DIFFUSION THROUGH VAPOR-BARRIER GAPS IN HOUSE WALLS

Thesis for the Degree of M. S.
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IN HOUSE WALLS

by

DAVID ALAN NORMAN

AN ABSTRACT

Submitted to the College of Agriculture
Michigan State University of Agriculture and
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1959

Approved by

Merle L Esmay

ABSTRACT

The vapor flow rate through gaps in a vapor barrier has been obtained by a conformal-mapping solution of the two-dimensional diffusion equation, the effects of convection being neglected. The flow rate in grains/hour-foot of crack length for a crack of width f in an impermeable material of thickness g is given approximately by

$$\frac{D\ (p_2 - p_1)\ \pi}{2\ \ln(4y/f) + \pi g/f}$$

where D is the permeability of the surrounding medium in grains/hour-foot-inch of mercury, and p_2 and p_1 are the partial vapor pressures (in Hg) at a distance y on either side of the barrier. An analogous formula is given for a lap. Since p_2 and p_1, and the expression for flow rate vary slowly with y at distances far from the gap, the point of measurement of p_2 and p_1 is not critical. The expressions obtained are found to be consistant with published measurements. Calculations by the expressions obtained show that some gaps occurring commonly in practice may allow a damaging amount of vapor to pass through the barrier.

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THE NEED FOR A STUDY OF GAPS IN A VAPOR BARRIER

The purpose of a vapor barrier in a house wall is to prevent an excessive amount of water vapor from entering the wall from the inside of the house. The vapor-barrier material might be a paint coating, a polyethylene or aluminum foil, or an asphalt-coated paper. Figure 1 shows typical ways of installing vapor-barrier material as an integral part of blanket insulation.

In spite of the name vapor barrier, some vapor passes into the wall through the vapor-barrier material or through gaps. The amount of vapor passing per unit time, wall area, and vapor-pressure difference is the permeance of the vapor barrier.

Experience has shown that a permeance of about one <u>perm</u>, defined as one grain per hour-square foot-inch of mercury, is the maximum value allowable in a vapor barrier that will prevent condensation. A vapor barrier of proper permeance for a given type of construction, and given inside and outside temperatures and vapor pressures, can be designed with the aid of available data for the heat conductance and vapor permeance of the materials on both sides of the vapor barrier and the results of this thesis.

Both the vapor flow rate through a gap and the vapor pressure near a gap need consideration. The flow rate is important when there is a lot of gap length per square foot as is the case when the vapor barrier is made up of

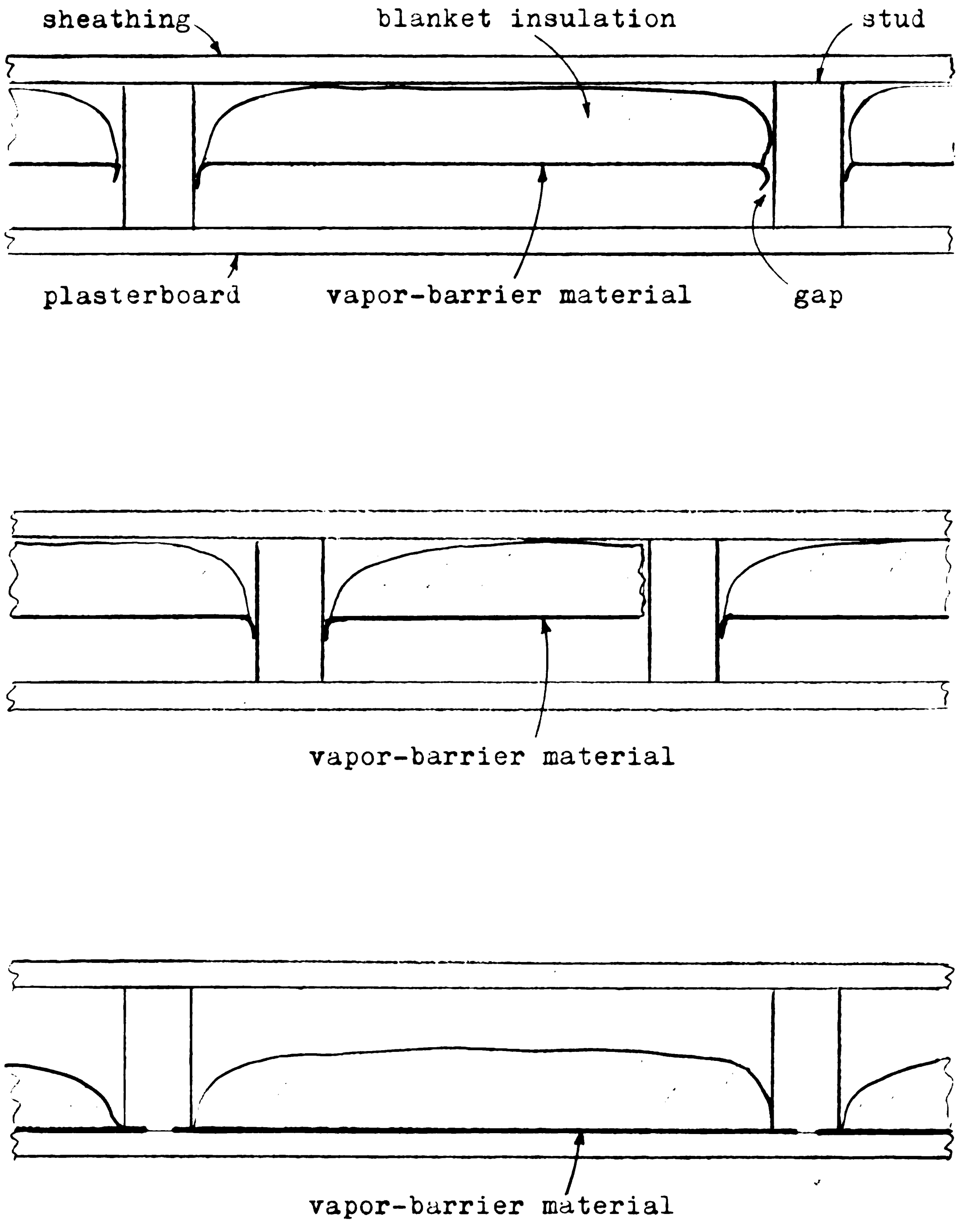

Figure 1. Cross-sections of a wall showing typical vapor barriers

narrow strips. For a 16-inch stud spacing there are about 18-inches of gap length for each square foot of wall. Even though the average permeance of a vapor barrier may be acceptable, a gap may cause local condensation depending on the closeness of a cold surface and how the vapor pressure changes with distance from the gap.

The harmfulness of vapor passing through gaps has been shown by experiment. Dill (Ref. 1) reported on tests of walls with eight arrangements of insulation and vapor barriers. Conditions were 70°F and 30%RH (70 degrees Fahrenheit and 30 percent relative humidity) on one side of the wall and -5°F on the other. The tests were run 100 hours. Frost was gathered from the sheathing and weighed.

A wood-fiber fill insulation arrangement, with a 0.56-perm vapor barrier turned and sealed against the frame with Scotch tape, had more than three times as much frost as a similar arrangement with no vapor barrier. With the vapor barrier the frost occurred near the corners of the panels. Without the vapor barrier the frost was evenly distributed over the sheathing, and the insulation and sheathing accumulated more moisture. A rock-wool fill arrangement had similar results. A double thickness of one-inch blankets enclosed by a 8.87-perm envelope placed between the studs with an air space on both sides had no frost on the sheathing or siding.

An actual weather test at Pennsylvania State College was reported by Reichel (Ref. 2). A test house of 48 panels of 22 different constructions was built outdoors. Inside conditions were 70°F and 40%RH. The siding of all the panels had three coats of conventional exterior white house paint. Observations were made of paint blistering, mold growth on the sheathing, and moisture in the sheathing and siding.

Paint blistering started on one panel on January 16 after one month of exposure. This panel had a 0.32-perm barrier on a one-inch blanket with a 5/16-inch gap between the barrier and the top and bottom plates. By March this panel had more blistering than the other panels which had no barrier or where the barrier flaps were attached to the studs and plates. Most of the blisters had water between the first and second paint layers.

No mold occurred where a vapor barrier was used. Heavy mold occurred where fill insulation only was used.

Moisture in the sheathing and siding was generally higher when no barrier was used. The highest siding moisture content occurred in the panel with a gap at the top and bottom of the vapor barrier.

Other experiments have found the flow rate through various slits, cracks, laps, and holes (Ref. 3, 4, and 5). The flow-rate data cannot, in general, be used to design gaps.

All these tests show that gaps in vapor barriers can allow passage of damaging amounts of water vapor. The tests results however are not very helpful in designing minimum dimensions for gaps. For this, theory is needed. Then perhaps vapor-barrier failures can be better analyzed and engineers can be more specific about what gaps may be allowed.

LIMITING THE THEORETICAL PROBLEM

The vapor-flow rate through gaps in vapor barriers is a function of many variables. Some of the variables such as the dimensions of the gap, the pressure difference across the vapor barrier, and the permeability of the medium surrounding the gap, will be considered mathematically. Cases which have a vapor source or sink will not be considered. Some of the other variables will be assumed to have a negligible effect on the flow rate, such as the end effect for a long narrow gap, the variation of the vapor pressure with time, or the variation of the permeability due to temperature or relative humidity. A factor left undetermined will be the amount of vapor flowing due to a difference in air buoyancy caused by temperature and vapor-pressure differences. This flow by convection could conceivably exceed the flow by diffusion. The total flow rate for a wall is probably at least as much as that caused by diffusion and therefore gaps should be designed at least to limit diffusion to a safe amount. An experimental check of derived vapor-diffusion equations for a gap in air could best be made by having the lightest air above a horizontal vapor barrier.

By considering only diffusion, without any vapor sources or sinks, or any variation in the vapor pressure with time or with the direction along the length of the gap, the vapor pressure, p, will be a harmonic function and the vapor flow rate can be found by solving Laplace's equation in two dimensions, that is,

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = 0,$$

subject to the boundary conditions appropriate to the gap.

The gaps chosen, the slit, crack, and lap, are realistic and yet mathematically manageable.

A SLIT IN A THIN VAPOR BARRIER

The slit, of width f, can be drawn and the boundary conditions stated as follows:

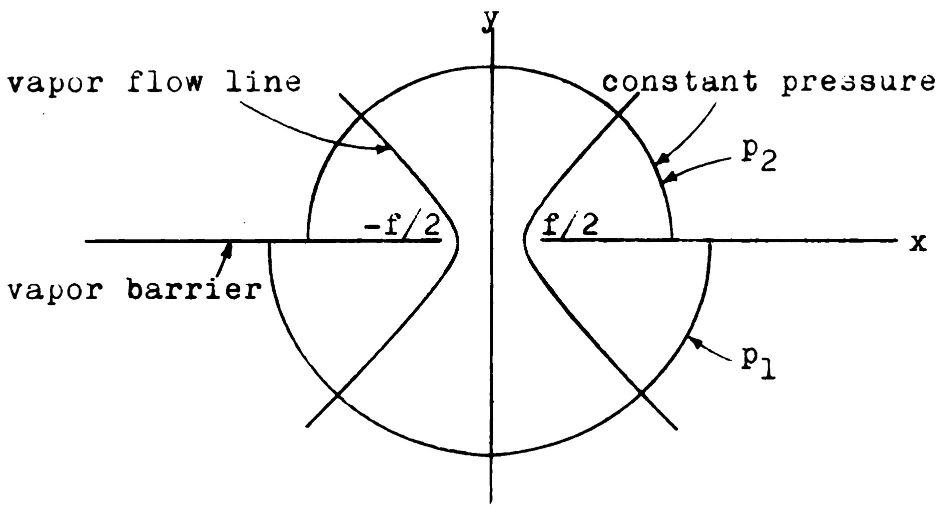

Figure 2. Vapor flow through a slit

1) $\dfrac{\partial^2 p}{\partial x^2} + \dfrac{\partial^2 p}{\partial y^2} = 0$ (all x and y);

2) $\dfrac{\partial p}{\partial y} = 0$ ($x < -f/2$, $x > f/2$, $y=0$);

3) the vapor pressures, p_1 and p_2, are specified for two of the constant-pressure curves appearing in the solution of the problem.

The problem can be solved using conformal mapping. The slit, constant vapor pressure and vapor flow lines as shown in the complex z-plane are mapped by an analytic function into simpler lines in the w-plane.

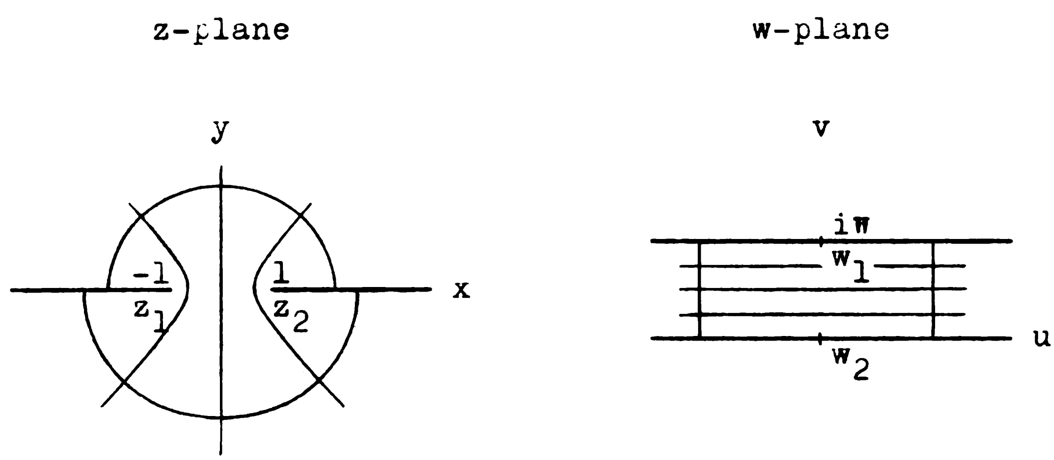

Figure 3. Mapping of a slit into an infinite strip

The pressure, gradient, and vapor flow rate are simple functions of u and v, and these may be transformed by the analytic function into the less simple functions of x and y.

Some of the properties of conformal mapping described by Churchill (Ref. 6) are:

1) A harmonic function, $H(u,v)$, remains harmonic under a conformal transformation, $z = F(w)$, where $F(w)$ is analytic and $dz/dw \neq 0$.

2) A boundary condition $H = c$, a constant, in the w-plane transforms to $H = c$ in the z-plane.

3) If the normal derivative, dH/dn, along some curve in the w-plane equals zero, then $dH/dn = 0$ along the corresponding curve in the z-plane.

4) The absolute value of the gradient of $H(x,y)$ equals the product of the absolute values of dw/dz and the gradient of $H(u,v)$.

5) Rate of flow across corresponding curves in the z-plane and the w-plane is the same.

The analytic functions $z = \sinh w$, $z = \cosh w$, $z = \sin w$, and $z = \cos w$, one or more of which are usually illustrated and discussed in a book on complex-variable theory, could be used to solve this problem. The function $z = \cosh w$ will give the particular orientation shown in Figure 3. A transformation function for a lap, which may not be listed, will be derived later by means of the Schwarz-Christoffel transformation.

If the mapping function is

$$z = \tfrac{1}{2}f \cosh w = \tfrac{1}{2}f \cosh(u + iv)$$

$$= \tfrac{1}{2}f(\cosh u)(\cos v) + \tfrac{1}{2}fi(\sinh u)(\sin v),$$

then

$$x = \tfrac{1}{2}f(\cosh u)(\cos v),$$

$$y = \tfrac{1}{2}f(\sinh u)(\sin v),$$

$$\frac{x^2}{\frac{1}{4}f^2\cosh^2 u} + \frac{y^2}{\frac{1}{4}f^2\sinh^2 u} = 1,$$

and

$$\frac{x^2}{\frac{1}{4}f^2\cos^2 v} - \frac{y^2}{\frac{1}{4}f^2\sin^2 v} = 1.$$

Lines where u is a constant are semiellipses in the z-plane and lines where v is a constant are hyperbolas in the z-plane. Figure 4 is an example of the constant-pressure and flow lines for vapor diffusing through a slit.

In the w-plane the pressure p is related to u and the points where the pressure is known by

$$\frac{p - p_1}{u - u_1} = \frac{p_2 - p_1}{u_2 - u_1} \quad \text{or,}$$

$$p = u\,\frac{p_2 - p_1}{u_2 - u_1} - u_1\,\frac{p_2 - p_1}{u_2 - u_1} + p_1.$$

For an ellipse of constant pressure the major axis is f cosh u, and the foci are at f/2 and -f/2. The variable u in the above equation may be replaced by a function of x and y for

$$u = \cosh^{-1} \frac{1}{f}\left[\sqrt{(x+\tfrac{1}{2}f)^2 + y^2} + \sqrt{(x-\tfrac{1}{2}f)^2 + y^2}\right].$$

For $x = 0$, $v = \pi/2$, $y = \frac{1}{2}f \sinh u$ and

$$u = \sinh^{-1} 2y/f = \ln\left[2y/f + \sqrt{(2y/f)^2 + 1}\right].$$

In Figure 5 the pressure, p, is plotted versus y, the distance perpendicular to the vapor barrier measured from the center of the slit, for three values of the slit width f.

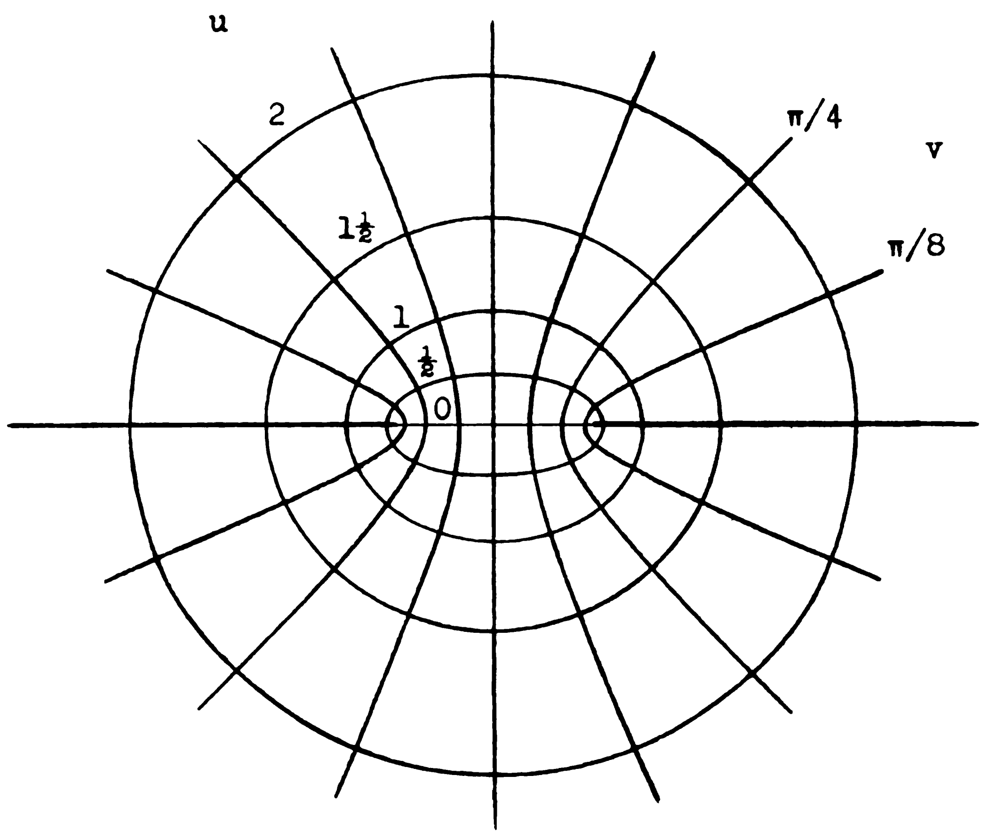

Figure 4. Flow and constant-pressure lines for a slit

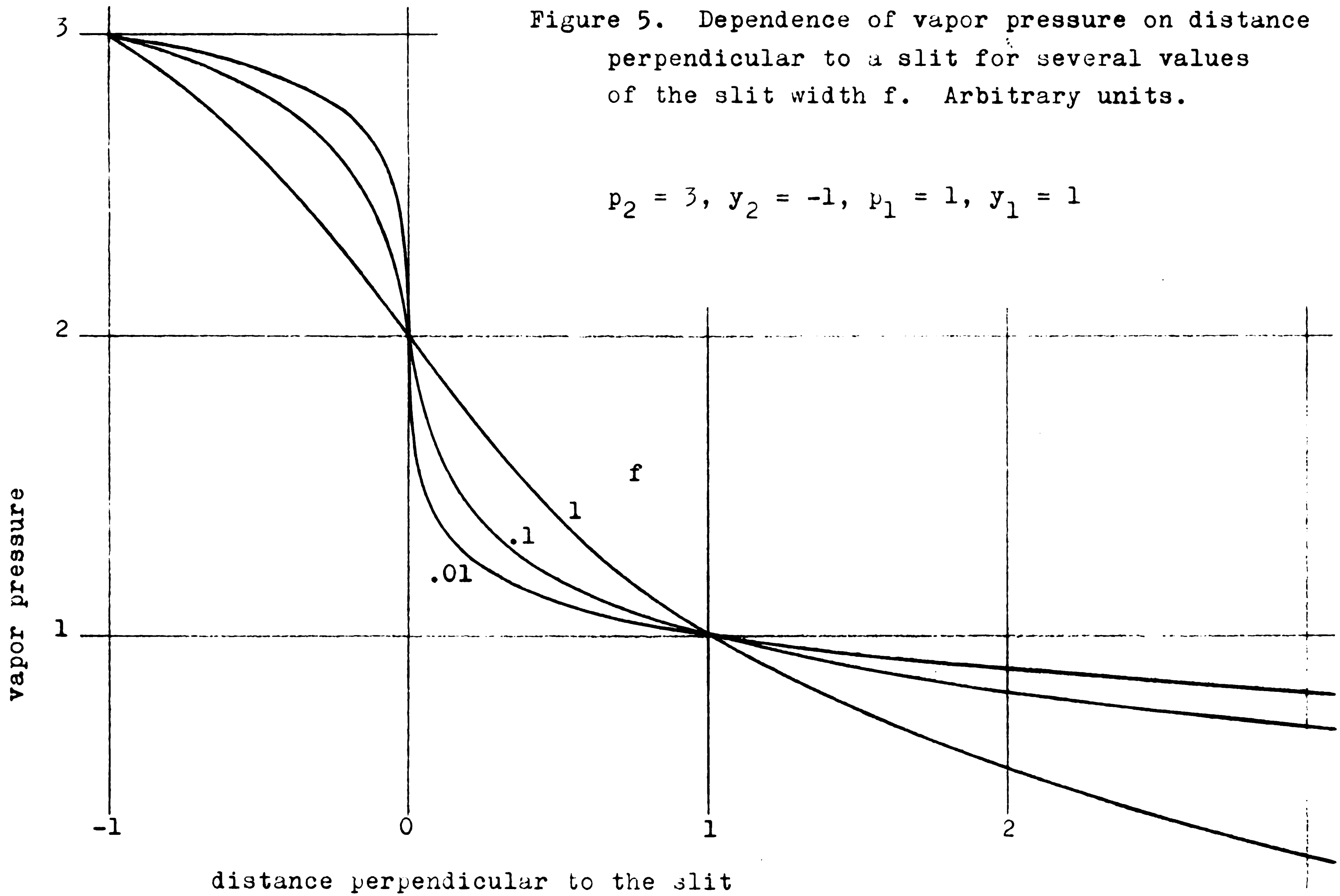

Figure 5. Dependence of vapor pressure on distance perpendicular to a slit for several values of the slit width f. Arbitrary units.

$p_2 = 3$, $y_2 = -1$, $p_1 = 1$, $y_1 = 1$

The vapor pressure changes rapidly near a narrow slit and then slowly farther away. If the point at which a particular pressure occurs is known only generally, on either side of a narrow slit, a point can be assumed with little error in the calculated pressure at any other point. The vapor in a stud wall air space on one side of a vapor barrier having only a narrow slit will be at about the same pressure, except near the slit. Thus the required permeance of a vapor barrier, calculated for plane flow through the wall, will be correct for a vapor barrier when some of the vapor flows through a narrow slit.

The absolute value of the gradient,

$$\left|\frac{\partial p}{\partial x} + i\,\frac{\partial p}{\partial y}\right| = \left|\frac{p_2 - p_1}{u_2 - u_1}\right| \left|\frac{dw}{dz}\right|$$

$$= \left|\frac{p_2 - p_1}{u_2 - u_1}\right| \left|\left[\tfrac{1}{2}f \sinh w\right]^{-1}\right|$$

$$= \left|\frac{p_2 - p_1}{u_2 - u_1}\right| \left|\left[z^2 - \tfrac{1}{4}f^2\right]^{-\frac{1}{2}}\right|,$$

becomes infinite as z approaches $\frac{1}{2}f$ or $-\frac{1}{2}f$, so that vapor flows faster near the edges of the slit than near the center. This suggests that reducing the width of the slit will not reduce in direct proportion the vapor flow through the slit.

The vapor flow rate in the w-plane depends on the permeability of the medium, the gradient, and the width of the flow region. Thus (with sample units)

$$\frac{M}{L} = D\,\frac{p_2 - p_1}{u_2 - u_1}\,\pi, \text{ where}$$

$\frac{M}{L}$ = flow rate per unit length of slit (gr/hr-ft),

D = permeability of the medium (gr-in/hr-ft^2-in Hg),

$\frac{p_2 - p_1}{u_2 - u_1}$ = gradient (in Hg/in), and

π = width (ft).

Flow rate across corresponding curves in the two complex planes is the same. In the z-plane this can be expressed in terms of the slit width, f, and the vapor pressures, p_1 and p_2, at the points y_1 and y_2 taken perpendicular to the vapor barrier from the center of the slit. Thus

$$\frac{M}{L} = \frac{\pi\, D\,(p_2 - p_1)}{\ln\left[\frac{2y_2}{f} + \sqrt{\frac{4y_2^2}{f^2} + 1}\right]\left[\sqrt{\frac{4y_1^2}{f^2} + 1} - \frac{2y_1}{f}\right]}\,.$$

If $y_2 = -y_1$ and both are called y, and if 1 is negligible compared to $(2y/f)^2$ then

$$\frac{M}{L} = \frac{\pi\, D\,(p_2 - p_1)}{2\,\ln(4y/f)}\,.$$

A CRACK IN A THICK VAPOR BARRIER

Flow rate through a crack in a vapor barrier of thickness g, as shown in Figure 6, can be approximated by adding a resistance for the nearly plane flow between the edges of the vapor barrier to the resistance of the nearly hyperbolic flow on either side. An exact solution is discussed in Appendix C.

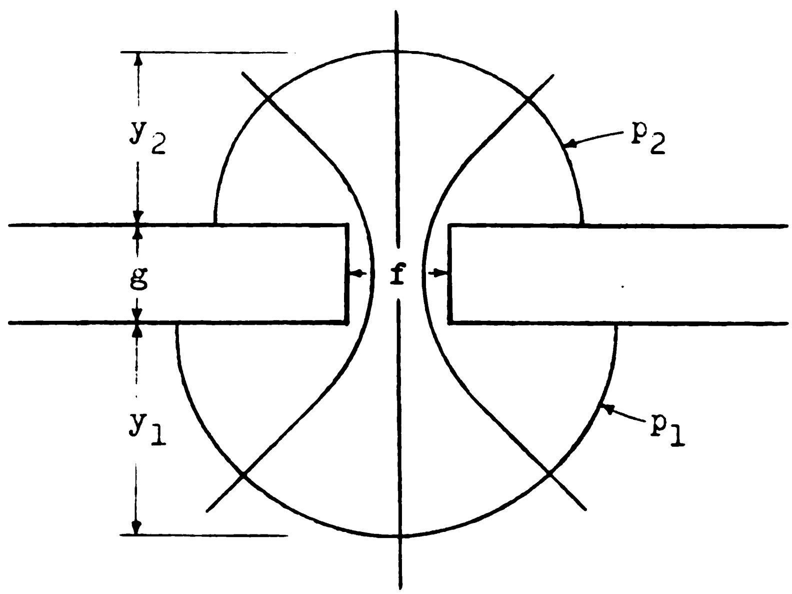

Figure 6. Vapor flow through a crack

Let V be defined by the equation $M/L = (V/L)(p_2 - p_1)$. Then

$$\frac{L}{V} = \frac{u_2}{\pi D_2} + \frac{g}{fD_3} + \frac{u_1}{\pi D_1} ,$$

where u_2 and u_1 may be found for y_2, y_1, and f by

$$u = \ln \left[2y/f + \sqrt{(2y/f)^2 + 1} \right].$$

D_1, D_2, and D_3 are the permeabilities of the materials on either side of and between the edges of the crack. If they are all equal they can be called D. Then if y_2 equals y_1 and they are called y, the specific flow rate for a crack,

$$\frac{V}{LD} = \frac{1}{\frac{2}{\pi} \ln \left[2y/f + \sqrt{(2y/f)^2 + 1} \right] + g/f} .$$

Figure 7 is a plot of this equation. For the range of variables of Figure 7 the error in V/LD will be less than about 2% (see Appendix C). To find the vapor flow rate through a crack of width f, in a vapor barrier of thickness g, when the pressures p_2 and p_1 are assumed at equal distances y on either side of the crack, find V/LD from Figure 7 for the known values of 2f/y and 2g/y. Then

$$M/L = (V/LD)(p_2 - p_1) .$$

Figure 7 makes finding the flow rates through slits and cracks easy and also aids in a general discussion of these gaps. First, the thickness of a thin sheet has little effect on the flow rate through a slit unless the slit width is about the same or less than the sheet thickness. Second, if it is necessary to make V/LD less than say 0.2, bringing the butt edges of a thin vapor barrier close together is not a very practical method. On the other hand, making g large is a good way to reduce flow. This can be done by lapping the sheets, as will be

p_2

$M/L = (V/LD)(D)(p_2 - p_1)$ where

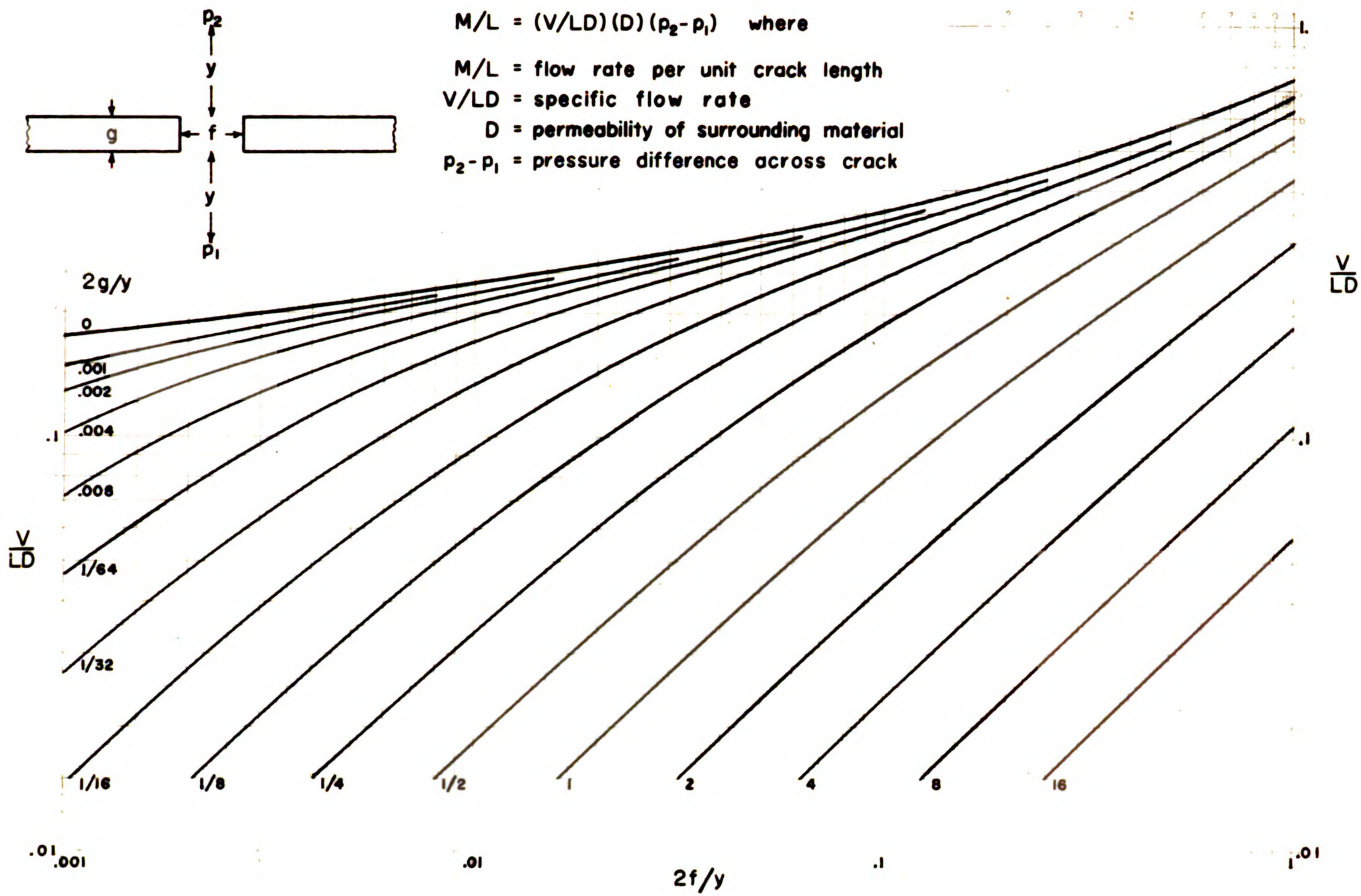

Figure 7. Dependence of V/LD on 2f/y with 2g/y as parameter. Dimensionless quantities.

shown theoretically correct in the next chapter, or by placing an inch or so of the vapor barrier flat against the stud, sill or plate so that there are no gaps of large f and small g. Third, for V/LD less than say 0.1 for a particular crack, y may vary over quite a wide range without changing V/LD much. For example, if f is 1/16-inch and g is 1-inch, then if

y = 1/8-inch, V/LD = 0.058, and if

y = 8-inches, V/LD = 0.050.

A LAP IN A THIN VAPOR BARRIER

Vapor flow through a lap, whose cross-section is shown in the z-plane of Figure 8, can be found by means of a conformal mapping into the upper half of the t-plane.

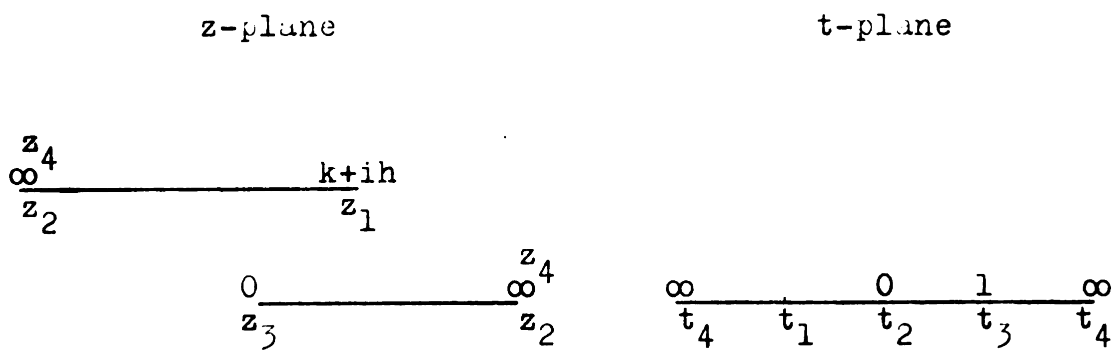

Figure 8. Mapping of a lap into the upper half-plane

The analytic function required can be found by the Schwarz-Christoffel transformation, by which the interior of a polygon is mapped into the upper half-plane.

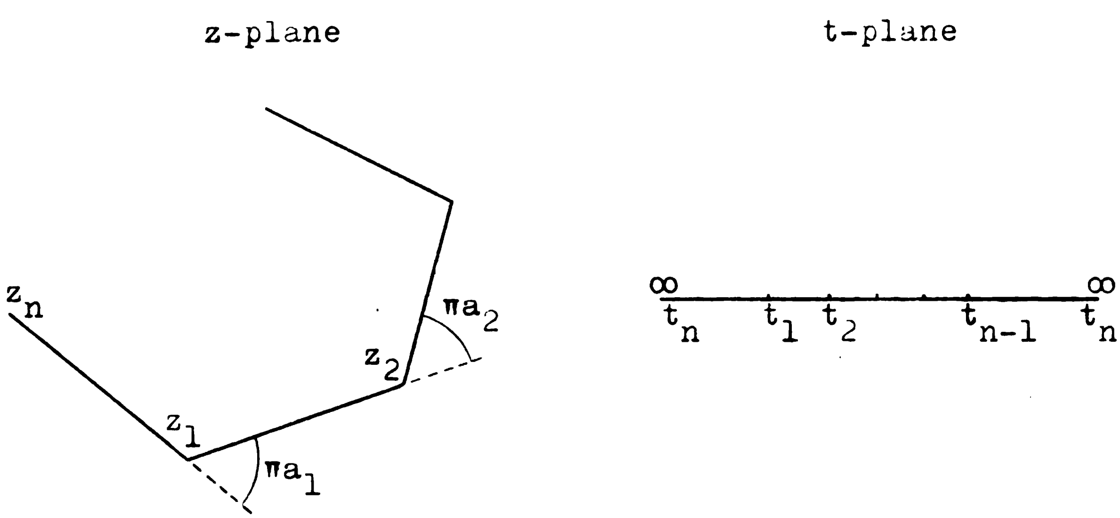

Figure 9. Mapping of a polygon into the upper half-plane

The Schwarz-Christoffel transformation is (Ref. 6)

$$z = A\int (t-t_1)^{-a_1} (t-t_2)^{-a_2} \; . \; . \; . \; (t-t_{n-1})^{-a_{n-1}} dt + B,$$

where

A and B are complex constants,

t_1, t_2, . . . t_{n-1} are points on the real axis of the t plane corresponding to

z_1, z_2, . . . z_{n-1}, the successive vertices taken so that the interior of the polygon is to the left when moving around the boundary, and

πa_1, πa_2, . . . πa_{n-1} are the exterior angles.

The image of z_n is $t_n = \infty$. Two of the constants t_1, t_2, . . . t_{n-1} can be chosen arbitrarily. The remaining n-3 constants and A and B must be determined to fit the polygon.

Now going back to Figure 8, the constants t_2 and t_3 are arbitrarily 0 and 1, t_4 is at ∞, and t_1 is to be determined. The exterior angles at z_1 and z_3 are both $-\pi$, and the angle at z_2 is 2π, which is the change in direction required to pass from the direction z_1z_2 at $y = h$, to the direction z_2z_3 at $y = 0$, when going around the boundary of the polygon at z_2.

Thus the required transformation is

$$z = A\int (t-t_1)(t)^{-2}(t-1)\, dt + B$$

$$= A\left[t - (t_1+1)\ln t - t_1 t^{-1}\right] + B$$

$$= A\left[t - (t_1+1)\ln |t| - i(t_1+1)\arg t - t_1 t^{-1}\right] + B.$$

When z is positive, real, and infinite, then t is positive, real, and infinite. Therefore the imaginary part of A and so also of B must be zero. When $z = 0$, $t = 1$, and

$$0 = A\left[1 - t_1\right] + B.$$

When $z = k+ih$, then $t = t_1$, $\arg t = \pi$, and so

$$k = A\left[t_1 - (t_1+1)\ln |t_1| - 1\right] + B, \text{ and}$$

$$h = A\left[-(t_1+1)\pi\right].$$

Thus

$$A = \frac{-h}{\pi(t_1+1)}, \qquad B = \frac{h(1-t_1)}{\pi(t_1+1)}, \text{ and}$$

$$z = \frac{h}{\pi}\left[\ln t - \frac{t}{t_1+1} + \frac{t_1 t^{-1}}{t_1+1} + \frac{1-t_1}{t_1+1}\right],$$

where t_1 is related to h and k by the equation,

$$\frac{\pi k}{h} = \ln |t_1| - 2 \frac{(-t_1+1)}{(-t_1-1)}$$

The function $t = e^w$ maps the upper half of the t-plane into the region $0 \leq v \leq \pi$ in the w-plane in which constant-pressure lines can be represented by u equals a constant and vapor-flow lines by v equals a constant. Expressing z in terms of w and t_1, which from here on is called t,

$$z = \frac{h}{\pi}\left[w - \frac{e^w}{t+1} + \frac{te^{-w}}{t+1} - \frac{t-1}{t+1}\right]$$

$$= \frac{h}{\pi}\left[u + iv - \frac{e^u(\cos v + i \sin v)}{t+1} + \frac{te^{-u}(\cos v - i \sin v)}{t+1} - \frac{t-1}{t+1}\right].$$

Thus,

$$x = \frac{h}{\pi}\left[u + \frac{\cos v}{t+1}(te^{-u} - e^u) - \frac{t-1}{t+1}\right] \quad \text{and}$$

$$y = \frac{h}{\pi}\left[v - \frac{\sin v}{t+1}(te^{-u} + e^u)\right].$$

Figure 10 is an example of flow through a lap. In this example k = 5 and h = 3, so t (or t_1 in Figure 8) which is very large negatively, can be found by the equation

$$|t| = e^{k\pi/h + 2} = e^{5\pi/3 + 2} = e^{7.236} = 1390.$$

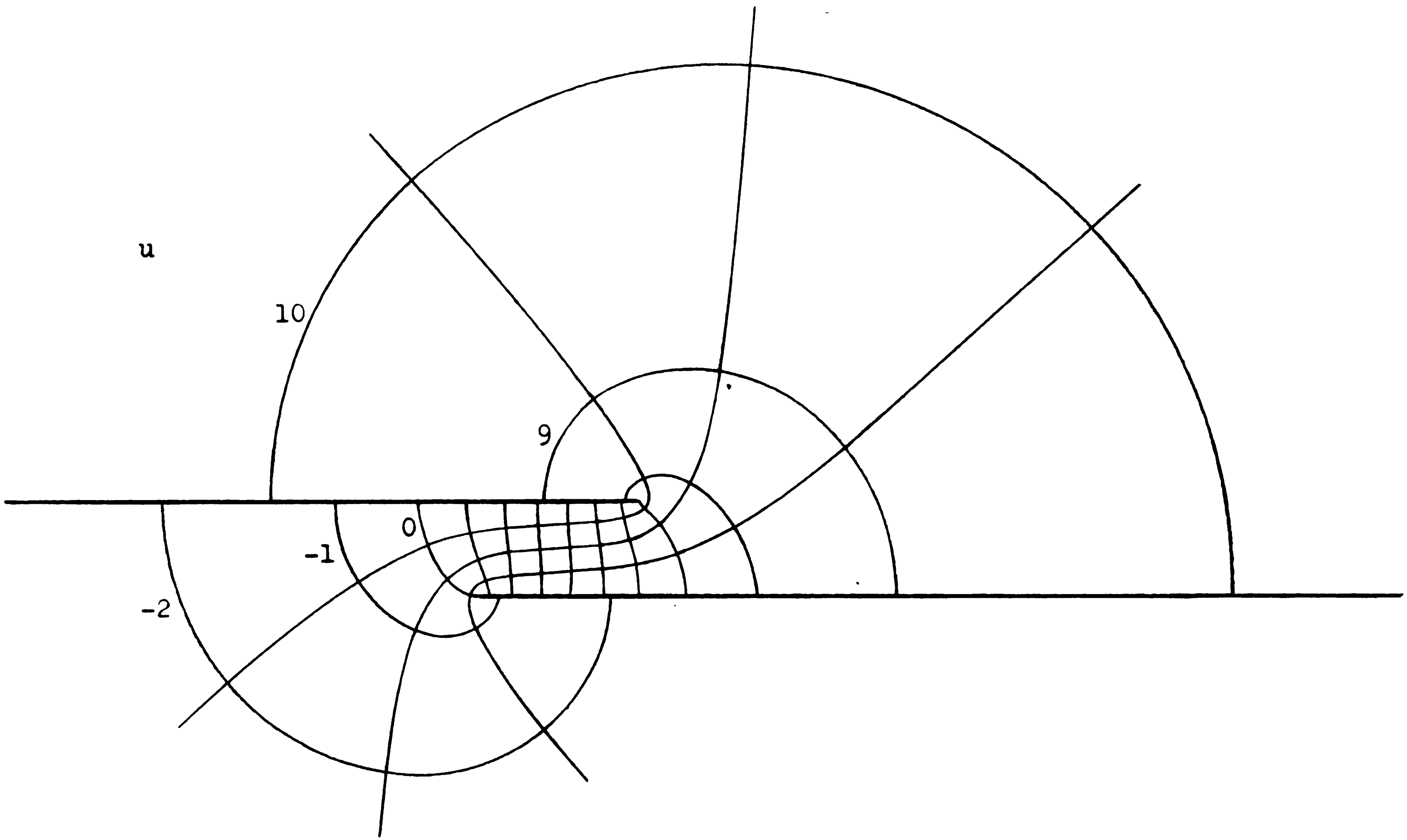

Figure 10. Flow and constant-pressure lines for a lap

Rate of flow through the lap is

$$\frac{M}{L} = D\,\frac{p_2 - p_1}{u_2 - u_1}\,\pi .$$

The points y_2 and y_1 can be taken along $v = \pi/2$ where

$$y = \frac{h}{2} + \frac{he^{u}}{\pi(-t-1)} - \frac{h(-t)e^{-u}}{\pi(-t-1)}.$$

To solve for $(u_2 - u_1)$ take y_2 and $-y_1$ greater than 3h and k/h greater than -1. Then the error in the following value for $(u_2 - u_1)$ will be less than 1%. Also take the x-axis at h/2. Then

$$y_2 = \frac{he^{u_2}}{\pi(-t-1)}, \qquad -y_1 = \frac{h(-t)e^{-u_1}}{\pi(-t-1)}$$

$$u_2 = \ln\frac{\pi(-t-1)(y_2)}{h}, \qquad -u_1 = \ln\frac{\pi(-t-1)(-t_1)}{(-t)h},$$

$$u_2 - u_1 = \ln\frac{\pi^2(-t-1)^2(y_2)(-y_1)}{(-t)h^2}.$$

Now a lap can be compared with a crack. Flow through a crack is

$$\frac{M}{L} = \frac{D(p_2 - p_1)\pi}{u_2 + u_1 + \pi g/f}.$$

If y_2 and y_1 are greater than 3f, then with error less than 1%

$$u_2 + u_1 + \frac{\pi g}{f} = \ln\frac{16(y_2)(y_1)}{f^2} + \frac{\pi g}{f}.$$

If $(y_2)(y_1)/(f^2)$ for a crack equals $(y_2)(-y_1)/(h^2)$ for a lap, then for equal flow rate through both gaps

$$\frac{\pi g}{f} = \ln \frac{\pi^2(-t-1)^2}{16(-t)},$$

where t is related to k and h by

$$\frac{\pi k}{h} = \ln |t| - 2\,\frac{(-t+1)}{(-t-1)}.$$

Figure 11 is a plot of $(g/f - k/h)$ versus k/h for equivalent cracks and laps. To the right of $k/h = 1$, $(-t)$ becomes very large and $(g/f - k/h)$ approaches

$$\frac{2}{\pi} \ln \frac{\pi}{4} + \frac{2}{\pi} = 0.48.$$

When $(g/f - k/h) = -k/h$, at $k/h = -0.8$, then $g/f = 0$. Negative values of g/f are not allowed.

When the equivalent g/f is found for a lap, then Figure 7 can be used to find the flow rate. Conversely, the dimensions of a lap may be found which will limit the flow rate to an acceptable level.

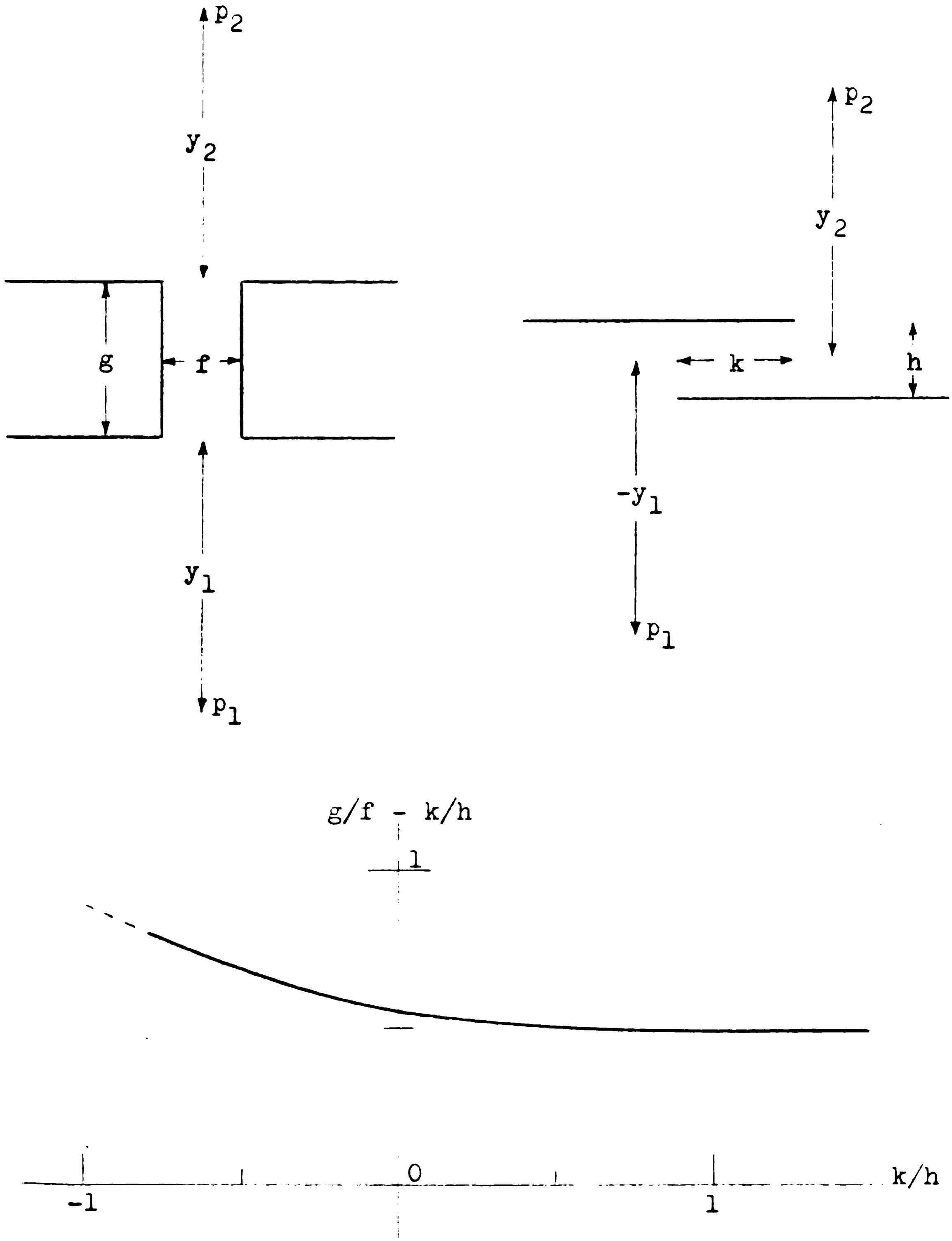

Figure 11. The relation between equivalent g/f and k/h

COMPARISON OF THE THEORY WITH PUBLISHED MEASUREMENTS

Flow through slits in aluminum foil was reported by Babbitt (Ref. 3). The foil was between two pieces of plasterboard or fiberboard or was backed on one side with plasterboard. Tests were made in a chamber used to test permeances of building materials. Air in the chamber was circulated with a fan so that the vapor pressure would be known at the surface of the material being tested. The units used by Babbitt have been changed to those used in the rest of this thesis. Calculated flow rates are compared with experimental flow rates in Table I.

For the tests of slits between two sheets of plasterboard, the permeability of the plasterboard, D, was 12.8-perm-in; the vapor pressure difference, $p_2 - p_1$, was 1.05-in Hg; and the thickness of the plasterboard, y, was 0.41-in. The theoretical flow rate is given by

$$\frac{M}{L} = \frac{\pi D (p_2 - p_1)}{2 \ln \left[2y/f + \sqrt{(2y/f)^2 + 1} \right]}$$

$$= \frac{\pi (12.8\text{-gr-in/hr-ft}^2\text{-in Hg})(\text{ft}/12\text{-in})(1.05\text{-in Hg})}{2 \ln \left[0.82/f + \sqrt{(0.82/f)^2 + 1} \right]}$$

For the slits between two sheets of fiberboard, D was 30-perm-in, $p_2 - p_1$ was 1.05-in Hg, and y was 0.51-in.

For the slits backed on only one side with plaster-

Table I. Flow rates through slits as reported by Babbitt (Ref. 3) compared with theoretical values

width, in	flow rate, gr/hr-ft experimental	flow rate, gr/hr-ft theoretical	exp./theo.
a slit between two sheets of plasterboard			
.009	.71	.33	2.1
.017	.85	.38	2.2
.028	.98	.43	2.3
.045	1.09	.48	2.2
.072	1.20	.56	2.2
.103	1.56	.63	2.5
.147	1.46	.73	2.0
a slit between two sheets of fiberboard			
.004	1.03	.64	1.6
.019	1.37	.87	1.6
.034	1.44	1.00	1.4
.045	1.42	1.07	1.3
.056	1.45	1.14	1.3
.065	1.65	1.19	1.4
.086	1.82	1.29	1.4
.120	1.78	1.44	1.2
.155	2.36	1.58	1.5
a slit backed on one side with plasterboard			
.016	1.31	.77	1.7
.031	1.51	.90	1.7
.047	1.67	1.01	1.7
.063	1.80	1.09	1.7
.079	1.90	1.17	1.6

board, $p_2 - p_1$ was 1.07-in Hg. Values for permeability and thickness were not stated but will be assumed to be 12.8-perm-in and 0.41-in as before. In these tests $y_1 = 0$, $y_2 = 0.41$-in, and

$$\frac{M}{L} = \frac{\pi(12.8)(1/12)(1.07)\text{-gr/hr-ft}}{\ln\left[0.82/f + \sqrt{(0.82/f)^2 + 1}\right]}.$$

The values of f/y would have to be increased about ten times, for the narrow slits, to make theoretical values in Table I agree with experimental values, so error in the measurement of f and y are probably not the cause of the discrepancy. From the variation in published measurements for the permeabilities of various building materials (see Appendix B), it is possible that the permeabilities of the fiberboard and the plasterboard could have been higher by a factor of 1½ or 2.

Vapor leakage was reported by Joy (Ref. 4) through cracks in painted plaster and through laps in sheet steel. The test cells were ones used to determine permeance of 12-in diameter building-material specimens. The air was static and at a temperature of 70.7°F. The vapor pressure was measured about three inches away from the specimens. Leakage, which includes diffusion and convection, was reported for a pressure difference of one inch of mercury.

A painted, 1/2-inch thick plaster panel, which had a permeance of 0.47-perm, was broken and reassembled with the two halves separated 1/16-in. Leakage reported was 2.04-gr/hr for the crack horizontal and 2.55-gr/hr for the crack vertical, which was stated to be more because of convection. The permeability of air (from Appendix A) is 143/12-perm-ft or 11.9-gr/hr-ft-in Hg. So

$$2f/y = 2(1/16)/3 = 0.042,$$

$$2g/y = 2(1/2)/3 = 0.33,$$

and from Figure 7,

$$V/LD = 0.088.$$

Thus for the crack,

$$\begin{aligned} M &= (V/LD)(L)(D)(p_2 - p_1) \\ &= (0.088)(1\text{-ft})(11.9\text{-gr/hr-ft-in Hg})(1\text{-in Hg}) \\ &= 1.05\text{-gr/hr}. \end{aligned}$$

Another way of calculating flow rate through this gap is to consider it as a slit in a thin vapor barrier (the paint) against solid plaster. The permeability of plaster is about 1.5-gr/hr-ft-in Hg (from Appendix B), $y_1 = 0$, $y_2 = 1/2$-in, $f = 1/16$-in, $4y/f = 32$, and

$$M = \frac{(\pi)(1)(1.5)(1)}{\ln 32} = 1.36\text{-gr/hr}.$$

Diffusion through the one-foot diameter panel, other than through the crack, would have been about $0.47\pi/4 = 0.37$-gr/hr.

In Joy's study of laps, two pieces of sheet steel were spaced 1/16-in apart, with either a 1/2-in or a 3-in lap. For a sample calculation using a 1/2-in lap, k/h = 8 and the equivalent g/f = 8.48. Then

$$M = \frac{D\ L\ (p_2 - p_1)}{(2/\pi)\ln(4y/f) + g/f}$$

$$= \frac{(11.9)(1)(1)}{(2/\pi)\ln 192 + 8.48} = (11.9)(0.085) = 1.01\text{-gr/hr.}$$

Or, if f is taken equal to h, then

$$2f/y = 2(1/16)/3 = 0.042,$$

$$2g/y = 2(g/f)(f)/y = 2(8.48)(1/16)/3 = 0.35,$$

and from Figure 7, V/LD = 0.085. Then

$$M = (11.9)(0.085) = 1.01\text{-gr/hr.}$$

Leakage measured and diffusion calculated compare as shown in the following Table.

Table II. Flow rates through laps as reported by Joy (Ref. 4) compared with theoretical values

k in	sheet steel orientation	vapor flow	leakage measured gr/hr	diffusion calculated gr/hr
	horizontal	up	0.83	
1/2	horizontal	down	0.91	1.01
	vertical	laterally	1.62	
	horizontal	up	0.23	
3	horizontal	down	0.23	0.23
	vertical	laterally	0.54	

The amount of vapor moved by convection shows up strongly in this experiment. In a house wall the higher air space and the temperature difference tend to increase convection, but the resistance offered by the insulation tends to decrease convection. Air may be kept from moving through gaps by placing the vapor-barrier material against or between solid materials.

APPLICATION OF THE THEORY

The design of gaps in vapor barriers depends on what vapor flow rate may safely be allowed. This depends on such things as the amount of gap length per square foot of wall area, the permeance of the vapor-barrier material, and the permeance and conductance of the rest of the wall, especially the part between the vapor barrier and the outside. The location of the gap is important, for the closer the gap is to a cold surface the less vapor can be allowed to flow.

A start is to assume that the vapor-barrier material is impermeable, the allowable permeance of the applied vapor barrier is one perm, the gaps are near the warm side of the wall, and the vapor barrier is applied in strips 16-inches wide between the studs, making about 18-inches of gap length per square foot of wall. Then the allowable vapor flow rate per foot of crack,

$$\frac{M}{L} = \frac{\text{1-gr}}{\text{hr-ft}^2}\,\frac{\text{ft}^2\text{ of wall}}{\text{1.5-ft of crack}} = \frac{\text{0.67-gr}}{\text{hr-ft}}$$

$$= \frac{V}{LD}\,D\,(p_2 - p_1) = \frac{V}{LD}\,\frac{\text{11.9-gr}}{\text{hr-ft-in Hg}}\,(\text{1-in Hg}).$$

Thus the allowable specific flow rate,

$V/LD = (0.67) \div (11.9) = 0.056.$

For ease of discussion while using Figure 7, the noncritical assumption can be made that y = 2-inches. The value of g or f may be chosen arbitrarily, and the other will then be determined.

Four types of gaps are shown in cross-section in Figure 12.

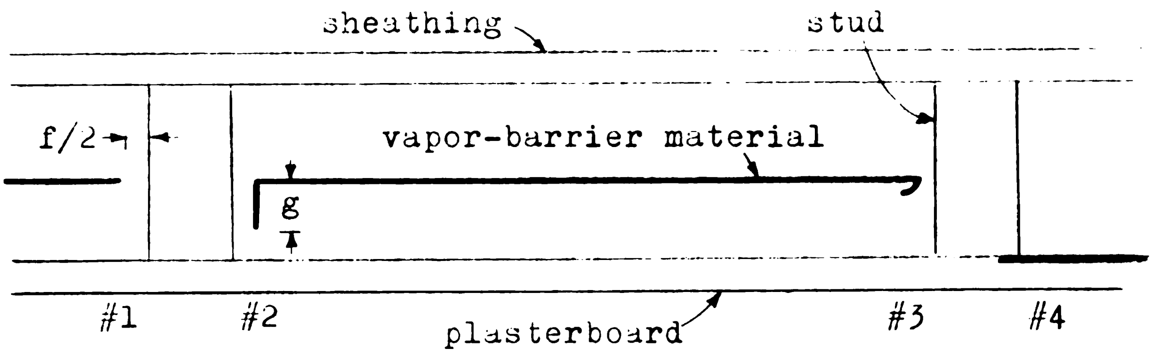

Figure 12. Four types of vapor-barrier gaps

Gap #1 can be considered as half of a slit, whose flow rate then, is half as much as that through a slit twice as wide as the distance f/2. The value of V/LD can be doubled and the resulting value of f divided by two, to give the distance from the vapor-barrier material to the stud. Thus V/LD = 0.112, f is less than 0.001-inch for a 0.002-inch thick vapor-barrier material, and the vapor-barrier material must be held less than 0.0005-inch away from the stud. This gap is impractical.

Gap #2 looks like half of a crack on the outside part and half of a lap on the inside. The crack and lap have nearly the same flow rate for g/f about 7 and so flow through gap #2 can be approximated by half of a crack. A practical size for g is 3/4-inch and so if V/LD = 0.112, then f = 0.12-inch and the tab on the vapor-barrier material must be 0.06-inch from the stud. This gap can be obtained by having the tab held flat against the stud by close spacing of a 3/4-inch wide staple placed horizontally.

Probably most vapor-barrier strips are applied with the thought in mind that gap #2 is being obtained but usually some gaps in a particular house are like gap #1 and #3. Gap #1 occurs where the stud space is too narrow for the width of strip being applied and so the strip is cut and no attempt is made to turn and staple a tab against the stud on one side. Gap #3 occurs when the vapor-barrier material is applied tightly between the studs, either as a general practice or when the stud spacing is slightly oversize.

For gap #1, if f/2 = 1/8-inch and y is assumed to be 2-inches, then one half of V/LD is 0.22 or about four times the value allowed for an 8-inch stud space. The permeance of the vapor barrier is 4-perm. If f/2 is reduced to 1/16-inch, the permeance is reduced to 3.4-perm; and if f/2 is 1/32 inch, the permeance of the vapor barrier for the 8-inch stud space is still 2.9-perm. The permeance of the warm side of the wall without a vapor barrier is about 8-perm.

Gap #3 fits the theory the least of any gap discussed thus far. It is not as bad as gap #1 but is probably not a great deal better.

Gap #1 sometimes occurs at the top and bottom of the stud space or around obstructions such as electric junction boxes. The vapor from these gaps may be restricted to a small sheathing area by the insulation and for this area of the stud space there would not be an effective vapor barrier.

The thing that makes gaps #1, 2, and 3 difficult is that they are in air. If the material around gap #4 is only five times less permeable than air, then V/LD will be 0.56. If y is assumed to be 1-inch then 2f will be 1/2-inch and the slit between strips of vapor-barrier material may be 1/4-inch wide. The permeability of wood is more than ten times less than air, so there is an easy opportunity to make gap #4 a very good one.

If a good gap is in air, the value for y, provided it is not chosen too close to the gap, does not affect the flow rate very much. If a gap is not in air, such as gap #4, the flow rate will be quite accurate if y is chosen as the distance fron the gap to air, possibly using different values for y and D on either side of the gap. If the value for y must be assumed for a poor gap in air, then accuracy will depend on good judgment.

Since the vapor pressure changes slowly at distances far from good gaps, the allowable permeance of these gaps in permeable vapor-barrier material is approximately the allowable permeance of the applied vapor barrier minus the permeance of the vapor barrier material. This fact, together with the possibility of convection, may require that gap dimensions be less than those calculated in this chapter.

SUGGESTIONS FOR FURTHER STUDY

An insulated wall with no vapor barrier and thus with a warm side permeance of about 8-perm, allows condensation and paint failure. Good, fair, and poor vapor barriers, with permeances of say 1, 2, and 4-perm, need to be observed in the light of the theory of this thesis, and compared with subsequent paint damage.

A poor vapor barrier may allow paint blistering, but what causes it? An interesting theory on how the pressures of 200 to 500-lb/in^2 necessary to separate paint from wood are obtained in wood siding was presented by Babbitt and tested by Kuzmak (Ref. 7 and 8). Such high hydrostatic pressures may be built up by condensed vapor under the paint coat when a temperature difference exists across the siding which acts as a membrane. An explanation is needed of how a one-perm vapor barrier is satisfactory, although theoretically at least, it allows condensation on or in the siding. It may be that as long as the flow rate is sufficiently low and the periods of very cold weather are sufficiently short, the paint will not blister.

The rate of vapor liberation in a house by several sources was reported by Hite (Ref. 9). The total amount liberated each hour and each day can vary widely. If the house depends on these sources for moisture, the relative humidity must go up and down. Perhaps the structure can now be protected against high inside vapor pressure and so a healthy winter indoor air can be maintained. It seems possible that this could be done without great expense because there are existing sources of moisture inside and a free low vapor pressure sink outside. A well-ventilated attic over a highly permeable, well-insulated ceiling can prevent too high a relative humidity, but a system is needed that will also protect against too low a relative humidity.

APPENDIX A. THE PERMEABILITY OF AIR

The coefficient of diffusion for water vapor in air at standard temperature and pressure (0°C and 1-atm) listed by Boynton (Ref. 10) is

$$K_o = 0.220\text{-cm}^2/\text{sec}.$$

If P_o/P is the ratio of standard to actual total pressure, and T/T_o is the ratio of actual to standard absolute temperature, then

$$K = K_o(P_o/P)(T/T_o)^{1.75},$$

and the flow rate,

$$M = K\ A\ dc/dn,$$

where A is the area perpendicular to the gradient of the concentration, dc/dn.

The value listed for K is for the diffusion of air and water vapor into each other. For vapor concentrations small compared with the concentration of air, K is correct for water vapor diffusing into stationary air (Ref. 11).

Vapor pressure can be used instead of concentration to find flow rate, by substituting p/RT for c, where R is the gas constant for water vapor. Then

$$M = K/RT\ A\ dp/dn.$$

Let D = K/RT be the permeability of air to water vapor. The units for D can be changed to perm-in, the units often used for building materials, as follows:

$$\frac{K_o}{R} = \frac{0.220\text{-cm}^2}{\text{sec}} \; \frac{\text{lb-T}}{85.8\text{-ft-lb}} \; \frac{3600\text{-sec}}{\text{hr}} \; \frac{\text{ft}^2}{(30.5)^2\text{-cm}^2}$$

$$\frac{7000\text{-gr}}{\text{lb}} \; \frac{\text{lb/in}^2}{2.036\text{-in Hg}} \; \frac{1728\text{-in}^3}{\text{ft}^3}$$

$$= \frac{59000\text{-gr-in-T}}{\text{hr-ft}^2\text{-in Hg}} = 59000\text{-perm-in-T.}$$

At $T = 460°R = 0°F$, and for standard total pressure,

$$D = K/RT = K_o/RT = 59000\text{-perm-in-T}/460\text{-T}$$

$$= 128\text{-perm-in.}$$

At $T = 530°R = 70°F$,

$$D = K/RT = (K_o/RT)(T/T_o)^{1.75}$$

$$= (59000/530)(530/460)^{1.75}\text{-perm-in}$$

$$= 143\text{-perm-in.}$$

APPENDIX B. THE PERMEANCE OF BUILDING MATERIALS

The permeance of vapor barriers is of significance mainly in comparison with the permeance of other building materials. If condensation is to be prevented at a point in a wall, the rate of vapor flow toward and away from the point must balance. Since the vapor-pressure difference across the siding is smaller than across the rest of the wall, the rest of the wall must have a lower permeance than the siding. The following list shows why about one perm is the maximum allowable permeance of an applied vapor barrier.

The unit for permeance is perm, which is a short name for grain/hour-square foot-inch of mercury. The permeability in perm-in is the permeance in perms multiplied by the thickness in inches. In the column headed by humidity, the relative humidity is given for the air on either side of the material as it was tested, first for the high vapor pressure side and then for the low. Some of the tests included a temperature difference across the material. The references selected give a description of the test.

Material	Thickness in	Humidity %	Permeance perm	Permeability perm-in	Ref.
air space, vapor flow down, 71°F	5.7	92-73		120.	4
rock wool	3.625	100-30		125.	12
	3.5	75-0		91.	13
	1.	75-50		26.	14
sawdust	2.	100-50		63.	14
fiberboard	.77	80-45	41.	32.	4
	1.06	75-0	18.	19.	13
	.49	75-0	30.	15.	13
plus asphalt on one surface	.49	75-0	4.0		13
fiberboard, one prime coat	.5	100-30	32.		12
plus two coats flat wall paint	.5	100-30	12.		12
plasterboard	.37	75-0	35.		13
gypsum wallboard	.375	50-0	20.		15
plus two coats rubber base paint	.375	50-0	.8		15
plus one coat primer and sealer, one coat oil base flat paint	.375	50-0	1.0		15
plus one coat interior aluminum one coat oil base flat paint	.375	50-0	1.1		15
plaster	1.34	75-0	13.	18.	13
3/4-inch plaster on wood lath		50-100	2.9		14
plaster on wood lath		100-30	11.		12
plus two coats aluminum paint		100-30	1.2		12
plaster on gypsum lath		100-30	19.		12
plus one coat primer, two coats flat wall paint		100-30	3.7		12
1/2-inch plaster on gypsum lath		40-85	20.		4

Material	Thickness in	Humidity %	Permeance perm	Permeability perm-in	Ref.
sugar pine wood	.25	91-23	4.3	1.1	4
	.25	90-78	19.	4.8	4
Douglas fir wood	.375	100-30	13.	4.8	12
white pine wood	.75	75-50	1.4	1.0	14
spruce wood	.56	75-0	1.7	1.0	13
pine wood	.80	75-0	.9	.8	13
	.51	75-0	3.3	1.7	13
plus one coat white enamel	.51	75-0	1.9		13
plus two coats white enamel	.51	75-0	.9		13
plus three coats white enamel	.51	75-0	.7		13
cedar bevel siding		75-50	1.8		14
plus three coats white lead paint		75-50	1.0		14
plywood	.25	75-50	3.7		14
plus two coats white lead paint	.25	75-50	1.9		14
plywood	.25	100-30	5.6		12
plus one coat asphalt paint	.25	100-30	2.7		12
plus two coats asphalt paint	.25	100-30	.7		12
plus one coat aluminum paint	.25	100-30	3.0		12
plus two coats aluminum paint	.25	100-30	1.3		12
plus one coat prime, one coat flat wall paint	.25	100-30	2.7		12
concrete	1.5	50-100	1.7		14
brick wall	4.	50-100	.8		14
concrete block wall	8.	30-30	2.		4
tile wall	4.	50-100	.12		14

Material	Weight lb/500-ft^2	Humidity %	Permeance perm	Ref.
red rosin sheathing paper		75-50	60.	14
rosin sized Kraft sheathing paper	20	100-30	68.	12
	40	100-30	55.	12
asphalt impregnated felt sheathing paper	75	89-6	2.3	4
asphalt saturated and coated sheathing paper	50	50-0	.15	16
	50	100-50	1.0	16
	35	100-30	1.0	12
	38	100-30	.22	12
	50	90-1	.35	4
aluminum foil, 0.00035-inch, on paper	16	50-0	.005	16
	16	100-50	4.4	16
0.0006-inch, on paper		99-1	.02	4
0.004-inch		100-1	.00	4
polyethylene film, 0.002-inch, 32°F			.015	17
77°F			.13	17
90°F			.18	17
100°F			.23	17
90°F			.27	15
0.004-inch, 90°F			.11	15

APPENDIX C. THE EXACT SOLUTION FOR A CRACK

An exact solution for the vapor flow rate through a crack can be found by using transformations derived by Davy (Ref. 18). Figure 13 shows corresponding points and lines for the transformations:

$$z = b - \frac{ais}{K} - \frac{ai}{2E\text{-}m,K}\left[2Z(s) + \frac{(\mathrm{cn}\ s)(\mathrm{dn}\ s)}{(\mathrm{sn}\ s)}\right],$$

$$t = \mathrm{ns}\ s$$

$$w = \ln\ t$$

Figure 14 indicates the error incurred when the flow rate equation for a crack found on page 17 is used. Several values of s along the negative imaginary axis were chosen for each of several values of m. Corresponding values of w, z, and b/a = g/f were found using numerical values from references 19 and 20. The values of exact/approximate were found by dividing the difference between values of w by the corresponding values of $u_2 + u_1 + \pi g/f$ assuming flow along straight lines between the edges of the crack and flow along semihyperbolas on either side of the crack.

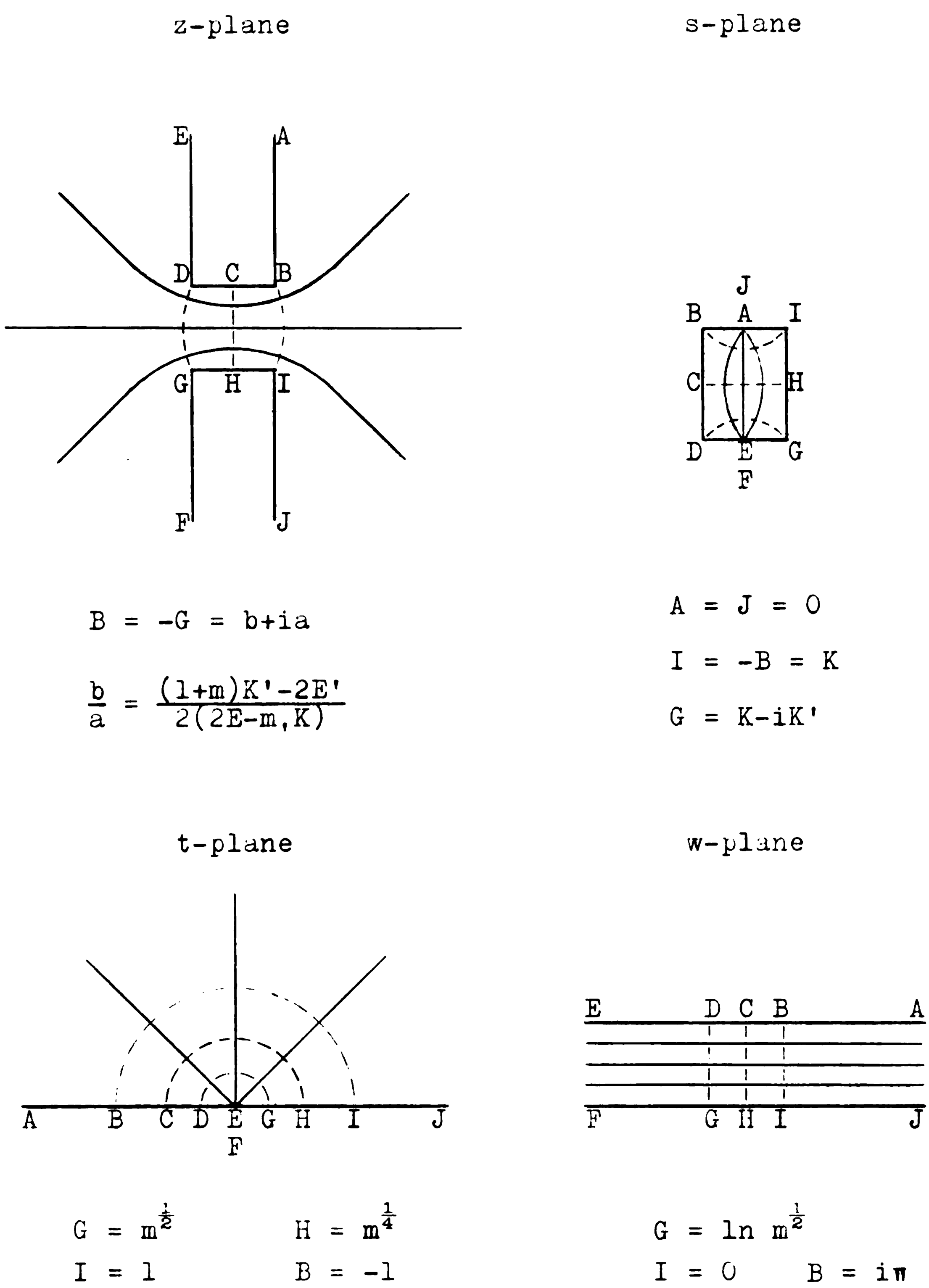

Figure 13. Mapping of a crack into a rectangle, the upper half-plane, and an infinite strip

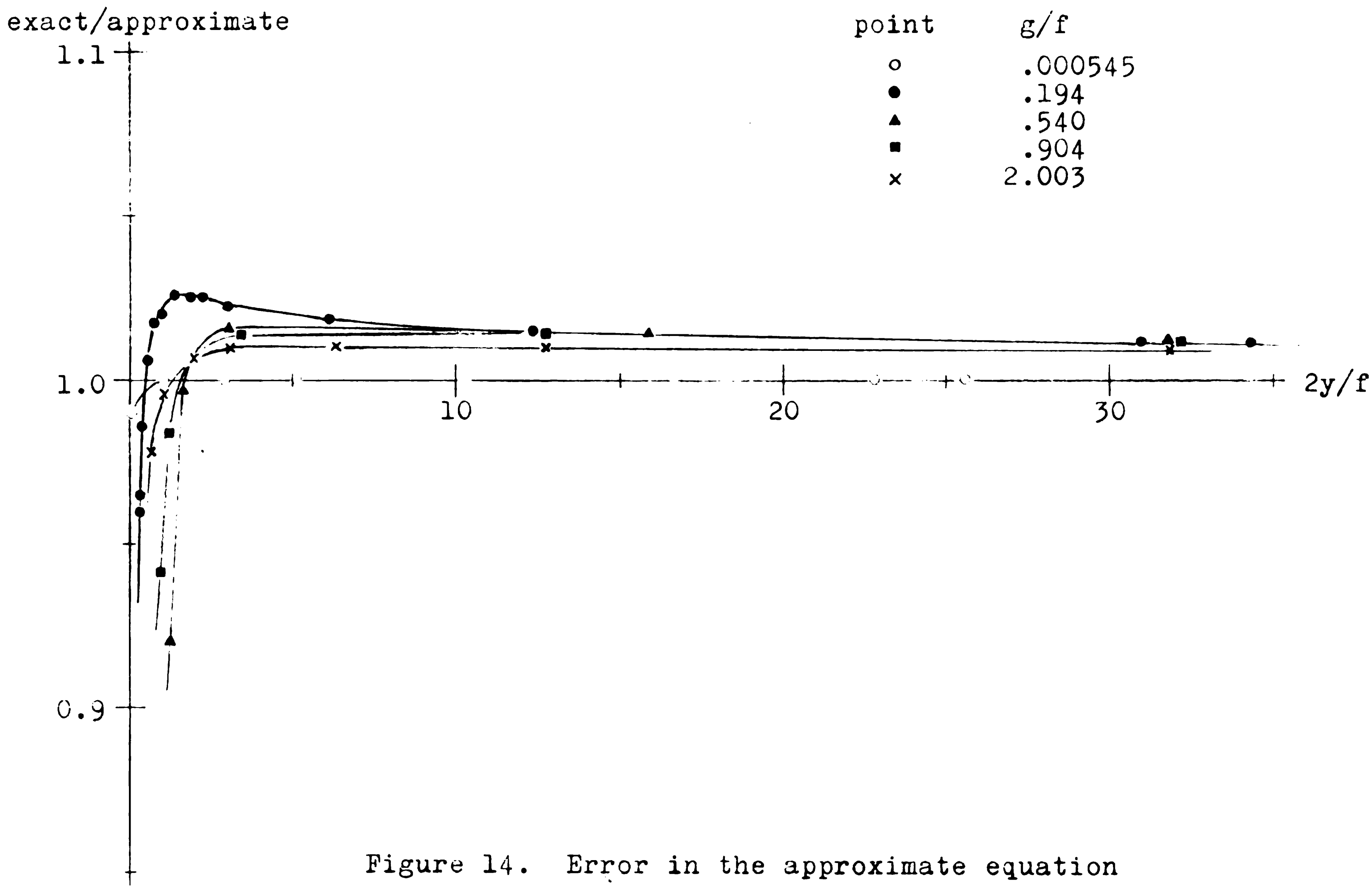

Figure 14. Error in the approximate equation for flow rate through a crack

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