A FUNCTIONAL STUDY OF COOPER'S E-72 LOADING DIAGRAM

Thesis for the Degree of B. S.
MICHIGAN STATE COLLEGE

William Alf Peterson

1943

A Functional Study of Cooper's E-72 Loading Diagram

A Thesis Submitted to

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of

AGRICULTURE AND APPLIED SCIENCE

by

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Candidate for the Degree of

Bachelor of Science

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Acknowledgment

The author wishes to thank Professor John E. Mever for his co-operation in the preparation of this thesis.

Introduction

The writer of this thesis developed graphs on Coopers E-72 Loading Diagrams for maximum shear, maximum bending moment, and maximum pier reaction using different length spans. The graphs will be very useful for faster design of railway bridges due to the already calculated maximums used for the various lengths of spans.

Stringers, floor beams, and girders are the three principal parts of a railroad bridge. The size of each part must be calculated to resist the force or forces acting upon it. The main feature of safe design is to use the maximum value whether it be shear in kips or bending moment of kip foot. The graphs and calculations cover only the maximum reaction when the train stands idle at the designed position. Other reactions caused by impact forces, centrifugal forces, wind forces, climatic forces, or any other existing forces will have to be handled in addition to those calculated.

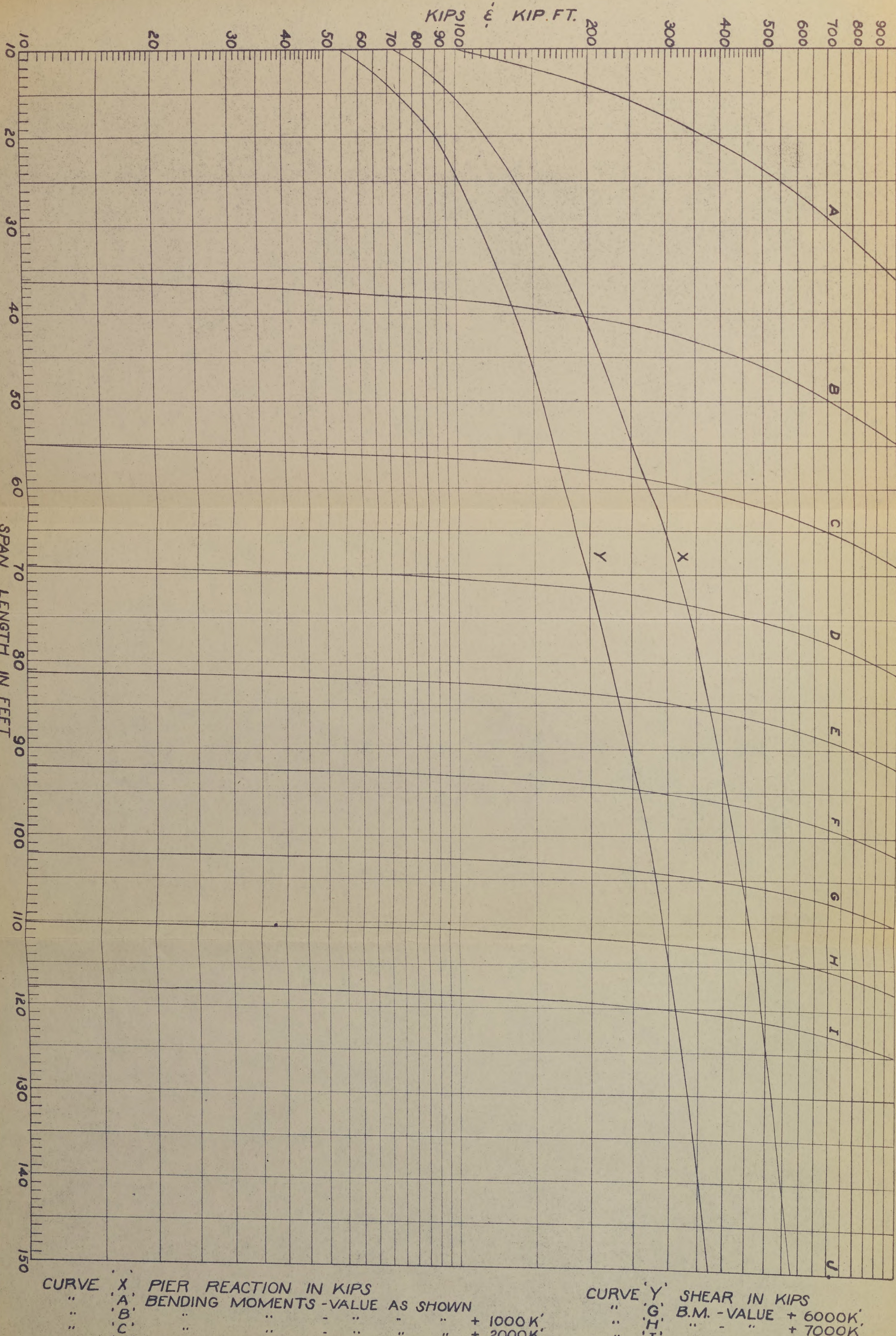
Rivet size, and number of, can be designed by knowing the maximum reactions. Hitch angles and pier construction as well as numerous other parts can be designed when using materials that resist tension, compression, shear, bearing, or torsion, in structures. Failure of any part or section might prove disasterous.

Reference examples on how these [illegible] were calculated for various span lengths, by using Cooper's E-72 Loading Diagram, will prove helpful if greater accuracy necessitates doing so.

COOPER'S E-72 DIAGRAM

SUM OF LOADS IN THOUSANDS OF POUNDS	18	54	90	126	162	185.4	208.8	232.2	255.6	273.6	309.6	345.6	381.6	417.6	441	464.4	487.8	511.2	
LOAD IN THOUSANDS OF POUNDS PER WHEEL	18	36	36	36	36	23.4	23.4	23.4	23.4	18	36	36	36	36	23.4	23.4	23.4	23.4	3600 #/, TRAIN LOAD
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	
DISTANCE IN FEET		8	5	5	5	9	5	6	5	8	8	5	5	5	9	5	6	5	5
SUM OF DISTANCES IN FEET		8	13	18	23	32	37	43	48	56	64	69	74	79	88	93	99	104	109
MOMENTS IN THOUSANDS OF FT. lb.		144	414	864	1494	2952	3879	5131.8	6292.8	8337.6	10,526.4	12,074.4	13,802.4	15,710.4	19,468.8	21,673.8	24,460.2	26,899.2	29,455.2

GRAPHS ON COOPER'S E-72 LOADINGS



CURVE 'X' PIER REACTION IN KIPS
" 'A' BENDING MOMENTS - VALUE AS SHOWN
" 'B' " " - " " " + 1000K'
" 'C' " " - " " " + 2000K'
" 'D' " " - " " " + 3000K'
" 'E' " " - " " " + 4000K'
" 'F' " " - " " " + 5000K'

CURVE 'Y' SHEAR IN KIPS
" 'G' B.M. - VALUE + 6000K'
" 'H' " - " + 7000K'
" 'I' " - " + 8000K'
POINT 'J' " - " +12,000K'

WILLIAM ALF PETERSON

DATA TABULATION

Span Length In Feet	Shear In Kips	Bending Moment in Kip Feet	Pier Reaction In Kips
10	[illegible]	101.25	[illegible]
11	[illegible]	[illegible]	[illegible]
12	[illegible]	[illegible]	[illegible]
13	66.46	171.00	[illegible]
14	[illegible]	[illegible]	[illegible]
15	[illegible]	[illegible]	[illegible]
16	[illegible]	[illegible]	[illegible]
17	[illegible]	[illegible]	[illegible]
18	[illegible]	[illegible]	109.50
19	[illegible]	[illegible]	[illegible]
20	[illegible]	[illegible]	[illegible]
21	[illegible]	[illegible]	[illegible]
22	[illegible]	[illegible]	[illegible]
23	[illegible]	[illegible]	[illegible]
24	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]
26	[illegible]	[illegible]	[illegible]
27	[illegible]	[illegible]	[illegible]
28	[illegible]	[illegible]	[illegible]
29	[illegible]	[illegible]	[illegible]
30	[illegible]	[illegible]	[illegible]
31	[illegible]	[illegible]	[illegible]

Span Length In Feet	Shear In Kips	Bending Moment in Kip Foot	Pier Reaction In Kips
32	[illegible]	[illegible]	[illegible]
33	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	[illegible]
35	[illegible]	[illegible]	175.6
36	[illegible]	[illegible]	[illegible]
37	[illegible]	[illegible]	[illegible]
38	[illegible]	[illegible]	[illegible]
39	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]
41	[illegible]	[illegible]	[illegible]
42	[illegible]	[illegible]	[illegible]
43	[illegible]	[illegible]	[illegible]
44	[illegible]	[illegible]	[illegible]
45	147.04	[illegible]	[illegible]
46	[illegible]	[illegible]	[illegible]
47	[illegible]	[illegible]	[illegible]
48	[illegible]	[illegible]	[illegible]
49	[illegible]	[illegible]	[illegible]
50	[illegible]	[illegible]	[illegible]
51	[illegible]	1766.40	[illegible]
52	[illegible]	[illegible]	[illegible]
53	[illegible]	[illegible]	[illegible]
54	[illegible]	[illegible]	[illegible]
55	[illegible]	[illegible]	[illegible]

Span Length In Feet	Shear In Kips	Bending Moment in Kip Feet	Pier Reaction In Kips
56	16[illegible].7[illegible]	2047.5[illegible]	[illegible]7.04
57	[illegible]	[illegible]	[illegible]1.74
58	17[illegible]	[illegible]	2[illegible]
59	17[illegible]	[illegible]9.70	[illegible]71.[illegible]
60	17[illegible]	[illegible].85	[illegible].7[illegible]
61	178.85	[illegible].40	280.57
62	1[illegible]0.26	247[illegible].50	2[illegible]4.69
63	1[illegible].54	[illegible]54[illegible].05	2[illegible]9.01
64	1[illegible].61	2619.[illegible]	[illegible].1[illegible]
65	1[illegible].[illegible]1	26[illegible].[illegible]	[illegible]97.65
66	1[illegible].[illegible]5	2771.10	301.97
67	191.5[illegible]	[illegible]46.75	30[illegible]
68	1[illegible]4.0[illegible]	[illegible]	[illegible]10.45
69	1[illegible]6.[illegible]	[illegible]7.90	[illegible]14.64
70	1[illegible].[illegible]	[illegible]075.50	31[illegible].67
71	[illegible]01.[illegible]	314[illegible].70	3[illegible]3.70
72	204.[illegible]	[illegible]7.[illegible]0	3[illegible].7[illegible]
73	[illegible]	[illegible]01.[illegible]	[illegible]9.6[illegible]
74	209.[illegible]	3[illegible].50	[illegible]34.65
75	211.[illegible]	3[illegible].50	[illegible].69
76	214.06	[illegible].60	[illegible].61
77	[illegible]16.70	[illegible]4.40	[illegible].61
78	[illegible]19.0[illegible]	[illegible]71[illegible]	[illegible].[illegible]5

Span Length In Feet	Shear In Kips	Bending Moment in Kip Feet	Pier Reaction In Kips
79	221.53	[illegible]	354.00
80	223.86	[illegible]	357.9
81	226.02	3975.[illegible]0	361.5[illegible]
82	228.42	4062.10	366.12
83	230.77	4151.60	3[illegible].73
84	233.06	4241.40	372.55
85	235.3[illegible]	4333.50	375.[illegible]5
86	237.75	4427.50	37[illegible].72
87	240.14	4519.10	381.60
88	242.40	4615.60	3[illegible]6.55
89	2[illegible]4.72	470[illegible].70	3[illegible]9.95
90	247.02	4806.90	39[illegible].41
91	249.31	4901.50	396.[illegible]6
92	251.61	49[illegible].00	400.31
93	253.95	5036.60	407.63
94	256.25	5133.50	407.09
95	25[illegible].49	5231.10	410.[illegible]5
96	260.69	5[illegible]90.00	414.00
97	263.02	5[illegible].10	417.17
98	26[illegible].44	5531.60	420.47
99	267.7[illegible]	5632.25	423.64
100	270.00	57[illegible].00	426.95
101	272.31	5[illegible]92.70	435.15
102	274.59	6004.70	4[illegible].15

Span Length In Feet	Shear In Kips	Bending Moment in Kip Foot	Pier Reaction In Kips
103	276.57	6139.13	436.31
104	278.74	6255.50	439.63
105	280.90	6367.70	442.80
106	28[illegible].[illegible]6	64[illegible]0.50	4[illegible]6.11
107	284.57	6600.00	44[illegible].27
108	287.35	6741.40	452.45
109	2[illegible]9.43	6[illegible]73.00	455.47
110	291.61	6998.25	458.64
111	293.74	7124.65	461.81
112	29[illegible].[illegible]6	7252.65	464.38
113	297.98	7378.15	467.36
114	300.09	7508.50	470.[illegible]
115	30[illegible].10	7641.00	47[illegible].76
116	304.23	7773.75	476.73
117	306.57	7901.00	479.06
118	30[illegible].46	[illegible]034.65	4[illegible]3.85
119	310.54	[illegible]169.75	4[illegible]6.45
120	312.68	8[illegible]05.55	4[illegible]9.[illegible]6
121	314.60	[illegible]455.55	49[illegible].77
122	316.76	[illegible]575.00	496.0[illegible]
123	31[illegible].38	[illegible]705.00	499.24
124	322.05	[illegible]31.15	502.56
125	342.9[illegible]	[illegible]995.55	505.72
150	377.21	12711.50	5[illegible].66

Span Length In Feet	Shear In Kips	Bending Moment In Kip Foot	Pier Reaction In Kips
175	4[illegible]1.9[illegible]	18,[illegible]54.50	6[illegible]9.0[illegible]
200	4[illegible].[illegible]1	[illegible]1,[illegible]71.00	75[illegible].2[illegible]
[illegible]	56[illegible].77	[illegible]1,666.[illegible]0	[illegible]7.[illegible]6

Reference Examples in Shear (Maximum)

Example I

Cooper's E-72 half track loading upon a 16' stringer

Trial 1

Load 1 at the left end of the stringer

$$\frac{414 + 90\ (3)}{16} = 42.75 \text{ Kips}$$

Trial 2

Load 2 at the left end of the stringer

$$\frac{1404 + 162 - (18 \times 24)}{16} = 76.31 \text{ Kips}$$

Trial 3

Load 3 at the left end of the stringer

$$\frac{14[illegible] + 16[illegible]\ (3) - (1[illegible]41 + 54 \times 21)}{16} = 74.25 \text{ Kips}$$

Therefore, with load 2 as it approaches the left end of the beam (to within an infinitesimal distance) a maximum shear will be developed on this 16 foot stringer. If each wheel load were tried as shown, you would find that wheel 11 would produce a shear equal to that of wheel 2, which is 76.5 kips.

Example 2

Cooper's E-72 half track loading upon a 10' span

Trial 1

Load 1 at the left end of the span

$$\frac{[illegible].6 + [illegible].6\ ([illegible])}{50} = 1[illegible].0[illegible]\ \text{Kips}$$

Trial 2

Load 2 at the left end of the span

$$\frac{[illegible]87.6 + [illegible].6\ ([illegible]) - 1[illegible]\ ([illegible])}{50} = 1[illegible].[illegible]\ \text{Kips}$$

Trial 3

Load 3 at the left end of the span

$$\frac{[illegible]87.6 + [illegible]7[illegible].6\ (7) - (1[illegible] + [illegible] \times [illegible]5)}{50} = 14[illegible].70\ \text{Kips}$$

During the first three trials, a supposed maximum was reached when load 8 approached the end of the span; yet if a combination of trials continue a maximum will be obtained when load 11 approaches the span's end.

$$\frac{[illegible],[illegible].[illegible] + [illegible]11.[illegible](5) + [illegible].[illegible]([illegible].6) - (10,[illegible].4 + [illegible]7[illegible].6 \times [illegible]0)}{50} =$$

157.00 Kips

It was found that with load [illegible] on the end of the various spans from 10' to 250' maximum was produced on all trials except spans whose lengths were between 4[illegible]' and [illegible]' when the maximum was obtained with load 11 over the end of the span.

Reference Examples in Bending Moments (Maximum)

Example 1

Cooper's E-72 half track loading upon a 12' stringer

Trial 1

Load 3 at the center of the stringer

$$\left(\frac{41 + 20 - 1\,(4)}{12}\right)6 \qquad = \quad 12[illegible] \text{ Kip foot}$$

Trial 2

Load 3 at the center of the stringer

$$\left(\frac{64 + 126 - 12\,(12)}{12}\right)(6) - 36\,([illegible]) = 144 \text{ Kip foot}$$

Trial 3

Load 4 at the center of the stringer

$$\left(\frac{L[illegible] + 10[illegible] - (144 + 54(1[illegible])}{12}\right)[illegible] - [illegible](6) = 14[illegible] \text{ Kip foot}$$

Although a maximum bending moment of 144 Kip foot has been found, a trial of absolute maximum bending moment must be calculated. If the span length were laid out on paper, the calculation would be simplified.

After drawing the twelve foot span, take the diagram and place as many wheels as possible on the span. Three wheels of either series 2-3-4; 3-4-5; 11-12-13; or 12-13-14. The resultant is equal to 10[illegible] Kips and acts through the center wheel, then by bisecting the resultant and the distance to the nearest wheel, which is the center wheel, and placing this bisector at the midpoint of the span, the left reaction can be computed, hence the bending moment. The bending moment will be, in this case, 144 Kip foot. This

absolute maximum bending moment happens to be the same as the calculations from Cooper's diagram, but Cooper's diagram does not always give the maximum bending moment by placing, by trial, various wheels at the center of the span.

A span 10 foot long is a perfect example to illustrate the difference between Cooper's diagram by placing a wheel at the center and by the absolute method.

Cooper's method

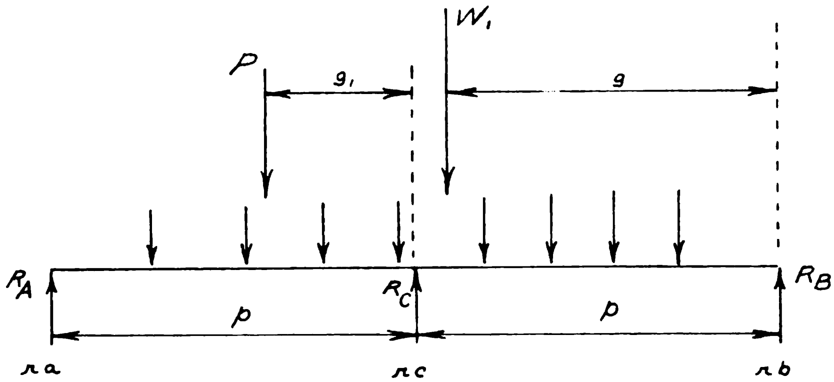
Taking load 2 at the center

$$\left(\frac{[illegible] - 18\ (10)}{10}\right)5 = 90 \text{ Kip foot}$$

Absolute maximum bending moment method

Taking loads 2 and 3, whose resultant is 72 kips and midway between both loads are 5 foot apart. Then bisecting the distance between load 2 and the resultant, which are 2.5 foot apart, and placing this bisector at the midpoint of the span. That places load 2 a distance of 3.75 foot from the left end of the span and load 3 a distance of 1.25 foot from the right end of the span. The left reaction can be calculated and by the shear diagram the absolute maximum bending moment can be calculated by the area method from the shear diagram. For a ten foot span the A.M.B.M. was found to be 101.25 Kip foot.

Principle solution of the development of maximum pier reaction



P = resultant of loads between A and C

W_1 = resultant of loads between two spans

g_1 = distance of P from C

g = distance of W_1 from B

$M_c = 0$

$r_a p - P g_1 = 0$

$r_a p = P g_1$

$M_b = 0$

$ra8[illegible] + rc[illegible] - W_1[illegible] = 0$

$8P_{1} + rc[illegible] = W_1[illegible]$

$rc[illegible] = W_1[illegible] - 8P_{1}$

$$rc = \frac{W_1[illegible] - 8P_{1}}{p}$$

If loads move X distance to the left left

The change in $rc = \dfrac{W_1X - 8PX}{PX}$

Rate of change in $rc = \dfrac{W_1X - 8PX}{PX} = \dfrac{W_1 - 8P}{p}$

Maximum reaction is when $W_1 = 8P$

Reference Examples in [illegible] Reactions

Example 1

Trial 1

Load 8 placed one foot to the right of the center support on two-13 foot spans

$$\frac{23(8 - 10 + 18)}{13} + \frac{16([illegible])}{13} + \frac{1[illegible]([illegible])}{13} = [illegible] K$$

Trial 2

Load 8 placed on the center support

$$\frac{[illegible](6 + 11 + 1[illegible])}{13} + \frac{[illegible](11)}{13} + \frac{1[illegible](2)}{13} = [illegible] K$$

Trial 3

Load 8 placed one foot to the left of the center support

$$\frac{36\ (7 + 12)}{16} + \frac{36\ (15 + 10)}{16} + \frac{18\ (2)}{16} = 101.2 \text{ KK}$$

The maximum reaction has been computed and the value will not be exceeded by any other position of Cooper's E-72 loading upon the spans of 16 foot

Example 2

Trial 1

Using two-50 foot spans

$$23.4 \left(\frac{5 + 45 + 50 + 44 + 39}{50} \right) = \frac{23.4\ (183)}{50}$$

$$+$$

$$36 \left(\frac{14 + 19 + 24 + 29 + 15 + 20 + 25 + 30}{50} \right) = \frac{36\ (17[illegible])}{50}$$

$$+$$

$$18 \left(\frac{37 + 7}{50} \right) = \frac{18\ (44)}{50} \qquad = 22[illegible].044 \text{ K}$$

Trial 2

Load 18 at C

$$23.4 \left(\frac{1[illegible] + 20 + 26 + 31 + 17 + 1[illegible] + 24 + 29}{50} \right) = \frac{411[illegible].4}{50}$$

$$+$$

$$36 \left(\frac{40 + 45 + 50 + 45 + 4}{50} \right) = \frac{6624}{50}$$

$$+$$

$$18 \left(\frac{37}{50} \right) = \frac{666}{50}$$

$$+$$

$$\frac{3.6\ (10)\ 5}{50} = [illegible].6 \qquad = [illegible]2[illegible].16[illegible] \text{ K}$$

Trial 3

Load 10 at C

$$23.4\left(\frac{2 + 7 + 13 + 18 + 26 + 31 + 37 + 42}{50}\right) = \frac{4118.4}{50}$$

$+$

$$36\left(\frac{27 + 32 + 37 + 42 + 2 + 7 + 12 + 17}{50}\right) = \frac{6336}{50}$$

$+$

$$18\left(\frac{50}{50}\right) = 18 \qquad = 227.08 \text{ K}$$

Trial 4

Load 13 at the center

$$23.4\left(\frac{20 + 25 + 31 + 36 + 8 + 13 + 19 + 24}{50}\right) = \frac{4118.4}{50}$$

$+$

$$36\left(\frac{45 + 50 + 45 + 40}{50}\right) = \frac{6480}{50}$$

$+$

$$18\left(\frac{32}{50}\right) = \frac{576}{50}$$

$+$

$$3.6\ (15)\ \frac{7.5}{50} = \frac{405.0}{50} \qquad = 231.59 \text{ K}$$

Load 13 one foot left of the center support

$$23.4\left(\frac{21 + 26 + 32 + 37 + 7 + 12 + 18 + 23}{50}\right) = \frac{4118.4}{50}$$

$+$

$$36\left(\frac{46 + 49 + 44 + 41}{50}\right) = \frac{6480}{50}$$

$+$

$$18\left(\frac{31}{50}\right) = \frac{558}{50}$$

$+$

$$3.6\ (16)\ \frac{8}{50} = \frac{460.8}{50} \qquad = 230.9 \text{ K}$$

Load 13 one foot right of the center support

$$23.4\left(\frac{19 + 24 + 30 + 35 + 9 + 14 + 20 + 25}{50}\right) = \frac{4118.4}{50}$$

$+$

$$36\left(\frac{44 + 49 + 46 + 41}{50}\right) = \frac{6480}{50}$$

$+$

$$18 \left(\frac{33}{50}\right) = \frac{594}{50}$$

$+$

$$3.6\ (14)\ \frac{7}{50} = \frac{352.8}{50} \qquad = 230.9\ K$$

A maximum pier reaction of 231.59 Kips has been computed when load 18 was placed directly over the center support of the two 50 foot spans.

Books Used for Guiding Reference

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Index

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Peterson

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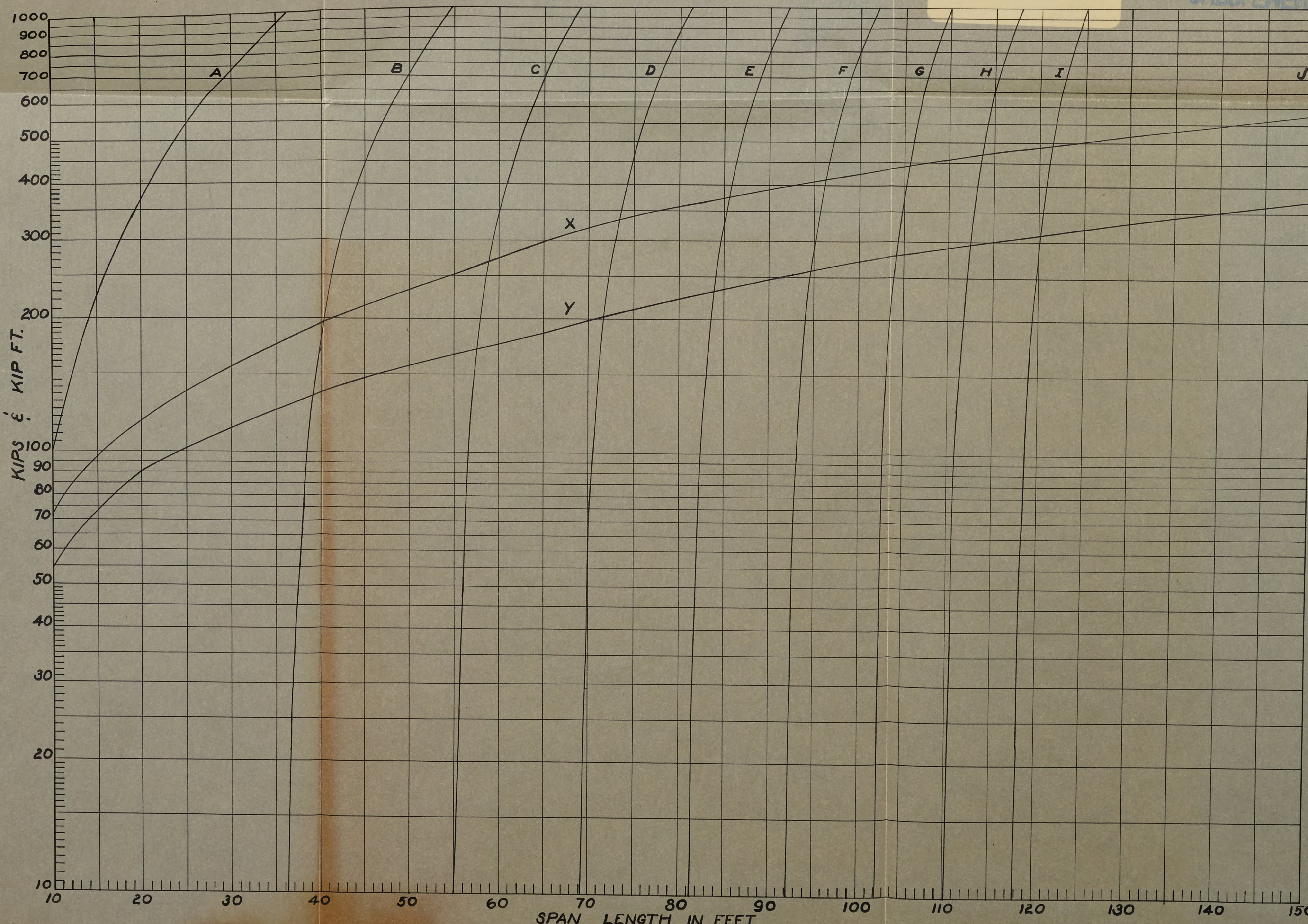
GRAPHS ON COOPER'S E-72 LOADINGS



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" 'D' " " - " " " + 3000 K'
" 'E' " " - " " " + 4000 K'
" 'F' " " - " " " + 5000 K'

CURVE 'Y' SHEAR IN KIPS
" 'G' B.M. - VALUE + 6000 K'
" 'H' " - " + 7000 K'
" 'I' " - " + 8000 K'
POINT 'J' " - " + 12,000 K'

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