A COMPARISON OF THE VOLUMES OF A GRAVITY DAM WITH AN ARCH DAM

Thesis for the Degree of B. S.
MICHIGAN STATE COLLEGE

Sooran A. Yavruian

1942

A Comparison of the Volumes of a Gravity Dam with an Arch Dam

A Thesis Submitted to

The Faculty of

MICHIGAN STATE COLLEGE

of

AGRICULTURE AND APPLIED SCIENCE

by

Sooran A. Yavruian

Candidate for the Degree of

Bachelor of Science

July 1942

THESIS

INTRODUCTION

Many types of dams have been built using less material and yet retaining the safety and permanence of a "gravity" dam. The most successful of these is the single arch dam where conditions permit its use; and so the purpose of this thesis is to determine the difference in material, e.i. the amount of concrete, under a given set of conditions, between a gravity dam and an arch dam.

The given conditions are:

1. A vertical walled canyon, 100 ft. in width, capable of resisting the arch thrust.
2. A good foundation, non-porous rock--nO uplift.
3. Coef. of friction- concrete on concrete .6
   concrete on rock .75
4. Height of dam 100 ft.
5. Top width 6.5 ft
6. No wind or ice pressure
7. Non-overflow type dam

DESIGN OF NON-OVERFLOW

SOLID-GRAVITY DAM

The general requirements for the stability of a gravity type dam are:

1. There shall be no tendency toward overturning
2. That the maximum pressure on any plane will not exceed a certain prescibed safe working stress.
3. That there will be no sliding on horizontal sections.

The first condition is met by keeping the resultant pressure in the middle third of the base of the section, X thus making the factor of safety against overturning 2 or greater. The second condition is met by spreading out the base from that required to fulfill the first condition, if necessary. The third condition is almost always met by the requirements of the first and second conditions.

Nomenclature--The symbols used will have the following significance:

W = The weight of the masonry of the dam above any horizontal section.

h = The height of the dam subjected to water pressure.

P.= The horizontal water pressure on the upstream face.

R = The resultant of P and W

e = The eccentricity of center of pressure on horizontal section.

p = Unit compression on upstream face.

p = Unit compression on downstream face.

f_s = The factor of safety against sliding

f_o = The factor of safety against overturning

b = The length of base

s = The unit shearing stress

z = The distance to R from the heel of the base of the horizontal section

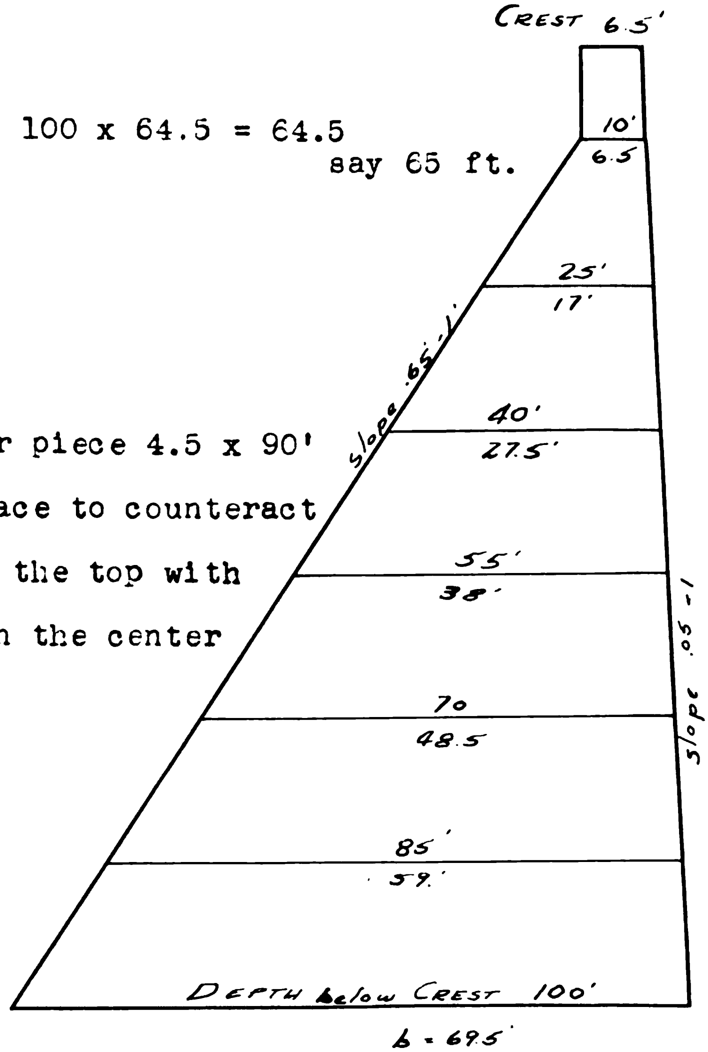
k = The distance of W from the heel of the base

y = The distance of P from the heel of the base

Determination of the Cross Section

$$b = h \times \sqrt{\frac{62.4}{150}} = 100 \times 64.5 = 64.5$$

say 65 ft.



Add triangular piece 4.5 x 90' to upstream face to counteract the effect of the top with of 6.5 ft., on the center of gravity.

Area of the cross section equals $\frac{1}{2}$ (65 x 100 - 6.5x10 - 4.5x90)

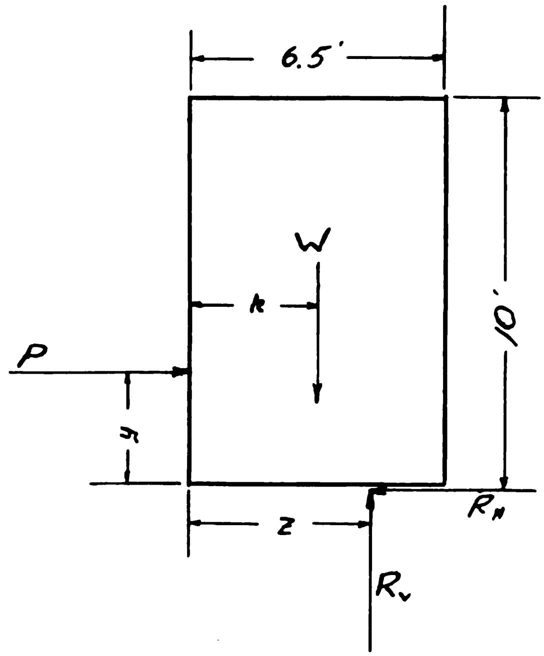
= 3,485 sq. ft.

The volume of dam = 3,485x100 - 348,500 cu. ft. or 12,910 cu. yd

STRESS TABLE FOR THE GRAVITY DAM DESIGN

Horiz'l plane ft below crest	Reservior full vert. comp. Min.	Max.	Max Comp	Shear Max.	Reservior empty Vert. Comp Min.	Max	Factor of safety sliding	Factor of safety overturning
10	0	28	40	3	14	14	1.88	2.00
25	6.5	25	36	8	1	28	1.11	2.22
40	4	39	55	9	2	42	1.03	2.27
55	6	53	76	17	3	56	1.02	2.30
70	6	67	96	22	4	70	1.01	2.31
85	7	81	115	28	5	85	1.01	2.32
100	8	97	139	31	6	98	1.25	2.32

Section 0' to 10' Reservoir full



$W = 6.5 x 10 x 150 = 9750\#$

$P = \frac{wh^2}{2} = \frac{62.4 x 10^2}{2} = 3,120$

$M_{heel} = 0$

$3120 \times 10/3 + 9750 x 3.25 = 9750z = 0$

$z = \frac{10,400 + 31,690}{9750} = 4.33'$

$e = 4.33 - 3.25 = 1.08'$

$p = \frac{9760}{6.5} \left(1 \pm \frac{6x1.08}{6.5}\right) = 1,500 (1 \pm 1)$

$p = 3000 \#/\square' = 28$ p.s.i.

$p = 0$

$f_s = \frac{9750 x .6}{3120} = 1.88$

$$f_o = \frac{2x150xb^2}{62.4h} = \frac{2x150x6.5^2}{62.4x10} = 2.02$$

$$s = \frac{P}{A} = \frac{3120}{6.5x144} = 3$$

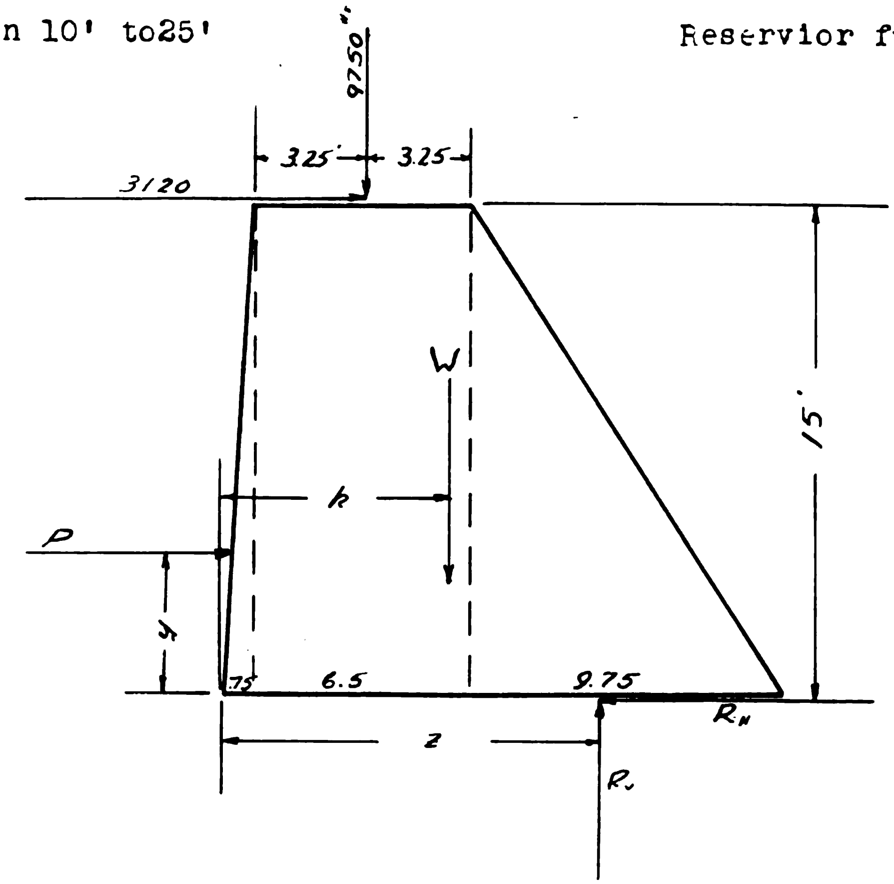
Reservoir empty

$$p = \frac{9760}{6.5} \left(1 \pm \frac{6x\cancel{2}.0\cancel{2}}{6.5}\right) = 1,500 \ (1 \pm 0)$$

$$p = 1,500^{\#\square'} = 14 \ p.s.i$$

$$p = 1,500^{\#\square'} = 14 \ p.s.i$$

Section 10' to25'   Reservior full



$$W = \left(\frac{.75x15}{2} + 6.5x15 + \frac{9.75x15}{2}\right) 150 =$$

$$850 + 14,625 + 10,975 = 26,450$$

$$P = \frac{62.4}{2} x (h^2 - h_1^2) = 31.2x(25^2 - 10^2) = 16,380 \#s$$

$$26,450 k = 850x\frac{.75x2}{3} + 14,625x4 + 10,925x10.5$$

$$k = 6.58'$$

$$16,380X + 3120x10 = 19,500x\frac{25x2}{3}$$

$$X = 17.94'$$

$$y = 25.00 - 17.94 = 7.06'$$

$$M_{heel} = 0$$

$$16,380x7.06 + 3,120x15 + 26,425x6.58 + 9,750x4 - 36,200z = 0$$

$$z = \frac{115,700 + 46,800 + 174,000 + 39,000}{36,200} = 10.37'$$

$$e = 10.37 - 8.5 = 1.87$$

$$p = \frac{36.200}{17} \left(1 \pm \frac{6x1.87}{17}\right) = 2130 \ (1 \pm 6.6)$$

$$p = 3540^{\#0'} = 25 \text{ p.s.i.}$$

$$p = 935^{\#0'} = 6.5 \text{ p.s.i}$$

$$f_s = \frac{36,200x.6}{19,500} = 1,11$$

$$f_o = \frac{2x150x17.02}{62.4x25} = 2.22$$

$$s = \frac{19,500}{17x12x12} = 8$$

Reservoir empty

$$9750x4 + 26,525x6.58 = 36,200x$$

$$x = 5.89$$

$$e = 8.5 - 5.89 = 2.61$$

$$p = \frac{36,200}{17} \left(1 \pm \frac{6x2.61}{17}\right) = 2130 \ (1 \pm .92)$$

$$p = 170^{\#0'} = 1 \text{ p.s.i.}$$

$$p = 4,100^{\#0'} = 28 \text{ p.s.i}$$

The Computations for the suceeding sections are done in a simalar manner.

DESIGN OF AN ARCH DAM

Hanna Method

from

<u>Design of Dams by Hanna and Kennedy</u>

Arch dams differ from gravity dams principally in that they transmit the water load to the sides, rather than to the bottom of the canyon.

In order that the material of the gravity dam and arch dam shall be similar, we shall assume the maximum allowable compressive stress in the concrete equal to the maximum stress in the gravity dam, which was 139 p.s.i., or 20,200 pounds per square foot, and the allowable shear 31 p.s.i. In any arch dam there will be some tension at the crown or at the abutments; so we will use the allowable tension of 50 p.s.i.

We will use a constant radius of extrados, 82' and a constant central angle of 75°.

The arch ring will be considered 'thin' if the radius of the extrados is more than 5 times the thickness of the arch ring, below 5 the arch ring is considered 'thick'.

<u>Nomenclature:</u>

N_a, M_a, V_a = thrust, moment and shear at the abutments.

N_h, M_h, V_h = thrust, moment and shear at the haunch

N_c, M_c, V_c = thrust, moment and shear at the crown

Nomenclature:

p = unit water pressure on upstream face of any arch ring

p' = unit water pressure on neutral axis of any arch ring

R = radius of extrados

r = radius of neutral axis

r_e = radius of neutral axis thick arch ring

t = thickness of arch ring radially to the neutral axis

ϕ_a = one-half central angle of extrados for loaded arch ring

ϕ_1 = angle between the crown radius and the radius of extrados through point g

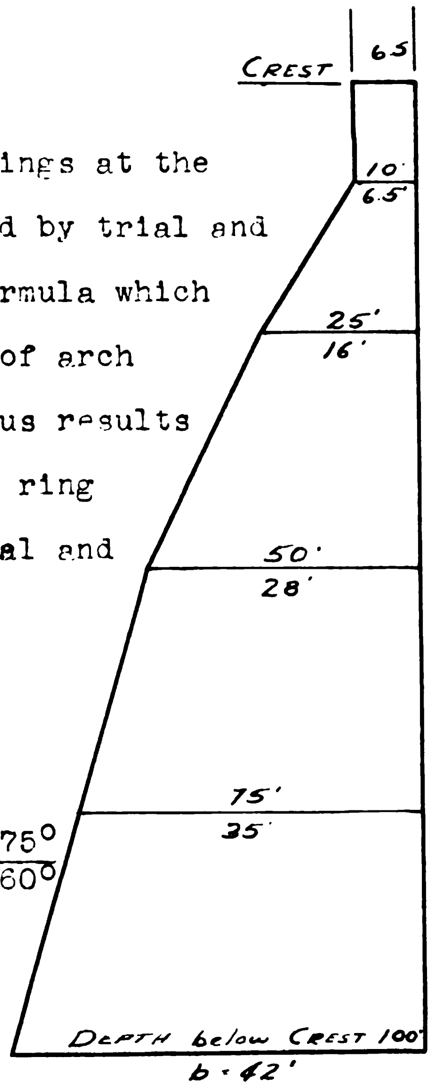
g = any stress point on neutral axis

f_e = unit stress at extrados

f_1 = unit shear at extrados

f_v = unit shear

Cross section of the Arch dam



The thicknesses of the arch rings at the various ellevations were found by trial and error. The 'thin cylinder' formula which was used in the calculations of arch dams in the past gave erroneous results in this case, and so the arch ring thicknesses were found by trial and error.

The volume of any section =

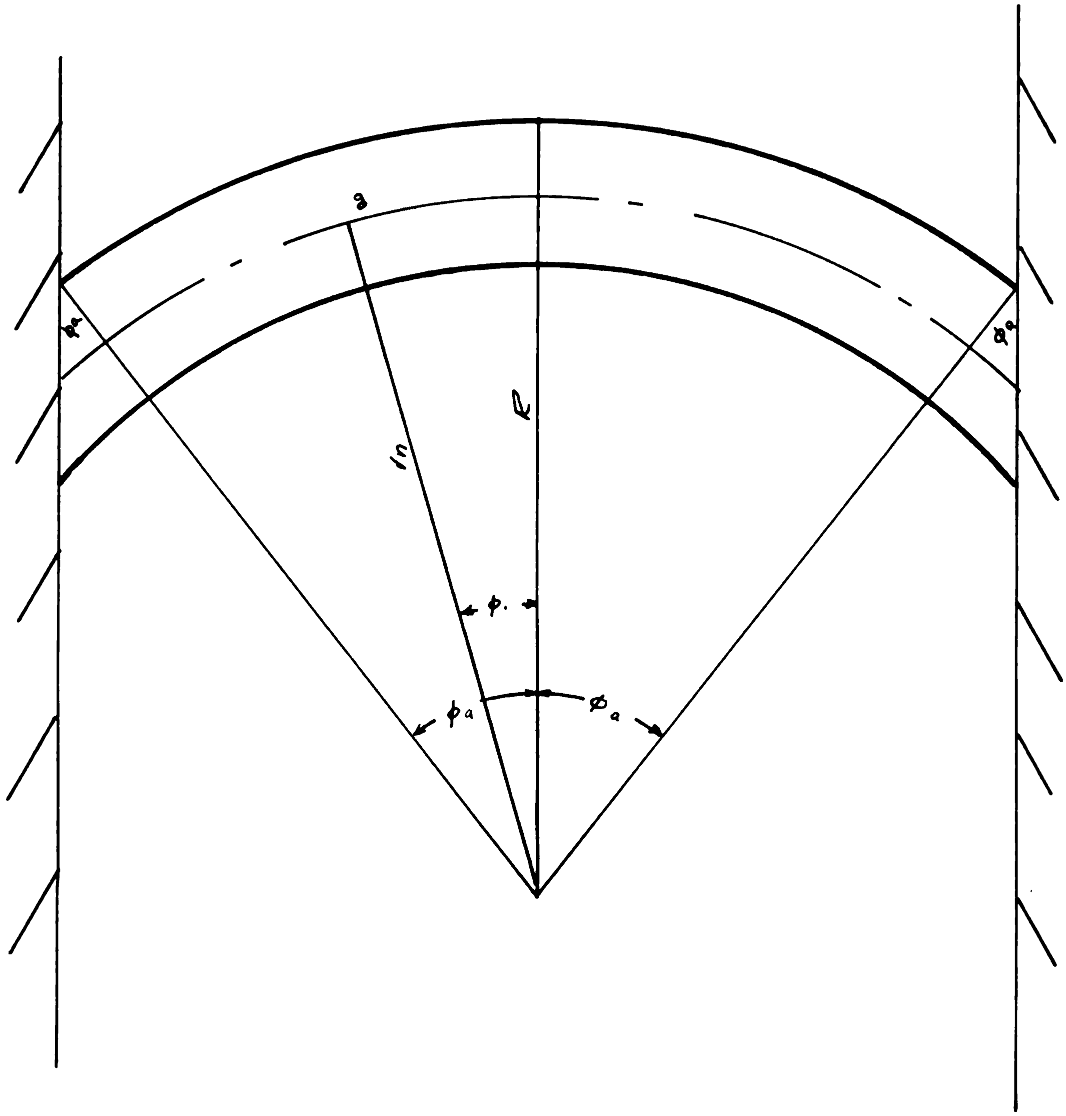
$$t - t' \div 2(r_n - r_n/2(2 \times \pi \times h)\frac{75^\circ}{360^\circ}$$

The total volume of arch ring =

$$\Big[\Big(10 \times \frac{6.5}{2}\Big)\Big(82 - \frac{6.5}{2}\Big) - \Big(\frac{16 + 6.5}{2}\Big)(25)\Big(82 - \frac{16 + 6.5}{4}\Big) - \Big(\frac{16 + 28}{2} \times 25\Big)\Big(82 - \frac{16 + 28}{4}\Big)$$

$$\Big(\frac{28 + 42}{2} \times 50\Big)\Big(82 - \frac{16 + 28}{4}\Big)\Big]\Big(2\pi \frac{75^\circ}{360^\circ}\Big) =$$

222,620 cu. ft. = 8,350 cu.yds.



Volume of the Arch wedge at abutments

$x = t \tan$

Area of wedge $= \frac{t^2 \tan\phi}{2}$. total area $= \frac{2\, t^2 \tan\phi}{2} = t^2 \tan$

Volume of wedge $=- .767\frac{(6.5^2 + 6.5^2)}{2} \times 10 + \frac{(6.5^2 + 16^2)\ .767}{2} \times 15$

$+ \frac{(10^2 + 28^2)\ (.767)}{2} \times 25 + \frac{(28^2 + 42^2)\ .767}{2} \times 50 = 60,995$ cu. ft.

Total volume of arch dam = total volume of arch ring - the

volume of the wedges = 222,620 + 60,995 = 283,615 cu. ft.

Horiz'l plane ft. below crest	Radius of Extrados	Thick of Arch Ring	Type of Arch Ring	Shearing Stress P.s.i.		
				Abut't	Haunch	Crown
10	82	6.5	thin	49.5	26.5	0
25	82	16.0	thin	22.0	11.0	0
25	82	16.0	thick	21.0	11.5	0
50	82	28.0	thick	35.0	18.0	0
75	82	38.0	thick	46.0	26.0	0
100	82	42.0	thick	53.0	29.5	0

Horiz'l plane ft below crest	Combined Compressive Stresses in Area P.s.i.					
	Crown		Haunch		Abutment	
	Ex	In	Ex	In	Ex	In
10	82.0	14.0	60.5	37.0	-15.0	114.5
25	77.0	-12.5	44.5	23.0	-52.0	126.0
25	70.0	-12.5	39.5	25.5	-46.5	137.0
50	62.0	-9.5	38.0	29.0	-38.0	139.0
75	58.5	-11.0	29.5	27.0	-34.0	138.0
100	46.0	-4.5	28.0	32.0	-25.5	141.0

STRESS TABLE FOR THE ARCH DAM

Arch ring 1 ft. high whose center is 25 ft below water surface

$R = 82$ ft.

$t = 16$ ft. $\qquad$ $R/t = 5.01$ arch ring can be considered as either 'thick or thin' We will use 'thin'.

$r = 74$ ft.

$$t/r = 16/74 = .216$$

$$K = 100.6$$

$$N_c = p'r \times \left(1 - \frac{Kt^2}{12r}\right) = 25 \times 62.4 \times 74 \times \left(1 - \frac{106.6 \times 16^2}{12 \times 74^2}\right) =$$

$$127{,}900\ (1 - .415) = 74.900$$

$$M_c = -r(p'r - N_c)\ \frac{\sin \phi_a}{\phi_a} - \cos \phi_1 = -74(53{,}000)\ \frac{\sin 37^\circ\ 30'}{37^\circ\ 30'} - \cos 0^\circ$$

$$= -74(53{,}000)\ (.07) = 274{,}200$$

$$V_c = -(p'r - N_c) \sin \quad = 53{,}000 \times \sin 0^\circ = 0$$

$$N_h = p'r - (p'r - N_c) \cos \quad = 127{,}900 - 53{,}000 \times \cos 18^\circ\ 45' = 77{,}700$$

$$M_h = -74(53{,}000)\ \frac{\sin 37^\circ\ 30'}{37^\circ\ 30'} - \cos 18^\circ\ 45' = -74(53{,}000)(-.017)$$

$$= 66{,}600$$

$$V_h = 53{,}000 \times \sin 18^\circ\ 45' = -17{,}000$$

$$N_a = 127{,}700 - 53{,}000 \times \sin 37^\circ\ 30' = 85.900$$

$$M_a = -74(53{,}000)\ \frac{\sin 37^\circ\ 30'}{37^\circ\ 30'} - \cos 37^\circ\ 30' = -74(53{,}000)(.137)$$

$$= -546{,}000$$

$$V_a = -53{,}000 \times \sin 37^\circ\ 30' = -53{,}000 \times 609 = -32{,}100$$

Stresses at Abutments

$$f = \frac{N}{t} \pm \frac{6M}{t^2}$$ (compressive stress formula)

$$f = \frac{85,900}{16} - \frac{-546,000x6}{16x16}$$

$$f_e = 5350 - 12,850 = -7,500$$

$$f_i = 5350 + 12850 = 18,200$$

$$f_u = \frac{3v}{2t} = \frac{3x32,100}{2x16} = 3000 \quad = 22 \text{ p.s.i.}$$

at haunch

$$f = \frac{77,700}{16} \pm \frac{66,600x6}{16x16}$$

$$f_e = 4,250 \pm 1,550 = 6,400$$

$$f_i = 4,850 - 1,550 = 3,300$$

$$f_v = \frac{3V}{2t} = \frac{3x17,000}{2x16} = 1600 \quad = 11 \text{ p.s.i.}$$

at crown

$$f = \frac{74,900}{16} \pm \frac{274,200x6}{16x16}$$

$$f_i = 4650 - 6450 = -1800$$

$$f_e = 4,600 + 6,450 = 11,100$$

$$f_v = 0$$

ALL PRECEEDING CALCULATION SIMILAR

Arch ring 1 ft. high whose center is 25 ft below water surface

$R = 82$ ft. $R/t = 5.01$ arch ring can be considered as either 'thick or thin"

$t = 16$ ft. We will use 'thick'.

$r = 74$ ft.

$$r_n = \frac{t}{\log_e\left(\frac{R}{R-t}\right)} = \frac{16}{\log_e 1.24} = 73.83'$$

$$c = 74.00 - 73.83 = .17$$

$$\frac{t}{r_n} = .2167 \qquad K' = 4.97$$

$$N_c = pR\left(1 - \frac{K'}{12}\right) = 62.4 \times 25 \times 82\left(1 - \frac{4.97}{12}\right) = 75{,}000$$

$$M_c = -r_n(pR - N_c)\left(\frac{\sin\phi_o}{\phi_o} - \cos\phi_o\right) = -73.83\ (52{,}900)(.07) = -273{,}800$$

$$V_c = 0$$

$$N_h = pR - (pR - N_c)\cos\phi_o = 127{,}900 - 52{,}900 \times .947 = 77{,}700$$

$$M_h = -73.83\ (52{,}900)(-.017) = 66{,}400$$

$$V_h = -(pR - N_c)\sin\phi_o = -52{,}900 \times .341 = -18{,}050$$

$$N_a = 127{,}900 - 52{,}900 \times .793 = 86{,}000$$

$$M_a = -73.83\ (52{,}900)(.137) = -535{,}000$$

$$V_a = -52{,}900 \times .609 = -32{,}200$$

Stresses at Crown

$$f_e = \frac{Nr_n}{Rt} + \frac{Mr_n(\frac{t}{2} + c)}{RI} = \frac{75,000x73.83}{82x16} + \frac{273,800x73.83x8.17}{82x\frac{16^3}{12}}$$

$$= 4200 + 5900 = 10,100$$

$$f_i = \frac{Nrn}{(R - t)t} - \frac{Mrn(\frac{t}{2} - c)}{(R-t)I} = \frac{75,000x73.82}{66x16} - \frac{273,800x73.83x7.83}{66x341}$$

$$= 5250 - 7050 = -1800$$

$$f_v = 0$$

At the haunch

$$f_e = \frac{77,700x73.83}{82x16} + \frac{66,400x73.83x8.17}{82x341}$$

$$= 4,300 + 1,400 = 5,700$$

$$f_i = \frac{77,700x73.83}{66x16} - \frac{66,400x73.83x7.83}{66x341}$$

$$= 5,400 - 1,700 = 3,700$$

$$f_v = \frac{3x18050}{2x16} = 1,700$$

At the Abutments

$$f_e = \frac{86,000x73.83}{82x16} + \frac{535,000x73.82x8.17}{83x341} = 6,700$$

$$= 4850 - 11,500 = -6,700$$

RESULTS AND CONCLUSIONS

The total volume of materials saved equals the volume of the gravity dam minus the volume of the arch dam

$$= 348{,}500 - 283{,}615 = 64{,}885 \text{ cu. ft.}$$

$$\text{The \% saving} = \frac{64{,}885}{348{,}500} = 18.6\%$$

The arch dam , not only represents a savings of 18.6% of concrete, but also has the greater safety factor of the two dams. And in conclusion, I believe that wherever possible an arch dam can be used advantageously.

finis