THE EFFECT OF INCLUDED PHASES ON THE GROWTH OF PLAGIOCLASE PORPHYROBLASTS

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ABSTRACT

THE EFFECT OF INCLUDED PHASES ON THE GROWTH OF PLAGIOCLASE PORPHYROBLASTS

by

Kim Kathleen Ryan

Burke (1968)* has developed a model for abnormal (porphyroblastic) grain growth that can be tested on natural metamorphic systems. His model states that when inclusion coalescence or dissolution is controlling the rate of normal grain growth, the grain size, D, is directly related to d/f where d is the average inclusion diameter and f is the inclusion volume fraction. The presence of inclusions in the matrix material during growth is critical in order to establish the proper conditions for the occurrence of porphyroblasts.

This study tests this model using samples of a metapelite containing plagioclase porphyroblasts. Correlation coefficients between D and d/f of plagioclase matrix grains are highly positive when these grains reach their optimum size. However certain matrix grains show high negative

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*Burke, J.E. 1968: Grain growth in Fulrath, R.M. and Pask, J.A. (Editors) Ceramic Microstructures. John Wiley & Sons, New York, pp. 681-700.

correlations and these are the precursor of porphyroblasts. Thus, if the grain has reached a particular size and the inclusions are no longer controlling the rate of growth, porphyroblasts develop with inclusion free outer margins.

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Kim Kathleen Ryan

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INTRODUCTION

The exaggerated or abnormal growth of certain minerals has long been an enigma to metamorphic petrologists. The purpose of this study is to test a model for abnormal grain growth proposed by Burke (1968) in order to ascertain why some minerals attain porphyroblastic size during growth in polycrystalline aggregates whereas others of the same species do not (Figure 1, 2). Porphyroblasts are grains in a metamorphic rock that exhibit this abnormal growth and are frequently five to ten times larger than the matrix grains (Spry, 1969).

Plagioclase commonly assumes porphyroblastic habit in metamorphic rocks (Crawford, 1966; Cooper, 1972). Various theories (Turner, 1948; Ohta, 1972) have been formulated to explain the development of plagioclase porphyroblasts, but none of them provide a satisfactory general model. Turner (1948) proposes that metasomatic processes form albitic porphyroblasts while Ohta (1972) formulates that they are developed through the coalescence of a mosaic of plagioclase matrix grains under high vapor pressure and temperature. Metallurgists and ceramists have studied exaggerated grain growth for many years. J. E. Burke (1968) proposed a general model for their development which was based to a large extent on the earlier work of Kingery (1962).

Figure 1.--Plagioclase porphyroblast with inclusions in center (about 50X). Crossed polars.

Figure 2.--Plagioclase matrix grain with inclusions in center (about 100X). Crossed polars.

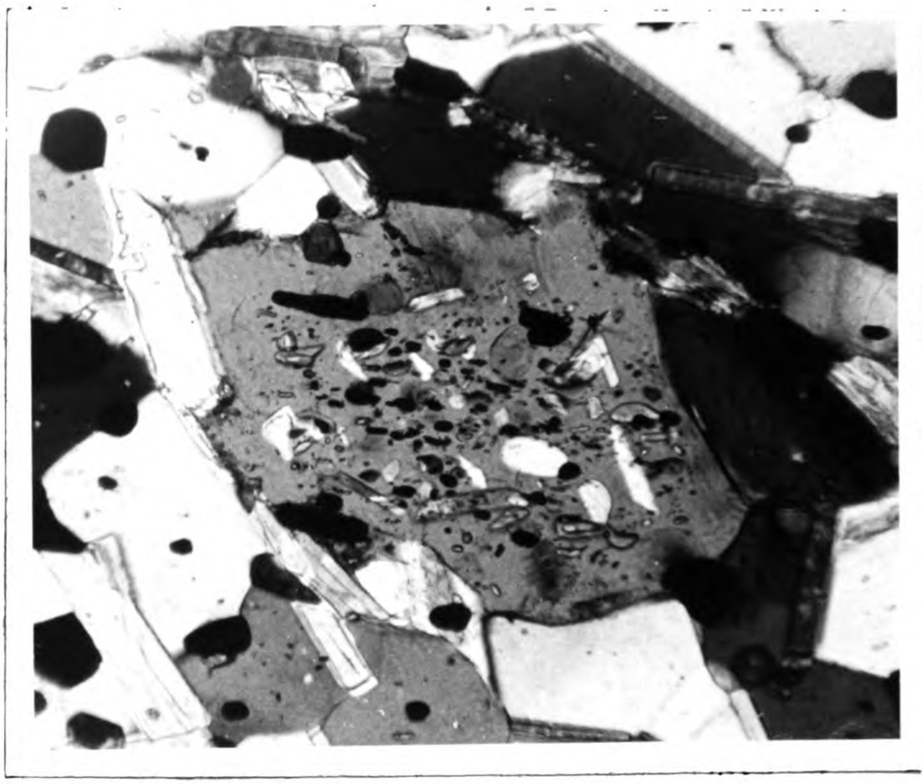
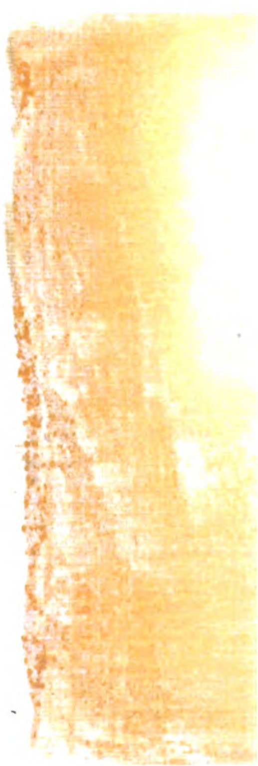


Fig 1



BURKE'S MODEL: GRAIN GROWTH AND THE EFFECT OF IMPURITIES

Burke's (1968) model for abnormal or porphyroblastic grain growth is dependent upon the coalescence or dissolution of included phases within a grain. The effect of the inclusions is to alter the energy relationships so that an imbalance is created and growth occurs.

During recrystallization in a polycrystalline solid the grain size increases once a critical temperature is reached. The grain boundaries migrate and the size distribution and relative orientation of the grains within the aggregate change. During growth, the neighboring grains are consumed as others enlarge. Impurities in the material radically alter the resulting texture.

Growth, impurity segregation at low temperatures or grain boundary migration at higher temperatures, occurs in order to minimize the energy of the system involved. The rate of normal or continuous growth of a grain can be stated as:

$$V = f(M,P)$$

where V is the velocity: M, the mobility: and P, the driving force.

The driving force of grain growth, P, is due to the difference in surface energies between two boundaries. These boundary energies are predominantly caused by lattice strain (Westbrook, 1967). Since boundaries are thought to be an equilibrium state, grain boundaries migrate towards their center of curvature (Harker and Parker, 1945). Thus P is given by:

$$P = \sigma(1/\rho_1 + 1/\rho_2) \qquad \text{(Hillert, 1965)}$$

where σ is surface free energy and ρ_1 and ρ_2 are the two principal radii of curvature.

Therefore the rate of growth can be represented by:

$$V = M\sigma(1/\rho_1 + 1/\rho_2).$$

The mobility of the boundary is dependent on temperature and the atomistic configuration of the boundary. Boundaries become mobile when an external energy source, usually temperature, is raised to a level which exceeds the activation threshold for the substance:

$$M = M_o \exp \frac{(-Q_b)}{(\ RT)} \qquad \text{(Turnbull, 1951)}$$

Q_b is the activation energy; R, the universal gas constant; T, temperature. Aust and Rutter (1962) have determined the boundary mobility for a substance, M_o, to be:

$$M_o = \frac{erb}{kT}$$

e, the Naperian base; r, the atomic jump frequency; b, the local distance of boundary movement per atomic jump; k, Boltzmann's constant; and T, temperature.

This rate of continuous grain growth, V = f(M,P) is profoundly changed by the presecence of included phases in the growing substrate. The salient effect of inclusions is to create a drag on the migrating boundary. This is caused by the tendency of moving boundaries to cohere to stationary boundaries.

Zener (1949) quantified the amount of coherence of drag as:

$$S = \sigma \cdot Z$$

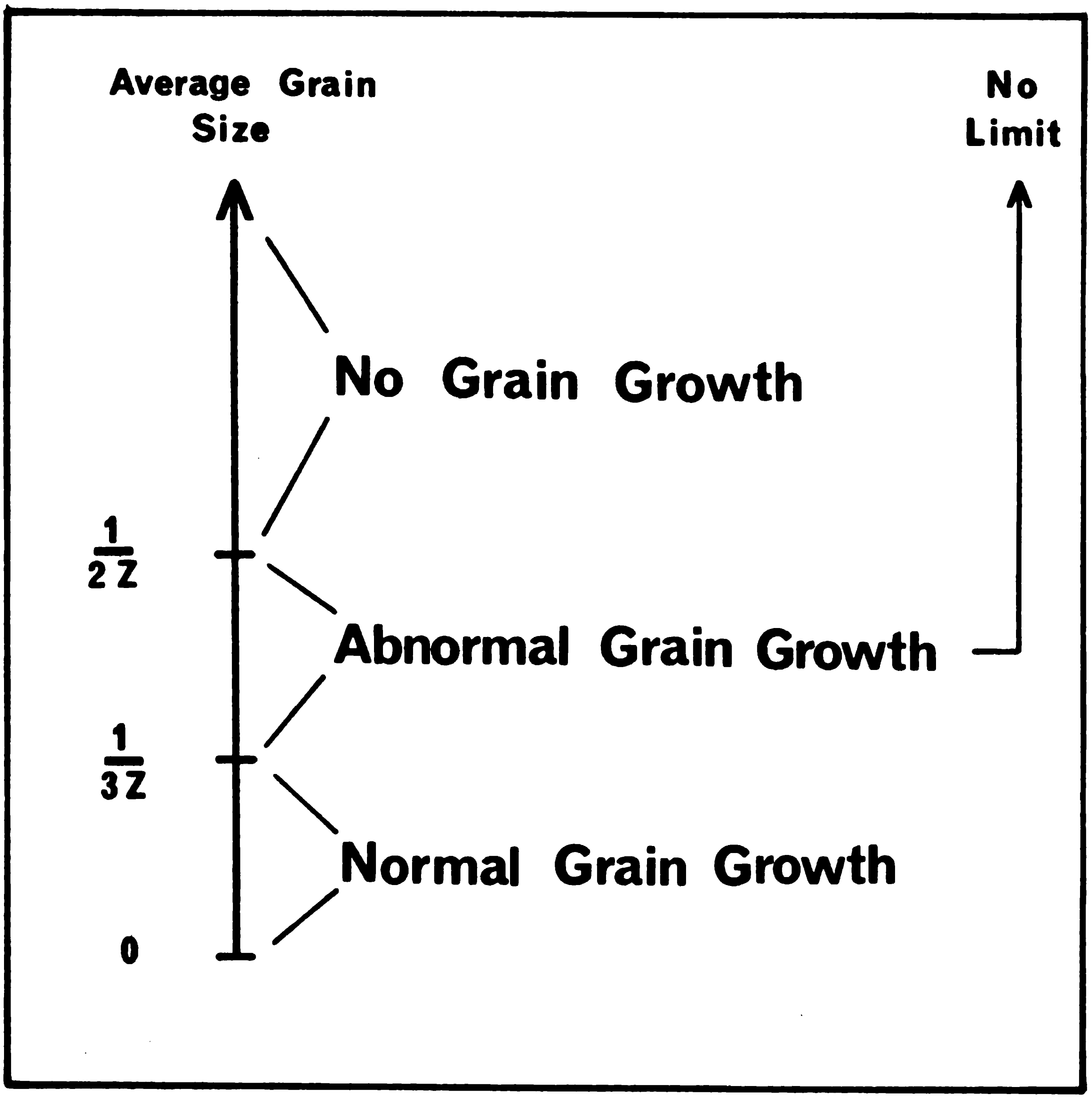
where S is the amount of drag; σ, surface free energy; and Z is related to the size and volume fraction of the dispersed (included) phase. The drag per unit area of a migrating boundary is inversely proportional to the particle radius for a given volume fraction of a dispersed phase (Cahn, 1966). The drag effect becomes critical during normal growth since the driving force decreases as the grain approaches its ultimate size.

Lücke and Detert (1957) proposed a theory which stated that the rate of grain boundary migration is controlled by the diffusion rate of the impurity atoms behind the boundary. However if the driving force of the boundary is great enough to overcome the impurity atmosphere, the boundary will abruptly breakaway from the impurities. This breakaway is dependent on the concentration of the impurities and temperature. Lücke and Detert's theory was modified by Gordon and Vandemeer (1962) and Cahn (1962) in order to better fit experimental data (Aust and Rutter, 1959). One of the principal modifications was that rather than an abrupt break at a critical temperature a transition zone was defined where abnormal growth may start.

The drag effect of inclusions can also be overcome in two other ways either through dissolution or coalescence. Turnbull (1951) states that the retardation of boundary migration by inclusions is less effective at higher temperatures due to coalescence. Beck et al. (1949) found that the initial volume of the dispersed phase, the temperature reached, and the resolution or coalescence of the dispersed phase greatly effected the ultimate grain size.

For abnormal growth to occur the presence of impurities in the matrix material is necessary during growth. As the ultimate grain size is approached, the inhibiting effect of the inclusions must decrease in order for further growth to occur. Hillert (1965) defines two grain size limits, $1/2Z$ above which no grain growth occurs and $1/3Z$ which is the limit for normal grain growth, where Z is equal to $3f/4r$ and f is the volume fraction of the inclusions and r is the average radius of the inclusions. The grain sizes inbetween the two limits are the critical ones for abnormal growth to begin (Figure 3). Hillert further states that to initiate abnormal

Figure 3.--Grain growth limits from Hillert (1965).


Average Grain Size
No Limit
No Grain Growth
1/2Z
Abnormal Grain Growth
1/3Z
Normal Grain Growth
0

growth the Z value must decrease either by increasing the particle size (r) through coalescence or by decreasing the volume fraction of the impurity (f) by dissolving the included phase. Hillert's theory, which is to a large extent a mathematical model, has been confirmed experimentally by the earlier work of Beck et al. (1949). In addition, Beck et al. (1949) state that favorable sites for growth (inclusion-free areas) must exist in some part of the grain.

Burke (1968) emphasizes the importance of included phases in order to establish the proper conditions for abnormal growth. Burke (1968) states that when inclusion coalescence or dissolution is controlling the rate of normal growth the grain size of the matrix material is approximately:

$$D \simeq 2d/3f = d/f$$

where D is the grain size; d, the inclusion diameter; and f, the volume fraction of the inclusions. D_1 is a critical average grain size where the uniform dispersion of the second phase prevents further growth:

$$D_1 = (4/3)(r/f)$$

(Cahn, 1966; Burke, 1968)

where r is the particle radius. As D approaches D_1, the rate of growth approaches zero. However as D approaches D_1, a highly curved boundary may have sufficient energy to continue migrating providing the concentration of the impurities was not sufficient to prevent growth. As a consequence of this condition local growth areas, free of inclusions, would develop. If a series of local growth events occurred around the periphery of the grain, the grain would have a large enough number of sides for continuous growth assuming a sufficient material supply was available (Von Neumann, 1952). Thus, abnormal growth would occur.

TEST OF BURKE'S MODEL: AREA SELECTED AND METHOD OF STUDY

Porphyroblastic and non-porphyroblastic rocks were collected from a pelitic schist belt of the Grenville Series located near Fernleigh in southeastern Ontario (Hounslow and Moore, 1967) in order to test Burke's model on the development of plagioclase porphyroblasts. The area has been regionally metamorphosed and the schist belt exhibits a metamorphic gradient from chloritoid grade on the southeast end to upper amphibolite on the northeast end. In the Fernleigh area well developed porphyroblasts of garnet, kyanite, staurolite, and plagioclase occur. Plagioclase was chosen for this study due to its known textural responsiveness to changes in metamorphic gradient (Ehrlich et al. 1972; Byerly and Vogel, 1973). The plagioclase porphyroblasts increase in size up to approximately five centimeters in diameter at Fernleigh as the metamorphic gradient increases.

In order to ascertain if normal growth of the matrix grains was being controlled by inclusion coalescence or dissolution, the matrix grain size D was compared to d/f by using standard petrographic techniques. Plagioclase matrix grains were used exclusively in this study because these grains reflect normal growth conditions (i.e. less than 1/3Z or over 1/2Z in size), and because the conditions that lead to abnormal growth, due to included phases, should be preserved in these grains. Porphyroblasts are of little value for determining the initial cause of run-away growth because they have exceeded the 1/3Z limit and did not stabilize. In order to reduce textural variation, the plagioclase matrix grains studied were from rocks collected only from the Fernleigh site.

Using a petrographic microscope a plagioclase matrix grain was randomly selected on a thin section. The stage orientation and the traverse direction of the count were randomized.

For each of thirty-three plagioclase matrix grains, the volume fraction of the inclusions, the diameters of the inclusions, and the diameter of the matrix grain were measured. The volume fraction of the inclusions, f, with in a plagioclase matrix grain was measured by a point counting technique described by Underwood (1970). A grid was inserted into one ocular and superimposed on the grain at the appropriate power so that unity during the count was not exceeded. The phases counted were either plagioclase or inclusions, the inclusions consist of quartz, magnetite, tourmaline, muscovite, and a biotitic mica. To insure that a sufficient number of measurements were taken with a ninety-five percent probability of obtaining a significant result at the one percent level, a total of 200 points were counted per matrix grain (Beyer, 1966). The volume fraction was computed by the formula:

$$f = P\alpha / P_T$$

where $P\alpha$ is the total number of points falling on inclusions and P_T is the total number of points counted.

The longest diameters of the inclusions, d, which fell beneath one of the random lines of the grid were measured with a micrometer. The total variance of the diameters was calculated after each ten measurements and found to stabilize at fifty inclusion diameters (Figure 4). After this only fifty diameters were measured and the mean diameter for 50 measurements was used in further computations.

The maximum diameter of the entire matrix grain, D_m, was measured optically using a micrometer.

As visual proof for the development of plagioclase porphyroblasts from matrix grains, photographs were taken to document the various stages of growth.

To detect compositional differences, preliminary microprobe analysis of a limited number of grains was performed.

Test of Model

In order to test the hypothesis that normal growth in plagioclase matrix grains was being controlled by inclusion coalescence or dissolution ($D_m = d/f$), a size-frequency diagram (Figure 5) (Table 1) was constructed for the matrix grains. Correlation coefficients, r, (Beyer, 1966) were

Figure 4.--Stabilization of variance of inclusion diameters. Selected examples demonstrate variation of curves.

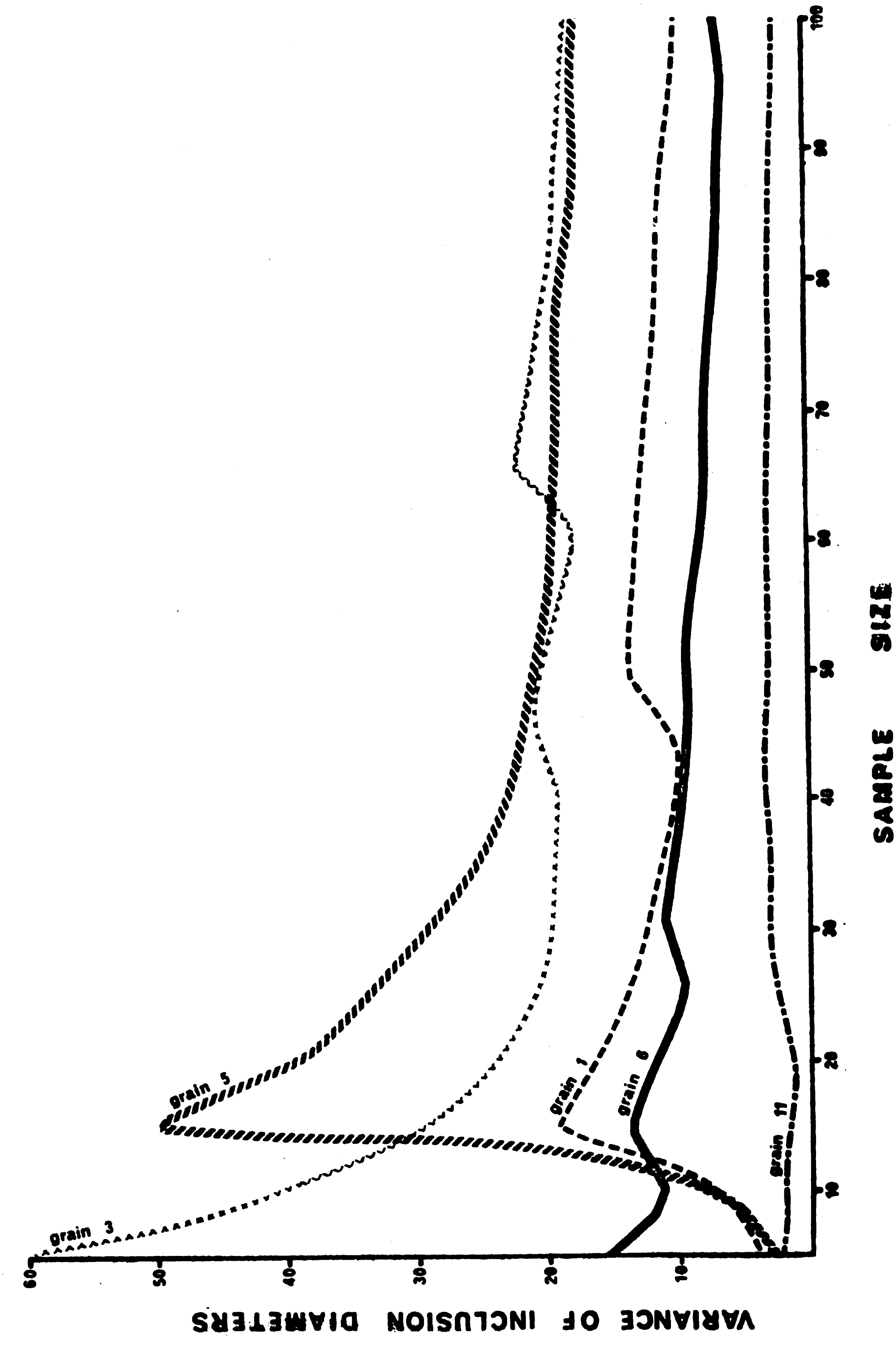

VARIANCE OF INCLUSION DIAMETERS
SAMPLE SIZE
grain 3
grain 5
grain 1
grain 6
grain 11
60
50
40
30
20
10
10
20
30
40
50
60
70
80
90
100

Figure 5.--Size-frequency diagram of plagioclase matrix grains.

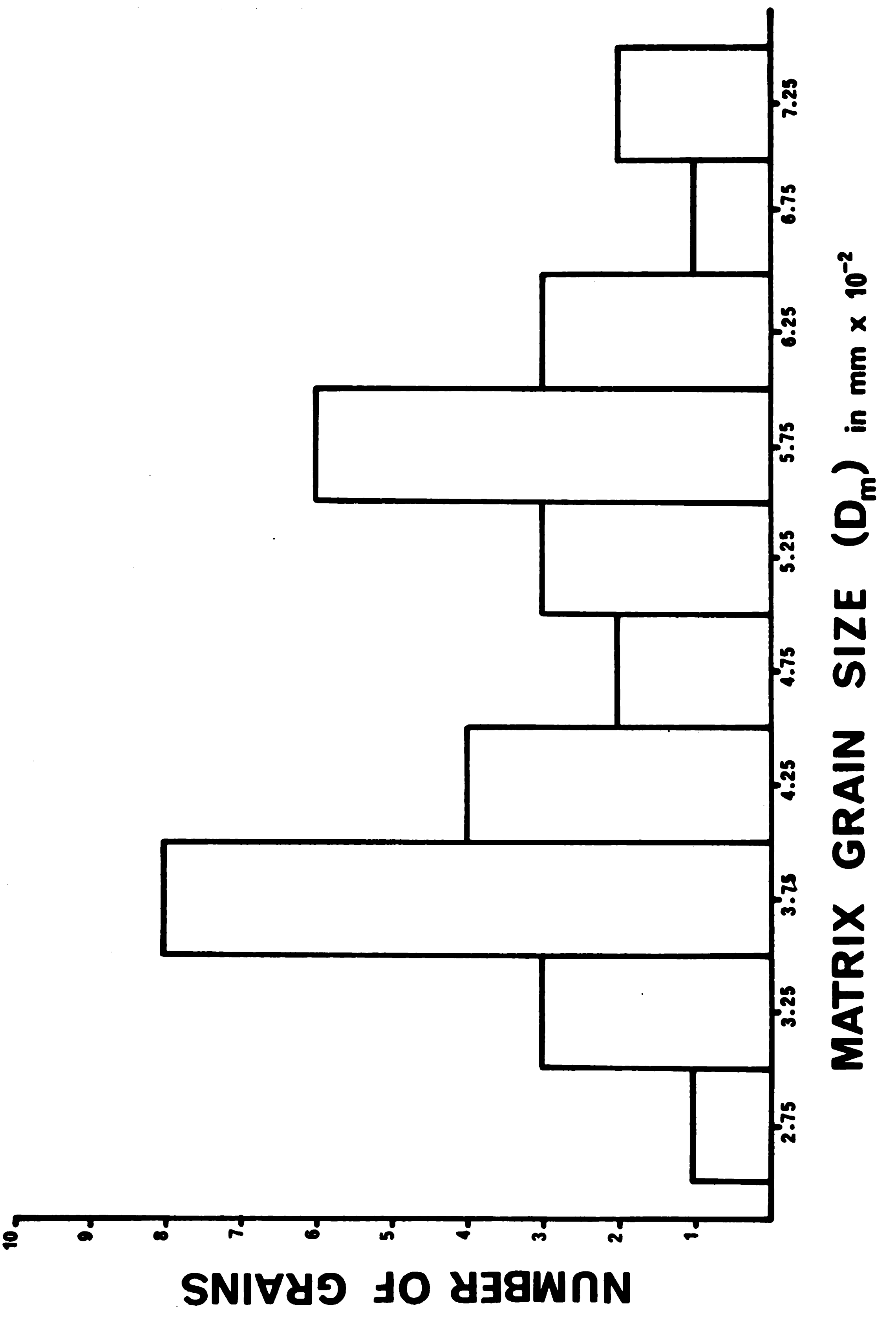

MATRIX GRAIN SIZE (D_m) in mm x 10^{-2}
NUMBER OF GRAINS
2.75
3.25
3.75
4.25
4.75
5.25
5.75
6.25
6.75
7.25
1
2
3
4
5
6
7
8
9
10

TABLE 1.--Ranking and Grouping of Grains according to Size Distribution.

Grain Number	Size(D_m) in mm	d/f	Groups	Group Numbers
4	.29585	.02161		1
20	.3298	.02725		
3	.3492	.03420		2
17	.3492	.03815		
8	.35405	.03194		
29	.36375	.03320		
14	.3686	.03244		
18	.3686	.04640		3
15	.38315	.03405		
26	.38315	.01318		
16	.38315	.04297		
23	.3880	.04618		
5	.40255	.02618		
22	.40255	.04020		
19	.4268	.02489		4
21	.4462	.03362		
24	.4947	.00909		
7	.4995	.01758		
9	.5044	.00988		
28	.5044	.01554		5
31	.5432	.03485		
12	.5626	.01687		
27	.5626	.01246		
2	.56745	.02350		6
1	.5723	.04349		
30	.5723	.03278		
10	.5917	.01869		
13	.6014	.03560		
33	.6014	.03341		7
32	.6208	.04024		
11	.6528	.01332		
25	.7275	.01559		8
6	1.1058	.06925		

calculated for each class interval to determine the degree of covariance between D_m and d/f (Figure 6) (Table 2).

Correlation coefficients were also calculated for the sample population (33 grains), for individual rock samples and by thin section sample (5 grains), but no significant differences in the magnitude of the coefficients were found.

As shown in Figure 6 and Table 2 moderately high negative correlations were found for groups four and seven. The grain sizes for these two groups fall into the upper size classes of the bimodal distribution. These negative correlations indicate that the grains which fall within these classes are no longer being controlled by inclusion coalescence or dissolution or in other terms they are potential grains for abnormal growth.

The high positive correlation for grain sizes which lie in the size interval 5.01 to 5.50 millimeters demonstrates that the growth of these grains is being controlled by inclusion activity and possibly have almost reached their most stable size under normal growth conditions.

Since the effect of inclusions does not become critical until the ultimate grain size is closely approached, the poor correlations for the other groups indicate that during growth another inhibiting factor stopped the growth of these grains. One cause is possibly that the initial volume of the impurities within the grain was sufficient to stop growth at a smaller size.

Figure 6.--D_m vs. d/f correlation diagram.

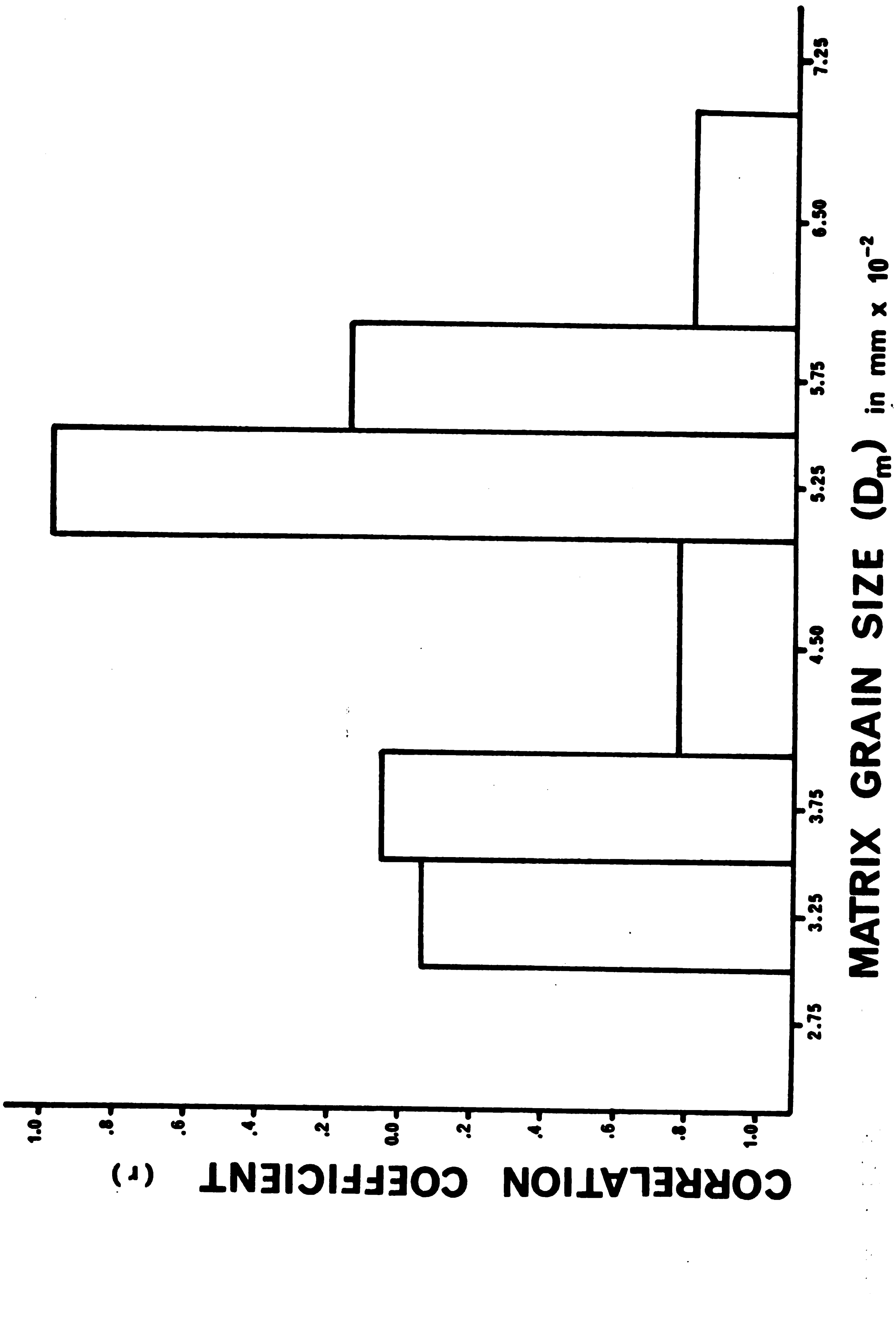

MATRIX GRAIN SIZE (D_m) in mm x 10^{-2}
2.75
3.25
3.75
4.50
5.25
5.75
6.50
7.25
CORRELATION COEFFICIENT (r)
1.0
.8
.6
.4
.2
0.0
.2
.4
.6
.8
1.0

TABLE 2.--D_m vs. d/f: Correlation Coefficients, r, per Size Grouping.

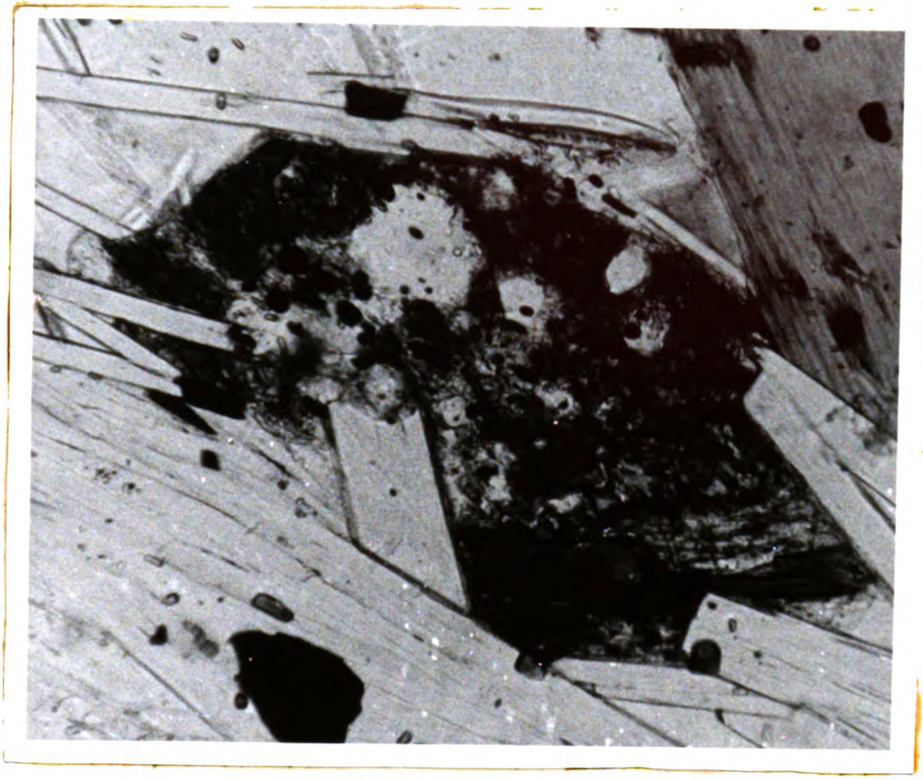
Group Number	r
1	---
2	-0.05582
3	0.04789
4	-0.78266
5	0.97635
6	0.14108
7	-0.81357
8	---

In Burke's model as the critical size (D_1) is approached growth will cease and the result should be inclusion filled grains. Figures 7a, b shows an inclusion filled matrix grain with relatively straight boundaries. However if the boundaries are sufficiently curved, the driving force may be large enough to override the inclusion drag effect and inclusion-free areas are produced and Figures 8a, b display a matrix grain which has developed this local inclusion-free growth areas. Figure 9 demonstrates that through a series of local growth events a relatively clear plagioclase rim has grown outwards from the inclusion filled center of a matrix grain. Notice the curved boundaries. Figure 10 shows coalescence of quartz within plagioclase matrix grains. Figures 11a, b are of a porphyroblast with a well developed plagioclase rim and with the majority of the inclusions concentrated in the center. It is clear that in many of the plagioclase porphyroblasts the grain boundary has broken away from the inclusions. The driving force for this breakaway may be highly curved, high-energy boundaries.

Preliminary microprobe analyses were run to detect compositional differences between plagioclase matrix grains and plagioclase porphyroblasts. The results are shown graphically in Figure 12 and visually in Figures 13a, b. The matrix grains are relatively homogeneous in composition while the porphyroblasts show a wide range of anorthite content. This may suggest that part of the driving force for

Figure 7a.--Inclusion-filled plagioclase grain with straight boundaries (about 100X). Plane light.

Figure 7b.--Same grain as 7a. Shows distribution of inclusions (about 100X). Plane light.



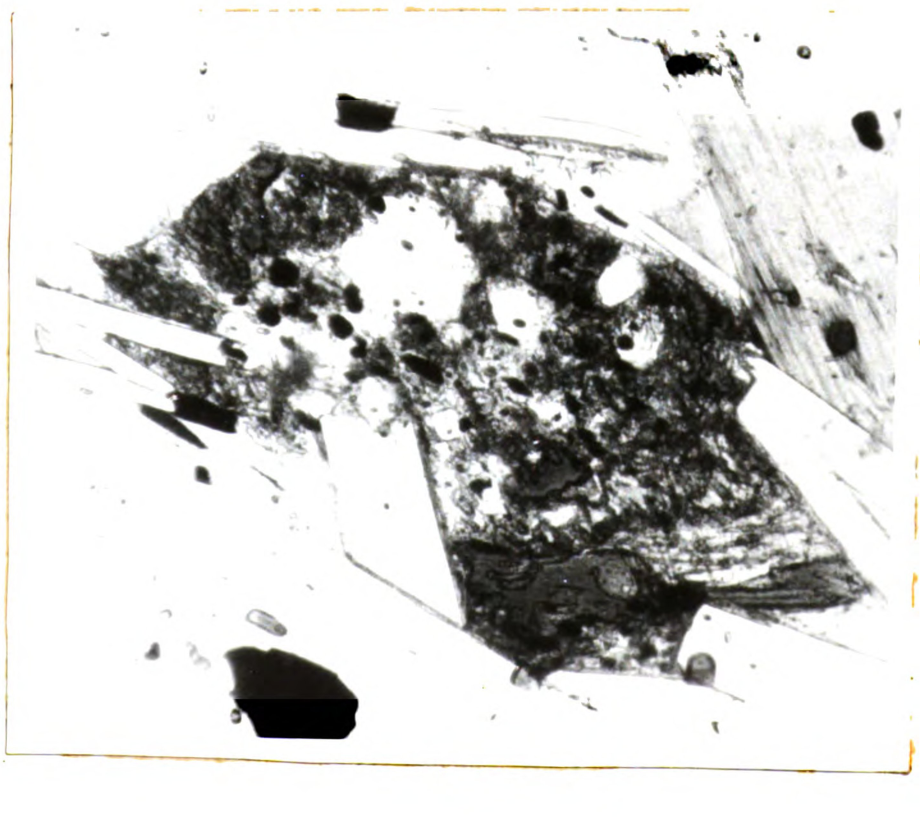
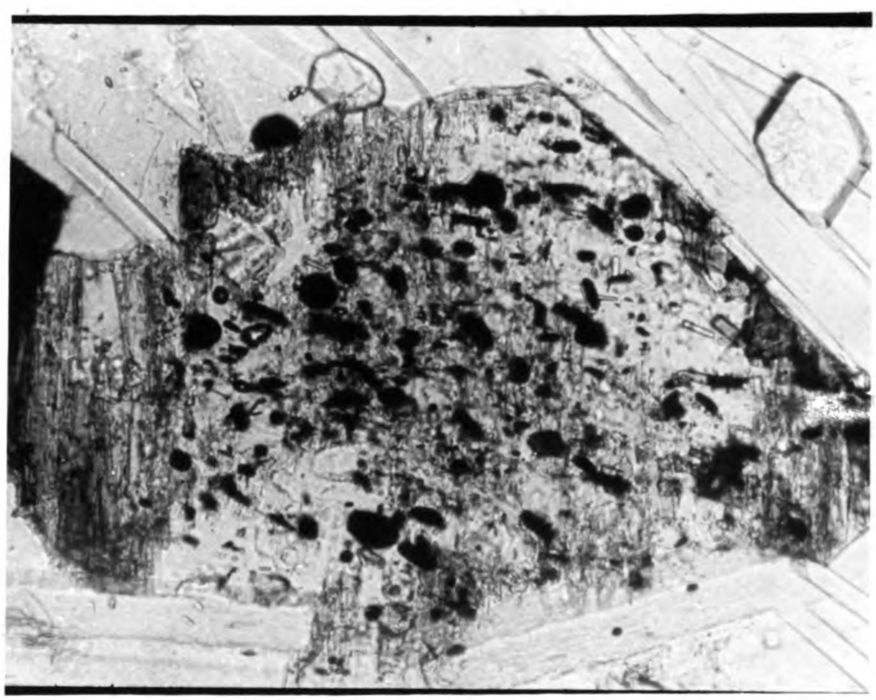


Figure 8a.--Plagioclase matrix grain with relatively inclusion-free areas (about 200X). Crossed polars.

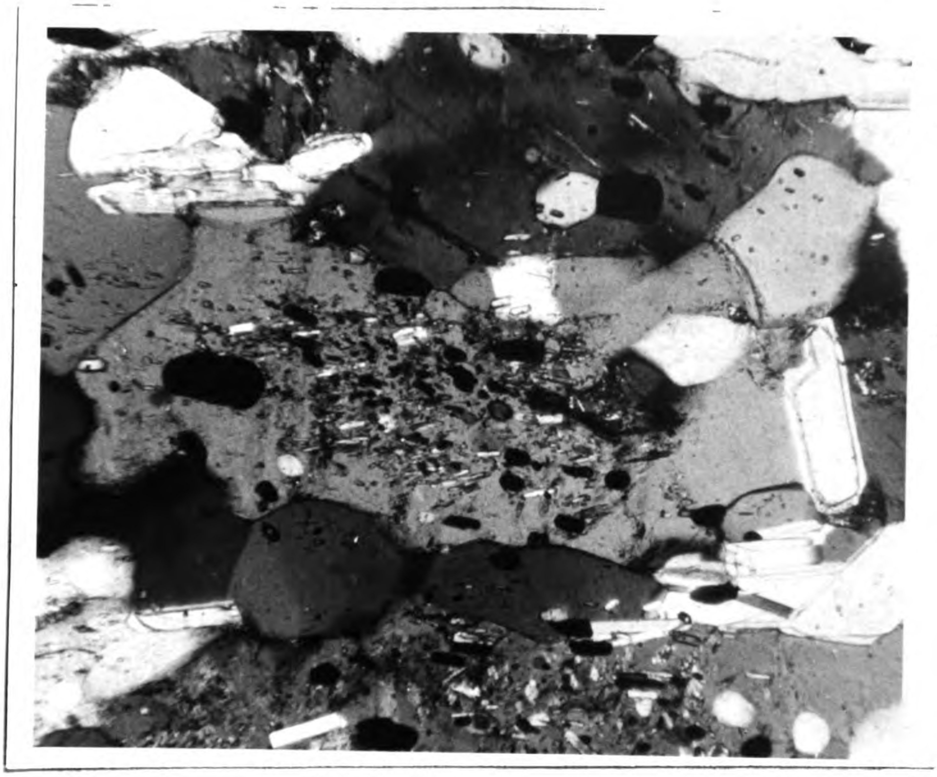
Figure 8b.--Same grain as 8a (about 200X). Plane light.




80

Figure 9.--Plagioclase matrix grain with inclusions in center and clear rim. Note curved boundaries (about 100X). Crossed polars.

Figure 10.--Plagioclase matrix grain. Note coalesced quartz inclusions in center (about 100X). Crossed polars.



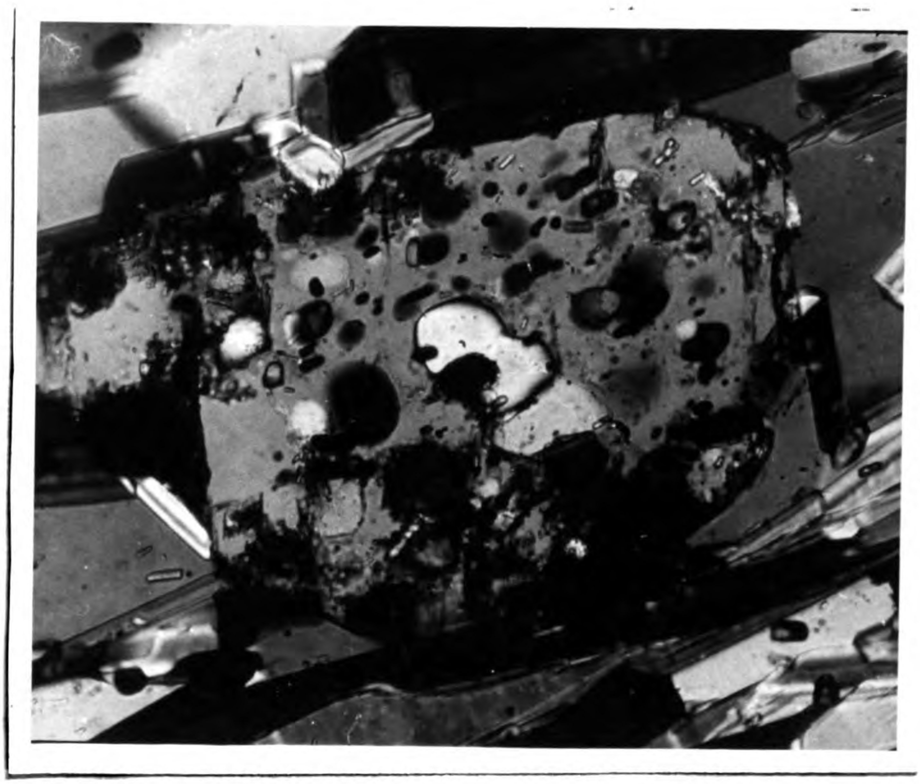
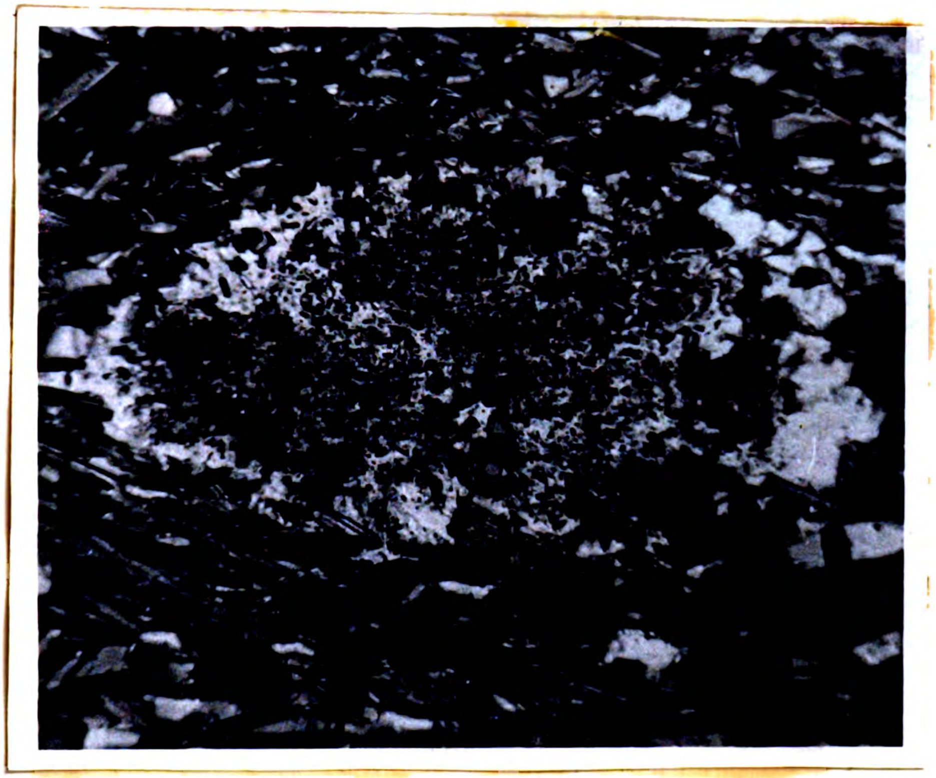
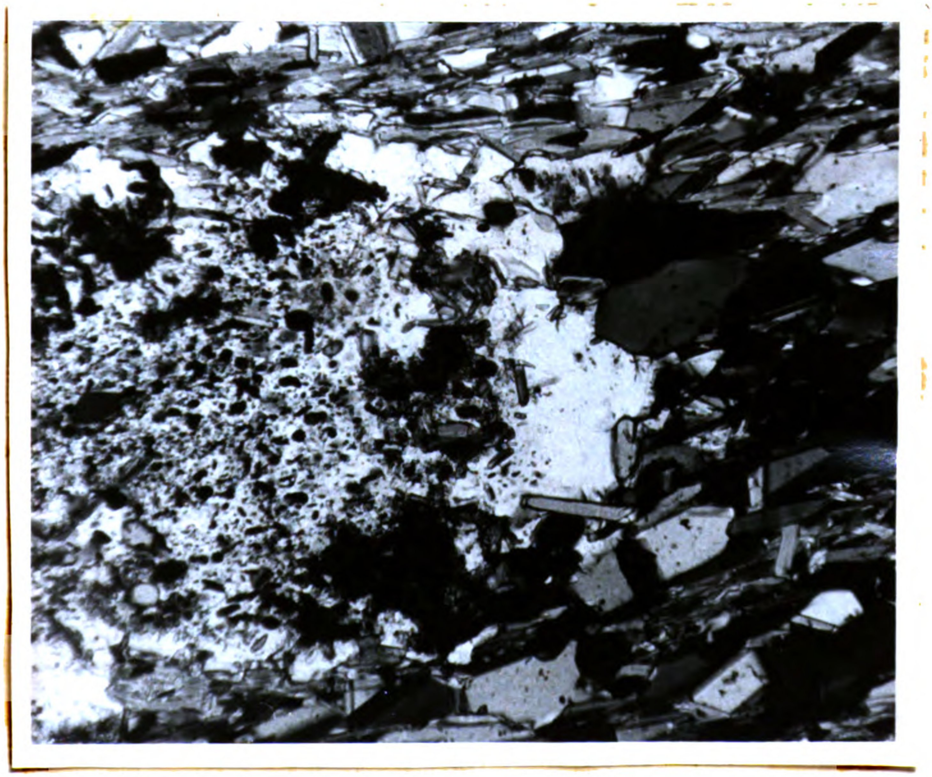


Figure 11a.--Plagioclase porphyroblast with well-developed plagioclase rim (about 50X). Crossed polars.

Figure 11b.--Enlargement of right half of 11a showing rim and cuspate boundaries (about 50X). Crossed polars.



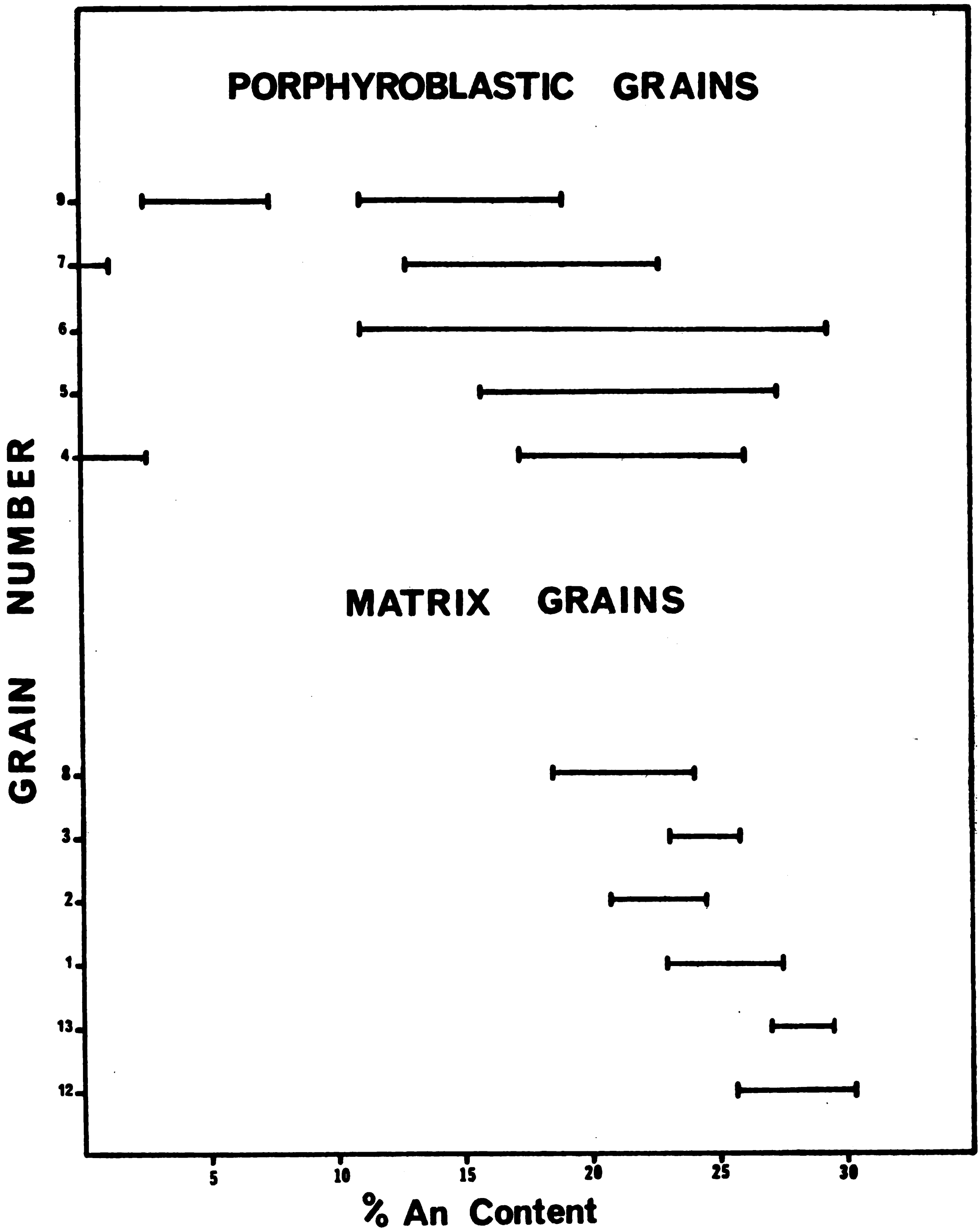


top

right

Fig 12 b

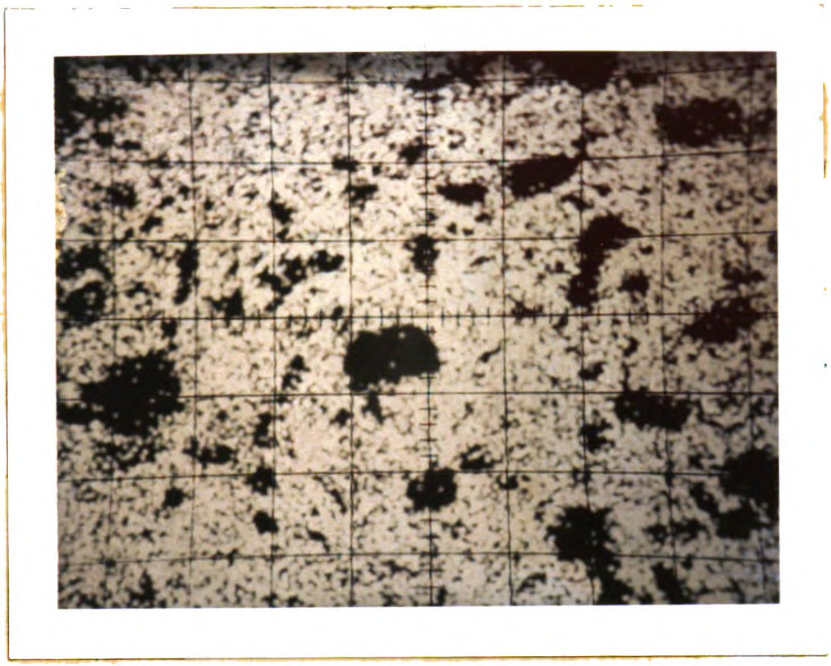
Figure 12.--Compositional ranges of an content between plagioclase matrix grains and plagioclase porphyroblasts.

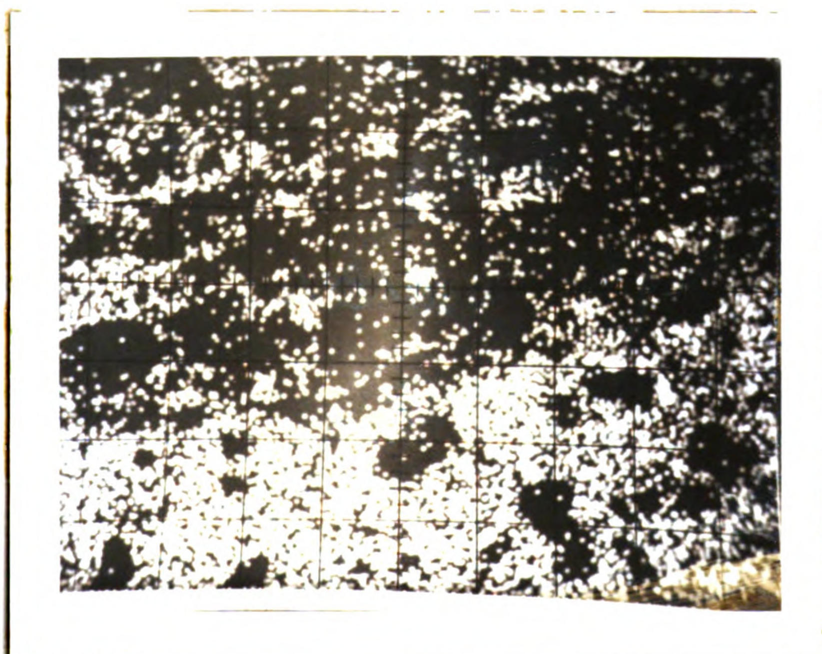


PORPHYROBLASTIC GRAINS
MATRIX GRAINS
GRAIN NUMBER
9
7
6
5
4
8
3
2
1
13
12
5
10
15
20
25
30
% An Content

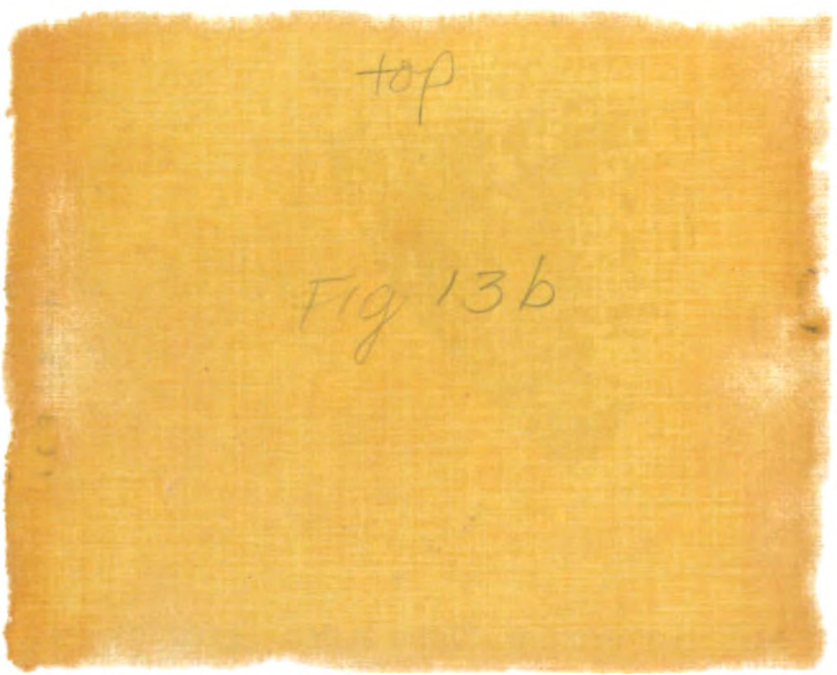
Figure 13a.--Na-Kα X-ray microprobe image. Note variable concentration of sodium. Dark areas are quartz grains.

Figure 13b.--Ca-Kα X-ray microprobe image. Same area as 13a. Note calcium deficiency in upper portion.






Top
Fig 13a


top
Fig 13b

the runaway growth of the porphyroblast is that the plagioclase actually consists of two or more distinct feldspars, one of which is acting as an included phase.

CONCLUSIONS

The results of this study indicate that Burke's model for abnormal growth is a viable one for the development of plagioclase porphyroblasts and that the role of included phases within a growing matrix grain is critical during normal growth. This study has shown that grains in which included phases are no longer controlling growth are likely candidates for porphyroblastic development. Once runaway growth is initiated, it is maintained either by highly curved, energetic boundaries or by the development of included phases of similar composition as the host.

The preliminary microprobe work has shed some light on the included phases (i.e. two plagioclases) in the porphyroblasts, but it is obvious that this work must be expanded to encompass the effects of impurities which will re-dissolve and diffuse through the grains at elevated temperatures. The microprobe data showed that high concentrations of anorthite and albite exist within the porphyroblasts (Figure 12a, b). Through dissolution these impurity concentrations could greatly effect the extent of growth and must be evaluated for our complete understanding of the formation of plagioclase porphyroblasts.

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