MAGNETIC LOSSES IN TRANSMISSION LINE TOWERS HAVING A CLOSED MAGNETIC PATH SURROUNDING ONE CONDUCTOR

Thesis for the Degree of M. S.

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1928

THESIS

Electric lines
7 Hysteresis
Title

Electrical engineering

MAGNETIC LOSSES IN TRANSMISSION LINE TOWERS HAVING A CLOSED MAGNETIC PATH SURROUNDING ONE CONDUCTOR.

A Thesis Submitted to

The Faculty of the

MICHIGAN STATE COLLEGE

By

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Candidate for the Degree

of

Master of Science.

June 1928.

THESIS

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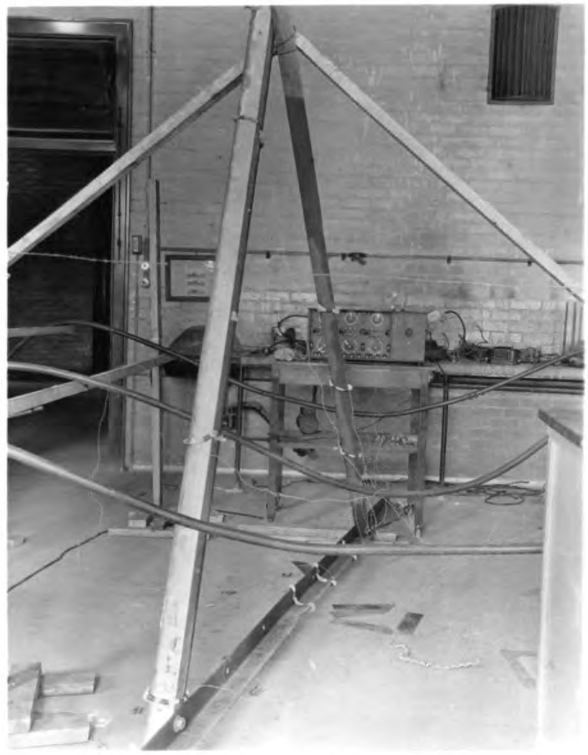
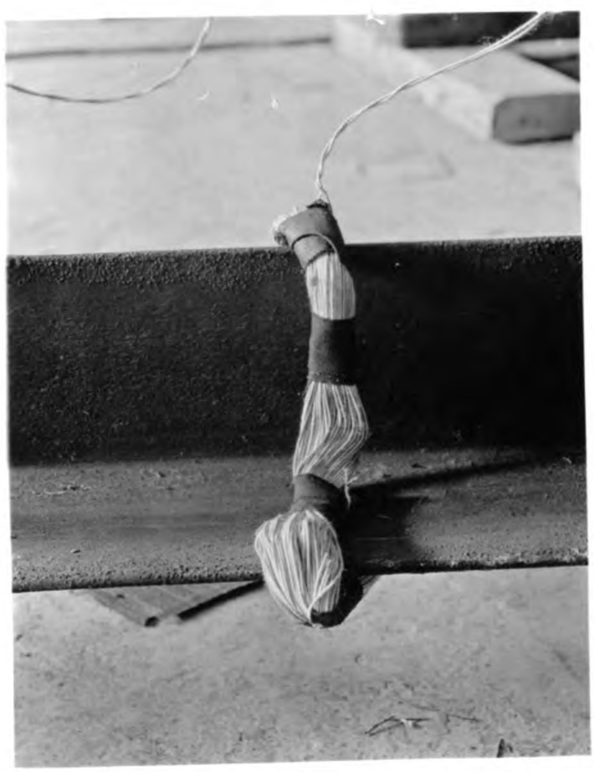




INTRODUCTION

Many transmission lines have one conductor passing through a closed loop of structural steel. Such is generally the case of lines in hilly countries, or where three or more transmission line circuits are carried on the same tower structure due to high cost of right of way, of transposition towers for double circuits, etc.

Up to the present day it is customary in the design of a line, not to pay any attention to the magnetic losses in the towers, but for the sake of determining the facts we carried out such an investigation. Therefore, it is the purpose of this paper to investigate the losses in the loop and derive a formula by which it would be possible to compute the losses due to any current at any distance. Although those losses do not amount to a great deal in one single tower, they should be taken into consideration when the line is long and more particularly when it is under much load. The amount of magnetic losses then is not negligible.

The losses causing heating in the structure of substation are similar to the losses in the closed loop of the tower. At some points the heating is maximum as illustrated at various points of the legs of the loop.

ACKNOWLEDGMENT

The author desires to express his indebtedness to the Commonwealth Power Corporation of Michigan for furnishing the necessary tower parts, to the Board of Water and Electric Light of the City of Lansing for lending transformers and supplying cables capable of carrying high currents. The author wishes also to express his appreciation to the staff members of the Electrical Engineering Department of the Michigan State College for the suggestions and help received from them, and to Mr. F. Mitchell for his collaboration.

MAGNETIC LOSSES IN TRANSMISSION TOWERS HAVING A CLOSED MAGNETIC PATH SURROUNDING ONE CONDUCTOR

The Problem:

Many transmission lines have one conductor passing through a triangular, rectangular or polygonal closed magnetic path formed by the structural steel of the tower. It is the purpose of this paper to determine the amount of the iron losses in the tower caused by induction. The triangular shaped magnetic path was chosen as it is the most commonly used.

Procedure:

A triangular shaped loop of structural steel was erected. A three phase transmission circuit was built. One conductor of the line went through the center of the loop, while the two other conductors were on the right and left side. The three conductors were in the same horizontal plane and at the same distance from the steel.

Many exploring coils of known number of turns were wound around the three legs of the loop at intervals. The induced voltage in each coil was measured and recorded. By this means the fluxes induced in the loop at different points due to different currents were calculated. By using Steinmetz' and the eddy current formulae the different losses were calculated and graphs plotted.

Apparatus used:

Due to the limitation in the available apparatus many schemes were planned for obtaining enough current in the line and for measuring the

losses. It was first thought that by means of current transformers it would be possible to obtain any desired current in the line. The secondary coils of three transformers were used as primaries, and the primary sides were connected to the line. Although a large amount of current was flowing in the secondaries (used as primaries) of the current transformers not enough current appeared in the line. This scheme was rejected after trying each of the following connections, Delta-Delta, Wye-Wye, Delta-Wye, and Wye-Delta. Another set of current transformers was used but satisfactory results were not obtained.

A constant current transformer core was available in the laboratory. On it were mounted five coils designed to stand 5000 volts when in series. The core has three legs. It was planned to use one coil on each leg and the whole as a three phase transformer. The secondary being formed by winding two turns of the cable around each leg.

This scheme was tried, about 1000 volts being applied on one coil of the primary. Enough current passed in the secondary line. The 1000 volts were obtained through a step up single phase transformer belonging to the laboratory. Two other similar transformers were necessary. Furthermore, they must be designed to carry 10 amps or their high voltage sides, as it takes 10 amps to energize the three phase coil and give the required current in the line. No transformers were found which could meet the requirements.

It was then found necessary to build a special transformer. The three legged core was used. Forty-five turns of No. 13 cable on each leg constituted the primary coils of the transformer and two turns of the line cable the secondary coils. The connection was made a Wye-Wye.

To change the current in the line the applied voltage was changed by changing the field of the supply alternator in the laboratory. The speed was always kept constant giving a constant frequency of 60 cycles.

The current in the line was measured by a step down current transformer. The free end of the line was short circuited to give the different high currents.

It was then necessary to find out a means to measure the voltages induced in the coils. The voltages induced in a one hundred turn coil was estimated not to exceed 1.5 volts. Low A.C. voltmeters were not available and those found on the market did not satisfy the requirements due to their low resistance. Voltmeters with high resistances allowing but a fraction of an ampere to flow in them was necessary, on account of the back ampere-turns which tend to oppose the inducing flux.

A.C. galvanometers or vacuum tube voltmeters could have been used if available.

All meters were shielded to protect their coils against the direct effect of magnetic lines around the conductors. Furthermore, the leads to the measuring instruments were run perpendicularly to the conductors and for more safety, twisted around each other.

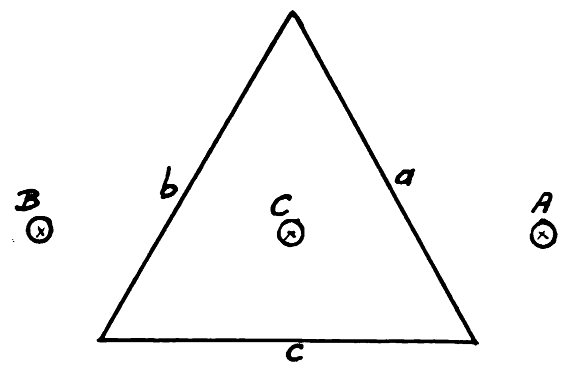
An oscillograph was used to measure the induced voltage. The wave on each vibrating element was examined and when found to be sinusoidal was accurately measured. To measure the wave it was included between two boundaries of light projected by the two other elements thus giving twice its maximum value. The distance between the two lines was then measured and recorded. The oscillograph was calibrated at different intervals, using the same leads as those used to record the wave.

Thus with the instruments available in the department it was possible to perform the tests and get accurate results.

In regard to the exploring coils, it was first thought possible to use some coils available in the department. When tried it was found that a large voltage was induced in the coils, much larger than was expected. It was discovered that this voltage was largely due to a direct induction from the line to the coils. Two similar coils were then used. One was put around a steel leg and the other outside of it. The two were under similar conditions with respect to the line. They were connected so as to oppose each other. The oscillograph was then used to register the difference in voltages, viz. the voltage due to the flux in the iron tower. This was found to be small.

Due to the fact that at different points on each leg the flux varies, it was decided to build very narrow exploring coils so as to give voltages in a narrow piece of steel. On the other hand, due to the direct induction from the line, the exploring coils were wound as thin as possible.

In this test the coils were wound with one hundred turns of very thin wire (B. & S. No. 27). They were then put around the legs of the loop and given the same shape, viz., they were bound in the contour of the legs. In this manner the influence of the flux from the lines was reduced to a minimum.



By examining the figure it is clearly seen that the fluxes in the three legs were unequal. Furthermore, as will be shown later, the flux in c was smaller than either that in a or b. On the other hand, due to the symmetry of the figure, when under balanced load the number of lines in a and b were equal.

From another point it was assumed throughout tnis test that leg a shielded both legs b and c against the magnetic effect of the current flowing in conductor a and that leg b shielded both legs a and c agsinst the current flowing in b. Thus the flux induced in leg c is due to conductor c only while that in a and b is due to a and c, and b and c respectively. The necessity for using different exploring coils at different points along the legs is seen from the fact that the flux was not uniform in any leg. This will be mathematically proven later.

Consider one leg a and let us study the magnetic effect in a due to current flowing in lines a and c.

CM is perpendicular and bisects the leg. (cy = 1/2 CQ).

AP is another perpendicular from a.

If the effect of c alone is considered, the maximum flux would be at point M, according to the formula

$$B = \mu \frac{12I}{r} \text{ lines per square centimeter}$$

where r is the distance in centimeters, I the current in amperes, and

μ the permeability.

If the effect of a is considered, maximum flux is at P.

Due to the fact that both fluxes are aiding and that R is at same distance from a and c the maximum flux would be at R.

It is to be noted that the flux at PO is largely due to currents in a. At O the flux is small.

At Q the flux is mostly due to c and is nearly equal to that on the corners of leg c.

That point P does not fall on O can be proven as follows:

AP = CM, being perpendicular and conductors a and c are at the same distance from QO.

We further have for the same reason CM = CY = aP = aS

If P falls at O then aO = aS. Therefore aO is perpendicular to YS and QO or in triangle aOS

$$aS = aO$$

$$\text{Angle } aSO = aOS = 90^\circ \quad \text{which is impossible.}$$

Therefore, P cannot fall on O, viz. the perpendiculars drawn from the outside conductors do not fall at the joints of the legs.

The value of z in terms of r is found as follows:

$$QM = r \tan 60$$
$$= r\sqrt{3}$$
$$= 1.73205\ r$$
$$MR = r \tan 30$$
$$= r\sqrt{\tfrac{1}{3}}$$
$$= 0.57735\ r$$
$$MP = 2\ MR$$
$$= 1.1547\ r$$

$$QP = 1.73205\ r + 1.547\ r$$

$$= r\ [2.88675]$$

$$Z = \sqrt{(2.88675\ r)^2 + r^2}$$

$$= r\ [\sqrt{9.333}\ ]$$

$$= 3.05504\ r$$

To find g in terms of r

$$Po = QM - MP$$

$$= 1.73205\ r - 1.1547\ r$$

$$= .57735\ r$$

$$\therefore\ g = \sqrt{r^2 + (0.57735\ r\ )^2}$$

$$= 1.1547\ r$$

$$aM = CP = \sqrt{\frac{4r^2}{3} + r^2}$$

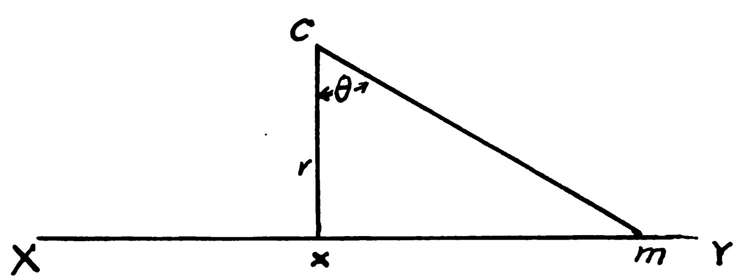
$$= r\sqrt{\frac{7}{3}}$$

$$= 1.5275\ r$$

$$CR = Ra = \frac{r}{\text{Cos } 30}$$

$$= 1.1547$$

Consider now this other figure. The flux at any point along XY can be found in terms of the flux at x and the angle θ. The flux at x is



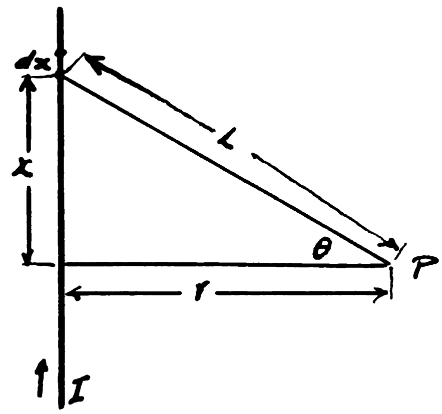
$$B = \mu\ \frac{.2\ I}{r}$$

At m it is $$B_m = \mu \frac{.2\ I}{Cm}$$

$$= \mu \frac{.2\ I \cos\theta}{r}$$

$$= B_x \cos\theta$$

Theoretical solution:



Assume a conductor carrying a current I abamperes flowing as shown in the figure. It is required to know the magnetic intensity at a point P outside of that conductor. To solve this problem consider an element of length dx at distance L centimeters from P. By Coulomb's law the effect of that element on a point L centimeters away is

$$dH = \frac{I\ dx}{L^2} \cos\theta$$

Therefore the effect of the total wire is

$$H = \int_{-}^{+} \frac{I \cos\theta}{L^2}\, dx$$

assuming the conductor to be very long.

From the figure we have

$$x = r \tan\theta$$

Therefore $$dx = r \sec^2\theta\, d\theta$$

Also $$L = \frac{r}{\cos\theta}$$

Inserting these values in the above expression

$$H = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{I \cos\theta\, r \sec^2\theta\, d\theta}{\frac{r^2}{\cos^2\theta}}$$

This becomes $$H = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{I \cos \theta}{r} d\theta$$

$$H = \frac{I}{r} [\sin \theta]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{2 I}{r}$$

If I is in amperes $$H = \frac{.2 I}{r} \quad \text{gausses}$$

If the point P is in a material having μ permeability

$$B = \mu \frac{.2 I}{r} \quad \text{lines per square centimeter}$$

This is one of the fundamental formula applicable here.

In alternating current if the waves are sinusoidal and time properly chosen the current at any instant may be expressed as

$$i = I_{max} \sin w t$$

where w is the angular velocity

$$w = 2\pi f$$

and t the time in seconds.

The instantaneous current i produces a flux Q. Due to the fact that i is varying Q is varying too. In any material which has a constant permeability the flux wave has the same shape as the current wave. Due to the fact that the flux density in the tower legs is low it can be assumed that the permeability of the steel within the boundaries used is constant and therefore the flux wave is sinusoidal.

Therefore $$Q = Q_m \sin w t$$

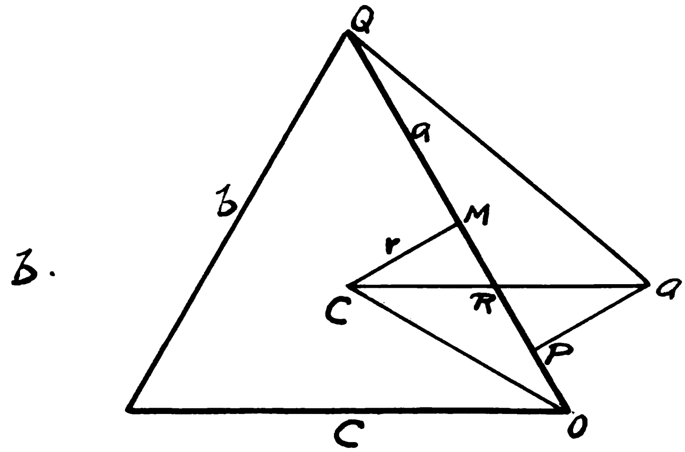
It was seen that $$Q = \mu \frac{.2 I}{r} A$$

where A is cross section area.

Therefore $$Q_m = \mu \frac{.2 I_m}{r} A$$

and $$Q = \mu \frac{.2\, I_m \sin w\, t}{r} A$$

In a balanced three phase system the currents are 120^0 apart.



Assuming the currents to be

$i_a = I_m \sin w\, t$

$i_b = I_m \sin (w\, t + 120)$

$i_c = I_m \sin (w\, t + 240)$

Therefore, the general formulae for the fluxes due to the three currents are:

$$Q_a = Q_m \sin w\, t = \mu \frac{.2\, I_m}{r} A \sin w\, t$$

$$Q_b = Q_m \sin (wt + 120) = \mu \frac{.2\, I_m}{r} A \sin (wt + 120)$$

and $$Q_c = Q_m \sin (wt + 240) = \mu \frac{.2\, I_m}{r} A \sin (wt + 240)$$

First, to find the fluxes at different points on leg a viz. at Q, M, R, P and O.

Q_b has no effect on leg a because leg b is acting as a shield to leg a against current in conductor b.

The flux at o due to current a is

$$Q_{a-o} = C \frac{.2\, I_m}{1.1547\, r} \sin w\, t$$

where $$C = \mu A$$

The flux at o due to C is

$$Q_{c-o} = C \frac{.2\, I_m}{2r} \sin (wt + 240)$$

Therefore the total flux at o is

$$Q_{t-o} = Q_{a-o} - Q_{c-o}$$

$$Q_{t-o} = C \frac{.2\ I_m}{r} \left[\frac{\sin wt}{1.1547} - \frac{\sin (wt + 240)}{2} \right]$$

Similarly the fluxes at P, R, M and Q are respectively

$$Q_{t-P} = C \frac{.2\ I_m}{r} \left[\sin wt - \frac{\sin (wt + 240)}{1.5275} \right]$$

$$Q_{t-R} = C \frac{.2\ I_m}{1.1547\ r} \left[\sin wt - \sin (wt + 240) \right]$$

$$Q_{t-M} = C \frac{.2\ I_m}{r} \left[\frac{\sin wt}{1.5275} - \sin (wt + 240) \right]$$

$$Q_{t-Q} = C \frac{.2\ I_m}{r} \left[\frac{\sin wt}{3.05504} - \sin (wt + 240) \right]$$

On the other hand

$$e = -N \frac{dQ}{dt} 10^{-8}$$

Therefore

$$e_o = -NC \frac{.2\ I_m}{r} 10^{-8} \frac{d}{dt} \left[\frac{\sin wt}{1.1547} - \frac{\sin (wt + 240)}{2} \right]$$

$$= -NC \frac{2\ I_m}{r} 10^{-9} \left[\frac{w \cos wt}{1.1547} - \frac{w \text{ Cos } (wt + 240)}{2} \right]$$

$$= -K \left[\frac{\cos wt}{1.1547} - \frac{\text{Cos } (wt + 240)}{2} \right]$$

where

$$K = NC \frac{2\ I_m}{r} 10^{-9}\ w$$

$$e_o = -K \left[\frac{\cos wt}{1.1547} - \frac{\text{Cos } wt \text{ Cos } 240 - \sin wt \sin 240}{2} \right]$$

$$= -K [.866 \text{ Cos } wt + .25 \text{ Cos } wt - .433 \sin wt]$$

$$= -K [1.116 \text{ Cos } wt - .433 \sin wt]$$

Its maximum value occurs when

$$-1.116 \sin wt = .433 \text{ Cos } wt$$

$$\tan wt = - .388$$

$$wt = 158^{o}\ 45'$$

and minimum when $wt = -21^{o}\ 15'$

Its minimum is then

$$e_{om} = -K [1.116 \times .93201 + (.433 \times .36244)]$$

$$e_{om} = -K\,[1.04012 + .156936]$$

$$= -1.19705\ K$$

and its maximum

$$e_{om} = -K\,[1.116 \times (-.93201) - (.433 \times .36244)]$$

$$= +\ 1.19705\ K$$

Similarly

$$e_P = -K\left[\text{Cos } wt - \frac{\text{Cos }(wt + 240)}{1.5275}\right]$$

$$= -K\,[\text{Cos } wt + .327233 \text{ Cos } wt - .56693 \sin wt]$$

$$= -K\,[1.327 \text{ Cos } wt - .566 \sin wt]$$

Its minimum occurs when

$$\tan wt = -.42652$$

$$wt = -23^\circ\ 6'$$

Its maximum when

$$wt = 156^\circ\ 54'$$

It is then

$$e_{om} = +K\,[1.327 \times .91982 + .566 \times .39234]$$

$$= K\,[1.2206 + .22206]$$

$$= 1.4426\ K$$

$$e_R = -K\,[.866 \text{ Cos } wt + .433 \text{ Cos } wt - .75 \sin wt]$$

$$= -K\,[1.299 \text{ Cos } wt - .75 \sin wt]$$

Its maximum value occurs when

$$\tan wt = -.577$$

$$wt = 150$$

It is then

$$e_{RM} = K\,[1.299 \times .86603 + .75 \times .5]$$

$$= [1.1249 + .375]\ K$$

$$= 1.5\ K$$

$$e_M = -K \left[\frac{\text{Cos wt}}{1.5275} - \text{Cos (wt + 240)}\right]$$

$$= -K\ [.6546 \text{ Cos wt} + .5 \text{ Cos wt} - .866 \text{ sin wt}]$$

$$= -K\ [1.1546 \text{ Cos wt} - .866 \text{ sin wt}]$$

Its maximum is when

$$\tan wt = .75$$

$$wt = 143^{\circ}\ 7'$$

It is then

$$e_{Mm} = K\ [.5196 + .9235]$$

$$= K\ [1.4431]$$

$$e_Q = -K \left[\frac{\text{Cos wt}}{13.05504} - \frac{\text{Cos (wt + 240)}}{2}\right]$$

$$= -K\ [.3273 \text{ Cos wt} + .25 \text{ Cos wt} - .433 \text{ sin wt}]$$

$$= -K\ [.5773 \text{ Cos wt} - .433 \text{ sin wt}]$$

It has its maximum value when

$$\tan wt = .75$$

$$wt = 143^{\circ}\ 7'$$

It is then

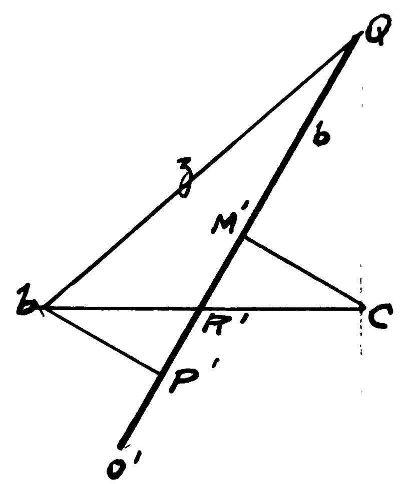
$$e_{MQ} = K\ [.5773 \times .79986 + .433 \times .60019]$$

$$= K\ [.4617 + .2598]$$

$$= .7215\ K$$

Leg b

Q_a has no effect on leg ''b''. The flux at O' due to current b is



$$Q_{b-o} = C\ \frac{.2\ Im}{1.1547r}\ \sin\ (wt + 120)$$

The flux at O' due to c is

$$Q_{c-o} = C\ \frac{.2\ Im}{2r}\ \sin\ (wt + 240)$$

Therefore the total flux at O' is

$$Q_{t-o'} = Q_{a-o'} - Q_{c-o'}$$

$$Q_{t-o'} = C\,\frac{.2\ Im}{r}\left[\frac{\sin(wt + 120)}{1.1547} - \frac{\sin(wt + 240)}{2}\right]$$

Similarly the flux is at P', R', M' and Q are respectively

$$Q_{t-P'} = C\,\frac{.2\ Im}{r}\left[\sin(wt + 120) - \frac{\sin(wt + 240)}{1.5275}\right]$$

$$Q_{t-R'} = C\,\frac{.2\ Im}{r\ 1.1547}\left[\sin(wt + 120) - \sin(wt + 240)\right]$$

$$Q_{t-M'} = C\,\frac{.2\ Im}{r}\left[\frac{\sin(wt + 120)}{1.5275} - \sin(wt + 240)\right]$$

$$Q_{t-Q} = C\,\frac{.2\ Im}{}\left[\frac{\sin(wt + 120)}{3.05504} - \frac{\sin(wt + 240)}{2}\right]$$

Consequently

$$e_{o'} = -NC\,\frac{.2\ Im}{r}\,10^{-8}\,\frac{d}{dt}\left[\frac{\sin(wt + 120)}{1.1547} - \frac{\sin(wt + 240)}{2}\right]$$

$$= -NC\,\frac{2\ Im}{r}\,10^{-9}\,w\left[\frac{\cos(wt + 120)}{1.1547} - \frac{\cos(wt + 240)}{2}\right]$$

$$= -K\,[.866\cos(wt + 120) - .5\cos(wt + 240)]$$

$$= -K\,[.866\,(-.5\cos wt - .866\sin wt) - .5\,(-.5\cos wt + .866\sin wt)]$$

$$= K\,[.433\cos wt + .75\sin wt - .25\cos wt + .433\sin wt]$$

$$= K\,[.183\cos wt + 1.183\sin wt]$$

It has its maximum when

$$.183\sin wt = 1.183\cos wt$$

$$\tan wt = \frac{1.183}{.183}$$

$$= 6.464$$

$$wt = 81^{o}\ 12.6'$$

It is then

$$e_{MO'} = K\,[.183 \times .15281 + 1.183 \times .98825]$$

$$= K\,[.02796 + 1.16909]$$

$$= K\,[1.19705$$

Similarly

$$E_{P'} = -K\left[\cos(wt + 120) - \frac{\cos(wt + 240)}{1.5275}\right]$$

$$= K\left[.5 \cos wt + .866 \sin wt - \frac{.5 \cos wt - .866 \sin wt}{1.5275}\right]$$

$$= K\,[.5 \cos wt + .866 \sin wt - .327233 \cos wt + .56693 \sin wt]$$

$$= K\,[172767 \cos wt + 1.33293 \sin wt]$$

It has its maximum when

$$.173 \sin wt = 1.3332 \cos wt$$

$$\tan wt = 7.7151$$

$$wt = 82^{o}\ 36'.4$$

It is then

$$E_{MP'} = K\,[.173 \times .12870 + 1.3329 \times .99168]$$

$$= K\,[.0222651 + 1.32191]$$

$$= K\,[1.344175]$$

$$E_{R'} = -K\,[.866 \cos(wt + 120) + .433 \cos wt - .75 \sin wt)]$$

$$= -K\,[.866\,(-.5 \cos wt - .866 \sin wt) + .433 \cos wt - .75 \sin wt]$$

$$= K\,[.433 \cos wt + .75 \sin wt - .433 \cos wt + .75 \sin wt]$$

$$= K\,[1.5 \sin wt]$$

This is maximum when

$$\sin wt = 1$$

$$wt = 90$$

It is then $E_{MR'} = 1.5\ K$

$$e_{t-M'} = -K\left[\frac{\cos(wt+120)}{1.5275} - \cos(wt+240)\right]$$

$$= K\ [.6546\ (.5 \cos wt + .866 \sin wt) - .5 \cos wt + .866 \sin wt]$$

$$= K\ [.3273 \cos wt + .5668 \sin wt - .5 \cos wt + .866 \sin wt]$$

$$= K\ [1.4328 \sin wt - .1727 \cos wt]$$

This is maximum when

$$1.4328 \cos wt + .1727 \sin wt = 0$$

$$\tan wt = -8.2964$$

$$wt = 96^{o}\ 52.4'$$

It is then

$$e_{mM'} = K\ [1.4328 \times .99281 + .1727 \times .11968]$$

$$= K\ [1.42249 + .02066]$$

$$= K\ [1.44315]$$

$$E_{Q} = -K\left[\frac{\cos(wt+120)}{3.05504} - \frac{\cos(wt+240)}{2}\right]$$

$$= K\ [.3273\ (.5 \cos wt + .866 \sin wt) + .5\ (-.5 \cos wt + .866 \sin wt)]$$

$$= K\ [.16365 \cos wt + .28344 \sin wt - .25 \cos wt + .433 \sin wt]$$

$$= K\ [-.08635 \cos wt + .71644 \sin wt]$$

This is maximum when

$$.08635 \sin wt + .71644 \cos wt = 0$$

or

$$\tan wt = -8.296$$

$$wt = 96^{o}\ 52.4'$$

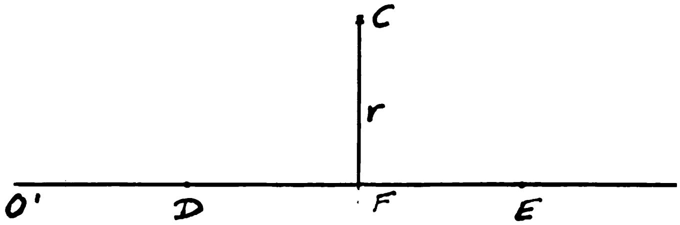
Its value is then

$$E_{mQ} = K\ [.08635 \times .11968 + .71644 \times .99281]$$

$$= K\ [.010334 + .711288]$$

$$= K\ [.721622]$$

Leg c



The flux at F is

$$Q_{c-F} = C \frac{.2\ I_m}{r} \sin (wt + 240)$$

Therefore the voltage at F is

$$e_{c-F} = NC \frac{.2\ I_m}{r}\ w \cos (wt + 240)$$

$$= -K \cos (wt + 240)$$

$$= -K [-.5 \cos wt + .866 \sin wt]$$

$$= K [.5 \cos wt - .866 \sin wt]$$

It has its maximum value when

$$-.5 \sin wt - .866 \cos wt = 0$$

$$\tan wt = -1.732$$

$$wt = 300$$

It is then

$$e_m = K [.5 \times .5 - (-.866 \times .866)]$$

$$= K$$

$$Fo = r \sqrt{3}$$

$$= 1.732\ r$$

$$FE = .866\ r$$

$$CE = \sqrt{r^2 + (.866\ r)^2}$$

$$= r \sqrt{\frac{7}{4}}$$

$$= \frac{r}{2} \sqrt{7}$$

$$= \frac{r}{2} (2.6457) = 1.3228\ r$$

$$e_{c-E} = K \left[\frac{.5 \cos wt}{1.3228} - \frac{.866}{1.3228} \sin wt\right]$$

$$= K [.3779 \cos wt - .6546 \sin wt]$$

It has its maximum when

$$wt = 300^0$$

It is then

$$e_{cE} = K [.3779 \times .5 + (.6546 \times .866)]$$

$$= K [.18895 + .56688]$$

$$= .75583\ K$$

$$e_o = K [\frac{.5 \cos wt}{2} - \frac{.866}{2} \sin wt]$$

$$= K [.25 \cos wt - .433 \sin wt]$$

It has its maximum when

$$wt = 300$$

It is then

$$e_{m-o} = K [.25 \times .50 + .433 \times .866]$$

$$= K [.125 + .365]$$

$$= K [.50]$$

$$= .2235\ K$$

It is to be noticed that there is a phase angle between the different voltages at different points on the lower halves of legs a and b. This difference in the phase angle on the same leg is very small, being only 15^0 on legs a and b. The upper halves have the same angle or a very small difference in phase angle. This constancy in the angles shows that the flux in the upper halves is almost a pulsating plux, viz. it is largely due to the current in conductor c.

Leg c has no difference in its voltage phase angles at different points. The voltages along c are in phase.

On the other hand, the voltage is proportional to the flux, thus

$$Q_m = DV$$

Therefore, the fluxes at different points is proportional to their voltages. The fluxes on leg a at o, P, R, M and Q are respectively

$$Q_{mo} = D\ [1.19705\ K]$$

$$Q_{mP} = D\ [1.4426\ K]$$

$$Q_{mR} = D\ [1.5\ K]$$

$$Q_{mM} = D\ [1.4431\ K]$$

$$Q_{m}Q = D\ [.7215\ K]$$

where D is a constant depending upon frequency and number of turns.

Similarly on leg b the fluxes at o', P', R', M', and Q' are respectively

$$Q_{mo'} = D\ [1.19705\ K]$$

$$Q_{mP'} = D\ [1.34475\ K]$$

$$Q_{mR'} = D\ [1.5\ K]$$

$$Q_{mM'} = D\ [1.44315\ K]$$

$$Q_{mQ'} = D\ [.7216\ K]$$

The average flux on a or b is

$$Q_m = \frac{DK}{S}\ [1.19705 + 1.4426 + 1.5 + 1.4431 + .7215]$$

$$= D\ [1.26085]$$

On leg c

$$Q_{mo} = D\ [.2235\ K]$$

$$Q_{mE} = D\ [.75583K]$$

$$Q_{mF} = D\ [1.000\ K]$$

$$Q_{mD} = D\ [.75583\ K]$$

$$Q_{m}o' = D\ [.2235\ K]$$

$$Q_m = \frac{DK}{S}\ [2.95866]$$

$$= D\ [.59173\ K]$$

Therefore the flux in each of the legs a and b is about

$\frac{1.26085}{.59173}$ times the flux in c

or $$Q_a = Q_b = 2.13\ Q_c$$

The hysteresis losses in a and b are $(2.13)^{1.6}$ times those in c,

or $$P_{ha} = P_{hb} = (2.13)^{1.6}\ P_{hc}$$
$$= 3.3528\ P_{hc}$$

The eddy current losses in a and b are each $(2.13)^2$ times those in c

$$P_{ea} = P_{eb} = (2.13)^2\ P_{ec}$$
$$= 4.5369\ P_{ec}$$

If all the losses were due to hysteresis alone, the total losses in each of the legs a and b would have been

$$P_{ta} = P_{tb} = 3.3528\ P_{hc}$$

viz. the losses in c would then be only $\frac{1}{7.7056}$ of the total losses or about 13%.

If all the losses were due to eddy current alone, the total losses in each leg (a and b) would have been

$$P_{ta} = P_{tb} = 4.5369\ P_{tc}$$

viz. the losses in c would then be $\frac{1}{10.0738}$ of the total losses or about 10%.

The actual losses in c are between those two values, viz. between 10 and 13% while from 43 to 45% of the total will be the losses in each of the legs a and b.

Leg c was not shielded by any line. Under present conditions the values of voltages at different points on leg c are different from what

was found previously. To calculate these voltages procede as follows:

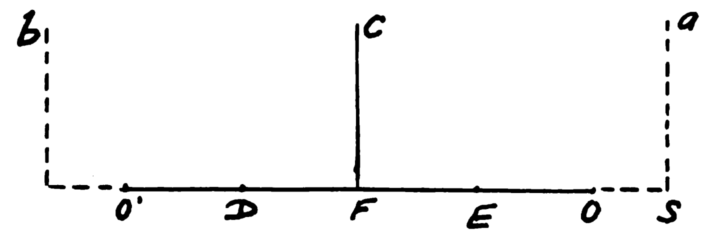
Ao = 1.1547 r

So = .57735 r

Eo = .866

Co = 2 r

CE = 1.3228 r



Therefore

$$AE = \sqrt{r^2 + (1.443\ r)^2}$$

$$= r\sqrt{1 + 2.082249}$$

$$= 1.755\ r.$$

$$AF = \sqrt{r^2 + (.57735 + 1.732)^2\ r^2}$$

$$= r\sqrt{1 + 5.331481}$$

$$= 2.516\ r.$$

$$AD = r\sqrt{1 + (.57735 + 2.598)^2}$$

$$= r\sqrt{1 + 10.080625}$$

$$= 3.33\ r.$$

$$AO' = r\sqrt{1 + (.57735 + 3.464)^2}$$

$$= r\sqrt{1 + 16.329681}$$

$$= 4.163\ r.$$

$$Q_o = C\left[\frac{\sin wt}{1.1547} + \frac{\sin (wt + 120)}{4.163} + \frac{\sin (wt + 240)}{2}\right]$$

$$Q_E = C\left[\frac{\sin wt}{1.775} + \frac{\sin (wt + 120)}{3.33} + \frac{\sin (wt + 240)}{1.3228}\right]$$

$$Q_F = C\left[\frac{\sin wt}{2.516} + \frac{\sin (wt + 120)}{2.516} + \sin (wt + 240)\right]$$

$$Q_D = C\left[\frac{\sin wt}{3.33} + \frac{\sin (wt + 120)}{1.755} + \frac{\sin (wt + 240)}{1.3228}\right]$$

$$Q_{O'} = C\left[\frac{\sin wt}{4.163} + \frac{\sin (wt + 120)}{1.1547} + \frac{\sin (wt + 240)}{2}\right]$$

Therefore

$$e_o = -K\left[\frac{\cos wt}{1.1547} + \frac{\cos (wt + 120)}{4.163} + \frac{\cos (wt + 240)}{2}\right]$$

$$= -K\,[.866 \cos wt + .2402 (\cos wt + 120) + .5 (\cos wt + 240)]$$

$$= -K\,[.866 \cos wt - .2403 (.5 \cos wt + .866 \sin wt) - .5 (.5 \cos wt - .866 \sin wt)]$$

$$= -K\,[.866 \cos wt - .1201 \cos wt - .2080 \sin wt - .25 \cos wt + .433 \sin wt]$$

$$= -K\,[.4959 \cos wt + .225 \sin wt]$$

It has its maximum value when

$$.4959 \sin wt = .225 \cos wt$$

$$\tan wt = .45372$$

$$wt = 204^{o}\ 24.6'$$

It is then

$$e_{om} = K\,[.4959 \times .9106 + .225 \times .41326]$$

$$= K\,[.45165 + .09298]$$

$$= .54463\ K.$$

$$e_E = -K\left[\frac{\cos wt}{1.755} + \frac{\cos (wt + 120)}{3.33} + \frac{\cos (wt + 240)}{1.3228}\right]$$

$$= -K\,[.5698 \cos wt + .3 \cos (wt + 120) + .7559 \cos (wt + 240)]$$

$$= -K\,[.5698 \cos wt + .3 (-.5 \cos wt - .866 \sin wt) + .7559 (-.5 \cos wt + .866 \sin wt)]$$

$$= -K\,[.5698 \cos wt - .15 \cos wt - .2598 \sin wt - .37795 \cos wt + .65461 \sin wt]$$

$$= -K\,[.0419 \cos wt + .3948 \sin wt]$$

This is maximum when

$$.0419 \sin wt = .3948 \cos wt$$

$$\tan wt = 9.42243$$

$$wt = 263^{o}\ 56.6'$$

It is then

$$e_E = K\,[.0419 \times .1055 + .2948 \times .99442]$$

$$= K\,[.00442 + .39259]$$

$$= .39701\ K$$

$$e_F = -K\left[\frac{\cos wt}{2.516} + \frac{\cos (wt + 120)}{2.516} + \cos (wt + 240)\right]$$

$$= -K\,[.3974 \cos wt + .3974\,(-.5 \cos wt - .866 \sin wt) + (.5 \cos wt - .866 \sin wt.]$$

$$= -K\,[.3974 \cos wt - .1987 \cos wt - .344148 \sin wt - .5 \cos wt + .866 \sin wt]$$

$$= K\,[.3013 \cos wt - .522 \sin wt]$$

This is maximum when

$$-.3013 \sin wt = .522 \cos wt$$

$$\tan wt = -\frac{.522}{.3013}$$

$$\tan wt = -1.7324$$

$$wt = 300$$

It is then

$$e_{Fm} = K\,[.3013 \times .5 + .522 \times .866]$$

$$= K\,[.15065 + .452052]$$

$$= K\,[.600702]$$

$$= .600702\ K$$

$$e_D = -K\left[\frac{\cos wt}{3.33} + \frac{\cos (wt + 120)}{1.755} + \frac{\cos (wt + 240)}{1.3228}\right]$$

$$= -K\,[.3 \cos wt + .5698 \cos (wt + 120) + .755 \cos (wt + 240)]$$

$$= -K\,[.3 \cos wt + .5698\,(-.5 \cos wt - .866 \sin wt) + .7559\,(-.5 \cos wt + .866 \sin wt)]$$

$$= -K\,[.3 \cos wt - .2849 \cos wt - .4934 \sin wt - .37795 \cos wt + .65461 \sin wt]$$

$$= K\,[.3629 \cos wt - .1612 \sin wt]$$

It has its maximum value when

$$-.3629 \sin wt = .1612 \cos wt$$

$$\tan wt = -.4442$$

$$wt = 336^{\circ}\ 3'$$

It is then equal to

$$e_{Dm} = K\,[.3629 \times .9139 + .1612 \times .40594]$$

$$= K\,[.33165 + .07543]$$

$$= .40708\ K$$

$$e_{O'} = -K\left[\frac{\cos wt}{4.163} + \frac{\cos (wt + 120)}{1.1547} + \frac{\cos (wt + 240)}{2}\right]$$

$$= -K\,[.2402 \cos wt + .866 \cos (wt + 120) + .5 \cos (wt + 240)]$$

$$= -K\,[.2402 \cos wt + .866\,(-.5 \cos wt - .866 \sin wt) + .5\,(-.5 \cos wt + .866 \sin wt)]$$

$$= -K\,[.2402 \cos wt - .433 \cos wt - .75 \sin wt - .25 \cos wt + .433 \sin wt]$$

$$= K\,[.4428 \cos wt + .317 \sin wt]$$

It is maximum when

$$.4428 \sin wt = .317 \cos wt$$

$$\tan wt = .7158$$

$$wt = 35^{\circ}\ 35.6'$$

It is then equal to

$$e_{OM} = K\,[.4428 \times .81315 + .317 \times .58200]$$

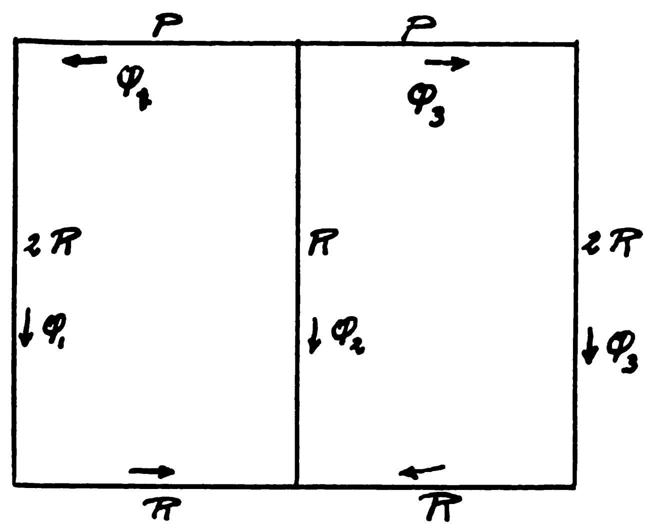
$$= K\,[.36006 + .18449]$$

$$= .54455\ K$$

We thus see that under these conditions we have a rotating flux in leg c. The voltages at different points of the legs reach their maxima at different times. This was found to be true experimentally.

Reasons why losses in legs a and b are unequal.

Although due to summetry the fluxes in legs a and b should be equal, in the test they were found to be unequal. This was due to the fact that the currents were not 120° apart as assumed in the theory. The impedances of the three current transformers were unequal in value and the fluxes in the potential transformer were not exactly 120° apart. To prove this last statement assume i_1, i_2, and i_3 to be the three instantaneous magnetizing currents, Q_1, Q_2, and Q_3 the instantaneous fluxes, 2R the reluctance of each one of the outside legs, R that of the inside leg, P and R the reluctances of each part of the yoke as shown in the figure.



Therefore

$$.4\pi\, n\, (i_1 - i_2) = (2R + R + P)\, Q_1 - RQ_2 \qquad (1)$$

$$.4\pi\, n\, (i_3 - i_2) = (2R + R + P)\, Q_3 - RQ_2 \qquad (2)$$

and $$Q_1 + Q_2 + Q_3 = 0$$

By adding $$.4\pi\, n\, (i_1 + i_3 - 2i_2) = (2R + R + P)(Q_1 + Q_3) - 2RQ_2 \qquad ($$

and knowing that $$i_1 + i_2 + i_3 = 0$$

therefore $$i_1 + i_2 = -i_3$$

and by substitution we get

$$.4\pi\, n\, (-3i_2) = (2R + R + P)\, (-Q_2) - 2RQ_2$$

$$.4\pi\, n\, (3i_2) = Q_2\, (4R + R + P)$$

Therefore $$.4\pi\ n\ i_2 = \frac{4R + R + P}{3}\ Q_2 \qquad (3)$$

Equation (3) shows that the flux Q_2 is in phase with the magnetizing current i_2 in the middle leg.

Substituting (3) in (1) we get

$$.4\pi\ n\ i_1 - \frac{4R + R + P}{3} Q_2 = (2R + R + P)\ Q_1 - RQ_2$$

which becomes

$$.4\pi\ n\ i_1 = (2R + R + P)\ Q_1 + \frac{R + R + P}{3} Q_2 \qquad (4)$$

This clearly shows that the magnetizing current is not in phase with the flux in leg 1.

By substituting (3) in (2) we similarly get

$$.4\pi\ n\ i_3 = (2R + R + P)\ Q_3 + \frac{R + R + P}{3}\ Q_2$$

which means the same thing as for magnetizing current i_1. The flux in leg 1 though is lagging the current while that in leg e is leading.

When the transformer is fully loaded, viz. when the total current input is large compared to the magnetizing current the flux in each leg becomes more in phase with the magnetizing current.

In the case under consideration the load current input in the primary never was more than ten times the magnetizing current. Therefore the secondary voltages never were 120^o apart.

To this add the effect of the impedances which produced another phase difference.

Hence the currents were not 120^o apart and the fluxes induced in legs a and b were not the same.

Procedure to obtain results from the oscillographic readings:

The oscillograph gave the height of the maximum voltage. The effective value was then computed. The resistance of the oscillographic element and the outside resistance in series with it was measured. From the voltage applied on the oscillograph and the resistance of the element and the rheostat in series with it the current flowing in the oscillograph and the exploring coil was computed.

The impedance of the coil was next measured. Its direct current resistance was taken. From these two values the reactance of the coil was computed.

The drop in voltage in the oscillographic circuit is ohmic. To the value of the voltage across the oscillograph was directly added the ir drop in the coil and vectorially the ix drop. The total voltage gave the induced voltage in the coil.

Knowing the induced voltage and referring back to the formula for the induced voltage in a transformer the maximum flux was found. Knowing the cross sectional area of the legs the maximum flux density was computed and thence the hysteresis and eddy losses.

To get the average losses the average flux was used.

It is to be noticed that losses-distance curves were plotted for one set of readings. The areas under these curves were measured by a planimeter and the average loss taken. This checked very closely with the results of average losses found by using the average flux.

Formulae used on computation:

a. The impedance of the coil is found by

$$Z = \frac{E}{I}$$

where Z is impedance in ohms

E the effective voltage (alternating current)

I the effective current (" ")

The direct current resistance is

$$R = \frac{E}{I}$$

where R is resistance in ohms

E direct durrent voltage in volts

I direct current in amperes

The reactance of the coil is then

$$X = \sqrt{Z^2 - R^2} \text{ ohms.}$$

(Many readings for E_{alt}, I_{alt}, E_{dir}, and I_{dir} were taken. The average values of Z and R were taken to compute X.)

b. The resistance of the element was found by applying direct current and taking readings of current and voltage.

$$R_e = \frac{E}{I}$$

c. The distance measured on the oscillograph is twice the maximum value. Half of that distance gives the maximum value. The effective voltage is

$$E_{eff} = .707\ E_m$$

where E_m denotes the maximum value.

The total induced voltage is

$$E_t = \sqrt{(e + ir)^2 + (ix)^2}$$

where e is the load voltage in phase with current

i the current flowing in the circuit

ir the drop across the ohmic resistance of the coil

r the ohmic resistance of coil

ix drop across inductance of coil

x reactance of coil.

d. The maximum can be calculated by the following formula

$$E_{eff} = 4.44\ N\ f\ Q_m\ 10^{-8} \text{ volts}$$

Therefore $$Q_m = \frac{E \times 10^{8}}{4.44\ N\ f} \text{ lines}$$

where Q_m is maximum flux

E the effective induced voltage

N number of turns in coil

f frequency in cycles per second.

The flux density is found thus

$$B_m = \frac{Q_m}{A}$$

where A is area of cross section in square centimeters

e. The hysteresis loss is computed by means of Steinmetz' empirical formula

$$P_h = K_h\ f\ B_m^{1.6}\ 10^{-7} \text{ watts per square centimeter}$$

where K_h is coefficient of hysteresis

f. The eddy loss is found thus

$$P_e = \frac{\pi^2}{6\ P}\ h^2\ f^2\ B_m^2\ 10^{-16} \text{ watts per square centimeter}$$

where P is resistivity the material

h thickness of the sheet

g. The total losses are the sum of the hysteresis and eddy current losses

$$P_t = P_h + P_e$$

Results and Computation:

a. Resistance and impedance of coil

At 60 cycle frequency the impedance of the exploring coil was found to be

$$Z = 7.556 \text{ ohms}$$

Its direct current resistance was

$$R = 7.22 \text{ ohms}$$

Its reactance is then

$$X = \sqrt{(7.556)^2 - V(7.22)^2}$$

$$= 2.24 \text{ ohms}$$

b. Resistance of the element and outside rheostat in series.

This was found to be

$$R = 9.5$$

c. Applied load voltage.

The voltage applied from the coil on the oscillograph gave a deflection of 3/8" (1st set of readings on leg a with 360 amp.) With a direct current voltage of 2 volts the deflection was 9/8". The maximum value of voltage was then

$$E_m = \frac{2 \times 8 \times 3}{9 \times 8 \times 2} = .333$$

Its effective value was then

$$E_{eff} = .707 \times .333$$

$$= .235 \text{ volts}$$

The current flowing in the oscillograph and the coil was therefore

$$I = \frac{.235}{9.5}$$

$$= .0248 \text{ amp.}$$

The ir drop in the resistance of the coil was

$$ir = .0248 \times 7.22$$

$$= .179 \text{ volts}$$

The ix drop was

$$ix = .0248 \times 2.24$$

$$= .0556$$

The total induced voltage was

$$E_t = \sqrt{(.235 + .179)^2 + (.0556)^2}$$

$$= \sqrt{.171396 + .003091}$$

$$= \sqrt{.174487}$$

$$= .417$$

d. Flux

The maximum flux was therefore

$$Q_m = \frac{.417 \times 10^8}{4.44 \times 60 \times 100}$$

$$= 1565 \text{ lines}$$

The area of the cross section being 3/8" or 2.418 sq. cm., the maximum flux density was

$$B_m = \frac{1565}{2.418}$$

$$= 648 \text{ lines per square centimeter}$$

e. Hysteresis losses

The coefficient of hysteresis for that sample of steel is

$$K_h = .015$$

The hysteresis loss was therefore

$$P_h = .015 \times 60 \times 648^{1.6} \times 10^{-7} \text{ watts per sq. cm.}$$

$$= 28.36 \times 10^{-4} \text{ watts per sq. cm.}$$

f. The eddy current loss

The resistivity of steel at ordinary temperature is

$$P = 19 \text{ microhm centimeter}$$

The eddy current coefficient is therefore

$$K_e = \frac{(3.14)^2 \times 10^6}{6 \times 19} \times (.315)^2$$

$$= 8.61 \times 10^3$$

.315 is the thickness of the steel in centimeter being 1/8 of an inch.

The eddy current loss is therefore

$$P_e = 8.61 \times 10^3 \times (60)^2 \times (648)^2 \times 10^{-16}$$

$$= 13.0 \times 10^{-4} \text{ watts per sq. cm.}$$

g. The total loss per square centimeter is

$$P_t = (28.36 + 13)\, 10^{-4}$$

$$= 41.36 \times 10^{-4} \text{ watts per sq. cm.}$$

Measurement of the Impedance of Coil

Frequency 60 cycle. Direct Current

I	E	Z ohm
.58	7.57	7.57
.63	4.75	7.54
.69	5.22	7.56
.8	6.05	7.56
.88	6.65	7.55
	Average	7.556

x = 2.24

I	V	R ohm
.4	2.92	7.3
.66	4.8	7.27
.5	3.6	7.2
.3	2.15	7.16
.23	1.65	7.17
		7.22

Resistance of element and resistance

I	V	R ohm
.1	.95	9.5
.2	1.94	9.7
.31	2.9	9.35
.396	3.75	9.46
.5	4.73	9.46
	Average	9.5

Direct Current Resistance of Coil

V	A	Res Coil		Average
2.92	.4	7.3		
4.8	.66	7.27		
3.6	.5	7.2	Coil	7.22
2.15	.3	7.16		
1.65	.23	7.17		
.95	.1	9.5		
1.94	.2	9.7		
2.9	.31	9.35	Set	9.5
3.75	.396	9.46		
4.73	.5	9.46		

DATA

			a			b			c		
I_a	I_b	I_c	1	2	3	1	2	3	1	2	3
360	360	360	3/8	11/16	13/16	3/4	13/16	1/2	5/16	3/8	1/4
			L = 19.5				f = 60				
		Res on 9									
310	310	310	5/16	9/16	11/16	9/16	5/8	7/16	1/4	5/16	3/16
		Res on 9					f = 60				
230	230	230	1/4	7/16	8/16	1/2	1/2	5/16	3/16	1/4	3/16
		Res on 9					f = 60				
440	440	440	7/16	13/16	1	19/16	1	1/2	5/16	7/16	1/4
		Res on 9					f = 60				

Phase Angle

$a - b = 60^{\circ}$

$a - c = 30^{\circ}$

It was noticed that there was a phase difference in the fluxes at different points on c, which amounts to around 60° between 1 and 2, 2 and 3. There was a very little angle between 2 and 3 on a and 1 and 3 on b, but no angle between 1 and 2 on a and 2 and 3 on b.

DATA

a		b		c		I_a	I_b	I_c	
max.	eff.	max.	eff.	max.	eff.				
.333	.235	.666	.470	.277	.195				
.611	.431	.722	.510	.333	.235	360	360	360	f = 60
.722	.510	.444	.314	.222	.157				
.277	.195	.527	.372	.222	.157				
.500	.353	.555	.392	.277	.195	310	310	310	f = 60
.611	.431	.388	.274	.166	.117				
.222	.157	.444	.314	.166	.117				
.388	.274	.444	.314	.222	.157	230	230	230	f = 60
.472	.333	.277	.195	.166	.117				
.388	.274	1.054	.744	.277	.195				
.722	.510	.888	.628	.388	.274	440	440	440	f = 60
.888	.628	.444	.314	.222	.157				

DATA

				a		$(e')^2$			
e	I	Ir	(e+Ir)	IX	$(IX)^2$	$(e+Ir)^2$	e'^2+IX^2	e_t	e_m
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.431	.0454	.327	.758	.101	.010010	.574564	.584574	.764	2870
.510	.0537	.388	.898	.120	.014400	.806404	.820804	.906	3410
.195	.0205	.148	.343	.0459	.002106	.117649	.119756	.346	1300
.353	.0372	.268	.621	.0834	.006955	.385641	.392597	.626	2355
.431	.0454	.327	.758	.101	.010010	.574564	.584574	.764	2870
.157	.0165	.119	.276	.0369	.0013616	.076176	.077538	.278	1045
.274	.0288	.208	.482	.0627	.0039312	.232324	.236255	.486	1825
.333	.0351	.253	.586	.0787	.0061436	.343396	.349590	.591	2220
.274	.0288	.208	.482	.0645	.0041602	.232324	.236484	.486	1825
.510	.0537	.388	.898	.120	.014400	.806404	.820804	.906	3410
.628	.0661	.477	1.105	.148	.021904	1.221025	1.242929	1.114	4190

$I = \frac{e}{r_e}$ e = e.m.f. recorded by oscillograph

r_e = resistance of element and outside resistance = 9.5

Total $e = \sqrt{(e+Ir)^2+(Ix)^2}$

r = 7.22

x = 2.24

DATA

a

ϕ_m	B_m	$\log B_m$	$1.6 \log B_m$	$B_m^{1.6}$	10^{-4} w hys.loss	B_m^2	Edd.L.	Total Losses w^{-4} w
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
2870	1190	3.07555	4.92088	83347	75.01	1416100	43.9	118.91
3410	1410	3.14922	5.038752	109450	98.50	1988100	61.6	160.10
Av.	1080	3.03743	4.859888	72450	65.20	1166400	36.2	101.4
1300	538	2.73078	4.369248	23400	21.06	289444	8.99	30.05
2355	975	2.98900	4.7824	60590	54.53	950625	29.5	84.03
2870	1190	3.07555	4.92088	83347	75.01	1416100	43.9	118.91
Av.	901	2.95472	4.72755	53400	48.06	811801	25.05	73.11
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
2220	920	2.96379	4.742064	55254	49.72	846400	26.25	75.97
Av.	702	2.84634	4.554144	35850	32.26	492804	15.3	47.56
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
3410	1410	3.14922	5.038752	109450	98.50	1988100	61.6	160.10
4190	1740	3.24055	5.18588	153060	136.75	3027600	93.9	230.65
Av.	1300	3.11394	4.982324	96020	86.41	1690000	52.4	136.81

DATA

b

e	I	Ir	e' Ir+e	Ix	$\overline{IX}^2$	$(e')^2$	$e'^2+\overline{IX}^2$	E	Φ_m
.470	.0495	.357	.827	.111	.012321	.683929	.696250	.834	3350
.510	.0537	.388	.898	.120	.014400	.806404	.820804	.906	3410
.314	.0331	.239	.553	.0742	.005506	.806404	.820804	.558	2100
.372	.0392	.283	.655	.0879	.007726	.429025	.436751	.660	2480
.392	.0412	.298	.690	.0924	.008537	.476100	.484637	.696	2618
.274	.0288	.208	.482	.0646	.004173	.232324	.236497	.486	1825
.314	.0331	.239	.553	.0742	.005506	.305809	.311315	.558	2100
.314	.0331	.239	.553	.0742	.005506	.305809	.311315	.558	2100
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.744	.0784	.565	1.309	.175	.030625	1.713481	1.744106	1.320	4960
.628	.0661	.477	1.105	.148	.021904	1.221025	1.242929	1.114	4190
.314	.0331	.239	.553	.0742	.005506	.305809	.311315	.558	2100

DATA

b

ϕ_m	B_m	log B_m	1.6 logB_m	B_m	10^{-4} w hys.Loss	B_m^2	Ed.L. 10^{-4}w	Total Losses w^{-4} w
3350	1390	3.14301	5.028816	106875	96.16	1932100	59.9	156.06
3410	1410	3.14922	5.038752	109450	98.50	1416100	43.9	142.40
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
Av.	1233	3.0910	4.9456	88200	79.38	1519289	47.2	126.58
2480	1050	3.01284	4.820544	66152	59.53	1060900	32.9	92.43
2620	1080	3.03342	4.853472	71364	64.22	1166400	36.2	100.42
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
Av.	955	2.9800	4.768	58600	52.74	912025	28.22	80.96
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
Av.	759	2.8802	4.60832	40600	36.54	576081	17.87	54.41
4960	2060	3.31387	5.302192	200530	180.47	4243600	131.5	311.97
4190	1740	3.24055	5.18488	153060	137.75	3027600	94.7	232.45
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
Av.	1556	2.1920	5.1072	128000	115.2	2420000	75.0	190.20

DATA

c

e	I	IR	e' IR+e	IX	$(IX)^2$	$(e')^2$	E^2	Induced E	e_m^c
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.274	.0288	.208	.482	.0645	.004160	.232324	.236484	.486	1825
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045

DATA

o

Φ_m	B_m	log B_m	1.6 logB_m	$B_m^{1.6}$	10^{-4} w hys.Loss	B_m^2	Ed.L. 10^{-4}	Total Losses 10^{-4}
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.30
Av.	539	2.7316	4.37065	23450	21.10	290521	9.02	30.12
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
775	320	2.50515	4.00824	10192	9.17	102400	3.82	12.99
Av.	430	2.6335	4.2136	16353	14.71	184900	5.74	20.45
775	320	2.50515	4.00824	10192	9.17	102400	3.82	12.99
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
775	320	2.50515	4.00824	10192	9.17	102400	3.82	12.99
Av.	357	2.5527	4.08432	12142	10.92	127449	3.96	14.88
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
Av.	575	2.7597	4.41552	26030	23.42	330625	10.22	33.64

DATA

Summary of Results

$L = 19.5''$

	a			b			c		
W_h	W_c	Total	W_h	W_c	Total	W_h	W_c	Total	$I_a=360$
$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$I_b=371$ $I_c=360$
28.36	13.0	41.36	96.16	59.9	156.06	21.06	8.97	30.03	
75.01	43.9	118.91	98.50	43.9	142.40	28.36	13.00	41.36	
98.50	61.6	160.10	45.44	23.5	68.94	14.88	5.82	20.70	
65.20	36.2	101.4	79.38	47.2	126.58	21.10	9.02	30.12	Av.

Average losses per cu. cen. of tower: Hys. 55.23, Edd. 30.81, Total 86.04

21.06	8.99	30.05	59.53	32.9	92.43	14.88	5.82	20.70	$I_a=310$
54.53	29.5	84.03	64.22	36.2	100.42	21.06	8.97	30.03	$I_b=321$
75.01	43.9	118.91	36.22	17.7	53.92	9.17	3.82	12.99	$I_c=303$
48.06	25.05	73.11	52.74	28.22	80.96	14.71	5.74	20.45	Av.

Hys. 38.50 Edd. 19.67 Total 58.17

14.88	5.82	20.7	45.44	23.5	68.94	9.17	3.82	12.99	$I_a=235$
36.22	17.7	53.92	45.44	23.5	68.94	14.88	5.82	20.70	$I_b=240$
49.72	26.25	75.97	21.06	8.97	30.03	9.17	3.82	12.99	$I_c=228$
32.26	15.3	47.56	36.54	17.87	54.41	10.92	3.96	14.88	Av.

Hys. 26.57 Edd. 12.38 Total 38.95

36.22	17.7	53.92	180.47	131.5	311.97	21.06	8.97	30.03	$I_a=440$
98.50	61.7	160.10	137.75	94.7	232.45	36.22	17.7	53.92	$I_b=430$
136.75	93.9	230.65	45.44	23.5	68.94	14.88	5.82	20.70	$I_c=440$
86.41	52.4	136.81	115.2	75.0	190.2	23.42	10.22	33.64	Av.

Hys. 75.01 Edd. 45.87 Total 120.88

DATA

I_a	I_b	I_c	a 1	a 2	a 3	a 4	b 1	b 2	b 3	b 4	c 1	c 2	c 3	c 4
470	475	475	5/16	1/2	13/16	7/8	7/8	7/8	11/16	3/8	7/32	7/16	3/8	1/4

B - C = 180 L = 24" $f = \frac{1200 \times 60}{1200} = 60$

R = 9

I_a	I_b	I_c	a 1	a 2	a 3	a 4	b 1	b 2	b 3	b 4	c 1	c 2	c 3	c 4
365	363	365	7/32	3/8	9/16	11/16	11/16	9/16	5/16	3/16	5/16	5/16	3/16	

f = 60 S = 1200

I_a	I_b	I_c	a 1	a 2	a 3	a 4	b 1	b 2	b 3	b 4	c 1	c 2	c 3	c 4
315	320	310	3/16	5/16	1/2	9/16	9/16	19/32	1/2	9/32	5/32	9/32	7/32	5/32

S = 1200

DATA

a		b		c					
max.	eff.	max.	eff.	max.	eff.	I_a	I_b	I_c	
.277	.195	.776	.548	.194	.137				
.444	.314	.776	.548	.388	.274	470	475	475	f = 60
.722	.510	.611	.431	.333	.235				L = 24"
.776	.548	.333	.235	.222	.157				
.194	.137	.611	.431	.166	.117				
.333	.235	.611	.431	.277	.195	365	363	365	f = 60
.537	.372	.500	.353	.277	.195				
.611	.431	.277	.195	.166	.117				
.166	.117	.500	.353	.138	.097				
.277	.195	.527	.372	.500	.353	315	320	310	f = 60
.444	.314	.444	.314	.194	.137				
.500	.353	.250	.177	.138	.097				

DATA

a

e	I	IR	e' e+IR	IX	$(IX)^2$	$(e')^2$	Induced E^2	E	Φ_m
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.314	.0331	.239	.553	.0741	.005506	.305809	.311315	.558	2100
.510	.0537	.388	.898	.120	.014400	.806404	.820804	.906	3410
.548	.0577	.417	.965	.1291	.016641	.931225	.947866	.973	3660
.137	.0144	.104	.241	.0323	.001043	.058081	.059124	.243	915
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.372	.0392	.283	.655	.0879	.007726	.429025	.436751	.660	2480
.431	.0454	.327	.758	.101	.010010	.574564	.584574	.764	2870
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.314	.0331	.239	.553	.0742	.005506	.305809	.311315	.558	2100
.353	.0372	.268	.621	.0834	.006956	.388641	.392597	.626	2355

DATA

Φ_m	B_m	log B_m	1.6 logB_m	a $B_m^{1.6}$	10^{-4} Hys.L.	B_m^2	Ed.L. 10^{-4}	Total Losses 10^{-4}w
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
3410	1410	3.14922	5.038752	109450	98.50	1988100	61.6	160.10
3660	1520	3.18184	5.090944	123230	110.90	2310400	71.6	182.50
Av.	1084	3.0350	4.8560	71800	64.62	1178000	36.5	101.12
915	378	2.57749	4.123984	13305	11.97	142884	4.43	16.40
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
2480	1030	3.01284	4.820544	66152	59.53	1060900	32.9	92.43
2870	1190	3.07555	4.92088	83347	75.01	1416100	43.9	118.91
Av.	811	2.9090	4.6544	45125	40.61	657721	20.4	61.01
775	320	2.50515	4.00824	10192	9.17	102400	3.82	12.99
1200	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
2355	975	2.98900	4.7824	60590	54.53	950625	29.5	84.03
Av.	676	2.7604	4.41664	26100	23.49	456976	14.12	37.61

DATA

b

e	I	IR	e' $IR+e$	IX	$(IX)^2$	$(e')^2$	$(L_t)^2$	Induced E	ϕ_m
.548	.0577	.417	.965	.129	.016641	.931225	.947866	.973	3660
.548	.0577	.417	.965	.129	.016641	.931225	.947866	.973	3660
.431	.0454	.327	.758	.101	.010010	.574564	.584574	.764	2870
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.431	.0248	.178	.609	.0556	.003091	.370881	.373972	.611	2300
.431	.0248	.178	.609	.0556	.003091	.370881.	.373972	.611	2300
.353	.0372	.268	.621	.0834	.006956	.385641	.392597	.626	2355
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.353	.0372	.268	.621	.0634	.006956	.385641	.392597	.626	2355
.372	.0392	.283	.655	.0879	.007726	.429025	.436751	.660	2480
.314	.0331	.239	.553	.0742	.005506	.305809	.311315	.558	2100
.177	.0186	.134	.311	.0417	.001738	.096721	.098459	.313	1175

DATA

b

Φ_m	B_m	log B_m	1.6 log B_m	$B_m^{1.6}$	10^{-4} Hys.L.	B_m^2	Ed.L. 10^{-4}	Total Losses 10^{-4}
3660	1510	3.17898	5.090944	123230	110.90	2280100	71.6	182.50
3660	1510	3.17898	5.090944	123230	110.90	2280100	71.6	182.50
2870	1190	3.07555	4.92088	83347	75.01	1416100	43.9	118.91
1865	770	2.88649	4.518384	41532	37.37	592900	18.4	55.77
Av.	1245	3.0952	4.95232	89600	80.64	55000	48.1	128.74
2300	950	2.97772	4.764352	58123	52.31	902500	28.0	80.31
2300	950	2.97772	4.764352	58123	52.31	902500	28.0	80.31
2355	975	2.98800	4.7824	60590	54.53	950625	29.5	84.03
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
Av.	853	2.9309	4.68944	48910	44.01	727609	22.59	66.60
2355	975	2.98900	4.7824	60590	54.53	950625	29.5	84.03
2480	1020	3.01284	4.820544	66152	59.53	1060900	32.9	92.45
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
1175	486	2.68664	4.298624	19888	17.89	236196	7.33	25.22
Av.	840	2.9243	4.67888	47730	42.95	705600	21.85	64.8

DATA

c

e	I	IR	e' IR+e	IX	$(IX)^2$	$(e')^2$	Induced $(E)^2$	E	ϕ_m
.137	.0144	.104	.241	.0323	.001043	.058081	.059124	.243	915
.274	.0288	.208	.482	.0645	.004160	.232324	.236484	.486	1825
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650
.353	.0372	.268	.621	.0834	.006956	.385641	.392597	.626	2355
.137	.0144	.104	.241	.0323	.001043	.058081	.059124	.243	915
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650

DATA

c

Φ_m	B_m	log B_m	1.6 logB_m	$B_m^{1.6}$	10^{-4} Hys.L.	B_m^2	10^{-4} Ed.L.	Total Losses 10^{-4}w
915	379	2.57864	4.123984	13305	11.97	143641	4.43	16.40
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
Av.	554	2.7435	4.3896	24525	22.07	306916	9.52	31.57
775	312	2.49415	4.00824	10192	9.17	97344	3.82	12.99
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
775	312	2.49415	4.00824	10192	9.17	97344	3.82	12.99
Av.	425	2.6284	4.20544	16046	14.44	180625	5.60	20.04
650	269	2.42975	3.8876	7720	6.94	72361	2.24	9.18
2355	975	2.98900	4.7824	60590	54.53	950625	29.46	83.99
915	379	2.57864	4.123984	13305	11.97	143641	4.43	16.40
650	269	2.42975	3.8876	7720	6.94	72361	2.24	9.18
Av.	473	2.6749	4.27984	19045	17.14	223729	6.94	24.08

DATA

Summary of Results

L = 24"

a			b			c			
W_h 10^{-4}w	W_c 10^{-4}w	Total 10^{-4}w	W_h 10^{-4}w	W_c 10^{-4}w	Total 10^{-4}w	W_h 10^{-4}w	W_c 10^{-4}w	Total 10^{-4}w	
21.06	8.97	30.03	110.90	71.6	182.50	11.97	4.43	16.40	I_a=470
45.44	23.5	68.94	110.90	71.6	182.50	36.22	17.7	53.92	I_b=475
98.50	61.6	160.10	75.01	43.9	118.91	28.36	13.0	41.36	I_c=475
110.90	71.6	182.50	37.37	18.4	55.77	14.88	5.82	20.70	
64.62	36.5	101.12	80.64	48.1	128.74	22.07	9.52	31.57	Av.

Hys. 49.78 Edd. 31.27 Total 81.15 Average losses per cu. cm. of tower

11.97	4.43	16.40	52.31	28.0	80.31	9.17	3.82	12.99	I_a=360
28.36	13.0	41.36	52.31	28.0	80.31	21.06	8.97	30.03	I_b=363
59.53	32.9	92.43	54.53	29.5	84.03	21.06	8.97	30.03	I_c=360
75.01	43.9	118.91	21.06	8.97	30.03	9.17	3.82	12.99	
40.61	20.4	61.01	44.01	22.59	66.6	14.44	5.60	20.04	Av.

Hys. 33.02 Edd. 16.20 Total 49.22 Average losses per cu. cm. of tower

9.17	3.82	12.99	54.53	29.5	84.03	6.94	2.24	9.18	I_a=322
21.06	8.97	30.03	59.53	32.9	92.43	54.53	29.46	83.99	I_b=322
45.44	23.5	68.94	45.44	23.5	68.94	11.97	4.43	16.40	I_c=322
54.53	29.5	84.03	17.89	7.33	25.22	6.94	2.24	9.18	
23.49	14.12	37.61	42.95	21.85	64.8	17.14	6.94	24.08	Av.

Hys. 27.86 Add. 14.30 Total 42.16 Average losses per cu. cm. of tower

DATA

			a					b					c				
I_a	I_b	I_c	1	2	3	4	5	1	2	3	4	5	1	2	3	4	5
285	285	285	3/16	7/32	5/16	7/16	3/8	3/8	7/16	7/16	9/32	3/16	5/32	3/16	1/4	5/32	1/8
350	350	350	5/32	1/4	3/8	17/32	7/16	9/16	9/16	1/2	5/16	1/4	5/32	1/4	5/16	7/32	5/32
445	445	450	1/4	3/8	9/16	3/4	9/16	11/16	3/4	11/16	7/16	5/16	3/16	11/32	11/32	1/4	5/32

For 285 285 285:

L 26.5 sp. 1200

Res. 9 f = 60

For 350 350 350:

L = 26.5 sp. 1200

Res. 9 f = 60

For 445 445 450:

L = 26.5 sp. 1200

Res. 9 f = 60

DATA

				a		b		c	
I_a	I_b	I_c		max.	eff.	max.	eff.	max.	eff.
			1	.166	.117	.333	.235	.138	.097
			2	.194	.137	.388	.274	.166	.117
285	285	285	3	.277	.195	.388	.274	.222	.157
			4	.388	.274	.250	.177	.138	.097
			5.	.333	.235	.166	.117	.111	.078
			1	.138	.097	.500	.353	.138	.097
			2	.222	.157	.500	.353	.222	.157
350	350	350	3	.333	.235	.444	.314	.277	.195
			4	.472	.333	.277	.195	.194	.137
			5	.388	.274	.222	.157	.138	.097
			1	.222	.157	.611	.431	.166	.117
			2	.333	.235	.666	.470	.305	.215
445	445	450	3	.500	.353	.611	.431	.305	.215
			4	.666	.470	.388	.274	.222	.157
			5	.500	.353	.277	.195	.138	.097

$L = 26.5$

$f = 60$

DATA

a

e	I	IR	$\frac{e'}{IR+e}$	IX	$(IX)^2$	$(e')^2$	Induced E^2	E	Φ_m
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.137	.0144	.104	.241	.0323	.001043	.058081	.059124	.243	915
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.274	.0288	.208	.482	.0645	.004160	.232324	.236484	.486	1825
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.333	.0351	.253	.586	.0787	.006194	.343396	.349590	.591	2220
.274	.0288	.208	.482	.0646	.004160	.232324	.236484	.486	1825
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.353	.0372	.268	.621	.0834	.006956	.385641	.392597	.626	2355
.470	.0495	.357	.827	.1110	.012321	.683929	.696250	.834	3135
.353	.0372	.268	.621	.0834	.006956	.385641	.392597	.417	1565

DATA

a

Φ_m	B_m	log B_m	1.6 logB_m	$B_m^{1.6}$	10^{-4} Hy.L.	B_m^2	Ed.L. 10^{-4}	Total Losses 10^{-4} w
775	312	2.49415	4.00824	10192	9.17	97344	3.82	12.99
915	379	2.57864	4.123984	13305	11.97	143641	4.43	16.40
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
Av.	526	2.7210	4.3536	22570	20.3	276676	8.57	28.87
650	269	2.34044	3.8876	7720	6.94	72361	2.24	9.18
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
2220	920	2.96379	4.742064	55254	49.72	846400	26.25	75.97
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
Av.	605	2.7818	4.45088	28240	25.41	366025	11.35	36.76
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
2355	975	2.98900	4.7824	60590	54.53	950625	29.5	84.03
3135	1300	3.11394	4.982304	96010	86.40	1690000	52.4	138.80
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
Av.	800	2.9031	4.64496	44150	39.73	640000	19.85	59.58

DATA

b

e	I	IR	e' IR+e	IX	$(Ix)^2$	$(e')^2$	Induced E^2	E	Φ_m
.235	.0248	.179	.414	.0556	.003091	.171396	.174487	.417	1565
.274	.0288	.208	.482	.0645	.004160	.232324	.236484	.486	1825
.274	.0288	.208	.482	.0645	.004160	.232324	.236489	.486	1825
.177	.0186	.134	.311	.0417	.001738	.096721	.098459	.313	1175
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.353	.0372	.268	.621	.0834	.006956	.385641	.392597	.626	2355
.353	.0372	.268	.621	.0834	.006956	.385641	.392597	.626	2355
.314	.0331	.239	.553	.0742	.005506	.305809	.311315	.558	2100
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.431	.0454	.327	.758	.101	.010010	.574564	.584574	.764	2875
.470	.0495	.357	.827	.111	.012321	.683929	.696250	.834	3135
.431	.0454	.327	.758	.101	.010010	.574564	.584574	.764	2875
.274	.0288	.208	.482	.0645	.004160	.232324	.236484	.486	1825
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300

DATA

b

Φ_m	B_m	log B_m	1.6 logB_m	$B_m^{1.6}$	10^{-4} Hy. L.	B_m^2	Ed.L. 10^{-4}	Total Losses 10^{-4}w
1565	648	2.81158	4.498528	31517	28.36	419904	13.0	41.36
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
1825	755	2.87795	4.60472	40250	36.22	236196	17.7	53.92
1175	486	2.68664	4.298664	19888 ·	17.89	236196	7.33	25.22
775	320	2.49415	4.00824	10192	9.17	102400	3.82	12.99
Av.	593	2.7731	4.43696	27340	24.60	351649	10.9	35.50
2355	975	2.98900	4.7824 ·	60590	54.53	950625	29.5	84.03
2355	975	2.98900	4.7824	60590	54.53	950625	29.5	84.03
2100	870	2.93952	4.703232	50496	45.44	756900	23.5	68.94
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
Av.	758	2.8797	4.60752	40500	36.45	574564	17.8	54.25
2870	1190	3.07555	4.92088	83347	75.01	1416100	43.9	118.91
3135	1300	3.88649	4.982304	96010	86.40	1690000	52.4	138.80
2870	1190	3.07555	4.92088	83347	75.01	1416100	43.9	118.91
1825	755	2.87795	4.60472	40250	36.22	570025	17.7	53.92
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
Av.	994	2.9974	4.79584	62500	56.25	988036	30.65	86.90

DATA

c

e	I	IR	e' e+IR	IX	$(IX)^2$	$(e')^2$	E^2	Induced E	e_m
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650
.078	.0082	.0592	.137	.0183	.000334	.018769	.019103	.139	523
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.195	.0205	.148	.343	.0460	.002116	.117649	.119765	.346	1300
.137	.0144	.104	.241	.0323	.001043	.058081	.059124	.243	915
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650
.117	.0123	.088	.205	.0275	.000756	.042025	.042781	.206	775
.215	.0226	.155	.370	.0506	.002560	.136900	.139460	.373	1400
.215	.0226	.155	.370	.0506	.002560	.136900	.39460	.373	1400
.157	.0165	.119	.276	.0370	.001369	.076176	.077545	.278	1045
.098	.0103	.0744	.172	.0231	.000533	.029584	.030117	.173	650

DATA

c

Φ_m	B_m	log B_m	1.6 log B_m	$B_m^{1.6}$	10^{-4} Hy.L.	B_m^2	Ed.L. 10^{-4}	Total losses 10^{-4} w
650	269	2.42975	3.8876	7720	6.94	72361	2.24	9.18
775	312	2.49415	4.00824	10192	9.17	97344	3.82	12.99
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
650	269	2.42975	3.8876	7720	6.94	72361	2.24	9.18
523	216	2.33445	3.73512	5434	4.89	46656	1.44	6.38
Av.	300	2.4771	3.96336	9190	8.27	90000	2.79	11.06
650	269	2.42975	3.8876	7720	6.94	72361	2.24	9.18
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
1300	538	2.73078	4.369248	23400	21.06	289444	8.97	30.03
915	378	2.57864	4.123984	13305	11.97	143641	4.43	16.40
650	269	2.42975	3.8876	7720	6.94	72361	2.24	9.18
Av.	377	2.5763	4.12208	13243	11.91	142129	4.41	16.32
775	312	2.49415	4.00824	10192	9.17	97344	3.82	12.99
1400	580	2.76343	4.421488	26392	23.75	336400	10.04	33.79
1400	580	2.76343	4.421488	26392	23.75	336400	10.04	33.79
1045	433	2.63649	4.218384	16535	14.88	187489	5.82	20.70
650	269	2.42975	3.8876	7720	6.94	72361	2.24	9.18
Av.	435	2.6385	4.2216	16656	14.99	189225	5.87	20.86

DATA

Summary of Results

$L = 26.5"$

a			b			c			
W_h	W_e	W_t	W_h	W_e	W_t	W_h	W_e	W_t	
$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	$10^{-4}w$	
9.17	3.82	12.99	28.36	13.0	41.36	6.94	2.24	9.18	
11.97	4.43	16.40	36.22	17.7	53.92	9.17	3.82	12.99	I_a=285
21.06	8.97	30.03	36.22	17.7	53.92	14.88	5.82	20.70	I_b=285
36.22	17.7	53.92	17.89	7.33	25.22	6.94	2.24	9.18	I_c=285
28.36	13.0	41.36	9.17	3.82	12.99	4.89	1.44	6.33	
20.3	8.57	28.87	24.60	10.9	35.50	8.27	2.79	11.06	Av.

Hys. 17.72 Edd. 7.42 Total 25.14 Average losses per cu. cm. of tower

6.94	2.24	9.18	54.53	29.5	84.03	6.94	2.24	9.18	
14.88	5.82	20.70	54.53	29.5	84.03	14.88	5.82	20.70	I_a=350
28.36	13.0	41.36	45.44	23.5	68.94	21.06	8.97	30.03	I_b=350
49.72	26.25	75.97	21.06	8.97	30.03	11.97	4.43	16.40	I_c=350
36.22	17.7	53.92	14.88	5.82	20.70	6.94	2.24	8.18	
25.41	11.35	36.76	36.45	17.8	54.25	11.91	4.41	16.32	Av.

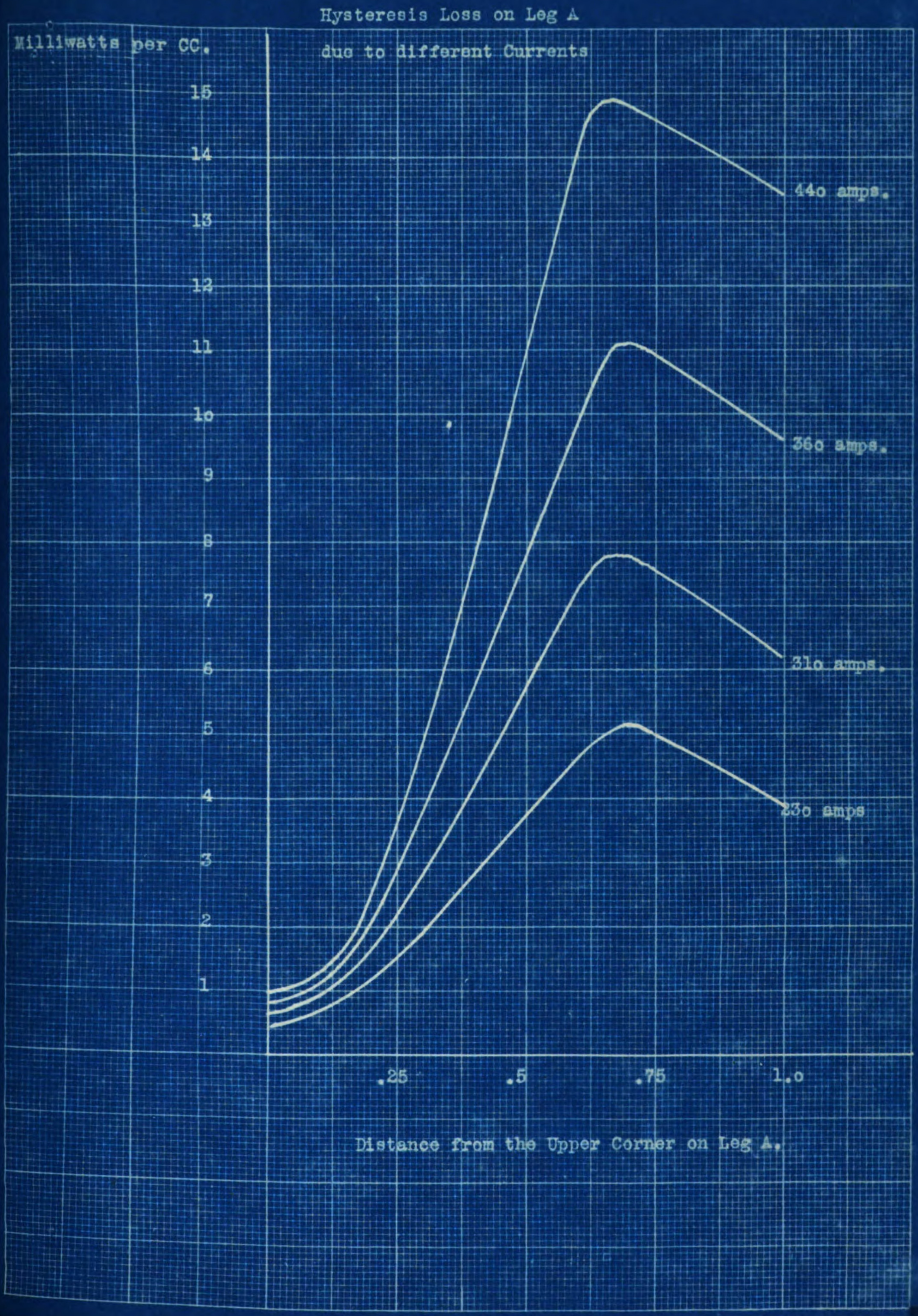
Hys. 24.59 Edd. 11.19 Total 35.78 Average losses per cu. cm. of tower

14.88	5.82	20.70	75.01	43.9	118.91	9.17	3.82	12.99	
28.36	13.0	41.36	86.40	52.4	138.80	23.75	10.04	33.79	I_a=445
54.53	29.5	84.03	75.01	43.9	118.91	23.75	10.04	33.79	I_b=445
86.40	52.4	138.80	36.22	17.7	53.92	14.88	5.82	20.70	I_c=445
28.36	13.0	41.36	21.06	8.97	30.03	6.94	2.24	9.18	
39.73	19.85	59.58	56.25	30.65	86.90	14.99	5.87	20.86	Av.

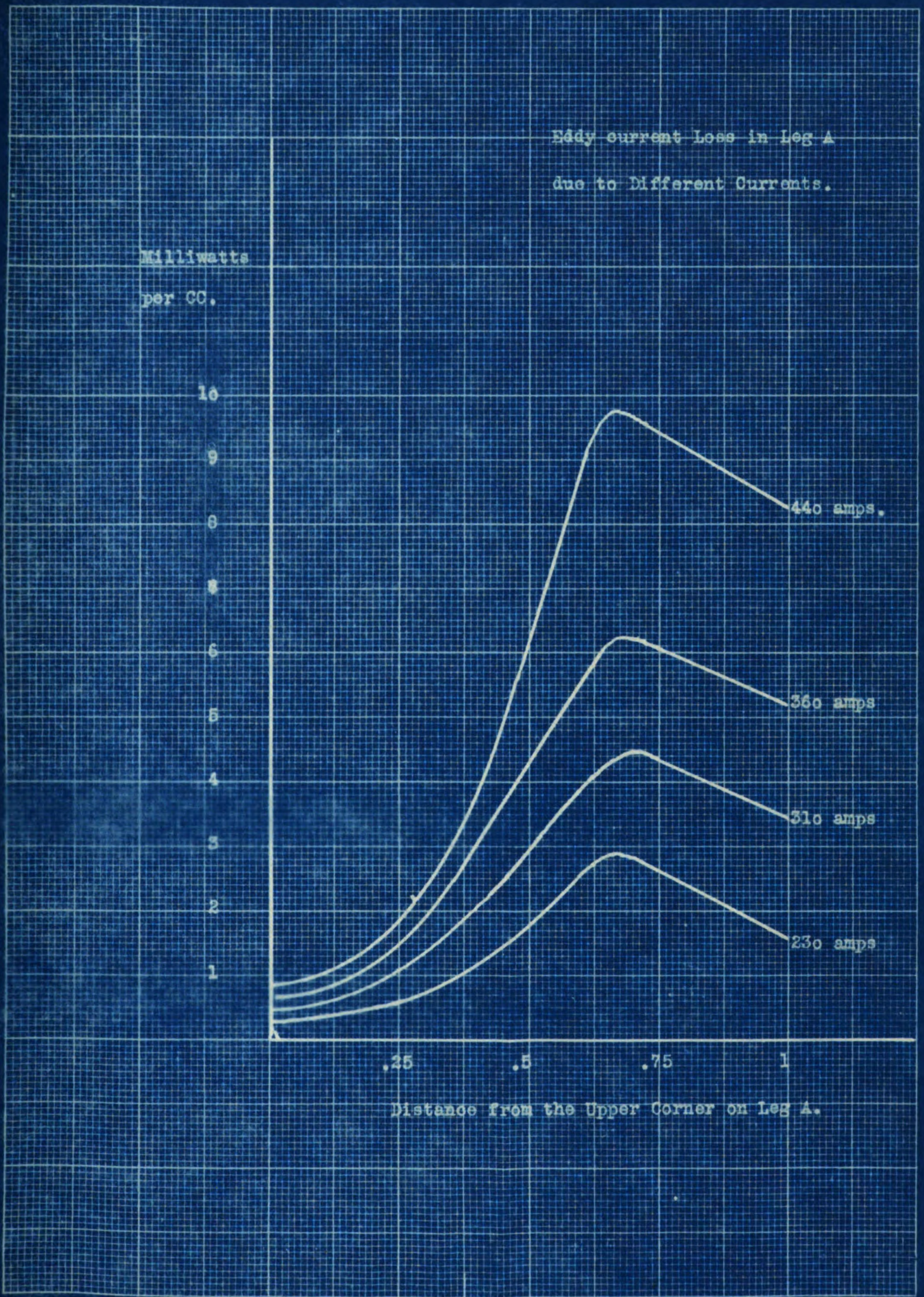
Hys. 36.99 Edd. 18.79 Total 55.78 Average losses per

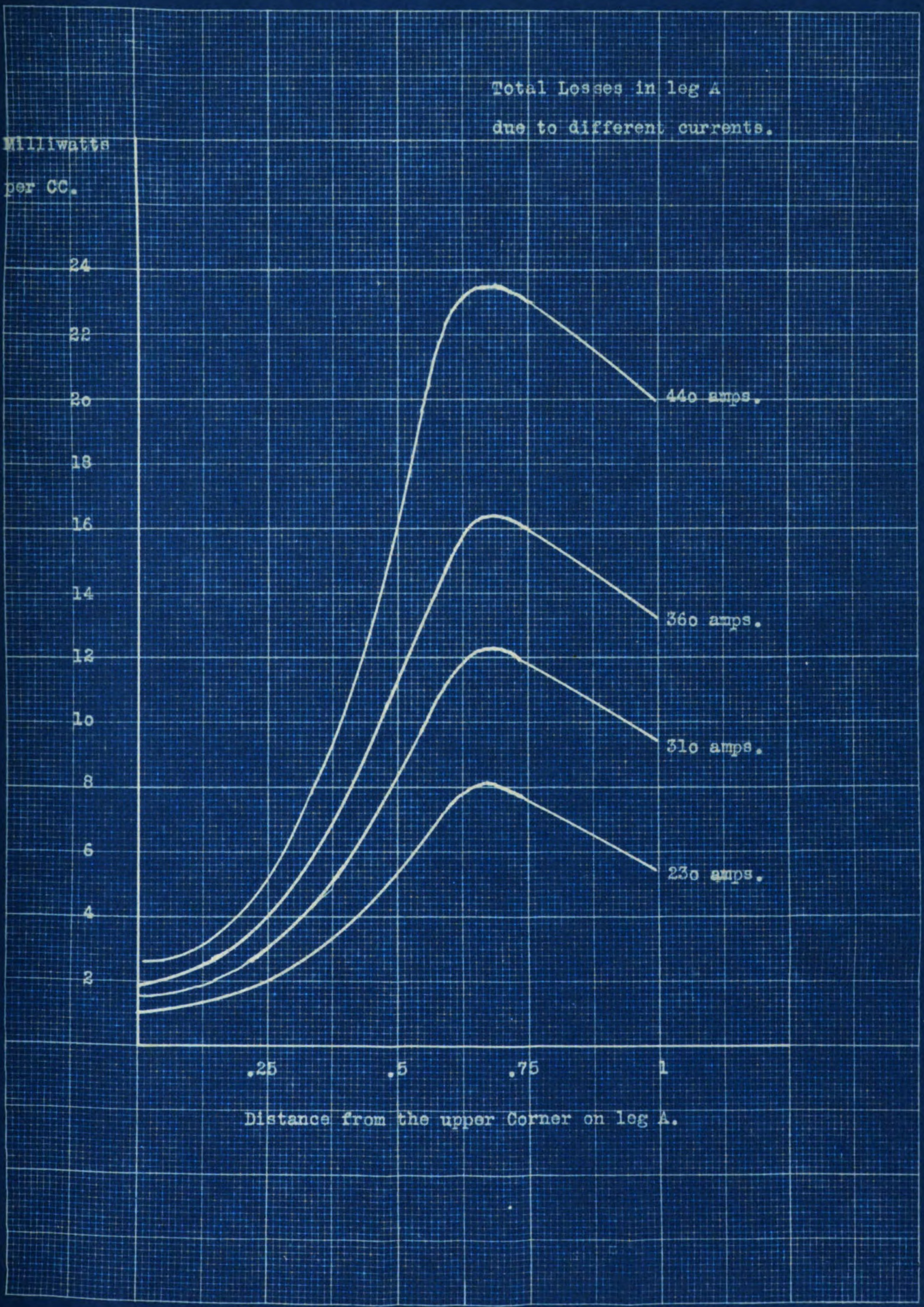
The following curves are loss distance curves. They show the distribution of losses at various points along the legs.

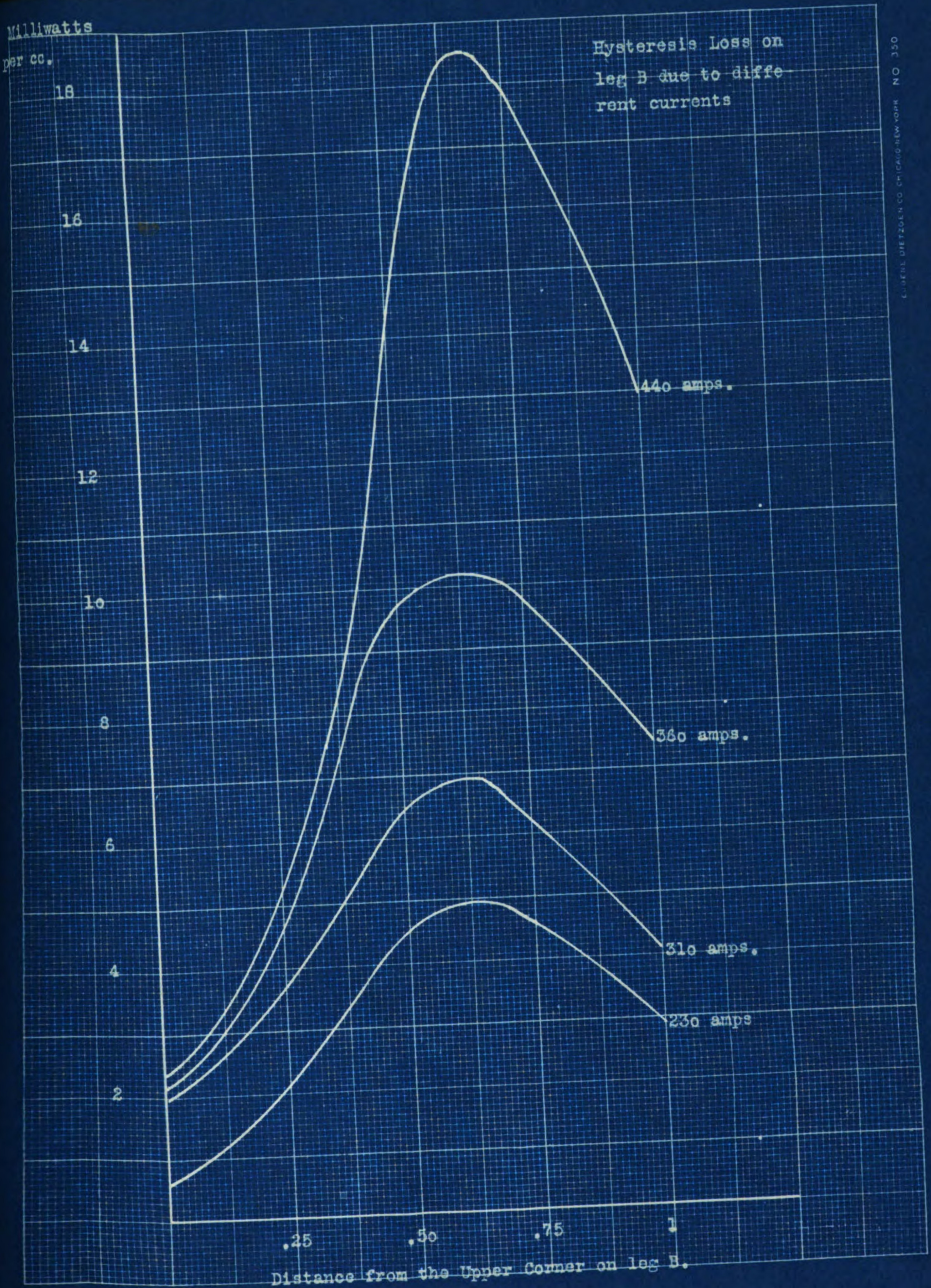
The area under each curve was measured and when divided by the length it gave the same losses as obtained by computation when using the average flux.

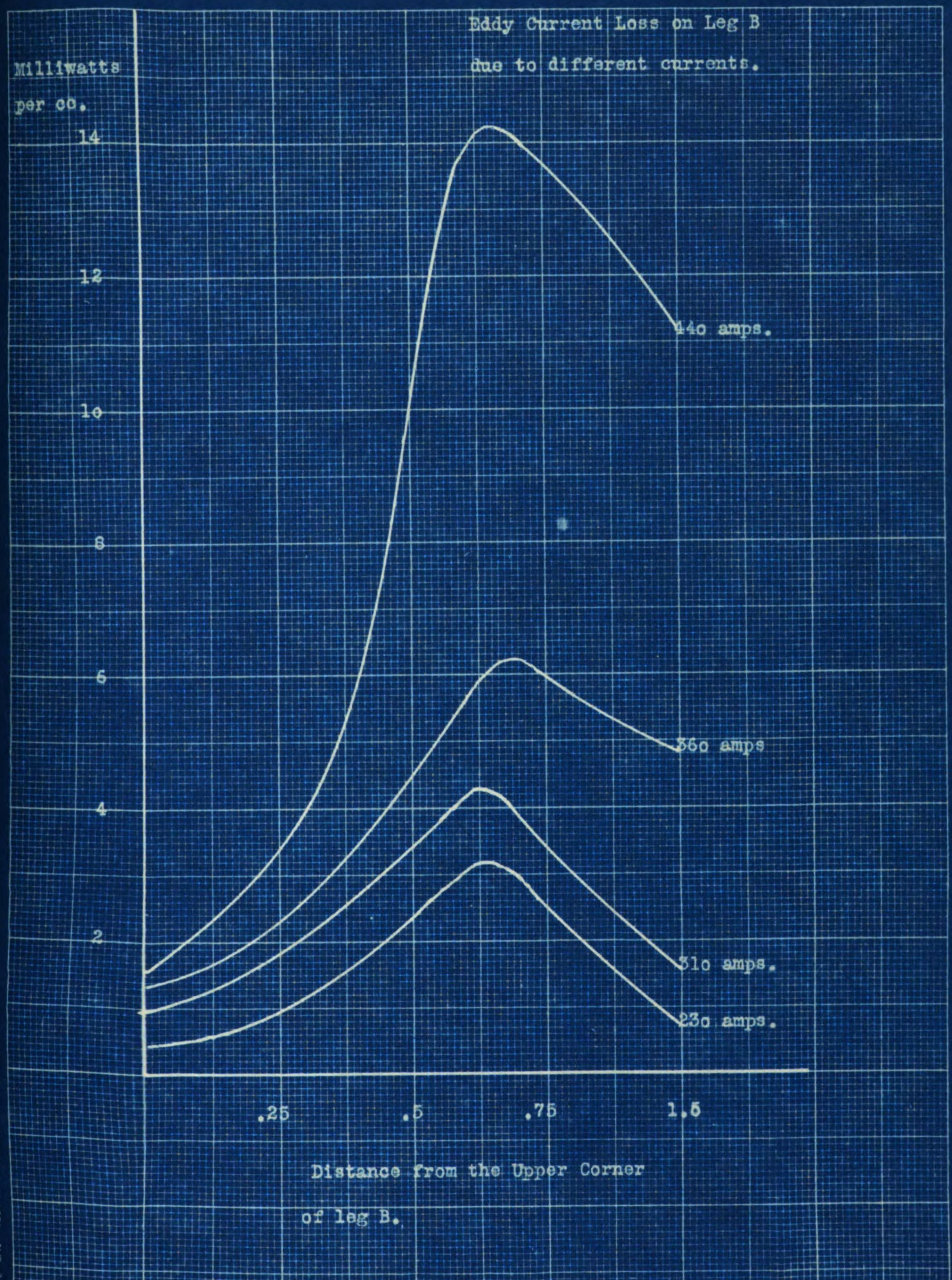


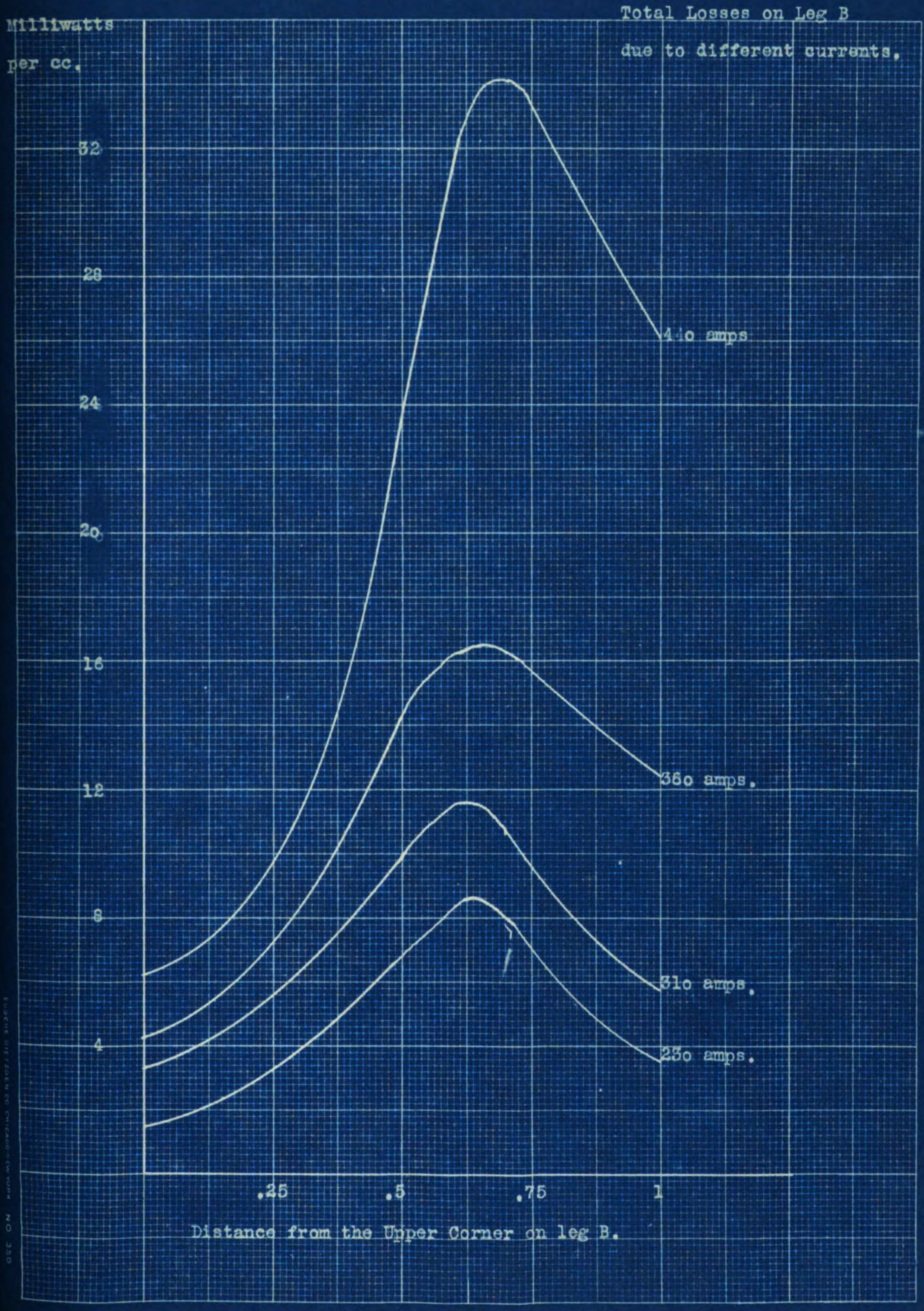
Hysteresis Loss on Leg A
due to different Currents
Milliwatts per CC.
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
440 amps.
360 amps.
310 amps.
230 amps
.25
.5
.75
1.0
Distance from the Upper Corner on Leg A.


Eddy current Loss in Leg A
due to Different Currents.
Milliwatts
per CC.
10
9
8
7
6
5
4
3
2
1
440 amps.
360 amps
310 amps
230 amps
.25
.5
.75
1
Distance from the Upper Corner on Leg A.


Total Losses in leg A
due to different currents.
Milliwatts
per CC.
24
22
20
18
16
14
12
10
8
6
4
2
440 amps.
360 amps.
310 amps.
230 amps.
.25
.5
.75
1
Distance from the upper Corner on leg A.

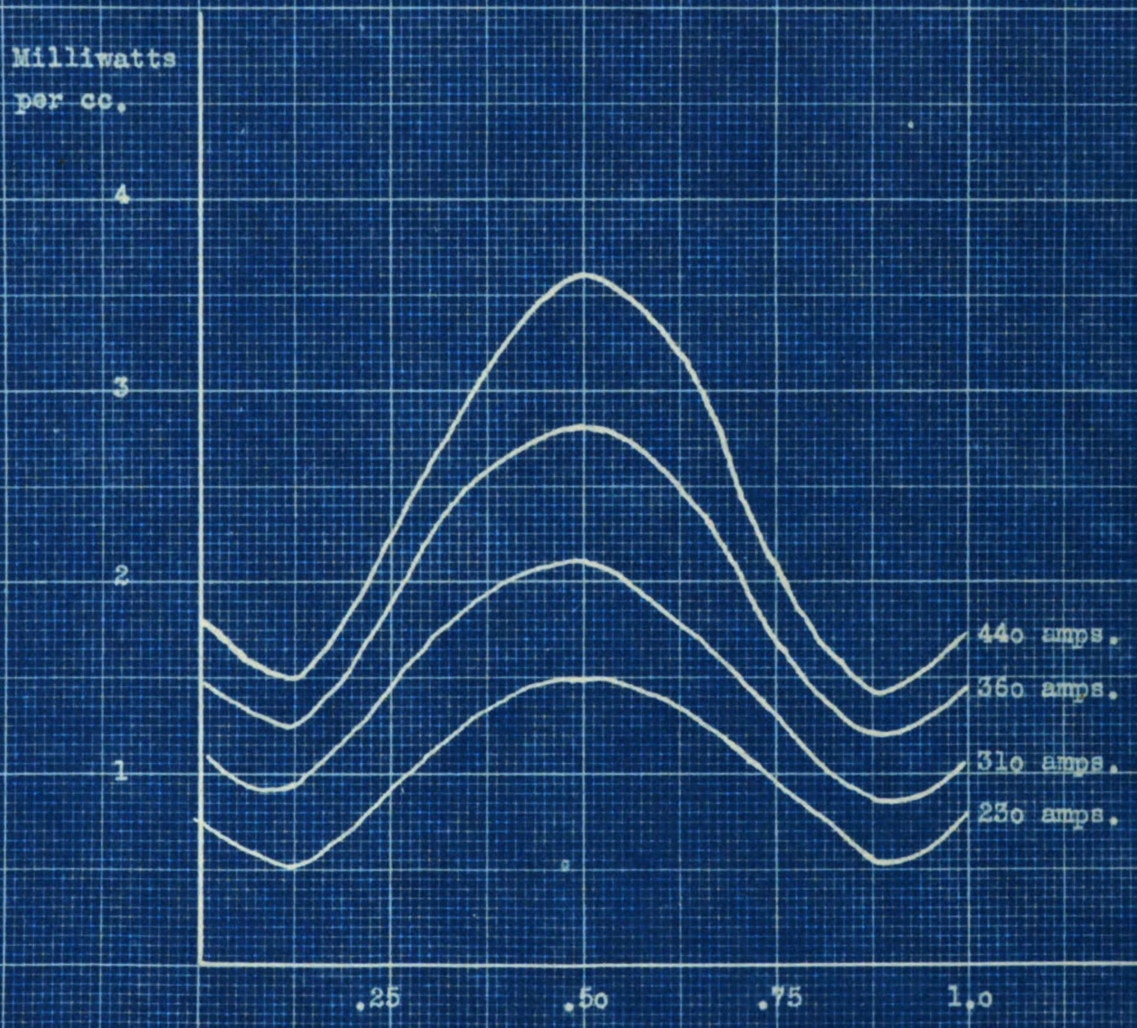

Milliwatts
per cc.
18
16
14
12
1o
8
6
4
2
Hysteresis Loss on
leg B due to diffe-
rent currents
44o amps.
36o amps.
31o amps.
23o amps
.25
.5o
.75
1
Distance from the Upper Corner on leg B.


Eddy Current Loss on Leg B
due to different currents.
Milliwatts
per cc.
14
12
10
8
6
4
2
440 amps.
360 amps
310 amps.
230 amps.
.25
.5
.75
1.5
Distance from the Upper Corner
of leg B.

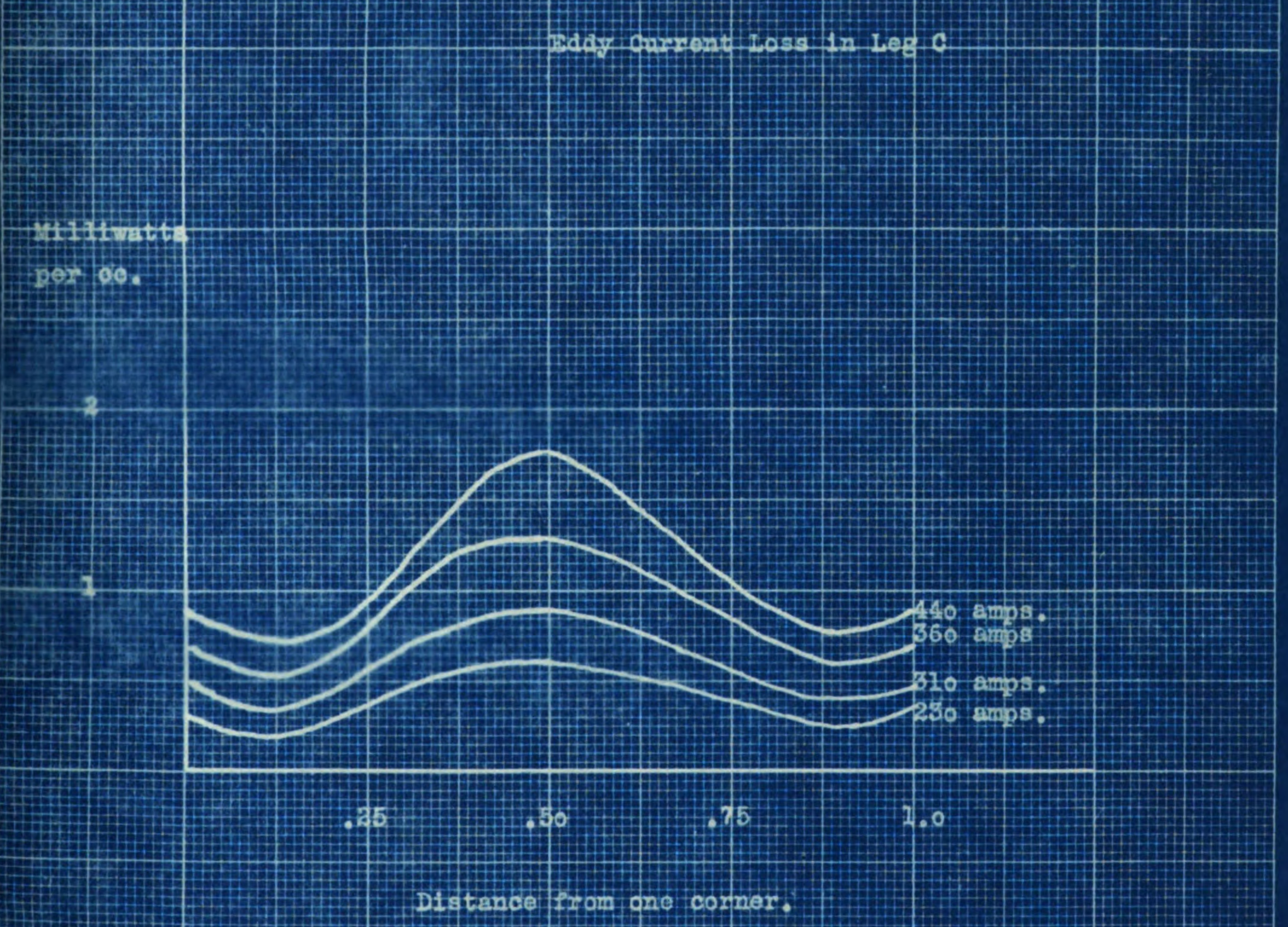


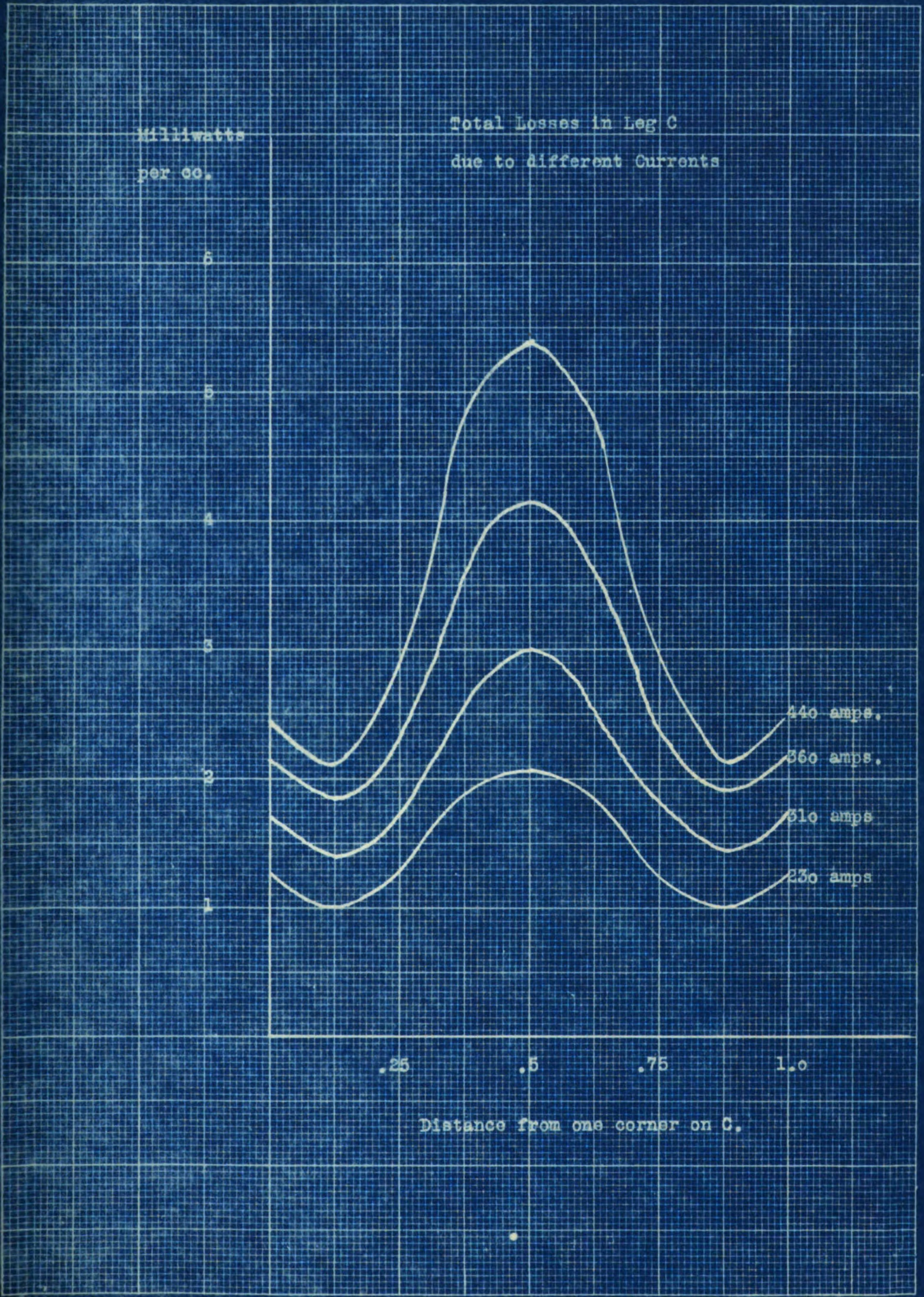
Total Losses on Leg B
due to different currents.
Milliwatts
per cc.
32
28
24
20
16
12
8
4
440 amps
360 amps.
310 amps.
230 amps.
.25
.5
.75
1
Distance from the Upper Corner on leg B.

Hysteresis Loss in Leg C at different points
due to different Currents,
conductor at 19.5 in.

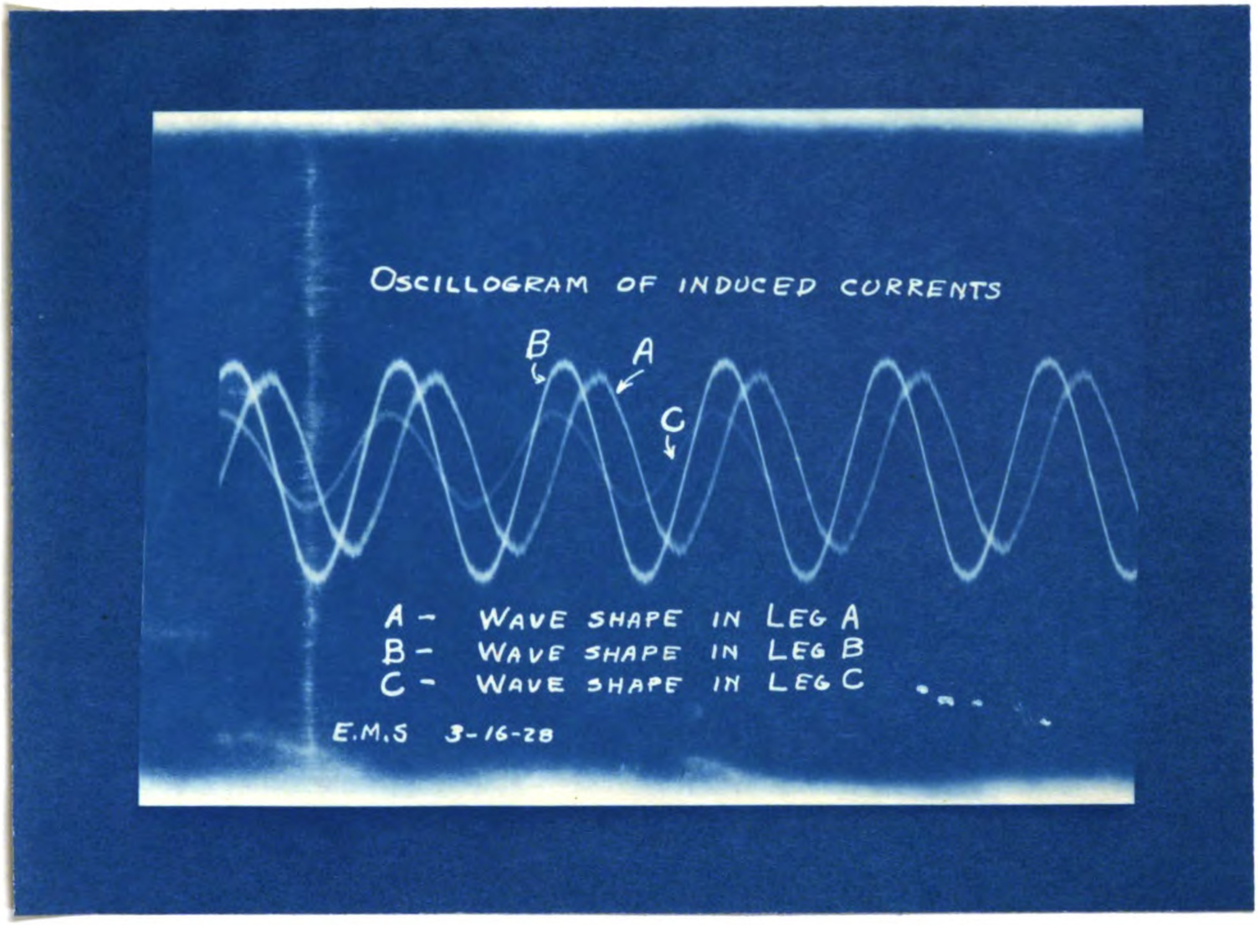


Distance from one corner expressed
as a fraction of the total length
of leg C.


Eddy Current Loss in Leg C
Milliwatts
per cc.
2
1
44o amps.
36o amps
31o amps.
23o amps.
.25
.5o
.75
1.o
Distance from one corner.



Milliwatts
per cc.
Total Losses in Leg C
due to different Currents
6
5
4
3
2
1
44o amps.
36o amps.
31o amps
23o amps
.25
.5
.75
1.o
Distance from one corner on C.



An oscillogram of the induced currents on the three legs.

Results from experiment

$$\left(\frac{360}{230}\right)^n = \frac{86}{38.95}$$

$$n\,(2.556303 - 2.361728) = 1.934498 - 1.591065$$

$$.194575\,n = .343433$$

$$n = 1.765$$

$$\left(\frac{440}{360}\right)^n = \frac{120}{86}$$

$$n\,(2.643453 - 2.556303) = 2.082785 - 1.934498$$

$$.087150\,n = .148287$$

$$n = 1.701$$

$$\left(\frac{440}{230}\right)^n = \frac{120}{38.95}$$

$$n\,(2.643453 - 2.361728) = 2.082785 - 1.591065$$

$$.281725\,n = .491720$$

$$n = 1.745$$

$$\left(\frac{475}{360}\right)^n = \frac{81}{49}$$

$$n\,(2.676694 - 2.556303) = 1.908485 - 1.690196$$

$$.120391\,n = .218289$$

$$n = 1.813$$

$$\left(\frac{475}{320}\right)^n = \frac{81}{42}$$

$$n\,(2.676694 - 2.507856) = 1.908485 - 1.623249$$

$$.168838\,n = .285236$$

$$n = 1.689$$

Average $n = (1.765 + 1.701 + 1.745 + 1.313 + 1.689)1/5$

$$= 1.742$$

Reducing losses to same currents at different distances

$$1.742\ (\log 360 - \log 285) = \log 86 - \log x$$

$$1.742\ (2.556303 - 2.454845) = 1.934498 - \log x$$

$$1.742\ (.101458) = 1.934498 - \log x$$

$$.176740 = 1.934498 - \log x$$

$$\log x = 1.757758$$

$$x = 57.247$$

$$1.742\ (\log 360 - \log 200) = \log 86 - \log 2$$

$$1.742\ (2.556303 - 2.301030) = 1.934498 - \log x$$

$$1.742\ (.255273) = 1.934498 - \log x$$

$$.444685 = 1.934498 - \log x$$

$$\log x = 1.489813$$

$$x = 30.89$$

$$1.742\ (\log 360 - \log 285) = \log 49.22 - \log x$$

$$1.742\ (2.556303 - 2.454845) = 1.692142 - \log x$$

$$.176740 = 1.692142 - \log x$$

$$\log x = 1.515402$$

$$x = 32.76$$

$$1.742\ (\log 360 - \log 200) = \log 49.22 - \log x$$

$$.444685 = 1.692142 - \log x$$

$$\log x = 1.247457$$

$$x = 17.67$$

$$1.742\ (\log 440 - \log 360) = \log x - \log 49.22$$

$$1.742\ (2.643453 - 2.556303) = \log x - 1.692142$$

$$1.742\ (.087150) = \log x - 1.692142$$

$$.151815 = \log x - 1.692142$$

$$\log x = 1.843957$$

$$x = 69.81$$

$$1.742\ (\log 360 - \log 285) = \log x - \log 25.14$$

$$.176740 = \log x - 1.400365$$

$$\log x = 1.577105$$

$$x = 37.77$$

$$1.742\ (\log 360 - \log 200) = 1.577105 - \log x$$

$$.444685 = 1.577105 - \log x$$

$$\log x = 1.132420$$

$$x = 13.56$$

$$1.742\ (\log 440 - \log 360) = \log x - 1.577105$$

$$.151815 = \log x - 1.577105$$

$$\log x = 1.728920$$

$$x = 53.57$$

The combination of those results are shown in the following table.

Reducing to Four Different Currents

I	Dis-tance	Losses 10^{-4} w	Dis-tance	Losses 10^{-4} w	Dis-tance	Losses 10^{-4} w
360	19.5	86	24	49.22	26.5	37.77
285	19.5	57.24	24	32.76	26.5	25.14
200	19.5	30.89	24	17.67	26.5	13.56
440	19.5	120.88	24	69.81	26.5	53.57

Four different currents were chosen in such a manner that the losses due to one of these currents are known at one distance while those due to the other are known for some other distance. By using the formula

$$\left(\frac{I_1}{I_2}\right)^{1.742} = \frac{L_1}{L_2}$$

the losses for the various currents were found at different distances.

From the Table we get

$$\left(\frac{19.5}{24}\right)^x = \frac{49.22}{86}$$

$$x(\log 24 - \log 19.5) = \log 86 - \log 49.22$$

$$x(1.380211 - 1.290035) = 1.934498 - 1.692142$$

$$.090186\ x = 2.42356$$

$$x = 2.687$$

$$\left(\frac{19.5}{24}\right)^x = \frac{17.67}{30.89}$$

$$.090186\ x = 1.247237 - 1.489818$$

$$.090186\ x = .242581$$

$$x = 2.689$$

$$\left(\frac{19.5}{24}\right)^x = \frac{32.76}{57.24}$$

$$.090186\ x = 1.757758 - 1.515402$$

$$.090186\ x = .242356$$

$$x = 2.687$$

$$\left(\frac{19.5}{26.5}\right)^x = \frac{37.77}{86}$$

$$x[1.423246 - 1.290035] = 1.934498 - 1.577105$$

$$.133211\ x = .357393$$

$$x = 2.683$$

$$\left(\frac{19.5}{26.5}\right)^x = \frac{25.14}{57.24}$$

$$.133211\ x = 1.757758 - 1.400365$$

$$.133211\ x = .357393$$

$$x = 2.683$$

$$\left(\frac{19.5}{26.5}\right)^x = \frac{13.56}{30.89}$$

$$.133211\ x = 1.489813 - 1.132420$$

$$.133211\ x = .35739$$

$$x = 2.683$$

$$\frac{13.56}{17.67} = \left(\frac{24}{26.5}\right)^x$$

$$1.247457 - 1.132420 = (1.423246 - 1.380211)x$$

$$.115037 = .043035$$

$$x = 2.673$$

From the preceding results it is seen that x has a constant value.

Therefore $x = (2.687 + 2.689 + 2.687 + 2.663 + 2.683 + 2.673)\frac{1}{6}$

$$x = 2.683$$

Deduction of laws

From results it is seen that for a constant distance

$$\left(\frac{I_1}{I_2}\right)^m = \frac{L_1}{L_2} \qquad (1)$$

or more exactly

$$\left(\frac{I_1}{I_2}\right)^{1.742} = \frac{L_1}{L_2} \qquad (2)$$

and for a constant current

$$\left(\frac{D_1}{D_2}\right)^x = \frac{L_2}{L_1} \qquad (3)$$

or more exactly

$$\left(\frac{D_1}{D_2}\right)^{2.683} = \frac{L_2}{L_1} \qquad (4)$$

Laws (2) and (4) give the relation between currents, distances, and losses

Thus

$$\left(\frac{D_1}{D_2}\right)^x = \left(\frac{I_2}{I_1}\right)^n \qquad (5)$$

or

$$D_2^x\, I_2^n = D_1^x\, I_1^n \qquad (6)$$

showing that the same loss is obtained when current I_1 becomes I_2 if D_1 becomes D_2. The value of D_2 is found from (5) or (6).

From the deduced laws let us find the losses due to a unit current at a unit distance. First find losses at distance 19.5'' due to unit current.

From (2)

$$\left(\frac{I_1}{I_2}\right)^{1.742} = \frac{L_1}{L_2}$$

$$\left(\frac{1}{200}\right)^{1.742} = \frac{x}{30.89 \times 10^{-4}}$$

$$\log 200 = 2.301030$$

$$1.742 \log 200 = 4.008394$$

$$(200)^{1.742} = 10148$$

$$\frac{1}{10148} = \frac{x}{30.89\ 10^{-4}}$$

$$x = 3.045\ 10^{-7}$$

Then find losses at distance of 1 inch

From (4)

$$\left(\frac{19.5}{1}\right)^{2.683} = \frac{x}{3.045\ 10^{-7}}$$

$$\log 19.5 = 1.290035$$

$$2.683 \log 19.5 = 3.461163$$

$$(19.5)^{2.683} = 2891.8$$

$$2891.8 = \frac{x}{3.045\ 10^{-7}}$$

$$x = 8.805531\ 10^{-4}$$

$$\left(\frac{1}{360}\right)^{1.742} = \frac{x}{86}$$

$$\log 360 = 2.556303$$

$$1.742 \log 360 = 4.453079$$

$$(360)^{1.742} = 28384$$

$$\frac{1}{28384} = \frac{x}{86}$$

$$x = 3.029\ 10^{-7}$$

Then at 1 inch distance it is

$$\left(\frac{19.5}{1}\right)^{2.683} = \frac{x}{3.029\ 10^{-7}}$$

$$2891.8 = \frac{x}{3.029 \times 10^{-7}}$$

$$x = 8.759\ 10^{-4}$$

$$\left(\frac{1}{285}\right)^{1.742} = \frac{x}{57.24\ 10^{-4}}$$

$$\log 285 = 2.454845$$

$$1.742 \log 285 = 4.276339$$

$$(285)^{1.742} = 18895$$

$$\frac{1}{18895} = \frac{x}{57.24\ 10^{-4}}$$

$$x = 3.029$$

Then at 1 inch distance it is

$$\left(\frac{19.5}{1}\right)^{2.683} = \frac{x}{3.029\ 10^{-7}}$$

$$x = 8.759\ 10^{-4}$$

The average x is then 8.759×10^{-4}

Therefore, if it is desired to find the losses due to a current I, D inches from the tower use the following formula

$$L_T = 8.759\ 10^{-4}\ \frac{(I)^{1.742}}{(D)^{2.683}}$$

As a check assume $I = 440$

$D = 24''$

$$L_T = 8.759\ 10^{-4}\ \frac{(440)^{1.742}}{(24)^{2.683}}$$

$\log 440 = 2.643453$

$1.742 \log 440 = 4.604895$

$(440)^{1.742} = 40262$

$\log 24 = 1.380211$

$2.683 \log 24 = 3.703106$

$(24)^{2.683} = 5047.8$

$$L_T = 8.759\ 10^{-4}\ \frac{40262}{5047.8}$$

$= 8.759 \times 7.97 \times 10^{-4}$

$= 69.80 \times 10^{-4}$ as checked with 69.81×10^{-4}