A FEEDBACK CONTROL SYSTEM FOR VOLTAGE REGULATION

Thesis for the Degree of M. S.

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This is to certify that the

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A FEEDBACK CONTROL SYSTEM FOR VOLTAGE REGULATION

By

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W. C. Peterson

TABLE OF CONTENTS

I. Purpose

The primary purpose of this thesis is the design and analysis of a control system intended to maintain the output voltage of a DC generator at a fixed value, or any arbitrary fixed value within the voltage rating of the machine. It is required that the deviation from the desired fixed value shall not exceed 0.1% under steady state conditions.

II. Introduction

A feedback control system is best suited to accomplish the above stated requirements. As the name implies, such a system is characterized by a circuit arrangement in which the output quantity, in this case the output voltage of the generator, is fed back to the input, to obtain a continuous comparison with a fixed reference quantity. The control elements respond to the magnitude of difference between reference and output quantities and in turn supply field current to the generator so as to tend to maintain the output at a fixed value.

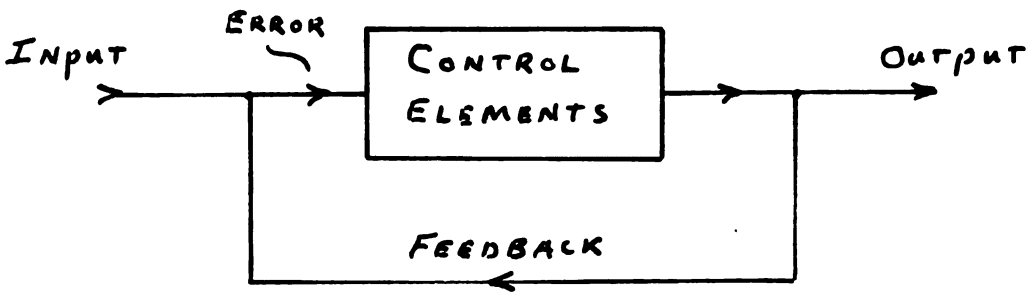
Since the purpose of the system to be described is to maintain an output quantity at a fixed level, it is called a regulating system. The system is incidentally also capable of responding to arbitrary changes in the level of the reference quantity, but such performance is not ordinarily required. Since it is capable of such performance, the system may be properly designated a servo system.

A system designed to maintain the rotary or translatory position of an output member in correspondence with the arbitrary position of some input member is called a servomechanism. In this case the output member might be a gun turret, and the input member a hand wheel, as an example.

The above discussion is intended to bring out the close relations existing between regulating systems and servomechanisms; each system is in fact a type of feedback control system.

The methods for synthesis and analysis of all types of feedback control systems are much the same, differing only in detail having to do with the end result desired.

Figure 1 shows a simple block diagram of a feedback control system having unity feedback from output to input.



Simplified Block Diagram of Feedback Control System

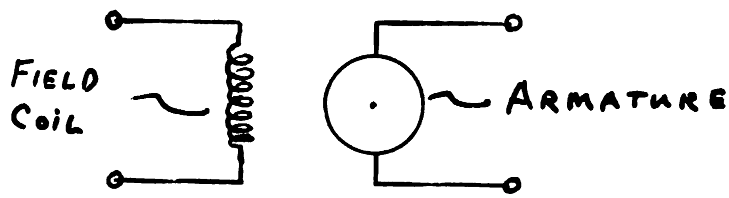
Fig. 1

III. The DC Generator

The output device in this system is a DC generator. The rating is 3 KW, 250 volts, 12 amperes. The specific machine used had the serial number 2426655, manufactured by the General Electric Company. The generator was driven by a synchronous motor at the constant speed of 1800 RPM.

This machine was used as a separately excited generator, the field current being supplied by an amplidyne generator. The generator was equipped with interpoles for improvement of commutation. Field poles were of laminated steel, but part of the magnetic circuit of the machine, namely the generator frame, was made of solid rolled steel.

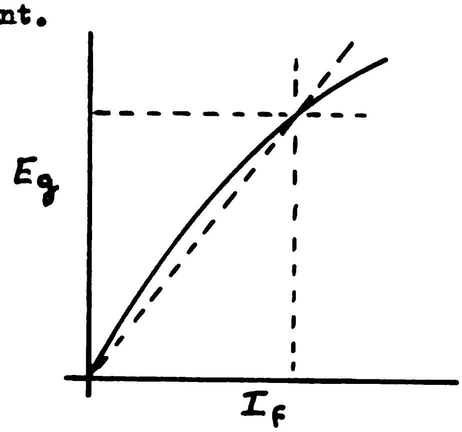
Figure 2 is an elementary diagram of the generator as used in this system.



Elementary Diagram of D.C. Generator

Fig. 2

A test made at steady state on any DC generator shows a definite correspondence between field current, I_f and generated, or no load output voltage E_g, neglecting hysteresis effects in the core steel. Figure 3 shows positive field current resulting in positive generated volts; the relations for minus values of I_f and E_g are identical but in the third quadrant.



Typical Magnetization Curve of D.C. Generator

Fig. 3

The curve is observed to be quite linear at the lower values of voltage and current. The bend at higher values is the result of core material approaching magnetic saturation.

For purposes of control system analysis, a linear relation between E_g and I_f is desired. The machine used was tested at steady state and found to have a saturation characteristic similar to that shown in Figure 3. The straight (dotted) line was then drawn through the rated voltage point to establish an optimum linear relation. This gave the relation

$$\left.\frac{E_g}{I_f}\right|_{\substack{\text{steady state}\\ \text{no load}}} = 602$$

or since field resistance was 364 ohms

$$\left.\frac{E_g}{E_f}\right|_{\substack{\text{steady state}\\ \text{no load}}} = 1.654$$

Since we are mainly interested in the transfer characteristic under load, the machine was loaded to full load and a similar relationship was found between output voltage under load and field voltage, giving

$$\left.\frac{E_o}{E_f}\right|_{\substack{\text{steady state}\\ \text{full load}}} = 1.39 = K_c$$

In order to predict the performance of a feedback control system, under conditions of arbitrary input quantities or disturbances, it is desirable to know the frequency response of each element in the system.

We must then establish a relationship for the generator, between a sinusoidal field current and the corresponding generated voltage. We have seen above that generated voltage may be expressed as $E_g = KI_f$, assuming linearity, for the case of steady DC operation. In addition the field current at steady state may be expressed as

$$I_f = \frac{E_f}{R_f}$$

Under conditions of sinusoidal input, most writers [1,2,3] assume that a constant correspondence between I_f and flux, and therefore between I_f and E_g exists just as in the steady state DC case. It is however recognized that the field circuit contains inductance which must be considered under sinusoidal conditions. The relation for field current is given as,

$$I_f\ (j\omega) = \frac{E_f\ (j\omega)}{R_f + j\omega L_f}$$

where L_f is a constant field inductance.

The assumed relation for generated volts is

$$E_g\ (j\omega) = KI_f\ (j\omega).$$

. . . .

[1] James, Nichols and Phillips. Theory of Servomechanisms. First Ed. 1947, p106, New York: McGraw-Hill.

[2] Chestnut and Mayer. Servomechanisms and Regulating System Design. Vol. 1, 1951, p174, New York: John Wiley and Sons.

[3] Brown and Campbell. Principles of Servomechanisms. 1948, p127 New York: John Wiley and Sons.

There is reason to doubt that the last relation given is correct, since the generator core is of steel, resulting in hysteresis and eddy current losses under sinusoidal conditions. Therefore the field current is not entirely a magnetising current, but must contain a loss component.

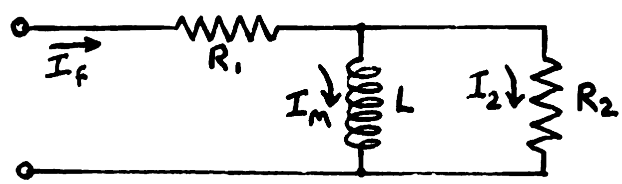
In order to test this relation experimentally, oscillograms were taken of sinusoidal field current and the resulting sinusoidal generated voltage. The result showed a phase shift between I_f and E_g, varying with frequency. Specific values were

21.8 degrees at 4.95 cycles per second

29.4 degrees at 9.55 cycles per second

31.6 degrees at 19.4 cycles per second.

The field circuit had a DC resistance of 364 ohms. However, tests made on the field circuit at varying frequencies up to 20 cps indicated that the effective resistance increased to approximately 2000 ohms at the upper frequency. In addition the effective inductance decreased with frequency. From this result it is evident that the field circuit cannot be exactly represented as a fixed resistance in series with a fixed inductance. It was found that Figure 4 is a more accurate equivalent circuit for the field. Here R_1 is the DC resistance and R_2 and L are fixed values determined from sinusoidal tests.



Equivalent Circuit for Field of D.C. Generator

Fig. 4

In Figure 4, I_f is total field current, I_m is magnetizing current, and I_2 a loss current, R_2 may be considered the resistance of the eddy current paths in the core steel, referred to the field circuit, in the same sense that core losses in a transformer[4] are referred to the primary circuit as an equivalent shunt resistance. However, in the present case, the representation, to be accurate, must hold for a range of frequencies.

From the power standpoint it is seen that for a constant I_m ($j\omega$), the voltage across L, and therefore across R_2 must equal $jI_m\omega L = K_1\omega$. The dissipation in R_2 is equal to

$$I_2^2R_2 = \left(\frac{K_1\omega}{R_2}\right)^2 R_2 = \frac{(K_1\omega)^2}{R_2}$$

Therefore the loss in R_2 is proportional to frequency squared. It is known that eddy current loss is proportional to frequency squared, therefore this equivalent circuit should be reasonably correct from this standpoint if losses are predominantly due to eddy currents. Hysteresis losses vary directly with frequency and cannot be represented in the same fashion.

The total field impedance for Figure 4 is

$$Z = R_1 + \frac{R_2 j\omega L}{R_2 + j\omega L} = \frac{R_1R_2 + j\omega L\,(R_1 + R_2)}{R_2 + j\omega L}$$

. . . .

4 Bryant and Johnson. Alternating Current Machinery. First Ed. 1935, p108, New York: McGraw-Hill.

Field current will be

$$I_f = \frac{E_f}{R_1}$$

Magnetizing current is

$$I_m = \frac{R_2}{R_2 + j\omega L} \times I_f = \frac{R_2}{R_2 + j\omega L} \times \frac{E_f}{R_1}$$

$$= \frac{E_f R_2}{R_1 R_2 + j\omega L\ (R_1 + R_2)}$$

$$= \frac{E_f}{R_1 + j\omega L\ (R_1/R_2 + 1)}$$

Now if $R_2 \gg R_1$, the denominator is simply $R_1 + j\omega L$. This condition could be approached by making the core losses small by proper design, such as use of very thin core steel laminations.

Generated voltage can be accurately expressed as

$E_g\ (j\omega) = KI_m\ (j\omega)$.

This relation is independent of the values of R_1 and R_2.

The last expression for I_m gives for very low and very high frequencies,

as $\omega \rightarrow 0$, $I_m \rightarrow \frac{E}{R_1}$ so $E_g \rightarrow \frac{KE}{R_1}\ \angle 0^\circ$

as $\omega \rightarrow \infty$, $I_m \rightarrow \frac{ER_2}{j\infty}$ or $0\ \angle -90^\circ$

so $E_g \rightarrow 0\ \angle -90^\circ$.

At the two limits given, the value of E_g assumes the same values as above when determined using the relations from the references 1, 2 and 3 previously cited. The same values will not necessarily be obtained by the two methods for intermediate values of frequency.

Representation of the field circuit as in Figure 4 unfortunately complicates system calculations, since the effective R and L values are no longer constant. In addition, it appears that satisfactory results can be obtained by using the conventional series R and L field representation, therefore this method will be used in further calculations. However, it is the writers' belief that the relation $E_g(j\omega) = KI_f(j\omega)$ should not be assumed without qualification as done in the references cited.

The value of field inductance L was determined by applying sinusoidal voltages at 20 cps to the field circuit, with a series R and C path connected in parallel with the field. The R and C values were adjusted to resonance at this frequency with equal currents in both branches. Then if R_e and L_e are equivalent series values for the field, R_2 and L could be determined from the following relations derived on the basis of Figure 4:-

$$R_e = R_1 + \frac{R_2\omega^2L^2}{R_2^2 + \omega^2L^2}$$

$$L_e = \frac{LR_2^2}{R_2^2 + \omega^2L^2}$$

at resonance, $\omega L_e = \frac{1}{\omega C}$

or $L_e = \frac{1}{\omega^2 C}$.

The values determined on this basis were $R_2 = 9620$ ohms, $L = 37.25$ Henries.

If desired, tests may be made at two frequencies, e.g. 10 and 20 cps, and then R_2 and L may be calculated from the R_e expression above,

or the L_e expression alone.

Using the conventional method, the transfer function relating output voltage and field voltage for the generator may be written as

$$K_cG_c\ (j\omega) =$$

$$\frac{E_o}{E_f}\ (j\omega) = \frac{I_f}{E_f}\ (j\omega) \times \frac{E_o}{I_f}\ (j\omega) = \frac{K_{c1}}{R_f + j\omega L_f}$$

$$= \frac{K_c}{1 + j\omega \frac{L_f}{R_f}} = \frac{K_c}{1 + j\omega T_c}$$

$$K_cG_c\ (j\omega) = \frac{1.39}{1 + j\omega \frac{37.25}{364}} = \frac{1.39}{1 + j\omega\ .1025.}$$

Here the value of K_c is the value obtained from steady state test since $\lim_{\omega \to 0} K_cG_c\ (j\omega) = K_c$.

The above expression for $K_cG_c\ (j\omega)$ neglects the generator armature time constant. By actual measurement, the value of this time constant was found to be .000805 at full load, at which load the time constant would be a maximum. This value was assumed to be negligible and was therefore neglected.

IV. The Amplidyne Generator

An amplidyne generator was used to furnish field current to the generator. Its own control field current was in turn supplied by an electronic amplifier. The particular amplidyne used was serial number MXW-2076, rated 250 volts, 1 ampere, manufactured by the General Electric Company.

The amplidyne generator [5,6] may be described as a two stage power amplifier of the rotating type. The effect of two stages of amplification through the use of one rotating armature is obtained by making use of the phenomenon of armature reaction, a phenomenon detrimental to the operation of the usual DC machine.

In a simple DC generator, such as the one used in this system, described above, the armature revolves in a flux field set up by field ampere turns. At no load, no current flows in the armature. However, when a load is connected to the generator brushes, a current flows in the armature circuit, resulting in a new magnemotive force proportional to the number of turns, and to the current in the armature. This new MMF acts in a direction in quadrature with the MMF due to field ampere turns. In the simple DC generator, this results in a distortion of the flux field, referred to as armature reaction. In this case it is a harmful effect, which must be at least partially neutralized to obtain satisfactory generator operation.

In the amplidyne, the brushes set in the position corresponding to the brushes in the simple generator, are short circuited, so the cross MMF, or armature reaction MMF becomes a large value, much larger in fact than the MMF due to field ampere turns. Another set of brushes is placed on the commutator, in quadrature with the short circuited brushes. Voltages are induced in the armature due to rotation within the flux set up

. . . .

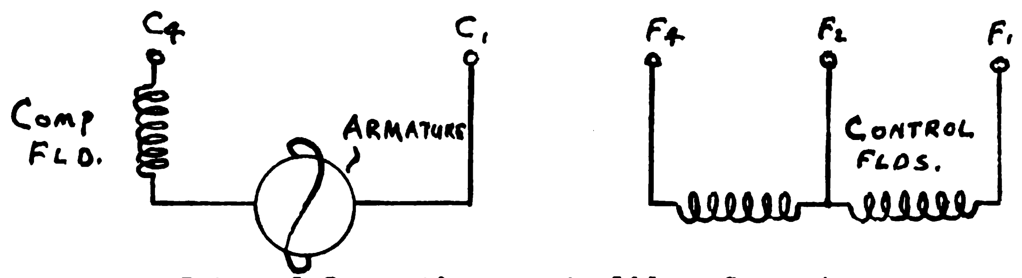
5 Alexanderson, Edwards and Bowman. "The Amplidyne Generator, a Dynamoelectric Amplifier for Power Control", GE Review, Vol. 43, p104, March 1940.

6 Fisher, Alec. "The Design Characteristics of Amplidyne Generators". AIEE Transactions, Vol. 59, p939, 1940.

by the cross MMF mentioned. Current is supplied to a load connected to these brushes.

A new MMF is of course set up by the load current flowing in the armature. This new MMF has a direction tending to oppose the original field MMF, and therefore must be neutralized. This is done by allowing the load current to flow through compensating windings, wound on the same poles as the original field (also called the control field), with polarities such that the control field MMF and compensating field MMF are additive.

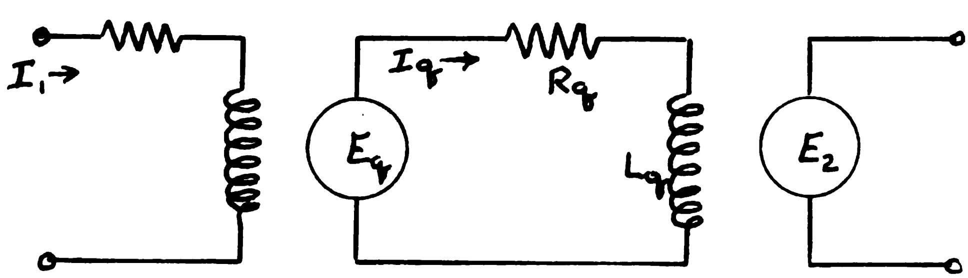
Figure 5 shows the internal connections of the amplidyne.



Internal Connections of Amplidyne Generator

Fig. 5

Under open circuit, or no load conditions, the amplidyne circuit may be drawn in equivalent form as a pair of simple DC generators, in cascade, as in Figure 6.



Equivalent Circuit for Amplidyne Generator

Fig. 6

The same diagram will apply for load conditions, when a load is connected at the output terminals, if perfect compensation exists. If less than perfect compensation exists, a rigorous analysis must include the effect of a negative feedback from load current to input current. Overcompensation would be equivalent to positive feedback. At no load, no armature reaction effect can exist in the final stage, therefore no feedback need be considered whether compensation is perfect or not.

Considering a current I, in the control field at no load, the magnetization curve of the first stage describes the relation between E_q and I_1. The design of the magnetic circuit is such that negligible core saturation takes place through a range of operation corresponding to rated values. Actual test on the amplidyne verified this statement. The steady state relation between E_q and I_1 is therefore linear and may be expressed as

$$\left.\frac{E_q}{I_1}\right|_{\text{steady state}} = K_1.$$

The same statements as to saturation are true for the relation between E_2 and I_q, therefore

$$\left.\frac{E_2}{I_q}\right|_{\text{steady state}} = K_2.$$

These constant relations will be assumed to hold for the sinusoidal case as well, for the same reasons as previously stated (Page 9). That is $\frac{E_q}{I_1}(j\omega) = K_1$ and $\frac{E_2}{I_q}(j\omega) = K_2$.

However, as a matter of record, a phase shift was found to exist between E_q $(j\omega)$ and I_1 $(j\omega)$. The existence of this shift had also been shown by Fisher (reference 6). No shift was found to exist between E_2 $(j\omega)$ and I_q $(j\omega)$, from tests made by the writer.

The quadrature or short circuit path current I_q, is related to the quadrature voltage E_q by the quadrature axis impedance, or

$$K_q G_q \ (j\omega) = \frac{I_q}{E_q} (j\omega) = \frac{1}{R_q + j\omega L_q} = \frac{1/R_q}{1 + j\omega \frac{L_q}{R_q}} = \frac{K_q}{1 + j\omega T_q}$$

where T_q is the time constant of the short circuit path.

It is desired to write the transfer function between E_2 $(j\omega)$ and I_1 $(j\omega)$ for conditions in which the amplidyne output current flows through the generator field. It should be noted that this condition exists whether the output generator is loaded or not, although generator field current must vary to some degree between these conditions. The question as to whether the amplidyne is completely compensated or not must then be considered. Tests made at steady state showed that the amplidyne was somewhat undercompensated; in other words, the drop in output voltage as the amplidyne was loaded was greater by an amount corresponding to the degree of undercompensation, than that predicted on the basis of armature resistance alone.

This undercompensation would have the effect of slightly decreasing the total phase shift through the amplidyne, therefore system analysis neglecting this effect should give conservative results as to phase shift. For this reason, the internal feedback effect on phase shift was neglected. However, the effect on magnitude was determined exactly in the steady state test made to determine the constant K_b.

The amplidyne transfer function for load conditions can then be written as

$$K_bG_b\ (j\omega) = \frac{E_2}{I_1}(j\omega) = \frac{E_2}{I_q}(j\omega) \times \frac{I_q}{E_q}(j\omega) \times \frac{E_q}{I_1}(j\omega)$$

$$= \frac{K_1K_2K_q}{1 + j\omega T_q} = \frac{K_b}{1 + j\omega T_q}$$

$$= \frac{K_b}{1 + (\omega T_q)^2} \Big|\ \tan^{-1}(-\omega T_q)$$

The value of the constant K_b may be determined from steady state test under load since

$$K_bG_b\ (j\omega)\Big|_{\text{steady state}} = K_b = \lim_{\omega \to 0} \frac{K_1K_2K_q}{1 + j\omega T_q}$$

Test indicated that $K_b = 3.3 \times 10^4$ Volts/Amp.

The above constant corresponds to a voltage gain of 19 for the amplidyne, and a power amplification of 1745, considering an amplidyne field resistance = 1740 ohms.

The value of $T_q = \frac{L_q}{R_q}$ was determined by taking an oscillogram of E_2 ($j\omega$) and I_1 ($j\omega$) under no load conditions at a frequency of 9.68 cps. The measured phase shift was -39.8 degrees, resulting in a time constant T_q = .0157 from $\theta = \tan^{-1}(-\omega T_q)$.

The amplidyne control field constants were determined in the same manner as outlined on page 9 for the DC generator field. The values in this case are, referring to Figure 4,

L = 108 Henries, R_2 = 13700 ohms.

The amplidyne output armature inductance was neglected in writing the amplidyne transfer function, since this inductance is small compared with that of the generator field which is connected to the amplidyne output in the complete system circuit.

V. Direct Coupled Electronic Amplifier

An additional amplifying element ahead of the amplidyne generator was desired to increase the amplification of the error signal and thereby obtain a high system accuracy. An electron tube type of amplifier can perform this function very well, since power and current input to the amplidyne control field are very low, less than 0.1 watt at 5 milliamps. at full excitation. The amplifier must be of the direct coupled type to respond to DC signal voltage, and should have good linearity at normal signal levels to permit system analysis by linear methods. In addition, it should have relative freedom from drift and erratic changes in gain. Such a device was not available, therefore the design of a suitable amplifier was undertaken. Figure 7 shows the circuit of the amplifier finally developed, which met the above requirements.

Since the amplidyne control field has a mid-tap, a three terminal connection was used between amplifier output stage and the field. A pair of 6L6 tubes was used for this output stage, connected in differential amplifier[7] connection. Using this arrangement, the no-signal tube currents cancel since they flow in opposite directions toward the field mid-tap. A common cathode resistor results in a fixed bias since current

. . . .

[7] Valley and Wallman. Vacuum Tube Amplifiers. Vol. 18, Radiation Laboratory Series, MIT, p New York: McGraw-Hill.

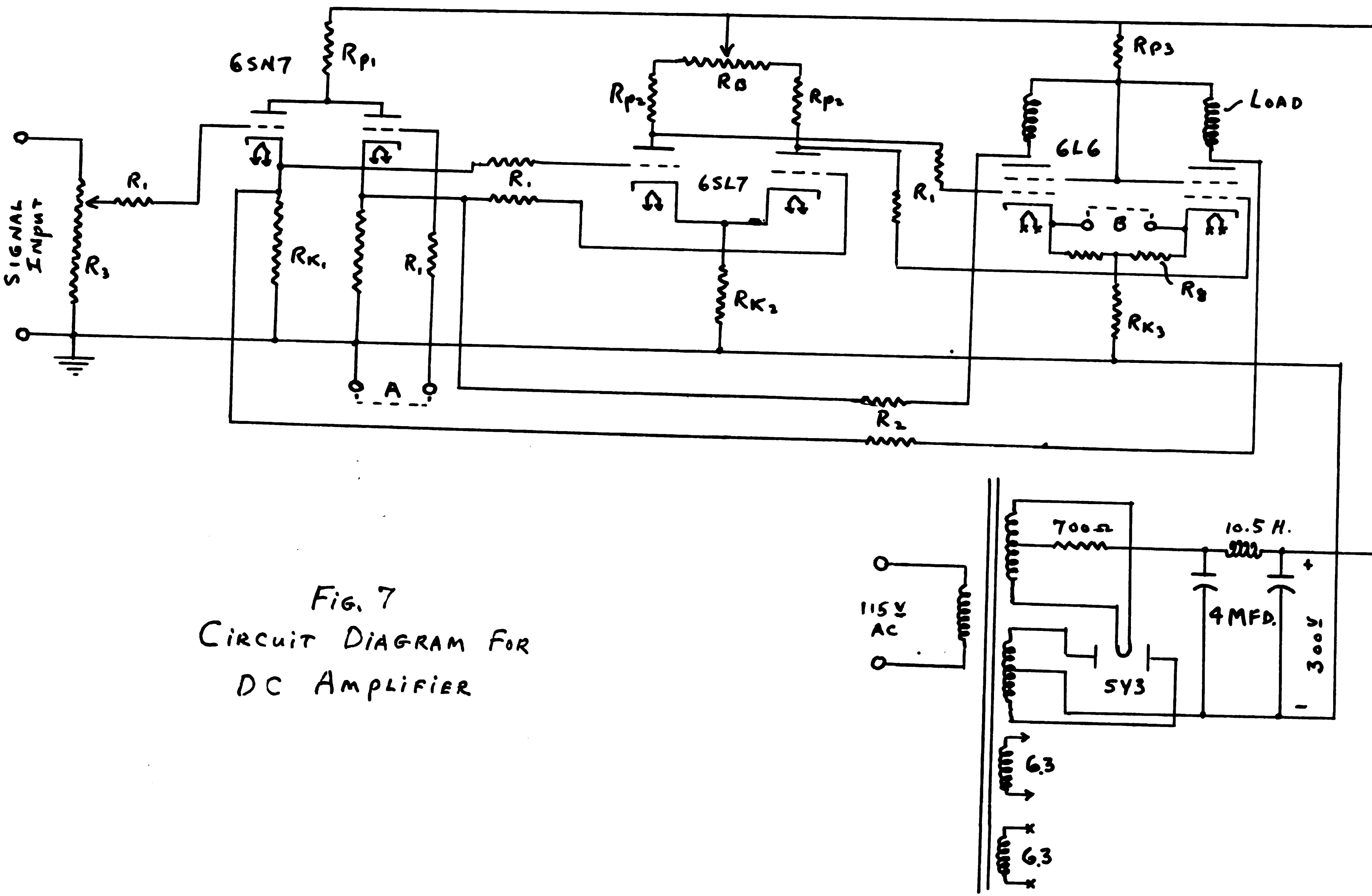


Fig. 7
Circuit Diagram For
DC Amplifier

increments due to signal are (+) in one tube and (-) in the other and of equal magnitude.

A double triode tube, a 6SL7, was used as a voltage amplifier stage ahead of the pair of 6L6's. A differential amplifier connection was also used in this case.

It was decided to incorporate negative feedback in the amplifier to improve linearity and freedom from zero drift. Feedback arrangements in direct coupled amplifiers are complicated by the necessity for direct coupling, and the consequent lack of isolating elements, such as transformers.

To achieve isolation of feedback signal from amplifier input signal, an additional stage was added at the input end, consisting of a 6SN7 double triode connected as a cathode follower[8]. The feedback signal was then from output outer terminals to the 6SN7 cathode resistors.

The circuit permitted single ended input at the gain potentiometer R_3, so the cathode terminal could be grounded to minimize hum and stray pick-up effects.

Values of tube constants were determined for the particular operating points in each case by reference to manufacturer's characteristic curves. Values of circuit components were

R_3	=	20000 ohms.	R_{k3}	=	1500 ohms.
R_1	=	5000 ohms.	R_{p3}	=	850 ohms.
R_{p1}	=	71900 ohms.	R_2	=	200000 ohms.
R_{k1}	=	1500 ohms.	R_{p2}	=	570000 ohms.
R_{k2}	=	7500 Ohms.	R_b	=	100000 ohms.

(R_3 and R_b are the potentiometers for gain and zero balance adjustment, respectively.)

. . . .

8 Ibid.

The above values, and a 300 volt plate supply, gave grid bias values of 4.2 volts, 5.25 volts, and 98 volts respectively for stages 1, 2 and 3.

In the following analysis to determine the amplifier transfer function, currents and voltages are incremental values resulting from input signal. The current i_3 is net current change through the entire amplidyne field. The voltage e_2 is voltage change between 6L6 grids, and e_1 the corresponding voltage change at the 6SL7 grids. The voltage e_s is input signal with R_3 at maximum gain setting. The current i_1 is current change in 6SN7 plates.

The gain of the 6SL7 stage is a simple constant

$$K_2 = \frac{e_2}{e_1} = \frac{\mu 2}{1 + \frac{r_{p2}}{R_{p2}}} = \frac{50}{1 + \frac{50000}{620000}} = 46.2$$

Considering the output stage as a current source, where Z_L is impedance of entire amplidyne field,

$$\frac{e_1}{2} = i_1 R_1 \quad \text{so} \quad e_1 = 2 i_1 R_1$$

$$e_2 = K_2 e_1 = 2K_2 i_1 R_1$$

$$i_3 = g_{m3} \frac{e_2}{2} \times \frac{2r_{p3}}{Z_L + 2r_{p3}}$$

$$= \frac{2g_{m3}K_2 i_1 R_1 r_{p3}}{Z_L + 2r_{p3}}$$

$$e_3 = i_3 Z_L = \frac{2g_{m3}K_2 i_1 R_1 r_{p3} Z_L}{Z_L + 2r_{p3}}$$

$$i_1 = \frac{\mu_1 \left(\frac{e_s}{2} - i_1 R_1\right)}{r_{p1} + R_1} - \frac{e_3}{2(R_1 + 200000)}$$

$$= \frac{\mu_1 \left(\frac{e_s}{2} - i_1 R_1\right)}{r_{p1} + R_1} - \frac{g_{m3} K_2 i_1 R_1 r_{p3} Z_L}{(R_1 + 200000)(Z_L + 2r_{p3})}$$

$$i_1 \left[1 + \frac{\mu_1 R_1}{r_{p1} + R_1} + \frac{g_{m3} K_2 R_1 r_{p2} Z_L}{201500 \times (Z_L + 2\, r_{p3})}\right] = \frac{\mu_1 e_s}{2(r_{p1} + R_1)}$$

$$i_1 \left[2.82 + \frac{5700 \times 10^{-6} \times 46.2 \times 1500 \times 200000\, Z_L}{201500 \times (Z_L + 40000)}\right] = \frac{e_s}{1650}$$

$$\frac{i_1}{e_s} = \frac{Z_L + 40000}{(42.02\, Z_L + 113000)\, 1650}$$

$$\frac{i_3}{i_1} = \frac{2 g_{m3} K_2 R_1 r_{p3}}{Z_L + 2r_{p3}} = \frac{15.8 \times 10^6}{Z_L + 40000}$$

Therefore, the amplifier transfer function relating amplidyne field current to signal volts is

$$\frac{i_3}{e_s} = \frac{i_1}{e_s} \times \frac{i_3}{i_1} = \frac{227.3}{Z_L + 2685}$$

As a function of $(j\omega)$, this is

$$K_a G_a\,(j\omega) = \frac{i_3}{e_s}(j\omega) = \frac{227.3}{(R_1 + 2685) + j\omega L_1}$$

The steady state value as $\omega \to 0$ in the above expression results in a voltage gain of 89.5 which checks DC experimental values.

R_1 and L_1 are constants of the amplidyne field, previously determined as R_1 = 1740 ohms and L_1 = 108 Henries giving

$$K_a G_a\,(j\omega) = \frac{227.3}{4425 + j\omega 108} = \frac{.0514}{1 + j\omega\, .0244}$$

VI. Composite Open Loop Transfer Function and Steady State System Analysis

Having determined the transfer functions for individual control components, we may now write the composite transfer function for the block labelled "control elements" in Figure 1. This is sometimes referred to as an open loop transfer function, since it is a total characteristic of the control elements when the system loop is open; however, when the feedback loop is closed, it represents the ratio of output to error. The composite transfer function KG $(j\omega)$ is the product of the transfer function of the separate cascaded components. Then,

$$KG\,(j\omega) = K_aG_a\,(j\omega) \times K_bG_b\,(j\omega) \times K_cG_c\,(j\omega)$$

$$= \frac{.0514}{1 + j\omega\,.0244} \times \frac{3.3 \times 10^4}{1 + j\omega\,.0157} \times \frac{1.39}{1 + j\omega\,.1025}$$

$$= \frac{2360}{(1 + j\omega\,.0244)\,(1 + j\omega\,.0157)\,(1 + j\omega\,.1025)}$$

Dimensionally this is a numeric, since it is the ratio of output volts to error volts.

Assuming that the feedback loop can be closed, and that the system will then function satisfactorily, the steady state error may be calculated. At steady state the value of KG $(j\omega)$ is

$$\lim_{\omega \to 0} KG\,(j\omega) = K = 2360.$$

The relation between input, output and error is

Error = Input minus output, due to unity negative feedback,

or $E = I - O$

but $\frac{O}{E} = K$ so $O = KE$.

then $E = I - KE$

$E(1 + K) = I$

and $E = \frac{I}{1 + K} = \frac{I}{2360} = .00043\ I.$

This indicates that error volts under load should equal 0.043% of input or reference volts which is well within the desired 0.1% error. However, the assumption that the system will function satisfactorily is not necessarily valid. Additional analysis based on frequency response is necessary to show whether or not the system will be stable, due to the presence of time lags in the control components.

VII. Frequency Response of the Closed Loop System

The algebraic manipulations carried out in the last section for steady state conditions can also be performed with the sinusoidal quantities.[9] Thus, if the symbol for output volts is $E_3\ (j\omega)$, that for error volts $E\ (j\omega)$ and that for reference or imput volts $E_2\ (j\omega)$,

$$\frac{E_3}{E}\ (j\omega) = KG\ (j\omega)$$

or $E_3\ (j\omega) = E\ (j\omega)\ KG\ (j\omega)$

$$E\ (j\omega) = E_2\ (j\omega) - E_3\ (j\omega)$$

$$= E_2\ (j\omega) - E\ (j\omega)\ KG\ (j\omega)$$

then $E\ (j\omega)\left[1 + KG\ (j\omega)\right] = E_2\ (j\omega)$

$$\frac{E}{E_2}\ (j\omega) = \frac{1}{1 + KG\ (j\omega)}$$

which is the relation between error and input volts.

Similar manipulations lead to $\frac{E_3}{E_2}\ (j\omega) = \frac{KG\ (j\omega)}{1 + KG\ (j\omega)}$

which is the relation between output and input volts.

. . . .

9 Brown and Campbell. Op. Cit. p140.

At any particular frequency, e.g. ω_4, KG (jω) itself reduces to a magnitude and an angle which may be then plotted as a vector[10], OP, as illustrated in Figure 8. The expression for

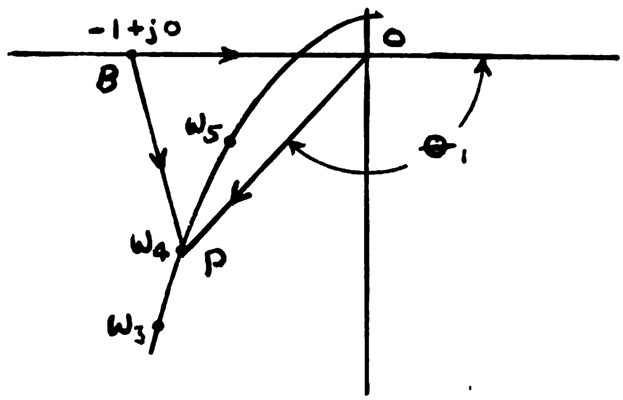


Illustration of Output Vector Locus Plot

Fig. 8

$\frac{E_3}{E_2}$ (jω) becomes

$$\frac{E_3}{E_2}(j\omega) = \frac{|OP| \underline{\theta_1}}{1 + |OP| \underline{\theta_1}}$$

From this relation, $|OP| \underline{\theta_1}$ may be considered to represent E_3, the output vector, and $1 + |OP| \underline{\theta_1}$ to represent E_2, the input vector. But the vector BP drawn from the (-1 + j0) point to P is exactly $1 + |OP| \underline{\theta_1}$, so this is the input vector E_2. Also from the relation

$$E_2 (j\omega) = E (j\omega) + E_3 (j\omega),$$

it is evident that the vector BO, which has the constant magnitude of unity and the constant phase angle of zero degrees, represents the error voltage E.

. . . .

10 Brown and Campbell. Op. Cit. p152.

The locus of the tips of the output vectors for the range of frequencies from zero to infinity results in an output vector locus plot, such as the curve through ω_3, ω_4, ω_5. This is sometimes referred to as a Nyquist diagram.

The Nyquist stability criterion[11] states that any feedback system will be unstable if the output vector locus plot passes through or encircles the (-1 + j0) point on the co-ordinates. This is apparent also from the fact that as the locus passes through this point, input has become equal to zero and the output to input ratio has become infinity. Whether the system will be stable if the locus passes below the (-1 + j0) point depends on the character of the roots of the open loop transfer function. That is, the function KG(s) must have no poles in the right hand half of the (s) plane. The function KG(s) is the transfer function in LaPlace notation, and may be formed by substituting (s) for (jω) in the expression for KG (jω).

The concept of phase margin is of importance in discussing stability. Referring to Figure 8, the phase margin is defined as the angle equal to 180 degrees minus θ, at the frequency where the vector OP has unity magnitude. Therefore in accordance with the Nyquist criterion, the phase margin must be positive for the system to be stable.

To bring out explicitly the relations between frequency and both amplitude and phase, it is desirable to plot attenuation and phase diagrams for the system under discussion. The attenuation diagram is a plot of amplitude in decibels versus radian frequency; the phase diagram a plot

. . . .

11 Brown and Campbell. Op. Cit. p170.

of phase angle versus radian frequency, using a semi-logarithmic frequency scale. The amplitude in decibels is calculated from the relation

Amplitude in decibels = 20 x $\log_{10}$ of (amplitude expressed as a numeric).

To plot these curves for the composite transfer function KG (jω), we first separate terms and set

$$K = 2360$$

$$G_1 (j\omega) = \frac{1}{1 + j\omega\ .0244} = \frac{1}{1 + j\frac{\omega}{41}}$$

$$G_2 (j\omega) = \frac{1}{1 + j\omega\ .0157} = \frac{1}{1 + j\frac{\omega}{63.7}}$$

$$G_3 (j\omega) = \frac{1}{1 + j\omega\ .1025} = \frac{1}{1 + j\frac{\omega}{9.75}}$$

Amplitude expressed in decibels may be added arithmetically to find total amplitude.

The value of K in decibels is

$$K\ (Db) = 20 \log_{10} 2360 = 67.5\ Db.$$

The magnitude of G_1 is $|G_1| = \frac{1}{\sqrt{1 + (\frac{\omega}{41})^2}}$.

For frequencies such that $\frac{\omega}{41} << 1$, $|G_1| \rightarrow 1$ and for frequencies such that $\frac{\omega}{41} >> 1$, $|G_1| \rightarrow \frac{41}{\omega}$, so these limits may be considered asymptotes to the amplitude curve. These asymptotes cross at $1 = \frac{41}{\omega}$

or $\omega_1 = 41$ radians/sec.

At frequencies less than $\omega_1 = 41$, the asymptote is unity or zero Db. At frequencies greater than ω_1, the asymptote is a straight line with a negative slope of 20 Db per decade.

Similarly for G_2, the "break" frequency is

$$\omega_2 = 63.7$$

and for G_3, $$\omega_3 = 9.75.$$

The attenuation rate is also 20 Db per decade for G_2 and G_3.

The phase angle for G_1 is, by rationalizing the expression for $G_1 (j\omega)$

$$\theta_1 = -\tan^{-1} \left(\frac{\omega}{41}\right).$$

As $\omega \to 0$, $\theta_1 \to$ zero degrees,

as $\omega \to \infty$, $\theta_1 \to (-90)$ degrees, and

at the break frequency, $\left(\frac{\omega}{41}\right) = 1$, so $\theta_1 = -\tan^{-1} 1 = (-45)$ degrees.

Zero and (-90°) are asymptotes to the phase angle curve, but other points in addition to -45° must be calculated to establish the shape of this curve. For all curves of this type however, $\tan \theta = 2$ at two times the break frequency, so $\theta = -63.45^\circ$; also $\tan \theta = 1/2$ at one-half the break frequency, so $\theta = -26.55^\circ$. In addition, these angles are complementary.

Attenuation and phase curves are drawn for G_1, G_2 and G_3 in Figure 9. By adding amplitudes at specific frequencies, and adding the value for the K term, and adding angles at specific frequencies, the composite curves of Figure 10 are obtained. These curves represent the amplitude A_T and phase θ_T for $KG (j\omega)$.

To apply the Nyquist criterion, we read the phase angle from Figure 10 at the frequency where A_T = zero Db. This angle is -251 degrees.

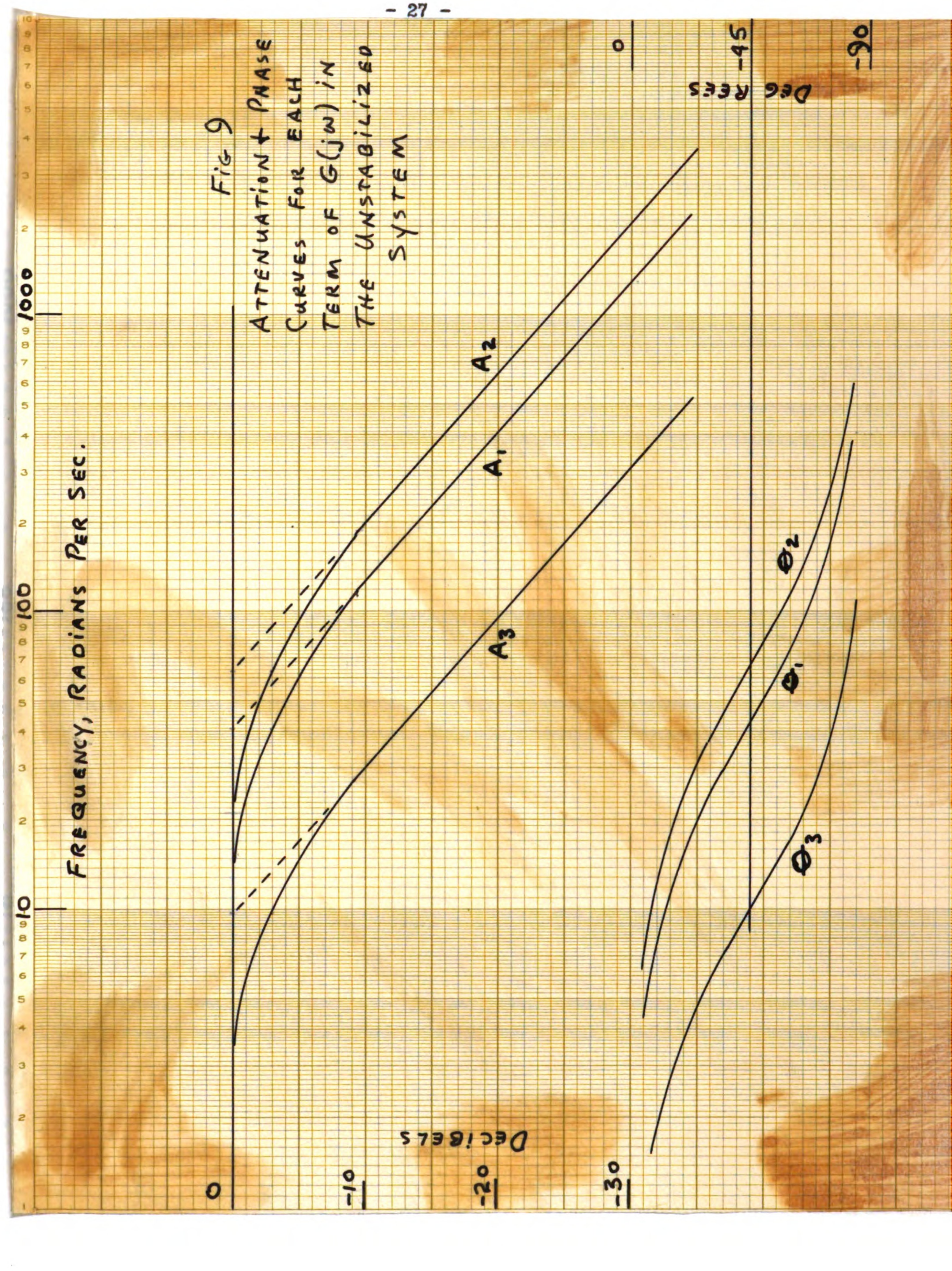


Fig 9
Attenuation + Phase Curves for each Term of G(jw) in The Unstabilized System
Frequency, Radians Per Sec.
Decibels
Degrees
0
-10
-20
-30
0
-45
-90
10
100
1000
A2
A1
A3
Φ2
Φ1
Φ3

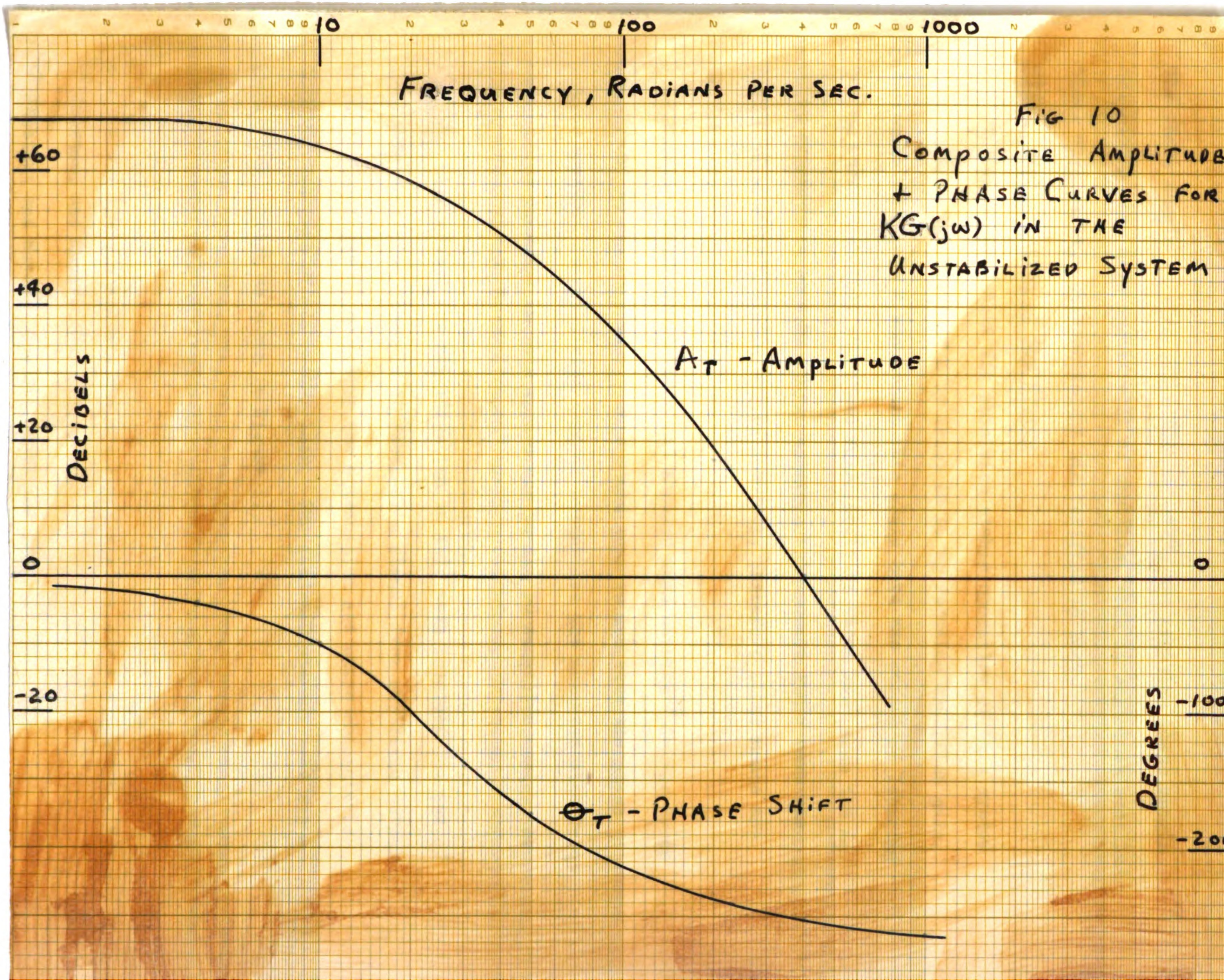


FIG 10
COMPOSITE AMPLITUDE + PHASE CURVES FOR KG(jw) IN THE UNSTABILIZED SYSTEM

The phase margin is then 180 - 251 = -71°, the negative phase margin indicating that this system will be unstable, that is sustained oscillations will exist. This statement was verified by test.

VIII. Stabilization of the System

The simplest method for obtaining stable operation of this system is to reduce the value of K by about 50 Db by reducing amplifier gain. Reference to Figure 9 shows that this would have the effect of lowering the A_T curve such that the phase angle would be about 160° when the A_T curve crosses the zero Db line. However, this method would have the effect of greatly reducing system accuracy, therefore other means are called for.

Another method would be to add a phase lead network[12] in the error path ahead of the amplifier. This would tend to decrease the phase angle, however, it would also tend to accentuate the unwanted high frequency noise inherent in this system due to the DC generator commutator ripple.

The best method for stabilization in this case involves the use of frequency sensitive elements to feedback a signal from amplidyne output to amplifier input[13], to modify the attenuation and phase characteristics of this part of the system loop. The feedback element used was a double R-C network as shown in Figure 11.

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[12] Chestnut and Mayer. Op. Cit. p255.

[13] Chestnut and Mayer. Op. Cit. p273 and 280.

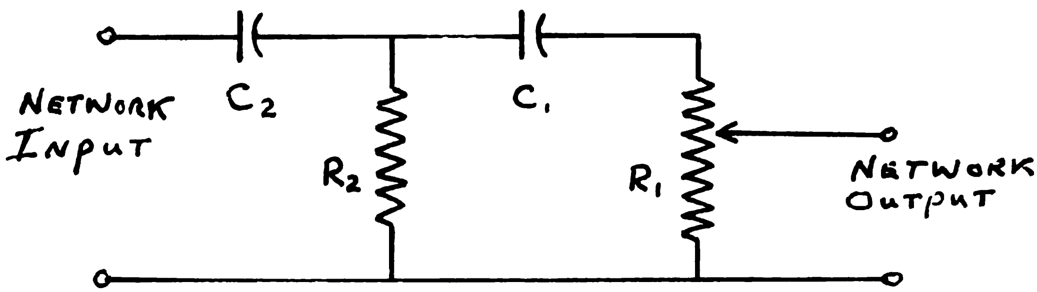


Diagram of Double R-C Feedback Network.

Fig. 11

The actual connection to amplifier input was between the second 6SN7 grid and ground, at the point marked A in Figure 7, with polarities to give negative feedback.

The optimum R and C values were determined by a combination of analytical and experimental methods to yield the end result of a control transfer function allowing stable system operation, and at the same time giving the simplest possible form for the transfer function to simplify system analysis.

It was found desirable, in addition, to add a phase lag network ahead of the amplifier to attenuate and minimize the masking effect of the commutator ripple fed back from the generator output. This network had the form shown in Figure 12.

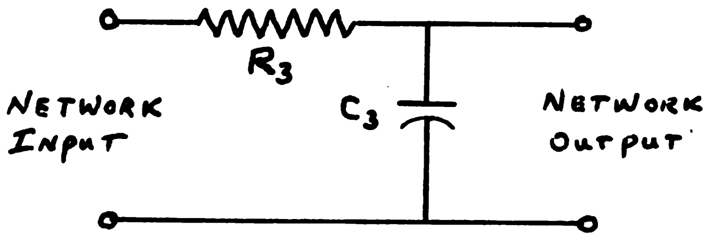
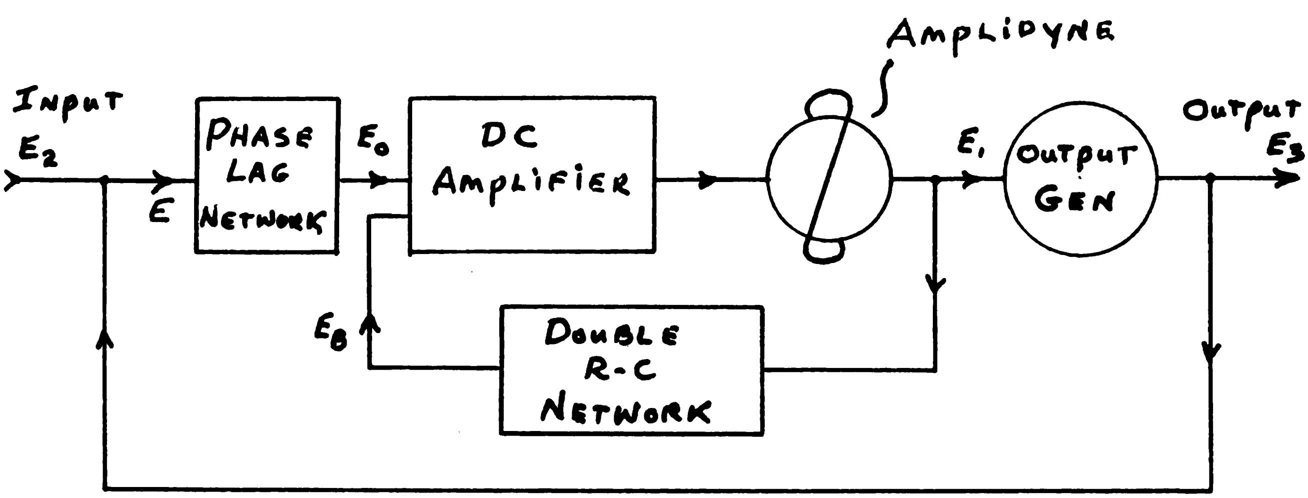


Diagram of Phase Lag Network.

Fig. 12

The block diagram for the stabilized system then took the form shown in Figure 13.



Block Diagram for Stabilized System.

Fig. 13

The minor loop including amplifier, amplidyne and double R-C network must be reduced to an equivalent series transfer function for purposes of system analysis. The constants for Figure 11 were

$R_1 = 6000\ \Omega$ (tap at $420\ \Omega$)

$C_1 = 15.8$ mfd.

$R_2 = 500\ \Omega$

$C_2 = 15$ mfd.

Letting $T_1 = R_1C_1$, $T_2 = R_2C_2$, $T_{21} = R_2C_1$, the transfer function of this network is, in LaPlace notation,

$$\frac{E_B}{E_1}(s) = \frac{42}{600} \times \frac{T_1T_2s^2}{T_1T_2s^2 + (T_1 + T_2 + T_{21})s + 1}$$

$$= \frac{.07\ s^2}{s^2 + 155.4s + 1420} = F_B.$$

The transfer function of amplifier and amplidyne combined, with minor loop open, is $K_{11}G_{11}(s) = K_AK_BG_A(s)G_B(s)$

$$= \frac{1695}{(1 + .0244s)(1 + .0157s)}.$$

Relations with minor loop closed are,

$$E_B(s) = F_B\ E_1(s)$$

$$E_1(s) = \left[E_o(s) - E_B(s)\right] K_{11}G_{11}(s)$$

$$= \left[E_o(s) - F_B\ E_1(s)\right] K_{11}G_{11}(s)$$

$$E_1(s)\left[1 + F_BK_{11}G_{11}(s)\right] = E_o(s)\ K_{11}G_{11}(s)$$

therefore the relation between $E_1(s)$ and $E_o(s)$ with minor loop closed is

$$\frac{E_1}{E_o}(s) = \frac{K_{11}G_{11}(s)}{1 + F_BK_{11}G_{11}(s)}$$

$$= \frac{\dfrac{443 \times 10^4}{(s + 41)(s + 63.7)}}{1 + \dfrac{.07 \times 443 \times 10^4\ s^2}{(s^2 + 155.4s + 1420)(s + 41)(s + 63.7)}}$$

$$= \frac{443 \times 10^4(s^2 + 155.4s + 1420)}{s^4 + 260.1s^3 + 33.03 \times 10^4s^2 + 55.53 \times 10^4s + 3.72 \times 10^6}.$$

It is necessary to factor numerator and denominator to facilitate an attentuation and phase response study. The method credited to Porter[14] was found the most useful for factoring the quartic equation in the denominator.

The factored form is

$$\frac{E_1}{E_0}(s) = \frac{443 \times 10^4 (s + 9.75)(s + 145.7)}{(s^2 + 1.67s + 11.3)(s^2 + 258.4s + 33 \times 10^4)}.$$

The transfer function for the phase lag network of Figure 12 is, for high load impedance,

$$\frac{E_o}{E}(s) = \frac{\frac{1}{sC_3}}{R_3 + \frac{1}{sC_3}} = \frac{1}{1 + sR_3C_3}$$

$$= \frac{1}{1 + .00244s} = \frac{410}{s + 410}$$

with R_3 = 1220 ohms, C_3 = 2 mfd.

The new composite transfer function for the control elements is,

$$KG(s) = \frac{E_o}{E}(s) \times \frac{E_1}{E_o}(s) \times \frac{E_3}{E_1}(s).$$

The last ratio is the transfer function of the generator in LaPlace notation, or

$$K_3G_3(s) = \frac{1.39}{1 + .1025s} = \frac{13.58}{s + 9.75}.$$ Then,

$$KG(s) = \frac{246.5 \times 10^8 (s + 9.75)(s + 145.7)}{(s + 410)(s^2 + 1.67s + 11.3)(s^2 + 258.4s + 33 \times 10^4)(s + 9.75)}$$

. . . .

14 Chestnut and Mayer. Op. Cit. p131.

From this we obtain the new control transfer function $KG(j\omega)$ (noting that the $(s + 9.75)$ terms will cancel) as follows,

$$KG(j\omega) = \frac{2360 \times (1 + j\frac{\omega}{145.7})}{(1 + j\frac{\omega}{410})(1 - \frac{\omega^2}{11.3} + j\frac{\omega}{6.78})(1 - \frac{\omega^2}{33 \times 10^4} + j\frac{\omega}{1276})}$$

Writing the factors separately,

$$K = 2360$$

$$G_1 = 1 + j\frac{\omega}{145.7}$$

$$G_2 = \frac{1}{1 + j\frac{\omega}{410}}$$

$$G_3 = \frac{1}{1 - \frac{\omega^2}{11.3} + j\frac{\omega}{6.78}}$$

$$G_4 = \frac{1}{1 - \frac{\omega^2}{33 \times 10^4} + j\frac{\omega}{1276}}.$$

The factors G_3 and G_4 result from the corresponding terms in the expression for $KG(s)$, each term having two conjugate complex roots.[15]

Factor G_3 will have straight line attenuation asymptotes, one along the zero Db line, the other at a slope of (-40) Db per decade, with the break frequency $\omega_3 = \sqrt{11.3} = 3.36$. The exact shape of the attenuation curve near the break frequency, and the shape of the corresponding phase curve can be found most readily by reference to published curves such as in reference 15. However, the damping factor must be known. For factor G_3 this will be,

. . . .

[15] Chestnut and Mayer. Op. Cit. p310.

$$d_3 = \frac{3.36}{2 \times 6.78} = .248.$$

For factor G_4, break frequency ω_4 and damping factor d_4 are determined in the same manner. They are ω_4 = 575 radians/sec.

d_4 = .226

The attenuation rate for factor G_1 is (+20) Db per decade, and break frequency is ω_2 = 145.7 radians/sec.

The attenuation rate for factor G_2 is (-20) Db per decade, and break frequency is ω_6 = 410 radians/sec.

The phase angle curve for factor G_2 is constructed as described in section VII. The phase angle curve for factor G_1 is similar, except the angles are positive.

As before, the K term represents on the decibel scale

$$K(Db) = +67.5 \text{ Db.}$$

Figure 15 shows the attenuation curves corresponding to each term of G ($j\omega$). Dotted lines are the asymptotes, and solid lines the exact curves. The phase angle curves for each term are shown in Figure 16. Symbols A_1 and θ_1 represent attenuation and phase curves for factor G_1, and so on. Finally, the composite attenuation or amplitude A_T, and phase θ_T are shown in Figure 17, for KG ($j\omega$).

The composite attenuation curve indicates that when amplitude is zero Db, frequency is 230 radians/ sec. Reference to the phase curve shows that at this frequency, the phase angle is (-164) degrees so

$$\text{Phase margin} = 180 - 164 = +16 \text{ degrees.}$$

The positive phase margin indicates that the system is stable. Test of the system also showed stable operation, extending through all values

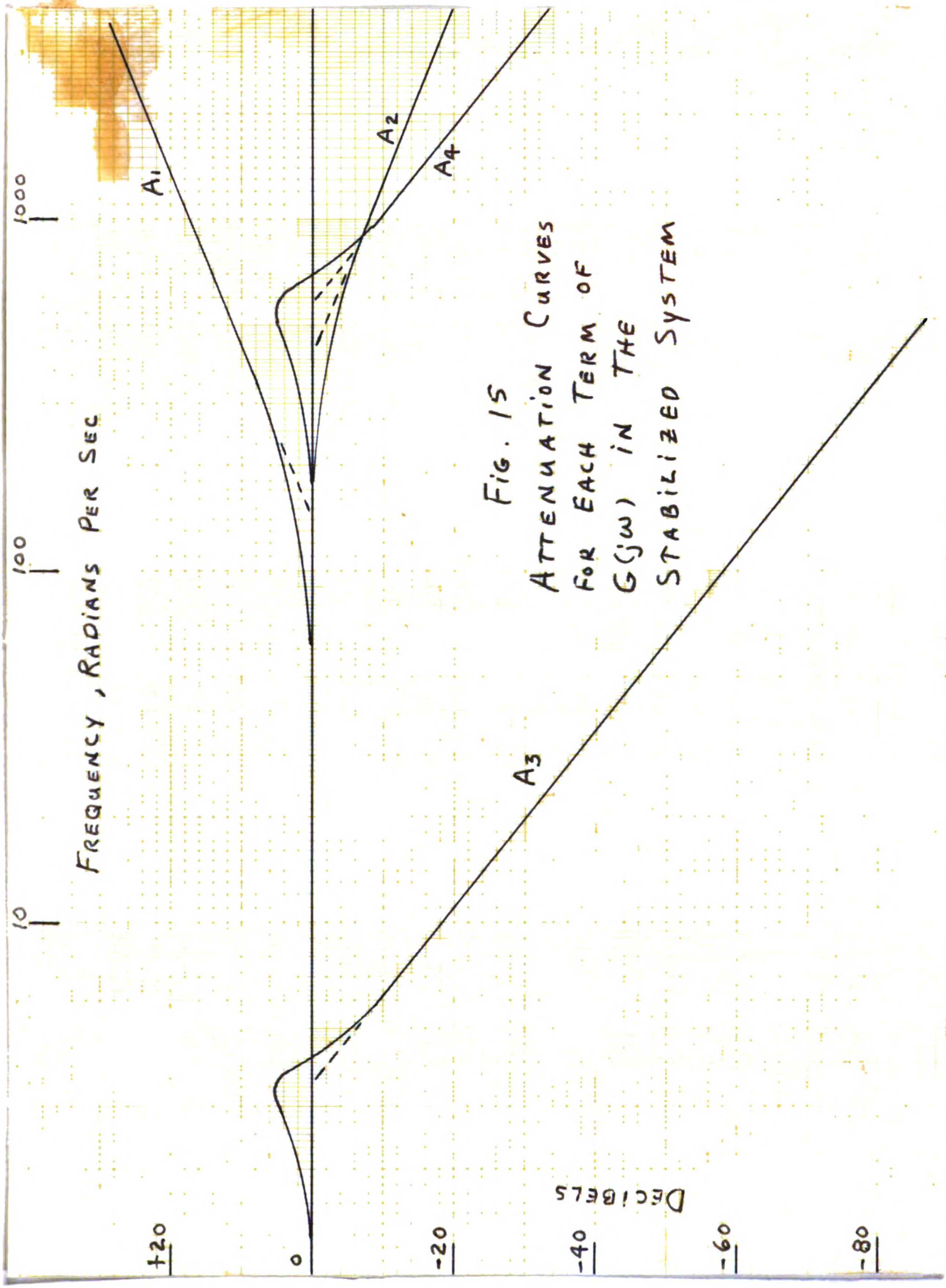


Fig. 15
Attenuation Curves for each Term of $G(j\omega)$ in the Stabilized System

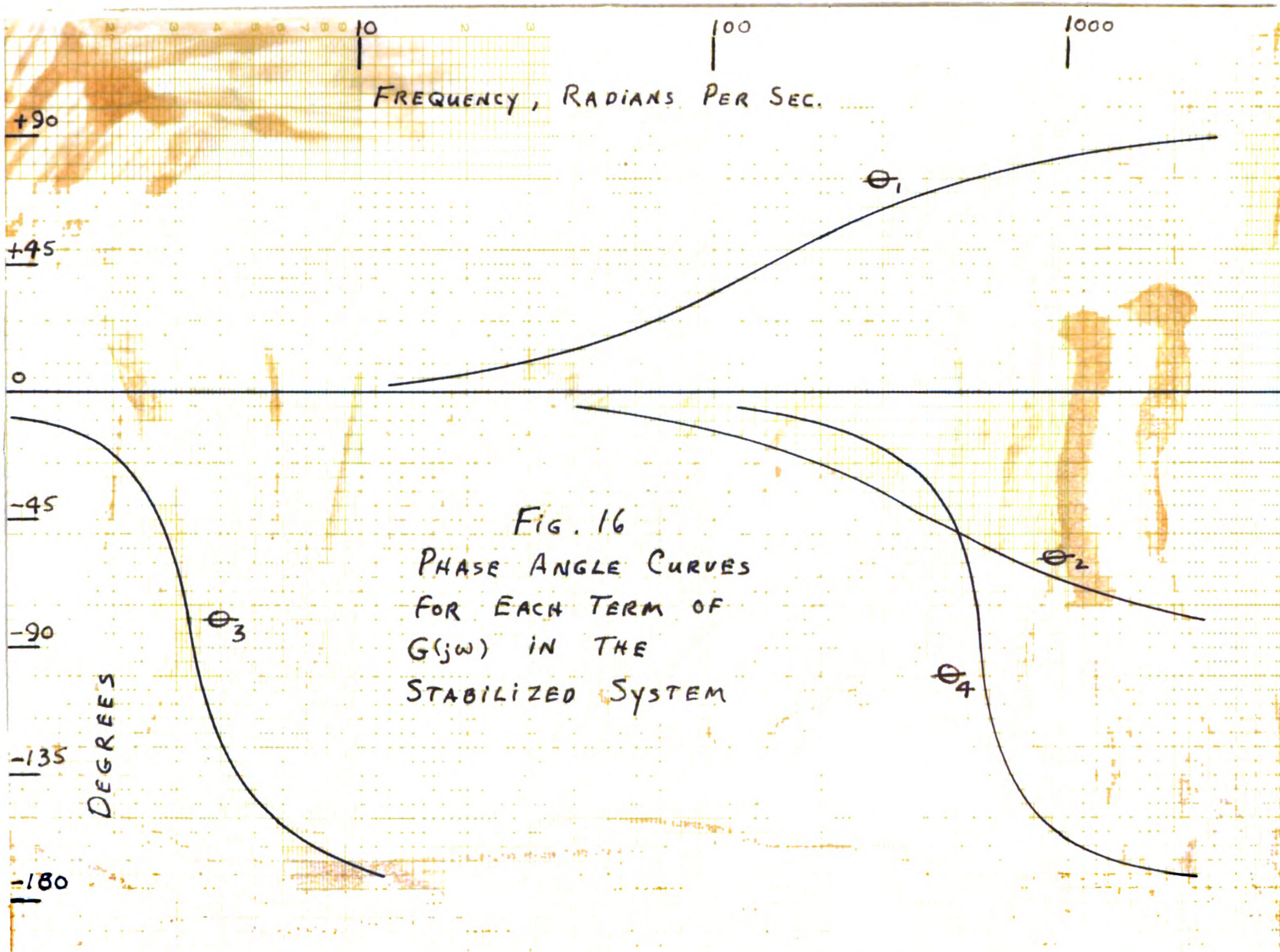


FIG. 16
PHASE ANGLE CURVES FOR EACH TERM OF G(jw) IN THE STABILIZED SYSTEM

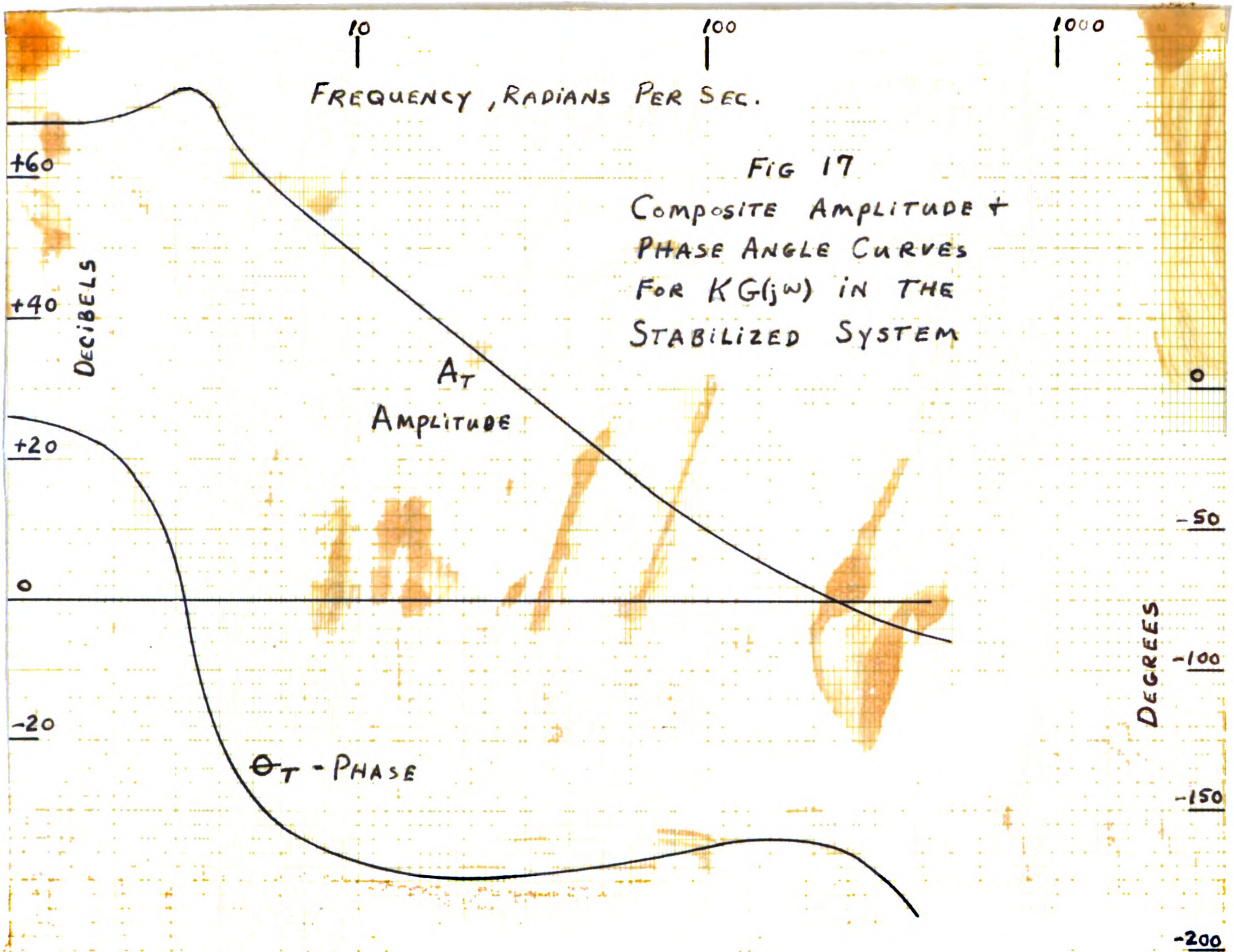


FIG 17
COMPOSITE AMPLITUDE + PHASE ANGLE CURVES FOR $KG(j\omega)$ IN THE STABILIZED SYSTEM

of load from zero to approximately 20% overload, and for all values of input voltage from zero to 250 volts. Sudden changes in load or input voltage caused only momentary oscillations which were quickly damped out.

It is of interest to note that, according to a theorem of Bode,[16] restricted to minimum phase systems, the attenuation and phase shift curves are definitely related to each other. Without attempting to state or completely explain the theorem, one practical result is that the slope of the attenuation curve at zero Db is usually a good indication of the phase shift at this point. A slope of 40 Db per decade is usually found to be an upper limit for this slope if phase margin is to be positive. In this case, the original system having excessive phase shift, had a slope, or attenuation rate of (-64) Db per decade, whereas the stabilized system had a slope of approximately (-23) Db per decade.

Steady state tests for error volts in the final, stabilized system showed an error voltage of 0.09 volts at no load, and 0.19 volts at full load (12 amperes) when input volts was set at 250 volts. Therefore,

$$\text{No load error} = \frac{.09}{250} = .00036 \text{ or } .036\%.$$

$$\text{Full load regulation} = \frac{.10}{250} = .0004 \text{ or } .04\%.$$

$$\text{Full load error} = \frac{.19}{250} = .00076 \text{ or } .076\%.$$

These values are well within the limit of 0.1% error established at the beginning of this paper.

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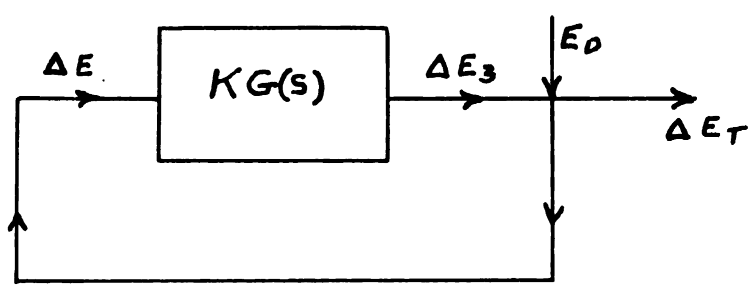
16 Chestnut and Mayer. Op. Cit. p297.

IX. Transient Response of the System

The usual function of a voltage regulator is to maintain the output voltage of a system at a fixed value, or at an accurate correspondence with a fixed reference or input voltage. Disturbances in such a system result generally from the sudden addition of load on the output device. Therefore the transient response of the system following a disturbance at the output is of major interest. For purposes of analysis, we may redraw the system block diagram as shown in Figure 18, in which deviations from steady state values are emphasized.[17]



Simplified Block Diagram for System Subject to Output Disturbance

Fig. 18

E_D is a step function voltage disturbance introduced at the output terminals. It will be considered negative in value, due to a voltage drop, and therefore has the value $-|E_D|\ U(t)$ or in LaPlace notation $-\frac{|E_D|}{s}$, where $|E_D|$ is magnitude of disturbance.

ΔE, ΔE_3, and ΔE_T are incremental values of control input (error volts), control output and terminal voltage respectively, resulting from the step disturbance. It is assumed that E_D has a small enough value

. . . .

[17] Gardner and Barnes. Transients in Linear Systems. Vol. 1, 1942, p192, New York: John Wiley and Sons.

so that the control components will not saturate, therefore we may assume linearity and the control transfer function previously developed should apply.

No input or reference voltage is shown in Figure 18, since the incremental values referred to are independent of the value of the reference voltage. This results from the fact that the principle of superposition applies in a linear system.

The relations existing are

$$\Delta E(s) = -\Delta E_T, \text{ due to unity negative feedback.}$$

$$\Delta E_3(s) = \Delta E(s)\, KG(s)$$

$$= -\Delta E_T(s)\, KG(s)$$

but

$$\Delta E_T(s) = \Delta E_3(s) + E_D(s)$$

$$= -\Delta E_T(s)\, KG(s) + E_D(s)$$

$$\Delta E_T(s)\left[1 + KG(s)\right] = E_D(s)$$

therefore

$$\Delta E_T(s) = \frac{E_D(s)}{1 + KG(s)} = \frac{-|E_D|}{s\left[1 + KG(s)\right]}.$$

The control transfer function for the unloaded system may be written

$$KG(s) = \frac{KA(s)}{B(s)} = \frac{292.5 \times 10^8\,(s + 145.7)}{(s + 410)(s^2 + 1.67s + 11.3)(s^2 + 258.4s + 33 \times 10^4)}$$

then

$$\Delta E_T(s) = \frac{-|E_D|}{s\left[1 + KG(s)\right]} = \frac{-|E_D|}{s\left[1 + \frac{KA(s)}{B(s)}\right]}$$

$$= \frac{-|E_D|\, B(s)}{s\left[B(s) + KA(s)\right]}$$

therefore, $\Delta E_T(s)$ =

$$\frac{-|E_D|(s + 410)(s^2 + 1.67s + 11.3)(s^2 + 258.4s + 33 \times 10^4)}{s\left(s^5 + 670.1s^4 + 43.71 \times 10^4 s^3 + 135.9 \times 10^6 s^2 + 294.8 \times 10^8 s + 426 \times 10^{10}\right)}$$

The inverse LaPlace of this expression will yield the time expression for change in terminal voltage. As a preliminary to finding the inverse LaPlace, the denominator must be factored. In this case, the most straightforward method to accomplish this is to find first one real root, which we know exists in a polynomial of odd degree, and then find the remaining roots by the method of Porter previously mentioned.

Had two of the terms in the expression for KG(s) not cancelled, the denominator in $\Delta E_T(s)$ would have been a sixth degree polynomial, and much more difficult to factor. The (s + 9.75) term in the transfer function of the double R-C network was the result of choosing R and C values to yield such a term, in order to cancel the (s + 9.75) term in the generator transfer function and at the same time properly stabilize the system.

The factored expression is

$$\Delta E_T(s) = -|E_D| \frac{A_1(s)}{B_1(s)}$$

$$= \frac{-|E_D|(s + 410)(s^2 + 1.67s + 11.3)(s^2 + 258.4s + 33 \times 10^4)}{s(s + 259)\left[(s + 68)^2 + 267.5^2\right]\left[(s + 137)^2 + 444^2\right]}$$

The denominator may be also written,

$B_1(s)$ =

$$s(s + 259)(s + 68 + j267.5)(s + 68 - j267.5)(s + 137 + j444)(s + 137 - j444).$$

The inverse LaPlace of $\Delta E_t(s)$ will be of the form

$$\mathcal{L}^{-1} \Delta E_T(s) = -|E_D| \left[C_1 + C_2 \varepsilon^{-\gamma t} + C_3 \varepsilon^{-\alpha_1 t} \sin\beta_1 t + C_4 \varepsilon^{-\alpha_2 t} \sin\beta_2 t\right]$$

where $\gamma = 259$

$\alpha_1 = 68, \quad \beta_1 = 267.5$

$\alpha_2 = 137, \quad \beta_2 = 444.$

The constants and phase angles must now be evaluated by partial fraction methods.

$$C_1 = \left[s\frac{A_1(s)}{B_1(s)}\right]_{s=0}$$

$$= \frac{410 \times 11.3 \times 33 \times 10^4}{259 \times 7.61 \times 10^4 \times 21.6 \times 10^4} = 3.59 \times 10^{-4}$$

$$C_2 = \left[(s + 259)\frac{A_1(s)}{B_1(s)}\right]_{s=-259}$$

$$= \frac{151 \times 6.65 \times 33 \times 10^8}{1.08 \times 2.119 \times 259 \times 10^{10}} = -.56$$

$$C_3 = 2j\left[(s + 68 - j267.5)\frac{A_1(s)}{B_1(s)}\right]_{s=-68+j267.5}$$

$$= \frac{(-2.28 - j7.84) \times 10^{12}}{(-328.3 + j31) \times 10^{10}} = \frac{+j(7.84 - j2.28) \times 10^2}{(328.3 - j31)}$$

$$= \frac{816 \angle -16.25^\circ}{330 \angle -5.4^\circ} = 2.475 \angle 79.15^\circ$$

$$C_4 = 2j\left[(s + 137 - j444)\frac{A_1(s)}{B_1(s)}\right]_{s=-137+j444}$$

$$= \frac{(-14 - j1316) \times 10^{10}}{(11.25 + j6.14) \times 10^{12}} = \frac{-j(1316 - j14) \times 10^{10}}{(11.25 + j6.14) \times 10^{12}}$$

$$= \frac{1317 \angle -90^{\circ}}{1280 \angle 28.6^{\circ}} = 1.03 \angle -118.6^{\circ}.$$

The time function of terminal voltage change then is

$$\Delta E_T(t) = \mathcal{L}^{-1} \Delta E_T(s)$$

$$= - |E_D| \Big[3.59 \times 10^{-4} - .56\varepsilon^{-259t} + 2.475\varepsilon^{-68t} \sin (267.5t + 79.15^{\circ}) + 1.03\varepsilon^{-137t} \sin (444t - 118.6^{\circ}) \Big].$$

The value of this expression at time equal zero is nearly $(-|E_D|)$, which is correct from physical reasoning since the control system cannot respond instantaneously to any disturbance. After time 0.01 sec., the third term, having the constant 2.475, accounts for nearly all of the remaining transient.

Figure 19 is a plot of the transient time function $\Delta E_T(t)$, plotted in per unit values with magnitude $|E_D|$ as a base. The curve shows an initial overshoot of approximately unity, and an oscillation of about 2-1/2 cycles which disappears in approximately 0.07 sec. If $|E_D|$ is assumed equal to one volt, the ordinates may be read directly in volts.

As a check on the actual transient performance of the system, an oscillogram was taken of terminal voltage resulting from the sudden application of load. In order to obtain a trace from which magnitudes could be read, it was necessary to apply a large disturbance. This was done by suddenly connecting a load resistance of 21.8 ohms to the initially unloaded system. After the transient had died out, the measured load current was 11.5 amperes, or nearly full load on the output generator. On the basis that the generator armature impedance is

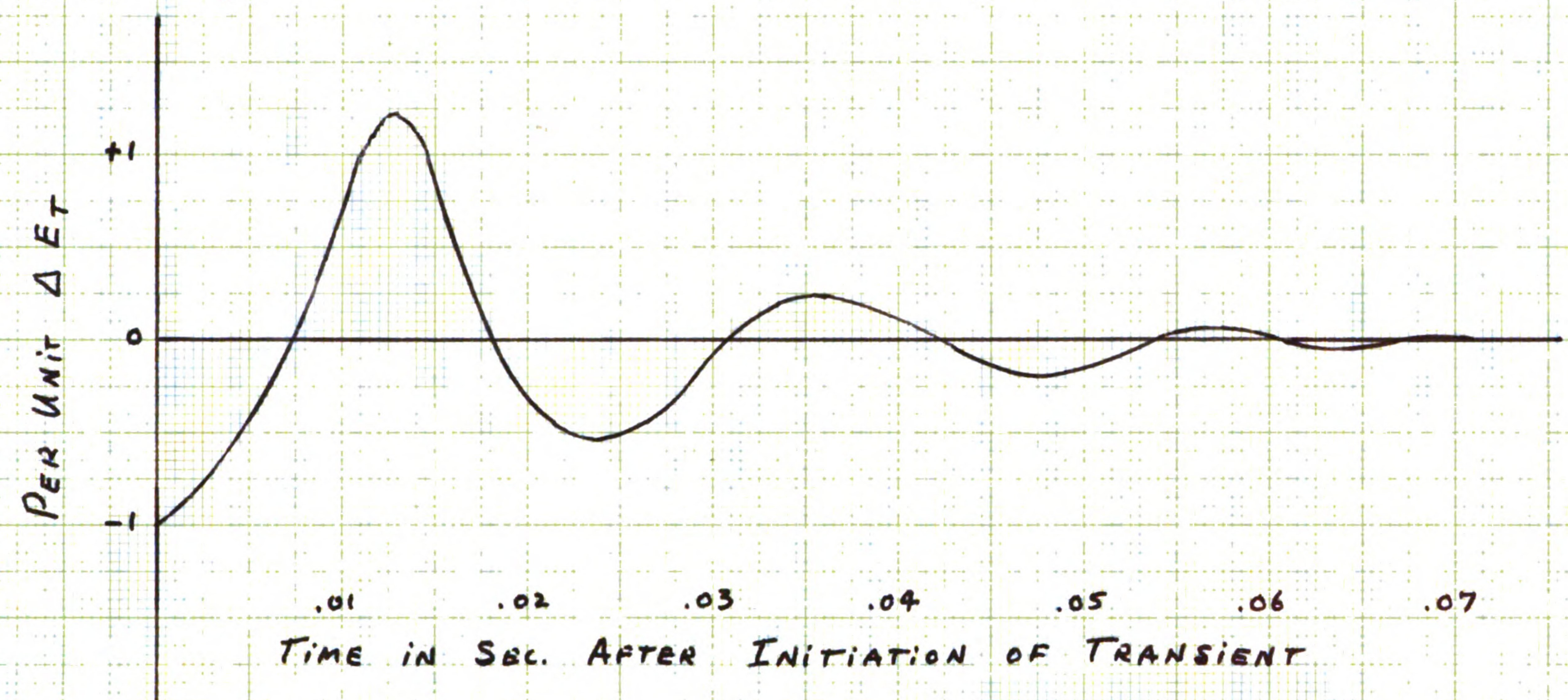


Fig. 19
Calculated Transient Response
After Step Disturbance $E_D = -U(t)$
Per Unit Base = $|E_D|$

entirely resistive, the magnitude of disturbance E_D may be calculated as

$$|E_D| = 11.5 \times 1.356 = 15.6 \text{ volts},$$

where armature resistance = 1.356 ohms.

The oscillogram, shown on page 47, gives the following results:-

1. The terminal voltage drops to a very low value (theoretically zero) at the instant of connecting the load resistance, due to the presence of inductance in the armature circuit. At this instant, the current and voltage relations may be expressed by the equation

$$E_g = iR_a + iR_L + L\,di/dt$$

where R_a is armature resistance, R_L the load resistance, and L the armature inductance. The current initially increases very rapidly, as shown by the shape of the current trace, making the term $L\,di/dt$ account for nearly all of the generated voltage at this instant. Therefore, the term iR_L = terminal voltage, is negligible. The current trace shows an exponential current rise, rather than the step function assumed in the calculations, where armature inductance was neglected.

This effect due to armature inductance has no important effect on the response of a practical control system, and therefore may be properly considered a separate phenomena. In the case of the control system under discussion, the magnetic circuits of the amplidyne and output generator become saturated when error reaches a value of approximately 0.4 volts. Error values larger than this can therefore produce no additional controller output, therefore the initial terminal voltage drop can be neglected for output disturbances of 0.4 volts or larger.

It may be noted that the transient due to armature inductance

disappears in less than 0.01 second, in the present case.

2. The initial rate of recovery, after the inductance effect is past, is less than the calculated rate. This is again due to saturation in the control elements. The result is that the factor K in the control transfer function KG $(j\omega)$ is no longer fixed in value but actually varies from a very low value up to the calculated value of 2360, as error voltage varies from a very high value, to values less than 0.4 volt.

3. The duration of the transient is approximately 0.15 seconds. This represents very satisfactory performance, although the value is larger than the duration of 0.07 sec. determined from the calculated transient response.

4. The oscillogram brings into view the ripple content of output generator terminal voltage, which is a characteristic of the generator itself. This ripple is made up principally of a 120 cycle wave resulting from the four pole construction of the machine, and the rotational speed of 1800 RPM, plus a higher frequency commutator ripple.

The control system can do nothing to minimize this ripple content since the control elements in cascade act as a low pass filter. Referring to Figure 17, it may be seen that radian frequencies in excess of 230 radians per sec. are attenuated. This corresponds to a frequency of 36.6 cycles per sec., which is considerably lower than the 120 cycles per sec. mentioned. It will be recalled that the phase lag network of Figure 12, page31 , was included in the system for the purpose of blocking this ripple from the amplifier input. Even if this had not been done, the other control elements would have acted as an effective low pass filter.

It might appear that the generator output voltage is not very effectively regulated as long as this ripple is present. However, for most applications, the ripple has no practical importance as long as the DC component of output voltage is held constant. The DC component is here referred to in the sense that the actual output voltage can be analyzed by methods of Fourier series into the sum of a DC, or constant, term plus a series of sine and cosine terms. The controller responds principally to the DC component due to the low cut-off property mentioned previously.

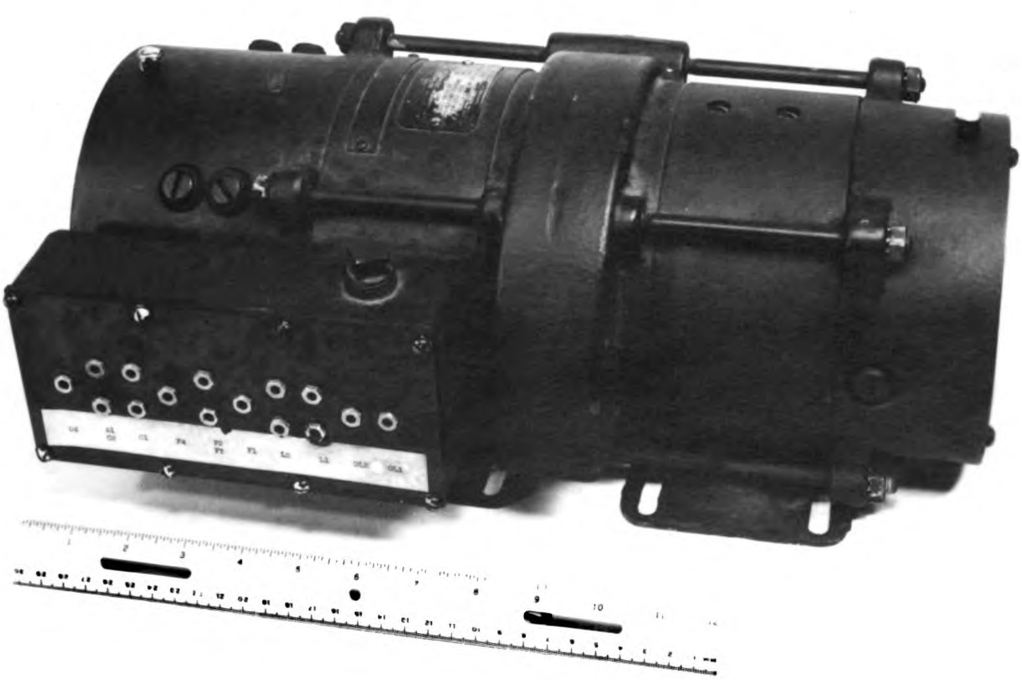
In these special cases where the ripple would be objectionable, a specially designed, low ripple generator might be used, or else filtering of the output current might be resorted to. If an output filter is used, the feedback signal should preferably be taken from the output of the filter, that is directly across the load as before. In such a case, the transfer function of the filter must be determined and included in the overall control transfer function KG(s).

X. Summary and Discussion

The steps in the development of a feedback control system have been described and an analysis of the system has been presented. The test results show that the system performs satisfactorily under both steady state and transient conditions.

Certain assumptions are necessary in the analysis of a system of this type, in order to apply linear analytical methods. For example, straight line magnetization curves are assumed for the rotating elements, although the actual curves are known to be non-linear.

View of Direct Coupled Electronic Amplifier and Power Supply



View of Amplidyne Generator with Driving Motor

Simplifications and approximations are necessary but must be carefully applied to minimize resultant errors.

Many of the present difficulties in working with feedback control systems have to do with measurements of magnitudes, phase angles and frequencies in the low frequency range between 1 and 20 cycles per sec., particularly where low energy levels are involved. These quantities can be measured by use of the magnetic oscillograph but this method is time consuming due to the need for setting up and adjusting the oscillograph, and developing the oscillograph negatives. The cathode ray oscillograph may be used for magnitude measurements, to limited accuracy, but does not permit direct frequency and phase measurements. This device is, however, quite useful as a null indicator in connection with bridge circuits and phase shifting networks. The development of suitable indicating instruments for direct measurement of magnitude, frequency and phase at low frequencies would greatly facilitate work on feedback control systems.

The generation of voltages at low, and at the same time adjustable, frequencies is a necessary preliminary to measurements at these frequencies. The work of the writer was greatly simplified in this respect due to the availability of an R-C oscillator capable of supplying low frequency voltages.[17] The development of this oscillator was carried out as a research project at Michigan State College.

The principles of the control system described in this paper may be readily extended to the voltage control of AC generators, and to

. . . .

[17] Smollett, Roy John. An R-C Oscillator. Unpublished M.S. thesis. Michigan State College, 1950, 42 numb. leaves, 17 figures.

the control of current at the output rather than voltage, for either AC or DC systems. In addition, the same principles may be applied to servomechanisms, with modifications having to do with the necessity for controlling the position or speed of a mechanical load, rather than a voltage or current.

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