"SOME FUNDAMENTAL RELATIONSHIPS OF
TRAFFIC FLOW ON A FREEWAY"

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By

Donald P. Ryan

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ABSTRACT

SOME FUNDAMENTAL RELATIONSHIPS OF TRAFFIC FLOW ON A FREEWAY

by Donald P. Ryan

This study was made to determine the relationships of speed versus volume, speed versus density, and volume versus density for a freeway. The Edsel Ford Freeway in Detroit was selected so that internal and external factors affecting traffic flow would be at a minimum. The primary factor affecting traffic flow would be internal friction. Speed, volume, and density are the characteristics of traffic flow for which data were gathered and analyzed.

Studies have been made in the past on this topic by Greenshields, Olcott, Barnett, and others, each having a different set of results and relationships. There is some agreement that the speed-volume relationship is parabolic, but with a tendency for the upper portion of the curve to be linear. It is believed that the speed-density relation is slightly curvilinear. The volume-density relation is believed to be parabolic with the vertex upward.

A set of linear regression equations have been developed for the area of non-critical flow for the three interrelationships. The method of analysis consisted primarily of a chronological plot of data and the F-linearity test. Other statistics used to substantiate the findings were the determination of correlation

coefficients, the standard error of estimate, and confidence limits for slopes and standard deviations.

The speed-volume relation has an acceptable set of regression equations with good linearity but with low correlation (35 - 40%). The average regression equation is $S = 50.2 - 0.275\ V$. The regression coefficients are significantly non-zero. The chronological plot of lane 1 under congestion illustrates the need for separating non-critical and critical flow and the likeliness of the cutoff line.

The speed-density relation has an acceptable set of regression equations with good linearity and fairly good correlation (35 - 75%). The average regression equation is $S = 58.0 - 0.363D$. All regression coefficients are significantly non-zero. The chronological plot of lanes 1 and 3 illustrate the need for separating non-critical and critical flow and the need for a cutoff line.

The volume-density relationship has an acceptable set of regression equations with very good linearity and good correlation (75 - 95%). The average regression equation is $V = 5.7 - 0.472D$. The regression coefficiants are significantly non-zero. The chronological plot of lanes 1, 3, and average illustrate the need for separating non-critical and critical flow and the need for a cutoff line.

The results of the speed-volume analysis differ with Greenshields results in that the rate of speed decrease with corresponding volume increase is much higher in his analysis and would not be a good estimate of Freeway Flow. The speed-density results differ with Greenshields in that his relation is curvilinear with much lower values. Olcott's equations, $S = 54.7 - 0.473D$ (fast lane) and $S = 29.6 - 0.200D$ (slow lane), also differ in values and rates of decreases of speed. The linear volume-density relation determined for this analysis is for non-critical flow whereas others have used all data and a parabolic relation. In general, the linear relationship, as derived in this study, explains the relations in non-critical flow and the area between it (density cut off at 55 vpm and speed cut off at 40 mph) and critical flow.

ACKNOWLEDGMENT

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CHAPTER I

INTRODUCTION

The fundamental characteristics of traffic that are always present in a traffic stream are speed, volume, density, and headway. These characteristics have certain interrelationships which are believed to vary with the geometric design of roadways, with a change in one or more of the characteristics or with a change in one of the four frictions.[7]

The geometric design of a facility will have considerable influence on the extent to which the four frictions will be present. Three of the frictions, internal, medial, and marginal, can be minimized through the geometric design of roadway. However, internal friction is affected by changes in speed, volume, and density.

In non-critical flow, internal friction is relatively low, but in critical flow this friction builds up until congestion is complete and all traffic comes to a halt. This occurrence takes place during peak flows at a time when the facility should be functioning flawlessly. Because of these breakdowns in operation, research is needed to determine what is happening up to the point of critical flow.

The purpose of this paper is to determine the relationships of speed versus volume, speed versus density, and volume versus density in the area of non-critical flow and to determine the line or area between non-critical flow and critical flow.

[7]This number indicates the author reference in the bibliography unless a footnote is given.

The interrelationships of the fundamental characteristics of traffic flow are determined and tested by statistical methods and through the use of one-minute incremental chronological plots of data.

CHAPTER II

REVIEW OF LITERATURE

One method of analysis for studies of speed, volume, density, and stream gap is the statistical approach of Dr. Greenshields.[6] With his applications to theoretical maximum capacity (volume), relationship of speed and volume, and the spacing of vehicles was obtained from a 1934 photographic study. In these photographic observations it was assumed that vehicle spacing of vehicles traveling in groups was at a minimum and was approximately equal to $d_s = 1.1 \cdot S + 21$; where d_s[1] equals spacing in feet; S equals speed in miles per hour; and 1.1 is a factor for the reaction time of .75 seconds. This linear equation produces an ideal flow of 4000 vph. This rate can only be produced momentarily. Hence, the volume formula is not given because of its impracticability. However, it does point to two significant facts:

A. The volume increases with speed but apparently approaches a maximum point at about 40 mph, where the constant 21 becomes insignificant.

B. The minimum spacing depends primarily on "reaction-perception-judgment" time.

He used these relationships along with statistical applications for plotting curves in the obtaining of a speed-volume relation. This relation was found to be linear, S = 43 - 0.54V, where S equals speed

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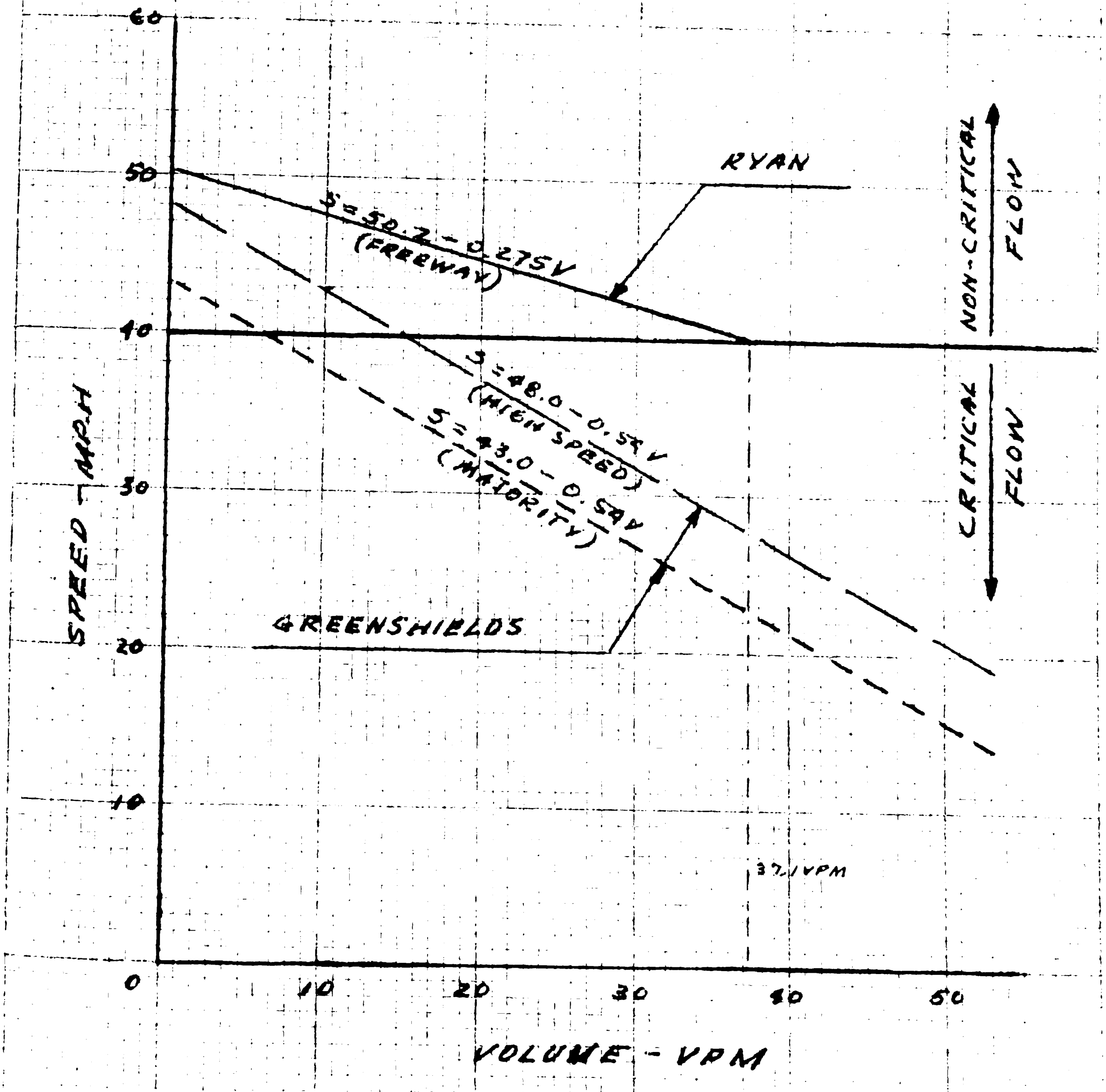
[1]See Page 84 of the Appendix for a complete list of all symbols and abbreviations.

in miles per hour and V equals volume in vehicles per minute. See Figure 1. By saying that V = D · S, (D being density), the equation becomes $S = \frac{43}{1 + 0.35D}$. This equation yields a line with a slight curve. And here Dr. Greenshields says that, for all practical purposes, the line may be assumed to be straight with only slight error. That statement has been discussed pro and con with no definite decision. However, these curves were established on a limited number of observations and further only with two and four-lane highways in rural areas.

Dr. T. W. Forbes[5] used a method of "spaced aerial photography" to obtain data for his analysis of speed, volume, density, and headway. The films obtained in the field were developed and an analysis made by analyzing the films. The samples were then used as the basis for longer-ranged estimates. From the analysis, Forbes found a reduction in speed when the volume per lane increased from 600 vph to 1900 vph. He also concluded that there was no maximum volume point, and for a six-lane divided highway, the average speed of each lane (of the three in one direction) varied under similar conditions. This statement is contrary to what some others have found. The curves of the speed-volume relation show a definite trend away from the straight line relation Greenshields says exist. These disagreements on the relations of traffic flow are not surprising as the facilities tested were not similar in basic design characteristics.

A paper entitled "On Kinematic Waves" by M. J. Lighthill[10] and G. B. Whitham offers a theory of traffic flow except that this is related to "long crowded roads", whereas previous studies were concerned

Figure 1: A comparison of Greenshields results for the speed-volume relation.



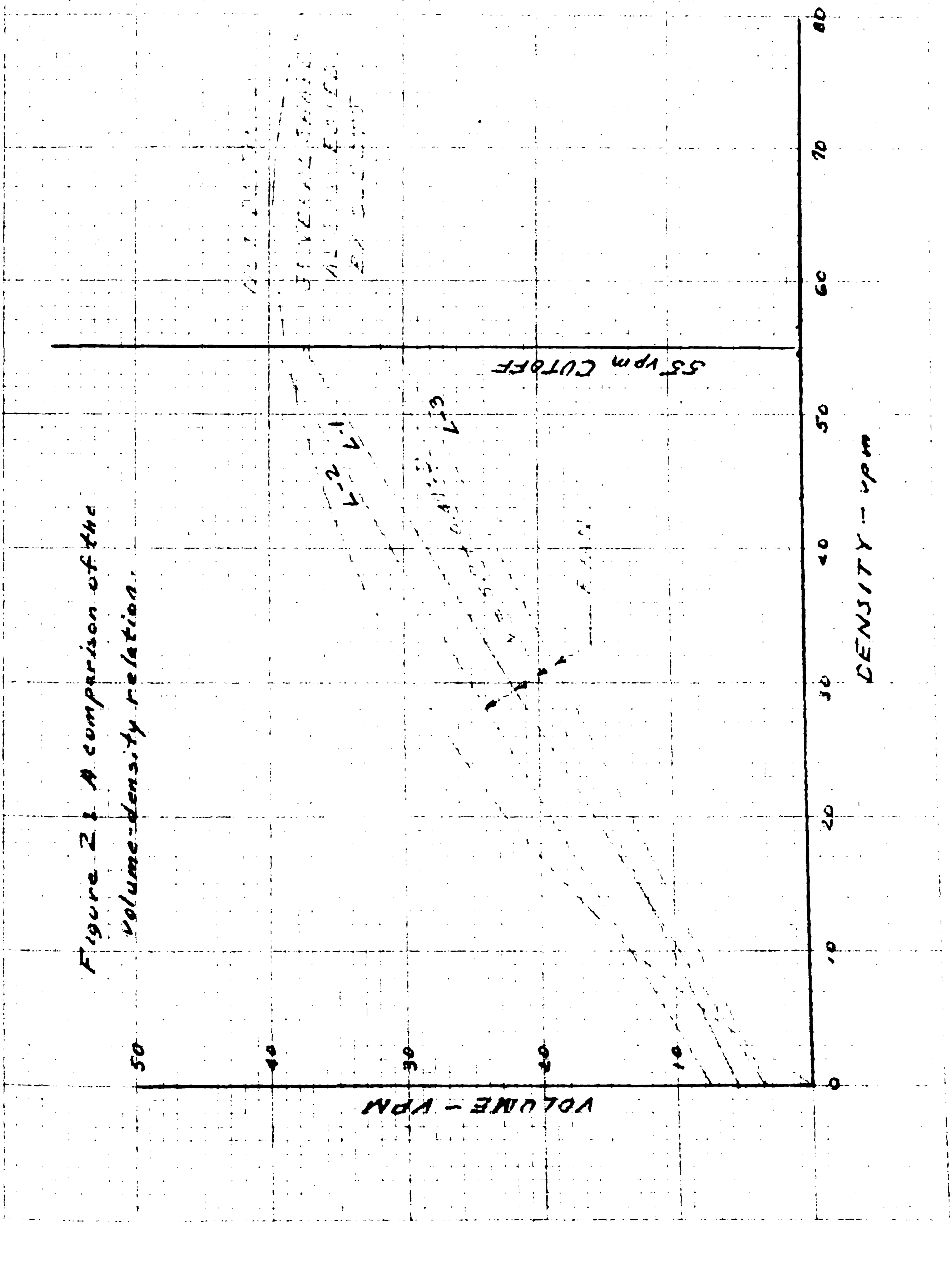


Figure 2: A comparison of the volume-density relation.

with a stationary point study. One of the main points of the report is that Lighthill does not agree with the relation that speed is linearly related to density. The linear relation was determined from a limited number of points contained in a large intermediate range with no points in high and low areas. Lighthill contends that this is a rather drastic interpolation and that this intermediate range probably does not lie in a straight line.

Lighthill also compared the flow-concentration curve (volume-density) of his study with that of Greenshields. Greenshields concluded the relation of volume and density to be a parabolic curve that went through the origin, with the vertex upwards, having a maximum value of approximately 2200 vph. However, Lighthill's curve had no one particular curve shape. The curve had a maximum of approximately 1500 vph, but the curve was such that this could occur through a greater range in concentration. Lighthill concludes that traffic composition, geometric design, culture along the highway, and human characteristics may alter the relation at any given time. This was also concluded by T. W. Forbes who further implied that human characteristics could change within a country. Lighthill also concluded that the difference in the lengths of vehicles would have some affect on "concentration" relations.

Some of the more recent ideas have also used or tested Greenshields' linear and parabolic relations. One of these is by E. S. Olcott.[13]

In this study, tunnels were used as testing sites in the determination of the effect of speed and spacing on tunnel capacity. The tunnels are somewhat ideal in that the frictional aspects regarding the one-way two-lane flow of traffic exist only internally and, furthermore, only within lanes because lane changing is not permitted. Because there are more lanes feeding traffic to and from the tunnel, toll booths have little effect upon the flow of traffic.

Olcott tested the hypothesis that speed and density are approximately lineally related as did Greenshields. The general equation, $Y = c + mx$, was used for the relation with Y being average speed, x average density, c the Y intercept, and m the slope of the line. Using this relation, calculations were made for the data and linear equations derived.

The calculation of this speed-density relation yielded an equation where $S = 29.6 - 0.208\ D$ for the slowest lane with the highest percentage of commercial vehicles and an equation $S = 54.7 - 0.473\ D$ in the lane with highest speed and no commercial vehicles. This is significant in that this report of the Ford has a similar situation. See Figure 3.

Olcott further found that the critical density was 65 vpm at an optimum speed of 21 mph. This is significant as the values obtained in this report are different.

By using the same basic equation and finding the speed-volume relation, a parabolic curve may be plotted. See Figure 4. The apex of the parabola is the optimum density. Points lying above the optimum are those volumes which occur at higher than average speeds. Mr. Olcott furthur indicated that, when critical density is exceeded, for the same

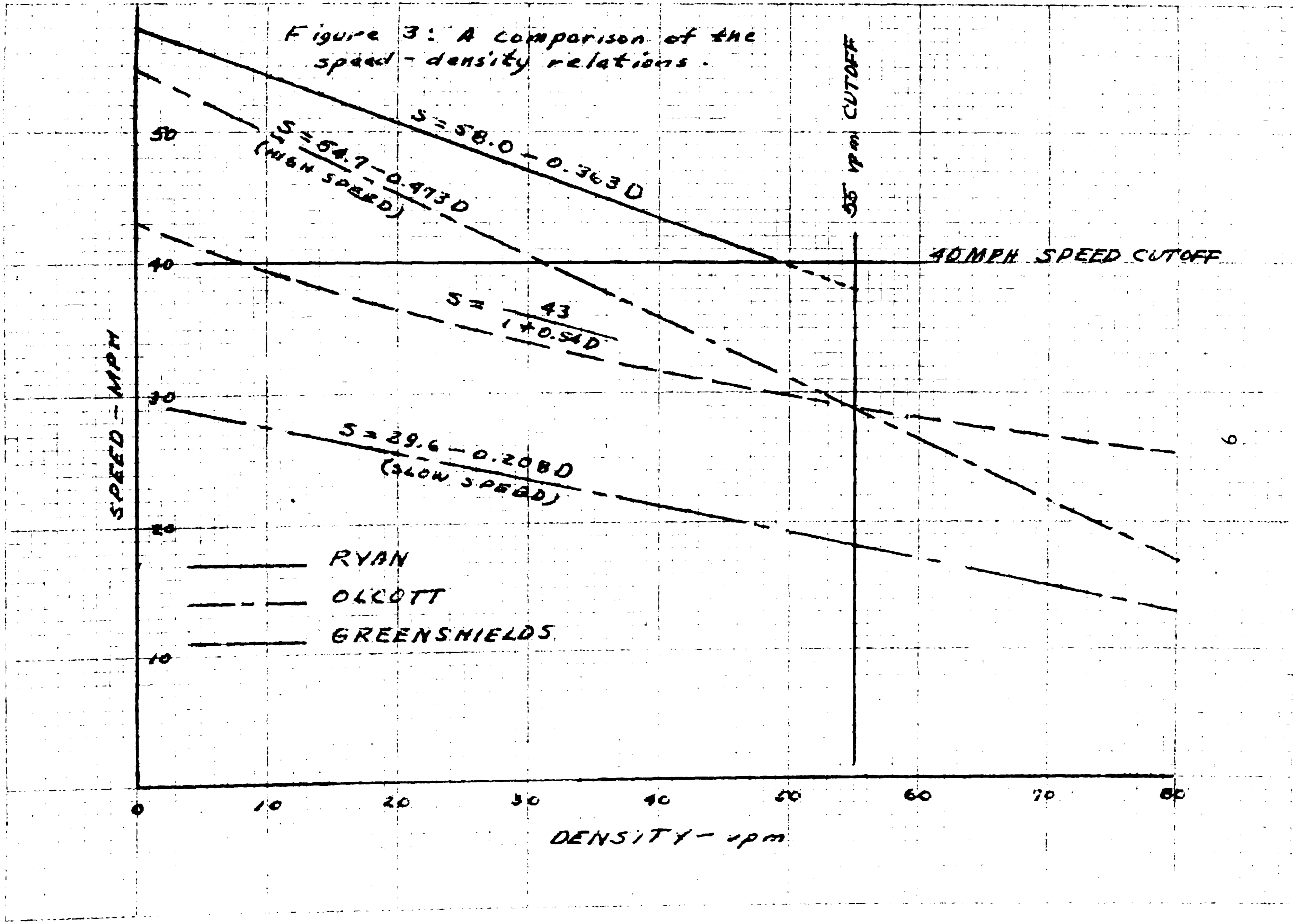
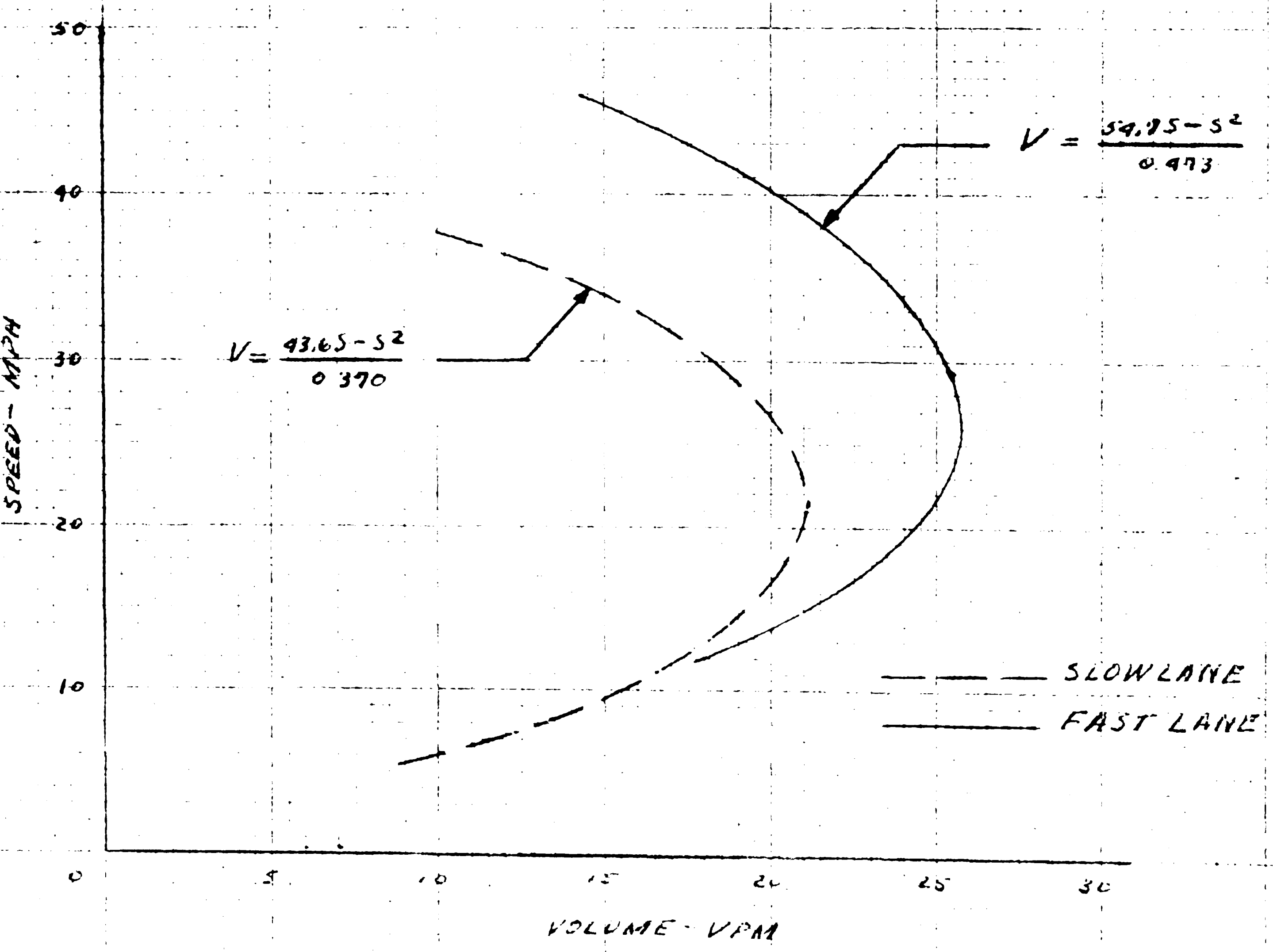


Figure 3: A comparison of the speed-density relations.

Figure 4: Olcott's parabolic speed-volume curves.



volume a lower average speed is expected. This may be due partly to human characteristics which determine what is a safe or unsafe headway.

It may be concluded from Olcott's report that the basic relations of Greenshields are satisfactory for the tunnel study. The only difference being in the numerical relations; that is, different slopes on the speed-density curve or peak values on speed-volume curve. Even with those similarities, Olcott feels that further study is necessary for a complete understanding.

A second traffic flow study in tunnels was made by Edie and Foote[4] and they were trying to relate volume and concentration by considering the effect of bottlenecks. They searched for a model to guide them in the study. The model they chose was the basic relationships of Greenshields. They thought the linear relation of speed and density, and the parabolic relation of speed and volume, would be the best for initial data collection.

However, during the analysis, they found that the kinematic wave theory was quite useful in interpreting observed flow concentration data under more conditions, but it did not predict exactly the patterns observed upstream and downstream from a bottleneck. A difference in results and predicted values occurred, and they thought this was caused by single-lane flow without passing. Another cause may be driver reactions and behavior.

They further concluded that congested flow at most sections of the tunnel had relatively little to do with tunnel environment, and they concluded that Greenshields' model indicates that its usefulness in interpreting flows is limited. With these limitations in mind, they indicated a need for further research on the overall problem of traffic flow and its applications in tunnel flow.

A paper by J. Barnett[2] indicated that "too much traffic" (density) is one of the determinants in traffic flow breakdown. The operational point of a highway is similar to the critical stressing point of steel, beyond the yield point failure occurs.

Barnett further emphasizes that the speed-volume relation is not parabolic but is a combination of linear and parabolic. Volumes, under peak conditions are not 1400 but 2000 vph at an optimum speed of 40 mph.

So far, the previous research in traffic flow indicates that Greenshields' provided the most logical basis to the relationships, but his theories have been quite controversial. The controversy stems primarily from the fact that he was the first to do an extensive study into this area. However, some controversy does concern those portions of the curves where his data was insufficient.

CHAPTER III

METHOD OF STUDY

Description of Test Area

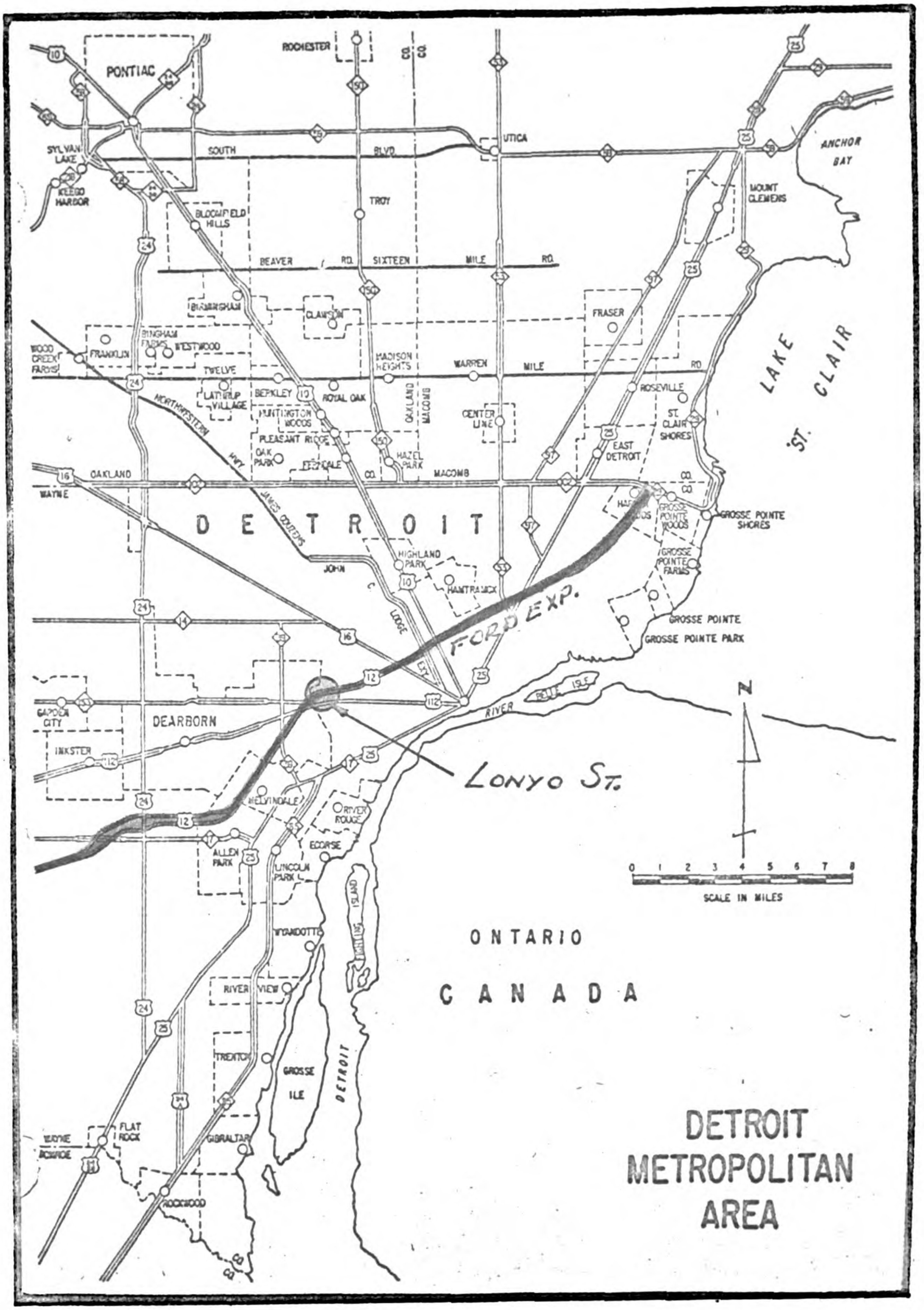
The study site of this research project was located on the Edsel Ford Freeway at Lonyo Street in Detroit. The Ford is a six-lane, divided, and grade-separated depressed freeway, located approximately two miles north of the CBD and is some 15 miles long. It is a part of Interstate Route 94 - Chicago to Detroit.

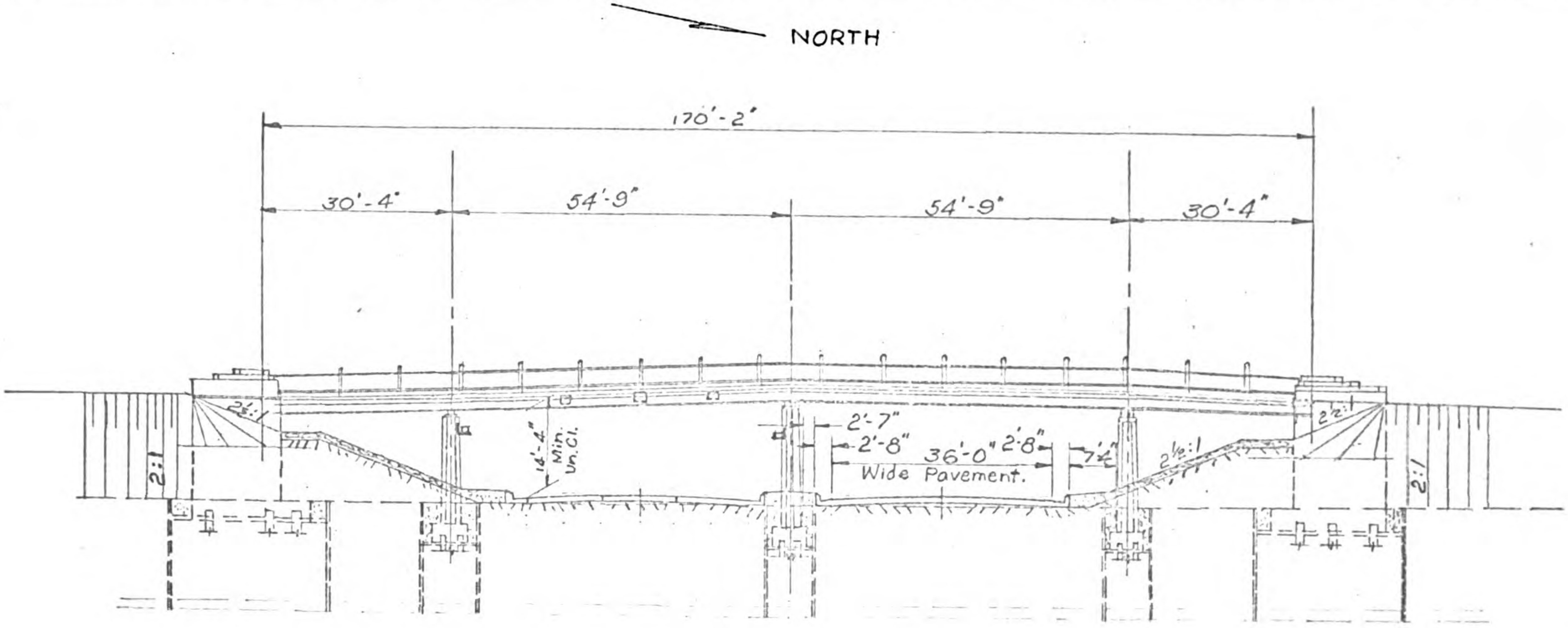
Data were collected on the three eastbound lanes of the Ford by using electronic radar equipment suspended over the roadway. The pieces of equipment were mounted on special brackets fastened to steel beams of a bridge. The radar detectors were mounted on the far side of the structure such that they were invisible to approaching traffic. (downstream traffic.)

There are two on-ramps in the vicinity of the study area: one is on the west side of the bridge approximately a quarter of a mile distant. The other ramp is on the east side of the bridge approximately 600 feet distant. In general, this section of freeway is typical in design of modern freeways. See Figure 7 for ramp east of Lonyo.

Figure 5 is a map of the Detroit area showing the study site in relation to the Ford and the CBD. Figure 6 is a cross-section of the Ford at the Lonyo overpass giving roadway widths and the positioning of the equipment.

Figure 5: Map of Detroit area showing study site location





SCALE: 1" = 30'

FIGURE 6:
INSTALLATION OF TRAFFIC
COUNTING AND SPEED DETECTORS
EDSEL FORD EXPRESSWAY CROSSING
AT
LONYO AVE.

Figure 7: Ramp details --- entrance to Ford.

Ramp Entrance, Edsel Ford Expressway, Detroit. View shows Michigan Avenue in relation to entrance.

Ramp, Edsel Ford Expressway, Detroit. View shows the Ramp is void of any frictional features and is long enough to provide for attaining speed of expressway and proper merging when headway is ample.

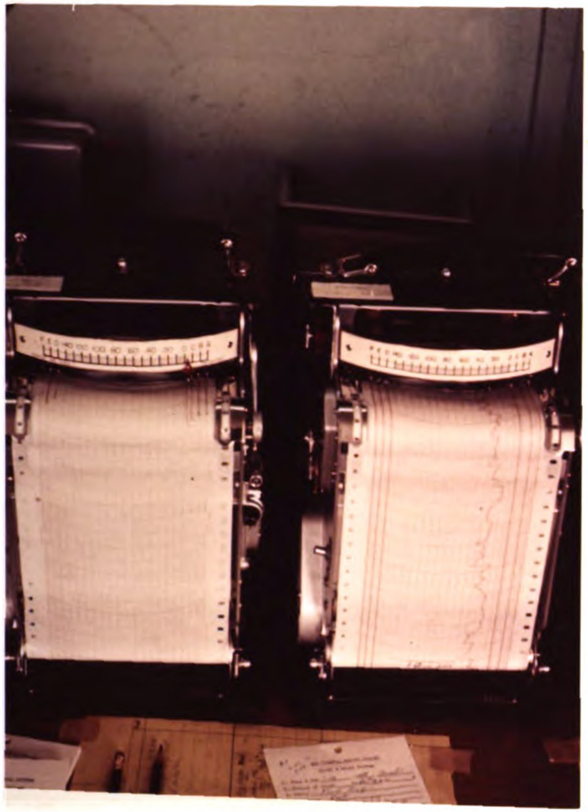
Instrumentation:

At the conclusion of a preliminary investigation to determine the type of instrumentation that would best yield the desired information, a decision was made to use the Esterline-Angus 20 Pen Recorder as the core unit. Additional units were needed to complete the instrumentation necessary for the collection of volume, density, and headway data. These units consisted of a modified model of the RC Streeter-Amet "Ametron" digited recorder, modified models of the MC-11 Electro-Matic Electronic Cycle Computer each connected with an Esterline-Angus Graphical Recorder; modified models of the S-2A Electro-Matic Radar Speed Meter and accompanying Esterline-Angus Graphical Speed Recorders; and three model RD-1A Electro-Matic Overhead Radar Vehicle Detectors.

Each vehicle that passed the "point" of field study, was detected by the overhead radar detector. This information was then sent over telephone wires to the receiving station. Here the information went to a repeater relay where the information was sent to two recording devices--a 20-pen recorder and a Volume-Density Computer. See Figures 8 and 9.

The 20-pen recorder contains a one-hundred foot roll of graph paper that can be geared to operate at various constant speeds. There are twenty pens built into the instrument and each is actuated by a six-volt relay. The pens make longitudinal lines on the moving paper rolls except when a pen is actuated and then the pen moves laterally approximately one-eighth of an inch. Each relay is wired to two terminal

Figure 8: **Data rec** ording equipment.



Volume-Density Recorder: Recorder gives a graphical plot of Volume of traffic as related to the time of day.

Speed Recorders: Recorder gives a graphical plot of speeds of vehicles as related to the time of day.

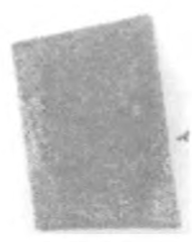
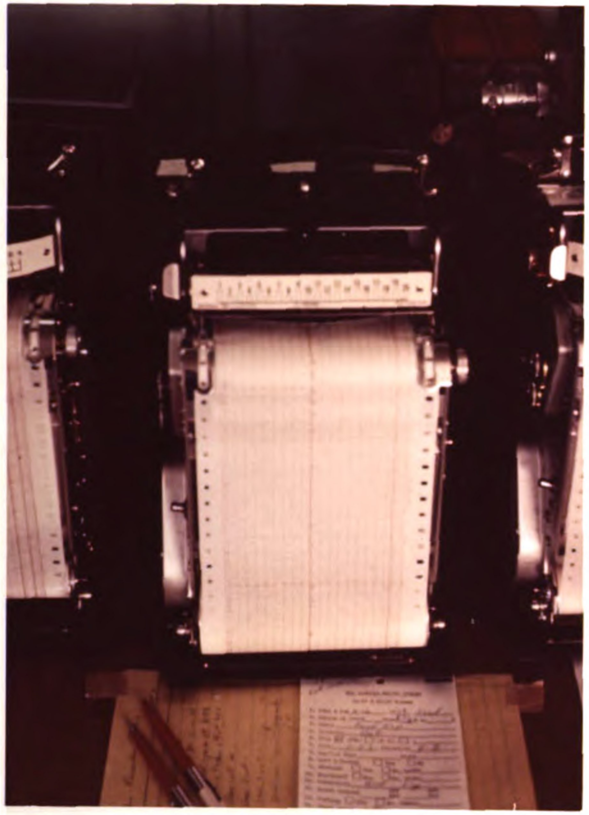


Figure 9: Data recording equipment.



Twenty-Pen Recorder: Individual cars and time spacing for each lane are plotted on chart which moves at rate of six inches per minute. Time intervals to 1/10 may be obtained.

Modified R.C. Recorder: Vehicles are counted for one-minute periods and the total is printed out, resets to zero and counts next minute. Has approximate error of two percent.

posts on the panel in the rear of the machine. See Figure 9.

In this study, one pen for each lane was used. Their actuating relays were connected to telephone wires. The telephone wires were connected to Radar Vehicle Detectors. The radar detecting devices, in turn, sent an electrical impulse to the 20-Pen Recorder each time a vehicle passed beneath a detector. The 20-Pen Recorder then plotted this impulse on the moving paper. The impulse appeared as a lateral line, thus distinguishing it from the normal longitudinal lines which was a plot of time. By counting the lateral lines for a given time period, volume was obtained.

The speed and volume density recording devices operated in a manner similar to the 20-Pen Recorder. That is, information was sent over telephone lines to be plotted by the recording instrument by making a lateral line across the paper. For these recorders, the length of the lateral line varied, depending upon the speed or volume of vehicles.

A line that is plotted on a graph requires the interpretation of its value by a person whereas a digital recorder prints the exact value. The information printed by the one digital recorder was volume of traffic for one lane for one minute of time. This was the character of the Streeter Amet R.C. Digital Recorder as compared with the Esterline-Angus graphical recorder. However, each recorder served its purpose satisfactorily.

The detecting devices operated on the radar principle; that is, the sending and receiving of electrical impulses. The speed detector measures the time difference between signals (impulses) received and converts this information to electrical current. This current is received by the recorder and plotted on a scale that reads MPH instead of an ampere scale. The overhead vehicle detector differs in that received impulses causes the electrical circuit to be broken for as long as a vehicle is within the range of impulses. This break in the current is recorded on the 20-Pen graphical recorder as a lateral line. Hence, the distance between lateral lines will be time spacing or headway. This same impulse was also sent to the volume density computer. However, each impulse was not plotted individually as in the 20-Pen or speed recorders, but the impulse was sent to a condensor where additional impulses could be added, and released at specified intervals of time. Hence, the information derived from the volume density computer was data integrated over short periods of time. The plot of which would show graphically the volume of traffic in relation to time.

Before any data could be collected for use, all of the equipment had to be thoroughly tested. The detectors had to be installed at the study location and the recorders or receiving units installed at the receiving station. The detectors transmitted the information over telephone lines to the recorders. The use of telephone lines for the transmission of data was also a factor which necessitated the preliminary testing and also daily testing when the units were actually obtaining data for the research project. See Figure 10 for the field and receiving installations.

The field installation consisted of placing the overhead radar detectors and special mounting brackets above the roadway. The brackets were clamped rigidly to the bottom flange of an exterior beam of the

Figure 10: Office and field equipment installations.

Recording Equipment: Recording devices are of three types; Two volume-density, one twenty-pen, and two speed recorders. Cable behind equipment is a Bell Telephone line used for transmitting data from field (one mile distant) to office.

Lonyo Bridge Location: View shows overhead vehicle detectors on downstream side of bridge. Speed detectors are in the shadows. The detecting devices were not readily discernible when approaching the bridge and did not affect the traffic flow in any way.

steel beam bridge. The speed detectors were placed in a similar manner at first until extensive testing finally yielded data that was satisfactory when the detectors were near the ground. This made it necessary to enclose the speed detectors so that they would not be discernible from the traveled roadway. The enclosure was a box with an opening in one side just large enough for the detector to emit and receive its impulses. The box was then located approximately 30 inches above the ground and eight feet from the side of the road. With all equipment installed and tested for satisfactory operation, it was then assumed that the data collected would be the best possible because it would be void of most human errors. See Figure 11 for speed detector mounting.

The data that was collected by the detectors would not be complete with just one set up. This was due to equipment limitations. The radar speed detector could be adjusted to discern individual cars in one lane of traffic. However, only two speed detectors were available for the three-lane highway. This meant that the information desired for each car (speed and spacing) would have to be obtained in steps. The first step was to secure the speed and spacing data (includes volume) for lanes 1 and 3 and spacing (includes volume) only on lane 2. When this had been completed, the speed equipment was moved so that speeds were detected in lane 3 by one detector and in lanes 2 and 3 by the other detector. By subtracting the speeds recorded in lane 3 from the speeds of 2 and 3, the speeds of lane 2 were then discernible.

Figure 11: Sensitivit y controls and speed detector mounting.

Sensitivity controls and test meter for adjusting overhead detectors.

Speed detector mounting and box enclosures.

Data Collected:

It would be impossible to say that the data collected was completely void of errors, but it was assumed that the instrument errors were minor. The speed detectors were accurate to ± 2 mph for all speeds, which is better than Olcott's method of speed detection which had varying accuracy, depending upon the speed of the vehicles (the greater the speed the larger the error). The overhead detectors were capable of distinguishing cars that had as little as six feet headway. The recorders could record and be interpreted as accurately as the detectors could detect.

In obtaining data for this study, it was desirable to take samples during periods of peak traffic flow or, in other words, have sufficient volume to cause a decrease in speed and an increase in density, thus producing a backing up of traffic. Since the detecting units were mounted on the inbound traffic lane, the peak flow of traffic would be in the morning. A time was selected to include this peak flow of traffic and with flow conditions just prior and immediately following the peak. From volume count records obtained from the City of Detroit, Department of Streets and Traffic, the time of 6:45 a.m. was selected as the starting point and 8:00 a.m. as the ending point. The backup of traffic generally occurred between these times.

All data gathered for this study were classified by one-minute intervals. The belief has always been to use samples of data taken at five-minute or longer intervals. It is the belief of this author that much valuable information has been lost due to dampening effect caused by averaging data over a five-minute period. This is because during any given five-minute sample of data, the extremes could occur but, through averaging, they would be lost.

The entire installation of equipment was somewhat more desirable than Olcott's or Norman's.[1] Olcott collected his data in a tunnel with his observers adjacent to the flow of traffic. The observers were non-uniformed and this rarely occurs; hence, it may have had some effect upon the drivers. O. K. Norman used a rubber-hose and wire-strip assembly placed across the pavement. This definitely had some effect upon drivers as drivers fear "speed traps" and tend to slow down as soon as they see it. However, the equipment used in this study was very hard to discern. The overhead radar detectors were mounted above the roadway on the far side of the bridge and were not visible to the driver until after he had passed under them. The speed detectors, as mentioned earlier, were mounted in a box and positioned off the roadway approximately eight feet. The box did not have the appearance of being related to a speed trap; hence, there was no effect upon the drivers. The receiving equipment was a remote installation, approximately three-fourths of a mile away. This meant that there was no need for observers in the field. With this type of installation, it was thought to be the best possible for collecting data accurately and with the least effect upon the traffic.

The majority of vehicles studied were driven by people working in or around Detroit. It was, therefore, assumed that most drivers would be familiar with the highway.

The posted speed limits for the Ford Expressway are 55 mph maximum and 40 mph minimum "when conditions permit". Tractor-trailer units and buses were restricted to operating only on the outside lane. However, this was not fully complied with as an occasional truck or bus would be in the center lane passing slower trucks in the outer lane. The number of trucks or buses observed on the inner lane was negligible due to the high speed of the lane, the fact of a 45 mph maximum speed limit for trucks, and also the law restricting their use to the outer lane.

Method of Analysis:

In the past, many analyses have been made on speed, volume, and density data with results being partly linear and partly curvilinear relationships. Some of the questions that have been asked regarding the significance of relationships have not been answered. One question is: What is the significance of two entirely different relationships for one set of data? Therefore, which relation is more significant--critical flow where congestion prevails or non-critical flow where flow is relatively free? Furthermore, one might ask, what is critical flow or what is non-critical flow? Is there a grey area between critical flow and non-critical flow and can it be defined?

This analysis will be made on the basis of determining a linear relationship for data in the non-critical flow area and then with the aid of this relation determine the dividing line or area between critical flow and non-critical flow.

The main steps in the analysis of the data consisted of calculating several statistics, making a chronological plot of data, and then trying to correlate the two into a set of logical deductions. This will be done for each lane of data and then for a combined lane analysis.

The first step, calculating the statistics, involved the development of a correlation table[6] which shows numerically as well as graphically the two-way distributions of speed-volume, speed-density, and volume-density. The table produced the following statistics needed in the analysis: N (number of data pairs), ΣY (the sum of the Y values), ΣY^2 (the sum of all Y^2 values), ΣX (the sum of the X values), ΣX^2 (the sum of all X^2 values), and ΣXY (the product sum of all X times Y values). The types of tables used and above-listed statistics are found in Figures 32, 33, and 34 in the Appendix.

Once the above-listed statistics were calculated, other statistics and test thereof were made. The first two values determined are the means for X and Y data. These are given in Tables 1 through 4 as $\bar{X}$ and $\bar{Y}$. The means were calculated by the equations

$$\bar{X} = X_0 + (\Sigma X/N)\, i_x$$

and

$$\bar{Y} = Y_0 + (\Sigma Y/N)\, i_y$$

where X_0 and Y_0 are estimates of $\bar{X}$ and $\bar{Y}$ and where i_y and i_x = the class interval.

The next statistic to be calculated was the slope of the regression. This is the value shown in the Tables 1 through 4 as $m_{y/x}$. This value was obtained from the equation

$$m_{y/x} = \frac{i_y}{i_x} \quad \frac{N(\Sigma XY) - (\Sigma X)(\Sigma Y)}{N(\Sigma X^2) - (\Sigma X)^2}$$

The confidence limits for the slopes of the regressions were determined by the equation

$$\pm b = S_b \cdot t_{\alpha/2,\, n-2}$$

$$\text{where } S_b = S_{y/x} / S_x \sqrt{N-1}$$

given $S_{y/x}$ = the standard error of estimate (see Page 32)

S_x = the standard deviation in X direction

and where $t_{\alpha/2,n-2}$ = a value obtained from Table 3 of Crow[3]

The purpose of the slope confidence limits is to determine whether the slope $m_{y/x}$ is a valid estimate for the regression.

Regressions are determined so that an analytical relationship may be obtained. The relationship is hypothesized to be linear but is tested for linearity by the F-linearity test.

The F-linearity test indicates the relative linearity of data. The F test was made with the equation

$$F = \frac{M}{W} \quad \frac{(N-k)}{(k-2)}$$

$$\text{where } M = C - W,$$

$$C = \frac{1}{N}\left[N\Sigma Y^2 - \frac{(N\Sigma XY)^2}{(N\Sigma X^2)}\right],$$

$$\text{and } W = i_y^2\left[\Sigma Y^2 - \Sigma\left(\Sigma Y_i^2 / f_{xi}\right)\right]$$

Calculations of the F value should be less than the 95% level to be accepted. Values of F larger than the 95% level would be rejected, but this bit of information is just as useful as it helps explain the relation as much as acceptable F values. If the F-linearity test is rejected, a new lower cutoff line will be used and a re-calculation made. This will be repeated until an acceptable F test is made.

Next is the calculation of the correlation coefficient. This calculation is used to determine whether the regression coefficient is significantly non-zero. The null hypothesis is tested; that is, it is believed that the regression coefficient is zero. The equation used for determining the correlation coefficient is

$$r = \frac{N\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{\left[N\Sigma X^2 - (\Sigma X)^2\right]\left[N\Sigma Y^2 - (\Sigma Y)^2\right]}}$$

If resulting r values are larger than the 95% level values, the hypothesis is rejected and it can then be said that the coefficient is significantly non-zero. The closer the value of r to 1, in general, the greater the significance of non-zero and correlation of Y on X. From the regression coefficient squared--r^2--it may be seen that the regression of Y on X accounts for (r^2 x 100) per cent of the variance of Y. The larger the percentage the better the explanation of Y on X.

The last statistic used in the analysis is the standard error of estimate which determines the scatter, in the ordinate (Y) deviation, of the observed points about the regression line. This may be determined by either the equation

$$S_{Y/X} = \sqrt{\frac{\Sigma X^2 \Sigma Y^2 - (\Sigma XY)^2}{(N-2)\Sigma X^2}}$$

or the equation

$$S_{Y/X} = \sqrt{\frac{N-1}{N-2}\left(S_Y^{\,2} - m_{Y/X}^{\,2} S_X^{\,2}\right)}$$

where S_y^2 = variance of Y values

and S_y = standard deviation of Y values

A large value of $S_{y/x}$ means data is more widely scattered, whereas a small $S_{y/x}$ indicates the data to be relatively close to the regression line; and, as far as traffic flow is concerned, small values of $S_{y/x}$ are more desirable because this should indicate relatively smooth-flowing traffic. It is interruption to flow that causes the scatter of data and large error of estimate. The quantity $S_{y/x}^2$ estimates that part of the variance of Y left unexplained by the regression of Y on X.

Two statistics calculated which parallel the standard error of estimate are the standard deviations for the X and Y directions. These are given by the equations

$$S_Y = \sqrt{\frac{N\Sigma Y^2 - (\Sigma Y)^2}{N(N-1)}}$$

$$S_X = \sqrt{\frac{N\Sigma X^2 - (\Sigma X)^2}{N(N-1)}}$$

The squared values, S_y^2 and S_x^2, are the variances and may be used in the second equation given for the standard error of estimate. The standard deviations indicate the relative scatter of data may be correlated to the type of flow.

The second step in the analysis is the making of the chronological plot of data. This is a successive plot of data from the beginning of the study period to the end. The plot is made by taking two pairs of data and locating them on a graph and then drawing an arrow between them indicating which occurred first according to time. To help distinguish data in critical flow from non-critical flow, the points and arrows are shown in red. The cutoff line is shown in blue. The chronological plot gives an illustration of some of the statistics calculated in step one. The chronological plot will also help to point out the difference between data in critical and non-critical flow.

The correlation of data and logical deductions will comprise the chapters on "Analysis of Data", "Some Fundamental Relationships of Traffic Flow on a Freeway", and the "Conclusions".

CHAPTER IV

ANALYSIS OF FIELD DATA

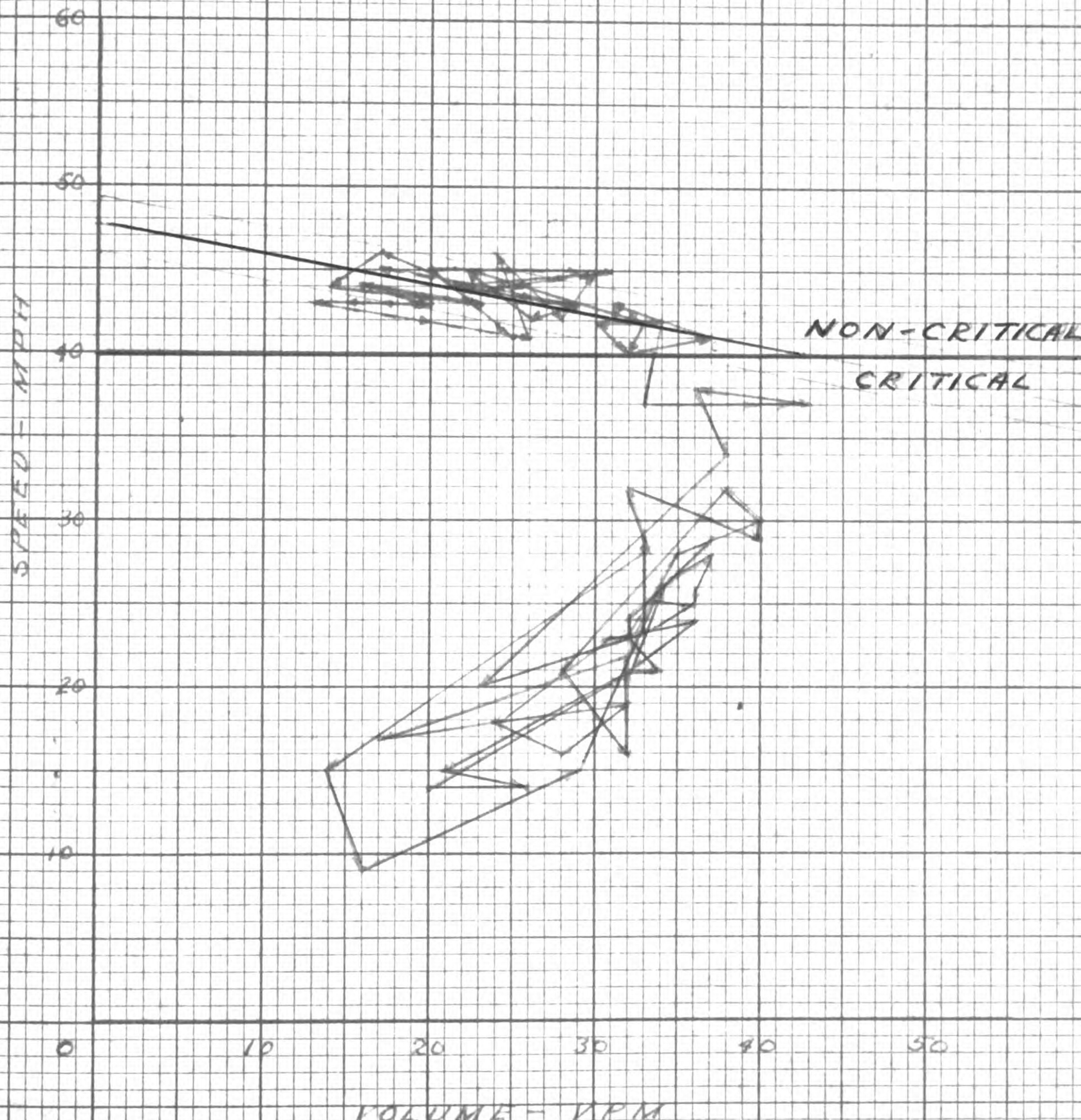
Analysis of Speed-Volume Relationship:

Speeds used in the regression analysis were limited to those above 40 mph--the 40 mph line being the dividing line between non-critical flow and critical flow. Critical flow is that movement of vehicles which occurs when too many vehicles are operating such that lane changing and passing are restricted and speeds are, therefore, necessarily reduced. Non-critical flow is that movement of vehicles which allows lane changing and passing as desired. There is one speed-volume line in Figure 12 which crosses the critical speed line with the remaining speed-volume lines in this critical area and they appear to be very erratic in occurrence.

The chronological plot of data also gives an indication to a greater range in speeds in the critical flow area as compared to the non-critical flow. The latter region of flow has a range of six mph while the critical region of flow has a range of some 31 mph.

Paralleling this line of thought is a comparison of average speed changes. Average speed increases per one-minute interval in non-critical flow are 1.94 mph whereas in critical flow, average increases are 3.55 mph. Average speed decreases are, respectively, 1.77 mph as compared with 4.43 mph. The significance of this comparison is that changes in speed and volume are more pronounced in the critical flow and, thus, reflect increases in internal friction to such a degree that it becomes difficult for the traffic stream to overcome this internal friction.

Figure 12: Chronological plot of data for Lane I, Feb. 13, 1958



However, when the fluctuation in speed is less pronounced, as is illustrated in Figure 13, the apparent effect of further congestion is not present and small increases in internal friction are soon dissipated and the lane speed soon increases back into the region of non-critical flow. Average speed increases and decreases are 2.15 and 1.7 mph for this lane which compares to volumes found for lane 1 in the region of non-critical flow.

Figure 15, which is a speed-volume plot of lane 3, illustrates data where average speed increases and decreases, 3.23 and 4.14, respectively, are similar to those of lane 1, but complete congestion does not occur as it did in lane 1. This may be due to the difference in densities between lane 1 and 3. Lane 1 has an average density of 61.4 vpm which is more than twice lane 3's density of 26.7 vpm. Density is related to headway in that headway is the reciprocal of density. This means that the headway for lane 3 is twice that of lane 1 and would, therefore, be more likely to accommodate the larger changes in speed with less effect to traffic flow in that stream. It would, therefore, seem from this analysis that density, or its reciprocal--headway--is a more critical characteristic of the three being studied.

For each of the three lanes, regression analyses were also made in order to determine an analytical relationship between the two characteristics.

Figure 15: Chronological plot of data for Lane 2, March 27, 1958

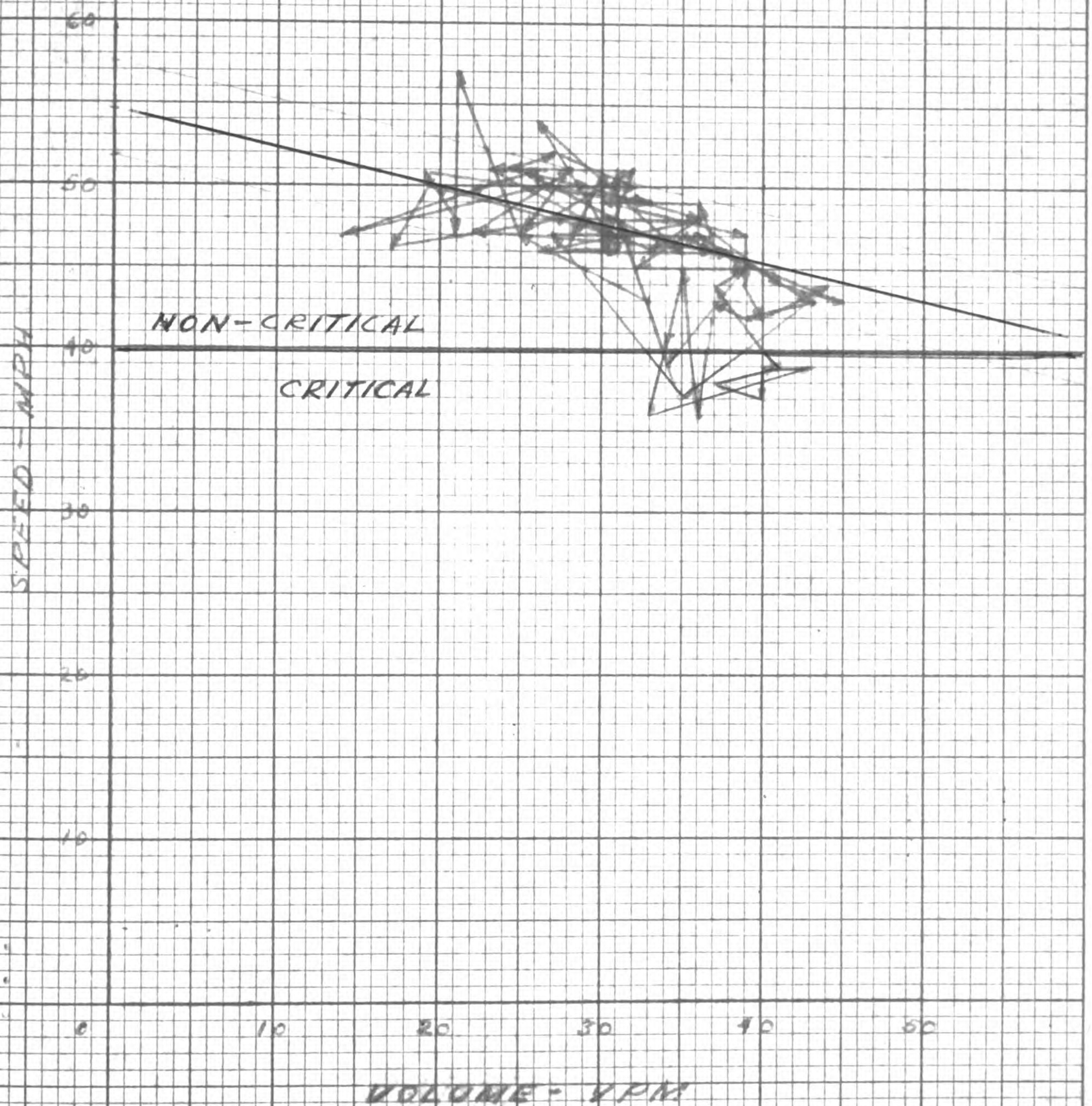


Figure 14: Chronological plot of data for Lane 2, March 28, 1958

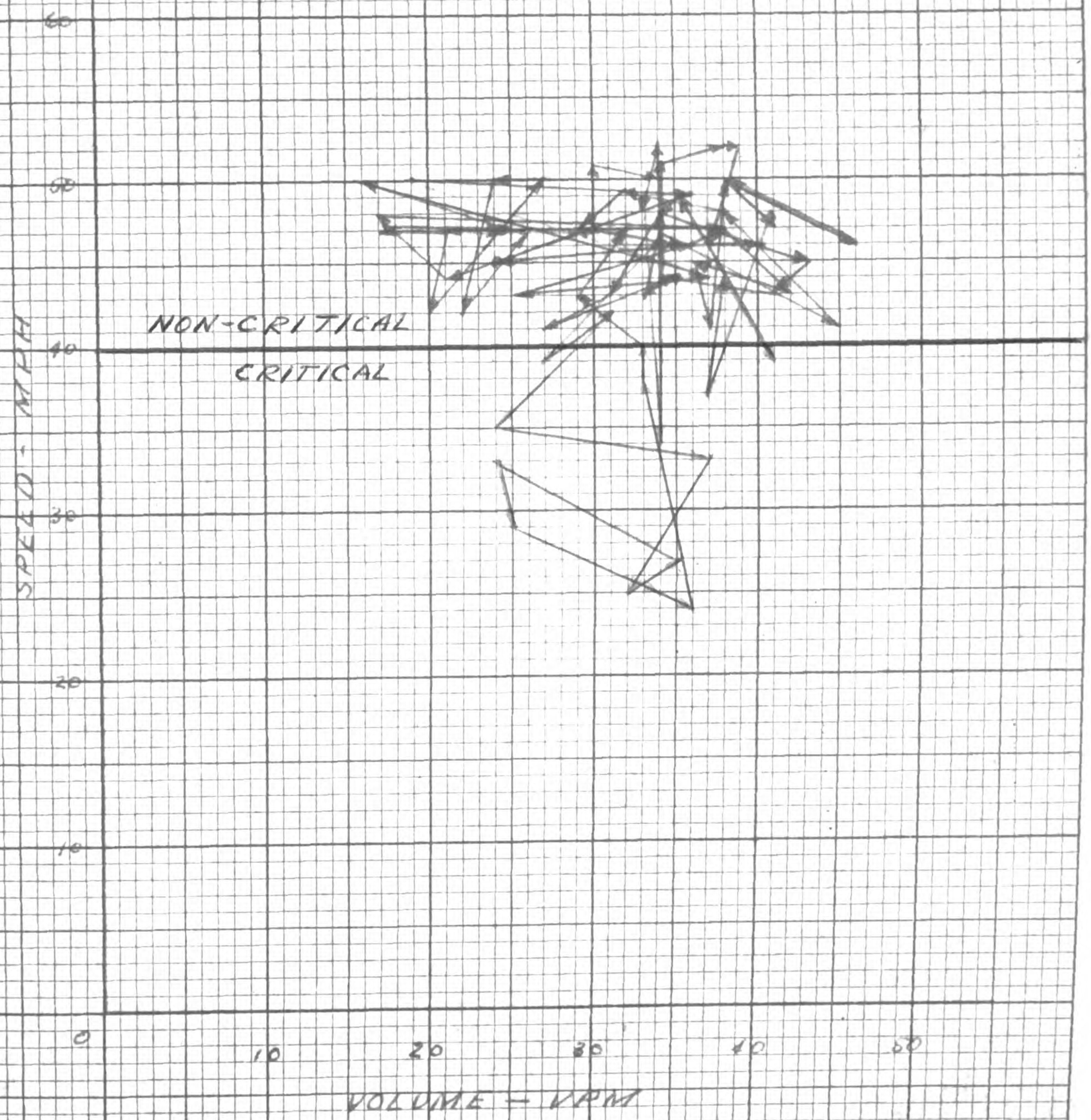
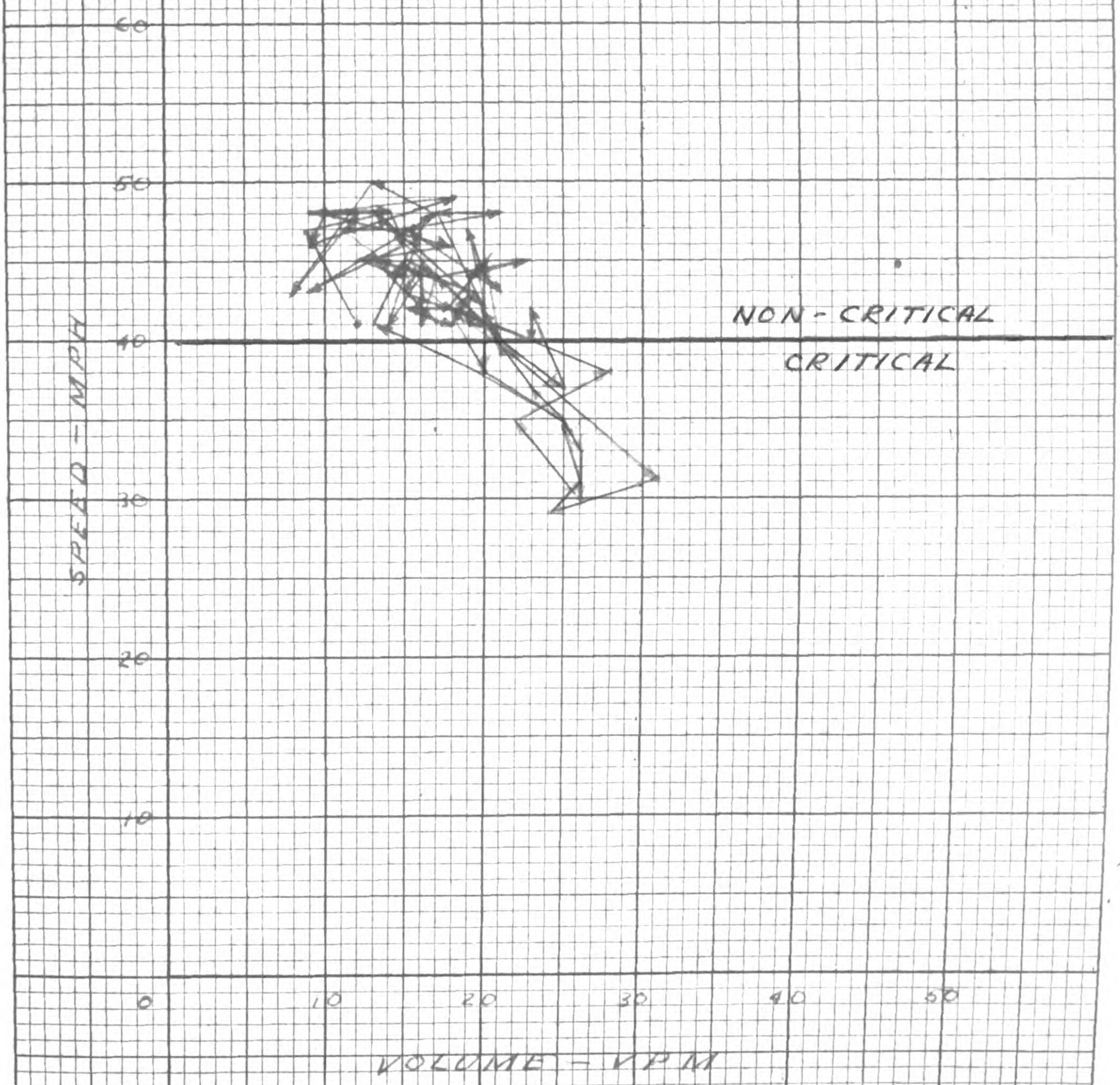


Figure 15: Chronological plot of data for Lane 3, March 28, 1958



Values for the slopes of the regression line for lane 1 were (-)0.17 and (-)0.13; for lane 2, (-)0.27 and (+)0.01; and for lane 3, (-)0.27 and (-)0.23. A calculation of the 95% confidence limits for lane 1 indicate that all the slopes lie within the limits except for the lane 2 Friday data which had a positive slope. These limits are $-0.32 < m < -02$ for lane 1 and $-0.40 < m < -0.06$ for lane 3. The significance of the confidence limits is that they show that the slopes of all but one of the regression lines lie within the 95% confidence limits.

It is the author's belief that the positive slope for the speed-volume relationship of lane 2 occurred by chance and not by a normal distribution of data. The trend of the line under normal circumstances has been that the slope of the regression is negative. This can be further substantiated because of the many factors which influence the relationship such as the increase in internal friction as volume increases; or as volume increases, density increases; or as density increases, speed decreases; and as speed increases, the desired headway increases. See Figure 14 for plot of data.

Therefore, excluding the lane 2 regression from this analysis, it can be said that all regression slopes are satisfactory and for lane 1 the slope is approximately -0.172, -0.231 for lane 2, and -0.252 for lane 3.

Each of the regressions were tested for linearity of relationships. The results of this test indicated at the 95% level there was no basis for rejecting the hypothesis that the relationship of speed-volume was linear. The results of the F-linearity test and the values of F at the 95% level are shown in Table 1. The results further indicate that there is some basis for saying that speed and volume are linearly related in the area of non-critical flow and not in the area of critical flow.

The factor which points out the previous statement is the chronological plot of speed-volume data for lane 1. This plot indicates quite forcefully the need to handle the critical flow by itself and not to group it with all the data with intent of trying to fit just one regression to it. Therefore, treating the data separately, we can say that the area of non-critical flow is linear and a tendency for critical flow to be non-linear.

The square of the correlation coefficient as determined do not offer too much help in explaining the variance of Y on X. Only 30% of the regression of Y on X is explained by the analysis. However, the null hypothesis that the population of speeds and volumes have zero correlation was rejected in all cases except lane 2 which had the positive slope. It is significant that there is some correlation even though the r^2 percentage for each analysis is relatively low.

The next items to be tested were the standard deviations for speed and volume and standard error of estimate. Standard deviations of volume for lane 1 range from 2.11 to 2.73 with a confidence interval of $1.63 < s_x < 2.84$. Standard deviations of volume for lane 2 range from 2.02 to 2.08 with a confidence interval of $1.96 < S_x < 2.79$. And, the

TABLE 1

SPEED-VOLUME RELATION STATISTICS

STATISTIC	LANE 1		LANE 2		LANE 3	
	THUR	FRI	THUR	FRI	THUR	FRI
N	30	47	63	60	49	56
$\bar{X}$	25.7	28.4	30.8	32.0	13.1	20.9
$\bar{Y}$	43.0	48.0	47.3	46.4	43.8	44.4
$m_{y/x}$	-0.179	-0.166	-0.231	0.010	-0.273	-0.231
C	47.6	52.7	54.6	46.1	48.6	49.2
F	0.355	0.935	1.07	1.06	1.04	0.463
$F_{.95}$	2.42	2.11	2.02	2.08	2.25	2.22
r	-0.621	-0.523	-0.632	0.026	-0.650	-0.514
$r_{.95}$	0.349	0.282	0.245	0.250	0.276	0.260
$S_{Y/X}$	0.82	1.98	1.37	1.31	1.11	1.20
$S_{Y/X}^2$	0.673	3.91	1.88	1.72	1.24	1.44
S_Y	0.607	0.866	0.845	1.02	0.80	0.876
S_X	2.11	2.73	2.30	2.56	1.91	1.94
CUTOFF	---	---	---	---	---	---

standard deviations of volume for lane 3 range from 1.91 to 1.94 with a confidence interval of $1.590 < S_x < 2.39$. The significance of this determination is that the confidence interval of lanes 1 and 2 indicates that the standard deviations of volume for these lanes are within the confidence limits of both lanes and that each standard deviation could be a valid estimate of the other. The confidence limits of lane 3 indicate that the estimate of standard deviation of either lane 1 or 2 would not necessarily indicate the standard deviation of lane 3. This, however, should follow as the average and range of volume values for lanes 1 and 2 are much different from those of lane 3.

The standard error of estimates range from 0.82 to 1.97 with all values being larger than their respective standard deviation for the Y direction. This would indicate that the actual deviation in the Y direction is less than the estimate of error about the regression in the Y direction. This would be a result of a greater variance in the X value or volume values.

One thing that should be noted in the chronological plot of speed-volume data is the fact that critical flow (congestion) does not always occur immediately following peak flow. There is some delay after peak non-critical flow before congestion occurs. This can be seen in Figure 8. Furthermore, peak minute volumes do not necessarily occur in non-critical flow but are more likely to occur during critical flow when speeds are between 35 and 40 mph for lanes 1 and 2 and 30 to 35 mph for lane 3. Figure 1 and Figure 4 also indicate that critical flow occurred on lanes 1 and 2 when the speeds were less than 45 mph and when volumes were from 30 to 40 vehicles per minute. Lane 3 had slightly different criteria in that the volume range was from 15 to 25 vehicles per minute.

Figures 15 through 19 illustrate the various flows of traffic. High and low volumes, high and low density, and high and low headways.

<u>Analysis of Speed-Density Relationships</u>

Speeds were designated as non-critical above 40 mph and as critical below 40 mph for the speed-volume and speed-density analysis. This cut-off line may be seen in Figure 20 which is a speed-density plot of lane 1.

In the chronological plot of the speed-density data, the use of the 40 mph line as the criteria for determining critical flow and non-critical flow resulted in a series of speed-density plots below 40 mph and above a density of 55 vehicles per mile. This series of points were treated as being points in critical flow as were those points below 40 mph in the speed-volume plot. This is the series of red lines shown on Figure 20.

In this plot of data for lane 1, as in the speed-volume plot, the range of speeds is an indication of the type of flow. The range in non-critical flow is six mph while the range in critical flow is 31 mph. The range of densities is 35 vpm for non-critical flow to 57 vpm for critical flow. Not only is the range in density higher in critical flow, but the average minute change in density is also greater. The average density increase in non-critical flow is 7.78 vpm while in critical flow it is 10.45 vpm. The average density decrease in non-critical flow is 7.64 vpm while in critical flow the decrease is 13.45 vpm. These are 34 and 76% increase in the density changes in critical flow. This means that at a critical density of 55 vpm, a

Figure 16: Illustrations of varying densities.

Light Volume, Light Density. Edsel Ford Expressway, Detroit. View is looking West, upstream.

High Volume, High Density. Edsel Ford Expressway, Detroit. Traffic is approaching critical flow conditions. View is looking West, upstream.

Figure 17: Illustrations of varying densities.

High Density, Low Volume, Low Speed. Edsel Ford Expressway, Detroit. Note the headway caused by the truck in relation to other vehicles.

High Density, Low Volume, Low Speed. Edsel Ford Expressway, Detroit. Note the bus in lane two and the headways caused by it in relation to other vehicles.

Figure 18: Illustrations of varying densities.

Density: Edsel Ford Expressway, Detroit. Note the density in lane two. Traffic is free flowing and average speed is approximately 45 MPH.

Denaity: Density in lanes two and three have reached an operating maximum and traffic is forced to reduce its speed to zero for short periods of time.

Figure 19: Illustrations of small headways at high speeds.

Headway: Edsel Ford Expressway, Detroit. Note the minimum headway between these cars, actually less than twenty feet, at speeds between 50 and 60 MPH. Time spacing of these cars is approximately 0.3 seconds.

Headway: Edsel Ford Expressway, Detroit. Note the minimum headway between these cars, actually less than 25 feet at speeds between 50 and 60 MPH in both lanes. Car in the center lane has little effect on spacing.

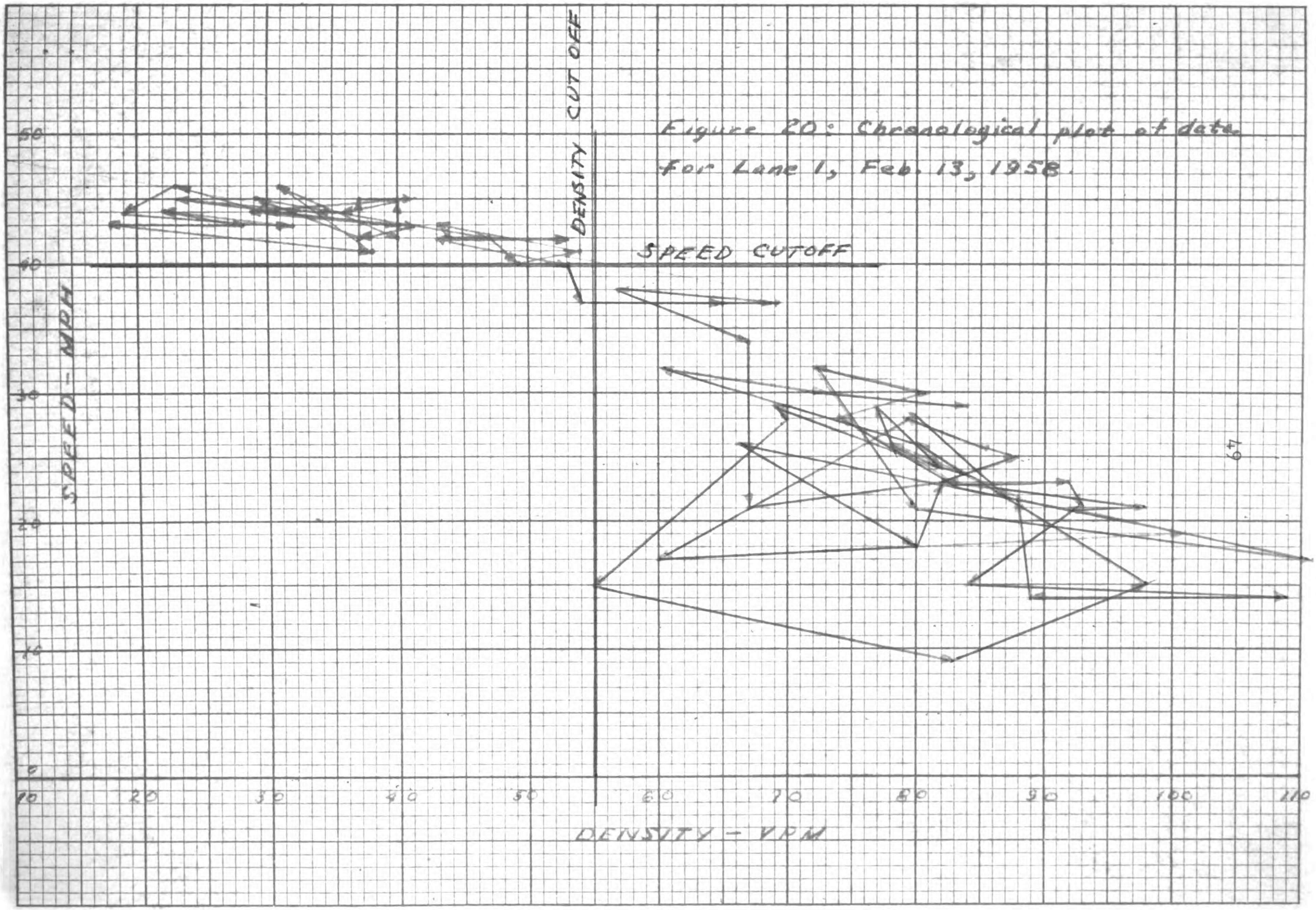


Figure 20: Chronological plot of data
For Lane 1, Feb. 13, 1958.

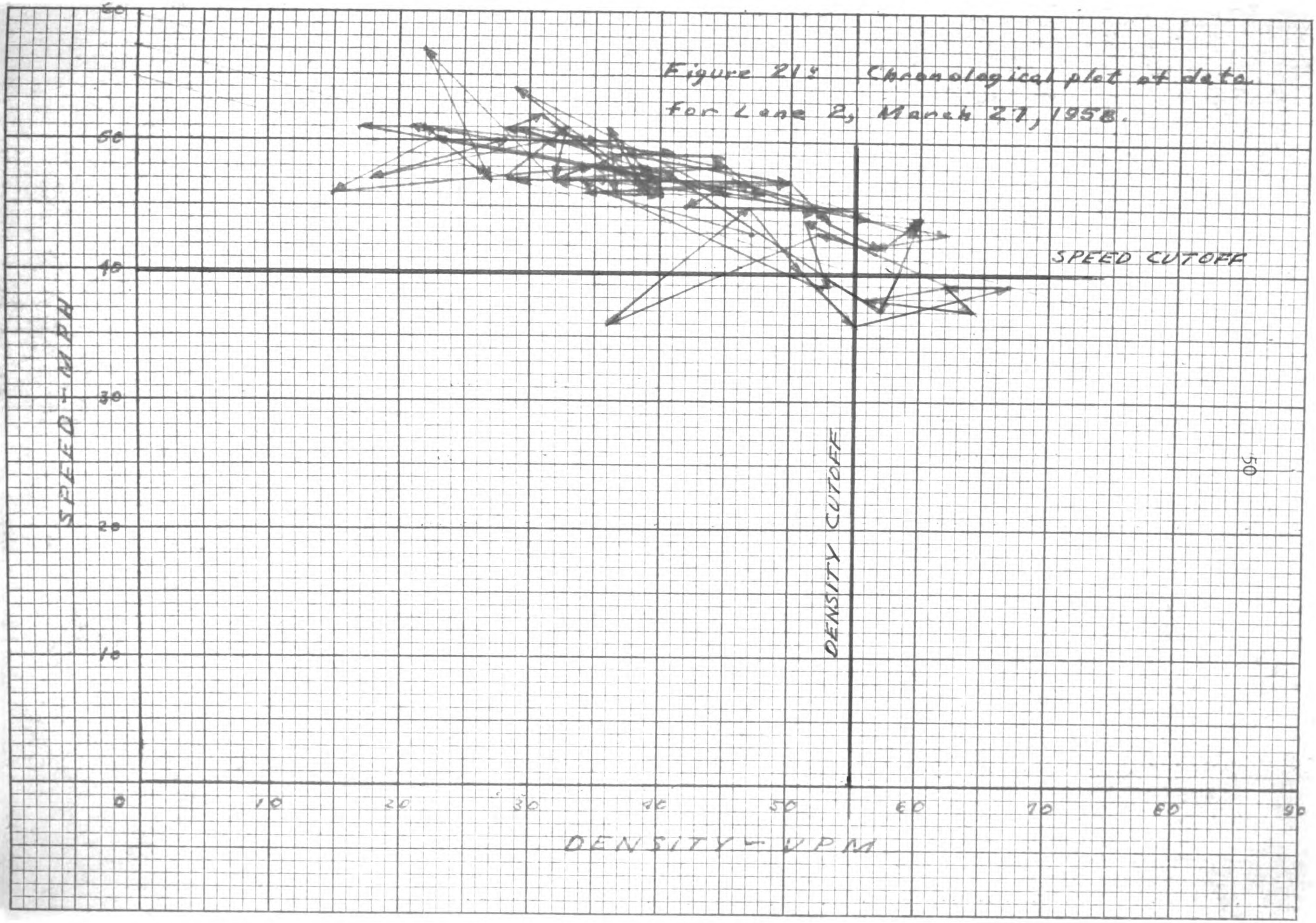
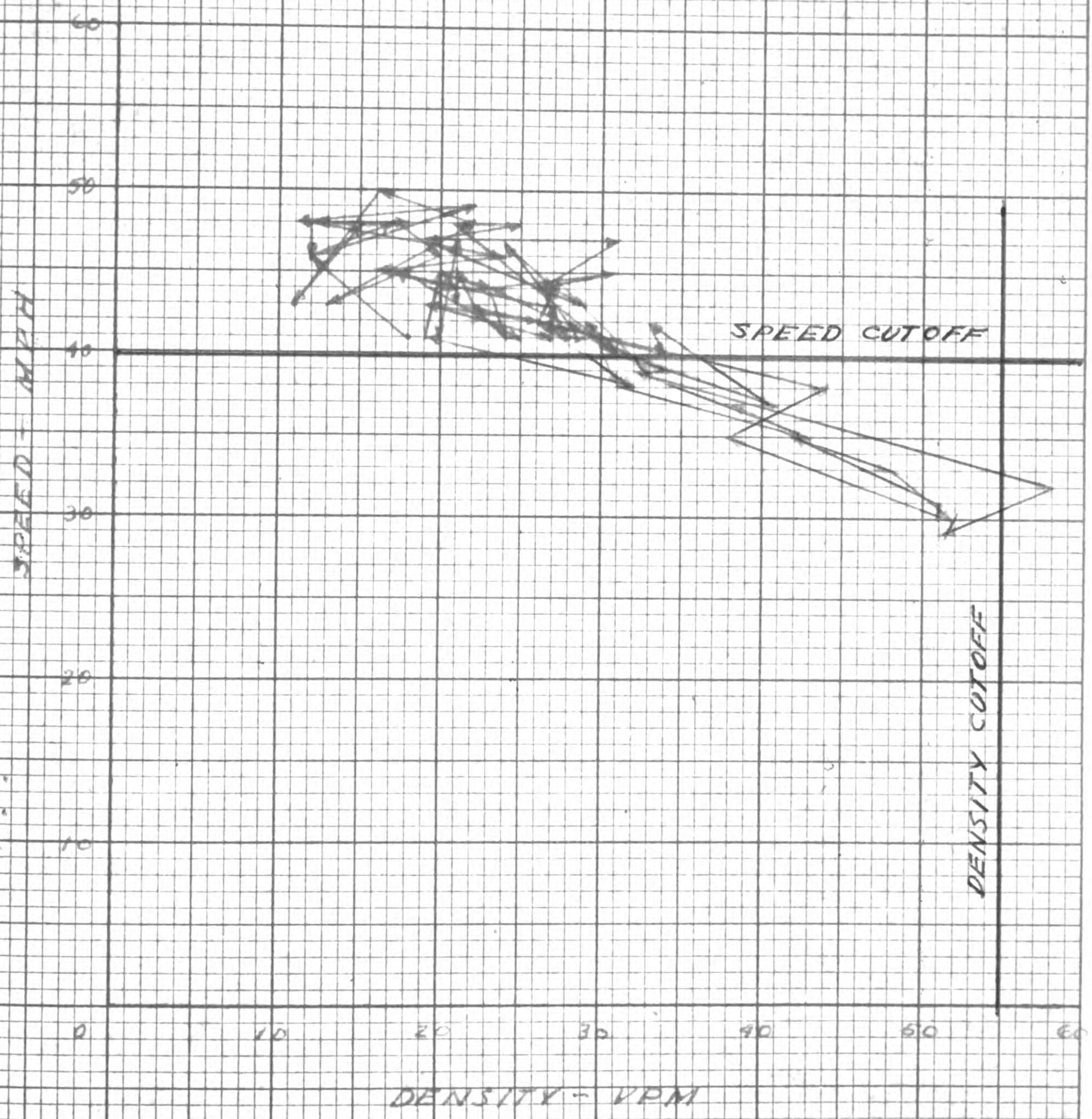


Figure 21: Chronological plot of data For Lane 2, March 27, 1958.

Figure 22: Chronological plot of data for Lane 3, March 28, 1958.



10.45 vpm increase reduces the headway per vehicle from 96 feet to 80.7 feet. This is almost a car length reduction in headway for the average density change. However, the maximum density change was from a density of 60 vpm to a density of 100.5 vpm--a 40.5 vpm density change. This is a decrease in headway from 88 feet to 52.7 feet. Such density changes could not take place at higher speeds--this occurred at 18 mph, due to drivers' desired time headway between cars. At 18 mph, the time headway is 1.99 seconds which is close to the two-second headway that is normally considered as average. The maximum density attained was 111 vpm or a headway of 47.6 feet.

It should be noted that the chronological plot of data indicates that maximum density does not occur with minimum speed.

It was pointed out in the speed-volume analysis that with small changes in speed there was little effect on traffic flow in the critical flow area. This is also true in the speed density relationships. It can be seen in Figure 21, for lane 2, that there are only a few points in the below 40 mph and above 55 vpm group. The speed change and density changes are similar to those of lane 1 for critical flow. Average density increases in critical flow are 7.1 vpm with decreases being 8.3 vpm. In non-critical flow, the average density increase is 9.2 vpm with average decreases being 7.8 vpm. This is not a significant difference between critical flow and non-critical flow.

A look at Figure 22, lane 3's speed-density plot, gives a somewhat different picture of the relationship. The critical speed line of 40 mph separates a series of points in the critical flow area that have densities less than the 55 vpm critical density of lane 1, in all instances except one where the density is 58.1 vpm. This series of points would indicate that for this lane critical density should be less than 55 vpm. The plotted data indicates that between 30 to 35 vpm is the critical density area. Furthermore, it can be seen in Figure 22 that fluctuations in density are greater in the area from 30 vpm and up than in the non-critical flow region.

For each of the three lanes, regression analyses were made in order to determine an analytical relationship between the speed and density characteristics.

Values for the slopes of the regression line for lane 1 are (-) 0.391 and (-) 0.159; for lane 2, (-) 0.271 and (-) 0.310; and for lane 3, (-) 0.323 and (-) 0.427. A calculation of the 95% confidence limits for lane 1 indicate that all the slopes lie within the limits. The limits are $(-)\ 0.719 < m < (-)\ 0.063$. Calculations for the other lanes give limits which also encompass all of the calculated slopes. This would imply, then, that any slope could be an estimate of the slope of the regression line for the speed-density relation. See Table 2 for these and other statistics.

One thing that is pointed out in the regression calculations is the data for lanes 1 and 3 yielded a rejection in the F-linearity test when all densities were used. Densities went as high as 105 vpm in lane 1 and 72 vpm for lane 3. A cutoff at 69 vpm was used for lane 1's

TABLE 2

SPEED-DENSITY RELATION STATISTICS

STATISTIC	LANE 1		LANE 2		LANE 3	
	THUR	FRI	THUR	FRI	THUR	FRI
N	31	44	74	74	73	73
$\bar{X}$	36.8	35.1	42.0	45.3	26.7	25.7
$\bar{Y}$	42.9	43.3	45.8	43.8	45.6	42.4
$r_{Y/X}$	-0.142	-0.150	-0.271	-0.310	-0.323	-0.427
C	43.1	53.6	57.2	57.8	54.2	53.8
F	0.122	1.55	0.481	1.255	1.79	1.34
$F_{.95}$	2.37	1.82	2.03	1.66	2.08	2.15
r	-0.594	-0.614	-0.774	-0.593	-0.746	-0.871
$r_{.95}$	0.349	0.223	0.228	0.223	0.228	0.223
$S_{Y/X}$	1.272	1.621	1.20	4.83	3.553	1.35
$S_{Y/X}^2$	1.615	3.32	1.44	23.3	12.62	1.82
S_Y	0.662	0.713	1.46	2.42	1.62	1.58
S_x	3.22	2.08	2.15	2.30	3.74	1.95
CUTOFF	55	55			55	

second trial regression calculation. This also was rejected by the F-linearity test. A third trial calculation was made with a cutoff at 55 vpm. The F-linearity test was accepted this time. The first trial regression calculation for lane 3 was made using all densities but was rejected. The next trial was made at a cutoff of 55 vpm, which was accepted by the F-linearity test.

Because lane 1's Thursday and Friday data were quite similar, it was believed that the Thursday data should also give some indication as to the possible critical density of 55 vpm. In the first calculation of all the statistics for lane 1 on Thursday, all of the data were used. The resulting F-linearity test was acceptable--1.234 calculated with the 95% level of acceptance being 1.80. The correlation coefficient calculated to be $r = -.907$ with $r^2 = .820$. When the data was re-used with a cutoff at 55 vpm, the results were different. The F-linearity test yielded an acceptable value = 0.1215 with a 95% level of acceptance equal to 2.37. This result indicates that this data has a greater degree of linearity than when all data were used. This fact helps to point out that there is some change in the relationship of speed and density for values above 55 vpm. However, the statistic coefficient of correlation, which explains the variation of Y on X is less for the calculations using the 55 vpm cutoff than for the calculations using all the data. The coefficient of correlation for 55 vpm cutoff is $r = -0.694$ with $r^2 = 0.481$ and, for all data, $r = -0.907$ with $r^2 = 0.822$. These results indicate that the variance of Y on X is explained more where all data were used than where data were cut off at a density of 55 vpm. The important thing that is indicated by the calculations is that there is some reason to believe that above 55 vpm the relationship of speed and density changes. These facts, along with the analysis of the chronological plot, give a good indication that the critical area of density is 55 vpm and greater.

The next items tested were the standard deviations of speed and density. The standard deviations of speed for lane 1 were 0.662[1], 3.542,[2] and 0.713 with confidence intervals of $0.525 < S_y < 0.836$, $3.05 < S_y < 4.22$, and $0.538 < S_y < 0.905$, respectively. The first two data are for Thursday. The first of these two is for the cutoff at 55 vpm and is the reason for being so much smaller. The third datum is for Friday and is similar to Thursday as it was calculated for a cutoff of 55 vpm also.

The standard deviations of speed for lane 2 speeds are 1.46 and 2.42 with confidence intervals of $1.25 < S_y < 1.75$ and $2.03 < S_y < 2.89$, respectively. No cutoff of density was used for these calculations.

The standard deviations of lane 3 speeds are 1.617 and 1.58 with confidence limits of $1.39 < S_y < 1.93$ and $1.35 < S_y < 1.89$, respectively. These are similar even though the first datum is a product of a cutoff at 55 vpm. This is because the cutoff involved only one datum being cut off.

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[1]Standard deviations for lane 1 calculations with densities cut off at 55 vpm.

[2]Standard deviations for lane 1 calculation using all data.

The standard deviations of density offer little in the way of explaining the relationship of speed and density and are, therefore, not covered in this analysis.

The standard error of estimates for the speed-density relation range from 1.20 to 4.83. This would indicate a greater scattering of points and more than likely less correlation of data.

The correlation coefficients were calculated for all data as were r^2 values which indicate the per cent of variance of Y explained by X. Values of r range from 0.593 to 0.907 with corresponding r^2 values of .351 and .821. The latter values indicate that from 35 to 82% of the variation in Y can be explained by the variation of X. This probably is a result of the greater scattering of data. When density data were cut off at 55 vpm, the per cent of variation of Y on X was less than when all data were used. Approximately 50% of the variation was explained with a cutoff at 55 vpm but up to 80% could be explained using all data. Table 3 gives all the values of r and also the 99% level of acceptance for r. The minus sign on all the coefficients indicate a negative correlation between speed and density. This means that as density increases, speed decreases.

Analysis of Volume-Density Relationships

In the two previous portions of the analysis, the attempt was made to explain the variation of speed by variations of volume and of density. This analysis will attempt to explain the variation of volume by the variation of densities.

The chronological plot of data was made using 55 vpm as the line between critical and non-critical flow. Figures 23, 24, and 25 are plots of volume-density data for lanes 1, 2, and 3, respectively.

In the plot of volume versus density for lane 1 Thursday data, the range and fluctuations of density are again an indication that the flow of traffic has a different character for densities above 55 vpm. The range in density for non-critical flow is 36 vpm and in critical flow 57 vpm. The average increase and decrease in density changes are 10.45 and 13,45 vpm, respectively, for critical flow. For non-critical flow, the averages are smaller with values of 7.78 and 7.64 vpm, respectively. The difference in these changes gives some indication that the flow characteristics are different from normal flow. See paragraph three of the Analysis of Speed-Density Relationships for additional density analysis. The ranges of density and the apparent average increases and decreases may be seen in Figure 23 which helps to emphasize this fact of different flow characteristics prevailing in critical flow. The figure also shows the relative linearity of volume versus density up to 55 vpm.

The range in volumes is not too different in critical flow (30 vpm) from non-critical flow (24 vpm), nor are the average volume increases and

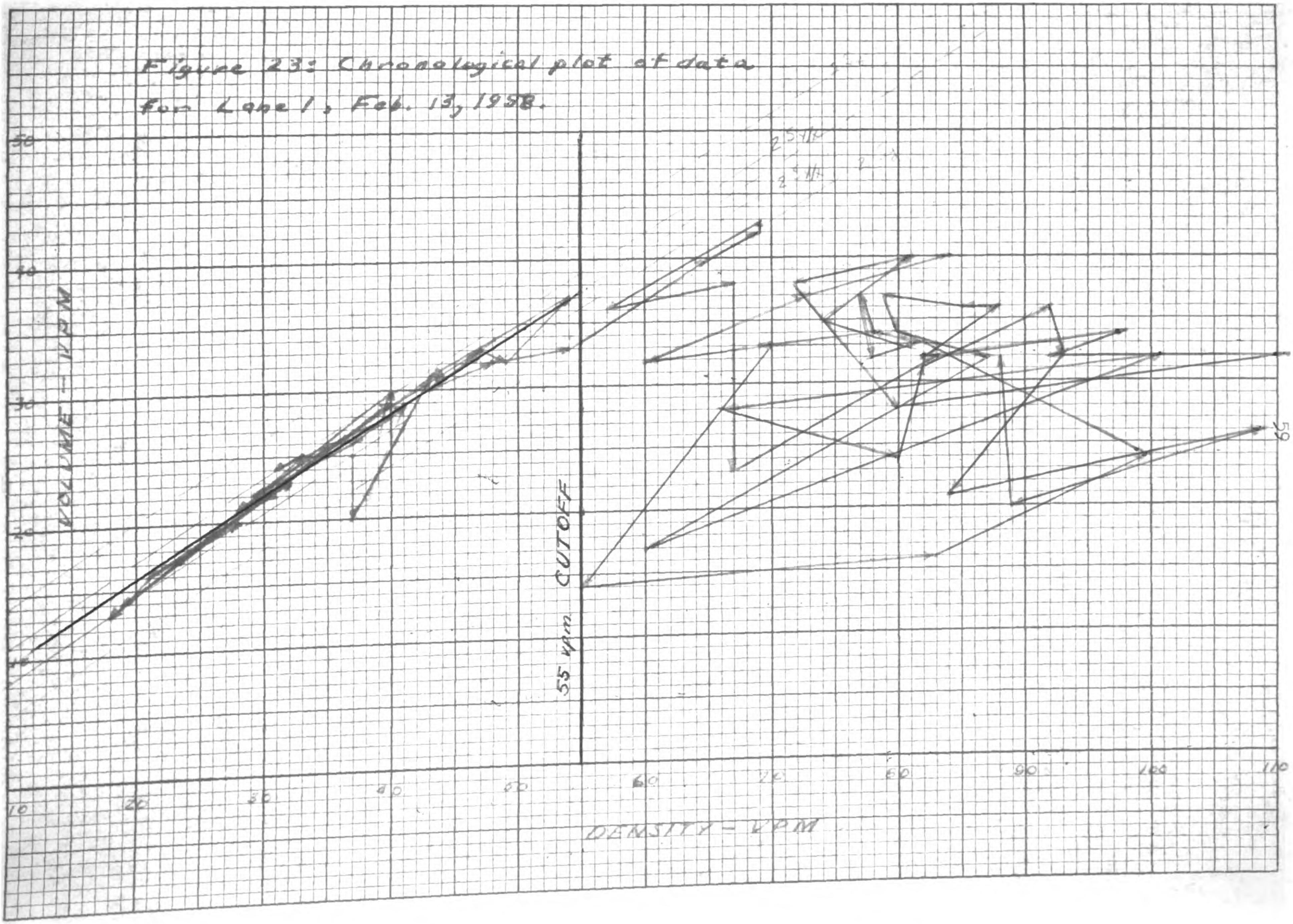


Figure 23: Chronological plot of data For Lane 1, Feb. 13, 1958.

decreases widely different. These average increases and decreases are, respectively, 4.18 and 6.93 vpm for critical flow. The one thing that is pointed out here is that average decreases are larger than average increases in critical flow while average increase in critical flow are larger than average decreases in non-critical flow. This may be caused by the greater friction in congestion. Another factor may be the density characteristic differences of non-critical flow and critical flow. The non-critical flow may have the ability to absorb these changes by drivers accepting smaller headways. However, in critical flow, there may not be enough headway to absorb the desired increase in speed or increase in density.

Lane 2's volume-density plot does not show as vividly the relationship as lane 1's plot, but there is a trace of the general tendency. Figure 24 shows the plot of lane 2 and it can be seen that above a density of 50 vpm there is a tendency for the character of flow to change. The average increase in critical flow drops two vpm from non-critical flow while average decreases in density increase by half a vehicle per mile. It must also be pointed out here that to attain an acceptable F by the F-linearity test, densities had to be cut off at 55 vpm.

Figure 25 depicts the plot of volume-density data for lane 3. The densities, used in the calculation of linearity of data, had to be cut off at 45 vpm in order to attain an acceptable F-linearity test. A close visual inspection of the plotted data indicates a gentle curve primarily above 30 vpm. It is felt that a higher degree of linearity

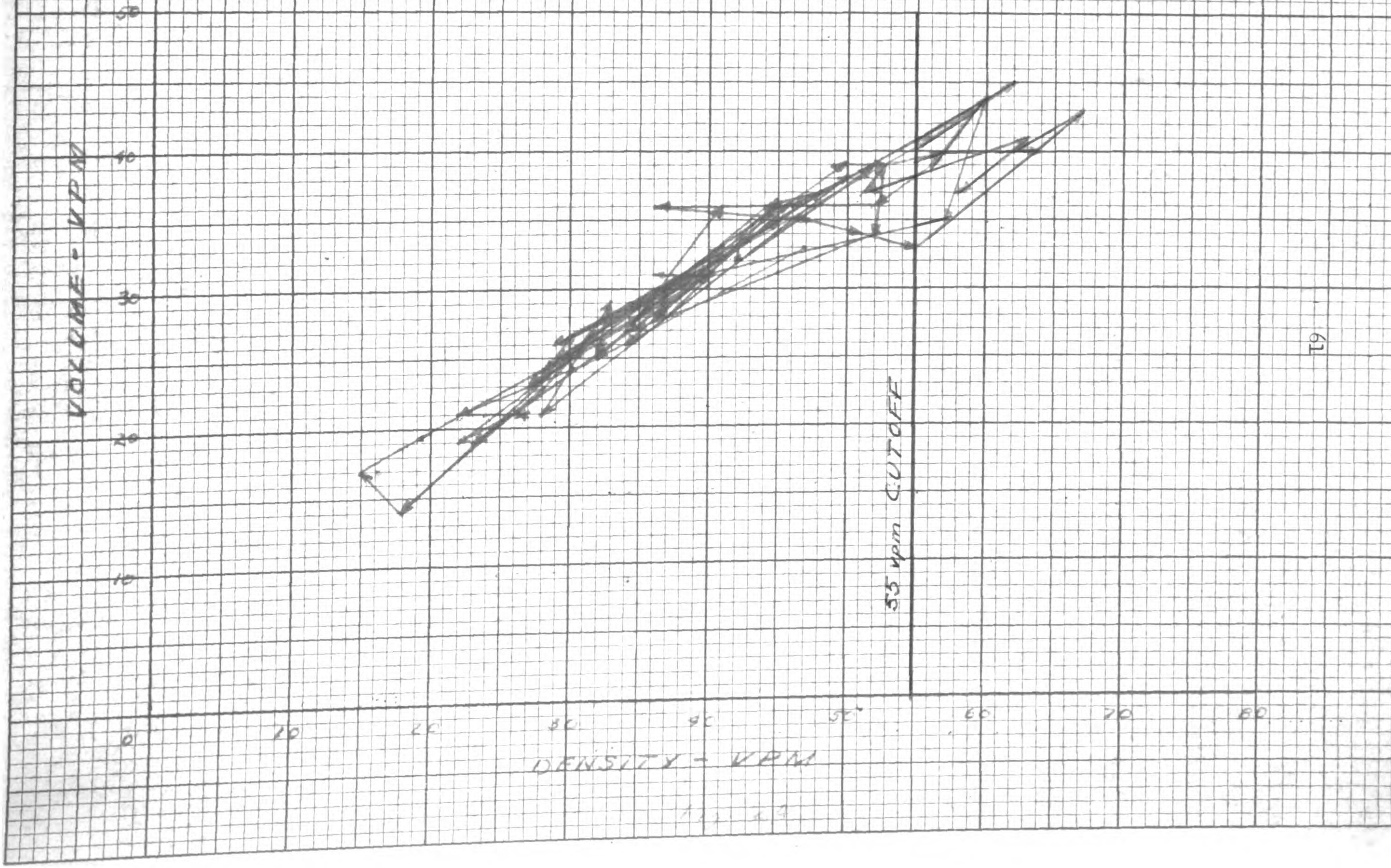


Figure 24: Chronological plot of data for Lane 2, March 27, 1958.

Figure 25: Chronological plot of data for Lane 3, March 28, 1958.

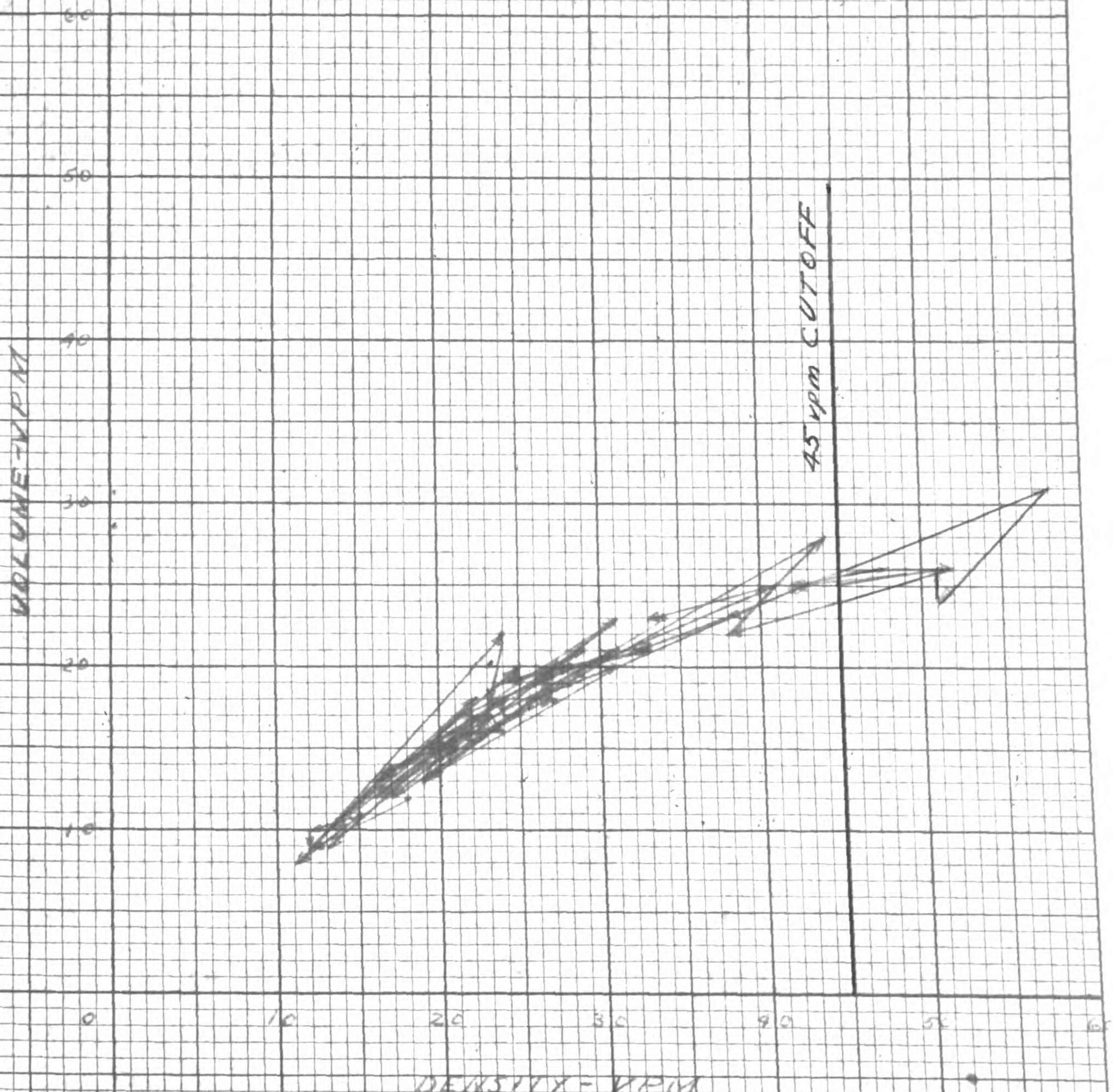


Fig. 25

could be attained if the density were cut off between 30 and 35 vpm. This belief is also supported by the analysis of speed-volume data. Another factor affecting or creating this lower density is the fact that most of the trucks use this lane.

For each of the three lanes, regression analyses were made in order to determine an analytical relationship between volume and density characteristics.

Values for the slopes of the regression line for lane 1 are 0.314 and 0.605; for lane 2, 0.534 and 0.574; and for lane 3, 0.497 and 0.570. The calculations for the 95% confidence limits of slope values are for lane 1, $0.106 < m < 0.602$ and $0.419 < m < 0.971$; for lane 2, $0.505 < m < 0.603$ and $0.440 < m < 0.708$; and for lane 3, $0.443 < m < 0.561$ and $0.497 < m < 0.621$. The first set of limits for lane 1 is for data taken when the facility was fully congested and with a cut-off of 55 vpm. The other limits are for data taken when the facility was operating under peak conditions but not fully congested and, in most cases, with a density cutoff of 55 vpm. The exception may be noted in Table 3. When lane 1 Thursday data was recalculated for a cutoff of 55 vpm, the slope value was determined as being 0.612. The confidence limits for the slope values hold within a lane but do not necessarily include all other data. The value of (c) also fits this last statement in that the Y intercepts within lanes are very close, but this is not true when comparing lane against lane. Therefore, for lane 1 a regression equation is $V = 0.612\,D + 3.7$, for lane 2 it is $V = 0.534\,D + 7.6$, and for lane 3 it is $V = 0.497\,D + 3.3$.

TABLE 3

VOLUME-DENSITY RELATION STATISTICS

STATISTIC	LANE 1		LANE 2		LANE 3	
	THUR	FRI	THUR	FRI	THUR	FRI
N	31	45	61	70	63	63
$\bar{X}$	36.2	35.4	30.2	43.3	25.0	24.2
$\bar{Y}$	25.9	27.7	29.9	32.0	15.8	17.0
$m_{y/x}$	0.612	0.695	0.504	0.574	0.497	0.559
C	3.7	3.1	7.6	7.1	3.3	3.5
F	0.874	1.563	1.855	1.544	1.325	1.570
$F_{.95}$	2.32	2.35	2.28	2.04	1.97	2.00
r	0.956	0.913	0.955	0.961	0.941	0.957
$r_{.95}$	0.325	0.291	0.250	0.234	0.237	0.237
$s_{y/x}$	0.628	0.928	0.771	1.280	0.804	0.457
$s_{y/x}^2$	0.394	0.847	0.590	1.635	0.646	0.209
s_y	2.10	2.54	1.99	2.55	1.67	1.57
s_x	1.97	2.12	1.96	2.29	3.08	2.68
CUTOFF	55	55	55	75	45	45

The regression analysis did indicate in five of the six analyses that the use of all data would not give an acceptable F-linearity test. Lanes 1 and 2 indicated a density cutoff of 55 vpm while lane 3 data required a cutoff at 45 vpm. Lane 1 Thursday data and lane 2 Friday data had density cutoffs at 65 and 75 vpm, respectively. It was believed that a higher degree of linearity could be attained if the cutoff were lowered to 55 vpm. Recalculations of data were made for this assumption and the results proved it in all respects. The calculated F values were 0.8[illegible]4 for the 55 vpm cutoff and 1.21 for the 65 vpm cutoff. This comparison indicates the data of 55 vpm cutoff to be more linearly related than for the data of the 65 vpm. Secondly, the coefficient of correlation for the 55 vpm was 0.955 as compared to the 0.841 for the 65 vpm. This higher coefficient of correlation gives an r^2 value equal to 0.913. It can be said then that 91% of the variance of volume is explained by the variance of density. Thirdly, the standard error of estimate is smaller for the 55 vpm than the 65 vpm--approximately one-third as large. These facts indicate that the 55 vpm cutoff would be a better choice for the line between non-critical flow and critical flow.

Due to lane 3's relative similarity of problem on a cutoff line it is believed that similar recalculations with a cutoff at 35 vpm would result in an increase in linearity and correlation of data. The chronological plot, which also helps to point this out, was discussed earlier in this analysis.

As in the previous analyses, the coefficients of correlation were determined for all data. These values are given in Table 3 along with the 97% acceptance value for r and the values of r^2. These values ranged from a low of 0.411 for lane 1 to a high of 0.947 for the same lane. This means that for the Thursday data only 41% of the variance of volume could be explained by the variance of density. This was with a cutoff of density values at 65 vpm. However, with a cutoff of 55 vpm, the r^2 value becomes larger--0.913 or 91% of the variance of the volume being explained by variance of density. This is close to the calculated value of 0.947 or 95% for lane 1's Friday data which also used a cutoff of 55 vpm. The only low squared correlation coefficient is for lane 2 Friday data, but this was with a density cutoff of 75 vpm and not the 55 vpm cutoff used for most other data.

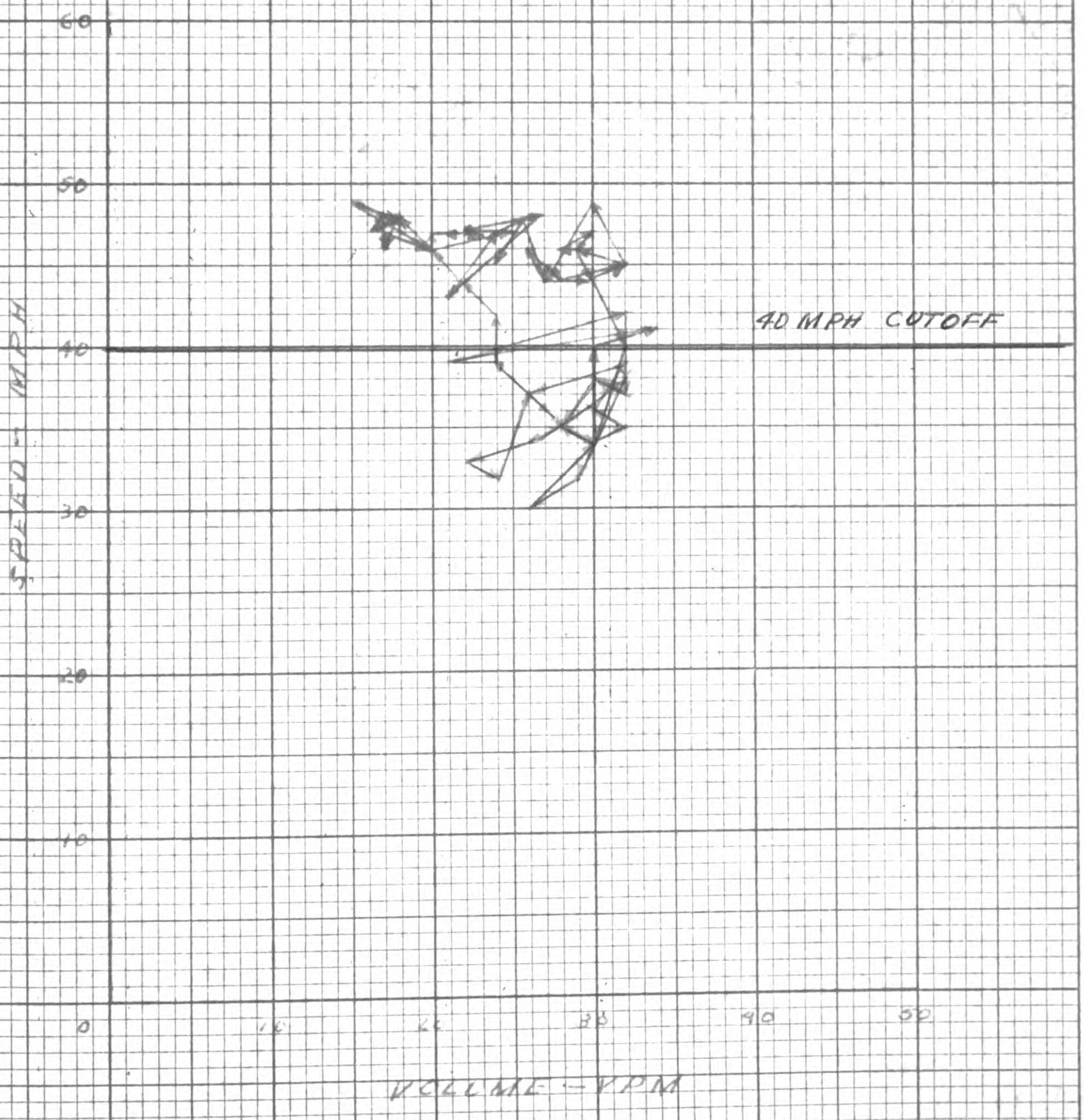
The standard deviations of either S_y (volume), S_x (densities), or the standard error of estimate $S_{y/x}$ (volumes/densities) do not aid in explaining the relation of volume versus density. However, it does give an indication concerning the cutoff value. When the cutoff values are above 55 vpm, the standard error of estimate is greater than 1.0 while with cutoffs of 55 vpm or less, the standard error of estimate values are less than 1.0.

Combined Average Data Analysis

The combined lanes analyses were made in the same general manner as the individual lane analysis. In general, the results of the analysis and general trends are little different from those of the individual lane results.

The chronological plots of data indicate the same cutoff lines for speed and density. These are shown in Figures 26, 27, and 28. The statistics for the analyses are summarized in Table 4.

Figure 26: Chronological plot of data for lane averages.



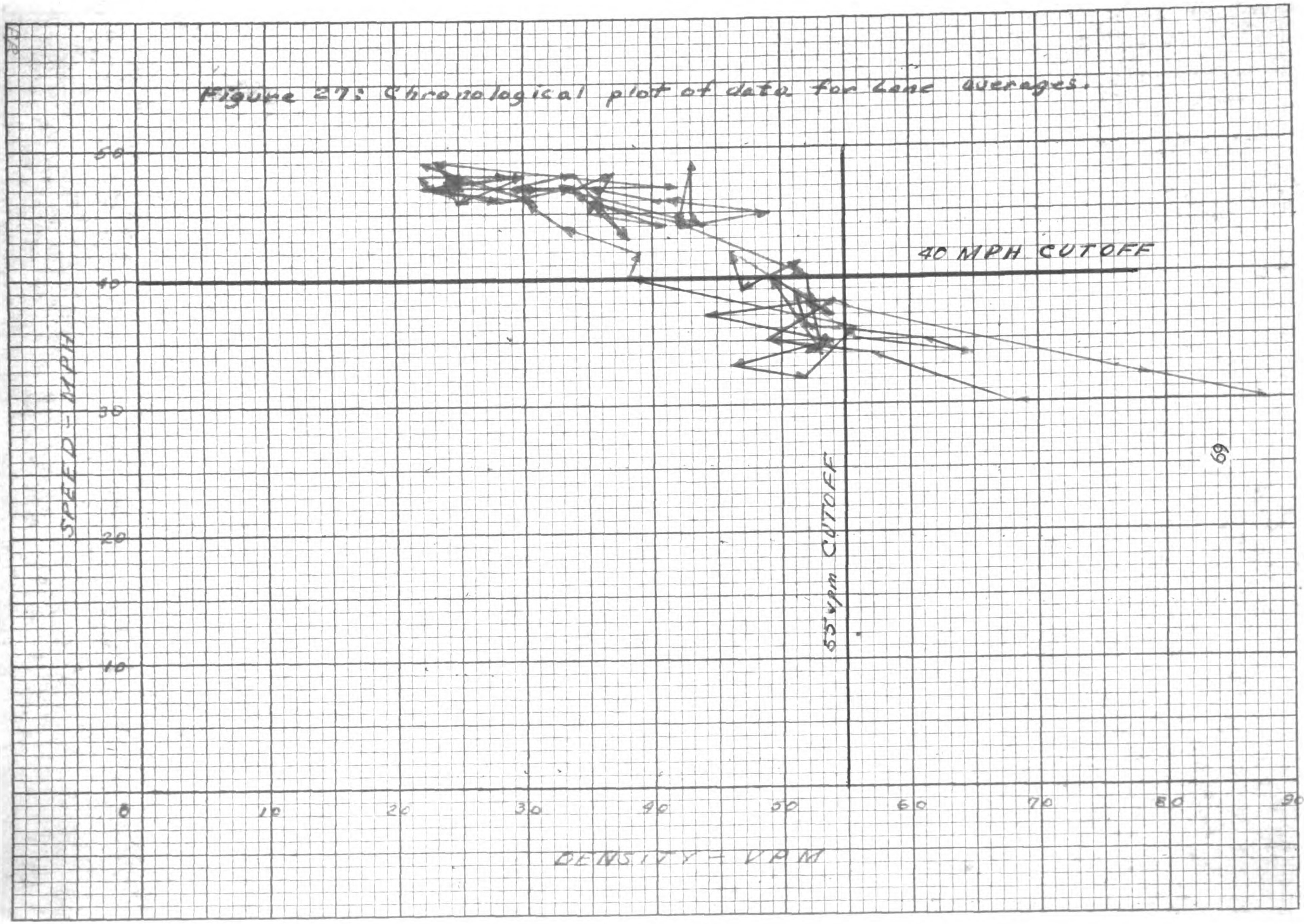


Figure 27: Chronological plot of data for Lane averages.
40 MPH CUTOFF
55 vpm CUTOFF
SPEED - MPH
DENSITY - VPM
0
10
20
30
40
50
60
70
80
90

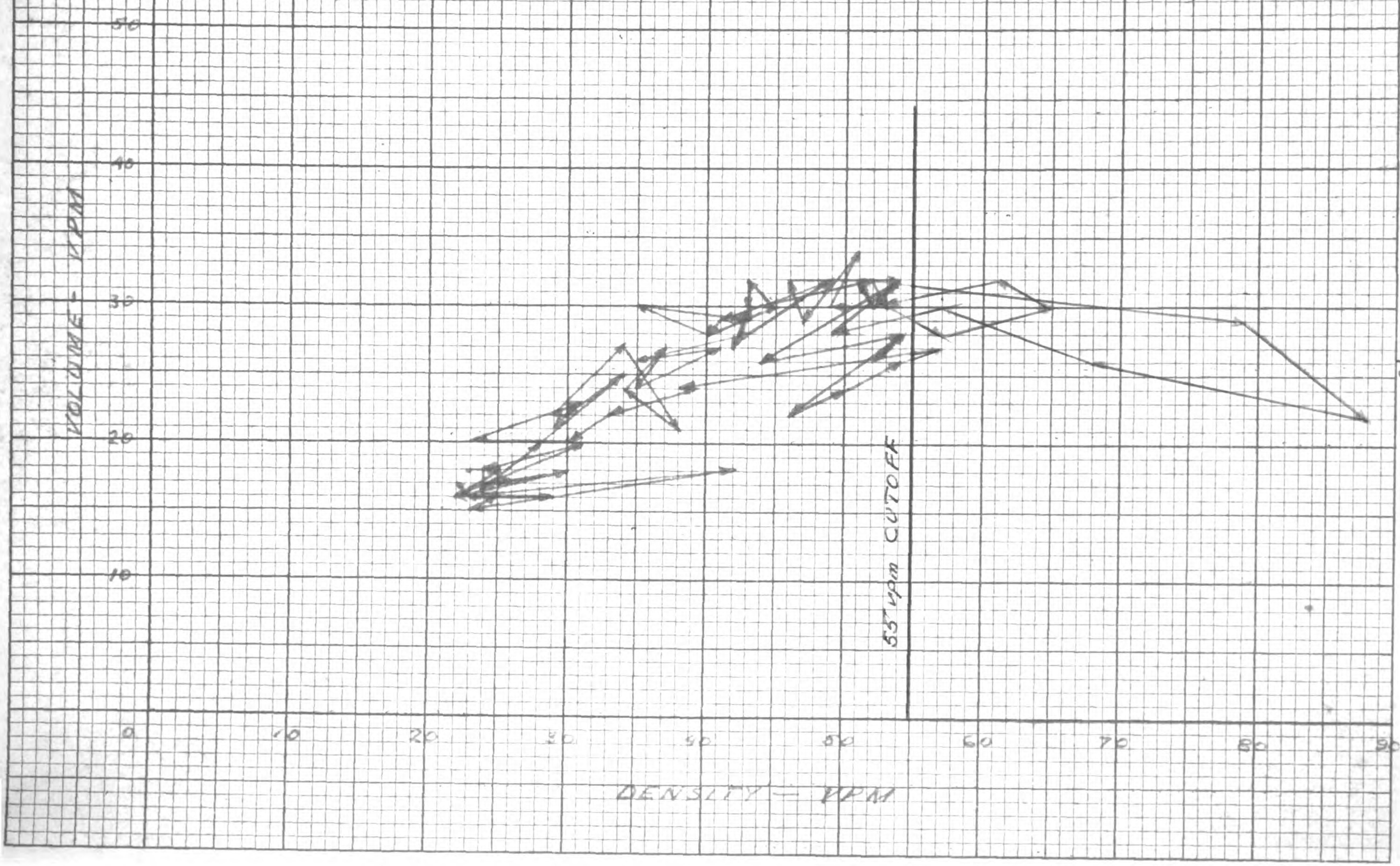


Figure 28: Chronological plot of data for Lane averages.

TABLE 4

ALL FUNDAMENTAL RELATION STATISTICS - AVE.

STATISTIC	SPEED/VOLUME		SPEED/DENSITY		VOLUME/DENSITY	
	THUR	FRI	THUR	FRI	THUR	FRI
N	37	51	74	62	52	66
$\bar{X}$	21.7	23.1	43.8	33.2	35.6	37.8
$\bar{Y}$	23.0	24.3	42.1	43.6	23.0	24.3
$m_{y/x}$	-0.275	-0.227	-0.363	-0.351	0.472	0.459
C	50.2	51.1	50.2	57.4	5.7	7.0
F	0.012	1.17	1.483	0.94	1.125	0.894
$F_{.95}$	2.36	2.32				
r	-0.674	-0.593	-0.896	-0.793	0.072	0.060
$r_{.95}$	0.325	0.273	0.228	0.245	0.271	0.241
$S_{y/x}$	1.114	1.404	3.31	3.45	1.056	1.125
$S_{y/x}^2$	1.236	1.97	10.93	11.83	1.112	1.265
S_y	0.78	0.713	1.98	1.62	1.93	1.03
S_x	1.91	1.37	2.91	3.94	3.57	2.10
CUTOFF	55	55	--	55	--	--

CHAPTER V

SOME FUNDAMENTAL RELATIONSHIPS OF TRAFFIC FLOW ON A FREEWAY

The purpose of this research was to determine the fundamental relationships of speed versus volume, speed versus density, and volume versus density. The method of analysis used to determine these relationships centered on the use of a chronological plot of data and the F-linearity test.

Relationship of Speed versus Volume

Through a combination plot of data and statistical testing, it was determined that a cutoff line for speeds at 40 mph would be used to differentiate between non-critical flow (above 40 mph) and critical flow (below 40 mph). The F-linearity test was used throughout the study in determining the cutoff lines.

The chronological plot of data indicated a greater range in speeds in critical flow than for non-critical flow--31 mph versus six mph--when a lane develops congestion (eg. lane 1). This plot also suggests that the data in the critical area should be handled separately and not with the non-critical data. If the data were not handled separately, the results could be quite misleading. Furthermore, the data in critical flow offers little in the explanation of the relationship because it is erratic and inconsistent. The one thing that is pointed out in all the plots is that congestion does not necessarily occur immediately following maximum volume in non-critical flow. This is somewhat contrary to the relations that have been purported by the parabolic curve of the speed-volume plot. The parabolic curve of Figure 4 implies that maximum volume occurs just before congestion or critical flow. This is not so according to the chronological plots shown in Figures 12 thru 15. It is believed that this is caused by the so-called 'Wave Action' of slowdowns which have occurred downstream from the study point.

The two plots of speed-volume data for lanes 2 and 3 are not as emphatic about the relationship as the lane 1 plot, but they still tell a similar story. All lanes indicate average speed increases are larger in non-critical flow while average speed decreases are larger in critical flow. The reason for this is that in non-critical flow, densities are low and can accommodate the speed increases while critical flow is at a density where there is little room for accommodating large changes in speed. However, speed decreases in non-critical flow are smaller because there is more maneuver area while a decrease in critical flow limits the maneuver area too much and causes larger decreases, thus producing an overall average decrease larger than that of non-critical flow. This relation also applies to volume increases and decreases.

The linear relation of speed and volume in non-critical flow was statistically proven quite acceptable by the F-linearity test, even though the correlation coefficients were low (30%) in most instances. This low correlation is quite surprising because of the relatively high correlation (90%) between speed and density and volume and density. The regression relation of speed versus density in non-critical flow is negative in that as volume increases, speed decreases. In critical flow, the trend is for speed to decrease with decreases in volume, but this is with a substantial increase in density.

The regression equation for the average of the three lanes is S = -0.275 V + 50.2. Greenshields indicated a regression of S = -0.009 + 43 which is quite different from this analysis. His slope does not fit the 95% confidence limits either and his Y intercept is 17% less. It would also appear that the slope of Greenshields' regression equation would not test significantly non-zero which would indicate the line to be almost horizontal and not in accordance with the general trend.

The calculations do not indicate too much difference between lane relationships, but the chronological plots do indicate a difference between lane 3 and lanes 1 or 2. A comparison of Figure 15 with Figures 12 and 13 will show the difference quite readily.

Relationship of Speed versus Density

The cutoff lines for the speed-density relation were determined by both the F-linearity test and the chronological plot of data. The density cutoff line is 55 vpm for lanes 1 and 3 and not specified for lane 2.

The range in speeds shown in the chronological plot of data are the same as for the speed-volume analysis. This plot of lane 1 also suggests that the data in the critical area should be handled separately. This, however, could not be verified by the F-linearity test which did not reject the hypothesis of linearity of data when all densities were used. Lane 2 did not substantiate the rejection of linearity at densities above 55 vpm. Lane 3, however, is linear only for values less than 55 vpm. Lane 1 calculations for all data yielded an acceptable F-linearity test with a -.927 correlation coefficient. However, the 55 vph cutoff calculation produced greater linearity but with a smaller correlation coefficient, -0.694. The standard error of estimate is also smaller which means a less scattering of data--1.272 compared to the high 6.97 for all data.

The chronological plots help to substantiate the cutoff lines of both speed and densities. The average increases and decreases of speed and density further support the position of the cutoff line as they did in the speed-volume relation.

The linear relation of speed and density in non-critical flow was statistically proven quite acceptable by the F-linearity test. The correlation coefficients are larger for this relationship than for the speed-volume, which would also indicate a better correlation of data. The r^2 values indicate that up to 70% of the variance of speed can be explained by the variance of density.

The regression equation for the average of the three lanes is $S = -0.363\ D + 53.0$. Greenshields indicated a regression of $S = \frac{43}{1 + 0.94\ D}$ which gives a slight curve to the line, but he then suggested that a linear relation would be satisfactory with only slight error. Olcott, however, found a speed-density regression for tunnel flow of $S = -0.473\ D + 54.7$ for the high-speed lane and $S = -0.223\ D + 29.6$ for the slow-speed lane with high percentage of commercial vehicles. The high-speed lane on the Ford has a regression of $S = -0.15\ D + 53.6$

and a slow-speed lane regression of $S = -0.427\ D + 53.3$. It would, therefore, appear that tunnel flow is not related to freeway flow nor is the flow analyzed by Greenshields. There is the possibility that the estimate of slope by Olcott would be acceptable but there is a need for a different Y intercept. Further analysis on Y intercepts might lead to a set of acceptable confidence limits for the different speed controls on facilities.

It was previously pointed out that the speed-density regressions are different for high-speed lanes and low-speed lanes. This further supports the contention implied that the main difference in relationships is the character of prevailing speeds.

<u>Relationship of Volume versus Density</u>

The cutoff lines for the volume-density relation were also determined by both F-linearity test and the chronological plot of data. The density cutoff lines are 55 vpm for lanes 1 and 2 and 45 vpm for lane 3. There is also a good indication that the lane 3 cutoff would be better at 35 vpm. The average increases and decreases in density change for lane 1 support the cutoff at 55 vpm.

The chronological plot of data is more emphatic about the change in character of flow in the critical region and is substantiated by the F-linearity test which rejects linearity above 55 vpm.

The correlation coefficients for this relation indicate a high degree of correlation--the average r is 0.941. The variance of density,

TABLE 5

REGRESSION EQUATIONS FOR FUNDAMENTAL RELATIONS

LANE		SPEED/VOLUME	SPEED/DENSITY	VOLUME/DENSITY
1	THUR	S=-0.1[illegible]V+47.6	S=-0.1[illegible]D+4[illegible].1	V=0.612D+3.7
	FRI	S=-0.1[illegible]V+52.7	S=-0.15[illegible]+53.6	V=0.6[illegible]5D+3.1
2	THUR	S=-0.231V+54.6	S=-0.2[illegible]1D+57.2	V=-.5[illegible]4D+7.6
	FRI	S=0.010V+41.1	S=-0.310D+57.8	V=0.57[illegible]D+7.1
3	THUR	S=-0.273V+4[illegible].6	S=-0.323D+54.2	V=0.49[illegible]D+3.3
	FRI	S=-0.231V+4[illegible].2	S=-0.42[illegible]D+53.8	V=0.35[illegible]D+3.5
AVG	THUR	S=-0.2[illegible]V+50.2	S=-0.3[illegible]D+58.0	V=0.4[illegible]2D+5.7
	FRI	S=-0.227V+51.1	S=-0.3[illegible]1D+57.4	V=0.45[illegible]D+7.0

r^2, accounts for about 83% of the variance in volume. The scatter of points is small as the standard error of estimate, in general, is less then 1.0.

The regression equation for the three-lane average is V = 0.472 D + 5.7 for non-critical flow up to the critical flow line. This is not compared to Greenshields' or Olcott's relation as they did not use the linear relation. Olcott did discuss the area of critical flow which he said was at a density of 65 vpm and a speed of 21 mph. The density of 65 vpm and a speed of 21 mph indicate a volume of 1365 vph which would be below the design capacity of 1500 vph used when the Ford was planned. His results do differ considerably from this analysis in that critical flow is 55 vpm at a speed of 40 mph and this is almost twice that of the tunnel flow. Greenshields indicated that maximum volume is 2200 vph which does compare with the maximum of 2250 vph for this analysis. Lighthill, however, indicated the maximum to be only 1500 vph. Barnett indicates that peak flow occurs at a volume of 2000 vph and at a speed of 40 mph which would agree with the results of this analysis. The general indication is, therefore, for a maximum flow of 2200 vph at a speed of 40 mph with a critical density of 55 vpm.

Proposed Correlation Potential

It was pointed out in the speed-volume analysis the correlation of speed and volume was low--r = -0.59; in the speed density analysis about medium--r = -0.72; and in the volume-density the correlation was high--r = 0.94. There would seem to be a pattern in this set of figures.

A possible explanation of this would be that one of the characteristics does not indicate traffic flow as well as the others. If a weighted value were given to speed (S), volume (V), and density (D) according to the correlation produced, the characteristic which produces the smallest weighted value should then be the one with the least correlation potential. The table below gives the weighted values according to the correlation yielded in the analysis. Each characteristic of the relation with the lowest correlation was given a weight of 1. The correlation value between the low and high was given a weight of 2. The relation with the highest correlation was given a weight of 3.

TABLE 6

WEIGHTED VALUES FOR CORRELATION POTENTIAL

RELATION	CHARACTER 1	CHARACTER 2	WEIGHTED VALUE S	WEIGHTED VALUE V	WEIGHTED VALUE D
S/V (r=.59)	S	V	1	1	-
S/D (r=.72)	S	D	2	-	2
V/D (r=.94)	V	D	-	3	3
TOTAL			3	4	5

The results of the weighted values indicate that speed has the lowest correlation potential of the three characteristics with density the greatest potential of correlation. If this could be further substantiated, then it might be best to base the explanation of traffic flow on the volume-density relationship.

CHAPTER VI

CONCLUSIONS

1. There is a good indication that the cutoff lines between critical and non-critical flow are as follows:
    - A. 40 mph for the speed-volume relation.
    - B. 55 vpm for the speed-density relation.
    - C. 55 vpm for lanes 1 and 2 and 45 vpm for lane 3 for the volume-density relation.
2. The F-linearity test indicates that a linear relation is acceptable below the above cutoffs for the speed-volume, speed-density, and volume-density relationships.
3. The chronological plot of data substantiates the need for separating critical flow from non-critical flow and is generally very useful in the explanation of relationships.
4. The correlation of data is not the same for all relationships:
    - A. Speed-volume correlation coefficient r= -0.674
    - B. Speed-density correlation coefficient r= -0.886
    - C. Volume-density correlation coefficient r= 0.872
5. There is reason to believe that speed has a low potential for correlation with volume and density. The best indicated correlation is that of volume and density.
6. There is a difference in slope of the regression equations for high-speed lanes and low-speed lanes.

7. Under full congestion, the speed range in critical flow is several times the range in non-critical flow.

8. Average speed increases are larger than decreases in non-critical flow. Average speed decreases are larger than increases in critical flow. Average speed decreases of critical flow are larger than speed increases of non-critical flow.

9. Further research should be performed in this area with samples under full congestion, under peak flow but with no congestion, of freeways with similar operating characteristics, and with more data on both sides of the peak period of flow.

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APPENDIX

SYMBOL AND ABBREVIATION SHEET

The symbols and abbreviations listed below are used throughout the report.

Δb = confidence limits for slope $m_{y/x}$

a = Y axis intercept

D = density of vehicles

d_s = distance spacing of vehicles

F = F-linearity test calculated value

$F_{.95}$ = 95 percent level of acceptance value

i_x = class interval in X direction

i_y = class interval in Y direction

k = number of paired data

m or $m_{y/x}$ = slope of the regression Y on X

mph or MPH = miles per hour --- speed

N = the number of samples

r = the correlation coefficient

S_b = the standard error of regression coefficient b

S = speed --- miles per hour

S_x = standard deviation in the X direction

S_x^2 = variance in the X direction

S_y = standard deviation in the Y direction

S_y^2 = variance in the Y direction

$S_{y/x}$ = standard error of estimate

$t_{\alpha/2, n-2}$ = the t distribution, two-tailed test with n-2 d.f.

V = volume of vehicles

vpm = vehicles per mile --- density

vph or VPH = vehicles per hour

VPM = vehicles per minute --- volume

X = variable in the X direction

$\bar{X}$ = mean of data in the X direction

ΣX = sum of the values in the X direction

ΣX^2 = sum of the X^2 values

ΣXY = sum of the X times Y values

Y = variable in the Y direction

$\bar{Y}$ = mean of data in the Y direction

ΣY = sum of the values in the Y direction

ΣY^2 = sum of the Y^2 values

Figure 29: Volume-Density sample chart plot for Lanes 2 and 3, Ford Expressway. Chart indicates that for a volume of 1200 VPH, graph would read 100%. Chart speed is 12 inches per hour.

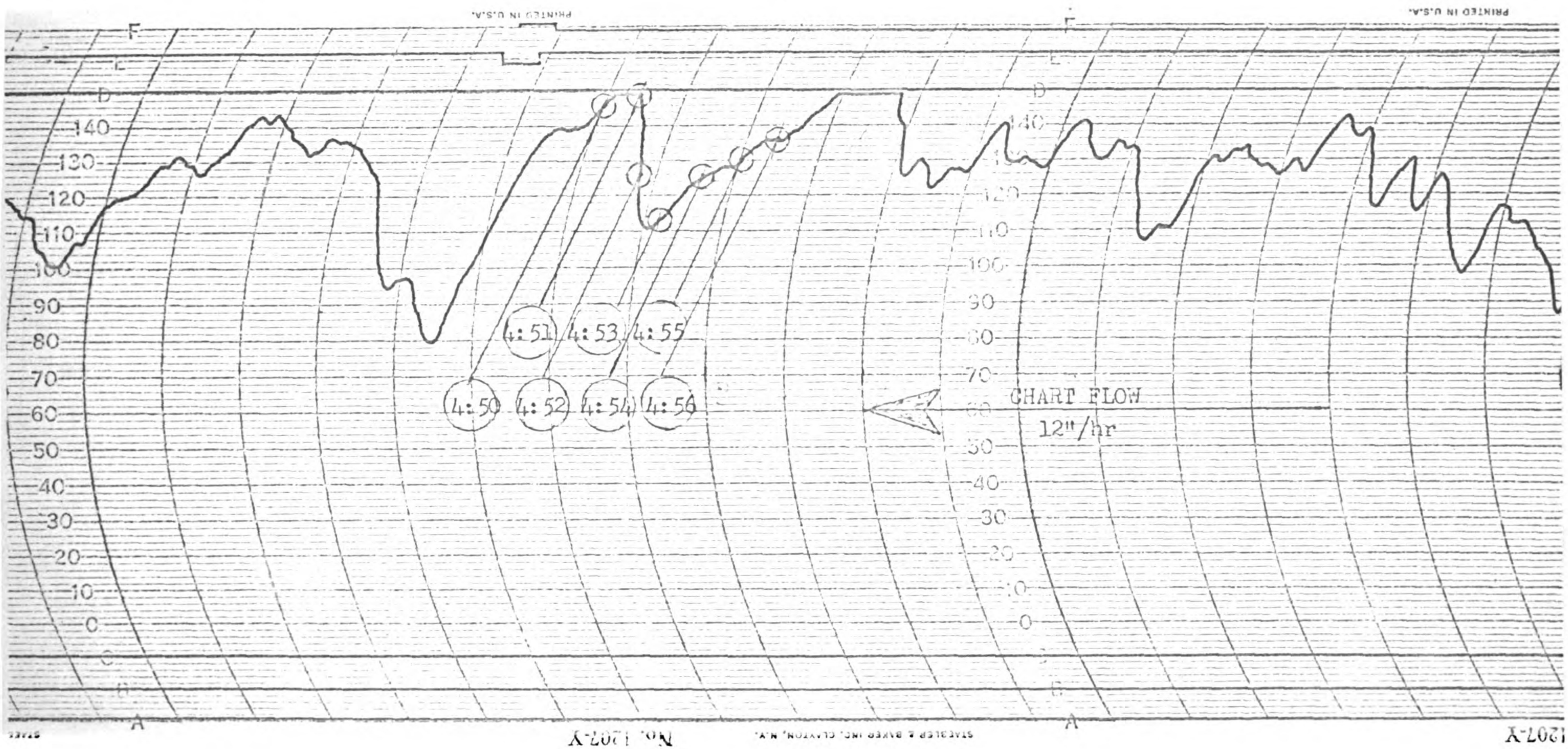

Figure 30: Speed chart for Lanes 2 and 3, Ford Expressway. This sample speed chart illustrates the speed recordings. Chart speed is $1\frac{1}{2}$ inches per minute.

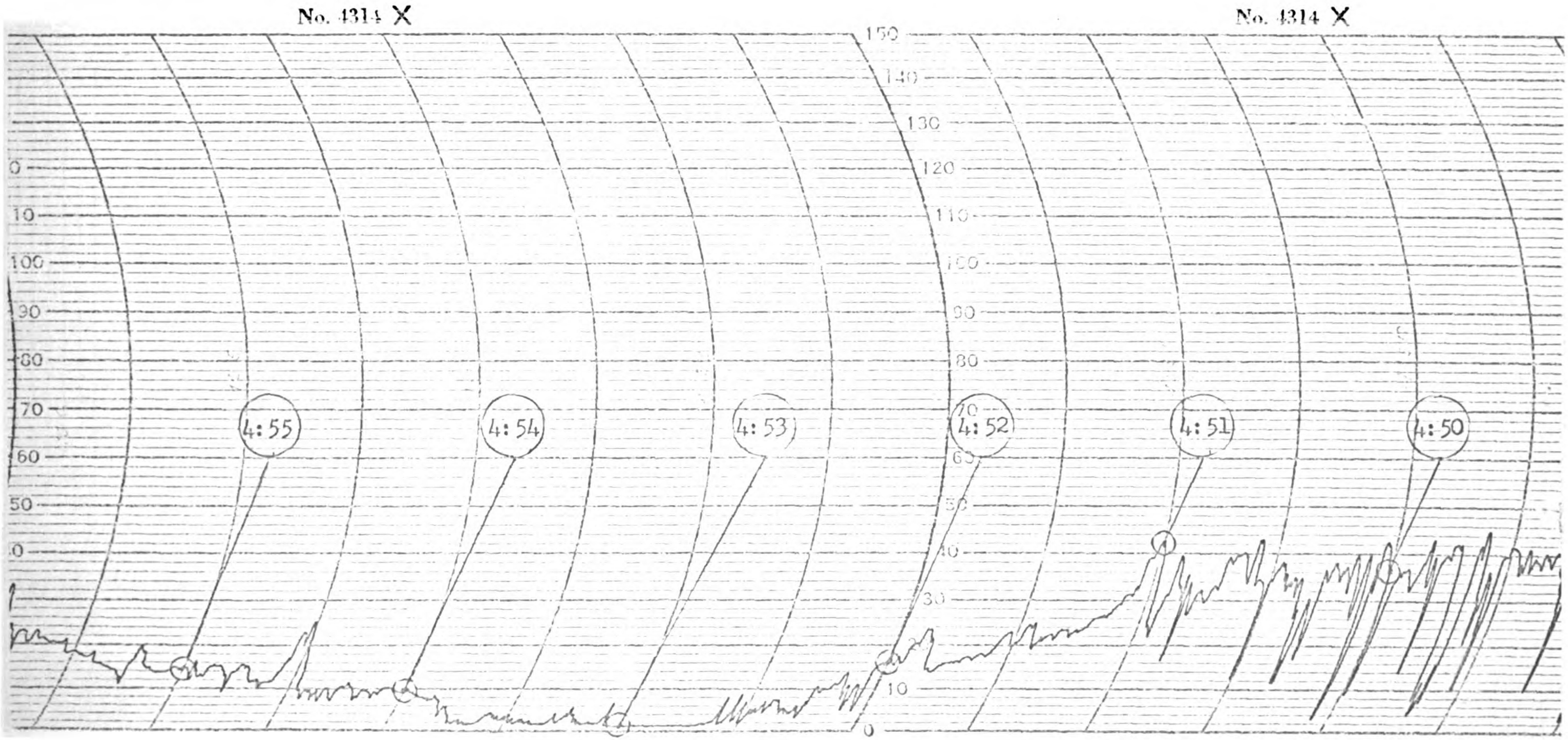

Figure 31: Twenty-pen recorder sample chart for Lanes 1, 2 and 3, Ford Expressway. The chart is a plot of vehicles that have passed the study point and the lane in which they traveled. Chart speed is 12 inches per minute.

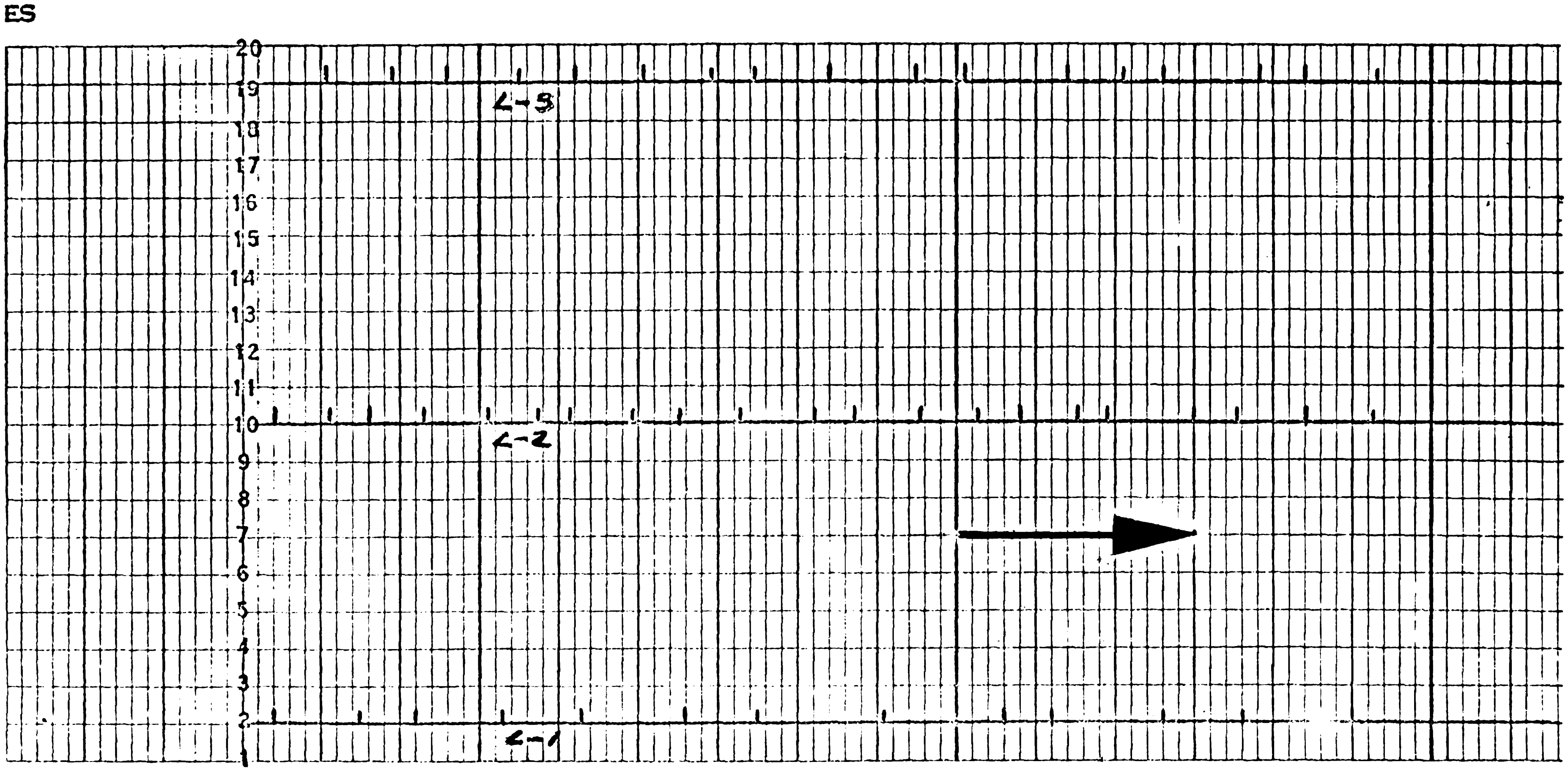

Figure 32: Speed-Volume Correlation Table.

V/N	ΣY	ΣY^2	ΣX	ΣXY
ΣX				
ΣX^2				
ΣY				
ΣXY				

Figure 33: Speed-Density Correlation Table.

DENSITY / SPEED	15-	20-	25-	30-	35-	40-	45-	50-	55-	60-	65-	70-	75-	80-	85-	90-	95-	100-	105-	110-	f_y	Y	$f_y(Y)$	$f_y(Y^2)$	Σ_x
58-60																									
55-57																									
52-54																									
49-51																									
46-48																									
43-45																									
40-42																									
37-39																									
34-36																									
31-33																									
28-30																									
25-27																									
22-24																									
19-21																									
16-18																									
13-15																									
10-12																									
7-9																									
4-6																									
1-3																									

N/N

ΣY ΣY^2 ΣX ΣXY

ΣYX ΣY ΣX^2 ΣX

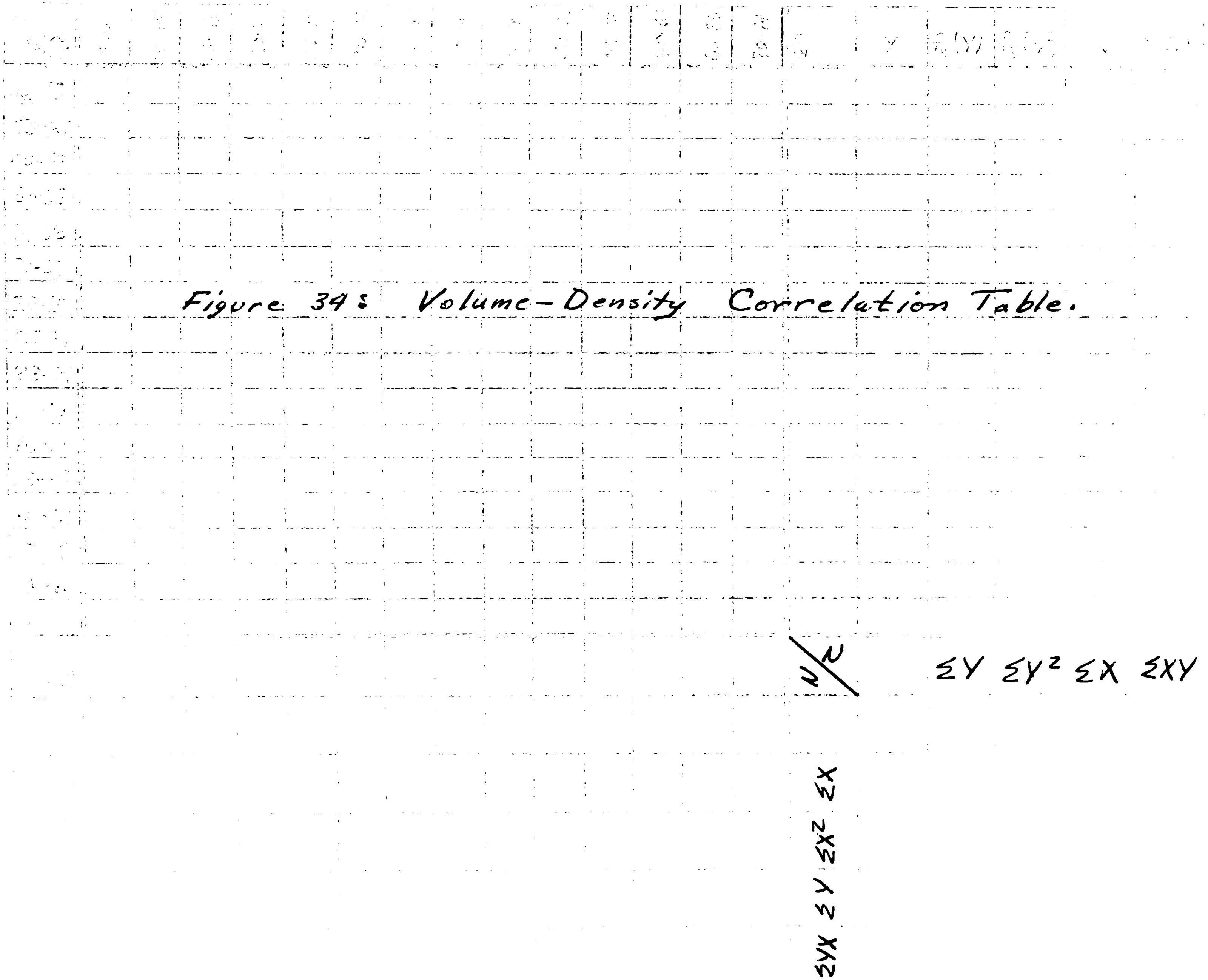

Figure 34: Volume-Density Correlation Table.