ANALYSIS AND DESIGN OF ONE-HINGED, PLYWOOD BOX-BEAM, RIGID FRAMES

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Ram Daur Misra

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By

Ram Daur Misra

ABSTRACT

Submitted to the Colleges of Agriculture and Engineering of
Michigan State University of Agriculture and
Applied Science in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
AGRICULTURAL ENGINEERING

Department of Agricultural Engineering

Approval Merle L. Esmay

ABSTRACT

The overall objectives of this study were to make a stress analysis, design and develop design tables for one-hinged plywood box-beam rigid frames.

Castigliano's theorem was used to analyze the frame for a uniformly distributed snow load, a wind load and a load over one-half portion of the frame.

The equations derived in the frame analysis were used to draw design diagrams. A set of design tables for snow and wind loads was prepared. Michigan State's Mistic Computer was used for necessary calculations.

A frame with a span of 32 ft., side wall height 16 ft., and roof slope 4 in 12 was selected for design and analysis. The frame spacing was selected at 10 ft. on centers. A design load of 30 psf was used.

Moduli of elasticity of the box-beam as well as its component materials (construction grade Douglas fir lumber and A-D grade plywood) were determined separately. An E value of 1.788×10^6 psi was used in the analysis.

A model analysis was made to predict the prototype behavior of the model frames. Half scale true models were constructed and tested. Loads were applied through hydraulic cylinders with a motor driven two-way pump. Cylinders were mounted on heavy structural

steel I beams in such a way that equal loads were applied at every foot of rafter projection.

The vertical crown deflection and the bending stress at the heels were measured and compared with the corresponding calculated values. Ames dial gages were used to measure deflection. Strain was measured with SR-4 electric resistance strain gages.

A 7" x 10" photostress sheet plastic was mounted at the haunch of one test model to determine the stress distribution. A sequence of isochromatic (locus of same principal stress difference) and isoclinic (locus of same principal stress direction) pictures for two different loads were taken. Shear-difference method was used to separate the principal stresses.

Model frame tests were substantiated by four haunch specimen tests. A steel strap was used to connect outer column and rafter flanges at the haunch in one of the tests. Three plywood face grain orientation were used: (1) parallel to the column axis, (2) parallel to the rafter axis, and (3) 30° to the column axis.

From the results of the investigation the following conclusions were made.

1. The design information presented in appendix can be used, with confidence, for the design of one-hinged frames.

2. The dimensions of the haunch plywood web can be determined by using $\frac{P}{A} - \frac{Mc}{I}$ rather than $\frac{P}{A} + \frac{Mc}{I}$ for equating normal

stress. A and I are the section properties of the plywood web only.

3. Haunch web face grain orientations did not have any significant effect on the strength of haunch joint. Therefore, an economical layout of the haunch web is possible.

4. Making plywood dimension parallel to column larger than that parallel to rafter, improves frame stiffness.

5. A definite economy in material can be obtained by tapering the rafter as the moment decreases parabolically.

6. Model analysis provides an economical and accurate method of predicting structural behavior of full scale frames.

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INTRODUCTION

Rigid frames of steel, concrete and wood have been used for some time in institutional, commercial and industrial buildings. Their acceptance and popularity in modern farm buildings is rapidly increasing with advancing structural technology. They are proving economical in clear span roof construction for farm buildings.

Rigid frame structures possess certain other advantages over conventional trussed construction, such as: increased clear head room, simplicity in erection and attractive appearance. Increased clear headroom can be profitably utilized in storage structures. In hay storage buildings clear head room can materially reduce the storage cost per ton. Mechanical equipment for processing can be easily installed and removed when desired. A more efficient light and air distribution is possible in closed buildings.

The possible types of rigid frames in use are: no-hinged, one-hinged (crown), two-hinged (heels) and three-hinged (crown and heels). Two and three-hinged frames are commonly used. The three-hinged frame is statically determinate and hence is easier to design. Korn (1953), however stated that three-hinged frames are more expensive than two-hinged frames.

One-hinged frames have not been widely used to date because of lack of suitable and economical devices for fixing the heel joints against rotation. If a suitable anchorage device can be designed they offer a promising application in farm buildings.

Nelson and others (1960) listed the following possible advantages of construction with one-hinged frames:

1. Greater stiffness and therefore greater load carrying ability for a given frame cross-section.
2. Elimination of heel to foundation joint. This joint can be a critical design detail under lateral or uplift loads.
3. Elimination of continuous foundation.
4. Increased simplicity and reduced time for frame erection.
5. Elimination or reduction of site preparation prior to erection of frame.

The best utilization of physical and mechanical properties of one, or a combination of two or more, structural materials to produce better farm buildings has always been the basic objective of farm structures design. Plywood, with its inherent physical properties and structural soundness, is being utilized increasingly for box-beams in modern building construction. These box-beams consist of solid and/or laminated upper and lower flanges connected together with plywood webs. The webs may be nailed, bolted, glued or glued and nailed to the flanges. Vertical spacers are used to separate the flanges along the beam length, to prevent web buckling, and to distribute the concentrated loads. The flanges bear most of the bending stresses while webs bear the shearing stresses.

The concept of plywood box-beams for the economical design of one-hinged frames for farm building design was investigated in this study.

Objective

The general objective of this investigation was to make theoretical and measured stress analyses for one-hinged plywood box-beam rigid frames. Design tables were also developed.

REVIEW OF LITERATURE

This investigation included the theoretical analysis, design, testing and prediction of prototype behavior of plywood box-beam, one-hinged gable frames. One half scale true models were tested for prediction of prototype and development of design criteria.

The review of literature covered the following main parts:

1. Structural Frame Analysis
2. Design Loads for Buildings
3. Design of Plywood Box-beams
4. Use of Glue in Construction
5. Model Analysis
6. Stress Analysis

Structural Frame Analysis

A number of theories have been developed for analyzing rigid frames and are presented in most of the indeterminate structures texts. In the analysis of all indeterminate structures the use of some deformation characteristics in addition to equilibrium equations is made. For analyzing one-hinged rigid frames, Castigliano's theorem provides a convenient method. This theorem states that with any system of loads the deflection of a linearly elastic body at any given load and in the direction of that load is equal to the partial derivative of the total strain energy with respect to the given load, i. e.

$$\delta_p = \frac{\partial U}{\partial P}$$

where δ_p = deflection in the direction of load P

U = total strain energy

Shermer (1957) states that in frame analysis where deformation is primarily flexural, the energy due to shear and axial stresses is generally ignored as its contribution to the total elastic energy is very small.

Design Loads for Buildings

There is considerable information available in the literature on the design loads for buildings.

The U. S. Division of Housing Research, HHFR (1952) recommends the following design loads for various slopes and regions.

Location	Roof Loads (psf of horizontal roof projection)			
	Slope			
	3 in 12 or less	6 in 12	9 in 12	12 in 12 or more
Southern States	20	15	12	10
Central States	25	20	15	10
Northern States	30	25	17	10
Great Lakes, New England and Mountain Areas	40	30	20	10

Barre and Sammet (1955) recommend a minimum of 20 pounds per square foot for snow loads. These figures should be increased according to the National Bureau of Standards (1957) in the northern part of the United States. In Michigan these figures vary from 20 pounds per square foot in the southern part of the state to 35 pounds per square foot at the extreme northern part of the upper peninsula.

A number of recommendations for the design of buildings to withstand wind loads are in existence. Powell and Norman (1960) showed considerable differences between various recommendations for shape factors. The differences ranged from 0. 7 to 0. 8 on windward side and 0. 4 to 0. 5 on leewardside, on the walls of closed buildings. The recommended pressure distribution differed even more.

Esmay (1961) states that in designing for wind stresses the following should be kept in mind:

1. Velocity pressure is never taken as less than 20 psf.
2. Wind may come from any direction; hence any surface member must be designed for the maximum force coefficients, both positive and negative.
3. Although a building may be normally closed, windows or doors may be left open, resulting in an increase of negative pressure on the leeward side.

Neubauer and Walker (1961) state that when snow loads are maximum, a critical wind load will seldom occur, and such a load may be neglected, or applied at a fractional value.

Giese and Henderson (1950) state that, because live loads in farm buildings can be determined rather closely, the high factor of safety is not required for farm construction.

Design of Plywood Box-beams

A detailed procedure for plywood box-beam design is given in TECHNICAL DATA ON PLYWOOD (1948). A summary of design procedure is listed below.

(1) Design information (bending moments shears) should be calculated using appropriate formulas and a trial section should be selected for checking.

(2) The maximum bending stress is computed from the formula

$$f = \frac{Mc}{FI}$$

where f = bending stress (psi)

M = bending moment (in-lb)

c = distance of extreme fibre from neutral axis

F = form factor

I = net moment of inertia

(3) The maximum horizontal shear is computed from the formula:

$$v = \frac{VQ}{It}$$

where v = shear stress on the plane under consideration, psi

V = shear force on the cross-section, lbs

Q = statical moment about the N. A. of all fibers whose grain is parallel to the beams axis and lying outside the plane under consideration, in^3

I = moment of inertia

t = total thickness of plywood webs, in

I and Q are computed from the gross dimensions.

(4) The maximum flange-web shear stress is found from the formula

$$S_A = \frac{VQ_A}{Id_A}$$

where Q_A = statical moment of flange area only in in.^{3}

d_A = flange depth in in.

(5) Actual deflection may be checked against the allowable by applying appropriate deflection formulas, taking shear into account.

(6) Lateral support should be provided at points depending on the width of the compression flange and the stress existing in it.

(7) Bearing and intermediate stiffeners should be provided and spaced so as to develop full shear strength of the web.

(8) Butt joints occurring in flange members should be properly spaced. Splice plates of appropriate size should be provided for the web butt joints.

Haunch Design

Haunch design is most critical in solid wood rigid frames and is even more so with box-beams. The literature on this subject was limited mainly to solid wood design.

Curtis (1959) conducted tests on 88 rigid joints formed with plywood gusset plates, glued onto solid framing members. On the basis of this study he proposed the following design procedure:

1. The maximum stress in the gusset is calculated by formula:

$$f = \frac{M}{s} + \frac{P}{A}$$

where M = maximum bending moment, in-lb

$s = \frac{td^2}{6}$, d = a + b (Symbols are indicated in Figure 1)

t = thickness of effective plies, in

P = axial load, lb

A = effective cross-sectional area, in^2

= dt = (a+b)t

2. Shear "s" in plane parallel to the glue line is calculated by formula

$$s = \bar{s}_t + \bar{s}_p$$

where s = resultant shear

st = rolling shear $= \frac{Tc}{J}$

T = M = twisting moment

$c = (\frac{l}{2})^2 + (\frac{a}{2})^2$ (See Figure 1)

J = polar moment of inertia

$= \frac{bh(b^2+h^2)}{12}$ for rectangle

s_p = component of shear stress required to resist load P

$= \frac{P}{A}$, A = 2al

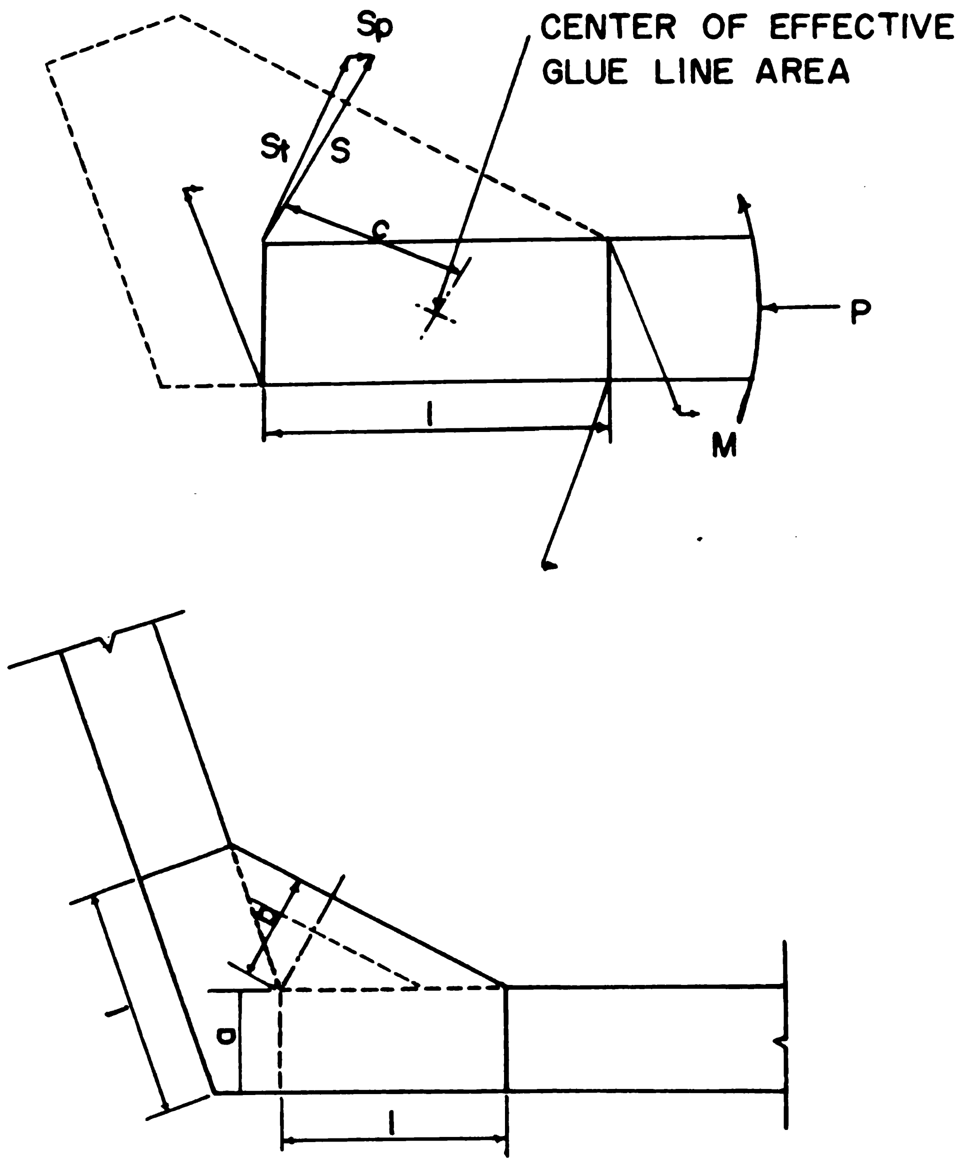


FIGURE 1 DETAILS OF JOINT (CURTIS 1959)

Custis' results are summarized as follows:

1. Nailed and glued gusset plates form satisfactory rigid knee joints with straight dimension lumber members
2. Required size and thickness of plywood gussets can be determined analytically by the procedure suggested. Maximum shear parallel to glue bond should be less than 90 psi.
3. Joints loaded to produce tension in unsupported edge of the gusset are at least as strong as those loaded to produce compression in the unsupported edge of the gusset.
4. An orientation of the grain that was approximately parallel to the direction of the maximum fibre stress in the gusset was found to be best.

The Use of Glue in Construction

Considerable work has been done in development of glue and glueing techniques for wood structures. They have proven effective for farm construction.

Giese (1940) states that one of the important reasons for using glue is that the bearing area of the joints may be increased to almost any size, thus making possible a more uniform stress distribution throughout the structure.

Casein and resorcinol resin have been two principal types of glues, used most commonly in farm construction (Boyd). The basic difference in the two glues is their ability to withstand the direct action of water. The resorcinol resin glue is waterproof while casein is only water resistant.

Skinner (1946) found that joints made with resorcinol-resin glue and Douglas fir lumber at 16 percent moisture content were approximately as strong as similar joints made from wood at 7 percent moisture content. Joints made at 30 percent and tested at 22 percent moisture content were approximately 70 percent as strong as those made from the drier wood. These tests indicated the possibility of making glue joints successfully with wood of various moisture conditions in field.

Giese and Henderson (1945) state that casein glue is of ample durability if protected from direct action of water. The authors further recommend, as practical shear design stresses, 430 and 215 psi parallel and perpendicular to grain of wood respectively.

Model Analysis

Model analysis has been used extensively in the field of structural engineering, in research and actual design of structures. Its application in agricultural engineering research has been increased in recent years.

Nelson and others (1960) carried out model analysis of one-hinged rigid frames of solid and laminated lumber. Their results were in close agreement with that of the prototype. They concluded that the models of arches and fixed end arch anchorages were productive research tools, when designed and operated in accordance with similitude principles.

Aldrich and Boyd (1959) employed model analysis for evaluating behavior of rigid plastic arches under load. They developed prediction equations for stress and deflection for true and distorted models. On the basis of the results of distorted models, they concluded that the use of structural models permits investigation of hypotheses concerning mechanical phenomena in structures without the expense and difficulties encountered with full scale units. The measured and estimated values did not differ by more than 10 percent.

Stress Analysis

Photostress

Photostress is a photoelastic technique of stress analysis. The specimen to be stress analyzed is coated with a special transparent plastic that exhibits temporary birefringence when strained. This birefringence is directly proportional to the intensity of strain. When a polarized light is passed through the strained plastic, black and colored fringe patterns corresponding to the direction and intensity of principal strains can be observed and measured through a polariscope. For details on photostress fundamentals reference should be made to classic photoelasticity texts. For instance Photoelasticity, Vol. 1 by M. M. Frocht (1941).

Post and Zandman (1961) made an extensive study of effects of Poisson's ratio and coating thickness on the accuracy of Photostress technique. They made the following conclusions on the basis of their experimental investigation.

1. For the case of plane stress problems and equal Poisson's ratio of structure and coating, the influence of coating thickness on birefringence developed along free boundaries is almost identically zero.
2. For unequal Poisson's ratio and singly connected structures in plane stress, birefringence developed along free boundaries is almost exactly independent of coating thickness.
3. For singly connected structures in plane stress, very thick coatings behave essentially as independent bodies subjected to prescribed end displacements, i. e. as photoelastic models. In this case birefringence is independent of Poisson's ratio.
4. In order to minimize effects of dissimilar Poisson's ratio and local reinforcement, thin coatings are preferable. For most engineering problems 1/8-in. coating should be adequately thin.

Boyd (1954) used SR-4 strain gages for stress analyzing a wooden fink truss. He concluded that SR-4 strain gages are useful in determining quantitatively the strain in wood. The results of his laboratory tests indicated the possibility of obtaining accuracies within 10 percent.

THE INVESTIGATION

Preliminary Consideration

Modulus of Elasticity Determination

The modulus of elasticity in bending was determined for the combination of dimension lumber and plywood in the form of a box-beam. A leg of one of the test frames was cut and used as a simply supported beam. Loads at two points equally spaced from the center were applied through hydraulic cylinders by a hand pump.

Two SR-4 strain gages were mounted in the center of the top and bottom sides of the beam. They were connected in adjacent arms of the bridge. This arrangement of the gages provided double sensitivity and an average strain. Pressure was measured by a bourdon tube pressure gage. The bending moment at the center was found by using conventional methods and stress was calculated from the following equation.

$$\sigma = \frac{Mc}{FI}$$

Where M = bending moment in in-lb

c = distance of outer fiber from neutral axis in inches

F = form factor (obtained from "Technical Data on Plywood" (1948)

I = the net moment of inertia, inches^4

E then was calculated using the following linear relationship

$$E = \frac{\sigma}{e}$$

Where e = strain in in/in.

Figure (2) shows the load-strain relationship obtained from the tests. The calculation and the value of E obtained is also shown.

To substantiate the tests made on the box-beam, the moduli of elasticity of solid dimensional lumber and plywood were determined separately. One specimen of each lumber and plywood was tested in bending in a Tinius Olsen testing machine. Electric resistance strain gages were mounted to measure strain. The values of "E" thus obtained were 2.5×10^6 and 1.61×10^6 psi for Douglas fir construction grade lumber and A-D grade plywood respectively.

The "E" value determined for the lumber was rather high but it compares very closely (2.55×10^6 psi) with some of the tests made by Mielock (1959) for the same grade of lumber. The low moisture content of the lumber contributed to the high modulus of elasticity.

Since the modulus of elasticity (1.778×10^6 psi) determined in box-beam tests falls within the range of that found separately for lumber and plywood, it was used in the design analysis in this study.

Frame Analysis

A one-hinged rigid frame is an indeterminate structure with a second degree of indeterminacy. Two deformation conditions in addition to the equilibrium equations must be considered for complete analysis.

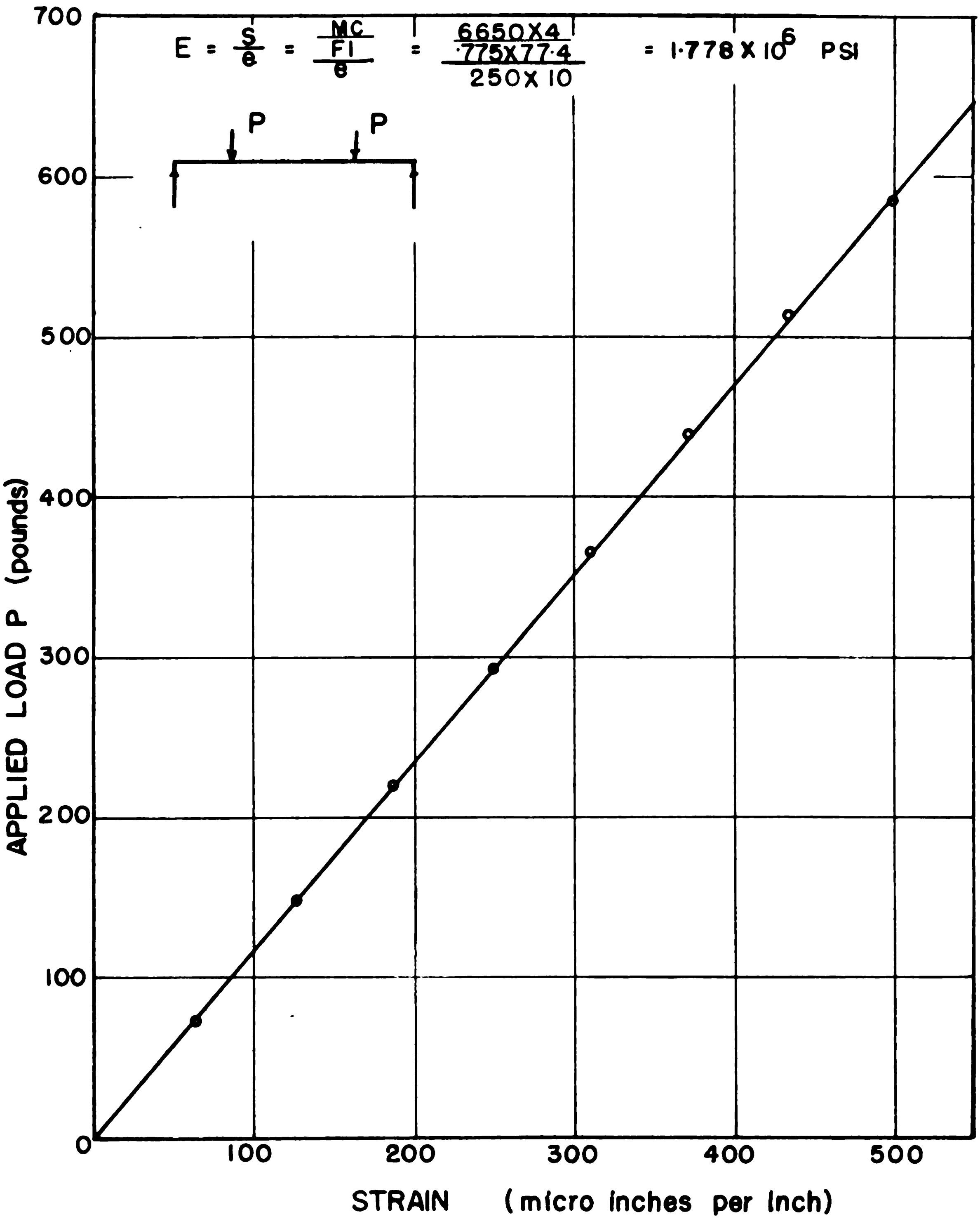


FIGURE 2 MODULUS OF ELASTICITY DETERMINATION FOR DOUGLAS FIR LUMBER AS USED IN THE FLANGES OF PLYWOOD BOX-BEAMS

For symmetrical loadings as with uniformly distributed load, the degree of indeterminacy is reduced by one. Wind loads and loads only over half of the frame require the consideration of two deformation conditions.

In analyzing the frame the use of Castigliano's theorem was made. This theorem states that for a linearly elastic body, the deflection in the direction of the load at the point of application of a load is equal to the partial derivative of the total strain energy with respect to that load, or

$$\delta p = \frac{\partial U}{\partial P}$$

Considering only bending energy (axial and shear energies are much smaller compared to the bending) and the deformation condition that, the horizontal deflection of the hinged crown must be zero, we have from Castigliano's theorem:

$$\delta c_H = \frac{\partial U}{\partial H} = \Sigma \int M \frac{\partial M}{\partial H} \frac{ds}{EI} = 0 \qquad 1$$

refering to the free body diagram Figure (3) gives:

In BC

$$M = Hx\mathrm{Sin}\ \theta - wx\ \mathrm{Cos}\ \theta \frac{x\ \mathrm{Cos}\ \theta}{2}$$

$$= Hx\frac{f}{s} - \frac{wx^2}{2}\left(\frac{L}{2s}\right)^2$$

therefore $\frac{\partial M}{\partial H} = x\frac{f}{s}$

In BC

$$M = H x \sin\theta - w x \cos\theta \frac{x \cos\theta}{2}$$

$$= H x \frac{f}{s} - \frac{w x^2}{2} \left(\frac{L}{2s}\right)^2$$

therefore $\frac{\partial M}{\partial H} = x \frac{f}{s}$

In BA

$$M = H(x+f) - \frac{wL}{2} \frac{L}{4}$$

$$\frac{\partial M}{\partial H} = (x+f)$$

substituting in equation 1 gives

$$2\left[\int_o^s \left\{ H x \frac{f}{s} - \frac{w x^2}{2} \left(\frac{L}{2s}\right)^2 \right\} \frac{xf}{s} \frac{dx}{EI_2} + \int_o^h \left\{ H(x+f) - \frac{WL^2}{8} \right\} (x+f) \frac{dx}{EI_1}\right] = 0$$

performing the integration and simplifying gives:

$$H = \frac{\frac{WL^2}{8}\left[\frac{fs}{4} + \frac{I_2}{I_1} h \left(\frac{h}{2} + f\right)\right]}{\frac{f^2 s}{3} + \frac{I_2}{I_1} h \left(\frac{h^2}{3} + f^2 + fh\right)}$$

$$M_A = H(h+f) - \frac{wL^2}{8} = M_E$$

$$M_B = H f - \frac{wL^2}{8} = M_D$$

Shears can be found by differentiating moment equations.

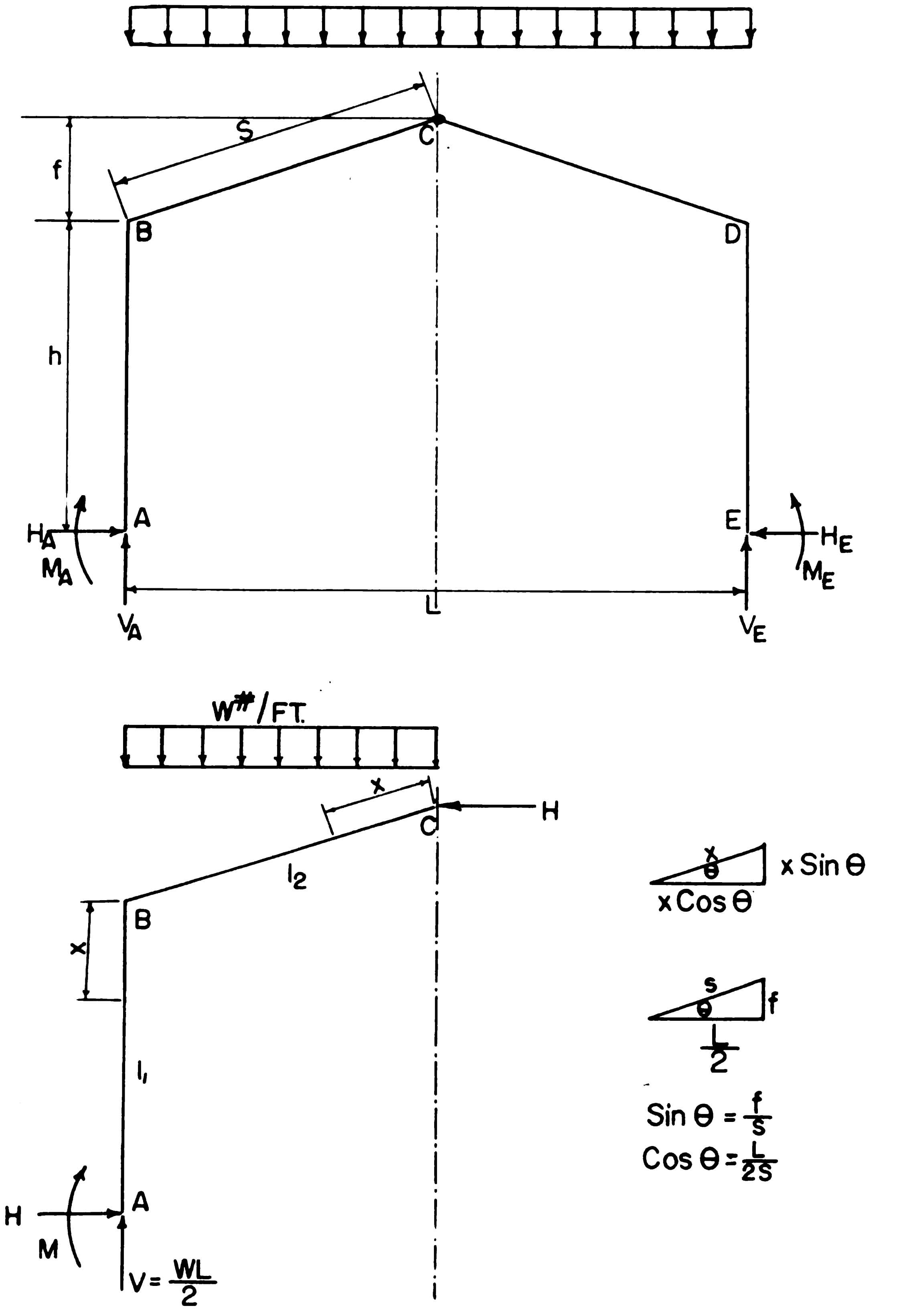


FIGURE 3 FREE BODY OF ONE-HINGED RIGID FRAME

The analyses for the cases of wind load and load only on half portion of the frame are included in the Appendix.

A set of design tables prepared by using Michigan State University's Mistic Computer is also included in the Appendix.

Frame Design

Design Assumptions

Certain necessary assumptions were made prior to designing. A frame with a 32 ft. span and column height of 16 ft. with roof slope of 4 in 12 was selected. These dimensions would be especially suitable for a hay storage structure. The spacing of frames was selected at 10 feet on centers. The total live and dead load of 30 psf was used for the design. The dead load of the structure was estimated at 5 psf, which leaves 25 psf for the anticipated live load. It was further assumed that a 25 psf snow load and a critical wind load would not occur at the same time. The wind load was therefore neglected.

Design Diagrams

The first step in the design of any structure consists of drawing design diagrams (bending moment, shear force and direct force). These diagrams for the particular frame selected were prepared for a constant moment of inertia and are shown in Figure (4). The equations derived in the theoretical frame analysis were used to calculate

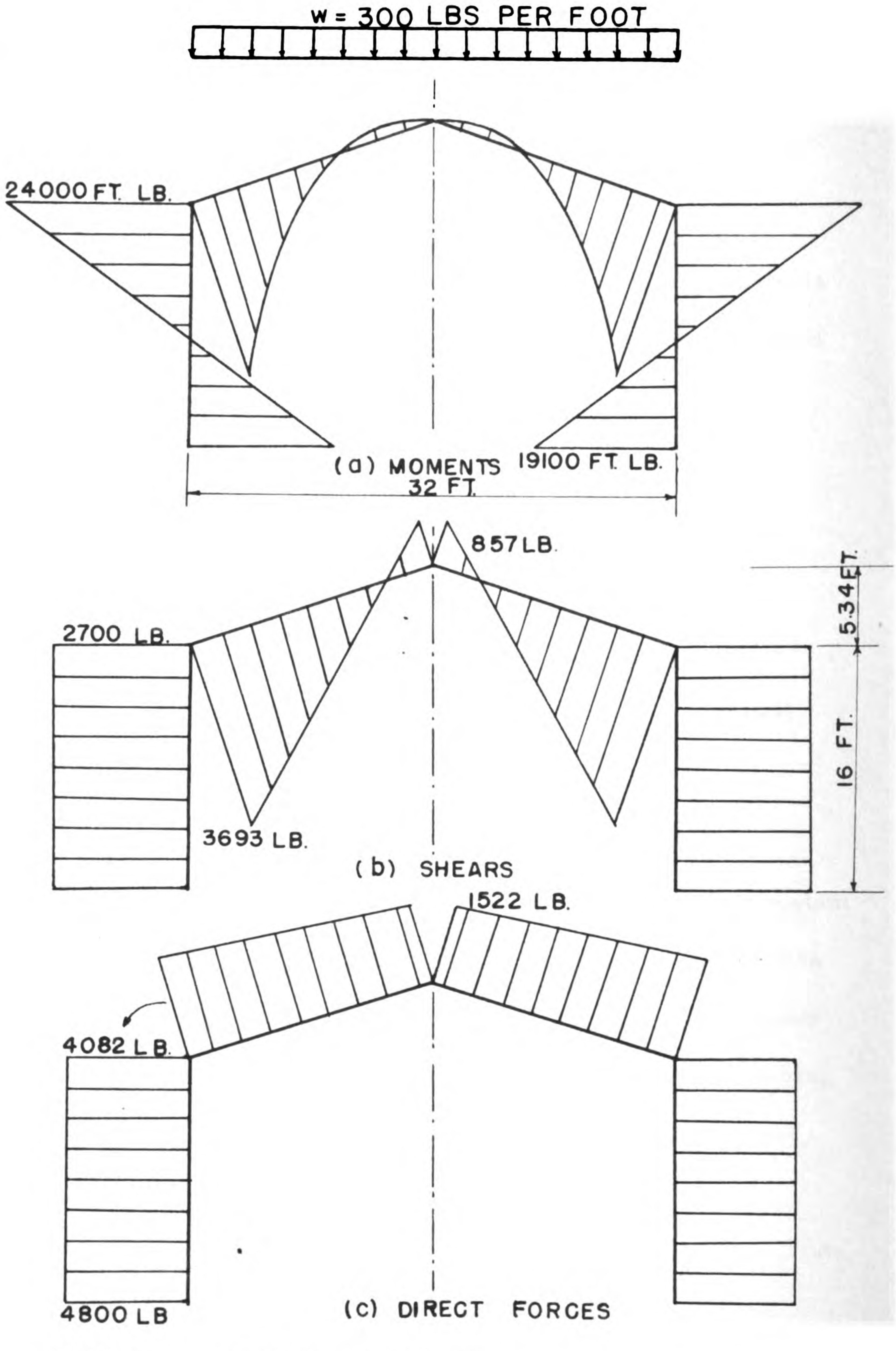


FIGURE 4 DESIGN DIAGRAMS

bending moments at various points. Shears and direct forces were calculated conventionally.

Design Procedure

The bending moment at the heel was taken as the design criteria for the column portion of the frame. The following equation was used to calculate the normal stress:

$$\sigma = \left(\frac{P}{A} + \frac{Mc}{FI}\right)$$

Where σ = normal stress in pounds per square inch

P = axial load in lbs.

A = effective cross-sectional area of the section in in.^{2}

c = distance of the outer fiber from the neutral axis (inches)

F = form factor

I = net moment of inertia of the section in in.^{4}

The cross-section of the column member was maintained constant as the bending moment varied from a positive maximum at the heel to a negative maximum at the haunch. The column flanges were spliced in the center near where the point of contraflexure occurred. Splicing was necessary as the length of the lumber required was not commercially available.

The bending moment in the rafter decreases parapolically with a maximum at the haunch to a minimum of zero at the hinged crown. Therefore the rafter was tapered accordingly. The stresses at four equidistant points on the rafter were checked and found within allowable limits.

To get a perfect hinged action at the crown, the upper flanges of both the rafters were made in a form of half lap joint. A 5/8-in. diameter bolt was used.

Figure (5) shows the design details of the prototype frame.

Haunch design. The review of literature did not indicate any rational method of plywood box-beam haunch design. It was, therefore, assumed (to be on the safe side) that all bending stresses at the haunch were taken by plywood webs. The section considered for design was that suggested by Curtis (1959) for gusset design. The following equation was used to calculate the normal stress.

$$\sigma = \left(\frac{P}{A} + \frac{Mc}{I}\right)$$

where all the dimensions are the same as indicated on page 23.

The first frame was designed with the above assumptions. Normal stress was kept within allowable limit of 2190 psi for the plywood.

The test on the preliminary model frame indicated that the haunch was over designed. Therefore, the design was revised. This time normal stress σ, was calculated using the following formula instead of the one mentioned before:

$$\sigma = \left(\frac{Mc}{I} - \frac{P}{A}\right)$$

This approach was used to arbitrarily compensate for the over assumption

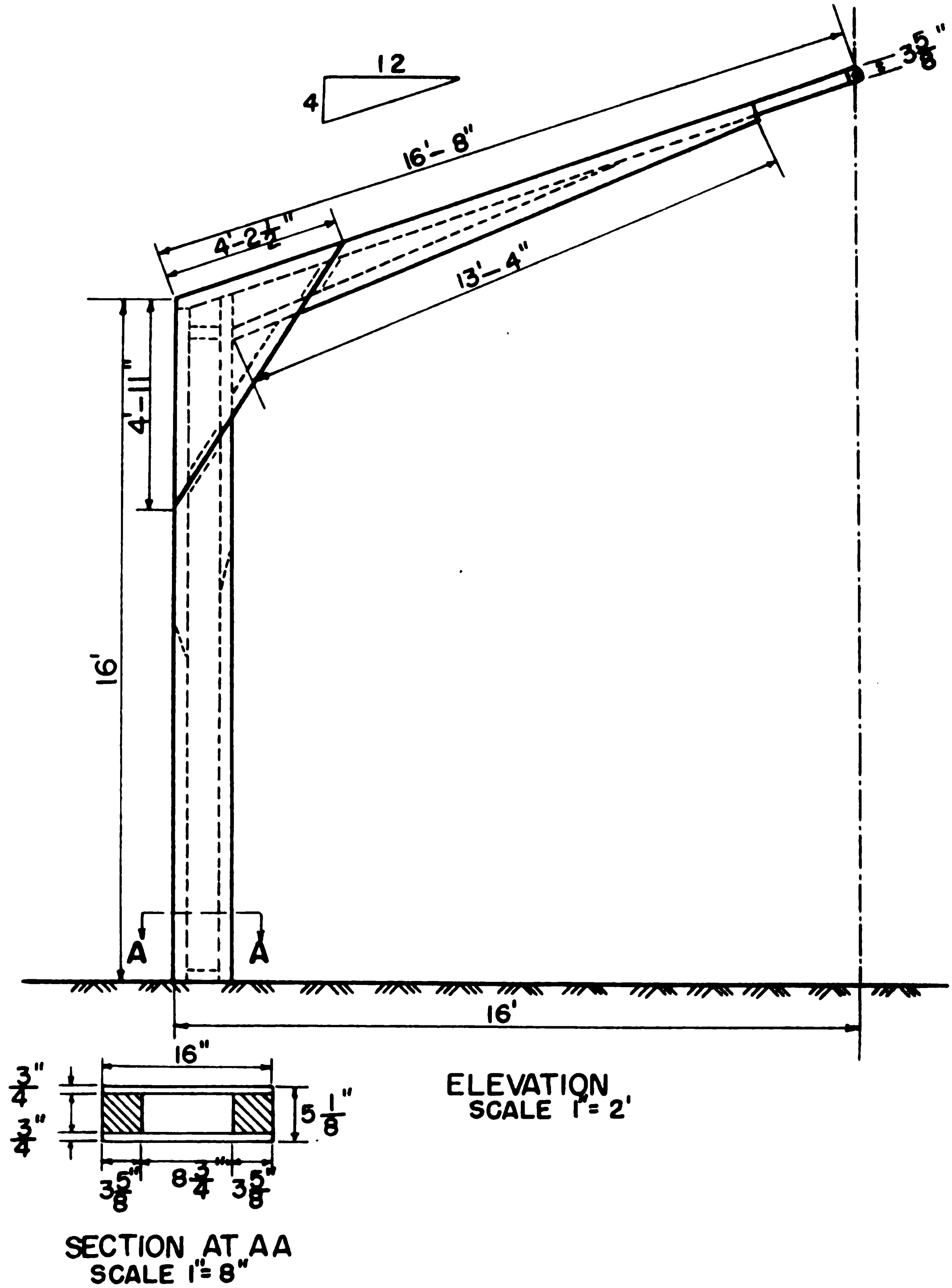


FIGURE 5 DESIGN DETAILS OF PROTOTYPE FRAME

that all the bending stress at the haunch was taken by the plywood web. By using the tension side as the design criteria this was equivalent to using a higher allowable stress value than normally recommended.

Web buckling in compression was minimized through the use of blocks of the same cross-section as the flange. Figure (6) shows details of haunch joint.

The shearing stresses in the plane parallel and perpendicular to the face plies were calculated from equation in the "Technical Data on Plywood." The values of shearing stresses used were 210 and 40 psi for horizontal and rolling shears respectively.

The new frame design was re-analyzed to see if there was any appreciable effect on the allowable stresses. Numerical integration for the tapered rafter indicated that the design information for $I_2/I_1 = 1$ was close enough for practical purposes.

Model Analysis

Model analysis was used in this investigation as the testing facilities available did not permit the loading of full scale frames. As the validity of model analysis has been well established, the testing of the full scale structures, involving additional cost was unnecessary.

The objectives of model analysis were:

1. Stress analysis of the model.
2. Determination of deflection characteristics of the model.

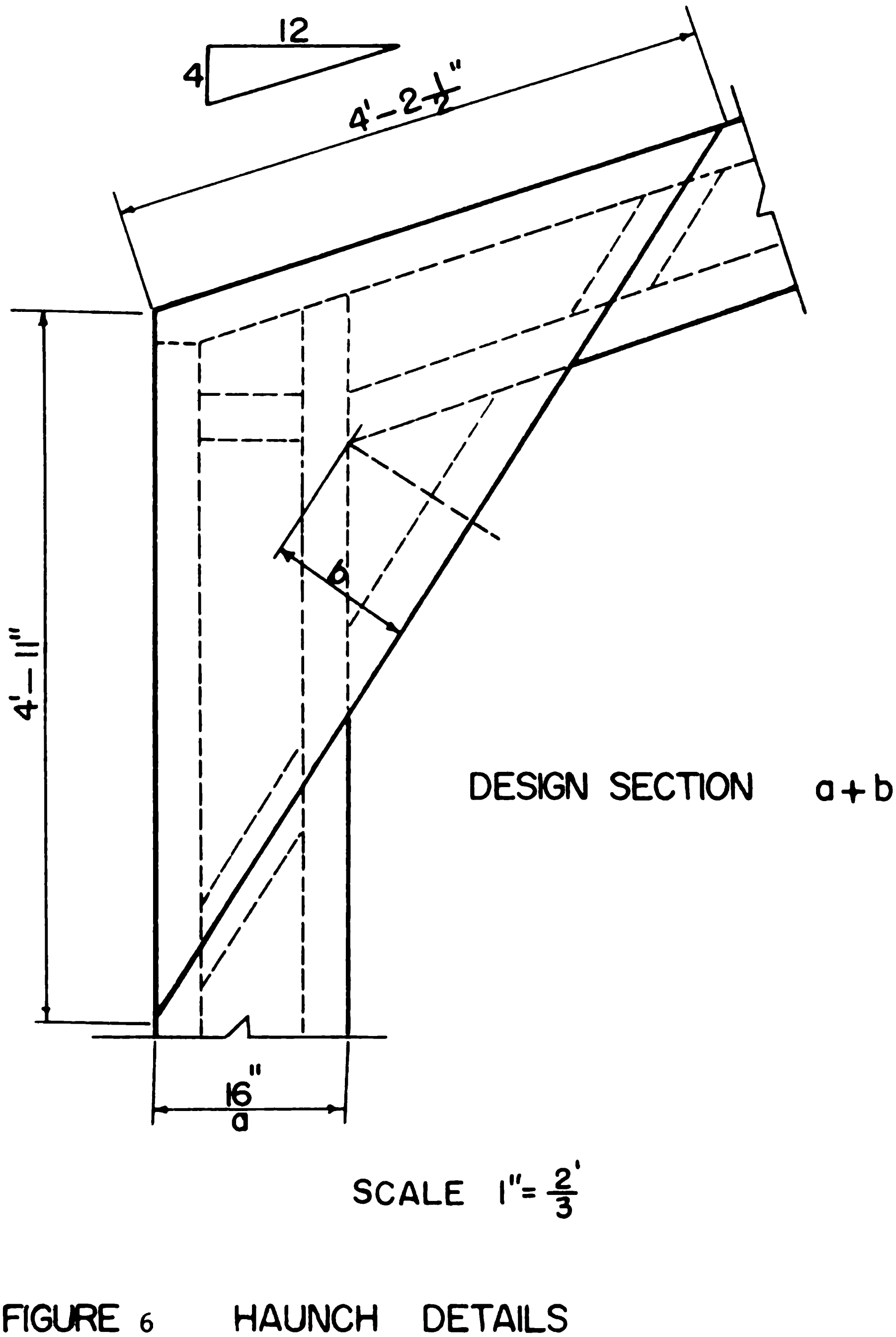


FIGURE 6 HAUNCH DETAILS

3. Determination of critical loads and modes of model failure.

Procedure for Model Analysis

In model analysis the first step consists of developing the prediction equations for the behavior of the structure under load. For structural models, stresses and deflections best describe the performance of a structural member. Hence prediction equations must be developed for stresses and deflections.

In most cases dimensional analysis is used to obtain the prediction equation. The theorem generally known as Buckingam Pi theorem was used. This theorem states that the number of dimensionless and independent quantities required to express a relationship among the variables in any phenomena is equal to the number of quantities envolved minus the number of fundamental dimensions in which these quantities may be expressed.

In equation form:

$$r = n - m$$

Where r is number of Pi terms.

n is number of quantities.

m is number of fundamental dimensions.

The procedure for dimensional analysis was listed as:

(a) Listing all variables that enter the problem.

(b) Describing each variable in terms of fundamental units.

(d) Determining the groups of dimensionless terms.

After dimensionless groups have been determined a problem then, is set up such that one Pi term is expressed as a function of remaining terms, i. e.

$$\pi_1 = f(\pi_2, \pi_3, \ldots \pi_r)$$

It should be noted that the only restrictions on Pi terms are that they be dimensionless and independent. The equation above is a general prediction equation.

Following this procedure Aldrich and Boyd (1959) developed the prediction equations for stresses and deflections. The variables considered and their respective dimensions were:

Variables	Dimensions
S = stress	$F\text{-}L^{-2}$
L = length of span	L
λ = any length (layout)	L
η = any length (cross-section)	L
w = load per foot of beam	$F\text{-}L^{-1}$
E = modulus of elasticity	$F\text{-}L^{-2}$

There were 6 variables and 2 fundamental quantities; therefore, 6 - 2 = 4 Pi terms.

$$\pi_1 = \frac{SL}{W}; \ \pi_2 = \frac{\lambda}{L}; \ \pi_3 = \frac{\eta}{L}; \ \pi_4 = \frac{w}{EL}$$

$$\pi_1 = f(\pi_2, \pi_3, \pi_4)$$

The same terms apply to the model.

$$\pi_{1m} = \frac{Sm\,Lm}{Wm}; \quad \pi_{2m} = \frac{\lambda m}{Lm}; \quad \pi_3 = \frac{\eta m}{Lm}; \quad \pi_4 = \frac{Wm}{EmLm}$$

and

$$\pi_{1m} = f(\pi_{2m}, \pi_{3m}, \pi_{4m})$$

The design conditions become:

$$\frac{\lambda}{L} = \frac{\lambda m}{Lm}; \quad \frac{\eta}{L} = \frac{\eta m}{Lm}$$

The operating conditions are:

$$\frac{w}{EL} = \frac{wm}{EmLm}$$

and the prediction equation for stress is

$$\frac{SL}{w} = \frac{SmLm}{wm}$$

Using similar procedure the following final equations were derived for deflection:

For the operating conditions:

$$\frac{w}{EL} = \frac{wm}{EmLm}$$

For design conditions:

$$\frac{\lambda}{L} = \frac{\lambda m}{Lm}; \quad \frac{\eta}{L} = \frac{\eta m}{Lm}$$

And the prediction equation for the deflection is

$$\frac{d}{L} = \frac{dm}{Lm}$$

A half size scale for both, layout and cross-section was used. Thus, the prediction equations become

$$S = Sm$$

$$d = 2d_m$$

The model was analyzed the same way as the prototype. Crown deflection was calculated using Castigliano's theorem. The integral in the tapered rafter portion was evaluated numerically by considering a finite number of sections. The relation thus obtained was:

$$d_{cv} = 6.45 \times 10^{-3} w$$

Where d_{cv} = vertical deflection of the crown, in.

w = load per foot run of the horizontal projection of rafters.

EXPERIMENTAL INVESTIGATION

Experimental Apparatus

The farm structures testing laboratory at Michigan State University was used for conducting the tests. Some modifications were made in the truss loading equipment in order to load the one-half scale rigid frames. The anchorage equipment consists of a reinforced concrete floor in which steel inserts are placed at two feet intervals. Two double I beams are placed in the floor along one side of the building. They are spaced such that extended bolt heads catch on the flanges for anchorage of reaction supports to the floor.

The fixity at the column bases was attained with two channel sections welded on a 3/8" x 4" x 12" flat iron and bolted to the floor on either side of the column. Two flat iron pieces (1/4" x 1 1/4" x 11") were used to connect the fixed supports at the top. Two braced 'T' sections were later added to improve fixity. Figure (7) shows the reaction supports.

Hydraulic cylinders, 1.985 in. diameter and a stroke length of about 6 in. were used to apply loads. Cylinders were mounted on steel I beams for application of loads to top chords of frames. They were strong enough to resist the backward thrust without any accountable deflection. The steel I beams were supported on brackets which were bolted to movable bars. These bars were fastened to the floor inserts

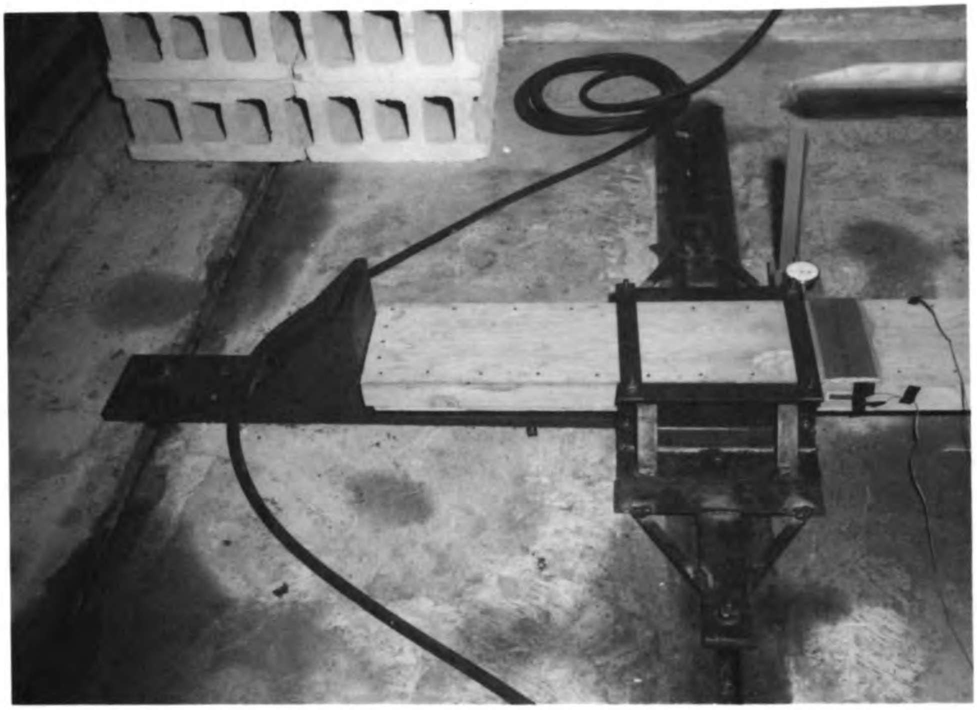


FIGURE 7 Fixed end reaction supports for model frame tests

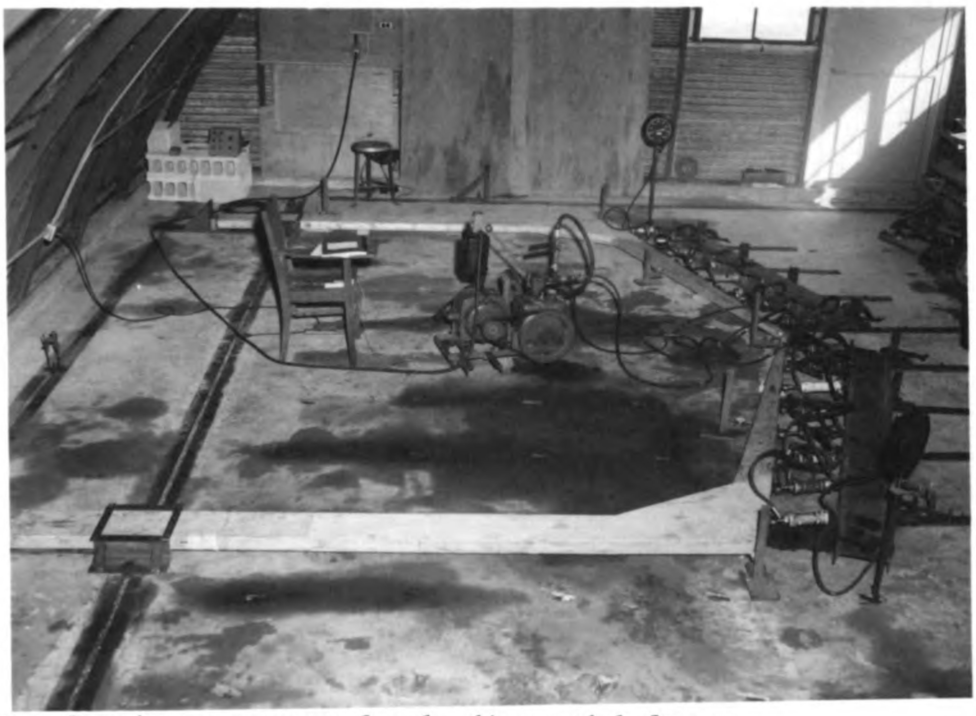


FIGURE 8 Testing apparatus for loading model frames

and spaced at 2 feet on centers. The spacing of the cylinders was one foot on center. This is equivalent to 2 feet spacing on the prototype frames and is an acceptable approximation to a uniformly distributed load.

Figure (8) shows complete loading set up for model frames.

The pressure in the cylinders was supplied with a motor driven two-way hydraulic pump. The pump was connected in the center of the hydraulic line to equalize any possible pressure drop to both sides of the test frame.

Two bourdon tube pressure gages, connected at either end of the hydraulic line were used to measure the pressure. These gages were frequently checked for calibration. An average area of 2.94 in.^{2} for each cylinder was used in converting the pressure from psi to load per foot. No hold-down brackets were required to keep the rigid frames in position during the test procedures.

Model Fabrication

The flanges of the one-half scale model frames were constructed from 4" x 4" Douglas fir construction grade lumber ripped into four equal parts. Webs were made of 3/8" interior A-D grade plywood. The exact size of plywood pieces was cust from 4' x 8' sheets. The model frames were fabricated in the laboratory after the ripped lumber was kept at least 24 hours at about 70° to assure equilibrium moisture content.

Dry mix powder, grade A Casein glue was used. The manufacturer's specifications for mixing and applying the glue were followed. Four penny nales, spaced approximately 3 in. apart were used to apply pressure for the glue set. Each model frame was allowed to set at least 48 hours before any load was applied.

Instrumentation

Both deflections and strain readings were taken during the test loadings. Deflections were measured with Ames dial indicators. Electric resistance strain gages were used to measure strains.

Deflection Measurement

Three Ames dial indicators with a deflection range of one inch, and calibrated in thousandths of an inch, were used to measure vertical crown and horizontal haunch deflections. Several other gages were used to check deflection at numerous points.

Strain Measurement

Type A-1, SR-4 electric resistance strain gages, one inch in length, resistance of 120 Ohms and gage factor of 2.03 were used. Strain gages were mounted on both flanges of each column as close to the base as possible. On a few tests strain gages were mounted at two or three points along the column and the rafter length.

The bending stresses at these points were desired, therefore strain gages were connected in adjacent arms of the bridge. This arrangement also provided double sensitivity.

Photostress technique of stress analysis was used to determine the stress distribution at the haunch. A 7" x 10" Photostress sheet plastic of type 'S,' thickness 0.12" and 'K' factor 0.086 was bonded with a reflective cement at the haunch. A large-field static Photostress meter with a mounted camera was used. A sequence of pictures of isochromatic and isoclinic patterns were taken.

Figure (9) shows the instrumentation used and the position of photostress plastic at the haunch.

Testing Procedure

The test load was applied with a hydraulic pump powered with an electric motor. Previous to all test loadings the pump was operated for 10 to 15 minutes with the bi-pass valve open. This warmed the hydraulic fluid and provided far more uniform loading of the test frames.

The loading of test frames was as slow as practicable. A load increment of 15 pounds per lineal foot was applied to the frame between strain and deflection readings. Beyond a test load of about 220 plf, it was difficult to maintain the uniform increments of loading. The minimum load applied to the model frames was 70 plf as compared to a design load of 150 plf.

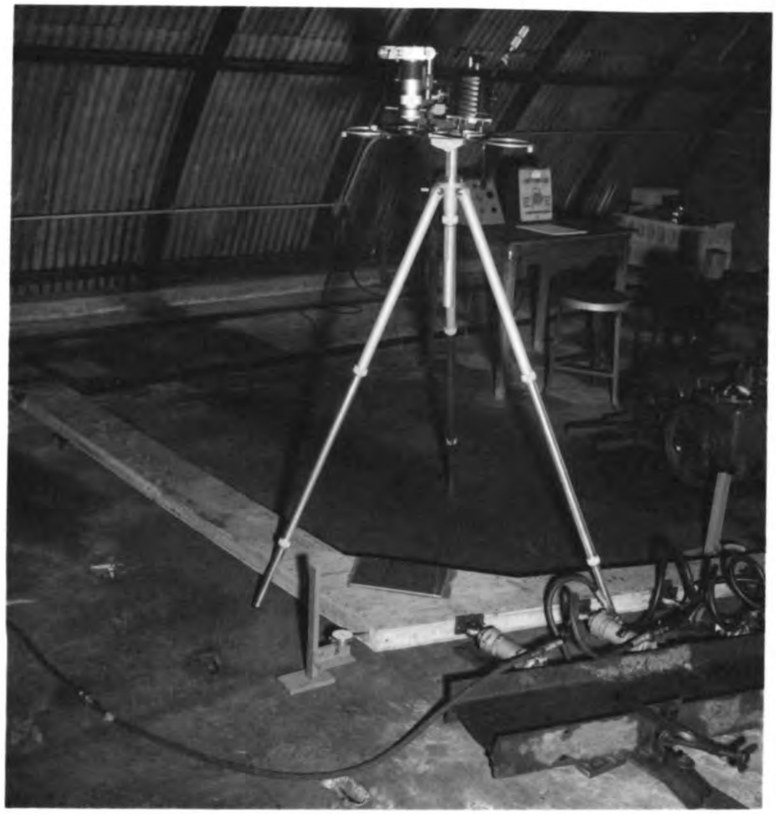


FIGURE 9 Instrumentation for photostress analysis of haunch joint

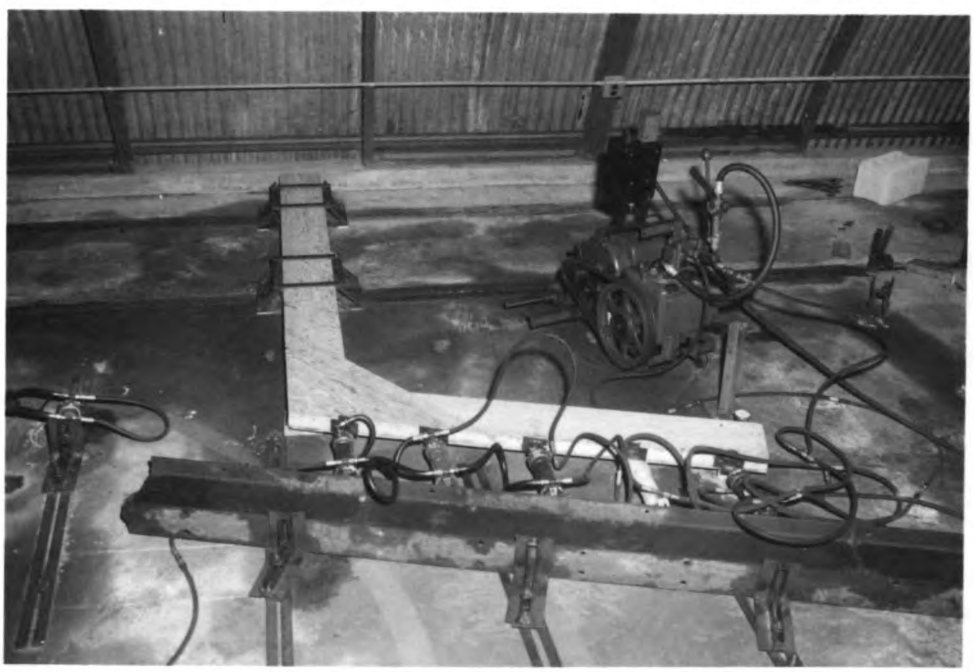


FIGURE 10 Haunch specimen 4 in place and ready for testing

After each increment of loading the model frames were allowed to come to equilibrium. Then deflection and strain readings were taken. Usually it required about five minutes to reach equilibrium.

Plotting Test Results

The deflections and stresses of different runs (usually 2 to 3) for each test were plotted against the load per foot of span. A best fit straight line was drawn through the plotted points. The lines were corrected so as to pass through the origin. This correction was necessary as the slack in the testing apparatus and the readjustment of different structural components of the frame made an initial shift in the measurement.

Haunch Tests

Purpose

The purpose of the haunch tests was to further verify and confirm theoretical frame analysis. Specifically the strength characteristics of the haunch joint were studied when the outer column and rafter flanges were connected with a steel strap. Figure (11) shows the details of haunch specimens.

Specimens

The haunch specimens had approximately the same cross-section dimensions as the model frames tested. The size of plywood web at the haunch was slightly changed. The layout dimensions of the specimens were determined by the capacity of the loading apparatus. Three plywood haunch web orientations were tested; namely, (1) parallel to the column axis, (2) parallel to the rafter axis, and (3) 36^{o} to the column axis.

Nailing and glueing procedures were exactly the same as for the model frames.

Testing Apparatus

Haunch specimens were loaded in a horizontal position similar to the model frames. Five cylinders spaced one foot on centers were used to apply loads. Figure (10) shows the testing apparatus.

Loading Pattern

The load was applied in increments of 75 plf. The haunch was gradually loaded to failure. The load at failure and types of failure in each case were recorded.

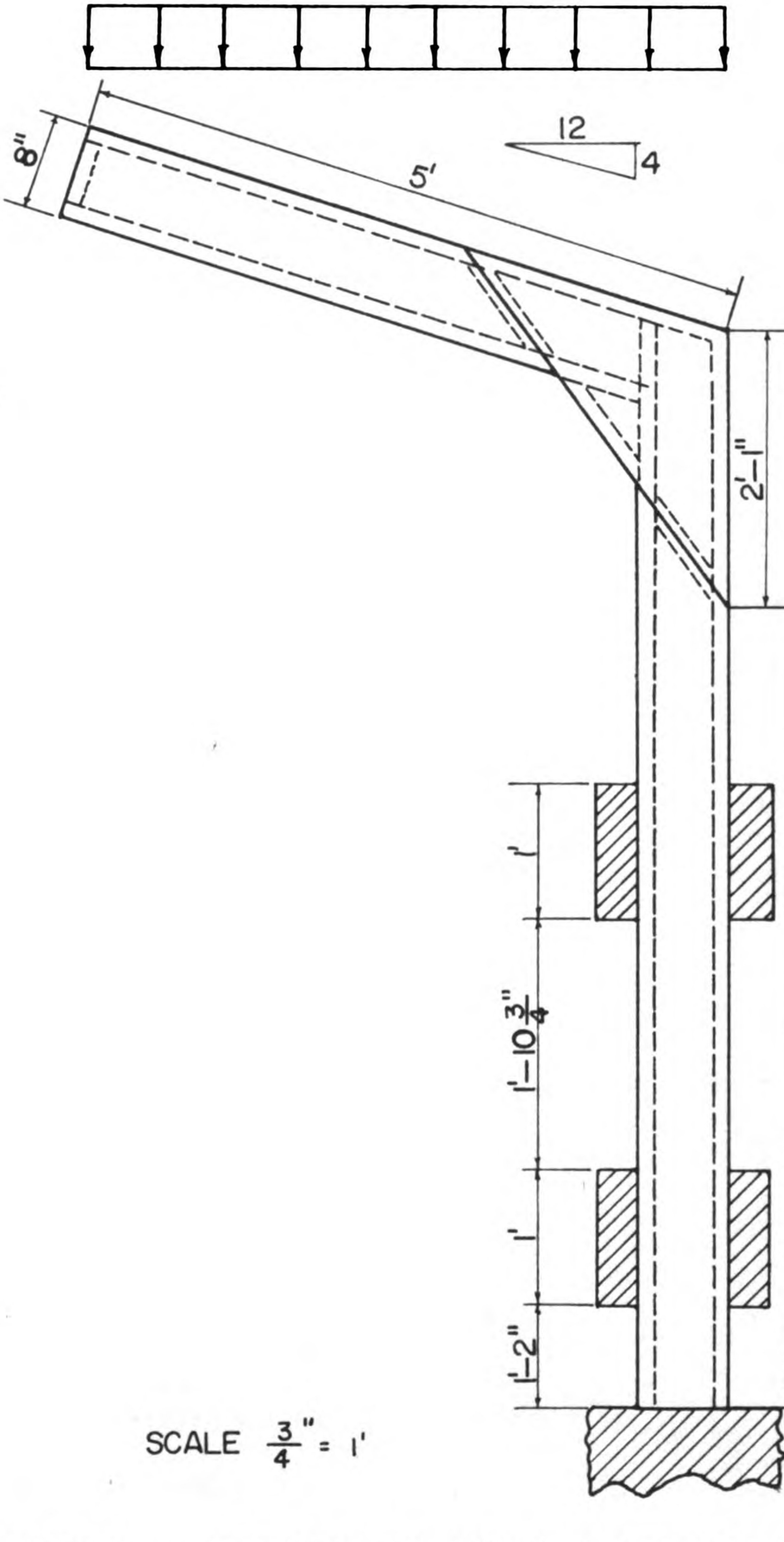


FIGURE 11 DETAILS OF HAUNCH SPECIMEN

RESULTS AND DISCUSSION

A total of five, one-half scale model frames were tested. The model frames were exactly the same except that the haunch plywood face grain directions were varied. Three plywood face grain orientations were analyzed, namely, (1) parallel to the column axis, (2) parallel to the rafter axis, and (3) 30^{o} to the column axis.

Comparisons of Model Test Results

In both of the tests made on the model frames with the grain direction parallel to the column axis, the failure occurred at 353 pounds per foot of span. This was equivalent to 706 plf or 70. 6 psf for full scale frames spaced ten feet on centers. Figures (12) and (13) show the load deflection characteristics while Figures (14) and (15) show the load stress curves.

Two model frames with haunch plywood face grain direction parallel to the rafter axis were tested (Tests 3 and 4). The larger side of the triangular web at the haunch was placed adjacent to the rafter. Only one of these frames was loaded to failure. The other frame had photostress plastic mounted and failure was not desired. The failure in the one test occurred at the haunch at 338 plf. The load deflection and load stress curves are shown in Figures (16) and (17) and (18), (19) respectively.

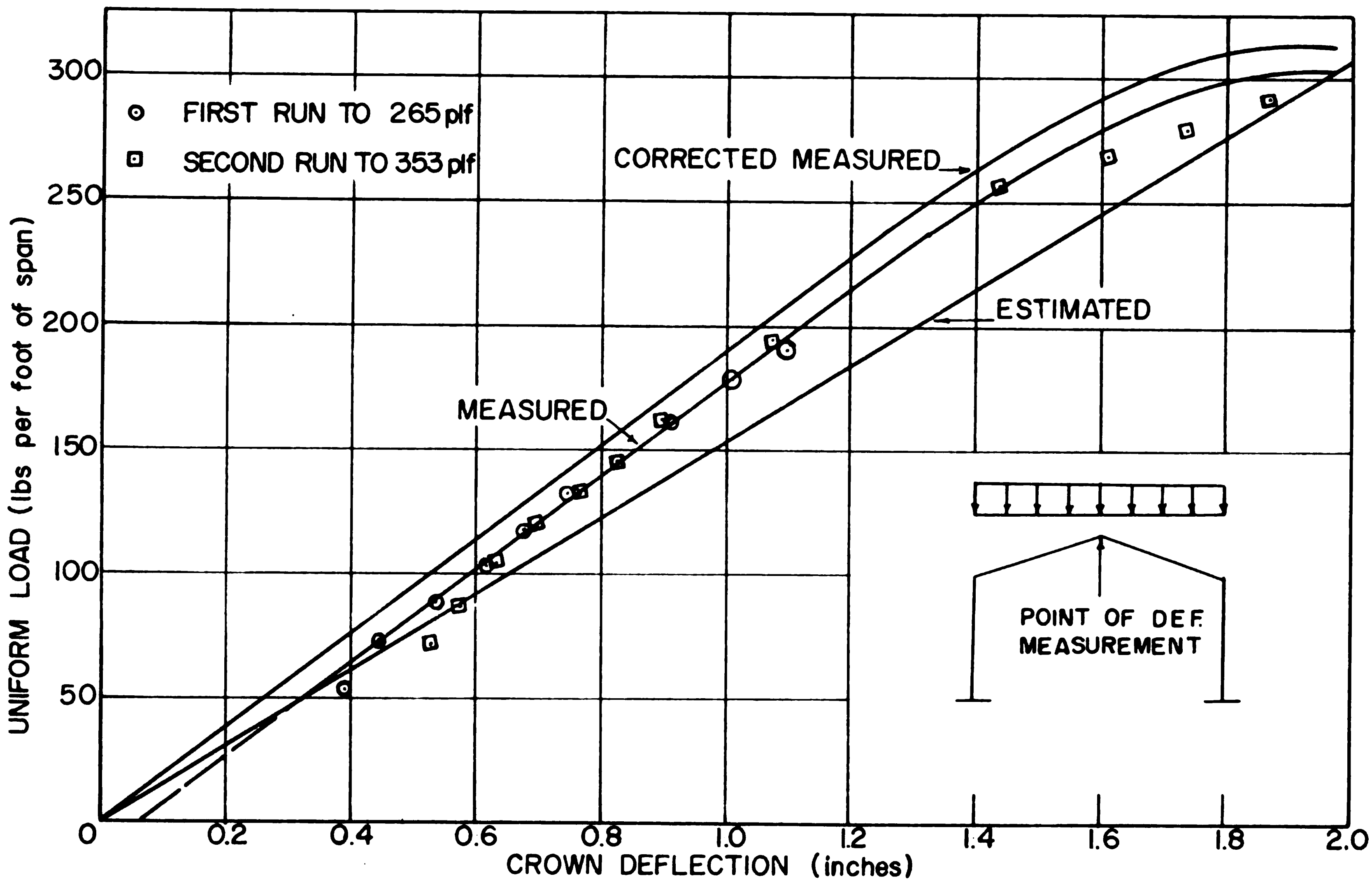


FIGURE 12 LOAD DEFLECTION CURVE FOR MODEL TEST I

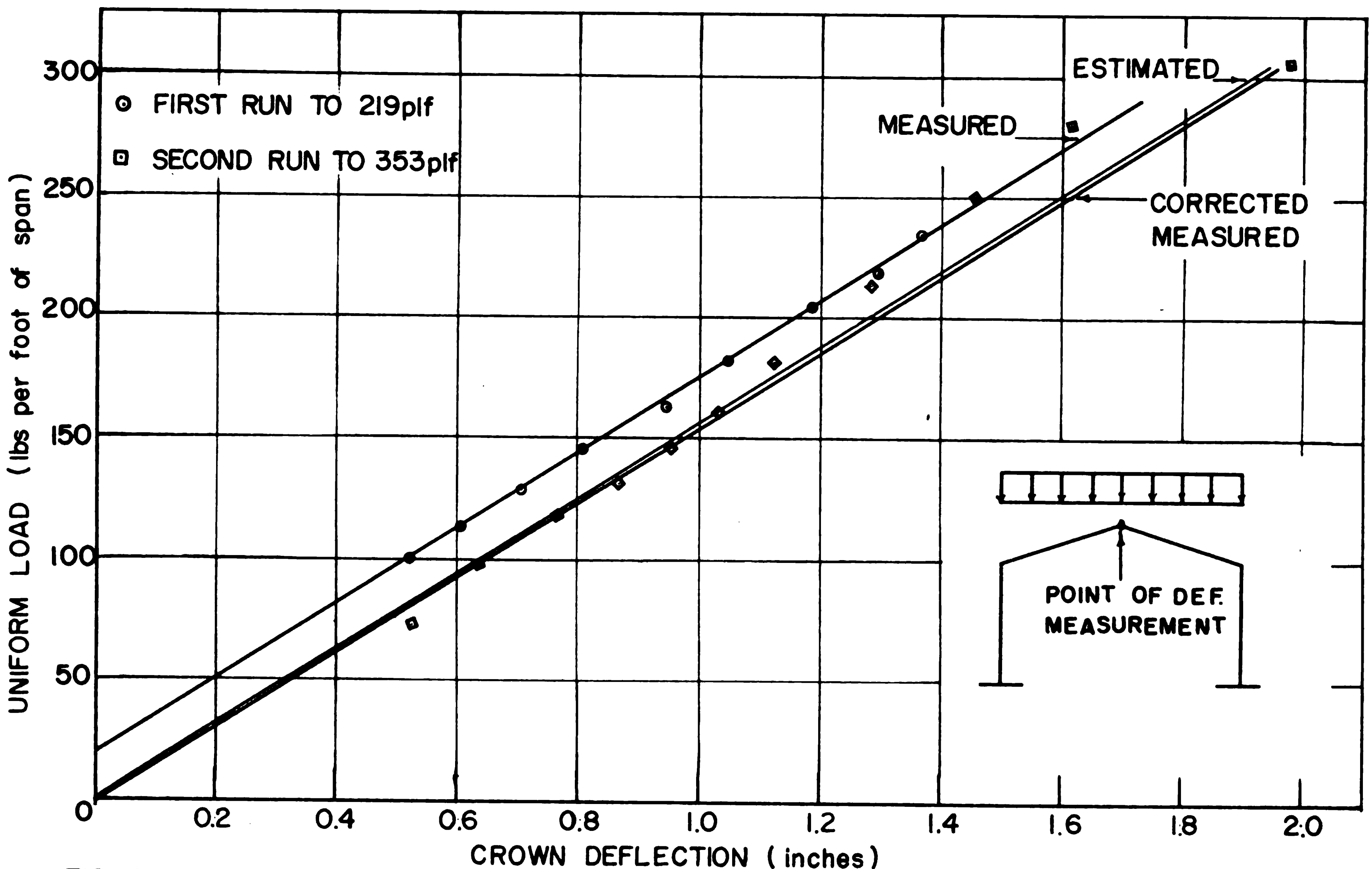


FIGURE 13 LOAD-DEFLECTION CURVE FOR MODEL TEST 2

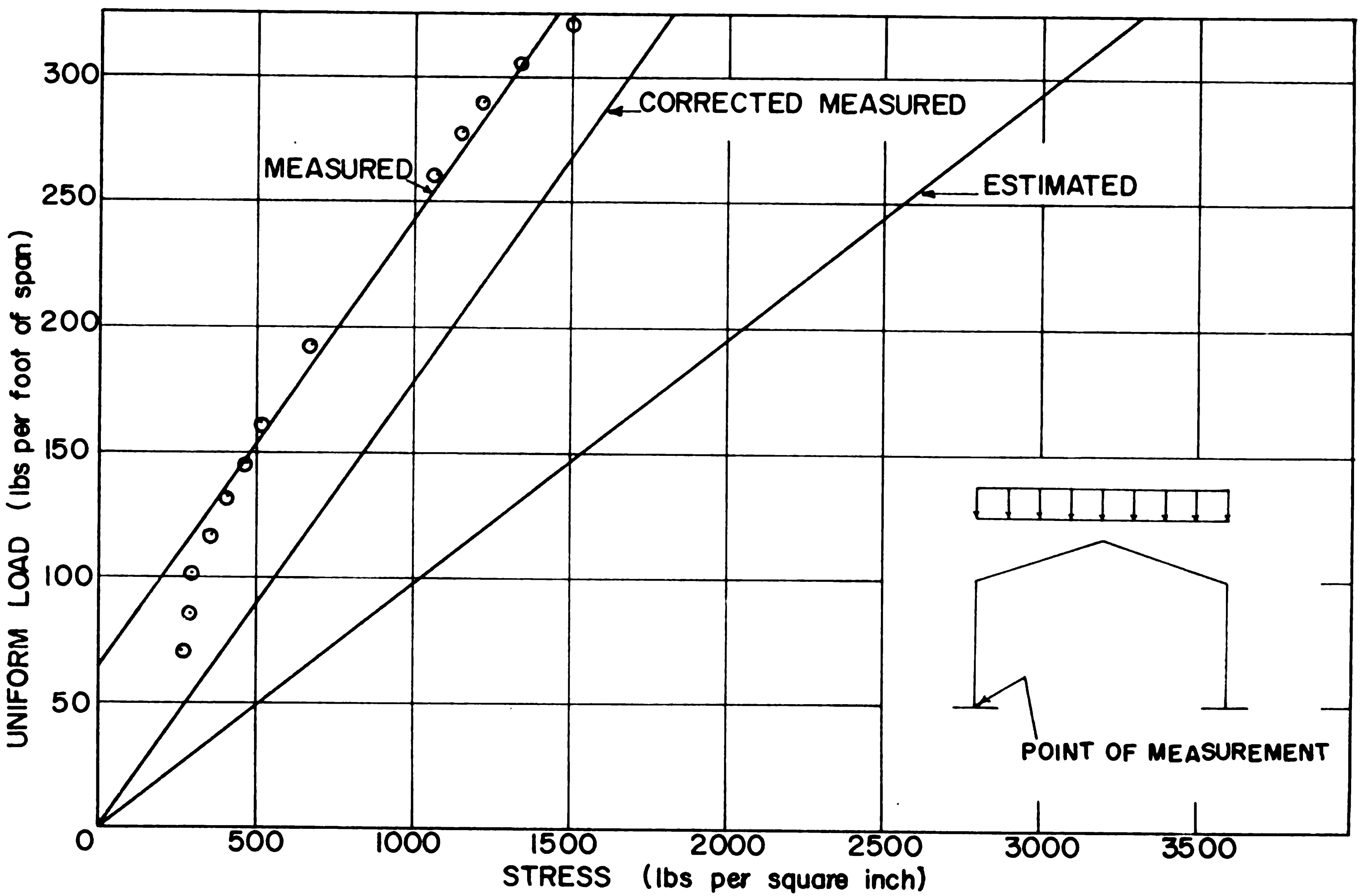


FIGURE 14 LOAD STRESS CURVE FOR MODEL TEST I

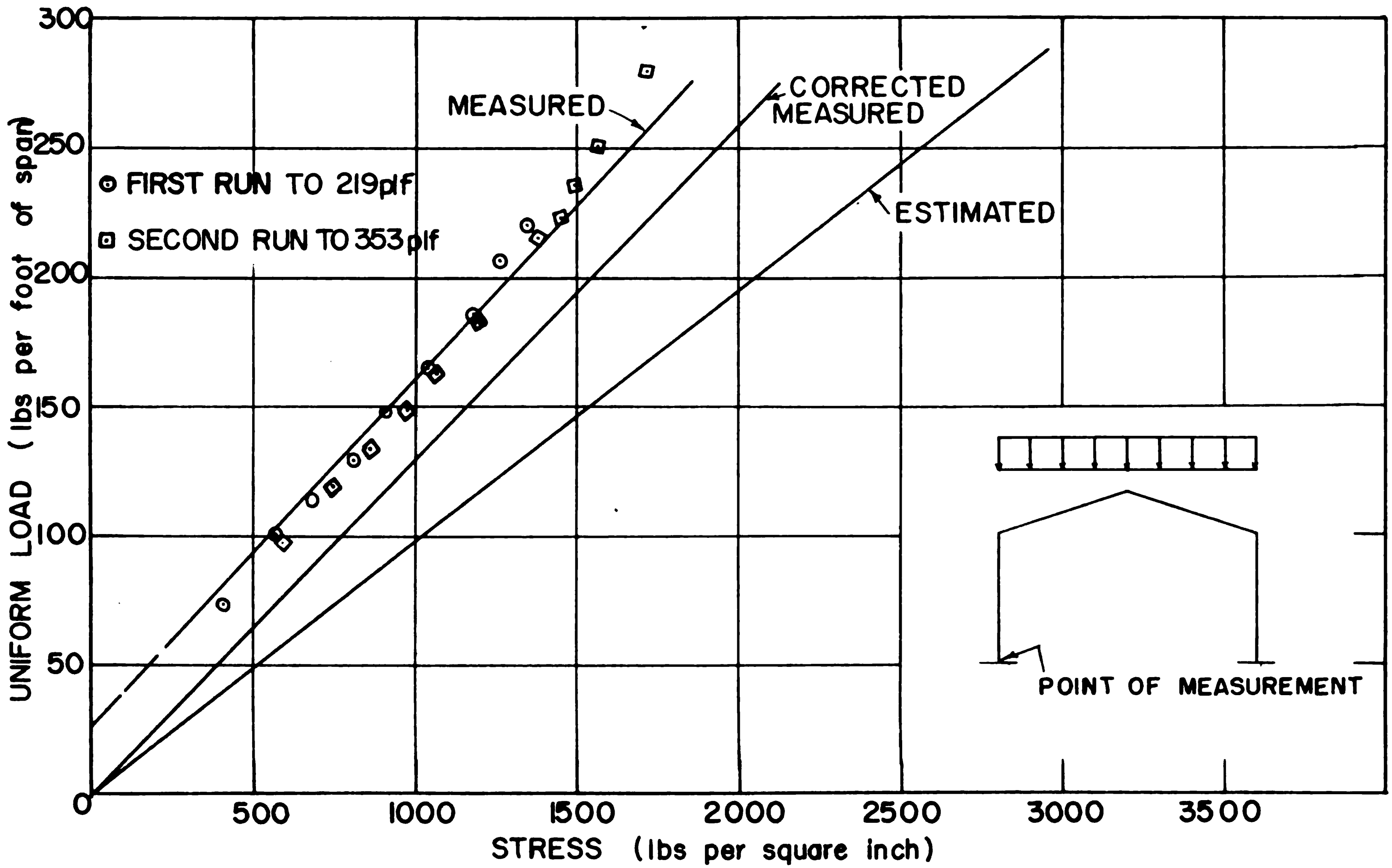


FIGURE 15 LOAD STRESS CURVE FOR MODEL TEST 2

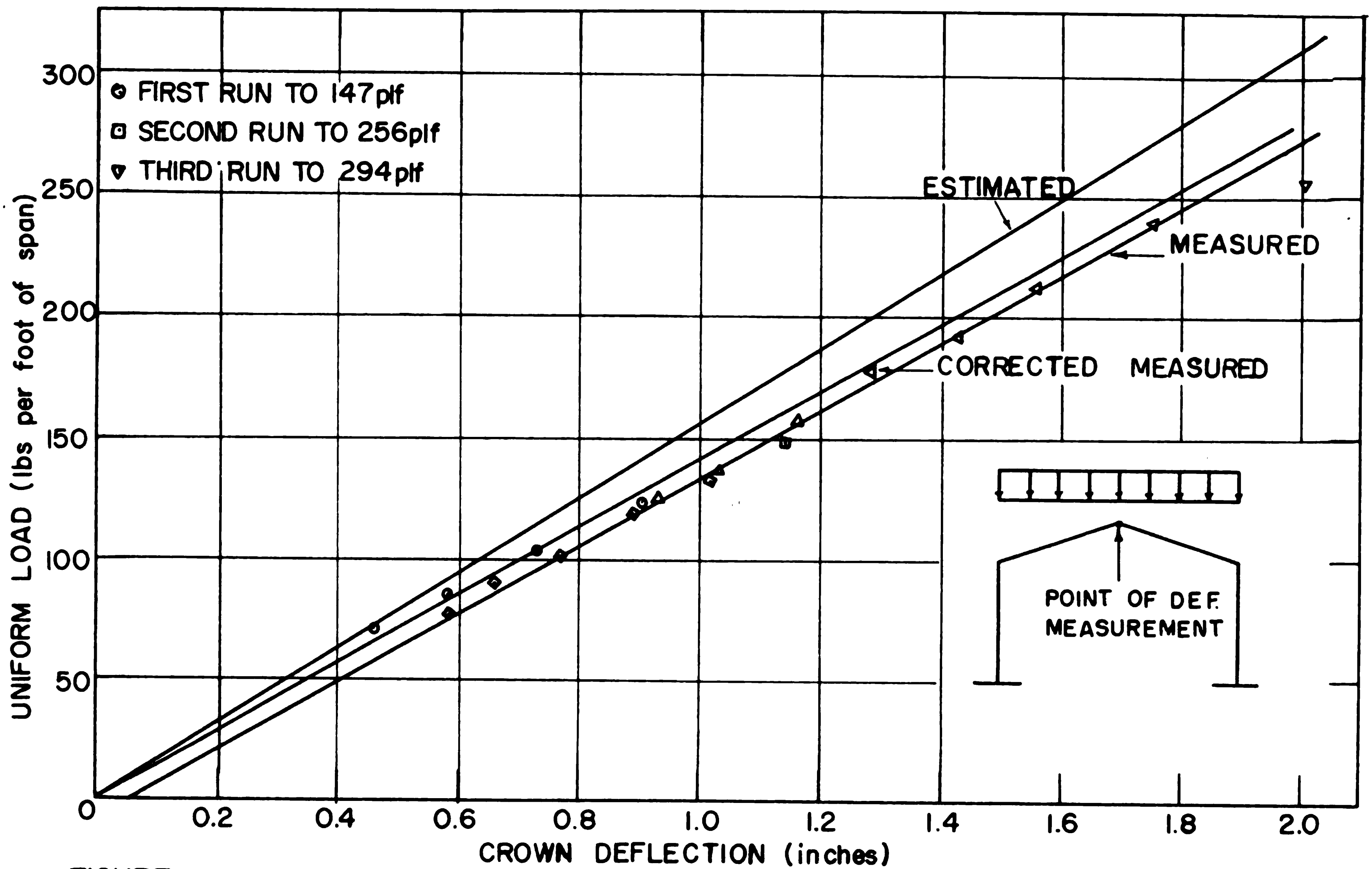


FIGURE 16 LOAD DEFLECTION CURVE FOR MODEL TEST 3

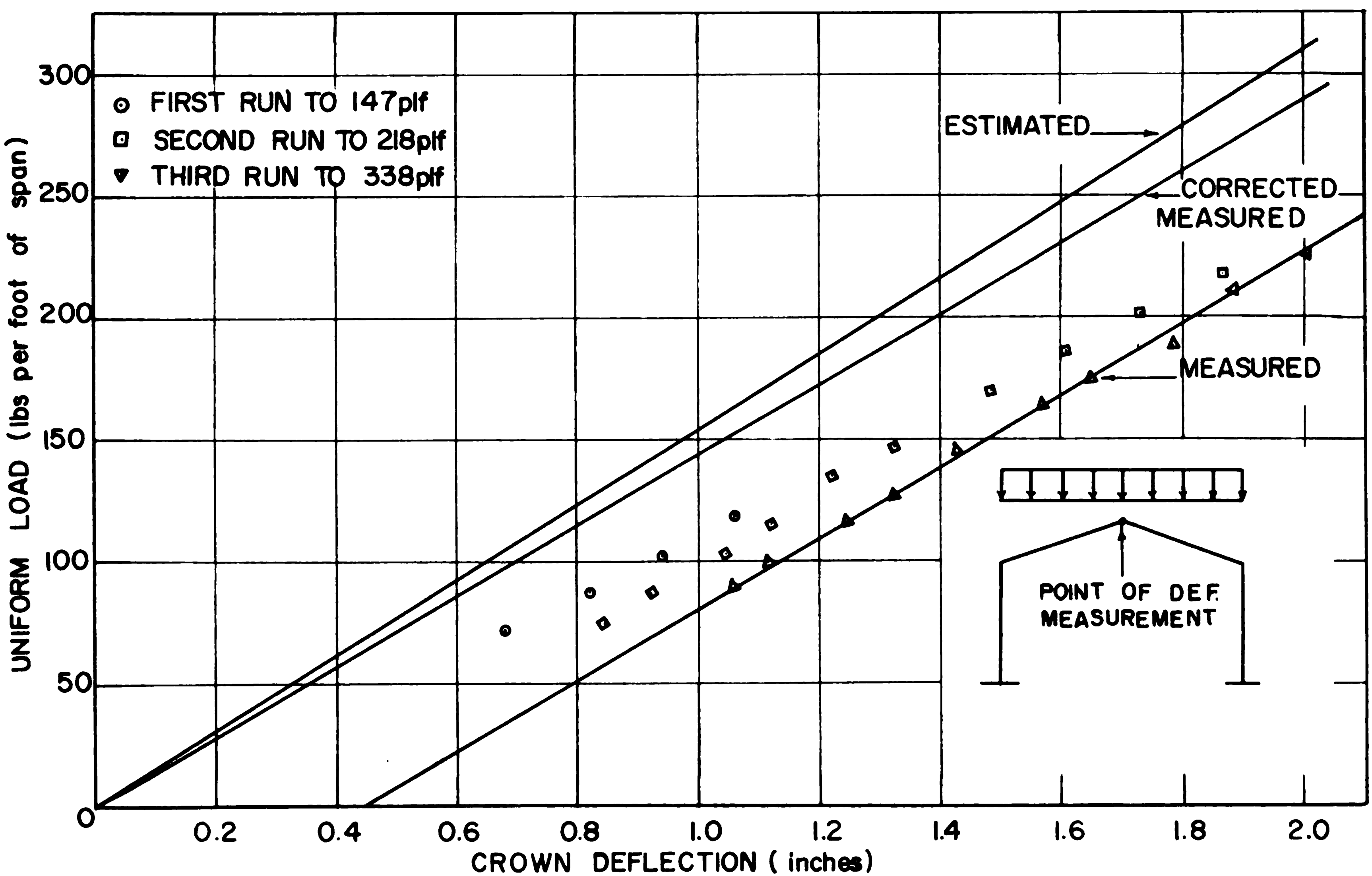


FIGURE 17 LOAD DEFLECTION CURVE FOR MODEL TEST 4

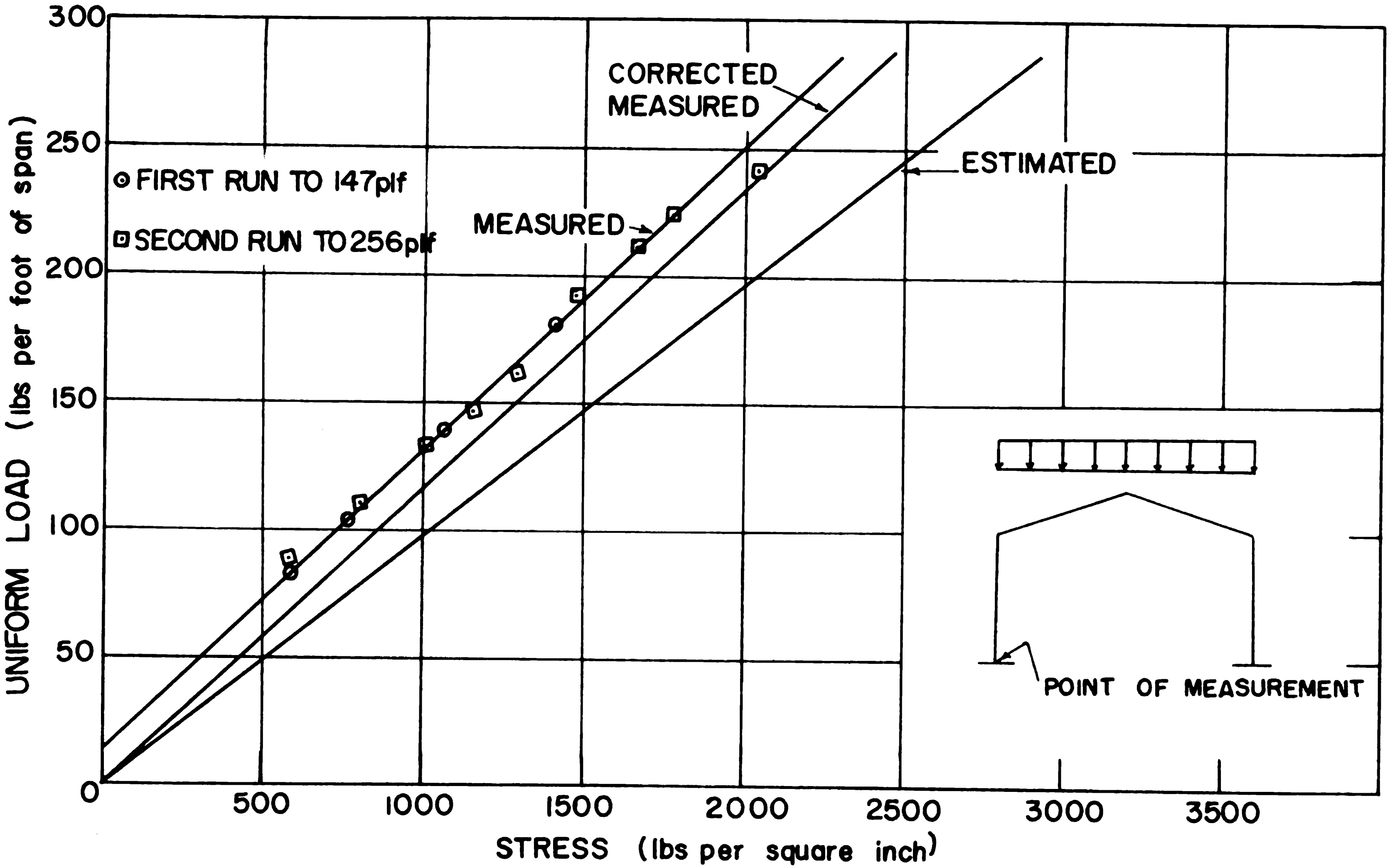


FIGURE 18 LOAD STRESS CURVE FOR MODEL TEST 3

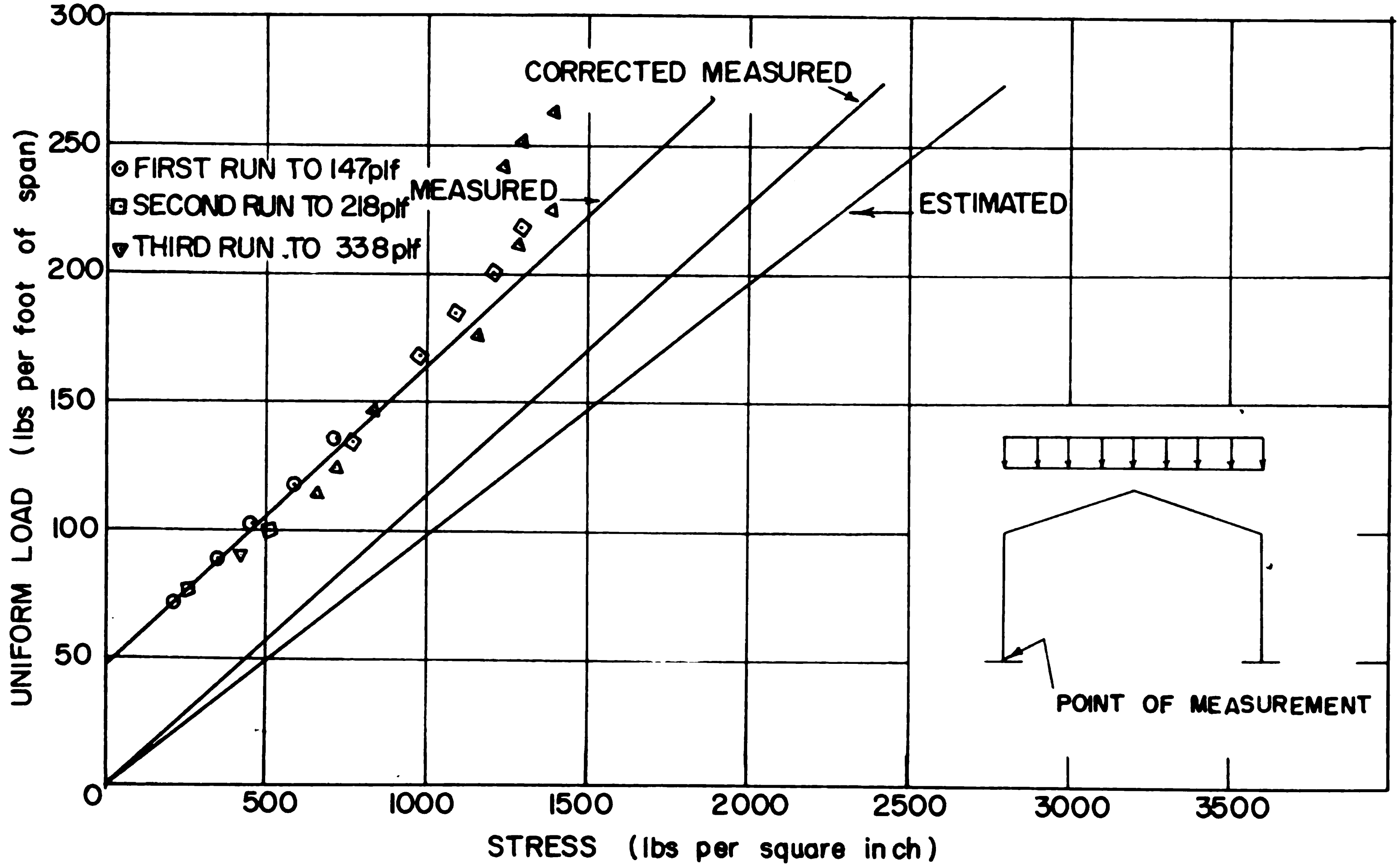


FIGURE 19 LOAD STRESS CURVE FOR MODEL TEST 4

The model frame tested with face grain direction 30° to the column axis and larger gusset side parallel to the column axis failed at 368 plf (736 plf on the prototype frame). The load deflection and load stress curves are shown in Figures (20) and (21).

Load deflection and load stress curves for all model tests are summarized in Figures (22) and (23) respectively.

The test results are summarized in Table 1. No major difference was evident between the loads at failure and the types of failure for the model tests with varying plywood face grain direction at the haunch. The load factors of safety ranged from 2. 25 for face grain direction parallel to the rafter axis to 2.46 for the frame with face grain direction 30° to the column axis.

A closer examination of load deflection curves (Figure 22) reveals that the frames having the large side of the triangular haunch plates parallel to the column axis (Test Nos. 1, 2 and 5) deflected less.

Types of Failure

The failure of one-half scale model frames was at the haunch in combined bending and shear for all but one frame. Test No. 2 with orientation of haunch plywood face grain parallel to the column axis failed at the hinged crown. This was an unusual and unexpected failure caused by an improperly fabricated hinged connection.

Test No. 1, with the same grain direction as 2, failed at the

Table 1. Summary of model frame tests

M_B = -238w in-lb

H = 4.55w

Test No.	Orientation of face grain at the haunch	Maximum load (lb/ft)	Maximum theoretical moment (in.-lb)	Type of failure	$\frac{\text{Max. load}}{\text{design load}}$ = load F.S.
1	11 to column axis	353	84000	Combined bending and shear at the haunch. Shear between plywood veneers as well as glue line	2.35
2	11 to column axis	353	84000	Crown failed in crushing and bending as two rafter members at the crown could not rotate	2.35
3	11 to rafter axis		-- -		
4	11 to rafter axis	338	80500	Combined bending and shear at the haunch (see Fig. 26)	2.25
5	30^{o} to column axis	368	85250	Combined bending shear and buckling (see Fig. 27)	2.46

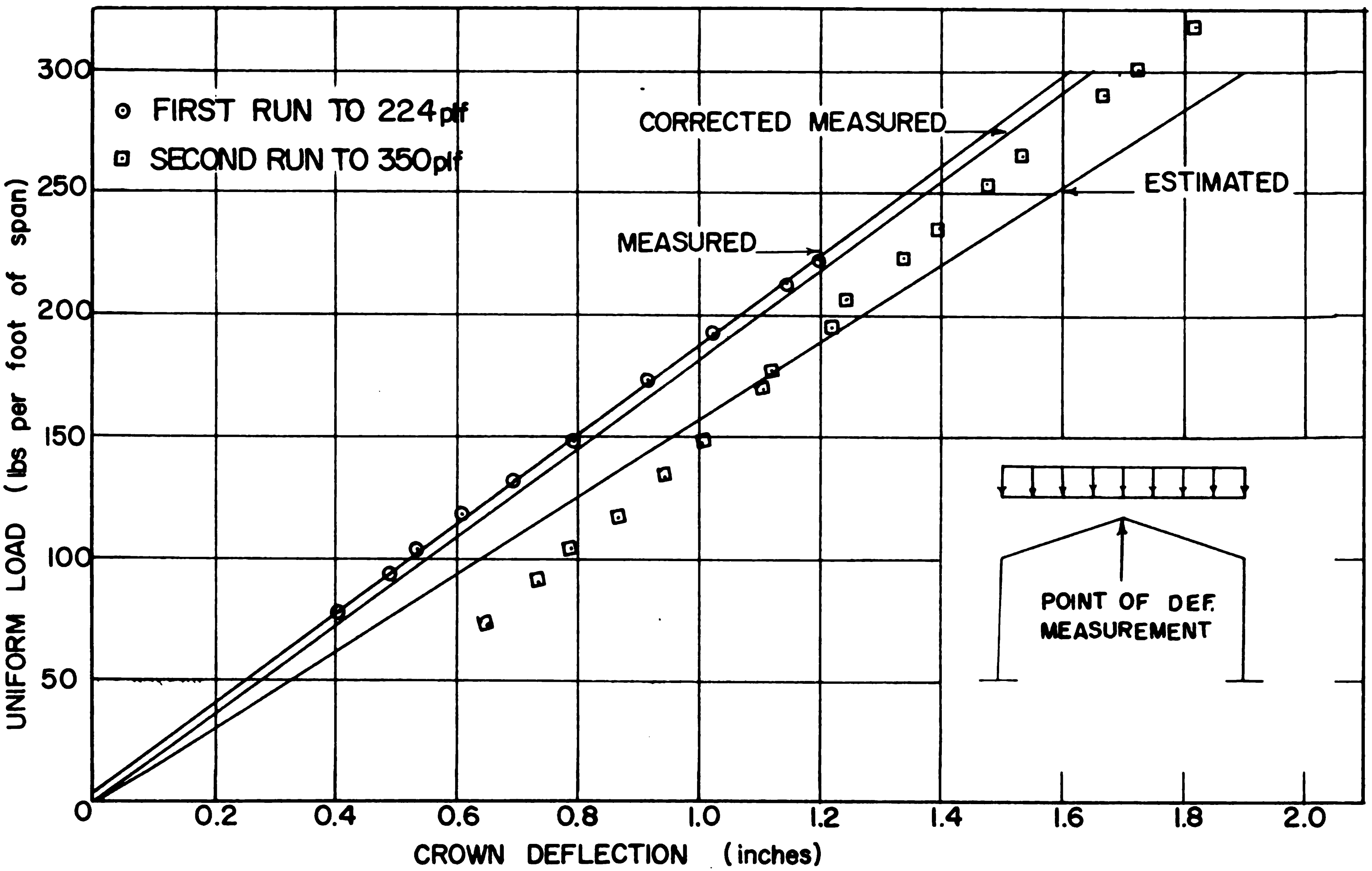


FIGURE 20 LOAD DEFLECTION CURVE FOR MODEL TEST 5

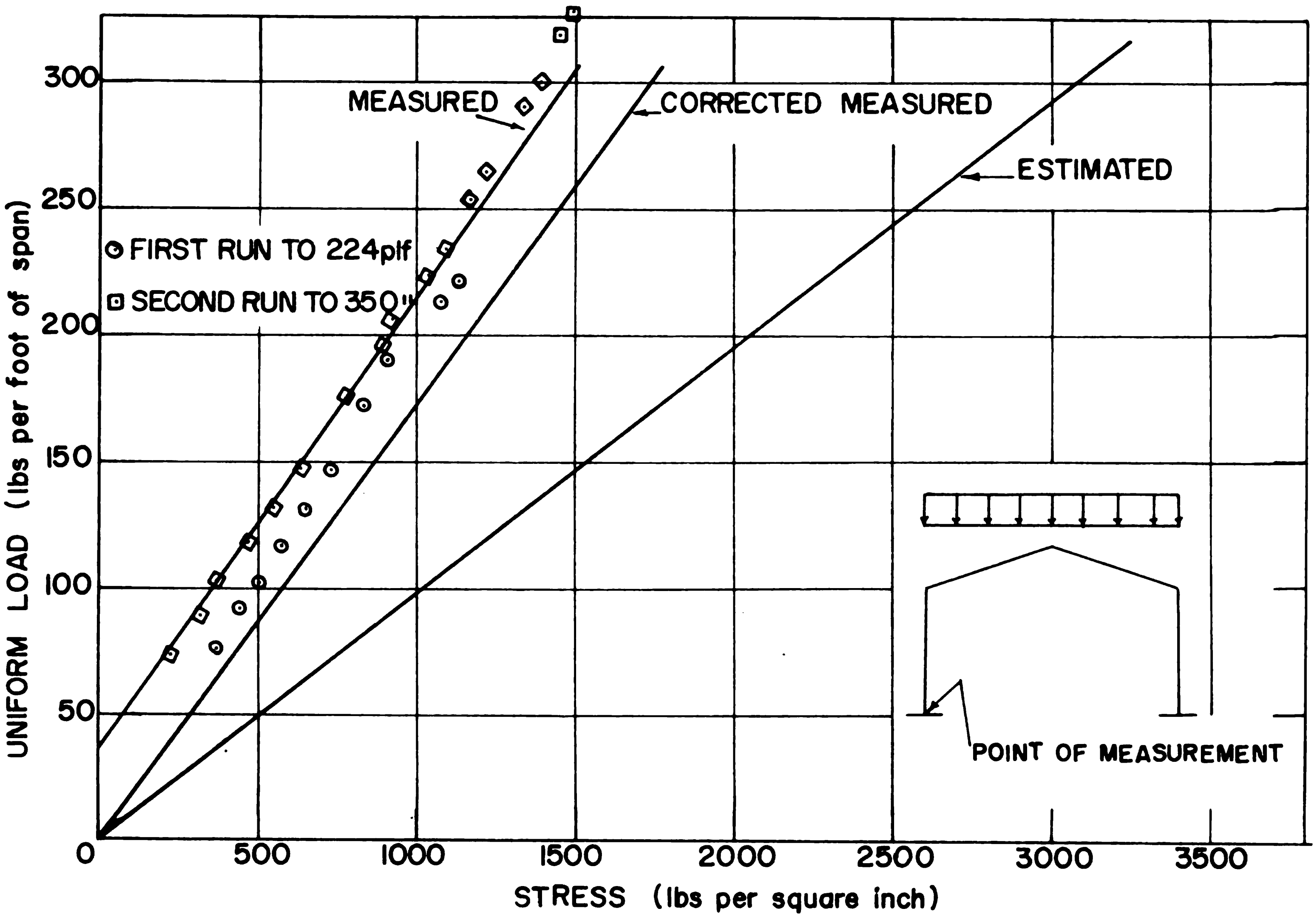


FIGURE 21 LOAD STRESS CURVE FOR MODEL TEST 5

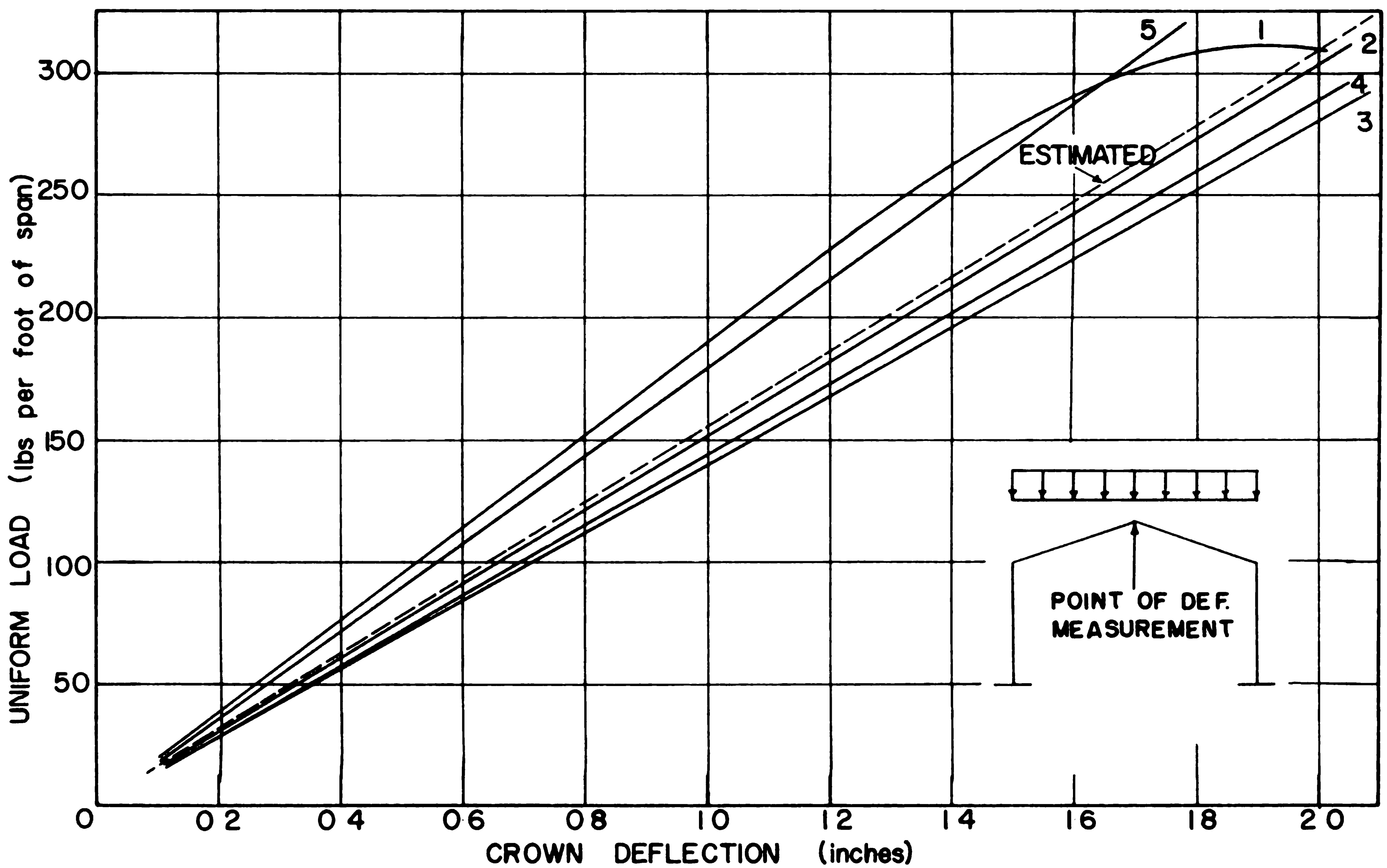


FIGURE 22 COMPARISION OF ESTIMATED AND MEASURED DEFLECTIONS

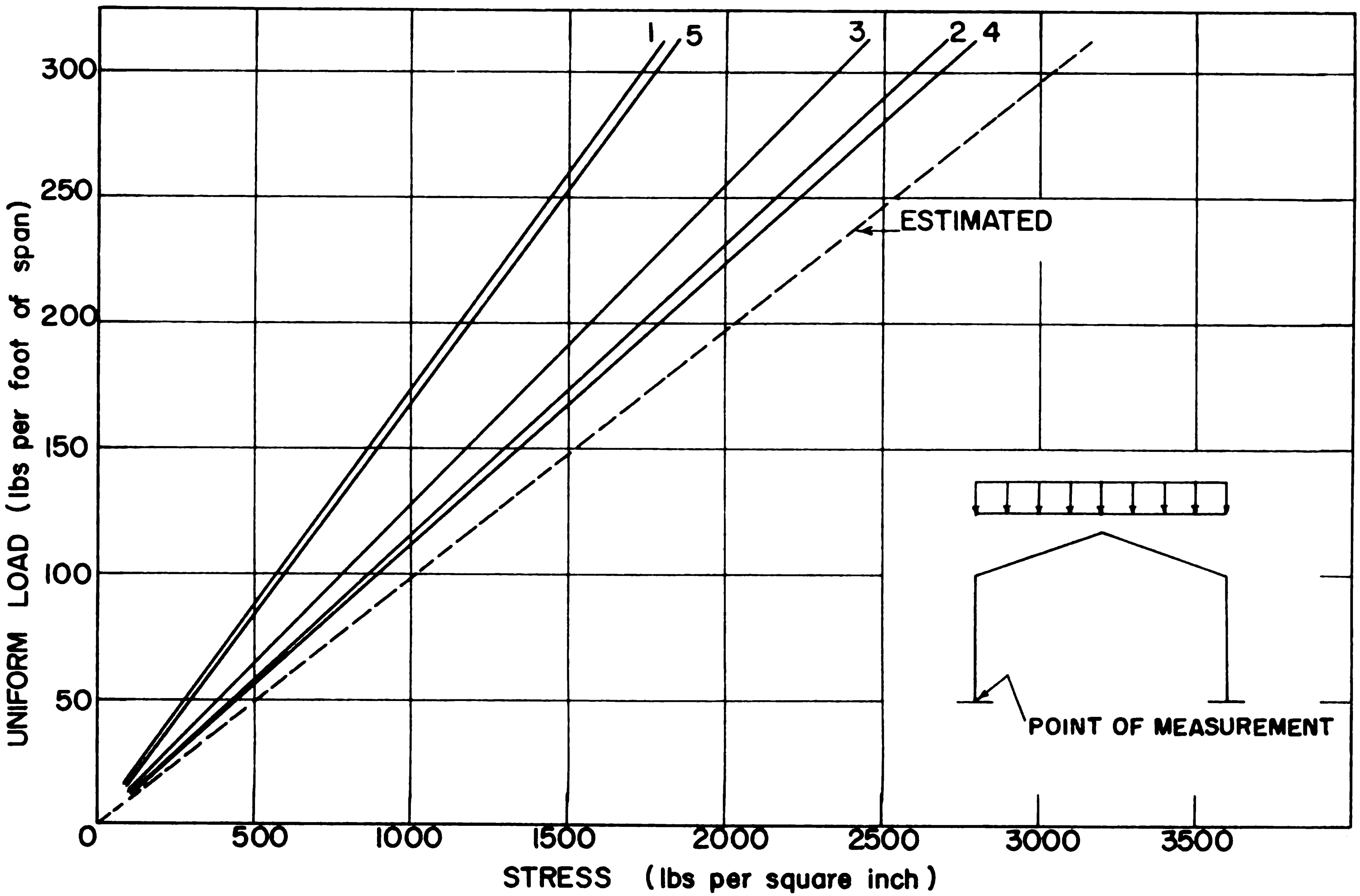


FIGURE 23 COMPARISION OF ESTIMATED AND MEASURED STRESSES

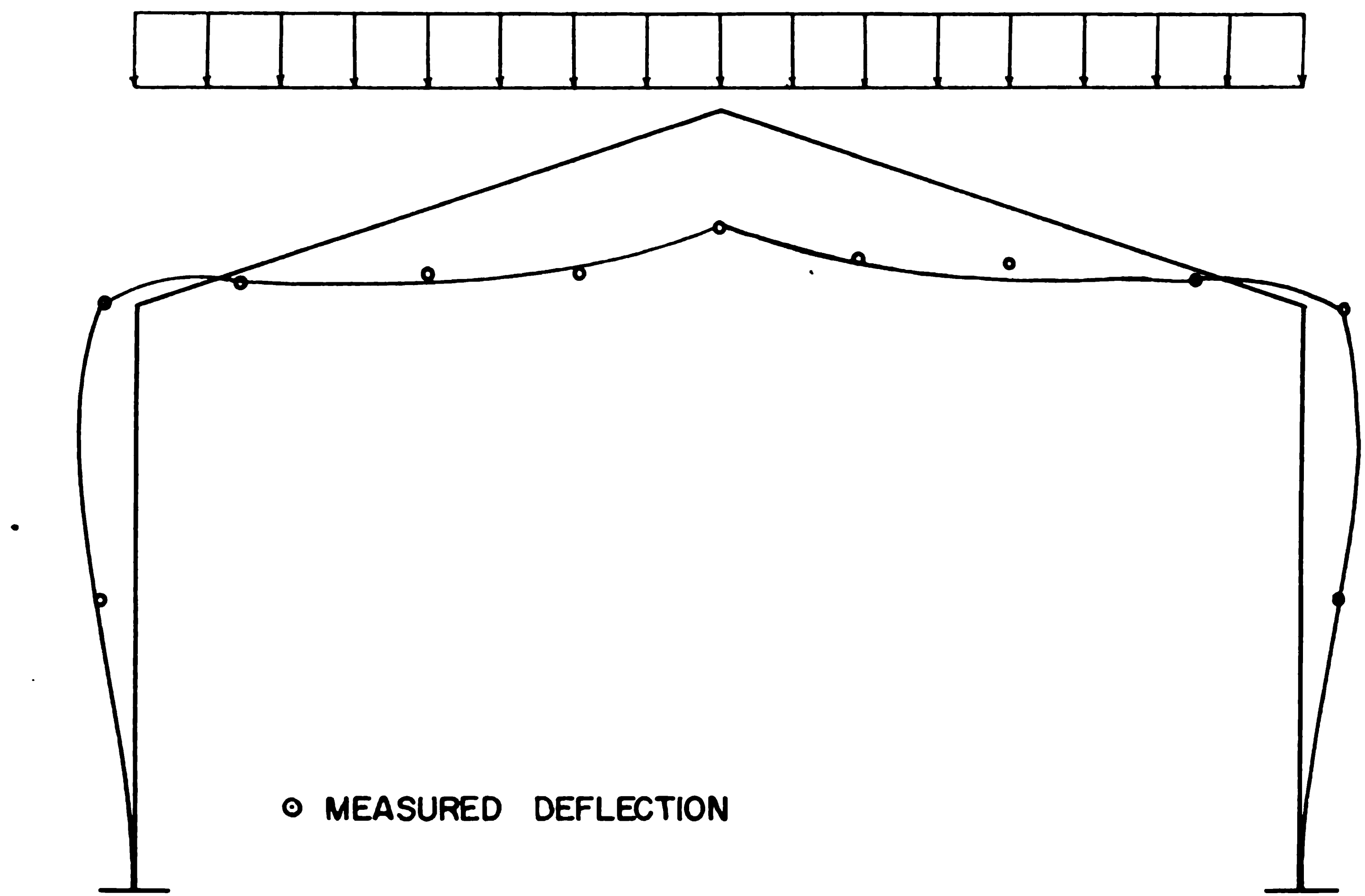


FIGURE 24 A TYPICAL DEFLECTED FRAME (Deflection Scale is 12 Times the Layout Scale)

haunch in buckling of webs. This was accompanied by a bending failure on the tension side of the haunch. Cracks appeared in the plywood webs in the face grain direction. Figure (25) shows the failure.

The failure in test No. 3, with face grain orientation parallel to the rafter axis is shown in Figure (26). This was a typical bending and shear failure. The failure started with rolling shear in the web and the flange glue line. The plywood web failed finally in excessive tension perpendicular to the face grain. Both haunches failed simultaneously with similar patterns.

In model test No. 5, with the face grain direction 30° to the column axis, many types of failures were apparent. Figure (27a) shows cracks on the tension side along the face grain and a buckling failure on the compression side. A rolling failure at the corner was also detected. Figure (27b) shows the tendency of the outer column flanges to be pushed out at the haunch because of the discontinuity of the member.

Haunch Test Results

Table 2 summarizes the performance of the test specimens.

The maximum bending moment (moment at failure) for the haunch tests compared closely with that for model frames (Table 1). This further supported the validity of the analysis.

FIGURE 25 Compression failure of unsupported web member at haunch

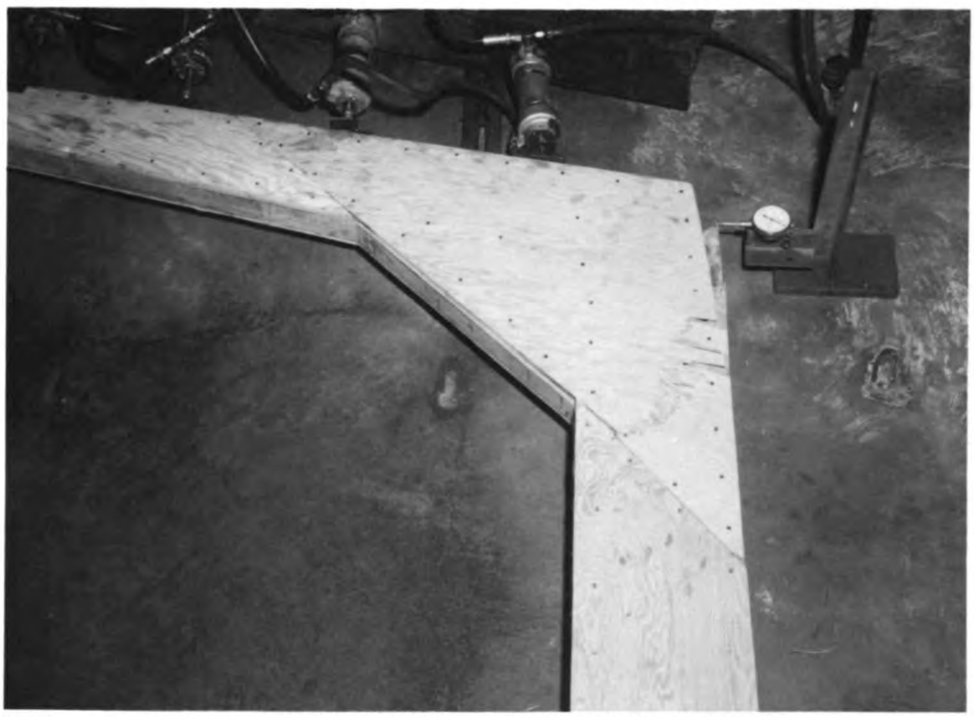
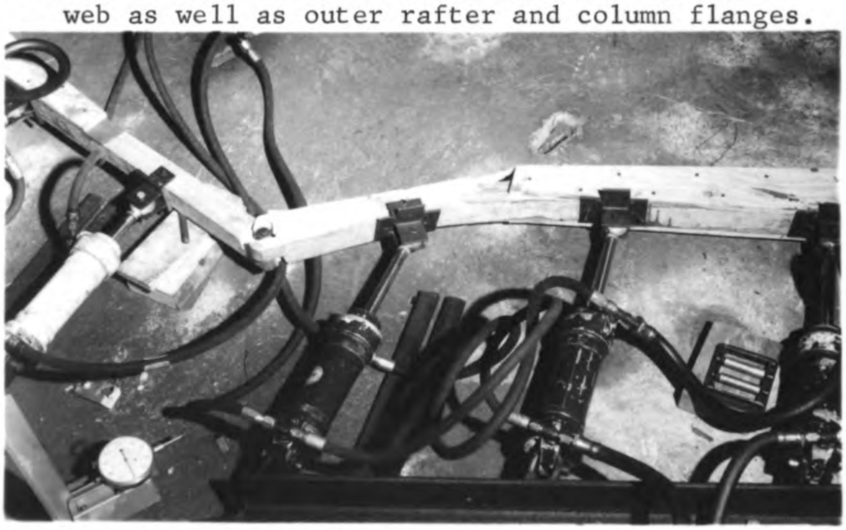


FIGURE 26 Bending and shear failure of haunch joint having a web stiffner

A Bending failure. Note cracks on both tension and compression side.

B Shear failure. Note shearing between plywood web as well as outer rafter and column flanges.



C Hinged crown failure.

FIGURE 27 - TYPICAL FAILURES IN MODEL TEST 5. ALL FAILURES OCCURRED SIMULTANEOUSLY.

Table 2. Summary of haunch test specimens

Test No.	Haunch description	Max. B. M. at failure (in. -lb.)	Type of failure	Relative strength
1	Outer column and rafter flanges connected with steel straps. Grain direction of face plies 11 to column axis.	89490	Outer column flange failed in tension, the web at the haunch also failed in shear between plywood veneers.	100
2	No steel straps used. Grain direction of face plies 11 to column axis.	73400	Shear failure in plywood veneers and in the plane 11 to the glue line. Outer column flange came out. Cracks also appeared along the face grain direction because of excessive bending.	82
3	No steel straps used. Grain direction of face plies 11 to the rafter axis.	75500	Combined bending and shear failure of the web.	84. 4
4	No steel strap, grain direction of face plies 36° to the column axis.	96750	Combined bending and shear. Outer column flange came off and web failed on the tension side. This was exactly the same type of failure as in model test 4.	108. 2

A tension failure occurred in the test No. 1 with steel strap. Excessive tension was evidenced in the outer column flange near the haunch and below the steel strap. Failure in the other specimens tested was in the plywood web in combined bending and shear. This indicated that the steel strap helped flanges develop full bending stress.

There was no major difference between the loads at failure for the specimens without steel straps. This was true although the failure in the haunch specimen with plywood face grain oriented 36° to the column axis, occurred at considerably higher moment. The maximum applied moment was even more than for the haunch with steel strap. This was due partly to a better grade of plywood (A-A Int., A-D Int. in the rest) at the haunch and partly due to better glue joint.

Stress Distribution at the Haunch

The stress distribution at the haunch was investigated using photostress technique. A 7" x 10" photostress sheet plastic (0.120 in. thick) was bonded with reflective cement at the haunch. A sequence of isochromatic and isoclinic pictures were taken for two loads.

Figures (28 and 29) show a typical isochromatic pattern at 147 plf and a 338° isoclinic for the same load. Nails and stress concentration around the nails are clearly visible.

FIGURE 28 An isochromatic pattern as seen through the Photostress meter at a load of 147 plf. Neutral axis in second fringe from the top.

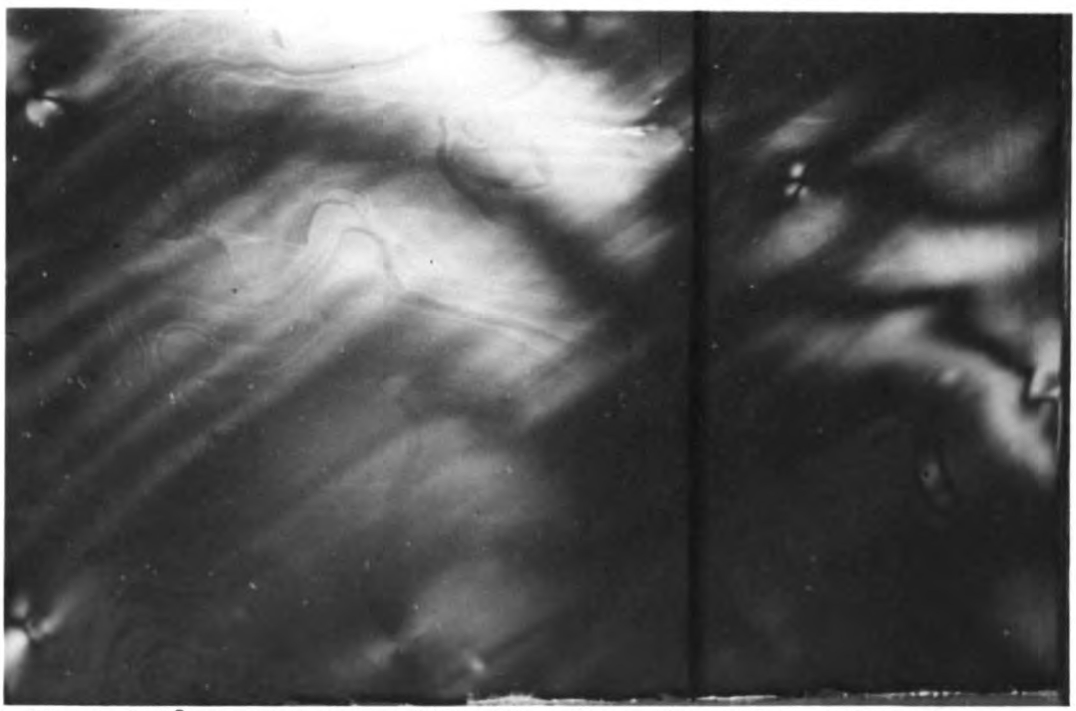


FIGURE 29 A 338^{o} isoclinic at 147 plf. Note the effect of grain direction and stress concentration around the nails.

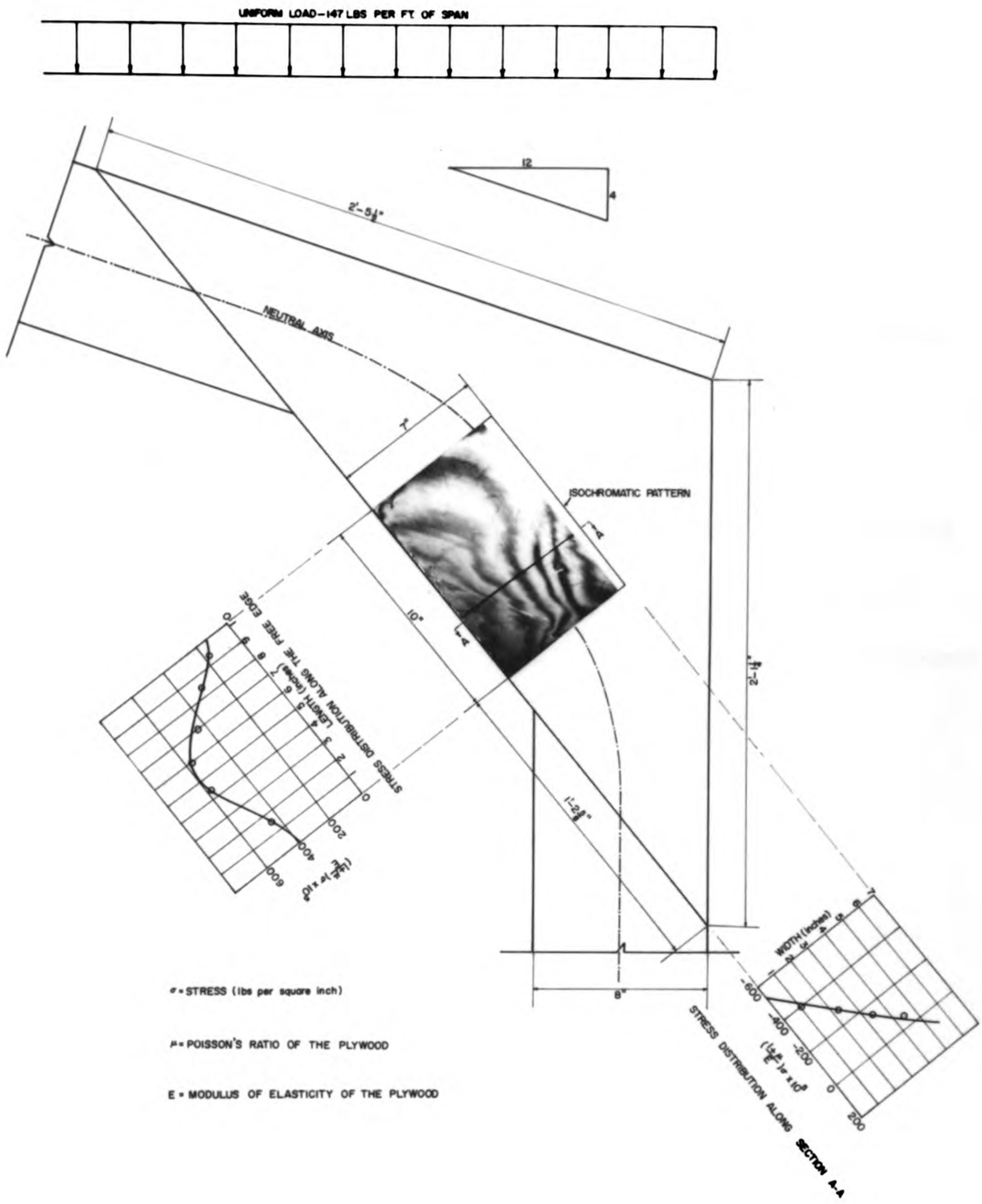


FIGURE 30 DISTRIBUTION OF STRESSES AT THE HAUNCH

The location of photostress plastic and the resulting stress distribution are shown graphically in Figure (30). Stresses along the free edge were calculated and plotted against the length of the sheet plastic. The values shown are principal stresses parallel to the free edge. Shear difference method was used to calculate stresses at interior points. Stresses plotted at section A-A are normal stresses parallel to the free edge.

The distribution shown is expressed as functions of E and μ (modulus of elasticity and Poisson's ratio) of the plywood. Numerical values of E and μ were not used as they varied at different points in the plywood because of its orthotropic nature. However, if it is desired to determine the stresses quantitatively using constant numerical values of E and μ, they can be determined as follows.

From stress distribution along free edge at point 3

$$\frac{1+\mu}{E}\,\sigma \times 10^{5} = 530$$

taking $\mu = 0.449$ in tangential direction for Douglas fir, and

$$E = 1.778 \times 10^{6} \text{ psi}$$

gives

$$\sigma = \frac{1.778 \times 10^{6}}{1 + 0.449}\, 10^{-5} \times 530$$

$$= 6560 \text{ psi}$$

The values of σ thus obtained using constant numerical values of E and μ were very high for the load applied. This was attributed to the fact that due to anisotropic and nonhomogeneous characteristics of plywood, E and μ were different at every point and is in fact, erroneous to assume constant values.

It is evident from the stress distribution along the free edge that the stresses decrease more rapidly towards rafter side from point 5 than the column side. Neutral axis on the column side is also closer to the free edge than it is on the rafter side. This was due to the larger dimension of the triangular web along the rafter. This provided a larger bearing section closer to the rafter and hence smaller stress. For a better design, the larger gusset dimension should be placed along the column side.

Evaluation of Model Analysis

The measured and estimated variables are presented in Table 3. In tests 2, 3 and 4 the measured deflection was more than estimated. This is an indication of some rotation at the column bases and a performance similar to a three-hinged frame.

In tests 1 and 2 the smaller deflections than estimated were due to some unintentional fixity of hinged joint at the crown. In each of these two frames the triangular plywood haunch web plates had their larger side parallel to the column axis. This probably provided some additional frame stiffness.

Table 3. Estimated and measured values of crown deflection (d) and fiber stress (s) at the heel

	Crown deflection (d) in in.			Heal stress s in $\#/in.^2$		
	Estimated d in in.	Measured d in in.	Percent difference	Estimated s in $\#/in^2$	Measured s_2 in $\#/in^2$	Percent difference
1	$0.645 \times 10^{-2} w$	$0.525 \times 10^{-2} w$	18.6	10.2w	5.70w	-
2	"	$0.65 \times 10^{-2} w$	0.775	"	7.80w	23.5
3	"	$0.71 \times 10^{-2} w$	10.09	"	8.60w	15.70
4	"	$0.68 \times 10^{-2} w$	5.42	"	8.90w	12.9
5	"	$0.555 \times 10^{-2} w$	13.95	"	5.80w	-

The stresses in each test were found less than the estimated. This was partly due to some rotation at the heels and partly due to instrumentation difficulties. Certain uncontrollable factors such as change in the environmental conditions of the test laboratory, change in the moisture content of wood and its effect on the bonding and response of strain gages, accounted for some variation between measured and estimated stresses.

The greatest difference in deflection for all tests between measured and estimated was 18.6 percent in the first test while the least was 0.775 percent in the second test. The greatest difference in the stress was 23.5 percent in the second test. The least was 12.9 percent in the fourth test. A larger difference in the estimated and measured stresses was found for tests 1 and 5. This was partly due to improper functioning of strain measuring equipment, and partly due to some rotation at the heels. Therefore, data for these tests were not considered.

The difference between measured and estimated deflections for three out of five tests were within 10 percent. The stresses for the same tests were within 23.5 percent with the limitations and difficulties encountered it would seem that the analysis was valid and its results can be projected for the design of prototype frames. The design table presented in appendix can be used for the design of one-hinged frames.

The magnitude of deflection and stress that can be expected in prototype frames can be predicted from the results of model analysis. Referring to model analysis, the prediction equations for crown deflection and heel stress are:

$$d = 2d_m \quad \text{and} \quad s = s_m$$

The crown deflection and heel stress for the full scale frame predicted from the test values for model test 4 gives:

$$d = 2(0.680 \times 10^{-2} w_m) \text{ and}$$

$$s = 8.90\, w_m$$

Since $w = 2w_m$

we have

$$d = 0.68 \times 10^{-2} w \text{ and}$$

$$s = 4.45\, w$$

For a normal design load of 300 plf (30 psf) on the full scale frame

$$d = 0.680 \times 10^{-2} \times 300 = 2.04 \text{ in}$$

$$s = 4.45 \times 300 = 1339 \text{ psi}$$

It should be remembered that stress 's' used here is only bending stress as the axial stress ($\frac{P}{A}$) was not measured. If the axial stress is also added to the bending stress obtained above then the total stress at the heels would be close to the allowable stress (1500 psi) for the grade of lumber used.

CONCLUSIONS

The following conclusions are based on the analysis and results as discussed and presented in this thesis:

1. The stress tables presented in the appendix are satisfactory for the design of one-hinged rigid frames. The design information presented under $\frac{I_2}{I_1} = 1$ are close for tapered rafter design.

2. The assumption that all stresses at the haunch are taken by the plywood webs was found to be too conservative and resulted in over design of the haunch. Using $\frac{P}{A} - \frac{Mc}{I}$ rather than $\frac{P}{A} + \frac{Mc}{I}$ to calculate normal stress at the haunch, arbitrarily compensated for the above assumption and provided a better balanced design.

3. Haunch web face grain orientations did not have an appreciable effect on the overall strength of the model frames. The factors of safety ranged from 2.25 to 2.45 for face grain direction parallel to the rafter axis and 30° to the column axis respectively. This permits a more economical layout of the haunch web.

4. The results of photostress analysis and the deflection characteristics of model frames indicated that frame stiffness can be improved by making plywood web dimension parallel to column larger than that parallel to the rafter portion.

5. A definite economy in material can be obtained by tapering the rafter portions of the frame as the bending moment decreases

parabolically from a maximum at the haunch to a minimum of zero at the crown.

6. Nail-glueing method of fabrication in the laboratory was satisfactory.

7. Model analysis provides an economical and accurate method of predicting the structural behavior of full scale frames. (The agreement between measured and estimated values obtained was 0.775% to 18.6% for deflection and 12.9% to 23.5% for stress). Therefore prototype testing can be materially reduced.

SUMMARY

The major objective of this investigation was to make theoretical and measured stress analysis for one-hinged plywood box-beam rigid frames for farm use. Design tables were also developed.

The frame was analyzed using Castigliano's theorem. A frame with 32 ft span, 16 ft column height and 4 in 12 slope was designed for 10 ft spacing. The design load used was 30 psf. Major consideration in the study was given to its suitability for model analysis.

Five one-half scale model frames with three different plywood face grain orientations--(1) parallel to the column axis, (2) parallel to the rafter axis, and (3) 30° to the column axis at the haunch were tested. Four haunch specimen tests were also made to find the effect of connecting outer column and rafter flanges with a steel strap.

The vertical crown deflection and bending stress at the heels were measured and compared with the corresponding estimated values. The load at failure and types of failure were compared.

A 7" x 10" photostress sheet plastic was mounted to determine stress distribution at the haunch.

The test results of the model frames are shown in Table 3. A satisfactory agreement between the measured and estimated variables was obtained. The analysis and design method used was satisfactory. The design tables presented in the appendix can be used for the design

of one-hinged frames. For box-beam frames, values given for $\frac{I_2}{I_1} = 1$ are close enough for the type of design adapted.

SUGGESTIONS FOR FURTHER STUDY

1. Investigate the possibilities of developing suitable and economical anchorage devices for one-hinged plywood box-beam frames.

2. Develop standard designs of one-hinged box-beam frames for common farm buildings.

3. Investigate the possibility of using other building materials such as steel and concrete.

4. Investigate the creep characteristics of plywood box-beam frames.

5. Determine the extent of variation of moduli of elasticity and Poisson's ratio in plywood.

6. Investigate the possibility of improving haunch strength by using laminated lumber to have continuous flange.

APPENDIX

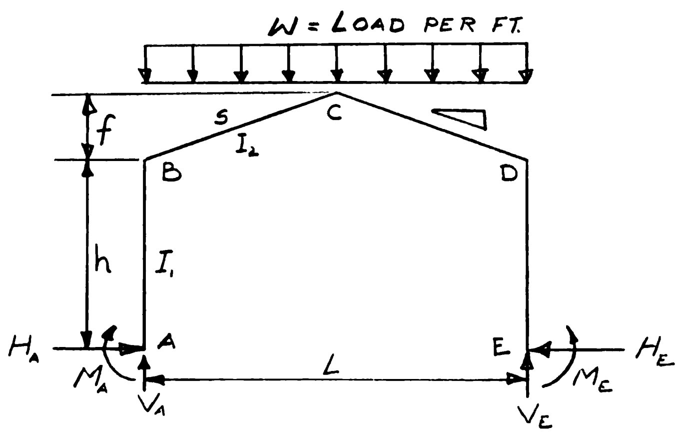
Single Span One hinged Rigid Frames
DESIGN TABLES – SPAN 20 FT. - 50 FT.

The tables that follow are essentially self explanatory.

Values shown are based upon a uniformly distributed load of one pound per foot and a roof slope of 4 in 12. Other values are proportional.

The equations derived for the following three cases can be used for any frame dimensions.

Case I: Uniform Vertical Roof Load



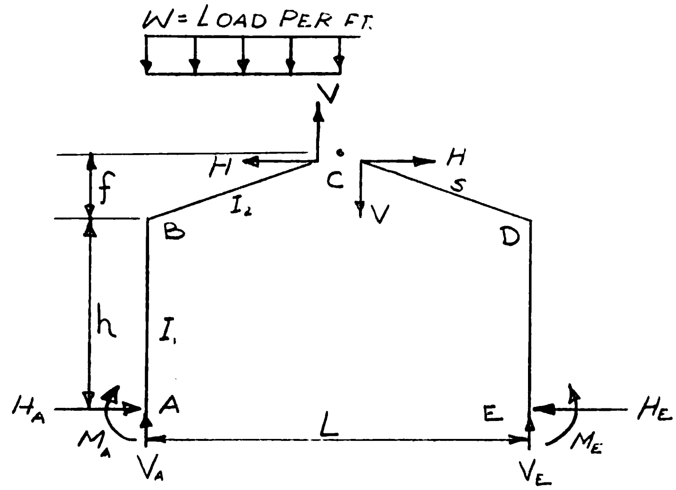
$$V_A = V_E = V = \frac{wL}{2}$$

$$H_A = H_E = H = \frac{\frac{wL^2}{8}\left[f\frac{s}{4} + \frac{I_2}{I_1}h\left(\frac{h}{2} + f\right)\right]}{f^2\frac{s}{3} + \frac{I_2}{I_1}h\left(\frac{h^2}{3} + f^2 + fh\right)}$$

$$M_A = M_E = H(h + f) - \frac{wL^2}{8}$$

$$M_B = M_D = Hf - \frac{wL^2}{8}$$

Case II: Uniform Vertical Roof Load on Half Span



$$V = \frac{\frac{wL}{8}\left(\frac{S}{4} + \frac{I_2}{I_1}h\right)}{\frac{s}{3} + \frac{I_2}{I_1}h}$$

$$H = \frac{\frac{wL^2}{16}\left[f\,\frac{s}{4} + \frac{I_2}{I_1}h\left(f + \frac{h}{2}\right)\right]}{f^2\,\frac{s}{3} + \frac{I_2}{I_1}h\left(f^2 + \frac{h^2}{3} + fh\right)}$$

$$V_A = \frac{wL}{2} - V$$

$$V_E = V$$

$$H_A = H_E = H$$

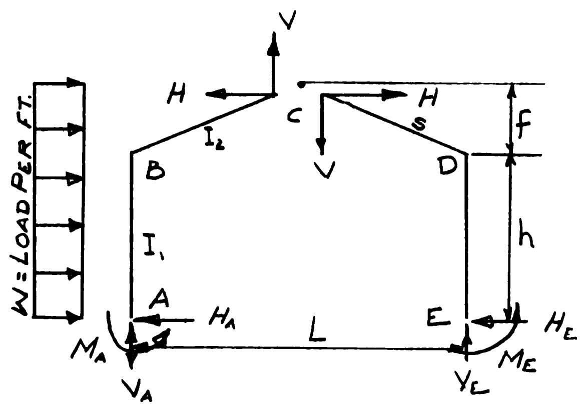
$$M_A = \frac{VL}{2} + H\,(f+h) - \frac{wL^2}{8}$$

$$M_E = \frac{VL}{2} - H\,(f+h)$$

$$M_B = \frac{VL}{2} + Hf - \frac{wL^2}{8}$$

$$M_D = \frac{VL}{2} - Hf$$

Case III: Uniform Wind Load



$$V = \frac{\frac{wh}{2}\left(f^2 + \frac{h^2}{3} + fh\right)}{L\left(\frac{s}{3} + h\right)}$$

$$H = \frac{\frac{w}{4}\left[f^3\,\frac{s}{3} + \frac{I_2}{I_1}\,h\left(f^2\,\frac{h}{2} + f^3 + \frac{h^3}{4} + f\,\frac{h^2}{3} + fhs + 2f^2 s\right)\right]}{f^2\,\frac{s}{3} + \frac{I_2}{I_1}\,h\left(f^2 + \frac{h^2}{3} + fh\right)}$$

$$V_A = -V_E = V$$

$$H_A = w(h+f) - H$$

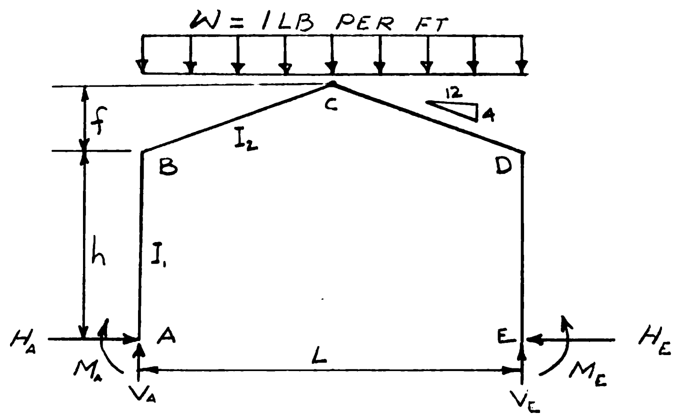
$$H_E = H$$

$$M_A = \frac{VL}{2} + H(h+f) - \frac{(f+h)^2}{2}\,w$$

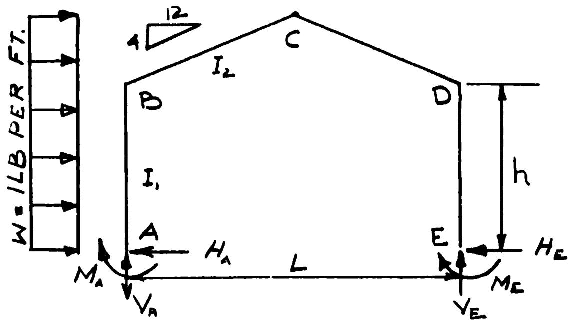
$$M_B = \frac{VL}{2} + Hf - \frac{f^2 w}{2}$$

$$M_D = \frac{VL}{2} - Hf$$

$$M_E = \frac{VL}{2} - H(f+h)$$



	$\frac{I_2}{I_1} = 1$				$\frac{I_2}{I_1} = \frac{1}{2}$			
h	M_A	$-M_B$	H_A	V_A	M_A	$-M_B$	H_A	V_A
6	23.33	23.81	7.86	10	27.01	22.50	8.25	10
8	24.66	28.04	6.59	10	28.41	26.94	6.42	10
10	25.18	31.20	5.64	10	28.60	30.35	5.89	10
				L=22				
6	27.46	27.14	9.10	11	31.69	25.53	9.54	11
8	29.42	32.24	7.71	11	34.01	30.80	8.10	11
10	30.26	36.15	6.64	11	34.60	34.99	6.96	11
12	30.61	39.18	5.82	11	34.51	38.26	6.06	11
				L=24				
6	31.69	30.52	10.37	12	36.42	28.63	10.84	12
8	34.45	36.52	8.87	12	39.88	34.70	9.32	12
10	35.69	41.33	7.69	12	41.03	39.71	8.07	12
12	36.36	44.93	6.77	12	41.18	43.70	7.07	12



	$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$					
h	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$
6	7.33	11.75	-3.71	22.63	6.18	0.68	10.07	10.77	-2.73	19.89	6.48	0.68
8	18.17	15.24	-0.26	25.51	8.18	1.03	20.21	14.64	0.34	23.48	8.36	1.03
10	31.69	19.54	3.69	28.42	10.12	1.44	33.26	19.14	4.08	26.85	10.24	1.44
L=22												
6	5.98	13.21	-5.50	26.31	6.20	0.66	9.58	11.84	-4.14	22.72	6.57	0.66
8	16.80	16.80	-1.90	29.64	8.20	0.98	19.54	15.94	-1.04	26.90	8.43	0.98
10	30.55	21.15	2.33	32.75	10.17	1.37	32.68	20.58	2.79	30.62	10.32	1.37
12	46.70	26.34	6.81	36.14	12.09	1.81	48.42	25.94	7.22	34.42	12.20	1.81
L=24												
6	4.57	14.75	-7.48	30.16	6.22	0.64	9.14	12.92	-5.65	25.58	6.68	0.64
8	15.28	18.48	-3.77	34.02	8.22	0.94	18.83	17.29	-2.59	30.46	8.52	0.94
10	29.17	22.89	0.53	37.41	10.20	1.31	31.96	22.09	1.33	34.62	10.40	1.31
12	45.63	28.13	5.31	40.93	12.15	1.73	47.90	27.56	5.87	38.66	12.29	1.73

	$\frac{I_2}{I_1} = 1$				$\frac{I_2}{I_1} = \frac{1}{2}$			
h	M_A	$-M_B$	H_A	V_A	M_A	$-M_B$	H_A	V_A
				L=26				
6	35.99	33.97	11.66	13	41.12	31.82	12.16	13
8	39.70	40.86	10.07	13	45.96	38.66	10.58	13
10	41.46	46.42	8.79	13	47.83	44.49	9.23	13
12	42.31	50.86	7.76	13	48.33	49.26	8.13	13
				L=28				
8	45.12	45.27	11.30	14	52.16	42.68	11.85	14
10	47.51	51.70	9.92	14	54.95	49.34	10.43	14
12	48.73	56.91	8.80	14	55.93	54.90	9.24	14
14	49.36	61.16	7.89		56.05	59.49	8.25	14
				L=30				
8	50.68	49.73	12.55	15	58.44	46.75	13.15	15
10	53.83	57.06	11.09	15	62.32	54.27	11.65	15
12	55.49	63.09	9.88	15	63.91	60.61	10.38	15
14	56.39	68.06	8.89		64.36	65.96	9.31	15
				L=32				
10	60.36	62.48	12.28	16	69.89	59.17	12.90	16
12	62.56	69.37	10.99	16	72.24	66.39	11.55	16
14	63.78	75.09	9.92	16	73.11	72.52	10.40	16
16	64.47	79.82	9.02	16	73.21	77.70	9.43	16

	$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$					
h	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$
						L=26						
6	3.13	16.37	-9.64	34.14	6.25	0.62	8.79	13.99	-7.26	28.48	6.80	0.62
8	13.62	20.27	-5.86	38.64	8.24	0.92	18.13	18.68	-4.27	34.12	8.60	0.92
10	27.57	24.76	-1.39	42.39	10.23	1.26	31.16	23.67	-0.30	38.80	10.48	1.26
12	44.29	30.05	3.58	46.08	12.20	1.65	47.21	29.28	4.35	43.17	12.37	1.65
						L=28						
8	11.85	22.16	-8.15	43.46	8.25	0.89	17.45	20.10	-6.08	37.86	8.69	0.89
10	25.80	26.75	-3.53	47.65	10.26	1.22	30.31	25.32	-2.10	43.14	10.56	1.22
12	42.70	32.10	1.61	51.58	12.23	1.59	46.38	31.07	2.65	47.90	12.45	1.59
14	62.11	38.29	7.22	55.71	14.17	2.01	65.19	37.52	8.00	52.62	14.34	2.01
						L=30						
8	10.02	24.15	-10.63	48.46	8.27	0.87	16.82	21.54	-8.01	41.65	8.79	0.87
10	23.87	28.87	-5.90	53.16	10.27	1.18	29.42	27.01	-4.04	47.60	10.64	1.18
12	40.89	34.28	-0.58	57.40	12.26	1.54	45.46	32.94	0.77	52.83	12.53	1.54
14	60.56	40.52	5.22	61.69	14.22	1.94	64.40	39.51	6.23	57.86	14.42	1.94
						L=32						
10	21.81	31.10	-8.47	58.90	10.29	1.15	28.55	28.75	-6.13	52.16	10.73	1.15
12	38.88	36.59	-2.99	63.52	12.29	1.49	44.47	34.87	-1.27	57.93	12.61	1.49
14	58.77	42.88	2.99	68.03	14.26	1.88	63.47	41.58	4.29	63.32	14.50	1.88
16	81.12	50.01	9.44	72.76	16.20	2.30	85.45	49.00	10.44	68.73	16.38	2.30

	$\frac{I_2}{I_1} = 1$				$\frac{I_2}{I_1} = \frac{1}{2}$			
h	M_A	$-M_B$	H_A	V_A	M_A	$-M_B$	H_A	V_A
				L=34				
10	67.08	67.97	13.50	17	77.60	64.16	14.18	17
12	69.91	75.73	12.14	17	80.87	72.21	12.76	17
14	71.53	82.25	10.98	17	82.28	79.16	11.53	17
16	72.47	87.75	10.01	17	82.89	85.09	10.48	17
				L=36				
12	77.51	82.16	13.30	18	89.74	78.09	13.98	18
14	79.60	89.52	12.08	18	91.80	85.68	12.69	18
16	80.85	95.77	11.04	18	92.58	92.56	11.57	18
18				18				18
				L=38				
12	85.33	88.67	14.50	19	98.80	84.01	15.23	19
14	89.97	96.88	13.20		101.65	92.62	13.88	19
16	89.58	103.91	12.09		102.92	100.13	12.69	19
18								
				L=40				
14	96.61	104.32	14.35	20	111.78	99.42	15.08	20
16	98.65	112.16	13.18	20	113.62	107.76	13.84	20
18				20				20
20	100.73	124.82	11.28	20	114.38	121.40	11.79	20

	$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$					
h	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$
						L=34						
10	19.64	33.43	-11.26	64.86	10.31	1.12	27.69	30.52	-8.35	56.81	10.82	1.12
12	36.70	39.02	-5.63	69.91	12.31	1.45	43.44	36.86	-3.47	63.18	12.69	1.45
14	56.75	45.37	0.53	74.69	14.29	1.82	62.43	47.33	2.17	69.00	14.58	1.82
16	79.38	52.55	7.16	79.58	16.24	2.23	84.25	51.25	8.44	74.70	16.47	2.23
						L=36						
12	34.37	41.57	-8.49	76.55	12.33	1.42	42.38	38.90	-5.82	68.54	12.77	1.42
14	54.51	47.99	-2.16	81.65	14.32	1.77	61.31	45.95	-0.12	74.86	14.66	1.77
16	77.38	55.22	4.66	86.74	16.29	2.16	83.22	53.62	6.62	80.90	16.55	2.16
						L=38						
12	31.91	44.23	-11.56	83.42	12.34	1.39	41.32	40.98	-8.31	74.01	12.86	1.39
14	52.10	50.73	-5.07	88.90	14.34	1.73	60.13	48.23	-2.56	80.87	14.74	1.73
16	75.14	58.02	1.93	94.24	16.32	2.10	82.07	56.05	3.90	87.31	16.63	2.10
						L=40						
14	49.51	53.56	-8.20	96.42	14.36	1.69	58.92	50.56	-5.17	87.02	14.82	1.69
16	72.69	60.94	-1.02	102.05	16.35	2.05	80.83	58.55	1.36	93.92	16.71	2.05
20	126.76	78.14	14.74	113,69	29,24	2,88	133.05	76.57	16.32	107.40	20.48	2.88

	$\frac{I_2}{I_1} = 1$				$\frac{I_2}{I_1} = \frac{1}{2}$			
h	M_A	$-M_A$	H_A	V_A	M_A	$-M_A$	H_A	V_A
				L=42				
14	105.50	111.83	15.52	21	122.14	106.28	16.32	21
16	108.30	120.51	14.28	21	124.68	115.44	15.01	21
18				21				21
20	110.69	134.64	12.27	21	126.16	130.62	12.84	21
				L=44				
14	114.60	119.42	16.72	22	132.71	113.19	17.56	22
16	117.69	128.95	15.42	22	136.04	123.19	16.20	22
20	121.02	144.60	13.28	22	138.38	139.94	13.92	22
				L=46				
14	123.91	127.06	17.43	23	143.43	120.15	18.83	23
16	127.62	137.47	16.57	23	147.67	130.98	17.42	23
20	131.72	154.70	14.32	23	151.04	149.35	15.02	23
24	133.55	168.13	12.57	23	151.13	163.87	13.12	23
				L=48				
14	133.38	134.77	19.15	24	154.28	127.17	20.10	24
16	137.79	146.07	17.74	24	159.53	138.82	18.65	24
20	142.77	164.92	15.38	24	164.10	158.83	16.15	24
24	145.06	179.94	13.53	24	164.71	174.82	14.15	24

	$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$					
h	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$
						L=42						
14	46.78	56.58	-11.56	104.19	14.38	1.66	57.68	52.95	-7.92	93.30	14.90	1.65
16	70.05	64.00	-4.21	110.16	16.38	2.01	79.51	61.12	-1.33	100.69	16.79	2.01
20	124.60	81.30	11.93	122.16	20.30	2.80	131.95	79.40	13.84	114.82	20.57	2.80
						L=44						
14	43.92	59.68	-15.12	112.19	14.40	1.62	56.44	55.37	-10.82	99.67	14.99	1.62
16	67.22	67.18	-7.62	118.54	16.40	1.96	78.15	73.75	-4.18	107.62	16.87	1.96
20	122.20	84.58	8.89	130.99	20.34	2.74	130.70	82.30	11.17	123.49	20.65	2.74
						L=46						
14	40.94	62.88	-18.90	120.40	14.42	1.60	55.22	57.83	-13.85	106.13	15.08	1.60
16	64.24	70.48	-11.24	127.19	16.42	1.93	76.75	66.43	-7.19	114.68	16.95	1.93
20	119.54	88.00	5.62	140.16	20.38	2.67	129.33	85.39	8.34	130.37	20.73	2.67
24	184.81	108.88	24.36	153.95	24.24	3.54	192.73	106.96	26.27	146.03	24.49	3.54
						L=48						
14	37.87	66.19	-22.88	128.82	14.43	1.57	54.13	60.31	-17.01	112.66	14.17	1.57
16	61.10	73.90	-15.09	136.09	16.44	1.89	75.34	69.16	-10.35	121.86	17.03	1.89
20	116.67	91.55	2.13	149.65	20.41	2.62	127.86	88.35	5.32	138.47	20.81	2.62
24	182.53	112.53	21.23	163.71	24.29	3.45	191.59	110.26	23.50	154.65	24.58	3.45

	$\frac{I_2}{I_1} = 1$				$\frac{I_2}{I_1} = \frac{1}{2}$			
h	M_A	$-M_B$	H_A	V_A	M_A	$-M_B$	H_A	V_A
				L=50				
14	143.00	142.53	20.40	25	165.22	134.24	21.39	25
16	148.19	154.23	18.93	25	171.59	146.71	19.89	25
20	154.14	175.25	16.47	25	177.54	168.37	17.30	25
24	156.96	191.50	14.52	25	178.79	185.87	15.19	25

	$\frac{I_2}{I_1} = 1$						$\frac{I_2}{I_1} = \frac{1}{2}$					
h	$-M_A$	M_A	M_D	$-M_E$	H_A	$V_A=V_E$	$-M_A$	M_B	M_D	$-M_E$	H_A	$V_A=V_E$
L=50												
14	34.72	69.59	-27.06	137.42	14.45	1.54	52.88	62.81	-20.28	119.25	15.26	1.54
16	57.84	77.43	-19.16	145.21	16.45	1.86	73.92	71.92	-13.65	129.13	17.12	1.86
20	113.58	95.23	-15.98	159.46	20.44	2.57	126.29	91.49	2.14	146.75	20.89	2.57
24	179.97	116.30	17.88	173.84	24.34	3.38	190.28	113.64	20.54	163.54	24.66	3.38

REFERENCES

Aldrich, R. A. (1958). Adapting glass reinforced plastics to a rigid frame design. Thesis for degree of Ph. D., Michigan State Univ., East Lansing (unpublished).

Aldrich, R. A. and J. S. Boyd (1959). Glass-fibre plastics in farm building construction. Ag. Eng. 6:336-339.

Anderson, P. (1953). Statically Indeterminate Structures. The Ronald Press Co., New York.

Barre, H. J. and L. L. Sammet (1955). Farm Structures. John Wiley and Sons Inc., New York.

Boyd, J. S. (1954). Secondary stresses in trusses with rigid joints, special application to glued wooden trusses. Thesis for degree of Ph. D., Iowa State Univ., Ames (unpublished).

Curtis, J. O. (1959). Design of nailed and glued plywood gussets for lumber rigid frames. Paper No. 59-410, presented at annual meeting of ASAE.

Douglas Fir Plywood Association (1948). Technical Data on Plywood, Tacoma, Washington.

Esmay, M. L. (1961). Structural requirements of farm building, p. 544. Agricultural Engineers Handbook. McGraw-Hill Book Co., New York.

Frocht, M. M. (1941). Photoelasticity. Vol. 1. John Wiley and Sons, Inc., New York.

Giesie, H. (1940). The structural application of glue in framing farm buildings. Part 1. Ag. Eng. 22:47-50.

_______, and S. M. Henderson (1945). The structural application of glue in framing farm buildings. Part II, Ag. Eng. 31:455-457.

Heteny, M. (1950). Handbook of Experimental Stress Analysis. John Wiley and Sons, Inc., New York.

Korn, M. P. (1953). Steel Rigid Frames Manual: Design and Construction. J. W. Edwards, Ann Arbor, Michigan.

Mielock, P. J. (1959). Analysis and design of thirty foot single slope wood trusses. Thesis for degree of M. S., Michigan State Univ., East Lansing (unpublished).

Murphy, G. (1950). Similitude in Engineering. The Ronald Press Co., New York.

National Bureau of Standards (1957). American standard building code requirements for minimum design loads on buildings and other structures. Government Printing Office, Misc. Pub. M-179, Washington, D. C.

Nelson, G. L., J. I. Fryear, G. W. A. Mahoney, W. F. Lott, and C. E. Rice (1960). Development of a one-hinged frame for light building construction. Paper No. 60-900, presented at the winter meeting of ASAE.

Neubauer, L. W. and H. B. Walker (1961). Farm Building Design. Prentice-Hall Inc., Englewood Cliffs, N. J.

Perry, W. L. (1958). Design and testing of rigid frames with nailed plywood gusset plates. Douglas Fir Plywood Association Lab Report No. 76, Tacoma, Wash.

Post, D. and F. Zandman (1961). Accuracy of birefringent coating method for coating arbitrary thicknesses. pp. 21-32. Experimental Mechanics, Jan. 1961.

Shermer, C. L. (1957). Fundamentals of Statically Indeterminate Structures. The Ronald Press Co., New York.

Skinner, R. E. (1946). Glue as a fastner in framing farm buildings. Thesis for degree of M. S., Iowa State Univ., Ames (Unpublished).

U. S. Division of Housing Research HHFA (1952). Snow load studies. Housing Research Paper 19, Washington, D. C.

Wangaard, F. F. (1950). The Mechanical Properties of Wood. John Wiley and Sons Inc., New York.

Zandman F. (1959). Photostress Principles and Applications. Instruments Division, The Budd Company, Phoenixville, Pa.