A DESIGN OF A PHOTO-ELASTIC STRESS ANALYSIS MACHINE

THESIS FOR THE DEGREE OF B. S.

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Strains + stresses

Civil engineering Bridges + roofs

A Design of a Photo-Elastic

Stress Analysis Machine

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Introduction

Great progress has been made in the science of design in spite of the severe handicaps under which the designer works. Heretofore, the designer has had access to several methods of design but the inherent weaknesses of most of the methods are readily apparent. For instance, in judging the safe loads to be applied to a structure the designer has access to service records or destruction tests. The lack of service records of new structures with increased loads is a severe handicap. Destruction tests are inadequate in that they seldom indicate methods of improvement. Improvement could be made only by constructing numerous types of details and destroying them under load. During the construction of the Mid-Hudson Bridge, there was some doubt as to the strength of a certain detail. To prove its strength, an exact replica of the detail had to be constructed and destroyed under load.

The designer also uses, in conjunction with the foregoing methods, current information regarding the stress fields of the structure, physical data of the materials used, and the theories of strength best suited to the problem. The use of these items involves the computation of stresses by analytical methods. In problems involving any but the most simple stress distribution systems encounters insurmountable mathematical difficulties. It is only in a very limited number of cases that strict mathematical formulae may be applied. These are based on theories of elasticity, temperature stress, etc., and are efficacious only when the assumptions of load conditions are reproduced in the field.

In the final analysis, the failure of the methods of the designer is borne out by the fact that the designer doubts the accuracy of his work and must apply a safety factor to insure the stability and safe functioning of the structure.

The study of the effects of loads on structures by photo-elastic methods provides a practical, complete and accurate analysis of stress distribution. The following discussion presents a design and method of use of an apparatus for stress analysis by photo-elasticity.

CONSTRUCTION

In the design of this Photo-Elastic apparatus two major considerations were kept constantly in mind, namely, simplicity and economy. Several assumptions concerning the optical apparatus were necessary in order to permit omitting optical apparatus as used in more complete and complicated set-ups. These assumptions are upheld in some cases by actual experimentation and in others by theory.

It was assumed that the effect of temperature was negligible in comparison with the range of visible rays. No attempt has been made in this design to eliminate heat radiations by means of a water-cooler. The effect of heat on the polarizing unit is also neglected due to the type of prism used. Neglect of temperature consideration is immaterial as borne out by the experiment. See Ref. 1.

Another presumption was necessary in view of the economical design. This was concerning the optical accuracy and grade of glass used in the lenses. The conclusion reached was that a glass of the grade of good watch glass was sufficiently accurate for the purpose of this apparatus. All results obtained from the apparatus are purely relative. Each ray of light passing through the optical parts is acted upon within the limits of accuracy noticeable to the eye. The primary consideration in the construction of the apparatus is to keep the optical axes of the various parts on the same line as slight variations would cause a leakage which would be detrimental to the production of a clear image.

The Lamp House - Plate II

The lamp house and source of light may need an explanation. In order to secure an accurately controlled light source of sufficient intensity, a 500 watt bulb and parabolic mirror combination was utilized. The light source is fixed, care being taken to have the source on the optical axis of the apparatus. The parabolic mirror is adjustable only on the horizontal plane. This permits focusing the light on the first condenser. See Fig. 1. The lamp house is just large enough to admit the mirror. This eliminates all side or vertical displacement. The bulb should be silvered on the side opposite the mirror, the silvered area being large enough to prohibit direct light from the source from entering the condensing lens. The addition of the silvered area also increases the amount of light by approximately 50 percent. The interior of the lamp house is finished in a dull black to eliminate reflection interference.

Due to the variety of shapes and sizes of bulbs, it was decided to allow sufficient room for the bulb but not to hold the builder to a specific design. It would be a simple matter to insert a socket in the base of the lamp house of sufficient height to bring the source of light up to the optical axis of the instrument.

The Polarizing Unit - Plate III

The Condenser Mount I and the polarizing unit were made rigid with respect to each other for a specific reason. It was felt that elimination of minor adjustments would facilitate the operation of the machine as a whole. The spacer tube should be constructed of just sufficient length to bring the condenser and nicol prism in exact focus when the polarizing element is in place. The condenser lens is held in place by a retaining ring and the spacer tube. The polarizing mount of this unit should be rotatable through ninety degrees in either direction. This necessitates a snug fit of the nicol prism mounting in the condenser tube. For experimental purposes it would be possible to calibrate the tube in degrees of rotation.

Due to the small dimensions of the nicol prism and difficulty of machining a part to fit the prism the following arrangement was deemed necessary. The prism holder is a brass casting machined to the specified dimensions. A hole five eighths of an inch in diameter should be drilled through the casting with the center of the hole exactly on the optical axis. The condensing lens should be mounted in position and light projected through it from the lamp house. The nicol prism should then be cemented into position with plaster of paris. Theoretically, the center of the prism should be six and one half inches from the face of the lens. Accurate setting of the nicol prism is essential and care should be exercised in this phase of construction.

The quarter wave plate attachment needs no adjustment. It should be constructed to fit snugly enough to allow no excessive play yet should be capable of removal without disturbing the adjustment of the various other units.

Condenser and Analyser

The analyser unit is essentially the same as the polarizing unit. The respective positions of the quarter-wave plate and the nicol prism being reversed. This necessitated placing the knurled shoulder on the opposite side of the casting from the quarter-wave plate. In this unit, also, the quarter-wave plate is removable and the nicol prism mount is rotatable. Provision has been made, in the condenser tube, for the removal of the quarter-wave plate.

Condenser Mount III

Condenser mount III was designed as a single unit for compactness. The lenses are fixed in their respective tubes yet are adjustable relative to each other. The retaining rings of all lenses are identical.

The Tension Frame

The operation of this Photo-Elastic Apparatus necessitates using a piece of material cut from the same sheet as the model as a standard. This operation will be discussed in detail later. The standard must have known loads applied, the loads being equivalent tensile loads. The Tension Frame used in this design has been simplified greatly. To eliminate bulky apparatus and complicated construction the following method was devised. The load is applied by a hand wheel and screw arrangement through a movable chuck to the standard. The lower chuck has no vertical movement but it can be rotated. The loads are measured by a strain gauge appliance fastened to the two chucks, measuring the amount by which the standard elongates under the load. A statement to the effect that, "Most isotrapic materials, such as celluloid, elongate in a straight line ratio nearly to the point of failure", Ref. II, was the basis of the device. It will be necessary to calibrate the device and if large differences are found in the Moduli of Elasticity of the different standards, a coefficient must be applied to each standard. The calibration of the device is a simple matter. The method of procedure would be as follows, the apparatus could be up-ended, known weight applied to the movable chuck and the position of the gauge noted for each load. Thus every division passed over by the indicator would represent a load of so many pounds, etc. The calibration unit is placed in a heavy ring to obviate any displacement due to the bending of the frame. The entire unit is rotatable about 2 axes -- one vertical and one horizontal. This permits placing of the standard to correspond with any direction of lines of stress in the model.

Theory

The results obtained from the apparatus will be more clearly understood by the operator if he has a knowledge of the theory underlying this method of analysis.

The fundamental principles upon which the theory of Photo-Elasticity is based are stated by Prof. Coker as follows;

1. "The distribution of stress through any loaded isotropic elastic structure is independent of the material of which the structure is made and depends solely on the form of the structure and the way in which it is made."

2. "A transparent isotropic material, such as glass or celluloid, acquires doubly refracting properties when stressed differently in different directions, and the degree to which these properties are produced depends on the differences between the principle stresses in the material."

By "Isotropic", in the first principle, it is meant that the material has the same physical properties in every direction. This class includes most materials used in construction, including, concrete and steel. This property is also the basis for assuming that the materials used in the construction of the model is stressed in exactly the same manner as if the structure were built of steel or concrete.

The doubly refracting properties, principle 2., of isotropic materials when stressed have been thoroughly investigated. Ref. II. The amount of refraction depends solely on the differences in the principal stresses. Any element in a stressed material may be thought of as being acted upon by three principal stresses which are mutually perpendicular. They are referred to as P, Q, and R stresses. The P and Q stresses alone are considered in this work. The elimination of the R stress is made possible by using plate structures and considering only the stresses acting in the plane of the structure.

A discussion of the phenomenon of polarization of light is deemed unnecessary except in regard to the action of a stressed specimen on the light.

The effect of a quarter-wave plate on plane polarized light is to retard one vibration a quarter of a wave length with respect to the next vibration. This is known as circularly polarized light and may be thought of as having a horizontal and a vertical component vibrating a quarter of a wave length apart in time and phase.

When this circularly polarized light is projected through a stressed specimen, the same effect is produced as with the retardation plate except that the directions of vibrations are the same as the lines of principal stress or P and Q for any point in the specimen. If the principal stresses P and Q differ in intensity, the relative retardation of the vibrations is proportional to the difference $(P - Q)$.

Thus it can be seen that if a mono chromatic light is used, the color would be produced when the difference between P and Q is great enough. The color would be totally extinguished where P and Q are equal. This is a condition of zero stress. If the difference between P and Q is sufficient to produce a retardation of one wave length of the light, black will again result.

To apply this principle to ordinary light, it is first necessary to state that different colors are retarded different amounts by the same stress. Passing ordinary light through a stressed specimen will give rise to the typical "interference colors of the crossed Nicol arrangement". It will be noticed that with celluloid used as the specimen a definite series of colors will be obtained.

With the production of the series of colors we have also obtained a measure of the intensity of the difference (P - Q). A discussion of the method used to obtain (P + Q) will appear later.

When the quarter wave plates are removed the colors no longer give a measure of (P - Q) except where the direction of the principal stresses is at 45 degrees to the plane of the crossed Nicols. When the directions of stress coincide with this plane of polarization, the light is all cut out and dard bands are super-imposed on the image. These dark bands correspond to the locus of points of the same principal stress direction. Therefore, by rotating the prisms, keeping them always in the crossed position, a map of the directions of the principal stresses may be procured. For example, with the Nicols in a normal position a set of isoclinics marked 0 degrees may be drawn connecting the darkest points on the image. If the prisms are rotated 5 degrees, a new set of lines marked 5 degrees are drawn. In this connection it may be necessary to enumerate several fundamental rules governing the charting.

1. The curvature of a stress line varies continuously if at all.

2. Parallel stress lines correspond to uniform stress; convergent lines to increasing stress and diverging lines to diminishing stress.

3. No two stress lines of any <u>one</u> system can intersect or merge into each other.

4. Along any free boundary of a structure, one system of stress lines is tangential and the other is at right angles to the boundary. Ref. III.

A further discussion of the charting of stress lines will be made under the topic of use of the instrument.

In the foregoing discussion it was stated that a measure of the intensity of (P -Q) was obtained. It now becomes necessary to obtain a quantitative measure of the difference between these principal stresses. An accurate measure, in pounds, can be obtained by the following method:

Place a standard, cut from the same material as the model, in a calibration member or tension frame. Adjust the standard to coincide with the image of the model. From the nature of the loading of the model it will be simple to decide which of the principal stresses is tension. The bar of celluloid is set at right angles to this direction of maximum tension. A load is applied to the standard until the super-imposed images of the two pieces are dark. Note the amount of tension applied to the standard. The tension applied may be called "T". This tension T is the force necessary to cancel the effect of the stressed model on the polarized light, or, in other words, is the difference between the principal stresses (P - Q); or,

$$P - Q = T. \quad (1)$$

From the definition of two-dimension stress it can be seen that the stress perpendicular to an unloaded boundary of a model is necessarily zero. From this it can be seen that a principal stress may be obtained directly from the above equation. However, this is a special case and not applicable to internal areas. It now becomes necessary to evolve a new equation to be used in conjunction with the above.

When a load is applied to a material, there is a change of thickness of the material proportional to the sum of the principal stresses. By the use of Poisson's Ratio for the material and calibration of the change in thickness, the sum of the principal stresses or $(P + Q)$ could be computed. The exact value of Poisson's Ratio might be difficult to obtain so an alternative method is suggested.

A simple tension member, the standard previously mentioned, is loaded until its change of thickness is equal to the change of thickness of the model. Therefore $(P + Q)$ of the standard equals $(P\ \ Q)$ of the desired part of the model equals T', the tension applied to the standard,

or,

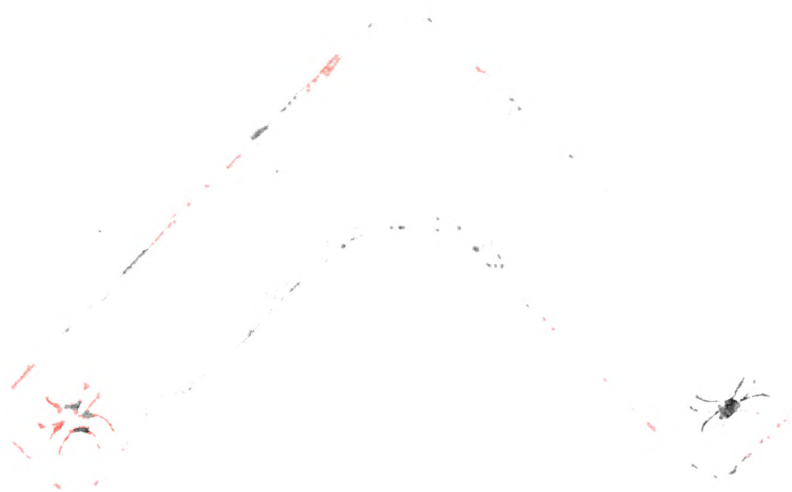
$$P + Q = T'. \qquad (2)$$

The $P + Q$ terms of these equations are identical allowing a simultaneous solution of the equations, from which,

$$P = \tfrac{1}{2}(T' + T) \qquad (3)$$

$$Q = \tfrac{1}{2}(T' - T) \qquad (4)$$

The quantitative values of $P + Q$ together with the charts showing principal stress directions and concentrations complete the results for ordinary problems and form the basis for further experimental studies.



The above sketch is an example of the types of images obtained with the Photo-Elastic Stress Analysis Machine. The colors are the interference colors of the crossed Nicol arrangement.

Procedure

The following procedure, or use of the apparatus, is generally used only when the fullest information concerning the stresses in the structure is desired. In many instances, modifications of these suggestions will suffice for the problem at hand. For instance, if a model is to be examined for the purpose of finding the weak points in the design, all that is necessary is to examine the model with circularly polarized light. The highly stressed areas are clearly recognizable by the colors of the image. The weak portions of the design will be immediately apparent.

To obtain complete information it is necessary first to determine the directions of the Principal Stresses P Q. A model, cut from a sheet of celluloid or Pyralin $\frac{1}{4}$" thick, to the exact reduced scale dimensions of the structure to be examined, is mounted in the beam of plane polarized light as shown on Plate I. This means that the quarter wave plates are removed from the apparatus. The polarizing axes of the polarizing and analyzing units are mutually perpendicular, preferably, one horizontal and one vertical.

The image of the model will appear on the screen with black lines and areas. These lines and areas represent portions of the model in which the condition of stress is such that no effect is made on the polarized light. The principal stresses may be parallel and perpendicular to the polarizing axis or they may be equal to each other or zero. With the axes of the polarizer and analyser in this position, the darkest areas are outlined and the central part of the dark bands are marked. The polarizer and analyser are then rotated to a new position and a new set of lines drawn on the above chart. Each line is marked with its degree of rotation, such as 0 deg., 5 deg., 10 deg., etc. These lines are the so-called isoclinics previously mentioned. In interpreting the information of the charts, the general case is assumed in which one of the principal stresses is perpendicular to the axis of the polarizer. If at any point on a line marked 5 deg., another line is drawn inclined at 5 deg. to the horizontal, this second line will represent the direction of one of the principal stresses for the corresponding point on the model. These second lines may be placed so as to form smooth curves. It is necessary to adhere strictly to the rules stated in the discussion of the theory of the apparatus. The stress line diagram now obtained is a reproduction of the directions of only one principal stress. The stress diagram of the other principal stress is obtained by drawing another system of lines which intersect the first system perpendicularly. This complete diagram is the stress system of the structure.

Knowing the stress directions, it is now necessary to find the difference in principal stresses (P - Q). The model is set up as before with the polarizer and analyser in their original positions. The quarter wave plates are then set in with their optic axes mutually perpendicular and inclined at forty-five deg. to the horizontal. An estimate as to the intensity of stress may be obtained by a study of the image. The color of the fringes depends on the difference (P - Q). To find the quantitative value of (P - Q), mount the standard in the calibration unit, adjust to the desired position, apply the load of the image of the portion of the model under observation is dark, and note the deflection of the scale. From this deflection compute the tensile load applied. This is the value of "T."

To compute the value of (P + Q), measure the thickness of the portion of the model under consideration, without load. Apply the desired load and again measure the same portion of the model. Apply a load on the calibration unit such that the standard changes in thickness to correspond exactly with the change in the model. The tensile load applied is T'. From the equations (3) and (4),

$$P = \frac{1}{2}(T' + T)$$

$$Q = \frac{1}{2}(T' - T)$$

With the values of P, and Q and the directions in which they act the stresses of the structure due to the known load have been completely analyzed.

A permanent record of the images may be obtained by using Finlay plates, C. W. Beck, 305 E. Fourth St., N. Y. C., for the colored images, and ordinary photographic negatives for the dark band images.

Bibliography

Ref.

I Byrne and Oberhauer - Masters thesis - University of Montana.

II Prof. Coker - General Electric Review.

III Prof. Coker - Photo-Elastic Apparatus Adam Hilger, Limited - London.

General.

R. V. Baud - Westinghouse, Electric & Mfg. Co.

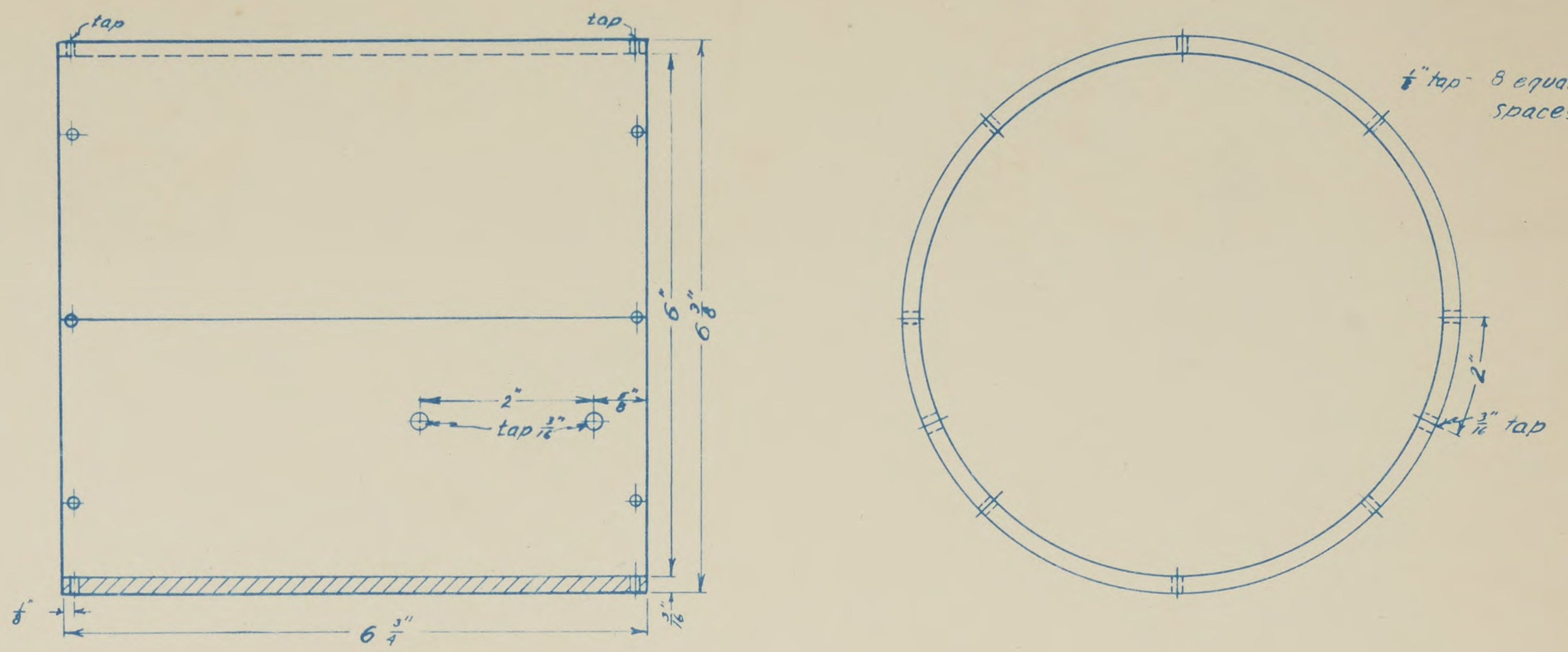
Eastman Kodak Co., Rochester, N. Y.

Baush & Lomb Optical Co., Rochester, N. Y.

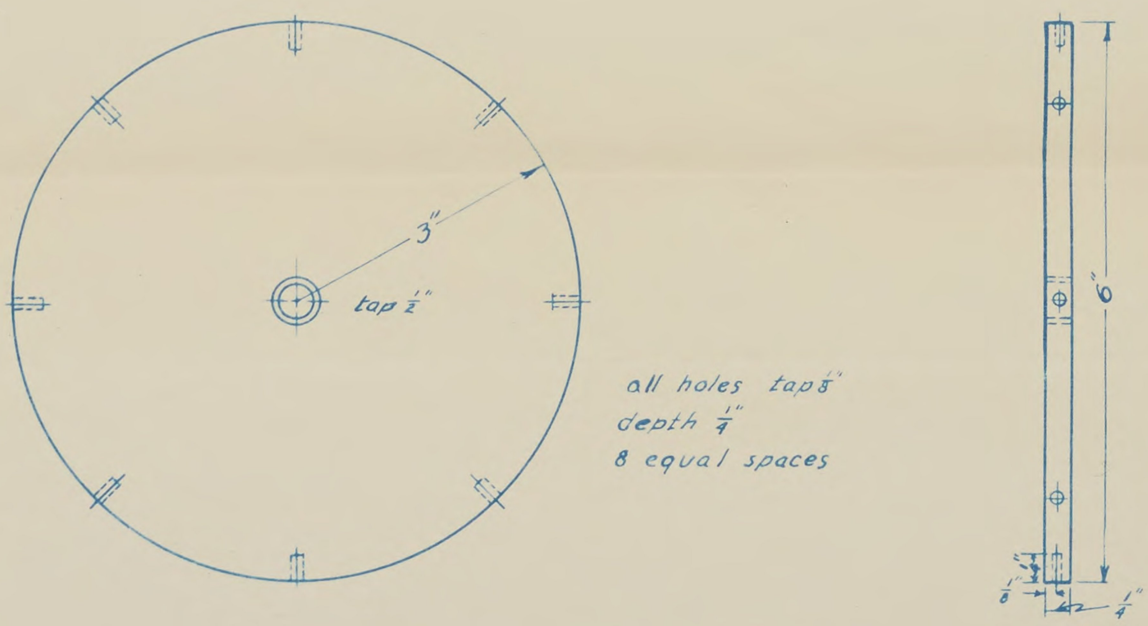
Du Pont Viscoloid Co., Wilmington, Del.


OPTICAL APPARATUS ASSEMBLY
LAMP MOUNT
POLARIZER
MODEL
CONDENSER MOUNT
CALIBRATION MEMBER
CONDENSER & ANALYZER
SCREEN
PHOTO-ELASTIC APPARATUS
DESIGNED C.W. NICHOLAS
PLATE I

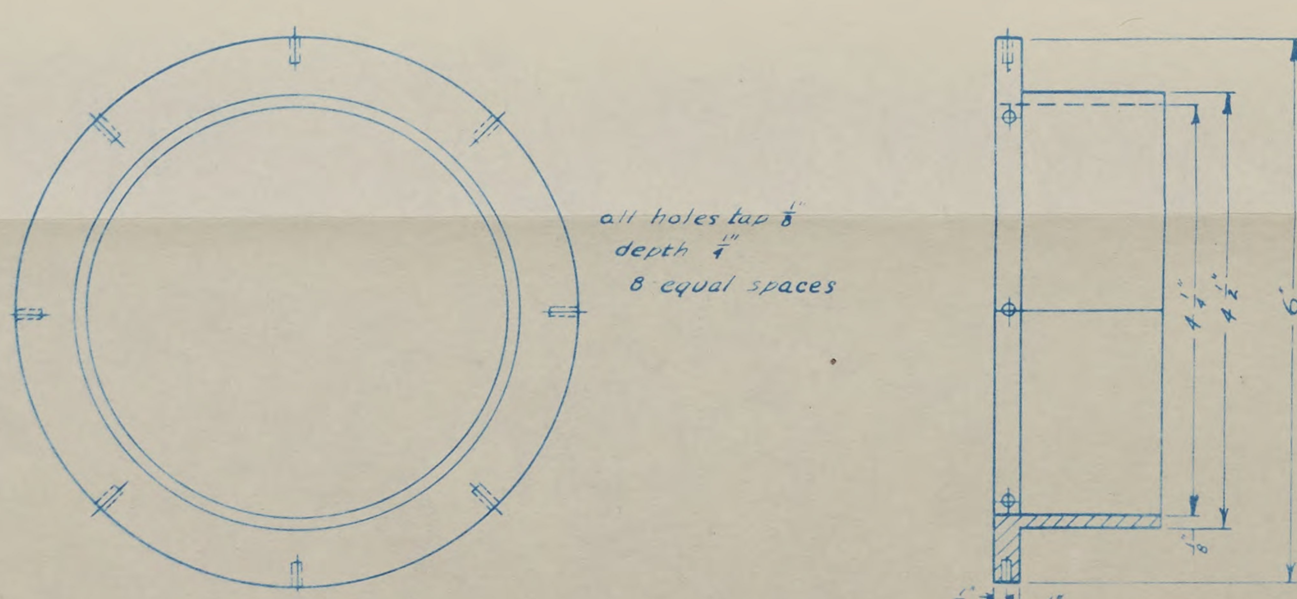
LAMP-HOUSE DETAILS



Lamp-house Casing



Rear-End piece



Front End piece

PHOTO-ELASTIC APPARATUS

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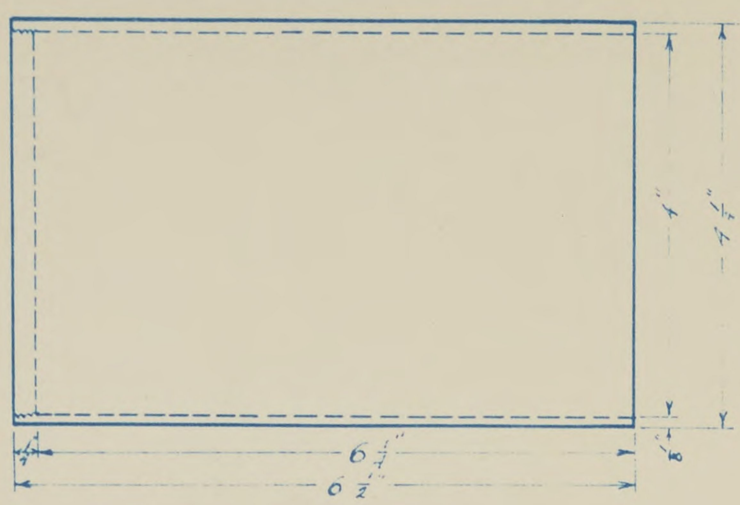
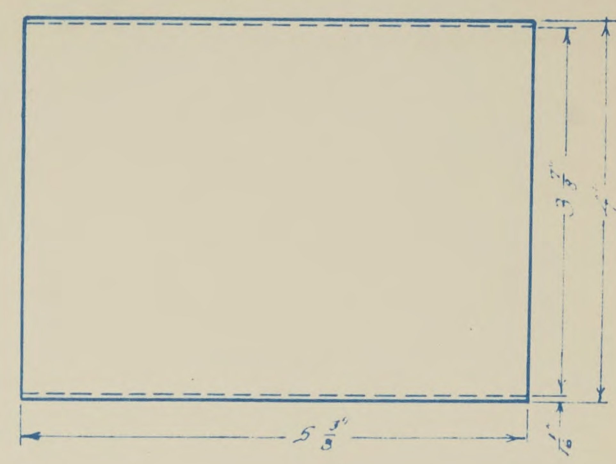
PLATE II

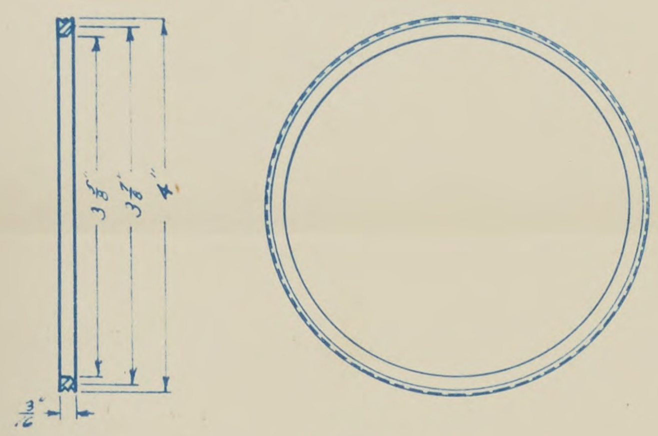
POLARIZING UNIT DETAILS



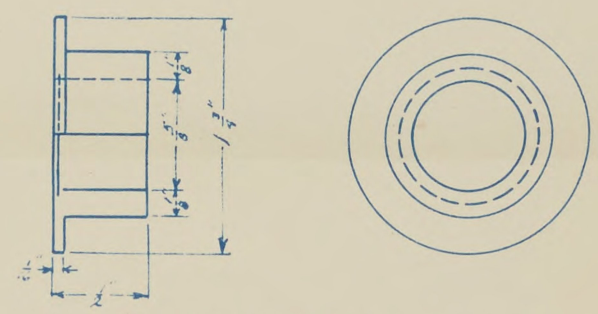
Condenser Tube-I



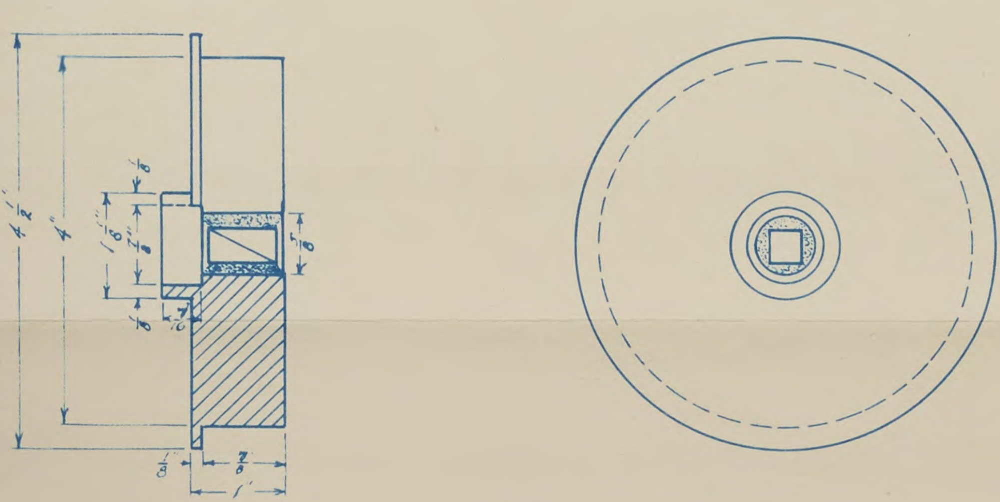
Spacer



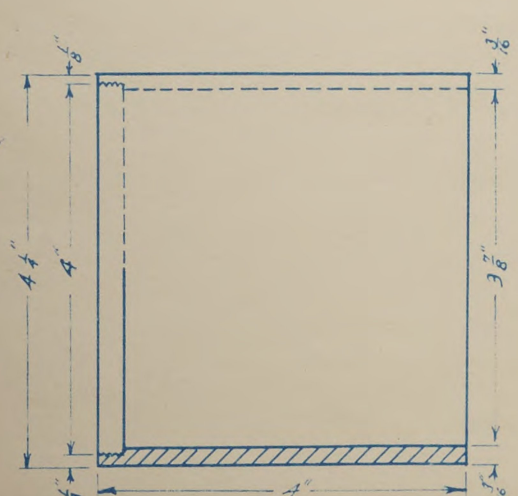
Retaining Ring



Quarter wave plate mount



Nicol prism Mount.



Condenser Mount II

PHOTO-ELASTIC APPARATUS

THESIS

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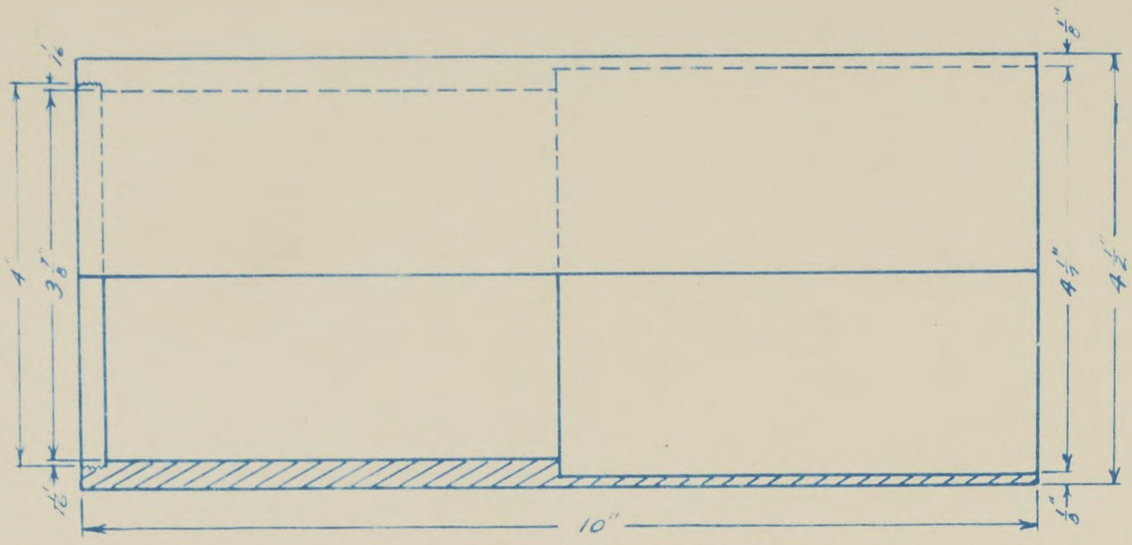
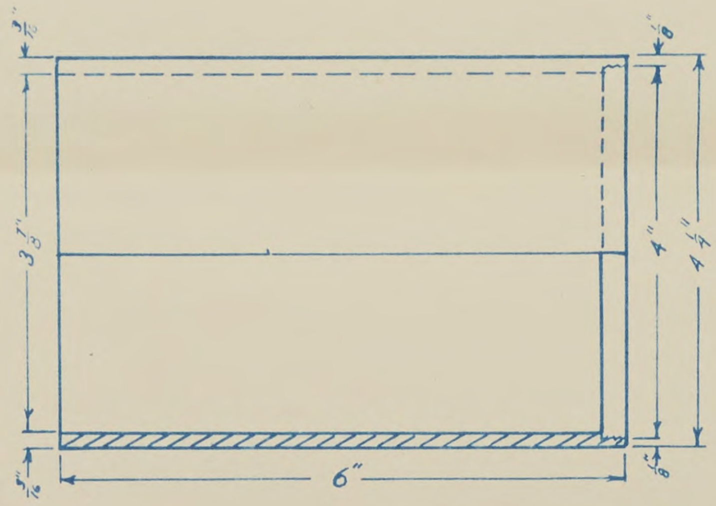
PLATE III

CONDENSER MOUNT III

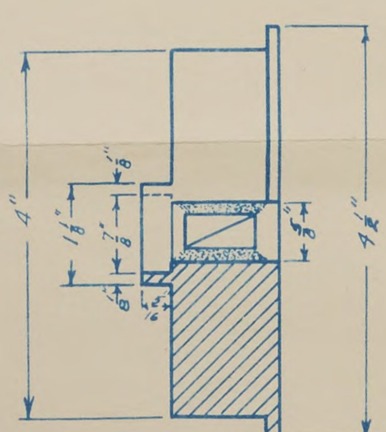


All retaining rings are as shown on Plate III

Outer Tube



Inner Tube



Note:

All optical casings and castings are of brass or bronze.

Analyzer Mount

PHOTO-ELASTIC APPARTUS

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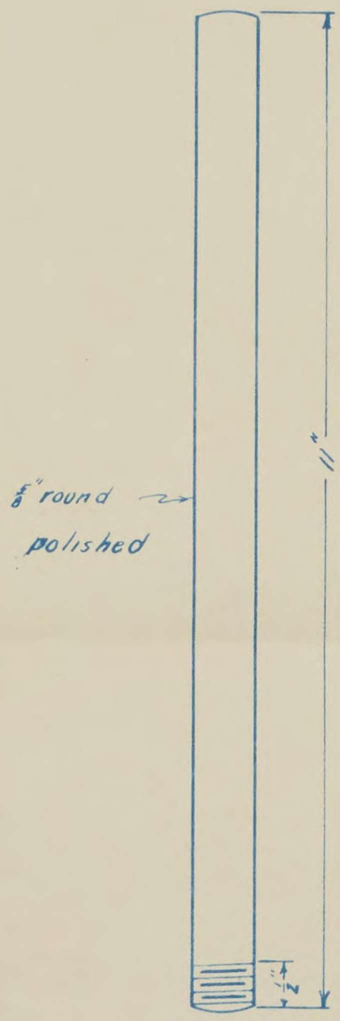
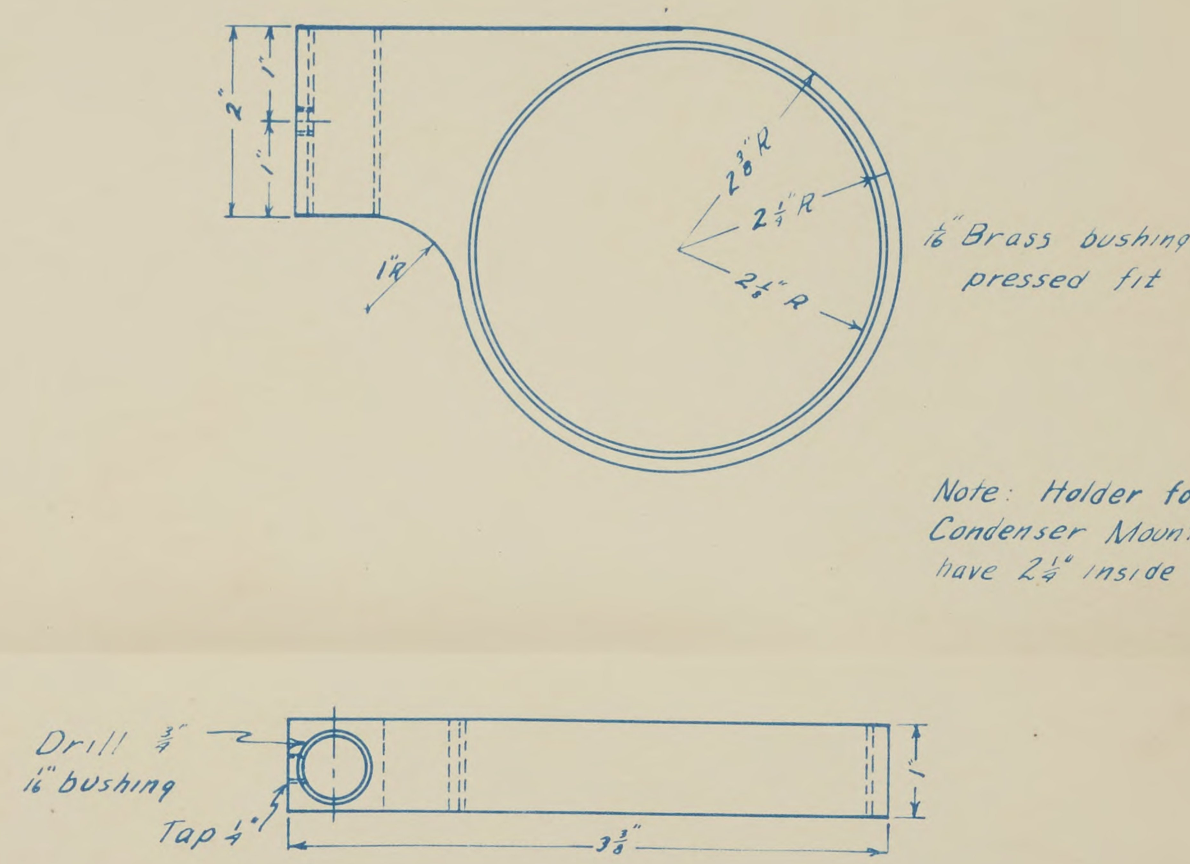
PLATE IV

DETAILS OF SUPPORTS

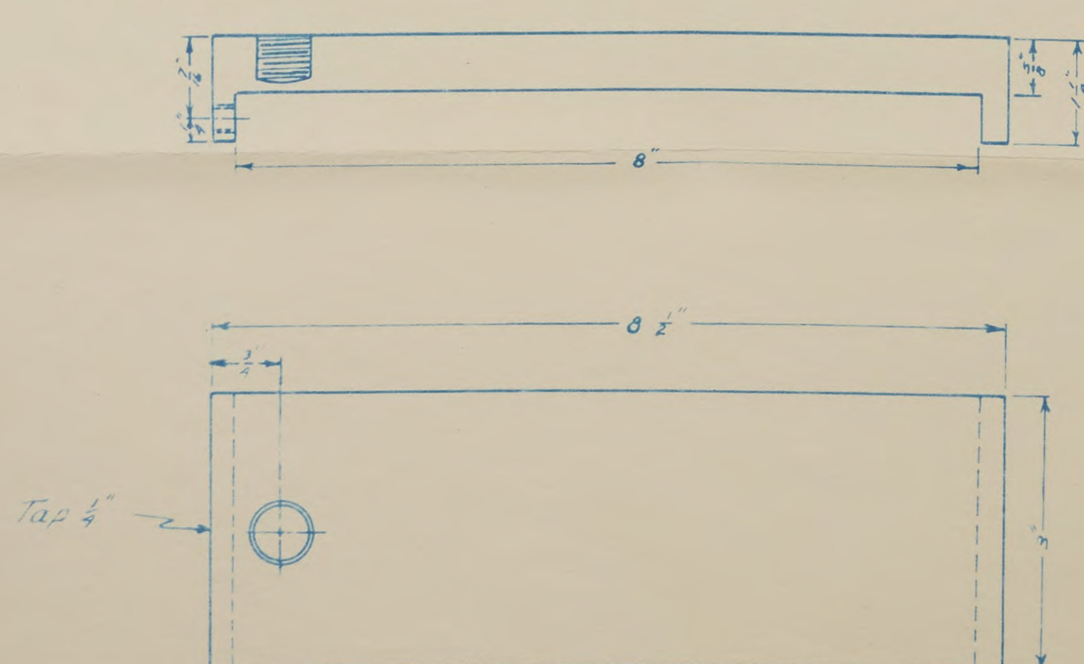


Holder Rod



Note: Holder for Condenser Mount III must have 2 1/4" inside radius.

Supporting Holder



Holder Base

PHOTO-ELASTIC APPARATUS

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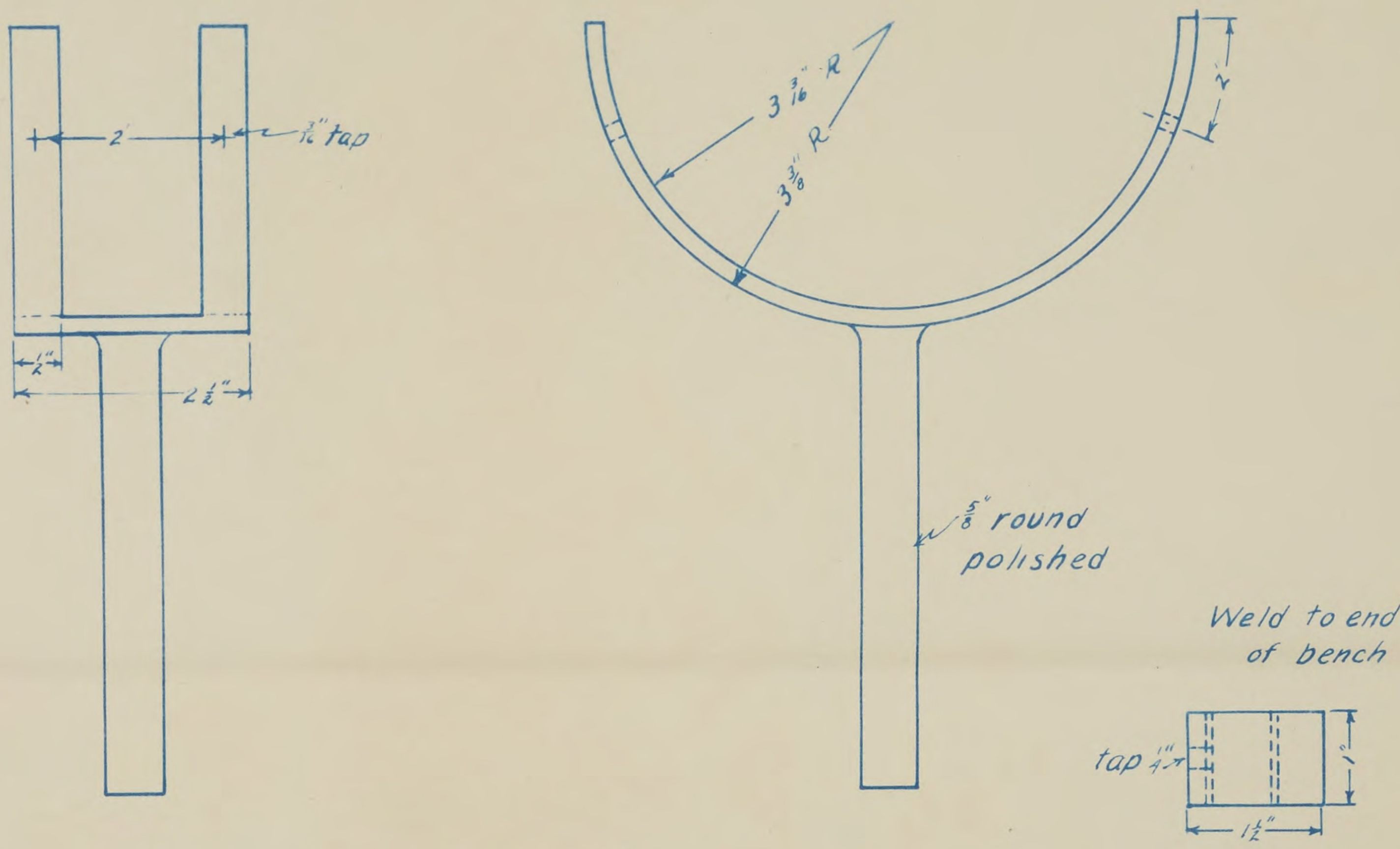
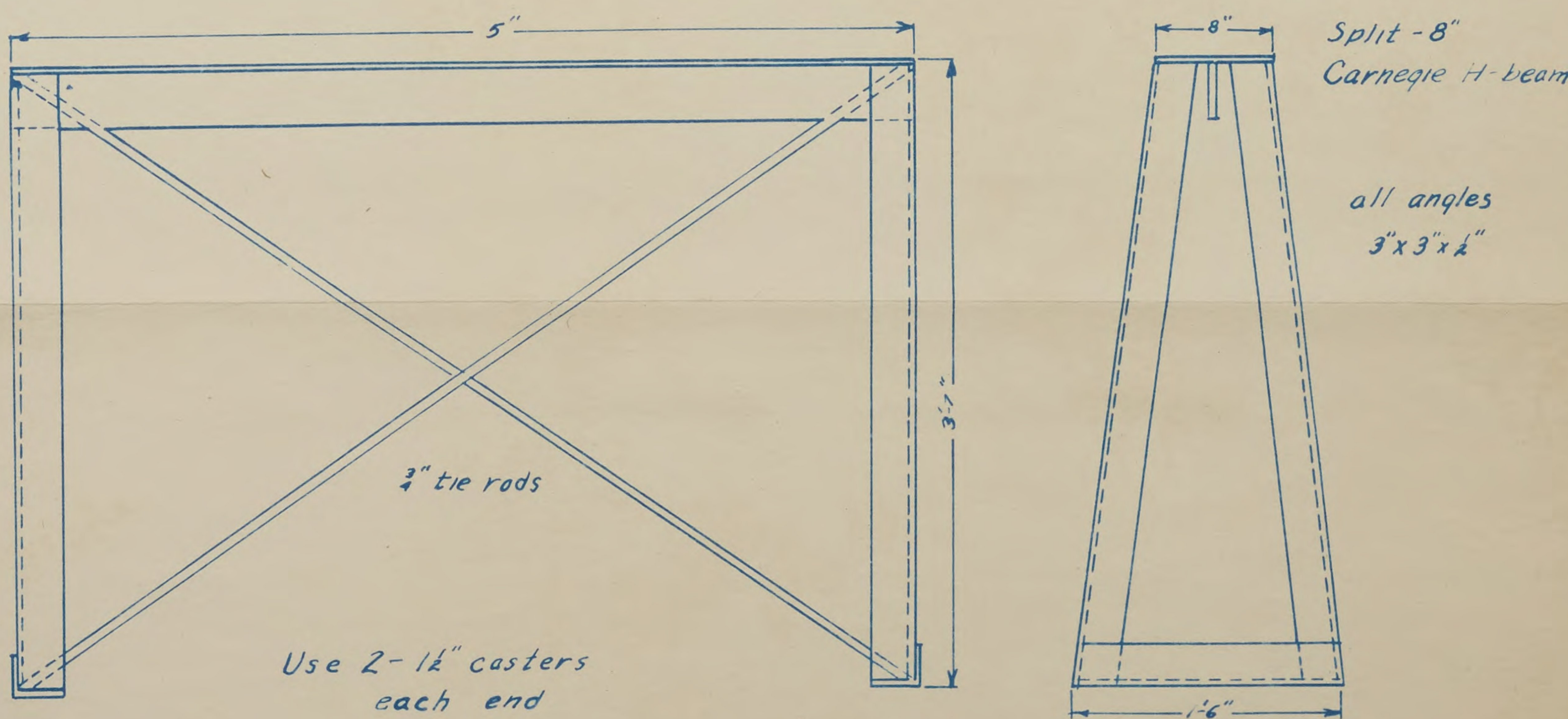
PLATE V

DETAILS OF SUPPORTS



Lamp-house Holder



Bench

PHOTO-ELASTIC APPARATUS

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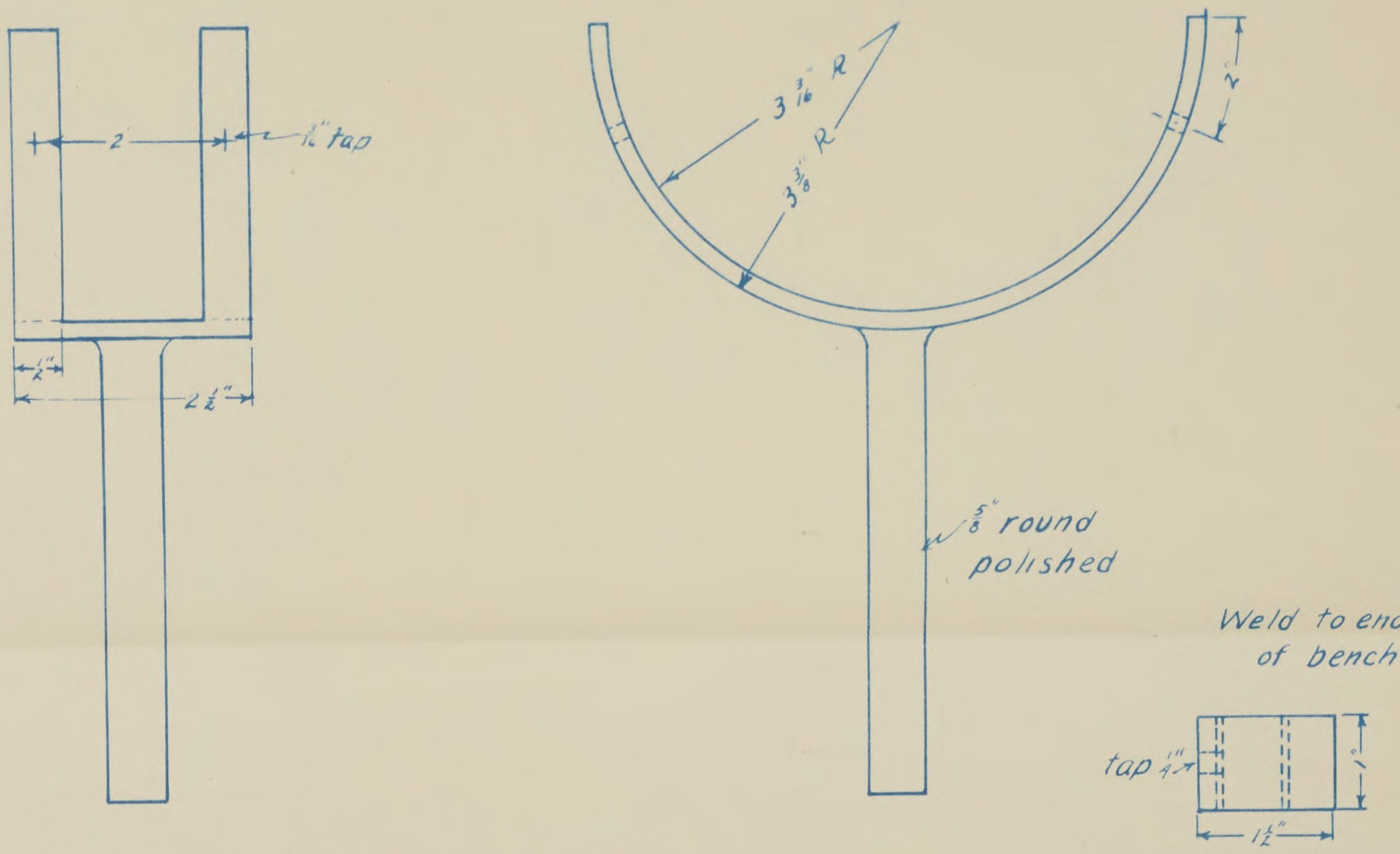
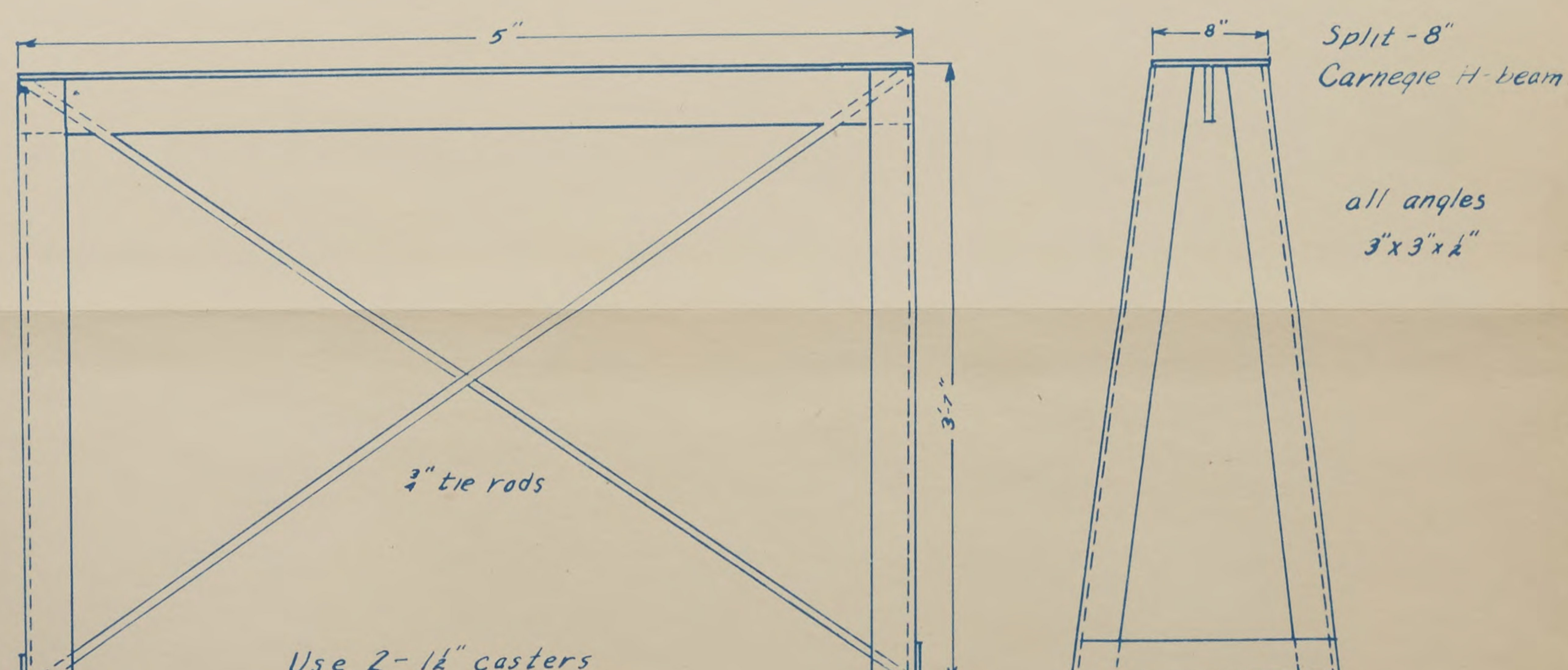
PLATE VI

DETAILS OF SUPPORTS



Lamp-house Holder



Bench

PHOTO-ELASTIC APPARATUS

THESIS

DESIGNED C.W. NICHOLAS

PLATE VI