SOME PROBLEMS IN THE DESIGN AND OPERATION OF ELECTRICAL MACHINERY AND THEIR SOLUTION

Thesis for the Degree of E. E.

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THESIS

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SOME PROBLEMS IN THE DESIGN AND OPERATION OF ELECTRICAL MACHINERY AND THEIR SOLUTION

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THESIS

Part I

DERIVATION OF A GENERAL "EMF-FLUX" EQUATION FOR ELECTRICAL MACHINES

In part one of this thesis I wish to show how a very general form of the equation, showing the relation between the magnetic flux in maxwells and the generated volts in electrical machines can be derived for use in practical design work; and also to show how, by simple and logical changes, the general equation can be adapted to several different types of electrical machines.

All the usual types of rotating electrical machines have alternate north and south magnetic poles distributed evenly around their air gaps. The width of the field of influence of all the poles is usually equal, and the average magnetic strength of the north poles is equal to the average magnetic strength of the south poles but, of course, opposite in polarity. The distance between two successive magnetic poles is called the pole pitch. This pole pitch can be expressed in electrical degrees, the distance between the center of one magnetic pole and the center of the next succeeding magnetic pole being 180 electrical degrees, and the distance between any point on a magnetic pole of given polarity and a corresponding point on the next succeeding magnetic pole of the same polarity being equal to 360 electrical degrees. (On a 2 pole machine a mechanical degree equals one electrical degree; on a 4 pole machine 1 mechanical degree equals 2 electrical degrees; on a 6 pole machine 1 mechanical degree equals 3 electrical degrees, etc.)

The pole pitch may also be measured as a circumferential distance in inches if it is indicated what circumference is intended; that is to say, whether on the surface of the rotor, the inside surface of the stationary member or stator or the mean circumference in the air gap. Inches circumferential distance will usually be used here except when the term pole pitch is used in a general way.

The useful flux per pole of an electrical machine is that magnetic flux which leaving that member of an electrical machine where the magneto-motive force exists, would pass entirely within a one turn full pitch coil directly on the surface of the other member of the machine. A full pitch coil, for any cylindrical rotor type of machine, is a coil whose parallel sides lie just one pole pitch apart.

Now to derive the general "voltage-flux" equation first consider a single turn full pitch coil rotated at a uniform rate in the magnetic field of a machine as shown in diagram 1. (The diagram is a general representation of a two pole machine and a similar diagram could be made for any even number of poles.)

See blue print, next page.

The space between the two large circles in diagram 1 represents the air gap of the machine. The two small circles a-1 and b-1 represent the two sides of a one turn full pitch coil. The magnetic field is represented vertically in the iron parts of the machine and radially in the air gap. In the position of the

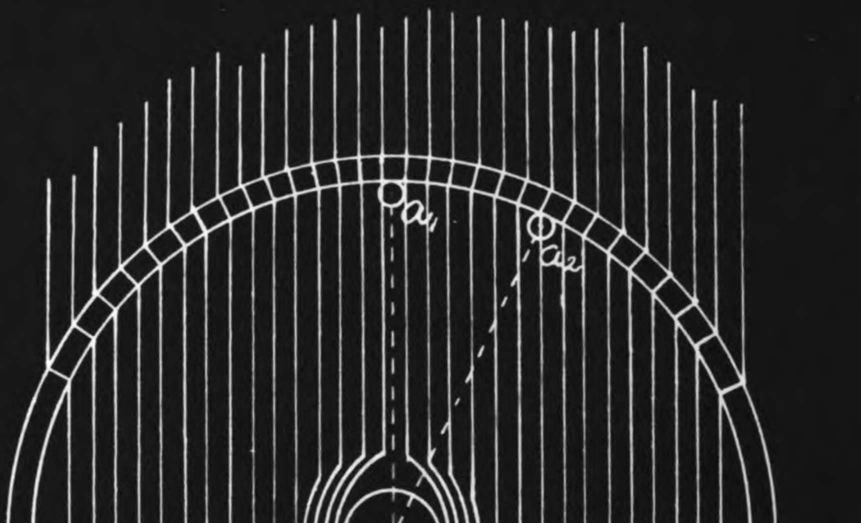
a1
a2
b2
b1
DIAGRAMATIC
RERRESENTATION OF
THE MAGNETIC LINES
OF FORCE

DIAGRAM 1.

coil shown at a-1, b-1, the plane of the coil is in the axis of the magnetic field that is parallel to the direction of the field. It is plain that for this position, namely with the sides of the one turn full pitch coil at the centers of the magnetic poles, there is zero magnetic flux passing through the coil.

Now if either the magnetic field or the coil are rotated so that the relative position of the coil with respect to the field is as shown at a-2 and b-2 there will be magnetic flux passing through the coil, that is the amount of magnetic flux within the one turn full pitch coil has changed. Therefore a voltage has been generated in the coil. The direction of the voltage, of course, is such that if a current were allowed to flow in response to this voltage, the magnetomotive force of this current would tend to oppose the change of flux taking place. Now if the flux density in the air gap is uniform between the positions a-1, b-1 and a-2, b-2 it is evident that the total change of flux within the coil will be proportional to the amount of movement between these positions. Hence, if the rotation is uniform the rate of change, and therefore the voltage generated, will be proportional to the density of the magnetic field. That is to say for uniform rotation the voltage generated is proportional to the density of the magnetic field through which the sides of the coil are passing. This is so not because the sides of the coil are "cutting lines of magnetic force", but because the rate at which the magnetic flux within the coil is changing is proportional to the density of the magnetic field through which the sides of the coil are passing.

Electrical engineers speak of "field forms" by which is meant the way the magnetic field of an electrical machine varies across the circumferential space occupied by a magnetic pole, or a curve plotted to show the variation of the magnetic density over this space occupied by a magnetic pole. The density of the magnetic field is never constant over a whole pole pitch in rotating electrical machines, that is to say the "field form" is never rectangular, Neither is the field ever a perfect triangle or a perfect trapezoid.

In some specially constructed synchronous machines the no load field form may have the shape of the sine curve. In polyphase induction machines the average field form set up by the currents in the primary windings during a cycle comes so close to a "sine field" that no error great enough to effect the practical design of induction machines is made by assuming that all the constants of a "sine field form" apply to polyphase induction machines. In the usual direct current machine or salient pole synchronous machine the density of the no load field may be constant or nearly so over the central part of a pole piece but it decreases, first slowly then more rapidly as we pass from the center of the pole piece towards the edges or "tips" of the pole piece, and the magnetic density drops off rapidly beyond the pole tips and passes through zero to the reverse polarity midway between the main poles.

However, regardless of the "field form" it is still true that the voltage generated in the coil in diagram 1, by uniform rotation, is at every instant proportional to the magnetic density of the field through which the sides of the coil are passing at that instant. The logic of this statement will be apparent if a smaller and smaller movement is considered at a time until an infinitely small element of the motion is considered; for the magnetic density of any field form can be considered as constant for an infinitely small element of the rotation, which elementary portion of the rotation will take place in infinitely short time so that the rate of change of flux through the coil and the voltage which it generates in the coil are at each instant proportional to the density of the magnetic field through which the sides of the coil are passing at the given instant.

From the above reasoning it is evident that with any single full pitch coil in a rotating electrical machine the voltage wave will have the same shape as the shape of the field form. (More is to be said about field form later.)

Then if the field form is sine shaped the voltage wave generated by a relative rotation of such a field and a full pitch coil will be a sine wave. And since the resultant of a number of equally spaced sine curves is also a sine curve, it follows that the resultant voltage wave generated by a group of equally spaced full pitch coils will also be a sine voltage wave.

However, most electrical machines have coils which are less than full pitch, that is, the two parallel sides of the coil lie a little less than one pole pitch apart. It will be found that the voltage generated at any instant in such a coil is proportional to the sum of the two magnetic densities through which the two sides of the coil are passing at the given instant. The easiest way to understand that this is so is to imagine the coil as consisting of two coils each having one imaginary side of infinitely small dimensions passing back through the center of the shaft or axis of rotation. In this case these two imaginary halves of the coil are out of the magnetic field and it is evident that the voltage generated in each half of the coil by uniform rotation will be proportional to the density of the magnetic field through which the outer or real sides of the coil are passing, at each instant, and if the two imaginary halves of the coil were properly connected in series the total voltage at each instant would be proportional to the sum of the magnetic densities through which the two real sides of the coil were passing at the given instant.

To carry this reasoning still farther it is evident that the two voltages generated in the two imaginary halves of the "short pitched" coil will be out of phase by the amount by which the short pitched coil falls short of being a full pitch coil. So that the voltage generated in a coil which is less than full pitch can be considered as the vector sum of two voltage waves which are out of phase by the same amount which the coil falls short of being a full pitch coil. The amplitudes of these two voltage waves generated by the two sides of the coil are, of course, one half the amplitude of the voltage wave which would be generated by the complete turns of a full pitch coil having the same number of turns.

Since a full pitch coil could also be separated into two imaginary halves in the same manner as the above mentioned coil, which was less than full pitch, it is evident that the voltage of a full pitch coil can be considered as the sum of the two equal voltage waves generated by the two sides of the coil.

These considerations show why it is sometimes more convenient for a designer of electrical machines to consider the voltage generated in each side, or conductor, of a single turn of a coil rather than to consider the voltage generated in a complete turn.

The above reasoning also shows that all that is necessary to obtain the voltage wave generated in a group of coils (such as a phase group of coils) in any winding, is to plot a voltage wave each ordinate of which is proportional to the sum of the magnetic densities through which each conductor of the group of coils is passing at the instant which that ordinate of the voltage wave is to represent. This is the simplest and most direct way of obtaining the voltage wave generated in any given winding when acted on by the field forms found in commercial electrical machines and it

is the one used by designing engineers in originally determining their design constants for the various types of windings and for various "field forms."

Before proceeding with the derivation it is best to choose symbols for and to define the various factors which will enter into the "EMF-flux" equation; hence let:

E_1 = The volts to be generated per phase or per winding.

ϕ = The useful flux per pole in maxwells.

C_p = The ratio of the average magnetic density to the maximum magnetic density of the magnetic field form.

C_w = Ratio of the effective volts per conductor of the given winding, to the maximum volts which would be generated by a single conductor rotated in the same field.

N_1 = Number of effective series conductors per phase or per winding.

f = Line or primary frequency in cycles per second.

Sin $\frac{a_1}{2}$ = Sine of one half the pitch angle of the coils in electrical degrees, that is, the sine of one half the angle spanned by the coils in electrical degrees, one pole pitch being 180 electrical degrees.

R.P.S = Revolutions per second of relative motion between the winding being considered and the magnetic field.

P = The number of poles.

T_1 = Number of effective series turns per phase or per winding: Then

$$N_1 = 2 \times T_1$$

T_c = Turns in one coil

N_c = Number of effective series conductors in one coil = $2\ T_c$

(Equations will be identified by numbers in a circle following the equations thus (1), (2) etc.)

Before proceeding with the derivation of the EMF-flux equation it should be remembered that the voltage generated in all the usual types of electrical machines, even in the direct current machine, is alternating in its nature and that the voltage of the usual D.C. machine is uniform and constant in direction only because the voltage of each coil is rectified by the commutator and the instantaneous voltage of all coils between positive and negative brushes is summed up by the connection to the commutator and brushes. So that the voltage in a single full pitch coil of any of the usual types of electrical machines can be considered as alternating in its nature and passing through one cycle when the coil or field moves over a space of two pole pitches.

Now all of the useful flux from a magnet pole passes through a full pitch coil when its sides are exactly midway between the magnetic poles, that is when the center of the coil is exactly in line with the center of a magnetic pole. Ninety electrical degrees later when the sides of the coil are exactly on the radial line passing through the center of the magnetic poles, that is when the center of the coil is exactly midway between magnetic poles the flux passing through the coil will be zero. This movement corresponds to one half a pole pitch or one fourth a cycle, that is four times that much movement is required for one cycle. Therefore the average voltage generated in a single full pitch coil will be:

$$\text{Ave. volts per coil} = \frac{\phi \times 4f \times T_c}{10^8} \qquad (1)$$

$$\text{But since } N_c = 2T_c \text{ ; } T_c = \frac{N_c}{2} \qquad (2)$$

$$\text{Ave. volts per coil} = \frac{\phi \times 2f \times N_c}{10^8} \qquad (3)$$

This, for a single full pitch coil, will be the average of a voltage wave which has the same shape as the field form for, as previously explained, the voltage wave shape generated by a single full pitch coil is the same shape as the field form.

Diagram 2 below shows the "field form" or distribution of magnetic density in the air gap over one pole pitch of a usual type of electric machine such as a salient pole synchronous machine.

See blue print, next page.

As previously defined, the field form factor or C_p factor of this field form will be the ratio of its average density to its maximum density. For a practical field form which is neither a sine curve or a straight sided geometrical figure this C_p constant is usually found by taking the ratio of the average of a large number of ordinates to the maximum ordinate. The C_p factor for this field form has been so determined and is equal to .67.

Since as previously explained the voltage wave generated by a single full pitch coil when rotated relative to such a magnetic field form will have the same shape as the field form it is evident that to get the maximum value of such a voltage wave the average value

FIELD FORM OF A 49.5 KVA 1200 RPM SINGLE PHASE 60 CYCLE
240 VOLT ALTERNATOR

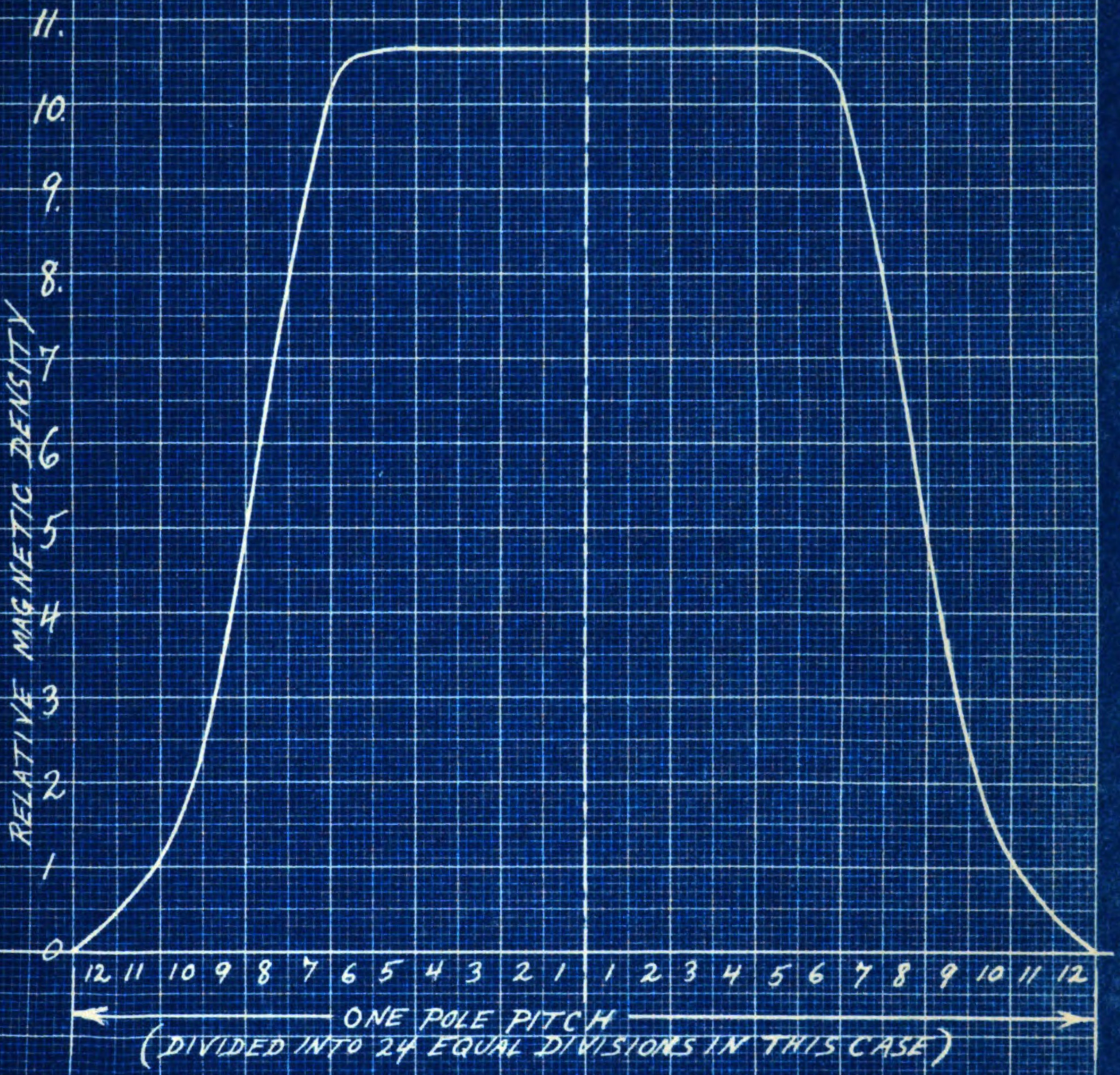

DIAGRAM 2

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of the voltage wave should be divided by the field form factor C_p; then since

$$\text{Ave. volts per coil} = \frac{\phi \times 2f \times N_c}{10^8} \quad (3)$$

$$\text{Maximum volts for a single full pitch coil} = \frac{\phi \times 2f \times N_c}{10^8 \times C_p} \quad (4)$$

Now having this formula for the maximum volts generated in a single full pitch coil all that is necessary to obtain the effective or "root mean square" value of the voltage generated in this full pitch coil is to multiply the above formula by the C_w factor. See previous definition of C_w.

We have then:

$$\text{Effective volts for one full pitch coil} = \frac{\phi \times 2f \times N_c \times C_w}{10^8 \times C_p} \quad (5)$$

For a single full pitch coil C_w will be the same as the ratio of the root mean square value of the field form to the maximum value of the field form. It is evident that this must be so since the voltage wave for such a coil has the same shape as the field form itself.

If there are several equally spaced full pitch coils connected together in series, as for example the phase group for one pole of a polyphase A. C. winding, the voltage wave generated in each individual coil will have the same shape as the field form, but the resultant voltage generated in the whole group of several coils in series will be the vector sum of several such voltage waves one for each coil and displaced in phase by a number of electrical degrees equal to the spacing of the coils from each other.

It is evident that the C_w factor for a winding of several equally spaced coils will not be the same as for a single full pitch coil and will not be equal to the ratio of the "root mean square" value of the field form to the maximum value of the field form.

The method of working out the voltage wave generated in a phase group of four equally spaced coils connected in series as well as the method of working out the C_w factor for such a winding when acted on by the field form shown in diagram 2 will now be given.

Taking a case of equally spaced coils with 12 coils in a pole pitch and considering the voltage generated in 4 of these coils in series (which would be like a polar phase group of a three phase winding), the voltage wave shape generated in the group of coils when acted on by the field form shown in diagram 2 may be determined as follows: The voltage wave that is generated in each individual coil has the same shape as the field form and since the four coils are in series and spaced one twelveth of a pole pitch or 15, electrical apart, the resultant voltage wave will be the sum of four voltage waves of the same shape as the field form and which are out of phase with each other by 15 electrical degrees.

This is shown in diagram 3 below.

See blue print, next page.

Now to find the C_w factor for these four equally spaced coils in this particular field form, this resultant voltage wave is simply spaced off equally into a considerable number of division, say 48 divisions,

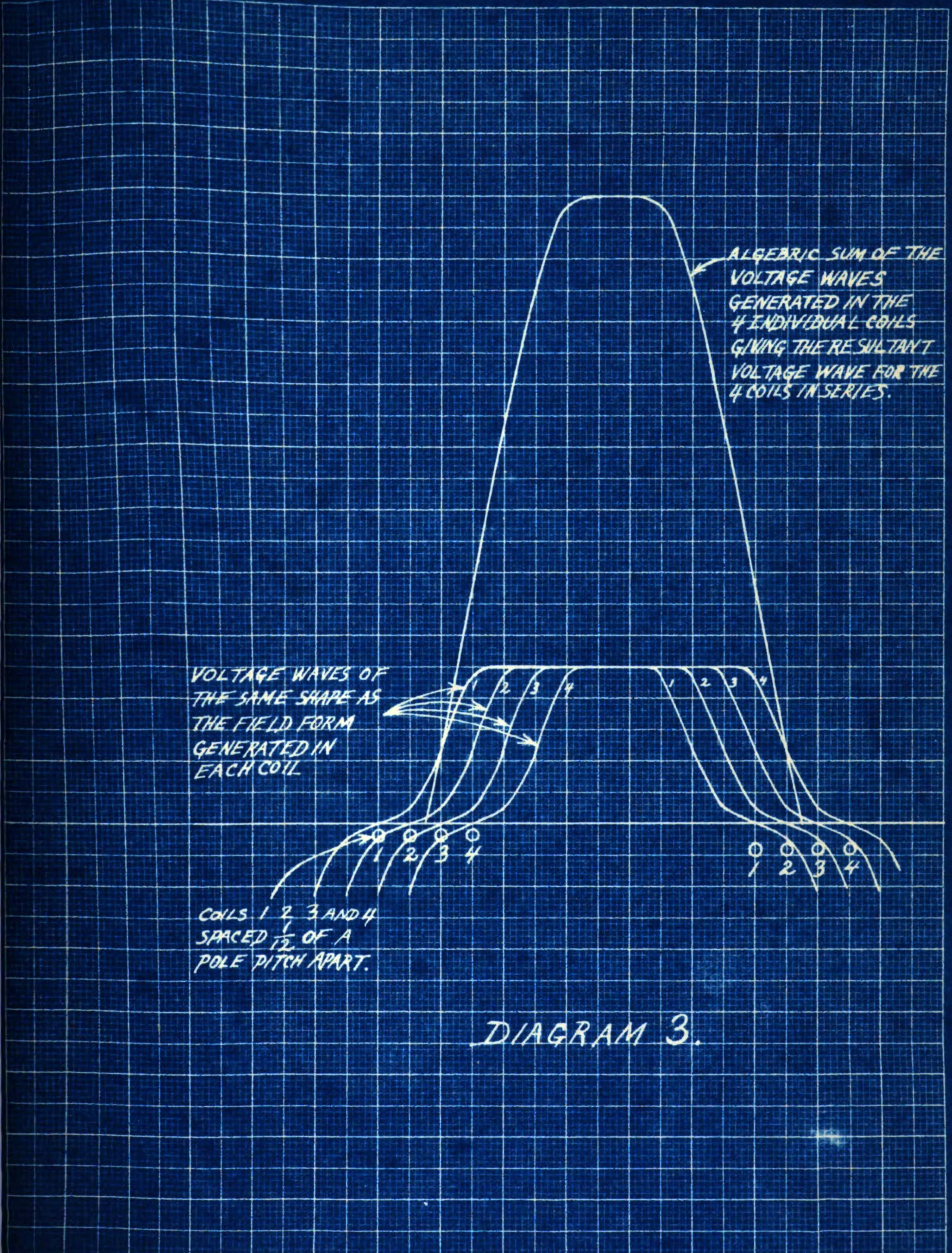

ALGEBRIC SUM OF THE VOLTAGE WAVES GENERATED IN THE 4 INDIVIDUAL COILS GIVING THE RESULTANT VOLTAGE WAVE FOR THE 4 COILS IN SERIES.
VOLTAGE WAVES OF THE SAME SHAPE AS THE FIELD FORM GENERATED IN EACH COIL
1 2 3 4
1 2 3 4
1 2 3 4
1 2 3 4
COILS 1 2 3 AND 4 SPACED 1/12 OF A POLE PITCH APART.

DIAGRAM 3.

the ordinate of the curve is read at the center of each division, these ordinates are squared and then the average of the squared values found. The square root of the average of the squared values is found, which is evidently the "root mean square" or effective value of this voltage wave. Then dividing this effective value of the voltage wave by the number of effective conductors in series in the whole group of coils, the effective volts per conductor of the winding is obtained. Then the ratio of this value of effective volts per conductor of the winding to the maximum volts which would be generated in one conductor alone will be, by definition, the C_w factor for these four equally spaced coils covering one third of a pole pitch when acted on by this field form. ((In the graphical method of solution shown in diagram 3 the ordinates of the voltage wave are only relative values and do not represent actual volts; also for the purpose of working out this C_w factor it is just as well to assume one turn coils, that is one conductor in each coil side. When this is done the value of C_w is the same as the ratio of the root mean square of the resultant voltage divided by the number of coils to the maximum of one of the separate voltage waves generated in each coil, the coils being full pitch coils.

The C_w factor for this particular field form, (diagram 1) has been worked out in just this way for a winding of equally spaced coils covering one third of a pole pitch and this C_w factor is equal to .710.

Now if we call the number of coils per pole in any one winding the "polar group of coils" or "polar phase group of coils" and let:

E_g = the effective or root mean square voltage generated in one polar group of coils and,

N_g = the number of conductors in series in one polar group of coils

It is evident that by working out the C_w factor in the manner previously described for the particular "polar group of coils", that is for evenly spaced coils covering the percentage of the pole pitch which the given "polar group of coils" covers, that the formula for E_g follows directly from the formula for effective volts for one full pitch coil by simply substituting N_g for N_c and using the correct value of C_w, that is:

Effective volts for one full pitch coil

$$= \frac{\phi \times 2f \times N_c \times C_w}{10^8 \times C_p} \qquad (5)$$

and

$$E_g = \frac{\phi \times 2f \times N_g \times C_w}{10^8 \times C_p} \qquad (6)$$

In the first case C_w is of course for a single full pitch coil in the given field form and in the second case C_w must be the correct one for a winding of evenly spaced coils covering the percentage of the pole pitch covered by this particular "polar group of coils."

That is the later equation above becomes general for a polar group of full pitch coils as long as the proper value of C_w is used.

Now if E_1 = the effective or root mean square volts for the winding or per phase winding as the case may be, and

N_1 = the effective series conductors per winding or per phase winding:

it is evident that for a full pitch winding all we need to do to get the total effective voltage for all the polar groups which are in series in a winding or one phase winding is to substitue these values of E_1 and N_1 instead of E_g and N_g in the above formula for E_g and we have

$$E_1 = \frac{\phi \times 2f \times N_1 \times C_w}{10^8 \times C_p} \quad (7)$$

This is the general equation for the effective A. C. voltage generated in a winding or phase winding except that it is still for a winding having full pitch coils only.

It has been demonstrated many times that when a winding is made up of equally spaced coils each of which spans less than a pole pitch, that is of coils which are less than full pitch, the resultant effective or root mean square voltage decreases directly as the sine of one half the pitch angle of "span" of the coils expressed in electrical defrees. This is true for any number of coils covering any fractional part of one pole pitch and for any ordinary field form which is found in electrical machines.

Now if

α = the pitch angle of the coils in electrical degrees, a full pitch coil spanning 180 electrical degrees and,

Sine $\frac{\alpha}{2}$ = the pitch factor or "chord factor" of the winding, which for a full pitch coil = 1.

Since the effective voltage varies as the sine of one half the pitch angle of the coils of a winding the general formula for the A. C. voltage generated in a winding becomes:

$$E_1 = \frac{\phi \times 2f \times N_1 \times C_W \times \text{SINE}\frac{\alpha_1}{2}}{10^8 \times C_P} \quad (8)$$

Usually E_1 is known from the circuit with which the machine is to operate, and ϕ is assumed by the designer to suit the rating of the machine which he is designing. For this reason, it is more convenient for the designer to solve this general A. C. voltage equation for the useful flux per pole, ϕ, instead of the voltage per winding or per phase E_1. When so transposed the equation becomes:

$$\phi = \frac{E_1 \times 10^8}{2 \times N_1 \times f \times \text{SINE}\frac{\alpha_1}{2}} \times \frac{C_P}{C_W} \quad (9)$$

This is the general equation for useful flux per pole which will apply to all the usual types of A. C. machines, both induction and synchronous.

In this general "flux" equation E_1 and f are determined from the given characteristics of the electric circuit with which the machine is to operate. N_1 and sine$\frac{\alpha}{2}$ are determined by the designer to suit the rating of the machine he is designing. The usual method of determining C_W has already been described and it is a long process.

The writer has, however, developed a very quick method of determining C_w and the ratio C_p/C_w directly from C_p. For all the field forms found in ordinary electrical machines this short method has been found to give results which are quite as accurate as the longer method. This quick method will be described in the following discussion of field forms and field form factors.

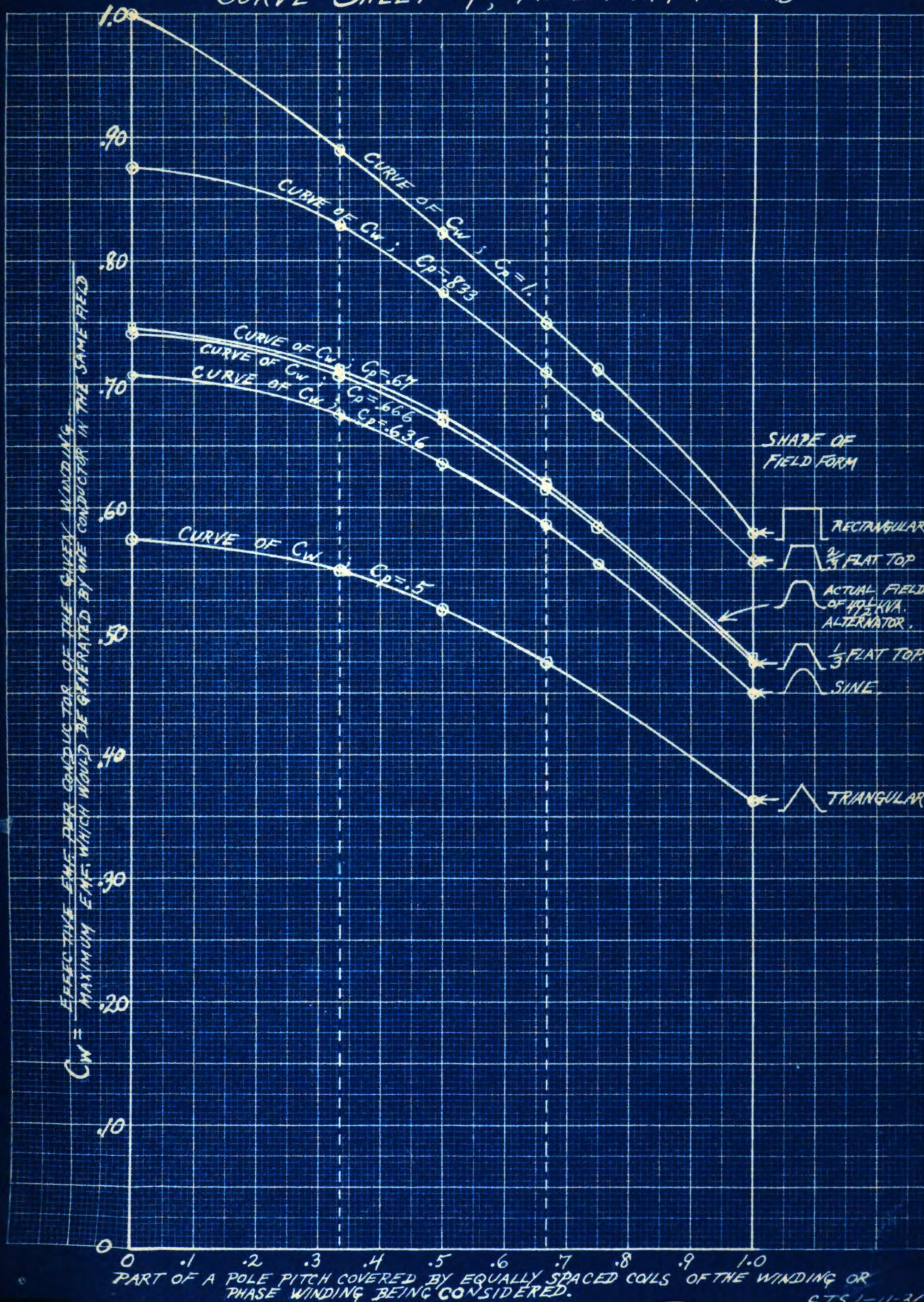
Diagram 4 shows the curves of the values of the C_w factor for various geometrically shaped field forms, a sinusoidal field form and one taken from the design of an actual alternator. These C_w factors were all worked out by the long method previously described and are plotted against the percentage of one pole pitch covered by a winding of equally spaced coils. The value of C_p which applies to each field form is given along the curve which applies to that particular field form.

See blue print, next page.

In diagram 4 the calculated points for the various field forms are shown on the curves by small circles or squares around these points. It should be noted that the curves when drawn through these calculated points are smooth and consistent and that for increasing values of C_p the value of C_w increases for any given part of the pole pitch covered by the winding. Note also how closely the curve of C_w for the actual alternator field form follows the curve of C_w for the 1/3 flat top field.

It was consistency of this data as well as the

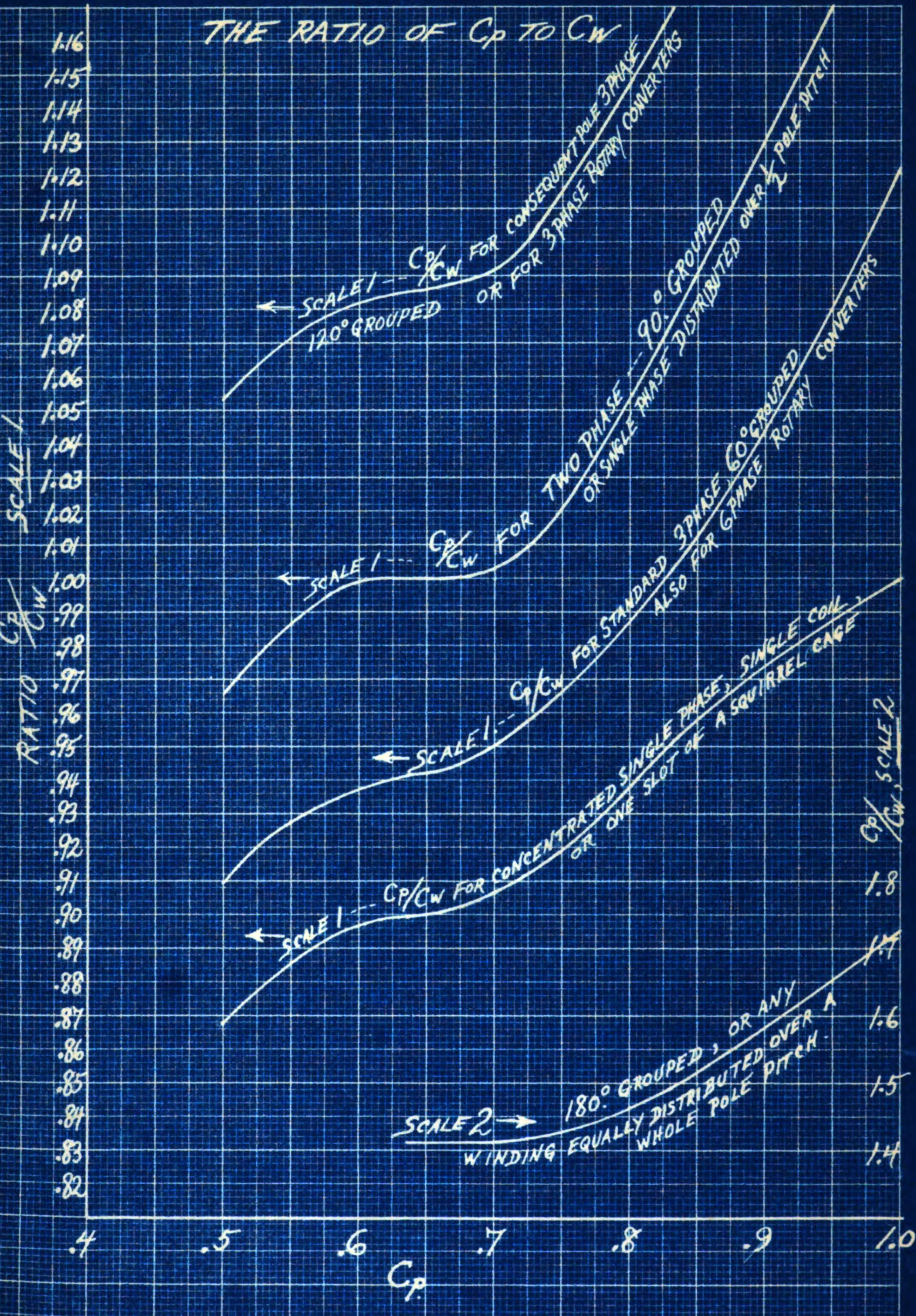
CURVE SHEET 4, "FIELD FORM FACTORS"



results of many actual designs which led the writer to devise the following curves for determining the ratio of C_p/C_w directly from the value of C_p as calculated from the field form. This set of curves of the ratio C_p/C_w plotted against C_p is shown on curve sheet 5 and is obtained by reading across the curves of curve sheet 4 on various vertical ordinates. Any given vertical ordinate on curve sheet 4, or course, corresponds to a given distribution of the winding; for example the .333 ordinate represents the distribution of winding corresponding to the usual type of three phase winding where the polar phase group of coils is distributed over 1/3 of a pole pitch. This grouping of the coils is often called "60 degree grouping" since the group of coils covers 60 electrical degrees.

Having curve sheet 5 for determining the ratio C_p/C_w directly from C_p, the designer is in a position to use the general equations (8) and (9) as soon as the shape of the field form is determined and its C_p factor is figured. The usual procedure is to assume a value of useful flux per pole ϕ which will be very near what will be required for the given size and rating of the machine, then to solve the general equation (9) for the effective series conductors per winding or per phase, N_1; then choose an actual number of series conductors N_1 as near to this required value as possible considering the number of slots etc. The general equation (9) is then used again to determine the final value of useful

CURVE SHEET 5



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flux per pole, ϕ. If the winding is to be less than full pitch the quantity sine$\frac{\alpha_1}{2}$ will be less than unity and this factor must also be taken into account in making these solutions.

The general formulas (8) and (9) are in the proper form for use in designing the usual forms of salient pole synchronous machine.

For the polyphase induction motor the field form constants from curve sheet 4 for a sine shape field can be used, since the average shape of the field form set up by the windings of these machines comes very close to a sine field. Substituting the value $C_p = .636$ for a sine field in equation (8) and (9) for the polyphase induction motor there results:

$$E_1 = \frac{\phi \times 2f \times N_1 \times C_w \times \text{SINE}\frac{\alpha_1}{2}}{10^8 \times .636} \qquad (10)$$

or

$$\phi = \frac{E_1 \times 10^8}{2 \times N_1 \times f \times \text{SINE}\frac{\alpha_1}{2}} \times \frac{.636}{C_w} \qquad (11)$$

These equations (10) and (11) can be used in this form for the polyphase induction motor reading the value of C_w on the curve for the sine field on curve sheet 4 and for 60°, 90°, or 120° winding distribution depending on what kind of a polyphase winding is being used.

However, these equations can be further simplified by introducing what is usually called the distribution factor.

The distribution factor is usually represented by C_d and is equal to the ratio of the effective volts for a given number of turns used in the distributed winding to the effective volts in a concentrated winding of the same number of turns. This is evidently the same thing as the ration of C_w for the distributed winding to C_w for a concentrated or single coil winding in the same field, and since the value of C_w for a concentrated winding in a sine field is equal to .707 (see curve sheet (4) there results for the sine field form:

$$C_d = \frac{C_w}{.707} \quad \text{or} \quad C_w = .707 \times C_d \qquad (12)$$

Substituting this value of C_w in equations (10) it becomes:

For the polyphase induction motor:

$$E_1 = \frac{\phi \times 2f \times N_1 \times C_d \times \text{SINE}\frac{\alpha_1}{2} \times .707}{10^8 \times .636} \qquad (13)$$

It will be noticed that the $\frac{.707}{.636}$ in this formula is equal to 1.11, which is commonly called the "form factor of the sine field". It is evident that this should be the form factor since .707 is equal to the ratio of the effective volts to the maximum volts for the case of a concentrated winding in a sine field, while .636 is the ratio of the average volts to the maximum volts with the same winding in a sine field. Combining the numerical factor 2 with the numerical factor 1.11 the equation for the polyphase induction motor becomes:

$$E_1 = \frac{\phi \times 2.22 \times f \times N_1 \times C_d \times \text{SINE}\frac{\alpha_1}{2}}{10^8} \qquad (14)$$

or

$$\phi = \frac{E_1 \times 10^8}{2.22 \times N_1 \times f \times C_d \times \text{SINE}\frac{\alpha_1}{2}} \qquad (15)$$

Now in the polyphase induction motor since ϕ is the total useful flux per pole it is evident that it is the maximum value of flux which would pass through a full pitch coil at the instant when the axis or center line of that coil coincided with the center of a pole of the rotating magnetic field. For this reason it can be compared directly with the maximum value of the useful flux of a transformer, and since in the usual type of transformer the winding is always concentrated and there is no such thing as a winding less than a full pitch winding; the values of C_d and Sine $\frac{\alpha}{2}$ can both be said to equal one in this case.

Hence for the transformer the equations become:

$$E_1 = \frac{\phi \times 4.44 \times f \times T_1}{10^8} \qquad (19)$$

and

$$\phi = \frac{E_1 \times 10^8}{4.44 \times T_1 \times f} \qquad (20)$$

Which is the usual form of the "EMF-flux" equation for the transformer.

The voltage ratio or transformer ratio between the secondary and the primary windings of a polyphase induction motor can be easily derived from a consideration of equation (10), which is an expression for the voltage per primary winding in terms of flux per pole, conductors per winding, etc.

The voltage ratio or transformation ratio of a polyphase induction motor is usually called $\mathcal{T}$, and is equal to the ratio of the standstill open circuit secondary voltage per phase winding to the primary voltage per phase winding. If we let E_2= this open circuit secondary voltage per phase winding, then

$$\gamma = \frac{E_2}{E_1} \qquad (21)$$

If we indicate other secondary quantities by a sub 2 and primary quantities by a sub 1 and remember that at standstill the frequency in the secondary of the motor is the same as the primary frequency, that is the rotating magnetic field is then acting on the secondary at synchronous speed, it is evident from equation (10) that:

$$E_2 = \frac{\phi \times 2f \times N_2 \times C_{w2} \times \text{SINE}\frac{\alpha_2}{2}}{10^8 \times .636} \qquad (22)$$

and that:

$$\gamma = \frac{E_2}{E_1} = \frac{N_2 \times C_{w2} \times \text{SINE}\frac{\alpha_2}{2}}{N_1 \times C_{w1} \times \text{SINE}\frac{\alpha_1}{2}} \qquad (23)$$

This is the usual form of the transformation ratio or voltage ratio for the polyphase slip-ring rotor induction motor in which N_2 would be the effective series conductors in a secondary phase winding, α_2 the pitch angle of the secondary coils in electrical degrees, and C_{w2} the C_w factor for the secondary winding, which is usually equal to .675 since the secondaries of slip ring motors usually have three phase windings with each phase winding distributed over one third of a pole pitch, (see curve sheet 4). The quantities in the denominator of this equation are primary winding quantities as previously explained and C_{w1} is taken from curve sheet 4 for the sine field form and for .333, .5 or .666 part of a pole pitch covered by the phase winding, depending on whether the primary winding is the usual three phase, two phase, or a "consequent pole" connected three phase winding. Once the value of $\mathcal{T}$ is determined the primary volts per phase multiplied by $\mathcal{T}$ gives the open circuit standstill secondary volts per phase.

For the polyphase induction motor with a squirrel cage secondary or rotor, $N_2 = 1$ since each rotor bar is considered as a separate phase and hence has but one conductor. For the squirrel cage rotor $C_{W2} = .707$ since each bar is a concentrated winding acted on by a sine field. (See curve sheet 4.) For a squirrel cage rotor in which the slots and bars are parallel to the shaft, that is for a rotor which is not "spiralled" sine $\frac{\alpha_2}{2} = 1$. Then if we let $\mathcal{T}_c$ equal the transformation ratio for a motor with a squirrel cage rotor we have, for a motor with an unspiralled rotor

$$\mathcal{T}_c = \frac{1 \times .707 \times 1}{N_1 \times C_{W_1} \times \text{SINE} \frac{\alpha_1}{2}} = \frac{.707}{N_1 \times C_{W_1} \times \text{SINE} \frac{\alpha_1}{2}} \qquad (24)$$

If the squirrel cage rotor is spiralled the voltage generated at one end of a rotor bar is out of phase with that generated at the other end by an amount equal to the angle of the spiral expressed in electrical degrees, just as the voltage generated in one side of a "short pitch" coil is out of phase with the voltage generated in the other side of the coil by a phase angle equal to the number of electrical degbees by which the coil falls short of being a full pitch coil.

Hence if m = the number of electrical degrees which the squirrel ca-e rotor is spirallled.

Then for the squirrel cage rotor $\alpha_2 = 180 - m$ (25)

and $$\sin\frac{\alpha_2}{2} = \sin\left(\frac{180-m}{2}\right) \quad (26)$$

Using this value instead of 1 in equation (24)

$$T_c = \frac{.707 \times \sin\left(\frac{180-m}{2}\right)}{N_1 \times C_{W_1} \times \sin\frac{\alpha_1}{2}} \quad (27)$$

and if the primary volts per phase are multiplied by this value of T_c the total effective volts generated in a rotor bar at standstill are obtained.

For the case of the direct current machine it is evident that the voltage generated between positive and negative brushes is equal to the sum of the instantaneous voltages being generated in all the coils between positive and negative brushes. This is of course the same thing as the average voltage per coil times the number of coils in series between the positive and negative brushes.

Referring back to formula (3) the

$$\text{average volts per coil} = \frac{\phi \times 2f \times N_c}{10^8} \quad (3)$$

in which as s tated before ϕ is the useful flux per pole, N_c is the series conductors per coil and f is the frequency in the armature, the voltage in the separate coils of a D.C. armature being really an alternating voltage and being rectified by the commutator to give the "D. C." voltage outside the armature.

Now if

E_d = the D. C. voltage between positive and negative brushes.

S_c = the total coils in the armature

n = the number of parallel circuits into which S_c coils are divided by the connection to the commutator and brushes.

Then the number of coils in series between positive and negative brushes $= \frac{S_c}{n}$. Now since the voltage between positive and negative brushes is equal to the average volts per coil multiplied by the number of coils in series between positive and negative brushes we will get this voltage by multiplying formula (3) by $\frac{S_c}{n}$, then

$$E_d = \frac{\phi \times 2f \times N_c}{10^8} \times \frac{S_c}{n} \qquad (28)$$

But $\frac{N_c \; S_c}{n}$ = the number of series conductors between positive and negative brushes which should be called N_1 for the D. C. machine, hence:

$$\frac{N_c \; S_c}{n} = N_1 \qquad (29)$$

Substituting this value of N_1 in equation (28) it becomes:

$$E_d = \frac{\phi \times 2f \times N_1}{10^8} \qquad (30)$$

But since R. P. M. $= \frac{120 \times f}{P}$, where P is the number of poles,

$$f = \frac{RPM \times P}{120} \qquad (31)$$

Substituting this value of f in equation (30) it becomes

$$E_d = \frac{\phi \times P \times N_1}{10^8} \times \frac{RPM}{60} \qquad (32)$$

Which is the standard "EMF-flux" equation for D. C. machines, E_d being the generated "D. C." voltage between positive and negative brushes.

Part II

DETERMINATION OF THE ACTUAL CURRENT IN THE ROTOR BARS AND IN THE END RINGS OF POLYPHASE MOTORS AND ALSO THE PRIMARY EQUIVALENT OF THE RESISTANCE OF THE ROTOR BARS AND END RINGS OF SUCH SQUIRREL CAGE WINDINGS

In order to determine the actual value of current in the bars of a squirrel cage rotor we must first derive an expression for this current in terms of the primary equivalent of the secondary current.

Taking first a general case of a motor having any number, Ph_2 of phases in the secondary; while the primary has a number of phases ph, also for this general case let:

E_1 = the primary volts per phase

E_2 = the normal secondary volts per phase (that is the open circuit standstill secondary volts per phase.)

I_2 = the current in each secondary phase winding.

I_2^1 = the primary equivalent, in each primary phase winding, of the secondary current, I_2.

Now the primary equivalent I_2^1 of the secondary current I_2 is the vector difference between the total current per primary phase at the given load and the exciting or no load current per primary phase. Hence this primary equivalent of the secondary current is easily found.

It is evident that "KVA" represented by this primary equivalent of the secondary current together with the primary voltage and the primary number of phases is equal to the "KVA" in the secondary circuit; that is to the "KVA" represented by the secondary current, the normal secondary voltage and the secondary number of phases.

Hence:

$$E_1 \times I_2^1 \times Ph_1 = E_2 \times I_2 \times Ph_2 \quad (33)$$

But if $\mathcal{T}$ is the ratio of the normal secondary voltage to the primary voltage that is if

$$\mathcal{T} = \frac{E_2}{E_1} \quad (21)$$

(See part I of this thesis)

Then

$$E_2 = \mathcal{T} \times E_1 \quad (34)$$

Substituting this value for E_2 in equation (33) we get

$$E_1 \times I_2^1 \times Ph_1 = \mathcal{T} \times E_1 \times I_2 \times Ph_2 \quad (35)$$

Dividing both sides of this equation by E_1

$$I_2^1 \times Ph_1 = \mathcal{T} \times I_2 \times Ph_2 \quad (36)$$

Then

$$I_2 = \frac{1}{\mathcal{T}} \times \frac{Ph_1}{Ph_2} \times I_2^1 \quad (37)$$

This is the general equation for the secondary current in the secondary of a polyphase induction motor, and gives the secondary current per phase in terms of the voltage transformation ratio, the number of primary and secondary phases and the primary equivalent of the secondary current. Equation (37) is used as it is for the slip ring motor which has a phase wound rotor or secondary. For the squirrel cage rotor motor each rotor slot or bar is considered as a "phase"

and hence the current per phase is, in this case, the current per rotor bar, and the transformation ratio is T_c where

$$T_c = \frac{.707 \times SINE\left(\frac{180-m}{2}\right)}{N_1 \times C_{W_1} \times SINE\frac{\alpha_1}{2}} \qquad (27)$$

as explained and derived in part 1 of this thesis.

Then is we let:

I_b = the effective current in a bar, or per slot, of a squirrel cage winding corresponding to a given primary current,

S_2 = the number of rotor slots (which takes the place of Ph_2 in the case of a squirrel cage rotor.)

Substituting these squirrel cage quantities in the general formula (37) we get

$$I_b = \frac{1}{T_c} \times \frac{Ph_1}{S_2} \times I_2' \qquad (38)$$

The equation gives the actual current in the rotor bar (or per rotor slot) for any given load current and corresponding value of I_2^1 in the primary.

Now having the actual effective value of the current I_b in the rotor bar it is easy to figure the (I^2R) loss in the bar part of the rotor winding. All that is necessary to do this is to figure the resistance of the rotor bars as if they were all connected together in one long bar and to multiply this resistance by $(I_b)^2$.

Now that we have an expression for the actual effective current in the rotor bars, the next step is to find an expression for the current in the rotor end rings; which it will be comparatively easy to do.

As explained in part I of this thesis, the average "field form" set up by the currents in the primary winding of a polyphase motor is very close to a sine shaped field form. Therefore the voltage wave generated in each rotor bar of a squirrel cage rotor of such a motor, will be approximately a sine voltage wave. This means that the current in each rotor bar will vary approximately according to the sine law. Also, since the rotor slots are evenly spaced the sine current waves of the different bars will be out of phase with each other by a phase angle corresponding to the spacing of the rotor slots in electrical degrees. This being the case the instantaneous values of the currents in the bars covering a pole pitch vary as the sine of the angle of the displacement of each bar in electrical degrees from the position of zero bar current. This is shown diagramatically in diagram 6 below.

Since the voltages and currents in the rotor bars are in opposite directions over adjacent pole pitches of the rotor because adjacent poles of the magnetic field are of opposite polarity, it is quite evident that the currents flowing into the end ring from half the bars under a pole of the magnetic field will join together and flow one way

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in the end ring toward the adjacent pole of the magnetic field. While the bar currents flowing into the end ring from the other half of the magnetic pole under considera-

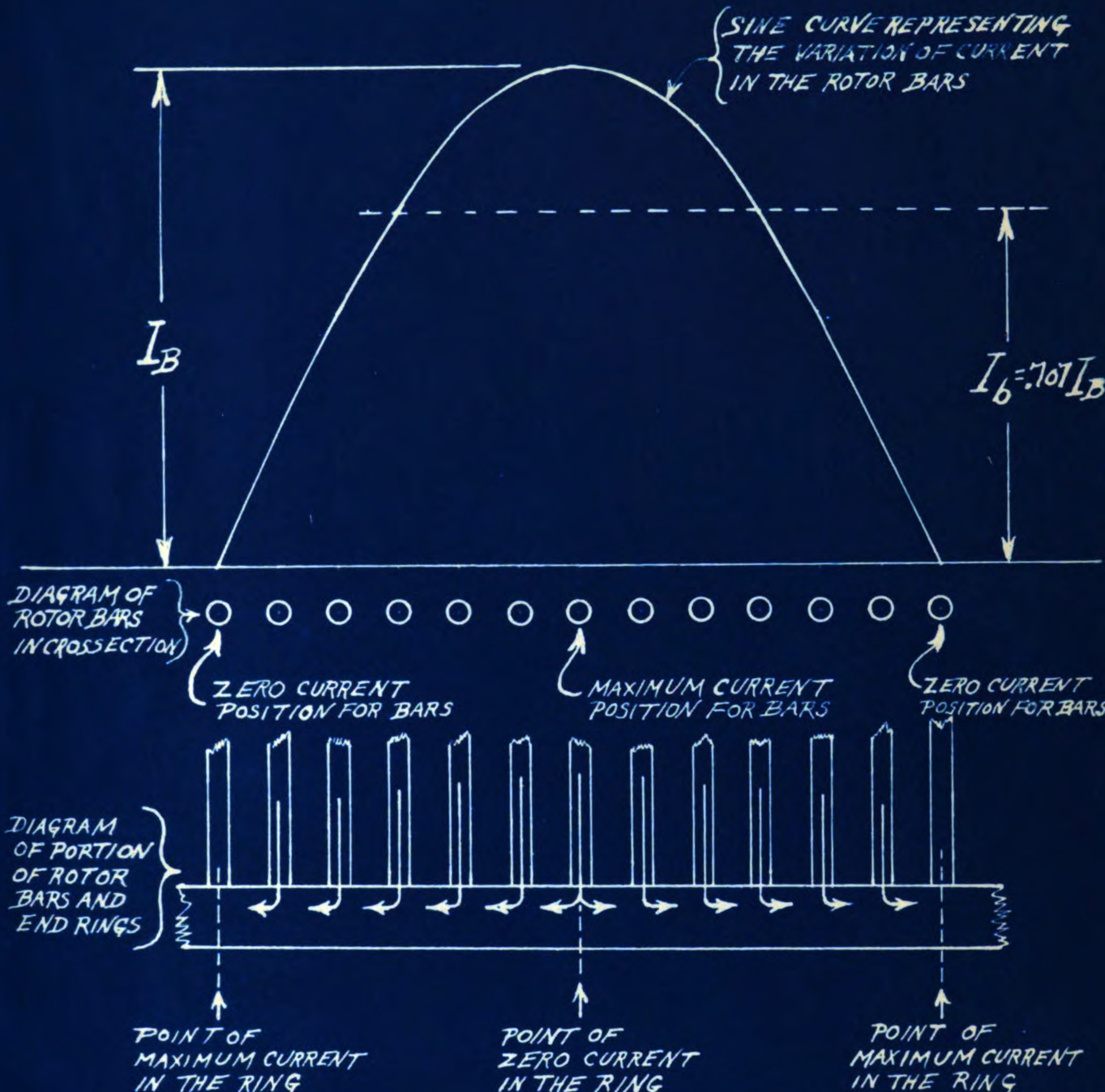

SINE CURVE REPRESENTING THE VARIATION OF CURRENT IN THE ROTOR BARS
I_B
$I_b = .707 I_B$
DIAGRAM OF ROTOR BARS IN CROSSECTION
ZERO CURRENT POSITION FOR BARS
MAXIMUM CURRENT POSITION FOR BARS
ZERO CURRENT POSITION FOR BARS
DIAGRAM OF PORTION OF ROTOR BARS AND END RINGS
POINT OF MAXIMUM CURRENT IN THE RING
POINT OF ZERO CURRENT IN THE RING
POINT OF MAXIMUM CURRENT IN THE RING

DIAGRAM 6.

tion will join together and flow the other way in the ring toward the adjacent pole on the other side of magnetic pole being considered. From these considerations it is also evident that the maximum current in the ring will occur at a point in the ring which is midway between a north and south magnetic pole at that particular instant, and that the current will be zero in the end ring at points which are in axial alighment with the center of a magnetic pole of the field.

This is shown diagramatically in diagram 6.

Hence the maximum current in the end ring is equal to the average current in a rotor bar times the number of bars in one half of a pole pitch.

Also since as explained before the instantaneous currents in the rotor bars vary according to the sine law over a pole pitch, it is evident that the current at different points, along the end ring, at any instant, will vary according to the sine law, and that the instantaneous current at any point in the end ring will be approximately proportional to the sine of the displacement of the given point in electrical degrees from the point of zero current in the ring. (The larger the number of rotor bars or slots per pole the more nearly this is true.)

To put these statements into the form of an equation let

I_B = the maximum current in a bar

I_b = the effective current in a bar

I_R = the maximum current in one end ring.

I_r = the effective current in one end ring

S_2 = the number of rotor slots or bars

P = the number of poles for which the primary is wound or connected.

Then first we have since the currents vary according to the sine law:

$$I_b = .707 I_B \text{ and } I_r = .707 I_R \tag{39}$$

and

$$\text{AVE. CURRENT IN THE BAR} = .636 I_B \tag{40}$$

Then:

$$I_R = \frac{1}{2} \times \frac{S_2}{P} \times .636\ I_B \tag{41}$$

But from equation (39) above

$$I_B = \frac{I_b}{.707} \quad \text{and} \quad I_R = \frac{I_r}{.707} \tag{42}$$

substituting these values of I_B and I_R in equation (41) we get

$$\frac{I_r}{.707} = \frac{S_2}{2P} \times .636 \times \frac{I_b}{.707} \tag{43}$$

or multiplying through by .707

$$I_r = \frac{S_2}{2P} \times .636\ I_b \tag{44}$$

which is an equation for the effective current in the end rings in terms of the effective current in the rotor bar.

Now having the effective value of current in the end rings all that is necessary to figure the corresponding (I^2R) loss in the end ring part of the squirrel cage winding is to figure the resistance of the end rings at both ends of the rotor as if both ends were stretched out into one long piece of metal of the same cross section as the end rings have, and to multiply this resistance by $(I_r)^2$.

The I_b in equation (44) will of course be the one corresponding to the desired value of primary load current and its component I_2^1 which is the primary equivalent of the secondary current for the given load.

The expressions for the effective current in the rotor bars and end rings and the method suggested for figuring the (I^2R) loss in the rotor bars and the end rings provide one method for figuring this loss in the squirrel cage rotor. However it is often more convenient for the designer to have a "primary equivalent" winding which will be expressed in the same terms as resistance in the primary phase winding and which he can treat in his calculations as if this primary equivalent of the secondary resistance were actually in the primary phase. Then if we let

R_2^1 = the primary equivalent, per primary phase winding, of the secondary resistance

It is evident that

$$Ph_1 \times (I_2^1)^2 \times R_2^1 = I_b^2 \times (\text{Res. of all Bars}) + I_r^2 \times (\text{Res. of Rings}) \qquad (45)$$

The best way to use this equation to solve for R_2^1 is to assume that $I_2^1 = 1$ for in this case I_b from equation (38) becomes

$$I_b = \frac{1}{f_c} \times \frac{Ph_1}{S_2} \qquad \text{(For the case when } I_2^1 = 1) \qquad (46)$$

and I_r from equation (44) becomes

$$I_r = \frac{S_2}{2P} \times .636 \times I_b \quad \begin{matrix} \text{(Where } I_b \text{ is the value)} \\ \text{(from equation (46) \quad)} \\ \text{(when } I_2^1 = 1 \quad \text{)} \end{matrix} \tag{47}$$

Then for $I_2^1 = 1$ equation (45) becomes

$$R_2^1 = \frac{(I_b)^2 \times \text{Res. of all Bars}}{Ph_1} + \frac{(I_r)^2 \times (\text{Res of Rings})}{Ph_1} \tag{48}$$

I_b and I_r being in this case figured from equation (46) and (47); that is being the values corresponding to $I_2^1 = 1$.

Now all the designer has to do to get (I^2R) loss in the squirrel cage rotor is to figure the primary equivalent I_2^1 of the secondary current for each load for which he wishes to make a calculation and use this to figure the rotor (I^2R) loss from the following equation:

$$\text{Secondary } (I^2R) \text{ loss} = (I_2^1)^2 \times Ph_1 \times R_2^1 \tag{49}$$

Using R_2^1 as figured from equation (48) and using the number of primary phases Ph_1 since R_2^1 as derived in equation (48) is the primary equivalent, per primary phase winding, of the resistance of the secondary squirrel cage winding.

Part III

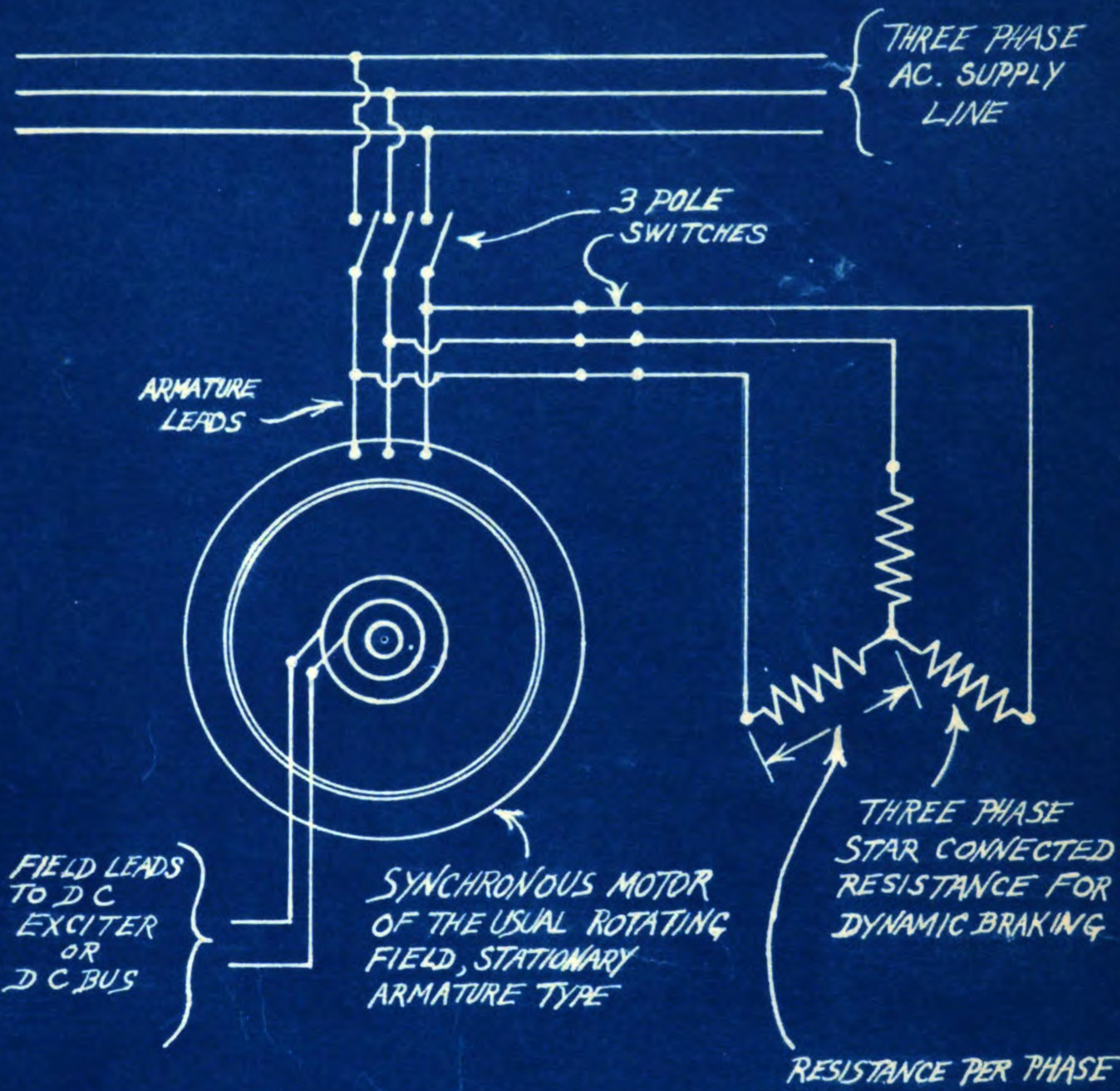
DYNAMIC BRAKING OF SYNCHRONOUS MOTORS AND THE PREDETERMINATION OF THE CORRECT VALUE OF THE EXTERNAL RESISTANCE PER PHASE TO GIVE THE MAXIMUM BRAKING TORQUE AT THE BEGINNING OF THE BRAKING PERIOD.

The connections necessary in order to use dynamic braking for stopping a synchronous motor are as follows:

The motor being in operation driving its load and having its normal or full load field excitation, the three armature terminals (a three phase machine being considered) must be disconnected from the three phase A.C. supply lines and then a star connected three phase resistance of the correct value must be immediately connected to the three phase armature terminals. These changes of connections being made with suitable switching apparatus, either manually or electromagneticly operated. The field excitation of the synchronous machine being kept at its normal value during all the braking period.

These connections are shown diagramatically in diagram 7 below.

It is very evident that as soon as the synchronous motor is disconnected from the A.C. supply line and connected to the dynamic braking resistance, as shown in diagram 7, the synchronous machine ceases to operate as a motor and begins to operate as a generator; being driven by the stored energy in its own rotor and in the machine which the synchronous motor was driving.



CONNECTIONS FOR DYNAMIC BRAKING OF A SYNCHRONOUS MACHINE

DIAGRAM 7.

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At the very beginning of this braking period the frequency of this synchronous machine (now acting as a generator) will be approximately equal to the normal line value since the speed of the machine has not yet decreased at that time. The voltage of the machine just before the dynamic braking resistance is connected to its armature terminals will be the no load voltage of the machine acting as a generator and operating at full speed and with a field excitation equal to the full load field current of the machine as a synchronous motor. It is evident that this voltage may be 20 to 35% higher than the A.C. supply line voltage, espeically if the synchronous motor has been operating at a leading power factor, and so has an over-excited field.

The amount of braking torque which will be exerted on the synchronous machine to slow it down in speed will depend on the amount of electrical energy delivered to the resistance by the machine acting as a generator. This amount of electrical energy will depend on the total generated voltage, or no load voltage, generated in the armature of the synchronous machine for the given field current; the synchronous impedance per phase in the armature of the synchronous machine and the resistance per phase in the star connected dynamic braking resistance which is connected to the armature terminals.

The total voltage generated in the armature of the synchronous machine during this dynamic braking process can be considered as made up of two components, one component being the synchronous impedance drop in the armature of the machine and the other component the resistance drop, which latter is mostly used up in forcing the current through the star connected resistance connected to the armature terminals, since the resistance of the armature is relatively very low. These two components of the total generated voltage are at right angles to each other in phase relation, or at least very nearly so.

The data which it is necessary to have, either from test or from design calculations, in order to be able to work out the required resistance for dynamic braking of a synchronous motor, is enough data to plot the following curves:

The no load saturation curve, usually plotted in percent of normal voltage as ordinates against field current in percent of the no load full voltage value as abscissa; the zero power factor full load current saturation curve, and the 100% power factor full load current saturation curve.

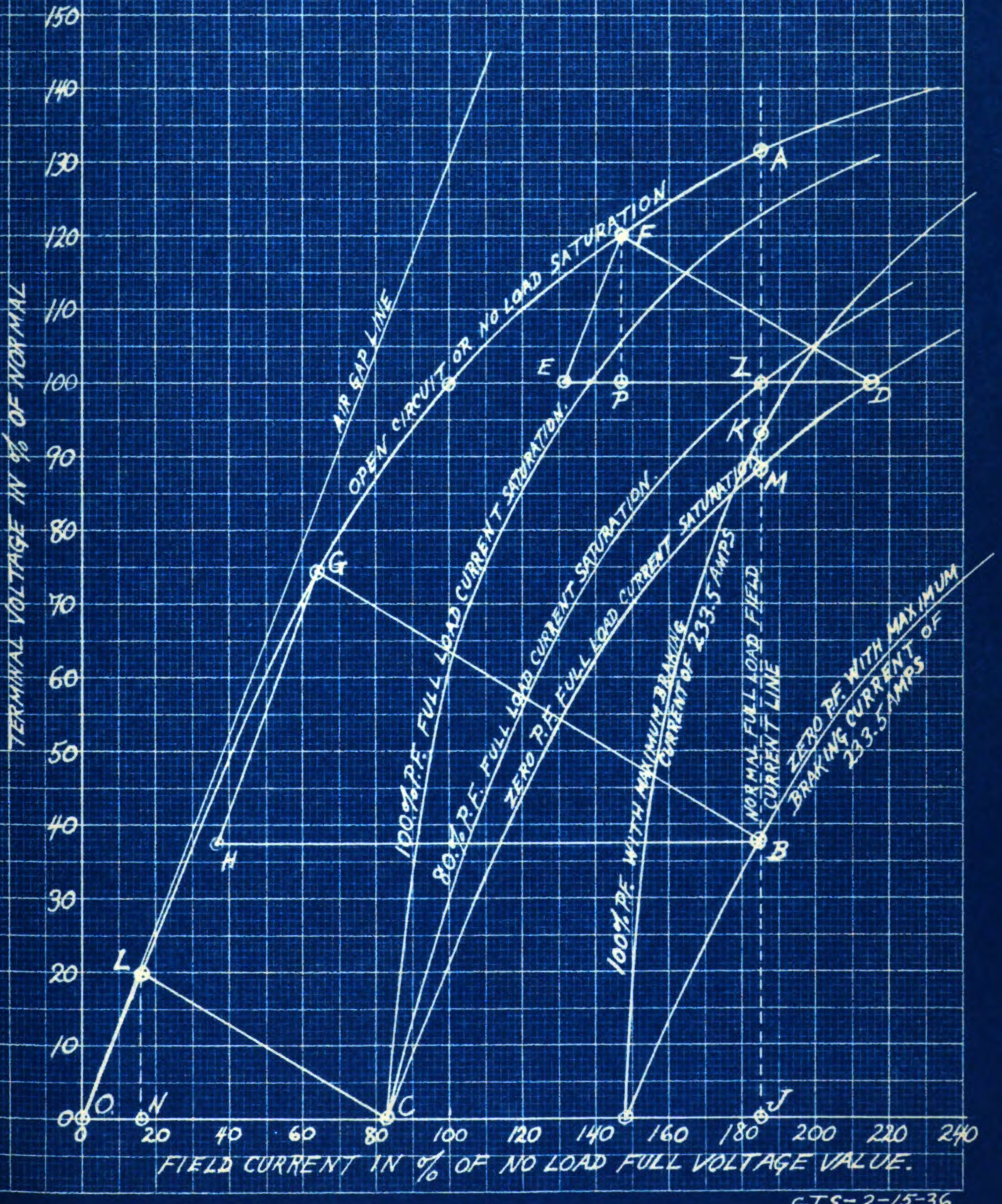
These curves for a 500.HP, 450 RPM, 3 phase, 60 cycle, 2200 volt, 80% leading power factor synchronous motor are shown on curve chart 8.

On this curve chart 100% field current is taken as that

CHARACTERISTIC CURVES OF A
500. HP, 450 RPM, 3 PH, 60 CY. 2200 VOLT, 80% LEADING POWER FACTOR
SYNCHRONOUS MOTOR

FULL LOAD AMPS. PER PHASE = 131. = I,
NORMAL PHASE VOLTAGE = 2200. ÷ √3 = 1270. VOLTS

CURVE CHART 8.



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field current which will give full voltage as a generator on no load. Since this particular machine is rated on the basis of 80% power factor at full load current, the full load field current will be that field current which will give full voltage, or 100% voltage, when the machine is operating with full load armature current and 80% power factor, as represented by the 100% voltage point on the full load 80% power factor saturation curve, that is by point Z on curve chart 8. This is found to be 185% field current, and this is the field current which is assumed to be flowing in the field circuit during the period of dynamic braking.

This valueof field current is indicated on curve chart 8 by a vertical line marked full load field current.

The total generated voltage in the armature at the beginning of the braking period will be the voltage value shown on the no load saturation curve on this vertical line corresponding to 185% field current. This total generated voltage is found to be 131.5% of full voltage for this particular machine. See point A on curve chart 8.

This machine is a star connected machine, as most alternators and synchronous motors are, and is wound for a 2200 line so that the normal voltage per phase is $2200 \div \sqrt{3}$ or 1270 volts.

Hence the total generated voltage per phase at the beginning of the braking period, being 131.5% of normal, will be equal to 1.315 x 1270 = 1670 volts.

As previously mentioned this total generated voltage is made up of two components, when dynamic braking by means of external resistance is used. One of these components is the synchronous impedance drop in the armature and the other component is the resistance drop or I_R drop in the external resistance.

Since the synchronous impedance of the armature acts like reactance, these two components are approximately at right angles to each other in phase relation.

The problem now is to choose the value of the external resistance per phase so that the maximum possible amount of electrical energy will be delivered to the external resistance, for when this is done the maximum amount of braking torque will be exerted to stop the synchronous machine.

It has been found that this electircal energy has a maximum value when the external resistance per phase is made equal to the synchronous impedance per phase of the armature of the synchronous machine. Since this synchronous impedance acts like a reactance this is similar to the case of an electric circuit having a fixed reactance and variable resistance, and being acted on by an alternating current voltage.

In this case the amount of electrical energy in the electric circuit becomes a maximum when the resistance is made equal to the fixed reactance. This latter fact can be demonstrated as follows:

In a circuit acted on by an A.C. voltage and containing a given reactance, X, and some variable value of resistance, R, the impedance Z of the circuit will be

$$Z = \sqrt{X^2 + R^2} \qquad (50)$$

and with an impressed A.C. voltage, E, the current, I, will be

$$I = \frac{E}{\sqrt{X^2 + R^2}} \qquad (51)$$

But the electrical power expended in such a circuit is always equal to I^2R or

$$\text{WATTS} = I^2R \qquad (52)$$

and from equation (51) above

$$I^2 = \frac{E^2}{X^2 + R^2} \qquad (53)$$

hence,

$$\text{WATTS} = \left(\frac{E^2}{X^2 + R^2}\right) \times R \qquad (54)$$

or

$$\text{WATTS} = \frac{E^2R}{X^2 + R^2} \qquad (55)$$

Taking the first derivative of this expression with respect to the variable R and equating this first derivative to zero to solve for the value of R which will give the

maximum value of watts we get:

$$\frac{(X^2 + R^2)\ E^2 - (E^2R) \times 2R}{(X^2 + R^2)^2} = 0 \qquad (56)$$

or

$$(X^2 + R^2)\ E^2 = 2R^2E^2 \qquad (57)$$

then dividing both sides by E^2

$$X^2 + R^2 = 2R^2 \qquad (58)$$

or

$$R^2 = X^2 \qquad (59)$$

from which $R = X$, for the condition of maximum power in watts.

And it has been found by trial that the braking torque at the beginning of the braking period, by this method of braking, is a maximum when the external resistance per phase is made equal to the synchronous impedance of the armature per phase.

Now, referring again to curve chart 8, since the no load voltage of 1670 volts corresponding to full load field current, as shown at point A, is to be divided into two equal components which are approximately at right angles to each other in phase, one of these components being the synchronous impedance drop in the armature and the other the external (IR) drop per phase; both of these components will be equal to .707 x 1670 = 1180 volts. That is:

SYN. IMPEDANCE DROP = IR DROP = .707 x 1670 = 1180 volts (60)

That is to say, the external resistance per phase must be so chosen that the (IR) drop per phase in the external resistance will be 1180 volts and the synchronous impedance drop in the armature of the synchronous machine per phase will be 1180 volts with the current which will flow when the external resistance is connected.

But since the synchronous impedance of the armature depends on the current being supplied by the synchronous machine, the current is an unknown quantity and the best solution which the writer has found for this problem is the following graphical one.

First the point K is laid off on the ordinate of curve chart 8 corresponding to full load field current and at a percent voltage corresponding to 1180 volts, that is at 93% voltage since 1180 volts is 93% of the normal full phase voltage of 1270 volts.

Now since the external resistance connected to the machine constitutes a 100% power factor load, it is evident that the point K is a point on a 100% P.F. load current curve for some value of current.

Now the synchronous impedance drop of a synchronous machine for any given load current and given field excitation, is equal to the distance between the no load saturation curve and the zero power factor saturation curve for the given load

current, this distance being measured along the vertical ordinate corresponding to the given field excitation and being expressed in percent of normal voltage. Hence the synchronous impedance drop for full load current and full load field excitation for this machine is equal to the distance AM on curve chart 8. Point M being the point on the zero power factor full load current saturation curve corresponding to full load field current. In percent of normal voltage this full load synchronous impedance drop, AM, equals 131.5 - 88.5 or 43% which is equal to .43 x 1270 or 546. volts per phase.

Now, as we have already determined, the synchronous impedance drop for maximum braking torque is to equal 1180 volts or 93% of normal voltage, see equation (60). Next then, we proceed to lay off a distance AB on the full load field line equal to 93% , or in other words we make JB equal to 131.5 - 93 or 38.5% of normal voltage. It is evident then that the point B is a point on the zero power factor saturation curve corresponding to the still unknown braking current at the beginning of the braking period.

On curve chart 8 the base of the triangle, OCL or EDF, that is OC or ED is equal to the percent field excitation required to give full load armature current on short circuit. The height of this triangle LN or FP is equal to the reactance drop of the armature per phase expressed in percent of normal voltage and the distance NC or PD is equal to the demagnetizing effect of the armature current at full load current and zero powerfactor, expressed in percent of the field current at full voltage and no load. These quantities being determined either from test or from design calculations, as the case may be.

As is well known to electrical engineers if this triangle OCL or EDF is moved so as to keep its vertex L or F on the no load saturation curve and its base OC or ED parallel to the position OC, the vertex D of the triangle will follow along the zero power factor full load current saturation curve.

But as stated a few paragraphs above, the point B is a point on the zero power factor saturation curve of a load current which is the still unknown current at the beginning of the braking period. If then, a triangle HBG is drawn similar to the triangles OCL and EDF and having one vertex at B and being drawn of the correct size so that its vertex G will fall on the no load saturation curve, it is evident that if this triangle is moved so as to keep its base HB parallel to OC and its vertex G on the no load saturation curve, its vertex B will describe the zero power factor saturation curve of a load current corresponding to the unknown braking current; in the same way that the vertex

D of the triangle EDF followed the zero power factor full load current saturation curve when moved in a similar manner.

Furthermore, just as the height or base of the triangles OCL and EDF are proportional to the full load current, so the height or base of this similar triangle HBG are proportional to the unknown braking current at the beginning of the braking period. Then all we need to do to find this braking current is to multiply the full load current by the ratio of HB : ED, for example,

But full load current equals 131. amperes, and the line HB scales 148. units long while line ED scales 83. units long. Therefore:

$$\text{INITIAL BRAKING CURRENT } \frac{HB}{ED} \times I_1 = \frac{148}{83} \times 131. = 233.5 \text{ Amps}$$

This initial braking current is amperes per phase and is the same in the armature winding of the synchronous machine and in each "leg" or phase of the external resistance, since both the armature and the external resistance are connected in star.

Now since we have previously determined the voltage drop per phase across the external resistance to be 1180 volts, and have now determined the current at the beginning of the braking period to be 233.5 amperes per phase it is easy to find the required resistance per phase to give the maximum braking torque or effort at the beginning of the braking period. If we call this resistance R_B then for this machine;

$$R_B = \frac{1180}{233.5} = 5.07 \text{ OHMS PER PHASE} \quad (61)$$

The watts (I^2R) loss developed in each phase of the resistance by this braking current of 233.5 amperes will be $(233.5)^2 \times R_B$ and the total watts (I^2R) loss in all three phases at the beginning of the braking period will be 3 times this quantity. If we call these total watts W_B then for this machine;

$$W_B = 3 \times (233.5)^2 \times 5.07 = 830,000. \text{ WATTS} \quad (62)$$

This is equivalent to 1110. HP of braking energy. Then the maximum braking torque T_B at the beginning of the braking period can be found from the usual formula giving the relation between torque, T, horsepower, HP, and speed, RPM, namely,

$$T = \frac{HP}{RPM} \times 5,250. \quad (63)$$

Then in this case

$$T_B = \frac{1110}{450} \times 5,250 = 12,950; \text{ (LBS. AT ONE FOOT RADIUS)} \quad (64)$$

The flywheel effect or WR^2 in pounds times the radius of gyration in feet squared of this motor is equal to 9,000/ lbs. x $(\text{Ft.})^2$.

If this same maximum braking torque were to be exerted throughout the entire braking period from synchronous speed to standstill, that is from 450 RPM to zero speed, the time required to stop the motor alone could be figured from the well known formula,

$$t = \frac{WR^2 \times RPM}{307. \times T} \qquad (65)$$

which gives the relations between time in seconds, t, WR^2 in pounds feet squared, speed in RPM and accelerating or retarding torque, T, in pounds at one foot radius. Then in this case the time in seconds to stop the motor alone would be

$$t = \frac{WR^2 \times RPM}{307 \times T_B} = \frac{9,000. \times 450}{307. \times 12,950.} = 1.02 \text{ seconds} \qquad (66)$$

The average speed during this braking time would be $\frac{450}{60} \times 1/2$ or 3.75 revolutions per second. Hence the number of revolutions required to stop would be, in this case

$$\text{REV. TO STOP} = t \times R.P.S. = 1.02 \times 3.75 = 3.825 \text{ REVOLUTIONS} \qquad (67)$$

Of course, it must be remembered that this braking time and revolutions to stop are for the motor alone, having a WR^2 = 9000. If the WR^2 of the driven machine was, say, twice as great as the motor, that is if the total WR^2 was equal to 27,000., the number of seconds as well as the number of revolutions to stop would be three times as great as figured above.

This braking time and revolutions to stop are based on the assumption that the maximum braking torque T_B is going to be exerted throughout the whole braking period from synchronism to standstill. In order that this may be true the external resistance must be decreased during the braking period in such a way that the watts expended in the external resistance will dedrease directly as the speed decreases. In making such a change in the external resistance it must also be remembered that the total generated voltage of the synchronous machine, with constant field current, will decrease directly with the speed.

The resistance of the armature winding can be taken into account, with sufficient accuracy, by subtracting the armature resistance per phase, which is comparatively small, from the figured external resistance to be connected per phase. This can be done with sufficiently accurate results because the component of voltage representing the (IR) drop in the armature winding and the component of voltage representing the (IR) drop in the external resistance, both being in phase with the current are in phase with each other.

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