14,275**	**10,573**
Households	**5,022**	**4,053**

Note: Reported Wealth and Income measures are in 1992 dollars.

Table 2. Summary Statistics for Married Households

	Wealth Sample				**Allocation Sample**			
	Mean		**Median**		**Mean**		**Median**	
	Husband	**Wife**	**Husband**	**Wife**	**Husband**	**Wife**	**Husband**	**Wife**
Age	58	54	58	55	58	55	58	55
White	0.80	0.81			0.84	0.85		
Black	0.10	0.10			0.08	0.08		
Other	0.02	0.02			0.02	0.02		
Hispanic	0.07	0.07			0.05	0.05		
Less than High School	0.21	0.19			0.17	0.15		
High School	0.32	0.39			0.33	0.40		
Some College	0.21	0.23			0.22	0.24		
College	0.12	0.10			0.13	0.10		
Post Grad	0.14	0.09			0.15	0.10		
High Risk Aversion	0.60	0.63			0.60	0.63		
Medium-High Risk Aversion	0.13	0.15			0.14	0.15		
Medium-Low Risk Aversion	0.10	0.10			0.10	0.10		
Low Risk Aversion	0.14	0.10			0.14	0.10		
Plan for Next Few Months	0.13	0.15			0.12	0.13		
Plan for Next Year	0.10	0.11			0.09	0.10		
Plan for Next Few Years	0.29	0.31			0.30	0.31		
Plan for 5-10 Years	0.35	0.32			0.36	0.33		
Plan for 10+ Years	0.12	0.11			0.12	0.11		
Word Recall	0.58	0.65	0.59	0.71	0.60	0.66	0.71	0.71
Pr(Live to 75)	0.64	0.67	0.70	0.75	0.65	0.68	0.70	0.75
Working	0.68	0.59			0.69	0.61		
Retired	0.25	0.11			0.26	0.11		
Unemployed	0.02	0.02			0.02	0.02		
Disabled	0.09	0.06			0.07	0.05		
Series 7 Correct	2.44	2.26	3	2	2.54	2.36	3	2

Table 2 (cont'd).

Current Income	$56,086	$43,750	$59,317	$46,498
Log(Current Income)	10.58	10.69	10.68	10.75
Household Size	2.16	2	2.16	2
Permanent Income	$56,914	$46,670	$59,988	$49,218
Permanent Log Income	10.64	10.72	10.73	10.77
Net Worth	$304,024	$140,111	$330,922	$159,045
Financial Wealth	$65,593	$10,887	$73,111	$15,200
Stocks			0.25	
Bonds			0.02	
CDs			0.09	
Checking			0.63	
Observations	**18,603**		**16,690**	
Households	**6,608**		**6,048**	

Note: Reported Wealth and Income measures are in 1992 dollars.

Table 3. Average Wealth and Proportion of Financial Wealth Allocated by Pr(Live to 75), Single Households

	Net Worth	**Financial Wealth**	**Stocks**	**Bonds**	**CDs**	**Checking**	**Fraction of Sample**
$0 \le P75 < 0.1$	$41,804	$7,063	0.08	0.01	0.06	0.86	0.0594
$0.1 \le P75 < 0.2$	$70,412	$14,898	0.13	0.01	0.07	0.79	0.0282
$0.2 \le P75 < 0.3$	$107,678	$24,816	0.16	0.01	0.08	0.75	0.0355
$0.3 \le P75 < 0.4$	$103,123	$17,245	0.13	0.02	0.09	0.76	0.0210
$0.4 \le P75 < 0.5$	$129,572	$35,292	0.14	0.03	0.08	0.75	0.0217
$0.5 \le P75 < 0.6$	$116,788	$28,681	0.15	0.01	0.09	0.75	0.2487
$0.6 \le P75 < 0.7$	$176,562	$57,327	0.19	0.02	0.08	0.70	0.0824
$0.7 \le P75 < 0.8$	$177,824	$44,265	0.21	0.02	0.10	0.68	0.0760
$0.8 \le P75 < 0.9$	$185,234	$51,814	0.21	0.02	0.09	0.68	0.2026
$0.9 \le P75 < 1$	$347,801	$168,255	0.24	0.02	0.07	0.67	0.0162
$P75 = 1$	$111,057	$26,632	0.15	0.01	0.01	0.77	0.2085

Note: Wealth figures are in 1992 dollars.

Table 4. Average Wealth and Proportion of Financial Wealth Allocated by Husband's Pr(Live to 75), Married Households

	Net Worth	**Financial Wealth**	**Stocks**	**Bonds**	**CDs**	**Checking**	**Fraction of Sample**
$0 \le P75 < 0.1$	$150,509	$30,026	0.17	0.01	0.09	0.73	0.0524
$0.1 \le P75 < 0.2$	$195,464	$31,120	0.18	0.01	0.07	0.73	0.0255
$0.2 \le P75 < 0.3$	$226,670	$48,087	0.21	0.02	0.1	0.67	0.0367
$0.3 \le P75 < 0.4$	$200,316	$41,090	0.21	0.02	0.11	0.67	0.0259
$0.4 \le P75 < 0.5$	$238,363	$45,413	0.22	0.02	0.09	0.68	0.0241
$0.5 \le P75 < 0.6$	$273,417	$57,287	0.22	0.02	0.1	0.66	0.2490
$0.6 \le P75 < 0.7$	$336,502	$80,045	0.3	0.03	0.09	0.58	0.1067
$0.7 \le P75 < 0.8$	$439,344	$116,920	0.31	0.03	0.09	0.57	0.0795
$0.8 \le P75 < 0.9$	$364,742	$76,434	0.3	0.03	0.09	0.59	0.2026
$0.9 \le P75 < 1$	$404,967	$91,239	0.39	0.03	0.05	0.53	0.0143
$P75 = 1$	$306,326	$61,103	0.23	0.02	0.09	0.65	0.1833

Note: Wealth figures are in 1992 dollars.

Table 5. Average Wealth and Proportion of Financial Wealth Allocated by Wife's Pr(Live to 75), Married Households

	Net Worth	**Financial Wealth**	**Stocks**	**Bonds**	**CDs**	**Checking**	**Fraction of Sample**
$0 \le P75 < 0.1$	$118,553	$18,608	0.14	0.01	0.08	0.77	0.0395
$0.1 \le P75 < 0.2$	$180,629	$48,247	0.18	0.01	0.1	0.71	0.0174
$0.2 \le P75 < 0.3$	$177,559	$33,642	0.16	0.01	0.1	0.74	0.0280
$0.3 \le P75 < 0.4$	$183,728	$44,373	0.22	0.01	0.11	0.66	0.0176
$0.4 \le P75 < 0.5$	$159,264	$28,859	0.19	0.02	0.12	0.68	0.0200
$0.5 \le P75 < 0.6$	$264,519	$58,833	0.22	0.02	0.1	0.66	0.2395
$0.6 \le P75 < 0.7$	$307,643	$68,165	0.26	0.02	0.1	0.62	0.0954
$0.7 \le P75 < 0.8$	$434,637	$97,380	0.31	0.03	0.08	0.58	0.0841
$0.8 \le P75 < 0.9$	$370,027	$83,681	0.3	0.03	0.09	0.59	0.2436
$0.9 \le P75 < 1$	$450,742	$115,980	0.35	0.03	0.07	0.55	0.0205
$P75 = 1$	$310,064	$57,644	0.25	0.02	0.09	0.64	0.1944

Note: Wealth figures are in 1992 dollars.

Table 6. Wealth Holding Estimation Results, Single Households

	Net Worth	Financial Wealth
Age	-51,213 (29,894)*	-14,279 (13,684)
Age^2	1,153 (590)*	350 (276)
Age^3	-8 (4)**	-3 (2)
Black	-45,779 (17,874)**	-5,396 (6,116)
White	19,166 (18,174)	17,318 (6,211)**
Hispanic	-23,964 (18,216)	-625 (6,337)
High School	17,680 (8,268)**	8,859 (2,833)**
Some College	58,107 (12,235)**	23,101 (4,975)**
College	111,375 (23,695)**	50,635 (10,958)**
Post Grad	165,583 (28,868)**	79,858 (13,114)**
Male	25,972 (10,429)**	12,850 (4,692)**
Divorced/Separated	-2,113 (10,033)	-3,394 (4,784)
Widowed	46,520 (11,615)**	9,966 (5,101)*
Working	248 (13,335)	7,630 (8,387)
Retired	36,706 (10,668)**	16,949 (5,008)**
Unemployed	3,935 (21,743)	-5,174 (6,628)
Household Size	-4,772 (1,845)**	-2,668 (769)**
Word Recall	26,900 (12,737)**	9,479 (4,822)**
Series 7 Correct	4,466 (1,924)**	2,328 (856)**
Medium-High Risk Aversion	6,102 (12,438)	-1,879 (3,570)
Medium-Low Risk Aversion	2,723 (15,650)	2,850 (8,821)

Table 6 (cont'd).

Low Risk Aversion	29,291 (13,970)**	9,374 (5,480)*
Plan for Next Year	6,775 (10,052)	860 (4,315)
Plan for Next Few Years	26,510 (6,049)**	9,814 (2,362)**
Plan for 5-10 Years	50,030 (9,069)**	22,221 (4,045)**
Plan for 10+ Years	94,892 (15,926)**	38,520 (8,780)**
Pr(Live to 75)	256 (95)**	124 (44)**
Current Income	-25,613 (9,518)**	-23,819 (6,887)**
Current Income^2	330 (153)**	228 (97)**
Permanent Income	31,176 (7,194)**	11,922 (3,949)**
Permanent Income^2	267 (62)**	108 (35)**
Current Income*Permanent Income	-74 (301)	19 (185)
Observations	**14,275**	
Households	**5,022**	

Note: Robust standard errors reported in parentheses. ** p<0.05 and * p<0.1. A full set of year dummies is included in all models. Reported coefficients for Pr(Live to 75) are for a one-percentage point difference (0.01).

Table 7. Average Partial Effects (Reported as Percentages) for Allocation of Financial Wealth , Single Households, Estimated by Single Equation Fractional Probit

	Stocks	**Bonds**	**CDs**	**Checking**
Age	-0.117 (0.092)	0.002 (0.022)	0.117 (0.067)*	-0.027 (0.105)
Black	-2.054 (2.616)	-0.167 (0.688)	0.989 (2.086)	0.787 (3.103)
White	3.830 (2.387)	0.360 (0.594)	1.555 (1.781)	-5.819 (2.819)**
Hispanic	0.676 (3.269)	0.168 (0.907)	-0.390 (2.187)	-0.421 (3.635)
High School	4.729 (1.511)**	0.830 (0.462)*	0.806 (0.994)	-4.637 (1.563)**

Table 7 (cont'd).

Some College	9.291 (1.711)**	-0.038 (0.399)	-0.609 (1.077)	-6.789 (1.740)**
College	11.068 (2.203)**	0.911 (0.639)	0.396 (1.330)	-10.372 (2.192)**
Post Grad	10.005 (2.164)**	1.019 (0.595)*	-0.705 (1.251)	-9.451 (2.167)**
Male	0.257 (0.961)	0.005 (0.231)	-1.227 (0.656)*	0.866 (1.089)
Divorced/Separated	-0.820 (1.257)	0.227 (0.285)	-0.451 (0.926)	1.319 (1.420)
Widowed	-0.349 (1.373)	0.405 (0.326)	-0.970 (0.986)	1.263 (1.537)
Working	0.642 (1.131)	-0.086 (0.296)	0.742 (0.848)	-0.601 (1.314)
Retired	1.915 (1.201)	0.710 (0.377)*	0.768 (0.932)	-2.906 (1.378)**
Unemployed	-1.392 (2.081)	1.511 (1.021)	2.082 (1.738)	-1.538 (2.533)
Household Size	-0.783 (0.400)*	-0.199 (0.127)	-0.686 (0.295)**	1.617 (0.451)**
Word Recall	1.630 (1.800)	-0.255 (0.441)	-2.310 (1.273)*	1.139 (2.053)
Series 7 Correct	0.116 (0.284)	0.065 (0.090)	-0.189 (0.214)	-0.033 (0.328)
Medium-High Risk Aversion	-1.369 (1.113)	-0.125 (0.275)	-0.012 (0.802)	1.499 (1.265)
Medium-Low Risk Aversion	1.450 (1.269)	0.193 (0.330)	-1.427 (0.806)*	-0.069 (1.431)
Low Risk Aversion	2.607 (1.114)**	0.372 (0.341)	-3.175 (0.739)**	-0.146 (1.277)
Plan for Next Year	0.291 (1.464)	0.699 (0.539)	1.885 (1.133)**	-2.390 (1.650)
Plan for Next Few Years	4.222 (1.171)**	0.459 (0.363)	1.920 (0.860)	-5.943 (1.303)**
Plan for 5-10 Years	5.163 (1.203)**	0.317 (0.353)	1.005 (0.829)	-6.049 (1.307)**
Plan for 10+ Years	5.750 (1.499)**	1.134 (0.544)**	-0.973 (0.980)	-6.096 (1.619)**
Pr(Live to 75)	0.0133 (0.0136)	-0.0046 (0.0035)	-0.0037 (0.0093)	-0.0037 (0.0154)
Log(Current Income)	0.539 (0.440)	-0.010 (0.139)	0.067 (0.283)	-0.620 (0.508)
Permanent Log Income	1.199 (0.756)	0.240 (0.214)	-1.145 (0.541)**	-0.391 (0.873)

Table 7 (cont'd)

Log(Net Worth)	5.031 (0.319)**	0.693 (0.129)**	1.900 (0.165)**	-6.884 (0.329)**
Observations		**10,573**		
Households		**4,053**		

Note: Average partial effects reported. Robust standard errors are in parentheses. ** p<0.05 and * p<0.1. A full set of year dummies is included in all models. Reported Average Partial Effects for Pr(Live to 75) are for a one-percentage point difference (0.01).

Table 8. Wealth Holding Estimation Reults, Married Households

	Net Worth		**Financial Wealth**	
	Husband	**Wife**	**Husband**	**Wife**
Age	-81,003 (37,969)**	-122,988 (54,066)**	-10,009 (14,784)	-4,413 (14,993)
Age^2	1,953 (738)**	2,480 (1,071)**	262 (293)	118 (323)
Age^3	-14 (5)**	-16 (7)**	-2 (2)	-1 (2)
Black	-98,615 (38,207)**	18,022 (33,643)	-32,990 (18,756)*	-6,577 (18,957)
White	6,275 (36,598)	32,419 (32,772)	322 (17,389)	-13,212 (19,586)
Hispanic	-51,752 (38,673)	51,415 (37,185)	-6,556 (16,973)	-16,710 (19,363)
High School	-11,679 (14,385)	21,975 (13,271)*	-3,068 (3,803)	5,762 (3,783)
Some College	-27,431 (20,164)	55,406 (23,091)**	-1,573 (5,098)	14,839 (5,294)**
College	54,901 (39,712)	84,406 (36,931)**	37,505 (12,501)**	38,063 (15,272)**
Post Grad	8,167 (36,482)	67,598 (38,935)*	30,609 (10,834)**	41,897 (13,844)**
Medium-High Risk Aversion	-18,926 (19,420)	-2,707 (17,566)	4,303 (7,331)	-300 (6,772)
Medium-Low Risk Aversion	-48,768 (18,796)**	23,058 (21,725)	-6,619 (5,551)	3,323 (7,363)
Low Risk Aversion	-7,599 (18,355)	46,826 (27,255)*	-1,033 (6,077)	-240 (6,792)
Plan for Next Year	-35,539 (21,740)	-12,209 (16,122)	-6,624 (9,458)	-785 (5,215)
Plan for Next Few Years	-21,460 (20,062)	20,647 (15,212)	-7,968 (7,436)	9,455 (5,234)*

Table 8 (cont'd)

Plan for 5-10 Years	-25,427 (21,058)	14,505 (17,162)	-8,661 (7,321)	7,219 (4,579)
Plan for 10+ Years	49,614 (27,072)*	59,879 (23,574)**	24,764 (10,742)**	27,512 (8,563)**
Word Recall	-14,951 (37,053)	-15,527 (50,025)	-13,304 (12,258)	4,178 (9,782)
Series 7 Correct	14,161 (4,998)**	821 (5,921)	4,867 (1,646)**	2,327 (1,644)
Household Size	-11,193 (5,887)*		-4,319 (2,196)**	
Working	-43,525 (13,399)**	-98,558 (17,784)**	-13,889 (5,442)**	-31,661 (6,289)**
Unemployed	-64,020 (20,379)**	-80,664 (22,084)**	-19,344 (6,320)**	-18,303 (8,064)**
Retired	15,281 (15,312)	19,732 (24,190)	11,103 (5,732)*	3,034 (9,127)
Pr(Live to 75)	241 (203)	425 (193)**	28 (62)	-22 (65)
Current Income	11,292 (5,504)**		-2,335 (1,303)*	
Current Income^2	21 (138)		132 (20)**	
Permanent Income	46,914 (6,340)**		13,071 (1,764)**	
Permanent Income^2	-281 (117)**		-55 (19)**	
Current Income*Permanent Income	83 (381)		-145 (57)**	
Observations	**18,603**			
Households	**6,608**			

Note: Robust standard errors reported in parentheses. ** p<0.05 and * p<0.1. A full set of year dummies are included in all models. Reported coefficients for Pr(Live to 75) are for a one-percentage point difference (0.01).

Table 9. Average Partial Effects (Reported as Percentages) for Allocation of Financial Wealth, Married Households, Estimated by Single Equation Fractional Probit

	Stocks		**Bonds**		**CDs**		**Checking**	
	Husband	**Wife**	**Husband**	**Wife**	**Husband**	**Wife**	**Husband**	**Wife**
Age	-0.175 (0.091)*	0.167 (0.079)**	0.041 (0.027)	0.029 (0.022)	0.128 (0.055)**	0.077 (0.047)	-0.012 (0.098)	-0.273 (0.085)**
Black	-2.420 (4.658)	-5.679 (4.267)	-0.299 (0.769)	1.241 (1.126)	-1.809 (2.718)	2.880 (3.423)	4.746 (4.736)	1.714 (4.734)
White	-0.713 (3.121)	-0.592 (2.884)	-0.065 (0.881)	0.688 (0.636)	1.372 (2.121)	-0.726 (1.936)	-0.762 (3.292)	0.572 (3.028)
Hispanic	-3.732 (3.390)	-6.245 (3.064)**	-0.751 (0.803)	0.179 (1.055)	0.337 (2.607)	-2.656 (1.765)	3.419 (3.761)	8.234 (3.366)**
High School	4.215 (1.307)**	5.521 (1.364)**	0.663 (0.456)	0.989 (0.488)**	0.994 (0.804)	2.955 (0.855)**	-4.253 (1.358)**	-8.104 (1.365)**
Some College	8.674 (1.502)**	5.378 (1.539)**	0.957 (0.518)*	0.869 (0.559)	0.016 (0.860)	2.447 (1.017)**	-7.612 (1.544)**	-7.538 (1.578)**
College	8.929 (1.718)**	8.163 (1.955)**	1.854 (0.667)**	1.939 (0.785)**	-1.858 (0.899)**	2.275 (1.276)*	-7.693 (1.749)**	-11.581 (1.983)**
Post Grad	6.747 (1.745)**	7.056 (2.012)**	1.863 (0.687)**	1.742 (0.761)**	-0.282 (0.998)	0.837 (1.185)	-6.994 (1.845)**	-9.431 (2.096)**
Medium-High Risk Aversion	1.471 (0.974)	-0.764 (0.911)	0.306 (0.276)	0.170 (0.268)	0.470 (0.611)	0.108 (0.577)	-2.204 (1.061)**	0.373 (0.994)
Medium-Low Risk Aversion	1.683 (1.144)	-0.637 (1.053)	-0.218 (0.315)	-0.084 (0.308)	-0.788 (0.676)	-1.254 (0.646)*	-0.568 (1.291)	1.769 (1.194)
Low Risk Aversion	2.788 (1.039)**	-0.053 (1.144)	0.161 (0.288)	0.018 (0.326)	-1.622 (0.604)**	-0.418 (0.686)	-1.599 (1.120)	0.230 (1.257)

Table 9 (cont'd).

Plan for Next Year	0.270 (1.494)	-1.711 (1.340)	-0.326 (0.386)	-0.285 (0.383)	0.421 (0.931)	0.095 (0.870)	-0.053 (1.568)	1.885 (1.462)
Plan for Next Few Years	2.076 (1.189)*	-1.620 (1.107)	-0.159 (0.345)	0.033 (0.359)	1.204 (0.749)	-0.004 (0.681)	-2.892 (1.256)**	1.661 (1.180)
Plan for 5-10 Years	1.996 (1.157)*	-0.153 (1.124)	-0.124 (0.337)	-0.084 (0.351)	0.292 (0.716)	0.090 (0.700)	-1.986 (1.229)	0.224 (1.202)
Plan for 10+ Years	3.167 (1.434)**	-0.772 (1.297)	-0.448 (0.342)	0.184 (0.415)	1.164 (0.913)	0.967 (0.883)	-4.121 (1.506)**	-0.460 (1.434)
Word Recall	4.500 (1.753)**	1.829 (1.703)	-0.404 (0.542)	0.393 (0.579)	-0.006 (1.178)	-0.319 (1.139)	-3.745 (1.942)*	-1.550 (1.899)
Series 7 Correct	0.624 (0.319)*	0.374 (0.278)	-0.020 (0.101)	-0.081 (0.091)	-0.035 (0.195)	0.242 (0.188)	-0.674 (0.334)**	-0.559 (0.302)*
Household Size	-0.827 (0.426)*		-0.031 (0.114)		0.395 (0.279)		0.499 (0.462)	
Working	0.247 (1.024)	0.218 (0.801)	-0.101 (0.289)	-0.194 (0.228)	0.195 (0.664)	0.636 (0.512)	0.104 (1.139)	-0.101 (0.871)
Unemployed	0.833 (2.332)	0.647 (2.253)	-0.742 (0.511)	0.229 (0.827)	-1.642 (1.352)	0.213 (1.466)	1.748 (2.478)	-0.980 (2.428)
Retired	2.846 (1.052)**	0.984 (1.090)	0.315 (0.311)	-0.044 (0.294)	1.188 (0.665)*	-0.141 (0.651)	-3.879 (1.145)**	-0.726 (1.178)
Pr(Live to 75)	0.0262 (0.0121)**	0.0224 (0.0131)*	0.0031 (0.0036)	0.0036 (0.0040)	-0.0130 (0.0075)*	-0.0189 (0.0081)**	-0.0146 (0.0131)	-0.0037 (0.0139)
Log(Current Income)	1.055 (0.492)**		0.123 (0.156)		0.300 (0.300)		-1.531 (0.549)**	
Permanent Log Income	2.300 (0.813)**		0.050 (0.222)		-1.209 (0.458)**		-1.359 (0.885)	

Table 9 (cont'd).

Log(Net Worth)	6.670 (0.389)**	0.968 (0.112)**	1.256 (0.159)**	-8.115 (0.400)**
Observations		**16,690**		
Households		**6,048**		

Note: Average partial effects reported. Robust standard errors are in parentheses. ** p<0.05 and * p<0.1. A full set of year dummies included in all models. Reported Average Partial Effect for Pr(Live to 75) is for a one-percentage point difference (0.01).

Table 10. Sensitivity Analysis, Removing Permanent Income, Single and Married Households, Wealth Holding and Asset Allocation

Single Households

Wealth Holdings

	Net Worth	Financial Wealth
Pr(Live to 75)	339 (100)**	156 (50)**

Asset Allocation

	Stocks	Bonds	CDs	Checking
Pr(Live to 75)	0.0142 (0.0136)	-0.0044 (0.0035)	-0.0046 (0.0093)	-0.0040 (0.0154)

Married Households

Wealth Holdings

	Net Worth	Financial Wealth
Husband Pr(Live to 75)	356 (208)*	66 (63)
Wife Pr(Live to75)	522 (200)**	12 (66)

Asset Allocation

	Stocks	Bonds	CDs	Checking
Husband Pr(Live to 75)	0.0274 (0.0121)**	0.0032 (0.0036)	-0.0135 (0.0075)*	-0.0152 (0.0131)
Wife Pr(Live to75)	0.0230 (0.0131)*	0.0036 (0.0040)	-0.0193 (0.0081)**	-0.0040 (0.0139)

Note: We remove all variables that include our measure of Permanent Income. ** p<0.05, * p<0.10. Average Partial Effects (reported as percentages are shown for Stocks, Bonds, CDs, and Checking. Reported coefficients and Average Partial Effects for Pr(Live to 75) are for a one-percentage point difference (0.01).

Table 11. Sensitivity Analysis, Treating Pr(Live to75)=1 differently, Single Households, Wealth Holding and Asset Allocation

	Treatment 1: Removing Observations	**Treatment 2: Removing Households**
Net Worth		
Pr(Live to 75)	394 (124)**	466 (141)**
Financial Wealth		
Pr(Live to 75)	182 (54)**	143 (43)**
Observations	**11,176**	**8,022**
Households	**4,461**	**3,241**
Stocks		
Pr(Live to 75)	0.0320 (0.0174)*	0.0501 (0.0209)**
Bonds		
Pr(Live to 75)	-0.0022 (0.0046)	0.0016 (0.0059)
CDs		
Pr(Live to 75)	0.0043 (0.0117)	-0.0019 (0.0139)
Checking		
Pr(Live to 75)	-0.0351 (0.0195)*	-0.0473 (0.0231)**
Observations	**8,369**	**6,117**
Households	**3,566**	**2,588**

Note: In Treatment 1 we drop observations where the household reports a survival probability of one. In Treatment 2 we drop households that ever reported a survival probability of one. ** p<0.05, * p<0.10. Average Partial Effects (reported as percentages) are shown for Stocks, Bonds, CDs, and Checking. Reported coefficients and Average Partial Effects for Pr(Live to 75) are for a one-percentage point difference (0.01).

Table 12. Sensitivity Analysis, Treating Pr(Live to 75)=1 differently, Married Households, Wealth Holding and Asset Allocation

	Treatment 1: Removing Observations	Treatment 2: Removing Households
Net Worth		
Husband Pr(Live to 75)	56 (253)	-171 (314)
Wife Pr(Live to 75)	698 (253)**	497 (337)
Financial Wealth		
Husband Pr(Live to 75)	-5 (81)	13 (98)
Wife Pr(Live to 75)	129 (97)	215 (141)
Observations	**12,436**	**7,610**
Households	**5,272**	**3,252**
Stocks		
Husband Pr(Live to 75)	0.0359 (0.0161)**	0.0288 (0.0203)
Wife Pr(Live to 75)	0.0326 (0.0170)*	0.0602 (0.0208)**
Bonds		
Husband Pr(Live to 75)	0.0053 (0.0047)	0.0121 (0.0061)**
Wife Pr(Live to 75)	0.0012 (0.0053)	0.0055 (0.0067)
CDs		
Husband Pr(Live to 75)	-0.0212 (0.0101)**	-0.0249 (0.0126)**
Wife Pr(Live to 75)	-0.0262 (0.0106)**	-0.0363 (0.0132)**
Checking		
Husband Pr(Live to 75)	-0.0002 (0.0002)	-0.0123 (0.0220)
Wife Pr(Live to 75)	-0.0039 (0.0180)	-0.0253 (0.0225)
Observations	**11,183**	**6,753**
Households	**4,812**	**2,912**

Note: In Treatment 1 we drop observations where any member of the household reports a survival probability of one. In Treatment 2 we drop households where any member ever reported a survival probability of one. ** p<0.05, * p<0.10. Average Partial Effects (reported as percentages) are shown for Stocks, Bonds, CDs, and Checking. Reported coefficients and Average Partial Effects for Pr(Live to 75) are for a one-percentage point difference (0.01).

Table 13. Means and Medians of Relevant Variables

	Mean		Median	
	Husband	**Wife**	**Husband**	**Wife**
Pr(Live to 75)	0.65	0.69	0.70	0.75
Age	58	55	59	55
Word Recall	0.61	0.67	0.71	0.71
Income	$59,578		$47,000	
Non-Financial Wealth	$268,579		$131,590	
Allocation to Stocks	0.26		-	
Allocation to Bonds	0.02		-	
Allocation to CDs	0.09		-	
Allocation to Checking	0.62		-	
Observations	**14,514**			
Households	**3,872**			

Note: All dollar values are stated in 1992 dollars. In the right panel all households with only one time observation are dropped

Table 14. Proportion of Variation Between Households for Relevant Variables

	Husband	**Wife**
	Independent Variables	
Pr(Live to 75)	0.66	0.65
Age	0.59	0.76
Word Recall	0.43	0.40
Income	0.65	
Non-Financial Wealth	0.82	
	Dependent Variables	
Allocation to Stocks	0.61	
Allocation to Bonds	0.42	
Allocation to CDs	0.52	
Allocation to Checking	0.61	
Observations	**14,514**	
Households	**3,872**	

Note: Results obtained by running an ANOVA using the Household as the categorical variable.

Table 15. Average Partial Effects from Pooled Linear Fixed Effects OLS Estimation on Entire Unbalanced Panel

	Stocks	Bonds	CDs	Checking
Husband Age	-0.001	0.004	-0.008	0.005
	(0.007)	(0.003)	(0.005)	(0.008)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.000
	(0.001)	(0.001)	(0.001)	(0.002)
Husband Word Recall	0.006	-0.000	0.002	-0.008
	(0.002)**	(0.001)	(0.001)	(0.002)**
Wife Age	-0.004	-0.003	0.009	-0.002
	(0.007)	(0.003)	(0.005)*	(0.008)
Wife Pr(Live to 75)	-0.003	0.001	0.000	0.002
	(0.001)**	(0.001)**	(0.001)	(0.002)
Wife Word Recall	0.002	0.001	-0.002	-0.001
	(0.001)	(0.001)	(0.001)*	(0.002)
Log(Income)	0.011	0.001	0.004	-0.017
	(0.005)**	(0.002)	(0.003)	(0.005)**
Log(Non-Financial Wealth)	-0.007	0.001	0.001	0.005
	(0.004)*	(0.002)	(0.003)	(0.005)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 16. Average Partial Effects from Single Equation Pooled Bernoulli Quasi-Maximum Likelihood Estimates for Each Balanced Panel Subset, Averaged Across Balanced Panel Subsets

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.001	0.004	-0.007	0.004
	(0.019)	(0.006)	(0.012)	(0.020)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.001
	(0.004)	(0.001)	(0.002)	(0.004)
Husband Word Recall	0.004	0.000	0.002	-0.006
	(0.004)	(0.002)	(0.003)	(0.005)
Wife Age	-0.003	-0.002	0.009	-0.005
	(0.019)	(0.006)	(0.012)	(0.020)
Wife Pr(Live to 75)	-0.003	0.001	0.000	0.002
	(0.004)	(0.001)	(0.002)	(0.004)
Wife Word Recall	0.001	0.001	-0.002	0.000
	(0.004)	(0.001)	(0.003)	(0.005)
Log(Income)	0.012	0.002	0.003	-0.016
	(0.012)	(0.004)	(0.007)	(0.014)
Log(Non-Financial Wealth)	-0.006	0.002	0.001	0.005
	(0.012)	(0.004)	(0.007)	(0.013)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 17. Average Partial Effects from Pooled Multinomial Quasi-Maximum Likelihood Estimates for Each Balanced Panel Subset, Averaged Across Balanced panel Subsets

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.001	0.004	-0.006	0.003
	(0.019)	(0.006)	(0.012)	(0.020)
Husband Pr(Live to 75)	0.000	-0.001	-0.001	0.001
	(0.004)	(0.001)	(0.002)	(0.004)
Husband Word Recall	0.004	-0.000	0.002	-0.006
	(0.004)	(0.002)	(0.003)	(0.004)
Wife Age	-0.003	-0.002	0.009	-0.004
	(0.019)	(0.006)	(0.012)	(0.020)
Wife Pr(Live to 75)	-0.003	0.001	0.000	0.002
	(0.004)	(0.001)	(0.002)	(0.004)
Wife Word Recall	0.000	0.001	-0.002	0.001
	(0.004)	(0.001)	(0.003)	(0.004)
Log(Income)	0.009	0.001	0.003	-0.013
	(0.012)	(0.004)	(0.007)	(0.012)
Log(Non-Financial Wealth)	-0.006	0.001	0.000	0.005
	(0.012)	(0.004)	(0.007)	(0.013)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 18. Procedure 2: Classical Minimum Distance applied to Multinomial Quasi-Maximum Likelihood Estimates for Each Year Within Balanced Panel Subsets

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	0.027	0.004	-0.005	-0.026
	(0.013)**	(0.003)	(0.007)	(0.013)**
Husband Pr(Live to 75)	0.005	-0.000	0.000	-0.005
	(0.002)*	(0.001)	(0.002)	(0.003)*
Husband Word Recall	0.009	-0.000	-0.000	-0.008
	(0.003)**	(0.001)	(0.002)	(0.003)**
Wife Age	-0.026	-0.003	0.003	0.026
	(0.013)**	(0.003)	(0.007)	(0.013)**
Wife Pr(Live to 75)	-0.007	-0.001	0.003	0.005
	(0.003)**	(0.001)	(0.002)	(0.003)*
Wife Word Recall	-0.001	0.000	-0.001	0.002
	(0.003)	(0.001)	(0.002)	(0.003)
Log(Income)	0.023	-0.007	-0.003	-0.013
	(0.008)**	(0.002)**	(0.005)	(0.008)
Log(Non-Financial Wealth)	0.003	0.000	-0.004	0.001
	(0.008)	(0.002)	(0.005)	(0.009)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effects for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 19. Comparing Average Partial Effect Estimates from Pooled Single Equation Bernoulli Quasi-Maximum Likelihood Estimates for Each Balanced Panel Subset, Averaged Across Balanced Panel Subsets to Procedure 2 Average Partial Effect Estimates

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	0.137	0.972	0.913	0.153
Husband Pr(Live to 75)	0.267	0.939	0.602	0.182
Husband Word Recall	0.237	0.762	0.332	0.679
Wife Age	0.228	0.952	0.588	0.117
Wife Pr(Live to 75)	0.348	0.141	0.324	0.393
Wife Word Recall	0.698	0.743	0.890	0.759
Log(Income)	0.344	0.023**	0.370	0.805
Log(Non-Financial Wealth)	0.466	0.637	0.534	0.782

Note: p-values are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Standard errors used to compute the test statistic are from Table 4: Weighted Average of the standard errors for Average Partial Effects estimated by Single Equation Bernoulli Quasi-Maximum Likelihood for each balanced panel subset.

Table 20. Comparing Average Partial Effect Estimates from Pooled Multinomial Quasi-Maximum Likelihood Estimates for Each Balanced Panel Subset, Averaged Across Balanced Panel Subsets to Procedure 2 Estimated Average Partial Effects

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	0.133	0.955	0.935	0.145
Husband Pr(Live to 75)	0.229	0.922	0.519	0.129
Husband Word Recall	0.219	0.815	0.348	0.641
Wife Age	0.219	0.902	0.592	0.122
Wife Pr(Live to 75)	0.342	0.139	0.310	0.428
Wife Word Recall	0.735	0.793	0.918	0.759
Log(Income)	0.241	0.026**	0.410	0.989
Log(Non-Financial Wealth)	0.435	0.758	0.576	0.752

Note: p-values are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Standard errors used to compute the test statistic are from Table 5: Weighted Average of the standard errors for Average Partial Effects estimated by Multinomial Quasi-Maximum Likelihood for each balanced panel subset.

Table 21. Comparing Average Partial Effect Estimates from Pooled Linear Fixed Effects OLS on Entire Unbalanced Panel to Procedure 2 Estimated Average Partial Effect Estimates

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	0.027**	0.985	0.708	0.019**
Husband Pr(Live to 75)	0.116	0.705	0.377	0.053*
Husband Word Recall	0.311	0.776	0.209	0.934
Wife Age	0.079*	0.921	0.391	0.030**
Wife Pr(Live to 75)	0.150	0.005**	0.136	0.224
Wife Word Recall	0.253	0.729	0.632	0.372
Log(Income)	0.132	0.000**	0.110	0.610
Log(Non-Financial Wealth)	0.223	0.552	0.328	0.688

Note: p-values are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Standard errors used to compute the test statistic are from Table 6: Standard errors for Average Partial Effects estimated by Procedure 2 (Classical Minimum Distance applied to Multinomial Quasi-Maximum Likelihood estimates for each year within each balanced panel subset).

Table 22. Means and Medians of Relevant Variables

	Mean		Median	
	Husband	**Wife**	**Husband**	**Wife**
Pr(Live to 75)	0.65	0.69	0.70	0.75
Age	58	55	59	55
Word Recall	0.61	0.67	0.71	0.71
Income	$59,578		$47,000	
Non-Financial Wealth	$268,579		$131,590	
Allocation to Stocks	0.26		-	
Allocation to Bonds	0.02		-	
Allocation to CDs	0.09		-	
Allocation to Checking	0.62		-	
Observations	**14,514**			
Households	**3,872**			

Note: All dollar values are stated in 1992 dollars. In the right panel all households with only one time observation are dropped

Table 23. Proportion of Variation Between Households for Relevant Variables

	Husband	**Wife**
	Independent Variables	
Pr(Live to 75)	0.66	0.65
Age	0.59	0.76
Word Recall	0.43	0.40
Income	0.65	
Non-Financial Wealth	0.82	
	Dependent Variables	
Allocation to Stocks	0.61	
Allocation to Bonds	0.42	
Allocation to CDs	0.52	
Allocation to Checking	0.61	
Observations	**14,514**	
Households	**3,872**	

Note: Results obtained by running an ANOVA using the Household as the categorical variable.

Table 24. Average Partial Effects from Pooled Linear Fixed Effects OLS estimation on entire unbalanced panel

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.001	0.004	-0.008	0.005
	(0.007)	(0.003)	(0.005)	(0.008)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.000
	(0.001)	(0.001)	(0.001)	(0.002)
Husband Word Recall	0.006	-0.000	0.002	-0.008
	(0.002)**	(0.001)	(0.001)	(0.002)**
Wife Age	-0.004	-0.003	0.009	-0.002
	(0.007)	(0.003)	(0.005)*	(0.008)
Wife Pr(Live to 75)	-0.003	0.001	0.000	0.002
	(0.001)**	(0.001)**	(0.001)	(0.002)
Wife Word Recall	0.002	0.001	-0.002	-0.001
	(0.001)	(0.001)	(0.001)*	(0.002)
Log(Income)	0.011	0.001	0.004	-0.017
	(0.005)**	(0.002)	(0.003)	(0.005)**
Log(Non-Financial Wealth)	-0.007	0.001	0.001	0.005
	(0.004)*	(0.002)	(0.003)	(0.005)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 25. Average Partial Effects from Single Equation Pooled Bernoulli Quasi-Maximum Likelihood Estimates on Entire Unabalanced Panel, Ignoring Unbalanced Nature of the Panel

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.001	0.004	-0.008	0.005
	(0.007)	(0.003)	(0.005)	(0.008)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.000
	(0.001)	(0.001)	(0.001)	(0.002)
Husband Word Recall	0.006	-0.000	0.002	-0.008
	(0.002)**	(0.001)	(0.001)*	(0.002)**
Wife Age	-0.004	-0.002	0.008	-0.002
	(0.007)	(0.003)	(0.005)	(0.008)
Wife Pr(Live to 75)	-0.003	0.001	0.000	0.002
	(0.002)**	(0.001)**	(0.001)	(0.002)
Wife Word Recall	0.002	0.001	-0.002	-0.001
	(0.002)	(0.001)	(0.001)	(0.002)
Log(Income)	0.011	0.001	0.004	-0.016
	(0.005)**	(0.002)	(0.003)	(0.005)**
Log(Non-Financial Wealth)	-0.007	0.002	0.001	0.005
	(0.005)	(0.002)	(0.003)	(0.005)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 26. Average Partial Effects from Single Equation Pooled Bernoulli Quasi-Maximum Likelihood Estimates for Each Balanced Panel Subset, Averaged Across Balanced Panel Subsets

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.001	0.004	-0.007	0.004
	(0.019)	(0.006)	(0.012)	(0.020)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.001
	(0.004)	(0.001)	(0.002)	(0.004)
Husband Word Recall	0.004	0.000	0.002	-0.006
	(0.004)	(0.002)	(0.003)	(0.005)
Wife Age	-0.003	-0.002	0.009	-0.005
	(0.019)	(0.006)	(0.012)	(0.020)
Wife Pr(Live to 75)	-0.003	0.001	0.000	0.002
	(0.004)	(0.001)	(0.002)	(0.004)
Wife Word Recall	0.001	0.001	-0.002	0.000
	(0.004)	(0.001)	(0.003)	(0.005)
Log(Income)	0.012	0.002	0.003	-0.016
	(0.012)	(0.004)	(0.007)	(0.014)
Log(Non-Financial Wealth)	-0.006	0.002	0.001	0.005
	(0.012)	(0.004)	(0.007)	(0.013)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 27. Average Partial Effects from Pooled Bernoulli Quasi-Maximum Likelihood Estimates on Entire Unbalanced Panel Adjusting for Unbalanced Nature of the Panel

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	0.000	0.015	-0.007	0.005
	(0.007)	(0.011)	(0.005)	(0.008)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.000
	(0.002)	(0.002)	(0.001)	(0.002)
Husband Word Recall	0.006	-0.001	0.001	-0.008
	(0.002)**	(0.002)	(0.001)	(0.002)**
Wife Age	-0.004	-0.010	0.006	-0.002
	(0.007)	(0.011)	(0.005)	(0.008)
Wife Pr(Live to 75)	-0.004	0.005	0.0001	0.002
	(0.002)**	(0.002)**	(0.001)	(0.002)
Wife Word Recall	0.002	0.003	-0.001	-0.001
	(0.002)	(0.002)	(0.001)	(0.002)
Log(Income)	0.011	0.008	0.004	-0.016
	(0.005)**	(0.007)	(0.003)	(0.005)**
Log(Non-Financial Wealth)	-0.007	0.007	-0.001	0.005
	(0.005)	(0.008)	(0.003)	(0.005)
χ^2 Test Statistic for ψ_{gr}	1.34	6.75	52.16	3.80
p-value	0.931	0.240	0.000**	0.579
χ^2 Test Statistic for ω_{gr}	1.45	4.74	5.26	2.83
p-value	0.919	0.448	0.385	0.726
Observations		**14,514**		
Households		**3,872**		

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 28. Average Partial Effects from Pooled Multinomial Quasi-Maximum Likelihood Estimates for Entire Unbalanced Panel, Ignoring Unbalanced Nature of the Panel

	Stocks	Bonds	CDs	Checking
Husband Age	-0.001	0.004	-0.008	0.005
	(0.007)	(0.002)	(0.005)*	(0.008)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.002)
Husband Word Recall	0.006	-0.000	0.002	-0.007
	(0.002)**	(0.001)	(0.001)*	(0.002)**
Wife Age	-0.003	-0.002	0.009	-0.004
	(0.007)	(0.002)	(0.005)*	(0.008)
Wife Pr(Live to 75)	-0.004	0.001	0.000	0.002
	(0.002)**	(0.001)**	(0.001)	(0.002)
Wife Word Recall	0.002	0.001	-0.002	-0.001
	(0.002)	(0.001)	(0.001)*	(0.002)
Log(Income)	0.008	0.000	0.002	-0.009
	(0.005)*	(0.002)	(0.003)	(0.005)**
Log(Non-Financial Wealth)	-0.008	0.001	0.000	0.007
	(0.005)*	(0.002)	(0.003)	(0.005)
Observations		**14,514**		
Households		**3,872**		

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 29. Average Partial Effects from Pooled Multinomial Quasi-Maximum Likelihood Estimates for Each Balanced Panel Subset, Averaged Across Balanced Panel Subsets

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.001	0.004	-0.006	0.003
	(0.019)	(0.006)	(0.012)	(0.020)
Husband Pr(Live to 75)	0.000	-0.001	-0.001	0.001
	(0.004)	(0.001)	(0.002)	(0.004)
Husband Word Recall	0.004	-0.000	0.002	-0.006
	(0.004)	(0.002)	(0.003)	(0.004)
Wife Age	-0.003	-0.002	0.009	-0.004
	(0.019)	(0.006)	(0.012)	(0.020)
Wife Pr(Live to 75)	-0.003	0.001	0.000	0.002
	(0.004)	(0.001)	(0.002)	(0.004)
Wife Word Recall	0.000	0.001	-0.002	0.001
	(0.004)	(0.001)	(0.003)	(0.004)
Log(Income)	0.009	0.001	0.003	-0.013
	(0.012)	(0.004)	(0.007)	(0.012)
Log(Non-Financial Wealth)	-0.006	0.001	0.000	0.005
	(0.012)	(0.004)	(0.007)	(0.013)
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 30. Average Partial Effects from Multinomial Quasi-Maximum Likelihood Estimates on Entire Unbalanced Panel, Adjusting for the Unbalanced Nature of the Panel

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.001	0.003	-0.006	0.004
	(0.007)	(0.003)	(0.005)	(0.007)
Husband Pr(Live to 75)	0.001	-0.000	-0.001	0.001
	(0.001)	(0.001)	(0.001)	(0.002)
Husband Word Recall	0.006	-0.000	0.001	-0.007
	(0.002)**	(0.001)	(0.001)	(0.002)**
Wife Age	-0.002	-0.002	0.006	-0.002
	(0.007)	(0.003)	(0.005)	(0.008)
Wife Pr(Live to 75)	-0.004	0.001	0.001	0.002
	(0.002)**	(0.001)*	(0.001)	(0.002)
Wife Word Recall	0.001	0.001	-0.001	-0.001
	(0.002)	(0.001)	(0.001)	(0.002)
Log(Income)	0.008	0.001	0.003	-0.012
	(0.005)*	(0.002)	(0.003)	(0.005)**
Log(Non-Financial Wealth)	-0.007	0.001	-0.001	0.007
	(0.005)	(0.002)	(0.003)	(0.005)
χ^2 Test Statistic for ψ_{gr}	2.13	17.40	69.65	-
p-value	0.830	0.004**	0.000**	
χ^2 Test Statistic for ω_{gr}	0.80	11.78	6.35	-
p-value	0.977	0.038**	0.274	
Observations	**14,514**			
Households	**3,872**			

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effect for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Table 31. Average Partial Effects from Multinomial Quasi-Maximum Likelihood Estimates Using the Multinomial Logit Functional Form on Entire Unbalanced Panel, Adjusting for Unbalanced Nature of the Panel

	Stocks	**Bonds**	**CDs**	**Checking**
Husband Age	-0.000 (0.000)	0.003 (0.000)**	-0.008 (0.006)	0.005 (0.007)
Husband Pr(Live to75)	0.001 (0.000)**	-0.000 (0.000)**	-0.001 (0.001)**	0.000 (0.001)
Husband Word Recall	0.006 (0.000)**	-0.000 (0.000)**	0.001 (0.001)*	-0.007 (0.001)**
Wife Age	-0.004 (0.000)**	-0.002 (0.000)**	0.008 (0.008)	-0.001 (0.008)
Wife Pr(Live to 75)	-0.004 (0.000)**	0.001 (0.000)**	0.001 (0.001)	0.002 (0.001)**
Wife Word Recall	0.002 (0.000)**	0.001 (0.000)**	-0.001 (0.001)**	-0.001 (0.001)*
Log(Income)	0.011 (0.000)**	0.002 (0.000)**	0.003 (0.007)	-0.016 (0.009)*
Log(Non-Financial Wealth)	-0.008 (0.000)**	0.002 (0.000)**	-0.000 (0.006)	0.007 (0.008)
χ^2 Test Statistic for ψ_{gr} p-value	2.42 0.789	6.27 0.281	13.91 0.016**	-
χ^2 Test Statistic for ω_{gr} p-value	1.52 0.911	6.75 0.240	4.86 0.434	
Observations		**14,514**		
Households		**3,872**		

Note: Standard errors in parentheses are robust to heteroskedasticity and serial correlation. ** p-value < 0.05, * p-value < 0.10. Reported average partial effects for Husband and Wife Pr(Live to 75) and Husband and Wife Word Recall represent a 10 percentage point change.

Figure 1. Average Structural Function for the Proportion of Financial Wealth Allocated to Stocks Against Husband Word Recall

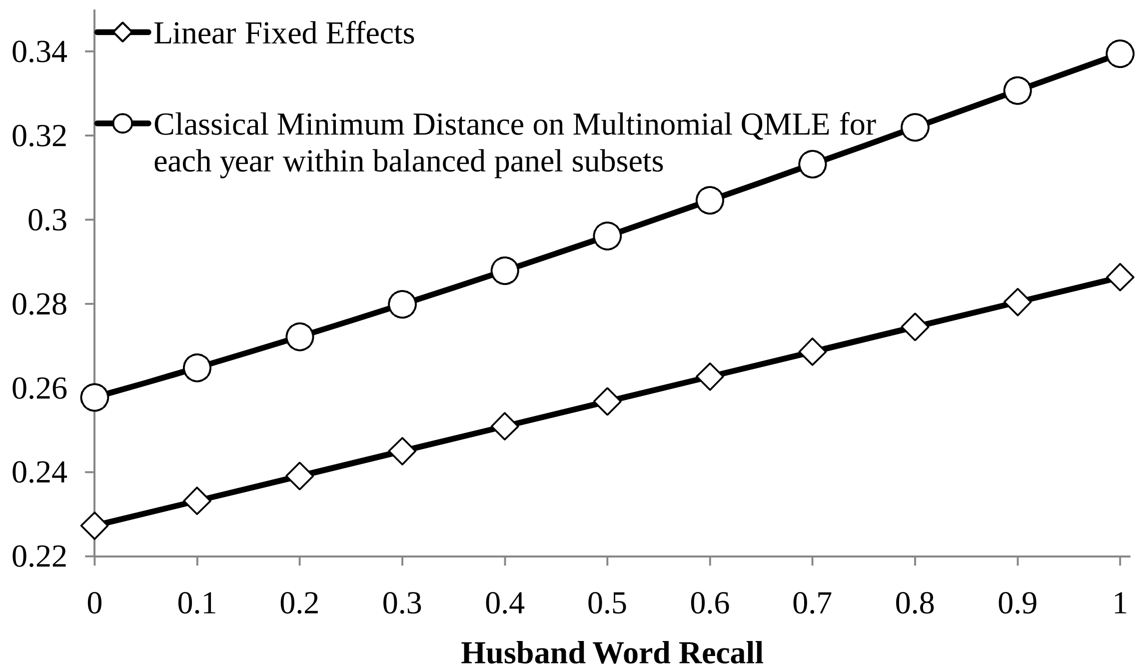


Figure 2. Average Structural Function for the Proportion of Financial Wealth Allocated to Bonds Against Husband Word Recall

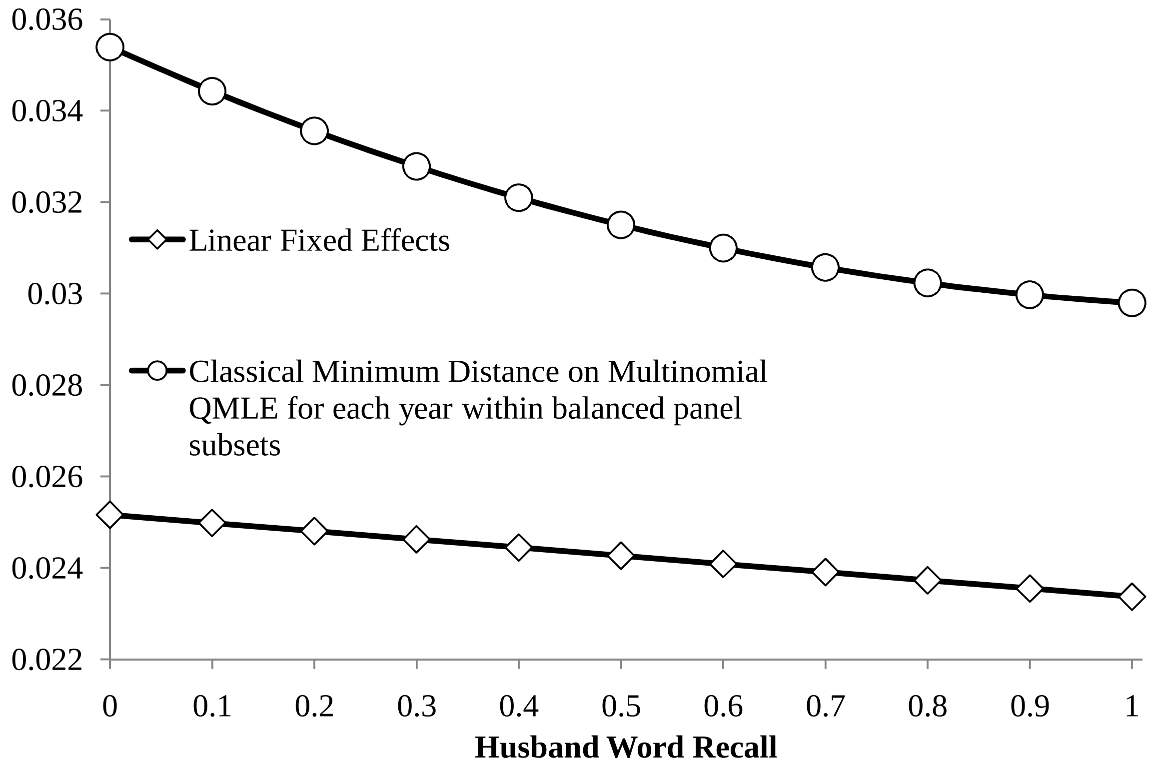


Figure 3. Average Structural Function for the Proportion of Financial Wealth Allocated to CDs Against Husband Word Recall

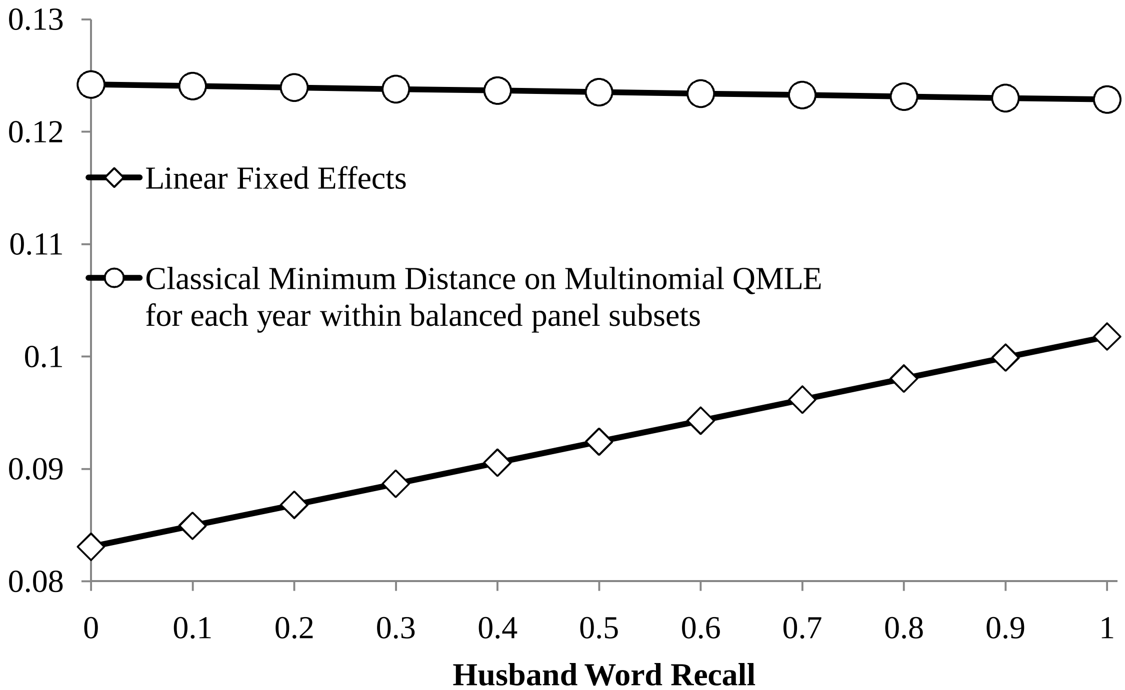


Figure 4. Average Structural Function for the Proportion of Financial Wealth Allocated to Checking Against Husband Word Recall

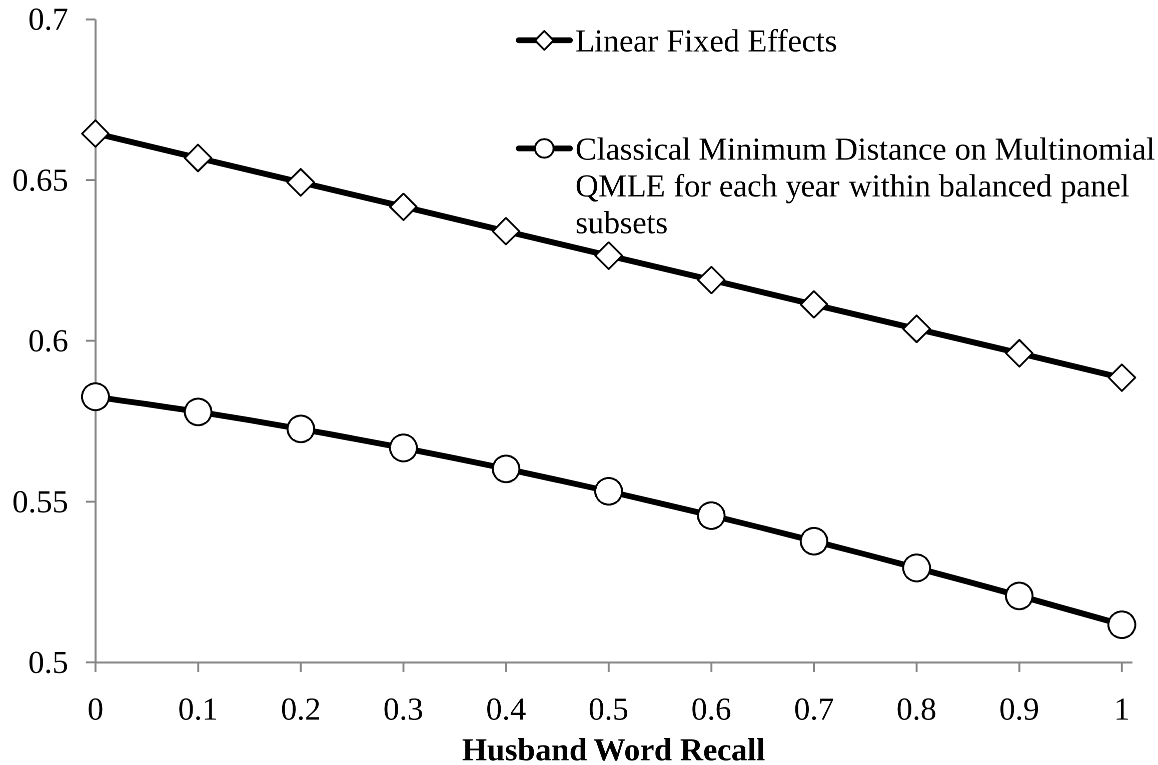


Figure 5. Average Structural Function for the Proportion of Financial Wealth Allocated to Stocks Against Wife Word Recall

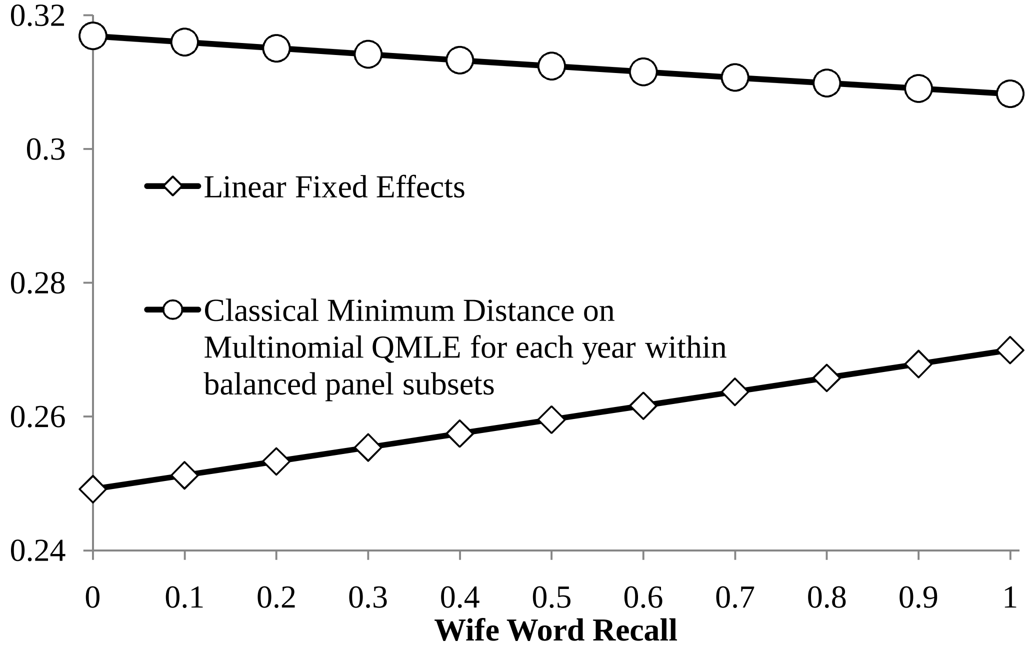


Figure 6. Average Structural Function for the Proportion of Financial Wealth Allocated to Bonds Against Wife Word Recall

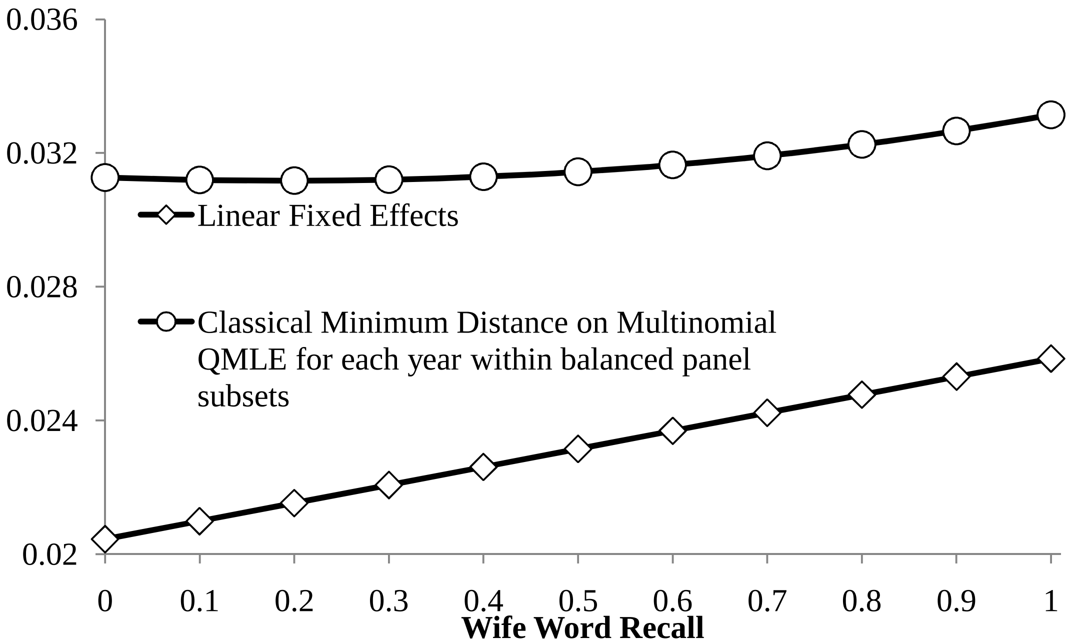


Figure 7. Average Structural Function for Proportion of Financial Wealth Allocated to CDs Against Wife Word Recall

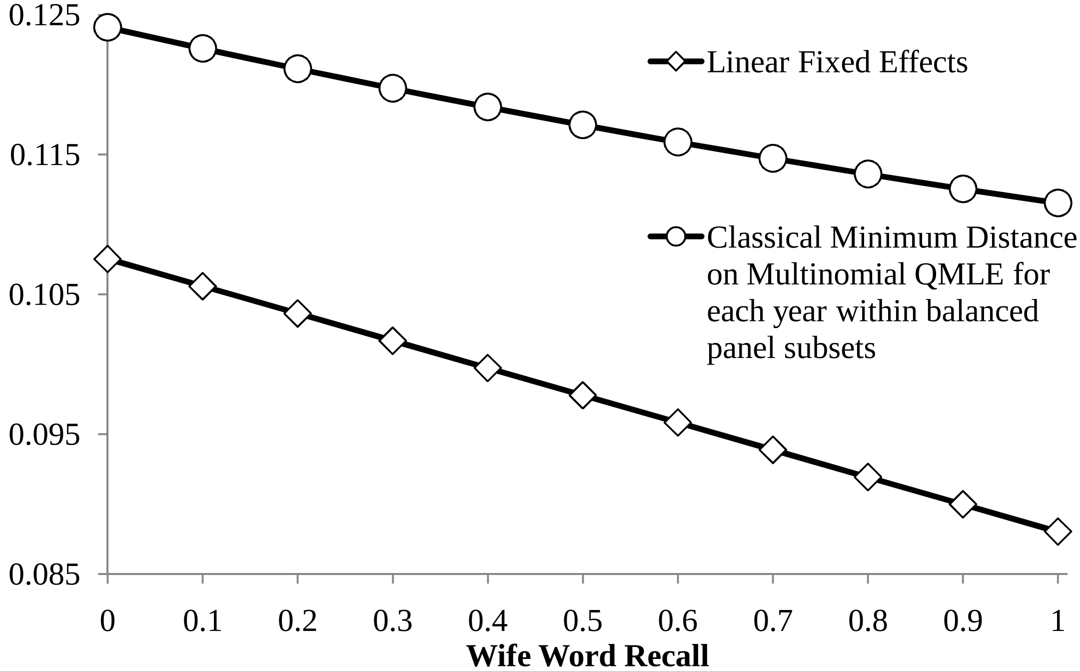
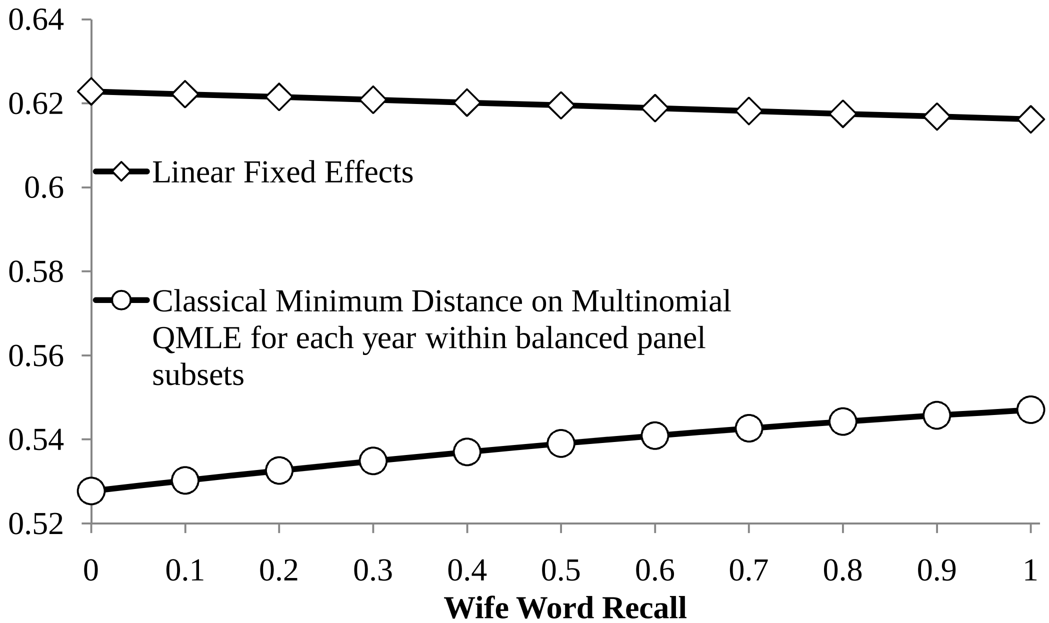


Figure 8. Average Structural Function for Proportion of Financial Wealth Allocated to Checking Against Wife Word Recall



BIBLIOGRAPHY

BIBLIOGRAPHY

Altonji, J.G. and Doraszelski (2005), "The Role of Permanent Income and Demographics in Black/White Differences in Wealth," *Journal of Human Resources*, 40 (1), pp. 1-30.

Barsky, R.B., F.T. Juster, M.S. Kimball and M.D. Shapiro (1997), "Preference Parameters and Behavioral Heterogeneity: An Experiemental Approach in the Health and Retirement Study," *The Quarterly Journal of Economics*, 112 (2), pp. 537-579.

Berkowitz, M.K. and J. Qui (2006), "A Further Look at Household Portfolio Choice and Health Status," *Journal of Banking and Finance*, 30, pp. 1201-1217.

Bernheim, B.D., J. Skinner, and S. Weinberg (2001), "What Accounts for the Variation in Retirement Wealth Among U.S. Households?," *American Economic Review*, 91, pp. 832-857.

Bloom, D.E., D. Canning, M. Moore, and Y. Song (2006), "The Effect of Subjective Survival Probabilities on Retirement and Wealth in the United States," Working Paper 12688, National Bureau of Economic Research.

Blundell, R. and J.L. Powell (2003), "Endogeneity in Nonparametric and Semiparametric Regression Models, with Richard Blundell," in *Advances in Economics and Econometrics: Theory and Applications*, Eighth World Congress, Volume 2, eds. M. Dewatripoint, L.P. Hanses, and S.J. Turnovsky, Cambridge: Cambridge University Press, pp. 312-357.

Chamberlain, G. (1980), "Analysis of Variance with Qualitative Data," *Review of Econometric Studies*, 47, pp. 225-238.

Chamberlain, G. (1982), "Multivariate Regression Models for Panel Data," *Journal of Econometrics*, 18, pp. 5-46.

Chamberlain, G. (1984), "Panel Data," in *Handbook of Econometrics*, Volume 2, ed. Z. Griliches and M.D. Intriligator, Amsterdam: North Holland, pp. 1247-1318.

Cocco, J., F. Gomes, and P. Maenhout (2005), "Consumption and Portfolio Choice over the Life Cycle," *The Review of Financial Studies*, 18 (2), pp. 491-553.

Delevande, A., M. Perry, and R. Willis (2006), "Probabilistic Thinking and Social Security Claiming," Working Paper 2006-129, Michigan Retirement Research Center.

DeNardi, M., E. French, and J.B. Jones (2009), "Life Expectancy and Old Age Savings," *American Economic Review*, 99 (2), pp. 110-115.

Elder, T. (2012), “The Predictive Validity of Subjective Mortality Expectations: Evidence from the Health and Retirement Study,” *Demography*, forthcoming.

Gan, L., G. Gong, M. Hurd, and D. McFadden (2004), “Subjective Mortality Risk and Bequests,” Working Paper 10789, National Bureau of Economic Research.

Gourieroux, C., A. Monfort, and A. Trognon (1984), “Pseudo Maximum Likelihood Methods: Theory,” *Econometrica*, 52, pp. 681-700.

Hurd, M. and K. McGarry (1995), “Evaluation of the Subjective Probabilities of Survival in the Health and Retirement Study,” *Journal of Human Resources,* 30, pp. S268-S292.

Hurd, M. and K. McGarry (2002), “The Predictive Validity of Subjective Probabilities of Survival,” *Economic Journal*, 112, pp. 966-998.

Hurd, M., J. Smith, and J. Zissimopoulous (2002), “The Effects of Subjective Survival on Retirement and Social Security Claiming,” Working Paper 9140, National Bureau of Economic Research.

Kédzi, G. and R. Willis (2003), “Who Becomes a Stockholder? Expectations, Subjective Uncertainty, and Asset Allocation,” Working Paper 2003-039, Michigan Retirement Research Center.

Koch, S.F. (2010), “Fractional Multinomial Response Models with an Application to Expenditure Shares,” Working Paper 2010-21, University of Pretoria.

Lillard, L. and R. Willis (2001), “Cognition and Wealth: The Importance of Probabilistiv Thinking,” Working Paper 2001-007, Michigan Retirement Research Center.

McArdle, J., J. Smith, and R. Willis (2009), “Cognition and Economic Outcomes in the Health and Retirement Survey,” Working Paper 15266, National Bureau of Economic Research.

Mullahy, J. (2010), “Multivariate Fractional Regression Estimation of Econometric Share Models,” Working Paper 16354, National Bureau of Economic Research.

Mundlak, Y. (1978), “On the Pooling of Time Series and Cross Section Data,” *Econometrica*, 58, pp. 809-837.

Papke, L. and J. M. Wooldridge (1996), “Econometric Methods for Fractional Response Variables with an Application to 401(k) Plan Participation Rates,” *Journal of Applied Econometrics,* 11, pp. 619-632.

Papke, L. and J. M. Wooldridge (2008), “Panel Data Methods for Fractional Response Variables with an Application to Test Pass Rates,” *Journal of Econometrics*, 145, pp.121-133.

Perry, M. (2005), “Estimating Life-Cycle Effects of Subjective Survival Probabilities in the Health and Retirement Study,” Working Paper 2005-103, Michigan Retirement Research Center.

Rosen, H. S. and S. Wu (2004), “Portfolio Choice and Health Status,” *Journal of Financial Economics*, 72, pp.457-484.

Sahm, C. (2007), “Risk Tolerance and Asset Allocation,” Dissertation, University of Michigan.

Salm, M. (2006), “Can Subjective Mortality Expectations and Stated Preferences Explain Varying Consumption and Savings Behaviors among the Elderly?,” IZA DP No. 2467, IZA Discussion Paper Series.

Sivakumar, A. and C. Bhat (2002), “Fractional Split-Distribution Model for Statewide Commodity-Flow Analysis,” *Transportation Research Record*, 1790, Paper No. 2142.

Smith, V. K., D. Smith, D.H. Taylor, Jr., and F. Sloan (2001), “Longevity Expectations and Death: Can People Predict Their Own Demise?,” *American Economic Review*, 91, pp. 1126-1134.

Wooldridge, J. M. (2002), *Econometric Analysis of Cross Section and Panel Data*, MIT Press, Cambridge, MA.

Wooldridge, J. M. (2006), “Cluster-Sample Methods in Applied Econometrics: An Extended Analysis,” Unpublished Manuscript, Michigan State University.

Wooldridge, J. M. (2010), “Correlated Random Effects Models with Unbalanced Panels,” Unpublished Manuscript, Michigan State University.

Yaari,M. (1965), “Uncertain Lifetime, Life Insurance and the Theory of the Consumer,” *Review of Economic Studies*, 32, pp. 137-150.