# EXPLORING TEACHERS' PRACTICES OF RESPONDING

Amanda Milewski

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of requirements
for the degree of

DOCTOR OF PHILOSOPHY

Mathematics Education

2012

**ABSTRACT**

EXPLORING TEACHERS' PRACTICES OF RESPONDING

By

Amanda Milewski

In the two decades since the introduction of the Professional Standards for Teaching Mathematics (NCTM, 1991) describing effective mathematics instruction, researchers have found U.S. teachers still lack the ability to foster productive mathematical discourse. Three instructional practices are key to teachers' ability to support rich discourse in their classrooms: *posing* rich tasks, *interpreting* students' ideas, and *responding* to students' mathematical thinking. Abundant research and resources exist to help teachers learn the practices of *posing* and *interpreting*. Likewise, adequate research exists to demonstrate that a typical teacher's practices of *responding* is overly-evaluative, as well as what alternative *responding* practices might look like. Professional resources designed to support teachers in learning better *responding* practices, however, are scant.

The purpose of this qualitative case study is to understand the extent to which a professional learning experience, which was designed to help teachers learn about the practices of *responding*, influenced the ability of three mathematics teachers to envision and enact alternative *responding* practices. To understand change in the participants' imagined *responding* practices, I administered a survey on three occasions across the

year-long experience. To measure change in participants' enacted practices of *responding*, I collected five videos from each participant across the experience. This research describes and explains how the three teachers' imagined and enacted practices changed during and following participation in professional development and action research.

A major contribution of this work is its observation frame, constructed to highlight changes to the breadth and focus of teachers' *responding* practices. I found many similarities in participants' post-professional-development and post-action-research changes including: a shift from focusing on mathematical products, a shift toward focusing on mathematical processes, a shift away from evaluative *responding* moves, and a shift toward *responding* moves that encourage student reflection of peers' ideas. I also found a few differences among the three participants, particularly in the action research cycle, where teachers aimed to sustain changes they had made following the professional development intervention. From their initial changes, one teacher enhanced her changes, another mostly maintained her changes, and a third teacher reversed many of her changes during the action research. At the end of the action research cycle, however, the three teachers' practices of *responding* were remarkably similar to one another, and all represented practices that differed dramatically from their baseline data. These results and their implications are discussed in relation to future iterations of this professional development as well as continued lines of research that could easily develop from this work.

## DEDICATION

I dedicate this work to my son, Cruz, and my parents.

Cruz, this work began well before you were born. As I neared its end, I decided I could not wait any longer to meet you. When I did finally meet you this work became so much less important in the light of the joy you brought to my life. So I took a break to enjoy you! At some point, I began to work on this again because I knew the importance of "finishing well" and wanted to teach that to you. You were truly my inspiration to finishing this work! I love you so much!

Mom and Dad, you know as well as I the depth of my gratitude. I would have never finished this work without your help with caring for Cruz as well as your consistent emotional support. I couldn't have asked for better parents!

**TABLE OF CONTENTS**

APPENDICES

## LIST OF TABLES

## LIST OF FIGURES

# Chapter 1: Introduction to the Study

## 1.1: Background: What's the Problem?

Principles and Standards for School Mathematics (NCTM, 2000) is one of the guiding documents for those interested in studying or enacting practice within mathematics education. This document makes a wide variety of recommendations about how school mathematics should be taught. The Teaching Principle states: "Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well" (p. 11). This teaching principle is discussed further in the document to give users a clearer picture of what is necessary for teachers to attain this principle.

> Teachers establish and nurture an environment conducive to learning mathematics through the decisions they make, the conversations they orchestrate, and the physical setting they create. Teachers' actions are what encourage students to think, question, solve problems, and discuss their ideas, strategies, and solutions. The teacher is responsible for creating an intellectual environment where serious mathematical thinking is the norm. More than just a physical setting with desks, bulletin boards, and posters, the classroom environment communicates subtle messages about what is valued in learning and doing mathematics. Are students' discussion and collaboration encouraged? Are students expected to justify their thinking? If students are to learn to make conjectures, experiment with various approaches to solving problems, construct mathematical arguments and respond to others' arguments, then creating an environment that fosters these kinds of activities is essential. (p. 18)

More recently the Common Core State Standards for Mathematics (CCSSM) have been adopted by many states. Soon these states will assess students according to the CCSSM. The CCSSM Standards for Mathematical Practice require students "construct viable arguments and critique the reasoning of others" (p. 6, CCSSM).

Within the description of this standard, mathematically proficient students are expected to be able to "justify their conclusions, communicate them to others, and respond to the arguments of others" (p. 6-7). The standard ends with the statement, "[s]tudents at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve arguments" (p. 7).

It is not difficult to see the parallel between these standard statements from the CCSSM and recommendations from NCTM's Principles and Standards document (2000). They both focus on the need for students to be involved with making conjectures, justifying their thinking, and respond to arguments from peers. The major difference between these two documents is the NCTM documents were recommendations for teaching and learning mathematics. The CCSSM, on the other hand, forwards these ideas as assessable standards that students, teachers, schools, and districts will be held accountable for. In many places, this difference has accelerated the conversation around the need for the mathematics teachers to change their instructional practices to better support students working toward these goals. The CCSSM stays silent about the teacher's role in working toward her students obtaining these standards. Thankfully, NCTM does not.

In an earlier document, the National Council of Teachers of Mathematics (NCTM, 1991) released the Professional Standards for Teaching Mathematics that explicated the teacher's role in classroom discourse. This earlier document explains that "the teacher of mathematics should orchestrate discourse by -

- posing questions and tasks that elicit, engage, and challenge each student's thinking;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;
- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate" (p. 86).

This vision for teacher practice stands in stark contrast to what was and is still typical for teacher practice. In 1988, for example, Ball pointed out that many teachers enter the profession believing that the purpose of questions is to "elicit right answers" (Ball, 1988, p. 44). Ball elaborates in a later paper entitled *What's all this talk about discourse?* (1991) by describing the typical teacherly reflex to a correct answer as to either move on, praise the student, or affirm the answer and restate the student's response for others to benefit (Ball, 1991, p. 44). Similarly, Davis (1997) found that teachers often take an evaluative stance towards students' responses.

According to Stigler and Hiebert (1999), dialogue in U.S. mathematics classrooms is dominated by IRE type interactions (Cazden, 2001; Mehan, 1985). IRE is defined to be a classroom interaction in which the teacher initiates (**I**) dialogue by asking a question, a student offers a response (**R**) to the question, and then the teacher evaluates (**E**) the student's response. Within the IRE structure the teacher's primary enacted role is to pose questions and to evaluate student thinking. When these findings are compared with NCTM expectations for the teacher's role in discourse, it is not hard to see that

much about the richness of discussion is impossible with the overly evaluative stance taken by most teachers.

It is not by accident that the mathematics education community has concerns about an excessively evaluative stance toward student understanding as many bodies of research point to various pitfalls inherent in such a stance. For example, motivation literature has demonstrated that praise can have negative effects on students' beliefs about intelligence, disposition towards learning, motivation to learn, and efficacy of learning (Mueller & Dweck, 1998). These negative effects are because an inappropriate use of praise, primarily praise that is strictly evaluative, can cause individuals to believe that intelligence is an innate quality that is fixed (Mueller & Dweck, 1998). Therefore, students come to believe that school is either about demonstrating how smart one is or hiding one's lack of intelligence (Dweck & Leggett, 1988). With this belief, students can easily fall into a pattern of learned helplessness believing there is nothing they can do about their lack of understanding since it is a result of a fixed, unchangeable characteristic (Turner et al., 2002).

The formative assessment literature also addresses some other issues around evaluative feedback. Formative assessment is defined to be assessment that is *for* the purpose of learning rather than assessment *of* learning (called summative assessment). Studies have shown that students can make learning and achievement gains when teachers make improvements to their formative assessment practices (Black & Wiliam, 1998; Fuchs & Fuchs, 1986). To make improvements to one's practices of formative assessment, teachers need to give descriptive feedback to students' thinking (Sadler,

1989). Evaluative feedback such as "great" or "not quite" is not considered actionable because such feedback fails to provide students with the reason that thinking was evaluated as good or incorrect. For this reason, this type of evaluative feedback (praise or corrective) is not descriptive enough to be effectively used *for* learning (Hattie & Timperley, 2007). Additionally, making decisions based on the IRE structure usually leaves teachers with a limited view of both how the individual as well as the class is thinking. Therefore, the IRE structure is not conducive for providing teachers with appropriate evidence with which to make instructional decisions, which is the basis for good formative assessment practices.

Instead of supporting the traditional mode of IRE, reform documents and researchers recommend that after posing a question and receiving a student's answer, the teacher's role is to orchestrate a discussion in which students are pressed to communicate their reasoning to their peers (Kazemi, 1998; Sherin, 2000; Stein, 2007). Furthermore, the authority of determining the correctness a solution should rest with the entire classroom community rather than simply the teacher or textbook (Hufferd-Ackles et al., 2004). In the following passage, Franke et al. (2007) describes various expectations of the teacher's role in discourse.

> Teachers are expected to pose problems but not provide answers (Lampert, 1999), stop or slow down the discussion to provide access to more students (Rittenhouse, 1998), model the academic discourse for the students (Ball, 1993; Lampert, 1990; Rittenhouse, 1998), comment and elaborate on students' ideas (Rittenhouse, 1998) and question student reasoning so as to foster certain habits of mind (Lampert, 1990; Lampert, Rittenhouse & Crumbaugh, 1996; Rittenhouse, 1998). Thus, as Ball (1993) pointed out, the teacher is responsible for the students' learning of mathematical content and, at the same time, for fostering a discourse environment that both supports students and helps to create, among them, new identities that

include a favorable disposition towards mathematics. It is no wonder IRE remains prevalent. (p. 231)

As Franke's last sentence implies, there are good reasons why this type of reform to a teacher's practices does not come easily to teachers (Smith, 1996). One reason is these sorts of changes to teacher practices are contrary to traditional modes of interacting with students. A second reason is the expectations placed on teachers are too great. Furthermore, teachers may not have frequent opportunities to learn to adjust their practices in ways that are akin with reform recommendations.

### 1.2: Purpose: In Search of a Solution!

Even when teachers get a sense they might need to change their pedagogy, the move towards more reform-minded practices is not an easy journey for teachers. Smith (1996) describes how the traditional mode of "teaching by telling" provides a foundation of efficacy from which teachers draw upon to do their work. He goes on to show how "deeply reform departs from the telling model" (p. 388) making reform efforts difficult to achieve. If we are to expect change in teachers' practice, we need to find ways to help them learn how to teach mathematics differently.

Smith's study goes on to suggest a different platform for teachers' sense of self-efficacy within this reform-minded vision for teaching mathematics. He suggests new "moorings for self efficacy" in the model of reform teaching include choosing worthwhile and engaging tasks, anticipating student reasoning, and orchestrating discussions around students' thinking (p. 397). Since then, others have picked up on Smith's suggestion to consider these three practices. Nicol (1999), who worked to help

prospective mathematics teachers learn to teach differently, finds that teachers "often experienced tensions with the kinds of questions posed and the reasons for posing them, with what they were listening for, and with how they responded to students' thinking and ideas" (p. 52). Similarly, Crespo (2007) is currently in the midst of a National Science Foundation Career grant entitled: *Examining prospective teachers' learning of three mathematics teaching practices – posing, interpreting, and responding – during teacher preparation.* In both Nicol and Crespo's work we hear echoes from Smith's study about the importance of the practices of posing tasks, interpreting what students are saying, and responding to students' thinking.

In this dissertation, I examine the mathematical teaching practices of responding to students' thinking. However, I also argue that the practices of posing mathematical tasks and interpreting students' thinking are foundational to the practices of responding. I posit that it is difficult for teachers to make advances in their practices of responding without first attending to their practices of posing meaningful tasks and interpreting students' thinking. Without careful problem posing, the nature of students' thinking may be limited by a narrow task, which makes responding appropriately a challenge. Likewise, without careful attention to students' thinking, teachers may misunderstand what a student aims to communicate for a variety of reasons (Ball, 1997) and respond inappropriately as a result.

The purpose of this study is to describe and explain the extent to which a professional development can help teachers learn to imagine and enact alternative ways of responding to students' thinking. By responding, I refer to what a teacher does

when reacting to a student's mathematical thinking. The professional development designed and studied in this present work includes a variety of experiences for teacher participants including: reading research articles, looking at records of practice, analyzing other teachers' practices of responding, reflecting on their own practices and decisions, and participating in a cycle of action research. The hypothesis was that teachers participating in these professional development experiences would become more adept at noticing, envisioning, and enacting alternative ways of responding to students' thinking.

### 1.3: Research Questions: Might Professional Development Help?

According to Crespo et al. (2007), responding is defined according to both a teacher's imagined practices and enacted practices. In imagined responding practice, a "teacher has a collection of hypothetical responding strategies for students' correct, incorrect, and novel work" (p. 210). Additionally, a teacher "can identify and construct a range of different types of responses when presented with hypothetical teaching scenarios" (p. 210). In enacted responding practices, a "teacher has a repertoire (a set of well practiced and deliberate) of responses (possibly a subset of their imagined response strategies) to students' correct, incorrect, and novel work" (p. 210). Some of the responding strategies given as examples for the above definition include asking students to restate what another student has said, asking students to agree or disagree with peers' thinking, and encouraging students to build upon their peers' ideas.

In both imagined and enacted responding practices, Crespo and colleagues differentiate a novice performance from an expert performance using several key

characteristics. The breadth of a teacher's repertoire of responding strategies (in both imaginary and enacted practices) is one key characteristic that can be used to distinguish between novice and expert performances. Another key characteristic that distinguishes an expert performance is the degree to which a teacher's responses are balanced between a focus on the process and product of students' mathematical thinking as opposed to a novice's performance that primarily focuses on the product (Crespo, 2007). Building on this definition of the practices of responding, this present work answers the following questions:

> Question 1) What are the outcomes on teachers' imagined and enacted practices of responding during and following participation in professional development designed to focus specifically on the practices of responding?
>
> Question 2) What are the outcomes on teachers' imagined and enacted practices of responding during and following participation in a cycle of action research designed to focus specifically on the practices of responding?

### 1.4: Rationale: Why should you care?

With this study, I will help the field to further understand the outcomes of professional development and action research on teachers' practices, with a particular focus on responding to students. This is important for three reasons. First, in this work I provide methods for others interested in studying teachers' instructional practices of responding. Second, I will use the results from this study to make revisions to future iterations of this professional learning experience to benefit of future cohorts. Last, results of this study can be used by those in the field to understand more about how teacher's imagined practices and enacted practices change across time.

The description of this study will help others' efforts to measure change within teachers' instructional practices of responding. All too often we measure change to teachers' instructional practices by solely considering test scores of students in the teachers' classrooms. While this is one measure, the measure is far enough removed from the change we are attempting to make that it is hard to draw any conclusions about whether the goals were actually accomplished. For example, students' scores could improve because the system incorporated more interventions for struggling learners at the same time teachers were attempting to change their practices. I believe one of the best ways to know whether an effort changed teachers' instructional practices is by measuring change in teachers' instruction. In this study, I provide new ideas to the field about ways we can do. Broadly, my research methodology will be helpful for others trying to study outcomes from professional development designed to help teachers' change instructional practices.

More specifically, one of my most significant offerings is observational frames I created to conduct the research. For the action research and dissertation research, my participants and I needed a taxonomy of responding moves in order to observe and measure changes within their practice. When we turned to the literature, we found little agreement across the literature on teachers' responding moves. Our shared research needs and the lack of direction from the literature created the impetus to collaborate and create an observational frame that would articulate these practices for our use. I believe this taxonomy will be very useful for teachers and researchers alike in

this new era of the Common Core State Standards that has made aspects of mathematics instruction, such as responding, an object of intense scrutiny.

A second contribution that the results of this study provide is a means for me to make changes to future iterations of this workshop. The data gives me the ability to reflect analytically on the design of the professional development. This reflection helps me to identify successful and unsuccessful elements within the teachers' experiences. In this way, a portion of this study is about me, the author, studying and improving my own practices as a mathematics consultant attempting to help teachers make changes to their instructional practices. This, however, also has broader implications for others trying to help teachers to make changes to their instructional practices. As I share this journey of learning about my own practices within this professional development, the reader can learn something about their own practices.

A final reason for this study can help others to understand more about how the practices of responding change as teachers moved from less expert performances to more expert performances. Others have suggested hypotheses for how the practices of responding grow across experience (Crespo et al., 2007). This study provides a significant contribution of new cases for the field to consider. As these new cases are considered against those previous suppositions, the hypotheses can be refined to better represent the reality of how we can expect teachers' responding in both imagined and enacted practices. Understanding how these two types of practices change will be helpful for the field. On the surface imagined practices seem to play a prerequisite role to enacted practices. In other words, a teacher must first be able to imagine doing

something before they are able to actually enact it. A general assumption in the field is that change within imagined practices will lead to change in enacted practices. This assumption is apparent from the ways that both teacher educators and professional developers talk about their work. Many syllabi and professional development descriptions include goals such as "reimagining" practices or creating a "new vision" for the classroom. For this reason, the present work pays attention to change in both imagined and enacted practices.

### 1.5: Summary: Where are we headed?

In this work, I provide a description of a professional development and action research cycle that helped three teachers shift away from overly evaluative practices typical in teaching and learning. This practice remains prevalent in many mathematics classrooms in spite of recommendations made in iterations of the NCTM reform documents. In this present work, I examine a professional development effort designed to help teachers move closer to the recommendations from NCTM by considering and enacting alternative ways of responding to students' mathematical thinking.

I examine whether teachers' imagined and enacted practices of responding change during and following the professional development and action research cycles. I also explore how teachers' imagined practices of responding are reflected in their enacted practices of responding. This study has implications for those interested in creating and delivering professional development designed to transform teachers' instructional practices. This study also has implications for those who want to understand how "imagining" plays a role in teachers' ability to enact particular sorts of practices.

Finally, this present work helps inform future iterations of this professional development.

I now describe the organization of the remainder of this dissertation. In Chapter Two, I review the literature that was used when considering the design of the professional development. In the third chapter, I describe the design of the professional development and research methodology. In Chapter 4, I analyze the data. In the last chapter, I discuss my findings and conclusions.

# Chapter 2: Literature Review

## 2.1: Introduction

In the first chapter, I defined the problem explored within this dissertation. In short, a disconnect exists between the vision laid out in the NCTM reform documents for the ways teachers should respond to students' ideas and what is typical for classroom practice. The vision of NCTM is that teachers would be proficient in orchestrating rich mathematical discussions by exploring students' ideas and encouraging others to build on those ideas (NCTM, 1991, 2000). In Chapter One, I also discussed the purpose of this dissertation. In short, I am exploring how professional development can be part of the solution to the problem summarized above. Therefore, I designed a professional development program with the purpose of helping teachers consider and enact alternate responding practices.

From empirical research, we see that teachers are usually too evaluative in their stance towards students' thinking to be able to effectively pull off this sort of rich discourse (Ball, 1988, 1991; Davis, 1997; Stigler & Hiebert, 1999). This overly evaluative stance frequently used by teachers has a negative impact on learners' motivation. Inappropriate use of praise, for example, can undermine students' self-efficacy, beliefs about intelligence, disposition towards learning, and motivation to learn (Dweck & Leggett, 1988; Mueller & Dweck, 1998; Turner et al., 2002). This evaluative stance towards students' ideas also impacts teaching in undesirable ways. Formative assessment literature warns that an evaluative response is not actionable enough to

enable students to make changes in the future (Black & Wiliam, 1998; Fuchs & Fuchs, 1986; Sadler, 1989). Furthermore, teaching with an overly evaluative stance leaves a teacher with too little information about student thinking to make instructional decisions akin with recommendations from formative assessment literature (Hattie & Timperley, 2007).

The purpose of this chapter is to review the literature used to inform the design and research of the professional development intended to help teachers to change their practices of responding. The literature review is broken up into two sections. In the first section, entitled "Theory about professional development designed to change teachers' practice," I discuss the research that helped me define the theoretical considerations for developing and studying this professional development. In the second section, entitled "Empirical examples of professional development designed to change teachers' practice," I examine two contrasting pieces of empirical research that helped me think about the design elements of the professional development.

## 2.2: Literature Review: What Can We Learn From Others?

### *2.2.1: Theory about PD Designed to Change Teachers' Practices*

In this section, I explore some of the existing theoretical perspectives on the design and study of professional development intended to help teachers change their practices. To begin, I consider a paper by Richardson (1990) that compares two different perspectives on how professional development can be designed to bring about significant and worthwhile change within teachers' practices. Next, I examine Shulman's (1986) work about the various forms of knowledge for teaching and ways

teachers can obtain these types of knowledge. Third, I discuss a chapter by Ball and Cohen (1999) who create a new vision of what professional development will need to look like in order to help teachers make changes to their practice. Finally, I describe a conceptual framework proposed by Desimore for studying the effects of professional development.

Richardson (1990) considers three different perspectives on the work of helping teachers to make *Significant and Worthwhile Change in Teaching Practice.* Richardson begins this article by reviewing two bodies of literature: teacher change and learning to teach. These two bodies of literature take contrasting perspectives on how to help teachers make changes to their practices. In the teacher-change literature, Richardson explains that change is defined "as teachers doing something that others are suggesting they do. Thus, the change is deemed as good or appropriate, and resistance is viewed as bad or inappropriate" (Richardson, 1990, p. 11). Richardson posits that within the teacher-change literature "even the more recent work that is more sensitive to teachers' norms and beliefs, fails to question the reforms themselves" (Richardson, 1990, p. 11). This perspective is juxtaposed with views found within the learning-to-teach literature. Richardson says the "learning-to-teach research, in contrast, focuses more on individual teacher's cognitions, beliefs, and other mental processes than on behaviors" (Richardson, 1990, p. 12). Further, the learning-to-teach literature focuses on questions about what teachers know as they move through various stages of their careers and how they come to know those things.

Each of these views on how teachers change their practices has disadvantages toward understanding how to help teachers actually make changes. Richardson points out that the teacher-change literature has a weakness because it positions the impetus for change as coming from someone outside the classroom. The approach within that body of literature is too inflexible to allow teachers to be a real part of the change process. For example, when teachers fail to make changes according to what is being prescribed, they are frequently described as either a resistant obstacle to reform or "a pawn in the system with little power to make autonomous decisions concerning the appropriateness of a given practice for his or her classroom" (Richardson, 1990, p. 12). Within the argument that views the resistant teacher as an obstacle, teachers and reformers are at odds with one another. Within the explanation that views the unreformed teacher as a weak chess piece, the organization, rather than the teacher, is to blame for a lack of change. Neither of these views from the teacher-change literature gives enough credence to the autonomous nature of teachers to control changes to their practices.

The learning-to-teach literature is not without its drawbacks either. Richardson points out this individualistic and idiosyncratic approach to teacher change has an inability to be guided by standards, which in turn leads to an elusive sense of effectiveness (Richardson, 1990, p. 13). In other words, the learning-to-teach approach to teacher change considers all changes as equal in terms of effectiveness. The only goal is that teachers change, rather than change in particular ways according to standards. Richardson interprets this lack of prioritization within the teacher-change literature as a

weakness. She argues that the literature fails to provide teachers with the focus they need to make changes that advantage learning and teaching in the classroom. In effect, the learning-to-teach literature gives us plenty of evidence that teachers can and do change. But how can we be sure that the changes teachers make are worthwhile and significant?

In the latter portion of her article, Richardson answers this question by suggesting a third perspective that brings together these two different bodies of literature. This new perspective includes some key ideas that are worth considering. First, in this new view of change one must acknowledge that teachers are in control of change. In working with teachers in the change process, Richardson suggests that teachers need help understanding the "pedagogical and moral implications of their decisions" (Richardson, 1990, p. 13). Second, the focus of change should be on helping teachers to concentrate on practical knowledge rather than classroom actions. Richardson argues this point because practical knowledge acts to inform teachers' classroom actions. Richardson describes this practical knowledge as knowledge that helps teachers judge and act within situations similar to the past.

A third element of Richardson's perspective is that teachers "themselves must be involved in making judgments about what change is worthwhile and significant" by taking opportunities to enter a dialogue about "practices and ways of thinking outside an individual teacher's own experiences" (Richardson, 1990, p. 14). Richardson suggests that one promising method for involving teachers in making these judgments is through the use of empirical research on teaching and learning. Lastly, this new

perspective on teacher change demands that one pays attention to the context in which teachers are attempting to make changes. Teachers should be understood as situated within cultures and norms of school buildings that include colleagues, administrators, students, families, and communities. Richardson posits that in attempting to make changes, one should consider the opportunity for the collective construction of new standards and values for practices. The ideas from Richardson's perspective were key for the creation of the present professional development effort.

Next, we shift to the work of Shulman who considers different representations for the knowledge for teaching. In his paper about knowledge for teaching, Shulman (1986) addresses questions such as what teachers know and how they come to know it. Immediately, we can see some leaning towards what Richardson has described as the learning-to-teach perspective. However, in the ways that Shulman aims to answer these questions, he also seems to be interested in the ways we can move teachers along a trajectory towards more principled changes.

Shulman begins to explore the knowledge for teaching by distinguishing between the domains of knowledge and the forms for representing these domains. The domains of knowledge, as defined by Shulman, are content knowledge, pedagogical knowledge, and pedagogical content knowledge. Briefly, these three types of knowledge for teaching mathematics can be understood by considering what a teacher must know about mathematics, about teaching in general, and about teaching mathematics specifically. Shulman also defines three representational forms that all three of the domains of knowledge for teaching can take on: propositional knowledge,

case knowledge, and strategic knowledge. He explains that "much of what is taught to teachers is in the form of propositional knowledge" (Shulman, 1986, p. 10), which he describes as "lengthy lists of research-based behaviors for teachers to practice" (Shulman, 1986, p. 11). He goes on to describe three different kinds of propositional knowledge: principles, maxims, and norms. A principle usually comes from empirical research such as teachers' knowledge about the pedagogical benefits of wait time (Rowe, 1986). A maxim, on the other hand, comes off as more of a practical idea such as "Never smile until Christmas" (Shulman, 1986, p. 11). The final type of propositional knowledge, called norms, embody the moral and ethical aspects of teaching such as the importance of not embarrassing a student in front of his peers.

Next, Shulman argues that although propositional knowledge is powerful, it is difficult to recall and use in the classroom setting directly. Therefore he argues for the development of something called case knowledge, which parallels the representations of knowledge in in the fields of medicine and law. Case knowledge, Shulman describes, "is knowledge of specific, well-documented, and richly described events" (Shulman, 1986, p. 11). He says to create a case is to theoretically argue that "it is a 'case of something,' or to argue that it is an instance of a larger class" (Shulman, 1986, p. 11). In some sense, the cases are narrated forms of some larger theoretical principles worth knowing. Shulman suggests three "representations" for case knowledge that are parallel to the three types of propositional knowledge: prototypes to exemplify theoretical claims, precedents to illuminate maxims, and parables to demonstrate norms. Finally, he argues that strategic or practical knowledge are best formed by

consideration of both propositional and case knowledge. Shulman describes strategic knowledge as forming when "principles collide and no simple solution is possible" (Shulman, 1986, p. 13). In other words, when a teacher confronts situations in which principles seem to contradict one another, they must resolve that situation, and in so doing they form strategic knowledge. So in many senses, Shulman's work moves beyond the learning-to-teach literature by providing a vehicle for teacher change through these representational forms of teacher knowledge.

Shulman's argument for the use of cases has certainly been picked up within the field of education. As stated by Shulman in the conclusion of his work, "the ultimate test of understanding rests on the ability to transform one's knowledge into teaching" (Shulman, 1986, p. 14). In many ways, Shulman set the field moving forward with a new method of using cases to transform theoretical teaching knowledge into practical teaching knowledge. Shulman's argument of a need for more than just propositional representations of knowledge resonates with the purposes of this present work. The aim of the designed professional development is to move teachers through a trajectory that ultimately helps them change their practices in ways that are significant and worthwhile when considering what we know about learning and teaching mathematics. Therefore, Shulman's ideas have served as one of the theoretical anchors for this work. Connected to Shulman's suggestion for the case method of educating teachers is Ball and Cohen's work around records of practice.

In Ball and Cohen's chapter *Developing Practice, Developing Practitioners* (1999), they argue two points that are useful for this dissertation. The first point is about what

teachers must know to be able to enact practices that are akin to the ideas of reform. Along with a list that could be categorized broadly as content knowledge, pedagogical knowledge, and pedagogical content knowledge, Ball and Cohen describe another kind of knowledge that does not fall neatly into one of these three categories: the ability to learn in and from practice. To have this kind of knowledge, teachers must be able to build up knowledge from their moment-to-moment experiences in the classroom. Teachers would also have to be able to use that knowledge to improve their practice. Additionally, teachers would have to know how to learn by experimenting in various situations with students. Ball and Cohen argue a stance of inquiry helps teachers to learn while teaching. The authors explain that "the best way to improve both teaching and teacher learning would be to create the capacity for much better learning about teaching as a part of teaching" (Ball & Cohen, 1999, p. 11-12). Even though much can be learned from experiences outside the classroom, we must find ways for teachers to learn practices from within where the knowledge of teaching is situated.

The second point from Ball and Cohen that is useful for this present work is that learning from moment-to-moment experiences of classroom interaction can be incredibly difficult if one is expected learn in real time. The demands of the classroom often do not allow for deep, in the moment reflection that is required for teacher learning. Because of this, Ball and Cohen argue that better learning opportunities exist with using of records of practice such as lesson plans, students' work, or videotapes of classroom interactions. They explain "one reason that records of practice are so important to changing the discourse of practice, and hence improving teaching, is that

in conversations about them, teachers could hardly avoid grappling with standards" (Ball & Cohen, 1999, p. 18). This discussion of standards creates a shared vision for what we mean by practice. This argument is akin to Shulman's (1986) suggestion to move towards the case method of representing knowledge for teaching. Additionally the use of video records of practices can help teachers learn observation tools that will allow them to develop more refined ways to analyze teaching and learning. Learning such methods would help teachers gain freedom from learning through the apprenticeship of observation (Lortie, 1975) by moving to more complex ways of understanding practice. These two points from Ball and Cohen, namely the need to learn from practice and the usefulness of records of practice, are key theoretical elements that helped to shape the design of the present professional development.

Desimore (2009) proposes a conceptual framework for studying professional development in order to *Improv[e] Impact Studies of Teachers' Professional Development.* In her article, Desimore posits that "recent research reflects a consensus about at least some of the characteristics of professional development that are critical to increasing teacher knowledge and skills and improving their practice, and which hold promise for increasing student achievement" (p. 183). She summarizes these key features as professional development that has a content focus, teachers actively learning, coherence with teachers' knowledge and beliefs, sufficient duration, and collective participation. She also suggests a basic model that represents "an operational theory of how professional development works to influence teacher and student outcomes" (p. 184). This model begins with teachers experiencing an effective professional development

that includes the five characteristics mentioned above. Next, the professional development works to build new knowledge, skills, attitudes and beliefs. Third, the teachers carry their new knowledge, skills, attitudes, or beliefs back to the classroom to change their instruction or approach. Finally instructional changes fosters an increase in students' learning. Desimore's identification of the key characteristics and hypothesis about the mechanism for change were both elements infused within the design and study of this professional develop.

In this section, I explored some important theoretical principles that are useful in considering the design and study of the professional development at hand. From Richardson's (1990) work, I highlighted the importance of not falling into an all-too-idiosyncratic and individualized perspectives of the learning-to-teach literature or the inflexible and disabling views expressed by teacher-change literature. Instead, in this work I aim to adopt the perspective that teachers are ultimately in control of change. Therefore teachers need opportunities to learn about, consider which, and decide upon the changes that they believe are worthwhile and significant. From Shulman (1986), I learned teachers need more than propositional knowledge to change their practices. So taking Shulman's advice, I aim to include opportunities for teachers to experience propositional knowledge, case knowledge, and strategic knowledge in their efforts to alter their practices of responding.

Ball and Cohen's work (1999) helped me to understand the potential of experience itself to be a teacher of teachers. To harness this potential, however, teachers need opportunities to develop an appropriate stance and tools to learn from practice

using records of practice as a primary vehicle. Therefore, a large part of the professional development designed for this effort included the use of records of practice and observation tools to help teachers become better at reflecting on practice. Finally, Desimore's work gave me a frame for the design and study of this professional development. The key characteristics from her posited consensus were each present in the professional development experience. Also the mechanism model that she suggested was extremely helpful for structuring the professional development in a way that supports teachers' changes.

*2.2.2: Empirical Examples of PD Designed to Change Teachers' Practices*

In this next portion of the dissertation, I examine two contrasting pieces of research about teaching practice for examples of how reform recommendations have played out in classrooms. I selected these contrasting examples because they represent a story of a successful implementation of reform principles as well as a story demonstrating a lack of success of reform principles penetrating the classroom. These two contrasting examples provided me with practical ideas about how to design a professional development experience to support teachers' implementation of new instructional practices. I begin by considering the successes of Cognitively Guided Instruction as a tool for teachers to implement practices of reform.

Cognitively Guided Instruction (CGI) is a professional development and research program that examines broadly the question of how teachers' knowledge about children's mathematical thinking affects teachers' instructional practices (Carpenter et al., 1989). The project is considered a success because researchers found

that when teachers experienced a professional development designed to familiarize them with research about student thinking, the teachers chose to adjust their instructional practices in line with reform recommendations (Carpenter et al., 1989, p. 525). Further, lower achieving students of CGI-trained teachers were found to make larger gains than lower achieving students of non-CGI-trained teachers (Carpenter et al., 1989, p. 526). A long-term focus on students' thinking through classroom assessment has been demonstrated to have an impact on teachers' beliefs and instructional practices (Fennema et al., 1996).

Not all efforts to change one's practices according to reform recommendations have gone as well as in the CGI research. One of the more notable examples, within mathematics education research, of reform recommendation gone badly is the effort in California to reform education by changing the state's framework and standards with only brief attention to helping teachers understand the changes. The case study of Mrs. Oublier (Cohen, 1990) illustrates the problem. Cohen writes about how one teacher, Mrs. Oublier, tries to employ the California State Department of Education's new framework into her practice. What he finds is a teacher's attempts to reorganize her classroom results in inappropriate implementation of the state's reform recommendations. He describes, for example, an activity that is meant to move students towards reasonable estimation. In the activity, Mrs. Oublier has two students demonstrate how to estimate the length of a desk with paperclips. The result of the activity is that students across the classroom provide slightly bizarre responses to the task. What is more peculiar, however, is that all students' responses are counted as

correct, and no discussion of correctness ensues. About Mrs. Oublier, Cohen concludes "her relatively superficial knowledge of this subject insulated her from even a glimpse of many things she might have done to deepen students' understanding" (p. 322). About the reform efforts in California, Cohen (1990) posits "if we take the framework's arguments seriously, then Mrs. O should be helped to struggle through to a more complex knowledge of mathematics and a more complex practice of teaching mathematics" (p. 327).

Above we see an example of two very different results of attempts to reform mathematics classrooms. Some of the differences in results between the two efforts can be understood by considering differences in the two professional development experiences teachers received. In the CGI professional development, teachers were given guiding principles along with instructions to consider implementation of reform in their classrooms (Carpenter et al., 1989, p. 138). These principles connect very well with what Richardson (1990) talks about as opportunities to learn practical knowledge rather than simply classroom actions. In order for teachers to reform their practices, they need to understand the propositional knowledge that underpins the change. Additionally, teachers were assigned to four weeks of continuous professional development over the summer designed to help them implement those aspects of reform that the project was most focused on (Carpenter et al., 1989, p. 138). Finally, teachers were given a variety of activities such as lectures, discussions, readings, planning, watching videotapes of children doing math, talking to participants or researchers, and looking at curriculum materials to consider how to best implement the

principles of reform within their own classrooms (Carpenter et al., 1989, p. 139). In other words, teachers were treated as professionals who needed significant time and a broad range of activities, including records of practice, to understand the core principles of the program.

In the case of Mrs. Oublier, the principles of change from the old to the new framework were not addressed within the professional development she attended. "The new mathematics framework seemed to recognize some problems that students would have in learning new mathematics, but from Mrs. O's perspective the state did not act as though it recognized the problems of teacher's learning" (Cohen, 1990, p. 327). We have little information about the amount of time that Mrs. Oublier spent in professional development. Cohen's description, however, makes it clear that Mrs. O attended at least three different types of workshops: one from the state, one from a textbook company, and one during the summer. From the descriptions we have these three different types of workshops seem disjointed at best since Cohen concluded, "that whatever she has learned from the workshops, new materials, and new policies, it did not include a new view of mathematics" (Cohen, 1990, p. 324). Possibly, the disjointed nature of the workshops was enough to cause more confusion than clarity about the principles of change. Finally, Mrs. Oublier's workshop failed to give her a variety of activities to consider how she might implement the reform changes in her classroom. "The state acted as though it assumed that fundamental instructional reform would occur if the teacher is told to do it" (Cohen, 1990, p. 327). This attitude of simply being "told to do it" does not resonate with either Richardson's (1990) ideas about recognizing teachers'

control over the decision to change nor did it honor Shulman's (1986) ideas about teachers' need for more than just propositional knowledge. Some key factors of the CGI experience, such as helping teachers to understand the principle key to reform, the use of records of practice to develop case knowledge, and realization of the teachers' control of change are all noticeably absent from Mrs. Oublier's professional development experience.

Besides the differences above, other differences between these two experiences give us some hints about what else might be necessary for reform to be successful in the classroom. Having a long-term continuous experience to consider reform recommendations may be a key factor to successes seen within CGI. Mrs. Oublier's disjointed professional development experiences may have caused more confusion and loss of focus than the long-term commitment we see with the CGI intervention. Finally, allowing teachers freedom to explore these changes through a variety of experiences could be a key factor that bolstered CGI 's success. Just as it is ineffective for teachers to 'tell' children how to do mathematics, telling teachers how to change their instruction may not be enough support for them to act on those changes.

From this section, we have gained a few ideas for how one might develop a successful professional development program. From the theoretical literature, I learned some key elements of a professional development designed to change instructional practices include attending to teachers need to learn of principles that support reform, develop case knowledge through records of practice, and make choices about changes

they will make. From the empirical literature, I learned the importance of a sustained professional development program that includes a wide variety of experiences for teachers to explore new ideas.

### 2.3: Summary

In this chapter, I reviewed the literature that was key in the design of the study's professional development created to assist teachers in envisioning and enacting new practices of responding. I began by taking a look at some of the theoretical considerations for professional development that is intended to help teachers change their instructional practices. Richardson (1990) juxtaposed the teacher-change literature with the learning-to-teach literature, and this contrast that helped me to understand why neither of these perspectives on changing teachers' practices alone is adequate. Next, I examined Shulman's (1986) work, with specific focus on "representational" forms of knowledge for teaching. Shulman posits that the representations we typically use to transmit teacher knowledge, propositional forms, are not rich enough to transform teachers' practice. Shulman's suggestion for the field to take a step towards establishing more balance between propositional, case, and strategic representations of knowledge was a key consideration in the formation of the professional development within this work. Third, Ball and Cohen's (1999) work helped me to consider how practice itself serves a vital tool for teacher learning. In other words, practice can serve as a teacher of teachers if we can figure out how to harness its power. This view of practice helped me to understand that practice must play a key role in the design of this professional development. Finally, Desimore (2009) helped me to understand how

critical some of the components were and helped me to place particular emphasis on those activities that served to create those components of content focus, active learning, coherence, duration, and collective participation. Additionally the mechanism model she suggested helped me to develop the structure for the professional development, More specifically that model helped me to understand the importance of beginning by addressing knowledge, skills, beliefs and attitudes and then moving the teachers into an action research cycle to address the desired change to their instruction.

In the next section, I turned my focus towards some empirical examples of professional development designed to help teachers transform their instructional practices. To do this, I contrasted the case of Mrs. Oublier with research on CGI. By looking at these two pieces of research side-by-side, I learned about the importance of sustained professional development that attends to teachers' learning needs. CGI was built to meet teachers' needs for understanding broader principles behind reform, to have time to explore these ideas with support, and to have a variety of experiences with which to think about these ideas. The lessons I learned from comparing these two different types of professional development experiences were foundational to the ways I ultimately designed the professional development for this present work.

From the empirical examples and theoretical ideas about professional development found in these two sections, I designed a professional development program aiming to assist teachers in making changes to their practice. The details of the professional development's design are discussed at greater length in chapter 3. In

chapter 3, I also outline the research methodology used to study changes to teachers' practices during and after participation in the professional development and action research cycles. In the fourth chapter, I discuss results from the data analysis about the teachers' imagined and enacted practices of responding that was collected before, during, and after the year-long professional development experience. Finally, in chapter 5, I discuss my findings and explore implications of this work.

# Chapter 3: Study Design and Research Methodology

## 3.1: Introduction

The purpose of this chapter is to describe the design of the study and research from this present work. By way of introduction, I begin with a brief overview of the study, reviewing the dissertation's purpose and problem along with the research questions from chapter 1. Next, I shift my focus to the design of the overall study. In this portion of the chapter, I describe the ways research participants were selected by defining the greater professional development work with which they were involved. I also describe in detail the design and goals for the professional development being researched within this present work. Last, I focus on the data collection and analysis methods used.

### *3.1.1: Problem and Purposes Overview*

In chapter 1, I outlined the problem to be examined in this present work. Briefly, the NCTM reform documents (NCTM, 1991, 2000) describe a vision of how teachers should orchestrate mathematical discussions in ways that allow for students' ideas to be explored and built upon. This vision is not typically carried out with consistency in mathematics classrooms across the U.S. Teachers' typical responding practices are far too evaluative to allow for rich mathematical discourse to take hold (Ball, 1988, 1991; Davis, 1997; Stigler & Hiebert, 1999).

In the first chapter, I also outlined the purpose of this present work. In short, this study explores how professional development can help teachers move beyond what is typical in their practices of responding. I discussed the need to develop experiences for teachers to reconsider their responding practices to enable them to support students' thinking in ways that are closer to NCTM's vision of reform.

*3.1.2: Research Questions*

In attempt to understand more about how professional development can help increase teachers' expertise in their responding practices, this dissertation poses the following two research questions:

> Question 1) What are the outcomes on teachers' imagined and enacted practices of responding during and following participation in professional development designed to focus specifically on the practices of responding?
>
> Question 2) What are the outcomes on teachers' imagined and enacted practices of responding during and following participation in a cycle of action research designed to focus specifically on the practices of responding?

## 3.2: Participants

During the 2008-2009 school year, the Michigan Department of Education commissioned teacher-leaders and consultants from around the state to form teams to focus on improving formative assessment practices. Two coaches from the Macomb Intermediate School District (MISD), one being the author (a mathematics consultant) and the other a science consultant, gathered a team of seven volunteer teachers and one administrator from one district's middle school and high school. The seven teachers included two high school mathematics teachers, two high school science teachers, two middle school mathematics teachers, and one middle school science teacher. The

administrator was a former middle school mathematics teacher. In this way, the design of the team was already incorporating some of Richardson's (1990) ideas about the importance of situating teachers' work within school communities as well as Desimore's (2009) ideas about collective participation and duration.

This group's work during the 2008-2009 school year was organized around the concept of a video club. Specifically, each teacher collected two different videos of her classroom practice and agreed to share her video in exchange for feedback from peers. At the conclusion of the 2008-2009 school year, the seven teachers and administrator requested that the MISD consultants consider keeping the team together for one additional year for continued work on improving practices.

The professional development and action research cycle, described in more detail in the next section, were designed in response to the team's request for extended learning. The nature of this group was one where teachers voluntarily participated, rather than being forced by an outside governing body to change their practices. This voluntary participation was an important starting point given Richardson's (1990) ideas of helping teachers make their own decisions to change. Furthermore, the group was actively seeking more help in the same direction as the first year suggesting that had at least in part already achieved some level of coherence, one of the critical component of effective professional development as suggested by Desimore (2009). This current study includes only the three mathematics teachers because of the focus of this dissertation. Of the three mathematics teachers, one is a teacher of 35 years of

experience (who plans to continue working an additional 3 years), and two who have been teaching between 15 and 20 years.

The group's composition had some advantages. First, because every team member already had two classroom videos collected the year prior to the study, those artifacts served as a starting place for our work. Second, since this group had been working together in a very similar situation (sharing of classroom video), issues such as establishing norms, trust, and confidence had already been handled successfully. Finally, the team was composed of teachers who all had more than ten years of experience. This served as an advantage for the research because these teachers' practices of responding were likely more established and stable than if we had completed this study with a team of brand new teachers.

As with any collection of people, this group is not without its limitations. The first limitation is that this group is inherently different from many groups of teachers in professional development because of their shared history. For example, the team's willingness to jump into an entire year of intimately sharing practices came about much easier the second time around. We cannot ignore these teachers' previous, apparently positive, experiences with sharing video. Beginning this sort of intensive professional experience with a group of strangers would likely vary results greatly. The opportunity to work with a consistent group of teachers for an extended period of time is rare, even though Desimore (2009) lists duration as a critical component for effective professional development, and would be difficult to replicate in other professional development efforts. Another point worth noting is the team's unique quality of having every

member come from the same school district where many of them have worked together for over 10 years in the same department. Like the team dynamics, the building dynamics is not a common circumstance for groups of teachers involved in professional development contexts together even though collective participation is another critical component according to Desimore (2009). The results of this work might be very different if we had a team of teachers, each from different districts, coming together in a professional context to share their classroom and grow their practices of responding. Our established group, however, is important to study because it helps us to understand something about the importance of long-term, sustained professional development that is situated inside building- and district-level communities.

### 3.3: Professional Development Design

In response to the team's request to continue working on improving practices, the author developed a set of experiences aimed at broadening teachers' practices of responding (both imagined and enacted), and enhancing teachers' skills in analyzing teaching practices. This professional development included nine sessions, beginning with a focus on helping teachers change their imagined practices of responding and concluding with helping teachers change their enacted practices of responding.

To address teachers' imagined practices, the first four professional development sessions focused on helping teachers gain knowledge about responding practices. Beginning with a focus on helping teachers gain knowledge is consistent with Desimore's (2009) suggested mechanism for teacher change. To address teachers' imagined practices, teachers engaged in a variety of activities to help them reimagine

the nature of classroom interactions. This is important because in order to begin imagining changing their practices, teachers must first capture a new vision of teaching. Teachers explored these new instructional practices by reading practitioners' articles and examining records of practice (videos). More is said about the goals of this new vision in the next section.

The practitioners' articles primarily came from three NCTM professional journals: *Teaching Children Mathematics*, *Mathematics Teaching in the Middle School*, and *Mathematics Teacher*. Readings helped to incorporate ideas from both Shulman's (1986) propositional knowledge and Richardson's (1990) second principle about the importance of including practical knowledge rather than simply classroom actions. It also helped maintain a focus on content, as Desimore (2009) suggested would be critical. The videos came from a variety of sources including the TIMMS project, *Developing Mathematical Ideas* professional development series, and a casebook entitled *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning* (Boaler & Humphrey, 2005). My use of records of practice aligned this work with recommendations from Ball and Cohen (1999) and Shulman (1986) about the importance of building up case knowledge. I have included a separate reference list to highlight the practitioners' readings and videos used within the professional development.

Another major component of the first four sessions was time for teachers to consider their own practices in light of ideas from session readings, videos, and discussions. This aligns with Desimore's (2009) critical component of professional

development, active learning, because teachers were connecting their learning to their own classroom practices. To do this, each conversation about records of practice and articles included reflection time for teachers to consider their own practices related to the topics. Within these times of reflection, teachers had opportunities to reflect on what they considered worthwhile and significant as suggested by Richardson (1990). This also captures Desimore's critical component of coherence. Additionally, teachers analyzed classroom video using a variety of observation frames that focus on different components that make up or support responding to students' thinking. Teachers practiced using these frames on records of practice we viewed collectively. Teachers worked in pairs to analyze one another's videos (from the previous year) according to these frames. In this way, each teacher received peer-feedback about her own responding practices. This helped to reinforce Desimore's critical component of collective participation since teachers were sharing, viewing and providing constructive feedback to one another in regards to their instructional practices. This focus on observation tools was because of Ball and Cohen's (1999) suggestion to help teachers learn new tools that enable them to learn from and within practice (their own and others').

The final key element to the first four sessions is that teachers were asked to keep a concept or decision map. The idea for the decision maps came from a recent work by Herbel-Eisenmann (2009). In her work with teachers, Herbel-Eisenmann described how she "gave teacher-researchers large sheets of paper and asked them to arrange their stick-on notes about the center of the page, which represented what was 'closest to their

hearts' in their teaching" (2009, p.16). She calls these "belief mappings" and describes how these were made by the teacher-researcher across a time of studying together to clarify their beliefs about their own practices. These belief mappings played a key role, which will be discussed in the next paragraph, in helping teachers identify a change that they wanted to make in their practice. This goal parallels Richardson's (1990) ideas about the importance of recognizing teachers' need to make decisions about what changes they find worthwhile and significant as well as Desimore's (2009) ideas about coherence. The only modification of this professional development task is that the mappings did not focus on beliefs but rather on concepts that teachers found important within the professional development. The idea was that decision maps would help teachers keep track of ideas from the first four sessions that they saw as most worthwhile and significant. For this reason, the mappings in this study will be referred to as decision mappings. Reasons for this renaming are made clearer below.

As mentioned previously, Herbel-Eisenmann (2009) used belief mapping to help her teacher-researchers make decisions about what type of change they wanted to focus on. In her book, Herbel-Eisenmann describes:

> We returned to Hopkins (2002) and began the process of identifying a performance gap, or the discrepancies "between behavior and intention" (p. 57) ... Hopkins contended that identifying a performance gap can be an important beginning point for cycles of action research. To help the teacher-researchers identify their performance gaps, the university researchers asked them to consider our analyses of their baseline discourse, the reading that they thought were most helpful to them, the journals that they had been writing since the beginning of the project, their belief mappings, and the Hopkins book. (p. 20)

Using ideas from Herbel-Eisenmann's book, I designed the fifth professional development session so that teachers identify a performance gap by comparing feedback they received from their peers to information on their decision mapping. Since this workshop was about helping teachers change their practices of responding specifically, I asked participants to keep their decision mappings tightly aligned to responding practices. For example, teachers may have felt their technology practices to be most central to changes they wanted to make while viewing ideas about responding practices as more peripheral. We asked teachers, however, to focus specifically on the part of the map that contained ideas that they encountered related to responding practices. So in some ways, this mapping served a smaller purpose than it did for Herbel-Eisenmann, while in other ways the focused nature of the decision map combined other elements that teacher-researchers in Herbel-Eisenmann's project were considering, such as readings and journal entries. Parallel to Herbel-Eisenmann's (2009) work, participants used the comparison of their peer feedback and their maps to identify performance gaps. In this way, teachers were actively engaged in their learning (Desimore, 2009) during and after each session in thinking about which practices, from those discussed, they considered most important to adopt. Mindful of this gap, teachers formed action plans for how they intended to improve their responding practices over the subsequent months.

In the months following session five, teachers independently collected and analyzed videotape of their own classroom in accordance with Desimore's mechanism model for teacher change (2009). Teachers also continued to read professional articles

of their choosing (from a list suggested by facilitators). These independent activities and decisions were included to acknowledge that teachers were in control of change to their practice (Richardson, 1990). As facilitators, our job during that time was to support changes that teachers were seeking to make. Teachers met monthly to discuss their progress as they worked to improve their practices. I hypothesized that during these conversations teachers would be grappling with some points of disconnect they were finding between research and their own practices. These conversations were intended to be a key time for teachers to develop what Shulman (1986) called strategic knowledge as they brought propositional and case knowledge into the reality of the classroom. In the ninth professional development session, teachers reported their finding to the group. A summary with specific session details is found in Table 1.

*Table 1: Summary of Professional Development Components*

| | Session 1 | Session 2 | Session 3 | Session 4 | Session 5 | Session 6-8 | Session 9 |
|---|---|---|---|---|---|---|---|
| Date | 10/21 | 10/27 | 11/11 | 12/18 | 01/06 | 01/26, 02/17, 03/08 | 03/29 |
| Type | After School | All Day | All Day | All Day | All Day | After School | All Day |
| Topic | Introductions & Overview | Posing tasks | Listening and interpreting students' thinking | Responding to students' thinking | Identifying a performance gap | Action research | Reflecting and preparing to share |
| Looking at Records of Practice | Shea's Numbers - Ogechi from Mathematics Teaching and Learning to Teach, 1990<br><br>US87 from TIMSS, 1985 | The Border Problem, Part 1 from Boaler & Humphrey, 2005 | Crazy Cakes from Schifter et al., 2001 | The Border Problem, Part 2 from Boaler & Humphrey, 2005 | Shea's Numbers - Ogechi from Mathematics Teaching and Learning to Teach, 1990<br><br>US87 from TIMSS, 1985 | Teachers considered video from their own classroom | Teachers shared video from their own classroom and reflected on video from others' classrooms. |

*Table 1: Summary of Professional Development Components (cont'd)*

| | Session 1 | Session 2 | Session 3 | Session 4 | Session 5 | Session 6-8 | Session 9 |
|---|---|---|---|---|---|---|---|
| Practitioners Articles | Reinhart (2000)<br><br>Humphrey (2005)<br><br>Herbel-Eisenmann (2009) | Herbel-Eisenmann & Breyfogle (2005)<br><br>Smith & Stein (1998)<br><br>Herbel-Eisenmann (2009)<br><br>Vacc (1993) | Breyfogle & Herbel-Eisenmann (2004)<br><br>Rowe (1986)<br><br>Bushman (1994) | Ball (1991)<br><br>Sherin et al. (2000)<br><br>Stein (2001)<br><br>Smith et al. (2009) | Hopkins (2002) | Individual teachers selected their own practitioner readings about related ideas | |

*Table 1: Summary of Professional Development Components (cont'd)*

| | Session 1 | Session 2 | Session 3 | Session 4 | Session 5 | Session 6-8 | Session 9 |
|---|---|---|---|---|---|---|---|
| Learning to use observational frames | | Each teacher analyzed her teaching partner's video on interaction types, tasks, and launch questions.<br><br>Each teacher made suggestions about how to improve the interaction, tasks, and launch questions | Each teacher analyzed her teaching partner's video on discussion questions, wait time, turn type, and turn length.<br><br>Each teacher made suggestions about how to improve discussion questions. | Each teacher analyzed her teaching partner's video on response type.<br><br>Each teacher made suggestions about alternatives to evaluative responses. | | Each teacher analyzed own video according to response type and one observation frame of their choosing | |
| Decision Making | Began Decision Mapping | Developed & Adjusted Decision Mapping | Developed & Adjusted Decision Mapping | Developed & Adjusted Decision Mapping | Each teacher presented Decision Mapping | | |

*Table 1: Summary of Professional Development Components (cont'd)*

| | Session 1 | Session 2 | Session 3 | Session 4 | Session 5 | Session 6-8 | Session 9 |
|---|---|---|---|---|---|---|---|
| Action Research Cycle | | | | | Each teacher received observation data from teaching partner<br><br>Each teacher identified a performance gap where their practice didn't align with their decision about what they want to change | Each teacher selected a focus area that they were trying to improve within their practice related to responding. They focused on this practice in a particular portion of their class (beginning, middle, end) | Each teacher shared how they made improvements and learned through collecting and analyzing their own video<br><br>Teachers prepared a presentation to share with colleagues about whole year-long experience |

### 3.4: Professional Development Goals

As part of the professional development experience, all three teachers participated in four sessions prior to attempting to make any changes to their instructional practices. The first session served to set the stage for the work we would do together. The subsequent three sessions were organized around ideas of posing high cognitive-demand tasks, listening to students, and responding to students' ideas. In the session about posing tasks, teachers read and discussed research articles that helped them understand how the written and enacted cognitive demand of a task would impact students' opportunities to learn mathematics (Stein & Smith, 1998). Teachers also read and discussed articles about how question types set their classes' focus on either mathematical products or processes (Herbel-Eisenmann & Breyfogle, 2005; Vacc, 1993).

After discussing those articles, teachers watched a video of a middle school teacher, Cathy Humphrey, implementing a task called the Border Problem from the *Connected Mathematics Project Curriculum* (Boaler & Humphrey, 2005). By watching this video, teachers had an opportunity to think about how ideas from the articles played out in a real classroom when a teacher attended to the same issues they were themselves considering.

Last, teachers examined their research partner's baseline video to give feedback about the tasks and questions used. More specifically, participants considered their partner's tasks to determine the balance between low and high cognitive demand (Stein & Smith, 1998). Participants also coded their partner's questions as open or closed, as well as factual or reasoning (Vacc, 1993). The intended goals for this session were to

help teachers realize the importance of achieving balance between 1) high- and low-cognitive demand tasks and 2) questions about mathematical products vs. mathematical processes.

In the session about listening to students, teachers read and discussed articles that presented the importance of wait time (Rowe, 1986), listening to students' ideas before telling students how to complete a task (Bushman, 1994), and using questions that would focus on students' mathematical thinking (Breyfogle & Herbel-Eisenman, 2004). The group had quite a lengthy conversation about Rowe's (1986) article; specifically, students are normally limited to roles that involve answering the teacher's solicitations and awaiting evaluation. Participants discussed the possibility for students to have opportunities to play different roles in classrooms, such as re-structuring a task, soliciting peers for ideas about the mathematics, and responding to or building on one another's ideas.

Next, teachers considered how ideas from the articles they discussed could be seen in a video of a fifth grade classroom that came from the *Developing Mathematical Ideas* professional development curriculum (Schifter et al., 2001). In this video, participants watched as a teacher and her students discussed a pair of students' solution to a task, called Crazy Cakes, from the *Investigations Curriculum*. Participants watched as a teacher encouraged students to explain and defend their mathematical ideas. The group noticed how the teacher took a passive role in clearing up students' confusions. Again, by observing a teacher intentionally attending to students' mathematical ideas, participants were able to identify ideas they read about in the research articles.

In the final portion of this session about interpreting students' ideas, teachers analyzed their partner's video for various cues that would help indicate whether a teacher was listening. These cues included 1) wait time after a question is asked 2) wait time after a student shares their idea 3) turn length for both student and teacher and 4) turn types for both student and teacher. This analysis provided the teacher with an overall picture of their practices of listening, including 1) average wait time 2) average turn length 3) ratio of teacher turns to student turns 4) percentage of time the teacher is talking 5) percentage of time the students are talking 6) percentage of various types of turns for the students 7) percentage of various types of turns for the teacher. Across the second session, the professional development goals included helping teachers to realize the importance of 1) wait time 2) the amount of time they talk versus that which their students talk and 3) roles students assume.

In the session about responding to students' ideas, teachers considered research articles that addressed the importance of using a wide breadth of responding techniques. The articles and discussion helped teachers to understand how an overly evaluative stance can stifle a classroom environment and provide little room for notions called for in reform, such as students learning to justify their thinking, critiquing the thinking of others, or building upon another's ideas. In the articles from this session (Ball, 1991; Sherin, 2000; Smith et al., 2009; Stein, 2001), teachers were introduced to new responding techniques such as revoicing (without evaluation), inviting other students to agree or disagree, inviting other students to restate a students' ideas, asking questions to uncover the mathematical ideas to name a few.

After discussion of the articles, teachers watched a video to provide an opportunity for teachers to consider responding practices in a classroom. Again, teachers watched a video from Cathy Humphrey's middle school classroom (Boaler & Humphrey, 2005). This lesson, however, was the second day of the Border Problem they had watched in session 1. The focus of the discussion in the video was the sharing of students' ideas about the Border Problem. This focus made the video an excellent resource to consider Cathy's practices of responding.

Lastly, participants considered their partner's practices of responding in the baseline video. This time, they were coding all instances of responding according to the frame found later in chapter 3 (table 4 from section 3.6). Participants also took several of their partner's evaluative interactions and rewrote them to include non-evaluative moves that could have been made instead. The goal of this third session on responding focused on helping teachers 1) shift away from evaluative responding moves to non-evaluative responding moves 2) increase their breadth of various types of non-evaluative responding moves and 3) shift away from a heavy product-focused responding practices towards a balance between product- and process-focused techniques.

When participants selected their personal action-research goals, the professional development project's goals may have guided that selection. Some goals that participants selected were aligned with professional development goals. In other cases, participants selected goals that were different from mine. This fact was considered good because it gave evidence that teachers' felt autonomous in selecting goals they felt were

worthwhile and significant, which is an important part of teachers making changes to their practices according to Richardson (1990). Similarities and differences between participants' goals and professional development goals were both expected to be an important part of teachers' success.

During this session, teachers were also asked to consider strategies that they would use to enact these new practices. One example of strategies that teachers developed was the use particular prompts (i.e. What do other think about that?) that would focus the discussion towards use of invitation, rather than evaluation. The facilitators also suggested ideas from a framework presented by Margaret Smith at a colloquium talk given at Michigan State University (Smith, 2007). Since that time Smith and colleague Mary Kay Stein have published a book entitled *5 Practices for Orchestrating Productive Mathematics Discussions* (2011) explicating these practices in detail. These 5 practices include teachers anticipating and monitoring students' thinking, selecting and sequencing students to share their thinking, and helping students make the connections between various strategies. While these practices are not synonymous with the practices of posing, interpreting, and responding, there are certainly places of overlap. For example, Smith and Stein's focus on the practice of connecting overlaps with some of ways the group talked about the practices of responding. Similarly, the practices of anticipating and monitoring, as well as the mechanisms suggested by the Smith and Stein to enable these practices, can greatly assist a teacher's practices of interpreting as well as responding.

## 3.5: Data Collection

Data collected for this study fall under two categories: teachers' imagined practices and teachers' enacted practices. In this section, I describe and justify how I collected data. Finally, I conclude by describing how data in these two areas are connected to the research questions in this present work.

### *3.5.1: Data collection of teachers' imagined practices*

The data-collection methods described in this section were developed to understand portions of the research questions related to changes in teachers' imagined practices of responding. According to Crespo et al. (2007), one might expect that as teachers become more sophisticated in their imagined practices of responding, we would see them move from a narrower to a broader collection of hypothetical strategies for responding. We would also expect teachers to achieve a greater balance between their focus on mathematical products and processes (p. 212). To observe changes to teachers' hypothetical responding strategies, all involved teachers participated in three iterations of a survey designed to measure the breadth and focus of teachers' imagined responding practices. The first iteration of the responding survey was administered at the start of the school year, before the professional development began. The second administration was in the middle of the school year, after the professional development had started and before the action research cycle began. Finally, a third iteration of the survey was administered after the action research cycle had finished.

The survey was broken up into three sections. The first portion of the survey began by asking teachers to imagine posing a problem to their class. Participants were

asked to respond in writing to various prompts (see appendix 1) about how they would pose the problem as well as what they anticipated students saying and doing during the time immediately following the posing of the problem. Additionally, participants were asked to describe three different things they could imagine themselves saying or doing after students had time to work on the problem. Finally, the first section of the survey concluded with teachers writing a comparison of how these three things they imagined saying or doing were different from one another.

The second portion of the survey contained three hypothetical solutions to the problem from the first section. In all three scenarios, teachers were asked to imagine that these solutions were produced and presented on the board by students in their classrooms. Teachers were asked to provide two different things they might say or do next, and what they hoped to accomplish by taking those actions. In the first scenario, the student's solution was incorrect. In the second and third scenarios, the solutions were correct and novel respectively.

In the final portion of the survey, participants were first notified that they would be watching two different classroom interactions; during these interactions, they were to imagine that the teacher in the video was a colleague who had asked for feedback about his or her responding practices. The two different classroom interactions were selected to represent different types of interactions. The first interaction represented a broader variety of responding techniques that were balanced between product- and process-focused whereas the second interaction followed the traditional IRE structure. Teachers were encouraged to take field notes as they watched these two interactions

unfold. At the conclusion of each video, teachers were prompted to give their imaginary colleague three ideas to consider about his or her responding practices. Finally, teachers were asked to note some of the major differences they noticed between these two interactions. Details of the exact wording of the survey questions can be found in appendix 1.

*3.5.2: Data collection of teachers' enacted practices*

Data-collection methods described in this section are aimed at measuring changes in teachers' enacted practices of responding. According to Crespo et al. (2007), teachers who have grown in their enacted practices of responding should use a broader repertoire of strategies for responding and a more balanced focus between mathematical processes and products (p. 212). Like Crespo, I measured changes to enacted practices by looking for changes to the breadth and focus of teachers' responding strategies. I say more in the next section about how I choose to operationalize these terms, breadth and focus, when I discuss the creation of the coding frames developed to measure these aspects of responding practices.

In order to observe changes to teachers' enacted responding practices, I first established participants' baseline responding practices. To do this, each research participant submitted two videos collected from the school year prior to the professional development. To establish what changes had occurred within teachers' enacted practices across this year-long experience, I requested participants to collect three additional classroom videos during the action research cycle in the second half of the year.

Teachers used all their collected videos, both previous and those collected for action research, to analyze their own practices. Separately from the teachers' analysis, I also used these videos to analyze teachers' responding practices. A summary of the data collection methods has been provided in Table 2.

*Table 2: Summary of Data Collection*

| | **Previous School Year** | **Before Session 1** | **Session 1-4** | **Before Session 5** | **Session 5** | **Session 6-9** | **After Session 9** |
|---|---|---|---|---|---|---|---|
| Imagined Practices | | First survey | | Second survey | | | Third survey |
| Enacted Practices | 2 baseline videos collected | | | | | Collection of 3 new videos | |

*3.5.3: Connection of data collection to research questions*

In this section, I briefly describe how the data I collected is connected to the study's research questions. To begin, the first research question is about how teachers' participation in the professional development changed their imagined and enacted practices of responding. To understand changes to teachers' imagined practices of responding, I compared teachers' first and second surveys. To observe changes in teachers' enacted practices of responding, I compared teachers' baseline videos to their first action-research video. As the action research cycle began, teachers began by collecting a video from their classroom. That video represents the first time, immediately following the professional development but not yet affected by the action research cycle, that the teachers attempted to make changes to their practices of responding.

The study's second research question focuses on changes in teachers' imagined and enacted practices as they participate in an action research cycle. To see changes to imagined responding practices, I compared teachers' second and third surveys. For changes to teachers' enacted practices, I compared their three action research videos. All of the above describe methods for understanding the study's first two research questions aimed at describing changes in teachers' imagined and enacted practices. A summary of connections between the research questions and data collection can be found in Table 3:

*Table 3: A summary of the connections between the data collected and research questions*

| | **Imagined Practices** | **Enacted Practices** |
|---|---|---|
| RQ1) How does participation in the professional development change . . . | Compared Survey 1 to Survey 2 | Compared baseline videos to first video from AR cycle |
| RQ2) How does participation in the action research cycle change... | Compared Survey 2 to Survey 3 | Compared three videos from AR cycle |

### 3.6: Data Analysis Methods

The purpose of this section is to describe how the data was analyzed to answer the research questions. I begin by describing how I developed the frames used to analyze imagined and enacted practices of responding. Next, I describe how I analyzed data about teachers' imagined practices as well as enacted practices.

*3.6.1: Developing the Coding Frames for Imagined and Enacted Responding Practices*

In this section, I describe how I developed the frames to analyze imagined and enacted practices of responding. As stated earlier, Crespo et al. (2007) describes how teachers' practices of responding change as they grow from novice to expert. In short, Crespo posits that teachers will be able to demonstrate a broader repertoire of responding techniques. Additionally, Crespo expects the focus of teachers' responding techniques to shift from mathematical products to a balanced focus between mathematical products and processes. This description from Crespo served as a basis for the structure of these frames.

I built a coding frame to observe teachers' breadth of responding practices that defined various types of responding moves a teacher could make. Little agreement exists across the literature about what we would call these various types of responding moves. Therefore, a list of names and definitions was developed as one of the products of the professional development so that teachers could use these ideas in their own action research.

After the frame was built within the professional development, I attempted to use this list to analyze video unrelated to this project. I observed one responding move within those videos that was not well described in the original frame, namely *ignoring*. I noticed that at times teachers chose to not respond to students' thinking at all. So I added a move named "ignoring" to the frame.

In the original frame, numerous categories had a common purpose of encouraging students to build on one another's ideas. This common purpose was not

surprising given the article teachers read by Sherin et al. entitled *Students building on one another's ideas* (2000). Originally, all of the following moves were distinct: *invitation to uncover*, *invitation to advance*, *invitation to revoice, invitation to evaluate*, *invitation to connect*, *invitation to add on*, *invitation to tell*, and *general invitation.* After coding the data, I noticed that many of those categories were never used or used so infrequently that it did not make sense for them each to have their own category. When looked at as a group, however, some interesting trends existed within the data. For that reason I collapsed all of these moves into one category, called *invitation*, making it easier to see what was changing about the ways teachers were *inviting* students to build on one another's ideas. For the same reason, I collapsed the categories of *comment*, *connect*, and *add on* into one category called *other*.

Table 4 illustrates the list of responding moves and their definitions developed within the professional development, including an additional move of *ignoring*:

*Table 4: A summary of the responding moves and their definitions used for coding teachers' responding practice*

| Move | Description | Example |
|---|---|---|
| **Revoice** | Restating or paraphrasing a student's ideas. Sometimes used to clarify, check for understanding, or amplify the students' idea. | So what I hear you saying is . . .<br><br>John is saying that . . . |
| **Invitation:** | Inviting students to interact with another student's idea in any of the following ways: | What do others think about Joe's idea? |
| General Invitation | Leaving it open to the students to decide how they will interact. | Did everyone understand what Suzanne's idea? |
| Invite to Uncover | Inviting students to make an uncovering move. | Okay, Kahlil, what question might you have for Sue? |
| Invite to Advance | Inviting students to make an advancing move. | Can anyone think of a question that might challenge Max's thinking here? |
| Invite to Revoice | Inviting students to make a revoicing move. | Can anyone restate what Caleb just said? |
| Invite to Evaluate | Inviting students to make an evaluation move. | How many of you agree with Cruz? |
| Invite to Connect | Inviting students to make a connecting move. | How is Nick's idea connected to what we learned last chapter? |

*Table 4: A summary of the responding moves and their definitions used for coding teachers' responding practice (cont'd)*

| Move | Description | Example |
|---|---|---|
| Invite to Add On | Inviting students to add on. | Does anyone have anything they would like to add on to Charlie's idea? |
| Invite to Tell | Inviting students to tell. | Can someone remind Julia what the slope formula is? |
| **Uncover** | Probing or clarifying questions that try to get at student understanding. | Walk us through your steps. How did you begin?<br><br>I didn't understand that. Can you go over it again? |
| **Advance** | Questioning to challenge a student beyond where they are currently at in order to push them along a trajectory. | Will your method always work?<br><br>What would happen if you used different numbers? |
| **Orient** | Refocusing the student to reconsider their response in light of the question being asked. | Okay, what was the original question and how does your idea help us advance towards it? |
| **Other:**<br><br>Comment | Teacher offers a non-evaluative comment about a student's idea. | I'd like you to notice how Andrew not only provided the answer, but also fully explained his thinking. |

*Table 4: A summary of the responding moves and their definitions used for coding teachers' responding practice (cont'd)*

| Move | Description | Example |
|---|---|---|
| Connect | Pointing students to recognize the relatedness of one's student's ideas to another idea previously discussed or known. | Joe's method is similar to Ann's method in that they both are using what they notice about the table to come up with the equation.<br><br>Isn't what Matt did just like what we talked about last week? |
| Add on | Progressing the student's idea towards completion. The idea here is to move an underdeveloped idea towards completion, not necessarily in a new direction. | Next, we would need to plug in the variable to the original equation. |
| Table | Deciding to put an idea on hold for later consideration. | That's a really interesting idea that you've brought up but we're not quite ready to think about that just yet. I would like to hold onto that idea and bring it back up at the end of the hour. |

*Table 4: A summary of the responding moves and their definitions used for coding teachers' responding practice (cont'd)*

| Move | Description | Example |
|---|---|---|
| **Tell** | Demonstrating or instructing students about an idea. | Alright, it seems that we have forgotten the Pythagorean Theorem. So let's go back and recall that the sum of the square of the sides of a right triangle is equal to the square of the hypotenuse.<br><br>You forgot the plus or minus with your answer. |
| **Ignore** | Teacher does not address idea. It is usually followed by an elicitation for another idea without coming back to original idea. | Okay, does anyone else have another idea? |
| **Evaluate** | Assessing a student's idea as correct or incorrect. | I disagree.<br><br>Good thinking. |
| **Revoice/Evaluate** | Restating or paraphrasing a student's ideas for the purpose of evaluating it as correct or incorrect. | Twenty-five. Okay – that's fine. |

Next, I built a coding frame to observe the focus of teachers' responding practices. I started by considering Crespo's distinction between a focus on mathematical products versus mathematical processes. In thinking more about how I would try to capture the essence of focus in a coding scheme, I turned to Herbel-Eisenmann and Breyfogle's (2005) work, *Questioning our Patterns of Questioning*. In this work, authors describe three types of interactions: IRE, funneling, and focusing. In an IRE (they call it IRF) interaction, the teacher asks a question, the student responds, and the teacher provides an evaluation or feedback. Herbel-Eisenmann and Breyfogle continue by describing, "funneling occurs when the teacher asks a series of questions that guide the students through a procedure or to a desired end. In this situation, the teacher is engaged in cognitive activity and the student is merely answering the questions to arrive at an answer, often without seeing the connection among the questions" (p.485). In contrast, "a focusing-interaction pattern requires the teacher to listen to students' responses and guide them based on what the students are thinking rather than how the teacher would solve the problem" (p. 486).

In order to expand the description of focus, I turned to one more research lens, provided by Hufferd-Ackles et al. (2004). In their study, the authors examined and described various levels of a math-talk community within classrooms. At the lowest level, we see a teacher that primarily elicits answer-focused responses with very little elicitation of students' thinking strategies. In the middle levels of a math-talk community, we see a teacher that primarily elicits answer-focused and procedure-focused responses while the students provide the "next step" in a procedure. In these

middle levels, the teacher may fill in the explanation of the procedure. Finally, at the highest level of a math-talk community, the teacher encourages students to describe their thinking more completely by requiring students to explain and defend their thinking. The teacher is open to discussing and comparing multiple strategies while the students contribute steps from their own thinking through fuller explanations and justifications of their decisions

Looking over these descriptions and then looking back at Table 4, certain responding moves seemed to carry purposes that would serve to forward a product- or process-focused interaction. For example, *inviting* another student to explain someone else's thinking seems to have much potential for opening up the conversation to focus on mathematical processes. On the other hand, *evaluating* an answer after a student has offered her mathematical thinking seems to serve to focus the conversation on the mathematical product. And of course, context seems to matter for determining the classification for some moves. For example, using an *orienting* move could be phrased in a way that is focused on the mathematical product (i.e. John, you haven't answered the original question) or mathematical process (i.e. John, what part of the original question made you think to try that method for solving?).

I classified moves that carried a clear focus regardless of context as either *process* or *product*. For those moves in which context mattered, I initially marked them as *context dependent* and went back to the data (video transcript or survey) to classify them accordingly. Classifying these responding moves into these two categories, *process* or *product*, allowed me to think about how teachers were using groups of these moves,

rather than looking at each move individually. These groups of moves helped me to understand more about how teachers were using these moves collectively to create a focus on *mathematical products* or *processes*. In Table 5, I clarify which moves were coded as *product*, *process*, or context dependent:

*Table 5: A summary of the focus for each of the responding moves*

| Move | Description | Example | Focus |
|---|---|---|---|
| **Revoice** | Restating or paraphrasing a student's ideas. Sometimes used to clarify, check for understanding, or amplify the students' idea. | So what I hear you saying is . . .<br><br>John is saying that . . . | Process |
| **Invitation:** | Inviting students to interact with another student's idea in any of the following ways: | | |
| General Invitation | Leaving it open to the students to decide how they will interact. | What do others think about Joe's idea?<br><br>Did everyone understand what Suzanne's idea? | Process |
| Invite to Uncover | Inviting students to make an uncovering move. | Okay, Kahlil, what question might you have for Sue? | Process |
| Invite to Advance | Inviting students to make an advancing move. | Can anyone think of a question that might challenge Max's thinking here? | Process |
| Invite to Revoice | Inviting students to make a revoicing move. | Can anyone restate what Caleb just said? | Process |

*Table 5: A summary of the focus for each of the responding moves (cont'd)*

| Move | Description | Example | Focus |
|---|---|---|---|
| Invite to Evaluate | Inviting students to make an evaluation move. | How many of you agree with Cruz? | Product |
| Invite to Connect | Inviting students to make a connecting move. | How is Nick's idea connected to what we learned last chapter? | Process |
| Invite to Add On | Inviting students to add on. | Does anyone have anything they would like to add on to Charlie's idea? | Context Dependent |
| Invite to Tell | Inviting students to tell. | Can someone remind Julia what the slope formula is? | Context Dependent |
| **Uncover** | Probing or clarifying questions that try to get at student understanding. | Walk us through your steps. How did you begin?<br><br>I didn't understand that. Can you go over it again? | Process |
| **Advance** | Questioning to challenge a student beyond where they are currently at in order to push them along a trajectory. | Will your method always work?<br><br>What would happen if you used different numbers? | Process |
| **Orient** | Refocusing the student to reconsider their response in light of the question being asked. | Okay, what was the original question and how does your idea help us advance towards it? | Process |

*Table 5: A summary of the focus for each of the responding moves (cont'd)*

| Move | Description | Example | Focus |
|---|---|---|---|
| **Other:** | | | |
| Comment | Teacher offers a non-evaluative comment about a student's idea. | I'd like you to notice how Andrew not only provided the answer, but also fully explained his thinking. | Process |
| Connect | Pointing students to recognize the relatedness of one's student's ideas to another idea previously discussed or known. | Joe's method is similar to Ann's method in that they both are using what they notice about the table to come up with the equation.<br><br>Isn't what Matt did just like what we talked about last week? | Context Dependent |
| Add on | Progressing the student's idea towards completion. The idea here is to move an underdeveloped idea towards completion, not necessarily in a new direction. | Next, we would need to plug in the variable to the original equation. | Context Dependent |
| Table | Deciding to put an idea on hold for later consideration. | That's a really interesting idea that you've brought up but we're not quite ready to think about that just yet. I would like to hold onto that idea and bring it back up at the end of the hour. | Context Dependent |

*Table 5: A summary of the focus for each of the responding moves (cont'd)*

| Move | Description | Example | Focus |
|---|---|---|---|
| **Tell** | Demonstrating or instructing students about an idea. | Alright, it seems that we have forgotten the Pythagorean Theorem. So let's go back and recall that the sum of the square of the sides of a right triangle is equal to the square of the hypotenuse.<br><br>You forgot the plus or minus with your answer. | Context Dependent |
| **Ignore** | Teacher does not address idea. It is usually followed by an elicitation for another idea without coming back to original idea. | Okay, does anyone else have another idea? | Context Dependent |
| **Evaluate** | Assessing a student's idea as correct or incorrect. | I disagree.<br><br>Good thinking. | Product |
| **Revoice/Evaluate** | Restating or paraphrasing a student's ideas for the purpose of evaluating it as correct or incorrect. | Twenty-five. Okay – that's fine. | Product |

*3.6.2: Analysis of enacted responding practices*

The purpose of this section is to describe techniques that I used to analyze both the baseline and action research videos to understand teachers' enacted practices of responding. In short, I used coding frames described in the previous section and summarized in Tables 4 and 5 to analyze video and transcript data. After applying this frame, I looked for changes to the types and focus of teachers' responding moves to understand teachers' enacted practices of responding.

Prior to analyzing any video from this project, I developed and refined the coding scheme using videos from other sources (i.e. TIMMS, various professional development resources) and describe that coding scheme more specifically now. To code a video, I began by transcribing the video. Next, I parsed the entire transcript according to the following four types of moves: structuring, soliciting, offering, and responding. These four moves were derived from Mary Budd Rowe's (1986) article *Wait time: Slowing down may be a way of speeding up!* In that work, Rowe describes the classroom as comprised of two players: the teacher and the students. Each of these players could, feasibly, have access to four types of moves. She defines the moves as follows:

> 1. Structuring: giving directions, stating procedures, suggesting changes
> 2. Soliciting: asking questions
> 3. Responding: answering solicitations, expanding on a structuring move, reporting data, or continuing a line of reasoning
> 4. Reacting: evaluating statements made by self or other player (p. 46).

Rowe shows how students are typically denied access to all moves except for responding. She shows increasing wait time is one way of opening up students' access to other moves.

In this study, I paid attention to what Rowe referred to as reacting, though I called it responding. However, I broadened reacting to include non-evaluative reacting as well. Because I broadened reacting to include non-evaluative moves, I decided to identify "continuing a line of reasoning" as one way of reacting to students' thinking. Therefore, I reshaped these categories into the following moves:

1. Structuring: giving directions, stating procedures, suggesting changes
2. Soliciting mathematical thinking: posing mathematical questions or tasks
3. Offering mathematical thinking: answering solicitations, offering thinking about solicitations, expanding on a structuring move, reporting data.
4. Responding to mathematical thinking: evaluative and non-evaluative responding to students' mathematical thinking

All research questions within this study are focused on the fourth category, responding to mathematical thinking. After I parsed the entire transcript according to these four categories, I focused on those instances of teacher responding - coding each according to the responding moves found in Table 4.

Once I developed and refined the coding schemes for the types of responding moves, I taught this coding scheme to another colleague, still using videos from outside of this study. After discussing this coding scheme to my colleague, I provided her with a video and transcript with instances of responding highlighted. She and I independently coded for the teachers' responding moves shown in Table 4. Last, we discussed the results and process of coding the video after both of us had completely

coded one fifteen-minute video. From this discussion, the original draft of the coding frame outlined in Table 4 was revised.

This process was iterated with two new videos (approximately 15 minutes in length) until together we reached a level of reliability of 84% agreement. Once we reached a sufficient level of reliability, I began the process of independently coding videos from the project's participants. After I completed the first participant, I selected a small portion (approximately 20%) of the participants' interactions for my colleague to code as well. These interactions came from different videos and were "de-sequenced" so the coder could not tell which ones came from the early baseline data or the later action research data. I checked my colleagues work against my own and found that we were still coding at about 76% agreement. I continued this process with the second and third participant. My colleague coded the second participant's data with 71% agreement and the third participant with 75% agreement. Since my colleague did not have access to knowing the actual order of the clips and is generally uninvolved in the results of the study, she provided a less biased perspective that enabled me to verify the reliability of my own complete analysis.

*3.6.3: Analysis of imagined responding practices*

In this section, I describe the methods I used to analyze the imagined practices survey (see appendix 1). After each survey administration, I de-identified the surveys and gave them to another party to transcribe. Next, I stored the original survey and its transcription, without further examination, until all three survey administrations were complete. This transcription and storage allowed me to not only de-identify but also

"de-sequence" the surveys. This "de-sequencing" was done to help reduce bias in my analysis caused by looking for change across the sequenced surveys. In other words, by not knowing which of the surveys came first, I limited my ability to see things in the data.

After all three surveys had been administered and transcribed, I began analyzing teachers' reactions to the second portion of the survey (questions 3a, 3b and 3c) to observe participants' responding practice in terms of breadth and focus. I remind the reader that in this portion of the survey participants were sharing their imagined responses to three different sets of hypothetical student thinking (incorrect, correct, and novel). As I looked at the surveys (in transcribed form) for instances of teacher responding, I coded each responding techniques for breadth using the responding moves outlined in Table 4 and for focus using Table 5. After the survey questions were coded, I explored differences in the ways that teachers responded to survey questions. Taking note of these differences, I formed hypotheses about which types of responding techniques represented expert practices and which represented more novice practices.

Crespo et al. postulated that teachers' responding moves would increase in breadth (2007). While looking over the "de-sequenced" surveys, I noticed differences in the types of moves participants used across these surveys. Two changes caught my attention specifically. The first was teachers were more evaluative in some surveys than others. Recall, that at this point, these surveys were "de-sequenced" and I did not know which one came first. I was taking note of these differences in order to try to blindly "re-sequence" the surveys based on what I was observing and expecting to happen with

teachers' practices. From this first difference, I posited that as teachers' responding practices grew from novice to expert, one could expect to see a move away from evaluative responding practices towards less evaluative responding practices. The second difference was teachers seemed to use *ignore* or *tell* more in some of the surveys while they favored *invitation* in others. From that observation, I postulated that teachers' shifting from novice to expert responding practices could be expected to shift from favoring the moves of *telling* or *ignoring* to techniques that involved *inviting* other students to comment or build on ideas offered by their peers.

Crespo et al. (2007) also postulated that as teachers' responding practices advanced the focus of their responding moves would shift from *product* to *process*. Therefore, before making final decisions about which order I thought the surveys went in, I looked at the focus with which teachers were responding and noticed differences across the surveys. Namely, in some of the surveys teachers seemed to be more *process-focused* and in other surveys they seemed to be more *product- focused*. Therefore, I formed my final hypothesis that participants would likely be making a shift from *product* to *process* as they progressed.

After forming these hypotheses, I used them to "re-sequence" the surveys for each participant. This blind "re-sequencing" of the surveys served as an important check of both validity and reliability of this portion of the study. One reason I included this blind "re-sequencing" of the surveys was to examine the validity of the hypotheses formed from my observations. A second reason for this deliberate "de-sequencing" and blind "re-sequencing" of surveys was to try to avoid bias created by seeing what I

wanted to see in the data thereby better ensuring reliability of my findings. Once I had "re-sequenced" the surveys, I compared my work with the actual sequence.

For each participant, the baseline survey, post-professional-development survey, and post-action-research survey were all blindly "re-sequenced". Using the hypotheses, I correctly placed seven of the nine surveys. The error was a reversal of one participant's second and third survey. The chances of this mostly correct placement happening randomly are 0.09% or 2/216. Calculation details for this percentage are described in more detail in appendix 2. In short, the hypotheses that I was using to place the surveys greatly informed my ability to place seven out of nine of those surveys in the correct order.

In summary, the imagined responding practices data was analyzed by first "de-sequencing" and de-identifying the surveys, analyzing the surveys for changes within responding practices, forming hypotheses about the noted differences, blindly "re-sequencing" the surveys according to the hypotheses, and comparing the blind "re-sequence" to the actual sequence of the surveys. The purpose of this analysis was to understand how teachers' imagined practices changed within the professional development experience with as little bias as possible.

### 3.7: Summary

In this chapter, I have discussed both the study and research design. In the discussion of the study design, I began by outlining how participants were selected. In short, the three participants were all secondary mathematics teachers from one district that were coming out of a previous year's work together in video clubs (that included

participation of other non-participants) where they learned to share their practices with one another. This work was birthed out of the teachers' request for more in-depth work on their classroom practices.

Continuing in the discussion about study design, I described how the professional development model grew out of ideas gathered from both empirical and theoretical research literature about professional development. Major components of the professional development were built around the notions that teachers needed activities to develop their imagined practices of responding (professional readings and watching various records of practices), a mechanism to reflect those imagined practices in order to decide which ones they considered significant and worthwhile (reflections and decision maps), and activities to reflect on and attempt to shift their enacted practices of responding (analyzing their baseline video and participating in a cycle of action research).

Next, I described and explained the research design. I began by describing the data I collected for the project and how that data was connected to the study's research questions about both imagined and enacted practices of responding. Finally, I concluded with a description of how the data was analyzed to understand how teachers' imagined and enacted practices of responding changed as they moved through this experience.

# Chapter 4: Analysis of Data

## 4.1: Introduction

The purpose of this chapter is to report the results of the research questions. I begin this chapter with a brief review of the problem and purposes that motivated this study. Next, I describe the organization of the data results. The remainder of the chapter is devoted to discussion of results from the study's three research participants.

### *4.1.1: Problem and Purposes Overview*

In the first chapter, I presented the problem being explored in this dissertation. In short, U.S. teachers typically fail to implement recommendations from NCTM (1991, 2000) to develop and maintain productive mathematical discourse. Responding through evaluation is particularly problematic and is the focus of this research. In this present work, I study a professional learning experience designed to help teachers change their responding to align with reform recommendations.

### *4.1.2: Organization of Results*

In the next three sections of this chapter (4.2, 4.3, 4.4) I present a case for each participant, Ann, Brenda, and Cecilia. Within each of these cases, I begin by describing the participant's background. Next, I examine changes within the participant's imagined practices, as measured by surveys. Following that, I explore changes made within the participant's enacted practices, as observed on video.

For each case, I use four graphs to consider either 1) changes from the baseline

data to post-professional-development (post-PD) data or 2) changes from the post-PD data to post-action-research (post-AR) data. When considering changes from the baseline to post-PD data, I first identify increases or decreases greater than 5%. I chose 5% because of the fact that there were up to ten total responding moves changing within a participants' practice. Since up to ten categories comprised 100% of an individuals practice, a 5% shift felt fairly large because of the relative size of these categories at any given point.

When discussing changes from the post-PD to post-AR data, I identify post-PD changes that are *strengthened, maintained*, or *reversed* through participation in the action research. I define *strengthened* as those changes that continue to grow in the same direction as observed in the post-PD data. In other words, teachers *strengthened* changes by furthering an increased or decreased use of a move by more than 5% as they moved through the action research.

I define *maintained* as those changes that participants sustained (former usage levels at plus or minus 5%) through the action research cycle. I define *reversed* as changes that participants made after the professional development which they abandoned as they progressed through the action research. In other words, changes that are *reversed* represent times a teacher initially increased or decreased the use of a responding move (by more than 5%) only to return to levels similar to the baseline data after the action research.

I use the distinctions of *strengthened, maintained*, or *reversed* because I think keeping an eye on the continuity of change across the year-long experience is more

interesting and important than simply stating whether moves increased or decreased within the action research. I believe noting changes that teachers furthered, sustained, or abandoned tells us something more about these teachers' journeys through the action research because it is connected to journey of change they began in earlier in the year.

Finally, I end the chapter (section 4.5) by examining commonalities across the three participants' data. In chapter 5, I draw conclusions about the two research questions, consider how my findings relate to others' research hypotheses, explore the implications of my findings, and make suggestions for future research.

## 4.2: Ann's Responding Practices Data

### *4.2.1: Background information about Ann*

The first participant, Ann, was a veteran high school mathematics teacher. At the time of this project, she was teaching Algebra 1 and Algebra 2. Over the course of her career, Ann has had 35 years of experience teaching every course from Pre-Algebra to Pre-Calculus. Ann decided to sign up for the professional learning team because she wants to finish her career strong. She loves teaching, but feels she still has much to learn.

In addition to this project, Ann was also completing an Online Masters in the Art of Teaching. She felt this project was helpful because it enabled her to experience some of the things she was reading about in her online courses. One of Ann's biggest frustrations was that her textbook materials and mathematics department were "traditional". Therefore her environment and colleagues were not supportive of the sort of reform that she was trying to enact. She also felt a frustration that students were

not able to communicate their thinking. She had come to believe that she needed to work to engage students in mathematical discourse daily but was not exactly sure how to do that.

At the conclusion of the first four sessions of professional development, Ann received a packet of data about her teaching that her research partner, Brenda, analyzed for her. Inside this packet, Ann noticed that she was reported to use evaluative responding 60% of the time. She identified this data as the primary frustration about her instructional practices that she wanted to change. With this frustration in mind, Ann decided that she would try to decrease her evaluative responding moves.

Across several conversations with Ann I recommended that she consider which responding moves she aimed to increase as she decreased *evaluation*. I had a concern that without keeping track of what was increasing, Ann might find herself just swapping one kind of evaluative move for another, such as *evaluative revoicing*. After those conversations, Ann adjusted her action research plan to include increasing her use of *invitation*, *uncover*, *revoice*, and *advance*. As she moved into the action research cycle, her primary point of focus was to decrease of *evaluation* and increase of non-evaluative moves, even though her stated goal included the specific increase of *invitation*, *uncover*, *revoice*, and *advance.*

Upon comparing Ann's goals with the professional development goals, I noticed a few differences. Ann's first written goal aligned exactly with the professional development goal of shifting away from evaluative responding moves towards non-evaluative responding moves. Ann's written goal of increasing *invitation, uncover,*

*revoice,* and *advance* was similar to the professional development goal of increasing the breadth of responding moves. Lastly, Ann's plan did not include a focus on shifting towards a more balanced focus between *product-* and *process-focused* techniques.

*4.2.2: Examining Ann's imagined practices data*

Here, I examine those findings that describe how Ann's imagined responding practices changed as she moved through the professional development and action research. The first finding is that Ann shifted the breadth of her imagined responding practices slightly from the baseline survey to the post-PD survey and post-AR survey. After participating in the professional development sessions, Ann increased her imagined use of *uncovering, advancing,* and *orienting* while eliminating *invitation, ignoring,* and *evaluating*. After participating in the action research, Ann *strengthened* her increased use of *orienting* and *maintained* her decision to eliminate *invitation* and *evaluation*. She also *reversed* her increased use of *uncovering* and *advancing* as well as her elimination of *ignoring*. The second finding is that Ann made dramatic shifts in the balance between *product-* and *process-focused* responding strategies. In Ann's baseline survey, she was fairly even-handed in her use of *product-focused* and *process-focused* moves. In the post-PD and post-AR survey, Ann was heavily focused on *mathematical processes*.

To put some context around these statements above, I consider an excerpt from Ann's surveys found in Table 5. On the far left side are the survey prompts. In the subsequent columns are Ann's baseline, post-PD, and post-AR survey responses to these prompts. Notice that the survey prompt includes a hypothetical student

presenting correct work on the front board. Ann was asked to imagine how she would respond to this student's work.

*Table 6: Excerpts from Ann's baseline, post-PD, and post-AR surveys*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Imagine you are going to ask you class to solve the following algebra problem**<br>**24 + 2x = 56 - 6x** | | | |
| **Imagine that a student shows the following strategy on the board.**<br><br>**24 + 2x = 56 - 6x**<br>**- 2x - 2x**<br>**24 = 56 - 8x**<br>**-56 -56**<br>**- 32 = -8x**<br>**4 = x**<br><br>**What is one thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?** | That is correct. | Did anyone solve it differently? I don't like to tell students right away if it's right or wrong because the thinking stops. Also, there are alternate ways to solve this. | How do we [know] 4 is the solution? I want to know if the student understands what it means to find a solution. Allows students to check using manipulation or graphing. |
| **How likely is it that you would respond in this way? Certain**<br>**Very Likely**<br>**Possible** | Very likely. | Very likely. | Certain. |

*Table 6: Excerpts from Ann's baseline, post-PD, and post-AR surveys (cont'd)*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Why?** | The answer is correct so I would not go over each step. | Students can see different strategies. Also just because one student got it right doesn't mean everyone did. | I have found that students assume if they have an answer it's correct. Want them to know what solving means. |
| **What is another thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?** | Can you tell me why you know it is correct? | How can you determine if your answer is correct? I want to know if students understand what is means to solve an equation. | Did anyone solve it differently and can show this method to the class? Some students may move variable to left, other may move [numbers] first, others may solve graphically. Want students to see there are many ways to get a solution. |
| **How likely is it that you would respond in this way?**<br>**Certain**<br>**Very Likely**<br>**Possible** | Very likely. | Very likely. | Very likely. |

*Table 6: Excerpts from Ann's baseline, post-PD, and post-AR surveys (cont'd)*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Why?** | I want them to be able to tell why they know x = 4 is the answer. | To [assess] if students understand what it means to solve. | Allows students to see that the order of adding opposite doesn't matter, can solve either side of equal sign and some students may use graphic methods. |
| **If you had to pick just one response, which of these would be most like what you can imagine saying next.**<br><br>**A) This is correct. It may have been easier if you had chosen to add the 6x instead of subtract the 2x.**<br>**B) Is there another way you could have done this?**<br>**C) Excellent, very nicely done.**<br>**D) What makes you confident your answer is right?** | C | B | D |

*Table 6: Excerpts from Ann's baseline, post-PD, and post-AR surveys (cont'd)*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Why would you say that?** | Because it is correct. I wouldn't [choose] A because I feel that criticizes the student and also [it is] a matter of opinion. That method made sense to that student's thinking. | I would want students to know there is more than one way to solve. I want students to use a method that is meaningful to them. | From their explanation I can determine if they understand what it means to solve. They may substitute 8 in or use a graph or use a table. So besides understanding solving. I'm also able to uncover different ways to check a solution. |

Across these survey excerpts, we begin to get a sense for changes to the breadth and focus of Ann's imagined practices of responding. In the baseline survey, Ann's first reaction was to *evaluate* the student's presented work with "That is correct." Ann's justification of this move reveals a *product-focused* stance as she saw no need to "go over each step" because "[the] answer is correct." Only when asked to consider an alternate response did she consider asking the student, "Can you tell me why you know it is correct?" Even in phrasing the *uncovering* move, Ann leaned towards *evaluation* as she revealed the answer was, in fact, correct. Ann's justification for this reaction was aimed at the student's mathematical *process* with, "I want them to be able to tell why they know 'x=4' is the answer". Her leaning towards *evaluation* is further confirmed when Ann selected "Excellent, very nicely done" as the most likely thing she could imagine saying. She revealed her *product-focus* through her justification of that selection, saying, "because it is correct." In this baseline data, Ann had reasonably heavy use of *evaluation* and a focus that included both mathematical *products* and *processes*.

In the post-PD survey, Ann's initial response to the student's thinking was to gather more solutions or strategies and avoid *evaluation* as evidenced by "I don't like to tell students right away if it's right or wrong". When we consider this quote in contrast with her decision to do just that on the baseline survey, the change happening in Ann's imagined responding practices begins to get clearer. When asked to consider an alternate responding technique, Ann focused on *uncovering* whether or not the student could justify her answer by asking "How can you determine if you answer is correct?" This time, however, the evaluative 'hint' implicit in the *uncovering* question from Ann's

baseline data was not present. Ann's *uncovering* here conceals whether or not she believes the answer is correct.

Ann's justification of the *uncovering* move therefore takes on a more *process-focused* stance with her explanation that she is interested in assessing "if students understand what it means to solve." Lastly, this shift away from using *evaluation* is confirmed as Ann selects "Is there another way you could have done this?" as her most likely response. Her justification is also more focused *process* than *product* with, "I would want students to know there is more than one way to solve. I want students to use a method that is meaningful to them." In this post-PD survey, Ann moves away from *product-focused* strategies and *evaluation* to favor an increased use of non-evaluative moves with a heavy focus on *mathematical processes*.

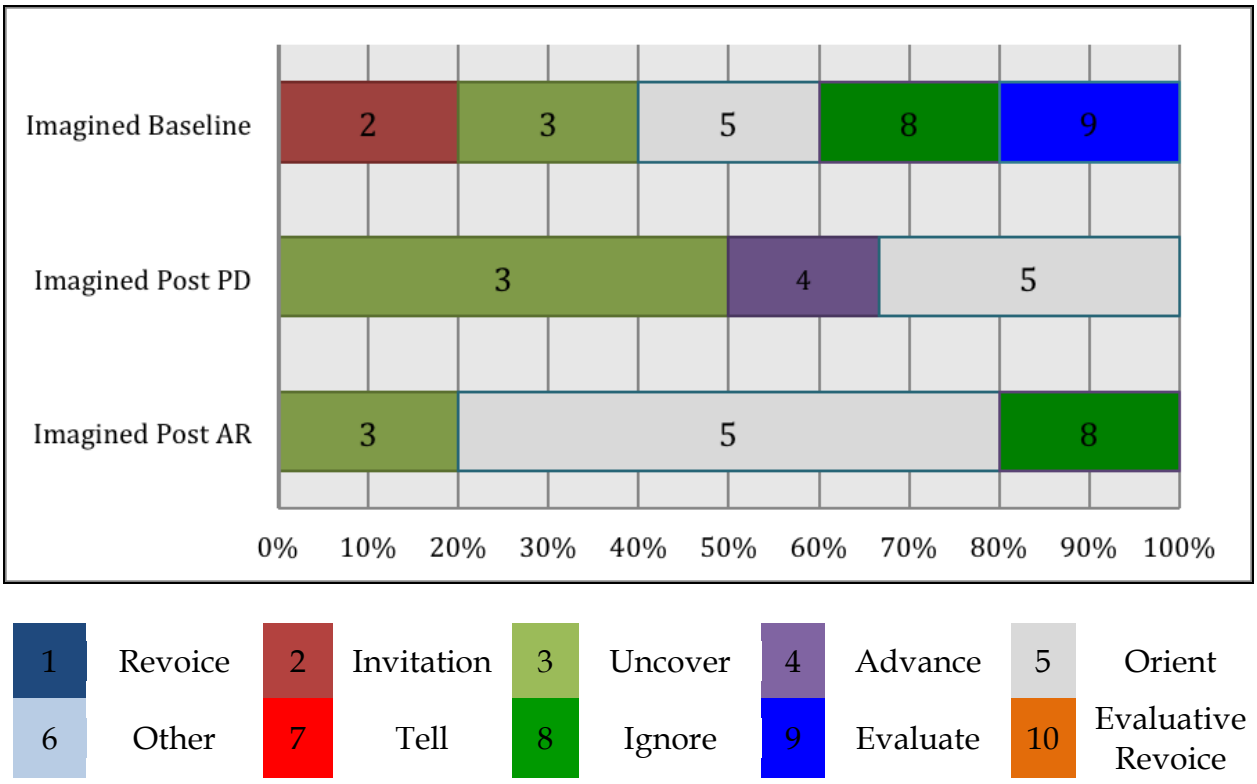
In the post-AR survey, Ann maintains some of these shifts away from *evaluation* and *product-focused* strategies. Ann's responses to the student's work on the post-AR survey are structurally very similar to her post-PD responses. Ann's responses to prompt one and two on the post-PD survey are almost identical to her responses to corresponding prompts on the post-AR survey. Ann's justifications for these responding moves are remarkably similar as well, emphasizing the importance of students' understanding of mathematics and students attending to multiple strategies in both surveys. These post-PD and post-AR justifications were more focused on *mathematical processes* than was evidenced in the baseline survey. For the last survey prompt, Ann selects a non-evaluative response of "What makes you confident your answer is right?" and focuses on *mathematical processes* - seeing this question as a

window into students' thinking.

The excerpts from these three administrations of the same survey help to illustrate that Ann was moving away from *evaluative* responding techniques in her imagined practices. She was not, however, increasing her use of new non-evaluative responding moves. Rather, she simply increased the same non-evaluative moves she was using in the baseline survey. She was also shifting away from a balanced focus between mathematical *products* and *processes* towards an increasing focus on mathematical *processes*.

The shifts in the breadth of Ann's imagined practices are summarized in Figure 1. Changes Ann made from the baseline survey to the post-PD survey data can be seen in the top two bars of the graph. Similarly, the bottom two bars demonstrate changes from the post-PD survey to the post-AR survey.

*Figure 1: The breadth of Ann's imagined practices of responding over time*




*Number of baseline moves = 5 Number of post-PD moves = 6 Number of post-AR moves = 5*

*For interpretation of the references to color in this and all other figures, the reader is referred to the electronic version of this dissertation.*

Although many aspects of Ann's imagined practices change, identifying which responding moves are increasing and which are decreasing over time clarifies the meaning of the picture. *Uncovering* had the largest growth, as Ann increased her imagined use from 20% to 50%. Ann also increased her use of *advancing* from 0% to 17%, and *orienting* from 20% to 33%. After participating in the action research, Ann *strengthened* her increased use of *orienting* to 60% and *reversed* her increased use of *uncovering* and *advancing*.

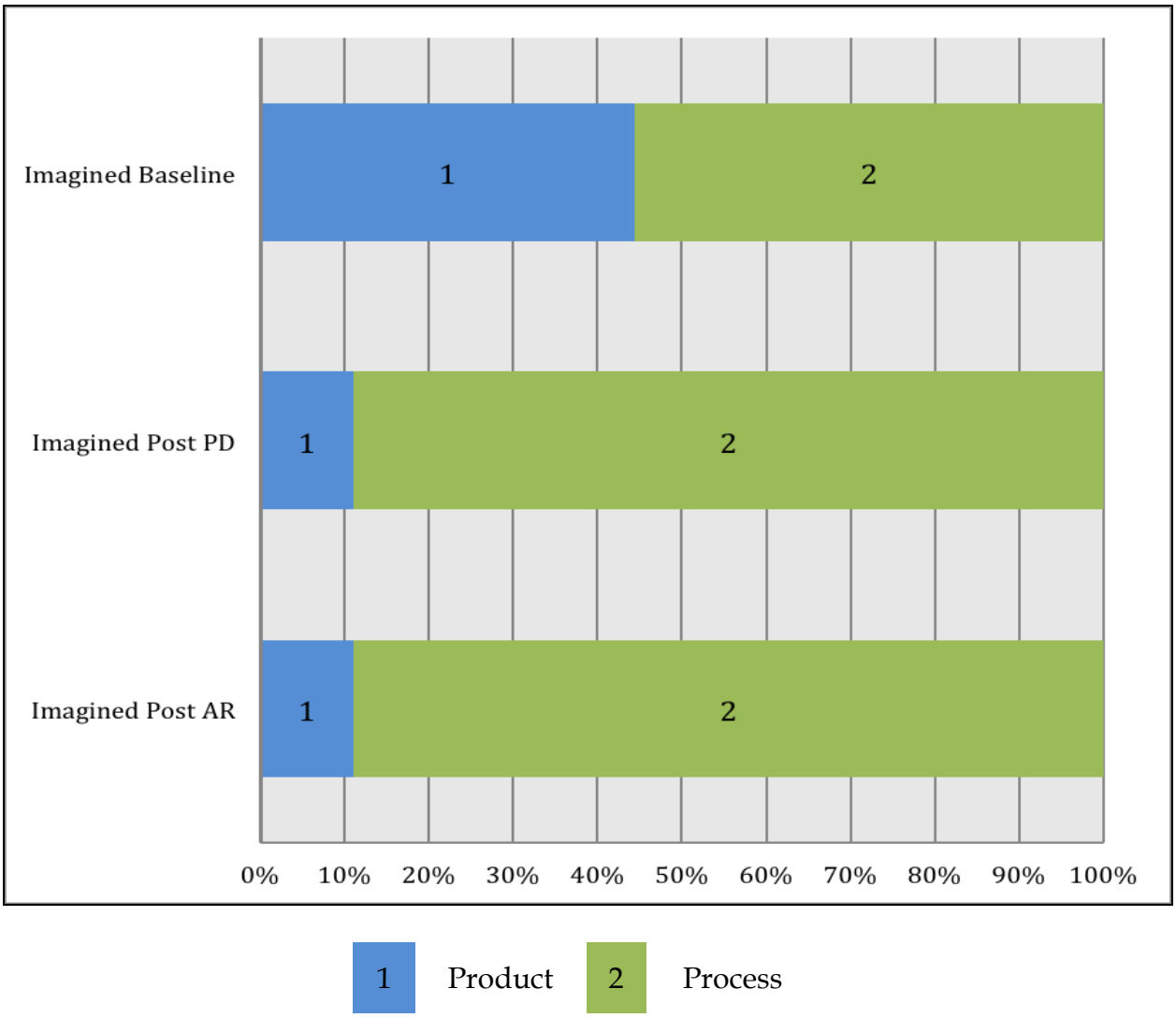
As Ann was increasing her imagined use of *uncovering, advancing,* and *orienting* in

the post-PD survey, she also eliminated *invitation, ignoring,* and *evaluating* from her imagined practices. In the baseline survey, each of these moves made up 20% of Ann's responses. After participating in the action research, Ann *maintained* her decision to eliminate *invitation* and *evaluation*. She *reversed* her elimination of *ignoring*, however, and returned to levels similar to the baseline data.

Upon comparing this imagined practices data with Ann's stated action research goals, we find that Ann accomplished her goal of decreasing *evaluation*, increasing *uncovering*, and increasing *advancing* following the professional development. At the conclusion of the action research cycle, however, the only imagined practice she *maintained* from her action research plan was to decrease *evaluation.* Ann's goal of increasing *invitation* was not realized either the post-PD or post-AR data, as she actually decreased and *maintained* her decreased use of *invitation*.

Shifts in the focus of Ann's imagined practices are summarized in Figure 2. In the baseline data, Ann focused on mathematical *processes* in 56% of her survey responses. On the post-PD survey, Ann used a *process-focused* technique 89% of the time. As Ann moved through the action research cycle, she *maintained* a strong *process-focused* stance in her imagined practices.

*Figure 2: The focus of Ann's imagined practices of responding over time*




*Number of baseline moves = 5 Number of post-PD moves = 6 Number of post-AR moves = 5*

Ann's action research goals did not include any statements about shifting from *product-* to *process-focused* strategies. There was, however, a professional development goal that was aimed at participants shifting from overly *product-focused* responding practices to a balanced focus between mathematical *products* and *processes*. When comparing Ann's changes to the professional development goal we see that Ann did in

fact shift away from *product-focused* strategies towards *process-focused* strategies. In doing so, however, she ended up with a less balanced responding practices that was heavily *process-focused*.

*4.2.3: Examining Ann's enacted practices data*

In this section, I focus on those findings that describe how the breadth of Ann's enacted practices changed. First, the breadth of Ann's responding practices changed dramatically from the baseline video to the post-PD video and she *maintained* most of these changes through the action research cycle. In the post-PD video, Ann increased her use of *invitation, revoicing,* and *uncovering*. She also decreased or eliminated *evaluative revoicing, telling, ignoring,* and *evaluating.* In the post-AR video, Ann *maintained* her increased use of *invitation* and *revoicing* and decreased use of *evaluative revoicing*, *ignoring,* and *evaluation.* The second finding is Ann made large shifts in the focus of her responding practices. In the baseline data, Ann's responding practices were heavily *product-focused*. In the post-PD and post-AR videos, Ann swings in the opposite direction with a heavy focus on *mathematical processes*.

To illustrate change in Ann's practices described above, I provide two portions of transcript from the beginning and end of Ann's year-long journey. First, to describe typical responding practices found in Ann's baseline data, I consider a portion of transcript from Ann's baseline video. This episode begins with a student, Anthony, sharing his thinking about a problem on the board. The problem is stated: "Solve [the] equation by completing the square: 4r^2 + 16r - 62 = 3."

1-2 **Anthony:** [Student gets up and comes to the front] "You want me to explain how I did it?

3 **Teacher:** Yeah, explain to the class how you did it.

4-5 **Anthony:** I didn't simplify it. I think it's right but . . . um, alright, I divided four.
[Puts a horizontal fraction bar under each term of 4r^2, 16r and 62, placing a four in the denominators]

6 So, I got r squared.

[Mumbles and writes on the board r^2 + 4r + ]

7 And I uh skipped a step

[Points at 62/4 and pauses and then turns to the teacher]

8 **Teacher:** Okay, well . . .

9-10 **Anthony:** What's…uh, negative sixty-four…uh, sixty-two divided by four?

11-12 **Teacher:** [Interrupting] It doesn't divide evenly. What about our properties of equality?

13 **Anthony:** Oh, it doesn't . . . I did it wrong?

14 **Teacher:** Well . . . properties of equality, you have to what?

[Wait time]

15 **Teacher:** Max?

16 **Max:** Whatever you divide one side by, you have to divide the other.

17 **Teacher:** Right, divide by one side, you have to divide by the other

[Anthony sits down as the teacher takes his place at the board. She erases his first step].

18 **Teacher:** Okay? What about something like this? Could I do this, do you think?

[writes 4r^2 + 16r = 65]

19 Can I do that?

[Wait time]

20 **Teacher:** Why can I do that?

[Wait time]

21 **Teacher:** Well I don't have to, but why can I do that? Is that legal?

22 **Student:** Yeah.

23-26 **Teacher:** It's legal because you add to one side what you did to the other. Okay, so I added sixty-two to both sides, okay. Now, instead of dividing, because you've got to divide all the way through if you're balancing I'm… I can factor.

[Writes 4(r^2 + 4r + ___) = 65+ ____ ]

27-34 Remember this? I did this trick the other day with you. This is okay. I can factor something outta here. Is this still equivalent to this? Okay? Do you see the difference between this and dividing through by four? And we can't pick and choose and divide what we want to divide by. No - not in an equation, right? But, factoring…it is just representing it in a different way. These two are still equivalent. Do you see that? They're still equivalent. Okay, anyone know what we're going to do next? Cause we're completing the square."

In this vignette, we see Ann interacting with Anthony as he is sharing his work on the board. At some point, it becomes obvious that Anthony is not using the expected strategy. He expresses uncertainty (lines 4-7) and Ann jumps in to *evaluate* his work (line 11-12). Anthony sheepishly reacts to this *evaluation* with "Oh, it doesn't . . . I did it wrong" (line 13). Next, she moves to involve another student, Max, in order to correct Anthony's work (line 15). Ann *evaluates* Max's thinking with "Right" and simultaneously *evaluates* Anthony's thinking by erasing his response (line 17). She continues to emphasize Max's work with an *evaluative revoicing* move stating, "Divide by one side, you have to divide by the other" (line 17). Anthony makes his way back to his seat as the teacher takes over the explanation and *tells* the class how to complete the problem (line 18-34), pausing once before completing the whole problem to be sure that

one student agrees, as indicated by "Yeah" (lines 20-22). All of this *evaluation, evaluative revoicing,* and *telling* leaves the classroom interaction with a rather unbalanced focus as the teacher drives the discussion towards the *mathematical product*. Ann's heavy use of *evaluation, evaluative revoicing,* and *telling,* in combination with overly *product-focused* techniques, have a way of steering the conversation in a direction that veers away from reform recommendations regarding mathematical discourse.

Now, contrast this interaction with a portion of transcript from Ann's action research cycle, nearly one year later. This episode begins with a problem on the board stated: "Simplify and find restrictions on variables."

$$\frac{(x + 5)\,(x + 1)}{(x + 1)\,(x - 3)}$$

A student, Christian, has already put his work up on the board. He has let the class know that although he knows how to simplify the problem, he is unsure about finding the restrictions on the domain. On the board he has written the following:

$$\frac{(x + 5)\,\cancel{(x + 1)}}{\cancel{(x + 1)}\,(x - 3)}$$

$$\frac{(x + 5)}{(x - 3)}$$

Initially he has stated he believes the restrictions to be $x = -5$ and $x = 3$. In other words, x cannot take on the values of either negative five or three.

1-2 **Teacher:** "Okay, so let's take this a little further. So, Christian - go on with what you were thinking now.

3-5 **Christian:** Um…well negative five plus five would cancel out to zero and

it's three minus three equals zero, but since you only need ... it's the bottom and you only find a restriction for that one.

6 **Teacher**: So, you want to change your answer now?

7 **Christian:** Just for the restriction to three.

8-14 **Teacher:** Okay, so the restriction is just three. Does anyone else have any opinions on that? Okay: so what Christian just said. Christian just said it's the denominator you have to look at. Okay, it's the denominator where the restrictions exist. We can get zero in the numerator no problem. It's a zero in the denominator. That's what Christian states. I want you all to look at this problem and…he is now saying this is the restriction.

[Erases -5, 3. Leaves 3 on the board]

15 How many of you agree with that? That that's the restriction.

[Wait time]

16 **Teacher:** Okay…don't be afraid to. The rest of you must disagree then.

[Wait time. Many students mumbling to one another quietly]

17-19 **Teacher:** Garima, do you have any thoughts on this? What do you think? Cause I hear you talking so I think you got something. You're thinking about something.

20 **Garima:** Umm…I don't really know; I kind of think so, but …

21 **Teacher:** Well tell me.

22-23 **Garima:** Well, based on the other problem you did, I kind of think so, but then, yeah, I don't know…

24 **Teacher:** You think its just three or you think its something else?

25 **Adam:** I think it is.

26 **Teacher:** You think it's three. Okay, Adam?

27 **Adam:** I think its three and negative one.

28 **Teacher:** Okay, why do you think its negative one?

29 **Adam:** Cause…uh, the first part of the denominator is x plus one. [inaudible]

30 **Teacher:** What do you guys think about that?

31 **Christian:** That was already cancelled out.

32 **Coe:** Yeah, but so was the three x minus one and the x plus four.

[Motioning to the last problem discussed where the class concluded with the following written on the board:

$$\frac{\cancel{(x+4)}\,(x-2)}{\cancel{(3x-1)}} \quad . \quad \frac{(2x+4)\,\cancel{(3x-1)}}{\cancel{(x+4)}}$$

$(x - 2)(2x + 4)$

Restrictions: $x = -1/3$ , $x = [-4]$

33-35 **Teacher:** Good point, Coe. Good point. I like that you guys are looking at the other ones there to draw some conclusions. So what do you think, Coe? What's your opinion on this?"

This vignette begins with Ann interacting with a student, Christian, who has shared his work on the board. Christian, like Anthony, has something incorrect about his work. This time, however, Ann lets him finish his work on the board, waiting to address his error with the class through a discussion. This restraint shows a shift away from a focus on *product* and toward *process*. Christian gets to talk about his ideas (lines 3-7) and Ann *invites* other students to react to those ideas by asking, "Does anyone else have any opinions on that?" (lines 8-9). When no one responds to Christian's thinking, Ann decides to *revoice* his statement, this time in a non-evaluative fashion with, "Christian just said it's the denominator you have to look at. Okay it's the denominator where the restrictions exist. We can get zero in the numerator no problem. It's a zero in the denominator. That's what Christian states" (lines 9-16).

The students are still hesitant, so she *invites* a specific student, Garima, who seems to be processing with her partner - "Garima, do you have any thoughts on this? What

do you think? Cause I hear you talking so I think you got something" (lines 17-19). With that *invitation*, Garima shares her thinking and notices differences in the way restrictions are handled in another problem (lines 20-23). With that start, another student, Adam, builds on Garima's idea without being *invited* by the teacher (line 25). Ann *revoices* his idea with "You think it's three. Okay, Adam?"(line 26) at which point Adam adjusts the teacher's *revoice* by adding that he thinks the restrictions are both three and negative one (line 27). Ann then makes an *uncovering* move with "Okay, why do you think it's negative one?" (line 28).

Adam justifies his thinking about negative one - explaining that x plus one from the original problem should be considered (line 29). Once Adam finishes his justification, Ann turns back to the class with an *invitation,* "What do you guys think about that?" (line 30). The conversation comes back to the original student, Christian, who is still unsure about including negative one in the list of restrictions (line 31). Without being *invited*, another student, Coe, adds his thinking to the conversation by agreeing with Adam and Garima (line 32). He combines these two students' ideas suggesting the original problem be used to identify restrictions since that follows the process from a previously completed example. Ann responds by *commenting* on students' interaction, rather than *evaluating* the idea. She says "Good point, Coe. Good point. I like that you guys are looking at the other ones there to draw some conclusions. So what do you think, Coe? What's your opinion on this?" (line 33- 35) The decisions to turn this idea back over to the students to discuss and restrict her *comments* to the ways the students are interacting as opposed to the correctness of the solution is a step

in the direction of valuing the *mathematical processes* rather than the *mathematical products*. The conversation continues like this for a few more turns until finally the teacher asks Coe put a final word on what the class had learned; namely, when finding restrictions one must look to the original problem.

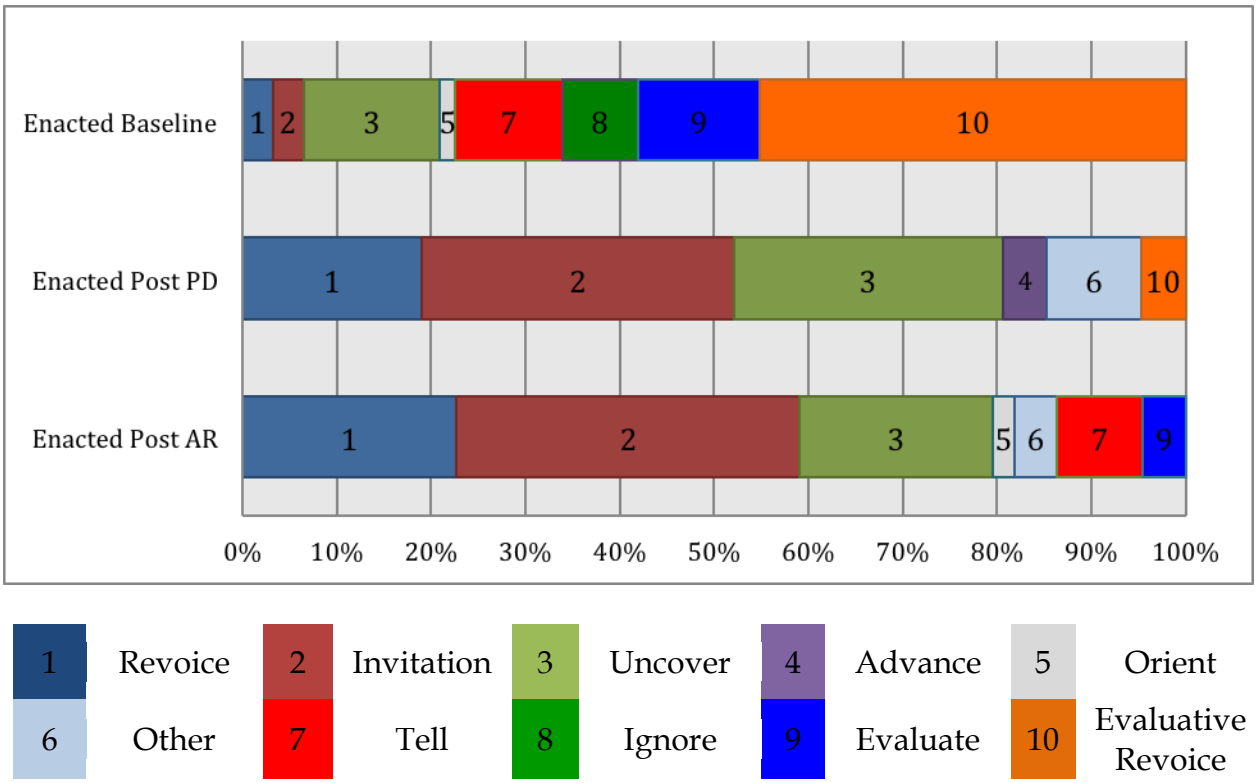
This interaction provides an illustration for shifts within Ann's responding practices across this year-long experience. Some of that shift is attributable to Ann's increased use of non-evaluative responding moves, specifically *revoicing, invitation*, and *uncovering.* An increased focus on *mathematical processes*, rather than *mathematical products*, also plays an important role in changing the interaction in Ann's classroom. These two vignettes typify Ann's baseline and action research practices of responding respectively. Next, I consider a summary of Ann's enacted practices of responding for both breadth and focus.

Figure 3 summarizes shifts in the breadth of Ann's responding practices. Most prominent was Ann's increased use of *invitation* from 3% to 33% after her participation in the professional development. Ann also increased her use of *revoicing* from 3% to 19% and *uncovering* from 15% to 29%. In the post-AR videos, Ann *maintained* her increased use of *invitation* and *revoicing* while she *reversed* her increased use of *uncovering.*

The largest decrease in Ann's responding practices was in her use of *evaluative revoicing,* from 45% in the baseline to 5% in the post-PD data. After the professional development, Ann also eliminated *evaluating, telling*, and *ignoring* which were formerly used 13%, 11%, and 8% respectively. In the post-AR video, Ann *maintained* her

decreased use of *evaluative revoicing* and elimination of *ignoring*. Ann did not *maintain* her elimination of *evaluating*, but did *maintain* her decreased use of *evaluating* from 13% in the baseline data to only 5% use in the post-AR data. Lastly, in the post-AR video, Ann *reversed* her decreased use of *telling* as she returned to levels of use similar to what was observed in the baseline video.

*Figure 3: The breadth of Ann's enacted practices of responding over time*



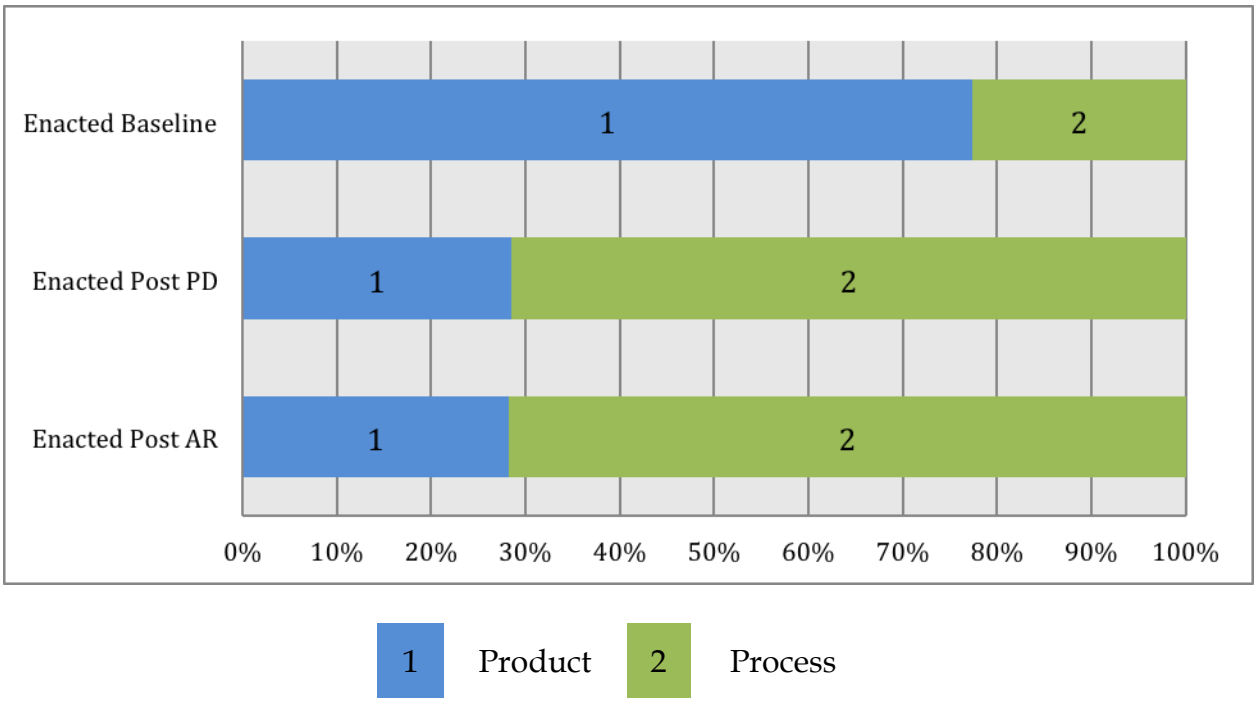

*Number of baseline moves = 62 Number of post-PD moves = 21 Number of post-AR moves = 44*

When this data is compared to Ann's action research goals in the post-PD data, Ann accomplished her goal of decreasing *evaluation*, increasing *revoicing*, increasing *invitation*, and increasing *uncovering*. Further, at the conclusion of the action-research-cycle, Ann accomplished her goal of *maintaining* a decreased use of *evaluation*, increased

use of *revoicing*, and increased use of *invitation*. Ann did not realize her goal of increasing *advancing* as she only increased its use from 0% in the baseline data to 5% in the post-PD data, only to revert back to 0% in the post-AR data.

In Figure 4, shifts to the focus of Ann's enacted practices are summarized. Within this summary of Ann's enacted focus data, we see she shifted from *product-* to *process-focused* strategies. In the baseline video, *process-focused* moves represented only 22% of Ann's overall responding strategies. After the professional development, Ann was using *process-focused* strategies 71% of the time. Ann's *maintained* her new focus on *mathematical processes* as she moved through the action research cycle.

*Figure 4: The focus of Ann's enacted practices of responding over time*




*Number of baseline moves = 62 Number of post-PD moves = 21 Number of post-AR moves = 44*

Upon comparing this finding with the professional development goal, Ann accomplished the professional development goal in part. Ann shifted away from an

overly *product-focused* responding practice. However, this shift landed Ann in a place that was heavily *process-focused*, rather than balanced between *products* and *processes*. Recall, this was true for her imagined practices data as well. More discussion about this comparison is found in chapter 5.

## 4.3: Brenda's Responding Practices Data

### *4.3.1: Background information about Brenda*

The second participant, Brenda, was a K-8 mathematics teacher. At the time of the project, she had recently been transferred to a middle school mathematics classroom after spending over a decade working as an elementary school teacher. Overall, Brenda had 14 years of teaching experience across many subject areas, from preschool to Grade 6. Brenda signed up for this professional learning opportunity because of the recent shift in her position. She wanted to learn more about how to apply what she knew about teaching all subjects in the elementary school to teaching mathematics in the sixth grade.

While taking this course, Brenda made a decision to pursue her Educational Specialist in Mathematics Education at a local university because she had gained an interest in research about teaching and learning mathematics. Brenda's greatest frustration within teaching was that the middle school's math textbook was "traditional" and differed dramatically from the elementary school's National Science Foundation (NSF) text she had been using previously. In the year prior, Brenda obtained permission to use a middle school NSF textbook that her middle colleagues had previously used, but abandoned in their last textbook adoption.

After her participation in the professional development, Brenda's teaching partner provided her with data on various aspects of her teaching. In this feedback, Brenda noticed three noteworthy pieces of data. The first data was that Brenda's students were only taking 35% of the turns in discussion, while Brenda was taking the remaining 65%. Further, Brenda was disturbed that her students only held 15% of the total discussion time, while her turns comprised 85%.

These two statistics provided an image of Brenda's classroom that she was not satisfied with. So she went to look for more evidence in her instructional practices that might be responsible for this imbalance. The third piece of data Brenda noticed was she rarely used *invitation* as a responding technique. Brenda reasoned that if she wanted her students to talk more and respond more to one another's ideas, she would need to begin asking them to do so. With these three pieces of data in mind, Brenda set two goals for her action research 1) to increase her use of *invitation* and 2) to increase her students' contribution.

When considering how Brenda's goals align with the professional development goals for responding, some similarities and differences emerge. The first goal of increasing *invitation* in particular is very specific compared to the professional development goal of increasing the breadth of non-evaluative responding moves. Brenda's goal is written as such because she felt *invitation* would play an important role in accomplishing her desired changes. So in this way, Brenda was aiming at increasing the breadth of her responding by adding one specific move, namely *invitation*.

Brenda's second goal is connected to ideas from the interpreting session. In that

session, teachers were to become more aware of the importance of providing students with more opportunities to access roles typically dominated by the teacher. Brenda's plan of *inviting* students to respond to one another's ideas as a means to engage them in dialogue, demonstrates Brenda's growing awareness of a connection between these ideas. I do not discuss Brenda's progress towards this goal as I proceed, mainly because this goal was not connected to topics I was studying. The reader may be interested, however, to know Brenda reported success in accomplishing this goal. She shared with our group that the percentage of discussion given for student turns was 15% in the baseline data, 58% after the professional development, and 47% after the action research cycle.

Lastly, two professional development goals were not included as part of Brenda's action research goals: 1) a shift away from an overly evaluative stance; 2) a shift away from a heavy *product-focus* toward a balance between *product-* and *process-focused* responding practices. Neither of these goals would have been appropriate for Brenda since she was neither overly evaluative nor heavily *product-focused* in her enacted baseline data. So it makes sense that neither of these goals were part of what Brenda chose to focus on.

*4.3.2: Examining Brenda's imagined practices data*

In this section, I consider changes to Brenda's imagined responding practices. The first finding is Brenda dramatically shifted the breadth of her imagined responding practices from the baseline to the post-PD. She *maintained* some of those changes while *reversing* others on the post-AR survey. From the baseline survey to the post-PD

survey, Brenda increased her use of *uncovering* and *invitation* while eliminating *orienting*, *telling*, and *ignoring*. In the post-AR survey, Brenda *maintained* her increased use of *invitation* and decreased use of *ignoring* and elimination of *telling*. She *reversed* her increased use of *uncovering* and elimination of *orienting*. The second finding is that Brenda made a large shift in the balance between her focus on *mathematical products* versus *processes*. Brenda's baseline data reveals that she leaned toward a *product-focus*; but following the professional development, Brenda focused heavily on *mathematical processes*. Brenda continued that shift, as she was even more *process-focused* following the action research cycle.

Survey excerpts in Table 6 help one to see the shifts in Brenda's imagined practices. Notice the survey prompt includes a hypothetical student presenting incorrect work on the board. Brenda is charged with considering how she would respond to this student's work.

*Table 7: Excerpts from Brenda's baseline, post-PD, and post-AR surveys*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Imagine you are going to ask you class to solve the following algebra problem**<br>**24 + 2x = 56 - 6x** | | | |
| **Imagine that a student shows the following strategy on the board.**<br><br>**24 + 2x = 56 - 6x**<br>**+ 6 + 6**<br>**30 + 2x = 56**<br>**-30 -30**<br>**2x = 16**<br>**x = 8**<br><br>**What is one thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?** | Did you substitute 8 for x? If not, try it to see if it works. If so, was the equation balanced? If needed, substitute 8 for x and see if it works. When it doesn't, ask for [students'] opinions as to why. What should we have done differently? I would want students to see where the mistake was made and learn from it. | Did anyone solve the problem in another way? Call on another student to come up to the board to share their strategy for solving. Then ask the class do you agree or disagree with these strategies? Why? Hopefully it will lead to a discussion about the misconceptions above (if the 2nd student solved correctly) | One thing I may do is ask if anyone solved the problem another way. Have that student share their strategy. Then have a discussion with the class about which solution is correct. (Hopefully this student solved it correctly) We would also discuss what went wrong in the problem above. I would also give this student the chance to correct his/her errors. |
| **How likely is it that you would respond in this way?**<br>**Certain**<br>**Very Likely**<br>**Possible** | Very likely. | Certain. | Very likely. |

*Table 7: Excerpts from Brenda's baseline, post-PD, and post-AR surveys (cont'd)*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Why?** | I think it is important for students to reflect on their own learning rather than me saying right or wrong. | Because I usually (when appropriate) stress that there is more than one way to solve a problem. Even though this one is wrong it can lead to a good discussion and hopefully clear up the misunderstanding above. | Because as the discussion ensues, this student would hopefully see where he went wrong without me being evaluative. |
| **What is another thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?** | Did anyone solve this problem a different way? Have students share alternate ways of solving. By doing this I would hope that the first student would see what he/she did incorrectly and be able to learn from it. | I can imagine students jumping in and being evaluative at which point I would ask one of them to come up and explain how he/she solved it. (As described on the previous page). | Invite other students to get involved in the conversation by asking if anyone agrees or disagrees with the way the problem was solved. Students who agree or disagree would have to justify their responses by showing or explaining how they solved the problem. |
| **How likely is it that you would respond in this way?**<br>**Certain**<br>**Very Likely**<br>**Possible** | Possible. | Possible. | Very likely. |

*Table 7: Excerpts from Brenda's baseline, post-PD, and post-AR surveys (cont'd)*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Why?** | Same as previous page. | [Relies] on the students' evaluative comments which I can see happening but no guarantees. | Because it gets the students involved in the discussion rather than being teacher directed or evaluative. |
| **If you had to pick just one response, which of these would be most like what you can imagine saying next.**<br><br>**A) That is incorrect. Can you find your error?**<br>**B) Tell me about what you were trying to do when you added 6.**<br>**C) This is incorrect because you can't add 6x to 24.**<br>**D) How do you know eight is the solution?** | B | B | B |
| **Why would you say that?** | Understand the students' thinking will help (hopefully) to address misconceptions. I think once the student realizes - $6x + 6 \neq 0$ he/she will be able to correct his/her mistake. | To uncover student thinking. | To uncover student thinking. It will help me to see what the misconception is. It may also help the student to self correct. If this does not happen it may lead to a whole class discussion to solve the problem correctly. |

Across these three surveys, Brenda's changes are less dramatic than Ann's. By looking closely, however, we can see slight changes. In the baseline survey, Brenda's reaction to the first prompt was to *tell* the student a solution to try, "Did you substitute 8 for x? If not, try it to see if it works." When asked to justify that decision, Brenda responded with, "I think it is important for students to reflect on their own learning rather than me saying right or wrong." Brenda's justification of her decision is *product-focused,* aiming at having the student determine the correctness of his solution, though Brenda is hoping that students would come to that realization independent of her.

In the second survey question, Brenda imagined responding to the student's work by re-soliciting. Re-soliciting can be technique for getting multiple strategies in place or generating discussion amongst students with different answers. In this case, however, there is no hint of coming back to the original student's work so this re-solicitation serves to *ignore* the original student's work. Brenda's hope, that shifting to another student's work would ensure "the first student would see what he/she did incorrectly," was a *product-focused* justification. In the final survey question, Brenda chose an *uncovering* strategy with a justification that was *product-focused* in nature. In the baseline survey, Brenda had a fairly large use of *ignoring* and *uncovering* and a smaller use of *invitation, orienting,* and *telling*. Brenda also frequently justified her decisions in terms of a focus on *mathematical products.*

In the post-PD survey, Brenda chose to re-solicit frequently with questions like "Did anyone solve the problem in another way?" Brenda used the same re-soliciting technique in her baseline survey; but post-PD, she re-solicited with intentions of coming

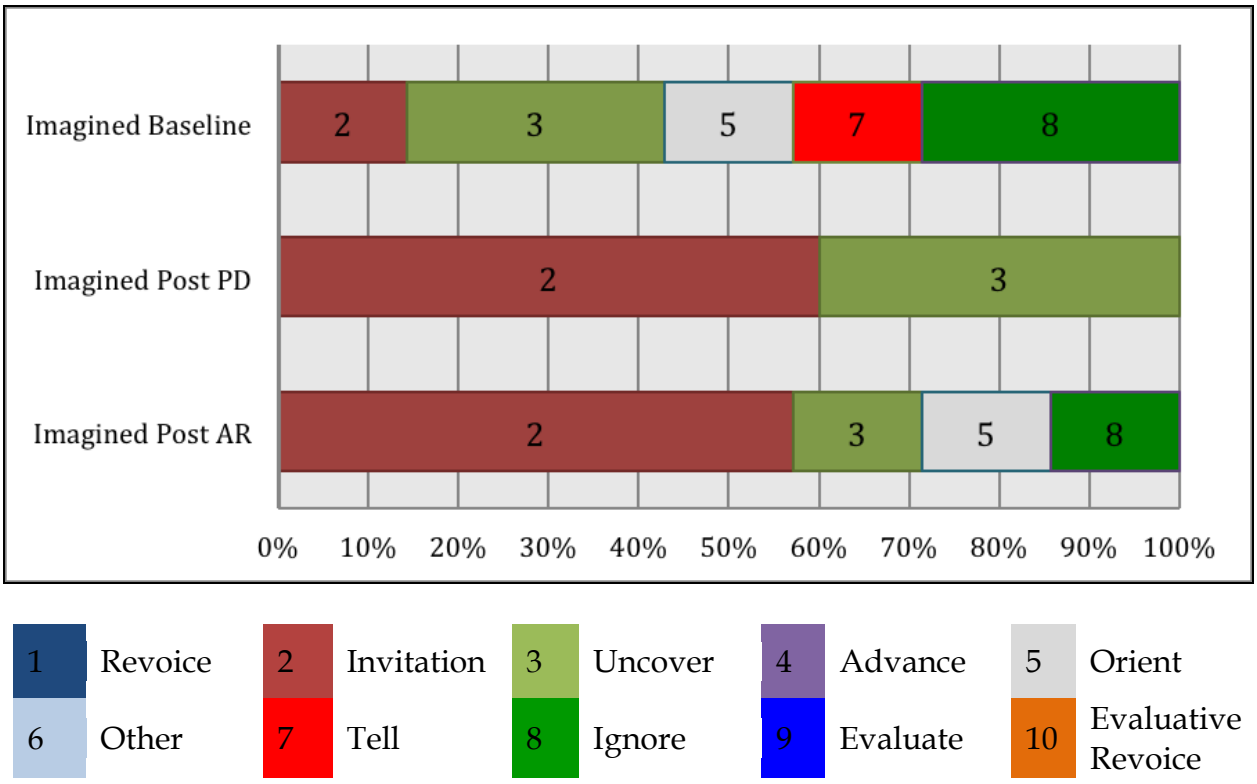
back to the original student's work. She explained how after she re-solicited, she planned to *invite* others to evaluate both students' work presented side by side. Brenda's justification for this decision to re-solicit reveals a focus on *mathematical processes* as she explains, "even though this one is wrong it can lead to a good discussion." In the second survey prompt, Brenda imagines her students taking initiative to respond *evaluatively* to the original student's work. She describes how she would welcome initiative by *inviting* students to share their work as justification for their *evaluation*. In the last survey question, Brenda selects *uncovering* again, as she did on the baseline, but her justification is *process-focused* as she emphasizes the importance of understanding students' thinking. In this second survey, we see slight shifts in Brenda's responding practices as she uses *invitation* and *uncovering* exclusively while all other moves are abandoned. A shift toward *mathematical processes* is also evident when one examines Brenda's justification for her decisions.

In the post-AR survey, Brenda maintains a heavier use of *invitation* and *process-focused* strategies. However, she abandons her increased use of *uncovering* in favor of bringing back *orienting* and *ignoring* in part. The responses to the first two survey prompts reveal Brenda's steadfast use of *invitation* as Brenda aims to get "other students to get involved in the conversation by asking if anyone agrees or disagrees." Brenda's justifications for those two questions are very much about getting "students involved in the discussion," as opposed to focusing on errors or obtaining answers. In the final survey prompt, Brenda chooses to *uncover* the student's thinking. Her justification, though, takes on a *product-focus* stance, mentioning a need for students to move toward

correctly solving the problem.

These three excerpts from Brenda's survey are intended to help describe changes she made. We see slight changes in Brenda's increased use of *invitation* and *uncovering*. We also see how Brenda's justification changed ever so slightly from *mathematical products* to *mathematical processes*, as her mention of solution correctness tapers and the importance of student understanding grows.

*Figure 5: The breadth of Brenda's imagined practices of responding over time*




*Number of baseline moves = 7 Number of post-PD moves = 5 Number of post-AR moves = 7*

Figure 5 summarizes changes in the breadth of the imagined practices data for Brenda. Within this graph, we see Brenda's use of *invitation* and *uncovering* grows considerably. Brenda imagined using *invitation* 14% of the time in the baseline survey

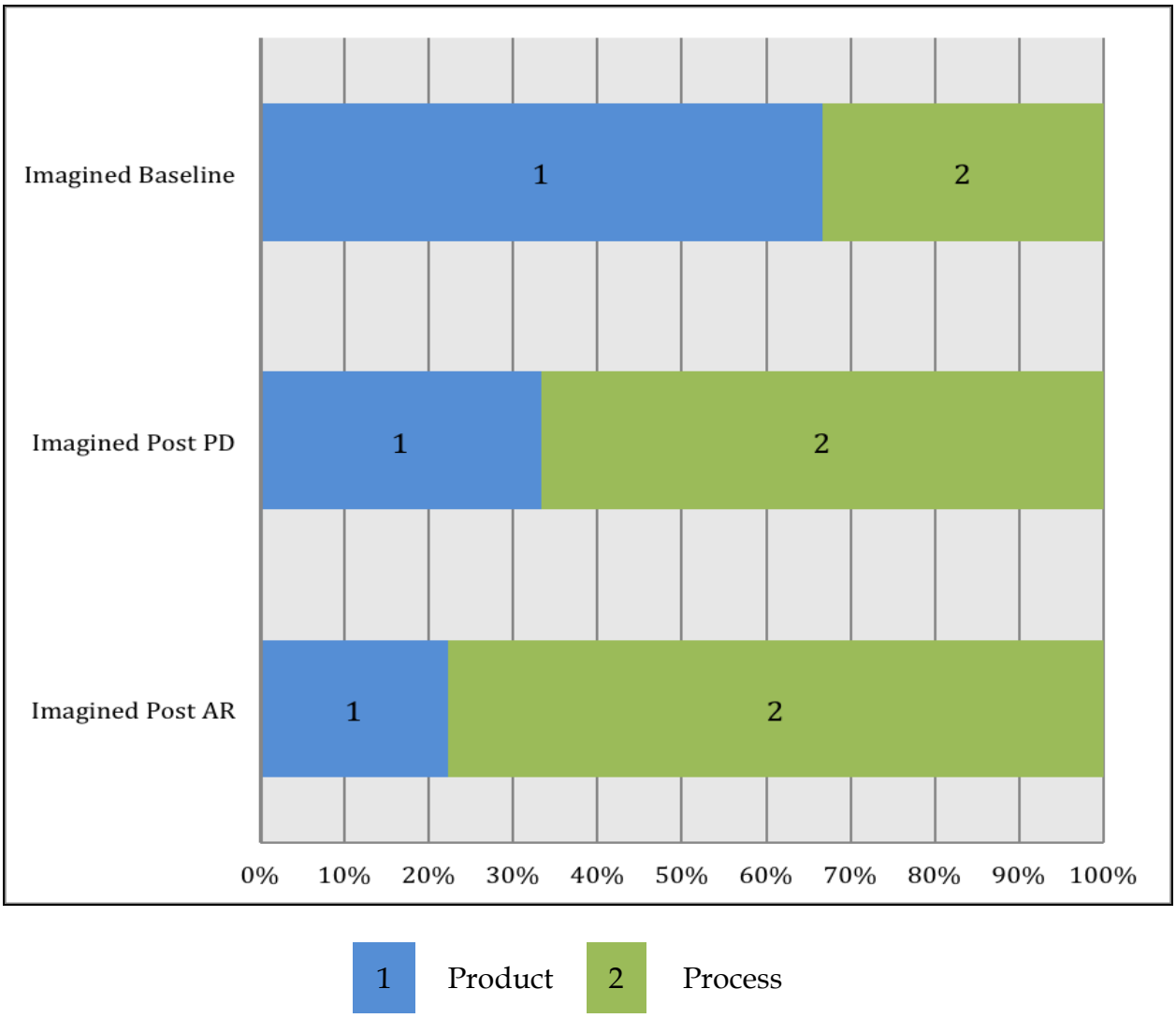
and 60% of the time in the post-PD survey. Similarly, Brenda imagined herself using *uncovering* only 29% of the time on the baseline survey and 40% of the post-PD survey. In the post-AR survey, Brenda *maintained* her decision to use *invitation* more but *reversed* her increased use of *uncovering*. In fact, Brenda did not only *reverse* an increased use of *uncovering*; her post-AR use of *uncovering* was at an all-time low of 14%.

Just as Brenda was shifting toward an increased use of *invitation* and *uncovering* on the post-PD survey, she surrendered her use of *orienting*, *telling*, and *ignoring*. In the baseline data, *orienting* and *telling* each represented 14% of the survey's responses while *ignoring* represented 29%. After the action research cycle, Brenda *maintained* her decision to eliminate *telling* and *maintained* a decreased use of *ignoring* of 14%. She *reversed*, however, her decision to eliminate *orienting* and restored its use to 14%.

When comparing Brenda's action research goals to changes in her breadth, we see Brenda accomplished her goal of increasing her use of *invitation,* both after the professional development and after the action research cycle. In fact, this change represents the single largest change in Brenda's imagined practices.

Changes Brenda made within the focus of her imagined practices are summarized in Figure 6. The data shows an obvious shift toward *process-focused* strategies. Brenda used *process-focused* responding techniques 33% of the time in the baseline survey and 66% of the time in the post-PD survey. Brenda *strengthened* that shift towards more *process-focused* strategies as she moved through the action research cycle with a 78% focus on *mathematical processes*.

*Figure 6: The focus of Brenda's imagined practices of responding over time*




*Number of baseline moves = 7 Number of post-PD moves = 5 Number of post-AR moves = 7*

Brenda did not include any goals that address shifting from *product-* to *process-focused* teaching within her action research plan. When considering the professional development goal, however, Brenda was successful at shifting away from a heavier *product-focus* stance to a more *process-focused* stance after the professional development. As she *strengthened* that resolve to include even more focus on *mathematical processes*

through the action research, I would note that at some point Brenda becomes unbalanced, as she is heavily *process-focused*. More discussion about shifts that Brenda made in her imagined practices of responding is found in chapter 5.

*4.3.3: Examining Brenda's enacted practices data*

In this section, I discuss how the breadth of Brenda's enacted practices changed. First, the breadth of Brenda's responding practices changed dramatically from the baseline to post-PD video. As she moved through the action research cycle, Brenda *maintained* some and *reversed* many of those changes. After the professional development, Brenda increased *invitation* and *uncovering* and decreased *revoicing, evaluation, telling* and *evaluative revoicing*. After the action research cycle, Brenda *maintained* her increased use of *invitation* and her decreased use of *telling*. She ultimately *reversed* all other post-PD changes by the end of the action research cycle. In the post-AR data, *ignoring* also grew beyond Brenda's baseline and post-PD use. The second finding is Brenda shifted the focus of her responding practices only to *reverse* that shift in the action research cycle. In the baseline data, Brenda's responding practices were fairly balanced between *mathematical products* and *processes*. Brenda was heavily *process-focused* after the professional development but she swung back and achieved greater balance after the action research.

I provide two portions of transcript to provide a clearer picture of Brenda's enacted changes. To demonstrate Brenda's typical baseline responding practices, I examine a transcript from her baseline video. This interaction starts with Brenda asking her students to consider how to fold a strip of paper - a fraction strip - into five equal

lengths in order to create fifths. Prior to this interaction, Brenda's students were working on creating fraction strips folded into thirds, sixths, twelfths, fourths, and eighths. A student, Aaron, offers his method of creating fifths for consideration by his classmates.

1 **Teacher:** "Alright, how about those fifths? That's the tricky one.

[Wait time, several students raise their hands]

2 **Teacher:** It's easy? Okay, Aaron, tell me how easy it is.

3 **Aaron:** I…I did I fold it up until I folded it up four times.

4-5 **Teacher:** Okay, but how did you know? Can I see yours? Okay, so Aaron took his…oh… but did you rip part off?

6 **Aaron:** Yeah.

7-9 **Teacher:** Ohhhh. [Brenda's face indicates sympathy] You know Aaron, that was a good strategy because they're even. But now if we hold it up to our other fraction strip, is it the same size?

[Student nods head "no" and puts his head down].

10 **Teacher:** No.

[Brenda's expression indicates sympathy].

11-14 **Teacher:** No. GOOD thinking though! You fold it. My question was going to be, how did you know you were going to end up at the end and it was going to be the right size? Well you didn't… it wasn't the right size was it?

15 **Aaron:** [Holds up his fingers to demonstrate]. It was a little bit left.

16 **Teacher:** It was a tiny bit left, so you just ripped it off?

17 **Aaron:** [Aaron nods his head "yes"].

[A couple of students giggle about the interaction]

[Aaron smiles sheepishly and then puts his head down on his arm]

18-19 **Teacher:** But now Aaron, what's our target? Why are we making fraction strips?

20 **Aaron:** We want to get our goal.

21 **Teacher:** To get our goal, which is what?

22-23 **Aaron:** [Reads from the board] To order all of the rational numbers and whole numbers on a number line.

24-27 **Teacher:** Yeah. And if we, if our fraction strips aren't the same size, when we go to put them on a number line… this is fifths… so if I wanted to put two fifths on a number line but my paper is not all the same size, am I going to put it in the right spot on a number line?

28 **Aaron:** No.

29-30 **Teacher:** No. No, that won't work will it? Good thinking though, Aaron. Clever. Anybody, working on the fifths, have another idea?"

In this interaction, Brenda is interacting with Aaron's idea about how to create fifths on his fraction strip. As Aaron talks about his idea, we see he understands the need to create five equal parts (line 3). Brenda makes a move to *uncover* his thinking by asking him, "How did you know, can I see yours?" (line 4) and in so doing, focuses on his *mathematical process*. When it becomes clear that Aaron simply ripped part of the fraction strip off to achieve his five parts (lines 5 - 6), Brenda moves to *evaluate* Aaron's thinking with "No. Good thinking though" (line 7-11) Aaron was not clear about the problem's constraints, namely the final strip of paper should be the same length strip as he started with. In light of Aaron's lack of clarity, Brenda emphasizes the *mathematical product* with *evaluation*, though remains somewhat balanced with her compliment on his thinking *process*.

Brenda proceeds to *tell* Aaron that he cannot tear the extra off of the fraction strip and emphasizes the need for this constraint by *orienting* Aaron toward the lesson's goal (lines 18-19). Rather than asking Aaron to connect the goal and constraint, Brenda

decided to *tell* how the *process* of tearing the fraction strip would not satisfy the goal of creating a *mathematical product* that would be useful to mark fractions on a number line (lines 24-27). Ann's baseline responding practices were spread evenly between *revoicing, uncovering*, *telling, evaluating,* and *evaluative revoicing.* This interaction illustrates how Brenda's use of *telling* and *evaluating* act as obstacles toward building a richer discourse where students take center stage. The interaction demonstrates Brenda had a fairly balanced focus between *mathematical products* and *processes*.

Contrast that portion of transcript from Brenda's baseline data with this excerpt taken from Brenda's action research cycle. This episode may appear to come from the same class period, as students are talking about fraction strips again. This interaction, however, happens approximately one year later with a new set of students working on the same task. In this transcript, students have just finished creating fraction strips for halves, fourths, thirds, and eighths. Brenda and her students have just finished labeling the eighths strip and she is asking her students a question about how this strip compares with other strips they have made.

1-2 **Teacher:** "Does anybody notice anything about those [inaudible] when you compare it to your other fraction strips?

3 **Dylan:** They go by twos.

4 **Teacher**: They go by twos. Yep, they do. What do you mean by two?

5 **Dylan:** Uhh…I don't know.

6 **Teacher:** What do you mean? Dylan: go ahead.

7 **D'wan:** They're all equivalent [inaudible]. They all multiples.

8-9 **Teacher:** Do you mean the two, the four and the eight - the denominators - are all multiples of two? Is that what you were trying to say D'wan?

10 **D'wan:** Yeah.

11-14 **Teacher:** Okay. That's true. What do you notice when you compare them to your other to fractions strips? And if you don't have yours out, I did draw mine on the board. They're not as evenly-spaced as yours but do you notice anything when you compare them?

[Wait time. Inaudibly speaking to a student near by. Picks up student's strip and is talking to him.]

15 **Teacher:** What do you notice? Justin, what do you notice?

16 **Justin:** The lines that line up are equivalent?

17 **Teacher:** Okay Justin, tell me two lines that line up that are equivalent?

18 **Justin:** One-fourth and two-eighths.

19 **Teacher:** How do you know they're equivalent?

20 **Justin:** Cause, one times two is two, and four times two is eight.

21 **Teacher:** Okay, that's…David?

22-23 **David:** I have something that each time you goes down the fraction strip you add two lines.

24 **Teacher:** What do you mean, David?

25 **David:** On the first one, you have a half that and two on the side of it.

26-27 **Teacher:** Okay. Okay, so you're saying we started with one and then we got two more. Okay.

[Wait time. Waiting for student to continue. Another student speaks inaudibly]

28 **Teacher:** Go ahead, Shelby.

29-33 **Shelby:** Every time we do it, you double it, like one-half and then one-half times two is two-fourths, and two-fourths times two is four-eights cause you double it every time. So like for the…for one-half when you go to one-fourth you add two lines then when you go down you add four lines. And then like next time you add, like . . .

34 - 35 **Teacher:** We don't have eight lines, but we do have eight equal sections, don't we…

36 **Shelby:** If you add…you double the sections.

37-39 **Teacher:** We double the amount of parts we divided in the two. Anybody have anything else they'd like to add? Or another idea on what you noticed?

[Wait time]

40 **Student:** They're all even numbers?

41 **Teacher:** What are even numbers?

[inaudible]

42 - 43 **Teacher:** Okay, the denominators - the two, the four and the eight - are all even numbers. Anything else you noticed?

[Wait time]

44 - 45 **Teacher:** Does anybody notice any other equivalent fractions up there? Dylan?

46 **Dylan:** Three-fourths and six-eighths

47 **Teacher:** Go ahead, Shelby.

48-50 **Shelby:** Everyone… where there is a line beneath it they're equivalent like one-fourth is equal to two-eighths and three-fourths is equal to six-eighths.

51 **Teacher:** And what about two-fourths?

52 **Shelby:** They're equal to [inaudible] eighth.

53 **Teacher:** And what about one-half?

54 **Shelby:** They're equal to two-fourths and four-eighths."

In this vignette, we watch Brenda interacting with several students around their observations about these four fraction strips students just created. Dylan starts by sharing his observation that "they go by twos" (line 3). Dylan's contribution is difficult to understand, so Brenda *revoices* his contribution (line 4) and follows up with an *uncovering* move (line 4). Dylan is uncertain how to further clarify his thinking (line 5) but Brenda persists (line 6). D'wan comes to Dylan's rescue by sharing his thoughts about the equivalence of the fractions and multiples in the denominators (line 7). Brenda, again, *revoices* and *uncovers* by asking D'wan, "Is that what you were trying to say D'wan?" (lines 8 – 9). D'wan gives her a short affirmation (line 10). Though the students are not terribly eloquent about their thinking, Brenda is consistently focusing on students' *process* by asking them to clarify their thinking.

Brenda, likely noticing that all observations seemed to be about one strip rather than comparisons of multiple strips, *reorients* students by restating the original solicitation (lines 11 - 15). At this point, Justin shares an observation that some of the lines across the strips seemed to be lining up (line 16). Brenda makes another *uncovering* move with "Okay Justin, tell me two lines that line up that are equivalent?" (line 17) and Justin provides an example (line 18). Brenda uses *uncovering* again to have Justin justify why "they're equivalent" (line 19). Justin justifies his claim about equivalent fractions explaining, "Cause, one times two is two, and four times two is eight" (line 20). This repetitive use of *uncovering* shows that Brenda decided to tune

into students' thinking *process* rather than hunting for an answer.

Just as Brenda was responding to Justin's idea, David jumps in noticing something new about how folding causes the number of sections to double (lines 22 - 23). Brenda *uncovers* (line 24) and David clarifies (line 25). Brenda *revoices* David's idea and then waits for others to consider (lines 26 – 27). Shelby attempts to combine Justin's idea about equivalent fractions and David's idea about doubling sections (lines 29 – 33). When she combined the ideas, however, Shelby began talking about David's ideas as doubling the number of lines rather than sections. Brenda responded quickly to Shelby with, "We don't have eight lines, but we do have eight equal sections, don't we…?" (lines 34 - 35). Though Brenda's *evaluation* may have been focused on correctness, something about *telling* Shelby she was counting sections, not lines, feels important. This common misconception that students have about counting lines as opposed to spaces is surely familiar for Brenda who is a middle school teacher. Attending to this misconception makes sense to ensure that everyone is attending to the same thing. Brenda's decision to *tell* in this instance feels very much like an attempt to focus on *mathematical processes.*

As Shelby sums up what she was sharing (line 36), Brenda *invites* others to add onto this idea (lines 37 – 39). Another student adds his observation about the denominators of the fraction strips all being even numbers (line 40). Brenda *revoices* this observation (line 41). She repeats her *invitation* to add onto Justin's idea about equivalent fractions (lines 42 - 45). Dylan shares an additional equivalent fraction of three-fourths and six-eighths (line 46). Shelby moves to add on to Dylan's idea and

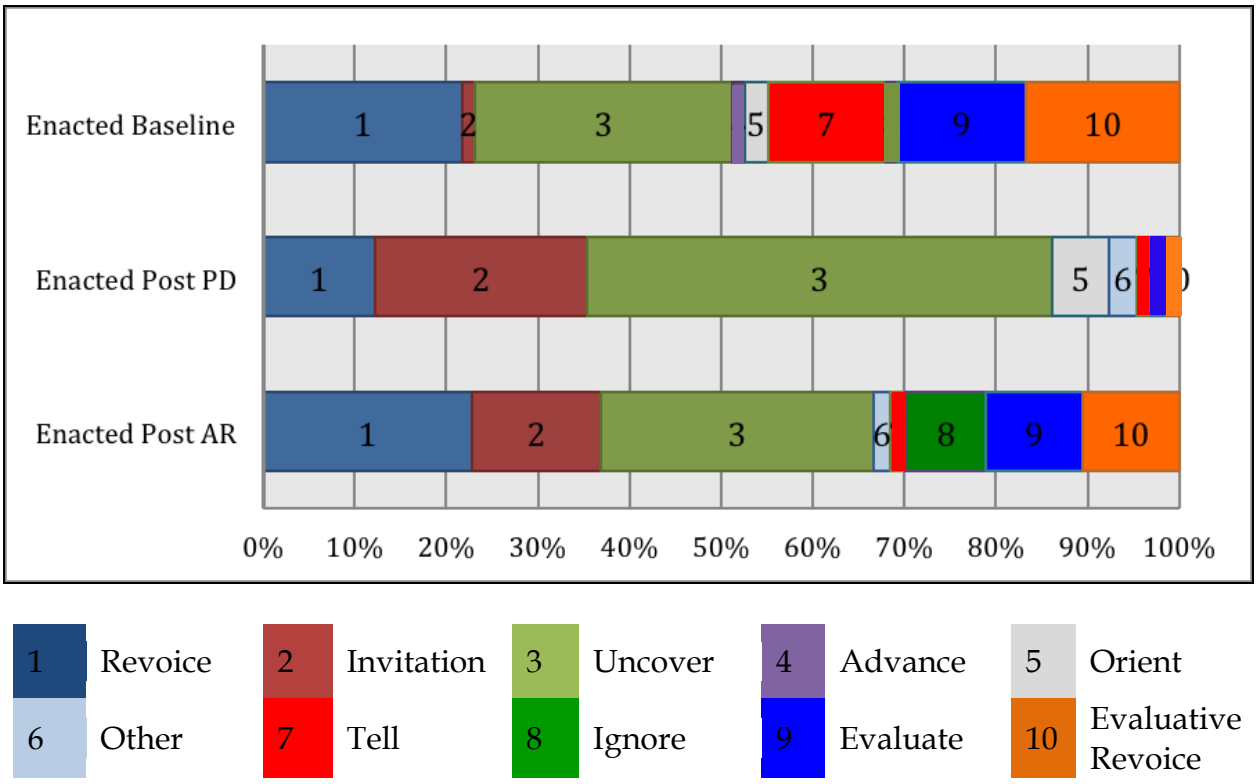
Brenda encourages her to share (line 47). Shelby exclaims, "Everyone… where there is a line beneath it they're equivalent like one-fourth is equal to two-eighths and three-fourths is equal to six-eighths" (line 48-50) and in so doing summarizes the idea that has been building across this interaction. Brenda moves to *uncover* Shelby's understanding by asking her, "And what about two-fourths?" (line 51). Shelby answers correctly (line 52). Brenda continues to push with another *uncovering* question about a different strip, halves, that has not been discussed yet (line 53). Similar conversation continues until the hour expires.

This interaction provides a typical look at how Brenda's practices shifted. Brenda's new use of *invitation,* increased use of *uncovering* and *revoicing,* and reduction of *telling* and *evaluating* in this scene made a world of difference in opening interaction to other students. One obvious change, though not part of my study, was the students' interaction in the second vignette was much greater than in the first vignette. This shift is not surprising given Brenda's action research plan focused on increasing students' contributions. In the first vignette, Brenda was talking with only one student while other students looked on. At times in the first vignette, students were even invited to share playfully in laughter about Aaron's creative methods of ripping the extra bit off the fraction strip. Brenda's baseline data did not have any instances of *inviting* students to respond intellectually to Aaron's ideas, however. In this second vignette, we see students listening, building upon, or adding onto one another's work as Brenda *invites* them to respond.

Finally, Brenda placed a greater emphasis on *mathematical process* in this second

interaction. In Brenda's shifted focus, however, there is arguably less balance between *process* and *product*. This imbalance may be somewhat responsible, though, for her students' apparent excitement about sharing their ideas. More discussion about the balance between *mathematical processes* and *products* is found in chapter 5. For now, it is enough to highlight these two vignettes as images useful for understanding Brenda's changing practices. Now, I turn to consider the summary of Brenda's enacted practices.

*Figure 7: The breadth of Brenda's enacted practices of responding over time*




*Number of baseline moves = 80 Number of post-PD moves = 68 Number of post-AR moves = 57*

Figure 7 summarizes the breadth of Brenda's enacted practices. After participation in the professional development, Brenda increased her use of *invitation* and *uncovering*. In the baseline video, Brenda used *invitation* 1% of the time while her post-PD video included 22% use of *invitation*. Brenda used *uncovering* 28% of the time

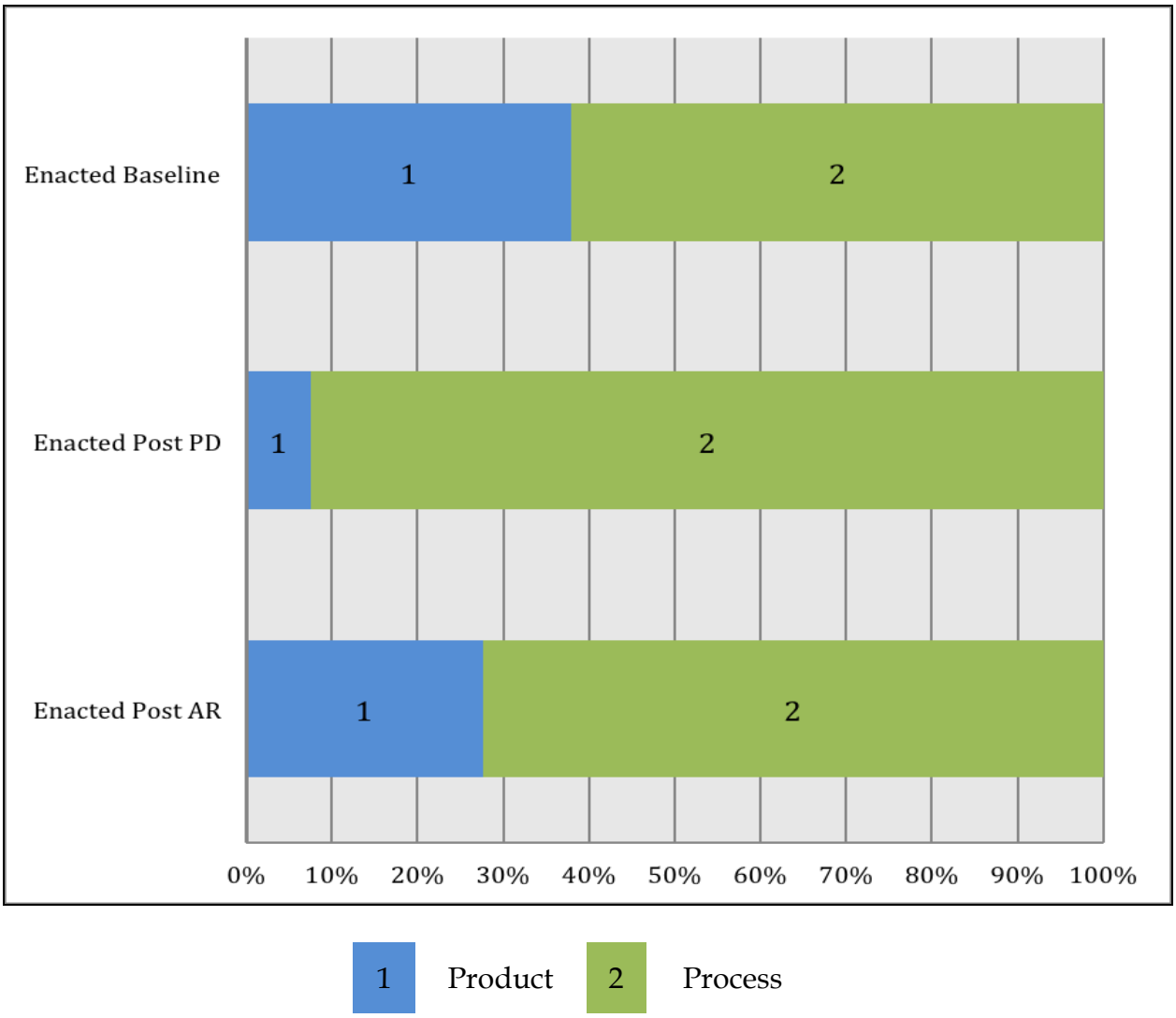
in the baseline and 49% of the time in the post-PD video. I would argue that Brenda *maintained* an increased use of *invitation* in her post-AR video, even though this maintenance is not entirely evident from Figure 7. Though Brenda's use of *invitation* drops from 28% to 14% across the action research, 14% use is still quite more frequent than was observed in the baseline video. For this reason, I argue that Brenda *maintained* an increased use of *invitation* through the action-research cycle. Figure 7 demonstrates clearly, however, that Brenda *reversed* her increased use of *uncovering* as she returned to 30% use in the post-AR data.

While Brenda was increasing *invitation* and *uncovering* in the post-PD video, she was also reducing or eliminating *revoicing, telling*, *evaluating*, and *evaluative revoicing.* Brenda reduced her use of *revoicing* from 21% in the baseline video to 12% in the post-PD video. In the baseline video, Brenda used *telling*, *evaluating*, and *evaluative revoicing* 13%, 14%, and 16% respectively. After the professional development, all of these moves were nearly eliminated, with each representing only 1% of the teacher's responding practices. Brenda was able to *maintain* a decreased use of *telling.* Brenda *reversed* her decisions to reduce *revoicing*, *evaluating*, and *evaluative revoicing* as her use of those moves returned to within 5% of the original use in the baseline data.

Finally, Brenda's use of *ignoring* in the post-AR video does not fit the categories of *maintaining, strengthening,* or *reversing.* Brenda hardly used *ignoring* in the baseline video (slightly more than 1% of the time) and eliminated its use after the professional development. This shift was too small to call this a considerable decrease. However, in the post-AR video, Brenda increased her use of *ignoring* to 9%.

When we compare Brenda's action-research goals to her data, we see she accomplished her goal of increasing *invitation* after the professional development. As Brenda moved through the action research cycle, her level of use of *invitation* dropped a bit, but she still *maintained* a much higher use than was seen in the baseline data. When considering the professional development goal of reducing *evaluation,* we see that Brenda accomplished this goal after the professional development, though this was not something she had aimed at in her action research. She did, however, *reverse* this decrease in *evaluation* after the action research cycle.

*Figure 8: The focus of Brenda's enacted practices of responding over time*




*Number of baseline moves = 80 Number of post-PD moves = 68 Number of post-AR moves = 57*

Figure 8 summarizes the focus data for Brenda's enacted practices. Brenda made a noteworthy shift towards *process-focused* strategies after participation in the professional development moving from 61% use in the baseline video to 92% use in the post-PD video. Following the action research, however, Brenda *reversed* that increased focus on *mathematical processes* with 72% use in her post-AR video.

Brenda partially accomplished the professional development goal related to focus by shifting toward a focus on *mathematical processes*. She was not, however, overly focused on *mathematical products* to begin with. So the post-PD shift in Brenda's practices left her in a place where she was less balanced between *products* and *processes.* After the action research, Brenda's practices ultimately return to levels more similar to where she began, and with this in mind we might argue this shift actually leads her back toward balancing *processes* and *products* in her responding practices.

## 4.4: Cecilia's Responding Practices Data

### *4.4.1: Background information about Cecilia*

The final participant in this study, Cecilia, is also a secondary mathematics teacher of 16 years. Within that 16 years, Cecilia taught middle school for two years; served as a mathematics specialist and supervisor for four years with Project SEED, a non-profit mathematics program that services a large urban school district; and spent the remaining ten years teaching high school math at her present building. A few years prior to this project, Cecilia had rejoined the high school staff after a six-year family leave enabled her to stay at home with her three small children. Before her extended leave of absence, Cecilia's specialty was teaching Geometry, Honors Geometry, and remedial courses. During her leave, remedial and honors courses had been eliminated as part of the No Child Left Behind legislation and another teacher had been hired to teach Geometry. Cecilia found it necessary to assume a role of teaching Algebra 1 and corresponding support courses. She felt she had a lot to learn about how to teach these new courses, but also felt a need to sharpen her instructional skills after years out of the

classroom. Cecilia's biggest frustration was the demands of a rigorous and overwhelming curriculum as well as a need to balance this curriculum with her commitment to students.

At the conclusion of the professional development, Cecilia's teaching partner, Ann, provided her with data on various aspects of her teaching. Cecilia was dissatisfied with the teacher-to-student-turn ratio reported in her packet. The ratio demonstrated Cecilia took three times as many turns as her students. She was also frustrated that her average turn-length was two and a half times longer than student's. Finally, her frustration was enhanced with the discovery that 79% of the discussion was devoted to her turns. Cecilia was set on changing her way of handling discussions. She therefore began writing her action research plan to "decrease the ratio of teacher turns to student turns."

As I sat down with her to discuss her action research plan, I expressed my concern that her plan did not include anything about her directly nor did it include changes she envisioned making that would decrease this ratio. She described how she simply needed to talk less. I suggested that talking less might not be enough to get students to talk more or differently about mathematics. Cecilia agreed and said she was planning on using different responding techniques to get her students more engaged. At this point, we turned to the list of responding moves that the group had generated in the responding session and talked about which moves she was planning on using more. Cecilia identified *invitation*; however, she was resistant to add this to her action research plan. She did not want to shift her focus from the number of turns each party, teacher

and students, was contributing.

I cautioned that simply leaving her statement, "to increase my use of *invitation*" as a strategy rather than a measureable goal, she would not have any data to prove whether or not she had truly increased her use of *invitation*. At this, Cecilia looked perplexed. I explained that if at the end of the action research cycle, she found out that her ratios or turn lengths had not changed much, she would be left wondering why. "Since there are two parties that impact the ratio," I explained, "you could end up saying, 'well, I know I *invited* the students more, but they did not cooperate.' The truth might be that you did not actually *invite* your students more, but rather just thought you did."

I felt it was important that Cecilia's action research plan be focused on measuring her own practices as opposed to practices that she and her students both have responsibility for. Like Ann, Cecilia ended up amending her action research to include some analysis of her practices as well. In our group conversations, however, Cecilia focused almost exclusively on progress she made toward her written goal about decreasing the ratio of teacher turns to student turns.

Upon comparing Cecilia's goals with those of the professional development for responding, we see some alignment. Cecilia's first goal of decreasing the ratio of teacher turns to student turns was not addressed in the responding session. Rather, that goal is more closely aligned with ideas from the interpreting session. These goals are still important in many ways because Cecilia was growing in awareness of her students' need to be heard. Again, because these ideas were not central to this study, I

do not discuss this goal from Cecilia as I move through the data. I will, however, quickly share data with respect to this goal that Cecilia reported as part of her action research project. The ratio of teacher to student turns was 3.4 to 1 in Cecilia's baseline data, 1.1 to 1 in her post-PD data, and 1.1 to 1 in her post-AR data. Similarly, the ratio of Cecilia's turn-length to her students' moved from 2.52 to 1 in the baseline data, 0.48 to 1 in her post-PD data, and 1.2 to 1 in her post-AR data. Cecilia was therefore successful in reducing and maintaining a reduced ratio of teacher-to-student turns to a point that she and her students were taking equal turns with approximately equal length.

Cecilia's second goal of increasing *invitation* was, like Brenda's, very specific when compared with the professional development goal of increasing the breadth of her non-evaluative responding moves. However, in her case a focused effort on increasing *invitation* made good sense given her desire to increase student sharing.

Finally, Cecilia's action research plan did not include two professional development goals. Specifically, Cecilia's plan did not include a stated effort to shift away from an overly evaluative or *product-focused* stance. Cecilia's enacted responding practices were fairly balanced in terms of her focus on *mathematical products* and *processes*, so it made sense for her to not include a goal about this. The breadth of her enacted practice, however, did include a fairly large percentage of *evaluative* moves. Therefore I comment on this goal as I move through the data, even though it was not a specific focus for Cecilia.

*4.4.2: Examining Cecilia's imagined practices data*

In this section, I examine how Cecilia's imagined practices of responding shifted. First, the breadth of Cecilia's imagined practices changed dramatically after the professional development. Some of those changes were *strengthened* or *maintained* as she moved through the action research, namely an increased use of *invitation* and *uncovering* as well as an elimination *evaluation* and *advancing*. There were also decisions that Cecilia made after the professional development that she ultimately *reversed* after participation in the action research. These include her increased use of *ignoring* as well as her decreased use of *orienting*. Second, Cecilia shifted her imagined practices toward a more *process-focused* stance after the professional development and *strengthened* that shift as she moved through the action research cycle.

To illustrate these changes Cecilia was making to her imagined practices, I include excerpts from her three surveys in Table 8. Notice in this excerpt, Cecilia is asked to imagine how she might respond to a hypothetical student's work that is both novel and correct. It is also important to note Cecilia's responses on the first two surveys make it evident that she believes this work to be incorrect. Cecilia's responses on the last survey make it less clear whether or not she came to understand this work as mathematically correct.

*Table 8: Excerpts from Cecilia's baseline, post-PD, and post-AR surveys*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Imagine you are going to ask you class to solve the following algebra problem**<br>**24 + 2x = 56 - 6x**<br><br>**Imagine that a student shows the following strategy on the board.**<br><br>**24 + 2x = 56 - 6x**<br>**12 + x = 28 - 3x**<br>**12 + 4x = 28**<br>**4x = 16**<br>**x = 4**<br><br>**What is one thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?** | Wow … You got me on this! 4 works … but the math is incorrect!! ☹ …<br><br>Well, I would ask the class what they think … ask them to look at the steps … see if we can see how the student got the numbers!! Mmm! | Did anyone do this problem a different way? – [Please] show on the board – Then have both [students] explain what they did each step. | What do you guys think? Agree … disagree? |
| **How likely is it that you would respond in this way? Certain**<br>**Very Likely**<br>**Possible** | Very likely. | Very likely. | Very likely. |
| **Why?** | This is tough for me … I would probably point out the math mistakes. | | See what class thinks … have them discuss. |

*Table 8: Excerpts from Cecilia's baseline, post-PD, and post-AR surveys (cont'd)*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **What is another thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?** | I would bring up the point that even if the number works when we check it … the math still has to be accurate … | Could you explain to us? What you did each step & why, then see if students can find the error. | How could we check to see if [it's] correct? |
| **How likely is it that you would respond in this way?**<br>**Certain**<br>**Very Likely**<br>**Possible** | Very likely. | Very likely. | Possible. |
| **Why?** | [Students] need to realize that each step has to be mathematically correct! | Uncover student thinking. | So [students] get in the habit of checking their work. |
| **If you had to pick just one response, which of these would be most like what you can imagine saying next.**<br><br>**A) Could you explain how you got from step one to step two?**<br>**B) This is correct, but your method doesn't always work. What if I changed the negative 6x to a positive 3x?**<br>**C) This is, by far, the best solution I have seen today.**<br>**D) How do you know four is a solution?** | A | D | A |

*Table 8: Excerpts from Cecilia's baseline, post-PD, and post-AR surveys (cont'd)*

| Survey Prompts | Baseline Response | Post-PD Response | Post-AR Response |
|---|---|---|---|
| **Why would you say that?** | Because there is a mistake there & I'd like to uncover student thinking. | 1st to see that they can verify … Then I would ask "A" … to get out that the math steps have to be accurate. | To uncover student thinking. |

In Table 7, I have included excerpts from Cecilia's baseline, post-PD, and post-AR surveys to get a sense of how her responding practices changed. In the baseline survey, Cecilia admits to not understanding this student's work. She is confident, though, that this work is incorrect. She responds by *inviting* other students to comment on the work. Her justification reveals a *product-focus* as she says "This is tough for me … I would probably point out the math mistakes." When asked to reconsider a second possible response, Cecilia suggests that she would likely *evaluate* the student's work with the comment, "I would bring up the point that even if the number works when we check it … the math still has to be accurate ... ". Her justification leans toward a *product-focus* as she explains that, "[students] need to realize that each step has to be mathematically correct!" In the final survey prompt, Cecilia chooses to *orient* the student back to her point of perceived error with "Could you explain how you got from step one to step two?" A focus on *mathematical products*, specifically the error, continues to come through in Cecilia's justification as she explains she choose to *orient*, "because there is a mistake there and I'd like to uncover student thinking." This excerpt above provides a slice of Cecilia's baseline data highlighting her use of *invitation, evaluation, orientation* and a focus that leans toward *mathematical products.*

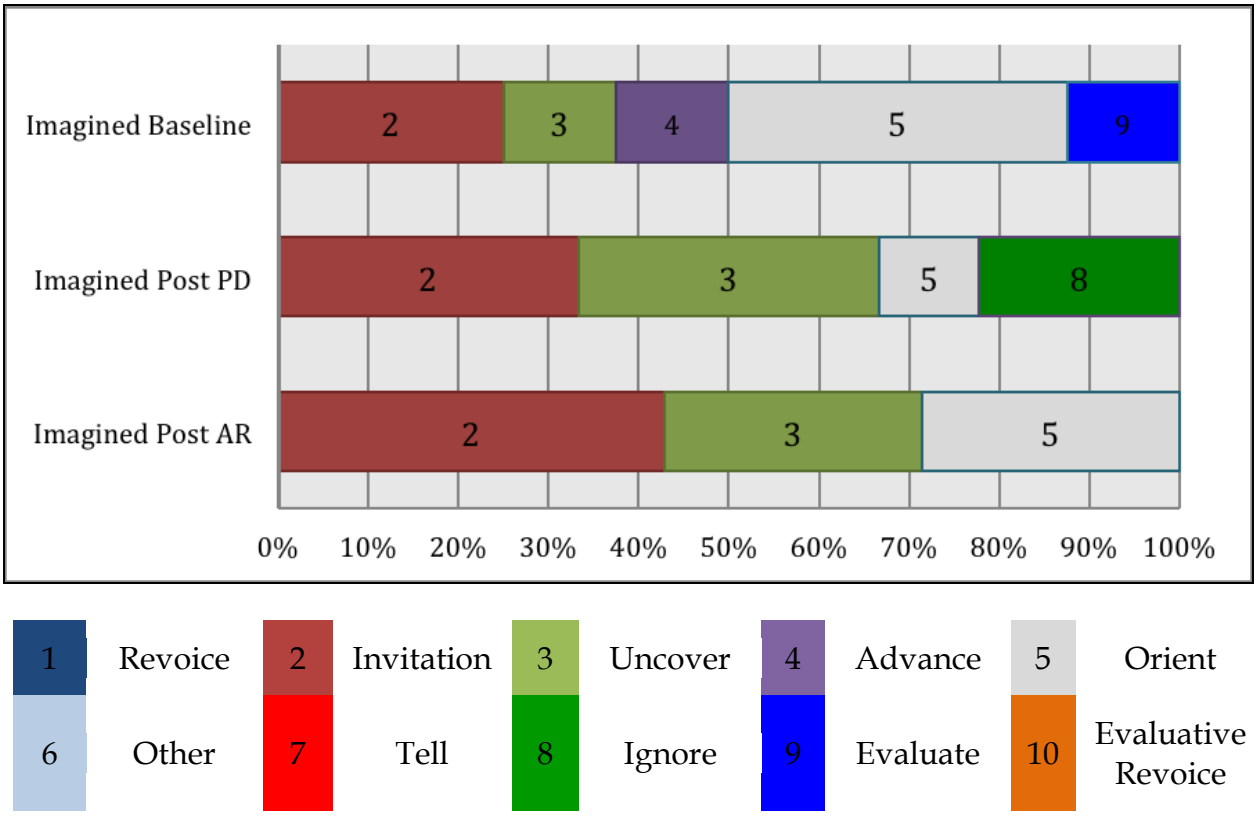
In the post-PD survey, Cecilia responded to student thinking by first re-soliciting for a second student's work and then *uncovering* by having, "both [students] explain what they did each step." When asked to select a second possible response, Cecilia selected an *uncovering* strategy with, "could you explain to us what you did in each step

and why?" Continuing, Cecilia evidently still believes this to be incorrect as she says she'd like to see "if students can find the error." Although her response is a bit *product-focused*, it differs from her baseline response because she resists being the one to *evaluate* the thinking in the post-PD survey. In the final survey question, she first makes an *uncovering* move with "how do you know four is a solution?" and explains her interest in the *mathematical process* by checking if the student can verify his answer. Here we see Cecilia's practices of *uncovering* growing to take the place of *evaluation* and *orienting*. Cecilia's justification reveals a shift toward more *process-focused* moves. In other places in the survey, Cecilia is also increasing her use of *invitation* and *ignoring*.

In the post-AR survey, we see Cecilia *strengthening* and *maintaining* some decisions while other decisions are *reversed*. In the first survey question, Cecilia *invites* other students to evaluate the student's thinking. She justifies this decision based on a *process-focused* stance, wanting to promote a discussion by hearing what others. In the second prompt, Cecilia *orients* the student back to the original question by asking, "How could we check to see if [it's] correct?" Her justification leans toward a *product-focus* as she explains the value of getting students "in the habit of checking their work." Finally, in the last prompt, Cecilia explains that she is trying to *uncover* student thinking with the question, "Could you explain how you got from step one to step two?"

These three excerpts were selected to help demonstrate changes seen in Cecilia's practices. Next, we consider the summary of Cecilia's imagined practices data.

*Figure 9: The breadth of Cecilia's imagined practices of responding over time*



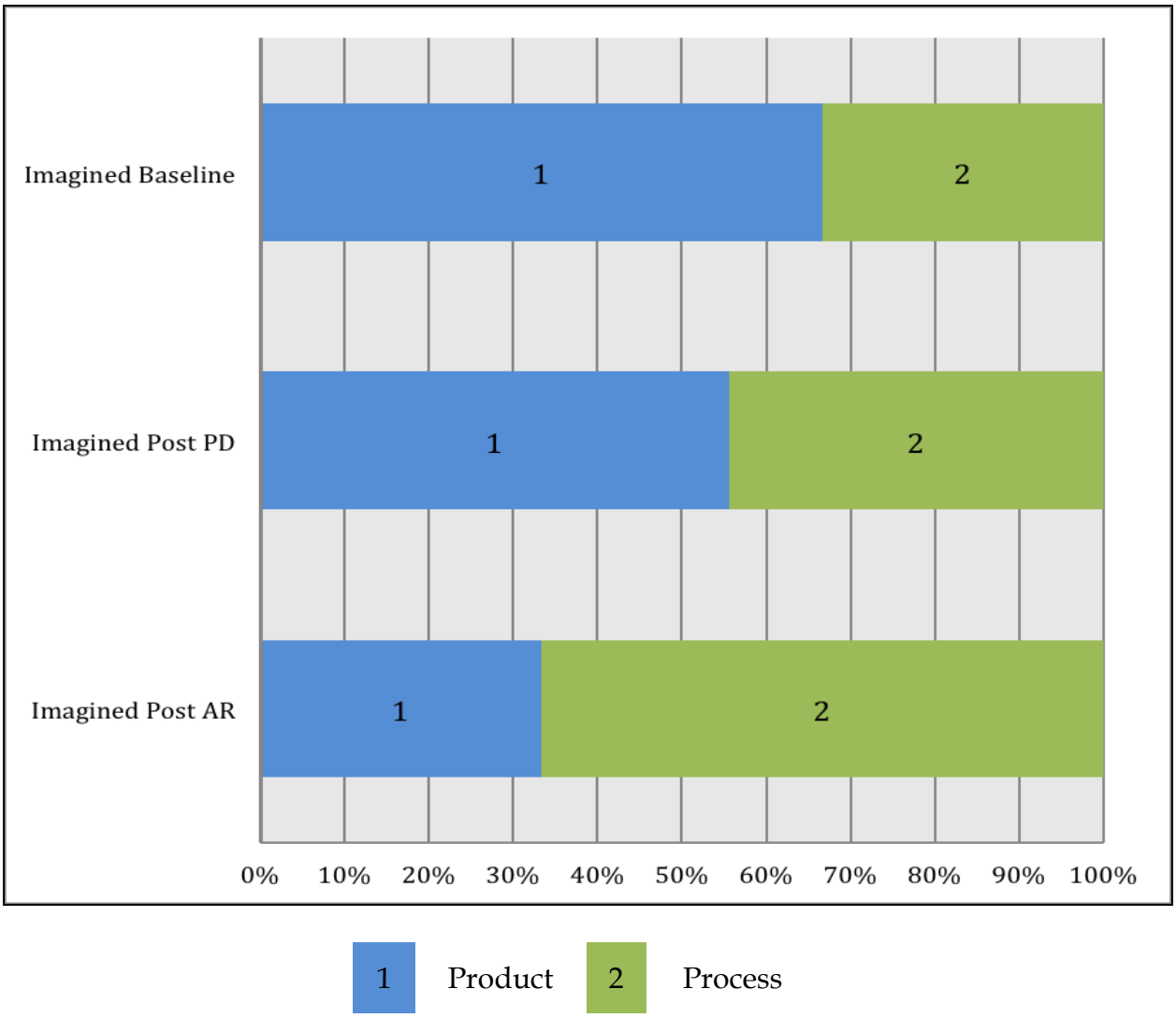

*Number of baseline moves = 8 Number of post-PD moves = 9 Number of post-AR moves = 7*

Figure 9 summarizes changes in the breadth of Cecilia's imagined practices. After the professional development, Cecilia increased *invitation*, *uncovering,* and *ignoring.* Cecilia used *invitation* 25% of the time in the baseline survey and 33% of the time in the post-PD survey. Cecilia used *uncovering* 13% of the time in the baseline survey and 33% of the time in the post-PD survey. *Ignoring* was not used at all in Cecilia's baseline survey and used 22% of the time in the post-PD survey. After participating in the action research, Cecilia *strengthened* her increased use of *invitation* to 43% use and *maintained* her use of *uncovering* while she *reversed* her increased use of *ignoring*.

As Cecilia imagined increasing some responses on the post-PD survey, other responses were decreased or eliminated after she completed the professional development. *Advancing* and *evaluation* each made up 13% of the baseline data; both were eliminated in the post-PD survey. *Orienting* made up 38% in the baseline survey and was reduced to 11% use on the post-PD survey. In the post-AR survey, Cecilia *maintained* her elimination of *advancing* and *evaluation* and *reversed* her decreased use of *orienting.*

Comparing this data with Cecilia's action research goals, we see that she accomplished her goal of increasing her imagined use of *invitation* following the professional development and the action research. Notice that even though reducing *evaluative* moves was not a stated objective for Cecilia, she still accomplished this professional development goal following the professional development and action research.

*Figure 10: The focus of Cecilia's imagined practices of responding over time*




*Number of baseline moves = 8 Number of post-PD moves = 9 Number of post-AR moves = 7*

Next, I examine Figure 10 that summarizes changes in the focus of Cecilia's imagined practices. This data demonstrates Cecilia's shift toward *process-focused* responding moves. Cecilia focused on *mathematical processes* 34% of the time in the baseline survey and 44% of the time in the post-PD survey. She *strengthened* that increased use of *process-focused* strategies across the action research cycle with 67% use.

When comparing Cecilia's changes to the professional development goal, Cecilia shifted away from responding practices that were heavily *product-focused*. In so doing, she ended up with responding practices that were heavily focused on *mathematical processes* and therefore no closer to achieving balance.

*4.4.3: Examining Cecilia's enacted practices data*

In this section, I discuss changes to the breadth of Cecilia's enacted practices change across this year-long experience. The first finding is Cecilia made substantial changes following the professional development and *strengthened* or *maintained* all of those changes following the action research. After the professional development, Cecilia increased her use of *revoicing, invitation,* and *uncovering*. She also decreased her use of *telling, evaluation,* and *evaluative revoicing*. Through the action research, Cecilia *strengthened* her increased use of *revoicing,* decreased use of *telling*, decreased use of *evaluation*, and decreased use of *evaluative revoicing*. She also *maintained* her increased use of *invitation* and *uncovering*. The second finding is Cecilia shifted toward a more *process-focused* stance following the professional development and *strengthened* that shift in the action research.

I illustrate the above findings with two portions of transcript from Cecilia's baseline and action research. I begin with a vignette from Cecilia's baseline video. In this vignette, Cecilia is standing at the front board with following problem written: (5x + 1) (3x + 2). During this interaction, Cecilia stays at the board and writes as the class moves through the problem together.

1-4 **Teacher: "**On the back, could you write this problem down, five x plus one, multiplied by three x plus two. Chapter nine, multiplying two binomials. Multiply it out using your distributive property and we will have somebody volunteer when you're done with the product.

[Waits 11 seconds]

5-6 **Teacher:** Double distribute… I see some pencils down. Maybe some of us already have an answer?

[Waits 8 seconds]

7-8 **Teacher:** Raise your hand if you'd like to share the product that you have and we'll double check it. Taylor, what'd ya get?

9 **Taylor:** Fifteen x squared plus thirteen x plus two.

10-11 **Teacher:** Fifteen x squared plus thirteen x plus two. If you double distribute, remember how we say we doubled?

[Waits 1 second]

12-14 **Teacher:** You're gonna get fifteen x squared plus ten x plus three x plus two. We have some like terms that combine to that middle term of thirteen x. Did anybody get anything different?

15 **Student:** No.

16 **Teacher:** Okay, does anybody have any questions?

[Waits 1 second]

17-18 **Teacher:** Okay, so that is what we did. Raise your hand if you know what we call our answer.

[Waits 0.5 second]

19-21 **Teacher:** I said it a couple of times, but I don't know if you heard me call it this. It starts with a 'P.' Raise your hand if you know it. One, two hands. Three.

[Waits 0.5 second]

22-23 **Teacher:** It starts with a 'P.' Answer to a multiplication problem. Four, five, okay…what is it? Everyone.

24 **Student:** Polynomial.

25-27 **Teacher:** Product! Right! This is the PRODUCT of these two numbers being multiplied together. Two numbers being multiplied together, starts with an F.

28 **Student:** Factor.

29 **Teacher:** Factors!

30-32 **Teacher:** Alright, so we have our product. Now, we are going to take the product and we are going to factor it. Meaning: I give you this, you come up with what two numbers multiply to it. So write down number two.

[Waits 1 second]

33-37 **Teacher:** You're going to write fifteen x squared plus thirteen x plus two. Now we're going to take our product and factor it. Now we're going to do this together, we're going to do the steps together. First thing we look for is a greatest common factor. Is there a greatest common factor amongst these terms?

[Waits 0.5 second]

38 **Teacher:** No. Nothing goes into 15, 13, and 2; so, do we share a letter?

39 **Student:** No.

40-41 **Teacher:** No, okay, so no GCF. If there isn't, it would be great if there were four terms, because then we could factor by…?

42 **Student:** Grouping.

43-44 **Teacher:** Grouping. So we try to re-write this problem. We will write this problem to have four terms."

In this vignette, we watch Cecilia interacting with her class. First, the class multiplies two binomials and then they factor the resulting trinomial back into two binomials. At the start of the vignette, Cecilia calls on Taylor to provide her answer for the multiplication of the two binomials (line 8). Once Taylor provides her answer (line 9), Cecilia *revoices* what Taylor shares in a way that lets her know it is correct (lines 10 - 11). Next, Cecilia proceeds to *tell* the class Taylor's process, "You're gonna get fifteen x

squared plus ten x plus three x plus two. We have some like terms that combine to that middle term of thirteen x" (lines 12 - 14). Next, Cecilia asks students to recall the name for the mathematical object just created through multiplication (lines 17 – 18). She waits a moment. When no one shares the answer she provides a hint, "It starts with a P" (lines 20 – 23). A student offers "polynomial" (line 24) and Cecilia, in her excitement and speed, mishears and incorrectly *revoices* and *evaluates* the student as she exclaims "Product! Right! This is the product of these two numbers being multiplied together" (line 25) She continues by asking the class to fill in the blank with, "Two numbers being multiplied together, starts with an F" (lines 26 – 27). This time a student offers the correct answer of "factor" (line 28) and again Cecilia *revoices* and simultaneously *evaluates* (line 29). We see Cecilia present this same sort of fill-in-the-blank question, students offering their thinking, and then Cecilia using an *evaluative revoice* again. This pattern of Cecilia initiating a question, her students offering an answer, and Cecilia responding with *evaluation* is fairly prevalent throughout the remainder of the transcript. Clearly, IRE is a comfortable pattern for Cecilia. Furthermore, the whole dialogue is driven toward finding answers. The patterns of an overly evaluative stance filled with *evaluation*, *evaluative revoicing*, and *telling,* mixed with a heavy *product-focus,* leaves little room for the kind of discourse called for in reform recommendations.

Compare this last interaction with this next portion of transcript from Cecilia's action research cycle. This vignette is preceded by an activity where Cecilia and her students generate a list of the powers of three (starting with $3^1$ and moving

consecutively through $3^{12}$). After the list was generated, she continued by asking every student, one by one, to provide an answer to $3^0$, without using a calculator. The class was split; some thought the answer was one, while others believed it was zero. Still others thought the answer might be three. At this point, Cecilia slowly begins to make her way to the back of the classroom. As Cecilia is making this move, one student claims the answer must be one because "anything to the power of zero is one." The vignette picks up here.

1-12 **Teacher:** "Have you guys heard that rule before? That anything to the power of zero is one. Maybe some of you have heard this. Here's the deal, you guys… chapter nine, we're starting today. We've got about ten rules we gotta learn: how to add, how to multiply, how to divide, all these different rules with exponents. It's good to know 'em, but the most important thing is 'why!' That's when you would remember. It's gotta make sense to us. Why is it one? Why is it one? We have a lot of people thinking it's one. One person tried to show us why it's one. I don't think most people were convinced, at least I know I wasn't, why it's one. I think why is it one…can anyone see something up here to make us think that it's one? And if not, it's okay, we can come back to it. But I'm wondering, can someone else maybe see? Remember when I told you about a pattern in the chart that might help us. Does anyone see another reason ...

13 **Student:** [overlapping with teacher] …cause...

14-15 **Teacher**: [overlapping with student] ...to convince us. Why is two to the zero one? Why is three to the zero one? Alright.

16-17 **Student:** Well I don't know if it's right but, every time you go to zero, one, two, three, four, five, you just add one.

18 **Teacher:** Are you talking about the exponents as you go?

19 **Student:** Yeah.

20 **Teacher:** So then that's going to lead us to write a one?

21 **Student:** I don't know.

22-23 **Teacher:** [Teacher shrugs her shoulders a bit] Okay, Tys…Germane…and then Terry.

24-27 **Germane:** Cause all the answers talk about…you divide mostly the patterns in the answers, they're all divided by three. Once you get by three, you divide by three, but there's another answer you could divide three by, one. [pointing as he is talking] So that's where you get one.

[Wait time, during which time students are first quiet and then conversation and a bit of giggling breaks out]

28 - 29 **Teacher:** Do you mind just coming up and showing us what you're thinking?

[Wait time; again a bit of conversation and giggling goes on. Germane comes to the front board]

30-31 **Teacher:** Because Germane is using the word divide and we were multiplying all this time so he threw in another operation with us so.

32-33 [Wait time while Germane prepares himself to speak. Lively commenting and giggling during wait time].

34-37 **Germane:** For each answer you get, you divide 'em by three so, once you get down to three, you divide three by one. You can't, there is no other, the only way is to divide three by one. Which is basically just three itself. So you divide one by three, I mean if you multiply one by three and that gives you the answer. [Puts marker down and proceeds to head back to seat. Room is now silent.]

38 - 39 **Teacher:** Okay, can you wait for just one minute. Does anyone have any questions for Germane?

40 **Terry:** Yeah, how did you get ...

41 **Teacher:** Terry, you have a question?

42-43 **Terry:** So, what you saying is like, one times three equals three. Three times threes equals nine. You multiplying by three as you go up.

44 **Germane** You could do it that way also.

45-47 **Terry:** Right. Doubling like 'cause one times three is three. Three times three is nine. Nine times three is twenty-seven. Twenty-seven times three is eighty-one.

48-49 **Germane:** Right. That's the pattern. [Begins drawing a series of loops on the board from one number to the next in the pattern]

50-51 **Teacher:** So as Ger … as he's writing that, we're multiplying up and then as Germane is suggesting, if we go down, instead of multiplying we're ...

52 **Student:** Dividing.

53 **Germane:** Dividing.

54 - 55 **Teacher:** So if we were to go down the chart, twenty-seven divided by three is ...

56 **Germane:** Nine.

57 **Teacher:** Okay. Nine divided by three is ...

58 **Germane:** Three and then one.

59 **Teacher:** Three divided by three is ...

60 **Germane:** One.

[Germane has been following along with conversation by answering questions, pointing at the board to help the teacher clarify his points. However he has faced the board the whole time, as opposed to looking directly at the class. Still facing the board, Germane sheepishly says] Can I go back to my seat?

[Class breaks out in laughter]

61 **Teacher:** Does anyone else have any questions for Germane?

**Germane:** [Germane quietly puts both hands in his pockets, twists his body side to side nervously and says quietly, with a smile, 'No']

62 **Student:** Yep, I don't [inaudible]. No, I'm just playing.

[Class giggles a bit]

63 **Teacher:** Alright Germane, good job. Thank you.

[Germane turns away from the camera and quietly returns to his seat with a sheepish smile on his face].

64 **Teacher:** Um. So what do you guys think now? Does that ...

[Discussion breaks out a bit and a bit of giggling as well. Teacher makes her way back to the front of the class.]

65-67 **Teacher:** So, going down the chart if we were dividing by three each time and three divided by three gives us one... not zero? Would that hold over here with the two chart?

[Waits 1 second]

68-70 **Teacher:** As we go up the chart, you don't notice how Joe mentioned that we were doubling or multiplying by two going up; what are we doing going down the chart?

71 **Student:** In half.

72 **Teacher:** We're dividing. So if we divided by two, eight divided by two?

73 **Student:** Four.

74 **Teacher:** Divided by two is two. Two divided by two [points at the one on the board]. Makes sense.

[Waits 2 seconds]

75-78 **Teacher:** Alright, I'm curious. One or zero? Let's just take a vote instead of me checking in with every single person. How many people are believing right now three to the zero, two to the zero is one based on what we talked about?

[Kids put hands up]

79-80 **Teacher:** Okay, put your hands down. How many people are still thinking it's zero?

[Kids put hands up]

81-84 **Teacher:** [Nodding her head yes] Okay we still have a couple people holding out. Okay. For the zeros, if someone could just explain, why zero? Where does the zero come from? Or if you need more time to think about it, we'll just let it go.

85 - 86 **Student:** I just still think it isn't one because three to the zero, there's no three.

[Teacher nods her head as the student is talking]

87 **Student:** It's zero.

88 **Teacher:** That's fine. We're going to think about this more."

In this vignette, we watch as Cecilia takes an idea that is clearly perplexing to students and asks them to consider if this "rule of anything to the power of zero is one" makes sense (lines 1 – 15). As students begin offering their ideas (lines 16 – 17) Cecilia makes a series of *uncovering* moves in response (lines 18, 20). Germane decides to share his idea (lines 24 - 27) and Cecilia invites him to come to the front board in order to *uncover* his ideas with the use of diagrams he is pointing to from his seat (lines 28 – 29). As Germane walks to the board, his body language communicates his discomfort with being in front of his peers. To fill some of that awkward dead space while Germane walks, Cecilia *revoices* some of his ideas (lines 30 – 31), but does so without any hints of evaluation. Once he gets to the board, Germane shares his thinking more fully (lines 34 – 37). Cecilia responds by *inviting* the class to question Germane about what he shared (lines 38 - 39).

Terry *revoices* Germane's ideas (lines 40 - 43) and Germane agrees with, "You could do it that way also" (line 44). Terry continues to expound on his way of looking at the problem (lines 45 – 47) and Germane responds with an affirmative *evaluation* and illustrates Terry's idea on the board (lines 48 – 49). Cecilia acknowledges Terry's idea

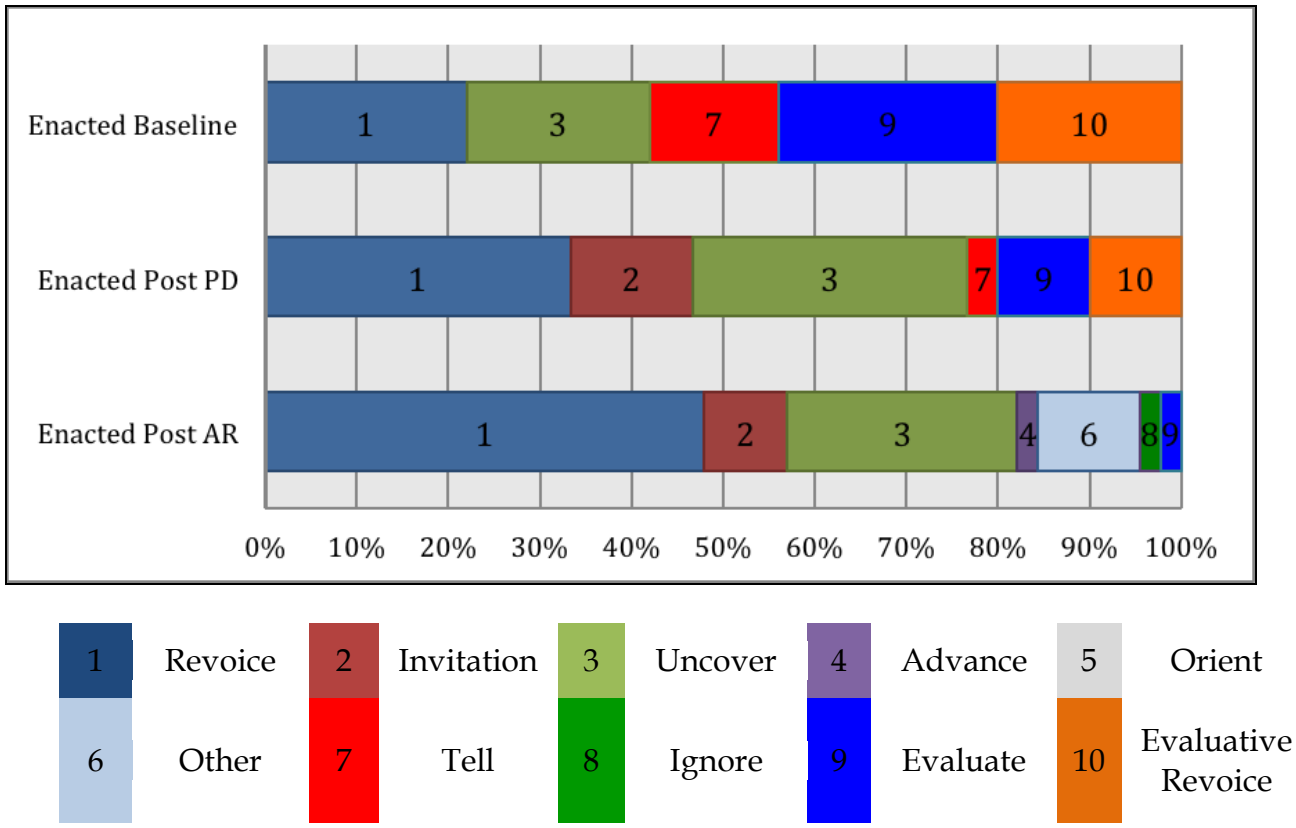
but brings the class back to Germane's original idea by *revoicing* it for the class to consider (lines 50 – 51). Part way through that *revoicing*, Germane jumps in to participate in the re-telling of his idea (lines 53 - 60). At the end of this collaborative *revoice,* Cecilia again *invites* the class to engage with Germane on his idea through questions (line 61). The class does not have any (line 62).

As Germane sheepishly takes his seat Cecilia says "Alright Germane, good job. Thank you" (line 63). On the surface, this statement may seem like an *evaluation,* but when considering Cecilia's next question of "Um. So what do you guys think now?" (line 64) it appears Cecilia's "good job" was simply a *comment* on his sharing rather than an *evaluation* of correctness. Cecilia wraps up the conversation by taking another vote to *uncover* students' thinking (lines 75 – 80), now that Germane has shared. At the end of this conversation, some students are still holding onto their belief that three to the zero is zero, rather than one (line 81). Cecilia responds to these students with an *uncovering* move of, "For the zeros, if someone could just explain, why zero? Where does the zero come from?" (lines 82 - 83). A final student shares why he believe the answer is zero rather than one (lines 85 - 86) and Cecilia leaves the question open assuring students, "We're going to think about this more" (line 88).

There is a dramatic shift in this interaction from that in the previous year. A shift toward responding moves such as *revoicing, invitation, uncovering,* while simultaneously moving away from *evaluation, telling,* and *evaluative revoicing,* made a noticeable difference in this second interaction. Furthermore, a focus on students' *mathematical*

*processes* took center stage in this interaction where *mathematical products* dominated the previous interaction. These two interactions together serve to provide a context to understand changes in Cecilia's responding practices. With this context as a backdrop, now consider the summary of Cecilia's enacted practices of responding for both breadth and focus.

*Figure 11: The breadth of Cecilia's enacted practices of responding over time*



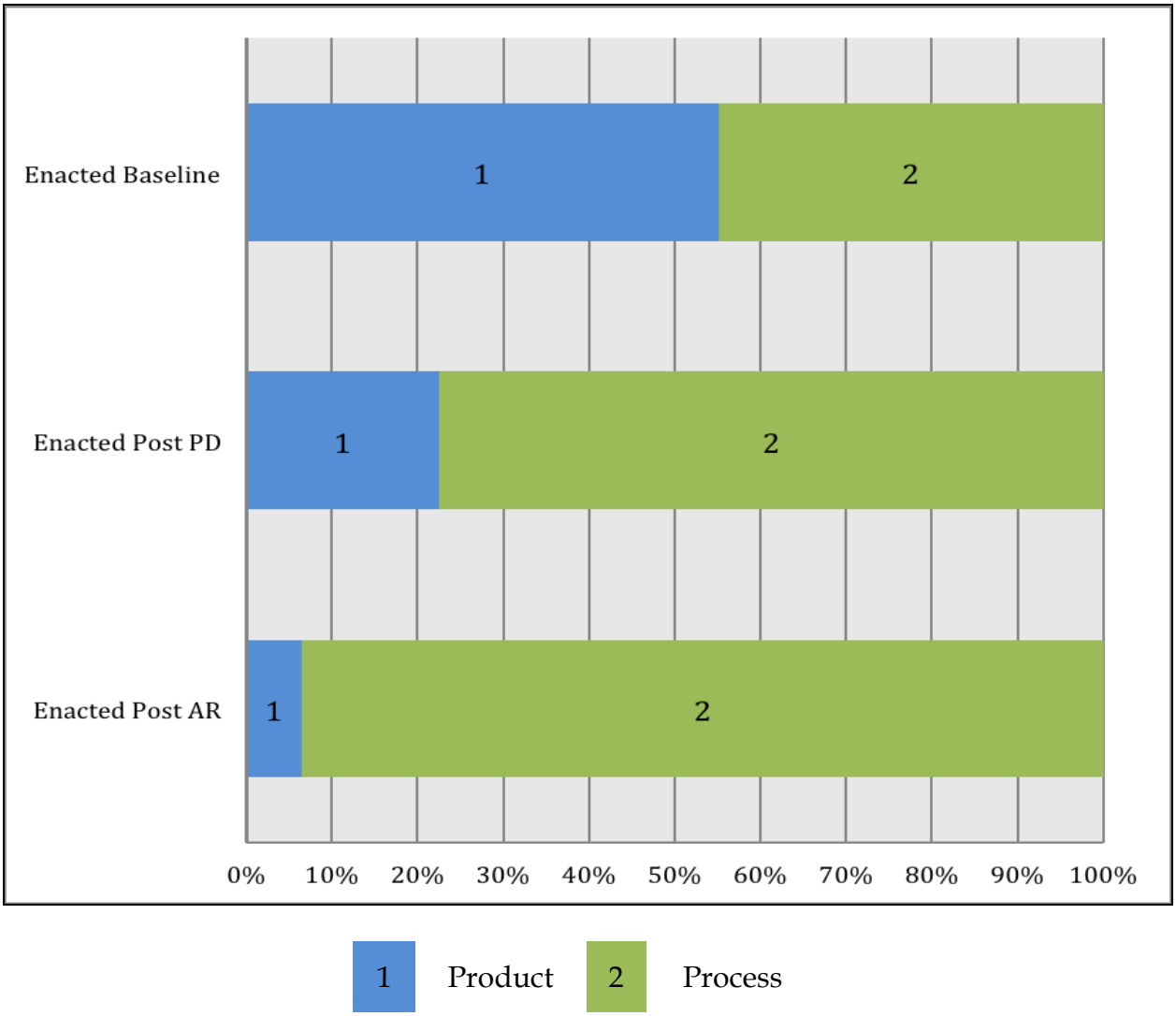

*Number of baseline moves = 50 Number of post-PD moves = 30 Number of post-AR moves = 88*

Figure 11 summarizes changes in the breadth of Cecilia's responding practices. Following her participation in the professional development, Cecilia increased *revoicing*, *invitation*, and *uncovering*. In Cecilia's baseline video, *revoicing, invitation,* and *uncovering*

represented 22%, 0%, and 20% of her responding practices, respectively. In the post-PD video, Cecilia increased her use of *revoicing* to 33%, *invitation* to 13%, and *uncovering* to 30% of her responding practices. After the action research cycle, Cecilia *strengthened* her increased use of *revoicing* to 44% use. She also *maintained* her increased use of *invitation* and *uncovering*

As Cecilia made more room in her enacted practices for *revoicing, invitation*, and *uncovering,* she reduced *evaluative revoicing*, *evaluation*, and *telling.* Cecilia used *evaluative revoicing* 20% in the baseline data and only 10% in the post-PD data. She also decreased her use of *evaluation* from 24% in the baseline data to 10% in the post-PD data. Lastly, she reduced *telling* from 14% to 3% in the post-PD video. As Cecilia moved through the action research cycle, she *strengthened* her decreased use of *evaluative revoicing* and *telling* by completely eliminating it. She also *strengthened* her decreased use of *evaluation* to only 2% use in the post-AR data. When comparing these results with Cecilia's stated goals in her action research plan, it is not difficult to see that she achieved her goal of increasing her use of *invitation* after the professional development, and *maintained* that increase after the action research. Cecilia also accomplished the professional development goal of decreasing her use of *evaluative* moves both after the professional development and after the action research, even through this was not one of her stated objectives.

*Figure 12: The focus of Cecilia's enacted practices of responding over time*




*Number of baseline moves = 50 Number of post-PD moves = 30 Number of post-AR moves = 88*

Figure 12 reveals a shift in Cecilia's focus toward *mathematical processes* and away from *mathematical products.* Cecilia began, in the baseline video, with a 45% focus on *mathematical processes.* After participating in the professional development, Cecilia increased her *process-focused* moves to 77%. Cecilia *strengthened* this increased usage of *process-focused* moves to 93% following her participation in the action research.

Comparing this data to the professional development goal helps one notice that Cecilia was successful at shifting away from a *product-focus* stance toward a more *process-focused* stance. However, she was not overly *product-focused* to begin with, and so the end result leaves Cecilia fairly unbalanced toward a focus on *mathematical processes.*

## 4.5: Summary

In this final section for chapter 4, I compare changes Ann, Brenda, and Cecilia made in their imagined and enacted practices throughout the professional development and action research. These comparisons are important for both practical and theoretical reasons that I discuss in chapter 5.

### *4.5.1: Common changes to participants' imagined practices*

In this section, I review changes made by all three participants within their imagined practices. First, I summarize similarities among participants' imagined practices. Table 9 represents changes all three participants made from the baseline to post-PD survey. By looking across all three participants, we can see responding moves that were commonly increased (**+**), decreased (**-**), or unchanged (⦸).

*Table 9: Summary of changes made by participant from their baseline survey to post-PD survey*

| | Ann | Brenda | Cecilia | Percent Agreement |
|---|---|---|---|---|
| 1=Revoice | • | • | • | 100% |
| 2=Invitation | - | + | + | 67% |
| 3=Uncover | + | + | + | 100% |
| 4=Advance | + | • | - | 0% |
| 5=Orient | + | - | - | 67% |
| 6=Other | • | • | • | 100% |
| 7=Tell | • | - | • | 67% |
| 8=Ignore | - | - | + | 67% |
| 9=Evaluate | - | • | - | 67% |
| 10=Revoice/Evaluate | • | • | • | 100% |
| Process-focus | + | + | + | 100% |

| | |
|---|---|
| + = Increase greater than 5% | - = Decrease greater than 5% |
| ⦸ = No change | • = No use in either survey |

After the professional development, all three participants increased their imagined use of *uncovering*. Two of the three participants increased their imagined use of *invitation*. Additionally, two of the three participants decreased their *evaluation*, while the third did not have any use of *evaluation* in either the baseline or post-PD survey. Two of three participants decreased their imagined use of *orienting* and a different set of two participants decreased their imagined use of *ignoring*. Lastly, after the professional development, participants shifted their focus toward *mathematical processes*.

Next, I consider which of these changes participants commonly sustained (*maintained* or *strengthened*) as the participants moved through the action research cycle. In Table 10, I have summarized this data. A word of explanation about reading this

table may be helpful. Changes to both post-PD and post-AR changes are recorded for each participant for each move so the reader can keep track of increases that were *strengthened* (+,+), *maintained* (+,⦸), and *reversed* (+,-); as well as decreases that were *strengthened* (-,-), *maintained* (-,⦸), and *reversed* (-,+). The table also includes shading to indicate the net effect of the two changes as increase ( ) or decrease ( ).

*Table 10: Summary of changes made by participant from their baseline survey to post-PD survey to post-AR survey*

| | **Ann** | | **Brenda** | | **Cecilia** | |
|---|---|---|---|---|---|---|
| | **Post-PD** | **Post AR** | **Post-PD** | **Post AR** | **Post-PD** | **Post AR** |
| 1=Revoice | • | • | • | • | • | • |
| 2=Invitation | - | ⦸ | + | ⦸ | + | + |
| 3=Uncover | + | - | + | - | + | ⦸ |
| 4=Advance | + | - | • | • | - | ⦸ |
| 5=Orient | + | + | - | + | - | + |
| 6=Other | • | • | • | • | • | • |
| 7=Tell | • | • | - | ⦸ | • | • |
| 8=Ignore | - | + | - | ⦸ | + | - |
| 9=Evaluate | - | ⦸ | • | • | - | ⦸ |
| 10=Revoice/Evaluate | • | • | • | • | • | • |
| Process-focus | + | ⦸ | + | + | + | + |

**= Net effect increase** **= Net effect decrease**

| | | |
|---|---|---|
| + + = Increase strengthened | +⦸ = Increase maintained | + - = Increase reversed |
| - - = Decrease strengthened | - ⦸ = Decrease maintained | - + = Decrease reversed |
| ⦸⦸= No change | ⦸+= New increase | ⦸- = New decrease |
| | • • = No use in any survey | |

In the post-AR survey data, two of the three participants sustained an increased use of *invitation.* Also, two of the three participants sustained a decreased use of *evaluation*, while the third participant never used any *evaluation* in any of the surveys. Additionally, all three participants sustained their increased focus on *mathematical processes.*

It is interesting to compare changes participants sustained to participants' stated goals. Recall Ann's goals of decreasing her *evaluative* moves while increasing *invitation*, *uncover*, *revoice,* and *advance.* Both Brenda's and Cecilia's goals included increasing their use of *invitation* along with some other goals about creating more equity for students' voices. With these goals in mind, it is not terribly surprising that Brenda and Cecilia shifted their imagined practices of responding to initially increase and sustain an increased use of *invitation*. Ann's primary goal, to decrease *evaluative* moves, is aligned with changes she was able to make and sustain as well. Connections between participants' stated goals and changes they were able to make and sustain are discussed at greater length in chapter 5.

In terms of participants' imagined focus, all three teachers made and sustained large shifts toward *mathematical processes* through this experience. Ultimately all three participants were heavily *process-focused* in the final survey and one could argue they had not achieved greater balance between *processes* and *products*. This "perceived" lack of balance is discussed in more detail in the last chapter.

*4.5.2: Common changes to participants' enacted practices*

In this section, I summarize the similarities among participants' enacted practices of responding. Table 11 reports changes all three participants made from their baseline video to their post-PD video.

*Table 11: Summary of changes made by participant from their baseline video to post-PD video*

| | **Ann** | **Brenda** | **Cecilia** | **Percent Agreement** |
|---|---|---|---|---|
| 1=Revoice | + | - | + | 67% |
| 2=Invitation | + | + | + | 100% |
| 3=Uncover | + | + | + | 100% |
| 4=Advance | ⦸ | ⦸ | ⦸ | 100% |
| 5=Orient | ⦸ | ⦸ | • | 67% |
| 6=Other | ⦸ | ⦸ | ⦸ | 100% |
| 7=Tell | - | - | - | 100% |
| 8=Ignore | - | ⦸ | ⦸ | 67% |
| 9=Evaluate | - | - | - | 100% |
| 10=Revoice/Evaluate | - | - | - | 100% |
| Process-focus | + | + | + | 100% |

| | |
|---|---|
| + = Increase greater than 5% | - = Decrease greater than 5% |
| ⦸ = No change | • = No use in either video |

After professional development, all three teachers increased *invitation* and *uncovering,* while two of the three teachers increased *revoicing*. Similarly, all three participants decreased *telling*, *evaluating*, *evaluative revoicing*. Last, every participant shifted her responding practices to include more focus on *mathematical processes.*

Next, I use Table 12 to highlight the changes participants sustained as they moved through the action research cycle.

*Table 12: Summary of changes made by participant from their baseline video to post-PD video to post-AR video*

| | Ann | | Brenda | | Cecilia | |
|---|---|---|---|---|---|---|
| | **Post-PD** | **Post AR** | **Post-PD** | **Post AR** | **Post-PD** | **Post AR** |
| 1=Revoice | + | ⊘ | - | + | + | + |
| 2=Invitation | + | ⊘ | + | ⊘ | + | ⊘ |
| 3=Uncover | + | - | + | - | + | ⊘ |
| 4=Advance | ⊘ | ⊘ | ⊘ | ⊘ | ⊘ | ⊘ |
| 5=Orient | ⊘ | ⊘ | ⊘ | ⊘ | • | • |
| 6=Other | ⊘ | ⊘ | ⊘ | ⊘ | ⊘ | + |
| 7=Tell | - | + | - | ⊘ | - | ⊘ |
| 8=Ignore | - | ⊘ | ⊘ | + | ⊘ | ⊘ |
| 9=Evaluate | - | ⊘ | - | + | - | - |
| 10=Revoice/Evaluate | - | ⊘ | - | + | - | ⊘ |
| Process-focus | + | ⊘ | + | - | + | + |

**= Net effect increase** **= Net effect decrease**

| | | |
|---|---|---|
| **++** = Increase strengthened | **+⊘** = Increase maintained | **+ -** = Increase reversed |
| **- -** = Decrease strengthened | **- ⊘** = Decrease maintained | **- +** = Decrease reversed |
| ⊘⊘= No change | ⊘**+**= New increase | ⊘**-** = New decrease |
| | • • = No use in any survey | |

In the post-AR video data, all three participants sustained an increased use of *invitation*. Two of the three participants sustained their increased use of *revoicing*. Two of the three participants sustained their decreased use of *telling*. Two other participants sustained their decreased use of *evaluating* and *evaluative revoicing*. Lastly, two of the three participants sustained an increased focus on *mathematical processes*.

Again, comparing participants' changes and action research goals produces some interesting findings. Three of the six changes that Ann made and sustained came directly from her action research goal, namely to decrease *evaluation*, increase *revoicing*, and increase *invitation.* Also, Ann's decreased use of *evaluative revoicing* is arguably connected to her goals and therefore was directed by her action research plan. One of three changes that Brenda made and sustained came directly from her action research plan, namely to increase *invitation*. Recall that Brenda did not use *evaluation* heavily in her baseline data and therefore did not have a goal to decrease it. This makes sense of her absence from the set of participants who made and sustained a decreased use of *evaluation* and *evaluative revoicing*. One of the eight changes that Cecilia made came out of her action research goal, namely to increase *invitation.* Recall that Cecilia did not have a goal of decreasing *evaluation* however, she did have high levels of evaluation in her baseline data and decreasing *evaluation* was an emphasis of the professional development. Last, here more than in imagined practices, the number of changes participants were able to make and sustain causes several questions to arise. Questions about this difference are discussed in more detail in chapter 5. For now, I conclude this chapter with a focus on commonalities across the participants.

*4.5.3: Conclusion*

In closing this chapter, I note there are many similar elements across Ann's, Brenda's, and Cecilia's data. Commonalities discussed above are important for both practical and theoretical reasons. Uncovering these commonalities is important for

practical reasons because they help us to understand the potential of this model of professional development. Helping teachers change their practices of responding through professional development is key to changing teaching and learning in classrooms. So much of a classroom environment hinges on the teacher's understanding of the impact of an overly evaluative stance. However, simply understanding the impact of these practices is not enough. Teachers need to re-imagine and enact new ways of responding to students' ideas. This professional development helped three teachers to do just that. The research on this change will help to replicate and improve the work that was started here with larger groups of teachers. This is discussed in more detail in the next chapter.

These commonalities have theoretical importance because they help us to consider hypotheses about how teachers' practices of responding change as they move from novice to expert. Prior to this work, Crespo et al. (2007) already had some hypotheses about how responding practices might grow from their work with pre-service teachers. However, the present study takes an in-depth look at three teachers. These pictures can be used to help sharpen the hypotheses from Crespo's work to give us a clearer picture of teacher learning related to instructional practices broadly and responding practices specifically. More discussion about the conclusions and implications of these findings are found in chapter 5. I also make suggestions for future research.

# Chapter 5: Findings, Conclusions, and Implications

## 5.1: Introduction

The purpose of this chapter is to use findings from chapter 4 to draw conclusions and suggest implications. In section 5.2, I review the purpose and problems of the study, reviewing the two research questions I aim to answer. Next, I provide a brief review of findings from the previous chapter in section 5.3. I also use Crespo's hypotheses (2007) as a point of comparison to summarize my findings. In section 5.4, I draw conclusions about the findings and suggest reasons for some of the more perplexing portions of data. Next in section 5.5, I consider the study's implications for reshaping the professional learning experience studied within this present work. In section 5.6, I examine the implications that these results have on a broader scale and make suggestions for future research.

## 5.2: Summary of the Study

In chapter 1, I outlined the problem and purposes of this present work. I identified a disparity that exists between U.S. teachers' practices of responding and the NCTM recommendations (1991, 2000) about mathematical discourse. Generally, teachers' practices of responding are too evaluative to be helpful in developing the kind of discourse these reform documents recommend. Therefore, the purpose of this study was to explore the potential for a professional learning experience to help teachers transform their responding practices to: 1) be less evaluative 2) increase the breadth of

their responding moves and 3) achieve greater balance between a focus on mathematical *products* and *processes*.

In chapter 2, I reviewed two bodies of literature that helped inform the design of the professional learning experience. In the first body of literature, I considered some of the theoretical perspectives on professional development intended to help teachers change their practices. Richardson's (1990), Shulman's (1986), Ball and Cohen's (1999), and Desimore's (2009) works were each seminal for the design of the professional learning experience. In the second body of literature, I considered some empirical examples of professional development that were designed to help teachers transform their practices. I considered lessons learned from two contrasting pieces of literature about the Cognitively Guided Instruction (CGI) program (Carpenter et al., 1999) and the case study about Mrs. Oublier (Cohen, 1990). I considered lessons learned within these two hallmark examples from the research field, and applied those lessons to the creation of this professional learning experience.

In chapter 3, I posed two research questions to understand the efficacy of the professional learning experience and to understand more about how teachers' practices of responding change. The research questions were formulated:

> Question 1) What are the outcomes on teachers' imagined and enacted practices of responding during and following participation in professional development designed to focus specifically on the practices of responding?
>
> Question 2) What are the outcomes on teachers' imagined and enacted practices of responding during and following participation in a cycle of action research designed to focus specifically on the practices of responding?

In chapter 3, I also discussed in detail the design elements of the professional development and research study. Some of the specific details include how this professional learning experience emerged following a request from a learning team of mathematics and science teachers working together in a school district to improve their instruction. These teachers requested another year of working together to continue their own professional growth.

Based on that request, I secured an agreement with the three mathematics teachers to study their instructional practices the following year. These three participants each had more than 10 years of classroom experience and taught mathematics to students in grades six through eleven. During that year-long experience, each research participant submitted two baseline videos from the first year of the project along with three additional videos taken as part of the action research cycle. Research participants also agreed to take a survey at the beginning, middle, and end of the year-long experience.

In the next section, I summarize the research findings from chapter 4 and draw conclusions from those findings to answer the research questions. I discuss implications of the results and make suggestions for future research in remaining sections.

## 5.3: Findings

In this section, I address the dissertation's two research questions. For each research question, I summarize the findings from chapter 4 related to the specific

question. I then compare those finding to Crespo's hypotheses (2007) paraphrased below:

> Hypotheses 1 - As teachers become more expert in their imagined and enacted practices of responding, one can expect them to have a broader range of strategies in their repertoire.
>
> Hypotheses 2 - As teachers become more expert in their imagined and enacted practices of responding, one can expect them to have a more balanced focus between mathematical processes and mathematical products.

### *5.3.1: Research Question 1*

The first research question for this study explores the outcomes of the professional development on teachers' practices. Data in chapter 4 show that after experiencing the professional development, teachers increased their imagined uses of *invitation* and *uncovering* while eliminating their use of *evaluating*. Teachers also made substantial shifts towards more *process-focused* strategies in their imagined practices. In addition, participants increased their enacted practices of *revoicing*, *inviting*, and *uncovering* while decreasing their use of *telling*, *evaluating*, and *evaluative revoicing*. Here too, the participants shifted their enacted practices toward *process-focused* techniques.

Crespo et al. (2007) posited that as teachers advance in their practices of responding, one could expect to see a greater breadth of responding strategies in their repertoire. This hypothesis is not supported by the post-PD imagined and enacted practices data. In the imagined practices data, the three participants narrowed their responding strategies. All three of the teachers' data represented fewer responding moves in the post-PD survey than in the baseline survey.

In the enacted practices data, participants did not demonstrate a narrowing or broadening of practices. Rather, participants' shifted away from certain types of moves in favor of other, previously-minimal moves. The responding moves of *revoicing, inviting,* and *uncovering* made up between 20% and 50% of the baseline data for all three participants and over 75% of their post-PD data.

Crespo (2007) also postulates that as teachers move from novice to expert teaching performances, we can expect a shift from a focus on *mathematical products* to a greater balance between *products* and *processes.* For the imagined and enacted practices, all three of the participants moved away from a *product-focused* stance towards a heavier *process-focused* stance. The supposition that this shift creates more balance is questionable. In looking across the focus data for all three participants, Crespo's hypothesis was accurate in that teachers did, in fact, shift away from *product-focused* strategies in favor of *process-focused* strategies. That shift, however, did not result in more balance as suggested.

*5.3.2: Research Question 2*

The second research question examines the outcomes of the action research on teachers' practices of responding. In teachers' imagined practices, the post-AR surveys revealed that participants *maintained* their increased use of *invitation* as well as their elimination of *evaluation.* Additionally, all three participants *maintained* or *strengthened* their increased use of *process-focused* strategies, and in so doing drifted further from a balanced focus between *mathematical products* and *processes*.

In the post-AR enacted practices data, teachers *maintained* or *strengthened* their increased use of *invitation, uncovering,* and *revoicing* as well their decreased use of *telling*, *evaluating*, and *evaluative revoicing*. Teachers also *maintained* or *strengthened* their shift towards *process-focused* responding moves in their enacted practices.

Again, considering Crespo's (2007) hypothesis, we might suppose that a teacher would use a greater breadth of responding strategies across the action research. The imagined data lacks evidence necessary to substantiate this claim. As with research question one, the data suggests that participants' growth within their imagined practices of responding produced a narrower set of strategies. Participants reduced their responding moves averaging four to five in the baseline survey to only three to four in the post-AR survey.

Similarly, participants' enacted practices fail to align with Crespo's postulate. Ann and Cecilia seemed to move toward certain responding moves while other, previously substantial moves, were eliminated or decreased. This trend is a result of Ann and Cecilia *maintaining* the decisions they made after the professional development. Brenda's shift was less obvious because she *reversed* many of her former decisions. Overall, Brenda made slight shifts that indicate the same movement toward particular moves and away other from other, previously-notable moves. Considering Crespo's (2007) hypothesis, I expected participants would have a balanced focus between *products* and *processes* at the conclusion of the action research. In both imagined and enacted post-AR data, participants continued to shift further in the

direction of *process-focused* strategies. This shift, however, exacerbated the unbalance between *mathematical products* and *processes* that already existed in the post-PD data.

## 5.4: Conclusions

In this section, I first discuss various aspects of the data were anticipated and draw probably conclusions for these aspects. Second I discuss probable conclusions for those results that were surprising.

### *5.4.1: Conclusions about expected data results*

To begin, I will draw conclusions about those results that matched my expectations. I was not surprised to find out that the participants shifted away from *telling*, *evaluation,* and *evaluative revoicing* toward *revoicing, invitation,* and *uncovering*. I was also not surprised that participants' moved away from *product-oriented* moves toward *process-oriented* moves. These shifts align with the ideas communicated across many of the research articles they read for the first four sessions. For example, in one article (Reinhart, 2000) the participants read one teacher's account of the shifts he made in his instructional practices. The impetus for these changes came about with the following realization:

> I concluded that a fundamental flaw existed in my teaching methods. When I was in front of the class demonstrating and explaining, I was learning a great deal, but many of my students were not! Eventually, I concluded that if my students were to ever really learn mathematics, *they* would have to do the explaining, and I, the listening. My definition of a good teacher has since changed from "one who explains things so well that students understand" to "one who get students to explain things so well that they can be understood." (pg. 478).

This passage captures a first hand account from a teacher who shifted his view of teaching from teaching as telling to something new. In the pages following this passage, the teacher goes on to describe his view of the teacher's role in this new practice by offering suggestions such as "never say anything a kid can say, "ask good questions", "use more process questions", "replace lectures with sets of questions", and "wait time is very important".

The ideas within this article weren't particularly unique to other articles that teachers read within the professional development. Across the first four sessions, teachers read 14 articles with similar ideas, varying only in grain size of issues rather than philosophy (some had a more generic focus on instructional practices while other focused on a particular set of practices germane to posing, listening, or responding). Of the 14 articles, ten of them were written as 1st or 3rd person accounts of actual classroom teachers who have made a journey toward reform and successfully changed their teaching. Similarly they watched five videos of teachers in classrooms. In four of those five videos, teachers implemented their practices in ways akin to the model called for in reform with very low use of *evaluation* and high use of non-evaluative practices such as *revoicing, uncovering,* and *inviting* as well as a focus on students' thinking rather than answers.

In the discussions around these articles and video , teachers were making connections to the stories from these teachers. It seemed they found themselves and their students in these classroom represented in articles and videos as evidenced by

statements like, “I struggle with that too” or “My students resist this change as well”. They also seemed to find ideas about how to tackle similar issues in their beliefs or classrooms because they would often say things like “I am writing that down as something to try”, “I never thought of it that way but I think that’s really true”, or “Wow! She never told the students if they were right or wrong! They figured it out for themselves. I think that’s a really big part of why her class is the way it is. I struggle with that”.

The articles and videos served to offer teachers’ the opportunity to begin deciding for themselves what changes they felt were worthwhile and significant as well as build practical knowledge about how to make those changes as suggested by Richardson (1990). The fact that many of these articles were written by or about real teachers where implementation was sometimes messy and challenging satisified Desimore’s (2009) requirement that professional development connect with teachers’ current knowledge and beliefs about classroom practices. It also served a portion of Desimore’s mechanism model for professional development by helping teachers build new knowledge, skills, beliefs, and attitudes because many of these teachers from the articles were further down the road of reform than the participants themselves. This also resonated with Shulman’s (1986) recommendations that teachers’ knowledge develop from cases rather than propositions. The discussion about the videos and articles furthered all of the above processes as teachers grappled with one another’s

ideas. In this way, the teachers were collectively constructing new standards and values for instructional practice as recommended by Richardson (1990).

Another portion of the professional development that likely encouraged participants' shifts was the analysis of their teaching partner's practices. According to Ball and Cohen's (1999) one way to help teachers learn from their own practice is by learning observational frames that help them analyze records of practice such as video. In using these observation frames on their peers' baseline video, teachers were taking an active role (Desimore, 2009) and beginning to build strategic knowledge (Shulman, 1986) to carry back to their classrooms.

During this portion of professional development, Cecilia shared the thought that "I don't even need to see the results of her (Ann's) analysis. From watching her video, I pretty much know where I am going to be at. I evaluate too much, I don't wait long enough, and my questions are all low-level and closed." Recall, these two teachers worked within the same department. They had a sense of how their practice compared with one another from their work the year prior in video clubs as well as their everyday interaction. Something about analyzing her partner's practices helped Cecilia reflect on her own practice and by doing so she was taking an active role in the ideas from the readings, videos, and discussion.

During the last PD session, while analyzing her partner's video Cecilia also communicated to the group "You know, so much has changed in my practice since I've started this that I don't feel like this data is even accurate anymore." Something within

the experience of analyzing her partner's video and awaiting data from her own practice was bringing about strategic knowledge about instructional decisions Cecilia made a half a year prior. I found this comment interesting because it was made months before the participants were officially making any changes to their classroom in the action research cycle.

Another activity that I believed helped teachers to collectively shift away from a focus on *products*, *telling*, *evaluation,* and *evaluative revoicing* toward a focus on *process, revoicing, invitation,* and *uncovering* was the session built for them to create their action research plans. This session gave participants time to compare their practice with their ideas of which changes would be significant and worthwhile. Richardson (1990) suggests that reflection and ultimately decision is a necessary part of the process of teacher change. The session included discussion times for participants to bounce ideas off of one another and collectively consider what types of changes they valued. Richardson also supports this idea as part of what it means to understand the situated nature under which teaching and changes to teaching takes place. Thinking about Desimore's mechanism model (2009) for professional development, the participants at this point in the year had experienced a professional development that carried all the characteristics to be effective, had been active building new knowledge, skills, beliefs, and attitudes. They were now ready to enact these new elements into their practice, The action research cycle gave them the prime opportunity to begin those shifts. For all of these reasons, I was not surprised at the collective shifts the participants made away

from *telling*, *evaluation,* and *evaluative revoicing* toward *revoicing, invitation,* and *uncovering* as well as a shift away from *products* toward *processes.*

*5.4.2: Discussion about perplexing breadth data results*

I was surprised by some of the results in the data and those surprises warrant further discussion. One surprise was the breadth of teachers' responding practices did not broaden as predicted by Crespo (2007). Notice a decreased variety of responding moves imagined by participants across this experience (Table 13).

*Table 13: The number of participants' imagined responding moves across the year*

| | **Types of Imagined Responding Moves in the Baseline Data** | **Types of Imagined Post-PD Responding Moves in the Baseline Data** | **Types of Imagined Post-AR Responding Moves in the Baseline Data** |
|---|---|---|---|
| **Ann** | 5 | 3 | 3 |
| **Brenda** | 5 | 2 | 4 |
| **Cecilia** | 5 | 4 | 3 |

Similarly, notice a decreased variety of responding moves enacted by participants (Table 14).

*Table 14: The number of participants' enacted responding moves across the year*

| | **Types of Enacted Responding Moves in the Baseline Data** | **Types of Enacted Post-PD Responding Moves in the Baseline Data** | **Types of Enacted Post-PD Responding Moves in the Baseline Data** |
|---|---|---|---|
| **Ann** | 8 | 6 | 7 |
| **Brenda** | 9 | 8 | 8 |
| **Cecilia** | 5 | 6 | 7 |

When considering the breadth graphs from chapter 4, a better description is that the participants' practices shifted from an emphasis on certain moves in the baseline data (*telling*, *evaluating,* and *evaluative revoicing)* toward other moves in the post-PD and post-AR data (*revoicing, invitation,* and *uncovering)* as discussed above. So the question stands: why the difference between what Crespo postulated and the data from this study?

Let's begin by considering the imagined practices data. The question here is why participants narrow their repertoire of imagined responding moves. One possible factor at play here could be the nature of the changes teachers were trying to make. In Crespo's work, she is studying the ways pre-service teachers change as they progress through a teacher education program. In this study, we have teachers making changes on a much shorter time cycle and the changes are being driven by an action research plan. Did the action research plan work to focus the teachers in such a way that they in turn narrowed their responding practices? Consider Brenda as an example. Her action research plan involved the increased use of invitation. In her baseline data, Brenda imagined using *invitation* 14% of the time along with using four other responding moves. In her post-PD survey, Brenda used *invitation* 60% of the time along with only one other responding technique. In the post-AR survey, Brenda *maintained* her high-level use of *invitation* and spilt the remaining 40% amongst three other moves. The main thrust of Brenda's imagined change is clearly to increase her use of *invitation*. This goal might have come through so strongly in her thinking about responding, that

the narrowing of her overall breadth of practices was not an issue in her mind. The literature that I reviewed for this present study does not indicate whether it is common for teachers to narrow as a result of focusing on particular practices. This could be an interesting question for future study.

Next, let's consider the enacted practices data. For this data, the question is why we see participants make a shift, rather than increase the breadth of their responding moves as predicted by Crespo. One possible reason could be teachers in this present work started in different places than the teachers in Crespo's work. This possibility seems likely given Crespo's postulate was developed primarily on a population of pre-service and new (in-service) teachers. In this present study, each teacher had accumulated more than ten years in the classroom.

The classroom experience of the participants in this work might have afforded them opportunities to explore a greater variety of responding moves. If this were the case, we would expect to see the participants starting with a broad repertoire within the baseline data. Consider Ann and Brenda. Both start with fairly a broad number of moves in their baseline data. Ann was using all the moves but one and Brenda all but two. Ann and Brenda would have had difficulty broadening their practices because the baseline data included so many already. So instead of adding new moves to their repertoire and thereby increasing the breadth of their responding practices, Ann and Brenda shifted the level of use of various responding moves that were already in their

repertoire. And being conscious of the moves may result in narrowing one's range of moves in order to be more deliberate in choosing how to respond.

In Cecilia's case, however, she began with a narrower set in her enacted practices of responding. We see that she did, in fact, steadily increase the breadth of her responding practices. Cecilia may not have had the same exposure to various responding moves given her time away from the classroom. Therefore, the increased breadth in her case mirrors something closer to that which was observed in Crespo's work.

In considering each of these possibilities, I find that we have something to learn about the ways we can describe or predict teacher change. What we take as novice and experienced, for example, might have an impact on what we can say about how teachers' practices change over time. The conditions under which change was achieved might also have a fairly large impact on the ways teachers implement those transformations. More work needs to be done with a wider variety of teachers in studies larger than this one to revise Crespo's hypothesis to include these differences.

A second surprising aspect of the data was teachers' focus data did not completely align with Crespo's (2007) hypothesis arguing that teachers' responding practices would shift from a *product-focused* stance toward one more balanced between *mathematical products* and *processes*. In some ways the data aligned with this claim; namely, there was a shift away from *product-focused* strategies toward *process-focused*

strategies. However, the teachers' practices did not achieve the balance predicted by Crespo.

One explanation is that a one-year window was too short for this balance to be achieved. These participants may have achieved more balance with additional time. From this perspective, I can imagine the percentage splits were heavier towards *process* because the teachers were trying to help their students transition to a new way of doing mathematics as well as learning a new approach themselves, one that placed more emphasis on *mathematical processes*. Possibly a heavier emphasis on *mathematical processes* is necessary in this sort of transition because a teacher is compensating for those students who had not made sense of these new ways of doing mathematics and for their own relative inexperience with teaching this way. Maybe Ann, Cecilia, and Brenda knew that even a slight focus on product would have undone all of their work trying to help students understand that they truly valued *mathematical processes.* Possibly, we would see teachers achieve more balance in a class where reform practices are operating for a longer period of time.

Another possibility is that Crespo's (2007) hypothesis needs to be revised to more clearly define what is meant by balance. It is not clear that "balance" translates into a 50/50 split between *mathematical processes* and *products*. If a 50/50 split is not what is meant, then this notion of "unbalance" within participants' data should be reconsidered. In reconsidering what "balance" might mean, I think it would be helpful to first consider the enacted practices data that represents the journey of these three

participants (Table 15). Notice Brenda's baseline practices were already beyond a 60/40 split between *mathematical processes* and *products* and Cecilia's baseline practices were close to a 50/50 split. As participants progressed, we see them shift towards a heavier focus on *mathematical processes*. Ann shifts to a 70/30 split and *maintains* that split through her action research. Brenda shifts to a 90/10 split and then *reverses* back to a 70/30 split. Cecilia shifts to a 75/25 split and then presses forward to a near 95/5 split.

*Table 15: The participants' percentages of process-focused moves across the year*

| | **Enacted Baseline Percentage of Process-Focused** | **Enacted Post-PD Percentage of Process-Focused** | **Enacted Post-AR Percentage of Process-Focused** |
|---|---|---|---|
| **Ann** | 23% | 71% | 72% |
| **Brenda** | 62% | 92% | 72% |
| **Cecilia** | 45% | 77% | 93% |

I do not argue that Cecilia's first transcript is an accurate example of balanced, even though it represents a near 50/50 split. Recall Cecilia's interaction with her students; it was far from feeling balanced between *process* vs. *product* with a series of fill-in-the-blank questions. Rather, the transcript was a good example of what Ball (1991) referred to as asking questions for the purpose of "getting right answers" (p. 4). And so we have an example of a classroom interaction (from the first vignette in section 4.4.3) for which the split between *process-focused* and *product-focused* was 55/45, and yet the reader will recall how that interaction failed to feel balanced.

Even within Brenda's first video with a 60/40 split, she missed opportunities for student learning because she pressed towards the *mathematical product.* Recall her

interaction with Aaron. Her decision to *tell* Aaron why ripping off a portion of the fraction strip could have been handled differently. If she had instead used *invitation*, for example, she may have gotten other students to engage with Aaron on his technique, shifting the focus of conversation toward students' *mathematical processing.* This alternative move would have provided students with an opportunity to learn more about the mathematics they were engaging.

On the other hand, I am not arguing that a 95/5 split should be considered balanced either. Recall Cecilia's final video. At the end of the hour, Cecilia is still unwilling to put closure on whether three to the zero power is equivalent to one or zero. She decided to leave that question open. And while I can think of some times one might want to do that, say at the beginning of a unit, I can also think of times that this sort of ambiguity might not be helpful.

I do not see benefit in declaring a particular percentage split to be balanced. I do, however, want to challenge the notion that balanced means a 50/50 split is ideal. We witnessed conversations that were a near 50/50 split, and those conversations left much to be desired in terms of mathematical discourse. I am proposing ultimately that whatever we mean by balanced, Ann's or Brenda's final video (with 70/30 splits) should fit that determination more than their baseline videos.

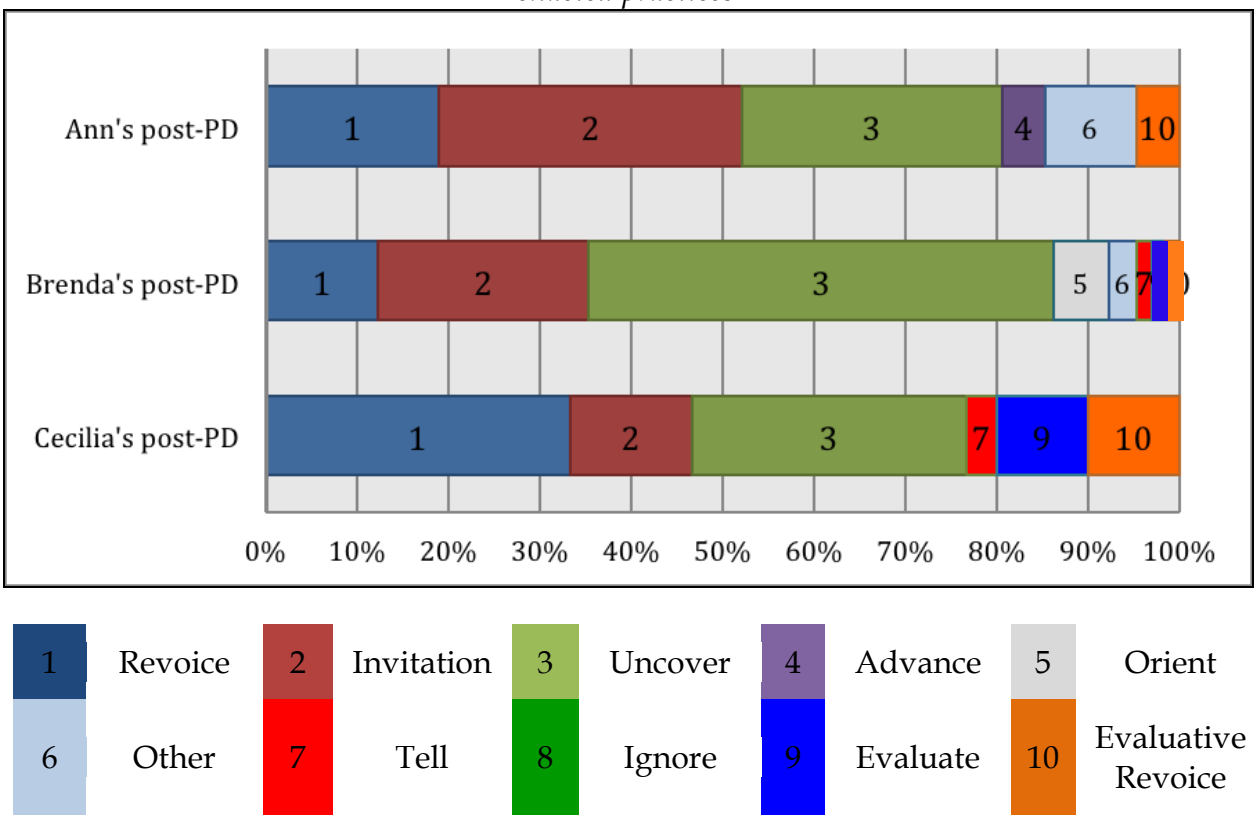
Recall how Ann allowed the conversation about the restrictions on variables in rational expressions to ensue between students for quite some time by focusing heavily on students' *mathematical processes*. At the end of the conversation, however, Ann

emphasized Coe's observation that summarized the correct mathematical notion. Consider Brenda's transcript as well. She stayed focused on *mathematical processes* throughout the conversation as students grappled to discover the big mathematical ideas behind various fraction strips they just created. Brenda made several moves that were simply about what students were *processing* as opposed to driving to a *mathematical product*. In the end, a student named Shelby was able to put a conclusion on the conversation by summarizing previous observations of several students. More longitudinal research would have to be done to know for sure whether this imbalance is a result of a short cycle of change or a need to adjust the definition for what we mean by balance.

*5.4.3: Discussion about the differences between the participants in the post-AR enacted data*

For the post-PD enacted data, participants were fairly homogeneous in terms of changes they made. They all shifted away from *product-focused* responding moves in favor of *process-focused* responding moves. Also, participants all made the same approximate number and types of shifts (generally increasing *revoicing, invitation,* and *uncovering* while decreasing *evaluation, telling, ignoring,* and *evaluative revoicing*). To be convinced of this, one simply needs to consider similarities in the profiles for these three teachers in their post-PD data (Figure 13).

*Figure 13: Comparison of Ann's (n=21), Brenda's (n=68), and Cecilia's (n=30) post-PD data for enacted practices*



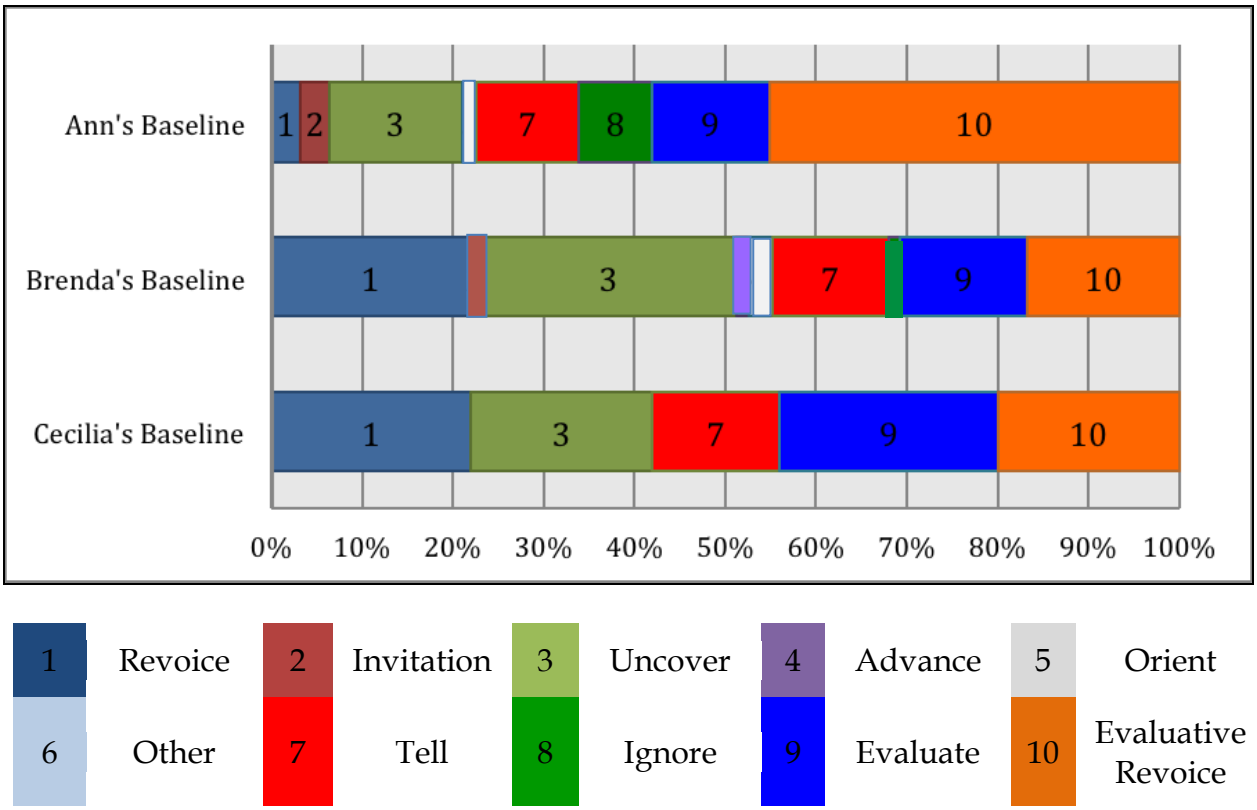

For the post-AR enacted data, however, the changes that participants made were quite different. These differences warrant further discussion. To understand this difference it helps to compare the percentages (table 16) of changes that these participants *strengthened*, *maintained*, or *reversed,* as they moved through the action research.

*Table 16: The percentages of post-PD enacted moves participants strengthened, maintained, and reversed in the action research cycle*

| | **Enacted Practices** | | | |
|---|---|---|---|---|
| | **Number of substantial post-PD changes made** | **Percentage of changes *strengthened*** | **Percentage of changes *maintained*** | **Percentage of changes *reversed*** |
| **Ann** | 9 | 0% | 78% | 22% |
| **Brenda** | 7 | 0% | 29% | 71% |
| **Cecilia** | 7 | 71% | 29% | 0% |

During the action research cycle, Ann *maintained* most of the post-PD changes she enacted. Brenda, on the other hand, tended to *reverse* her changes while Cecilia generally *strengthened* the changes she had made. Why the difference? One possible reason for this difference could be that Ann, Brenda, and Cecilia began in different places in their practices of responding.

*Figure 14: Comparison of Ann's (n=62), Brenda's (n=80), and Cecilia's (n=50) baseline data for enacted practices*

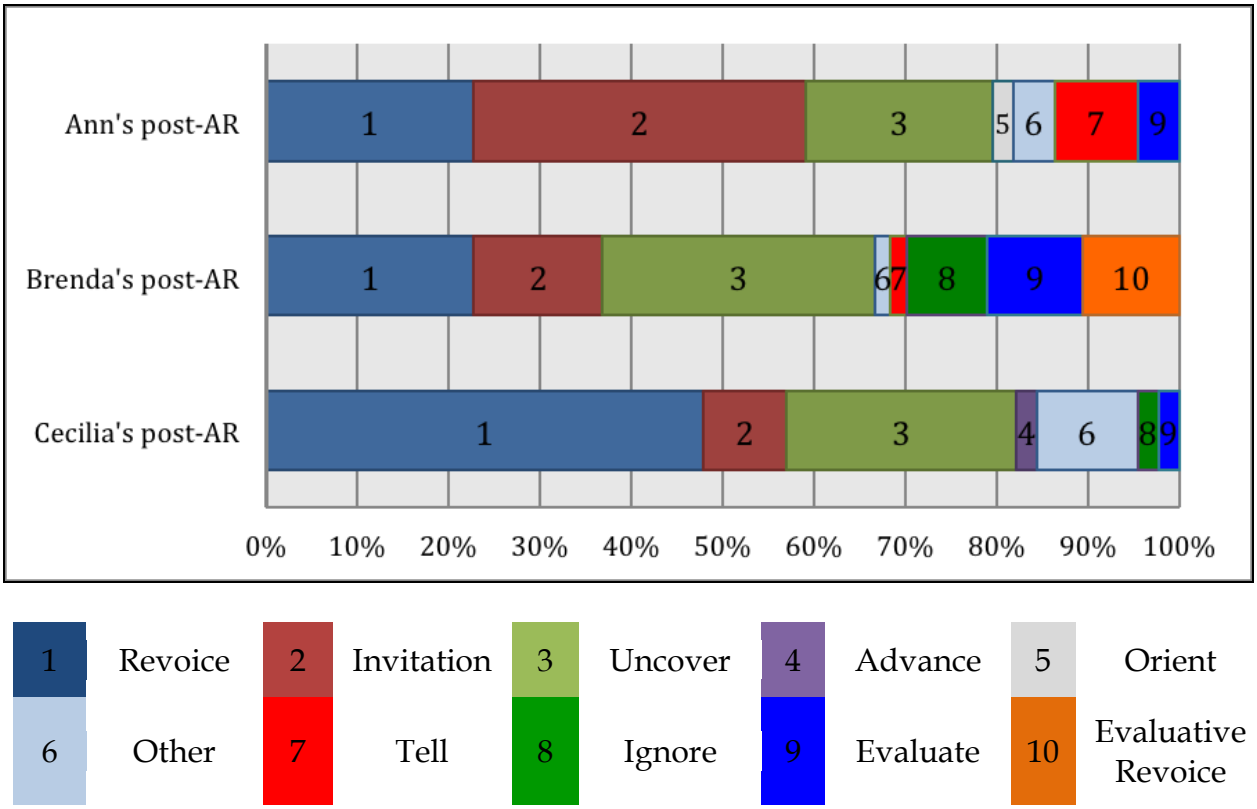



As seen in Figure 14, Brenda's baseline responding practices were very different from Ann's and somewhat different from Cecilia's. To observe this difference more easily, consider two groups of responding moves: 1) *revoicing*, *invitation*, and *uncovering* and 2) *ignoring, evaluating,* and *evaluative revoicing*. When comparing the first groups, Brenda's use of *revoicing, invitation,* and *uncovering* surpasses both Ann's and Cecilia's use. Brenda's use of this group is more than twice that of Ann's in the baseline data and more than ten percentage points greater than Cecilia. Next, considering the second group of *ignoring, evaluating,* and *evaluative revoicing*, we see that Brenda's baseline use

of these moves is less than half of Ann's baseline use and more than ten percentage points lower than Cecilia's use.

The fact that Brenda's starting place was so different than her peers could have played a factor in her *reversals.* In many ways, Brenda's initial practices were closer to ideas discussed in the year-long professional learning experience than Ann's or Cecilia's initial practices. This possibility is evident when considering the final data for these teachers' practices shown in Figure 15, particularly when considering the groups of practices mentioned above. Participants ended their experience with similar profiles of responding practices, namely a heavy use of *revoicing*, *invitation*, *uncovering* and a much diminished use of *telling*, *evaluating,* and *evaluative revoicing*. Maybe Brenda had fewer changes to make to her baseline data in order to look like this post-AR profile and therefore she could afford make more *reversals* than the other two participants.

*Figure 15: Comparison of Ann's (n=44), Brenda's (n=57), and Cecilia's (n=88) post-AR data for enacted practices*




A second possible reason for the difference across participants may be variations in the support they were receiving from within their building. Recall that Ann and Cecilia were both high school teachers, working in the same building, and had selected one another to be teaching partners. Brenda, on the other hand, had a teaching partner who went on maternity leave early in the year. That left her working in a building with no other participating mathematics teachers. Furthermore, she was the only one in her department trying to teach out of a reformed textbook focused on these instructional methods. In this way, Brenda was isolated in her practices in ways that Ann and Cecilia were not.

Though Brenda received support through action research meetings from myself, the other facilitator, and the other teachers, the support of a teaching partner was missing back in her building. This may have played a crucial role given Richardson's (1990) admonishment to recognize the importance of the situated nature of teachers trying to make changes. Ann and Cecilia had opportunities to collectively construct the ways they were going to *maintain* or *strengthen* changes they had made as they moved through the action research cycle. Brenda had less opportunity to do that since she was working in isolation for the most part.

More research with larger numbers of participants would need to be conducted to understand why these differences exist in this action research portion of the year-long professional learning experience. Next, I discuss implications from these conclusions and move to make suggestions for future research.

### 5.5: Implications

The findings of this research help me identify and begin addressing three problems for the future design and implementation of this professional development. The first problem is how the design of the action research cycle can be changed to help teachers make more consistent changes to their practices. When the post-PD changes align with the goals of the professional development, the action research cycle should help teachers to *maintain* or *strengthen* changes they made in the professional development. While the post-PD changes were aligned in this study, the action research did not uniformly help teachers maintain or strengthen changes. All three

participants had some amount of *maintenance* and *strengthening* of their post-PD changes. Cecilia's and Ann's experiences were preferable to Brenda's because they were able to *strengthen* and *maintain* respectively rather than *reverse* their decisions. The problem for future iterations of this professional learning experience is how the action research experience can better support a teacher like Brenda.

The first problem I identified from my data is a need to better support teachers whose starting place, like Brenda's, is closer to the ideal. Possibly the action research session conversations did not provide enough challenge to support Brenda in *strengthening* and *maintaining* changes she initially made. During the action research sessions, a significant amount of time was allotted for teachers to share how their action research plans were coming along as well as offering suggestions to one another.

In those conversations, the group saw Brenda as having her practices "all together," due mostly to their knowledge of her baseline videos. Participants had previously viewed one another's videos as part of the first year's initiatives to have teachers share their practices. Brenda's baseline practices were known by the group to be closer to the goals of the professional development when compared with other participants. Also, because Brenda had used these types of reformed techniques for years, there simply was not the same amount of drama that came with her sharing as with the other teachers. She was not struggling like other participants to help students shift into new roles because, in her class, students were already performing roles akin to

this reformed model. She also was not struggling to implement new, richer tasks because she had been using rich tasks for years.

Overall, when one considers Brenda's status in the group and the subdued nature of her sharing, it would not be surprising if the group did not provide Brenda with the same amount of challenge as other participants received. If this were the case, Brenda did not receive as many suggestions from her fellow teachers or facilitators about changes she could make in the month following the meeting. In this way, the support that Brenda received to *maintain* or *strengthen* changes was very possibly different simply because of her starting place was different.

A second problem that was identified was how to better support teachers like Brenda who find themselves participating in such an initiative without very much support in their building. This could very possibly be one reason why Brenda did not sustain the same amount of change as Ann and Cecilia. Brenda was able to utilize the science teacher in her building to videotape her lesson; however, her planning was done in isolation. Because of this difference, in future iterations, teachers will be strongly encouraged to sign up as teams consisting of at least two teachers in the same building. If this is not possible, then arranging time for teachers from different buildings to work more closely together through the action research cycle might be a suitable second alternative.

The third problem is the possibility of scaling this experience to a larger group of teachers. Though ideal to work with a small learning team, the demand to help more

teachers alter their instructional practices is ever increasing. That being the case, one of the most important steps for this present work is to consider how to scale this experience for a larger group of teachers. One key feature of this work was the participants' learning of new observational tools to reconsider their practices. The creation of a transcript for their partner's work, however, was overwhelming for participants and did not add much value to their knowledge of instruction. So in future iterations of this experience, teachers' baseline videos will be professionally transcribed in advance in order to make better use of their time when analyzing practices.

Another key aspect of the workshop was the one-on-one feedback each participant received during the analysis of her practices. In future iterations, the leader-to-participant ratio should be kept small in order to achieve similar results. Those leaders who provide one-on-one feedback to participants during their data analysis would not necessarily have to be the same individuals as the facilitators. Rather, someone needs to help participants with their uncertainty about this analytical stance towards practices. I can imagine teacher-leaders who are fluent with the observational frames helping participants overcome uncertainty with this new analytic stance.

A third key feature was that participants had ample time to reflect on and share their developing ideas with concepts. Participants were given time to consider these ideas individually, with partners, with a facilitator, and in a small group. That time was possible because of a low facilitator-to-teacher ratio. For this reason, future iterations of

this professional development model should maintain a low enough facilitator-to-teacher ratio to allow for some one-on-one and small-group discussions.

A final key feature was the facilitators' extensive knowledge about the participants and the changes they were attempting to make. In a larger group, this would be more difficult to track. With the findings from this study, I can imagine leveraging information from the survey to help with this. In other words, the survey results could be used to formatively assess participants' imagined practices to inform facilitators about which participants may need more support.

### 5.6: Future Research

In this section, I make three suggestions about possible avenues for future research that builds upon this dissertation. First, given the relative success that teachers within this study had in making some significant change to their responding practices, I think a replication study would be in order. In short, I suggest that future work include an implementation and study of this professional learning experience with a larger group of teachers. Obviously, the model is not perfect and some sensible changes, many suggested in the previous section, should be made.

A second suggestion for future research is trying to design a study to understand similarities and differences between imagined and enacted practices. In this study, the number of participants was simply too small to make any guesses about how imagined and enacted practices are related. In a larger study, however, this relationship could be studied and better understood in order to leverage powerful results within practice.

For example, one possible outcome is we learn that change within one's imagined practices is predictive of a change within one's enacted practices. If this could be shown reliably true, there would be several applications within the field. One such application could be a facilitator would use an imagined practices survey to anticipate changes teachers are making in their classrooms. The facilitator would use this information from participants' imagined practices to formatively assess participants' progress within their enacted practices. This information could be used to help a facilitator make better instructional decisions within the professional development.

A third suggestion for future research includes better understanding how teachers' prior experiences impact their ability to make and sustain changes. Brenda began this experience further along in her practices of responding. This can be understood by simply considering her levels of *evaluation, ignoring* and *evaluative revoicing* in contrast to her counterparts' levels. And yet, Brenda had the most difficulty with sustaining changes she had made to her responding practices as she moved through the action research. Did Brenda's prior knowledge impact her motivation or ability to sustain changes? Understanding how teachers' prior experience affects their ability to make changes to their instruction moves the field closer to understanding how to help support learning needs of all teachers in these efforts to transform teaching practices.

In closing, I believe the results from this study should motivate the field to pay more attention to helping teachers change their practices of responding. Conversation

about the importance of creating and implementing tasks is prevalent. It is accepted knowledge that the cognitive demand and implementation of a task make a significant difference in students' opportunities to learn mathematics. Consider the many projects that have been developed for the purpose of helping teachers rethink curriculum, develop meaningful tasks for the teaching and learning of mathematics, and implementing these reform curricula or tasks: Connected Mathematics Project, Mathematics in Context Project, University of Chicago School Mathematics Project, Quasar Project, Balanced Assessment for the Mathematics Curriculum, Mathematics Assessment Resource Service or MARS, The Shell Centre for Mathematics, Mathematics Assessment Project, and many more.

There has also been quite a bit of research, with seminal pieces dating back to Mary Budd Rowe's efforts in the 1980's, which help us understand the importance of teachers interpreting students' thinking. It is fairly well-charted territory that teachers' listening carries significant implications for teaching and learning. Now consider some of the professional development efforts designed to help teachers improve their instructional practices by focusing on students' mathematical thinking: Cognitively Guided Instruction, Developing Mathematical Ideas, Lesson Study, and many others.

Like the categories of posing and interpreting, we can identify a reasonable amount of research in the field about the importance of this notion of teachers' practices of responding. Looking within the literature about classroom discourse, we see much that touches on these ideas of shifting the ways teachers respond to students' ideas

(Cazden, 2001; Mehan, 1985). We get closer to the specificity of the ideas studied in this dissertation when we consider the discourse literature that focuses on teacher moves within classroom discourse (O'Connor, 1998; O'Conner & Michaels, 1993 & 1996). From this research, we get a clear picture of what is and is not ideal about teachers' practices of responding, namely an overly evaluative stance. However, it is difficult to name any widely available research-based efforts that help teachers to change their practices of responding.

Coming back to where we started, recall the vision set forth by the NCTM Professional Stands for Teaching Mathematics (1991) for orchestrating discourse. The first two bullets describing what a teacher's role should include read, "posing questions and tasks that elicit, engage, and challenge each student's thinking; listening carefully to students' ideas" (p. 86). Our field has responded to these two ideas by providing practitioners with both research and support to implement these changes to their instructional practices.

The next four bullets continue "asking students to clarify and justify their ideas orally and in writing; deciding what to pursue in depth from among the ideas that students bring up during a discussion; deciding when and how to attach mathematical notation and language to students' ideas; deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty" (p. 86). These four bullets speak to the need for teachers to be proficient at practices of responding to students' ideas. We have studied teachers'

proficiency of these practices and determined the IRE structure is disappointingly prominent. The question remains: how are we supporting teachers to move from what is typical to something new?

Recall that Smith (1996) suggested that, in order to make the shift from teaching as telling to instructional practices that reflect the ideals of reform, teachers will need new "moorings" of choosing worthwhile and engaging tasks, anticipating student reasoning, and orchestrating discussions around student thinking. Again, as a field we have certainly researched the practices of posing, interpreting, and responding and found they are indeed necessary practices for teachers trying to make this shift. We can identify many efforts that address the first two moorings. But where could a practitioner learn to implement the ideas prevalent within the research about their practices of responding? More efforts need to move toward charting this territory so that all mathematics teachers have access to research-based programs built to help teachers better meet the needs of their students by improving their practices of responding to students' ideas in less evaluative ways.

The need for this sort of shift has and will continue to grow in the coming years as the policy environment around mathematics instruction has changed dramatically with the introduction of the CCSSM. For more than two decades, NCTM has made the recommendation that teachers make these sorts of changes to their discourse in order to better support students' learning to communicate their thinking and critique their peers' thinking. The CCSSM has now moved this recommendation into the arena of

being an assessable standard with the inclusion of the Standards for Mathematical Practice. As a result, many who were unaware or uninterested in the recommendation from NCTM, have joined the conversation and are invested in providing the resources for mathematics teachers to change their instructional practices. This interest creates fertile ground for this initiative to grow because of its focus on helping teachers change their responding practices in ways that will help them better meet the needs of the CCSSM Standards for Mathematical Practice.

# APPENDICES

# Appendix 1: Survey Prompts

## Survey of Imagined Practices

*Responding Survey Part 1:*

**In the following portion of the survey you will be asked to consider several mathematics teaching scenarios and to imagine how you might act in the given situations. There are NO right or wrong answers. Instead we are interested in learning as much as we can about how you imagine handling the given scenarios. Please provide as much detail as you can in your response. Thank you!**

1a) Imagine you are going to ask your class to solve the following algebra problem. To be ready for the discussion about the problem with your students, work through the problem yourself.

Please describe how you'd solve this problem.

$24 + 2x = 56 - 6x$

1b) Now imagine you've posed the problem to your students.

$24 + 2x = 56 - 6x$

i) What are three things you expect to see and hear as you walk around the room that you would want to address with the class during the debriefing of the problem?

2a) Now imagine students' independent work comes to an end. It is clear the students' group discussion is coming to a close. Now you need to imagine what you will say after students have had sometime to work on the problem.

$24 + 2x = 56 - 6x$

i) Share at least three different ways you can imagine beginning the discussion after the students have worked on the problem. How are these ways different?

*Responding Survey Part 2:*

3a) Imagine that a student shows on the board the following strategy for solving.

$$\begin{array}{l} 24 + 2x = 56 - 6x \\ \quad +6 \qquad\quad +6 \end{array}$$

$$\begin{array}{l} 30 + 2x = 56 \\ -30 \qquad -30 \end{array}$$

$$\begin{array}{l} 2x = 16 \\ x = 8 \end{array}$$

i) What is one thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?

ii) How likely it is you would respond in this way?

Certain Very Likely Possible

iii) Why?

iv) What is another thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?

v) How likely it is you would respond in this way?

Certain Very Likely Possible

vi) Why?

vii) If you had to pick just one response, which of these would be most like what you can imagine saying next.

A) That is incorrect. Can you find your error?

B) Tell me about what you were trying to do when you added 6.

C) This is incorrect because you can't add 6x to 24.

D) How do you know eight is the solution?

viii) Why would you say that?

3b) Imagine that a student shows on the board the following strategy for solving.

$$
\begin{aligned}
24 + 2x &= 56 - 6x \\
- 2x \quad & \quad -2x \\
\\
24 &= 56 - 8x \\
-56 \quad & \quad -56 \\
\\
- 32 &= -8x \\
\\
4 &= x
\end{aligned}
$$

i) What is one thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?

ii) How likely it is you would respond in this way?

Certain Very Likely Possible

iii) Why?

iv) What is another thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?

v) How likely it is you would respond in this way?

Certain Very Likely Possible

vi) Why?

vii) If you had to pick just one response, which of these would be most like what you can imagine saying next.

A) This is correct. It may have been easier if you had chosen to add the 6x instead of subtract the 2x.

B) Is there another way you could have done this?

C) Excellent, very nicely done.

D) What makes you confident your answer is right?

viii) Why would you say that?

3c) Imagine that a student shows on the board the following strategy for solving.

$24 + 2x = 56 - 6x$

$12 + x = 28 - 3x$

$12 + 4x = 28$

$4x = 16$

$x = 4$

i) What is one thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?

ii) How likely it is you would respond in this way?

Certain Very Likely Possible

iii) Why?

iv) What is another thing you can imagine saying or doing next? What would you want to accomplish in saying or doing this?

v) How likely it is you would respond in this way?

Certain Very Likely Possible

vi) Why?

vii) If you had to pick just one response, which of these would be most like what you can imagine saying next.

A) Could you explain how you got from step one to step two?

B) This is correct, but your method doesn't always work. What if I changed the negative 6x to a positive 3x?

C) This is, by far, the best solution I have seen today.

D) How do you know four is a solution?

viii) Why would you say that?

*Responding Survey Part 3:*

**In this segment, you will be watching two different clips from two different classrooms. You will be asked to provide your reactions to how the teacher in those clips responds to students.**

**Directions:** Now we will watch a video of a classroom interaction. Imagine you are watching how one of your colleagues interacts with his or her class. During the clip you may use the space below to take field notes about how the dialogue unfolds. At the end of the clip, you will use these 'field notes' to give feedback to your colleague about his practices of responding to students.

4a) In giving feedback to your colleague about her practices of responding, what are some things you would want to draw their attention to? What can you imagine saying?

**Directions:** Now we will watch a video of a classroom interaction. Imagine you are really watching how one of your colleagues interacts with his or her class. During the clip you may use the space below to take field notes about how the dialogue unfolds. At the end of the clip, you will use these 'field notes' to give feedback to your colleague about his practices responding to students.

4b) In giving feedback to your colleague about his practices of responding, what are some things you would want to draw their attention to? What can you imagine saying?

**Directions:** Next, you will be asked to reflect on these two different classroom interactions. More specifically you will be asked to consider similarities and differences across the two clips.

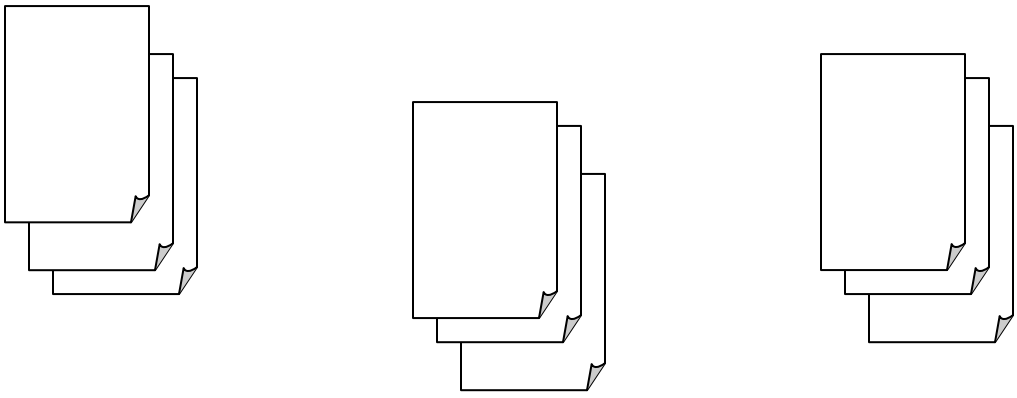
4c) What similarities or differences did you notice overall, between the two dialogues in these two classes?

## Appendix 2: Method for calculating the probability of randomly placing surveys in the correct sequence.

Calculating the probability of correctly re-sequencing 7 of the 9 de-sequenced surveys.

To begin, the researcher had the following situation: Each participant took one survey on three different occasions across the intervention (beginning, middle and end). The researcher had every survey transcribed so that the participant's identity and the time the survey was given the participant took the survey was indistinguishable. The researcher also had the names removed from the surveys so that the participants' identity was unknown. Each participant's surveys were piled and put in random order. The goal was for the researcher to correctly sequence the participants' survey based on looking for changes to participants' responding practices.

| Participant 1<br>Random Sequence | Participant 2<br>Random Sequence | Participant 3<br>Random Sequence |
|---|---|---|



In ordering the piles there are technically six ways any three surveys could be sequenced:

Assuming the correct order is 1,2,3 . . .
Ways to sort the piles

1,2,3 (Correct)
1,3,2 (Incorrect)
2,1,3 (Incorrect)
2,3,1 (Incorrect)
3,1,2 (Incorrect)
3,2,1 (Incorrect)

However, in thinking about the incorrect ways of sequencing, there are some that are "more wrong" than others. To define this "more wrong" succinctly, we will determine the levels of incorrect based on the distance a survey is from its correct position as follows:

If a survey is in the correct position, it will receive a distance of zero.
If a survey is one away from its correct position, it will receive a one.
If the survey is two spots away from its correct position, it will receive a two.

The total for the sequences will be arrived at by summing the total distances. For example, let's consider the last incorrect possibility listed above.

3, 2, 1

"3" is two positions away from its correct place; therefore it earns a score of two.
"2" is in the correct position, so it earns a score of zero.
"1" is also two positions away from its correct place, giving it a score of two.

Overall, this incorrect sequence will earn a score of 4.

Using this system, the sequences receive the following scores:

1,2,3 Score = 0
1,3,2 Score = 2
2,1,3 Score = 2
2,3,1 Score = 4
3,1,2 Score = 4
3,2,1 Score = 4

We will refer to those sequences with a score of 2 as a level 1 mistake. We will refer to those sequences with a score of 4 as a level 2 mistake. So what are the chances of a researcher randomly sequencing participants' surveys such that two of the participants' surveys are in the correct order and one participants' survey contains a level 1 mistake?

| | |
|---|---|
| Pile 1 – Correctly Sequenced | Probability 1 out of 6 |
| Pile 2 – Correctly Sequenced | Probability 1 out of 6 |
| Pile 3 – Level One Mistake | Probability 2 out of 6 |

Probability is (1/6) * (1/6) * (2/6) = 2/216

# REFERENCES

## Research References


Ball, D. (1988). Unlearning to teach mathematics. *For the learning of mathematics, 8(1),* 40-48.

Ball, D. (1991). What's all this talk about "discourse"? *Arithmetic Teacher, 39(3)*, 44-48.

Ball, D. (1997). What do students know? Facing challenges of distance, context, and desire in trying to hear children. In B.J. Biddle et al. (Eds.), *International Handbook of Teachers and Teachers*. (p. 769-818). Netherlands: Kluwer.

Ball, D. & Cohen, D. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In G. Sykes and L. Darling-Hammond (Eds.), Teaching as the learning profession: Handbook of policy and practice (pp. 3- 32). San Franciso: Jossey Bass.

Black, P. & Wiliam, D. (1998). Assessment and classroom learning. *Assessment in Education: Principles, Policy & Practice. 5(1)*, 7-74.

Boaler, J. & Humphrey, C. (2005). *Connecting Mathematical Ideas: Middle School Video Cases to Support Teaching and Learning.* Portsmouth: Heinemann.

Carpenter, T., Fennema, E., Peterson P., Chiang, C., Loef, M. (1989). Using Knowledge of Children's Mathematics Thinking in Classroom Teaching: An experimental study. American Educational Research Journal (26). pg. 499-531. Reprinted in Carpenter, T., Dossey, J., & Koehler, J. (Eds.). (2004). Classics in mathematics education research. Reston, VA: National Council of Teachers of Mathematics.

Cazden, C. (2001). *Classroom Discourse: The Language of Learning and Teaching.* Portsmouth: Heinemann.

Cohen, D. (1990). A revolution in one classroom: The case of Mrs. Oublier. *Educational Evaluation and Policy Analysis. 12(3)*, 311-329.

Crespo, S. (2002). Praising and correcting: prospective teachers investigate their teacherly talk. *Teaching and Teacher Education. 18(2002),* 739-758.

Crespo, S. (2007). *Studying elementary preservice teachers' learning of mathematics teaching: Preliminary insights.* In Lamberg, T. & Wiest, L. (Eds.)., Proceedings of the 29th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Stateline (Lake Tahoe), NV: University of Nevada, Reno.

Davis, B. (1997). Listening for differences: An evolving conception of mathematics teaching. *Journal for Research in Mathematics Education, 28,* 355-376.

Desimore, L.M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher, 38(3),* 181-199.

Dweck, C. & Leggett, E. (1988). A social-cognitive approach to personality. *Psychological Review, 95,* 256 - 273.

Fennema, E., Carpenter, T., Franke, M., Levi, L., Jacobs, V., & Empson, S. (1996). A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal of Research in Mathematics Education, 27(4),* 403-434.

Fuchs, L. & Fuchs, D. (1986). Effects of systematic formative evaluation: A meta-analysis. *Exceptional Children, 53(3),* 199-208.

Franke, M., Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practices. In F. Lester (Ed.), The Second Handbook of Research on Mathematics Teaching and Learning. Charlotte, NC: Information Age Publishing.

Hattie, J. & Timperley, H. (2007). The power of feedback. *Review of Educational Research, 77(1),* 81-112.

Herbel-Eisenmann, B. & Cirillo, M. (2009). *Promoting Purposeful Discourse: Teacher Research in Mathematics Classrooms.* Reston, VA:NCTM.

Hufferd-Ackles, K., Fuson, K.C., Sherin, M.G. (2004). Describing levels and components of a math-talk learning community. *Journal for Research in Mathematics Education, 35(2),* 81 - 116.

Kazemi, E. (1998). Discourse that promotes conceptual understanding. *Teaching Children Mathematics, 4(7),* 410-414.

Kazemi, E. & Hubbard, A. (2008). New directions for the design and study of professional development: Attending to the coevolution of teachers' participation across contexts. *Journal of Teacher Education, 59(5),* 428-441.

Lortie, D. (1975). *Schoolteacher: A Sociological Study.* London: University of Chicago Press.
Mehan, H. (1985). The structure of classroom discourse. In T. van Dijk (Ed.), Handbook of discourse analysis (Vol. 3, pp. 119 –131). London: Academic Press.

Mueller, C. & Dweck, C. (1998). Intelligence praise can undermine motivation and performance. *Journal of Personality and Social Psychology, 75,* 33-52.

National Council of Teachers of Mathematics (NCTM, 1991). Professional standards for teaching mathematics. Reston,VA: NCTM.

National Council of Teachers of Mathematics (NCTM, 2000). Principles and standards for school mathematics. Reston,VA: NCTM.

Nicol, C. (1999). Learning to teach mathematics: Questioning, listening, and responding. *Educational Studies in Mathematics, 37,* 45-66.

Richardson, V. (1990). Significant and worthwhile change in teaching practice. *Educational Researcher*, *19(7),* 10-18.

Rowe, M. (1986). Slowing down may be a way of speeding up! *Journal of Teacher Education, 37(1),* 43-50.

Sadler, D.R. (1989). Formative assessment and the design of instructional systems. *Instructional Science,18(2),* 119-144.

Sherin, M. (2000). Facilitating meaningful discussion of mathematics. *Mathematics Teaching in the Middle School. 6(2),* 122-125.

Shulman, L. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15(2),* 4-14.

Smith, J. (1996). Efficacy and Teaching Mathematics by Telling: A Challenge for Reform. *Journal for Research in Mathematics Education*, *27(4),* 387-402.

Stein, C. (2007). Promoting mathematical discourse in the classroom. *Mathematics Teacher, 101(4),* 285-289.

Stein, M.K, Engle, R.A., Smith, M.S., & Hughes, E.K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers beyond show and tell. *Mathematical Thinking and Learning. 10(4),* 313-340.

Stigler, J. & Hiebert, J. (1999). The Teaching Gap. New York: Free Press.

Turner, J, Midgley, C. Meyer, D. Gheen, M, Anderman, E. Kang, Y. (2002) The classroom environment and students' reports of avoidance strategies in mathematics: a multimethod study. *Journal of Educational Psychology*, 94(1), 88-106.

## Professional Development References


Ball, D. (1991). What's all this talk about "discourse"? *Arithmetic Teacher, 39(3)*, 44-48.

Bushman, L. (1994). Sometimes less is more. *Arithmetic Teacher, 41(7),* 378-380.

Dowling, D. (2009). Reading (article), writing (reflections) and a-risk-metic: Working to improve my practice. In Herbel-Eisenmann, B. & Cirillo, M. (Eds.), Promoting purposeful discourse: Teacher research in mathematics classrooms. (pp. 57-68). Reston,VA: NCTM.

Breyfogle, M. & Herbel-Eisenmann, B. (2004). Focusing on students' mathematical thinking. *Mathematics Teacher. 97(4),* 244-247.

Herbel-Eisenmann, B. & Breyfogle, M. (2005). Questioning our patterns of questioning. *Mathematics Teaching in the Middle School. 10(9),* 484-489.

Hopkins, D. (2002). A teacher's guide to classroom research. (2nd ed.). Buckingham, United Kingdom: Open University Press.

Humphreys, C. (2005). Opening the door to my classroom. In Boaler, J., & Humphreys, C. (Eds.), Connecting mathematical ideas: Middle school video cases to support teaching and learning. (p. 7-12). Portsmouth, NH: Heinemann.

Reinhart , S. (2000). Never say anything a kid can say! *Mathematics Teaching in the Middle School. 5(8),* 478-483.

Rowe, M. (1986). Slowing down may be a way of speeding up! *Journal of Teacher Education, 37(1),* 43-50.

Schifter, D., Bastable, V., Russell, S., & Wolek, C. (2001). *Developing Mathematical Ideas: Measuring Space in 1, 2, and 3 Dimensions.* 2002:Pearson

Sherin, M., Louis, D., Mendex, E. (2000). Students' building on one another's mathematical ideas. *Mathematics Teaching in the Middle School. 6(3),* 186-190.

Shindelar, A. (2009). Maintaining Mathematical momentum through "Talk Moves". In Herbel-Eisenmann,B & Cirillo, M. (Eds.), Promoting Purposeful Discourse: Teacher Research in Mathematics Classrooms. (pp. 165- 178). Reston,VA: NCTM.

Smith, M. (2007, November 14). *Orchestrating Productive Mathematical Discussion of Student Responses: Helping Teachers move Beyond "Showing and Telling".* Paper presented at Michigan State University Mathematics Education Colloquium, East Lansing, MI.

Stein, M. (2001). Mathematical argumentation: Putting umph into classroom discussions. *Mathematics Teaching in the Middle School. 7(2),* 110-112.

Smith, M., Hughes, E., Engle, R. & Stein, M. (2009). Orchestrating discussions. *Mathematics Teaching in the Middle School. 14(9),* 548-556.

Smith, M. & Stein, M. (2011). *5 Practices for Orchestrating Productive Mathematics Discussions.* Reston, VA: NCTM.

Smith, M. & Stein, M. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School. 3(4),* 268-275.

Vacc, N. (1993). Questioning in the mathematics classroom. *Arithmetic Teacher. 41(2),* 88-91.