SCATTERING FROM A SLOTTED CYLINDER

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ABSTRACT

SCATTERING FROM A SLOTTED CYLINDER

by John R. Short

It is well known that illuminating a metallic object by an electromagnetic wave induces currents on the object which, in turn, radiate to produce a scattered electromagnetic wave. This scattered field can be controlled by loading the surface of the object with lumped impedances. This thesis presents a theoretical and experimental study on the control of backscattering from a thick cylinder illuminated by a normally incident plane electromagnetic wave which has its electric field vector polorized perpendicular to the axis of the cylinder. The contról is achieved by loading the surface of the cylinder by a narrow, impedance backed longitudinal slot.

An exact theory is developed for the fields scattered by an infinitely long slotted cylinder illuminated by a plane wave as indicated above. A theoretical analysis is carried out to determine: (1) the optimum slot loading impedance required to force the backscattered electromagnetic field of the slotted cylinder to

vanish, and (2) the extent of the control over the backscattered field which can be obtained with a purely reactive loading.

It is verified experimentally that significant minimization of the backscattered field of a slotted cylinder may indeed be obtained through the use of a purely reactive loading. The experimental results agree excellently with the corresponding theoretical values.

This study should prove significant in the understanding of the modification of scattering from thick cylinders, which has practical application in the area of radar camouflage.

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I

INTRODUCTION

In recent years, the modification of the backscattering cross section of metallic objects has received considerable attention. Modification of the scattering from metallic spheres, plates, and loops has been studied by several authors[1-6]. Chen[7-9] and others[10, 11] have investigated the method for modifying the backscattering from a cylinder by impedance loading. These studies were concerned with thin cylinders where it was assumed that only an axial current was induced. When a cylinder is thick electrically in diameter and is illuminated by an E-M wave with its E field vector perpendicular to the cylinder's axis, a circumferential current is induced on the cylinder's surface and it, in turn, maintains a large scattered field[12].

The object of this research is to realize a method for controlling this circumferential induced current, thus controlling the scattering from the thick cylinder. This is accomplished by implementing an impedance backed longitudinal slot on the surface of the cylinder. A theoretical expression is derived for the fields scattered from such a slotted cylinder illuminated by a normally incident plane E-M wave whose E-field vector is perpendicular to the cylinder's axis. The backscattering cross section is then formulated in terms of the loading impedance, slot position, and other parameters. Optimum loading for minimum and maximum backscattering are obtained. The theory has been verified by an experimental investigation. It has been proved both theoretically and experimentally that the backscattering of a thick cylinder illuminated by an E-M wave with a perpendicular $\vec{E}$ field vector can be minimized drastically by a properly designed impedance backed slot.

II

THEORETICAL FORMULATION FOR SCATTERING FROM A SLOTTED CYLINDER

2.1 Formulation of the Problem

The geometry of the problem is as indicated in Figure 1. A perfectly conducting cylinder of infinite length and radius a has an impedance backed longitudinal slot cut on its surface. The slot is located at $\theta = \theta_o$ and has an angular width δ. The impedance Z backing the slot is lumped into the slot region on the cylinder's surface. A plane electromagnetic wave is incident normally upon the cylinder from the direction $\theta = \pi$ with its $\vec{E}$ field vector perpendicular to the cylinder axis. This incident wave induces a circumferential current which in turn reradiates a scattered electromagnetic field.

The scattered field from the slotted cylinder can be obtained by the superposition of the field scattered by an unloaded solid cylinder illuminated by a plane wave and the field radiated by a slotted cylinder excited by a potential difference impressed

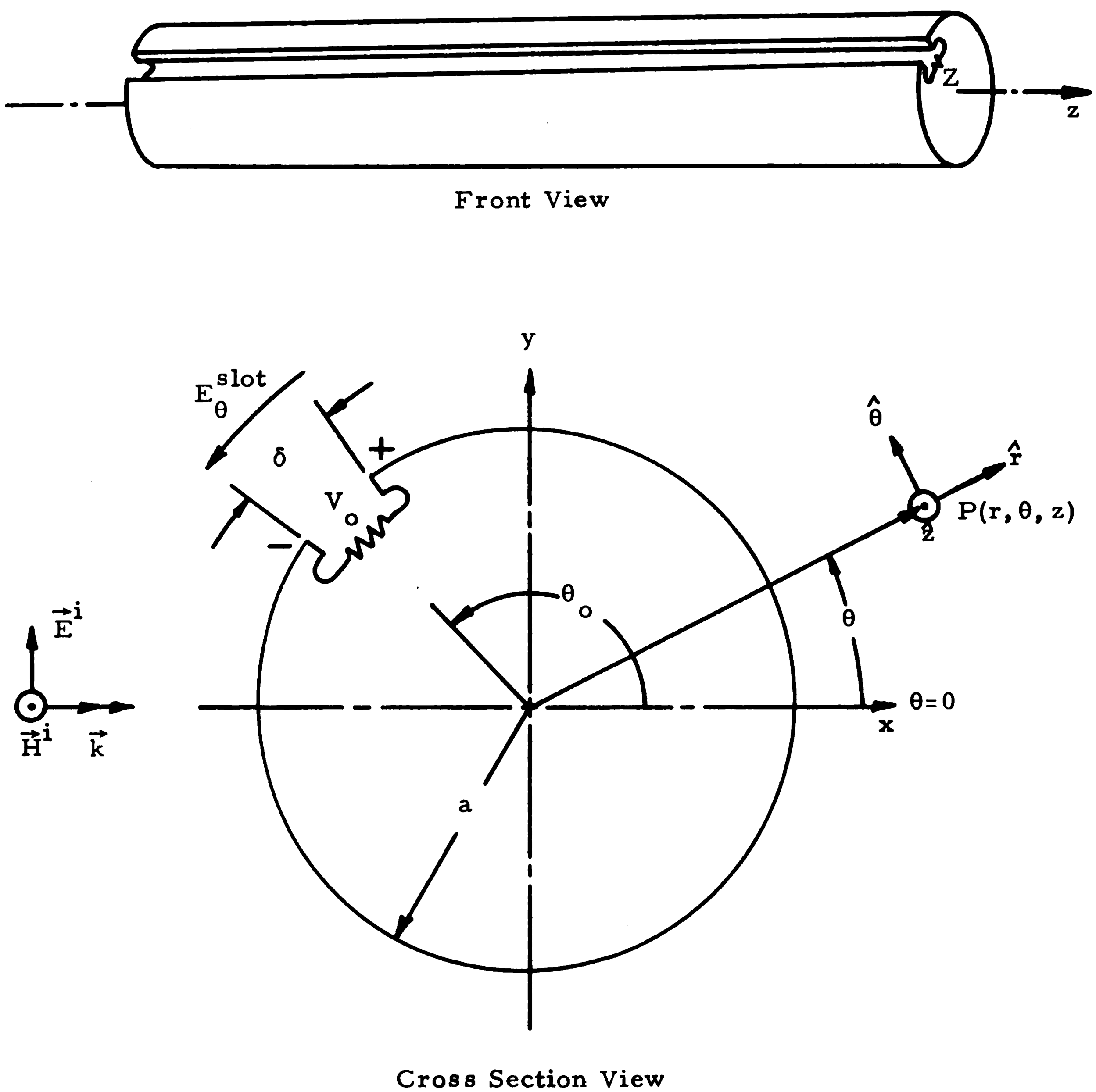


Figure 2.1: An Infinite Cylinder with a Longitudinal Slot Illuminated by a Plane EM Wave with its E Field Vector Perpendicular to the Cylinder Axis.

across the slot. A mathematical statement of this superposition is

$$\vec{E}^s = \vec{E}^c + \vec{E}^r \tag{2.1}$$

$$\vec{H}^s = \vec{H}^c + \vec{H}^r \tag{2.2}$$

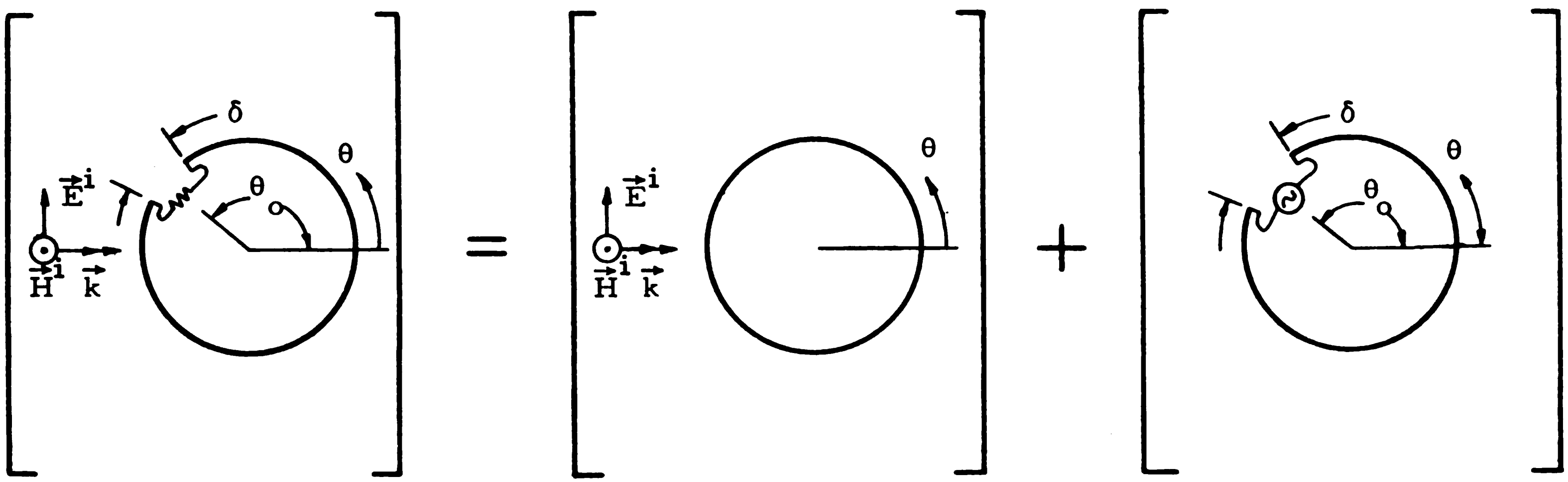
where $\vec{E}^s$ and $\vec{H}^s$ represent the fields scattered from a slotted cylinder illuminated by a normally incident plane wave, $\vec{E}^c$ and $\vec{H}^c$ represent the fields scattered by a solid cylinder illuminated by the same incident plane wave and $\vec{E}^r$ and $\vec{H}^r$ represent the fields radiated by a slotted cylinder with excitation applied across the slot. The excitation of the slot must be chosen in accordance with the total surface current on the illuminated slotted cylinder and the impedance backing the slot. This situation is indicated schematically in Figure 2.1.

2.2 Boundary Conditions

Since the slotted cylinder is assumed to be perfectly conducting, the tangential electric field must vanish at its surface except in the slot. If the slot is assumed to be very narrow (i.e., $ka\delta << 1$, where k is the free-space wave number) its electric field may be modeled as

$$E_\theta^{slot}(r = a^-) = \frac{ZK_\theta(\theta = \theta_o)}{a\delta} \text{ for } |\theta - \theta_o| < \frac{\delta}{2} \tag{2.3}$$

where the surface current K_θ is related to the total longitudinal



a) Illuminated slotted cylinder

b) Illuminated solid cylinder

c) Driven slotted cylinder

Figure 2.2: Superposition for an Illuminated Slotted Cylinder.

magnetic field at the surface of the cylinder as $K_\theta = -H_z$. The impedance Z of the longitudinal slot is defined as

$$Z = \frac{V_o}{K_\theta(\theta=\theta_o)} = \frac{E_\theta(\theta=\theta_o)a\delta}{-H_z(\theta=\theta_o)} \text{ (ohm meter)} \tag{2.4}$$

where V_o is the potential difference across the gap. The boundary condition at the surface of the illuminated slotted cylinder is,

$$E_\theta(r=a^+) = E_\theta(r=a^-)$$

which may be expressed as

$$E_\theta^i(r=a^+) + E_\theta^s(r=a^+) = \begin{cases} \frac{V_o}{a\delta} & \text{for } |\theta - \theta_o| < \frac{\delta}{2} \\ 0 & \text{for } |\theta - \theta_o| > \frac{\delta}{2} \end{cases} \tag{2.5}$$

where E_θ^i is the θ component of the incident electric field.

The superposition technique discussed in the previous section allows the boundary conditions at the surface of the illuminated slotted cylinder to be separated into the boundary condition for the illuminated solid cylinder,

$$E_\theta^i(r=a^+) + E_\theta^c(r=a^+) = 0 \tag{2.6}$$

and the boundary condition for the driven slotted cylinder as

$$E_\theta^r(r=a^+) = \begin{cases} \frac{V_o}{a\delta} & \text{for } |\theta - \theta_o| < \frac{\delta}{2} \\ 0 & \text{for } |\theta - \theta_o| > \frac{\delta}{2} \,. \end{cases} \tag{2.7}$$

Boundary conditions (2.6) and (2.7) define the scattering and radiating modes to be discussed in the following two sections.

2.3 Scattering from a Solid Cylinder

Consider a perfectly conducting cylinder of radius a which is illuminated by a normally incident plane electromagnetic wave with an $\vec{E}$ field vector perpendicular to the cylinder axis. The geometry of the problem is defined in Figure 2.2.b.

The incident plane wave can be represented by the following field expansions:[13]

$$H_z^i = e^{-jkx} = e^{-jkr\cos\theta}$$

$$= \sum_{n=0}^{\infty} \epsilon_{on}(-j)^n \cos(n\theta) J_n(kr) \qquad (2.8)$$

$$H_\theta^i = H_r^i = 0 \qquad (2.9)$$

$$E_\theta^i = \frac{j}{\omega\epsilon_o} \frac{\partial}{\partial r} H_z^i = j\zeta_o \sum_{n=0}^{\infty} \epsilon_{on}(-j)^n \cos(n\theta) J_n'(kr) \qquad (2.10)$$

$$E_r^i = -\frac{j}{\omega\epsilon_o} \frac{1}{r} \frac{\partial}{\partial\theta} H_z^i = \frac{j}{\omega\epsilon_o} \frac{1}{r} \sum_{n=0}^{\infty} \epsilon_{on}(-j)^n n \sin(n\theta) J_n(kr) \qquad (2.11)$$

$$E_z^i = 0 \qquad (2.12)$$

The Neumann factor ϵ_{on} is defined as

$$\epsilon_{on} = \begin{cases} 1 & \text{for} \quad n = 0 \\ 2 & \text{for} \quad n \neq 0 \end{cases}$$

and $J_n(kr)$ is the n th order Bessel function of the first kind. The impedance of free-space ζ_o is 120 π ohms, and k is the free-space wave number. The $e^{j\omega t}$ time dependence factor is implied.

The solution for the fields scattered by a perfectly conducting infinite cylinder illuminated by a plane wave are well known,[14] and are given by:

$$H_r^c = H_\theta^c = 0 \tag{2.13}$$

$$H_z^c = -\sum_{n=0}^{\infty} \epsilon_{on} (-j)^n \cos(n\theta)\, J_n'(ka)\, \frac{H_n^{(2)}(kr)}{H_n^{(2)'}(ka)} \tag{2.14}$$

$$E_r^c = \frac{1}{\omega\epsilon_o}\,\frac{1}{r} \sum_{n=0}^{\infty} \epsilon_{on} (-j)^{n+1}\, n \sin(n\theta) J_n'(ka)\, \frac{H_n^{(2)}(kr)}{H_n^{(2)'}(ka)} \tag{2.15}$$

$$E_\theta^c = \zeta_o \sum_{n=0}^{\infty} \epsilon_{on} (-j)^{n+1} \cos(n\theta) J_n'(ka)\, \frac{H_n^{(2)'}(kr)}{H_n^{(2)'}(ka)} \tag{2.16}$$

$$E_z^c = 0 \tag{2.17}$$

where $H_n^{(2)}(kr)$ is the nth order Hankel function of the second kind.

In the radiation zone (i.e., $kr \gg 1$), the scattered field behaves as an outward traveling cylindrical wave, which can be observed by replacing $H_n^{(2)}(kr)$ and its derivative by the leading terms of their asymptotic expansions for large kr. This procedure gives:

$$H_z^{cr} = -\sqrt{\frac{2}{\pi kr}}\; e^{-j(kr-\frac{\pi}{4})} \sum_{n=0}^{\infty} \epsilon_{on} \cos(n\theta) \frac{J_n'(ka)}{H_n^{(2)'}(ka)} \qquad (2.18)$$

$$E_\theta^{cr} = -\zeta_o \sqrt{\frac{2}{\pi kr}}\; e^{-j(kr-\frac{\pi}{4})} \sum_{n=0}^{\infty} \epsilon_{on} \cos(n\theta) \frac{J_n'(ka)}{H_n^{(2)'}(ka)} \qquad (2.19)$$

$$E_r^{cr} = 0$$

where the second superscript denotes radiation zone fields.

2.4 Radiation from a Driven Slotted Cylinder

Consider the separate but related problem of a perfectly conducting cylinder of infinite length and radius a having a longitudinal slot located on its surface at $\theta = \theta_o$. The slot has angular width δ which is assumed to be very small (i.e., $ka\delta << 1$) and is excited by a voltage V_o impressed across the slot. The geometry of the problem is indicated in Figure 2.2.c.

The boundary condition at the surface of the cylinder is given by equation (2.7) as

$$E_\theta^r(r=a) = \begin{cases} \frac{V_o}{a\delta} & \text{for } |\theta - \theta_o| < \frac{\delta}{2} \\ 0 & \text{for } |\theta - \theta_o| > \frac{\delta}{2} \end{cases} \qquad (2.7)$$

The magnetic field has only a z-component, and this field must satisfy the wave equation,

$$(\nabla^2 + k^2) H_z^r = 0 \,. \qquad (2.21)$$

Since the cylinder and slot are infinitely long, the radiated fields have no z-dependence, that is, $\frac{\partial}{\partial z} \equiv 0$. The wave equation for H_z^r thus becomes:

$$\left[\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + k^2\right] H_z^r = 0 \tag{2.22}$$

An appropriate solution for H_z^r is

$$H_z^r = \sum_{n=0}^{\infty} \left[a_n \sin(n\theta) + b_n \cos(n\theta)\right] H_n^{(2)}(kr) \tag{2.23}$$

where a_n and b_n are unknown coefficients which are to be evaluated by application of boundary condition (2.7).

The field H_z^r is symmetric with respect to the slot located at $\theta = \theta_o$. Let θ in equation (2.23) be replaced by θ', where

$$\theta' = \theta - \theta_o \tag{2.24}$$

such that $H_z^r(-\theta') = H_z^r(\theta')$. With this change of variable, equation (2.23) can be written in the following form

$$H_z^r = \sum_{n=0}^{\infty} A_n \cos(n\theta') H_n^{(2)}(kr) \tag{2.25}$$

where A_n is a new coefficient.

Other components of the radiated field can be obtained from equation (2.25) and Maxwell's equations as

$$H_\theta^r = H_r^r = 0 \tag{2.26}$$

$$E_\theta^r = \frac{j}{\omega\epsilon_o}\frac{\partial}{\partial r}H_z^r = j\zeta_o \sum_{n=0}^{\infty} A_n H_n^{(2)'}(kr)\cos(n\theta') \tag{2.27}$$

$$E_r^r = -\frac{j}{\omega\epsilon_o}\frac{1}{r}\frac{\partial}{\partial\theta}H_z^r = \frac{j}{\omega\epsilon_o}\frac{1}{r}\sum_{n=0}^{\infty} A_n H_n^{(2)}(kr)n\sin(n\theta') \tag{2.28}$$

$$E_z^r = 0 \tag{2.29}$$

The arbitrary coefficient A_n can be determined from equation (2.27) and boundary condition (2.7) to be

$$A_n = \frac{1}{j\zeta_o}\left(\frac{\epsilon_{on}}{\pi}\right)\left(\frac{V_o}{a\delta}\right)\frac{\sin(\frac{n\delta}{2})}{n}\frac{1}{H_n^{(2)'}(ka)} \tag{2.30}$$

The fields radiated from the driven slotted cylinder are now completely determined and are given by:

$$H_z^r = \frac{1}{j\pi\zeta_o}\left(\frac{V_o}{a\delta}\right)\sum_{n=0}^{\infty}\epsilon_{on}\frac{\sin(\frac{n\delta}{2})}{n}\cos(n\theta')\frac{H_n^{(2)}(kr)}{H_n^{(2)'}(ka)} \tag{2.31}$$

$$E_r^r = \frac{1}{\pi kr}\left(\frac{V_o}{a\delta}\right)\sum_{n=0}^{\infty}\epsilon_{on}\sin(\frac{n\delta}{2})\sin(n\theta')\frac{H_n^{(2)}(kr)}{H_n^{(2)'}(kr)} \tag{2.32}$$

$$E_\theta^r = \frac{1}{\pi}\left(\frac{V_o}{a\delta}\right)\sum_{n=0}^{\infty}\epsilon_{on}\frac{\sin(\frac{n\delta}{2})}{n}\cos(n\theta')\frac{H_n^{(2)'}(kr)}{H_n^{(2)'}(ka)} \tag{2.33}$$

In the radiation zone of the slotted cylinder the radiated field behaves as an outward traveling cylindrical wave. Replacing

$H_n^{(2)}(kr)$ and its derivative by the leading terms of the asymptotic expansion for large kr yields:

$$H_z^{rr} = \frac{1}{\zeta_o}\left(\frac{V_o}{a\delta}\right)\sqrt{\frac{2}{\pi^3 kr}}\; e^{-j(kr-\frac{\pi}{4})} \sum_{n=0}^{\infty} \epsilon_{on}(j)^{n-1} \frac{\sin(\frac{n\delta}{2})}{n} \frac{\cos(n\theta')}{H_n^{(2)'}(ka)} \tag{2.34}$$

$$E_\theta^{rr} = \left(\frac{V_o}{a\delta}\right)\sqrt{\frac{2}{\pi^3 kr}}\; e^{-j(kr-\frac{\pi}{4})} \sum_{n=0}^{\infty} \epsilon_{on}(j)^{n-1} \frac{\sin(\frac{n\delta}{2})}{n} \frac{\cos(n\theta')}{H_n^{(2)'}(ka)} \tag{2.35}$$

$$E_r^{rr} = 0 \tag{2.36}$$

2.5 Scattering from a Slotted Cylinder

The final problem to be considered is the superposition of the two preceding results in accordance with equations (2.1) and (2.2) to obtain the total fields scattered by an infinite, perfectly conducting, slotted cylinder illuminated by a normally incident plane wave.

The total scattered fields are

$$E_\theta^s = \sum_{n=0}^{\infty} \epsilon_{on} \frac{H_n^{(2)'}(kr)}{H_n^{(2)'}(ka)} \Big[\zeta_o(-j)^{n+1}\cos(n\theta)\, J_n'(ka) + \frac{1}{\pi}\left(\frac{V_o}{a\delta}\right) \frac{\sin(\frac{n\delta}{2})}{n} \cos(n\theta')\Big] \tag{2.37}$$

$$E_r^s = \sum_{n=0}^{\infty} \frac{\epsilon_{on}}{r} \frac{H_n^{(2)}(kr)}{H_n^{(2)'}(ka)} \left[\frac{(-j)^{n+1}}{\omega \epsilon_o} n \sin(n\theta) J_n'(ka) + \left(\frac{V_o}{a\delta}\right)\left(\frac{1}{\pi k}\right) \sin\left(\frac{n\delta}{2}\right) \sin(n\theta')\right] \quad (2.38)$$

$$E_z^s = 0 \quad (2.39)$$

$$H_\theta^s = H_z^s = 0 \quad (2.40)$$

$$H_z^s = - \sum_{n=0}^{\infty} \epsilon_{on} \frac{H_n^{(2)}(kr)}{H_n^{(2)'}(ka)} \left[(-j)^n \cos(n\theta) J_n'(ka) + \frac{j}{\pi \zeta_o} \left(\frac{V_o}{a\delta}\right) \frac{\sin(\frac{n\delta}{2})}{n} \cos(n\theta')\right] \quad (2.41)$$

while the total scattered fields in the radiation zone of the cylinder are:

$$E_\theta^{sr} = - \sqrt{\frac{2}{\pi kr}}\, e^{-j(kr - \frac{\pi}{4})} \sum_{n=0}^{\infty} \frac{\epsilon_{on}}{H_n^{(2)'}(ka)} \left[\zeta_o \cos(n\theta) J_n'(ka) + \frac{(j)^{n+1}}{\pi} \left(\frac{V_o}{a\delta}\right) \frac{\sin(\frac{n\delta}{2})}{n} \cos(n\theta')\right] \quad (2.42)$$

$$E_r^{sr} = 0 \quad (2.43)$$

$$H_z^{sr} = -\sqrt{\frac{2}{\pi kr}}\; e^{-j(kr-\frac{\pi}{2})} \sum_{n=0}^{\infty} \frac{\epsilon_{on}}{H_n'(ka)} \Big[\cos n\theta\, J_n'(ka) + \frac{(j)^{n+1}}{\zeta_o \pi} \left(\frac{V_o}{a\delta}\right) \frac{\sin(\frac{n\delta}{2})}{n} \cos(n\theta')\Big] \qquad (2.44)$$

It can be observed from the above expressions that the scattered field in the radiation zone is a outward traveling cylindrical wave, and that by adjusting V_o, which is related to the slot impedance Z and δ, the scattered fields may be made to vanish at any point in the radiation zone.

2.6 The Backscattering Cross Section

The backscattering cross section per unit length of the cylinder can be defined as follows with specific reference to an infinite cylindrical object:

$$\sigma_B^{(c)} = \frac{P^s_{omni}}{S^i} = \lim_{r\to\infty} 2\pi r \left| \left| \frac{\vec{E}^s}{\vec{E}^i} \right|^2 \right|_{\theta=\pi} \qquad (2.45)$$

where P^s_{omni} is the total power reradiated per unit length of an ideal omnidirectional scatterer that maintains the same field $\vec{E}^s$ at a radial distance r for all values of θ as that maintained by the actual scattering cylinder in the direction $\theta=\pi$.[12]

Equation (42) for the scattered electric field along with the above definition result in an expression for the backscattering cross section $\sigma_B^{(c)}$ of the slotted cylinder as

$$\sigma_B^{(c)} = \frac{4}{k}\left| \sum_{n=0}^{\infty} \frac{\epsilon_{on}}{H_n^{(2)'}(ka)} \left[\zeta_o (-1)^n J_n'(ka) - \frac{(j)^{n+1}}{\pi} \left(\frac{V_o}{a\delta}\right) \frac{\sin(\frac{n\delta}{2})}{n} \cos n(\pi - \theta_o) \right|^2 \qquad (2.46)$$

which is a function of ka, $\frac{V_o}{a\delta}$, and θ_o.

The voltage V_o which excites the slot can now be expressed in terms of the impedance Z of the slot and the total surface current on the illuminated slotted cylinder. The following result can be obtained from equations (2. 3) and (2. 4):

$$\begin{aligned} V_o &= Z\, K_\theta(\theta = \theta_o) \\ &= -Z\, H_z(r = a,\ \theta = \theta_o) \end{aligned} \qquad (2.47)$$

where H_z is the total longitudinal magnetic field given by

$$\begin{aligned} H_z &= H_z^i + H_z^s \\ &= H_z^i + H_z^c + H_z^r \ . \end{aligned} \qquad (2.48)$$

Note that H_z^i and H_z^c are independent of the driving voltage V_o while H_z^r is a function of V_o. With this in mind, the following two quantities may be defined:

$$\overline{H} = [H_z^i + H_z^c]_{\substack{r=a \\ \theta=\theta_o}}$$

$$= \sum_{n=0}^{\infty} \epsilon_{on}(-j)^n \cos(n\theta_o)[J_n(ka) - J_n'(ka) \frac{H_n^{(2)}(ka)}{H_n^{(2)'}(ka)}] \qquad (2.49)$$

and

$$\overline{Y} = \frac{H^r(r=a, \theta=\theta_o)}{V_o}$$

$$= \frac{1}{j\pi \zeta_o a \delta} \sum_{n=0}^{\infty} \epsilon_{on} \frac{\sin(\frac{n\delta}{2})}{n} \frac{H_n^{(2)}(ka)}{H_n^{(2)'}(ka)} \qquad (2.50)$$

The voltage V_o may now be solved for in terms of completely determined quantities. The above definitions and equation (2.47) give:

$$V_o = -\frac{Z\overline{H}}{1+Z\overline{Y}} \qquad (2.51)$$

Defining two more quantities

$$\overline{P} = \zeta_o \sum_{n=0}^{\infty} \epsilon_{on}(-1)^n \frac{J_n'(ka)}{H_n^{(2)'}(ka)} \qquad (2.52)$$

and

$$\overline{Q} = \frac{1}{\pi a \delta} \sum_{n=0}^{\infty} \epsilon_{on}(j)^{n+1} \frac{\sin(\frac{n\delta}{2})}{n} \frac{\cos n(\pi - \theta_o)}{H_n^{(2)'}(ka)} \qquad (2.53)$$

allows the backscattering cross section to be written in the following concise form.

$$\sigma_B^{(c)} = \frac{4}{k} \left| \overline{P} - \frac{Z\overline{H}\overline{Q}}{1+Z\overline{Y}} \right|^2 \tag{2.54}$$

If the problem is again considered from the viewpoint of superposition (as discussed previously in this section) $\overline{P}$ and $\frac{Z\overline{H}\overline{Q}}{1+Z\overline{Y}}$ represent the complex amplitudes of the contribution to the backscattering cross section by the scattering from an illuminated solid cylinder and the radiation by a driven slotted cylinder, respectively.

2.7 Optimum Impedance for Zero Backscattering

An optimum slot impedance that will cause the backscattered field to vanish in the radiation zone can be found by equating the backscattering cross section to zero and solving for the optimum slot impedance. This impedance is denoted as Z_{op} and is given by

$$Z_{op} = \frac{\overline{P}}{\overline{H}\overline{Q} - \overline{Y}\overline{P}} \tag{2.55}$$

which can easily be derived from equation (2.54).

Generally an active element is required to realize the above impedance function. In some cases it becomes physically impossible to realize this impedance at all. It may thus be more practical to consider the reduction of backscattering by using a purely reactive impedance.

2.8 Optimum Reactance for Maximum or Minimum Backscattering

In general, the backscattering cross section of a slotted cylinder cannot be reduced to zero with a purely reactive slot impedance. This does not, however, rule out the possibility of reducing $\sigma_B^{(c)}$ by a suitable choice of the loading reactance. An optimum reactance, X_{op}, for maximum or minimum backscattering can be determined by differentiating $\sigma_B^{(c)}$ with respect to X and setting the derivative equal to zero. The result of this procedure is

$$X_{op} = \frac{V \pm \sqrt{V^2 + 4W}}{2} \tag{2.56}$$

where

$$V = \frac{(E^2 + G^2) - (C^2 + D^2)(A^2 + B^2)}{(C^2 + D^2)(BE - GA) + D(E^2 + G^2)} \tag{2.57}$$

$$W = \frac{D(A^2 + B^2) + (BE - GA)}{(C^2 + D^2)(BE - GA) + D(E^2 + G^2)} \tag{2.58}$$

and the quantities on the right hand side of the above two equations are defined by

$$\overline{P} = A + jB \tag{2.59}$$

$$\overline{Y} = C + jD \tag{2.60}$$

$$(\overline{Y}\,\overline{P} - \overline{H}\,\overline{Q}) = E + jG \tag{2.61}$$

2.9 Numerical Results

The four functions $\overline{H}$, $\overline{Y}$, $\overline{P}$, and $\overline{Q}$ involved in the expressions for backscattering cross section $\sigma_B^{(c)}$ and slot impedance Z were numerically evaluated. Except in the case of the imaginary part of $\overline{Y}$, all the series converged very rapidly. The evaluation of the series $\overline{Y}$ (i.e., the magnetic field at the center of the slot of the driven slotted cylinder divided by the potential drop across the slot) is complicated by the slow convergence of its imaginary part. It was found that the rate of convergence was directly related to the gap width of the slot in the cylinder and that it was necessary to retain over thirty terms of this series to obtain the desired accuracy even when $\delta = 0.1$.

To investigate the effect of changes in slot width on the ability to control backscattering from the cylinder, the backscattering cross section $\sigma_B^{(c)}$ was calculated as a function of slot position θ_o and slot impedance Z for several values of the angular slot width δ (δ varied from 0.001 radians to 0.2857 radians). The effectiveness in minimizing or maximizing the scattering was found to be not noticeably affected by these variations in the gap width. The optimum slot reactance to minimize the backscattering cross section $\sigma_B^{(c)}$ when $\theta_o = 180^o$ and ka = 2.20 was found to range from 3.508 ohm-meter to 7.867 ohm-meter when the angular slot width δ varied from 0.001 radians to 0.2857 radians.

The effect of the electrical radius ka of the cylinder was next investigated. The angular width δ of the slot was fixed at 0.10 radians and the backscattering behavior of the slotted cylinder having radii ranging from ka = 1.0 to ka = 10.0 was considered. The optimum impedance for zero backscattering is indicated as a function of slot position θ_o in Figures 2.3 - 2.6 for various values of ka. It is observed that for ka $\geq$ 2.20 the resistive part of the slot impedance is nearly always negative. The maximum and minimum backscattering cross sections for purely reactive loading are given in Figures 2.7 - 2.12 as a function of slot position θ_o, for several values of ka. The corresponding values for optimum reactance are tabulated in Tables 2.1 and 2.2. It was found that when ka $\leq$ 5 the backscattering cross section could be minimized by at least 10 dB over an excursion in slot position from $\theta_o = 180^o$ of about forty degrees. The enhancement varied from zero to five dB, and depended greatly upon the slotorientation. In general, the control over the scattering was markedly decreased for ka > 5 and also when the slot is oriented in the shadow region. The backscattering cross section as a function of slot orientation for a fixed, purely reactive loading is indicated in Figure 2.13. The reactance is chosen to minimize the backscattering at $\theta_o = 180^o$. It was found for certain displacement of the slot from $\theta_o = 180^o$ that this reactance would enhance the backscattering and that in general as ka is increased the minimization of the backscattering as the slot is displaced from $\theta_o = 180^o$ becomes less effective.

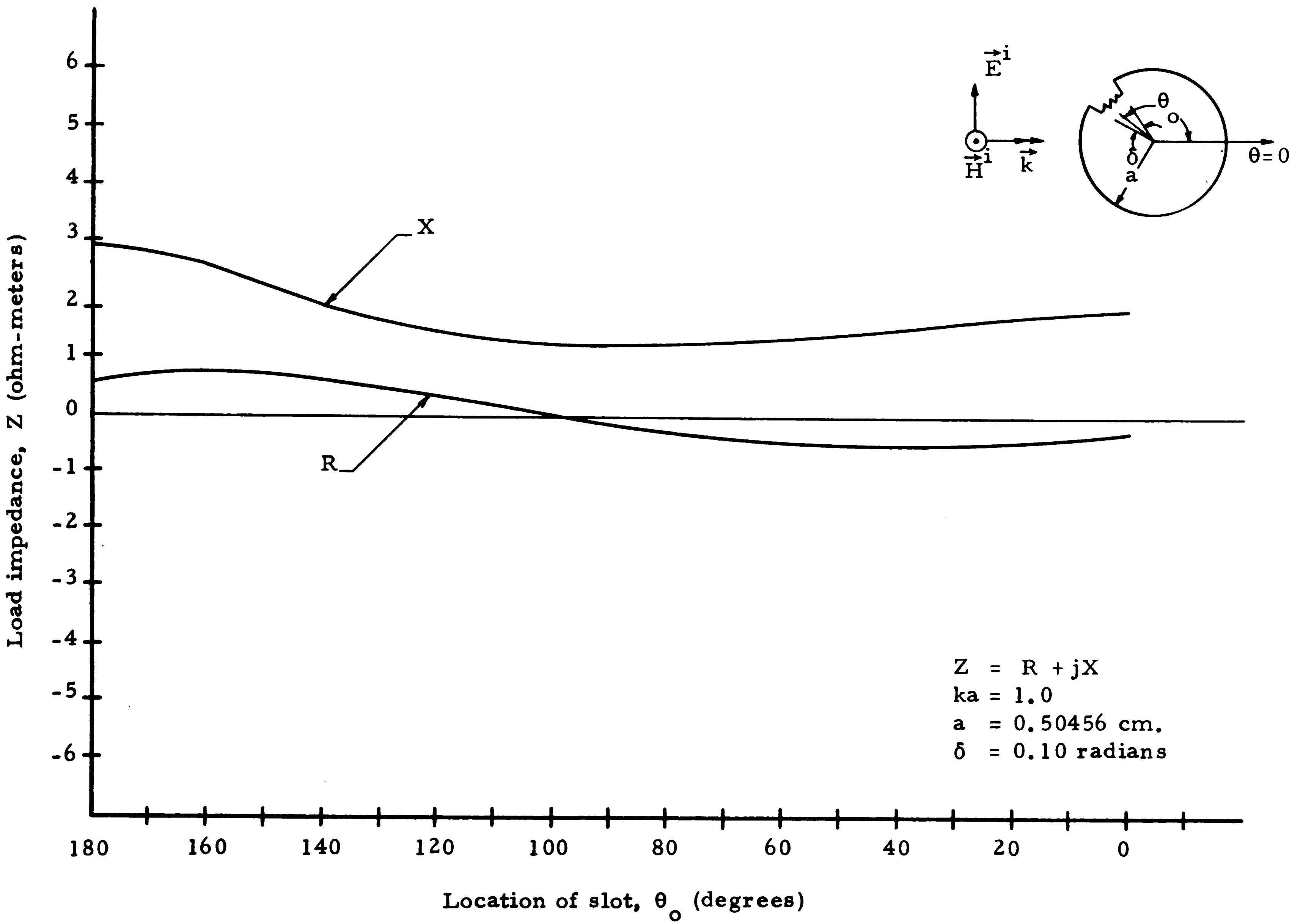


Figure 2.3: Optimum Slot Impedance for Zero Backscattering as a Function of Slot Position with ka = 1.0.

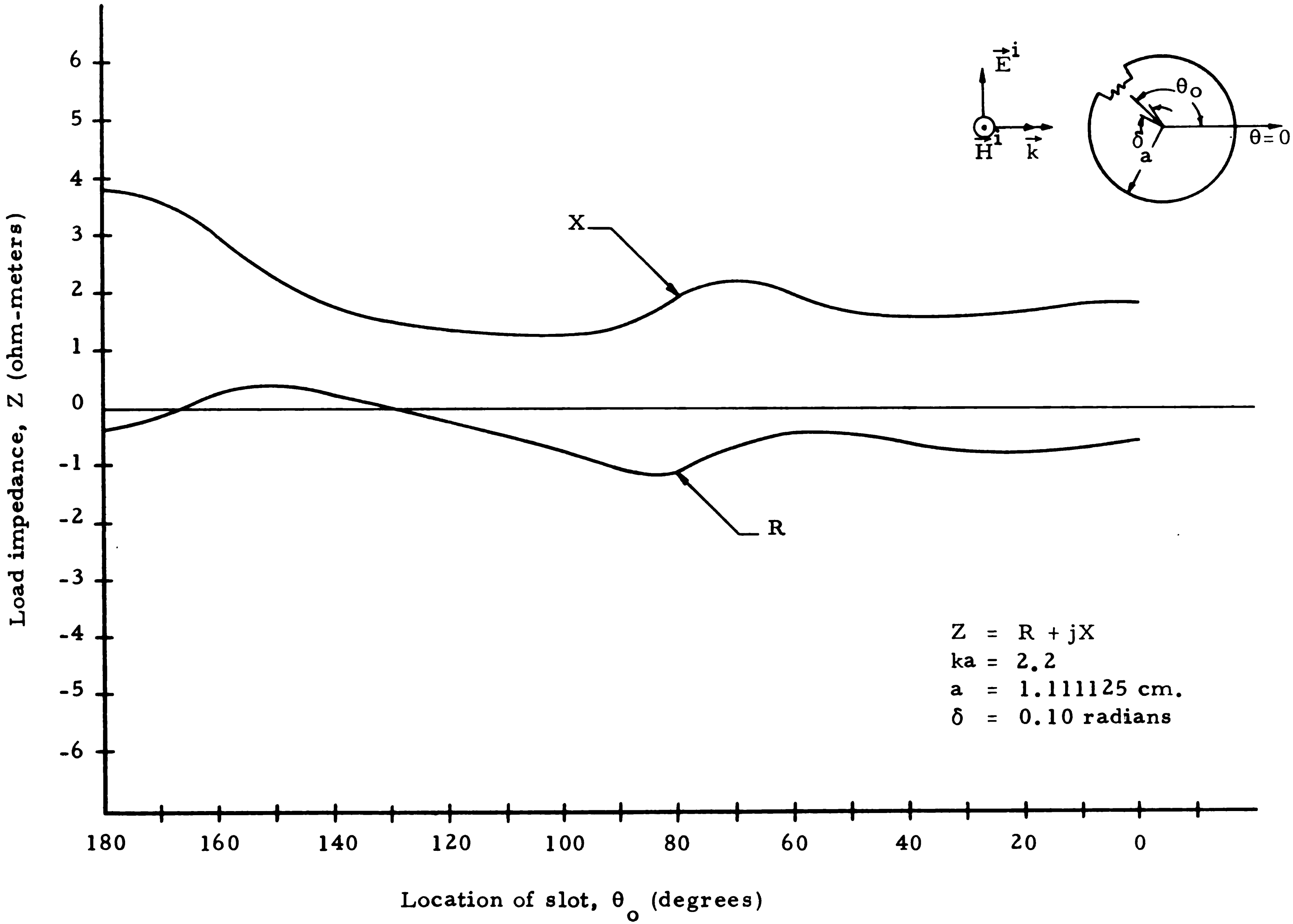


Figure 2.4: Optimum Slot Impedance for Zero Backscattering as a Function of Slot Position with ka = 2.2.

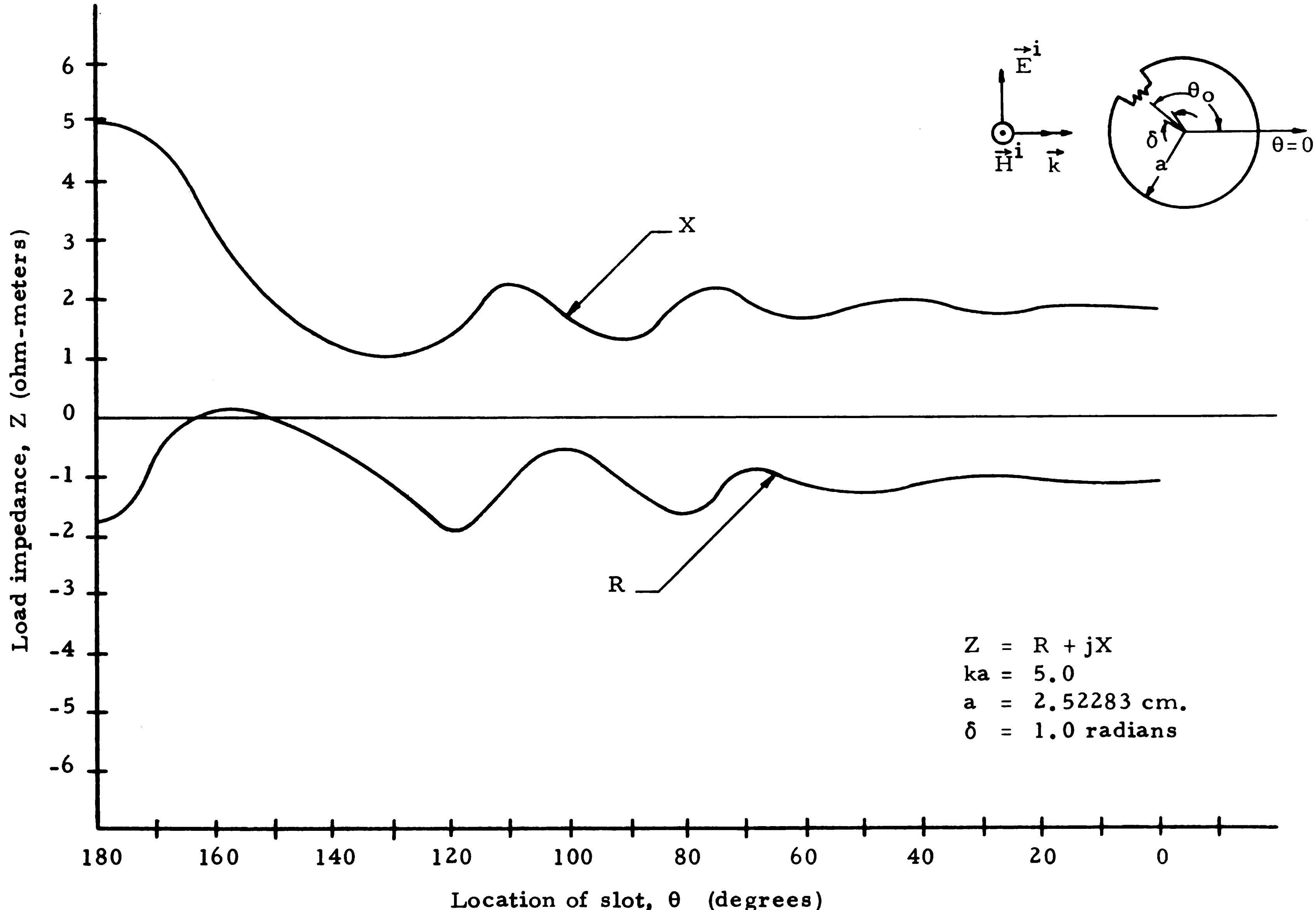


Figure 2.5: Optimum Slot Impedance for Zero Backscattering as a Function of Slot Position with ka = 5.0.

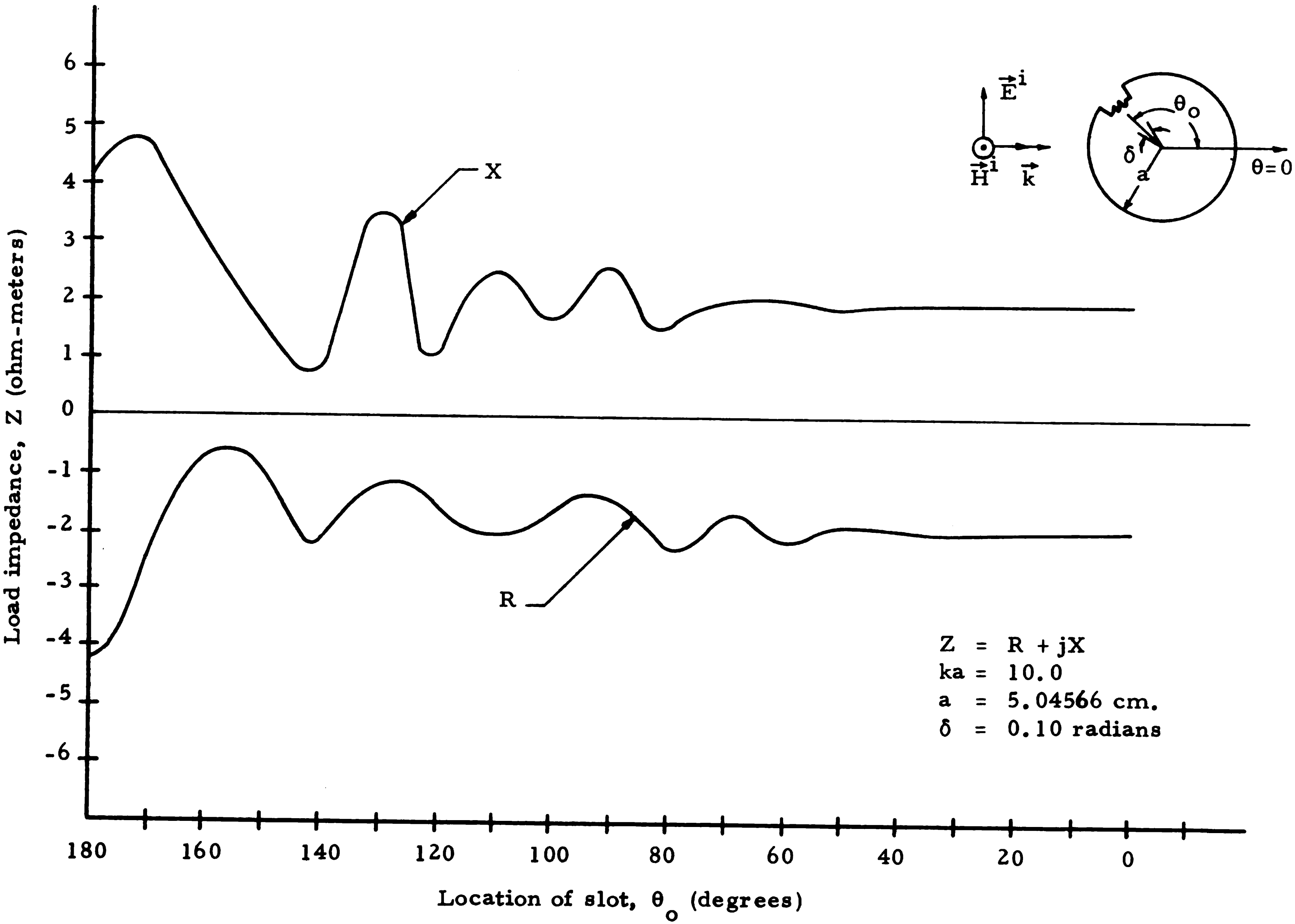


Figure 2.6: Optimum Impedance for Zero Backscattering as a Function of Slot Position with ka = 10.0

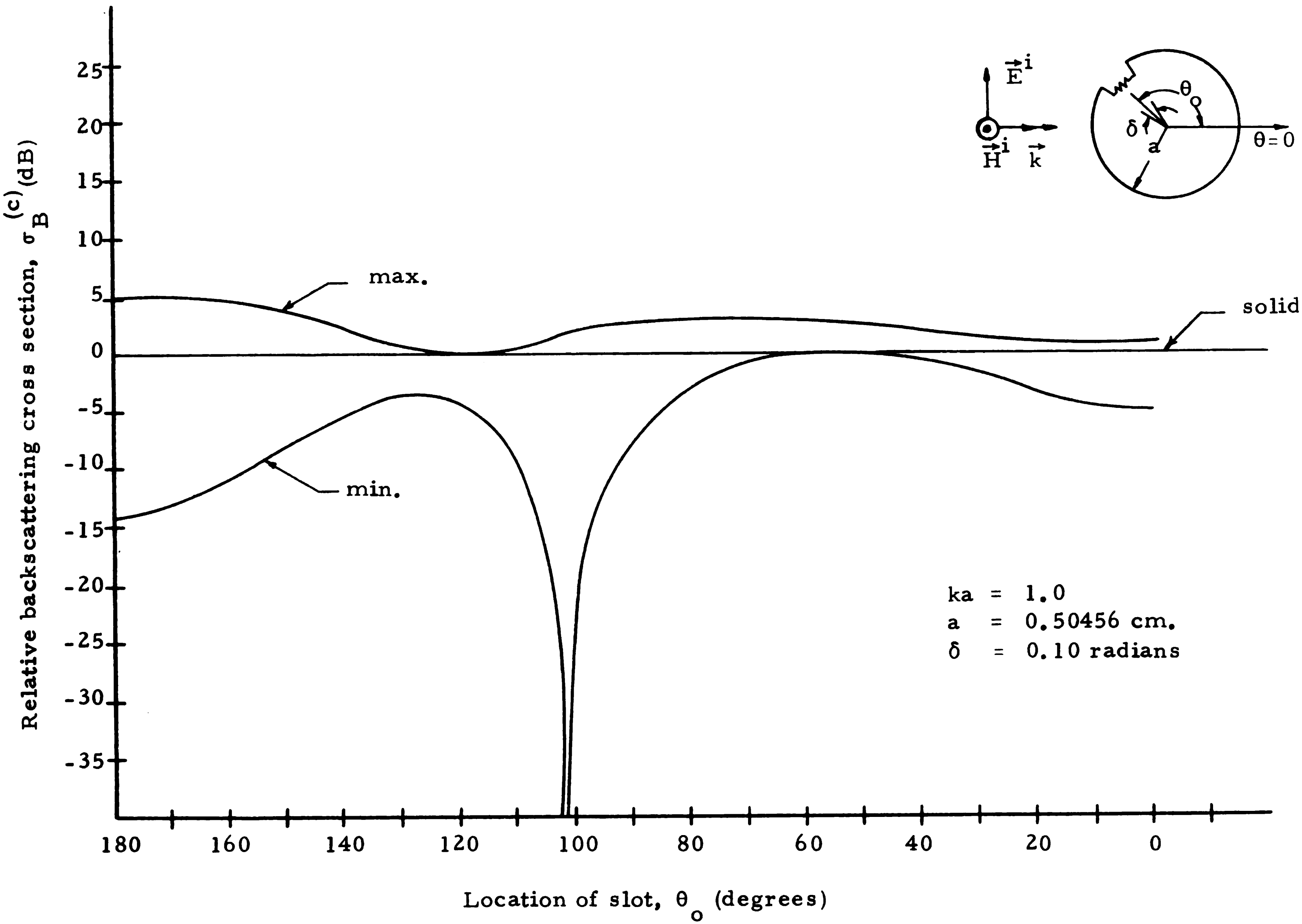


Figure 2.7: Optimum Minimum and Maximum Backscattering Cross Section as a Function of Slot Position for ka = 1.0.

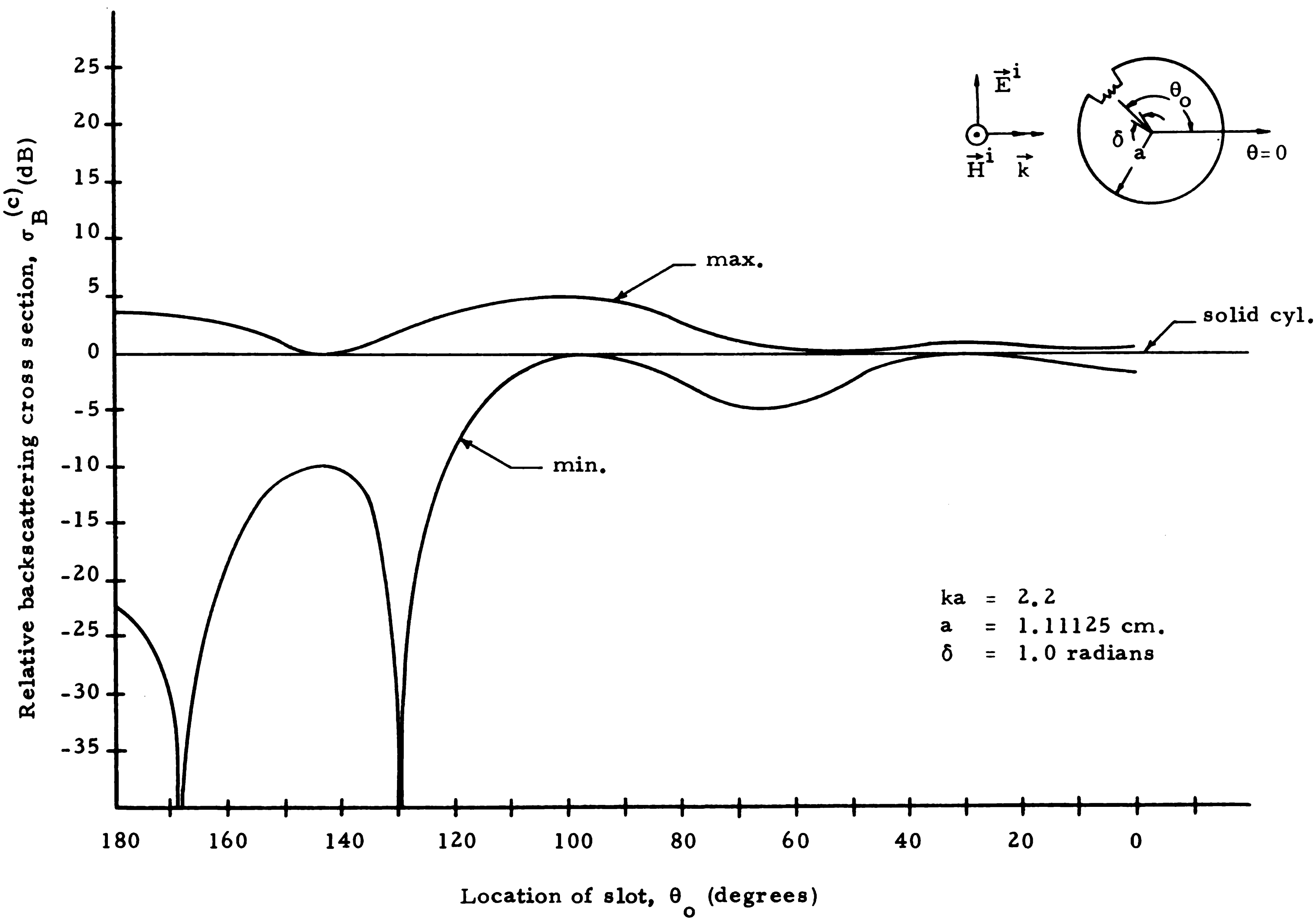


Figure 2.8: Optimum Minimum and Maximum Backscattering Cross Section as a Function of Slot Position for ka = 2.2.

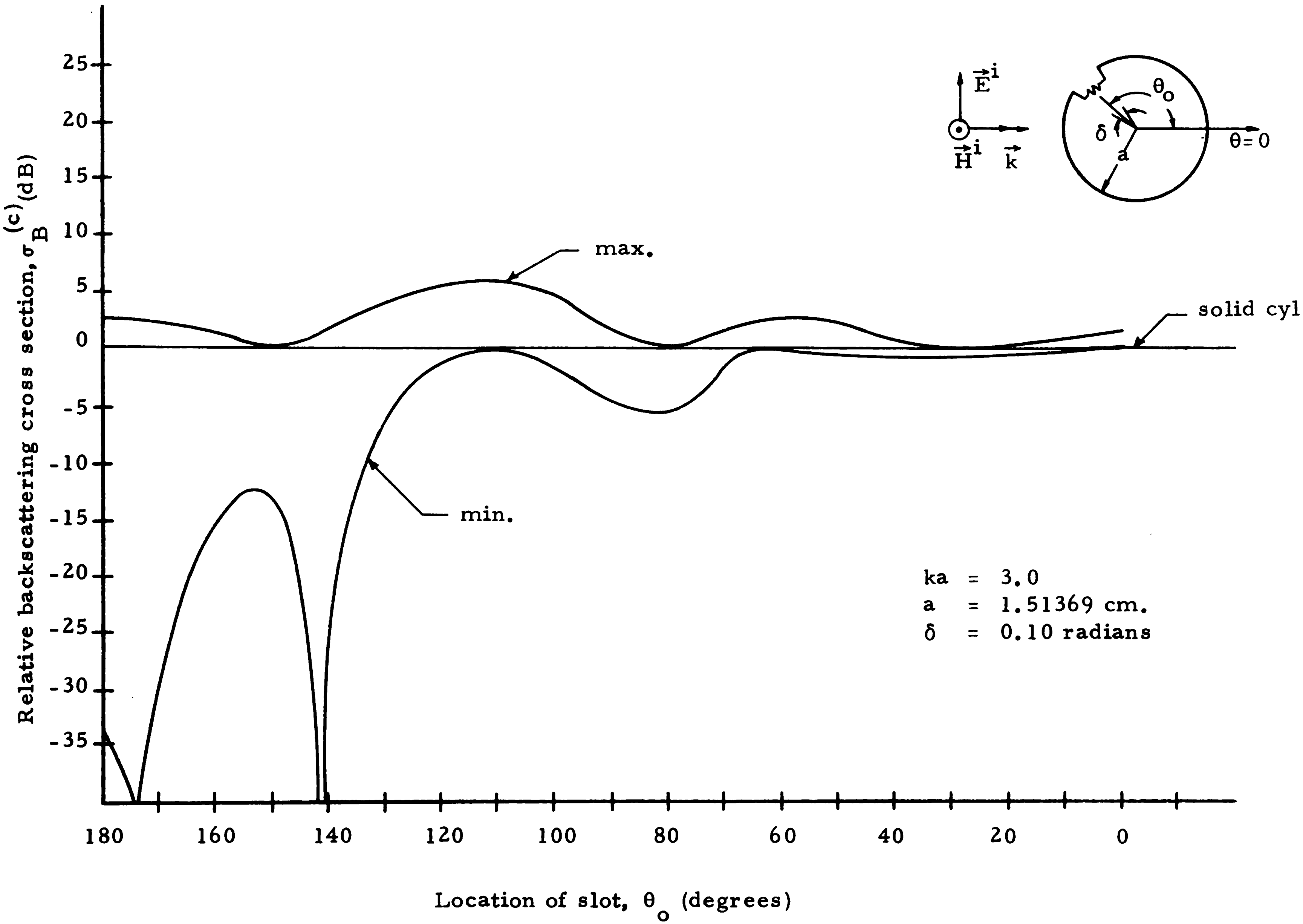


Figure 2.9: Optimum Minimum and Maximum Backscattering Cross Section as a Function of Slot Position for ka = 3.0.

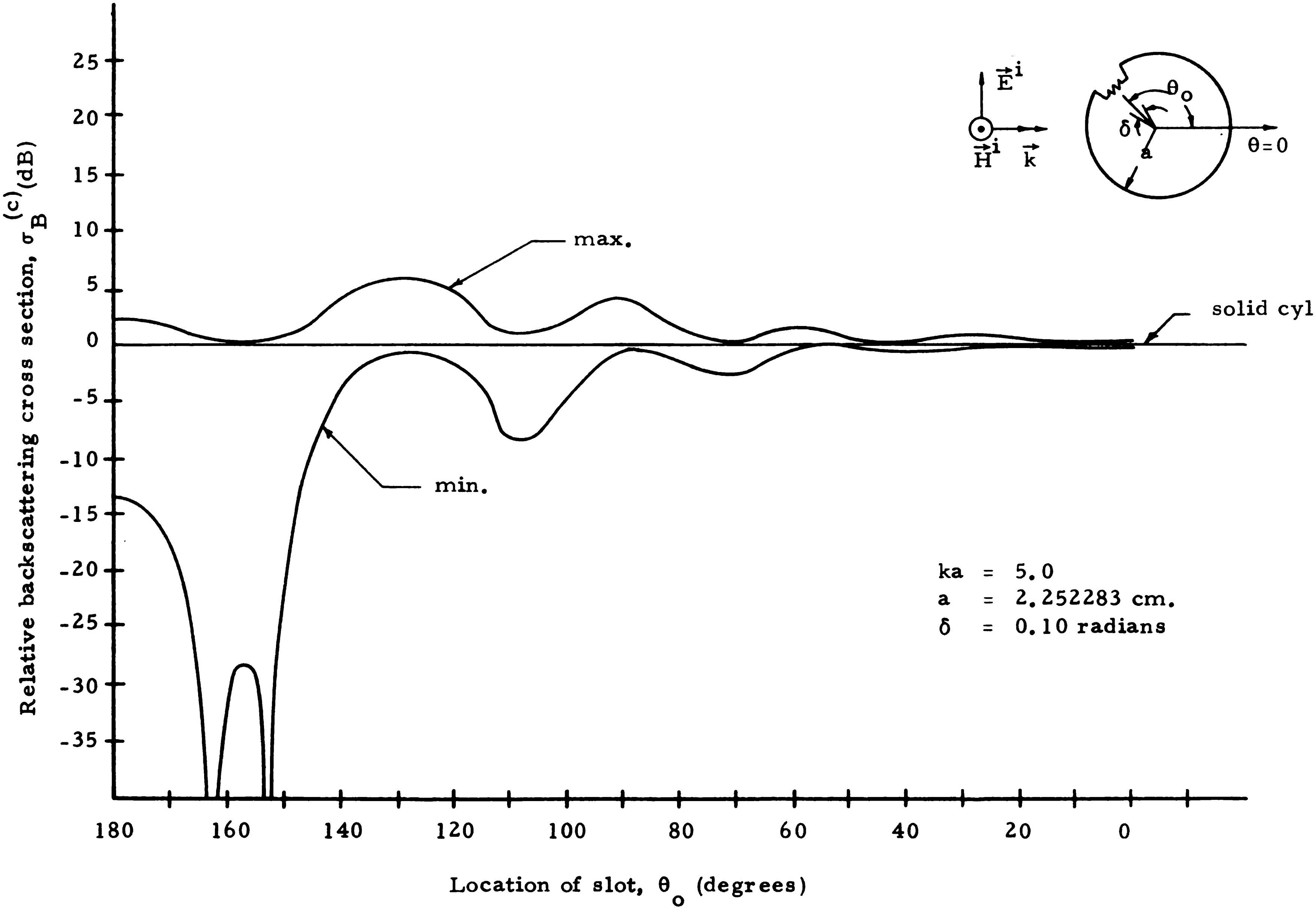


Figure 2.10: Optimum Minimum and Maximum Backscattering Cross Section as a Function of Slot Position for ka = 5.0.

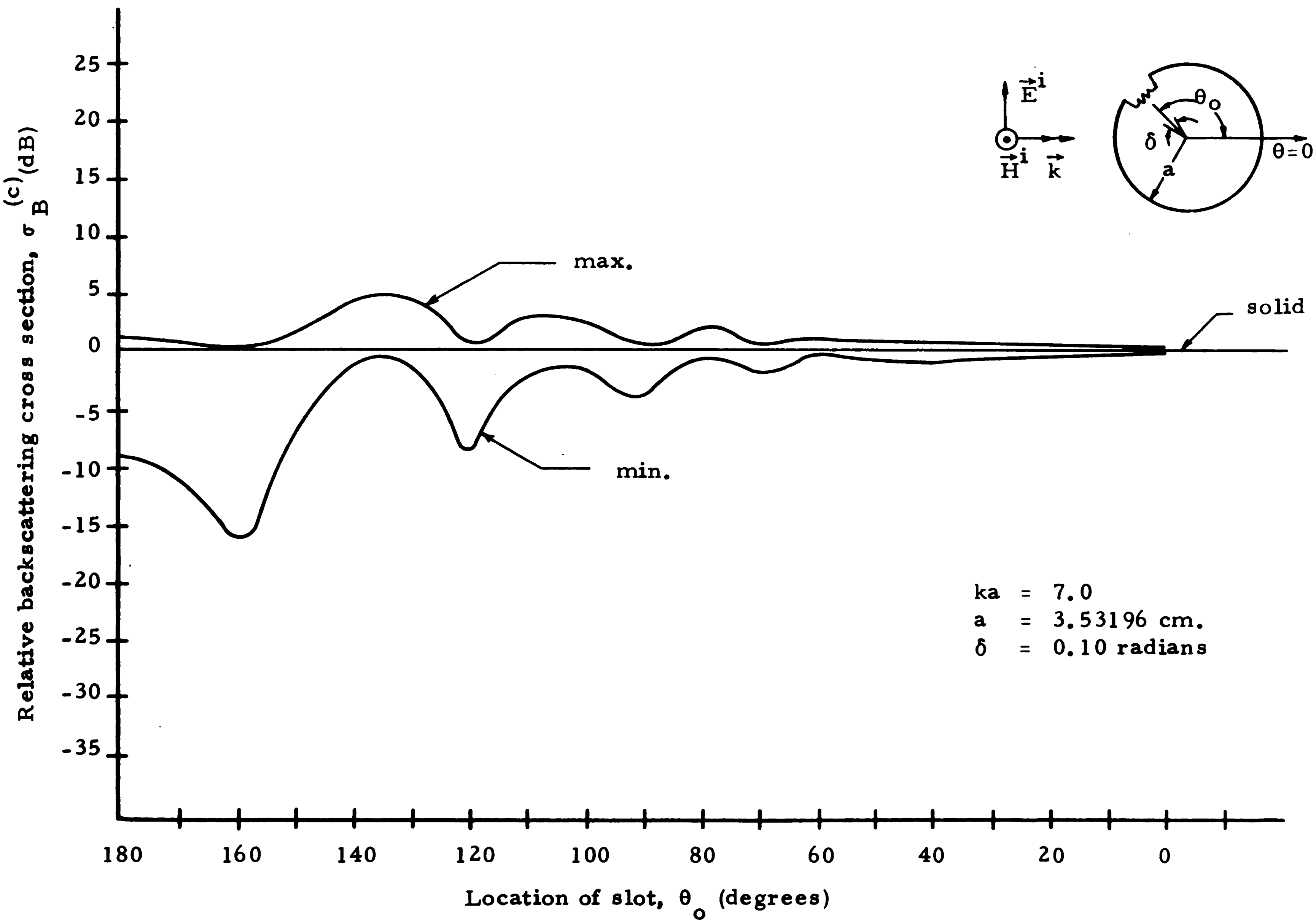


Figure 2.11: Optimum Minimum and Maximum Backscattering Cross Section as a Function of Slot Position for ka = 7.0.

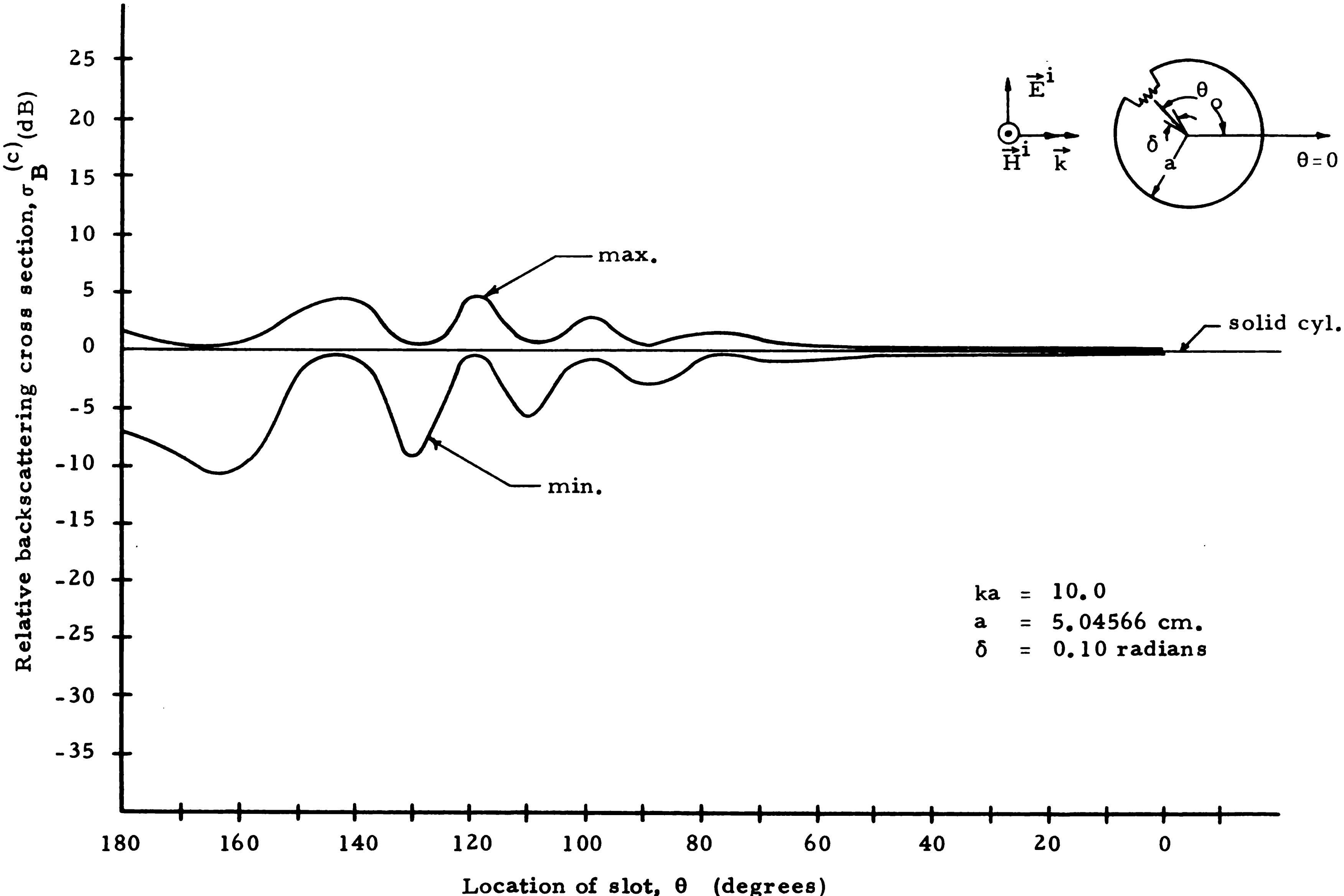


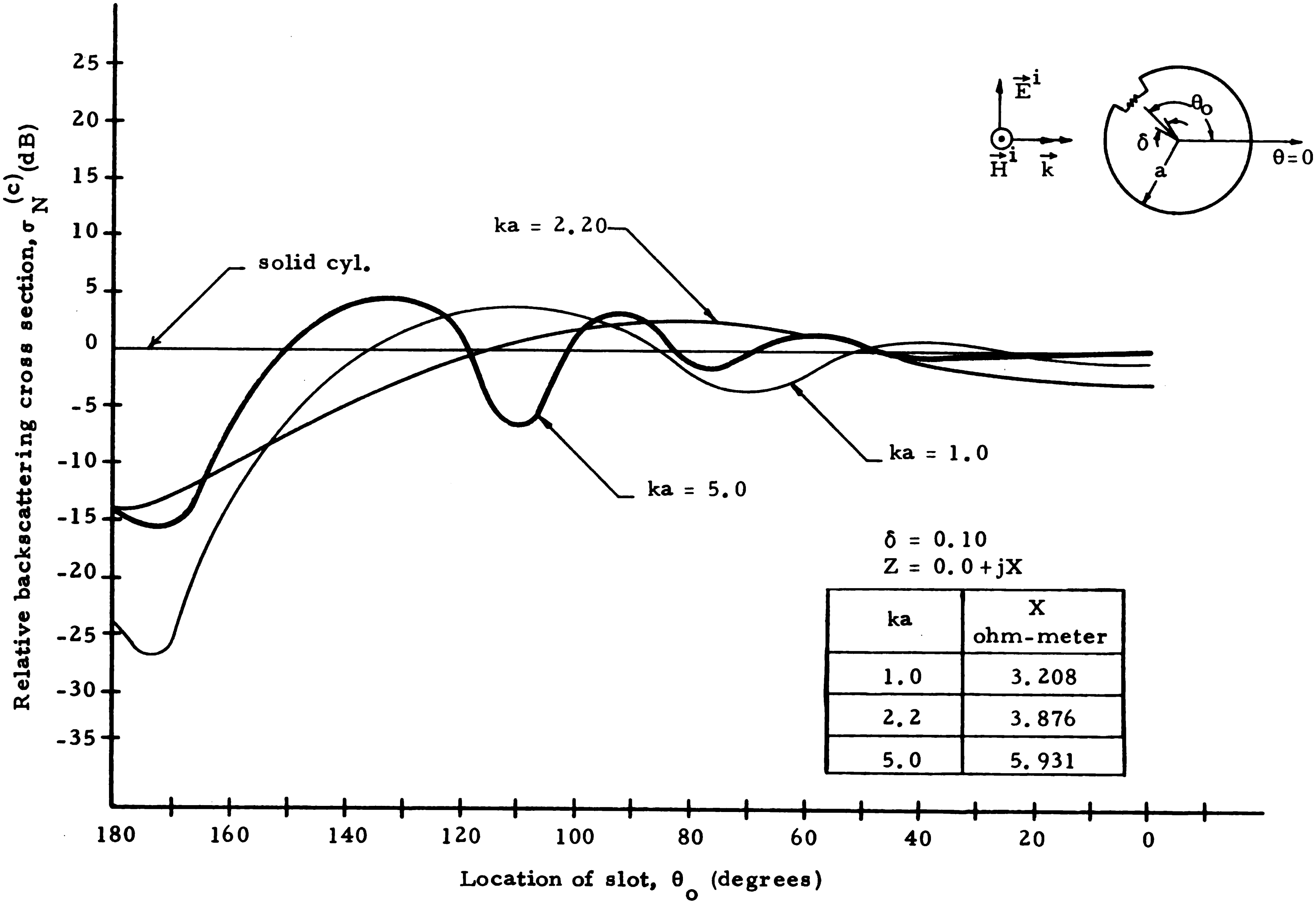
Figure 2.12: Optimum Minimum and Maximum Backscattering Cross Section as a Function of Slot Position for ka = 10.0.

Reactance, in Ohm-meters						
θ_o \ ka	1.0	2.2	3.0	5.0	7.0	10.0
180^o	3.208	3.876	4.023	5.931	7.736	10.663
170^o	3.145	3.655	3.705	4.898	5.512	6.390
160^o	2.970	3.125	2.966	3.195	3.148	2.990
150^o	2.715	2.475	2.148	2.019	1.860	1.384
140^o	2.393	1.837	1.621	1.348	0.832	-1.445
130^o	1.965	1.483	1.269	0.527	-2.732	3.978
120^o	1.515	1.255	0.830	-2.894	3.608	0.446
110^o	1.354	0.982	-0.069	3.874	1.221	5.028
100^o	1.276	0.461	-7.303	1.676	-8.243	0.118
90^o	1.204	-1.641	3.744	0.362	2.415	3.219
80^o	1.109	6.047	2.064	9.364	-0.195	-2.912
70^o	0.948	2.690	1.363	2.156	3.750	1.890
60^o	0.576	2.015	0.428	0.937	1.115	9.089
50^o	-1.458	1.674	-4.195	-129.927	6.220	0.459
40^o	4.045	1.328	4.729	2.244	2.000	3.101
30^o	2.412	-0.112	2.395	1.256	16.819	-2.363
20^o	2.078	3.280	1.037	-0.597	2.887	1.247
10^o	1.954	2.392	-1.765	1.821	-60.472	-9.235
0^o	1.920	2.265	-5.254	1.395	3.404	0.262

Table 2.1 Optimum Reactance for Minimum Backscattering Cross Section in Terms of Slot Position θ_o and Electrical Cylinder Radii ka.

Reactance, in Ohm-meters						
θ_o \ ka	1.0	2.2	3.0	5.0	7.0	10.0
180^o	1.308	1.379	1.364	1.487	1.516	1.552
170^o	1.304	1.353	1.311	1.363	1.289	1.122
160^o	1.289	1.260	1.081	0.745	0.018	-1.859
150^o	1.260	0.989	-0.060	-6.989	31.340	8.268
140^o	1.202	-0.844	11.292	4.804	4.037	3.109
130^o	1.016	5.615	3.514	2.973	2.437	0.062
120^o	-1.284	3.105	2.603	2.164	0.547	4.471
110^o	4.230	2.467	2.146	1.116	5.175	0.731
100^o	2.843	2.084	1.790	10.329	2.167	4.038
90^o	2.401	1.784	1.318	2.849	-2.936	-1.138
80^o	2.118	1.496	-0.473	1.653	3.027	2.776
70^o	1.890	1.115	4.014	-3.016	0.656	38.712
60^o	1.690	0.258	2.319	3.478	4.757	1.454
50^o	1.510	-8.045	1.836	1.862	1.396	4.492
40^o	1.348	3.479	1.443	-1.933	-36.079	-1.473
30^o	1.207	1.920	0.569	4.349	1.783	2.875
20^o	1.089	1.293	2.883	2.471	-0.510	7.121
10^o	1.008	0.921	1.929	50.761	1.976	2.336
0^o	0.977	0.784	1.815	5.110	0.354	4.208

Table 2.2 Optimum Reactance for Maximum Backscattering Cross Section in Terms of Slot Position θ_o and Electrical Cylinder Radii ka.



$\delta = 0.10$
$Z = 0.0 + jX$

ka	X ohm-meter
1.0	3.208
2.2	3.876
5.0	5.931

Figure 2.13: Relative Backscattering Cross Section as a Function of Slot Position for Various Values of ka with Fixed Purely Reactive Loading.

III

EXPERIMENTAL STUDY OF BACKSCATTERING BY A SLOTTED CYLINDER

3.1 Experimental Arrangement and Measurement Technique

A theoretical expression for the backscattering cross section of an infinitely long cylinder with an impedance backed longitudinal slot was obtained in the previous section. This result expressed the backscattering cross section of the slotted cylinder as a function of its radius, the frequency of the illuminating plane wave, and the impedance and position of the slot. The optimum slot impedance for zero backscattering was also found. It was discovered from this expression that the resistive part of the optimum impedance is negative for most slot orientations. An expression for the reactance of an optimum lossless loading to yield maximum and minimum backscattering was also derived.

To confirm these theoretical predictions on the backscattering behavior of the slotted cylinder, a series of experimental measurements was performed. Since realizing the necessary slot impedance for zero backscattering (containing a negative resistive part) physically would be extremely difficult, if not impossible, only a purely reactive load backing the slot was examined in this study.

The experimental model of the metallic scatterer consists of a cylindrical brass tube, 7/8 inch outside diamter and 36 inches long, with a 1/8 inch wide longitudinal slot cut on its surface (see Figure 3.1). The slot backing impedance is implemented by installing a parallel plane waveguide structure at the inner wall of the slotted cylinder. One end of the guide opens at the slot on the cylinder while the other end is short-circuited. The short location is adjustable such that the length of the guide can be varied, which in turn varies the slot impedance. The impedance of this structure may be approximated by that of a plane waveguide.[15]

Several methods are available for measuring the backscattering cross section of a metallic object. In this research, the source separation method is used. The principle of this method is to design the receiving system in such a manner that it does not respond to the incident or source field. A single antenna can then be used to radiate the illuminating E-M wave and subsequently receive the scattered wave.[16]

The X-band experimental arrangement which was used in this research is indicated in Figure 3.2.a. The experiment was conducted in an anechoic chamber (dimensions of 0.8m x 1.4m x 0.7m). A standard gain horn antenna (HP X890A) is projected into the chamber through one of its ends. The cylinder to be studied is mounted between the sides of the chamber and at

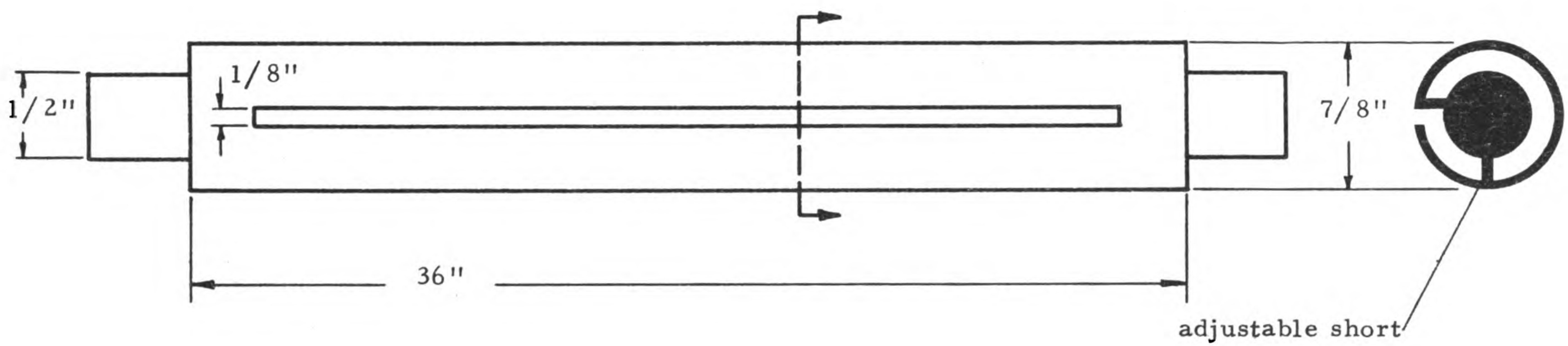
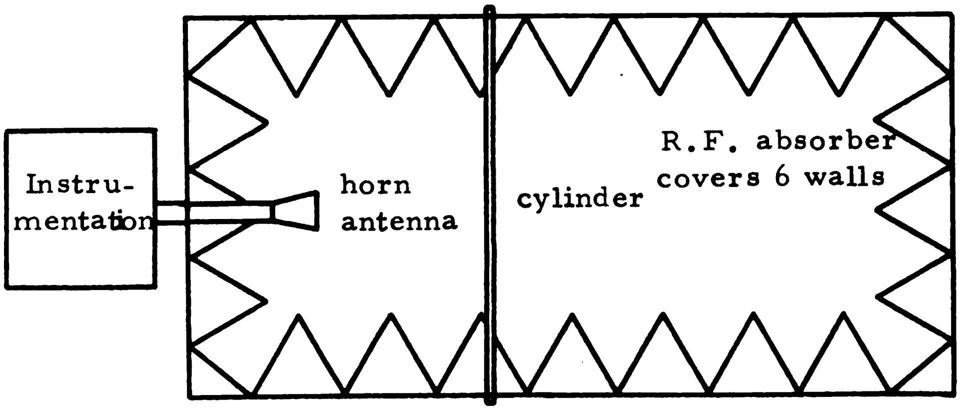
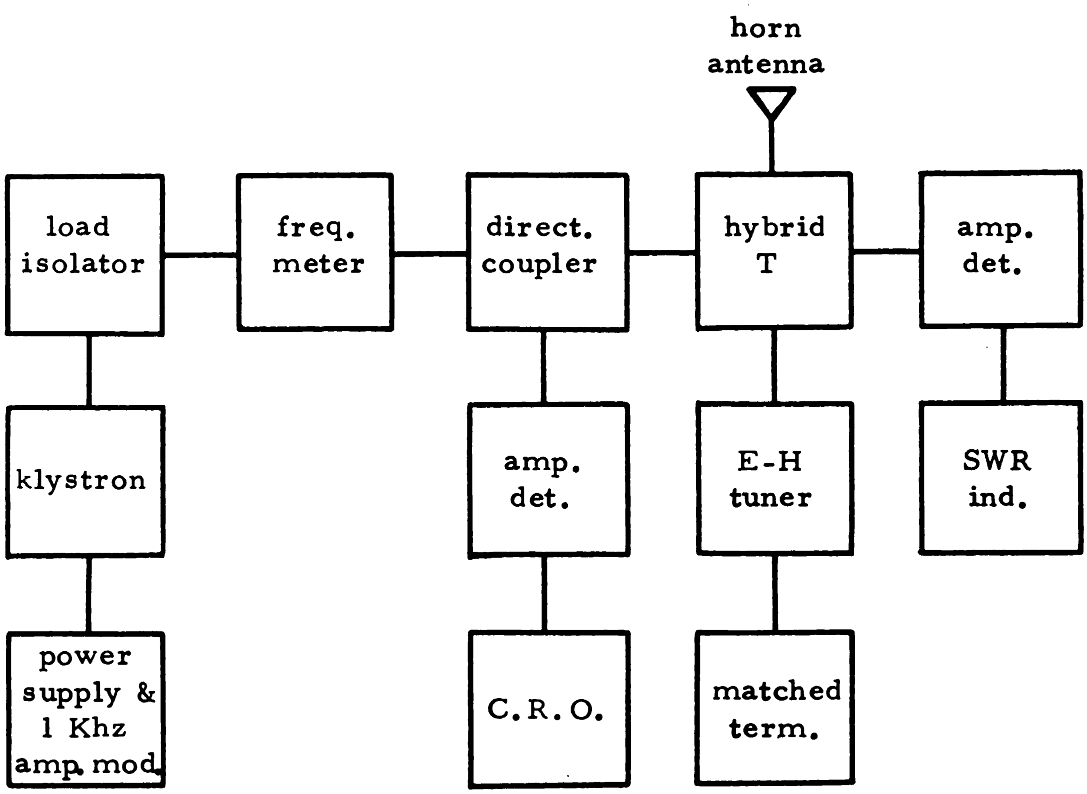


Figure 3.1: Experimental Model of Slotted Cylinder (Note not drawn to scale)



a) Anechoic Chamber



b) Block diagram of instrumentation

Figure 3.2: Experimental Arrangement.

a distance of 28 cm from the horn antenna. A block diagram of the instrumentation is shown in Fig. 3.1.b. The reflex klystron generator (FXR type X760A), modulated by a 1 Khz square wave generator (HP 715A power supply with internal modulation) is used as a RF source. The klystron is protected from reflected energy by the load isolator (Polytechnic Res. and Dev. Co. type 1203). The directional coupler (HP X752C) and associated detector (HP X485B) are used in conjection with a CRO to monitor the klystron output. Frequency is measured by the frequency meter (HP X532A). A four port hybrid junction (HP X845A) is used to separate the source signal driving the antenna and the scattered field signal received by the antenna. The two remaining ports of the junction are coupled to a matched load (HP X910A) through an E-H tuner (HP X880A), and to an amplitude detector (HP X485B). The detector output is then measured by a SWR indicator (HP 415B).

An E-M wave with a vertically polarized $\vec{E}$ field vector normally incident upon the cylinder can be implemented with the arrangement described above. When the scatterer is absent, the receiving system can be nulled by adjusting the E-H tuner. When the scatter is introduced, the reading on the SWR meter will then indicate the relative backscattering cross section. The scattering from a solid cylinder having the same radius as its slotted counterpart is used as reference.

It was found that this system was able to detect scattered fields of the order of 25 db. below the backscattering cross section of the solid cylinder while maintaining its stability for several hours.

The horn antenna does not illuminate the cylinder by a plane wave. The amplitude of the incident wave is greatest near the center region of the cylinder, and decreases with displacement in either direction parallel to its axis. The phase of the incident wave will also vary along the axial direction of the cylinder. It was found that by placing the cylinder about ten wavelengths in front of the horn antenna the consequences of the non uniform illumination and the end effects (arising from the finite length of the scattering model) were very small, while the detection system provided the desired sensitivity.

3.2 Experimental Results and Comparison to Theory

For comparison with the experimental data, the theoretical backscattering cross section for various slot impedances was computed from equation (2.54) with $ka = 2.20$ and $\delta = 0.2857$ radians.

The theoretical results discussed above and the corresponding experimental results are indicated in Figures 3.3 - 3.6, in which the backscattering cross section $\sigma_B^{(c)}$ of the slotted cylinder is plotted as a function of slot position θ_o. The backscattering cross section is given in db. and is normalized to that of a solid cylinder of the same radius. Figure 3.3 represents the maximum and minimum backscattering that can be obtained for a given slot position θ_o. The agreement between theory and experiment is excellent, and it is found that the backscattering cross section of the cylinder can be reduced by 25 db (i.e., to the noise level of the detection system) over an excursion in slot positions from $\theta_o = 180^o$ of nearly twenty degrees. Figures 3.4 - 3.6 represent the backscattering cross section $\sigma_B^{(c)}$ as a function of slot position θ_o when the slot impedance Z remains fixed. An appropriate slot impedance is chosen so the backscattering will be minimized for values of θ_o equal to 180^o, 170^o, and 150^o in Figures 3.4, 3.5, and 3.6, respectively. The agreement between theory and experiment is again excellent.

It can be concluded that the theory developed in Section II to precict the backscattering behavior of a slotted cylinder gives valid results.

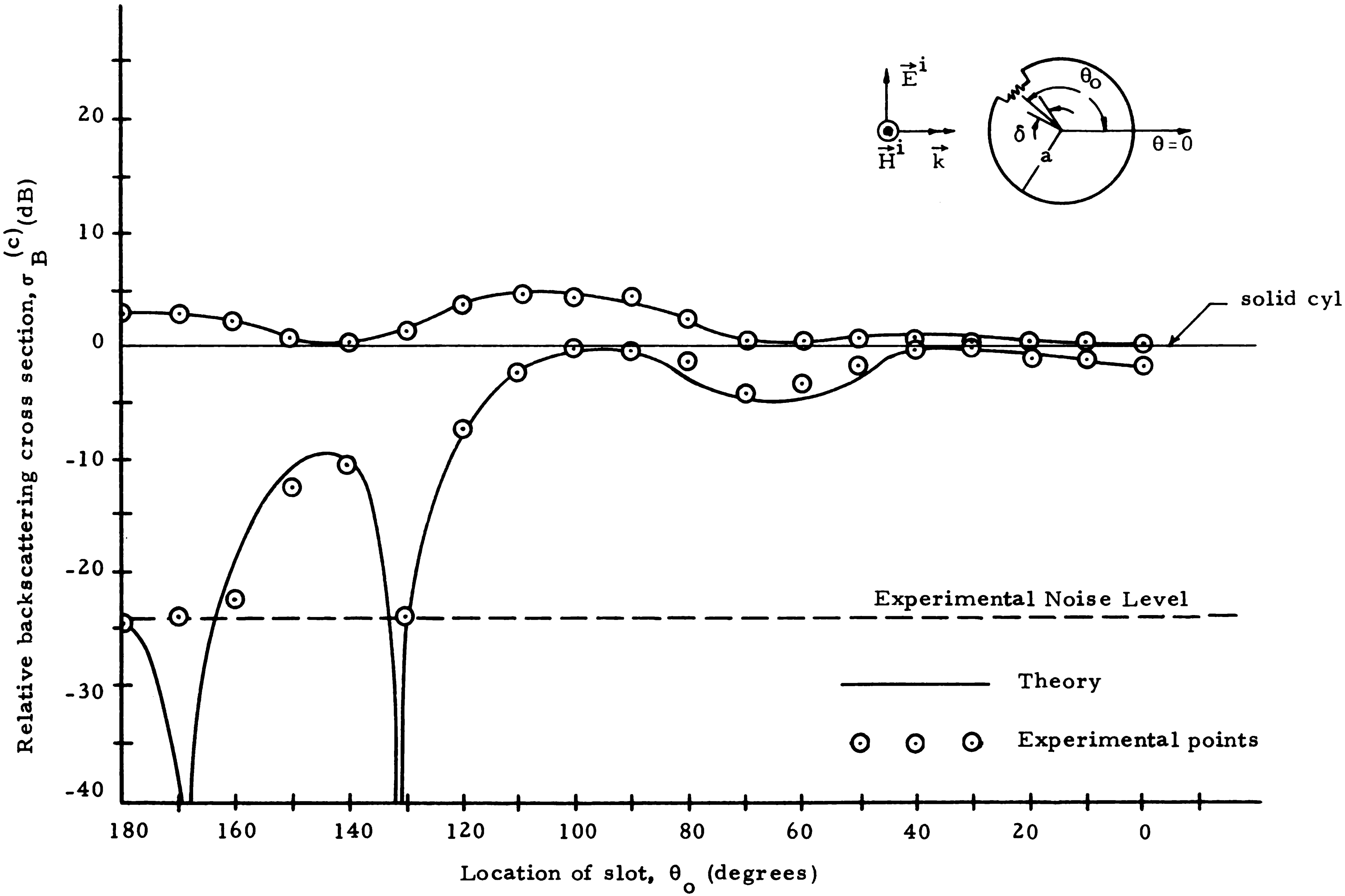


Figure 3.3: Optimum Minimum and Maximum Backscattering Cross Section as a Function of Slot Position.

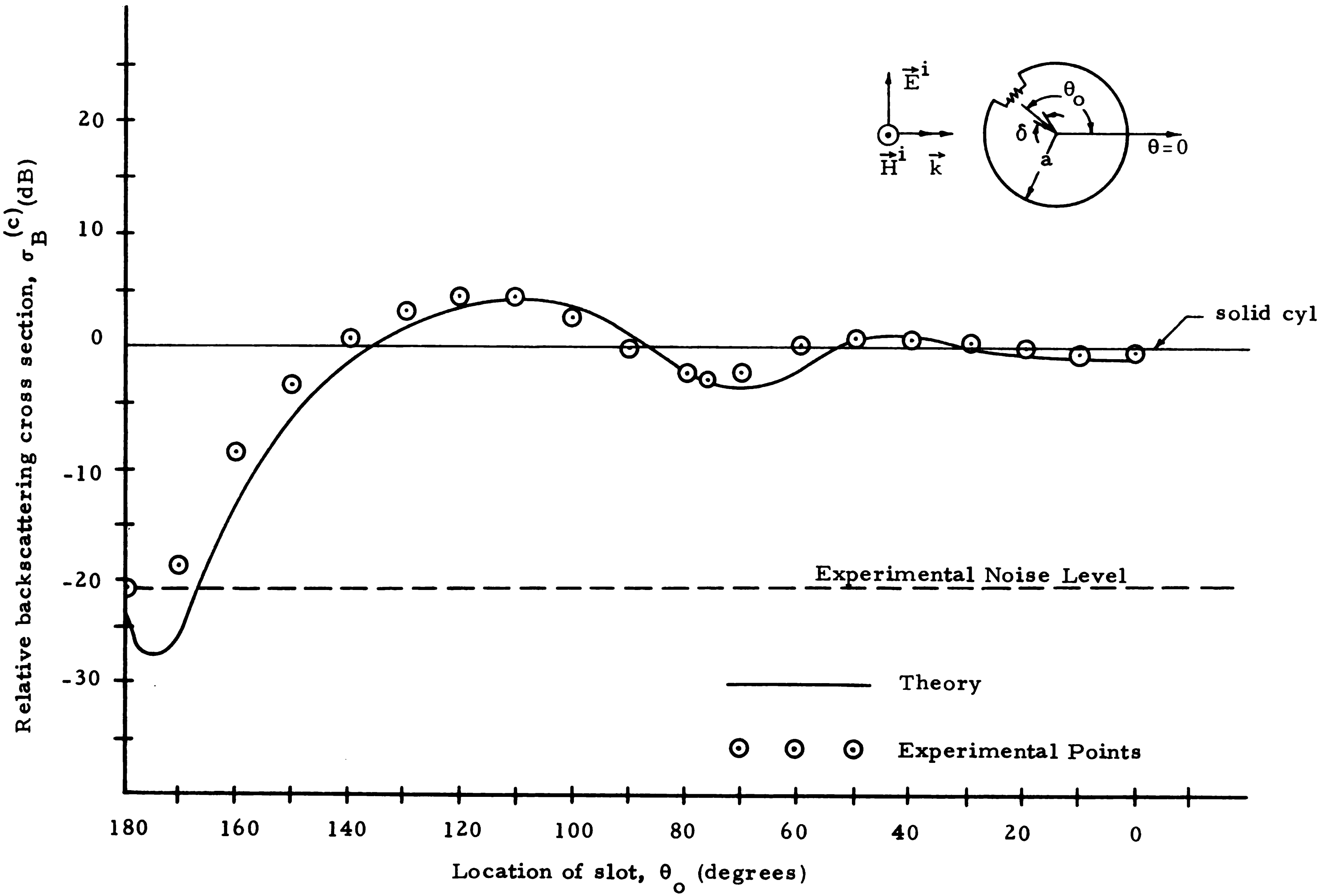


Figure 3.4: Relative Backscattering Cross Section as a Function of Slot Position, with Constant Impedance (Z = j 7.9 ohm-meters)

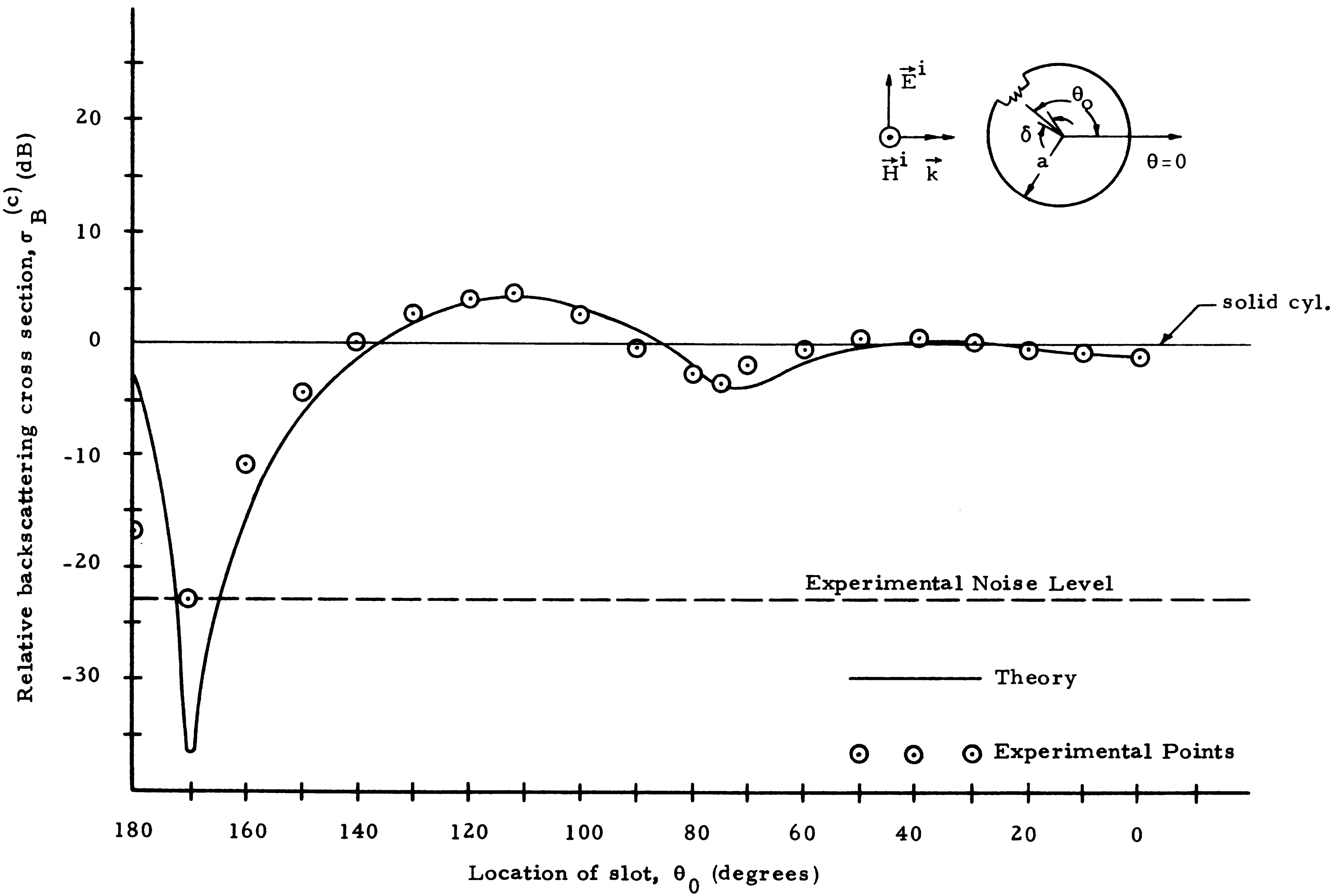


Figure 3.5: Relative Backscattering Cross Section as a Function of Slot Position with Constant Impedance ($Z = j\,7.0$ ohm-meters)

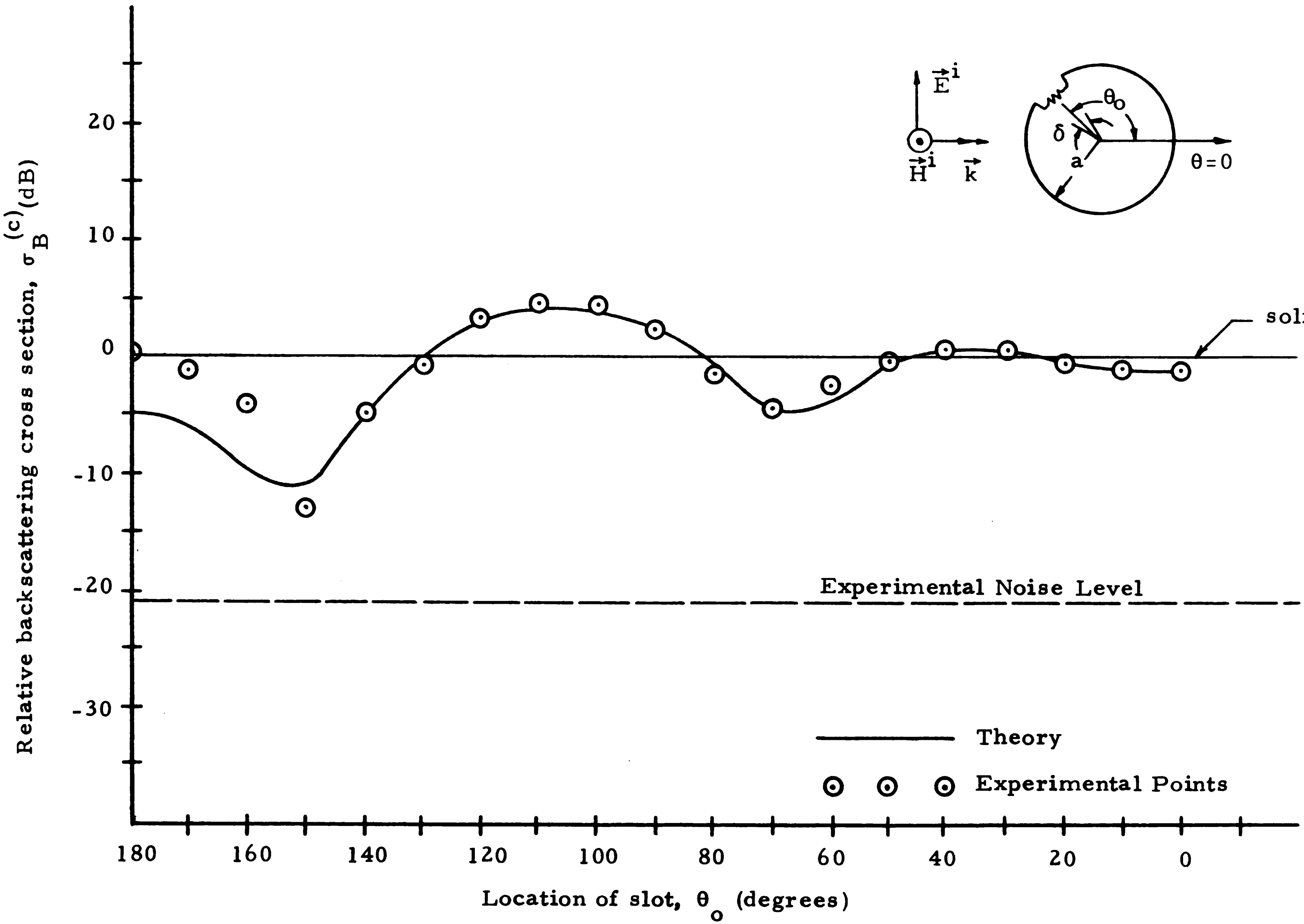


Figure 3.6: Relative Backscattering Cross Section as a Function of Slot Position, with Constant Impedance (Z = j 3.7 ohm-meters).

IV

CONCLUSION

In the preceding sections, the behavior of a metallic cylinder loaded by an impedance backed longitudinal slot was considered. Theoretically, the scattered field from an infinite cylinder loaded with an impedance-backed slot has been derived exactly. The optimum impedance which leads to zero backscattering and the optimum reactance which leads to minimum backscattering have been calculated. An experimental investigation has been conducted to verify the theory and an excellent agreement between theory and experiment was obtained.

The loading impedance required for some particular modification of the scattered fields is in general complex. The impedance required for zero backscattering generally has a negative resistive part, which requires an active loading impedance backing the slot. A purely reactive loading can also be very effective in modifying the scattering behavior, but its effectiveness is limited primarily to the case where the slot is located in the illuminated region of the cylinder. The ability to modify the scattering using a purely reactive loading also decreases as the electrical size of the cylinder increases. This suggests that it may be advantageous to implement two or more loaded longitudinal slots on the surface of an electrically thick cylinder to control the backscattering.

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