COMPARATIVE STUDY OF SIMULTANEOUS FAULTS

Thesis for the Degree of M. S.

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A. S. M. Nurullah

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This is to certify that the

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Comparative Study

of

SIMULTANEOUS FAULTS

investigated by

CLARKE COMPONENTS

in comparison with

SYMMETRICAL COMPONENTS

by

A. S. M. Nurullah

A Thesis

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INTRODUCTION

This thesis is mainly intended to investigate two simultaneous faults that occured in the power system by Clarke Components. Supposing the problem constitutes a paper mill which buys power from a big system to run a motor besides their own station capacity is thrown out because of a line to ground fault which occured in the 'system' with a simultaneous opening of a breaker in the motor circuit. (shown in Fig. 3.1)

The origin of this new method of attack came to the mind of Dr. J. Strelzoff, Professor of Electrical Engineering at Michigan State College, during his work in The Consumer Power Company, Jackson, Michigan. Though he solved the problem by the usual Symmetrical Components method, he had an idea that it might be possible to solve the same sort of problem with ease by using Clarke Components. Clarke Components were defined in 1938 by Miss Edith Clarke of the General Electric Company and have not been put to much use as yet. The author was entrusted to solve the problem stated above by Clarke Components as well as to draw a comparison with the Symmetrical Components.

In view of this, the Symmetrical Components method of solving the problem with a review of a few basic relations involved was used as well as the Clarke method, to show the simplicity of the Clarke method.

The reference books used have been listed in the bibliography and their numbers occur as exponents throughout the thesis. An appendix is provided at the back for ready reference of vitally important relations.

Chapter I

SYMMETRICAL COMPONENT

History:

The first paper indicating the possibilities of resolving an unbalanced system of currents into positive and negative sequence components of current was published in 1912 by L. G. Stokvis. A second paper dealing with third harmonic voltages in alternators was presented under the sponsorship or André Bondel at a meeting of the French Academy of Science in 1914. It is interesting to note that positive and negative sequence currents as they are now known were a by-product of Stokvis's main endeavor, which was to find a means of determining the magnitude of the third harmonic voltage produced by unbalanced line to line loads. A more detailed treatment of the resolution into positive and negative sequence currents of the unbalanced currents in the three phase ungrounded system was given[2] in 1915.

In 1918, Dr. C. L. Fortescue presented before the American Institute of Electrical Engineers a paper[3] which introduced the concept of zero sequence currents and voltages and provided a general method for the solution of unbalanced polyphase systems. In this paper Dr. Fortescue proved that, " a system of n vectors or quantities may be resolved when n is prime into n different symmetrical groups or systems, one of which consists of n equal vectors and the remaining (n-1) systems consists of n equi-spaced vectors which with the first mentioned groups of equal vectors forms an equal number of symmetrical n-phase systems...."

The method of symmetrical components is a general one, applicable to any polyphase system.

Review of a few basic relations:

In three-phase power systems, sinusoidal currents and voltages of fundamental frequency are represented for the purpose of calculation by vectors revolving at an angular velocity, $\omega = 2\pi f$ radians per second. The components which replace them must therefore be sinusoidal quantities of the same frequency, represented by vectors revolving at the same angular velocity. Since the angles between vectors, V_a, V_b, and V_c, and the somponents which are to replace them can be represented in the same vector diagram, with any current or voltage vector revolving at the same rate as reference vector.

Any three co-planar vectors V_a, V_b, and V_c, can be expressed in terms of three new vectors V_1, V_2, and V_3, by three simultaneous linear equations with constant coefficients. Thus:

$$V_a = C_{11} V_1 + C_{12} V_2 + C_{13} V_3 \qquad (1)$$

$$V_b = C_{21} V_1 + C_{22} V_2 + C_{23} V_3 \qquad (2)$$

$$V_c = C_{31} V_1 + C_{32} V_2 + C_{33} V_3 \qquad (3)$$

Where the choice of coefficient is arbitrary, except for the restriction that the determinant made up of coefficient must not be zero.

The purpose of expressing the original vectors in terms of three known vectors is to simiplify calculation and thereby to gain a better understanding of a given problem. With this thought in mind, two conditions should be satisfied in selecting systems of components to replace three phase current and voltage vectors.

(1) Calculations should be simiplified by the use of the chosen systems of components. This is possible only if the impedences (or admittances) associated with the components of current or voltages can be obtained readily by calculation or test.

(2) The systems of components chosen should have physical significance and be an aid in determining power system performance.

A system of three symmetrical vectors out of many possible systems is one in which three vectors are equal in magnitude and displaced from each other by equal angles. If V_a, V_b, and V_c are a set of voltage or current vectors, referring to phase a, b, and c respectively, of a three phase system, the three systems of three symmetrical vectors replacing V_a, V_b, and V_c are:

1. A system of three vectors equal in magnitude displaced from each other by 120°, with the component of phase b lagging the component of phase a by 120° and the component of phase c lagging the component of phase b by 120° as in Fig. 1(a).

2. A system of three vectors equal in magnitude displaced from each other by 120°, with the component of phase b lagging the component of phase a by 240°, and the component of phase c lagging the component of phase b by 240° as in Fig. 1 (b).

3. A system of three vectors equal in magnitude displaced from each other by 0° or 360° as in Fig. 1 (c).

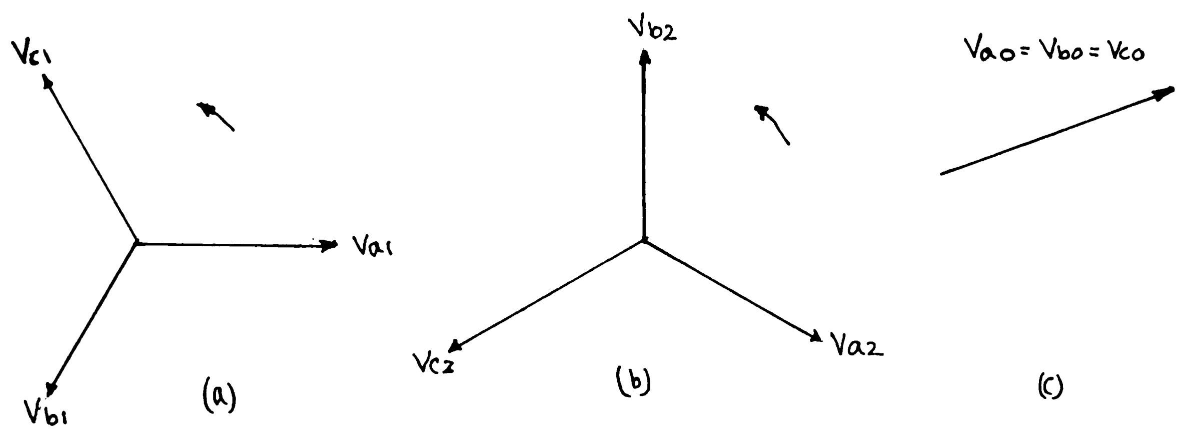


Figure 1. (a) positive, (b) negative and, (c) zero-sequence of vectors.

Three given vectors V_a, V_b and V_c are expressed in terms of their symmetrical components by the equations:

$$V_a = V_{a1} + V_{a2} + V_{ao} \tag{4}$$

$$V_b = V_{b1} + V_{b2} + V_{bo} \tag{5}$$

$$V_c = V_{c1} + V_{c2} + V_{co} \tag{6}$$

Phase A as reference.

$$V_{b1} = a^2 V_{a1} \quad ; \quad V_{c1} = a V_{a1}$$

$$V_{b2} = a V_{a2} \quad ; \quad V_{c2} = a^2 V_{a2}$$

$$V_{bo} = V_{ao} \quad ; \quad V_{co} = V_{ao}$$

Substituting these relations in (4) - (6), the results

$$V_a = V_{a1} + V_{a2} + V_{ao} \tag{7}$$

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0} \tag{8}$$

$$V_c = a V_{a1} + a^2 V_{a2} + V_{a0} \tag{9}$$

Again

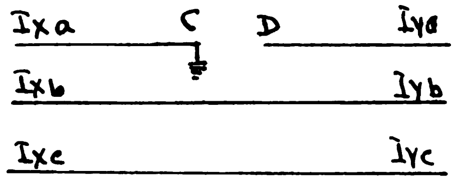
$$V_{a1} = \frac{1}{3}(V_a + aV_b + a^2 V_c) \tag{10}$$

$$V_{a2} = \frac{1}{3}(V_a + a^2 V_b + aV_c) \tag{11}$$

$$V_{a0} = \frac{1}{3}(V_a + V_b + V_c) \tag{12}$$

Any three phase systems can be represented by sequence networks. (This includes the equivalent sequence networks of generator, transformer; and transmission line etc.)

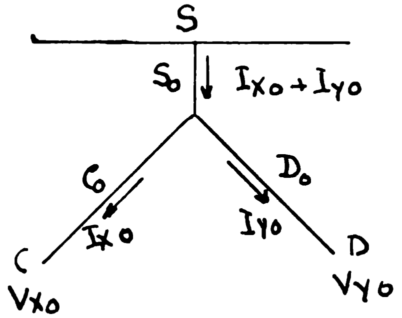
The method used by symmetrical somponent method in solving two simultaneous faults on the same phase--a line to ground and an open wire fault is presented here.*



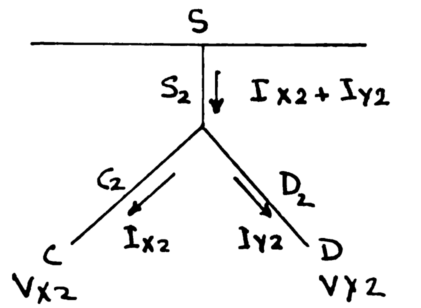
Solution by the equivalent wye method.

The equivalent circuit for replacing a single fault in the positive-sequence network is a single impedance connected in shunt (or series) with the positive-sequence network at the point of fault. The equivalent circuit for a double fault is a 3-terminal network. The 3-terminals are the two points of fault and the zero-potential bus. The simplest forms of 3-terminal networks are the wye and the delta. The equivalent wye circuit is derived by first replacing the negative and the zero-sequence networks by their equivalent wye and then combining these two into an equivalent wye which is then connected at the corresponding points of the positive-sequence network. From then the computation is similar to that for a simgle fault.

* Extract from Dr. Strelzoff's work.



Zero-sequence Wye

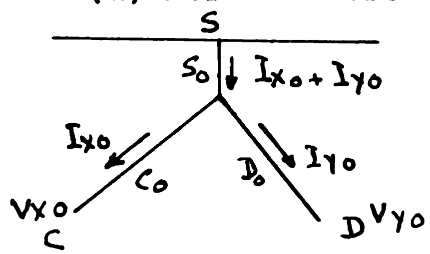


Negative-sequence Wye

The values of C_0, D_0, S_0, C_2, D_2, and S_2 may be obatined by standard me thod of reduction of a network to its equivalent wye.

For the zero-sequence and the negative-sequence networks reduced to their equivalent wye, we can write now the following equations:

(a) For the zero-sequence network:



$$0-V_{x0} = I_{x0}\ (C_0+S_0) + I_{y0}S_0 \qquad (1)$$

$$0-V_{y0} = I_{x0}S_0 \qquad + I_{y0}\ (D_0\ S_0) \qquad (2)$$

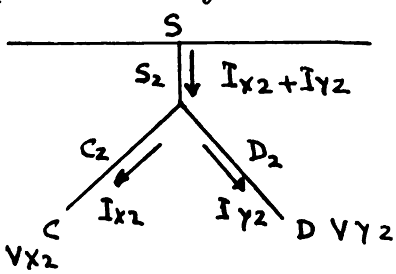
Solving the above equations for the currents I_{x0}, I_{y0}:

$$I_{x0} = -V_{x0}\frac{D_0+S_0}{D_{00}} + V_{y0}\frac{S_0}{D_{00}} \qquad (1a)$$

$$I_{y0} = V_{x0}\frac{S_0}{D_{00}} - V_{y0}\frac{C_0+S_0}{D_{00}} \qquad (2a)$$

$D_{00} = C_0D_0 + C_0S_0 + D_0S_0$

(b) For the negative-sequence network, the following two equations may be written:



$$0-V_{x2} = I_{x2}\ (C_2+S_2) + I_{y2}S_2 \qquad (3)$$

$$0-V_{y2} = I_{x2} \qquad + I_{y2}\ (D_2\ S_2) \qquad (4)$$

Solving the above equations for the currents I_{x2}, I_{y2}:

$$I_{x2} = -V_{x2}\frac{D_2+S_2}{D_{22}} + V_{y2}\frac{S_2}{D_{22}} \qquad (3a)$$

$$I_{y2} = V_{x2}\frac{S_2}{D_{22}} - V_{y2}\frac{C_2+S_2}{D_{22}} \qquad (4a)$$

where $D_{22} = C_2D_2 + C_2S_2 + D_2S_2$

In general this problem involves 12 unknowns; they are I_{x0}, I_{x1}, I_{x2}, V_{x0}, V_{x1}, V_{x2}, I_{y0}, I_{y1}, I_{y2}, V_{y0}, V_{y1}, V_{y2}. To solve it analytically we have the 4 equations 1, 2, 3, & 4 and the 6 equations determined by the respective fault conditions. There equations are:

(a) For the line-to-ground fault on phase a:

$$V_{xa} = 0 \qquad I_{xb} + I_{yb} = 0 \qquad I_{xc} + I_{yc} = 0$$

(b) For the open circuit on phase a:

$$I_{ya} = 0 \qquad V_{xb} - V_{yb} = 0 \qquad V_{xc} - V_{yc} = 0$$

In terms of symmetrical components, these equations are:

$$V_{x0} + V_{x1} + V_{x2} = 0 \qquad (5)$$

$$I_{y0} + I_{y1} + I_{y2} = 0 \qquad (6)$$

$$I_{x0} + I_{y0} = I_{x1} + I_{y1} \qquad (7)$$

$$I_{x2} + I_{y2} = I_{x1} + I_{y1} \qquad (8)$$

$$V_{x0} - V_{y0} = V_{x1} - V_{y1} \qquad (9)$$

$$V_{x2} - V_{y2} = V_{x1} - V_{y1} \qquad (10)$$

Hence we have 12 unknowns and only 10 equations. By eliminating negative- and zero-sequence currents and voltages from the 10 known equations, 2 equations containing only the 2 positive-sequence currents and the 2 positive-sequence voltages at points of fault, x and y remain. These equations will be of the form similar to those of equations (1) and (2), or (3) and (4).

$$V_{x1} = lI_{x1} + mI_{y1} \qquad (11)$$

$$V_{y1} = nI_{x1} + kI_{y1} \qquad (12)$$

For these equations to correspond to a set of equations involving an equavalent wye, it is necessary that $m = n$.

Equations (11) and (12) are derived as follows:

From equations (1) and (2) and (9)

$$E_{x0} - E_{y0} = E_{x1} - E_{y1} = - I_{xc} (C_0 + S_0) - I_{y0}S_0 + I_{x0}S_0 + I_{y0} (D_0 + S_0) = - I_{x0}C_0 + I_{y0}D_0$$

From equation (7)

$$I_{x1} + I_{y1} - I_{y0} = I_{x0}$$

Hence

$$E_{x1} - E_{y1} = - C_0 (I_{x1} + I_{y1} - I_{y0}) + I_{y0} D_0 = - C_0I_{x1} - C_0I_{y1} + I_{y0} (C_0 + D_0) \qquad (13)$$

Using equations (3) and (4) and (8)

$$E_{x2} - E_{y2} = E_{x1} - E_{y1} = -I_{x2} (C_2 + S_2) - I_{y2}S_2 + I_{x2}S_2 + I_{y2} (D_2 + S_2) - = - C_2I_{x2} + D_2I_{y2}$$

$$I_{x2} = I_{x1} + I_{y1} - I_{y2}$$

Also from equation (6) $\qquad -I_{y2} = I_{y0} + I_{y1}$

Hence

$$E_{x1} - E_{y1} = -C_2 (I_{x1} + I_{y1}) + (C_2 + D_2) I_{y2} = -C_2I_{x1} - C_2I_{y1} - (C_2 + D_2) (I_{y0} + I_{y1})$$

$$E_{x1} - E_{y1} = -C_2I_{x1} - (2C_2 + D_2) I_{y1} - (C_2 + D_2)I_{y0} \qquad (14)$$

Comparing equations (13) and (14), we write:

$$-C_0I_{x1} - C_0I_{y1} + I_{y0} (C_0 + D_0) = -C_2I_{x1} - (2C_2 + D_2) I_{y1} - (C_2 + D_2)I_{y0}$$

Solving for I_{y0}

$$I_{y0}\ (C_0+D_0+C_2+D_2) = I_{x1}\ (C_0-C_2) + I_{y1}\ (C_0-2C_2-D_2)$$

Let $C_0 + D_0 = Z_0 \qquad C_2 + D_2 = Z_2$

Then

$$I_{y0} = I_{x1}\frac{C_0-C_2}{Z_0+Z_2} + I_{y1}\frac{C_0-C_2-Z_2}{Z_0+Z_2} \tag{15}$$

Substituting the value of I_{y0} from equation (15) in equation (14) which for this purpose is rewritten as follows:

$$E_{x1}-E_{y1} = -I_{x1}C_2 - I_{y1}C_2 + I_{y2}\ (C_2 + D_2)$$

$$= -I_{x1}C_2 - I_{y1}C_2 - (I_{y0} + I_{y1})(C_2 + D_2)$$

$$= -I_{x1}C_2 - I_{y1}\ (C_2 + Z_2) - I_{y0}Z_2$$

$$E_{x1} - E_{y1} = -I_{x1}C_2 - I_{y1}(C_2+Z_2) - \frac{Z_2(C_0-C_2)}{Z_0+Z_2}I_{x1} - \frac{Z_2(C_0-C_2-Z_2)}{Z_0+Z_2}I_{y1}$$

$$E_{y1} = E_{x1} + I_{x1}\left[C_2 + \frac{Z_2(C_0-C_2)}{Z_0+Z_2}\right] + I_{y1}\left[C_2 + Z_2 + \frac{Z_2(C_0-C_2-Z_2)}{Z_0+Z_2}\right] \tag{16}$$

Using equations (5), (1) and (3)

$$-E_{x0}-E_{x2} = E_{x1} = I_{x0}\ (C_0 + S_0) + I_{y0}S_0 + I_{x2}(C_2 + S_2) + I_{y2}S_2$$

$$= (I_{x1} + I_{y1} - I_{y0})(C_0 + S_0) + I_{y0}S_0 + (I_{x1} + I_{y1} - I_{y2})$$
$$(C_2 + S_2) + I_{y2}S_2$$

$$= I_{x1}\ (C_0 + S_0 + C_2 + S_2) + I_{y1}(C_0 + S_0 + C_2 + S_2) -$$
$$I_{y0}C_0 - I_{y2}C_2$$

$$= I_{x1}\ (C_0 + S_0 + C_2 + S_2) + I_{y1}\ (C_0 + S_0 + C_2 + S_2) - I_{y0}C_0$$
$$+ (I_{y1}\ I_{y0})\ C_2$$

$$= I_{x1}\ (C_0 + S_0 + C_2 + S_2) + I_{y1}\ (C_0 + S_0 + 2C_2 + S_2) - I_{y0}$$
$$(C_0 - C_2)$$

Substituting equation (15) for I_{y0}

$$E_{x1} = I_{x1}\ (C_0+S_0+C_2+S_2) + I_{y1}\ (C_0+S_0+2C_2+S_2) - \frac{(C_0-C_2)^2}{Z_0+Z_2}I_{x1}$$
$$- \frac{(C_0-C_2)(C_0-C_2-Z_2)}{Z_0+Z_2}I_{y2}$$

$$E_{x1} = I_{x1}\left[C_0 + S_0 + C_2 + S_2 - \frac{(C_0 - C_2)^2}{Z_0 + Z_2}\right] + I_{y1}\left[C_0 + S_0 + 2C_2 + S_2 - \frac{(C_0 - C_2)(C_0 - C_2 - Z_2)}{Z_0 + Z_2}\right] \quad (17)$$

Substituting the value of E_{x1} from equation (17) in the equation (16)

$$E_{y1} = I_{x1}\left[C_0 + S_0 + C_2 + S_2 - \frac{(C_0 - C_2)^2}{Z_0 + Z_2}\right] + I_{y1}\left[C_0 + S_0 + 2C_2 + S_2 - \frac{(C_0 - C_2)(C_0 - C_2 - Z_2)}{Z_0 + Z_2}\right]$$

$$+ I_{x1}\left[C_2 + \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2}\right] + I_{y1}\left[C_2 + Z_2 + \frac{Z_2(C_0 - C_2 - Z_2)}{Z_0 + Z_2}\right]$$

Or
$$E_{y1} = I_{x1}\left[C_0 + S_0 + 2C_2 + S_2 + \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2} - \frac{(C_0 - C_2)^2}{Z_0 + Z_2}\right]$$

$$+ I_{y1}\left[C_0 + S_0 + 3C_2 + S_2 + Z_2 + \frac{Z_2(C_0 - C_2 - Z_2)}{Z_0 + Z_2} - \frac{(C_0 - C_2)(C_0 - C_2 - Z_2)}{Z_0 + Z_2}\right] \quad (18)$$

Equations (17) and (18) can now be written in the form

$E_{x1} = lI_{x1} + mI_{y1}$ (17a)

$E_{y1} = nI_{x1} + kI_{y1}$ (18a)

$$m = C_0 + S_0 + 2C_2 + S_2 - \frac{(C_0 - C_2)(C_0 - C_2 - Z_2)}{Z_0 + Z_2} = C_0 + S_0 + 2C_2 + S_2 - \frac{(C_0 - C_2)^2}{Z_0 + Z_2} + \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2}$$

$$n = C_0 + S_0 + 2C_2 + S_2 + \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2} - \frac{(C_0 - C_2)^2}{Z_0 + Z_2}$$

(19)

Or m=n, which is a necessary condition to be able to reduce the positive-sequence network to an equivalent wye. The remaining coefficients are:

$$l = C_0+S_0+C_2+S_2-\frac{(C_0-C_2)^2}{Z_0+Z_2} \tag{20}$$

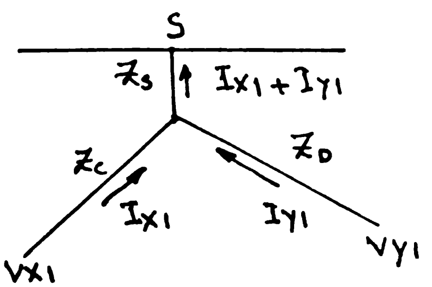
$$k = C_0+S_0+3C_2+S_2+Z_2+\frac{Z_2(C_0-C_2-Z_2)}{Z_0+Z_2}-\frac{(C_0-C_2)(C_0-C_2-Z_2)}{Z_0+Z_2}$$

$$= C_0+S_0+4C_2+S_2+D_2+\frac{(C_0-C_2-Z_2)(Z_2-C_0+C_2)}{Z_0+Z_2}$$

$$= C_0+S_0+4C_2+S_2+D_2-\frac{(C_0-C_2-Z_2)^2}{Z_0+Z_2} \tag{21}$$

Equivalent wye of the positive-sequence network.

Equations (17) and (18) ar (17a) and (18a) represent the following wye:



$$V_{x1} = I_{x1}\,(C+S) + I_{y1}S \tag{17b}$$

$$V_{y1} \quad I_{x1}S \quad + I_{y1}\,(S+D) \tag{18b}$$

Comparing (17a), (18a) with (17b), (18b)

$Z_c = l-m \qquad Z_d = k-m \qquad Z_s = m$

Then

$$Z_c = C_0+S_0+C_2+S_2-\frac{(C_0-C_2)^2}{Z_0+Z_2}-C_0-S_0-2C_2-S_2-\frac{Z_2(C_0-C_2)}{Z_0+Z_2}+\frac{(C_0-C_2)^2}{Z_0+Z_2}$$

$$Z_c = -C_2-\frac{Z_2(C_0-C_2)}{Z_0+Z_2} = \frac{-C_2Z_0-C_2Z_2-Z_2C_0+Z_2C_2}{Z_0+Z_2} = -\frac{C_2Z_0+C_0Z_2}{Z_0+Z_2}$$

(22)

$$Z_d = C_0 + S_0 + 4C_2 + S_2 + D_2 - \frac{(C_0 - C_2 - Z_2)^2}{Z_0 + Z_2} - C_0 - S_0 - 2C_2 - S_2 - \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2} + \frac{(C_0 - C_2)^2}{Z_0 + Z_2}$$

$$= 2C_2 + D_2 - \frac{(C_0 - C_2)^2}{Z_0 + Z_2} - \frac{Z_2^2}{Z_0 + Z_2} + \frac{2Z_2(C_0 - C_2)}{Z_0 + Z_2} - \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2} + \frac{(C_0 - C_2)^2}{Z_0 + Z_2}$$

$$Z_d = 2C_2 + D_2 - \frac{Z_2^2}{Z_0 + Z_2} + \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2}$$

$$= -\frac{Z_2^2}{Z_0 + Z_2} + D_2 + \frac{2C_2Z_0 + 2C_2Z_2 + Z_2C_0 - Z_2C_2}{Z_0 + Z_2}$$

$$= -\frac{Z_2^2}{Z_0 + Z_2} + D_2 + \frac{2C_2Z_0 + C_2Z_2 + Z_2C_0}{Z_0 + Z_2}$$

$$= \frac{-Z_2^2 + D_2Z_0 + D_2Z_2 + C_2Z_0 + C_2Z_0 + C_2Z_2 + Z_2C_0}{Z_0 + Z_2}$$

$$Z_d = \frac{C_2Z_0 + C_0Z_2}{Z_0 + Z_2} + \frac{(D_2 + C_2)Z_0 + (D_2 + C_2)Z_2 - Z_2^2}{Z_0 + Z_2}$$

$$Z_d = \frac{C_2Z_0 + C_0Z_2}{Z_0 + Z_2} + \frac{Z_2Z_0}{Z_0 + Z_2} \qquad (23) \qquad Z_c = -\frac{C_2Z_0 + C_0Z_2}{Z_0 + Z_2}$$

$$Z_s = C_0 + S_0 + 2C_2 + S_2 - \frac{(C_0 - C_2)^2}{Z_0 + Z_2} + \frac{Z_2(C_0 - C_2)}{Z_0 + Z_2}$$

Analytical solution.

First step. To determine the values of positive-sequence fault currents: I_{x1}, I_{y1}, (or fault voltages V_{x1}, V_{y1}). By analytical computations, we equate the following two sets of equations:

$$V_{x1} = V_x - I_{x1}(C_1 + S_1) - I_{y1}S_1 \qquad (22a)$$

$$V_{y1} = V_y - I_{x1}S_1 - I_{y1}(S_1 + D_1) \qquad (22b)$$

And

$$V_{x1} = (Z_c + Z_s)I_{x1} + Z_sI_{y1} \qquad (23a)$$

$$V_{y1} = Z_s I_{x1} + (Z_s + Z_d) I_{y1} \tag{23b}$$

Where V_x and V_y are the voltages existing under normal operations at the fault points, and C_1, S_1, and D_1 are determined in the same manner as C_2, S_2, and D_2. (If the negative reactances of the generators are the same as the positive-sequence values, then $C_1 = C_2$ etc.) Solving the above 2 sets of equations simultaneously results in

$$V_x = (C_1 + S_1 + Z_c + Z_s) I_{x1} + (S_1 + Z_s) I_{y1}$$

$$V_y = (S_1 + Z_s) \quad I_{x1} + (S_1 + D_1 + Z_s + Z_d) I_{y1}$$

The above 2 equations on solution yield the values of I_{x1} and I_{y1} which substituted into (23a,b) give the corresponding values of V_{x1}, V_{y1}. Knowing the values of I_{x1} and I_{y1} and using equation 15, the value of I_{y0} is then found.

Next, equation (7) yields the value of I_{x0}, and equation (6) the value of I_{y2}, and equation (8), the valus of I_{x2}. Knowing the values of I_{x2}, I_{y2}, I_{y0} and I_{x0} and using equations (1), (2), (3), & (4), the corresponding fault voltages are evaluated.

The current distribution in the positive-sequence network is obtained by superposition of the following values:

(a) The currents for normal operation before the faults occur.

(b) The currents due to the fault voltages V_{x1}, V_{y1}.

(c) The currents due to the fault currents I_{x1}, I_{y1}.

The current distribution in the negative - and zero-sequence networks are obtained by the superposition of the following values:

(a) The currents due to the fault voltages. (V_{x2}, V_{y2}, or V_{x0}, V_{y0})

(b) The currents due to the fault currents. (I_{x2}, I_{y2}, or I_{x0}, I_{y0})

Having found the current distributions in the positive-, negative-, and zero-sequence networks, the currents in the phases a, b, c are found by the usual procedure of symmetrical components method.

Chapter II

CLARKE COMPONENTS

History:

Clarke Components[9] or α, β, 0 components of current are not new. Components of current answering to the description of α, β, 0 although not so named were used in a method developed by Dr. W. W. Lewis, and published in 1917, to determine system currents and voltages during line to ground faults. In figure 2 of Dr. Lewis' paper, which is similar to Figure 2.1 of this chapter, phase currents are represented by arrows both in direction and magnitude, the number of arrows indicating relative magnitudes of currents in each circuit.

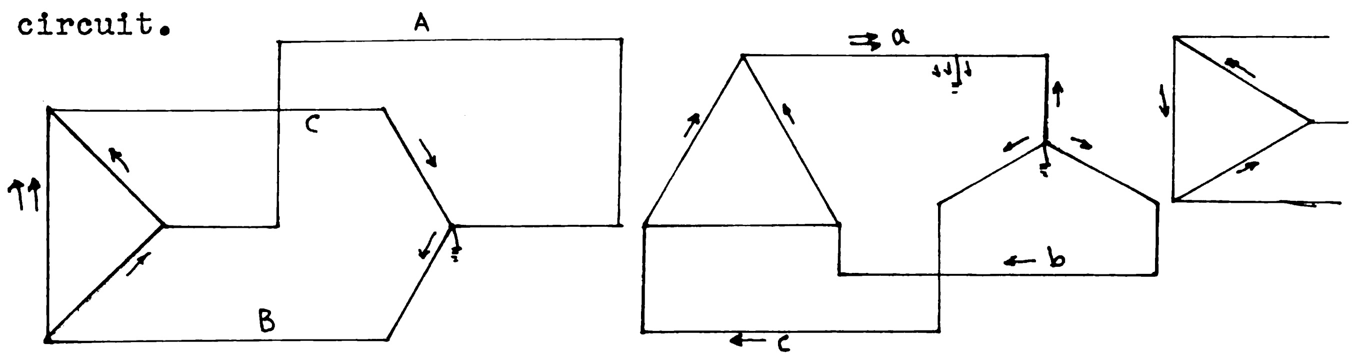


Figure 2.1 Phase currents represented by arrows in direction and magnitude, number of arrows showing relative magnitudes of currents in any circuit.

Currents in the Y-Δ transformer bank and in the line to the right of the fault are 0 currents; currents in the transmission line to the left of the fault are α currents; currents in second Y-Δ transformer bank and in the line at generator terminals are β currents; currents in the generator are α currents. In the method as used before symmetrical components were applied to unsymmetrical short circuits, each component of phase current met its respective impedences, but calculations were made with phase voltages and currents, not with component networks, and therefore it was time consuming if many circuits operated at different voltages were to be considered.

In problems involving unsymmetrical three-phase circuits, and in particular circuits with two of the phases symmetrical with respect to the third phase, the use of the components of current which flow in one phase and divided equally between the other two phases is a logical development. Such components, as yet unnamed, were used[4] in 1931, in paper,[4,5] both of which deal with transient conditions in rotating machines where development is materially simiplified by their use. Two papers have developed exclusively to these components. In one paper they are called α, β, 0 components and the system modified symmetrical components. In the other paper, entitled, "Two-Phase Co-ordinates of a Three-Phase System," by Dr. E. W. Kimbark, the components are called x, y, and z. Comparing these two set: x and α components are identical; y and β components differ only in sign; z components of voltage are 0 (zero-sequence) components of voltage, z components of currents are twice 0 (zero-sequence) components of currents and z impedences are one half 0 (zero-sequence) impedences.

Clarke Components[8] were introduced in 1938 by Edith Clarke, but their usefulness in the solution of the three phase circuit (unbalanced) is not yet fully appreciated. The paper by Camburn and Gross and another paper by Duesterhoeft are welcome because they indicate an increase of interest in Clarke Components.

There are still promising opportunities, however, for further work in exploring the application of these components to both steady state and transient problems.

Definition of Clarke Components:

With phase a as a reference phase in a Three-Phase System.

(1) α components in phase b and c are equal; they are opposite in sign and of half the magnitude of the component of phase a.

(2) β components in phase b and c are equal in magnitude and opposite in sign; in phase a they are zero.

(3) 0 (Zero) components are equal in three phases.

Relation between phase currents and voltages and their α, β, 0 components.

$$\begin{array}{cccc} V_a & V_b & V_c & \\ 1 & -\frac{1}{2} & -\frac{1}{2} & \alpha \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} & \beta \\ 1 & 1 & 1 & 0 \end{array}$$

or,

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix}$$

So, $V_a = V_\alpha \qquad\qquad\qquad + V_0$ (1)

$V_b = -\frac{1}{2}V_\alpha + \frac{\sqrt{3}}{2}V_\beta + V_0$ (2)

$V_c = -\frac{1}{2}V_\alpha - \frac{\sqrt{3}}{2}V_\beta + V_0$ (3)

The required condition i.e determinant must not be zero, is satisfied.

Again solving (1) - (3) $V_\alpha = \frac{2}{3}\left(V_a - \frac{V_b+V_c}{2}\right)$ (4)

$V_\beta = \frac{1}{\sqrt{3}}(V_b - V_c)$ (5)

$V_0 = \frac{1}{3}(V_a + V_b + V_c)$ (6)

The corresponding current equations are

$I_a = I_\alpha \qquad\qquad\qquad + I_0$ (7)

$I_b = -\frac{1}{2}I_\alpha + \frac{\sqrt{3}}{2}I_\beta + I_0$ (8)

$I_c = -\frac{1}{2}I_\alpha - \frac{\sqrt{3}}{2}I_\beta + I_0$ (9)

and

$I_\alpha = \frac{2}{3}\left(I_a - \frac{I_b+I_c}{2}\right)$ (10)

$I_\beta = \frac{1}{\sqrt{3}}(I_b - I_c)$ (11)

$I_0 = \frac{1}{3}(I_a + I_b + I_c)$ (12)

α, β, 0 one line diagrams:

When components of phase currents and voltages instead of phase quantities are used in calculations, each set of components is conviently represented by a seperate one line disgram or component network for which the components of currents and voltage in the three phases can be obtained. To draw component networks, it is necessary to determine:

1. references for the components of voltage,
2. components of generated voltage and,
3. impedences offered to the components of current or admittances associated with the components of voltage.

Generated α, β, and 0 voltages:

In a synchronous machine with generated voltages E_a, E_b, and E_c, the E_α, E_β, and E_0 voltages obtained by substituting E_a, E_b and E_c for V_a, V_b and V_c in (4) - (6) becomes:

$$E_\alpha = \frac{2}{3}\left(E_a - \frac{E_b+E_c}{2}\right) \qquad (13)$$

$$E_\beta = \frac{1}{\sqrt{3}}\left(E_b - E_c\right) \qquad (14)$$

$$E_0 = \frac{1}{3}\left(E_a + E_b + E_c\right) \qquad (15)$$

If the generated voltages are balanced

$$\begin{aligned} E_b &= a^2 E_a \\ E_c &= a E_a \\ E_\alpha &= E_a \\ E_\beta &= -jE_a \\ E_0 &= 0 \end{aligned} \qquad (16)$$

and currents in a balanced system:

In a symmetrical system operating under balanced conditions, the currents in phases b and c at any point of the system are $I_b = a^2 I_a$; $I_c = a\,I\,a$. Substituting these values for I_b and I_c in (10) - (12),

$$\begin{aligned} I_\alpha &= I_a \\ I_\beta &= -jI_a \\ I_0 &= 0 \end{aligned} \qquad (16a)$$

Equations (16) and (16a) show that generated voltages and load currents are present in both the α and β networks of a symmetrical system during normal operation. Because two networks must be considered instead of one, α, β, 0 components are not as convinient as symmetrical components for the study of symmetrical systems during normal operation or during three phase faults.

α, β and 0 networks:

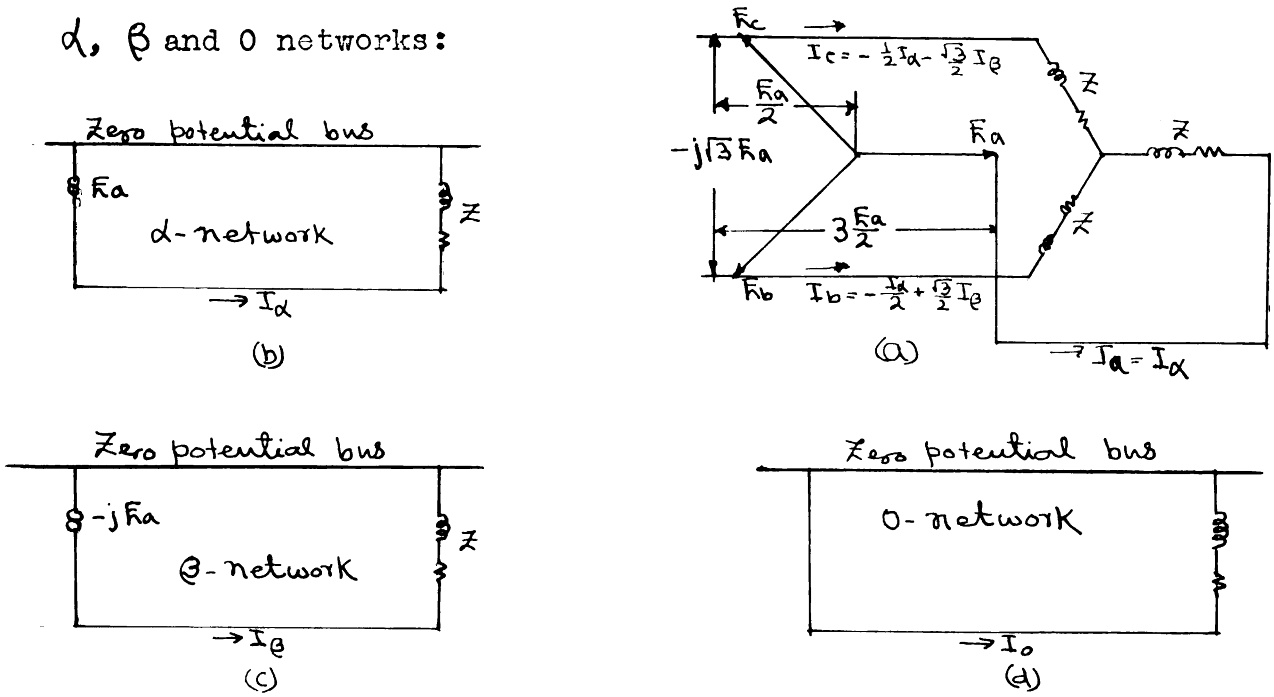


Figure 2.2 (a) Flow of α and β currents in balanced system with equal self impedences in three phases and balanced applied voltages. (b) α network for system (a). (c) β network for system (a). (d) 0-network for system (a).

Figure 2. shows a symmetrical three-phase system with balanced applied voltages and equal self impedences z in the three phases.

I_α , flowing in phase a and returning one half in each of phases b and c flows into a loop circuit. The voltage applied to this loop, as shown in the Fig. 2.2(a) is $E_a - (-E_a/2) = \frac{3}{2}E_a$. The α-loop impedence for a symmetrical three phase circuit of equal self-impedence Z in the three phases is $\frac{3}{2}$ Z. The current I_α in phase a is

$$I_\alpha = \frac{\frac{3}{2} E_a}{\frac{3}{2} Z} = \frac{E_a}{Z} \qquad (17)$$

The impedence met by I_α is Z. The equivalent circuit for phase a in the α system is shown in Fig. 2.2(b), with the applied voltage E_a and the self-impedence Z. In this equivalent circuit, voltages are referred to neutral, base voltage is line to neutral voltage, and base current is line current. Since the α currents and voltages in phases b and c at any point in the system are -1/2 those of phase a at the same point, it is unnecessary to have additional equivalent circuits for these phases. The equivalent circuit for phase a in the α system will be called the α network.

β currents, flowing in phase b and returning in phase c, flow in a loop circuit. The voltage applied to this loop, as shown in Fig. 2.2(a), is $-j\sqrt{3}\, E_a$. The β loop impedence for the symmetrical three-phase circuit of equal self-impedences Z in the three phases is 2Z. The β current flowing in phase b in the direction indicated by arrow is $\frac{\sqrt{3}}{2} I_\beta$.

Therefore $\frac{\sqrt{3}}{2} I_\beta = -j \frac{\sqrt{3}\, E_a}{2Z}$; and $I_\beta = -j \frac{E_a}{Z}$

The impedence met by I_β is Z. The equivalent circuit for the β system is shown in Fig. 2.2(c) with the applied voltage $-j\, E_a$ and the self-impedence Z. In this equivalent circuit, which will be called the β network, voltages are referred to neutral, base voltage is line to neutral voltage, and base current is line current. The β voltages and currents in phases b and c are the voltages and currents in the β network multiplied by $\frac{\sqrt{3}}{2}$ and $-\frac{\sqrt{3}}{2}$, respectively. The β network does not give directly the β voltages and currents in either phase b or phase c. This slight disadvantage is more than offset by the convenience of having the same line to neutral voltage and line current as base quantities in the β as in the α and 0 networks.

With a path for 0 currents through the circuit of equal self-impedences Z in the three phases, the impedence met by I_0 is Z. The 0 network for the system of Fig.2.2(a) is shown in Fig. 2.2(d).

α, β and 0 equivalent circuits to replace the various equipment, machines and transmission circuits of a three phase power system in the α, β and 0 networks can be determined when the α, β and 0 self and mutual impedences of the circuits are known. α, β and 0 impedences, just as positive, negative and zero-sequence impedences can be obtained by calculation or test. Before developing equivalent for use in the α, β, and 0 networks, relations between symmetrical components and α, β and 0 components will be established.

α, β and 0 components of voltages and current in terms of symmetrical components of voltage and current:

Symmetrical Components in matrix form;

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} V_{a_1} \\ V_{a_2} \\ V_{a_0} \end{bmatrix} \qquad (13)$$

Clarke Components in matrix form

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \begin{bmatrix} V_\alpha \\ V_\beta \\ V_0 \end{bmatrix} \qquad (19)$$

Similarly for current,

Symmetrical Components in matrix form

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{bmatrix} \begin{bmatrix} I_{a1} \\ I_{a2} \\ I_{a0} \end{bmatrix} \qquad (20)$$

and Clarke Components

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} & 1 \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 1 \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \\ I_0 \end{bmatrix} \qquad (20)$$

From the relations shown above, Symmetrical Components and Clarke Components can be related to each other.

Form (18) $V_a = V_{a1} + V_{a2} + V_{a0}$

and form (19)

$$V_a = V_\alpha + V_0$$

So

$$V_\alpha + V_0 = V_{a1} + V_{a2} + V_{a0}$$

$\because$ $V_0 = V_{a0}$ (can be visualised from (18) and (19))

$$V_\alpha = V_{a1} + V_{a2}$$

Now from (18)

$$V_b = a^2 V_{a1} + a V_{a2} + V_{a0}$$

from (19)

$$V_b = -\frac{1}{2} V_\alpha + \frac{\sqrt{3}}{2} V_\beta + V_0$$

So,

$$a^2 V_{a1} + a V_{a2} = -\frac{1}{2} V_\alpha + \frac{\sqrt{3}}{2} V_\beta$$

$$a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$$

$$a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$$

$$\therefore \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) V_{a1} + \left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) V_{a2} = -\frac{1}{2}(V_{a1} + V_{a2}) + \frac{\sqrt{3}}{2} V_\beta$$

or,

$$\frac{\sqrt{3}}{2} V_\beta = -j\frac{\sqrt{3}}{2} V_{a1} + j\frac{\sqrt{3}}{2} V_{a2}$$

$$V_\beta = -j(V_{a1} - V_{a2})$$

In similar way relations of two components of current can be found.

$$I_\alpha = I_{a1} + I_{a2}$$

$$I_\beta = -j(I_{a1} - I_{a2}) \qquad (21)$$

$$I_0 = I_{a0}$$

$$V_\alpha = V_{a1} + V_{a2}$$

$$V_\beta = -j(V_{a1} - V_{a2}) \qquad (22)$$

$$V_0 = V_{a0}$$

Conclusion: α components are positve plus negative components, β components are positive minus negative components turned backward by 90° and 0 components are same.

Symmetrical Components of voltage and current in terms of α, β, 0 components of voltage and current.

Solving simultaneous equations of (21)

$$
\begin{aligned}
V_{a1} &= \tfrac{1}{2}(V_\alpha + jV_\beta) \\
V_{a2} &= \tfrac{1}{2}(V_\alpha - jV_\beta) \\
V_{a0} &= V_0 \qquad\qquad (23) \\
I_{a1} &= \tfrac{1}{2}(I_\alpha + jI_\beta) \\
I_{a2} &= \tfrac{1}{2}(I_\alpha - jI_\beta) \\
I_{a0} &= I_0
\end{aligned}
$$

α, β and 0 self and Mutual Impedences and their relation with Symmetrical Component:

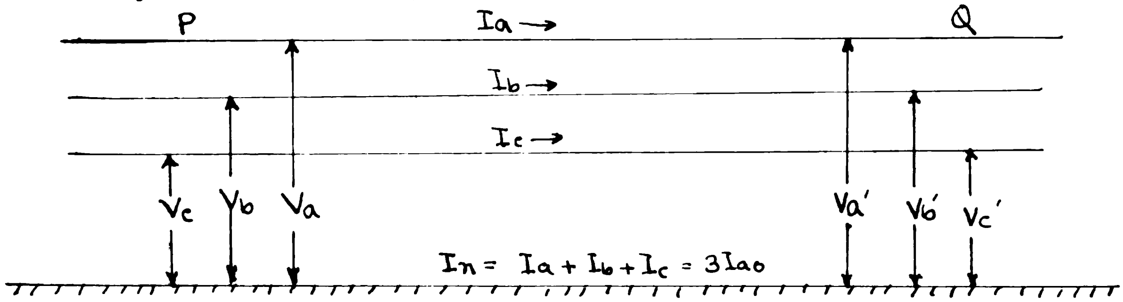


Fig. 2.3

Symmetrical Components:

The Symmetrical Component of voltage drop in an unsymmetrical three phase series circuit without internal voltages are expressed in terms of the symmetrical components of current flowing in the circuit and self and mutual impedences of the sequence networks. The basic assumptions that have considered are: (1) effect of saturation is negligible

(2) linear relations between currents and voltages assumed.

V_a, V_b, and V_c are phase voltages referred to ground at P and V_a', V_b' and V_c' are phase voltages referred to ground at Q.

So

$$\begin{aligned} v_a &= V_a - V_a' = I_{a1}Z_{a1} + I_{a2}Z_{a2} + I_{a0}Z_{a0} \\ v_b &= V_b - V_b' = a^2 I_{a1}Z_{b1} + aI_{a2}Z_{b2} + I_{a0}Z_{b0} \\ v_c &= V_c - V_c' = aI_{a1}Z_{c1} + a^2 I_{a2}Z_{c2} + I_{a0}Z_{c0} \end{aligned} \qquad (24)$$

Now

$$v_{a0} = V_{a0} - V_{a0}' = \frac{1}{3}(v_a + v_b + v_c)$$

$$\therefore v_{a0} = \frac{1}{3}\Big[I_{a1}(Z_{a1} + a^2 Z_{b1} + aZ_{c1}) + I_{a2}(Z_{a2} + aZ_{b2} + a^2 Z_{c2}) + I_{a0}(Z_{a0} + Z_{b0} + Z_{c0})\Big]$$

$$v_{a1} = V_{a1} - V_{a1}' = \frac{1}{3}(v_a + av_b + a^2 v_c)$$

$$\begin{aligned} v_a &= I_{a1}Z_{a1} + I_{a2}Z_{a2} + I_{a0}Z_{a0} \\ av_b &= I_{a1}Z_{b1} + a^2 I_{a2}Z_{b2} + aI_{a0}Z_{b0} \quad \because a^3 = 1 + j0 \\ a^2 v_c &= I_{a1}Z_{c1} + aI_{a2}Z_{c2} + a^2 I_{a0}Z_{c0} \quad \because a^4 = a \end{aligned} \qquad (25)$$

$$\therefore v_{a1} = \frac{1}{3}\Big[I_{a1}(Z_{a1} + Z_{b1} + Z_{c1}) + I_{a2}(Z_{a2} + a^2 Z_{b2} + aZ_{c2}) + I_{a0}(Z_{a0} + aZ_{b0} + a^2 Z_{c0})\Big]$$

Similarly,

$$\begin{aligned} v_{a2} &= V_{a2} - V_{a2}' \\ &= \frac{1}{3}\left[v_a + a^2 v_b + av_c\right] \\ &= \frac{1}{3}\Big[I_{a1}(Z_{a1} + aZ_{b1} + a^2 Z_{c1}) + I_{a2}(Z_{a2} + Z_{b2} + Z_{c2}) + I_{a0}(Z_{a0} + a^2 Z_{b0} + aZ_{c0})\Big] \end{aligned}$$

with positive-sequence currents only flowing in the circuit, I_{a2} and I_{a0} are zero, and equation (25) becomes

$$\begin{aligned} v_{a0} &= I_{a1}\frac{Z_{a1} + a^2 Z_{b1} + aZ_{c1}}{3} \\ v_{a1} &= I_{a1}\frac{Z_{a1} + Z_{b1} + Z_{c1}}{3} \\ v_{a2} &= I_{a2}\frac{Z_{a1} + aZ_{b1} + a^2 Z_{c1}}{3} \end{aligned} \qquad (26)$$

Equations (26) show that, with positive sequence currents only flowing in the circuit, voltage drops of all three sequences will occur between P and Q unless the co-efficients of I_{a1} are zero. Likewise, with only negative sequence currents or zero sequence currents flowing in an unsymmetrical circuit, voltage drops of all three sequences may be obtained.

The equations of (25) can be rewritten in much more compact form.

$$
\begin{aligned}
v_{a1} &= V_{a1} - V'_{a1} = I_{a1} Z_{11} + I_{a2} Z_{12} + I_{a0} Z_{10} \\
v_{a2} &= V_{a2} - V'_{a2} = I_{a1} Z_{21} + I_{a2} Z_{22} + I_{a0} Z_{20} \\
v_{a0} &= V_{a0} - V'_{a0} = I_{a1} Z_{01} + I_{a2} Z_{02} + I_{a0} Z_{00}
\end{aligned}
\tag{27}
$$

Where

$Z_{11} = \frac{1}{3}(Z_{a1} + Z_{b1} + Z_{c1})$ = self impedence to positive sequence currents.

$Z_{22} = \frac{1}{3}(Z_{a2} + Z_{b2} + Z_{c2})$ = self impedence to negative sequence currents.

$Z_{00} = \frac{1}{3}(Z_{a0} + Z_{b0} + Z_{c0})$ = self impedence to zero sequence currents.

$Z_{12} = \frac{1}{3}(Z_{a2} + a^2 Z_{b2} + a Z_{c2})$ = ratio of the positive sequence voltage drop produced by I_{a2} to I_{a2}.

$Z_{21} = \frac{1}{3}(Z_{a1} + a Z_{b1} + a^2 Z_{c1})$ = ratio of the negative sequence voltage drop produced by I_{a1} to I_{a1}. (28)

$Z_{10} = \frac{1}{3}(Z_{a0} + a Z_{b0} + a^2 Z_{c0})$ = ratio of the positive sequence voltage drop produced by I_{a0} to I_{a0}.

$Z_{01} = \frac{1}{3}(Z_{a1} + a^2 Z_{b1} + a Z_{c1})$ = ratio of the zero-sequence voltage drop produced by I_{a1} to I_{a1}.

$Z_{20} = \frac{1}{3}(Z_{a0} + a^2 Z_{b0} + a Z_{c0})$ = ratio of negative sequence voltage drop produced by I_{a0} to I_{a0}.

$Z_{02} = \frac{1}{3}(Z_{a2} + aZ_{b2} + a^2 Z_{c2})$ = ratio of the zero-sequence voltage drop produced by I_{a2} to I_{a2}.

Equations of (27) express the Symmetrical Components of voltage drop in an unsymmetrical three phase series circuit in which there are no internal voltages in terms of Symmetrical Components of current flowing through the circuit and the sequence self and mutual impedences defined by (28). Self impedences are indicated by Z with two like subscripts, mutual impedences by Z with two unlike subscripts. Z_{11}, Z_{22}, Z_{00} represent the positive, negative and zero sequence self impedences respectively of the circuit and are the impedences met by currents of positive, negative and zero sequence flowing in their respective networks.

Clarke Components:

$$v_{a1} = V_{a1} - V_{a1}' = I_{a1}Z_{11} + I_{a2}Z_{12} + I_{a0}Z_{10}$$

$$v_{a2} = V_{a2} - V_{a2}' = I_{a1}Z_{21} + I_{a2}Z_{22} + I_{a0}Z_{20} \qquad (27)$$

$$v_{a0} = V_{a0} - V_{a0}' = I_{a1}Z_{01} + I_{a2}Z_{02} + I_{a0}Z_{00}$$

Now from (21)

$$I_\alpha = I_{a1} + I_{a2}$$

$$I_\beta = -j(I_{a1} - I_{a2})$$

$$I_0 = I_{a0}$$

So, I_{a1}, I_{a2}, and I_{a0} of equations (27) in terms of α, β, 0 components are:

$$I_{a1} = \frac{1}{2}(I_\alpha + jI_\beta)$$

$$I_{a2} = \frac{1}{2}(I_\alpha - jI_\beta) \qquad (23)$$

$$I_{a0} = I_0$$

The equation (27) becomes

$$v_{a1} = \left[\tfrac{1}{2}(I_\alpha + jI_\beta)\right]Z_{11} + \left[\tfrac{1}{2}(I_\alpha - jI_\beta)\right]Z_{12} + I_0 Z_{10}$$

$$= \tfrac{1}{2} I_\alpha (Z_{11} + Z_{12}) + \tfrac{1}{2} j I_\beta (Z_{11} - Z_{12}) + I_0 Z_{10}$$

$$v_{a2} = \tfrac{1}{2} I_\alpha (Z_{21} + Z_{22}) + \tfrac{1}{2} j I_\beta (Z_{21} - Z_{22}) + I_0 Z_{20}$$

$$v_{a0} = \tfrac{1}{2} I_\alpha (Z_{01} + Z_{02}) + \tfrac{1}{2} j I_\beta (Z_{01} - Z_{02}) + I_0 Z_{00}$$

From (23) again

$$v_{a1} = \tfrac{1}{2}(v_\alpha + j v_\beta)$$

$$v_{a2} = \tfrac{1}{2}(v_\alpha - j v_\beta)$$

$$v_{a0} = v_0$$

$$\therefore v_\alpha = v_{a1} + v_{a2} \quad \text{and} \quad v_\beta = -j(v_{a1} - v_{a2})$$
$$= j(v_{a2} - v_{a1})$$

$$v_\alpha = \tfrac{1}{2} I_\alpha [Z_{11} + Z_{12} + Z_{21} + Z_{22}] + \tfrac{1}{2} j I_\beta [Z_{11} + Z_{21} - Z_{12} - Z_{22}] + I_0 [Z_{10} + Z_{20}]$$

$$v_\beta = \tfrac{1}{2} j I_\alpha [Z_{21} + Z_{22} - Z_{11} - Z_{12}] + \tfrac{1}{2} I_\beta [Z_{11} + Z_{22} - Z_{21} - Z_{12}] + I_0 j (Z_{20} - Z_{10})$$

$$v_0 = \tfrac{1}{2} I_\alpha (Z_{01} + Z_{02}) + \tfrac{1}{2} I_\beta j (Z_{01} - Z_{02}) + I_0 Z_{00}$$

The equations can be expressed in compact as follows:

$$v_\alpha = V_\alpha - V_\alpha' = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + I_0 Z_{\alpha 0}$$

$$v_\beta = V_\beta - V_\beta' = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + I_0 Z_{\beta 0}$$

$$v_0 = V_0 - V_0' = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + I_0 Z_{00} \qquad (29)$$

or,

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ v_0 \end{bmatrix} = \begin{bmatrix} Z_{\alpha\alpha} & Z_{\alpha\beta} & Z_{\alpha 0} \\ Z_{\beta\alpha} & Z_{\beta\beta} & Z_{\beta 0} \\ Z_{0\alpha} & Z_{0\beta} & Z_{00} \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \\ I_0 \end{bmatrix}$$

Where

$$Z_{\alpha\alpha} = \tfrac{1}{2}\left(Z_{11} + Z_{12} + Z_{21} + Z_{22}\right)$$

$$Z_{\beta\beta} = \tfrac{1}{2}\left(Z_{11} + Z_{22} - Z_{12} - Z_{21}\right)$$

$$Z_{oo} = Z_{00}$$

$$Z_{\alpha\beta} = j\tfrac{1}{2}\left(Z_{11} - Z_{22} + Z_{21} - Z_{12}\right)$$

$$Z_{\beta\alpha} = -j\tfrac{1}{2}\left(Z_{11} - Z_{22} + Z_{12} - Z_{21}\right) \qquad (30)$$

$$Z_{o\alpha} = \tfrac{1}{2}\left(Z_{01} + Z_{02}\right)$$

$$Z_{\alpha o} = \left(Z_{10} + Z_{20}\right)$$

$$Z_{o\beta} = j\tfrac{1}{2}\left(Z_{01} - Z_{02}\right)$$

$$Z_{\beta o} = -j\left(Z_{10} - Z_{20}\right)$$

Self and mutual impedances of the sequence networks in terms of α, β, o self and mutual impedances:

Proceeding in a manner analogous to that used to determine (30) or by solving (30) for Z_{11}, Z_{22} etc,

$$Z_{11} = \tfrac{1}{2}\left[Z_{\alpha\alpha} + Z_{\beta\beta} - j\left(Z_{\alpha\beta} - Z_{\beta\alpha}\right)\right]$$

$$Z_{22} = \tfrac{1}{2}\left[Z_{\alpha\alpha} + Z_{\beta\beta} + j\left(Z_{\alpha\beta} - Z_{\beta\alpha}\right)\right]$$

$$Z_{00} = Z_{oo}$$

$$Z_{12} = \tfrac{1}{2}\left[Z_{\alpha\alpha} - Z_{\beta\beta} + j\left(Z_{\alpha\beta} + Z_{\beta\alpha}\right)\right]$$

$$Z_{21} = \tfrac{1}{2}\left[Z_{\alpha\alpha} - Z_{\beta\beta} - j\left(Z_{\alpha\beta} + Z_{\beta\alpha}\right)\right] \qquad (31)$$

$$Z_{10} = \tfrac{1}{2}\left(Z_{\alpha o} + jZ_{\beta o}\right)$$

$$Z_{01} = \left(Z_{o\alpha} - jZ_{o\beta}\right)$$

$$Z_{20} = \tfrac{1}{2}\left(Z_{\alpha o} - jZ_{\beta o}\right)$$

$$Z_{02} = \left(Z_{o\alpha} + jZ_{o\beta}\right)$$

Symmetrical circuit—equal positive and negative sequence impedences:

In a circuit with equal positive and negative sequence self-impedences and no mutual impedances oftained from (30) are

$$Z_{\alpha\alpha} = Z_{\beta\beta} = Z_{11} = Z_1$$
$$Z_{00} = Z_{00} = Z_0 \qquad (32)$$
$$Z_{\alpha\beta} = Z_{\beta\alpha} = Z_{\alpha 0} = Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0$$

When there are no mutual impedances between the sequence networks, the positive negative and zero sequence self impedances are customarily indicated by Z_1, Z_2 and Z_0 respectively instead of Z_{11}, Z_{22}, and Z_{00}.

Unsymmetrical static circuit:

If $Z_{11} = Z_{22}$, $Z_{10} = Z_{02}$ and $Z_{20} = Z_{01}$. (30)

becomes

$$Z_{\alpha\alpha} = Z_{11} + \frac{1}{2}(Z_{21} + Z_{12})$$
$$Z_{\beta\beta} = Z_{11} - \frac{1}{2}(Z_{21} + Z_{12})$$
$$Z_{00} = Z_{00} \qquad (33)$$
$$Z_{\alpha\beta} = Z_{\beta\alpha} = j\frac{1}{2}(Z_{21} - Z_{12})$$
$$Z_{\alpha 0} = 2Z_{0\alpha} = (Z_{10} + Z_{20})$$
$$Z_{\beta 0} = 2Z_{0\beta} = -j(Z_{10} - Z_{20})$$

If $Z_{11} = Z_{22}$, $Z_{12} = Z_{21}$ and $Z_{10} = Z_{02} = Z_{20} = Z_{01}$ (30)

becomes

$$Z_{\alpha\alpha} = Z_{11} + Z_{12}$$
$$Z_{\beta\beta} = Z_{11} - Z_{12}$$
$$Z_{00} = Z_{00} \qquad (34)$$
$$Z_{\alpha\beta} = Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0$$
$$Z_{\alpha 0} = 2Z_{0\alpha} = 2Z_{10}$$

Symmetrical Components can also be represented in terms of Clarkes Components. From (33)

$$Z_{11} = Z_{22} = \frac{1}{2}(Z_{\alpha\alpha} + Z_{\beta\beta})$$
$$Z_{00} = Z_{00}$$
$$Z_{21} = \frac{1}{2}(Z_{\alpha\alpha} - 2jZ_{\alpha\beta} - Z_{\beta\beta}) \qquad (35)$$
$$Z_{12} = \frac{1}{2}(Z_{\alpha\alpha} + 2jZ_{\alpha\beta} - Z_{\beta\beta})$$
$$Z_{10} = \frac{1}{2}(Z_{\alpha 0} + jZ_{\beta 0}) = Z_{0\alpha} + jZ_{0\beta}$$
$$Z_{20} = \frac{1}{2}(Z_{\alpha 0} - jZ_{\beta 0}) = Z_{0\alpha} - jZ_{0\beta}$$

From (34)

$$Z_{11} = Z_{22} = \frac{1}{2}(Z_{\alpha\alpha} + Z_{\beta\beta})$$
$$Z_{12} = Z_{21} = \frac{1}{2}(Z_{\alpha\alpha} - Z_{\beta\beta})$$
$$Z_{00} = Z_{00} \qquad (36)$$

Modified 0 network:

In circuits in which $Z_{0\alpha} = \frac{1}{2} Z_{\alpha 0}$ and $Z_{0\beta} = \frac{1}{2} Z_{\beta 0}$, equations (27) are conveniently expressed in terms of a modified 0 network in which the voltage is 0 voltage, the current is $2I_0$ and the impedance is one-half the 0 impedance. Rewriting (27) in terms of $2I_0$,

$$\begin{aligned} v_\alpha &= V_\alpha - V_\alpha' = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} + (2I_0)\frac{Z_{\alpha 0}}{2} \\ v_\beta &= V_\beta - V_\beta' = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} + (2I_0)\frac{Z_{\beta 0}}{2} \\ v_0 &= V_0 - V_0' = I_\alpha Z_{0\alpha} + I_\beta Z_{0\beta} + (2I_0)\frac{Z_{00}}{2} \end{aligned} \qquad (37)$$

or

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ 2v_0 \end{bmatrix} = \begin{bmatrix} Z_{\alpha\alpha} & Z_{\alpha\beta} & Z_{0\alpha} \\ Z_{\beta\alpha} & Z_{\beta\beta} & Z_{0\beta} \\ 2Z_{0\alpha} & 2Z_{0\beta} & Z_{00} \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \\ 2I_0 \end{bmatrix} \qquad (37a)$$

or

$$\begin{bmatrix} v_\alpha \\ v_\beta \\ 2v_0 \end{bmatrix} = \begin{bmatrix} Z_{\alpha\alpha} & Z_{\alpha\beta} & Z_{\alpha 0} \\ Z_{\beta\alpha} & Z_{\beta\beta} & Z_{\beta 0} \\ 2Z_{0\alpha} & 2Z_{0\beta} & 2Z_{00} \end{bmatrix} \begin{bmatrix} I_\alpha \\ I_\beta \\ I_0 \end{bmatrix} \qquad (37b)$$

Equations (37b) apply to a modified 0 network in which currents are 0 currents, voltages are twice 0 voltages, and impedances are twice 0 impedances. Besides, one more fact is revealed from (37). Equations (37 can be used instead of (29) in case where $\frac{Z_{\alpha 0}}{2} = Z_{0\alpha}$ and $\frac{Z_{\beta 0}}{2} = Z_{0\beta}$, thereby giving receprocal mutual coupling between the α (or β) network and modified 0 network in which the current is $2I_0$, the impedances are one-half 0 impedances, and the voltages are 0 voltages. This impedence network is used by E. W. Kimbark[7].

α, β, 0 self and Mutual Impedances in terms of the Phase Impedances:

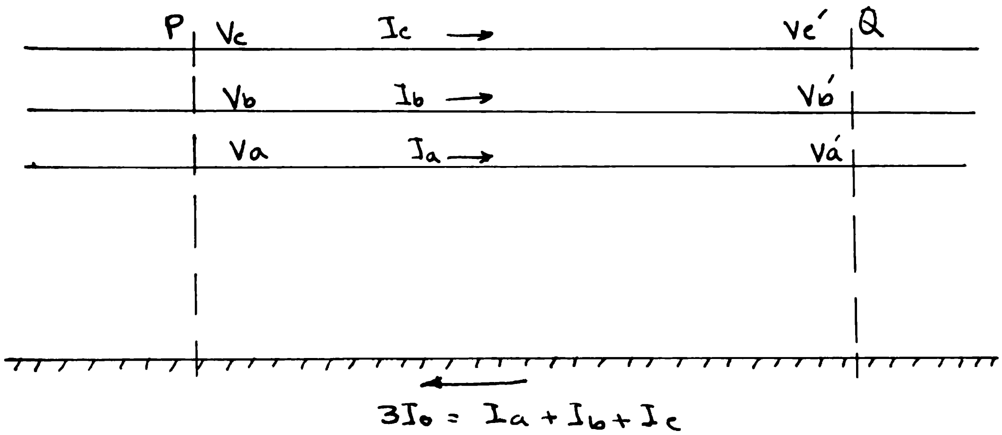


Fig. 2.4

Let Fig. 2.4 represent a general three phase satic circuit composed of bilateral circuit elements without internal voltages between points P and Q, with a return path for 0 currents. With phase voltages at P and Q referred to ground or to a neutral conductor at P and Q, respectively, the voltage drops v_a, v_b and v_c in phases a, b, and c in the direction of flow are

$$v_a = V_a - V_a' = I_a Z_{aa} + I_b Z_{ab} + I_c Z_{ac}$$

$$v_b = V_b - V_b' = I_a Z_{ab} + I_b Z_{bb} + I_c Z_{bc}$$

$$v_c = V_c - V_c' = I_a Z_{ac} + I_b Z_{bc} + I_c Z_{cc} \qquad (38)$$

Equations (38) are general equations expressing phase voltage drops in terms of phase currents after all other currents in the circuit have been eliminated. For example, in a three phase transmission circuit with a neutral conductor or ground wires, Z_{aa}, $Z_{ab} = Z_{ba}$ etc., may include the effects of neutral conductor or ground wires.

Replacing I_a, I_b and I_c in (38) by α, β, 0 components given by (7)-(9), v_a, v_b and v_c are expressed in terms of I_α, I_β and I_0. Substituting these equations for v_a, v_b and v_c in (4)-(6), and equating v_α, v_β and v_0 to the corresponding co-efficients in (29), the α, β, 0 self and mutual impedences in terms of phase impedences are

$$Z_{\alpha\alpha} = \frac{2}{3}\left[Z_{aa} + \frac{Z_{bb}+Z_{cc}}{4} - \left(Z_{ab} + Z_{ac} - \frac{Z_{bc}}{2}\right)\right]$$

$$Z_{\beta\beta} = \frac{1}{2}\left[\left(Z_{bb} + Z_{cc} - 2Z_{bc}\right)\right]$$

$$Z_{00} = \frac{1}{3}\left[Z_{aa} + Z_{bb} + Z_{cc} + 2(Z_{ab} + Z_{ac} + Z_{bc})\right]$$

$$Z_{\alpha\beta} = Z_{\beta\alpha} = \frac{1}{2\sqrt{3}}\left[Z_{cc} - Z_{bb} + 2(Z_{ab} - Z_{ac})\right] \qquad (39)$$

$$Z_{\alpha 0} = 2Z_{0\alpha} = \frac{1}{3}\left[2Z_{aa} - Z_{bb} - Z_{cc} + (Z_{ab} + Z_{ac} - 2Z_{bc})\right]$$

$$Z_{\beta 0} = 2Z_{0\beta} = \frac{1}{\sqrt{3}}\left[\left(Z_{bb} - Z_{cc} + Z_{ab} - Z_{ac}\right)\right]$$

Two phases with equal self impedances and equal mutual impedences with the third phase:

Let $Z_{bb} = Z_{cc}$ and $Z_{ac} = Z_{ab}$. Equations (39) then becomes

$$Z_{\alpha\alpha} = \frac{2}{3}\left[Z_{aa} + \frac{Z_{bb}}{2} - \left(2Z_{ab} - \frac{Z_{bc}}{2}\right)\right]$$

$$Z_{\beta\beta} = \frac{1}{2}\left[\left(Z_{bb} + Z_{cc} - 2Z_{bc}\right)\right]$$

$$Z_{00} = \frac{1}{3}\left[Z_{aa} + 2Z_{bb} + 2\left(2Z_{ab} + Z_{bc}\right)\right]$$

$$Z_{\alpha\beta} = Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0 \qquad (40)$$

$$Z_{\alpha 0} = 2Z_{0\alpha} = \frac{2}{3}\left[Z_{aa} - Z_{bb} + \left(Z_{ab} - Z_{bc}\right)\right]$$

Symmetrical circuit:

With all self impedences equal to Z_{aa} and all mutual impedences equal to Z_{ab}, (39) or (40) becomes

$$Z_{\alpha\alpha} = Z_{\beta\beta} = Z_{aa} - Z_{ab}$$

$$Z_{00} = Z_{aa} + 2Z_{ab} \qquad (41)$$

$$Z_{\alpha\beta} = Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = Z_{\alpha 0} = Z_{0\alpha} = 0$$

Unsymmetrical three phase self impedance circuit with finite 0 self impedance:

In a three phase series circuit between P and Q, let the self impedances of phases a, b, and c be Z_a, Z_b and Z_c, respectively and with no mutual impedance between phases. The α, β, 0 self and mutual impedances can be obtained by replacing Z_{aa}, Z_{bb} and Z_{cc} in (39) by Z_a, Z_b and Z_c respectively, and equating all mutual impedances between phases to zero. Then,

$$Z_{\alpha\alpha} = \frac{2}{3}\left(Z_a + \frac{Z_b + Z_c}{4}\right)$$

$$Z_{\beta\beta} = \frac{Z_b + Z_c}{2} \qquad (42)$$

$$Z_{00} = \frac{Z_a + Z_b + Z_c}{3}$$

$$Z_{\alpha\beta} = Z_{\beta\alpha} = \frac{Z_c - Z_b}{2\sqrt{3}}$$

$$Z_{\alpha 0} = 2 Z_{0\alpha} = \frac{2Z_a - Z_b - Z_c}{3}$$

$$= 2\left(Z_a - Z_{\alpha\alpha}\right) \qquad (42 \text{ cont.})$$

$$Z_{\beta 0} = 2 Z_{0\beta} = \frac{Z_b - Z_c}{\sqrt{3}}$$

Two phases with equal self impedances:

Let $Z_b = Z_c$, then (42) becomes

$$Z_{\alpha\alpha} = \frac{2}{3}\left(Z_a + \frac{Z_b}{2}\right)$$

$$Z_{\beta\beta} = Z_b$$

$$Z_{00} = \frac{1}{3}\left(Z_a + 2Z_b\right) \qquad (43)$$

$$Z_{\alpha\beta} = Z_{\beta\alpha} = Z_{\beta 0} = Z_{0\beta} = 0$$

$$Z_{\alpha 0} = 2 Z_{0\alpha} = \frac{2}{3}\left(Z_a - Z_b\right) = 2\left(Z_a - Z_{\alpha\alpha}\right)$$

Symmetrical self impedence circuit:

Let $Z_a = Z_b = Z_c = Z$. From (42) or (43)

$$Z_{\alpha\alpha} = Z_{\beta\beta} = Z_{00} = Z$$

Equivalent circuits to replace an actual circuit in the α, β, 0 network:

Synchronous machine with equal positive and negative sequence impedances:

From (32), the α and β self impedances are equal to Z, and the 0 self impedances to Z_0. There are no mutual impedances between the α, β, and 0 networks. With balanced generated voltages in the machine, the generated voltage in the α network from (16) is E_a; in the β network it is $-jE_a$. The α, β, and 0 equivalent circuits for a synchronous machine with balanced generated voltages and equal positive and negative impedances are chown in Fig. 2.5. Points T are the terminals of the machine to which the equivalent α, β and 0 circuits for the rest of the system are to be connected.

So in a three phase power system consisting of symmetrical circuit with equal positive and negative sequence impedances, the one line impedance diagrams for the α and β systems are the same as the positive sequence impedance diagram. Generated α voltages are positive sequence generated voltages; the α network, is the same as the positive sequence network. The β network differs from the positive sequence network only in its generated voltages, which are positive sequence voltages multiplied by -j.

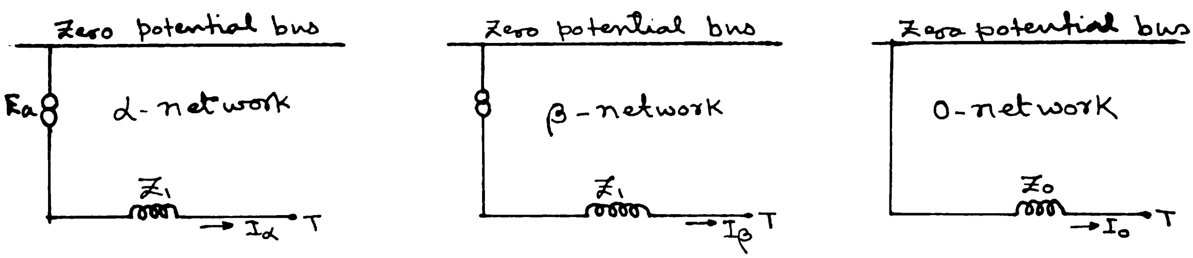


Fig. 2.5

Symmetrical circuit with unequal positive and negative sequence impedances:

From (30) with $Z_1 \neq Z_2$ and all sequence mutual impedances zero,

$$
\begin{aligned}
Z_{\alpha\alpha} &= Z_{\beta\beta} = \tfrac{1}{2}(Z_1 + Z_2) \\
Z_{00} &= Z_0 \\
Z_{\alpha\beta} &= -Z_{\beta\alpha} = j\,\tfrac{1}{2}(Z_1 - Z_2) \\
Z_{\alpha 0} &= Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0
\end{aligned}
\qquad (44)
$$

When the positive and negative sequence self impedances of a circuit are unequal and there are no sequence mutual impedances, the α and β self impedances are average of the positive and negative sequence impedances. There is no mutual coupling with the 0 network; but the α and β networks are coupled through non-reciprocal mutual impedences. Because of this non-reciprocal coupling between the α and β networks in rotating machines in which $Z_1 \neq Z_2$, the α, β, 0 components are not convenient for determoning fundamental frequency currents and voltages in systems in which the positive and negative sequence impedences cannot be assumed equal. However, if there is but one machine or group machines in which $Z_1 \neq Z_2$, (29) can be rewritten to give a reciprocal mutual coupling between the α network and a modified β network, from which an equivalent circuit can be obtained.

Modified β network:

Substituting $Z_{\alpha 0} = Z_{0\alpha} = Z_{\beta 0} = Z_{0\beta} = 0$ from (43) in (29), the α, β, and 0 components of voltage drop in the circuit in the direction of current flow are

$$v_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta} = I_\alpha \tfrac{1}{2}(Z_1 + Z_2) + jI_\beta \tfrac{1}{2}(Z_1 - Z_2)$$

$$v_\beta = I_\alpha Z_{\beta\alpha} + I_\beta Z_{\beta\beta} = -jI_\alpha \tfrac{1}{2}(Z_1 - Z_2) + I_\beta \tfrac{1}{2}(Z_1 + Z_2) \quad (45)$$

$$v_0 = I_0 Z_0$$

Rewriting v_α and v_β in (45) in terms of ($-I_\beta$), with $-Z_{\alpha\beta}$ replacing $Z_{\beta\alpha}$,

$$v_\alpha = I_\alpha Z_{\alpha\alpha} + (-I_\beta)(-Z_{\alpha\beta}) = I_\alpha Z_{\alpha\alpha} + (-I_\beta) Z_{\beta\alpha} \quad (46)$$

$$= I_\alpha (Z_{\alpha\alpha} - Z_{\beta\alpha}) + (I_\alpha - I_\beta) Z_{\beta\alpha}$$

$$v_\beta = I_\alpha Z_{\beta\alpha} + (-I_\beta)(-Z_{\beta\beta}) = (-I_\beta)(-Z_{\beta\beta} - Z_{\beta\alpha}) + (I_\alpha - I_\beta) Z_{\beta\alpha}$$

$$v_\alpha = I_\alpha Z_{\alpha\alpha} + I_\beta Z_{\alpha\beta}$$
$$= I_\alpha (Z_{\alpha\alpha} - Z_{\alpha\beta}) + (I_\alpha + I_\beta) Z_{\alpha\beta}$$
$$-v_\beta = I_\alpha(-Z_{\beta\alpha}) + I_\beta(-Z_{\beta\beta}) = I_\alpha Z_{\alpha\beta} + I_\beta(-Z_{\beta\beta}) \qquad (47)$$
$$= I_\beta(-Z_{\beta\beta} - Z_{\alpha\beta}) + (I_\alpha + I_\beta) Z_{\alpha\beta}$$

In (46) and (47), the mutual impedances between the α network and a modified β network are reciprocal. In (46), v_β has been retained, but I_β has been replaced by $(-I_\beta)$ flowing in the direction assumed as positive for I . In (47), I_β has been retained but v_β has been replaced by $-v_\beta$ measured in the direction of v_β .

Equivalent circuits for synchronous machine with $Z_1 \neq Z_2$

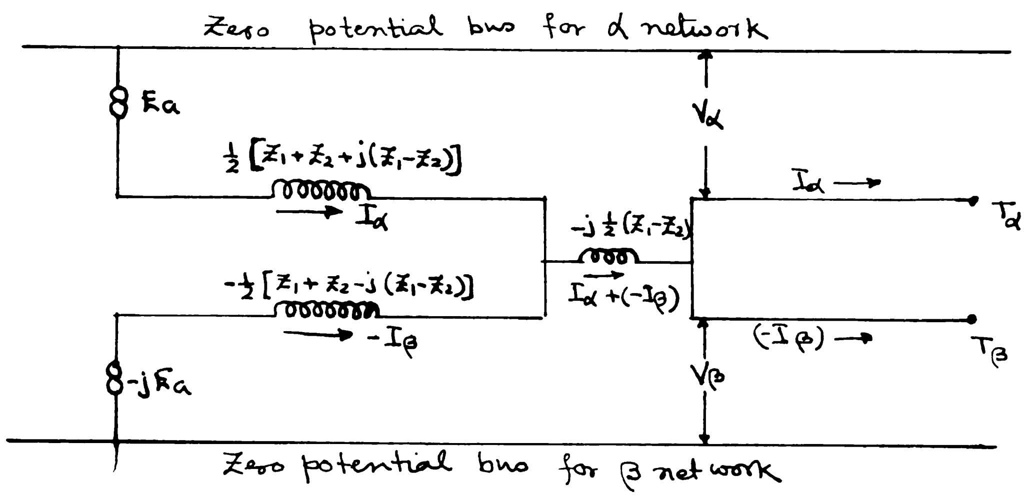


Fig. 2.6 (a). Current in the β network is $-I_\beta$.

Fig. 2.6 (a) satisfies the equation for α and β components of voltage drop in the direction of current flow given by (46) and (47).

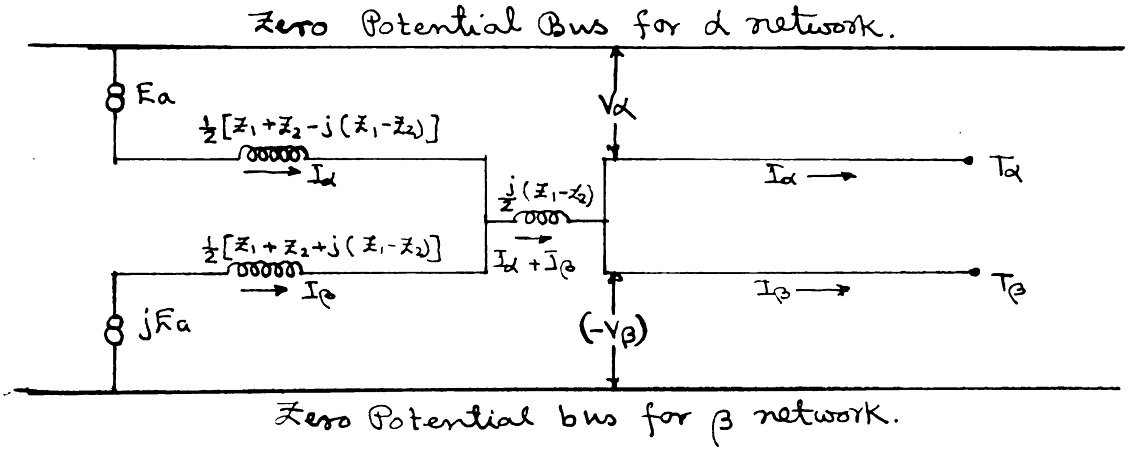


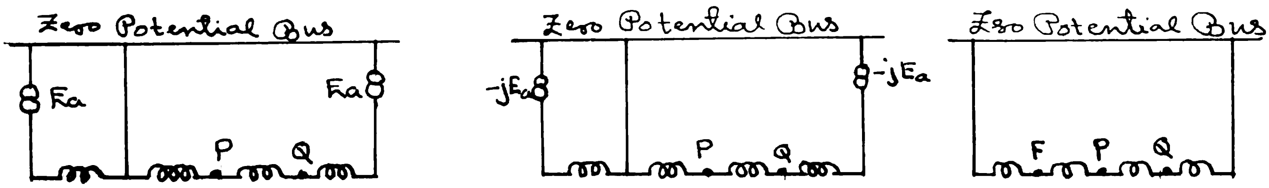
Fig. 2.6 (b)

In Fig. 2.6 (b) and equation (47), voltages in the β network are negative β voltages, giving a modified β network in which currents are β currents. The generated voltages in the modified β network of Fig. 2.6 (b) becomes $-(-jE_a) = jE_a$ as indicated. The points T_α and T_β are the terminals of the synchronous machine to which the α and β networks, respectively, for the system exclusive of synchronous machine are to be connected, after all impedences in the β network have been multiplied by -1. If the impedances in the β network include resistances, negative resistances will be present in the network, capacitive reactances will become inductive reactances and vice versa. The modification of β network presents no difficulties in a analytic solution. Zero network is the same as in Fig. 2.5.

Connections of α, β, 0 networks to represent an unsymmetrical circuit and a short circuit; (no intervening Δ - Y transformer bank)

Direct connections of the α, β, and 0 networks to

satisfy the fault conditions of Table I (Appendix) are shown in Fig. 2.7



Case I(a)-Three phase fault

Fig. 2.7

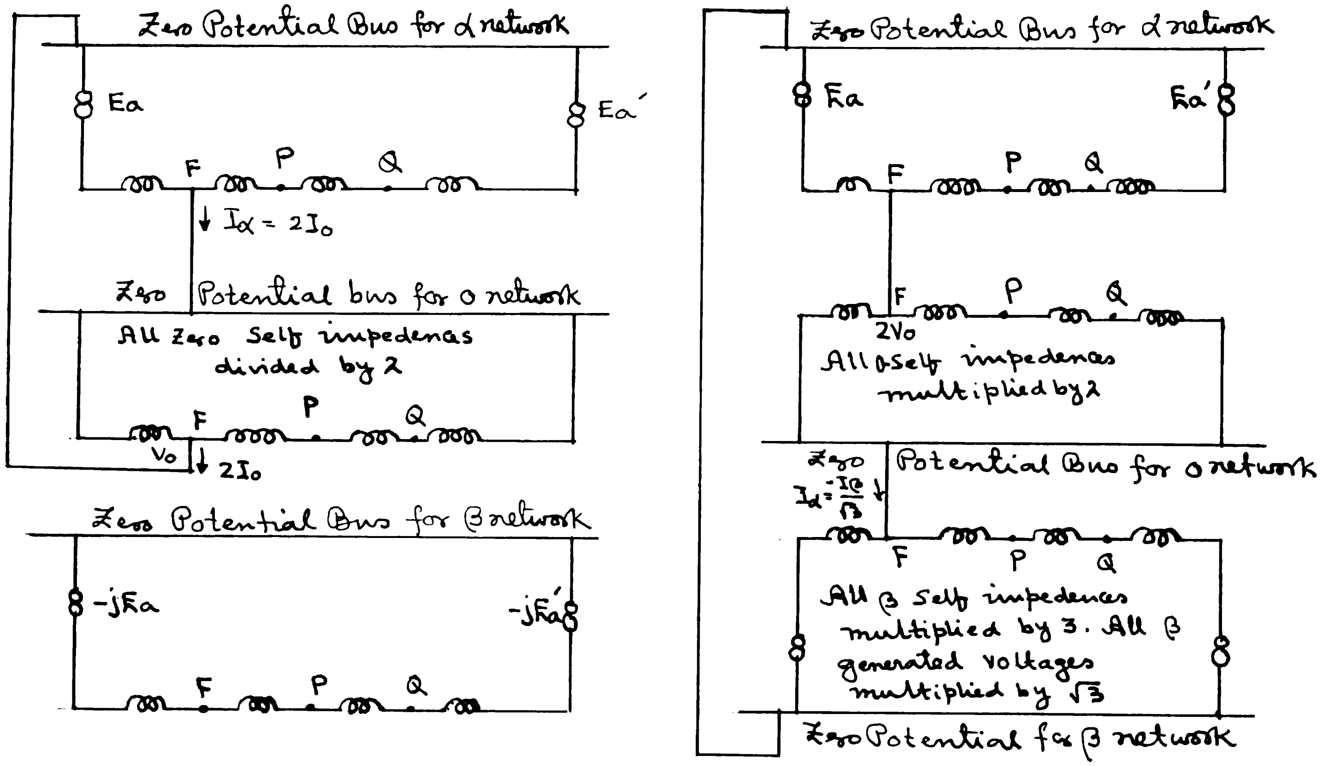
The fault is at F, the unsymmetrical circuit between P and Q. Mutual impedances between the component networks because of the unsymmetrical circuit are not indicated, but they may be present. For simplicity, two synchronous machines only are shown, but the system, exclusive of the unsymmetrical circuit, may be any symmetrical three phase system with equal positive and negative-sequence impecances. E_a and E_a' are the generated voltages in phase a of the two machines. Case I(b) is similar to I(a) except that F in the 0 network is shorted to the zero potential bus for the 0 network.

Case II(b) and II(c) involve modified β network.

The fault is (a) Line to ground - a and ground

(b) Line to ground - b and ground

(c) Line to ground - c and ground

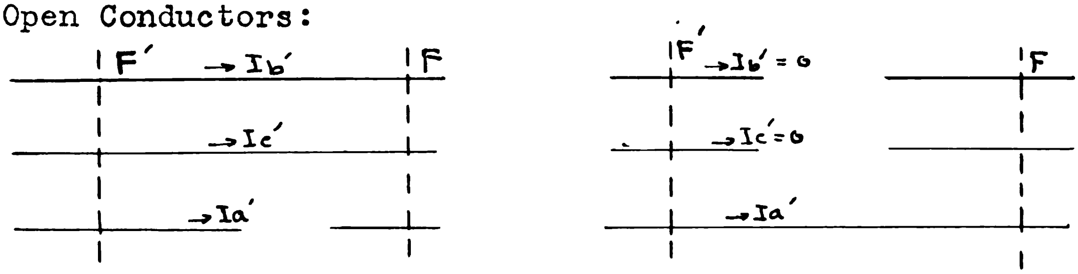


Case II(a) Case II(b)

Fig. 2.7

Other cases are not shown here because of limited space.

Open Conductors:



(a) (b)

Fig. 2.8

Fig. 2.8 (a) and (b) show one and two open conductors, respectively. Let v and I' with different subscripts represent voltage drop between F' and F respectively. Here v is a series voltage drop and I' used to indicate voltages to

ground at a fault and currents flowing from the phases into a fault. Table II (appendix) gives equations expressing relations between v_α, v_β and v_0, the components of voltage drop between F' and F, and between $I_\alpha{}'$, $I_\beta{}'$ and $I_0{}'$, the components of line current flowing from F' to F.

Conductor a open:

$$v_\alpha = 2v_0 \qquad I_\alpha' = -I_0'$$
$$v_\beta = 0$$

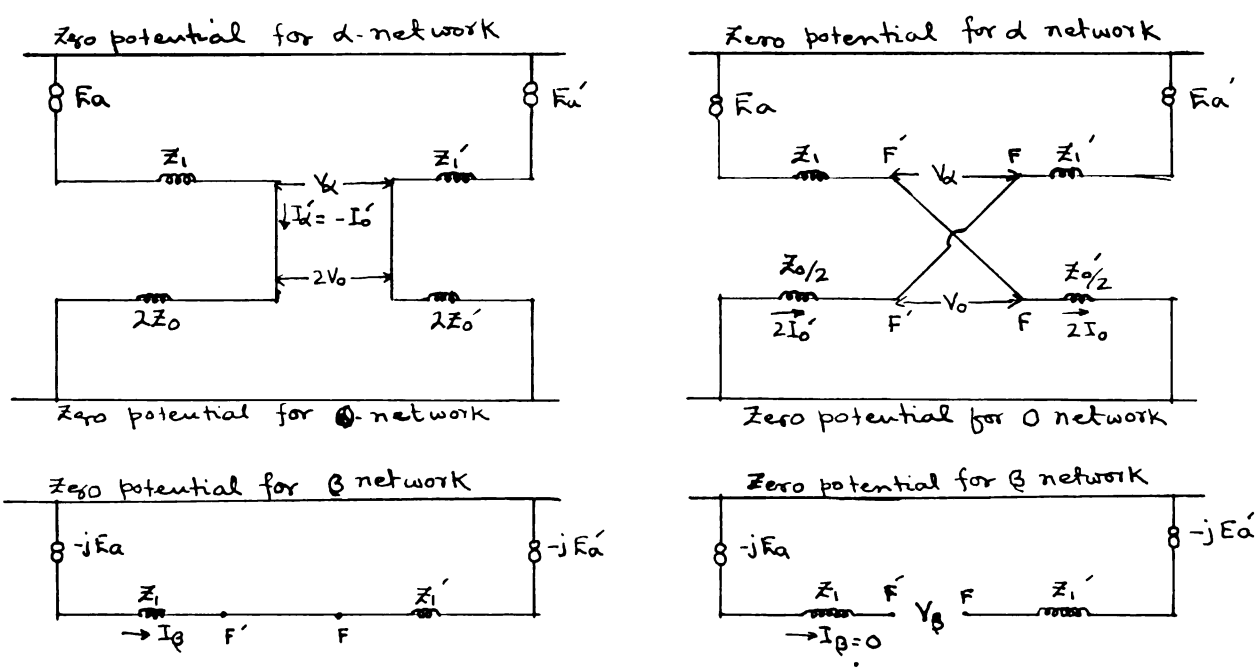
Conductor b open:

$$v_\alpha = -\frac{v_\beta}{\sqrt{3}} = -v_0$$

$$I_\alpha' - \sqrt{3}\,I_\beta' - 2I_0' = 0$$

Connection of the α, β and 0 networks to represent open conductors:

Fig. 2.8 (a) and (b) represent cases I (a) and II(a) in Table II.



(c) (d)

Fig. 2.8

Fig. 2.8(c) and (d) show direct connections of the α and 0 networks to satisfy Fig. 2.8(a) and (b) respectively. For simplicity two generators only are indicated. The system could be any system with equal positive and negative sequence impedances. Direct connection of the α and 0 networks is made possible by multiplying the 0 impedances in Fig. 2.8 (c) by 2 and dividing them by 2 in Fig. 2.8(d). In Fig. 2.8 (c), the currents in the 0 network are 0 currents but the voltages are twice 0 voltage; in Fig. 2.8(d) the voltages are 0 voltages but the currents are twice 0 currents. The network is unaffected by one open conductor (Phase a). It is open for two open conductors (Phases b and c).

Two simultaneous faults

open conductor and fault to ground through impedance on the same phase.

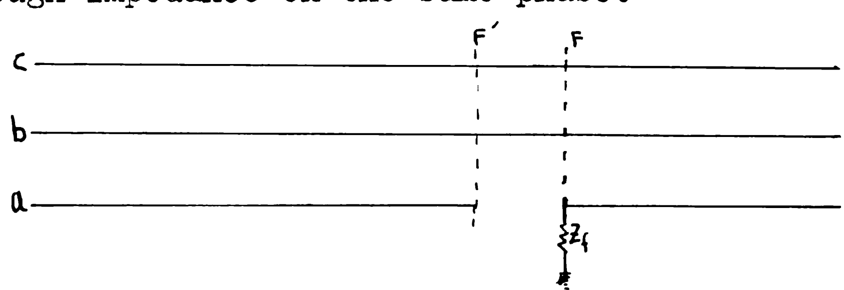


Fig 2.9(a)

The conductor is open between F' and F and the fault is at F on phase a through impedance Z_f to grounds as in Fig. 2(a). The conditions at the fault are:

$$I_b = I_c = 0;\ V_a = V + V_0 = I_a Z_f = (I_\alpha + I_0)\ Z_f \qquad (48)$$

From these equations and those of table II, case I(a), the following relations exists between the components of I_a and V_a at the fault and between the components of V_a and I_a' be-

tween F' and F:

$$I_\beta = 0 \qquad\qquad v_\beta = 0$$
$$I_\alpha = 2I_0 \qquad\qquad v_\alpha = 2v_0 \qquad\qquad (49)$$
$$V_\alpha = -V_0 + I_\alpha\left(\tfrac{3}{2}Z_f\right) \qquad\qquad I_\alpha' = -I_0'$$

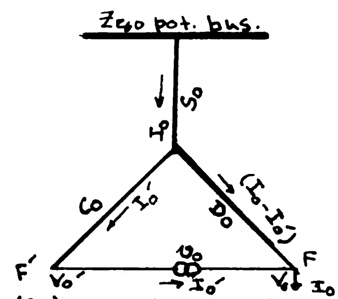
The β network is unaffected by the open conductor and fault.

The 0 network can be replaced by an equivalent Y with impedences C_0, D_0 and S_0, the identity of points F and F' being retained

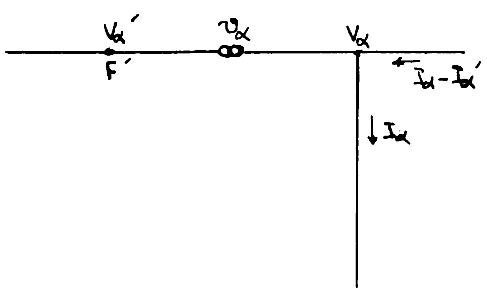
v_0= zer0 voltage drop between F' and F:

V_0 - zero voltage to ground at F

I_0 -]
I_0' =] zero current flowing into the fault.



2.9 (b) equivalent Y to replace 0 network.



V_α and V_α' = α voltages at F and F'

v_α = voltages drop between F' and F

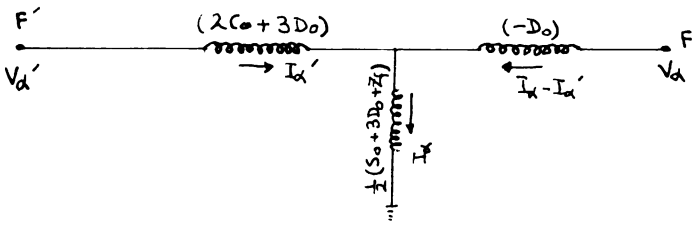
I_α and I_α' = α currents flowing into the fault from F' towards F.

Combining relations of 2.9(b) and 2.9(c)

$$V_0 = -I_0(S_0 + D_0) + I_0' D_0$$
$$v_0 = V_0' - V_0 = I_0 D_0 - I_0'(C_0 + D_0) \qquad\qquad (50)$$
$$V_\alpha' = V_\alpha + v_\alpha$$

Eliminating V_0, I_0, I_0', v_0 and v_α from simultaneous equations of (23) and (24), V_α and V_α' interms of I_α and I_α' are obtained and may be written

$$V_\alpha' = I_\alpha\left(\frac{S_0+3D_0+3Z_f}{2}\right) + I_\alpha'(2C_0+3D_0)$$
$$V_\alpha = I_\alpha\,\frac{(S_0+3D_0+3Z_f)}{2} + (I_\alpha - I_\alpha')(-D_0) \qquad (51)$$



(25)

The equivalent Y shown in figure 2.9(d) satisfies equation (35). It can be used to replace the fault and open conductor in the α network. For a fault through 0 impedences, $Z_f = 0$. If this equivalent circuit is substituded in the α network, between F' and F, the currents and voltages throughout the system can be determined. The β currents and voltages are uneffected by the fault and open conductor.

In the 0 network: from 49.

$$v_0 = \frac{v_\alpha}{2}\,;\quad I_0' = -I_\alpha'\,;\quad I_0 = \tfrac{1}{2} I_\alpha \qquad (52)$$
$$V_0 = -V_\alpha + I_\alpha\left(\tfrac{3}{2} Z_f\right)$$

Knowing the α components, with these relations and the 0 network, the 0 components of current and voltages throughout the system can be determined. The phase voltages and currents at any point are obtained by substituting the α, β, 0 components in (1) - (3) and (7) - (9), respectively.

The effect of Δ-Y transformer bank:

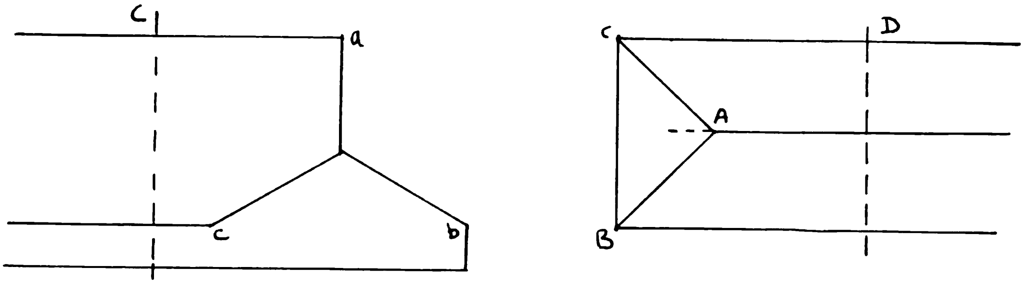


Fig. 2

The choice of the reference circuit is arbitrary. With the circuit C as reference, components of voltage and current at D can be related with C. Components at D are indicated by primed symbols.

$$I_\alpha = I_\beta'' ; \qquad V_\alpha = V_\beta''$$

$$I_\beta = -I_\alpha'' ; \qquad V_\beta = -V_\alpha'' \qquad (53)$$

Line reactance:

As α, β, 0 impedences are not available in reference books, symmetrical impedences are calculated for stranded copper conductor on gmd. basis, and they are found out from later's relation with former.

Chapter III

Solution by Clarke Components

Problem:

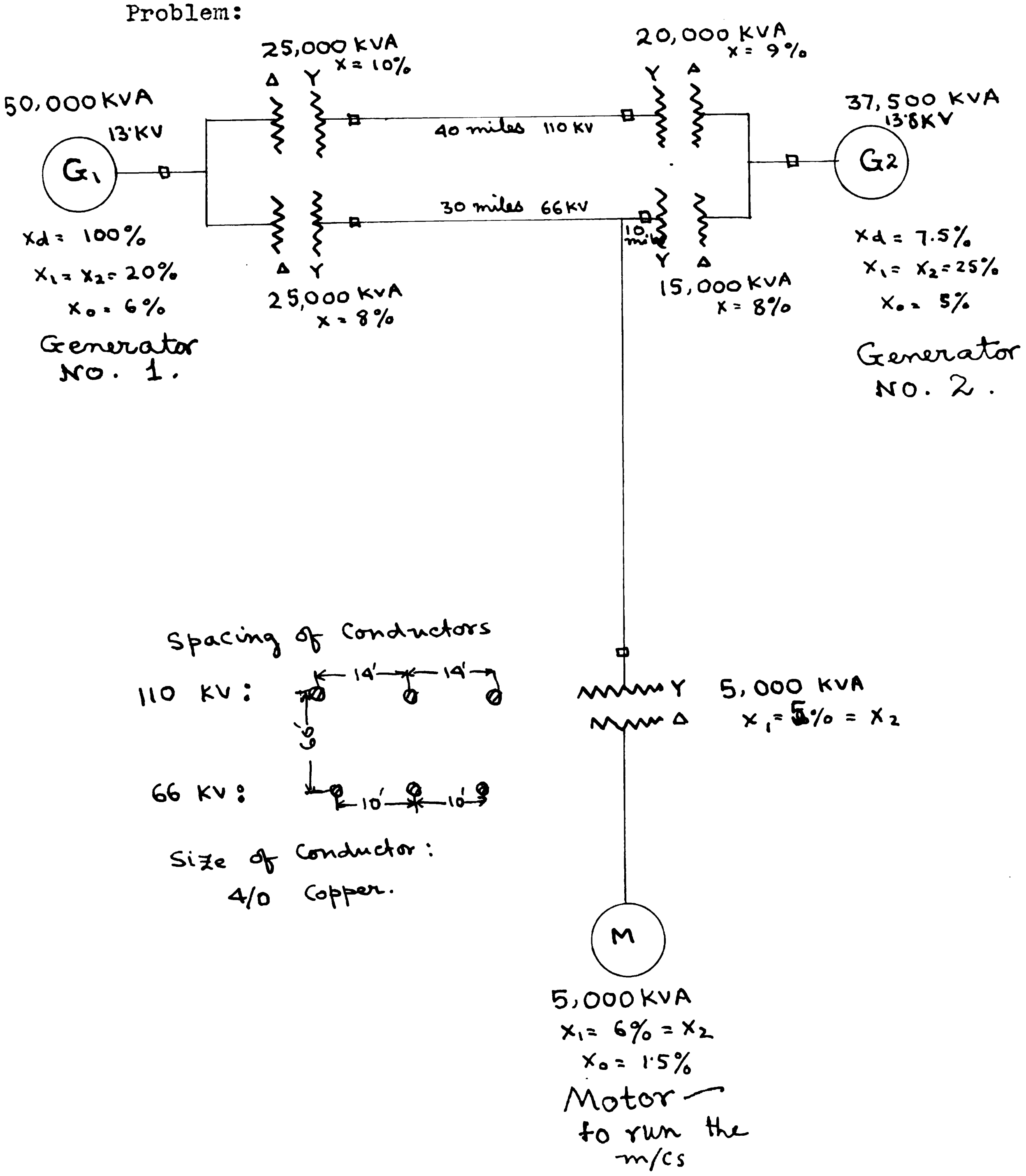


Fig. 3.1

Basis assumptions:

1. The fault currents are to be calculated using transient reactances.

2. A base of 50,000 kva for calculation.

3. All resistances can be neglected.

4. 100% synchronous impedence is used as reference.

5. The mutual impedence of the transmission line is neglected.

6. Saturation of transformer is neglected.

Terminology: X_1 = Positive sequence reactance

X_2 = Negative sequence reactance.

X_0 = Zero sequence reactance.

$E_a^{1,2,3}$ = voltage to neutral of phase a at generator or motor 1,2,3.

Generator equivalent circuit:

G_1

$X_1 = X_2 = 20\%$ $\therefore Z_{\alpha\alpha} = Z_{\beta\beta} = 20\%$

$X_0 = 6\%$ $Z_0 = X_0 = 6\%$

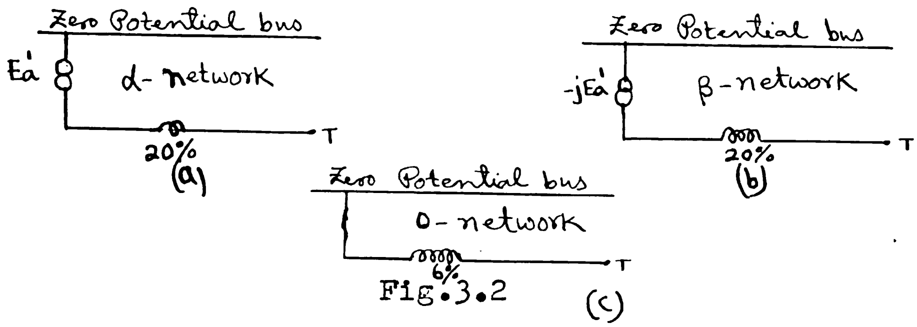


Fig.3.2

G_2

$$X_1 = 25\,\frac{50{,}000}{37{,}500} = 33{\cdot}3\% = X_2$$

$$X_0 = 5\,\frac{50{,}000}{37{,}500} = 6{\cdot}66\%$$

$$Z_{\alpha\alpha} = Z_{\beta\beta} = X_1 = X_2$$

$$Z_0 = X_0$$

$$X_d = \frac{75 \times 50{,}000}{37{,}500} = 100\%$$

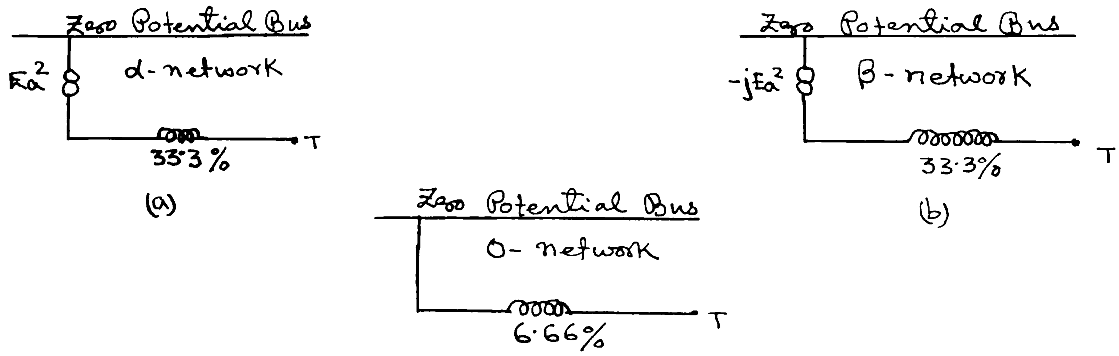
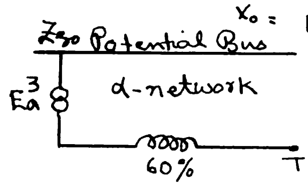


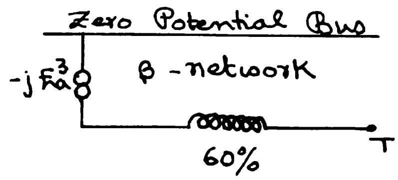
Fig. 3.3

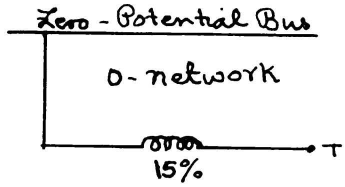
$$X_1 = 6\,\frac{50,000}{5,000} = 60\% = X_2$$

$$X_0 = 1·5\,\frac{50,000}{5,000} = 15\%$$

M







Transmission lines:

Reactance of 110kv line

For 4/0copper or conductor

$X_a = ·497$ ohms per mile

$X_d = \frac{1}{3}(X_d \text{ for } 14\text{ ft} + X_d \text{ for } 14\text{ ft} + X_d \text{ for } 28\text{ ft})$

$= \frac{1}{3}(·320 + ·320 + ·404) = ·348$ ohms per mile

$X_1 = X_2 = X_a + X_d = ·497 + ·307 = ·845$ ohms per mile.

$$X_1 = X_2 = \frac{·845 \times 40 \times 50,000}{110 \times 110 \times 10} = 14\% = Z_{\alpha\alpha} = Z_{\beta\beta}$$

$X_0 = \frac{1}{3}(Z_a + Z_b + Z_c) = Z_a \qquad [\because Z_a = Z_b = Z_c]$

$= X_a + X_e - \frac{2}{3}\left[X_d \text{ for } 14' + X_d \text{ for } 14' + X_d \text{ for } 28'\right]$

$= ·497 + 2.89 - 2 \times ·348$

$= 2.6$ ohms per mile

$$X_0 = \frac{2.6 \times 40 \times 50,000}{110 \times 110 \times 10} = 111\%$$

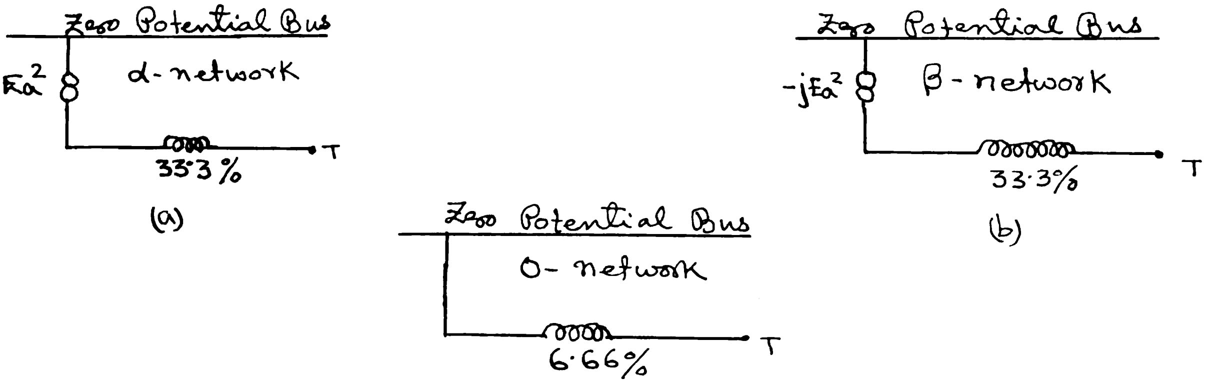
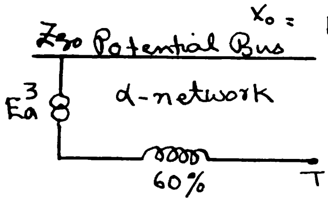


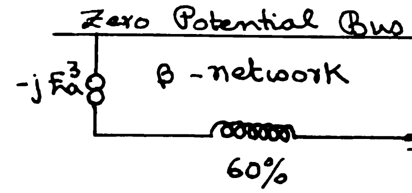
Fig. 3.3

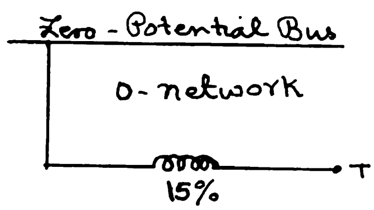
$X_1 = 6\,\frac{50{,}000}{5{,}000} = 60\% = X_2$

$X_0 = 1.5\,\frac{50{,}000}{5{,}000} = 15\%$

M







Transmission lines:

Reactance of 110kv line

For 4/0copper or conductor

$X_a = .497$ ohms per mile

$X_d = \frac{1}{3}(X_d \text{ for } 14\text{ ft} + X_d \text{ for } 14\text{ ft} + X_d \text{ for } 28\text{ ft})$

$= \frac{1}{3}(.320 + .320 + .404) = .348$ ohms per mile

$X_1 = X_2 = X_a + X_d = .497 + .307 = .845$ ohms per mile.

$$X_1 = X_2 = \frac{.845 \times 40 \times 50{,}000}{110 \times 110 \times 10} = 14\% = Z_{\alpha\alpha} = Z_{\beta\beta}$$

$X_0 = \frac{1}{3}(Z_a + Z_b + Z_c) = Z_a \qquad [\because Z_a = Z_b = Z_c]$

$= X_a + X_e - \frac{2}{3}\left[X_d \text{ for } 14' + X_d \text{ for } 14' + X_d \text{ for } 28'\right]$

$= .497 + 2.89 - 2 \times .348$

$= 2.6$ ohms per mile

$$X_0 = \frac{2.6 \times 40 \times 50{,}000}{110 \times 110 \times 10} = 111\%$$

Reactance of 66 kv line

For 4/0 copper conductor $X_a = \cdot 497$ ohms per mile.

$X_d = \frac{1}{3}(X_d \text{ for } 10 \text{ ft} + X_d \text{ for } 10 \text{ ft} + X_d \text{ for } 20 \text{ ft})$

$= \frac{1}{3}(\cdot 279 + \cdot 279 + \cdot 364) = \cdot 307$ ohms per mile.

$X_1 = X_2 = X_a + X_d = \cdot 497 + \cdot 307 = \cdot 804$ ohms per mile.

$$X_1 = X_2 = \frac{\cdot 804 \times 40 \times 50{,}000}{66 \times 66 \times 10} = 36\cdot 9\%$$

$Z_{\alpha\alpha} = Z_{\beta\beta} = X_1 = X_2 = 36\cdot 9\%$

$Z_0 = X_a + X_e - \frac{2}{3}(X_d \text{ for } 10 \text{ ft} + X_d \text{ for } 10 \text{ ft} + X_d \text{ for } 20 \text{ ft})$

$= \cdot 497 + 2.89 - 2 \times \cdot 307 = 2.65$ ohms per mile.

$$\therefore Z_0 = X_0 = \frac{2\cdot 65 \times 40 \times 50{,}000}{66 \times 66 \times 10} = 42\%$$

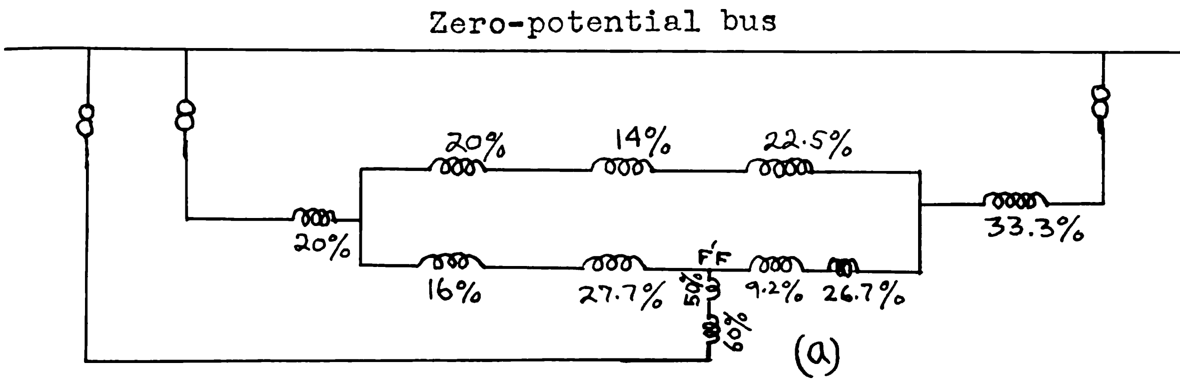
Transformers:

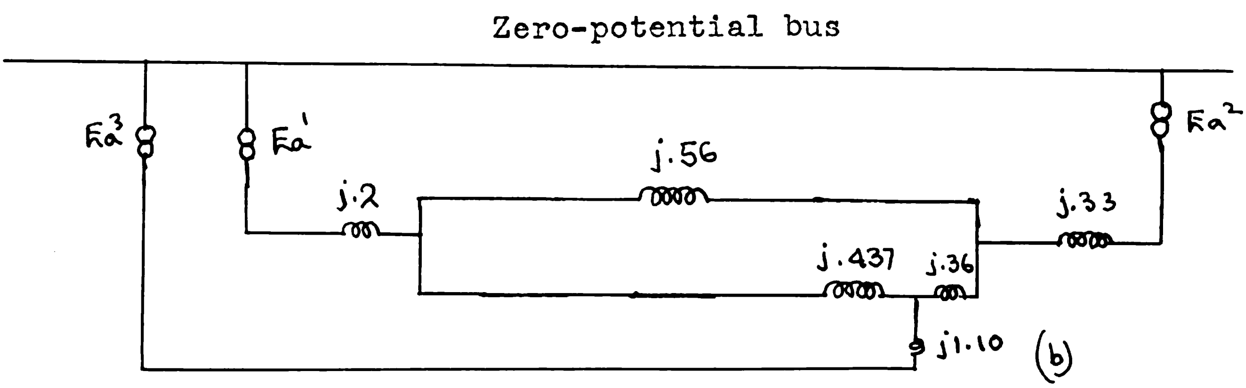
	on new base
1. 25,000 KVA $X = 10\%$	$X = 20\%$
2. 25,000 KVA $X = 8\%$	$X = 16\%$
3. 20,000 KVA $X = 9\%$	$X = 9\frac{50{,}000}{20{,}000} = 22.5\%$
4. 15,000 KVA $X = 8\%$	$X = 8\frac{50{,}000}{15{,}000} = 26.7\%$
5. 5,000 KVA $X = 5\%$	$X = 5\frac{50{,}000}{5{,}000} = 50\%$

$\because Z_a = Z_b = Z_c$; $Z_{\alpha\alpha} = Z_{\beta\beta} = Z_{00} = Z$

Equivalent circuits of the system:

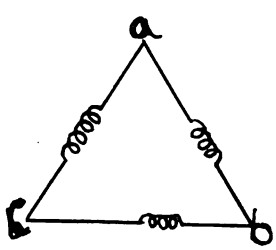
α-network



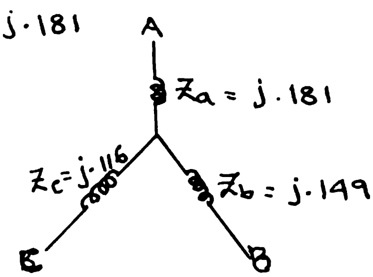


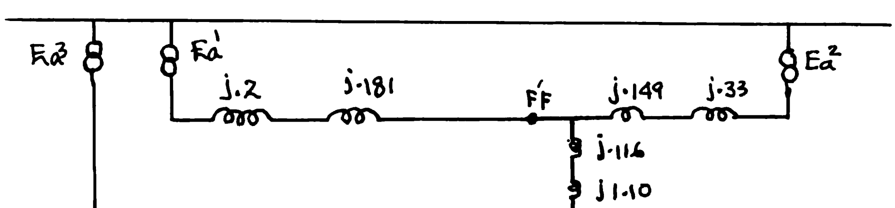
Sample calculation:

$$Z_a = \frac{j.56 \times j.437}{j.56 + j.36 + j.437} = -\frac{.243}{j\,1.357} = j.181$$



(c)





(d)

Fig. 3.5

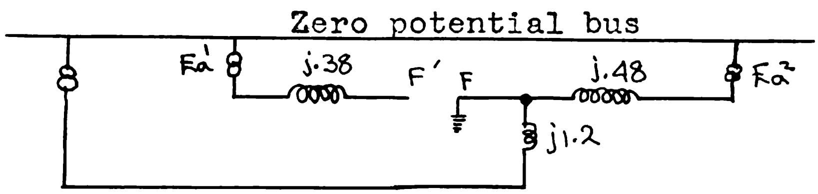
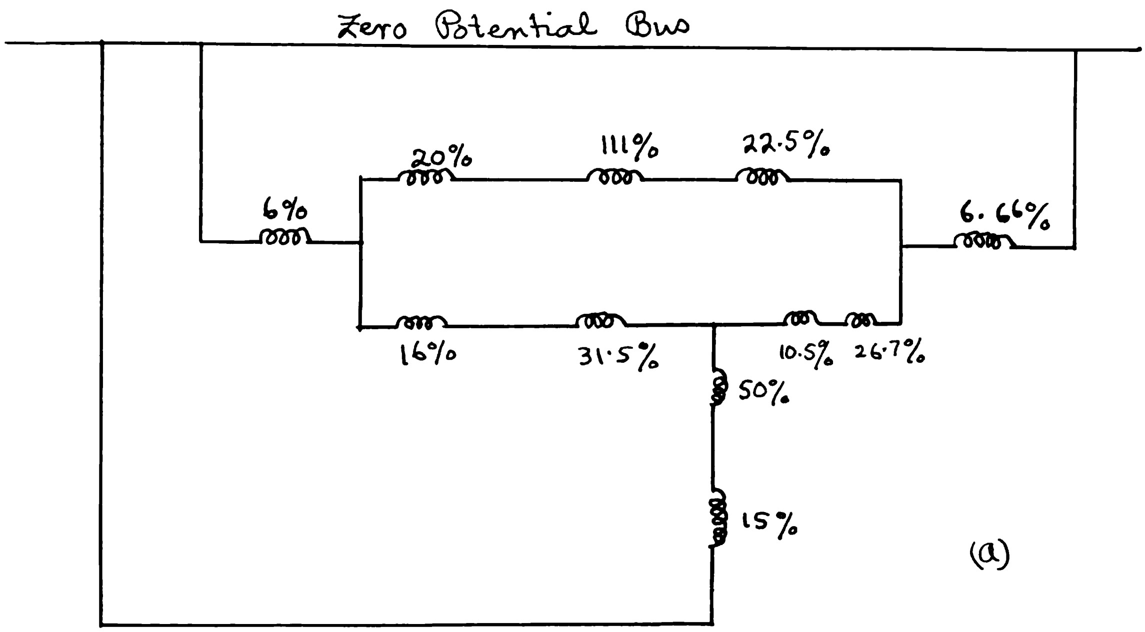


Fig. 3.5

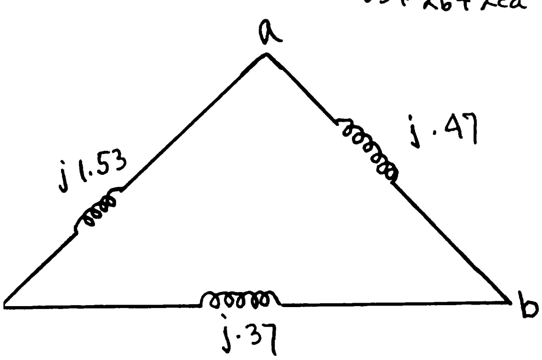
0-network

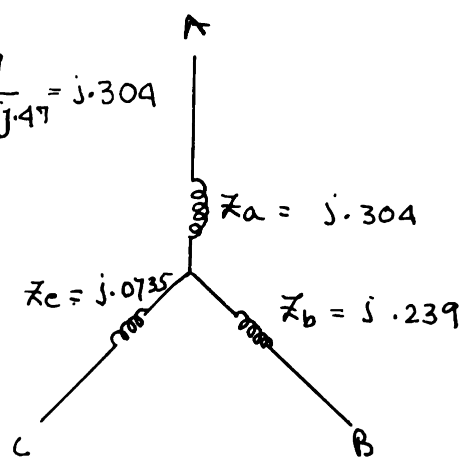


(a)

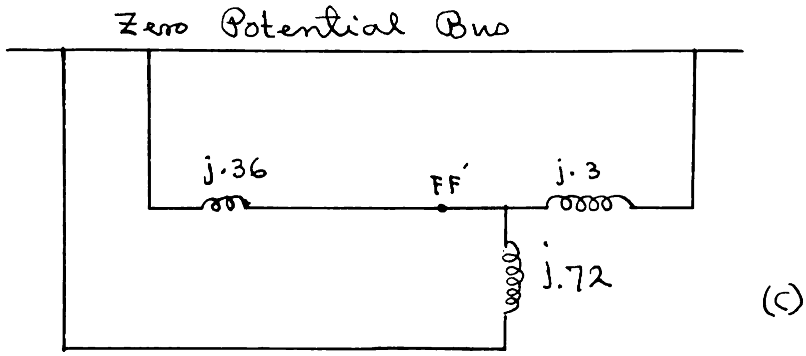
Sample calculation:

$$Z_a = \frac{Z_{ab} \times Z_{ca}}{Z_{ab} + Z_b + Z_{ca}} = \frac{j1.53 \times j.47}{j\,1.5 + j.37 + j.47} = j.304$$





(b)

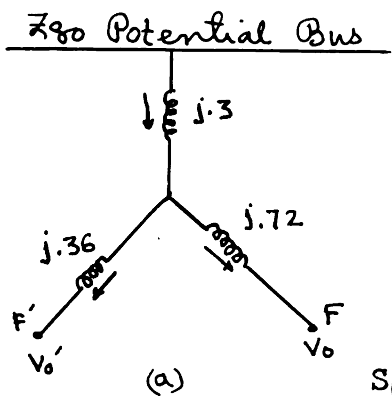


$j.304 + j.06 \cong j.36$

$j.239 + j.066 \cong j.30$

$j.0735 + j.65 = j.72$

Fig. 3.6



$S_0 = j.3$

$C_0 = j.36$

$D_0 = j.72$

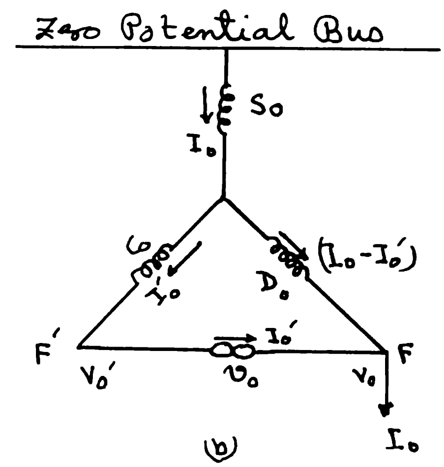
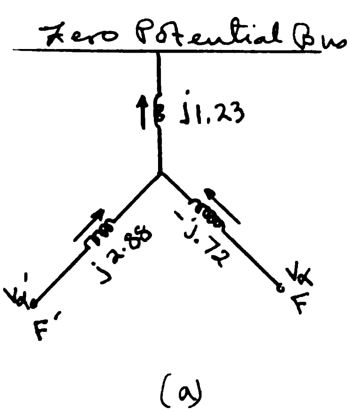


Fig. 3.7

α-network



$\frac{S_0 + 3D_0}{2} = j1.23$

$2C_0 + 3D_0 = j2.88$

$-D_0 = -j.72$

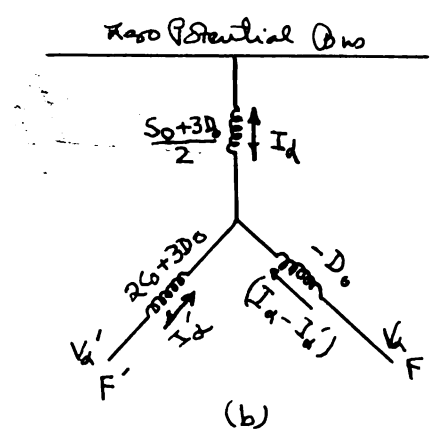


Fig. 3.8

Substituting Y-network of α in original α network.

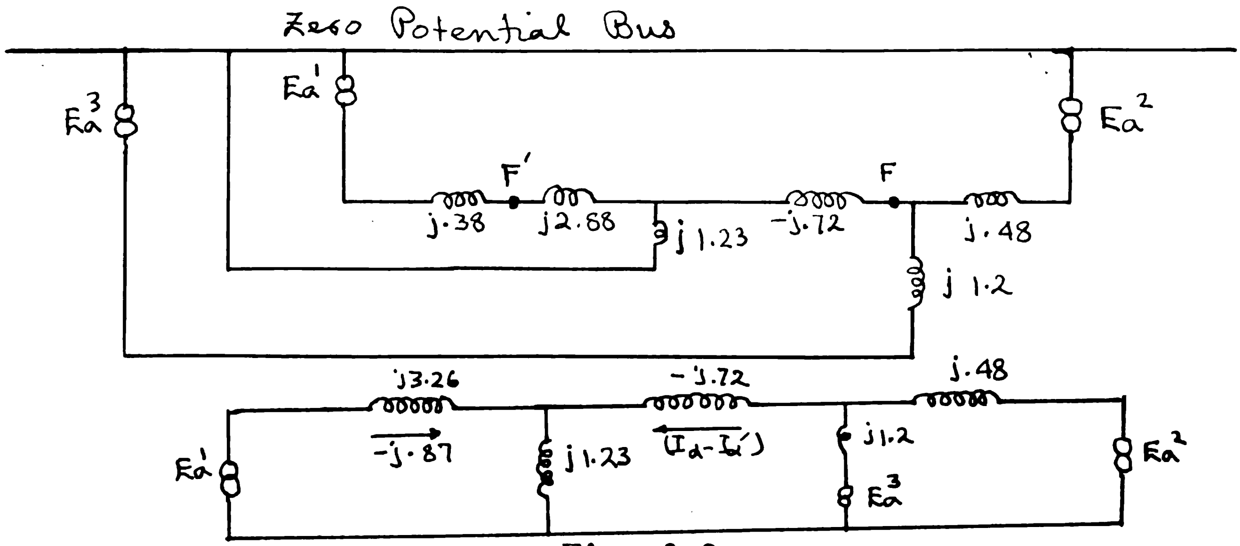


Fig. 3.9

Considering figure 3.8

base voltage 13.8kv

$V_\alpha' = 1 \qquad V_\alpha = .142$

$$V_\alpha' = I_\alpha\left(\frac{S_0 + 3D_0}{2}\right) + I_\alpha'(2C_0 + 3D_0)$$

$$V_\alpha = I_\alpha\left(\frac{S_0 + 3D_0}{2}\right) + (I_\alpha - I_\alpha')(-D_0)$$

or

$$1 = j1.23\, I_\alpha + j2.88\, I_\alpha'$$

$$.142 = j1.23\, I_\alpha + j.72\,(I_\alpha - I_\alpha')$$

Solution,

$I_\alpha = j1.22$

$I_\alpha' = -j.87$

$I_\alpha - I_\alpha' = j2.1$

Considering figure 3.9 (b)

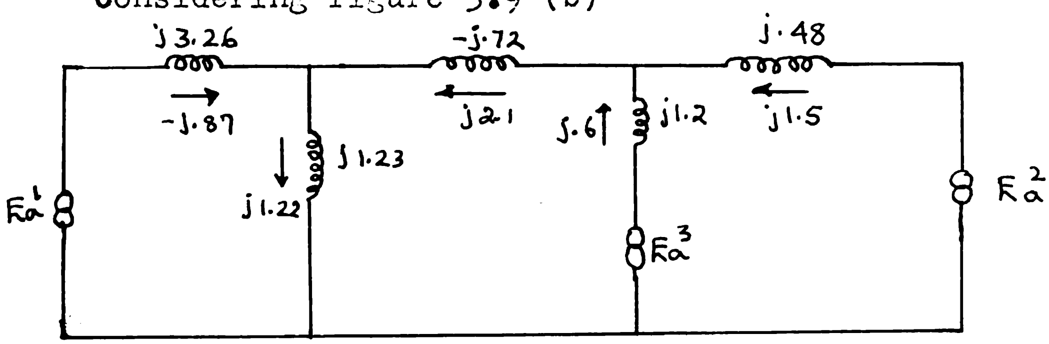


Fig. 3.10

0-network

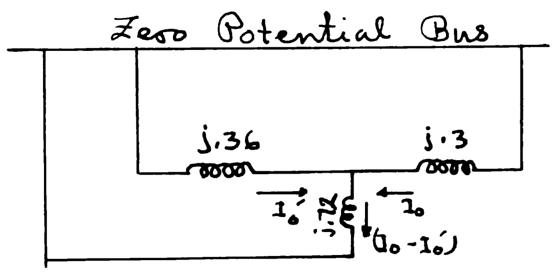


Fig. 3.11

$$I_0 = j\tfrac{1}{2}(1.22) = j.61$$

$$I_0' = -I_\alpha = -j1.22$$

$$(I_0 - I_0') = j.61 + j1.22 = j1.83$$

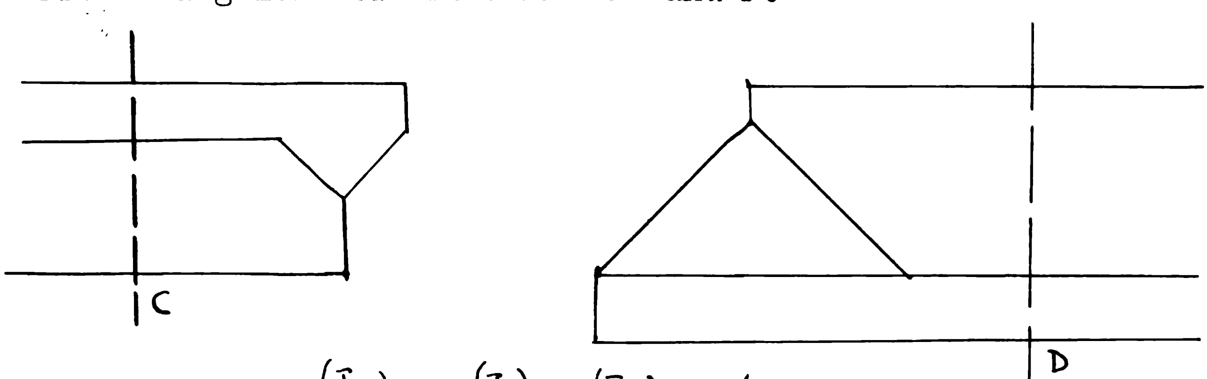
β network is not affected by the fault and does not contribute anything in the calculation and solution.

Hence it is not included here.

So far the effect of Δ-Y transformer bank is not considered.

All the relations developed are based on Y-Y bank.

Proceeding left of the fault F' and F.



$$(I_\beta)_C = -(I_\alpha)_D \; ; \; (I_\beta)_D = (I_\alpha)_C$$

I_0 does not change.

At G_1

Y

$I_\beta(C) = 0$

$I_\alpha(C) = -j.87$

Δ

$-I_\alpha(D) = 0$

$I_\beta(D) = -j.87$

$I_a = I_\alpha + I_0' = 0 - j1.22$

$I_b = -\frac{I_\alpha}{2} + \frac{\sqrt{3}}{2} I_\beta + I_0' = -.866 \times j.87 - j1.22 = -j1.973$

$I_c = -\frac{I_\alpha}{2} - \frac{\sqrt{3}}{2} I_\beta + I_0' = .866 \times j.87 - j1.22 = -j.467$

At G_2

Y

$I_\beta(C) = 0$

$I_\alpha(C) = j1.5$

Δ

$-I_\alpha(D) = 0$

$I_\beta(D) = j1.5$

$I_a = I_\alpha + I_0 = j.61$

$I_b = -\frac{I_\alpha}{2} + \frac{\sqrt{3}}{2} I_\beta + I_0 = .866 \times j1.5 + j.61 = j1.91$

$I_c = -\frac{I_\alpha}{2} - \frac{\sqrt{3}}{2} I_\beta + I_0 = -.866 \times j1.5 + j.61 = -j.59$

At M

Y

$I_\beta(C) = 0$

$I_\alpha(C) = j.6$

Δ

$-I_\alpha(D) = 0$

$I_\beta(D) = j.6$

$I_a = I_\alpha + \check{I}_0 = j1.83$

$I_b = -\frac{I_\alpha}{2} + \frac{\sqrt{3}}{2} I_\beta + \check{I}_0 = .866 \times j.6 + j1.83 = j2.35$

$I_c = -\frac{I_\alpha}{2} - \frac{\sqrt{3}}{2} I_\beta + \check{I}_0 = -.866 \times j.6 + j1.63 = j.31$

In the transmission line

(1) $-j.87 - j1.22 = -j2.2$

(2) $j1.5 + j.61 = j2.11$

(3) $j.6 + j1.83 = j2.43$

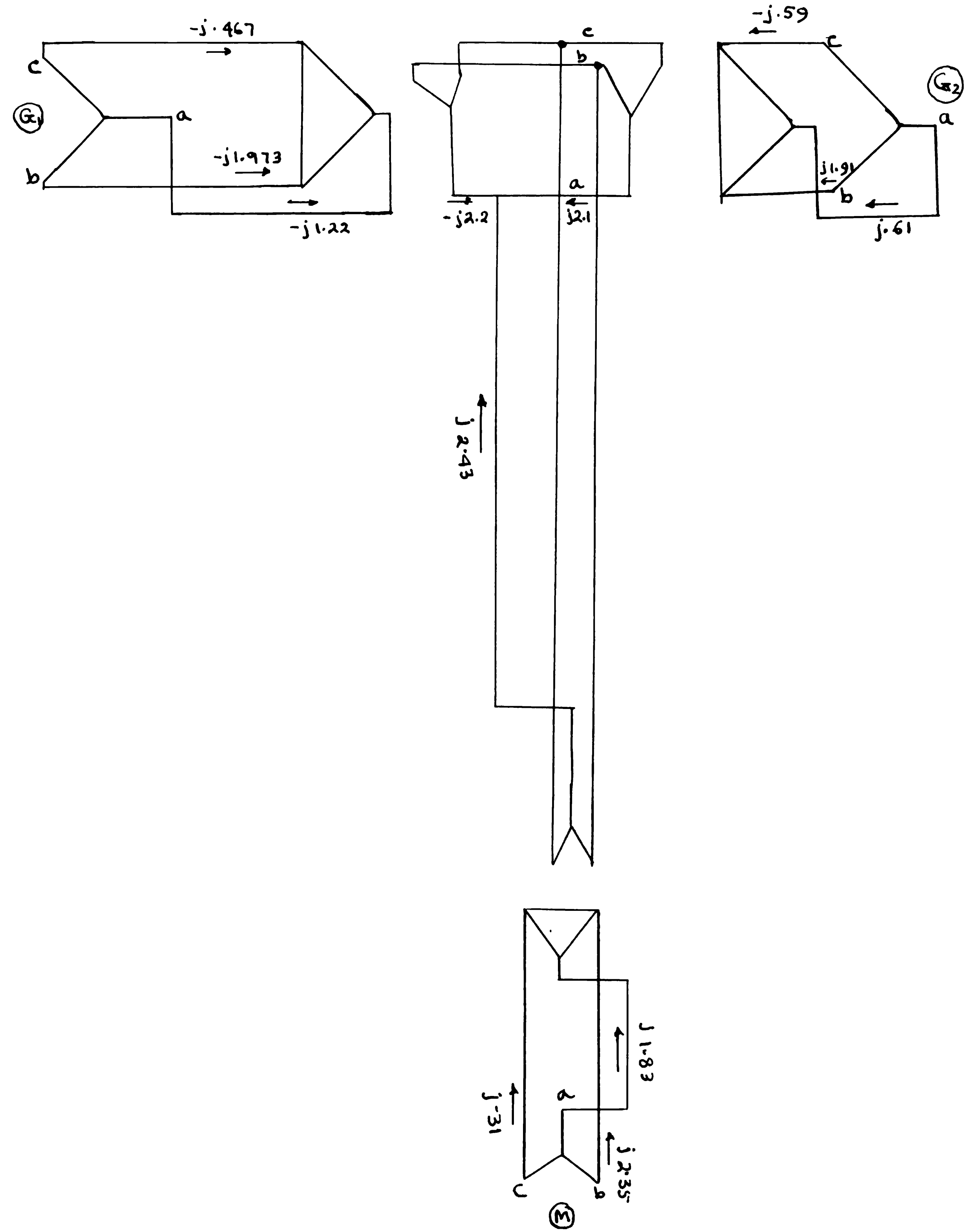


Fig. 3.12 Phase currents.

Chapter IV

Merits of Clarke Components System.

In power systems in which the positive and negative sequence impedences of rotating machines can be assumed equal, use of Clarke Components entails far less work in analytic solutions than the use of Symmetrical Components. They provide simpler equivalent circuit because of the mutual impedences between component networks resulting from unsymmetrical static circuits, if not zero, are receprocal. This is not the case with Symmetrical Components. When symmetrical components are to be used, the self and mutual impedences in the sequence network can often be derived from Clarke Components self and mutual impedences more simply than direct determination. Clarke Components provide a point of view which is helpful in visualizing a problem even if it will eventually be solved by Symmetrical Components. In an unsymmetrical circuit, where the impedences of two phases are equal, or two phases are symmetrical with respect to the third phase, Clarke Components give almost an immediate solution to many simple problems which require appreciable time by other methods.

Discussion on Symmetrical Somponents System and Clarke Components System:

It has been shown that there are twelve equations in the case of symmetrical components connecting the twelve unknown components of current and voltage of phase a at the two fault points--three for each fault point and two for each of the three sequence networks. Eight unknown negative and zero sequence components are eliminated, leaving four equations in terms of the four positive sequence components. Two of these four equations will be in terms of the negative and zero-sequence system impedences. The other two equations involve positive sequence quantities only. The twelve unknowns mentioned before are to be solved by ten available simultaneous equations and as a result, for other two left, an equivalent circuit has to be drawn.

$$V_{a1} = KI_{a1} + mI_{A1}$$

$$V_{A1} = nI_{a1} + lI_{A1}$$

constants k, m, n, and l are then evaluated.

On the other hand, in Clarke Components, we need nine equations as β-network is unaffected by the fault in question and these equations are straight forward. The evolution of constants in Y-circuit to represent the fault is quite sinple. Besides, the component networks are tied by simple relations. From these facts and properties discussed in Chapter 2, it can be concluded that Clarke Components are distinctively advantageous because they are less time consuming.

Appendix

Table I

Short Circuit on Three Phase System

Relations between α, β, and 0 components of voltages and currents at the fault.

Case	Type of Fault	Phases involved	Equations for Components of Voltage at Fault.	Equations for Components of Currents flowing into Fault.
I (a)	Three Phase	a, b, c	$V_\alpha = 0$; $V_\beta = 0$	$I_0 = 0$
(b)		a, b, c and ground	$V_\alpha = 0$; $V_\beta = 0$; $V_0 = 0$	
II (a)	Line to ground	a and ground	$V_\alpha = -V_0$	$I_\beta = 0$; $I_\alpha = 2I_0$
(b)	Line to ground	b and ground	$V_\alpha = \sqrt{3}V_\beta + 2V_0$	$I_\alpha = -\frac{I_\beta}{\sqrt{3}}$, $I_\alpha = -I_0$
(c)	Line to ground	c and ground	$V_\alpha = -\sqrt{3}V_\beta + 2V_0$	$I_\alpha = \frac{I_\beta}{\sqrt{3}}$; $I_\alpha = -I_0$
III (a)	Line to Line	b and c	$V_\beta = 0$	$I_\alpha = 0$; $I_0 = 0$
(b)	Line to line	a and b	$V_\alpha = \frac{V_\beta}{\sqrt{3}}$	$I_\alpha = -\sqrt{3}I_\beta$; $I_0 = 0$
(c)	Line to line	a and c	$V_\alpha = -\frac{V_\beta}{\sqrt{3}}$	$I_\alpha = \sqrt{3}I_\beta$; $I_0 = 0$
IV (a)	Two line to ground	b, c, and ground	$V_\beta = 0$; $V_\alpha = 2V_0$	$I_\alpha = -I_0$
(b)	Two line to ground	a, b, and ground	$V_\alpha = \frac{V_\beta}{\sqrt{3}}$; $V_\alpha = -V_0$	$I_\alpha = -\sqrt{3}I_\beta + 2I_0$
(c)	Two line to ground	a, c, and ground	$V_\alpha = -\frac{V_\beta}{\sqrt{3}}$; $V_\alpha = -V_0$	$I_\alpha = \sqrt{3}I_\beta + 2I_0$

Appendix

Table II

One or Two Open Conductors in a Three-Phase Circuit.

Relations between α, β, 0 components of series voltages v across openings and line current I' flowing through openings.

Case	Open Phases	Equations Relating Components of Voltages across openings	Equations Relating Components of line currents in opening
I(a)	a	$v_\beta = 0$; $v_\alpha = 2v_0$	$I_\alpha' = -I_0'$
I (b)	b	$v_\alpha = -\frac{v_\beta}{\sqrt{3}} = -v_0$	$I_\alpha' - \sqrt{3} I_\beta' - 2I_0' = 0$
I (c)	c	$v_\alpha = \frac{v_\beta}{\sqrt{3}} = -v_0$	$I_\alpha' + \sqrt{3} I_\beta' - 2I_0' = 0$
II (a)	b and c	$v_\alpha = -v_0$	$I_\alpha' = 2I_0'$; $I_\beta = 0$
II (b)	a and b	$v_\alpha + \sqrt{3} v_\beta - 2v_0 = 0$	$I_\alpha' = \frac{I_\beta'}{\sqrt{3}} = -I_0'$
II (c)	a and c	$v_\alpha - \sqrt{3} v_\beta - 2v_0 = 0$	$I_\alpha' = -\frac{I_\beta}{\sqrt{3}} = -I_0'$

Appendix

Table III

Function of Operator $a \left(= e^{\frac{2\pi j}{3}}\right)$

$$a = 1\underline{/120^\circ} = -0.5 + j0.866$$

$$a^2 = 1\underline{/240^\circ} = -0.5 - j0.866$$

$$a^3 = 1\underline{/360^\circ} = 1.0 + j0$$

$$a^4 = a = 1\underline{/120^\circ} = -0.5 + j0.866$$

$$-a = 1\overline{\backslash 60^\circ} = 0.5 - j0.866$$

$$-a^2 = 1\underline{/60} = 0.5 + j0.866$$

$$1 + a + a^2 = 0$$

$$a + a^2 = -1$$

$$a - a^2 = 0 + j1.732 = \sqrt{3}\underline{/90^\circ}$$

$$a^2 - a = 0 - j1.732 = \sqrt{3}\overline{\backslash 90^\circ}$$

$$1 - a = 1.5 - j0.866 = \sqrt{3}\overline{\backslash 30^\circ}$$

$$1 - a^2 = 1.5 + j0.866 = \sqrt{3}\underline{/30^\circ}$$

$$a - 1 = -1.5 + j0.866 = \sqrt{3}\underline{/150}$$

$$a^2 - 1 = -1.5 - j0.866 = \sqrt{3}\overline{\backslash 150^\circ}$$

Appendix

Symmetrical Component impedences:

$$Z_{11} = \frac{1}{2}\left[Z_{\alpha\alpha} + Z_{\beta\beta} - j\,(Z_{\alpha\beta} - Z_{\beta\alpha})\right]$$

$$Z_{22} = \frac{1}{2}\left[Z_{\alpha\alpha} + Z_{\beta\beta} + j\,(Z_{\alpha\beta} - Z_{\beta\alpha})\right]$$

$$Z_{00} = Z_{00}$$

$$Z_{12} = \frac{1}{2}\left[Z_{\alpha\alpha} - Z_{\beta\beta} + j\,(Z_{\alpha\beta} + Z_{\beta\alpha})\right]$$

$$Z_{21} = \frac{1}{2}\left[Z_{\alpha\alpha} - Z_{\beta\beta} - j\,(Z_{\alpha\beta} + Z_{\beta\alpha})\right]$$

$$Z_{10} = \frac{1}{2}\,(Z_{\alpha 0} + j\,Z_{\beta 0})$$

$$Z_{01} = (Z_{0\alpha} - j\,Z_{0\beta})$$

$$Z_{20} = \frac{1}{2}\,(Z_{\alpha 0} - j\,Z_{\beta 0})$$

$$Z_{02} = (Z_{0\alpha} + j\,Z_{0\beta})$$

Clarke Component impedences:

$$Z_{\alpha\alpha} = \frac{1}{2}(Z_{11} + Z_{12} + Z_{21} + Z_{22})$$

$$Z_{\beta\beta} = \frac{1}{2}(Z_{11} + Z_{22} - Z_{12} - Z_{21})$$

$$Z_{00} = Z_{00}$$

$$Z_{\alpha\beta} = j\,\frac{1}{2}\,(Z_{11} - Z_{22} + Z_{21} - Z_{12})$$

$$Z_{\beta\alpha} = -j\,\frac{1}{2}\,(Z_{11} - Z_{22} + Z_{12} - Z_{21})$$

$$Z_{0\alpha} = \frac{1}{2}\,(Z_{01} + Z_{02})$$

$$Z_{\alpha 0} = (Z_{10} + Z_{20})$$

$$Z_{0\beta} = j\,\frac{1}{2}\,(Z_{01} - Z_{02})$$

$$Z_{\beta 0} = -j\,(Z_{10} - Z_{20})$$

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