A STUDY OF THE CONCATENATION OF INDUCTION MOTORS

THESIS FOR THE DEGREE OF B. S.

Charles E. Slider

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THESIS

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Title Concatenation of
induction motors

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A STUDY OF THE CONCATENATION OF INDUCTION MOTORS.

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THESIS

To Professor Foltz,and members of the Electrical Engineering department faculty,to whom I am indebted for their co-operation in the obtaining of the data, this thesis is dedicated.

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A STUDY OF THE CONCATENATION OF INDUCTION MOTORS.

PART 1

Chapter I

Introduction and General Theory of the Induction Motor.

In the commercializing of alternating current as a means of distributing electrical energy, probably no one factor has been of greater importance than the invention and development of the multi-phase induction motor. It was invented by Mr. Nikola Tesla, in 1888, although the first successful motor was designed most probably by Mr. C. E. L. Brown at the Oerlikon works in Switzerland. Its simplicity of construction, ruggedness, and freedom from trouble has made it very popular, especially for service which requires only one or two speeds.

The Electrical and Magnetic Circuits.

The induction motor is essentially a transformer in which the secondary is free to revolve. There is also an air gap in the magnetic circuit of the induction motor. Due to the air gap, a much greater magnetizing current is required in the induction motor than in the transformer. This may be seen from the following equations:

$$(1) \quad \phi = \frac{F}{R}$$

Where, ϕ is the desired flux

F = the magnetizing force = $.4 \pi n I$

R = the reluctance = $\frac{l}{u a}$

Then for the same flux in the magnetic circuit of the motor as in the core of a similiar transformer, the following differences will exist. The reluctance of the magnetic path of the motor will be equal to the sum of the reluctances of the air gap and the iron path in series, and, as the permeability of the air gap is one and that of the iron may be several hundred, the reluctance of the air gap may easily enough be as great or greater than that of the core of either the motor or the transformer. If the flux is to remain the same, then an increase in the denominator must be accompanied by an increase in the numerator F. As all the quantities comprising F are fixed except I, then I must increase. The magnetizing component of the current is 90°out of phase with the power component and so the power factor is very low. Thus the air gap in the induction motor is made as small as practicable. The relatively large magnetizing component at no load and light load causes a low power factor, which gives rise to one of the greatest objections to induction motors running under these conditions. The vector sum of the power component and the magnetizing component is often called the exciting current and the magnetizing component simply the magnetizing current. The ratio of the magnetizing current to the diameter of the circle of current loci for any given motor, is called the leakage factor, and is denoted by a. Thus if the magnetizing current of a certain motor is 5 amps., and the diameter of the current loci circle(to the same scale)is 100 amps.,then the leakage factor will be .05. This is an important ratio because of its relation to many of the physical dimensions of the motor.It has been proven by experiment that the leakage factor varies directly as the air gap and inversely as the pole pitch.In experimenting with the air gap it was found that the diameter of the current loci circle remained practically the same regardless of the air gap,while the magnetizing current increased in the same ratio as the gap.The leakage factor also gives a means for estimating the maximum power factor which may be expected from a given motor.This is shown by figure 1

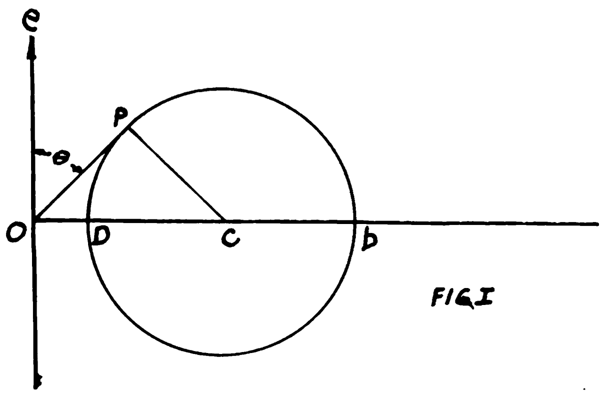


FIG.I

From trigonometry,the cosθ,or the maximum power factor will be;

$$\frac{C\ P}{O\ C}$$

or , $\cos\theta = \dfrac{1}{1 + 2a}$ where a is the leakage factor.

The maximum power factor occuring where the primary current O P is a tangent to the circle.In the design of a motor these different relations are an aid in determining the correct and most economical proportions.

The electrical circuit of the induction motor consists of two windings,(1) The stator or primary winding and(2) The rotor or secondary winding.

The stator winding must fulfill three requirements,(1)its conductors must be of sufficient crossection to carry both the load current and the exciting current.(2)it must have a sufficient number of turns to set up the required magnetic field.(3)The combination of the number of turns and the magnetic field must be such that a counter electro-motive force will be generated which is always slightly less than the impressed voltage.If all these conditions are satisfied,then the motor stator winding should perform its function properly.

The rotor carries the load current,and as it has no electrical connection to either the stator or the line,it must receive this load current by transformer action from the stator.There are two common types of rotor windings (1) the squirrel cage and(2)the wound rotor.

In the squirrel cage type the conductors are placed in slots in the outer edge of the rotor,the ends being welded to a short circuiting ring.Thus the phases cannot be opened and the same rotor may be used in machines having a different number of poles in the stator.In fact the stator will induce an opposite pole on the rotor for each one of its own.

The wound rotor has a winding placed on it which has the same number of poles as the stator in which it is going to be placed.It has many advantages as will be discussed later under torque.Due to the extra amount of labor required in winding,its first cost is much greater than that of the squirrel cage type.The later is more rugged,making a better motor for all around use.

Chapter II

The Revolving Field.

The operation of the induction motor depends entirely opon the revolving magnetic field.From a sine wave representation of a three phase alternating current,it can readily be seen that the resultant magnetic field at any instant will be zero. Now,if one of the waves be reversed(or in other words if one of the coils in a three phase winding be reversed) the resultant will be seen to be one wave of magnetizing current.This is shown by fig.2.

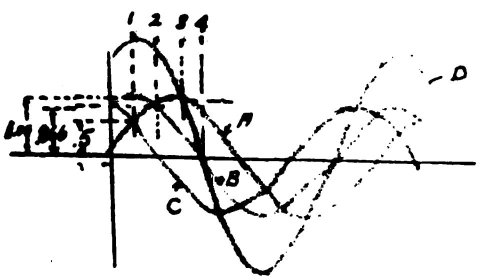
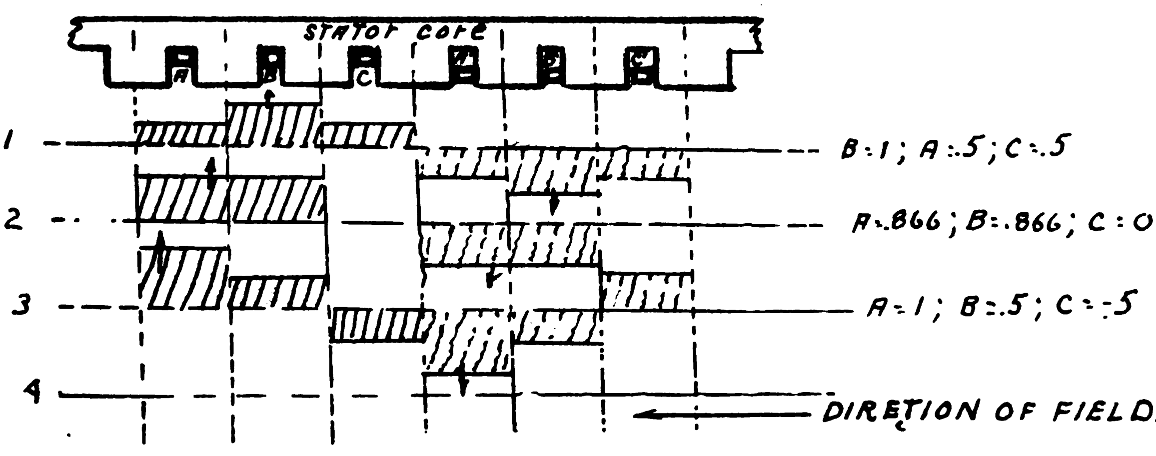


Fig.2

Then if these three coils are shown in the stator in the different time positions as follows;



The coils A,B,and C in the stator slots of figure three are carrying current as represented by the waves A,B,and C of fig. 2 respectively. It will be seen that the amplitude of each of the waves is transferred to the chart at the designated points. The crossectioned area represents the flux in both quantity and sign. The arrows on the crest of the waves indicate that the flux wave is moving to the left. In this way it produces the revolving field. From a study of fig.2, it is apparent that each pair of poles will represent 360 electrical degrees, or one complete revolution or cycle. Then if the frequency in cycles per. second be multiplied by 60 to change it to cycles per. minuite and this be divided by the number of pairs of poles, the speed of the motor will be obtained. Thus;

(3) $$\text{Speed in r.p.m.} = \frac{f \times \cancel{\text{pairs of poles}}\ 60}{\text{pairs of poles}}$$

(4) Transposing, $$f = \frac{\text{r.p.m.} \times \text{pairs of poles}}{60}$$

(5) or, $$\text{No. poles} = \frac{2 \times f \times 60}{\text{r.p.m.}}$$

In effect the flux wave travels as many electrical degrees in a ten pole motor as in a two pole, but it travels a smaller portion of a revolution in passing 360 electrical degrees, thus the speed depends opon the number of poles a given point would pass in going completely around the stator.

Chapter III

Slip, Current and Voltage Relations, and Torque.

When the rotor is at standstill, the frequency of the current in its coils is the same as that of the stator. The rotor now bears the same relation to the stator in this respect, as the secondary coil of a transformer does to the primary. The speed at which the stator flux rotates is called synchronous speed. As the rotor starts to turn, the difference between its speed and that of the rotating field becomes less, thus the frequency in the rotor becomes less, until at synchronous speed it would be zero. This relation will be a straight line function. The slip, which is the ratio of the difference between the rotor and synchronous speeds, and the synchronous speed thus;

$$(6) \quad \text{Slip} = \frac{\text{Syn. speed} - \text{Rotor speed}}{\text{Syn. speed}}$$

will vary in like manner. Then the frequency at any rotor speed will be,

(7) $f_2 = sf_1$, where f1 is the stator frequency and s is the slip expressed as a decimal. The resistance of the rotor remains practically constant.

The reactance will be $2\pi fL$, where L is the inductance in Henries. L remains constant, and so the reactance will vary as the frequency. From this we obtain;

(8) $X_2 = sX_1$, where X_1 is the reactance at standstill.

From the following equation for any induced voltage;

$$(9) \quad E = 4.44\, n\, f\, \phi_{max}\, 10^{-8}$$

it becomes apparent that, assuming constant flux, the induced rotor voltage will vary directly as the frequency. Assuming a ratio of transformation from the primary to secondary of one, the voltage induced in the rotor at any instant may be expressed by the following equation,

$$(10) \quad E_2 = E_1 s \text{ ,where } E_1 \text{ is the stator voltage.}$$

The current in the rotor, I_2, will then be,

$$(11) \quad I_2 = \frac{E_2}{Z_2}$$

$$\text{or} = \frac{s\ E_2}{(r_2^2 + s^2 X_2^2)^{\frac{1}{2}}}$$

From this equation it can be seen that with a small increase in slip near synchronous speed, the current will be greatly increased, and so within the rated capacity of the motor the slip will be comparatively small. Take for example the following small 110 volt motor;

Voltage induced/phase = 70

Resistance of rotor/Ø = 1 ohm

Reactance of " " 1 "

% slip = 1 and 4

$$\text{For 1 \% slip, } I_2 = \frac{70 \times .01}{(1 + .0001)^{\frac{1}{2}}} = .7 \text{ amp.}$$

$$\text{For 4 \% slip } I_2 = \frac{70 \times .04}{(1 + .0016)^{\frac{1}{2}}} = 2.8 \text{ amps.}$$

The later would be about full load for the motor.

The power factor in the rotor will be $\frac{r_2}{Z_2}$, or in full notation, $\cos\theta_2 = \frac{r_2}{(r^2 + s^2X^2)^{\frac{1}{2}}}$ (12)

The torque in any motor is proportional to $\phi_1\phi_2$, or to $\phi_1 I_2$. Applying this to alternating currents:

$$T \propto \phi_1 I_2 \cos\theta_2 \qquad (13)$$

From equation (12), it is seen that the maximum reactance is reached when $s = 1$, or in other words at standstill. Therefore at this time the power factor will be the smallest obtainable at any possible condition. According to equation (13), the torque will be decreased materially. As many conditions arise which require a large starting torque, some means must be used to supply this demand.

From equation (12), it is evident that if r_2 be increased, the power factor at starting will be increased. In the squirrel cage type rotor, the rotor bars are rigidly fastened to the short circuiting rings, and so the phases cannot be opened and resistance inserted. However in the wound rotor type the phases are left open and either brought out through slip rings to external rheostats, or resistances are placed in the rotor controlled by an automatic device which shorts the resistances out as soon as the rotor comes up to a set speed.

The ratio between r and X which will give max. torque at variable slip may be found by differentiating the general expression for torque,

$$T \propto \frac{s\,E_2\,r_2\,\phi_1}{r^2 + s^2X_2^2}$$

, and finding the maximum by setting the result equal to zero, and solving for r in terms of X.

Differentiating with respect to s and setting the result equal to zero,(replacing ϕ_1 and E_2,which are constant,by K)

$$\frac{dT}{ds} = \frac{(r_2^2 + s^2X_2^2)Kr_2 - Kr_2s(2sX_2^2)}{(r_2^2 + s^2X_2^2)^2} = 0$$

Discarding the denominator and collecting;

$$r_2^2 + s^2X_2^2 = 2\, s^2X_2^2$$

$$r_2^2 = s^2X_2^2$$

and extracting the squire root,

$$r_2 = sX_2 \qquad (14)$$

Then the power factor for maximum torque will be a constant, thus;

Substituting eq. (14) in (12)

$$\cos\theta \text{ for max. torque} = \frac{r_2}{(r_2^2 + r_2^2)^{\frac{1}{2}}}$$

$$= \frac{1}{2^{\frac{1}{2}}} = .707 \qquad (15)$$

At standstill the slip is unity. Substituting the full notation for torque as used in the general equation thus;

$$T \propto \frac{E_2r_2\phi_1}{r^2 + X^2}$$ (Sub. the notation for I_2 and $\cos\theta_2$)

Now if the equation s= 1 for standstill and r_2 therefore = X_2,be substituted in this equation,

$$T \propto \frac{\phi_1E_2}{2r_2}$$,which is the maximum torque at starting.

Chapter lV

Losses in Induction Motors.

The following in an induction motor must be found in order to determine the efficiency;

(1) Copper loss in the primary. - $I_1^2 r_1$

(2) " " " " secondary - $I_2^2 r_2$

(3) Iron loss in the magnetic circuit, due to hysteresis and eddy currents.

(4) Windage and friction.

Whether the motor windings are connected delta or wye, the copper loss will be = 3/2 I^2R (17), where R is the resistance between the terminals of the three phase winding, and I is the line current. Equation (17) applies to both the stator and rotor, but of course in the squirrel cage winding the phases cannot be opened and so the equivalent resistance is used.

If the rotor of a motor is open circuited and the line voltage applied to the stator, the wattmeters will read the sum of the iron loss and the I^2r loss in the stator. Then,

$$\text{Iron loss} = \text{Total loss} - I^2r \qquad (18)$$

Then if the rotor is shorted and the motor started, the difference between the total loss and that in eq.(18) will give the windage and friction loss. (except for a small I^2r loss in the secondary, which in this case is negligible)

PART 11

Methods of Finding Data for the Construction of Operating Characteristic Curves For the Induction Motor.

Chapter I

The Constant Magnetizing Current Circle Diagram.

In this discussion the data taken from the two motors M-8, and M-9, which constitute the concatenation set under consideration, will be used. The data taken from the name plates of the machines is as follows;

	M-8	M-9
Model, No.	69 A 415	73 A 586
Type	MT948 4-10-1800	MT596 6-10-1200
Form	CL	CL
Volts	220	220
Sec. Volts	170	165
Amps.	25.9	27.8
Sec. Amps.	28.	30.
No.	KF	KF
Speed F.L.	1725	1145

Both motors 3∅, 60 cycle - Built by General Electric

The first test run on the motors was the open circuit test. This gives the windage, friction and core loss. The data taken was as follows.

M-8

	I_1	I_2	I_3	W_1	W_2	E_{1-2}	E_{2-3}	E_{1-3}
Meter No.	6188	8889	22012	3443	1118	16759	21873	21556
Scale		0 - 10		0 - 1.5			0 - 300	
K		1		2			1	
	9.4	7.16	8.7	.277	.515	220	216	219
Av. Used		8.42					218.3	

M-9

Same meters and constants.

I_1	I_2	I_3	W_1	W_2	E_{1-3}	E_{2-3}	E_{1-3}
9.6	8.25	8.8	-.37	.595	220	220	220
Av.	8.88					220	

The next test taken was the short circuit test. This is taken at reduced voltage. It gives the data for the computation of short circuit current and the copper losses. Reduced voltage practically eliminates the iron losses by reducing ϕ_1. As the motor does not run there is no friction and windage loss.

M-8

	I_1	I_2	I_3	W_1	W_2	E_{1-3}	E_{2-3}	E_{1-2}
Meter No.	10029	22010	21518	207	208	3228	2561	18023
K	5	5	1	$2\frac{1}{2}$	$2\frac{1}{2}$	1	1	1
	3.8	3.8	18.2	.26	-.09	46.2	46.2	46.2
Av.		18.9					46.2	

M-9

(same meters)

4.00	3.8	19.5	-.115	.32	51.75	50.	51.4
Av.	19.5					51.05	

The data taken will now be worked ~~up~~ into shape for dimension -ing the circle diagrams.

M-8

After averaging the values in the short circuit data, the following figures are found,

$I = 18.9$ Amps. $W = \dfrac{.26 - .09}{.001} \times 2.5 = 425$ watts $E = 46.2$

The ratio between line voltage and the voltage applied is,

$\dfrac{220}{46.2} = 4.775$,and as the short circuit current at full line voltage will vary as this ratio,then I_{ss} will equal,

$4.775 \times 18.9 = 90.25$ amps.

The power varies as I^2,therefore the power at short circuit will be, $425 \times 4.775^2 = 9675$ watts.The power factor is;

$$\cos\theta_{ss} = \frac{425}{3^{\frac{1}{2}} \times 46.2 \times 18.9} = .2815$$

The resistance between terminals=.28 ohms.

Then the I^2R loss in the primary ~~and secondary~~ at short circuit is, $3/2 \times .28 \times 90.25^2 = 3414$ watts.From this the I^2R_s loss is found to be $9675 - 3414 = 6261$ Watts.

The following values are obtained from the open circuit data,

$I = 8.42$ amps. $W = (515 - 277) \times 2 = 476$ watts $E = 218.3$ v.

Then $\cos\theta_{oc} = \dfrac{476}{3^{\frac{1}{2}} \times 8.42 \times 218.3} = .1495$ or p.f.=14.95

Note: All wattmeter readings were checked for sign in case of open circuit by adding a mechanical load,and in case of short circuit by removing one wattmeter.

M-9

On short circuit the following average values were used,

$I = 19.5$ $W = (320 - 115)2.5 = 501$ watts $E = 51.05$ v.

Then the voltage ratio $= \dfrac{220}{51.05} = 4.32$,and the full voltage amps.$= 19.5 \times 4.32 = 84.25$ Amps.

The short circuit watts $= 501 \times 4.32^2 = 9340$ watts.

The power factor, $\cos\theta_{ss} = \frac{501}{3^{\frac{1}{2}} \times 51 \times 19.5} = .291$

The resistance between terminals is = .418 ohms(primary)

Then the copper loss in the primary is,

$3/2 \times .418 \times 84.25^2 = 4454$ watts, and the secondary copper loss is 9340 - 4454 = 4886 watts.

On open circuit the values were as follows,

I= 8.88 amps. W= (595 - 370)2=450 watts E=220 volts.

$$\cos\theta_{oc} = \frac{450}{3^{\frac{1}{2}} \times 220 \times 8.88} = .133$$

In giving the proceedure for the construction of the circle diagrams, the data taken on M-9 will be used, that of the other being constructed in the same manner.

The open circuit current of 8.88 amps. is first drawn in. This vector makes an angle with E, whose cos is .133. From a table this angle is 82.3 degrees. The easier way is to resolve the vector into its horizontal and vertical components and in that way locate the point. It is assumed in this diagram that the magnetizing current is constant, and, as the friction and windage in any given machine depend only on the speed, these will also be practically constant. These losses may then be represented by a straight line drawn from the end of the vector perpendicular to the E vector. The short circuit current is then drawn in, as in the diagram, at an angle whose cos is .291 and in the same manner as the open circuit vector. The locus of all currents will then be a semi-circle whose diameter lies on the no load loss line ov, the points o and Q being on the circumference. This determines the circle. The primary current will be a mimimum when equal to mo,and all other points of its extremity will lie on the circumference of the circle,the corresponding secondary current vector being drawn from o to the same points.The secondary copper loss is assumed to be zero at o,and so a line drawn from o to Q will be the limit of the secondary copper losses.The line sQ is the in phase component of the 84.24 amps.short circuit current,however as the product of this component and voltage gives the power,and this voltage is assumed constant,then if the line sQ be layed off in watts,the total length being the total short circuit loss,it will be a true measure in all current components.Thus if the 9340 watts be divided equally among the number of spaces on the graph paper,it will form a true scale of power in watts.

The Circle diagram of which the following is a copy,was drawn to a much larger scale and the recorded values taken from that,giving much greater accuracy.The point G is the division between the primary and secondary losses,and a line oG will divide the primary and secondary losses at any load. If a quadrant be drawn from the vector E to the x axis,and the distance out to its intersection with E be divided into 100 parts,then from trigonometry,a perpendicular dropped from the intersection of the circumference of the quadrant and any primary current vector,will indicate the power factor directly on the scale.

The loss line oQ is now extended to the line MT, intersecting at the point H. A perpendicular is then erected at the point H, and from a point N_1 on this line, a line drawn, parallel to MT, and intersecting the line oQ at N. Then if this line be divided into 100 parts, the point of intersection of a line drawn from point H, through any current value on the circle, and this scale, will designate the efficiency. The scale reads 0 at N, and 100 at N. This method of finding the efficiency is derived from similiar triangles thus;

$$FL:LH::HN_1:NN_1$$

and $$HL:PL::N_1n:HN_1$$

Multiplying these two equations ;

$$FL:PL::N_1N:N_1N$$

or Total loss: input:: N_1n:100

Thus N_1n represents per cent loss, and consequently Nn is per cent efficiency.

If a perpendicular be erected o, tangent to the circle, and from some point on this line, another line be drawn parallel to eG, this line may be divided into 100 equal parts and used as a slip scale. The point of intersection of the slip line and the secondary current will give the % slip. This proof is based on similiar triangles as in the proof in case of efficiency.

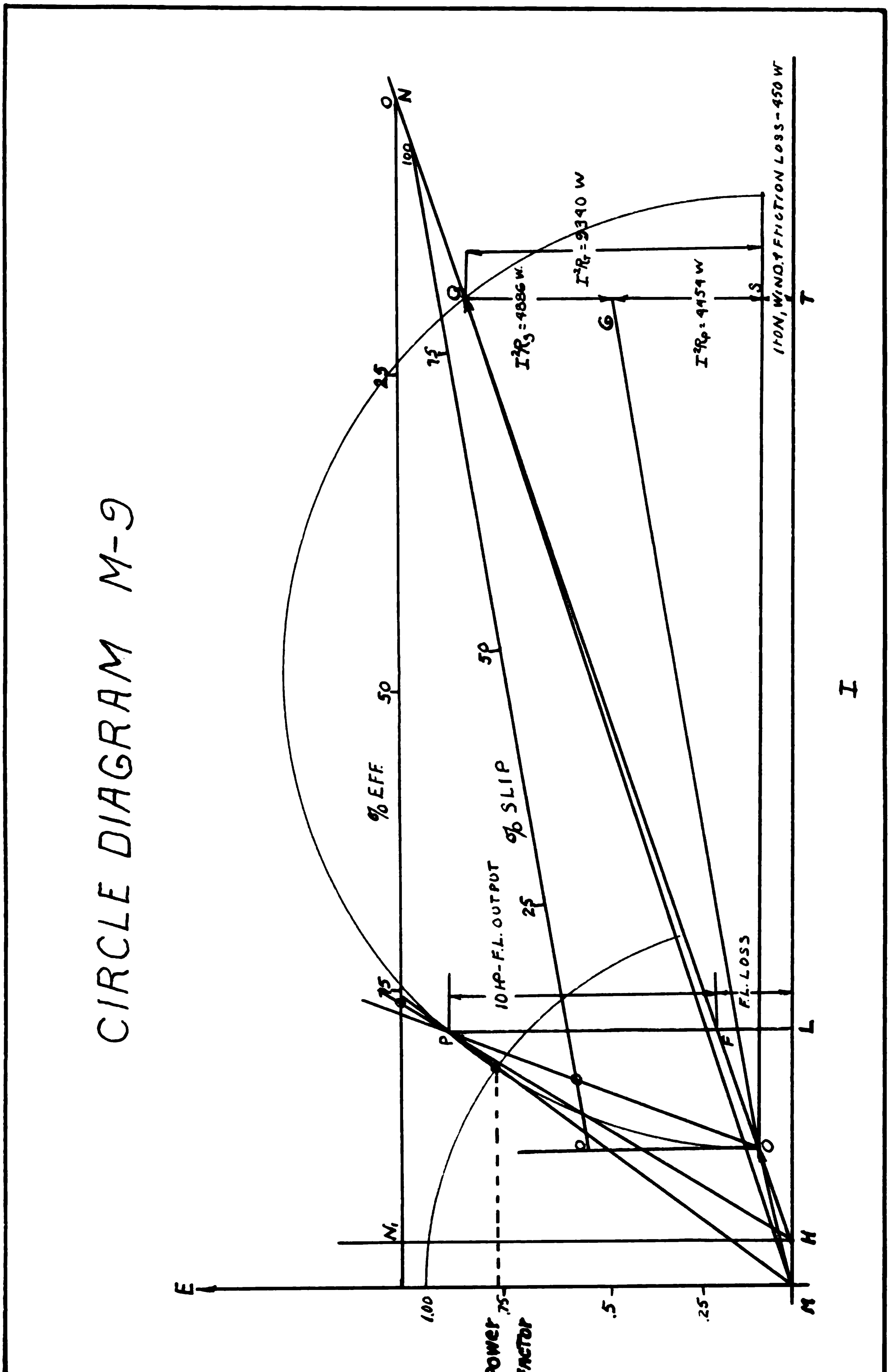
The large circle diagram was drawn tb the scale of 5 amps . to the inch in the case of current , the power comming out to be 2000 watts to the inch. Thus in the case of full load, as is illustrated in the accompanying diagram,the output PF must be equal to 7460 watts,which on the large diagram would be 3.73 inch spaces.(As stated before all illustrations refer to M-9) The line MP is the primary current for full load and is equal to 26.75 amps.At the same point the secondary current is 22.00 amps.,the slip 5.3%,the eff.85.7%,and the power factor 83%. A series of points were then chosen along the circumference, and the data obtained used in the plotting of performance curves. The following are the data sheets;

M-9

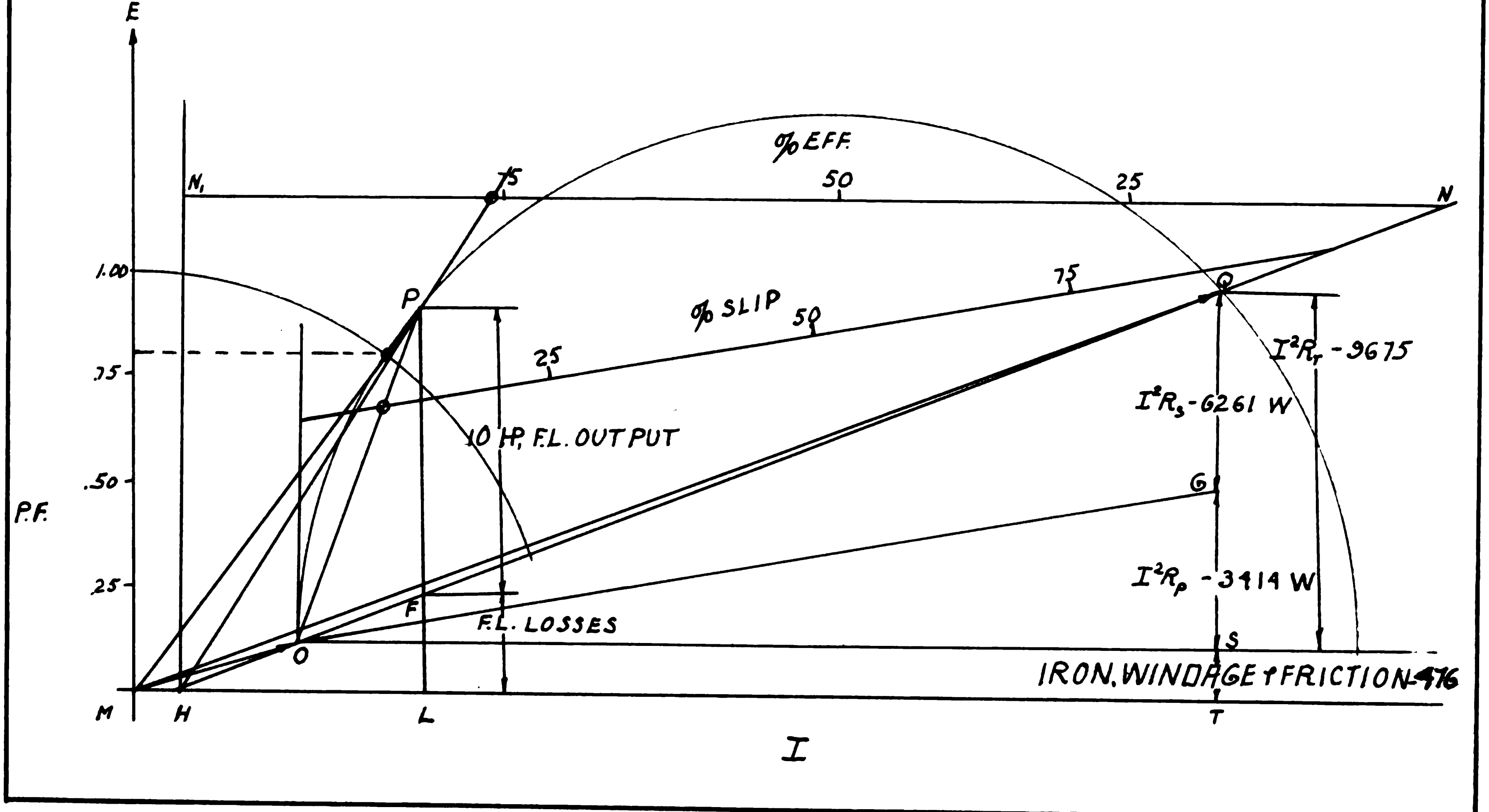
I_p	I_s	Cosθ	Slip	%syn.	%eff.	H.P Output
10.25	3.8	.49	.937	99.063	74.7	2.01
14.00	8.75	.713	2.00	98.00	84.4	4.56
18.75	14.00	.80	3.19	96.81	86.57	6.9
26.75	22.00	.83	5.3	94.7	85.7	10.00
30.00	25.00	.829	7.00	93.00	84.8	11.65
36.25	31.25	.815	7.65	92.35	82.7	13.12
44.25	38.5	.78	10.47	89.53	79.00	14.75
53.00	46.4	.73	10.93	89.07	74.00	15.3
64.5	57.00	.63	21.56	78.44	64.2	13.9
69.5	61.6	.568	26.56	73.44	57.8	12.00

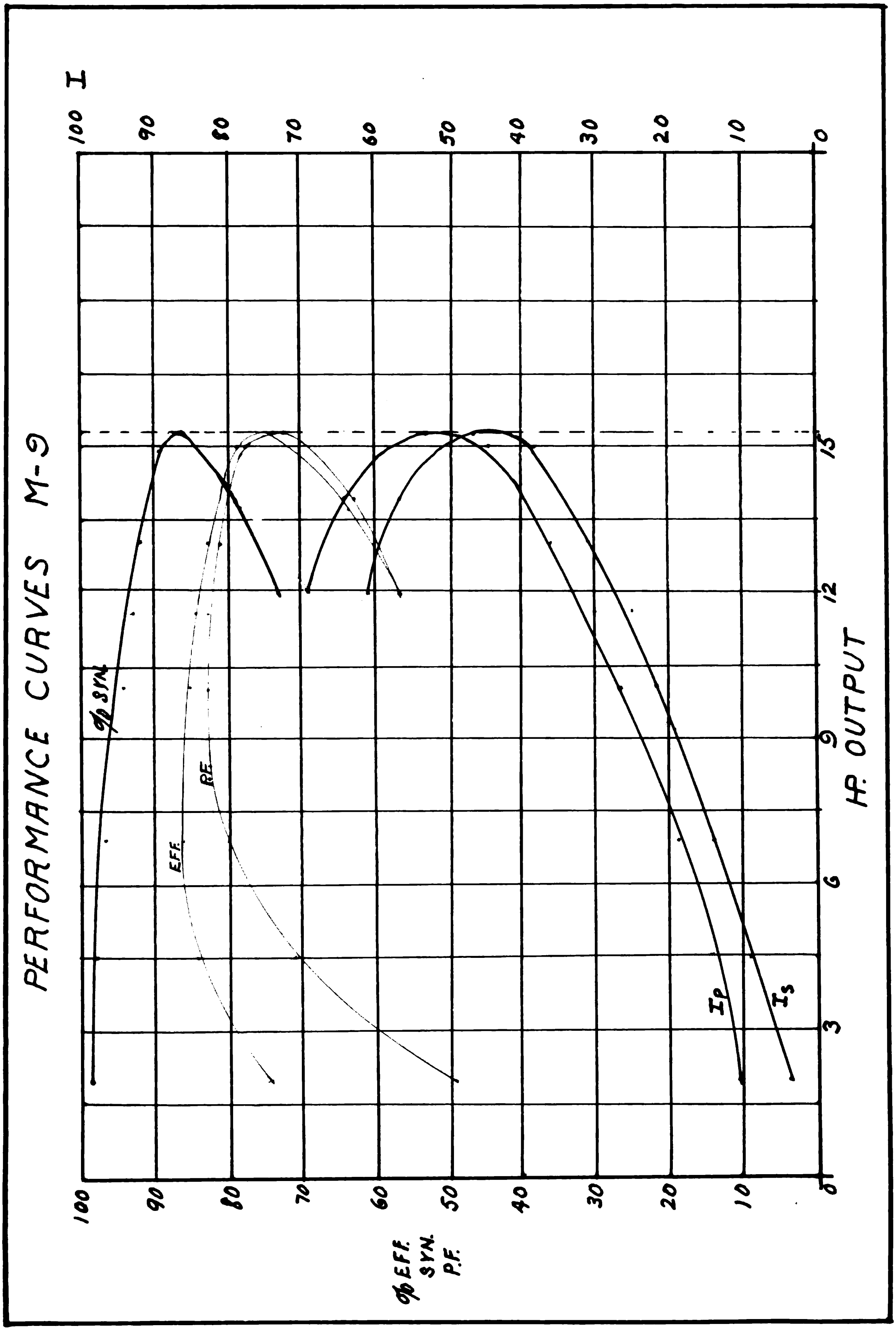
M-8

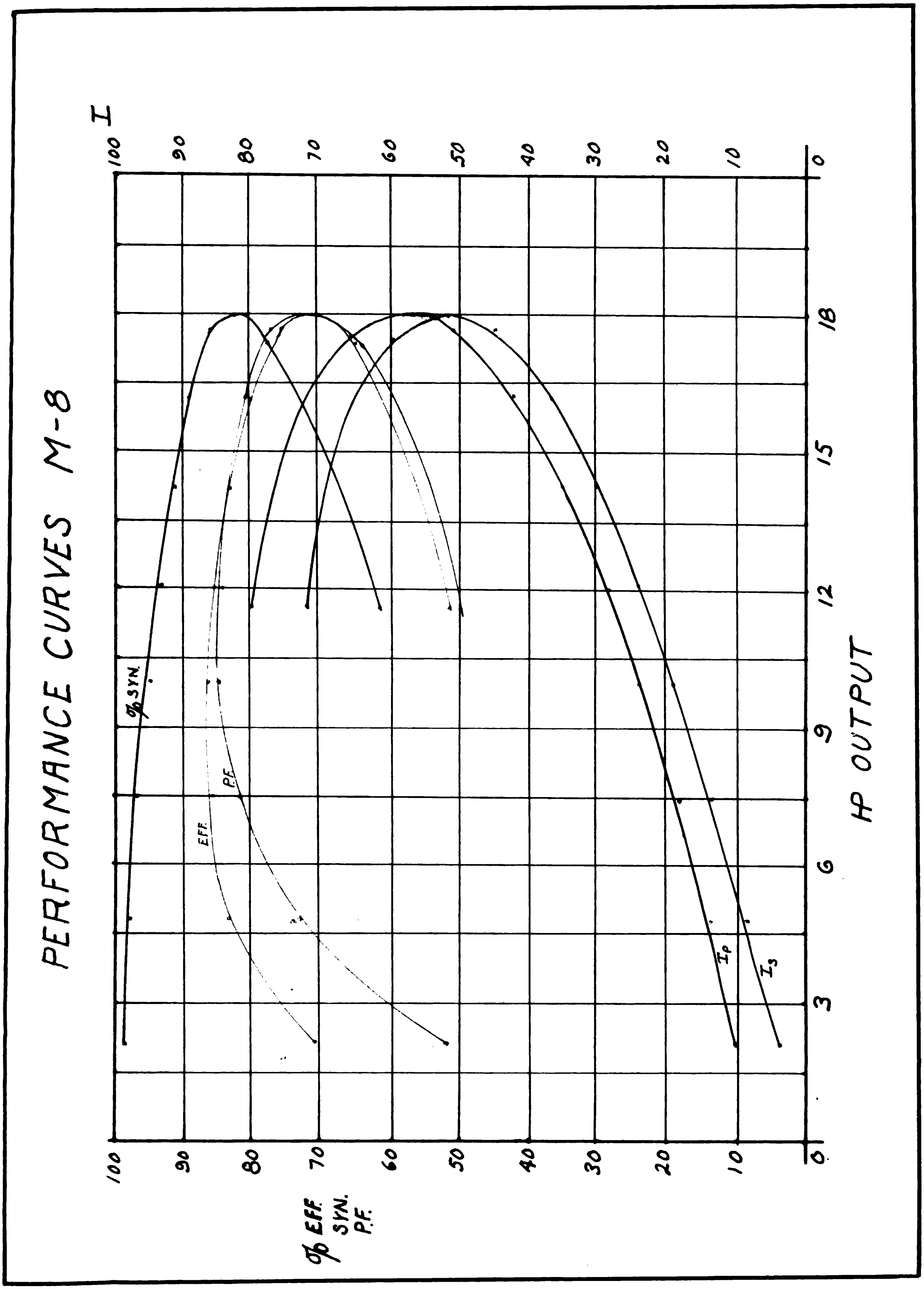
I_p	I_s	cosθ	slip	%syn	%eff.	H.P. Output
9.8	3.8	.52	1.125	98.875	71.41	2.09
13.5	8.75	.74	1.875	98.125	83.5	4.79
18.25	13.75	.82	3.1	96.9	86.00	7.46
24.25	19.75	.855	4.97	95.03	86.5	10.00
29.00	24.75	.85	6.3	93.7	85.6	12.12
35.5	30.75	.84	8.2	91.8	83.75	14.25
42.5	37.2	.82	10.35	89.65	81.25	16.17
51.00	45.2	.78	13.15	86.85	77.5	17.7
58.0	52.0	.74	16.7	83.3	73.00	18.00
67.0	60.0	.65	21.5	78.5	66.5	17.15
80.4	72.5	.50	38.00	62.00	51.7	11.43


CIRCLE DIAGRAM M-9
% EFF.
% SLIP
10 HP - F.L. OUTPUT
F.L. LOSS
$I^2R_S = 4886$ W.
$I^2R_r = 9340$ W
$I^2R_\phi = 4454$ W
IRON, WIND. & FRICTION LOSS - 450 W
POWER FACTOR
1.00
.75
.5
.25
I
E
M
H
L
T
N
N₁
P
Q
G
F
S
O

CIRCLE DIAGRAM M-8




PERFORMANCE CURVES M-9
H.P. OUTPUT
I
% EFF. SYN. P.F.
% SYN.
EFF.
P.F.
I_p
I_s
100 90 80 70 60 50 40 30 20 10 0
3 6 9 12 15


PERFORMANCE CURVES M-8
I
100
90
80
70
60
50
40
30
20
10
0
% SYN.
P.F.
EFF.
I_p
I_s
3
6
9
12
15
18
HP OUTPUT
% EFF.
SYN.
P.F.

Chapter II

The Brake Test.

Only enough points were taken in the brake run to serve as a check on the circle diagram, and to offer a comparison between the two methods. The following is the data together with the computed values of efficiency, power factor, and output.

M-9

I_p	W_t	E	T	HP	%Eff.	% P.F.
20.35	6100	218	30 lb-ft	6.7	82.	81.
26.00	7600	217	40 "	8.73	85.6	81.
30.25	9400	214	50 "	10.55	83.7	83.2
42.5	12400	212	70 "	13.6	81.6	79.5

In the above computations, the following formulas were used;

$$H.P. = \frac{T \times 6.2832 \times 1200 \times \%\ syn.}{33000 \times 100}$$

(The % syn. was taken from the circle diagram)

$$\%Eff. = \frac{H.P. \times 100}{W_t \times 1.34}$$

$$P.F. = \frac{W_t}{3^{\frac{1}{2}} \times E \times I_p}$$

The same points of output on the circle diagram compare as follows;

H.P.	I_p Bk.	I_p C.D	% Eff. Bk.	% Eff. C.D.	% P.F. Bk.	% P.F. C.D.
6.7	20.35	18.8	82.	86.	81.00	80.00
8.73	26.	23.	85.6	85.7	81.	83.
10.55	30.25	27.00	83.7	85.00	83.2	83.
13.6	42.5	38.00	81.6	82.8	79.5	82.00

M-8

Brake test data and computed results;

I_p	W_t	E	T	% Eff.	HP	% P.F.
16.15	4650	215	16	87.	5.44	77.5
27.6	9100	211	30	84.	10.25	86.
45.3	13200	206	44	84.5	14.95	81.7

The only difference in formulas is that of HP.

$$HP = \frac{T \times 6.2832 \times 1800 \times \%\ syn.}{33000 \times 100}$$

Placing the comparative values of the two different methods in adjacent columns,

	I_p		%Eff.		% P.F.	
HP	Bk.	C.D.	Bk.	C.D.	Bk.	C.D.
5.44	16.15	15.3	87.	85.	77.5	77.
10.25	27.6	23.3	84.	86.	86.	85.
14.95	45.3	38.00	84.5	84.	81.7	83.

Chapter III

Separation of Core Koss from Friction and Windage.

The resistances of the different windings will first be given in tabulated form;

The resistance of the stator windings was found by the IR drop method.The following are the averages of a number of readings. (These are the same as were used in the computations for the circle diagrams.

M-9	M-8
.209 ohms/ phase	.14 ohms/phase.

The resistances of the rotors were found by the wheatstone bridge method.The bridge was set up on the motor side of the set,and facing the motors,the phases were in a position denoted by the following view.The bars denote the slip rings.Brass strips were slipped under the brushes,thus assuring perfect contact.

/ / /
Z Y X

M-9			M-8		
X-Y	X-Z	Y-Z	X-Y	X-Z	Y-Z
.51	.5	.5	.45	.45	.45

As denoted in the view,these resistances are between terminals. Assuming that the rotor is wye connected,the resistance of the phases will be one half the terminal values. The data for the test with the secondary open ,and then with the secondary closed will next be given,leading to the separation of the losses.

M-9

Secondary closed-Motor running-No load.

I (average)	W_t	E
8.88	.45 K.W.	220

Secondary open - Not running.

I (av)	W_t	E
8.97	.307 K.W.	220

M-8

Secondary closed-Motor running-No load.

I (av)	W_t	E
8.42	.476	218.3

Secondary open- not running.

I (av)	W_t	E
8.73	.337	220

Seperation in M-9

The core loss in M-9 will be equal to the difference between the watts loss at open secondary and the I^2R loss at that current, that is $W_t - I^2R$. The current as seen above is 8.97 amps. and the resistance /Ø is .209 ohms. The core loss will then be,

$$W_c = 307 - (3/2 \times .209 \times 8.97^2)$$

$$= 307 - 25.3 = 281.7 \text{ watts core loss.}$$

On open circuit the losses consist of core loss, I^2R loss, and windage and friction. This is, of course with the motor running.

The core loss has been found, and the I^2R loss is found as before, using the closed secondary test current.

Thus; Windage and Friction = W_t - core loss - I^2R

or $W_{w,f}$ = 450 - 281.7 - $(8.88^2 \times 3/2 \times .209)$

= 450-281.7 - 24.67

= 143.63 watts Friction and windage.

Separation in M-8

The core loss equation is;

W_c = 337 - $(3/2 \times .14 \times 8.73^2)$

= 337 - 15.95

= 321.05 watts core loss.

The windage and friction loss equation is;

$W_{w,f}$ = 476 - 321.05 - $(3/2 \times .14 \times 8.42^2)$

= 476 - 321.05 - 14.85

= 140.1 watts Windage and Friction.

Tabulating these results;

	Core loss	Windage and friction.
M-9	281.7 watts.	143.63 watts
M-8	321.05	140.1 watts

Changing these values to H.P.

	Core Loss	Windage and Friction.
M-9	.377 hp	.1923 hp
M-8	.43 hp	.1877 hp

PART 111

A Method of Speed Control.

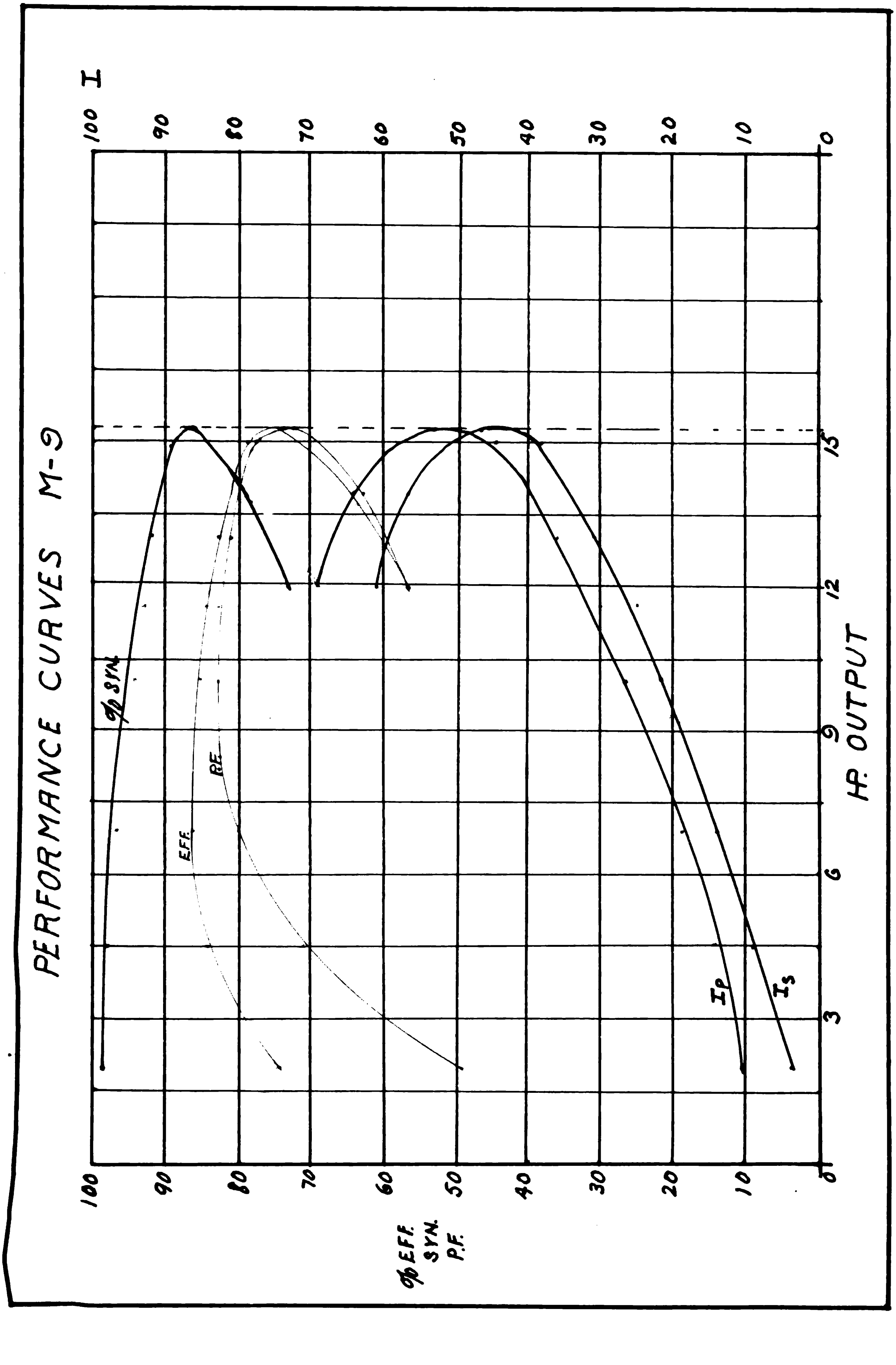
Concatenation.

Suppose that a certain six pole, wound rotor, 60 cycle induction motor is loaded to such an extent that its slip becomes 20%. Then the rotor current frequency will be .2 of 60 or 12 cycles. Now if this low frequency be fed to the stator of a second motor, say a 4 pole machine, the syn. speed of the second machine would be only 12/60 or .2 its normal speed. Thus if the motor were designed for 60 cycles, its speed would be only .2 x 1800, or 360 r.p.m. Of course it would be difficult to take a common motor and load it to 20% slip, as it would be producing about double its rated output. Not only that, it would require constant loading of both motors when feeding one machine from the other, which condition would not meet practical requirements.

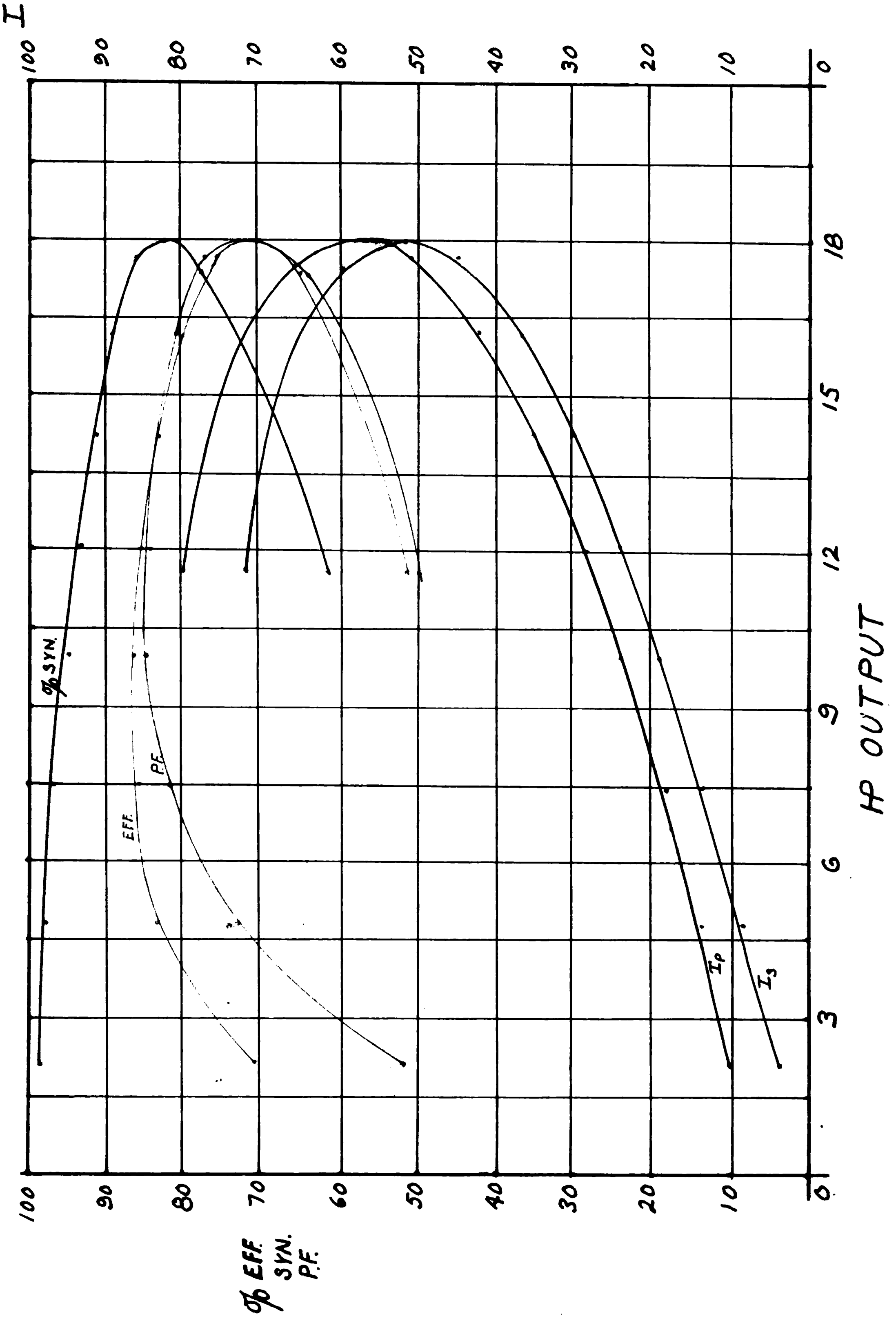
Now suppose that the rotor shafts of the two motors are coupled together, and neither motor loaded, but fed in the same manner as in the illustration stated above. In other words the motors are run in cascade or concatenation.

The formulas for concatenation speeds will first be derived for general cases and then applied to the concatenation set composed of M-9 and M-8.

~~We will~~ Take as an example two motors A and B, A having the greater number of poles. Then let A, the motor with the greater number of poles, be fed from the line, and B be fed from the rotor of A, the rotors being coupled together. Let the rotor of A feed the stator of B in such a manner that the direction of rotation


PERFORMANCE CURVES M-9
I
100
90
80
70
60
50
40
30
20
10
0
3
6
9
12
15
H.P. OUTPUT
% SYN.
E.F.F.
P.F.
I_p
I_s
% E.F.F.
SYN.
P.F.

PERFORMANCE CURVES M-8



Chapter II

The Brake Test.

Only enough points were taken in the brake run to serve as a check on the circle diagram, and to offer a comparison between the two methods. The following is the data together with the computed values of efficiency, power factor, and output.

M-9

I_p	W_t	E	T	HP	%Eff.	% P.F.
20.35	6100	218	30 lb-ft	6.7	82.	81.
26.00	7600	217	40 "	8.73	85.6	81.
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$$\%\ Eff. = \frac{H.P. \times 100}{W_t \times 1.34}$$

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H.P.	I_p Bk.	I_p C.D	% Eff. Bk.	% Eff. C.D.	% P.F. Bk.	% P.F. C.D.
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The only difference in formulas is that of HP.

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Placing the comparative values of the two different methods in adjacent columns,

	I_p		%Eff.		% P.F.	
HP	Bk.	C.D.	Bk.	C.D.	Bk.	C.D.
5.44	16.15	15.3	87.	85.	77.5	77.
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.209 ohms/ phase	.14 ohms/phase.

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X-Y	X-Z	Y-Z	X-Y	X-Z	Y-Z
.51	.5	.5	.45	.45	.45

As denoted in the view, these resistances are between terminals. Assuming that the rotor is wye connected, the resistance of the phases will be one half the terminal values. The data for the test with the secondary open ,and then with the secondary closed will next be given, leading to the separation of the losses.

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Secondary open- not running.

I (av)	W_t	E
8.73	.337	220

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The core loss in M-9 will be equal to the difference between the watts loss at open secondary and the I^2R loss at that current, that is $W_t - I^2R$. The current as seen above is 8.97 amps. and the resistance /ϕ is .209 ohms. The core loss will then be,

$$W_c = 307 - (3/2 \times .209 \times 8.97^2)$$

$$= 307 - 25.3 = 281.7 \text{ watts core loss.}$$

On open circuit the losses consist of core loss, I^2R loss, and windage and friction. This is, of course with the motor running.

The core loss has been found, and the I^2R loss is found as before, using the closed secondary test current.

Thus; Windage and Friction = W_t - core loss - I^2R

or $W_{w,f}$ = 450 - 281.7 - $(8.88^2 \times 3/2 \times .209)$

= 450-281.7 - 24.67

= 143.63 watts Friction and windage.

Separation in M-8

The core loss equation is;

W_c = 337 - $(3/2 \times .14 \times 8.73^2)$

= 337 - 15.95

= 321.05 watts core loss.

The windage and friction loss equation is;

$W_{w,f}$ = 476 - 321.05 - $(3/2 \times .14 \times 8.42^2)$

= 476 - 321.05 - 14.85

= 140.1 watts Windage and Friction.

Tabulating these results;

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PART 111

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Concatenation.

Suppose that a certain six pole, wound rotor, 60 cycle induction motor is loaded to such an extent that its slip becomes 20%. Then the rotor current frequency will be .2 of 60 or 12 cycles. Now if this low frequency be fed to the stator of a second motor, say a 4 pole machine, the syn. speed of the second machine would be only 12/60 or .2 its normal speed. Thus if the motor were designed for 60 cycles, its speed would be only .2 x 1800, or 360 r.p.m. Of course it would be difficult to take a common motor and load it to 20% slip, as it would be producing about double its rated output. Not only that, it would require constant loading of both motors when feeding one machine from the other, which condition would not meet practical requirements.

Now suppose that the rotor shafts of the two motors are coupled together, and neither motor loaded, but fed in the same manner as in the illustration stated above. In other words the motors are run in cascade or concatenation.

The formulas for concatenation speeds will first be derived for general cases and then applied to the concatenation set composed of M-9 and M-8.

~~We will~~ Take as an example two motors A and B, A having the greater number of poles. Then let A, the motor with the greater number of poles, be fed from the line, and B be fed from the rotor of A, the rotors being coupled together. Let the rotor of A feed the stator of B in such a manner that the direction of rotation of the magnetic fields of the two stators is the same. Then let

X = The concatenation speed.

With the motors running in cascade, the number of alternations of the current in the rotor of A (per. min.) will be,

(synchronous speed of A - X) x no. of poles in the stator of A, which is eq.(1).

As the speed varies as the no. of poles, the speed of the field of B will be;

$$(\text{syn. speed of A} - X)\ \frac{\text{no. poles A}}{\text{no. poles B}} \qquad (2)$$

But at no load there is no loss in the rotor of B, and consequently there can be assumed to be no slip. Then the field speed of B will be the concatenation speed of the set,

$$\text{or,} \quad X = (\text{syn. speed of A} - X)\ \frac{\text{no. poles A}}{\text{no. poles B}} \qquad (3)$$

This method of connection is called cummulative concatenation. Substituting the constants of M-9 and M-8 in the equation (3),

$$X = (\ 1200 - X\ )\ \frac{6}{4}$$

$$5/2\ X = 1800$$

$X = 720$ r.p.m., the speed of the set in cummulative concatenation. It may be seen that this is the speed of a motor having the same number of poles as the sum of these two motors or ten poles.

Now suppose that Motor B is fed from the line and motor A is fed from the rotor of B ,in such a way that the stator fields rotate oppositely.Then the connection is said to be differential concatenation.As the rotor in motor B is now rotating against the stator field,the number of alternations per minute will be;

$$(1800 + X)4$$

And the concatenation speed will be,

$$(1800 + X) \frac{4}{6} = X$$

Then, $1/3\ X = 1200$

$X = 3600$ r.p.m.,the speed in diffrential concatenation.

This is seen to be the speed of a motor of the number of poles equal to the difference of the number of poles of the two motors, or two poles.

From the above discussion and formulas it can be seen that if the motors have the same number of poles,the diffrential concatenation speed will be 0,and so the set has only 2 possible speeds, that of cummulative concatenation and that of either motor.Sometimes one stator is provided with all the coils,but in this case it is more efficient to regroup the coils instead of using concatenation as a method of speed control.

In the case of cummulative cascade,the frequency of the rotor current in A is $\frac{1200 - 720}{1200} \times 60 = 24$ cycles ,or 40% of the stator frequency.Then 40% is transmitted to the other machine and 60% is changed to mechanical power in A.

Concatenation Test Data.

In cummulative concatenation, a brake test gave the following data;

Ip	E	W	$3^{\frac{1}{2}}$E I	% P.F.	lb-ft T	Speed	H.P. in	out	% Eff.
15.9	218	1000	6000	16.6	0	723		0	0
16.5	216	2100	6180	34.	14	722	2.818	1.9	67.4
18.6	214	3600	6885	52.25	28	721	4.825	3.8	78.75
23.4	213	4750	8625	55.1	40	718	6.37	5.44	85.4
26.	209	6020	9400	64.	48	710	8.08	6.5	80.4
30.5	210	6430	11100	57.9	52	695	8.625	6.79	78.8

The breaking point is at about 55 lb-ft and 33 amps.

Using the same points and values of output, the following data was obtianed for the power transfer to M-8,

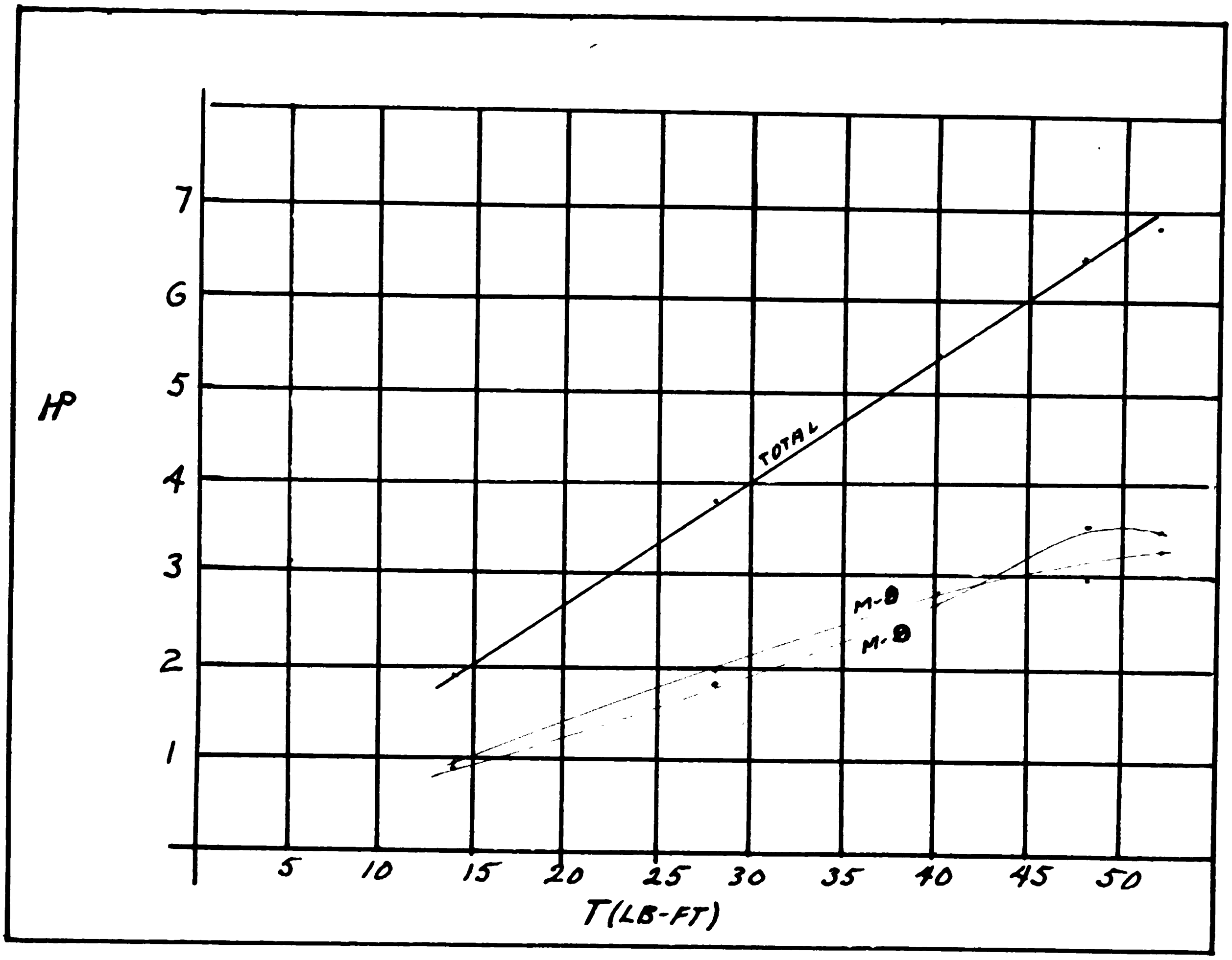
I_8	E_8	W_8	Frequency
6.9	83.	140	24.25
8.48	81.	730	24.5
12.1	79.	1360	24.75
16.25	75.	1750	25.
19.62	72.	2225	25.25
22.25	68.	2400	25.5

Now, if from the total input, the losses and friction and windage together with the power transfer to M-8, be subtracted, the rest will be the power delivered tb the shaft by M-9. Assuming that the no load losses are constant and equal to 450 watts, and that the copper losses are the sum of the losses in the primary and secondary, the following data is computed;

Total HP	(1) Total Input	Power Trans.	No load Loss	I^2R_p M-9	I^2R_s M-9	(2) Total Trans.	1-2	HP M-9	HP M-8
1.9	2100	730	450	165	54	1399	701	.94	.96
3.8	3600	1360	"	217	109	2236	1364	1.83	1.97
5.44	4750	1750	"	342	197	2739	2011	2.7	2.74
6.5	6020	2225	"	423	288	3386	2634	3.54	2.96
6.79	6430	2400	"	565	380	3795	2635	3.49	3.30

From an inspection of the H.P. output values for M-9 and M-8 it can easily be seem that the output does not divide as it was explained that it should in the theory. The stator of M-8 is fed with current of about 25 cycles, which greatly reduces the inductive reactance. Thus the power factor of this motor is greatly in excess of that of M-9. Although fed with current of much lower voltage than M-9, the impedance of its windings is much less and so a large current flows at high power factor. The two motors are of the same rating, and naturally their windings would be more evenly balanced than as if one were a large motor and the other a small one. As the rotor circuit of M-9 and the stator circuit of M-8 are in series, the increase of impedance tends to hinder the current in these circuits from rising very rapidly, as would be the case if the motors were operating independantly. This tends to favor M-8, but not M-9. Other factors which aid M-8 are(1); The voltage being low, the iron losses are much less; (2) The same may be said of the frequency; (3) The speed of M-8 is much less in proportion to that of M-9 (Relative to their normal speeds), that the power loss in its cooling fan is small.

As a reference, and as an aid in gaining a better idea of the division of load, the outputs have been plotted against torque as follows;



Differential Concatenation.

The following no load data was taken with the motors connected in differential concatenation;

I_p	E_p	W_t	% p.f.	Speed
27.2	208	4550	46.5	3500

No load test was run with the motors connected in this manner. Full load current was already flowing to the machines, and the input was well up toward the limit reached when loaded in cummulative. The set became very unstable when approaching the speed of a two pole motor, in fact if the resistance were cut out of the rotor circuit as quickly as it is ordinarily done, the set would stop. The resistance in the rotor circuit made a difference of about 500 rpm. in the speed of the set, that is;

Speed with Resistance in.	Resistance Cut.
3000 rpm.	3500 rpm.

There are several reasons why the no load losses should be so large. The great increase in the speed increases the power loss in the cooling fans. This power loss raises as the cube of the speed, thus it becomes 8 times as great in the 4 pole motor and 27 times as great in the 6 pole. The frequency in the rotor of the motor in which the rotor is revolving opposit-ely to the field, becomes excessive. As this is the rotor of M-8, the frequency is three times as great, and this also feeds the stator of M-9. An increase in frequency is accompanied by an increase in iron losses. Then again the motors are carrying full load current, which means a large I^2R loss.

General Discussion.

The theory that has been developed and applied to these motors in the foregoing pages has checked quite closely with the results of the brake tests on the machines. Probably the greatest variation was in the case of the division of load when in cummulative concatenation. The theory as stated may be found in Karapetoff's "Experimental Electrical Engineering," vol.2, page 384. He gives no proof, but merely gives it as a statement. As before stated, the motors in this case being of the same rating, and differing by only two poles, the windings will be more nearly balanced, which may account for the more nearly equal division of load.

The tests with the set running in differential concatenation prove that this is not a practical method of obtaining 3600 rpm, as the set consumes its full capacity in losses, and so is unable to carry any outside load. Furthermore, the set is not always self starting when connected in this way, although without load it will start in most rotor positions. Even if it did afford a small amount of power, the time required to bring it up to speed would make it too inefficient for modern shop practice. The study of the set is excellent, however, in that it brings out many points of interest to those dealing with induction motors, which would otherwise remain obscure.