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A MATHEMATICAL MODEL TO PREDICT THE LOSS IN
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DUE TO OFFSET

presented by

Kuen Woong Park

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of the requirements for the

M.S degree in Packaging

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**A MATHEMATICAL MODEL TO PREDICT THE LOSS IN COMPRESSION
STRENGTH OF CORRUGATED BOXES DUE TO OFFSET**

By

Kuen Woong Park

A THESIS

Submitted to
Michigan State University
In partial fulfillment of the requirements
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Abstract

A MATHEMATICAL MODEL TO PREDICT THE LOSS IN COMPRESSION STRENGTH OF CORRUGATED BOXES DUE TO OFFSET

By

Kuen Woong Park

Misaligned boxes in a stack reduce the stacking strength of the boxes. Most industry citations use the results developed in 1963 by McKee to predict the loss in box compression strength. These predictions are based on the assumption that each side of a box contributes about $1/12$ of the total compression strength and each corner contributes $1/6$. Actual tests prove these to be inaccurate. A formula for the loss in compression strength versus amount of offset is developed and fitted to the experimental data. A simplified equation was developed from this. The equation shows that for every 1% offset in any direction, the box loses 2.5% of its compression strength.

I dedicate this thesis to my family for their love.

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Kuen Woong Park

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KEY TO ABBREVIATIONS

ASTM	American Society for Testing and Materials
AVG	Average
CS	Compression Strength
CCS	Control Compression Strength
ECT	Edge crush test
FDA	Food and Drug Administration
L	Length
OCS	Offset Compression Strength
PRED	Predicted
RH	Relative Humidity
RSC	Regular Slotted Container
SSE	Sum of Squares Errors
STDEV	Standard Deviation
TAPPI	Technical Association of the Pulp and Paper Industry
W	Width

1. INTRODUCTION AND LITERATURE REVIEW

Corrugated board is the main material used to distribute and store many kinds of products. By the early 1900's, corrugated boxes were coming into common use, and they remain the most common type of distribution packaging. They are used as transport packages for a wide variety of products including fresh fruit and vegetables, consumer products, household appliances and industrial and military machines. Moreover, boxes are equally suited to all the different modes of transport. This versatility is largely due to the possibility of using different types of raw materials and thereby adapting the quality to each particular requirement and distribution system. Moreover, due to the trend toward standardization of container sizes and quality grades for typical products, it certainly improves efficiency.

Containers made from corrugated board provide protection from compression forces for products in transit or stacked in warehouses. Therefore, the stability of stacked corrugated boxes is important. A lack of information about the compression strength of stacked boxes can result in leaning stacks that could eventually collapse. Finally, it can result in excessive product damage and the possibility of human injury. The most important information about the stability of stacked boxes is the compression strength (CS) of the box.

Individual box CS is determined by the quality of corrugating mediums and linerboards as well as outside dimensions of the container. However, the stacked CS of corrugated containers is controlled by the stacking method. When we stack boxes in the

warehouse, the edges and/or corners can hang over other boxes. The type and extent of the overhang is occurred due to a lack of space and human error. Therefore, the prediction of the loss in container CS as a function of overhang is necessary to ensure stack stability in a warehouse.

Ever since the broadening of motor freight and rail carrier classifications in 1936 to include corrugated board shipping containers, manufacturers have sought predictive compression strength models for corrugated containers. Many researchers have tried to generate predictive equations that estimate the container compression strength without having to test every container.

In 1963, McKee developed a mathematical formula to predict the compression strength of corrugated boxes. The McKee study found that each side contributes $1/12^{\text{th}}$ to the box CS and each vertical edge contributes $1/6^{\text{th}}$. Therefore, the four vertical edges of the box make up about 66% of the CS and four sides about 33%. This conclusion does not take overhang into account. It may also not apply to boxes produced nowadays because raw materials and production methods of containers have changed. Moreover, because of limitations in computing tools at the time, the McKee equation is a simplification of a more general relationship with many constraints (1).

The purpose of this study was to investigate the loss in compression strength of corrugated board containers due to offset. Based on the data from the investigation, a mathematical model to predict the loss in CS will be developed. This study was initiated by the Fibre Box Association (FBA) through the Consortium of Distribution Packaging Research at Michigan State University. The purpose was to evaluate the data published in

“The Effect of Warehouse Mishandling and Stacking Patterns on the Compression Strength of Corrugated Boxes” that was produced by U. I. Ievans of Container Corporation of America in 1975.

1.1 Compression Strength

The compression strength of a box is the maximum top load that can be applied to it under specified conditions before it fails. It is expressed in pounds. It is used to determine how well the container will perform during transportation or stacking. In the ASTM standard, D 4169-05 (2): “Performance Testing of Shipping Container and Systems”, the ability of a package to withstand the compressive loads that occur during vehicle transport or warehousing can be estimated using a formula that takes into account an assurance level factor, the weight of the product and the height of the stack.

The compression strength test is described in ASTM D642-00 (3). Shipping containers with or without contents are tested an apparatus like that shown in Figure 1.

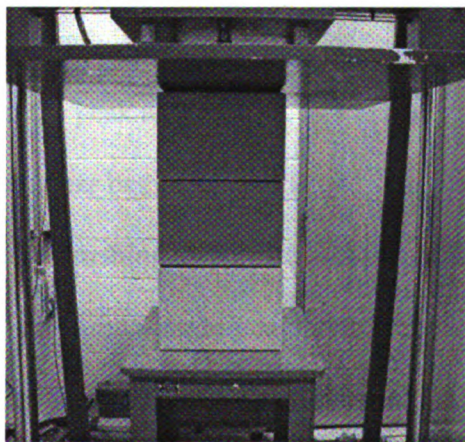


Figure 1. Compression Tester

This test is accomplished by placing the test container between two horizontal plates. The upper plate moves at constant velocity, about a half an inch per minute. When it squeezes the container, a load cell measures the force applied. The tester measures force (lbs) and deflection (inches). (See Figure 2)

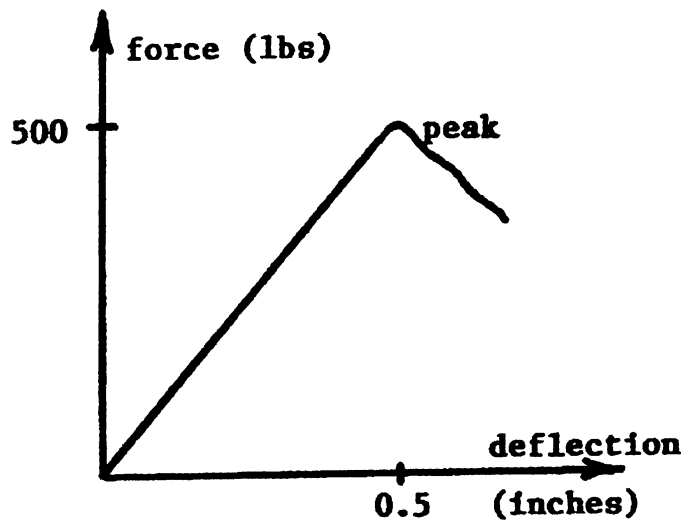


Figure 2. Stress and Strain Graph

The compression strength is the maximum force recorded, and it includes the corresponding deflection to the peak at the curve. In Figure 2, CS = 500 lbs at 0.5". This is the moment when the container begins to buckle and start losing its resistance. Typical compression damage includes crushing, deformation, stress cracking, and breakage.

Compression strength is measured by applying the load evenly over the top of the box. However, containers in warehouses usually do not have full support over their top surface. Without full support, a proportion of the stack strength is lost. This loss may lead to leaning stacks that could potentially collapse. This could produce secondary

damage. Once a stack starts to fall, it could cause a chain-reaction collapse (4).

The box CS is determined by various factors such as board properties, construction, and style of container. However, the various environmental factors result in the loss of CS too. Therefore, a safety factor is generally used to convert CS to real stacking strength. The safety factor depends on humidity, storage time, effect of stack misalignment, pallet stacking pattern, vibration, handling methods, and distance and type of transportation (4).

In 1977, levans studied the effect of humidity on compression strength of corrugated containers. The effect of cycling humidity during some period of time and its influence on the stacking strength was also investigated. He found that boxes collapse immediately at some critical moisture content. The rate at which the failure occurred depended on not only the contents of the box also the limits of the high and low extremes in cyclic humidity. A 26.7% reduction in compression strength occurred by changing the relative humidity from 50% to 85% (6).

Due to the effect of storage time, the boxes may have only about 80% of their original stacking strength after stored more than 30 days. The stacking pattern or types of pallets used. Misalignment of 2 cm in stacking method can result in a loss of CS of about 40%. The interlocking stacking pattern of the boxes results in more than 50% loss in stacking strength. Also, the use of pallets with an open under deck result in a loss of up to 65% loss in stacking strength. Drop due to handling mistake can also weaken the boxes considerably (7).

1.2 Prediction method of CS

In 1951, Kellicutt and Landt developed one of the earliest mathematical models for predicting the compression strength of corrugated boxes. They developed an equation relating the combined ring crush strength of the materials forming the board and the container's perimeter to the compression strength of the box. Based on the test, the following equation was proposed (8)

$$F = P_x \left[\frac{(a_x)^2}{\left(\frac{Z}{4}\right)^2} \right]^{\frac{1}{3}} Z J \quad (1)$$

F = compression strength of the box

$$P_x = P_{11} + P_{12} + i P_m$$

P₁₁ = ring compression strength of liner one in the cross machine direction

P₁₂ = ring compression strength of liner two

i = take up factor of the corrugated medium

P_m = ring crush strength of the corrugated medium

a_x = constant for the flute style

Z = perimeter of the box

J = constant based on the type of manufacture's joint and compensation for flaps

In 1956, Maltenfort also developed a linear equation based on the dimensions of the container and the edgewise compression strength of the board liner, as measured by the then newly developed Concora Liner Test (8) in order to predict the compression strength of single wall containers. After about 300 containers were tested, the following relationship was established each flute styles, A, B, and C (10):

$$F = 5.8L + 12W - 2.1D + O \quad (2)$$

F = compression strength of the box

L = container length

W = container width

D = container depth

O = constant based on flute style

for A flute O = 6.5(CLT-cd) + 365

for B flute O = 5.4(CLT-cd) + 212

for C flute O = 6.5(CLT-cd) + 350

CLT-CD = Compression strength from the CLT test in the cross machine direction

The main problem with these early models was their dependence on the strength of the paper used to form the corrugated board. These models were useful to the container manufactures that had access to data on component properties but not to the actual container users who had no way to know this information. Other investigations have shown that processing variations during the formation of corrugated fiberboard has a significant effect on the strength of the combined board (11).

McKee developed a formula based on the theoretical compression strength of edge supported plates. The “McKee formula”, developed by McKee, Gander and Wachutta at the Institute of Paper Chemistry, provides corrugated box designers with a mathematical model to predict compression strength of corrugated box using the following equation (1).

$$P = a (ECT)^b [\sqrt{D_x D_y}]^{(1-b)} Z^{(2b-1)} \quad (3)$$

P = compressive strength of the box

ECT = edge crush test value for the board

D_x , D_y = flexural stiffness of the board in the machine and cross directions

Z = perimeter of the box

a and b are constants

The values for “a” and “b” were determined to be 2.028 and 0.746 respectively. Due to the difficulty in measuring the flexural stiffness of corrugated board, the equation was further simplified using an empirically determined relationship between the flexural stiffness term and the thickness of the combined board. The simplified equation is

$$P = 5.87(ECT)\sqrt{h}\sqrt{Z} \quad (4)$$

P = compressive strength of the box

ECT = edge crush test value for the board

Z = perimeter of the box

h = thickness of the board

In order to get the ECT value of the board, a mini compression test is needed. The standard test for this is TAPPI T811 (12). For C-flute, a 2 inch wide, 1.25 inch tall rectangular sample of the board with its flutes vertical is placed in a miniature version of a compression tester and the force to begin crushing is measured. Finally, this force is divided by the 2 inch width in order to get the ECT value in lbs/in.

The McKee equation had many advantages over the earlier equation developed by Maltenfort, Kellicut and Landt. For the prediction of box compression strength, the use of board properties vs paper properties compensated for variations caused by the board forming process. Moreover, it allowed the user to independently confirm the mechanical specifications of the material (13).

In 1989, Kawanishi used a different method to model the compression strength of corrugated boxes. He developed a mathematical equation based on an analysis of the physical properties of the board. Also included in this model were terms to compensate

for container perimeter and print area. His equation is

$$BCS = 9.81 \times 3.79 \times 10^{-8} \times K^{0.379} \times b_{wL}^{0.650} \times b_{wB}^{1.20} \times TF^{-4.15} \times CC^{2.45} \times C_k^{3.43} \times BP_k^{0.565} \times BT^{0.0315} \times PR^{0.0602} \times MC_{SW}^{-1.10} \quad (5)$$

BCS = box compression strength

K = linerboard factor (3 for K linerboard, 2.5 for K' linerboard, 2 for B linerboard, factor for C linerboard not stated)

b_{wL} = total basis weight of linerboard (g.m⁻²)

b_{wB} = total basis weight of corrugated fiberboard (g.m⁻²)

TF = take-up factor

CC = average corrugation count (dimensions not stated)

C_k = corrugated fiberboard thickness (mm)

BP_k = box perimeter (cm)

BT = box type factor

PR = print ratio (dimensionless)

MC_{SW} = sidewall moisture content (% , basic not given)

The analysis of the physical properties of the board indicates that the most significant effect on the compression strength of the container is the moisture content of the sidewall (14).

In 1996, Rha developed the percent reduction in compression strength of the single wall corrugated containers as a function of lateral offset and diagonal offset. Through his investigation, he evaluated the effect of humidity and degree of offset. In order to determine the loss in compression strength as a function of offset, he tested three different sizes of containers at both normal and tropical conditions. He found that under normal conditions the reduction in compression strength was 27%, 40% and 45% when the contact area is reduced to 95%, 90%, and 85%. At tropical conditions, the reduction in compression strength decreased about 50% compared to normal conditions (15).

2. MATERIALS AND METHODS

2.1 Materials

Five different size single wall C-flute corrugated boxes were tested. They are labeled box A, B, C, D, E, and F in Table 1. All boxes were made from the same lot of 200 psi burst strength board except for box B, which had 275 psi burst strength. The box styles were regular slotted containers (RSC). All containers were erected, and the manufacture's joint and flaps were hot melt glued together both at the top and the bottom with the same glue in accordance with ASTM D642-00 (3).

Table 1.Boxes Tested

Box Type	Sizes of boxes (inches)	Burst strength (psi)
	L X W X H	
A	19 X 10 X 10	200
B	19 X 10 X 10	275
C	19 X 15 X 10	200
D	19 X 13 X 6	200
E	15 X 10 X 10	200
F	16 X 12 X 10	200

2.2 Conditioning and test methods

All corrugated boxes were conditioned following ASTM D 4332-05 (16)–

“Standard Practice for Conditioning Containers, Packages, or Packaging Components for Testing”. Standard warehouse atmospheric conditions were selected for this study. All sample corrugated boxes were pre-conditioned at 73°F 50% RH and 72 hours and then compression tests were conducted at ambient conditions within 30 minutes after removing these from the conditioning chambers. The ambient conditions ranged from 70-80°F and approximately 50% humidity.

ASTM D642-00 (3) was followed for compression testing of the corrugated boxes. The compression tester had a fixed platen and applied a load at a constant rate of 0.5 in/min, as recommended. When the box is glued shut, the box flaps do not form a flat level surface. They tend to bulge upward. Full contact with all four corners of the container is not established until this bulge is flattened out. ASTM D642-00 requires that a preload be applied to ensure definite contact before any deformation is recorded. A preload of 50 lbs for the single-wall container was applied in this study.

A Lansmont Corporation Compression Tester (Model No 152-30K) (16) was used for this study. This Model is designed to evaluate the performance of packages under compressive forces including individual shipping containers, pallets, unit loads, and large bulk containers. The maximum force at failure and corresponding deflection were measured for all the box types using various amounts of overhang.

2.3 Procedure

First, individual boxes were tested to establish individual box compression strength. Thirty samples for each box type were tested to establish average strength and

standard deviation in accordance with ASTM D642-00 (3). (See Table.2) All standard deviations are less than 10%, indication that the data is consistent.

Table 2. Individual Box CS

Box Type	AVG CS	STDEV
A	552.5	43.85
B	720.6	35.1
C	305.7	25.8
D	503.9	23.1
E	621.3	33.0
F	420.3	25.2

Next, a stack of 3 boxes was compression tested in accordance with the test plan shown in Table 3. The middle box was shifted the distance shown in the table to simulate overhang. An overhang of 0.5 inches on the length panel means that the length panel was pushed back 0.5 inches. (See Figure 4)

Table 3. Amounts of Overhang in 3 Box Stack

Overhang	Control	Length Panel			Width Panel			Two Adjacent Panels		
inches	0	0.5	1	1.5	0.5	1	1.5	0.5	1	1.5

Ten samples each were tested for no offset and the 3 different types of offset described above.

First, 3 empty boxes were stacked perfectly aligned as shown in Figure 3. This data was used as the ideal compression strength of a perfectly aligned stack and represents the “Control” value. Next, the middle box pushed back as shown Figure 4. The reduced compression strength of this misaligned stack represents the “Length Panel” offset value. Next, the middle was placed with on offset as shown in Figure 5. The reduced compression strength of this stack represents the “Width Panel” offset value. Finally, offsets on two adjacent panels were used as shown in Figure 6. This represents the “Two Adjacent Panel” offset value. Fresh boxes were used for each test.

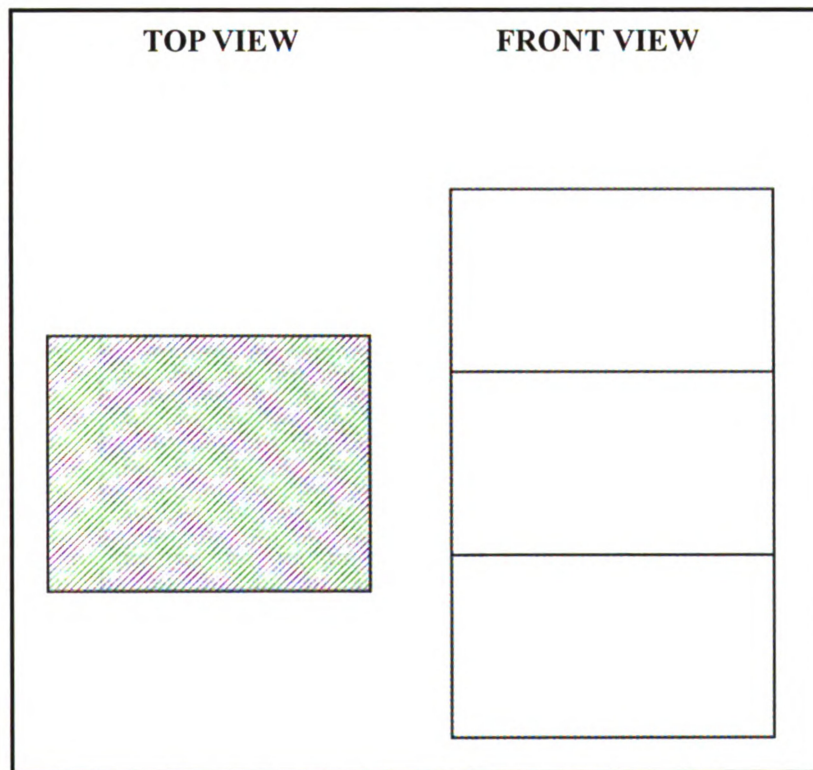
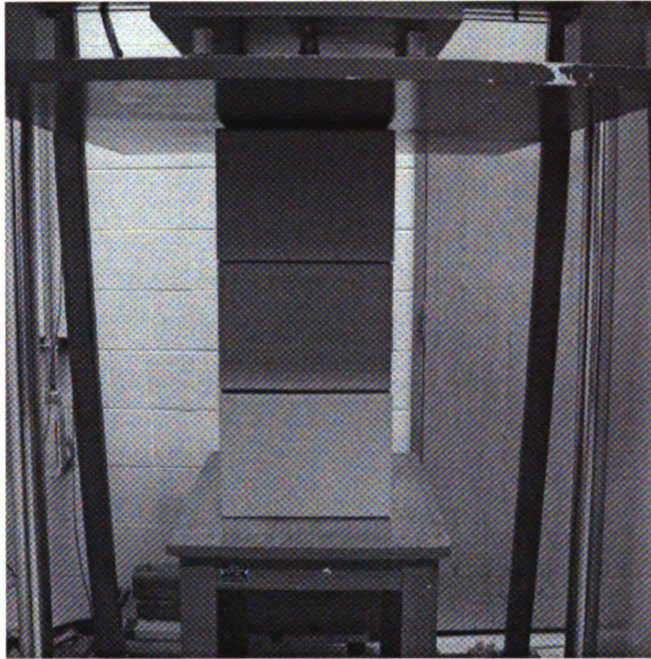


Figure 3. The Control Stack

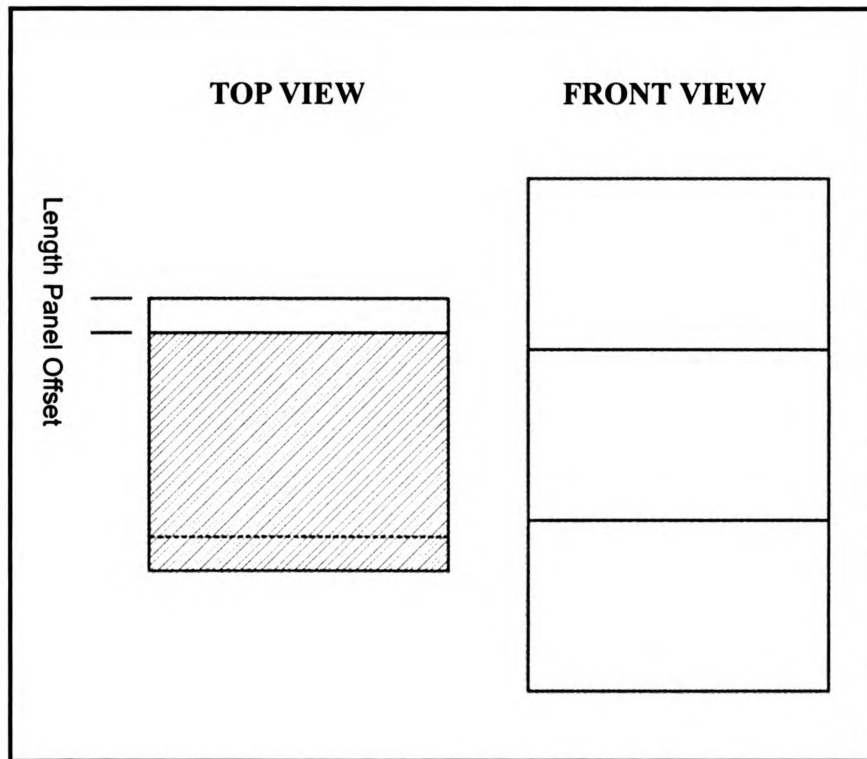


Figure 4. Length Panel Offset

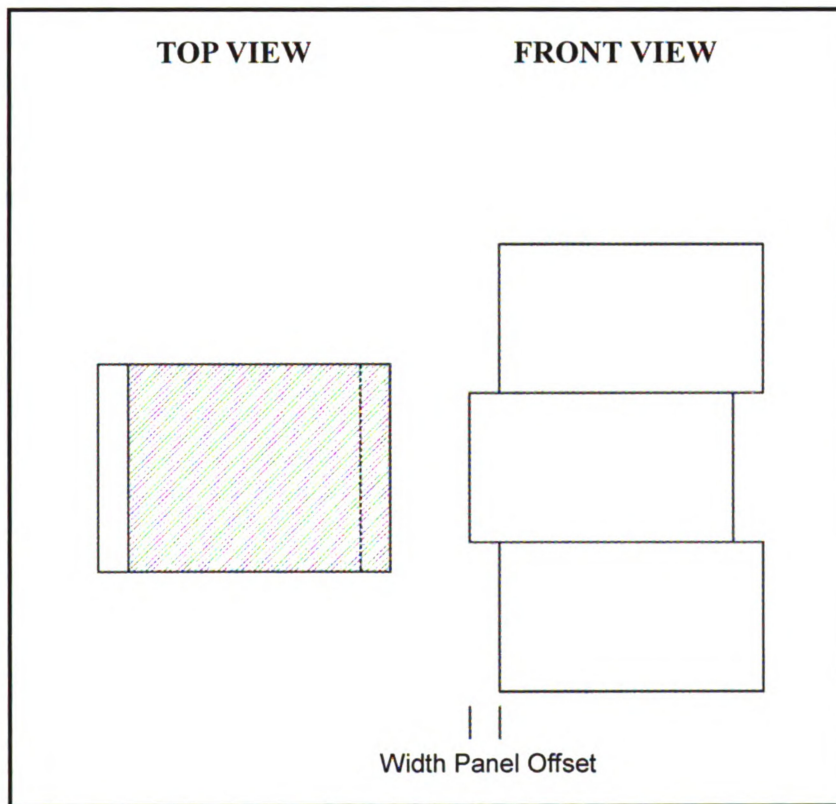


Figure 5. Width Panel Offset

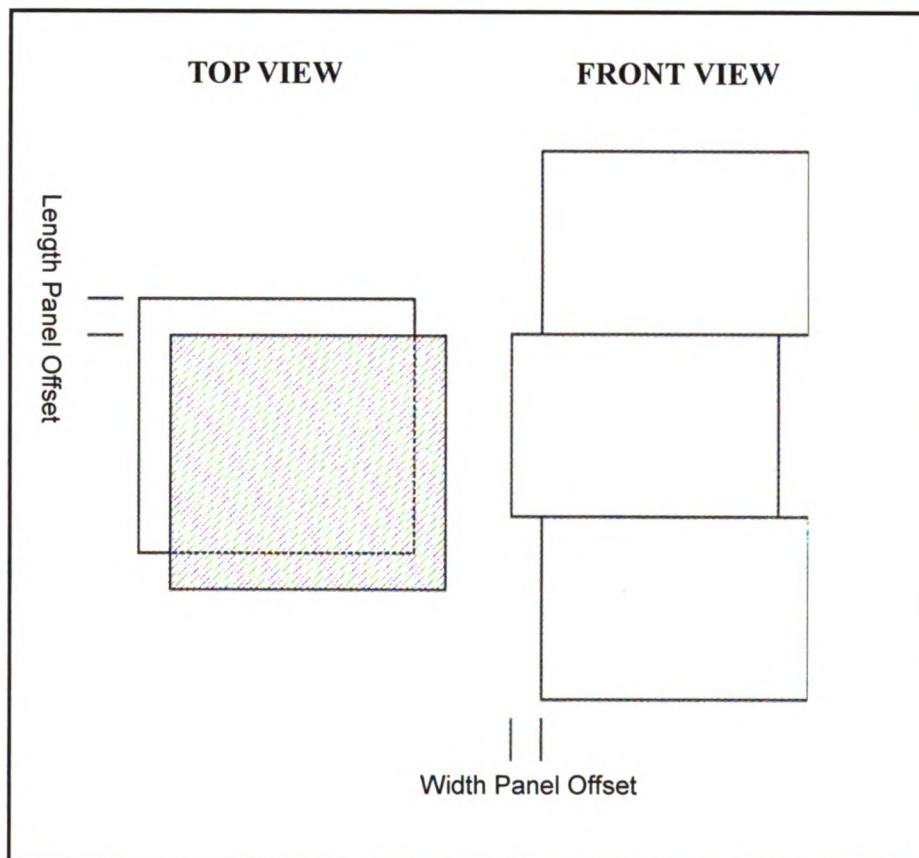
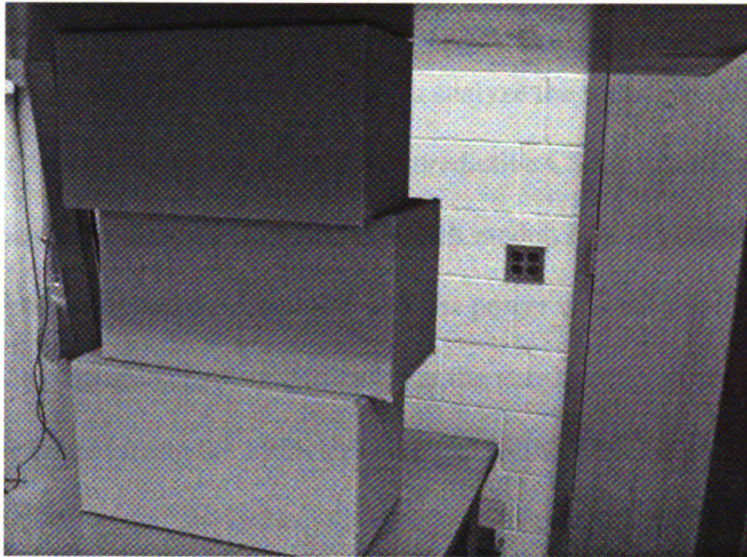


Figure 6. Two Panel Offset

2.4 Statistic Analysis

A paired sample T-test was performed to analyze the statistical differences between actual CS and CS values based on two predictive CS models to be developed for all boxes, and between predicted CS values for each model. Due to enough number of sample 54 (9 types offset and 6 different boxes), the power was enough to identify important predictors as statistically significance at the standard significant level ($\alpha=0.05$). This test was conducted using the SPSS software (SPSS Inc, Chicago, IL, USA).

3. RESULTS AND DISCUSSION

3.1 Results & Observation

Tables (9-a to 9-r) in Appendix A show all the results of the compression tests. All standard deviations are less than about 10%. This suggests that all the collected data is consistent. Observations can be drawn from the data.

- A larger offset resulted in a larger loss in CS.
- Two adjacent panel offsets resulted in a larger loss in CS than single panel offsets.
- A box has a larger burst strength resulted in a larger loss in CS
- No effect of loss in CS due to the difference of box dimensions.

3.2 McKee Equation

According to McKee, each side contributes $1/12^{\text{th}}$ and each vertical edge (corner) contributes $1/6^{\text{th}}$ of the container compression strength. For the 3 box stack where there was an offset on either the length or width panels, one side and two corners offered no support. Based on his report, reduction in CS should be

$$2 \times \frac{1}{6}(\text{corner}) + \frac{1}{12}(\text{side}) = \frac{5}{12} = 41.67\%$$

Therefore, 41.67% of the box CS should be lost. For stacks containers which have both length and width panel offsets, three corners and two sides offer no support. The reduction in CS should be

$$3 \times \frac{1}{6}(\text{corner}) + 2 \times \frac{1}{12}(\text{side}) = \frac{2}{3} = 66.67\%$$

Therefore, 66.67% of the box CS should be lost.

Using the collected CS data from the experiments, the actual reduction in compression strength of the boxes tested can be calculated. The percent reduction in compression strength for a particular offset is defined as:

$$\frac{\textit{CS for Control} - \textit{CS for particular offset}}{\textit{CS of Control}} \times 100\%$$

Tables (4-a to 4-f) show the average of percent reduction in each type of box compression strength for all offsets.

Table 4-a. Percent reduction in compression strength for BOX A

Type of Offset	Length Panel			Width Panel			Two Adjacent Panels		
	0.5"	1"	1.5"	0.5"	1"	1.5"	0.5"	1"	1.5"
Extent of Offset									
Percent CS Loss (%)	8.69	16.9	30.11	11.16	12.53	18.16	9.58	24.63	39.48
No. of samples	10	10	10	10	10	10	10	10	10

Table 4-b. Percent reduction in compression strength for BOX B

Type of Offset	Length Panel			Width Panel			Two Adjacent Panels		
	0.5"	1"	1.5"	0.5"	1"	1.5"	0.5"	1"	1.5"
Extent of Offset									
Percent CS Loss (%)	18.18	31.2	41.61	14.7	18.97	32.65	18.4	32.35	50.91
No. of samples	10	10	10	10	10	10	10	10	10

Table 4-c. Percent reduction in compression strength for BOX C

Type of Offset	Length Panel			Width Panel			Two Adjacent Panels		
	0.5"	1"	1.5"	0.5"	1"	1.5"	0.5"	1"	1.5"
Extent of Offset									
Percent CS Loss (%)	4.83	27.34	28.94	12.78	20.34	22.25	12.54	30.86	48.75
No. of samples	10	10	10	10	10	10	10	10	10

Table 4-d. Percent reduction in compression strength for BOX D

Type of Offset	Length Panel			Width Panel			Two Adjacent Panels		
	0.5"	1"	1.5"	0.5"	1"	1.5"	0.5"	1"	1.5"
Extent of Offset									
Percent CS Loss (%)	13.54	25.88	34.28	9.11	15.15	24.8	17.28	32.84	50.19
No. of samples	10	10	10	10	10	10	10	10	10

Table 4-e. Percent reduction in compression strength for BOX E

Type of Offset	Length Panel			Width Panel			Two Adjacent Panels		
	0.5"	1"	1.5"	0.5"	1"	1.5"	0.5"	1"	1.5"
Extent of Offset									
Percent CS Loss (%)	25.63	33.61	36.53	27.65	36.49	40.24	24.61	39.56	57.1
No. of samples	10	10	10	10	10	10	10	10	10

Table 4-f. Percent reduction in compression strength for BOX F

Type of Offset	Length Panel			Width Panel			Two Adjacent Panels		
	0.5"	1"	1.5"	0.5"	1"	1.5"	0.5"	1"	1.5"
Extent of offset									
Percent CS Loss (%)	17.58	36.04	41.52	15.18	25.87	34.27	22.45	40.48	49.56
No. of samples	10	10	10	10	10	10	10	10	10

Figures 7 and 8 show the comparison between predicted percent reduction in box compression strength (McKee) and the actual percent reduction for single panel offset and two panel offsets.

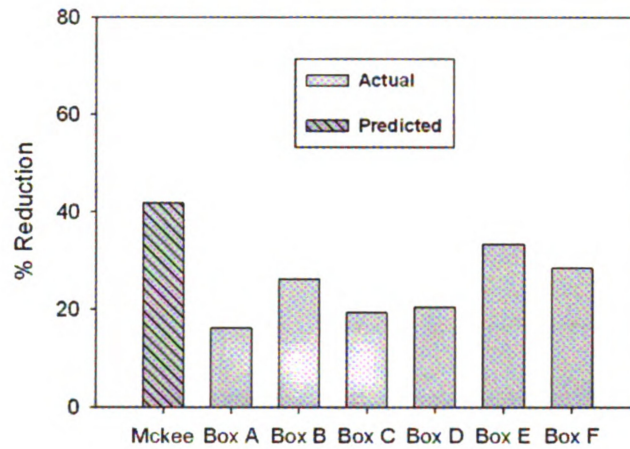


Figure 7. Percent Reduction in CS: McKee prediction vs Single Panel Offset

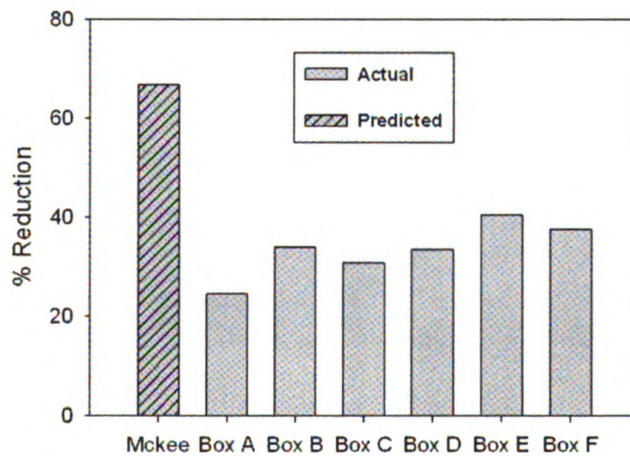


Figure 8. Percent Reduction in CS: McKee prediction vs Two Panel Offset

Based on the data, the predicted percent reduction in box compression strength is much higher than the actual percent reduction of box compression strength (Tables 4-a to 4-f). From the results, it was concluded that the loss in CS from his report did not fit the tested boxes.

Moreover, McKee did not consider the amount of offset or the contribution of the closed flaps to CS. Therefore, a different method is required in order to predict the reduced box compression strength.

3.3 Simple Stress Analysis

If we make the assumption that the box failure occurs when the stress (force divided by bearing area) reaches some critical amount, then this critical stress is

$$\frac{CCS}{L \times W} = \frac{460}{19 \times 10} = 2.42 \text{ psi} \quad (6)$$

where CCS is the control (no offset) compression strength. If the offset on the length side is “x” and on the width side is “y”, the bearing area is the block hatched regions: (L-x) × (W-y) shown in Figure 9.

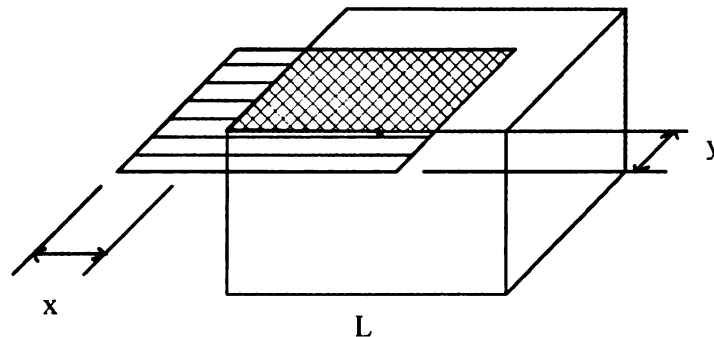


Figure 9. Stress on box

If the critical stress assumption is correct, it should fail when the measured offset compression strength (OCS) divided by the offset bearing area reaches 2.42 psi.

$$\frac{OCS}{(L-x) \times (W-y)} = 2.42 \quad (7)$$

Table 5 shows the percent error between actual CS and predicted CS for box A using equation (7). From this result, it can be concluded that the agreement between actual CS and predicted CS based on the critical stress theory is not good. This means the contact area alone between the load and the box does not account for the results.

Table 5. Percent Error for Critical Stress Theory

Box Type	x	y	actual CS	pred CS	%error
A	0	0.5	425.92	436.81	2.56
	0.5	0	414.41	447.70	8.04
	0.5	0.5	421.71	425.32	0.86
	0	1	371.94	413.82	11.27
	1	0	391.53	435.60	11.26
	1	1	337.30	392.04	16.23
	0	1.5	338.10	390.83	15.60
	1.5	0	395.92	423.50	6.97
	1.5	1.5	292.81	359.98	22.94
AVG			376.61	413.96	10.64

3.4 Fitting an Equation.

The critical stress theory was not expected to give good results because the surface area of box does not carry the load equally. The walls and corners carry more than the flaps. Therefore, it can be assumed that the stress is not uniform within this area.

From the critical stress theory,

$$\frac{CCS}{L \times W} = \frac{OCS}{(L-x) \times (W-y)} \quad (8)$$

From this equation, OCS can be described as:

$$OCS = CCS \left[1 - \frac{x}{L} - \frac{y}{W} + \frac{xy}{LW} \right] \quad (9)$$

Since $\frac{x}{L}$ and $\frac{y}{W}$ are small, $\frac{xy}{LW}$ is very small, so it can be ignored. Then, OCS can be described as

$$OCS = CCS \left[1 - \frac{x}{L} - \frac{y}{W} \right] \quad (10)$$

Since equation (10) doesn't fit the experimental data, the following equation will be tried:

$$OCS = CCS \left[1 - a \left(\frac{x}{L} \right) - b \left(\frac{y}{W} \right) \right] \quad (11)$$

Now fit this equation to the data by modifying “a” and “b” so that the sum of the squares of the errors (SSE) between the actual and predicted OCS can be minimized.

The SSE is

$$SSE = \sum_{i=1}^N \left\{ \frac{OCS_i}{CCS} - \left[1 - a \left(\frac{x_i}{L} \right) - b \left(\frac{y_i}{W} \right) \right] \right\}^2 \quad (12)$$

where N is number of test results, OCS_i is the experimental CS with offset x_i and y_i , and CCS is the control CS. Setting the derivative of SSE with respect to “a” and “b” equal to zero in order to minimize SSE gives:

$$\frac{d}{da}(SSE) = \sum_{i=1}^N 2 \left\{ \frac{OCS_i}{CCS} - \left[1 - a \left(\frac{x_i}{L} \right) - b \left(\frac{y_i}{W} \right) \right] \right\} \left(\frac{x_i}{L} \right) = 0 \quad (13)$$

$$\frac{d}{db}(SSE) = \sum_{i=1}^N 2 \left\{ \frac{OCS_i}{CCS} - \left[1 - a \left(\frac{x_i}{L} \right) - b \left(\frac{y_i}{W} \right) \right] \right\} \left(\frac{y_i}{W} \right) = 0 \quad (14)$$

From rearrangement of these equations, following equations are defined

$$a \times \sum_{i=1}^N \left(\frac{x^i}{L}\right)^2 + b \times \sum_{i=1}^N \left(\frac{x^i}{L}\right) \left(\frac{y^i}{W}\right) = \sum_{i=1}^N \left(1 - \frac{OCS_i}{CCS}\right) \left(\frac{x^i}{L}\right) \quad (15)$$

$$a \times \sum_{i=1}^N \left(\frac{x^i}{L}\right) \left(\frac{y^i}{W}\right) + b \times \sum_{i=1}^N \left(\frac{y^i}{W}\right)^2 = \sum_{i=1}^N \left(1 - \frac{OCS_i}{CCS}\right) \left(\frac{y^i}{W}\right) \quad (16)$$

From equations (15) and (16), 5 sums can be defined as

$$S_1 = \sum_{i=1}^N \left(\frac{x^i}{L}\right)^2 \quad (17)$$

$$S_2 = \sum_{i=1}^N \left(\frac{x^i}{L}\right) \left(\frac{y^i}{W}\right) \quad (18)$$

$$S_3 = \sum_{i=1}^N \left(\frac{y^i}{W}\right)^2 \quad (19)$$

$$S_4 = \sum_{i=1}^N \left(1 - \frac{OCS_i}{CCS}\right) \left(\frac{x^i}{L}\right) \quad (20)$$

$$S_5 = \sum_{i=1}^N \left(1 - \frac{OCS_i}{CCS}\right) \left(\frac{y^i}{W}\right) \quad (21)$$

Equations (15) and (16) are

$$aS_1 + bS_2 = S_4 \quad (22)$$

$$aS_2 + bS_3 = S_5 \quad (23)$$

The solution to equations (22) and (23),

$$a = \frac{S_3S_4 - S_2S_5}{S_1S_3 - S_2^2} \quad (24)$$

$$b = \frac{S_1S_5 - S_2S_4}{S_1S_3 - S_2^2} \quad (25)$$

The BASIC program in Appendix B does these calculations for each box. Table 6 shows the results.

Table 6. Fit results for “a” and “b”

Box Type	a	b
A	1.87	1.61
B	3.02	2.37
C	2.85	2.81
D	2.65	2.82
E	3.68	2.18
F	2.88	2.92

Since “a” and “b” for the six different boxes are similar, the same equation was fitted to all of the data. The best value of “a” and “b” for all 6 box types is $a=2.97$ and $b=2.32$. Table (7-a to 6-f) compare the actual CS to the predicted CS. The “Q” factor in the test 3 column will be discussed later

Table 7-a. Actual CS vs BASIC program results

Box Type		"a" and "b" for each box type			"a" and "b" for all Box types			"Q" for all Box types		
A	x y	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	pred CS	%error
	0 0.5	425.90	422.25	0.86	425.90	406.02	4.67	425.90	400.71	5.92
	0.5 0	414.40	436.62	5.36	414.40	423.37	2.16	414.40	428.43	3.39
	0.5 0.5	421.70	399.64	5.23	421.70	370.16	12.22	421.70	369.90	12.28
	0 1	371.90	385.27	3.60	371.90	352.80	5.13	371.90	342.18	7.99
	1 0	391.50	414.00	5.75	391.50	387.51	1.02	391.50	397.63	1.56
	1 1	337.30	340.04	0.81	337.30	281.08	16.67	337.30	280.57	16.82
	0 1.5	338.10	348.29	3.01	338.10	299.59	11.39	338.10	283.65	16.10
	1.5 0	395.90	391.38	1.14	395.90	351.64	11.18	395.90	366.82	7.34
	1.5 1.5	292.80	280.44	4.22	292.80	192.00	34.43	292.80	191.24	34.68
AVG		376.61	379.77	3.33	376.61	340.46	10.99	376.61	340.13	11.79

Table 7-b. Actual CS vs BASIC program results

Box Type	"a" and "b" for each box type				"a" and "b" for all Box types				"Q" for all Box types			
	B	x	y	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	pred CS	%error
		0	0.5	583.20	618.29	6.02	583.20	620.10	6.33	583.20	611.98	4.94
		0.5	0	608.00	645.58	6.18	608.00	646.59	6.35	608.00	654.32	7.62
		0.5	0.5	581.60	562.50	3.28	581.60	565.32	2.80	581.60	564.94	2.86
		0	1	474.70	535.21	12.75	474.70	538.82	13.51	474.70	522.60	10.09
		1	0	559.10	589.79	5.49	559.10	591.82	5.85	559.10	607.28	8.62
		1	1	466.80	423.63	9.25	466.80	429.28	8.04	466.80	428.51	8.20
		0	1.5	409.50	452.14	10.41	409.50	457.55	11.73	409.50	433.21	5.79
		1.5	0	472.30	534.00	13.06	472.30	537.05	13.71	472.30	560.23	18.62
		1.5	1.5	344.30	284.77	17.29	344.30	293.23	14.83	344.30	292.08	15.17
		AVG		499.94	516.21	9.30	499.94	519.97	9.24	499.94	519.46	9.10

Table 7-c. Actual CS vs BASIC program results

Box Type		"a" and "b" for each box type				"a" and "b" for all Box types				"Q" for all Box types			
C	x	y	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	%error
	0	0.5	446.70	425.48	4.75	446.70	433.14	3.04	446.70	429.52	3.85		
	0.5	0	409.40	434.23	6.07	409.40	432.74	5.70	409.40	437.91	6.97		
	0.5	0.5	410.50	390.31	4.92	410.50	396.48	3.42	410.50	398.03	3.04		
	0	1	341.10	381.56	11.86	341.10	396.88	16.35	341.10	389.64	14.23		
	1	0	373.90	399.06	6.73	373.90	396.08	5.93	373.90	406.43	8.70		
	1	1	324.50	311.22	4.09	324.50	323.56	0.29	324.50	326.67	0.67		
	0	1.5	333.60	337.64	1.21	333.60	360.62	8.10	333.60	349.76	4.84		
	1.5	0	365.00	363.89	0.30	365.00	359.43	1.53	365.00	374.94	2.72		
	1.5	1.5	240.50	232.14	3.48	240.50	250.64	4.22	240.50	255.30	6.15		
	AVG		360.58	363.95	4.82	360.58	372.17	5.40	360.58	374.24	5.69		

Table 7-d. Actual CS vs BASIC program results

Box Type			"a" and "b" for each box type			"a" and "b" for all Box types			"Q" for all Box types		
D	x	y	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	pred CS	%error
	0	0.5	433.60	447.05	3.10	433.60	456.80	5.35	433.60	452.34	4.32
	0.5	0	455.80	466.50	2.35	455.80	462.34	1.43	455.80	467.86	2.65
	0.5	0.5	414.80	412.06	0.66	414.80	417.63	0.68	414.80	418.70	0.94
	0	1	371.70	392.61	5.63	371.70	412.10	10.87	371.70	403.17	8.47
	1	0	425.50	431.51	1.41	425.50	423.17	0.55	425.50	434.22	2.05
	1	1	336.80	322.62	4.21	336.80	333.77	0.90	336.80	335.90	0.27
	0	1.5	329.60	338.16	2.60	329.60	367.40	11.47	329.60	354.01	7.41
	1.5	0	377.10	396.51	5.15	377.10	384.01	1.83	377.10	400.58	6.23
	1.5	1.5	249.80	233.18	6.65	249.80	249.90	0.04	249.80	253.09	1.32
AVG			377.19	382.25	3.53	377.19	389.68	3.68	377.19	391.10	3.74

Table 7-e. Actual CS vs BASIC program results

Box Type		"a" and "b" for each box type				"a" and "b" for all Box types				"Q" for all Box types			
E	x	y	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	%error
	0	0.5	208.20	249.42	19.80	208.30	247.55	18.85	208.30	244.32	17.29		
	0.5	0	202.50	245.64	21.30	202.50	252.30	24.59	202.50	256.21	26.52		
	0.5	0.5	211.00	215.05	1.92	211.00	219.86	4.20	211.00	220.53	4.51		
	0	1	185.80	218.83	17.78	185.80	215.11	15.77	185.80	208.63	12.29		
	1	0	177.80	211.28	18.83	177.80	224.60	26.32	177.80	232.42	30.72		
	1	1	169.20	150.11	11.28	169.20	159.71	5.61	169.20	161.05	4.82		
	0	1.5	177.70	188.25	5.94	177.70	182.66	2.79	177.70	172.95	2.67		
	1.5	0	167.30	176.91	5.75	167.30	196.91	17.70	167.30	208.63	24.71		
	1.5	1.5	120.10	85.16	29.09	120.10	99.57	17.09	120.10	101.58	15.42		
AVG			179.96	193.41	14.63	179.97	199.81	14.77	179.97	200.70	15.44		

Table 7-f. Actual CS vs BASIC program results

Box Type		"a" and "b" for each box type				"a" and "b" for all Box types				"Q" for all Box types			
F	x	y	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	pred CS	%error	actual CS	%error
	0	0.5	320.40	341.52	6.59	320.40	351.26	9.63	320.40	347.51	8.46		
	0.5	0	329.80	353.86	7.29	329.80	352.74	6.96	329.80	357.83	8.50		
	0.5	0.5	301.50	306.58	1.68	301.50	315.20	4.54	301.50	316.54	4.99		
	0	1	248.70	294.24	18.31	248.70	313.71	26.14	248.70	306.22	23.13		
	1	0	288.20	318.91	10.66	288.20	316.69	9.88	288.20	326.86	13.42		
	1	1	231.40	224.36	3.04	231.40	241.60	4.41	231.40	244.28	5.57		
	0	1.5	227.40	246.96	8.60	227.40	276.17	21.45	227.40	264.92	16.50		
	1.5	0	255.50	283.97	11.14	255.50	280.63	9.84	255.50	295.89	15.81		
	1.5	1.5	196.10	142.14	27.52	196.10	168.00	14.33	196.10	172.02	12.28		
	AVG		266.56	279.17	10.54	266.56	290.67	11.91	266.56	292.45	12.07		
ALL BOX	AVG		343.47	352.46	7.69	343.47	352.13	9.33	343.47	353.01	9.64		

The percent errors for all treatments are shown in Figure 9.

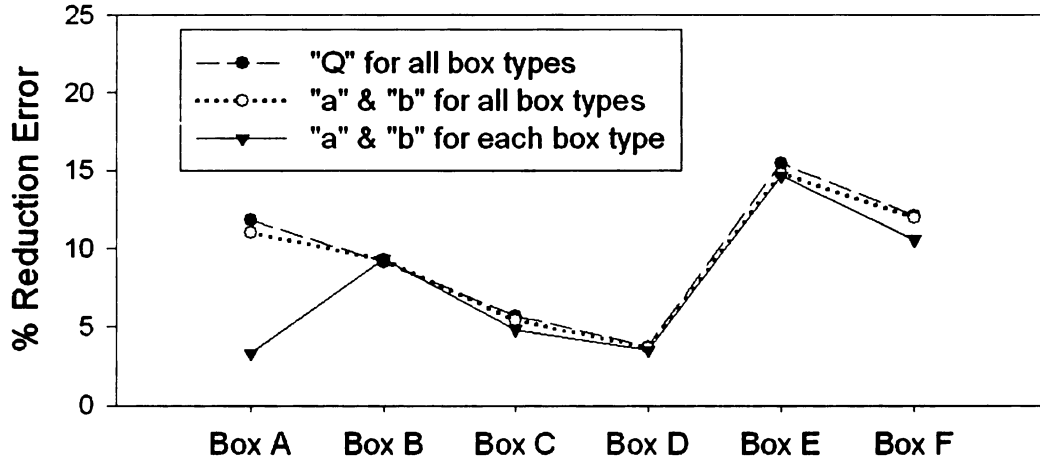


Figure 9. Percent Reduction error Comparison

3.5 Math model for one parameter

Based on the results of Table 6 and the value of “a” and “b” for all boxes together, “a” and “b” are approximately the same. Therefore, only one parameter can be used for simplification of the model,

$$OCS = CCS \left[1 - Q \left(\frac{x}{L} + \frac{y}{w} \right) \right] \quad (26)$$

Now fit this equation to the data by choosing “Q” so that the sum of the squares of the errors (SSE) between the actual and predicted CS is minimized. The SSE is

$$SSE = \sum_{i=1}^N \left\{ \frac{OCS_i}{CCS} - \left[1 - Q \left(\frac{x_i}{L} + \frac{y_i}{w} \right) \right] \right\}^2 \quad (27)$$

Setting the derivative of SSE with respect to “Q” equal to zero in order to minimize SSE,

$$\frac{d}{da}(SSE) = \sum_{i=1}^N 2 \left\{ \frac{OCS_i}{CCS} - \left[1 - Q \left(\frac{x_i}{L} + \frac{y_i}{W} \right) \right] \right\} \left(\frac{x_i}{L} + \frac{y_i}{W} \right) = 0 \quad (28)$$

From a rearrangement of this equation, the following equations can be obtained

$$Q \times \sum_{i=1}^N \left(\frac{x_i}{L} + \frac{y_i}{W} \right)^2 = \sum_{i=1}^N \left(1 - \frac{OCS_i}{CCS} \right) \left(\frac{x_i}{L} + \frac{y_i}{W} \right) \quad (29)$$

From equations (29), 2 sums can be defined as:

$$S_6 = \sum_{i=1}^N \left(1 - \frac{OCS_i}{CCS} \right) \left(\frac{x_i}{L} + \frac{y_i}{W} \right) \quad (30)$$

$$S_7 = \sum_{i=1}^N \left(\frac{x_i}{L} + \frac{y_i}{W} \right)^2 \quad (31)$$

Rearrangement and simplification gives

$$Q = S_6 / S_7 \quad (32)$$

The BASIC program gives $Q=2.55$. Tables 7-a to 7-f and Figure 9 show the actual CS and predicted CS for all treatments. The difference between actual CS and predicted CS based on the above model is less than the difference based on the critical stress assumption.

3.6 Result of statistical analysis

A verification of two models: pr1 (“a” and “b” for all box type) and pr2 (“Q” for all box type) are used to scale the P-values, which are used as a measure of difference between actual and predicted CS (pair 1 and pair 2), and two predicted CS (pair 3). Table 7 shows the P-value of both models. Both 0.071 and 0.051 are larger than 0.05, so that there was no statistically significant difference between actual CS and two predicted CS values based on two models. Also, 0.494 is much larger than 0.05; therefore, there was no

statistically significant difference between two models.

Table 7. Paired Sample T-Test

		Paired Differences						t	df	Sig. (2-tailed)
		95% Confidence Interval of the Difference								
		Mean	Std. Deviation	Std. Error Mean	Lower	Upper				
Pair 1	ActualCS - pr1	-8.65537	34.56702	4.70397	-18.09035	.77961	-1.840	53	.071	
Pair 2	ActualCS - pr2	-9.54148	35.14788	4.78302	-19.13501	.05204	-1.995	53	.051	
Pair 3	pr1 - pr2	-.88611	9.45302	1.28639	-3.46629	1.69407	-.689	53	.494	

4. CONCLUSIONS

This study developed a mathematical model to predict the loss in compression strength of boxes stacked with various amounts of offset. From Tables 7-a to 7-f, the average of percent error between actual OCS and the predicted OCS for this model is 9.64%. This result shows that this model predicts actual CS fairly well. This equation is

$$\frac{ocs}{ccs} = 1 - 2.55\left(\frac{x}{L} + \frac{y}{W}\right) \quad (33)$$

x = extent of offset in length panel

y = extent of offset in width panel

L = length of box

W = width of box

Step1: Find the $\left(\frac{x}{L} + \frac{y}{W}\right)$ value from the extent of offset.

Step2: Calculate the $\frac{ocs}{ccs}$ value using the equation (33) and find the offset box compression strength (OCS).

4.1 Practical use of this model

This equation can be used for practical situations. As an example, suppose that three corrugated containers (20 in x 24 in x 10 in) are stacked with two adjacent panel offsets. There exists a 0.5 in offset in the length panel and a 1.8 in offset in the width panel. What is the loss in CS?

Step1: Find the $\left(\frac{x}{L} + \frac{y}{W}\right)$ value:

$$\frac{x}{L} + \frac{y}{W} = \frac{0.5}{20} + \frac{1.8}{24} = 0.1$$

Step2: Calculate the $\frac{ocs}{ccs}$ value using the equation (33):

$$\frac{ocs}{ccs} = 1 - 2.55\left(\frac{x}{L} + \frac{y}{W}\right) = 1 - 2.55(0.1) = 0.745 \cong 0.75$$

So the loss in compression strength is about 25%.

Based on this analysis, the following industry rule of thumb can be proposed. For every 1% of offset in either the length or width direction, an empty box loses about 2.5% of its compression strength because $\frac{ocs}{ccs} = 1 - 2.55(0.01) = 0.975$.

Future work

After developing this model for predicting the offset compression strength for a box with offset, a few considerations arise for further study.

- Application of the model should be conducted with various corrugated containers using different flute types, double and triple wall types, different box styles and different box sizes for verification.
- This model should be verified with samples provided by the same manufacturer in different circumstances such as different RHs, temperatures, and times in storage.

5.APPENDICES

APPENDIX A. Experimental data for Box CS test

Table 9-a. Box A OCS for 0.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	445.8	1.24	390.6	1.30	432.4	1.17	426.0	1.69
2	452.3	1.10	460.9	1.52	359.1	1.23	463.1	1.61
3	454.2	1.10	409.3	1.50	533.4	1.18	437.2	1.73
4	562.2	1.17	426.4	1.87	393.1	1.23	416.8	1.61
5	563.6	1.13	443.8	1.68	381.1	1.22	418.9	1.68
6	431.4	1.42	461.0	1.54	447.6	1.26	418.0	1.61
7	458.6	1.11	435.6	1.55	438.5	1.06	441.2	1.67
8	408.6	1.11	423.3	1.37	364.5	1.22	395.0	1.47
9	439.6	1.12	403.2	1.90	380.9	1.23	390.5	1.59
10	447.8	1.36	404.6	1.44	413.2	1.23	410.6	1.52
Avg	466.41	1.19	425.87	1.57	414.38	1.20	421.73	1.62
Stdve	52.80	0.12	24.42	0.20	52.07	0.06	21.62	0.08

Table 9-b. Box A OCS for 1 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	314.5	0.88	349.2	1.07	386.6	0.89	326.4	2.03
2	480.1	1.11	382.6	1.06	423.5	0.94	324.7	2.85
3	550.8	1.22	369.3	2.31	337.2	0.81	330.5	2.00
4	471.1	1.12	389.7	2.16	349.7	1.04	335.8	2.14
5	413.1	1.10	366.7	1.06	424.5	1.00	339.1	2.20
6	429.1	1.09	357.9	1.41	420.5	1.14	346.1	2.17
7	530.7	1.09	372.8	1.93	362.9	0.89	345.6	2.10
8	416.9	1.27	395.0	1.37	423.1	0.90	347.9	2.38
9	429.0	1.34	378.3	1.90	407.8	0.78	334.8	2.10
10	439.7	1.12	357.3	1.44	378.7	1.10	341.8	2.23
Avg	447.50	1.13	371.88	1.57	391.45	0.95	337.27	2.22
Stdve	66.47	0.12	14.78	0.47	33.20	0.12	8.26	0.25

Table 9-c. Box A OCS for 1.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	382.7	0.88	307.3	1.58	363.3	0.89	297.2	2.75
2	437.8	1.11	352.5	1.70	370.6	0.89	307.5	3.02
3	417.1	1.22	349.5	1.41	378.7	0.86	283.2	2.33
4	589.3	1.12	332.5	1.80	444.0	0.94	299.6	3.42
5	588.7	1.10	312.5	1.86	384.1	0.84	267.1	2.63
6	540.8	1.09	345.8	2.01	390.4	0.88	307.2	2.76
7	467.7	1.09	354.7	1.93	445.1	0.82	278.0	2.99
8	488.1	1.27	347.1	2.57	367.3	0.87	280.4	3.44
9	474.9	1.34	334.0	1.45	376.9	0.80	271.5	2.18
10	450.6	1.12	345.4	1.54	438.7	0.95	335.9	3.55
Avg	483.77	1.13	338.13	1.79	395.91	0.87	292.76	2.91
Stdve	69.62	0.12	16.53	0.34	33.19	0.05	20.89	0.47

Table 9-d. Box B OCS for 0.5 inch offset in the length, width, and adjacent directions

Control		Length		Width		Adjacent		
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	727.7	1.25	622.8	1.32	627.8	1.11	648.5	1.96
2	774.6	1.15	586.9	1.59	575.2	1.19	557.6	1.88
3	685.1	1.16	612.9	1.61	650.3	1.24	588.7	1.88
4	752	1.17	586.2	1.73	600.9	1.28	550.5	1.98
5	760.9	1.15	584.1	1.51	604.4	1.78	594	1.87
6	742.1	1.25	564.2	1.8	551.5	1.06	581.2	1.86
7	698	1.06	571.6	2.26	610.4	1.12	598.2	1.85
8	715.1	1.07	599.4	1.67	556.3	1.87	574.6	1.78
9	545.2	1.25	549.2	1.61	615.2	1.15	511.8	1.82
10	727.5	1.19	555.1	1.37	688.1	1.21	611.2	1.93
Avg	712.82	1.17	583.24	1.65	608.01	1.3	581.63	1.88
Stdve	65.02	0.07	23.98	0.26	41.71	0.28	36.96	0.06

Table 9-e. Box B OCS for 1 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	670.3	1.26	459.7	1.23	592.9	1.15	372.8	1.83
2	726.7	1.4	455.2	1.93	616.3	1.09	497.8	2.54
3	622.8	1.19	484.6	1.84	556.4	1.06	474.5	2.59
4	668.8	1.19	465.9	1.22	584.5	1.13	430.5	2.28
5	694.6	1.22	493.1	1.94	603.6	0.95	496.4	2.54
6	739.4	1.29	492.1	1.3	563.4	0.99	507	2.41
7	646.2	1.08	471.4	1.41	522.1	0.91	459.1	2.39
8	731	1.28	533.5	1.38	485.3	1.13	454.4	2.3
9	730.5	1.21	449.9	1.83	486.9	1.25	472	2.42
10	669.4	1.24	441.4	2.19	579.4	1.09	503.2	2.3
Avg	689.97	1.24	474.68	1.63	559.08	1.08	466.77	2.36
Stdve	40.59	0.08	27.11	0.35	46.55	0.1	41.14	0.22

Table 9-f. Box B OCS for 1.5 inch offset in the length, width, and adjacent directions

Control		Length		Width		Adjacent		
Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(In)	
1	784.8	1.2	360.3	1.2	471	0.83	299.4	2.09
2	663.2	1.47	413.5	0.26	470.4	0.87	281	1.22
3	723.3	1.14	365.6	1.12	466.2	0.8	370.4	2.96
4	687.4	1.34	402	1.17	482.6	0.89	303.2	1.91
5	678.3	1.16	375.6	1.47	483.4	0.86	280.6	1.05
6	674.9	1.33	413.9	1.24	482.6	0.93	517.9	3.3
7	687.1	1.27	434.6	1.28	476.8	0.8	270	1.1
8	713.3	1.26	434.2	1.24	455.2	0.84	331.9	3.29
9	751.3	1.29	433.5	1.24	469.5	0.9	368.3	2.81
10	649.8	1.1	462.2	1.27	465.7	0.79	420	3.49
Avg	701.34	1.26	409.54	1.15	472.34	0.85	344.27	2.32
Stdve	41.93	0.11	33.64	0.33	9.08	0.05	77.87	0.97

Table 9-g. Box C OCS for 0.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	481.6	1.00	460.4	1.12	396.2	0.97	388.2	1.32
2	421.5	0.94	457.8	0.98	384.4	0.97	449.5	1.53
3	494.3	0.94	474.2	0.96	417.5	0.99	401.8	1.36
4	454.3	0.91	447.1	0.96	374.7	1.01	389.3	1.46
5	462.8	1.00	425.6	0.89	404	0.82	427.3	1.43
6	488.4	0.98	465.8	0.98	413	1.01	391.1	1.51
7	499.3	1.06	431.3	1.00	401.1	0.8	408.8	1.42
8	520.5	1.04	453.1	1.40	458.2	0.87	416.7	1.46
9	467.3	0.95	428.3	10.90	410.5	0.93	430.7	1.4
10	403.8	0.94	423.3	1.19	434.3	1.07	401.6	1.35
Avg	469.38	0.98	446.69	2.04	409.39	0.94	410.50	1.42
Stdve	35.76	0.05	18.38	3.12	24.00	0.09	20.36	0.07

Table 9-h. Box C OCS for 1 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(In)
1	481.6	1.00	334.7	0.94	365.4	0.81	334.9	1.97
2	421.5	0.94	366.4	0.94	357.8	0.76	330.8	2.12
3	494.3	0.94	342.6	1.02	359.3	0.95	330.2	20.8
4	454.3	0.91	365.0	0.95	362.9	0.93	333.5	2
5	462.8	1.00	395.7	0.95	385.8	0.8	303.2	2.03
6	488.4	0.98	340.6	1.28	351.6	0.75	321.1	2.22
7	499.3	1.06	256.4	0.86	364.3	0.73	334.8	1.96
8	520.5	1.04	281.8	0.95	401	0.84	319.2	2.11
9	467.3	0.95	368.3	1.46	421.5	0.96	308.0	2.23
10	403.8	0.94	359.1	1.13	369.3	0.83	329.7	2.12
Avg	469.38	0.98	341.06	1.05	373.89	0.84	324.54	3.96
Stdve	35.76	0.05	42.15	0.19	22.16	0.08	11.35	5.92

Table 9-i. Box C OCS for 1.5 inch offset in the length, width, and adjacent directions

Control		Length		Width		Adjacent		
	Force (lb)	Deflection (In)	Force (lb)	Deflection(In)	Force (lb)	Deflection(In)	Force(lb)	Deflection(In)
1	481.6	1.00	303.8	1.09	386.5	1	256.2	2.42
2	421.5	0.94	386.3	0.95	367.7	0.84	259.9	2.13
3	494.3	0.94	318.7	1.54	359.9	0.82	267.3	1.93
4	454.3	0.91	351.6	1.58	335.7	0.62	232.5	1.41
5	462.8	1.00	341.0	1.42	328.4	0.78	223.6	1.1
6	488.4	0.98	352.4	1.42	389.1	0.87	252.9	1.97
7	499.3	1.06	304.1	1.50	343.1	0.82	248.4	2.2
8	520.5	1.04	341.0	1.13	385.1	0.81	225.9	0.99
9	467.3	0.95	318.9	1.55	385.9	0.87	218.2	0.91
10	403.8	0.94	317.7	0.91	368	0.7	220.5	0.97
Avg	469.38	0.98	333.55	1.31	364.94	0.81	240.54	1.60
Stdve	35.76	0.05	25.85	0.26	22.62	0.10	18.31	0.59

Table 9-j. Box D OCS for 0.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	487.1	0.87	466.8	1.33	401.6	0.84	385.4	1.04
2	472.9	0.78	447.6	0.87	397.6	1.17	432.2	1.2
3	488	0.81	398.7	0.84	504.4	0.8	408.4	1.33
4	535.8	0.88	432.2	0.83	450.5	0.86	401.9	1.19
5	455.9	0.92	454.7	0.85	486.4	0.81	386.7	1.27
6	505.1	0.82	463.3	0.87	494.3	0.82	409.2	1.17
7	506.8	0.84	441.6	0.87	456.1	0.85	425.1	1.35
8	511.3	0.87	425.4	0.82	465.1	0.79	442.8	10.9
9	519.6	0.90	414.0	1.24	452.3	0.84	424.9	1.22
10	532.2	0.91	391.3	0.76	449.8	0.89	431.5	1.1
Avg	501.47	0.86	433.56	0.93	455.81	0.87	414.81	2.18
Stdve	25.54	0.05	26.14	0.19	35.38	0.11	19.63	3.07

Table 9-k. Box D OCS for 1 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	487.1	0.87	339.3	0.88	427.7	0.65	354.7	1.58
2	472.9	0.78	358.2	0.81	486.2	0.73	343.5	1.6
3	488	0.81	342.4	1.24	398.6	0.74	319.2	1.55
4	535.8	0.88	379.7	1.42	428.1	0.66	329.8	1.74
5	455.9	0.92	383.9	1.38	530.9	0.72	352.1	1.63
6	505.1	0.82	422.2	1.68	455.8	0.66	324.1	1.54
7	506.8	0.84	355.7	1.44	366.7	0.65	330.0	1.64
8	511.3	0.87	383.5	1.45	358.5	0.67	338.0	1.8
9	519.6	0.90	355.0	1.57	399.5	0.63	343.4	1.72
10	532.2	0.91	397.2	1.84	403	0.73	333.0	1.77
Avg	501.47	0.86	371.71	1.37	425.50	0.68	336.78	1.66
Stdve	25.54	0.05	26.20	0.32	53.31	0.04	11.67	0.09

Table 9-1. Box D OCS for 1.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	487.1	0.87	320.4	0.73	411	0.62	270.7	2.21
2	472.9	0.78	333.5	0.85	328	0.59	271.1	2.7
3	488	0.81	296.8	0.81	358.2	0.57	254.6	2.27
4	535.8	0.88	335.8	0.80	405.2	0.62	207.6	1.24
5	455.9	0.92	279.1	0.77	377.4	0.54	243.7	2.17
6	505.1	0.82	365.3	0.87	344	0.64	255.6	2.07
7	506.8	0.84	315.6	0.80	390.2	0.55	207.8	1.34
8	511.3	0.87	348.7	0.85	361	0.49	256.4	2.22
9	519.6	0.90	349.3	0.90	373.1	0.53	263.4	2.15
10	532.2	0.91	351.1	0.87	422.9	0.67	266.7	1.94
Avg	501.47	0.86	329.56	0.83	377.10	0.58	249.76	2.03
Stdve	25.54	0.05	26.77	0.05	30.45	0.06	23.67	0.44

Table 9-m. Box E OCS for 0.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	296.7	0.61	206.9	0.86	202	0.67	209.6	1.24
2	323.9	0.69	200.7	1.48	218.4	0.62	225.9	1.08
3	235.6	0.59	224.6	0.85	198.3	0.7	230.6	1.31
4	300.2	0.84	192.7	0.73	175.9	0.68	192.4	1.1
5	225.1	0.81	219.2	0.76	164	0.51	191.7	1.26
6	254.2	0.65	203.4	0.86	214.5	0.78	206.6	1.65
7	313.9	0.71	220.7	0.72	201	0.67	226.5	1.44
8	306.3	0.72	199.1	0.64	207.2	0.59	205.1	1.26
9	301.1	0.75	205.6	0.67	190.4	0.54	208.1	1.18
10	242.3	0.86	208.8	0.69	253.5	0.76	213.8	1.29
Avg	279.93	0.72	208.17	0.83	202.52	0.65	211.03	1.28
Stdve	36.47	0.09	10.31	0.24	24.44	0.09	13.49	0.17

Table 9-n. Box E OCS for 1 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	296.7	0.61	200.5	0.61	180.4	0.84	172.2	2.14
2	323.9	0.69	158.0	0.84	151.5	0.29	175.8	1.87
3	235.6	0.59	169.7	0.63	150.7	0.48	187.0	2.29
4	300.2	0.84	192.8	0.56	169.9	0.44	166.9	2.33
5	225.1	0.81	205.0	0.61	172.7	0.41	157.1	1.43
6	254.2	0.65	185.4	0.57	182	0.6	154.5	2.75
7	313.9	0.71	175.0	0.72	203.5	0.39	179.2	3
8	306.3	0.72	162.9	1.31	177.3	0.53	171.3	2.7
9	301.1	0.75	205.9	0.63	185.9	0.51	177.4	1.65
10	242.3	0.86	203.2	0.65	204	0.51	150.7	2.79
Avg	279.93	0.72	185.84	0.71	177.79	0.50	169.21	2.30
Stdve	36.47	0.09	18.30	0.22	18.11	0.15	11.79	0.53

Table 9-o. Box E OCS for 1.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	296.7	0.61	182.4	0.62	155.4	0.55	119.5	1.33
2	323.9	0.69	179.3	0.54	146.4	0.35	102.3	1.55
3	235.6	0.59	167.7	0.62	174.8	0.37	135.2	2.02
4	300.2	0.84	167.6	0.64	172.5	0.61	115.2	0.72
5	225.1	0.81	160.5	0.62	190.7	0.61	123.7	1.71
6	254.2	0.65	204.1	0.70	165	0.56	139.9	1.78
7	313.9	0.71	179.5	0.77	166.8	0.49	124.3	1
8	306.3	0.72	167.3	0.74	154.9	0.34	106.5	1.11
9	301.1	0.75	198.9	0.68	183.5	0.53	116.6	0.95
10	242.3	0.86	169.5	0.79	162.9	0.47	117.7	1.33
Avg	279.93	0.72	177.68	0.67	167.29	0.49	120.09	1.35
Stdve	36.47	0.09	14.33	0.08	13.54	0.10	11.52	0.41

Table 9-p. Box F OCS for 0.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	344.9	0.65	316.3	1.06	346.6	0.82	355.2	1.35
2	386.1	0.77	329.4	1.01	299.3	0.67	291.3	1.46
3	321.3	0.89	345.0	0.95	303.4	0.72	287.0	1.41
4	351.6	0.87	304.0	1.14	394.2	1.16	344.3	1.65
5	386.5	1.07	289.7	1.04	348.7	0.82	315.4	1.36
6	448	0.85	301.5	1.09	296.2	0.64	303.3	1.45
7	377.2	0.94	313.7	1.17	299.2	0.49	296.3	1.43
8	449.5	0.96	339.2	0.99	327	0.9	260.7	1.63
9	420	0.92	344.5	1.38	354.1	0.87	231.3	1.07
10	402.7	0.86	321.1	1.01	328.8	0.76	330.3	1.57
Avg	388.78	0.88	320.44	1.08	329.75	0.79	301.51	1.44
Stdve	42.65	0.11	19.01	0.12	31.76	0.18	37.53	0.17

Table 9-q. Box F OCS for 1 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	344.9	0.65	308.3	1.21	312	0.56	203.2	1.73
2	386.1	0.77	287.6	0.92	302.2	0.72	263.0	1.91
3	321.3	0.89	271.0	0.76	301.4	0.73	244.5	1.54
4	351.6	0.87	215.1	0.65	298.4	0.68	262.6	2.26
5	386.5	1.07	204.0	0.48	278.9	0.49	190.6	1.67
6	448	0.85	240.0	0.63	289.6	0.72	236.2	2.07
7	377.2	0.94	230.0	1.44	281.9	0.64	239.7	1.96
8	449.5	0.96	238.8	0.70	259.7	0.85	259.2	2.14
9	420	0.92	221.9	0.73	273.9	0.48	230.0	2.06
10	402.7	0.86	270.1	0.85	283.9	0.54	185.2	1.97
Avg	388.78	0.88	248.68	0.84	288.19	0.64	231.42	1.93
Stdve	42.65	0.11	33.98	0.29	15.65	0.12	29.07	0.22

Table 9-r. Box F OCS for 1.5 inch offset in the length, width, and adjacent directions

	Control		Length		Width		Adjacent	
	Force (lb)	Deflection (in)	Force (lb)	Deflection(in)	Force (lb)	Deflection(in)	Force(lb)	Deflection(in)
1	344.9	0.65	211.5	0.69	251	0.52	239.8	2.25
2	386.1	0.77	210.0	0.97	245.2	0.68	272.0	1.85
3	321.3	0.89	221.6	0.86	246.4	0.64	236.5	2.26
4	351.6	0.87	234.3	0.87	243.3	0.56	211.6	1.71
5	386.5	1.07	238.6	1.02	292.4	0.68	187.4	2.04
6	448	0.85	188.3	0.72	285.7	0.82	157.4	1.42
7	377.2	0.94	217.9	0.79	238.9	0.57	144.2	0.97
8	449.5	0.96	261.5	1.01	290.1	0.59	149.0	1.16
9	420	0.92	247.2	1.01	272.9	0.77	190.2	2.37
10	402.7	0.86	242.8	0.99	189.5	0.67	173.0	1.49
Avg	388.78	0.88	227.37	0.89	255.54	0.65	196.11	1.75
Stdve	42.65	0.11	21.53	0.13	31.20	0.09	42.94	0.49

APPENDIX B. BASIC Programs for calculating “a”, “b” and “Q”

(Box A.BAS)

```
10 'box A
20 L=19 : W=10 'length, width
30 CCS=(446.4+447.5+483.8)/3 'avg control CS
40 DIM X(9),Y(9),OCS(9) 'OSC=offset compression strength
50 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I 'experimental results
60 DATA 0,.5,425.9, .5,0,414.4, .5,.5,421.7
70 DATA 0,1,371.9, 1,0,391.5, 1,1,337.3
80 DATA 0,1.5,338.1, 1.5,0,395.9, 1.5,1.5,292.8
90 S1=0 : S2=0 : S3=0 : S4=0 : S5=0 : FOR I=1 TO 9
100 S1=S1+(X(I)/L)^2 : S2=S2+(X(I)/L)*(Y(I)/W) : S3=S3+(Y(I)/W)^2
110 S4=S4+(1-OCS(I)/CCS)*X(I)/L : S5=S5+(1-OCS(I)/CCS)*Y(I)/W
120 NEXT I
130 A=(S3*S4-S2*S5)/(S1*S3-S2^2) : B=(S1*S5-S2*S4)/(S1*S3-S2^2)
140 PRINT "S1=";S1;"      S2=";S2;"      S3=";S3
150 PRINT "S4=";S4;"      S5=";S5
160 PRINT "a=";A;"      b="B
170 PRINT "short          long          actual CS          pred CS"
180 FOR I=1 TO 9 : PRED=CCS*(1-A*X(I)/L-B*Y(I)/W)
190 PRINT X(I),Y(I),OCS(I),PRED
200 NEXT I
```

RUN

S1= 1.939058E-02 S2= 1.842105E-02 S3= .07

S4= 6.595661E-02 S5= .1472127

a= 1.871453 b= 1.610552

(Box B.BAS)

10 'box B

20 L=19 : W=10 'length, width

30 CCS=(712.8+690+701.3)/3 'avg control CS

40 DIM X(9),Y(9),OCS(9) 'OSC=offset compression strength

50 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I 'experimental results

60 DATA 0,.5,583.2, .5,0,608.0, .5,.5,581.6

70 DATA 0,1,474.7, 1,0,559.1, 1,1,466.8

80 DATA 0,1.5,409.5, 1.5,0,472.3, 1.5,1.5,344.3

90 S1=0 : S2=0 : S3=0 : S4=0 : S5=0 : FOR I=1 TO 9

100 S1=S1+(X(I)/L)^2 : S2=S2+(X(I)/L)*(Y(I)/W) : S3=S3+(Y(I)/W)^2

110 S4=S4+(1-OCS(I)/CCS)*X(I)/L : S5=S5+(1-OCS(I)/CCS)*Y(I)/W

120 NEXT I

130 A=(S3*S4-S2*S5)/(S1*S3-S2^2) : B=(S1*S5-S2*S4)/(S1*S3-S2^2)

140 PRINT "S1=";S1;" S2=";S2;" S3=";S3

150 PRINT "S4=";S4;" S5=";S5

```

160 PRINT "a=";A;"      b="B

170 PRINT "short      long      actual CS      pred CS"

180 FOR I=1 TO 9 : PRED=CCS*(1-A*X(I)/L-B*Y(I)/W)

190 PRINT X(I),Y(I),OCS(I),PRED

200 NEXT

```

RUN

```

S1= 1.939058E-02      S2= 1.842105E-02      S3= .07

S4= .1022515      S5= .2215104

a= 3.022726      b= 2.368981

```

(Box C.BAS)

```

10 'box C

20 L=19 : W=15 'length, width

30 CCS=(469.4+469.4+469.4)/3 'avg control CS

40 DIM X(9),Y(9),OCS(9) 'OSC=offset compression strength

50 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I 'experimental results

60 DATA 0,.5,446.7, .5,0,409.4, .5,.5,410.5

70 DATA 0,1,341.1, 1,0,373.9, 1,1,324.5

80 DATA 0,1.5,333.6, 1.5,0,365.0, 1.5,1.5,240.5

90 S1=0 : S2=0 : S3=0 : S4=0 : S5=0 : FOR I=1 TO 9

100 S1=S1+(X(I)/L)^2 : S2=S2+(X(I)/L)*(Y(I)/W) : S3=S3+(Y(I)/W)^2

```

```

110 S4=S4+(1-OCS(I)/CCS)*X(I)/L : S5=S5+(1-OCS(I)/CCS)*Y(I)/W
120 NEXT I
130 A=(S3*S4-S2*S5)/(S1*S3-S2^2) : B=(S1*S5-S2*S4)/(S1*S3-S2^2)
140 PRINT "S1=";S1;"      S2=";S2;"      S3=";S3
150 PRINT "S4=";S4;"      S5=";S5
160 PRINT "a=";A;"      b="B
170 PRINT "short          long          actual CS          pred CS"
180 FOR I=1 TO 9 : PRED=CCS*(1-A*X(I)/L-B*Y(I)/W)
190 PRINT X(I),Y(I),OCS(I),PRED
200 NEXT I

```

RUN

```

S1= 1.939058E-02      S2= .0122807      S3= 3.111111E-02
S4= 8.967774E-02      S5= .1222909
a= 2.847089      b= 2.806927

```

(Box D.BAS)

```

10 'box D
20 L=19 : W=13 'length, width
30 CCS=(501.5+501.5+501.5)/3 'avg control CS
40 DIM X(9),Y(9),OCS(9) 'OSC=offset compression strength
50 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I 'experimental results

```

```

60 DATA 0,.5,433.6, .5,0,455.8, .5,.5,414.8
70 DATA 0,1,371.7, 1,0,425.5, 1,1,336.8
80 DATA 0,1.5,329.6, 1.5,0,377.1, 1.5,1.5,249.8
90 S1=0 : S2=0 : S3=0 : S4=0 : S5=0 : FOR I=1 TO 9
100 S1=S1+(X(I)/L)^2 : S2=S2+(X(I)/L)*(Y(I)/W) : S3=S3+(Y(I)/W)^2
110 S4=S4+(1-OCS(I)/CCS)*X(I)/L : S5=S5+(1-OCS(I)/CCS)*Y(I)/W
120 NEXT I
130 A=(S3*S4-S2*S5)/(S1*S3-S2^2) : B=(S1*S5-S2*S4)/(S1*S3-S2^2)
140 PRINT "S1=";S1;"      S2=";S2;"      S3=";S3
150 PRINT "S4=";S4;"      S5=";S5
160 PRINT "a=";A;"      b="B
170 PRINT "short          long          actual CS          pred CS"
180 FOR I=1 TO 9 : PRED=CCS*(1-A*X(I)/L-B*Y(I)/W)
190 PRINT X(I),Y(I),OCS(I),PRED
200 NEXT I

```

RUN

```

S1= 1.939058E-02      S2= 1.417004E-02      S3= 4.142012E-02
S4= 9.141522E-02      S5= .1544904
a= 2.651684      b= 2.822684

```

(Box E.BAS)

```

10 'box E

20 L=15 : W=10 'length, width

30 CCS=(280!+280!+280!)/3 'avg control CS

40 DIM X(9),Y(9),OCS(9) 'OSC=offset compression strength

50 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I 'experimental results

60 DATA 0,.5,208.2, .5,0,202.5, .5,.5,211.0

70 DATA 0,1,185.8, 1,0,177.8, 1,1,169.2

80 DATA 0,1.5,177.7, 1.5,0,167.3, 1.5,1.5,120.1

90 S1=0 : S2=0 : S3=0 : S4=0 : S5=0 : FOR I=1 TO 9

100 S1=S1+(X(I)/L)^2 : S2=S2+(X(I)/L)*(Y(I)/W) : S3=S3+(Y(I)/W)^2

110 S4=S4+(1-OCS(I)/CCS)*X(I)/L : S5=S5+(1-OCS(I)/CCS)*Y(I)/W

120 NEXT I

130 A=(S3*S4-S2*S5)/(S1*S3-S2^2) : B=(S1*S5-S2*S4)/(S1*S3-S2^2)

140 PRINT "S1=";S1;"      S2=";S2;"      S3=";S3

150 PRINT "S4=";S4;"      S5=";S5

160 PRINT "a=";A;"      b="B

170 PRINT "short      long      actual CS      pred CS"

180 FOR I=1 TO 9 : PRED=CCS*(1-A*X(I)/L-B*Y(I)/W)

190 PRINT X(I),Y(I),OCS(I),PRED

200 NEXT I

RUN

```

S1= 3.111111E-02 S2= 2.333333E-02 S3= .07

S4= .1655119 S5= .2388215

a= 3.681633 b= 2.184524

(Box F.BAS)

10 'box F

20 L=16 : W=12 'length, width

30 CCS=(388.8+388.8+388.8)/3 'avg control CS

40 DIM X(9),Y(9),OCS(9) 'OSC=offset compression strength

50 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I 'experimental results

60 DATA 0,.5,320.4, .5,0,329.8, .5,.5,301.5

70 DATA 0,1,248.7, 1,0,288.2, 1,1,231.4

80 DATA 0,1.5,227.4, 1.5,0,255.5, 1.5,1.5,196.1

90 S1=0 : S2=0 : S3=0 : S4=0 : S5=0 : FOR I=1 TO 9

100 S1=S1+(X(I)/L)^2 : S2=S2+(X(I)/L)*(Y(I)/W) : S3=S3+(Y(I)/W)^2

110 S4=S4+(1-OCS(I)/CCS)*X(I)/L : S5=S5+(1-OCS(I)/CCS)*Y(I)/W

120 NEXT I

130 A=(S3*S4-S2*S5)/(S1*S3-S2^2) : B=(S1*S5-S2*S4)/(S1*S3-S2^2)

140 PRINT "S1=";S1;" S2=";S2;" S3=";S3

150 PRINT "S4=";S4;" S5=";S5

160 PRINT "a=";A;" b="B

170 PRINT "short long actual CS pred CS"

```
180 FOR I=1 TO 9 : PRED=CCS*(1-A*X(I)/L-B*Y(I)/W)
```

```
190 PRINT X(I),Y(I),OCS(I),PRED
```

```
200 NEXT I
```

```
RUN
```

```
S1= 2.734375E-02      S2= 1.822917E-02      S3= 4.861111E-02
```

```
S4= .1318399      S5= .1942944
```

```
a= 2.875955      b= 2.91843
```

```
(All Boxes.BAS)
```

```
10 'boxes A,B,C,D,E,F together
```

```
20 DIM X(54),Y(54),CCS(54),OCS(54),U(54),V(54),Z(54)
```

```
30 '***** box A data *****
```

```
40 L=19 : W=10 : AVG=(446.4+447.5+483.8)/3
```

```
50 FOR I=1 TO 9 : CCS(I)=AVG : NEXT I
```

```
60 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I
```

```
70 DATA 0,.5,425.9, .5,0,414.4, .5,.5,421.7
```

```
80 DATA 0,1,371.9, 1,0,391.5, 1,1,337.3
```

```
90 DATA 0,1.5,338.1, 1.5,0,395.9, 1.5,1.5,292.8
```

```
100 FOR I=1 TO 9 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I
```

```
110 '***** box B data *****
```

```
120 L=19 : W=10 : AVG=(712.8+690+701.3)/3
```

```

130 FOR I=10 TO 18 : CCS(I)=AVG : NEXT I

140 FOR I=10 TO 18 : READ X(I),Y(I),OCS(I) : NEXT I

150 DATA 0,.5,583.2, .5,0,608.0, .5,.5,581.6

160 DATA 0,1,474.7, 1,0,559.1, 1,1,466.8

170 DATA 0,1.5,409.5, 1.5,0,472.3, 1.5,1.5,344.3

180 FOR I=10 TO 18 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

190 '***** box C data *****

200 L=19 : W=15 : AVG=469.4

210 FOR I=19 TO 27 : CCS(I)=AVG : NEXT I

220 FOR I=19 TO 27 : READ X(I),Y(I),OCS(I) : NEXT I

230 DATA 0,.5,446.7, .5,0,409.4, .5,.5,410.5

240 DATA 0,1,341.1, 1,0,373.9, 1,1,324.5

250 DATA 0,1.5,333.6, 1.5,0,365.0, 1.5,1.5,240.5

260 FOR I=19 TO 27 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

270 '***** box D data *****

280 L=19 : W=13 : AVG=501.5

290 FOR I=28 TO 36 : CCS(I)=AVG : NEXT I

300 FOR I=28 TO 36 : READ X(I),Y(I),OCS(I) : NEXT I

310 DATA 0,.5,433.6, .5,0,455.8, .5,.5,414.8

320 DATA 0,1,371.7, 1,0,425.5, 1,1,336.8

330 DATA 0,1.5,329.6, 1.5,0,377.1, 1.5,1.5,249.8

340 FOR I=28 TO 36 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

```

```

350 '***** box E data *****
360 L=15 : W=10 : AVG=280
370 FOR I=37 TO 45 : CCS(I)=AVG : NEXT I
380 FOR I=37 TO 45 : READ X(I),Y(I),OCS(I) : NEXT I
390 DATA 0,.5,208.3, .5,0,202.5, .5,.5,211.0
400 DATA 0,1,185.8, 1,0,177.8, 1,1,169.2
410 DATA 0,1.5,177.7, 1.5,0,167.3, 1.5,1.5,120.1
420 FOR I=37 TO 45 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I
430 '***** box F data*****
440 L=16 : W=12 : AVG=388.8
450 FOR I=46 TO 54 : CCS(I)=AVG : NEXT I
460 FOR I=46 TO 54 : READ X(I),Y(I),OCS(I) : NEXT I
470 DATA 0,.5,320.4, .5,0,329.8, .5,.5,301.5
480 DATA 0,1,248.7, 1,0,288.2, 1,1,231.4
490 DATA 0,1.5,227.4, 1.5,0,255.5, 1.5,1.5,196.1
500 FOR I=46 TO 54 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I
510 '***** do best fit *****
520 S1=0 : S2=0 : S3=0 : S4=0 : S5=0 : FOR I=1 TO 54
530 S1=S1+U(I)^2 : S2=S2+U(I)*V(I) : S3=S3+V(I)^2
540 S4=S4+(1-Z(I))*U(I) : S5=S5+(1-Z(I))*V(I)
550 NEXT I
560 A=(S3*S4-S2*S5)/(S1*S3-S2^2) : B=(S1*S5-S2*S4)/(S1*S3-S2^2)

```

```

570 PRINT "S1=";S1;" S2=";S2;" S3=";S3
580 PRINT "S4=";S4;" S5=";S5
590 PRINT "a=";A;" b="B
600 PRINT "actual CS   pred CS   %error"
610 FOR I=1 TO 18 : PRED=CCS(I)*(1-A*U(I)-B*V(I))
620 PRINT OCS(I),PRED,100*(PRED-OCS(I))/OCS(I)
630 NEXT I:INPUT BOGUS
640 PRINT "actual CS   pred CS   %error"
650 FOR I=19 TO 36 : PRED=CCS(I)*(1-A*U(I)-B*V(I))
660 PRINT OCS(I),PRED,100*(PRED-OCS(I))/OCS(I)
670 NEXT I:INPUT BOGUS
680 PRINT "actual CS   pred CS   %error"
690 FOR I=37 TO 54 : PRED=CCS(I)*(1-A*U(I)-B*V(I))
700 PRINT OCS(I),PRED,100*(PRED-OCS(I))/OCS(I)
710 NEXT I

```

RUN

```

S1= .1360172   S2= .1048554   S3= .3311423
S4= .646653    S5= 1.078602
a= 2.967626    b= 2.317526

```

(All Boxes.Q.BAS)

```

10 'boxes A,B,C,D,E,F together

20 DIM X(54),Y(54),CCS(54),OCS(54),U(54),V(54),Z(54)

30 '***** box A data *****

40 L=19 : W=10 : AVG=(446.4+447.5+483.8)/3

50 FOR I=1 TO 9 : CCS(I)=AVG : NEXT I

60 FOR I=1 TO 9 : READ X(I),Y(I),OCS(I) : NEXT I

70 DATA 0,.5,425.9, .5,0,414.4, .5,.5,421.7

80 DATA 0,1,371.9, 1,0,391.5, 1,1,337.3

90 DATA 0,1.5,338.1, 1.5,0,395.9, 1.5,1.5,292.8

100 FOR I=1 TO 9 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

110 '***** box B data *****

120 L=19 : W=10 : AVG=(712.8+690+701.3)/3

130 FOR I=10 TO 18 : CCS(I)=AVG : NEXT I

140 FOR I=10 TO 18 : READ X(I),Y(I),OCS(I) : NEXT I

150 DATA 0,.5,583.2, .5,0,608.0, .5,.5,581.6

160 DATA 0,1,474.7, 1,0,559.1, 1,1,466.8

170 DATA 0,1.5,409.5, 1.5,0,472.3, 1.5,1.5,344.3

180 FOR I=10 TO 18 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

190 '***** box C data *****

200 L=19 : W=15 : AVG=469.4

210 FOR I=19 TO 27 : CCS(I)=AVG : NEXT I

220 FOR I=19 TO 27 : READ X(I),Y(I),OCS(I) : NEXT I

```

```

230 DATA 0,.5,446.7, .5,0,409.4, .5,.5,410.5

240 DATA 0,1,341.1, 1,0,373.9, 1,1,324.5

250 DATA 0,1.5,333.6, 1.5,0,365.0, 1.5,1.5,240.5

260 FOR I=19 TO 27 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

270 '***** box D data *****

280 L=19 : W=13 : AVG=501.5

290 FOR I=28 TO 36 : CCS(I)=AVG : NEXT I

300 FOR I=28 TO 36 : READ X(I),Y(I),OCS(I) : NEXT I

310 DATA 0,.5,433.6, .5,0,455.8, .5,.5,414.8

320 DATA 0,1,371.7, 1,0,425.5, 1,1,336.8

330 DATA 0,1.5,329.6, 1.5,0,377.1, 1.5,1.5,249.8

340 FOR I=28 TO 36 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

350 '***** box E data *****

360 L=15 : W=10 : AVG=280

370 FOR I=37 TO 45 : CCS(I)=AVG : NEXT I

380 FOR I=37 TO 45 : READ X(I),Y(I),OCS(I) : NEXT I

390 DATA 0,.5,208.3, .5,0,202.5, .5,.5,211.0

400 DATA 0,1,185.8, 1,0,177.8, 1,1,169.2

410 DATA 0,1.5,177.7, 1.5,0,167.3, 1.5,1.5,120.1

420 FOR I=37 TO 45 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

430 '***** box F data*****

440 L=16 : W=12 : AVG=388.8

```

```

450 FOR I=46 TO 54 : CCS(I)=AVG : NEXT I

460 FOR I=46 TO 54 : READ X(I),Y(I),OCS(I) : NEXT I

470 DATA 0,.5,320.4, .5,0,329.8, .5,.5,301.5

480 DATA 0,1,248.7, 1,0,288.2, 1,1,231.4

490 DATA 0,1.5,227.4, 1.5,0,255.5, 1.5,1.5,196.1

500 FOR I=46 TO 54 : U(I)=X(I)/L : V(I)=Y(I)/W : Z(I)=OCS(I)/CCS(I) : NEXT I

510 '***** do best fit *****

520 S1=0 : S2=0 : FOR I=1 TO 54

530 S1=S1+(1-Z(I))*(U(I)+V(I)) : S2=S2+(U(I)+V(I))^2

550 NEXT I

560 A=S1/S2

570 PRINT "S1=";S1;" S2=";S2

590 PRINT "Q=";A

600 PRINT "actual CS   pred CS   %error"

610 FOR I=1 TO 18 : PRED=CCS(I)*(1-A*U(I)-A*V(I))

620 PRINT OCS(I),PRED,100*(PRED-OCS(I))/OCS(I)

630 NEXT I:INPUT BOGUS

640 PRINT "actual CS   pred CS   %error"

650 FOR I=19 TO 36 : PRED=CCS(I)*(1-A*U(I)-A*V(I))

660 PRINT OCS(I),PRED,100*(PRED-OCS(I))/OCS(I)

670 NEXT I:INPUT BOGUS

680 PRINT "actual CS   pred CS   %error"

```

```
690 FOR I=37 TO 54 : PRED=CCS(I)*(1-A*U(I)-A*V(I))
```

```
700 PRINT OCS(I),PRED,100*(PRED-OCS(I))/OCS(I)
```

```
710 NEXT I
```

```
RUN
```

```
S1= 1.725256    S2= .6768703
```

```
Q= 2.548872
```

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