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TESTING AN ARTIFICIAL TRANSMISSION LINE FOR STEADY AND TRANSIENT BEHAVIOR FOR DIFFERENT TERMINATIONS

Thesis for the Degree of M. S.

MICHIGAN STATE COLLEGE

Theodore Peter Rykala

1949

THESIS

This is to certify that the

thesis entitled

"Testing an Artificial Transmission Line

for Steady and Transient Behaviour

for Different Terminations"

presented by

Theodore P. Rykala

has been accepted towards fulfillment

of the requirements for

M.S. degree in E.E.

J A Strelzoff

Major professor

Date May 19, 1949

O-169

TESTING AN ARTIFICIAL TRANSMISSION LINE FOR STEADY AND TRANSIENT BEHAVIOR FOR DIFFERENT TERMINATIONS

By

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A THESIS

Submitted to the School of Graduate Studies of Michigan State College of Agriculture and Applied Science in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE

Department of Electrical Engineering

1949

THESIS

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INTRODUCTION

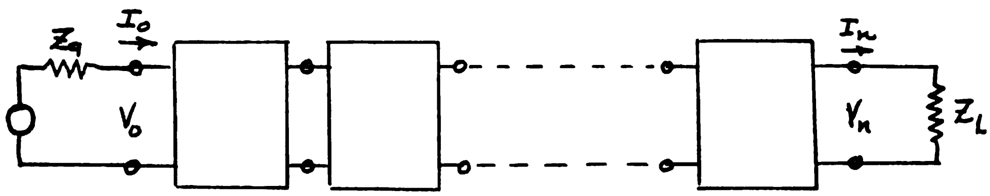
Transmission phenomena play an important part in various fields of endeavor. Whether in power, communications or other applications, the transfer of energy from one point to another embraces a large section of Electrical Engineering studies. It is the purpose of this paper to study the transfer phenomenon of a lumped parameter artificial line. Various types of circuits exist for the transmission of signals ranging from simple transmission lines to complicated networks of electronic devices. The prime requisite is to transmit a given signal in a manner whereby it can be recognized at the receiving end as the same signal which was placed on the input terminals.

All types of networks can be treated as a four-point terminal box. The effect of the system on a given signal can be studied using transient methods as most signal transmission requires that a network be in a transient state at all times. Response of a network to the step function and an impulse function provides a severe test for the system. These functions can be represented mathematically quite easily using Fourier methods and shown to contain the sum of a number of sinusoidal frequencies. The transient response can be completely specified in terms of steady state sinusoidal behavior.

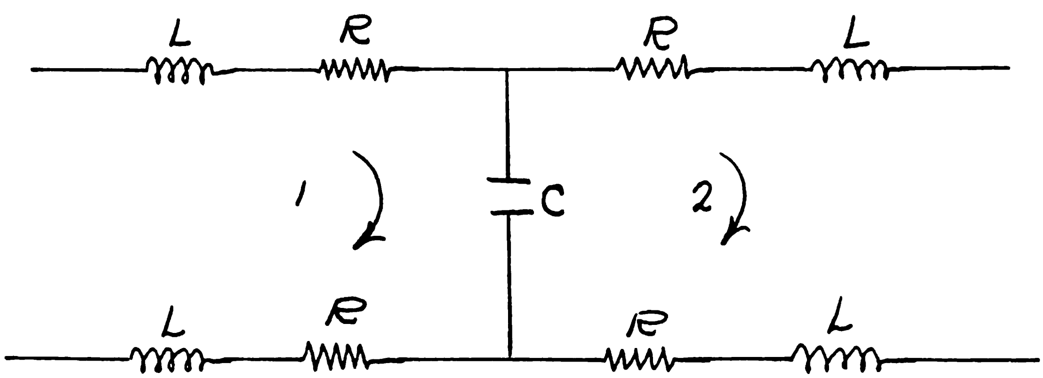
The ratio of the input voltage to the output voltage is a complex response function of the form $H(\omega)\,\epsilon^{j\beta}$. How this function behaves is the problem to be met. First, the ratio of voltages is to be determined; second, the variation of the response function with frequency is to be analyzed. Ladder networks are adaptable to this method of treatment, whereas some other transmission systems cannot be handled easily mathematically. In these networks, the transient effects must be handled experimentally.

SOLUTION OF CASCADE NETWORKS

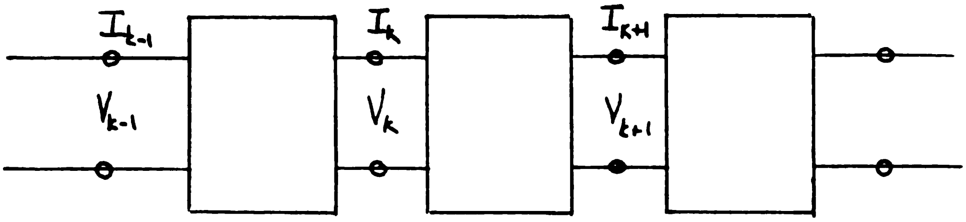
The artificial line is a number of like four-terminal networks or sections connected in cascade arrangement. There are forty sections in the line to be analyzed. This arrangement is shown below.



Each section contains inductance (L) and resistance (R) in series with capacitance (C) in parallel. This arrangement is called an H section and is represented below.



It is necessary at this point to define certain characteristics of the section. Y_{11} is the admittance of loop 1. Y_{22} is the admittance of loop 2 and is equal to the admittance of loop 1 in the symmetrical case. Y_{12} is the mutual admittance between loops. Taking a few sections in the interior of the line and extending the analytical results will give the over-all performance of the line.



Writing the equations for the current at junction k -

$$I_k = -Y_{21} V_{k-1} - Y_{22} V_k \qquad (1)$$

$$I_k = Y_{11} V_k + Y_{12} V_{k+1}$$

Rearranging the identities gives the following equation:

$$Y_{21} V_{k-1} + Y_{22} V_k + Y_{11} V_k + Y_{12} V_{k+1} = 0$$

$$V_{k-1} + \frac{Y_{22} + Y_{11}}{Y_{21}} V_k + \frac{Y_{12}}{Y_{21}} V_{k+1} = 0$$

However, due to the symmetrical conditions of each section,

$$Y_{22} = Y_{11}$$

and therefore,

$$V_{k-1} + \frac{2Y_{11}}{Y_{21}} V_k + V_{k+1} = 0 \qquad (2)$$

Considering the voltage in section k, the following equations can be written from Kirchhoff's Laws:

$$V_k = Z_{21} I_{k-1} - Z_{22} I_k \qquad (3)$$
$$V_k = Z_{11} I_k - Z_{12} I_{k+1}$$

or -

$$Z_{21} I_{k-1} - Z_{22} I_k - Z_{11} I_k + Z_{12} I_{k+1} = 0$$

dividing by Z -

$$I_{k-1} - \frac{Z_{11} + Z_{22}}{Z_{21}} I_k + \frac{Z_{12}}{Z_{21}} I_{k+1} = 0$$

The symmetrical properties of the section permit the following equation:

$$I_{k-1} - \frac{2 Z_{11}}{Z_{12}} I_k + I_{k+1} = 0 \qquad (4)$$

For the moment let us consider some of the properties of four-terminal networks. Input and output currents are obtained by the relationships

$$I_1 = Y_{11} E_1 + Y_{12} E_2 \qquad (5)$$
$$I_2 = Y_{21} E_1 + Y_{22} E_2$$

This is called the admittance method. The input and output voltages can be written by a system, as follows:

$$E_1 = Z_{11} I_1 + Z_{12} I_2 \qquad (6)$$
$$E_2 = Z_{21} I_1 + Z_{22} I_2$$

This system has impedance coefficients. Using another system of coefficients called general circuit coefficients, the input voltage and current can be written in terms of output current and voltage.

$$\begin{aligned} E_1 &= \mathcal{A} E_2 - \mathcal{B} I_2 \\ I_1 &= C E_2 - \mathcal{D} I_2 \end{aligned} \qquad (7)$$

The solution of these equations by the method of determinants will give the following relationships that exist between the various systems:

$$\begin{aligned} \frac{M_{11}}{Det} &= Y_{11} = \frac{Z_{22}}{|Z|} = \frac{\mathcal{D}}{\mathcal{B}} \\ -\frac{M_{12}}{Det} &= Y_{12} = -\frac{Z_{12}}{|Z|} = -\frac{1}{\mathcal{B}} \\ \frac{M_{22}}{Det} &= Y_{22} = \frac{Z_{11}}{|Z|} = \frac{\mathcal{A}}{\mathcal{B}} \end{aligned} \qquad (8)$$

Since the line sections are symmetrical -

$$\mathcal{A} = \mathcal{D}$$

It can now be shown that the coefficients of the I term and the V term in equations 2 and 3 are equivalent:

$$\frac{2Y_{11}}{Y_{12}} = \frac{2\,\frac{Z_{22}}{|Z|}}{-\frac{Z_{12}}{|Z|}} = \frac{2\,\frac{\mathcal{A}}{\mathcal{B}}}{-\frac{1}{\mathcal{B}}}$$

or $-\dfrac{2Y_{11}}{Y_{12}} = -\dfrac{2Z_{22}}{Z_{12}} = -2a$

Using this equality, equations 2 and 4 can be written in the following manner:

$$V_{k-1} - 2a V_k + V_{k+1} = 0 \qquad (2a)$$

$$I_{k-1} - 2a I_k + I_{k+1} = 0 \qquad (4a)$$

The above equations are of the same nature as the differential equations of the uniform line, considering only the solution to the first approximation.

$$\frac{\partial^2 E}{\partial x^2} - \gamma^2 E = 0$$

$$\frac{\partial^2 I}{\partial x^2} - \gamma^2 I = 0$$

The solutions for equations 2a and 4a can therefore be assumed to be of the exponential form which is the solution for the uniform line.

$$V_k = A_1 \epsilon^{-k\gamma} + A_2 \epsilon^{k\gamma}$$
$$I_k = B_1 \epsilon^{-k\gamma} + B_2 \epsilon^{k\gamma} \qquad (9)$$

In this solution γ plays the role of a propagation function per section in the same manner as the propagation function of a line per unit length. Determination of γ can be accomplished by substitution of this solution into equations 2a and 4a.

$$A_1 \epsilon^{-(k-1)\gamma} + A_2 \epsilon^{(k-1)\gamma} - 2aV_k + A_1 \epsilon^{-(k+1)\gamma} + A_2 \epsilon^{(k+1)\gamma} = 0$$

Combining terms -

$$(A_1 \epsilon^{-k\gamma} + A_2 \epsilon^{k\gamma}) \epsilon^{\gamma} - 2aV_k + (A_1 \epsilon^{-k\gamma} + A_2 \epsilon^{k\gamma}) \epsilon^{-\gamma} = 0$$

Substitution -

$$\left[\epsilon^{\gamma} - 2a + \epsilon^{-\gamma}\right] V_k = 0 \qquad (10)$$

Next substituting in equation 4a . -

$$B_1 \epsilon^{-(k-1)\gamma} + B_2^{(k-1)\gamma} - 2a I_k + B_1 \epsilon^{-(k+1)\gamma} + B_2^{(k+1)\gamma} = 0$$

$$(\epsilon^{\gamma} - 2a + \epsilon^{-\gamma}) I_k = 0 \qquad (11)$$

As the voltage and current cannot be zero at all times, it follows that the coefficients must be zero in equations 10 and 11.

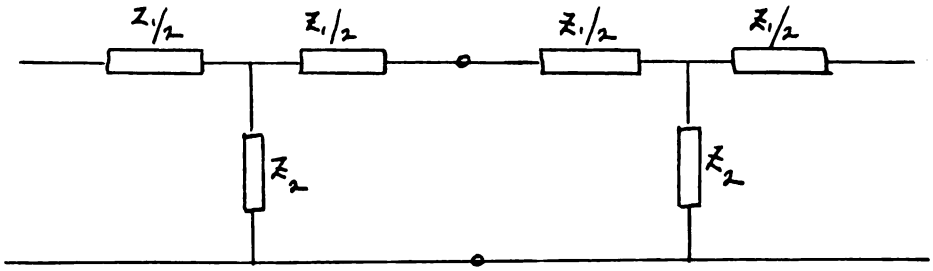
$$\epsilon^{\gamma} - 2a + \epsilon^{-\gamma} = 0$$

$$\epsilon^{\gamma} + \epsilon^{-\gamma} = 2a$$

This can be transformed into hyperbolic cosine-

$$\cosh \gamma = \frac{\epsilon^{\gamma} + \epsilon^{-\gamma}}{2} = a \qquad (12)$$

The H sections can be placed in a T arrangement by adding the impedance of the lower series arm to the upper arm.



Cosh γ can then be evaluated in terms of impedances.

$$\frac{\mathcal{A}}{\mathcal{B}} = \frac{Z_{11}}{|Z|}$$

$$\frac{1}{\mathcal{B}} = \frac{Z_{12}}{|Z|}$$

Hence,

$$\cosh \gamma = a = \frac{Z_{11}}{Z_{12}} = \frac{\frac{1}{2} Z_1 + Z_2}{Z_2}$$

or -

$$\cosh \gamma = 1 + \frac{Z_1}{2 Z_2} \qquad (13)$$

Boundary conditions permit the evaluation of the constants A_1, A_2, B_1 and B_2. Substituting the assumed solution for V_k into equation 3,

$$A_1 \epsilon^{-k\gamma} + A_2 \epsilon^{k\gamma} = Z_{11}(B_1 \epsilon^{-k\gamma} + B_2 \epsilon^{k\gamma}) - Z_{12}(B_1 \epsilon^{-(k+1)\gamma} + B_2 \epsilon^{(k+1)\gamma}) \qquad (14)$$

The coefficients of the term $\epsilon^{-k\gamma}$ on each side of the equation must equal each other if equation 14 is to hold.

$$A_1 = Z_{11} B_1 - Z_{12} B_1 \epsilon^{-\gamma} = B_1 (Z_{11} - Z_{12} \epsilon^{\gamma})$$

Likewise the coefficients of the term $\epsilon^{k\gamma}$ on each side of equation 14 are equal.

$$A_2 = Z_{11} B_2 - Z_{12} B_2 \epsilon^{\gamma} = B_2 (Z_{11} - Z_{12} \epsilon^{\gamma})$$

Then -

$$\frac{A_1}{B_1} = Z_{11} - Z_{12} \epsilon^{-\gamma} = Z_0$$

and -

$$\frac{A_2}{B_2} = Z_{11} - Z_{12} \epsilon^{\gamma} = -Z_0 \tag{15}$$

This is the characteristic impedance of the line, one being interpeted as the impedance looking into the sending end and the other looking into the receiving end of the line. In terms of the parameters of the T section -

$$Z_0 = Z_{12} \sqrt{\frac{Z_{11}}{Z_{11} Z_{12} - Z_{12}^2} - 1} \tag{16}$$

At the boundary of the network, the following conditions are present:

$$\begin{aligned} E_g &= V_0 + I_0 Z_g \\ 0 &= V_k - I_k Z_L \end{aligned} \tag{17}$$

At the input to the network, the voltage and current are given by substituting k equal to zero into equations 9a and 9b.

$$V_0 = A_1 + A_2$$
$$I_0 = B_1 + B_2$$

The voltage and current at the end of the line are obtained by setting k equal to n which is 40 for the particular structure under consideration.

$$V_n = A_1 \epsilon^{-n\gamma} + A_2 \epsilon^{n\gamma}$$
$$I_n = B_1 \epsilon^{-n\gamma} + B_2 \epsilon^{n\gamma}$$

Equation 17 can be written -

$$E_g = A_1 + A_2 + Z_g(B_1 + B_2) \qquad (18)$$
$$0 = A_1 \epsilon^{-n\gamma} + A_2 \epsilon^{n\gamma} - Z_L(B_1 \epsilon^{-k\gamma} + B_2 \epsilon^{n\gamma})$$

Substituting for the constants B_1 and B_2 -

$$\left(\frac{Z_g + Z_0}{Z_0}\right) A_1 - \left(\frac{Z_g - Z_0}{Z_0}\right) A_2 = E_g$$

$$-\left(\frac{Z_L - Z_0}{Z_0}\right) A_1 \epsilon^{-n\gamma} + \left(\frac{Z_L + Z_0}{Z_0}\right) A_2 \epsilon^{n\gamma} = 0$$

Using determinental methods for the solution of the above equations, the values of A_1 and A_2 are found to be:

$$A_1 = \frac{(Z_L + Z_0)\,\epsilon^{n\gamma}}{Z_0\,\Delta} E_g \qquad (19)$$

$$A_2 = \frac{(Z_L - Z_0)\,\epsilon^{-n\gamma}}{Z_0\,\Delta} E_g$$

where Δ is the value of the determinant of the above system of equations. The voltage and current equations for the kth section can be found in terms of the reflection coefficients. Defining the reflection coefficient (r_g) at the sending end -

$$r_g = \frac{Z_g - Z_o}{Z_g + Z_o} \tag{20}$$

and the reflection coefficient at the receiving end -

$$r_L = \frac{Z_L - Z_o}{Z_L + Z_o}$$

The voltage and current at any section is given by the equations -

$$V_k = \frac{E_g\left(Z_o \epsilon^{(n-k)r} + Z_o r_L \epsilon^{-(n-k)r}\right)}{(Z_g + Z_o)(\epsilon^{nr} - r_g r_L \epsilon^{-nr})}$$

$$I_k = \frac{E_g\left(\epsilon^{(n-k)r} - r_L \epsilon^{-(n-k)r}\right)}{(Z_g + Z_o)(\epsilon^{nr} - r_g r_L \epsilon^{-nr})} \tag{21}$$

where k is an integer between zero and n. The output voltage and current is obtained by setting k equal to n[4]

$$V_n = \frac{E_g Z_o}{Z_g + Z_o}\left[\frac{1 + r_L}{\epsilon^{nr} - r_g r_L \epsilon^{-nr}}\right]$$

$$I_n = \frac{E_g}{Z_g + Z_o}\left[\frac{1 - r_L}{\epsilon^{nr} - r_g r_L \epsilon^{-nr}}\right]$$

MATRIX SOLUTION OF LADDER NETWORKS [23]

Matrix algebra may be used to a great advantage in the solution of symmetrical networks thus making the problem easier. Artificial lines are very adept to this method as they consist of a group of symmetrical sections in cascade. The matrix form of the entire ladder network is -

$$\begin{bmatrix} V_0 \\ I_0 \end{bmatrix} = \begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix}^n \begin{bmatrix} V_n \\ I_n \end{bmatrix}$$

Raising a matrix to the n^{th} power is laborious if it had to be accomplished by direct matrix multiplication when n is a large integer. This difficulty can be overcome using theorems of matrix algebra. In the case of symmetrical sections the network can be represented -

$$\begin{bmatrix} \mathcal{A} & \mathcal{B} \\ \mathcal{C} & \mathcal{D} \end{bmatrix} = \begin{bmatrix} \cosh \gamma & Z_0 \sinh \gamma \\ \dfrac{\sinh \gamma}{Z_0} & \cosh \gamma \end{bmatrix}$$

and -

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix}^n = \begin{bmatrix} \cosh \gamma n & Z_0 \sinh \gamma n \\ \frac{\sinh \gamma n}{Z_0} & \cosh \gamma n \end{bmatrix}$$

The general voltage and current in the line is obtained by finding the inverse of the square matrix on the right of equation 22. In matrix form the output of a n section network is -

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}^{-n} \begin{bmatrix} V_0 \\ I_0 \end{bmatrix}$$

Performing the inversion gives -

$$\begin{bmatrix} V_n \\ I_n \end{bmatrix} = \begin{bmatrix} \cosh \gamma n & -Z_0 \sinh \gamma n \\ -\frac{\sinh \gamma n}{Z_0} & \cosh \gamma n \end{bmatrix} \begin{bmatrix} V_0 \\ I_0 \end{bmatrix}$$

PROPAGATION FUNCTION CHARACTERISTICS

The propagation constant and characteristic impedance play an important role in transmission phenomena and their variations with frequency determine the fidelity of signal or voltage transmission. Types of terminations also have effects on the fidelity of transmission, but these conditions can usually be changed in such a manner to eliminate their contribution to distortion.

γ is complex in nature having a real and an imaginary part.

$$\gamma = \alpha + j\beta$$

The real part α is the attenuation function and gives the amount of energy dissipated while passing through a unit length of line. The imaginary part β is the phase function per unit length of line. In the case of an artificial line the unit length is one section which approximates a given length of actual line. This particular artificial line approximates a pair of 104 mil wires spaced twelve inches apart, having a length of 6.32 miles. This propagation function is of the general form -

$$\gamma = \sqrt{ZY}$$

or-
$$\alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)} \qquad (25)$$

Squaring both sides of the equation -

$$\alpha^2 - \beta^2 + j\,2\alpha\beta = (RG - LC\omega^2) + j\,\omega(RC + LG)$$

Equating the reals and imaginaries -

$$\alpha^2 - \beta^2 = RG - LC\omega^2$$

$$2\alpha\beta = \omega(RC + LG)$$

The expression for attenuation α is-

$$\alpha = \left[\tfrac{1}{2}\left((RG - LC\omega^2) + \sqrt{(R^2 + L^2\omega^2)(G^2 + C^2\omega^2)}\right)\right]^{1/2}$$

The expression for phase shift β is- (26)

$$\beta = \left[\tfrac{1}{2}\left(LC\omega^2 - RG + \sqrt{(R^2 + L^2\omega^2)(G^2 + C^2\omega^2)}\right)\right]^{1/2}$$

These expressions for attenuation and phase shift are very cumbersome but it is possible to determine the effect of frequency variation. However, they will not give the entire story over a wide range of frequencies as the parameters R, G, L, and C do not remain constant.

Experimental data show that resistance variation with frequency in open wires runs from 250% to 400% over a frequency range of 500 to 50,000cps. The exact percentage depends upon the physical characteristics of the wire. Inductance variation decreases 4% in a frequency range of 0 to 50,000 cps. Capacitance which theoretically should not vary a measurable amount increases about 4% in a frequency range of 0 to 50,000 cps. Leakage parameter (G) varies the greatest amount; in a frequency range of 1000 to 50,000 cps, this variation is 5000%, almost a linear function of frequency. With good insulation, however the leakage is still negligible. [13]

These same characteristics exist in the artificial line as is shown in the table below. The parameter values are computed from impedance measurements obtained by bridge methods.

frequency	796 cps	1200 cps	2000
R	56.8 Ω	56.9 Ω	56.7 Ω
L	22.7 mh	22.6 mh	23.0 mh
G	1.5 μmho	1.5 μmho	1.4 μmho
C	.042 μf	.042 μf	.043 μf

It would seem that prediction of line behavior is a hopeless proposition with all of these variables involved. However, an engineering solution can be obtained by certain approximations, and the effect of various parameters can be determined by deviations from an ideal case. Most engineering formulation falls in this catagory. The ideal conditions for a transmission system are constant attenuation for all frequencies and linear variation with frequency for the phase function.

$$\alpha = \sqrt{RG}$$
$$\beta = \omega\sqrt{LC} \tag{27}$$

Equation 26 can be put into the following form:

$$\alpha = \sqrt{RG}\; f_1(x)$$
$$\beta = \omega\sqrt{LC}\; f_2(x) \tag{28}$$

where $f_1(x)$ and $f_2(x)$ are deviations from the ideal case.

$$f_1(x) = \frac{1}{\sqrt{2}}\left(1 - x^2 + \sqrt{1 + 2(m^2+n^2)x^2 + x^4}\right)^{\frac{1}{2}}$$

$$f_2(x) = \frac{1}{\sqrt{2}}\left(1 - x^{-2} + \sqrt{1 + 2(m^2+n^2)x^{-2} + x^{-4}}\right)^{\frac{1}{2}}$$

The terms in the above equations are defined as-

$$m = \frac{1}{2}\left(\sqrt{\frac{RC}{LG}} + \sqrt{\frac{LG}{RC}}\right)$$

$$n = \frac{1}{2}\left(\sqrt{\frac{RC}{LG}} - \sqrt{\frac{GL}{RC}}\right) \tag{29}$$

$$x = \frac{\omega}{\sqrt{\frac{RG}{LC}}}$$

The leakage parameter cannot be neglected at this point, even though it is small. It is quite evident that a relationship exists between the two error functions $f_1(x)$ and $f_2(x)$. This is shown by letting

$$y = \frac{1}{2}\left(x - \frac{1}{x}\right)$$

then-

$$x = \sqrt{y^2+1} + y$$

Equation 29 can now be written-

$$f_1(\alpha) = \alpha^{\frac{1}{2}}\left(\sqrt{m^2+y^2}-y\right)^{\frac{1}{2}}$$
$$f_2(\alpha) = \alpha^{-\frac{1}{2}}\left(\sqrt{m^2+y^2}+y\right)^{\frac{1}{2}} \qquad (30)$$

or -

$$f_1(\alpha)\cdot f_2(\alpha) = m \qquad (31)$$

The value of m for the artificial line is-

$$m = \frac{1}{2}\left(\sqrt{\frac{56.7\,(.042\times10^{-6})}{(22.7\times10^{-3})(1.5\times10^{-6})}} + \sqrt{\frac{(22.7\times10^{-3})(1.5\times10^{-6})}{56.7\,(.042\times10^{-6})}}\right)$$

$$m = 4.2$$

Conditions which give a distortionless line are $\frac{R}{L} = \frac{G}{C}$. This makes the value of m be one and the value of n be zero. Also -

$$f_1(\alpha) = f_2(\alpha) = 1$$

The equations for distortionless propagation function are then obtained by substituting these conditions into equation 28. It is apparent that distortion can be expected on the artificial line.

RESPONSE FUNCTION

The response function is a variable of frequency which is the outgrowth of the ratio of output voltage to the input voltage. Let us now consider one section of the artificial line which is terminated in the characteristic impedance. The response function of one section is -

$$\frac{V_{k+1}(t)}{V_k(t)} = \epsilon^{-\gamma} = \epsilon^{-(\alpha+j\beta)} = N(\omega)\,\epsilon^{-j\beta} \qquad (32)$$

For the distortionless case -

$$N(\omega) = \epsilon^{-\sqrt{RG}}$$

and

$$\epsilon^{-j\beta} = \epsilon^{-j\omega\sqrt{LC}}$$

The total effect of the line can then be determined by raising the amplitude function to the n th power and adding the phase characteristics of the n sections.

$$\frac{V_n(t)}{V_0(t)} = \epsilon^{-n\sqrt{RG}}\,\epsilon^{-j\omega n\sqrt{LC}} \qquad (33)$$

In any system having phase shift, the output signal may be small for a time, after which it increases rapidly giving rise to a time delay. This delay depends on the slope of the phase function versus frequency curve.

$$t_d = \left|\frac{\partial \beta}{\partial \omega}\right|$$

For the distortionless line, the time delay is constant.

$$t_d = n\sqrt{LC} \qquad (34)$$

$$t_d = 40\sqrt{(22.7\times10^{-3})(.042\times10^{-6})} = 123\times10^{-6}\ \text{sec}$$

The Fourier Transforms are strong tools in changing functions from frequency to time variables and reversing the procedure. This enables the manipulation of such equations as -

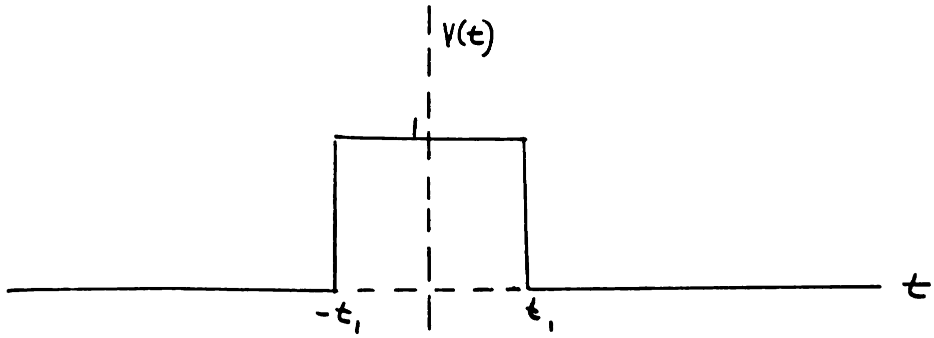
$$V_2(t) = A(\omega)\, V_1(t) \qquad (35)$$

which are often used in circuits. These transforms are-

$$V(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \epsilon^{j\omega t}\, V(\omega)\, d\omega$$

$$V(\omega) = \int_{-\infty}^{\infty} \epsilon^{-j\omega t}\, V(t)\, dt \qquad (36)$$

Applying these transforms to a pulse function will enable the prediction of a particular network. Pulse response of networks is proving to be very satisfactory in determining the transient behavior of the network. Consider the pulse of width t_1 applied at time equal to zero. Since the Fourier Transforms take in negative values to time, the function of positive time is reflected to the left of the origin. This does not have any physical significance but is necessary to be able to use this mathematical tool.



Applying the transform to the pulse of unit magnitude-

$$V(\omega) = \int_{-t_1}^{t_1} \epsilon^{-j\omega t_1} dt = \frac{2 \sin \omega t_1}{\omega} \tag{37}$$

This transform of the input voltage multiplied by the response function will give the transform of the output voltage.

$$V_n(\omega) = \epsilon^{-n\sqrt{RG}} \epsilon^{-j\omega t_d} \frac{2 \sin \omega t_1}{\omega} \tag{38}$$

Although in the distortionless line, all frequencies are transmitted, filter theory shows that a cut off frequency exists for the artificial line.

$$\omega_c = \frac{2}{\sqrt{LC}} = \frac{2}{\sqrt{(22.7 \times 10^{-3})(.042 \times 10^{-6})}} = 6.48 \times 10^4 \frac{rad}{sec} \tag{39}$$

Beyond this frequency, the amplitude response is zero. The transformation integral then becomes:

$$V_n(t) = \frac{\epsilon^{-n\sqrt{RG}}}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{2 \sin \omega t_1}{\omega} \epsilon^{j\omega(t-t_d)} d\omega$$

$$V_n(t) = \frac{\epsilon^{-n\sqrt{RG}}}{\pi} \int_{-\omega_c}^{\omega_c} \frac{1}{\omega} \left[\cos \omega(t-t_d) \sin \omega t_1 + j \sin \omega (t-t_d) \sin \omega t_1\right] d\omega \tag{40}$$

The second term of the integrand is an odd function and the first term is an even function. When the integration is performed, the odd functions drop out so they can be immediately eliminated. The first term can be rewritten -

$$\frac{1}{2\omega}\left[\sin\omega(t-t_d+t_1)-\sin\omega(t-t_d-t_1)\right]$$

then-

$$V_n(t)=\frac{\epsilon^{-n\sqrt{RG}}}{\pi}\int_0^{\omega_c}\frac{\sin\omega(t-t_d+t_1)}{\omega}\,d\omega-\frac{\epsilon^{-n\sqrt{RG}}}{\pi}\int_0^{\omega_c}\frac{\sin\omega(t-t_d-t_1)}{\omega}\,d\omega$$

$$V_n(t)=\frac{\epsilon^{-n\sqrt{RG}}}{\pi}\left[Si\,\omega_c(t-t_d+t_1)-Si\,\omega_c(t-t_d-t_1)\right] \qquad (41)$$

Cut off frequency plays an important role in fidelity of transmission. As the width of the pulse is decreased, the fidelity of transmission increases.

Reflections caused by terminations other than the characteristic impedance will give rise to greater echos. The response function is more complex in nature and great difficulty is encountered in solving the equation. In terms of the reflection coefficients, the response function at the k section is -

$$\frac{V_k}{E_g}=\frac{Z_0}{Z_g+Z_0}\left[\frac{\epsilon^{(n-k)\gamma}+r_L\epsilon^{-(n-k)\gamma}}{\epsilon^{n\gamma}-r_g r_L\epsilon^{-n\gamma}}\right] \qquad (42)$$

$$= H'(\omega)\,\epsilon^{j\theta'}$$

ERROR INTRODUCED IN LUMP LINES

The question arises as to the extent of the error introduced by lumping line parameters and over what range of frequencies the artificial line will approximate an actual line. Let us first consider the propagation function. In the actual line -

$$\gamma l = l\sqrt{(R+j\omega L)(G+j\omega C)} \qquad (43)$$

where l is the length of the line. The propagation function of the artificial line is given by the product of the number of sections in cascade and the propagation per section.

$$\gamma' l = 2n \sinh^{-1}\sqrt{\frac{Z_1}{4Z_2}} \qquad (44)$$

The series and shunt impedances of the symmetrical H section take the form-

$$Z_1 = \frac{l}{n}(R+j\omega L)$$

$$\frac{1}{Z_2} = \frac{l}{n}(G+j\omega C) \qquad (45)$$

The value of $\sqrt{\frac{Z_1}{4Z_2}}$ can be determined by substitution

$$\sqrt{\frac{Z_1}{4Z_2}} = \frac{1}{2}\sqrt{\frac{l}{n}(R+j\omega L)\frac{l}{n}(G+j\omega C)} = \frac{\gamma l}{2n} \qquad (46)$$

hence-

$$\gamma' n = 2n \sinh^{-1} \frac{\gamma l}{2n} \tag{47}$$

Using the series expansion for

$$\gamma' l = \gamma l \left[1 - \frac{1}{6}\left(\frac{\gamma l}{2n}\right)^2 + \frac{3}{40}\left(\frac{\gamma l}{2n}\right)^4 - - - - - \right]$$

$$\gamma' l = \gamma l \left[1 + \delta_\gamma \right] \tag{48}$$

The error in the propagation function is-

$$\delta_\gamma = -\frac{1}{6}\left(\frac{\gamma l}{2n}\right)^2 = -\frac{1}{6}\left(\frac{l}{n}\right)^2 (R+j\omega L)(G+j\omega C) \tag{49}$$

The characteristic impedance of the actual line is-

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \tag{50}$$

For a symmetrical section the characteristic impedance

is-

$$Z_0' = \sqrt{Z_1 Z_2} \sqrt{1 + \frac{Z_1}{4Z_2}} \tag{51}$$

Substituting equation 46 for

$$Z_0' = Z_0 \sqrt{1 + \left(\frac{\gamma l}{2n}\right)^2} \tag{52}$$

Using the binomial expansion for the radical-

$$Z_0' = Z_0 \left[1 + \frac{1}{2}\left(\frac{\gamma l}{2n}\right)^2 - \frac{1}{8}\left(\frac{\gamma l}{2n}\right)^4 - - - - - \right] \tag{53}$$

$$Z_0' = Z_0 \left[1 + \delta_z \right]$$

The error in the characteristic impedance is-

$$\delta_z = \frac{1}{2}\left(\frac{\gamma l}{2n}\right)^2 = \frac{1}{8}\left(\frac{l}{n}\right)^2 (R+j\omega L)(G+j\omega C) \tag{54}$$

The error in the characteristic impedance is of opposite sign and is three times the error in the propagation function. For a given frequency the error is proportional to the length squared and inversely proportional to the number of sections squared. As the number of sections is increased for a given length the error approaches zero. The limiting case where n is infinite is the smooth line.

Considering a given length and number of sections, the error increases with frequency. At high frequencies, the resistance and conductance are negligible, hence the error increases as the square of the frequency. The opposite sign of the errors indicates that some compensation exists; however, the highest essential frequency to be passed will determine the number of sections to be used, allowing a certain limit of error.

The error introduced at the highest frequency passed can be easily calculated. At 10,000 cps the resistance and leakage terms are small enough to be neglected. The error in the characteristic impedance is -

$$\delta_{3} = \frac{1}{8}\left(\frac{\ell}{n}\right)^2 \omega^2 LC$$

$$= \frac{1}{8}\left(\frac{253}{40}\right)^2 (2\pi \times 10^4)^2 (22.6 \times 10^{-3})(.042 \times 10^{-6}) \qquad (55)$$

$$= 18.7$$

The characteristic impedance is approximately 690 ohms for a wide range of frequencies in the audio range. Hence the percentage error for the artificial line is-

$$\frac{18.7}{690}\ 100 = 2.71\ \%$$

TRANSIENT SOLUTION OF A TRANSMISSION LINE

Methods of transient solution vary depending on the type of input voltage function. In general the Laplacian and Fourier transforms are superior to other methods of attack. However in the case of the unit step function, the results of Laplacian transforms lead to the Heaviside expansion theorem in the evaluation of the inverse of the Laplace. Since it is desired to compare the transient solution of the artificial to the actual transmission line, the operational methods of Heaviside will suffice.

The solution of the wave equation for the transmission line is - [10]

$$V_x = A_1 \epsilon^{\gamma x} + A_2 \epsilon^{-\gamma x}$$

$$I_x = \frac{1}{Z_0}\left[A_1 \epsilon^{\gamma x} - A_2 \epsilon^{-\gamma x}\right]$$

where the constants are evaluated by boundary conditions of the line terminations. These equations can also be written in terms of the parameters of the line. Using the following notations-

$$v = \frac{1}{\sqrt{LC}}$$

$$a = \frac{R}{2L}$$

$$b = \frac{G}{2C}$$

$$\rho = a + b$$

$$\sigma = a - b$$

The general transmission equations then become-

$$V_x = A_1 \epsilon^{\gamma x} + A_2 \epsilon^{-\gamma x}$$

$$I_x = \frac{1}{Lv}\sqrt{\frac{p+2b}{p+2a}}\left(A_1 \epsilon^{\gamma x} - A_2 \epsilon^{-\gamma x}\right) \qquad (5\text{-}6a)$$

where-

$$\gamma = \frac{1}{v}\sqrt{(p+\rho)^2 - \sigma^2}$$

Considering the boundary of the line terminated in a short circuit:

$$V_x = V_g \quad \text{at} \quad x = 0$$

$$V = 0 \quad \text{at} \quad x = l$$

Hence the equations of voltage and current at point x from the transmitter when the unit step voltage is applied become:

$$V_x = V_g \frac{\sinh \gamma(l-x)}{\sinh \gamma l} \, 1$$

$$I_x = \frac{V_g}{Lv}\sqrt{\frac{p+2b}{p+2a}} \; \frac{\cosh \gamma(l-x)}{\sinh \gamma l} \, 1$$

Solving these equations with Heaviside's expansion formula which is -

$$I_x = V\left[\frac{Y(0)}{Z(0)} + \sum \frac{Y(p)\,\epsilon^{pt}}{p\,Z'(p)}\right]$$

The determinental solution of -

$$Z(p) = 0 = \sinh \gamma l$$

is- $\gamma l = j\, s\pi \qquad s = 1, 2, 3 \cdots$

$$\gamma = \frac{js\pi}{l} = \frac{1}{v}\sqrt{(p+\rho)^2 - \sigma^2}$$

solving for p -

$$(p+\rho)^2 = \sigma^2 - \frac{s^2\pi^2 v^2}{l^2}$$

$$p_s = -\rho \pm \left[\sigma^2 - \frac{s^2\pi^2 v^2}{l^2}\right]^{1/2}$$

$$= -\rho \pm j\beta_s$$

Next taking the derivative of $Z(p)$

$$\frac{\partial Z(p)}{\partial p} = \cosh\gamma l \;\frac{\partial(\gamma l}{\partial p}$$

$$= \frac{l^2}{v^2}(p+\rho)\;\frac{\cosh\gamma l}{\gamma l}$$

For $p = 0$, the first term of the expansion is -

$$\frac{Y(0)}{Z(0)} = \frac{\sinh \frac{2\sqrt{ab}}{v}(l-x)}{\sinh \frac{2\sqrt{ab}}{v}\, l} = \frac{\sinh d(l-x)}{\sinh d l}$$

Substituting these equations in the expansion formula the developed solution for the voltage at point x is-

$$V_x = V_g\,\frac{\sinh d(l-x)}{\sinh dl} + V_g \sum \frac{j \sin s\pi\left(1-\frac{x}{l}\right)\left(\epsilon^{-\rho t}\epsilon^{\pm j\beta_s t}\right)}{\pm j\beta_s(-\rho \pm j\beta_s)\frac{l^2}{v^2}\cos(s\pi)\;\frac{1}{js\pi}}$$

which can be simplified to-

$$V_x = V_g\,\frac{\sinh d(l-x)}{\sinh dl} - \frac{2V_g v^2\pi}{l^2}\epsilon^{-\rho t}\sum s \sin\frac{s\pi x}{l}\,\frac{(\rho\sin\beta_s t + \beta_s\cos\beta_s t)}{\beta_s(\rho^2+\beta_s^2)}$$

This is the complete solution for the voltage. The first term gives the steady state condition and the summation term gives the transient state. The summation extends for all integral values of s from zero to infinity.

The current expression can be obtained in the same manner and is-

$$I_x = V_g \frac{\cosh \alpha(l-x)}{\sinh \alpha l} - \frac{2Ev^2}{l} \epsilon^{-\rho t} \sum \cos \frac{s\pi x}{l} \left\{ \frac{G\rho \sin \beta_s t + G\beta_s \cos \beta_s t}{\beta_s (\rho^2 + \beta_s^2)} - C \frac{\sin \beta_s t}{\beta_s} \right\} \qquad (63)$$

Applying the condition of negligible conductance or leakage, the above equations can be reduced to-

$$V_x = V_g \left(1 - \frac{x}{l}\right) - \frac{2V_g \pi v^2}{l^2} \epsilon^{-at} \sum \frac{s \sin \frac{s\pi x}{l} (a \sin \beta_s t + \beta_s \cos \beta_s t)}{\beta_s (a^2 + \beta_s^2)}$$

$$I_x = \frac{2Ev^2}{l} C \epsilon^{-at} \sum \cos \frac{s\pi x}{l} \frac{\sin \beta_s t}{\beta_s}$$

TRANSIENT SOLUTION OF AN ARTIFICIAL LINE

The general equations applicable to any artificial line are - [3]

$$i_p = \frac{V \cosh (n-k)\gamma}{Z_2 \sinh \gamma \sinh n\gamma} \tag{65}$$

$$\cosh \gamma = 1 + \frac{Z_1}{2 Z_2}$$

However, the input and output are the most considered formulas. For the artificial line of negligible leakage, the series and shunt impedances take the form-

$$Z_1 = Lp + R$$

$$Z_2 = \frac{1}{Cp}$$

and-

$$\cosh \gamma = 1 + \tfrac{1}{2} Cp (Lp + R) \tag{66}$$

The determinental solution is-

$$Z(p) = Z_2 \sinh \gamma \sinh n\gamma = 0$$

hence

$$\sinh \gamma = 0$$

or-

$$\gamma = \frac{j s \pi}{n} \qquad s = 1, 2, 3 \text{ ----}$$

then-

$$\cosh \gamma = \cos \frac{s\pi}{n} = 1 + \tfrac{1}{2} Cp (Lp + R)$$

solving for p

$$P_s = -\frac{R}{2L} \pm \frac{\sqrt{R^2C^2 - 8LC\left(1 - \cos\frac{s\pi}{n}\right)}}{2LC}$$

$$= -a \pm j\sqrt{4\nu^2 \sin^2\frac{s\pi}{2n} - a^2}$$

Taking the derivative of $Z(p)$

$$\frac{\partial Z(p)}{\partial p} = n Z_2 \sinh\gamma \cosh\gamma n \frac{\partial \gamma}{\partial p}$$

However,

$$\sinh\gamma\frac{\partial\gamma}{\partial p} = \frac{\partial(\cosh\gamma)}{\partial p} = LCp + \frac{1}{2}RC$$

then-

$$\frac{\partial Z(p)}{\partial p} = nZ_2\left(LCp + \frac{1}{2}RC\right)\cosh n\gamma$$

$$= \frac{nL}{p}(p+a)\cos s\pi$$

As p approaches zero, the value of $\cosh\gamma$ approaches one. The series expansion for

$$\cosh\gamma = 1 + \frac{\gamma^2}{2} + - - - - - -$$

substitution for γ^2 gives -

$$\cosh\gamma = 1 + \frac{1}{2}(LCp^2 + RCp)$$

and-

$$Z(p)_{p=0} = Z_2 \sinh\gamma \sinh n\gamma = \frac{n\gamma^2}{Cp}$$

Substituting into the expansion formula

$$I = \frac{V}{nR} + V\sum\frac{\epsilon^{pt}}{nL(p+a)\cos(s\pi)}$$

$$I = \frac{V}{nR} + \frac{2V\epsilon^{-at}}{nL}\sum\frac{\sin\sqrt{4\nu^2\sin^2\frac{s\pi}{2n} - a^2}\,t}{\cos(s\pi)\sqrt{4\nu^2\sin^2\frac{s\pi}{2n} - a^2}}$$

The frequency of the damped transient oscillation is-

$$\omega_s = \sqrt{4v^2 \sin^2\frac{s\pi}{2n} - a^2}$$

If n is very large

$$\sin\frac{s\pi}{2n} \simeq \frac{s\pi}{2n}$$

then the current takes the form of-

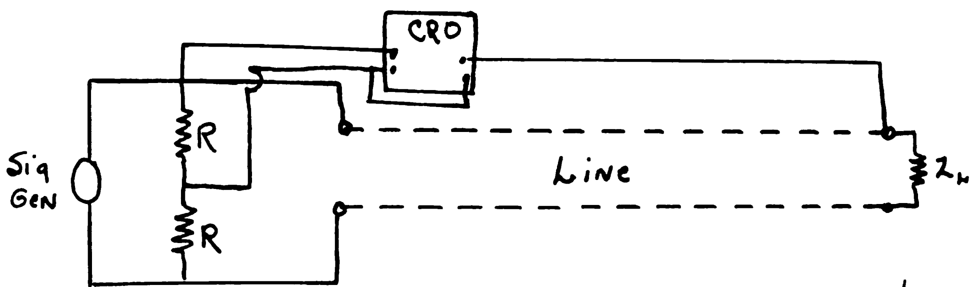
$$I = \frac{V}{nR} + \frac{2V_g \epsilon^{-at}}{nL} \sum \frac{\sin\sqrt{\frac{v^2s^2\pi^2}{n^2} - a^2}\; t}{\cos s\pi \sqrt{\frac{v^2s^2\pi^2}{n^2} - a^2}}$$

The summation on s is from zero to infinity. This equation is of the same form as that of the uniform line. Hence in all transient solutions, the artificial line approximates the uniform line if the number of sections is large.

RESPONSE FUNCTION MEASUREMENTS

The response function, a complex quantity relating the input voltage to the output voltage, is readily obtained by measuring the amplitude and phase shift. The amplitude response function of the artificial line was obtained by using an audio oscillator as a voltage source and making output voltage measurements with a vacuum tube voltmeter.

A DuMont type 274 cathode ray oscilloscope was used to determine the phase shift.



Circuit for Phase Shift Measurements

If two voltages of the same frequency are placed on the vertical and horizontal plates of the cathode ray tube, the resultant pattern on the screen will be a straight line if the two waves are in phase or 180 degrees out of phase. When a phase shift of 90 or 270 exists an ellipse will appear on the screen having its major axis in a vertical position. The angle of the major axis determines the phase shift. [1].

Voltage Measurements

Line Termination - 690 ohms resistance

Frequency	V_o	V_r	V_r/V_o
500 cps	10 v	1.5 v	.15
1000 cps	10	1.9	.19
2000 cps	10	2.1	.21
3000 cps	10	1.4	.14
4000 cps	10	1.7	.17
5000 cps	10	1.2	.12

Voltage Measurements

Line Termination - 1400 ohms resistance

Frequency	V_o	V_r	V_r/V_o
500 cps	10 v	1.6 v	.16
1000	10	2.0	.20
2000	10	3.0	.30
2500	10	2.2	.22
3000	10	1.4	.14
3500	10	1.8	.18
4000	10	3.2	.32
4500	10	2.4	.24
5000	10	1.5	.15

Phase Measurements

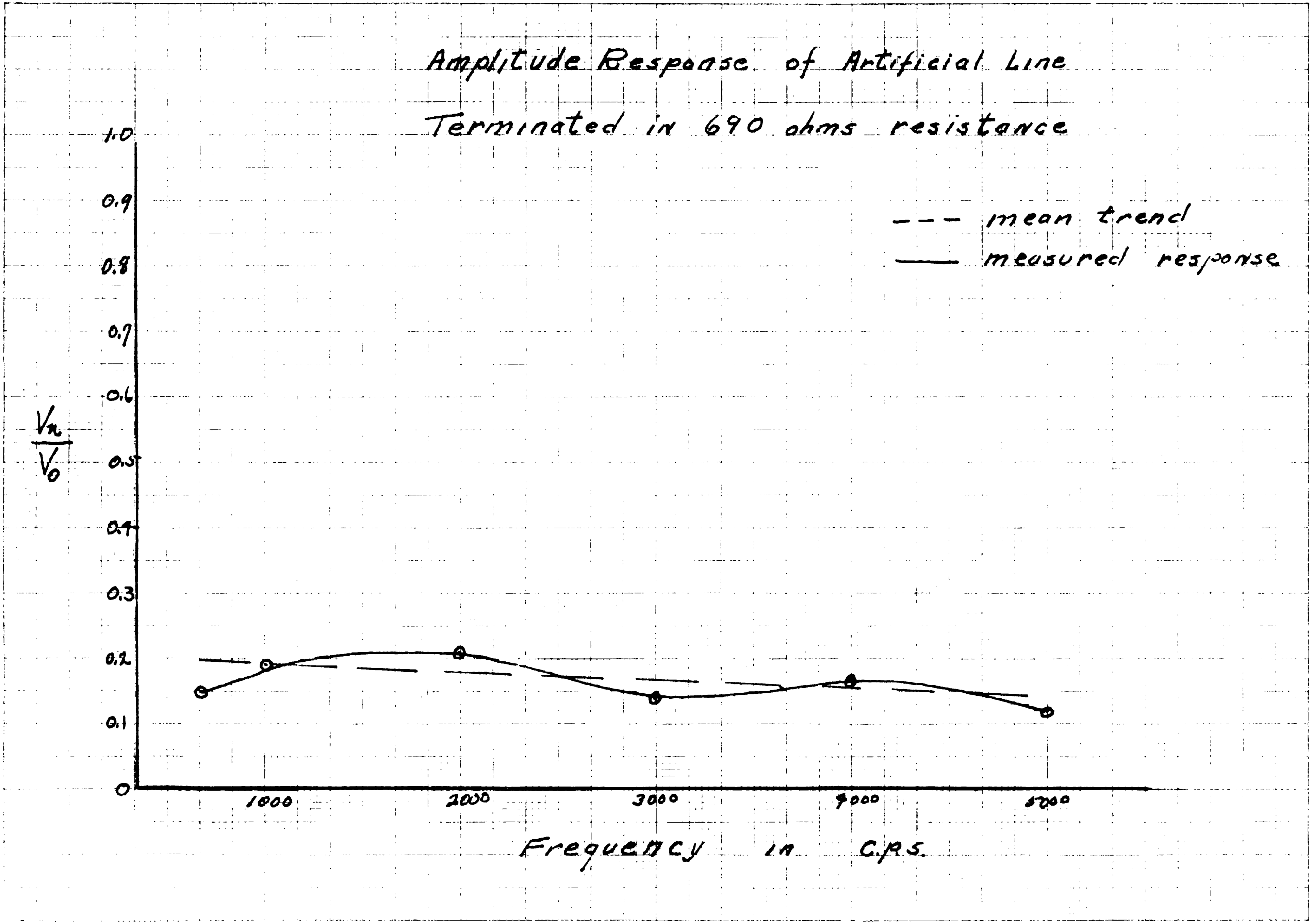
Line Termination - 690 ohms resistance

Frequency	Phase Shift
1000 cps	$.25\pi$
2000	$.50\pi$
3000	$.40\pi$
4000	$.75\pi$
5000	1.25π

Phase Measurements

Line Termination   1400 ohms resistance

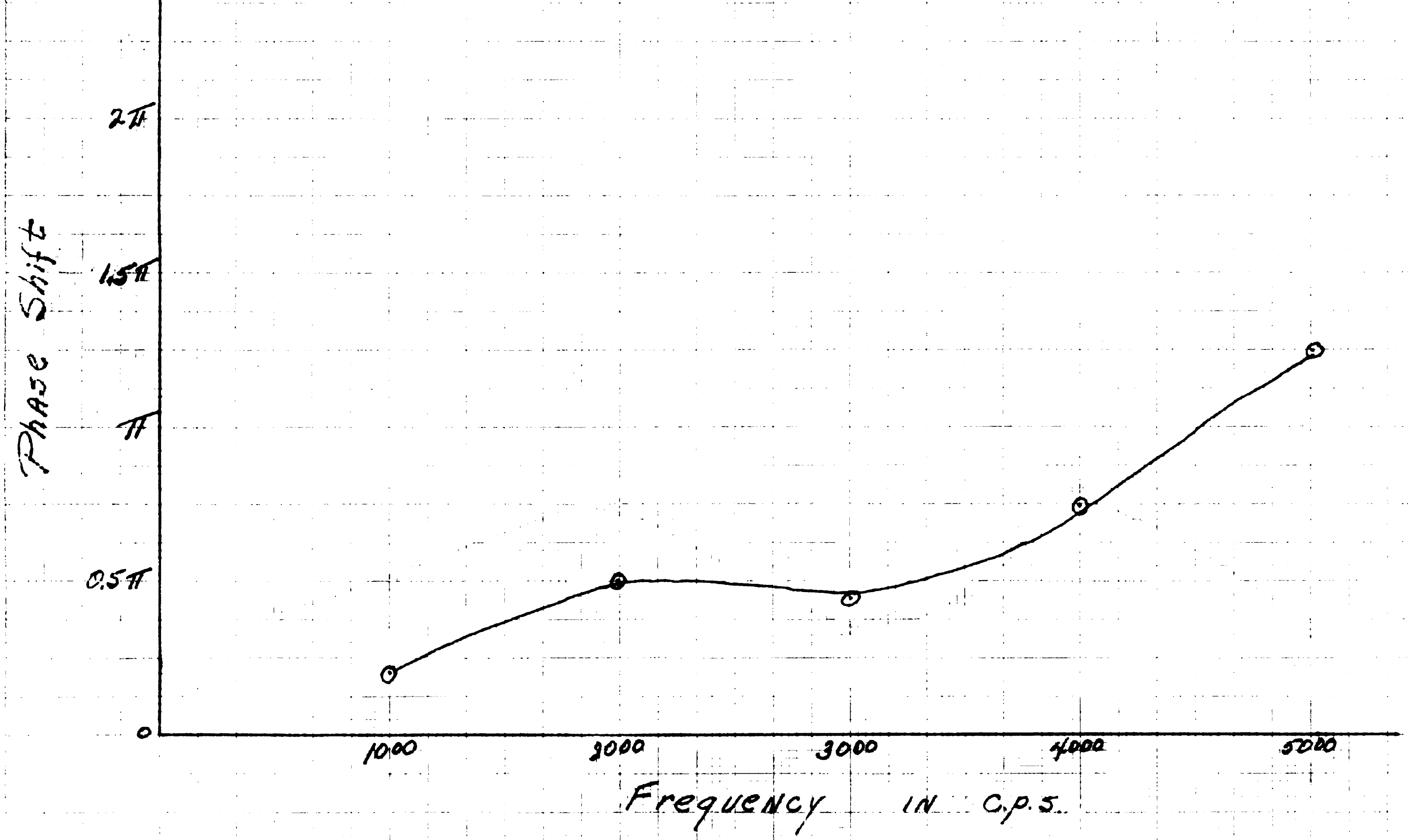
Frequency	Phase Shift
1000 cps	.40 π
2000	.8 π
3000	.4 π
4000	.8 π
5000	1.2 π

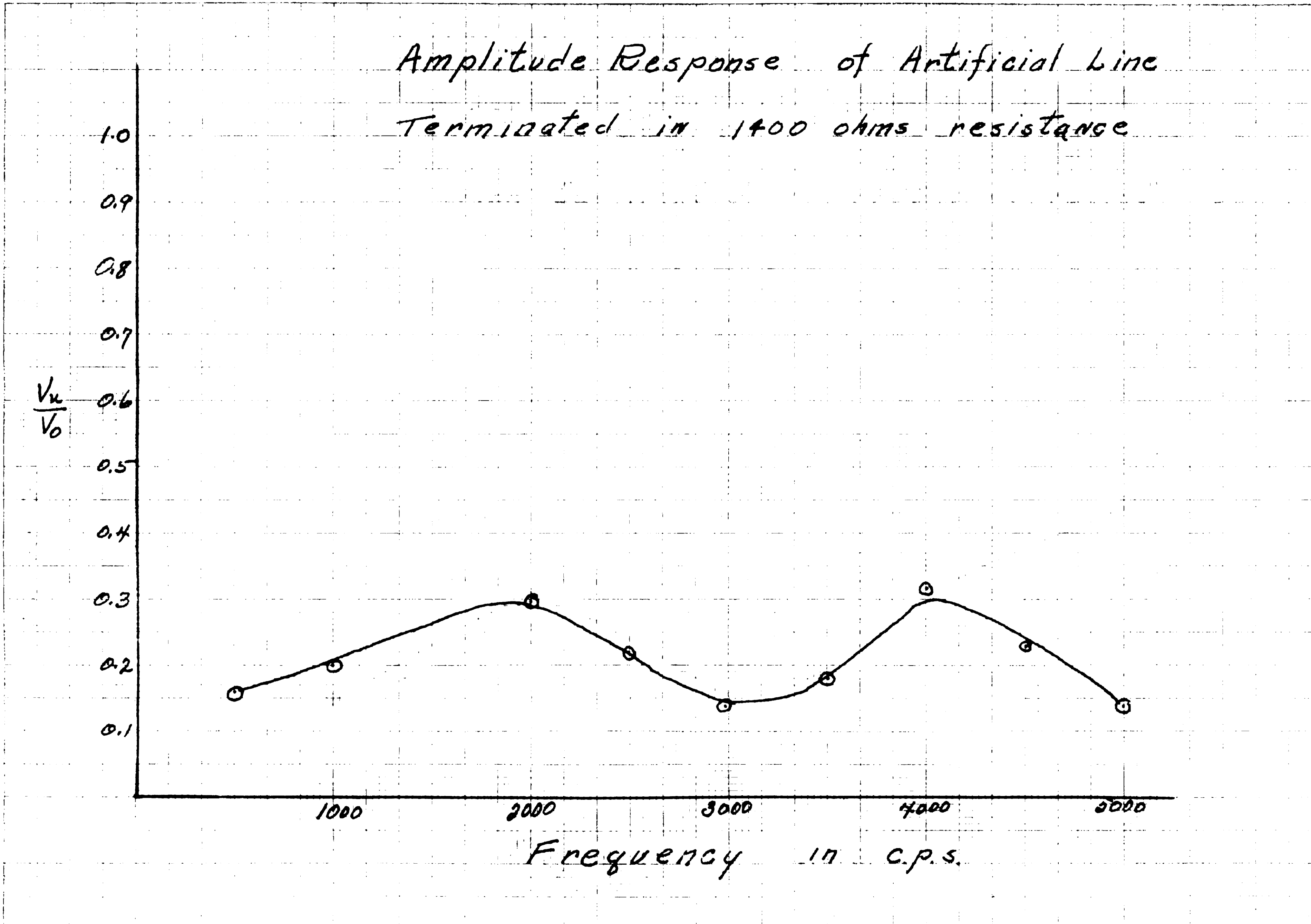


Amplitude Response of Artificial Line
Terminated in 690 ohms resistance
--- mean trend
— measured response
$\frac{V_n}{V_0}$
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
1000
2000
3000
4000
5000
Frequency in C.P.S.

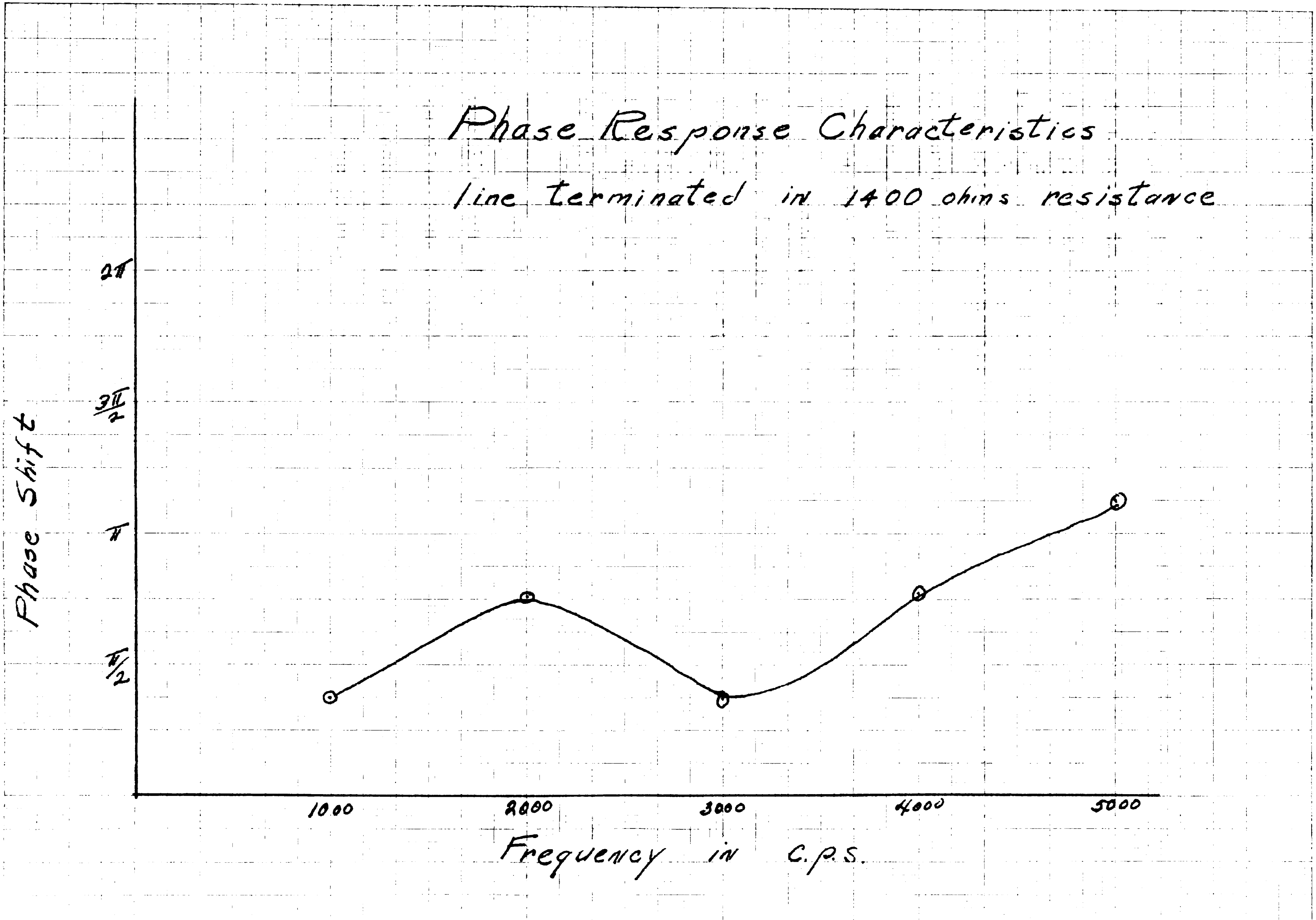
Phase Response Characteristics

Line terminated in 690 ohms resistance





Amplitude Response of Artificial Line
Terminated in 1400 ohms resistance
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
$\frac{V_k}{V_0}$
1000
2000
3000
4000
5000
Frequency in c.p.s.



Phase Response Characteristics
line terminated in 1400 ohms resistance
Phase Shift
2π
3π/2
π
π/2
1000
2000
3000
4000
5000
Frequency in C.P.S.

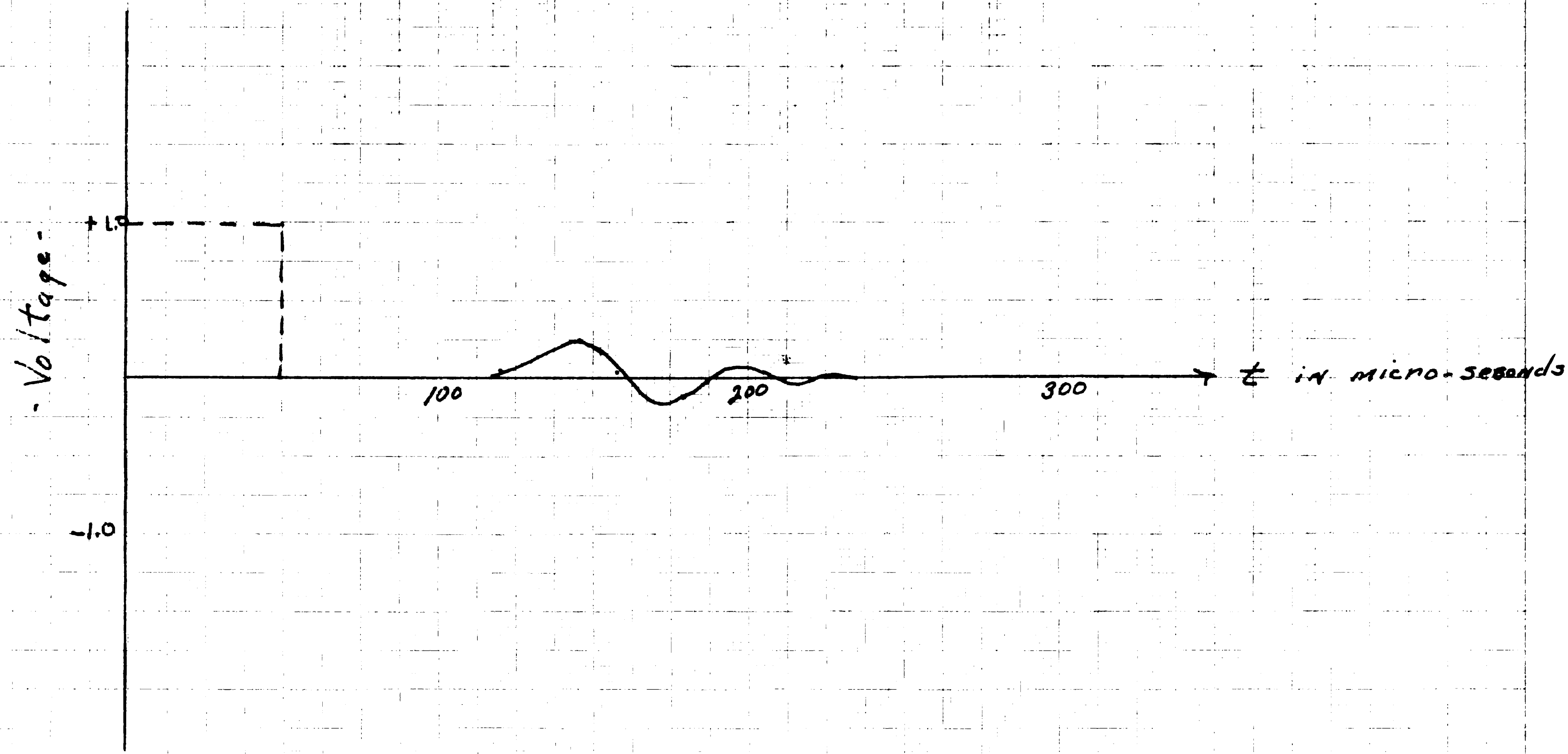
The amplitude response function with the line terminated in the characteristic impedance exhibits distortion in that it oscillates about a mean line at approximately 0.19. This is expected in that the ratio $\frac{R}{L}$ does not equal the ratio $\frac{G}{C}$ which is a necessity if transmission of a signal is to be perfectly distortionless. The fact that these curves indicate the existence of echos is shown by comparing the curves of the two terminations. With the termination of twice the characteristic impedance, the amplitude function deviates farther from a straight line as the termination causes greater reflection which will give rise to more echos. It would be possible to eliminate this distortion if a means for varying the inductance in each section of the line existed.

The phase function did not show as great an oscillation difference in the two terminations as the terminations contained no reactive components in their impedances. However, phase distortion is also present in the line.

The output of an impressed pulse function can now be computed with the use of equation 41. The mean amplitude for the frequency range of 1000 to 5000 cps is taken from the curves to be about 0.19. The general tendency for the curve is to decrease slightly but a flat response assumption will not introduce too great an error. Tables for the Si (x) function make plotting the response possible.

Computed Response To pulse of unit magnitude

Pulse width - 50 micro-seconds



CONCLUSION

Although experimental results of transient behavior have not been obtained in photographic form, the sinusoidal behavior of networks present the best method of determining pulse and step function responses. In order that fidelity of transmission can be observed, more equipment is needed. A multi-trace oscilloscope with provisions for micro-second timing is a necessity for thorough studies of transient behavior. Time delays and variations of pulse widths on fidelity can only be observed when both input and output can be viewed simultaneously. Equipment of this nature would be a great asset but it would also mean a large investment.

Various photographs have been obtained of transmission phenomena with respect to given types of response function curves and the results will apply to the response curves for the M. S. C. Artificial Line. This artificial line is well suited for the demonstration of transmission line theory.

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