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A STUDY OF AUTOMOBILE
RIDING QUALITY

Thesis for the Degree of M. S.
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Leonard G. Schneider
1938

THESIS

# A STUDY OF AUTOMOBILE RIDING QUALITY

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Submitted as Part of the Requirements for the Master of Science Degree.

by

Leonard Gustav Schneider

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## Acknowledgment

I desire to acknowledge the interest and aid of Professor G. W. Hobbs in the work herein outlined.

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## OBJECT

The object of this paper is the formulation of certain conclusions, the application of which will result in improved car riding quality.

Theory underlying the problem of ride will be investigated, and a verification of theoretical conclusions by means of experimentation on a model will be attempted.

## INTRODUCTION

In the early days of motoring, those fortunate enough to travel by automobile were concerned with certainty of arrival at a destination rather than with certainty of comfort in transit; it is, therefore, not unusual that efforts by manufacturers were directed along the line of improving reliability of the motor car. In our day one takes the reliability of a car as a foregone conclusion, and so it is not surprising that the popular thought has been directed to the next most desired quality of a car, namely, good riding quality.

For years no appreciable advance along this line was attempted on a sure foundation; it is true that some manufacturers tried larger springs with some improvement in ride, however, the prevailing opinion was "call in the tire and shock absorber men and let them work out our problem." As a result of this, cut and try methods were largely resorted to. The "absorber" men reasoned that the body should be more firmly fixed to the axle, and that high internal friction or so called "self damping" springs should be used to avoid body gallop. This resulted in not only a still more uncomfortable ride, but also a relatively unsafe one. The tire men, reasoning correctly, developed softer or "balloon" tires, finally some even proposing "donut" tires to get a maximum cushioning despite the inherent steering and wheeling difficulties imposed by the latter. However, it was soon evident that something deeper was at fault and thus experimental investigations were started.

Various organizations collected voluminous data by experiment, and from it modified the design of succeeding models; the basis for judgment, however, was the opinion of test drivers on the "feel" of the ride. For the purpose of argument, it will be granted that one is after a ride pleasant to the occupants; however, it is a well known fact that opinion varies greatly, and so it seemed that some scientific method of ride evaluation should be determined. At present the S.A.E. has a committee which is endeavoring to design instruments and set up procedures for testing; also some manufacturers during the past few years have set up certain standards as necessary to good riding quality by use of apparatus of their own devising. The results obtained thus are valuable, but are, of course, empirical to a great degree, and it would be interesting to know what actually are the underlying principles involved.

A few men have approached the problem theoretically; among these is Timoshenko, who is noted for his work in vibration. He gives a short but complete consideration to the problem in "Vibration Problems in Engineering", and it is hoped to use this as the basis in investigating the agreement of theory and practice.

## PROCEDURE

To check the correctness of facts gained from examination of theoretical material, it is necessary to run a test program on either an actual car or a model. The idea of working with a test model is fundamentally sound, especially in the case where an investigator's facilities or funds are limited. In view of the above, a test model was decided upon for the gathering of the experimental portion of the experiment.

The apparatus designed by the writer and described herein is not the first of its kind; the Chrysler Research Lab built the first in 1934; however, the two machines are of unsimilar construction, and the information obtained therefrom is used differently. The Chrysler machine was used only to get a practical weight distribution to give the least spring reaction, this information then being used to locate the engine in the Airflow models of 1934.

The apparatus used in this test consists of a driving motor and reduction gear box, model apparatus, and tape driving device (see Figs. 1, 2, and 3). The model consists of two parts or "towers", both similar in construction, and each supporting one end of the "body bar." The tower is the supporting system for a 6" diam. plywood wheel driven from the gear box by V belt, and so constructed that wedges can be fastened to its periphery; this is equivalent to the road. Resting on the "road wheel" is a 2" diam. plywood wheel having a yoke on its axle, and guided by slots in the tower frame. On the bottom of the yoke is a spring of desired tension, and it is fastened to one end of the body bar.

Fig. 1 - Model Apparatus

A - Car Model Wheel

B - Road Wheel

C - Yoke

D - Spring

E - Body Bar

F - Guide Drum

G - Backing Plate and Tape Guide

H - Body Bar Guide

I - Tower Frame

J - Pen

K - Weights

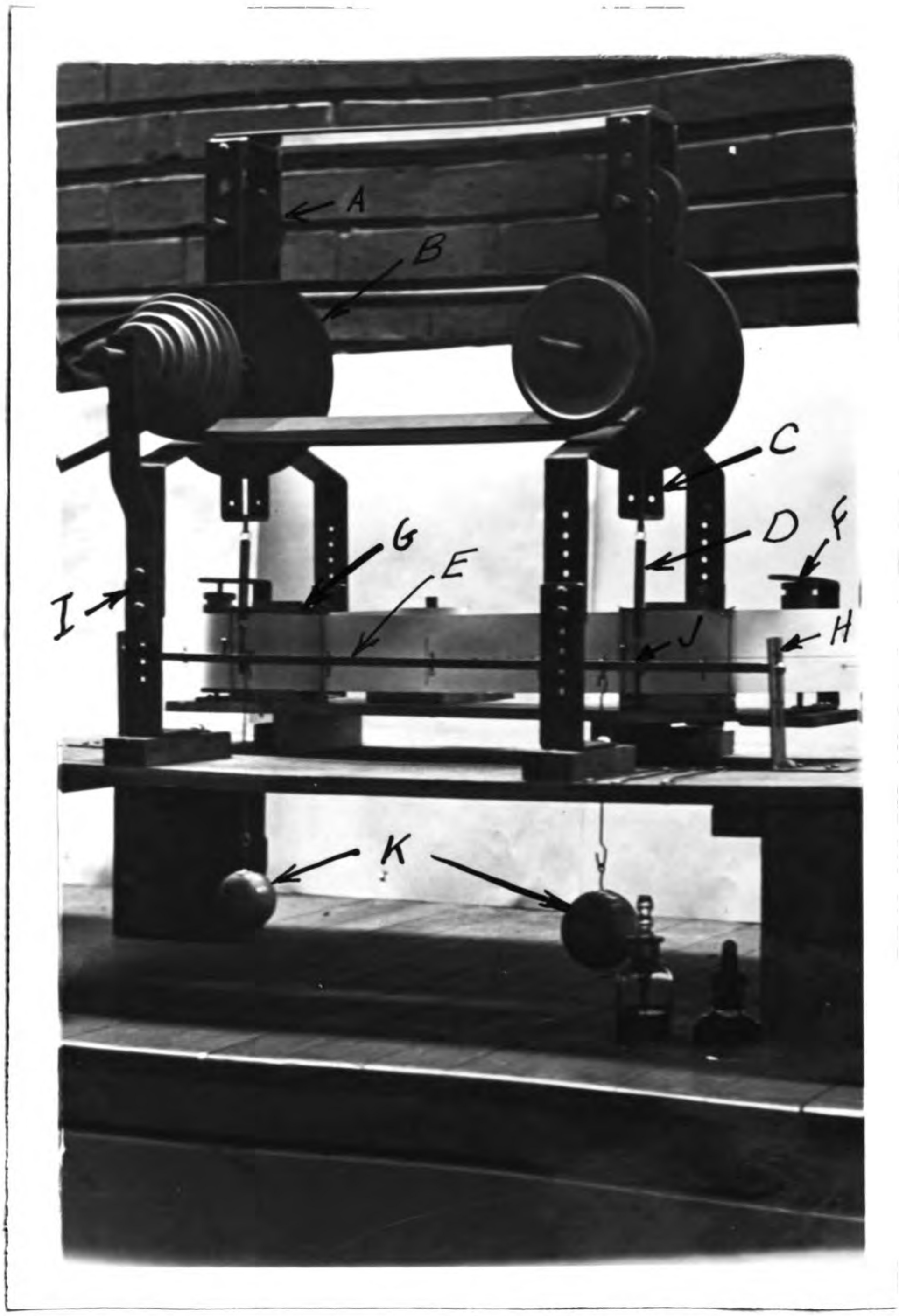


Fig. 1

Fig. 2 - Tape Driving Device

A - Driving Motor (Const. Speed DC)

B - Switch

C - Tape Guide

D - Driving Drum

E - Worm and Pinion Drive

F - Pinion and Gear Drive

G - Tape Takeoff Guide

H - Model Apparatus


Fig. 2

Fig. 3 - Driving Mechanism

A - A C Motor

B - Gear Box

C - Model Apparatus

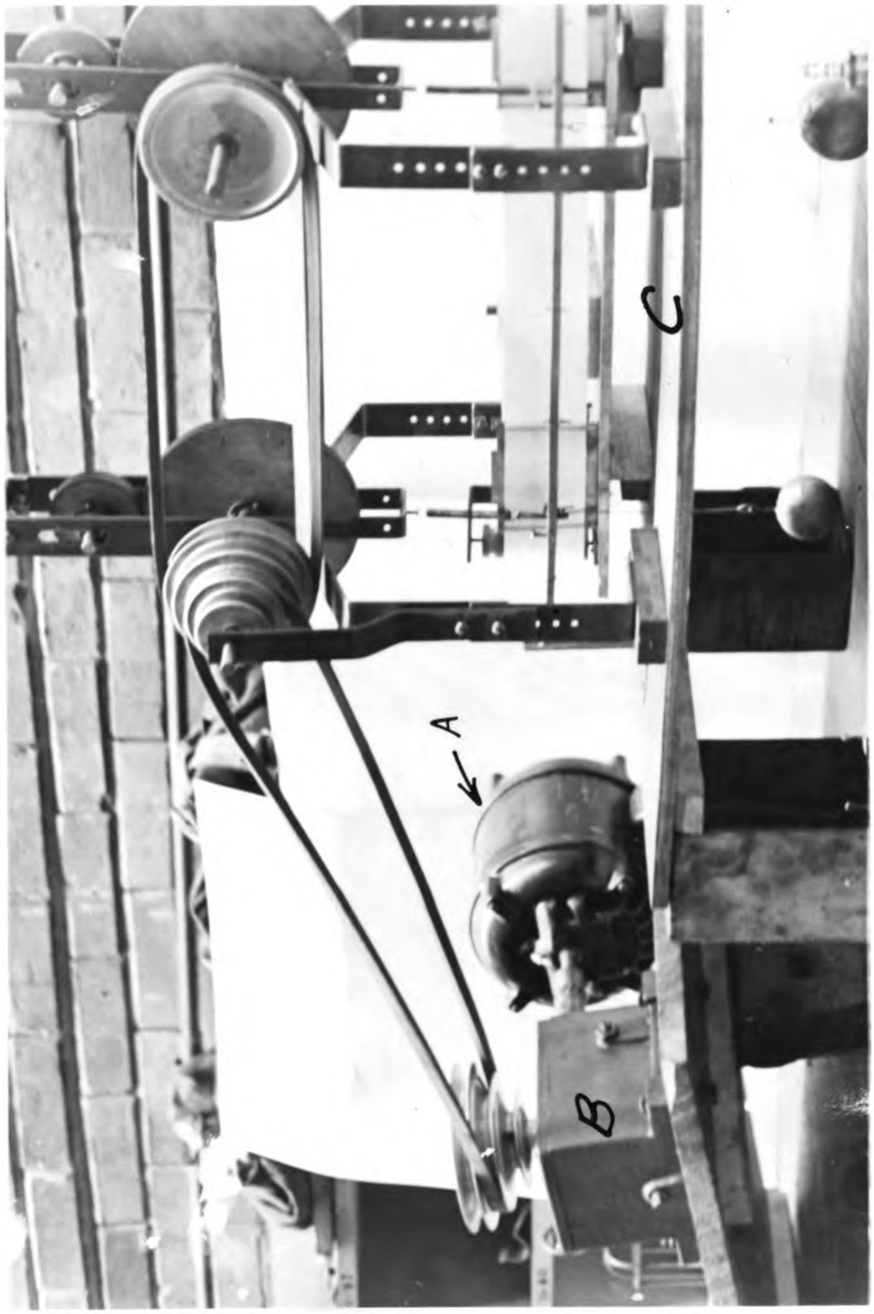


Fig 3

The body bar is loaded with weights of one-half pound, one pound, and two pounds in such a way that any weight combination and distribution may be obtained. At the point of attachment of the springs, the body bar carries recording pens, one to record front body movement in red ink, the other to record rear body movement in green ink on the moving paper tape.

The tape driving device was the source of considerable difficulty which necessitated considerable rebuilding before satisfactory operation was assured. The device used at first comprised a paper tape pulled against a backing plate through suitable guides and wound on a drum driven by a small constant speed motor with worm reduction gearing. With this model, results were obtained which would have been sufficiently accurate for a cursory examination. However, due to the fact that the radius of the paper drum was constantly changing (paper being wound around the drum), and the motor was apt to slow down very slightly if the tape had a tendency to stick, the abscissae scale of the tape was ever changing slightly and therefore the driving unit was redesigned and rebuilt.

The present unit as shown in the figure is made up of a larger constant speed motor, a worm reduction system, and a constant displacement type of driving drum.

In this manner difficulties are overcome, since first, the driving motor has sufficient power to always run at constant speed; second, by not rolling the tape or the drum, but using pins in the periphery of the drum to give a positive displacement, a constant abscissae scale is available as is verified by measurement.

The operation of the apparatus is as follows: wedges in any desired combination are placed on the road wheels, and the desired springs and weights placed on the body bar. The tape driving mechanism is started and the position of the tape adjusted so the body bar pens trace a clear line on the tape without friction. The main motor is started and revolves the road wheels, and as a result also the car wheels, since they roll on the road wheels. As a wedge strikes a car wheel, the wheel and all its complementary mechanism (yoke, spring, and end of body bar) are displaced, and as the wheel again runs on the road wheel, the spring and body bar continue to oscillate; the whole sequence of events being recorded on the tape. Ride analysis is made on the basis of tapes.

As will be brought out in the discussion, the problem of ride brings out a pecularity that the car body oscillates about two centers; these centers shift with a change in springing, and an attempt was made to investigate this also by use of the apparatus. A special body bar with stylus working on sensitized paper was made, but after numerous trials with graphical computations, it was found that no check could be obtained with the present set up. This phase of ride is therefore discussed only in theory.

Unsprung weight has some effect on riding quality, but due to a lack of time and the fact that this is a study in itself, only the factors of springs and weight distribution are considered. It would seem that weight distribution could be investigated by use of the same or similar apparatus.

## DISCUSSION

The problem of good riding quality in an automobile is one in vibration prevention. Any structure has a certain frequency of vibration to which it will respond; such frequency is called its natural frequency of vibration. It is possible to give a momentary impulse to the system which will cause the structure to oscillate in its given period, the oscillations gradually damping out. Now, the disturbing force may be of such nature that it gives impulses in step with the natural frequency. This is troublesome since the amplitude of vibration of the structure may become so great as to cause failure; also if one has a body vibrating at some frequency, it is possible to have a disturbing force acting just opposite to the vibration and thus damp it out, or if the period of the second disturbing force is different from the body oscillation period, alternate reinforcement and damping of oscillations may take place.

By an analysis of the situation, it becomes evident that in an automobile the latter case is generally the rule. Before vibration theory was fully understood, it was thought that a certain system of suspension could be developed which would be self damping, i.e. the vibration caused by spring action in passing over bumps from front to rear would be damped out due to action of one spring on another. That this is possible of accomplishment for a <u>given speed</u> no one will deny, and in the older cars where thirty-five M.P.H. was driving speed, such a design was perhaps advantageous in obtaining a better ride at this speed. Today, however, our cars are driven at road speeds of fortyfive to eighty which complicates the matter beyond solution by this means, for the time ensuing between a reaction on a front spring due to a bump and a reaction on a rear spring due to the same bump is very evidently determined by the speed of the car, while the rate of vibration of the car springs is fixed. Because of this complexity in the problem of ride quality, more fundamental ways of bettering riding quality must be resorted to.

In the trend of modern thought, the problem of ride must be analyzed from the viewpoint of vibration theory. Research by General Motors shows the following vibrations as affecting the problem:

First, main body vibration (60-150 cycles/min. for different cars.)

Second, tire vibration (250-350), in which car oscillates on tires only.

Third, vibration of unsprung masses (400-600), in which tires with accompanying unsprung mass leave ground and then bounce as a rubber ball.

The first factor only will be considered in this paper, the others each being problems in themselves. Elements affecting this factor are springs, weight distribution, and shock absorbers; the latter affecting the problem mechanically only.

From Newton's second law F = Ma the underlying principles of riding comfort and the accompanying principle of safety are drawn. The force delivered to a vehicle striking an obstruction depends on the acceleration given the vehicle, determined by the speed with which it

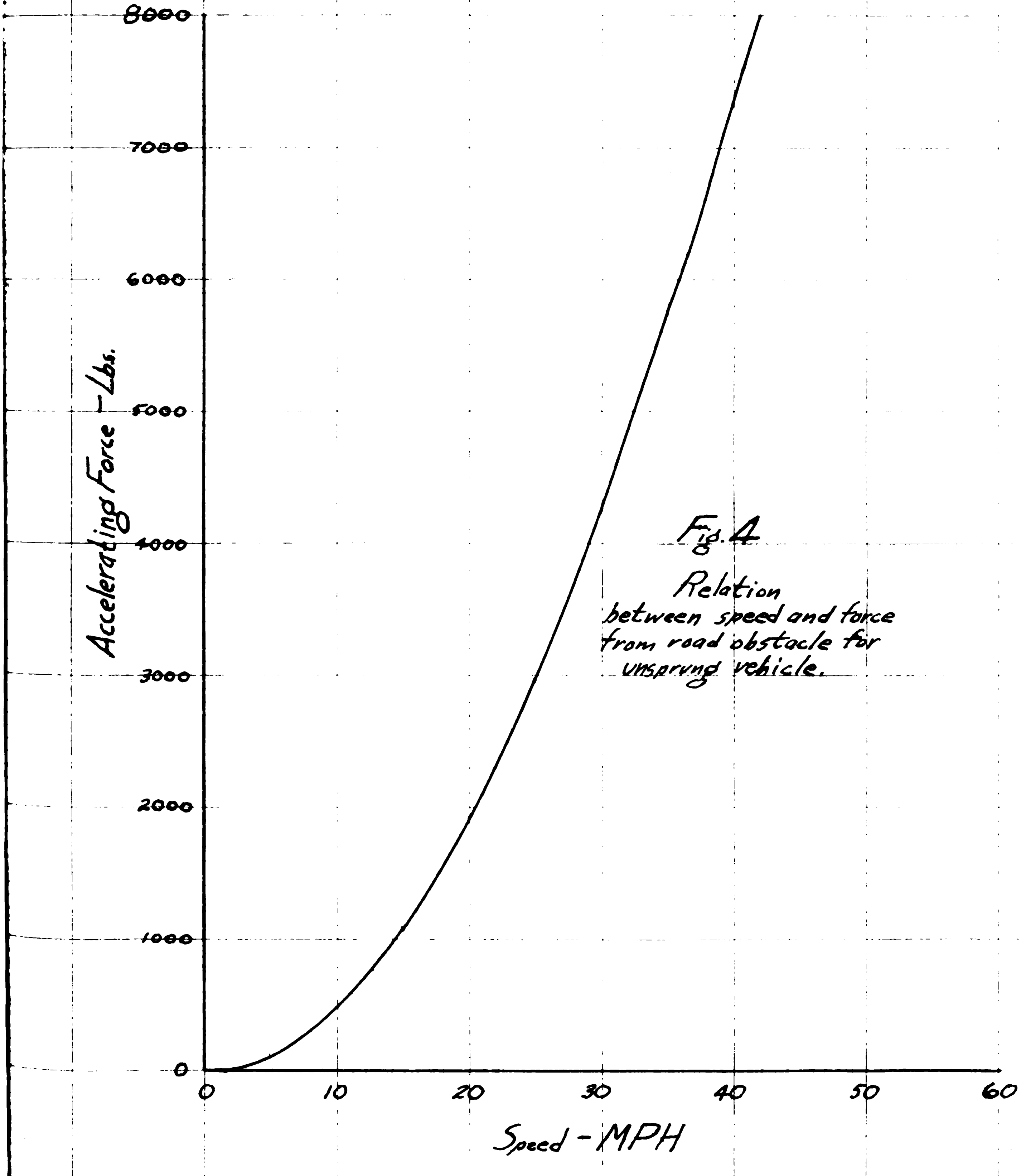


Fig. 4
Relation
between speed and force
from road obstacle for
unsprung vehicle.

travels on impact. In a vehicle having no springs, this force is enormous (see Fig. 4); in fact equaling the average (3400#) car weight at twenty-eight M.P.H., so that at speeds greater than this the car leaves the road resulting in a dangerous ride due to lack of control. Also a car subjected to large impact forces from the road will naturally have its life shortened due to damaged parts, and again we have sacrificed safety.

In a car with hard ride characteristics one, however, is not only sacrificing safety, but also comfort. Everyone has experienced the uncomfortable sensation induced by great acceleration or deceleration of a vehicle in motion; the picture in the vertical or "ride" plane is similar. The greater the acceleration in a vertical direction, the greater the discomfort of the passengers.

Consider a system composed of a weight suspended by a spring. Under the action of the weight W the spring is stretched a certain amount $\delta$ and the system is in equilibrium since $\delta K = W$ where K is the spring constant . If now the weight is displaced downward, the spring force is increased in proportion to the elongation of the spring; write the general equation

$$F = W + Kx$$

where

F = total spring force

x = displacement from center

If now the force displacing the spring downward is removed, vibrations will result because of spring force, and a problem in dynamics is set up. However, by applying D'Alemberts[1] principle, a problem in dynamics can be reduced to one in statics, therefore proceed as follows: in motion, forces acting on the body are (1) gravity W, downward +, (2) spring force W + Kx, upward -. Therefore,

$$\frac{W}{g}\frac{d^2x}{dt^2} = W - (W + Kx)$$

or
$$\frac{W}{g}\frac{d^2x}{dt^2} = -Kx$$

Simplify by dividing through by $\frac{W}{g}$ and let $\frac{Kg}{W} = m^2$

then
$$\frac{d^2x}{dt^2} + m^2x \mp 0$$

This is the equation of motion, and is a differential equation of the second order, the general solution being

$$x = C_1 \sin mt + C_2 \cos mt$$

Determine the constants by the boundary condition;

when $x = x_o$ $V = \frac{dx}{dt} = 0$ (at instant of release $t = o$)

substituting

$$C_2 = xo \text{ since } \cos mt\Big]_{t=o} = 1 \qquad \sin mt\Big]_{t=o} = 0$$

Now differentiate the general solution with respect to t

$$\frac{dx}{dt} = - mC_1 \cos mt + mx_o \sin mt$$

substituting in boundary conditions

$$C_1 = 0$$

Solution then becomes

$x = x_o \cos mt$ which is the equation of simple harmonic motion. In order for the motion to complete one complete cycle

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(1) See Kimball, or better Timoshenko - Engineering Mechanics - Dynamics.

t must equal $\dfrac{2\pi}{m}$

or $T = \dfrac{2\pi}{m}$

where T is period

but $f = \dfrac{1}{T}$

where f = frequency cycles/sec.

$$f = \frac{m}{2\pi}$$

Since $m^2 = \dfrac{Kg}{w}$

$$f = \frac{1}{2\pi}\sqrt{\frac{Kg}{w}}$$

but $\delta = \dfrac{W}{K}$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{g}{\delta}} \quad \text{or } f = \frac{3.13}{\sqrt{\delta}} \qquad g = \text{const} = 386 \text{ in./sec.}^2$$

or the frequency varies inversely as the square root of the deflection and depends only on it.

The acceleration in SHM is given by

$$a = \frac{4\pi^2}{T^2}\,x \qquad \text{Let } 4\pi^2 = \text{a const.}\ \beta$$

$$\therefore \quad a = \beta x\, f^2$$

or the acceleration varies directly as the frequency squared.

A basis is now available for setting down the fundamental principles of springing.

To obtain a low acceleration, the static deflection under load (and only the deflection) must be made large.

To do this it is necessary to either use softer springs or increase the load each carries, however, the first is to be preferred as will be shown now. Discomfort in ride due to a road obstruction may be divided into two parts, direct (that from body rise), and potential (that resulting from free oscillation of springs). It is evident that when a car passes over a bump the height of bump equals the spring deflection plus the body rise. If it were possible to keep the body rise at zero, no acceleration in a vertical direction would ensue, and therefore a ride of maximum comfort would result. Since this is impossible, it is necessary to use a spring of maximum deflection to keep body rise at a minimum, i.e. minimum direct effect. However, a spring having a large deflection will return to zero, then oscillate just as far in an opposite direction, drawing the body with it, therefore, the softer spring brings up the disadvantage of a greater potential effect. The obvious solution, of course, is the employment of a suitable damper or absorber to damp the free oscillation. The question now naturally arises;

What frequency is most suitable for comfort, and what deflection must therefore be used?

Experiment on human subjects alone can answer the question. General Motors engineers by use of the bouncing table set the table at 80 cycles/min.[2], and Chrysler engineers at 70 to 100. On the basis of previous theory, a frequency - deflection curve for springs might be constructed (see Fig. 5) to determine the required frequency, but the result would be slightly in error due to the fact that the system is supported by two springs rather than one. When this is

(2) See S.A.E. Journal, Vol. 29 P. 73

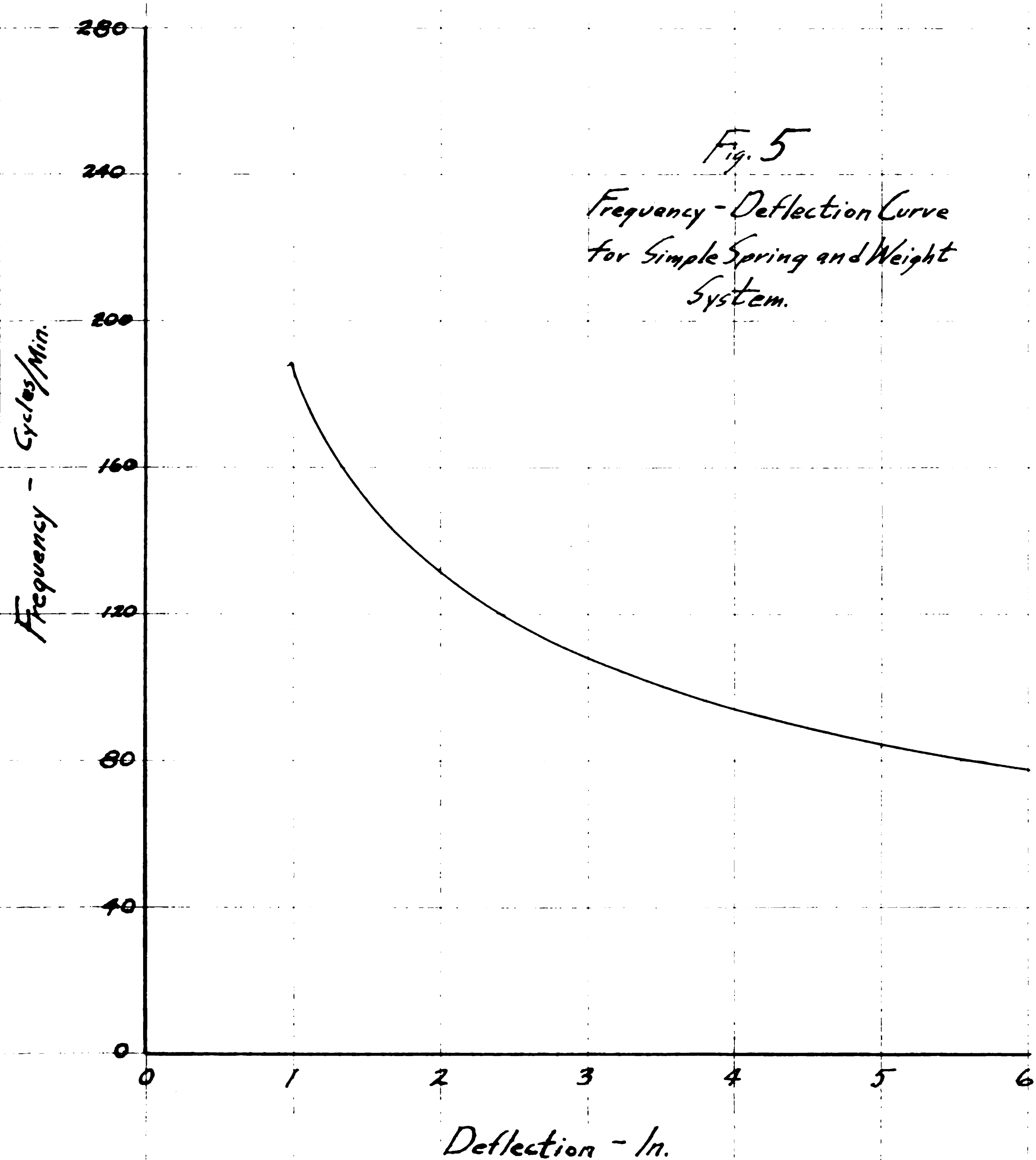
Fig. 5
Frequency - Deflection Curve for Simple Spring and Weight System.
280
240
200
160
120
80
40
0
Frequency - Cycles/Min.
0
1
2
3
4
5
6
Deflection - In.

true, the theory for simple suspension is not exactly fitting, although it will give approximate results. For this reason it is necessary to consider the problem according to Timoshenko's methods.

For the theoretical consideration of the problem of vibrations of vehicles, reference is made to Timoshenko's "Vibration Problems in Engineering"(3). Although study has enabled the writer to follow the reasoning behind most of Timoshenko's discussion, several of his steps in the solution are too widely separated, and due to inability to find suitable references, it was necessary to assume the author's correctness of solution on these points. The time and space necessary for a complete step by step presentation of the theoretical aspects are not justified in a work of this nature, therefore a brief sketch with results is given here.

Timoshenko considers the vehicle as having only a motion in one plane (as apparatus), therefore the problem involves two degrees of freedom, an up and down freedom of motion, and a rotative freedom of motion. By dealing with increase and decrease in energy, substituting in LaGrange's equations, the equations for free vibrations of the vehicle are obtained, and these are solved giving finally a frequency equation which has two roots as the solution.

(3) To those interested in reviewing the complete presentation, see "Vibration Problems in Engineering" - Timoshenko - P. 128. Also the following, which should be read as an introduction to the above.

"Vibration Prevention in Engineering" - Kimball.
"Engineering Mechanics - Dynamics" - Timoshenko (Chapters on Motion).
In "Vibration Problems in Engineering"
Articles 24, 25, 26, and briefly 28 and 29.

In the theory the coordinates are designated

$Z$ = vertical displacement of CG of body

$\theta$ = angle of rotation of body about CG

The equations of free vibration are

$$\frac{W}{g}\ddot{Z} = - K_1(Z - l_1\,\theta) - K_2(Z + l_2\,\theta) \tag{1}$$

$$\frac{W}{g} i^2\ddot{\theta} = l_1\,K_1(Z - l_1\,\theta) - l_2\,K_2(Z + l_2\,\theta)$$

($\ddot{Z}$ designating the second derivative of Z)

The frequency equation is

$$(a - p^2)\left(\frac{c}{i^2} - p^2\right) - \frac{b^2}{i^2} = 0 \tag{2}$$

whose roots are (above considered as equation in $p^2$)

$$p^2 = \frac{1}{2}\left(\frac{c}{i^2} + a\right) \pm \sqrt{\frac{1}{4}\left(\frac{c}{i^2} + a\right)^2 - \frac{ac}{i^2} + \frac{b^2}{i^2}} \tag{3}$$

$$= \frac{1}{2}\left(\frac{c}{i^2} + a\right) \pm \sqrt{\frac{1}{4}\left(\frac{c}{i^2} - a\right)^2 + \frac{b^2}{i^2}} \tag{4}$$

very evidently then there are two frequencies of vibration (therefore two modes), one of higher frequency on using the + sign in (4), and one of lower frequency on using the - sign.

Using equation 4, Timoshenko assumes the following data and solves for frequency;

$W$ = spring weight = 966#

$i^2$ = rad. of gy. = 13 ft.$^2$

$l_1$ = distance front spring to CG = 4'     Then

$l_2$ = distance rear spring to CG = 5'     $\delta_1$ = 4"

$K_1$ = front spring const. = 1600#/ft.     $\delta_2$ = 2.15"

$K_2$ = rear spring const. = 2400#/ft.

the results are

$$p_1^2 = 109 \qquad p_2^2 = 244$$

$$\text{or } p_1 = 10.5 \text{ rad./sec.} \qquad p_2 = 15.6 \text{ rad./sec.}$$

$$\text{then } f_1 = \frac{10.5 \times 60}{2} = 100 \text{ oscill./min.}$$

$$f_2 = \frac{15.6 \times 60}{2} = 149 \text{ oscill./min.}$$

Assume for the moment that the above springs are simply loaded, then by a previously proved equation

$$f = \frac{3.13}{\sqrt{\delta}}$$

$$f_1 = \frac{3.13}{2} = 1.56 \text{ oscill./sec. or } 94 \text{ oscill./min.}$$

$$f_2 = \frac{3.13}{1.466} = 2.14 \text{ oscill./sec. or } 128 \text{ oscill./min.}$$

By comparison with the above, it is seen that for conventional springing the higher frequency is about twelve per cent more than for the simple case and the lower about ten per cent more.(4)

Having considered the matter of theoretical frequency, it is now necessary to briefly consider the theory of the modes of vibration.(5) Timoshenko by mathematical manipulation (here is included much material that cannot be followed without a knowledge of higher mathematics) in one step arrives at an expression for locating two points about which the vehicle oscillates (two modes of vibration). One point or center is within the wheel base, the other outside; the two conditions

(4) G M by experiment (no data available) uses 15 and 10.

(5) See appendix 3.

are shown in Fig. 6. When motion is about the center within the wheel base, the car can be said to be pitching, and the oscillation is of the higher frequency; about the center without the wheel base, the car is bouncing, and oscillation is of lower frequency.

Upon inspection it is noted that with Timoshenko's assumptions (one spring stiffer than the other and weight distributed so $K_1 l_1 < K_2 l_2$) the center of bounce falls outside the end of the vehicle carrying the stiffer spring, and the center of pitch is between the CG and spring toward the end carrying the weaker spring.

Very evidently the location of the pivot points are dependent not on the spring deflections, but on the relative deflection thereof; the centers shifting with a charge in relative springing.

At this point it is well for the purpose of clearness to leave the author's discussion and refer briefly to a fact from mechanics. If the vehicle body be considered as a compound pendulum, the springs merely acting to support the body against the force of gravity, the idea of center of percussion may be applied.(6) This means that a body may be struck at a certain point such that the impact will produce only rotation about another point; the point of impact is the center of percussion, and the center of percussion and pivot center are interchangeable. If a and b denote the distances from CG to the two points, it can be proved that

$$i^2 = a\ b.$$

Returning now to the discussion, it is seen that the material

(6) See any text on Engineering Mechanics.

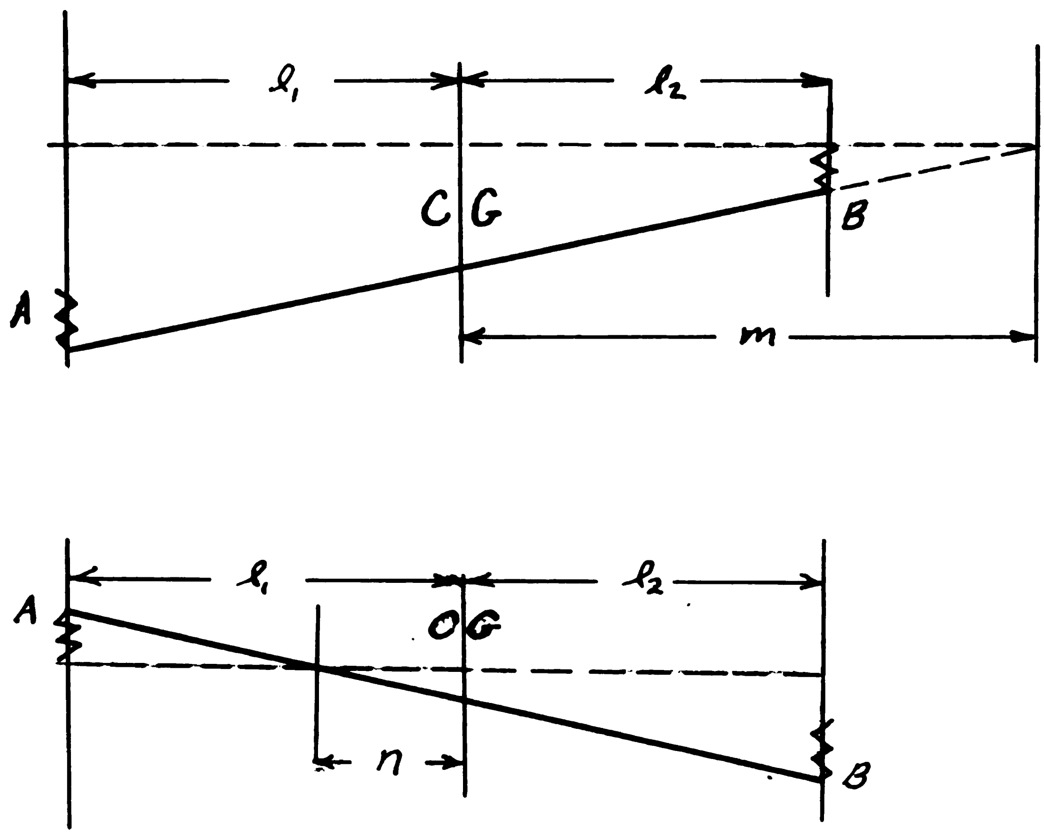


Fig. 6

above agrees exactly with the theory on centers of oscillation, and the conclusion may be drawn that if an impact is given to the pitch center, bounce will take place (i.e. rotation about bounce center) and vice versa.

Formerly it was standard practice to use stiffer front springs than rears due to steering difficulties necessitating small movement of the front axle. This resulted in the bounce center to the front of the car, and due to relation between relative spring deflections, the common location of this center was about at the front bumper; the pitch or rear center was slightly in front of rear axle. As already stated, to produce a pure motion of either type, the opposite center must be struck. If another point is struck, a combination motion is produced, but as the point of impact approaches either center, the motion becomes more and more nearly a motion characteristic of that center. Thus in the earlier cars the front axle, being very near the bounce center, on striking a bump gave mostly a pitching motion to the car, and the rear mostly bounce. Thus, the higher frequency pitching motion was the motion initially given and therefore predominant until modified by the rear reaction. These conditions at times gave rise to an annoying "neck snap" and thus two more theoretical considerations must be considered.

The expression for the product of m & n (distances from CG to pivot centers, see Fig. 6) is given as

$$mn = \left[\frac{b}{\left(\frac{1}{2}\left(\frac{c}{i^2} - a\right) - \sqrt{\frac{1}{4}\left(\frac{c}{i^2} - a\right)^2 + \frac{b^2}{i^2}}\right)}\right]\left[-\frac{b}{\left(\frac{1}{2}\left(\frac{c}{i^2} - a\right) + \sqrt{\frac{1}{4}\left(\frac{c}{i^2} - a + \frac{b^2}{i^2}\right)}\right)}\right] \qquad (5)$$

$$= i^2$$

Referring to Equation 5. If b = 0 (then $K_1 l_1 = K_2 l_2$ since b in the discussion is a const. equal to $\frac{(-K_1 l_1 + K_2 l_2)g}{W}$)

n = 0 and $m \rightarrow \infty$ (by applying L'Hospitals rule) so that the car's two centers have shifted so one is at CG, the other is at an infinite distance away. To fulfill this condition, the springs must have equal deflection.(7)

The motion of the car has now been modified so there is an up and down movement to the road (formerly bounce) and a pitching about the CG; thus with only one rotation center it is comparatively easy to provide an absorber of soft action to gently damp the action, thus giving a pleasant ride.

It should be noted that if the rear spring is softer than the front (as formerly) the above pleasant ride condition may be attained either by softening the front spring until $K_1 l_1 = K_2 l_2$ or stiffening the rear until the same condition is reached. The "type" of ride will be the same, but, as proved previously, the frequency of oscillation will be greater with the stiffer springs.

Should, however, $K_1 l_1 = K_2 l_2$ with the driver in the car, when the rear seat is loaded, the relation would no longer hold, since the rear spring would take almost all the additional weight and the desired ride would not be assured. It is, therefore, to be recommended that the car be designed so that $\delta_1 = \delta_2 (K_1 l_1 = K_2 l_2)$ under maximum load, then very evidently when the rear seat is empty $\delta_1 > \delta_2$ , which is the same as saying that the center of bounce is not at an infinite distance from the auto, but at some point nearer so that now

(7). See Appendix 2.

the car will again rotate about this center as well as the center of pitch. However, the bounce center is still sufficiently far from the car so that the ride will be almost as pleasant as for the design condition. This, therefore, seems to be the soundest plan of springing.

There remains now one more factor to consider, it is that of flywheel effect and axle interaction effect. Both of these effects are the result of change in radius of gyration, which relation is based on the equations for a compound pendulum just previously given. For this reason, this portion of the discussion is given here, although flywheel effect affects angular acceleration.

As weight is moved from the CG outward, the radius of gyration is increased, and the moment of inertia about the CG is increased, but

$$M = I\alpha$$

M = moment

I = moment of inertia

$\alpha$ = angular acceleration

So if I is increased, $\alpha$ must be decreased, thus less angular acceleration will be imparted to the car and passengers. This explains the practice of placing sand bags in the back of a coupe to increase riding comfort.

Axle reaction effect is based on the compound pendulum relation as previously given

$$i^2 = a\ b$$

If it would be possible to have the two centers of vibration at the axles, the axles (and springs) would be independent, thus the action of the front spring would produce only rotation about the rear, i.e. no interaction, and vice versa. Obviously, this would simplify the matter of ride regulation. This requires that

$$l_1 = a \quad \text{and} \quad l_2 = b$$

or

$$i^2 = a\,b = l_1\,l_2$$

The above condition is fulfilled when the sprung weight of the car is split into two weights concentrated over the axles; for instance the engine over the front, the body CG over the rear. Certainly the front is capable of attainment, but the second would result in such a grotesque appearing car as to be practically impossible. Therefore, it can be said that axle interaction can be theoretically prevented but due to present design cannot be actually achieved, although a very close approach can be made to it. Without a doubt it will be achieved in time.

The theoretical discussion is now complete and it would perhaps be desirable to summarize the conclusions arrived at.

1. Good ride requires low accelerations, and more specifically low angular acceleration to eliminate "neck snap."
2. Spring frequency depends on static deflection only, and soft springs must be used
3. Acceleration depends on spring frequency, which in turn is dependent on static deflection, therefore, soft springs should be used.

4. Required spring deflections may be taken from a frequency-deflection curve if allowance is made for the 10% greater frequency in the lower frequency of vibration and 12% greater in the higher.
5. The modes of vibration are pitch and bounce. The center about which the body bounces is outside the wheel base at the end of car carrying the stiffer spring and the center of pitch (higher frequency vibration) is within the wheel base between CG and end carrying softer spring. As the deflections are made more nearly equal, the pitch center approaches CG and bounce center moves away from car, until when deflections are equal, pitch center is at CG, and bounce center is at $\infty$. This results in best ride as far as type of ride is concerned.
6. If the radius of gyration squared equals the prod. of the distances from CG to axles (or springs), the action of one spring will not induce vibration in the other. This requires a concentration of weight over the axles.
7. Moving weight from the CG outward improves ride since it decreases angular acceleration, however, it should only be done until

   $i^2 = l_1 \, l_2$ (condition 6 above).

2-7 = Bump front only
8-11 = Bump front & rear

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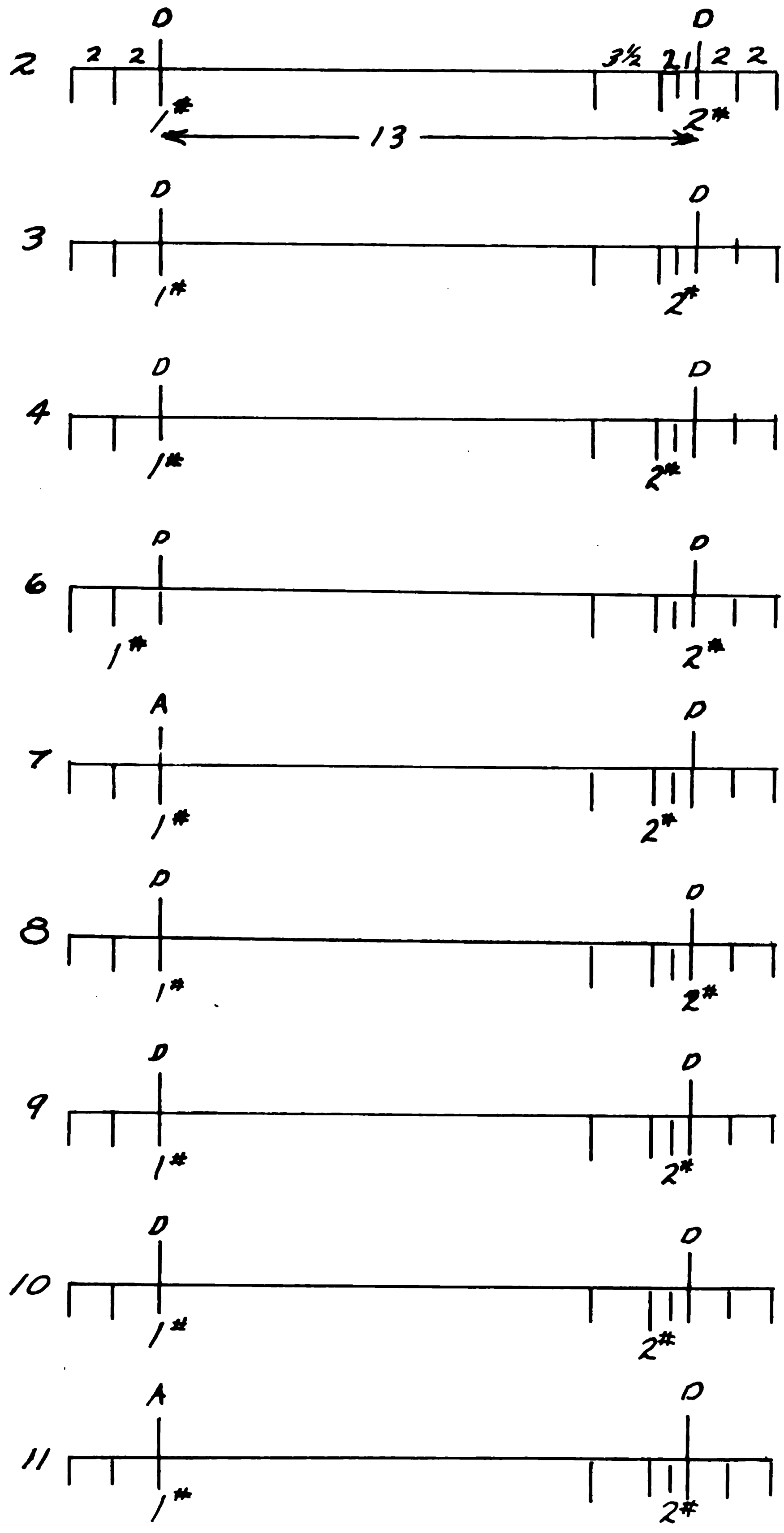
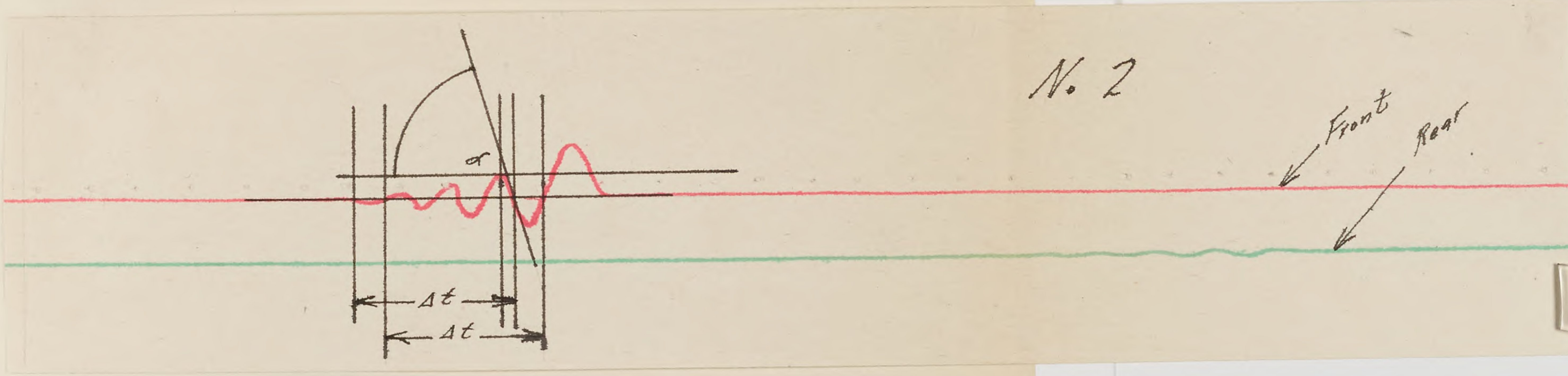
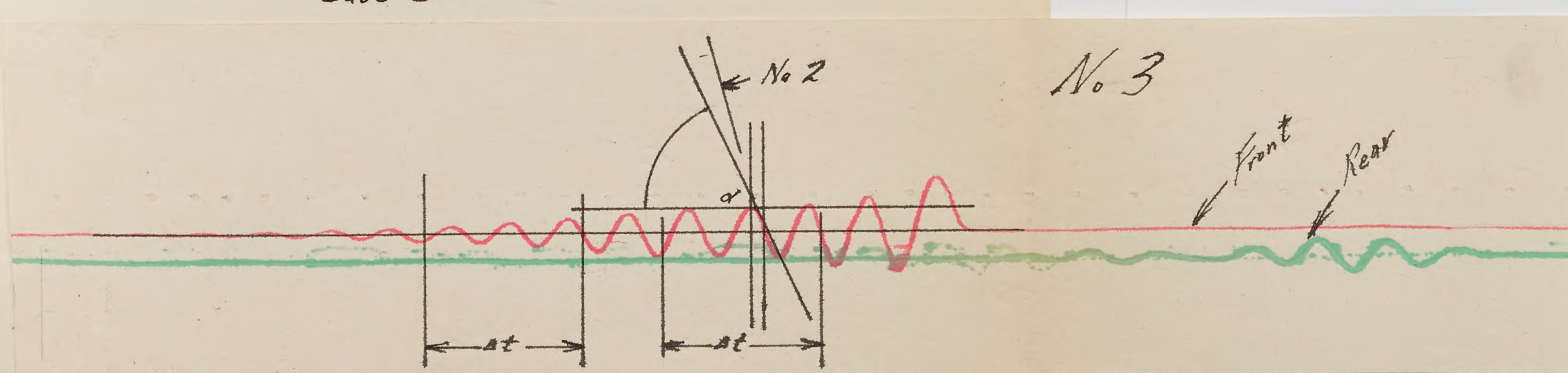


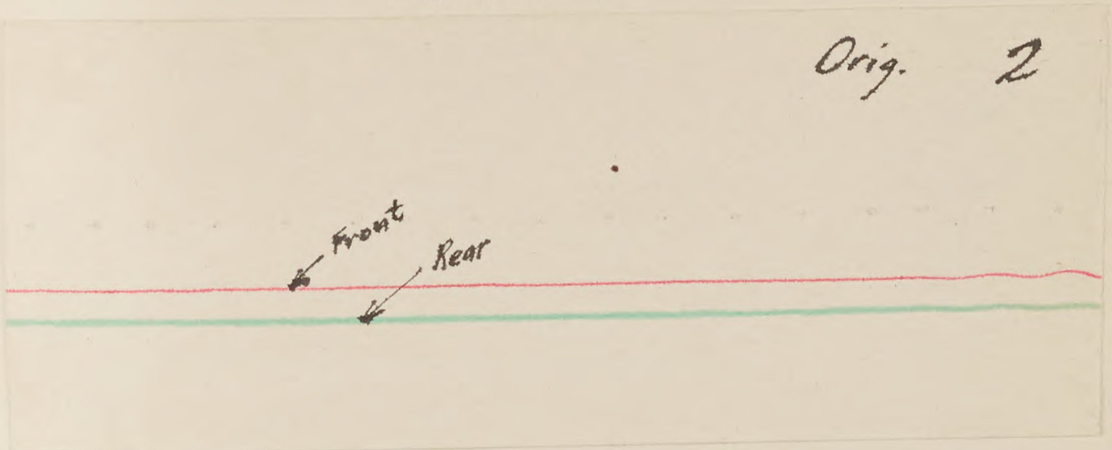



Case 2

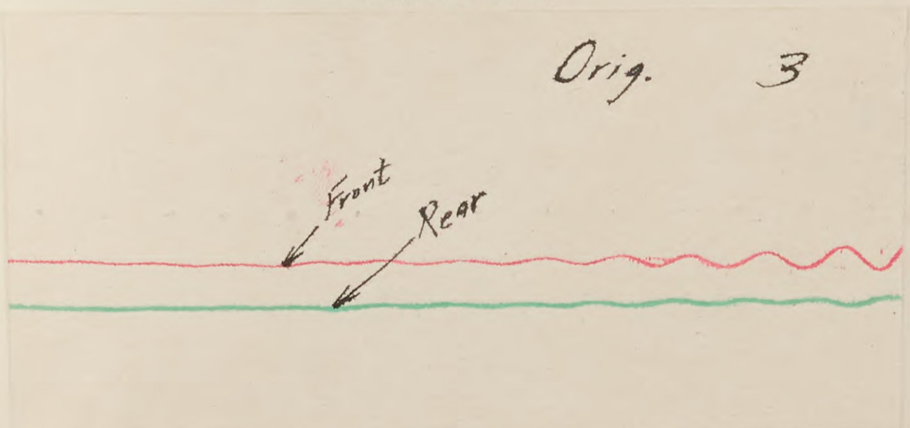
Case 3



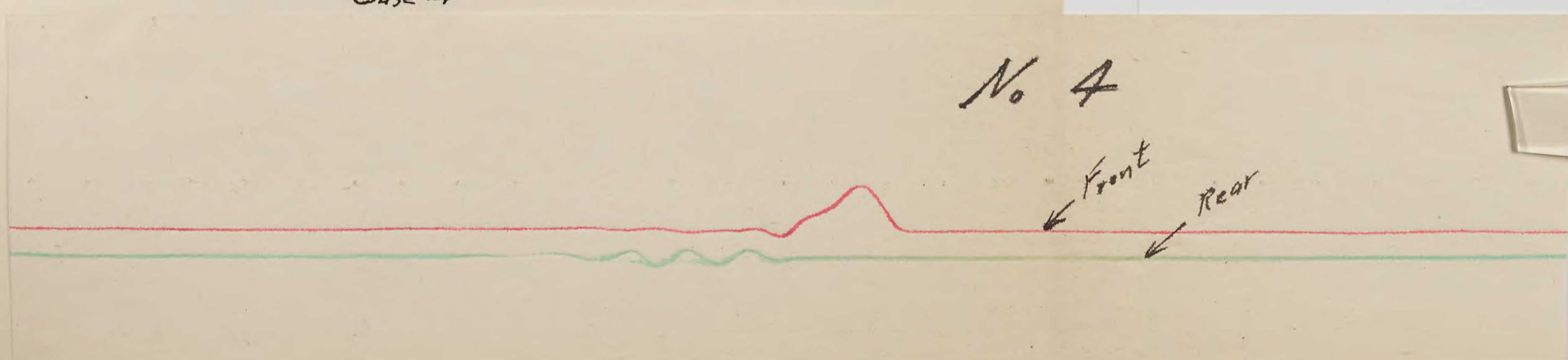
Case 2 (Or.)




Case 3 (Or.)




Case 4



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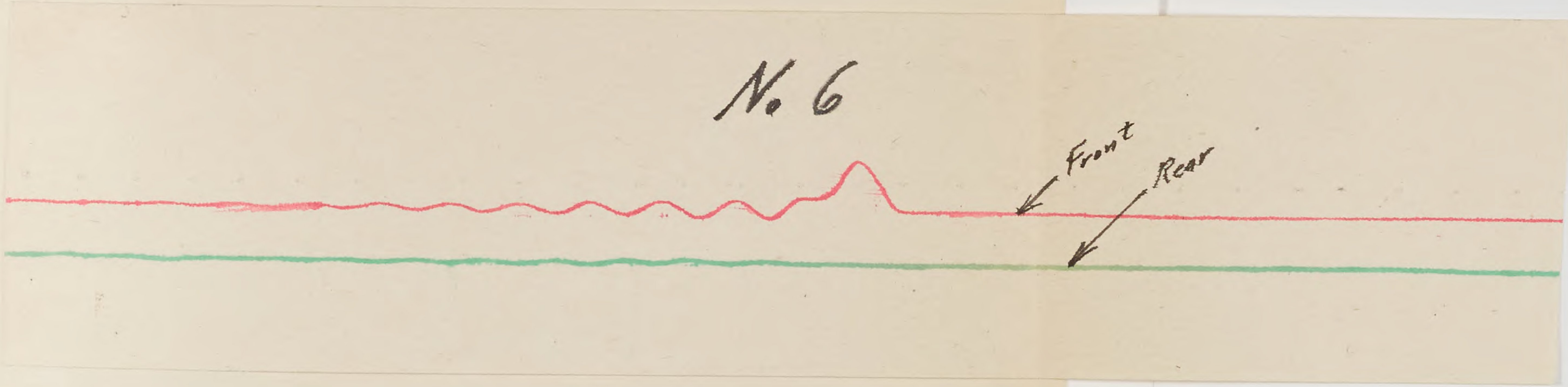
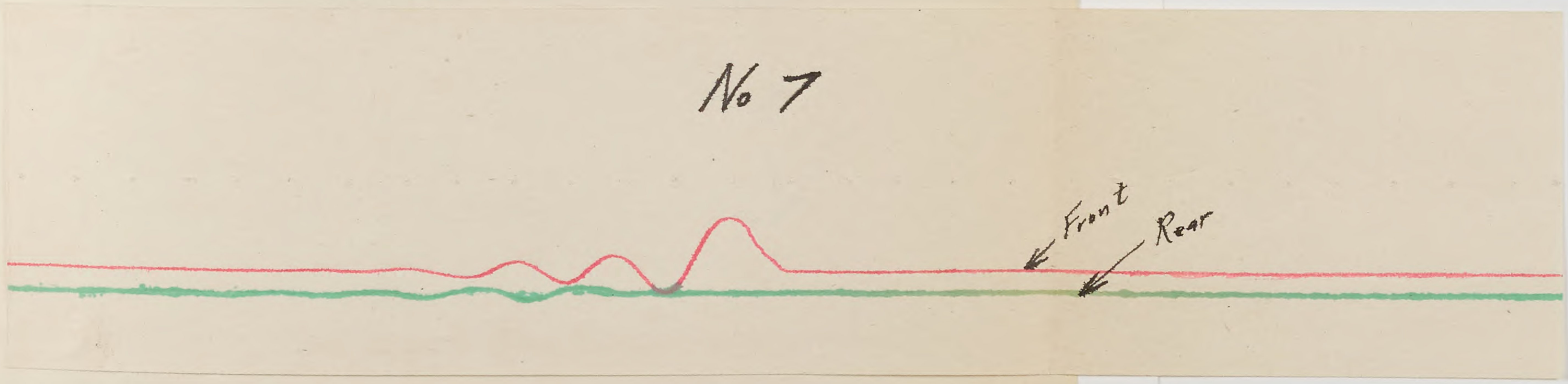
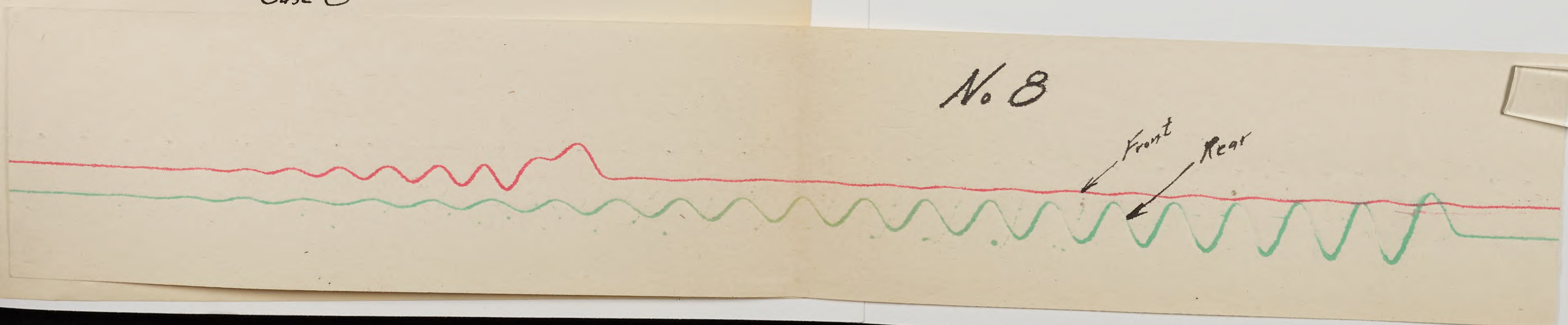
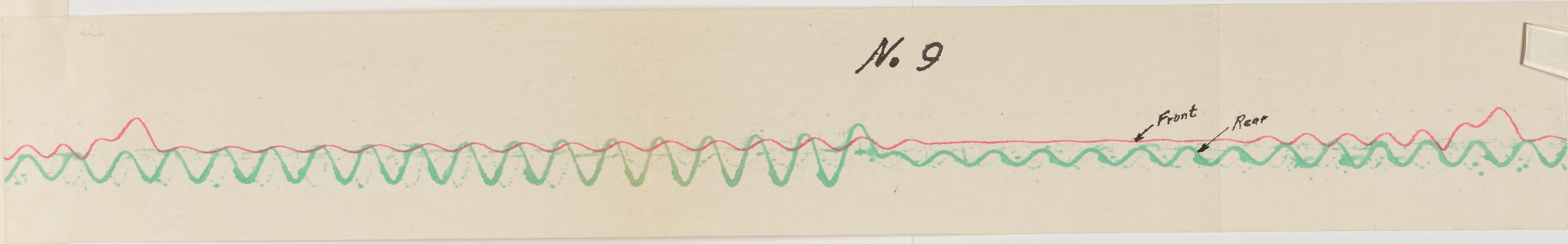


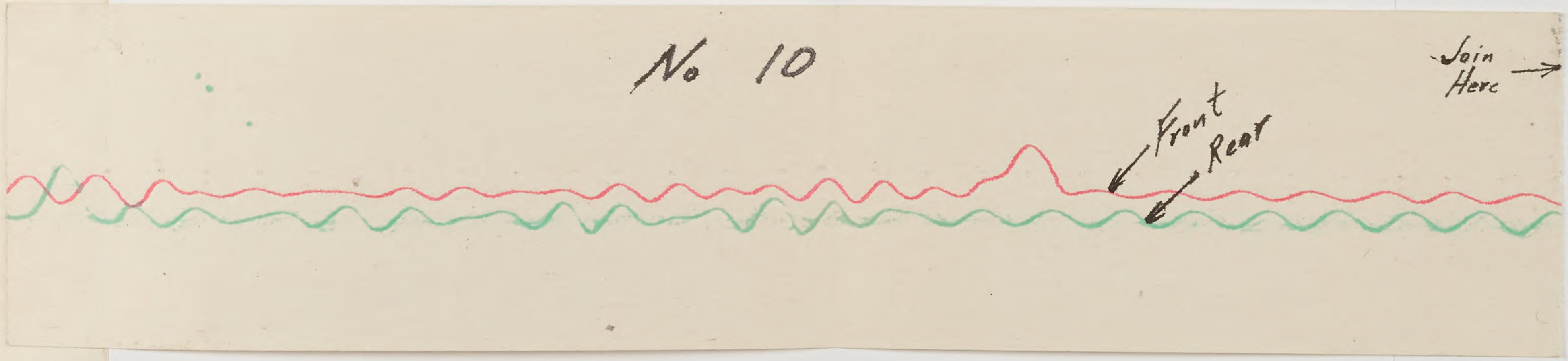

Case 7




Case 8




No 9
Front
Rear


No 10
Front
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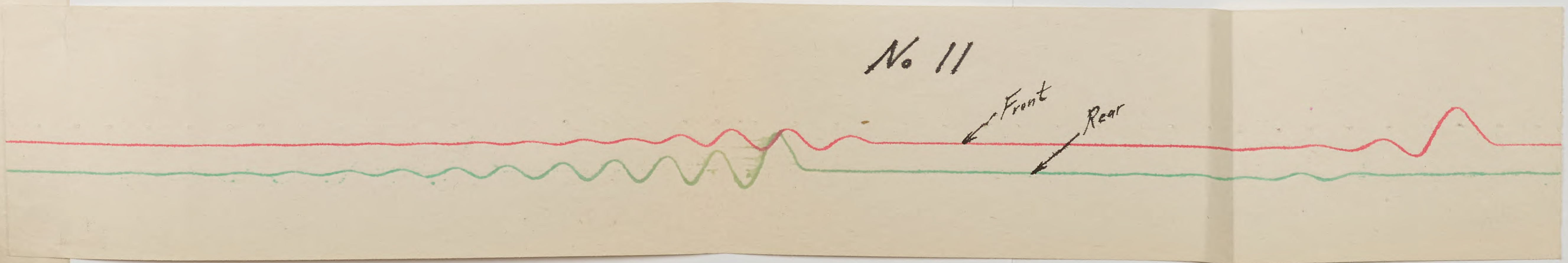
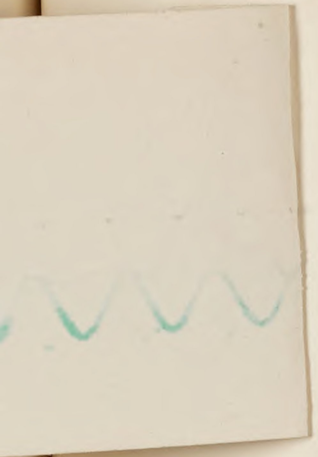
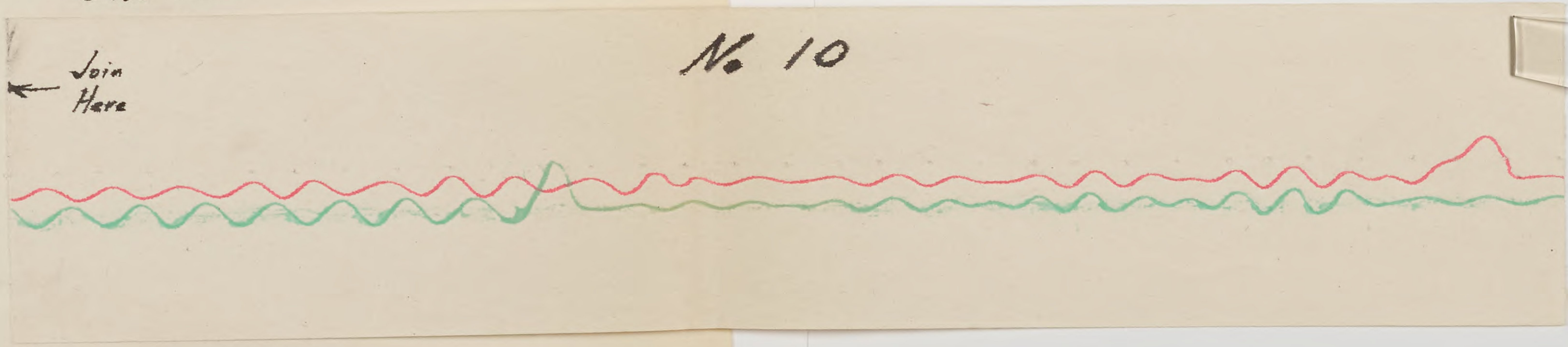
Join Here


No 11
Front
Rear

Case 9



Case 10

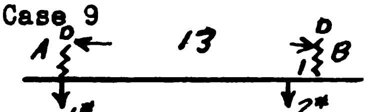



Case 11

A brief consideration will now be given the results obtained by experiment in order to attempt a check in the above conclusions based on theory.

1. The values for frequency correction should be checked. To check anyone frequency experimentally, the speed of the moving tape must be known; since the original plan of the paper did not include this phase, this data is lacking and therefore in checking, the ratio between frequencies for two springs for any assumed time $\Delta$ t is taken to the ratio of the true computed frequency of the same springs. This, although slightly more work, gives just as accurate a check as by tape speed computations.

Case 9




Spring D = 2#/in.

$\sum M_B = -13\,F_A + 13 + 2 = 0$

$F_A = \dfrac{15}{13}$ then $F_B = \dfrac{24}{13}$

$\delta_A = \dfrac{\frac{15}{13}}{2} = .577$ and $\delta_B = .924$

$f_A = \dfrac{3.13}{\sqrt{.577}} = \dfrac{3.13}{.758} = 4.13$ cy/sec. or 248 cy/min.

Case 11

Same loading - Spring A in front Spring A = 1#/in.

$F_A$ & $F_B$ = same

$\delta_A = \dfrac{15}{13} = 1.15$ $\delta_B = \dfrac{24}{13} = 1.84$

$f_A = \dfrac{3.13}{1.07} = 2.93$ cy./sec. or 176 cy/min.

Front springs have least deflection, bounce center to front and frequency must be corrected by value for pitch = 12%.

Correcting

$f_A = 278$

$f_A = 197$

Ratio $\frac{f_A}{f_A} = \frac{278}{197} = 1.4$

From curves for a time $\Delta$ t, which is any convenient distance

Case 9 $f = 2.75$ cycles for $\Delta t$

11 $f = 2$ " " $\Delta t$

Ratio $\frac{f}{f} = \frac{2.75}{2} = 1.38$

which agrees very closely (error 1.4%) with the 1.4 obtained by computation.

Therefore, frequency correction values as determined by theory are checked.

2. Acceleration varies directly with frequency. This is proved by use of tapes for Case 2 and 3.

First, by laying off a time interval $\Delta$ t, it is seen that for the same time interval $\Delta$ t, Curve 2 has three cycles, while Curve 3 has approximately two and three-quarter cycles, therefore, very evidently Curve 2 has greater frequency. Now, by laying off tangents to the curve, the velocity can be obtained since the curve is a displacement-time curve, so if the tangent be layed off for a given interval, the change in slope per unit of time will be the acceleration. This is done, the change in slope being $\alpha$, and it is seen by superimposing 2 on 3 that $\alpha_2$ is greater for the same time interval, therefore, Case 2 (higher frequency) has the greater acceleration.[8]

3. Frequency varies inversely with static spring deflection. This is most strikingly shown by a comparison of Cases 4 and 7. Here the only difference is in the softer springing used on the front in 7. The gently undulating curve in 7 contrasts sharply with the almost total lack of spring action in 4. Very evidently one cycle in 7, by inspection, takes much more space, i.e. time, than one in 4, therefore, the frequency of 7 is much lower. The effect of loading on the same spring can be seen from a number of curves, say 8 & 9. Here, using the rear curve for the direct spring action, it is seen that 8 has a lower frequency than 9; that this should be so is evident from the loading, since in Case 9 a portion of the 2# weight is carried by the front spring, which is not the case in 8. Also by a check of the red curves of 6 and 8, it is seen that 6 has a lower frequency than 8, as might be deduced from the fact that the forward weight in 6 has been moved out, increasing the deflection and therefore lowering the frequency.

4. Increase in radius of gyration improves riding quality, the maximum improvement being attained when $i^2 = ab = l_1 l_2$ (weight concentrated over axles.) This fact is very nicely brought out by a number of comparisons. In cases 2, 3, and 4, the loading was changed

(8). To verify results, two tapes of 2 and 3 were made at different speeds; the graphical constructions resulted in the same conclusions as given above.

slightly to give a change in i.[9] This is the case as shown in 2 (original). As the rear weight is moved inward, i.e. $i^2 < ab$ Case 3 (original) and 4, the induced reaction of the rear is shown by the green curve in which the waves become more pronounced indicating poor riding quality. A comparison of both 2 and 3 (new) leads to the same conclusion. It may be noticed that the new Case 2 tape shows a slight rear reaction. This is probably due to the fact that the weight hook directly below the spring on the apparatus was broken during the running of the two new tapes, and in order to conserve time the weight was hooked to the body bar as close to the spring as possible (approx. 1/8") instead of being directly below the spring as in 2 original.

In a like manner if the front weight be moved, the rear being over the axle, a reaction is again impressed on the rear as shown by Case 6; the curve here is very similar to 3 (original).

"Flywheel effect" may be shown by the following comparisons. Case 6 has the front shifted outward, increasing inertia and also static deflection, now by comparison with the red (front) curve of 8, the frequency is seen to be lowered. In case 8 and 9 (rear curves) the inertia and deflections are also increased, but the inertia is not

(9) A word of explanation should be inserted here in regard to loadings 2 and 3. The original set of tapes (all made under same speed conditions) included the small strips marked 2 and 3 preceded by the word original. The tapes were badly blotted when inking in the graphical computations and therefore it was necessary to discard all but the portion included (sufficient for purpose used). These tapes were rerun but at a different speed and therefore cannot be compared in scale with the remaining tapes. For comparison with the remaining tapes, the original 2 and 3 are used, the newer tapes being used for a comparison between themselves.

as great as in Case 6 since the weights in 6 are more removed from the CG than in 8. The change in deflection is the same in all cases as the weights are moved,[10] however, the change in frequency between 6 and 8 (front) is greater than between 8 and 9 (rear), therefore, since the additional decrease in frequency could not have been caused by deflection (since it would affect both comparisons equally if deflection changes are equal), it must be concluded that the increase in inertia ("flywheel effect") must have caused the additional change.

With the data at hand, very good and bad pictures of axle interaction are available. Cases 9 and 10 are types of undesirable ride, one being almost the opposite of the other. Case 11 shows a very desirable type of ride making use of a flexible front spring and a weight distribution designed to give about as near above axle concentrations as practically possible. In Case 9 a bad resonance condition is illustrated. Here the reaction of the front is in step with the oscillations produced by the rear bump, with the result that the rear oscillations are reinforced to such an extent that they do not die out until the rear system is again disturbed. Obviously this would be uncomfortable. Case 10 shows a case of the worst ride record obtained. Interaction here is very bad due to the rear weight being moved in so far; the constantly varying and irregular shape of the curves throughout their length testify of a ride that is constantly choppy and ever changing in characteristic. This in a car would

(10) See appendix 5 for proof.

added to obtain frequency in pitch.

7. Assuming a frequency value of 80 cycles/min. as a minimum for comfort, by use of the frequency-deflection curve and correction factors, it may be said that a car spring must have a minimum static deflection of 6 inches.

8. Weight should be concentrated over or as near over the springs (axles) as possible in order to eliminate axle interaction. An arrangement whereby the engine is over the front axle of the car and the CG of body is 1/10 of wheel base before rear axle will give a ride almost as fine as if the CG was over the axle. A distance of 1/4 to 1/5 of wheel base will give a fair ride.

Verified by theory but not experiment (appendix 4).

1. The centers of oscillation are similar to centers of percussion in action and vary with relative springing. The bounce center is outside the wheel base to end of stiff spring, and pitch center is within wheel base and toward end of soft spring.

2. For a most satisfactory ride, the static deflections of springs should be the same when the car is fully loaded, since then the car will have angular movement (pitch) about one center (CG) and move up and down in translation parallel to the road.

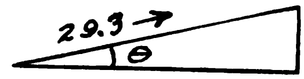
## APPENDIX 1

Computations for force - MPH curve.

Curb weight = 3400#

Rise 1" in 2' travel

20 MPH




$$\frac{60}{88} = \frac{20}{x}$$

$x = 29.3'/\text{sec.}$

Time for 2' = .0684 sec.

$V_x = 29.25$

$V_y = 1.22$

By impulse and momentum:-

Impulse = Change in momentum

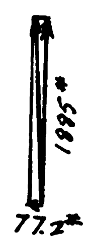
Tangent $\theta = .0417$ $\qquad \theta = 2^\circ 26'$

$$.0684\ F_x = \frac{3400}{32.2}(29.3 - 29.25)$$

$$F_x = 77.2\#$$

$$.0684\ F_y = \frac{3400}{32.2}(1.22)$$

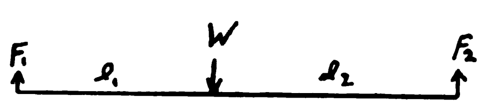
$$F_y = 1885\#$$




$$F = \sqrt{1885^2 + 77^2}$$

$$= 1888\#$$

## APPENDIX 2




$$- Wl_1 + (l_1 + l_2)\,F_2 = 0$$

$$F_2 = W\left(\frac{l_1}{l_1 + l_2}\right)$$

$$F_1 = W\left(\frac{l_2}{l_1 + l_2}\right)$$

$$K_1\ \delta_1 = F_1$$

$$K_2\ \delta_2 = F_2$$

Assume $\delta_1 = \delta_2$

$$\frac{F_1}{K_1} = \frac{F_2}{K_2}$$

$$\frac{K_1}{K_2} = \frac{F_1}{F_2}$$

$$\frac{K_1}{K_2} = \frac{\cancel{W}\left(\frac{l_2}{l_1 + l_2}\right)}{\cancel{W}\left(\frac{l_1}{l_1 + l_2}\right)}$$

$$\frac{K_1}{K_2} = \frac{l_2}{l_1} \qquad \text{or} \qquad K_1\, l_1 = K_2\, l_2$$

**Thus, to obtain this result, equal deflections must be assumed.**

.APPENDIX 3

Specifications for Thesis Model

Desoto

| | | |
|---|---|---|
| Engine and Transmission | | 757# |
| Steering Gear | | 28 |
| Radiator | | 46 |
| Bumpers | (32 apiece) | 64 |
| Battery | | 54 |
| Frame and Body | | 1802 |
| Gas Tank and Gas | | 124 |
| Spare Tire | | 29 |
| | | 2904# |

On model

Engine weight to include from above

| | |
|---|---|
| Engine and Transmission | 757# |
| Radiator | 46 |
| Battery | 54 |
| Total engine | 857# |

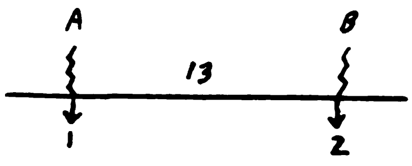
| | | |
|---|---|---|
| Body total | 2904# | Use front weight 1# |
| | 857# | Rear weight 2# (2000#) |
| | 2047# | |

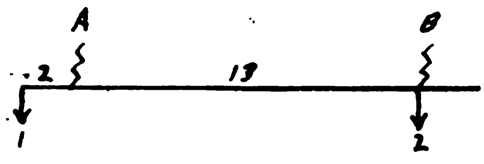
APPENDIX 4

An idea for possible checking of the centers of oscillation occurred to the writer too late for experimentation. The plan is as follows, and possibly is a method of proving the one theoretical point not checked by experiment.

Curves are taken as previously, except that in this case the two are made on the same zero displacement axis instead of having separate axes. A straight line is ruled on a piece of tracing paper, and thru the curves (along zero axis). A straight edge with stylus or point at each end is constructed so that the distance between styluses and pen points on model body bar are equal. Now by moving the tracing paper horizontally, the straight edge resting on it, at all times keeping the axis of paper and original axis coincident and placing the bar so one end traces along the front and the other along the rear curve, a series of lines may be laid off along the straight edge. The point of convergence of these lines should give the center of oscillations of the bar. The method obviously may require some revision or refinement on trial.

APPENDIX 5

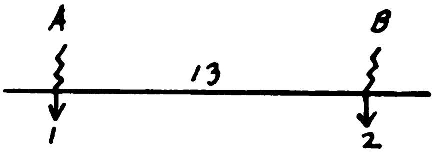


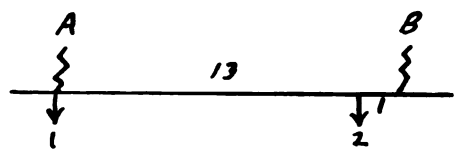



$\Sigma M_B = 13\ F_a = 13 = 0$ $\qquad$ $\Sigma M_B = 13\ F_a = 15 = 0$

$F_a = 1$ $\qquad$ $F_a = \frac{15}{13} = 1.15$

$1.15 - 1.00 = .15$ Difference




$\Sigma M_A = 13\ F_B - 26 = 0$ $\qquad$ $\Sigma M_A = 13\ F_B - 24$

$F_B = 2\#$ $\qquad$ $F_B = \frac{24}{13} = 1.85$

$2.00 - 1.85 = .15$ Difference

All springs are "D"

Scale = 2#/in.

To change $F_a$ and $F_b$ to deflection, divide by 2, but this is dividing by a constant and will not affect the relative results as far as the two subtractions are concerned, therefore, assume scale = 1 to simplify computation.

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4. "Analytical Mechanics for Engineers" - Seeley and Ensign.

5. "S.A.E. Transactions" - Vol. 20 - Part 2.

   " 21 - Part 2.

   " 27

   " 29

   " 31

Form 257-G

Sheet No. ..........

# MECHANICAL ENGINEERING LABORATORY
# MICHIGAN STATE COLLEGE

**Running Log of** Spring Calibration

**Observers** L. [illegible]

**Date** .........., 193....

| No. | Spring Used | #/in Defl. | |
|---|---|---|---|
| 1 | A | 1 | Satisfactory |
| 2 | a | 1 | |
| 3 | | | |
| 4 | B | 1 | Do not use. |
| 5 | b | 1 | |
| 6 | | | |
| 7 | C | 3½ | Unsatisfactory (Too Stiff) |
| 8 | c | 3½ | |
| 9 | | | |
| 10 | D | 2 | Satisfactory |
| 11 | d | 2 | |
| 12 | | | |
| 13 | E | 4 | (Too Stiff) |
| 14 | e | 4 | |
| 15 | | | |
| 16 | | | |
| 17 | | | |
| 18 | | | |
| 19 | | | |
| 20 | | | |
| 21 | | | |
| 22 | | | |
| 23 | | | |
| 24 | | | |
| 25 | | | |

**Remarks:**

Form 257-G

Sheet No.................................

# MECHANICAL ENGINEERING LABORATORY
# MICHIGAN STATE COLLEGE

Running Log of ........................................................................................................

........................................................................................................................

Observers { ............................................ { ............................................ Date.............................., 193......

No.

1

2

3 2 2 1 2 3½ 3½ 2 1 2 2

4

5

6 2 1 2

7 3 1 2

8 4 1 2

9 5 1 2

10 6 1 2

11 7 2 D

12 8 D+D 1 2 +R

13 9 1 2

14 10 1 2

15 1 2

16 1 2

17

18

19

20

21

22

23

24

25

Remarks: