# ESTIMATING INPUT-OUTPUT RELATIONSHIPS FOR WHEAT IN MICHIGAN USING SAMPLING DATA, 1952-54

Thesis for the Degree of M. S.

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Hsiang Hsing Yeh

1955

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By

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AN ABSTRACT

Submitted to the College of Agriculture of Michigan
State University of Agriculture and Applied
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requirements for the degree of

MASTER OF SCIENCE

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Approved by Dale E Hathaway

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## ABSTRACT

The purpose of this study is to test a technique of estimating input-output relationships for wheat in Michigan using sampling data. A better knowledge of these input-output relationships are needed to make recommendations to farm operators regarding the optimum amount of fertilizer of a given nutrient combination that they should use in order to maximize their profit from producing wheat.

The survey data was a portion of a Farm Management Survey Questionnaire collected from 411 farms by interviews and covered the crop years 1952-54. The input-output relationships were derived by fitting a Cobb Douglas function. This is an exponential equation, linear in logarithmic form and in that form is expressed as

$$\log X_1 = \log a + b_2 \log X_2 + b_3 \log X_3 + \ldots\ldots + b_{22} \log X_{22}$$

where $X_1$ is the dependent variable and $X_2 \ldots\ldots X_{22}$ are groups in independent variable inputs and the $b_i$'s are elasticities of $X_2 \ldots\ldots X_{22}$ with respect to dependent variable. The equation was fitted by the least square regression technique to find the $b_i$'s.

The fertilizer data were fitted in two different forms. In the first form, fertilizer application were classified into total pounds of nitrogen, total pounds of phosphorus, and total pounds of potassium. The variable for nitrogen top dressing as one independent variable. The second form of fertilizer application were classified into nitrogen applied at the planting time and nitrogen top dressing as two independent variables.

The data from the field survey were fitted in four different equations. The first two equations included five and six variables and the other two equations included seventeen and eighteen variables. They then were compared with data which has been available from controlled experiments on plots on the University experimental farms.

The tentative results of this study indicate that comparison of the most profitable rate of fertilizer application between survey and experimental data and the least cost combination which obtained from survey data might be as follows:

The most profitable rate of 3-12-12 fertilizer application obtained from the results of the five variable equation was approximately 220 pounds and from the seventeen variable equation was approximately 260 pounds while the most profitable rate of 0-12-12 fertilizer plus 10 pounds nitrogen top dressing in the six variable equation was approximately 48 pounds and of 0-30-10 fertilizer plus 10 pounds nitrogen top dressing in the eighteen variable equation was approximately 46 pounds. The 0-12-12 and 0-30-10 fertilizers were the least cost combination that were derived from six and eighteen variable equations. If these figures were compared with the experimental results, it appears that the application rate of 3-12-12 fertilizer indicated by the survey data might be slightly more than one-half those indicated by the experimental results. The most profitable rates of 0-12-12 and 0-30-10 fertilizer plus 10 pounds nitrogen top dressing was not available from experiments to compare with results from the survey data, but the results derived from the survey data appeared low. Nitrogen top dressing was used as an independent variable in the survey data and as a fixed variable in experimental design

The least cost combination for N-P-K fertilizer analysis were approximately 2-2-1 in the five variable equation and 4-10-1 in the seventeen variable equation while the least cost combination for N-P-K fertilizer plus nitrogen top dressing were approximately 0-1-1 in the six variable equation and 0-3-1 in the eighteen variable equation. If these combinations were compared with 1-4-4 in experimental data, it appears that an increase in nitrogen and a decrease in potassium in the fertilizer analysis used by farmers might be recommended in the future years. It also suggests that the future research projects for determining the optimum fertilizer applications should give more attention to least cost combinations than has been the case in the past.

Thus, while the results of this analysis indicate that the use of survey data to determine input-output relationships is feasible, further testing and methodological research is indicated before it can become a useful research tool.

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The author assumes responsibility for any errors in this manuscript.

TABLE OF CONTENTS

TABLE OF CONTENTS (Continued)

LIST OF TABLES

LIST OF TABLES (Continued)

LIST OF FIGURES

# CHAPTER I

## INTRODUCTION

Wheat is Michigan's most important cash grain crop. Since 1866, when the first crop estimates were made, the acreage of wheat has fluctuated between the high of 1,985,000 acres in 1882 and in 1884 to the low acreage of 661,000 in 1943. The acreage of wheat[1] has been reduced in recent years by the acreage allotment program. But, the yield of wheat per acre has had a definite upward trend. The first five-year (1866-70) average was 14.8 bushels per acre. The 1945-49 average was 26.3 bushels per acre. The average yield was 30.0 bushels per acre in 1954. The upward trend in part is due to improving some practices and increasing the rate of fertilizer application.

The objective of this study is to test a technique of estimating input-output relationships for wheat in Michigan using sampling data. A better knowledge of these input-output relationships are needed to make recommendations to farm operators regarding the optimum amount of fertilizer of a given nutrient combination that they should use in order to maximize their profit from producing wheat.

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[1] The data on acreages and yields were obtained from Michigan Agricultural Statistics, Lansing, Michigan.

Up to the present time there has been very little input-output data available for most Michigan crops. The data that has been available has come from controlled experiments on plots on the University experimental farms. Most of the experiments on crop response to fertilizer applications have not been designed to answer economic questions. Due to limitations of cost they generally have included a relatively limited range of applications of fertilizers. In nearly all cases they have been done using one or two different analysis of fertilizer which have assumed that the nutritive ratios tested were the least-cost combination under all conditions. In addition, the application of results from experimental plots to actual farm conditions makes several assumptions regarding the relationships between fertilizer applications and other production practices.

It was felt that survey data on input-output relationships on farms might provide one method of obtaining increased amounts of input-output data for some crops.

The survey data in this case was a portion of a Farm Management Survey Questionnaire collected from 411 farms in 1954 which included yield and input data for wheat for the crop years 1952-54 in four areas of the state.

It was felt that this input-output data from this survey might meet a desirable degree of reliability of results at a lower cost than the information could be obtained by other methods. In addition, the survey data is related to some actual farm conditions and if useable might overcome some of the problems of applicability of experimental data to farm conditions.

Some economic theories relevant to these problems are discussed and related to the farm business in Chapter II.

Chapter III describes the survey data used in this study and methodology employed in measuring the input-output relationship by fitting a Cobb-Douglas production function.

The effects of fertilizer application of various rates on total and marginal physical product of wheat from available experimental data are presented and discussed in Chapter IV. These data are used both as a check on the survey data and to indicate the optimum amount of fertilizer that should be used according to the experimental results currently available.

Chapter V deals with the fitting of the function to the survey data and an appraisal of the statistical results. Comparison of the results between experimental and survey data are indicated in this chapter.

The final chapter includes the general conclusions obtained from the research and suggests possible future research that might be done in this area.

# CHAPTER II

## THEORETICAL BACKGROUND

A statement of a useful classification which covers most of existing economic theory divides it into two broad categories. The first category of economic theory is the static economic theory covering micro and macro-static economic theory. The theory usually considered when the word static is used is a theory of a given number of exact relationships among the same given number of economic variables. An exact relationship, as used herein, is one which has a standard deviation of zero. In a theory of exact static relationships, the magnitudes of certain variables can and are permitted to change as the theory is used to explain the changes which occur when the value of one or a set of variables is changed. The second category of economic theory is the dynamic economic theory covering micro and macro dynamic economic theory. This theory is concerned with the time relationships of economic variables.

There have been various levels of development and integration achieved within and between these various categories. Both micro-static and macro-static economic theories have been rather well developed and integrated. On the other hand, the dynamic systems of theory have yet to achieve a comparable development and integration.

A major difficulty associated with static economic theory has been the nature of the assumptions made. By assuming as fixed most of the changes or variable factors, the system has been simplified at the expenses of reality. Technology, wants and preferences, asset distribution, and institutions have been posited as unchanged. At the same time, individuals have been assumed both rational and as having perfect knowledge.

Dynamic systems of economic thought have arisen when one or more of the static assumptions have been relaxed. Although the dynamic systems may be more realistic in their assumptions, the attendant complexities have inhibited their usefulness to a large extent.

One of the areas of static production theory is factor-factor and factor-product analysis. This area is concerned with problems of resource combination and allocation in the production of a single product. In this thesis, static factor-factor and factor-product theory is applied to both experimental data and sampling data by the joint use of mathmatics (as represented by the "Cobb-Douglas function") and statistics.

## The Production Function

The production function has been expressed as the concept of the factor-product relationship or the input-output relationship. This concept refers to the relationship between the input of factor and the output of product.

The problems for which static factor-factor and factor-product analysis are useful are one, the determination of proportions in which inputs should be combined to yield a maximum of a product for a given input or outlay; and, two, the determination of how much of the various inputs, combined in optimum proportions are required to yield a maximum profit.

A production function may be expressed as an algebraic equation of the form

$$Y = f \quad (X_1 \mid X_2, X_3, X_4, \ldots, X_n)$$

Its meaning is that output Y is dependent on a variable factor $X_1$ while $X_2$ is fixed along with resources $X_3$ through $X_n$. When some variable inputs are held constant and others are allowed to vary, a special relationship exists between input and the rate of output of the product. This relationship has been termed the Law of Diminishing Returns. It may be stated as follows:

"When output depends upon both variable and fixed inputs, the addition of a variable input to fixed inputs results first in output which increase at an increasing rate, then increase at a decreasing rate and finally tends to decrease."

The production function is divided into three stages. Stage II is a rational area of production with technical efficiency while both stage I and III are irrational area of production with technical inefficiency. These are shown graphically in Figure 1.

In stage II, marginal physical product decreases continuously and is always less than average physical product and greater than or equal to zero. The boundary of the rational area of production extends from the input denoting a maximum average physical productivity to the one defining the maximum total physical productivity. In other words, all

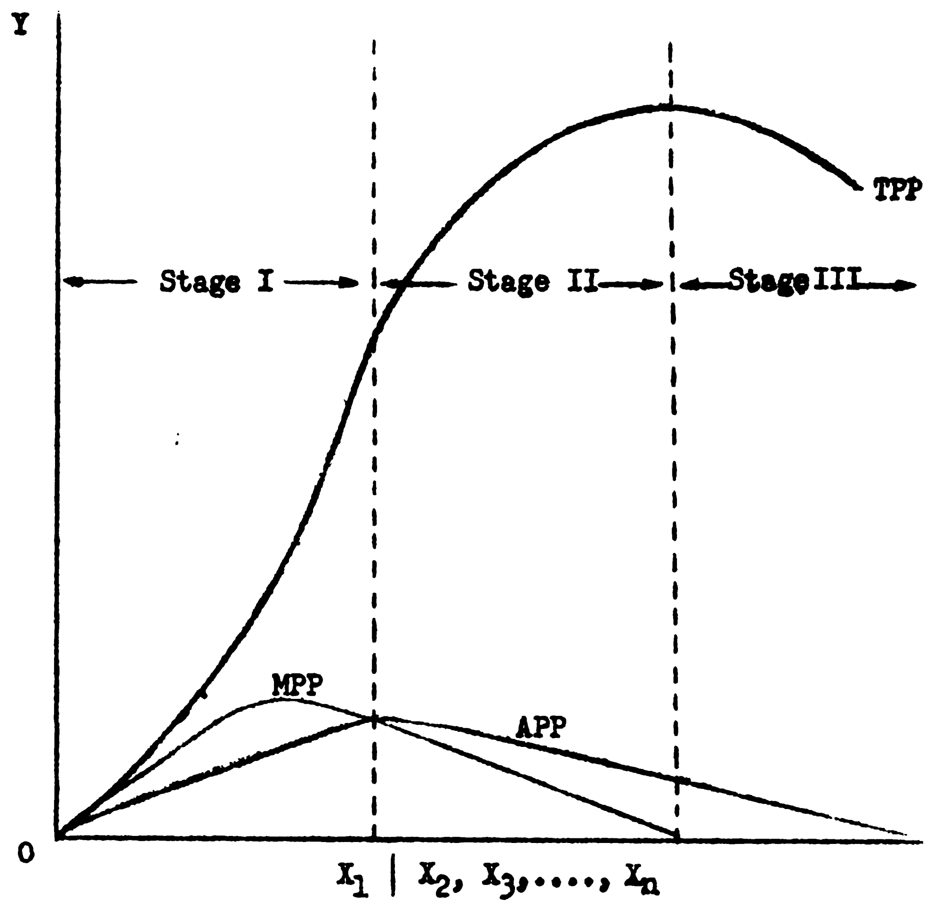


Figure 1. Relationship of Factor Input to Total, Marginal, and Average Physical Product, Stages of Production and Rational Use of Resources.

resource inputs between the one consistent with a maximum product per unit of variable factor and the one consistent with a maximum product from the fixed factor fall in stage II.

In stage I, marginal physical product is always greater than average physical product. The average physical productivity of the variable resource will increase continuously as more units are applied to the fixed factor. As production per unit of fixed factor is pushed to the limits of stage I, a greater product is forthcoming from the fixed factors as well as from each unit of the variable factor. Any level of resource use falling in stage I is uneconomic. In stage III, marginal physical product is less than zero, and additional input reduces total output. So, stage III is also an area of irrational production.

In most farm management work, it has been assumed that stage II is the area in which farms are being operated and that farmers cease application of variable resources to fixed factors before the upper limit of stage II has been reached.

A production function involving two variable inputs and an indefinite, but given, number of fixed inputs may be expressed in the following form as

$$Y = f \quad (X_1, X_2 \mid X_3, \ldots, X_n)$$

This equation states that the output of Y depends jointly upon the inputs of $X_1$ and $X_2$ in some definite manner when the inputs $X_3$ to $X_n$ are held constant. This function may be represented graphically as in Figure 2. The product contour lines $Y_1$, $Y_2 \ldots Y_{10}$ are called Iso-product curves because equal amounts of Y are produced by the various combination of $X_1$ and $X_2$ represented by a given line $Y_1$. Iso-product line $Y_2$ is greater than $Y_1$ until point E is reached indicating the highest possible output of Y. The dotted lines, OC and OD, are ridge lines and enclose both stage I and stage II since in that area all positive derivatives of Y as dictated by the function $Y = f \ (X_1, X_2 \mid X_3, \ldots, X_n)$ are in this area. The marginal physical productivity of both factors $X_1$ and $X_2$ are greater than zero falling within the area of the ridge lines while less than zero falling in outside of the ridge lines. Quantities of Y also exist beyond the ridge lines but production outside of the area requires greater amount of either $X_1$ and/or $X_2$ for the same output than the area enclosed by the ridge lines. Ridge lines, the Iso-product lines, the Iso-cost line and the scale line are shown in Figure 2. The line AB is known as an Iso-cost line because all combinations of $X_1$ and $X_2$ represented by it have equal total costs. In Figure 2 the Iso-cost line is tangent to Iso-product line $Y_4$. Here is the greatest output of Y for a given quantity of $X_1$, $X_2$ used at that point G.

At the point of tangency, the slopes of the two lines are equal. The slope of the Iso-product line is the rate of product substitution of $X_2$ for $X_1$ which is $\frac{MPP_{x_2}}{MPP_{x1}}$ and the slope of the Iso-cost line is the rate of outlay substitution which is $\frac{P_{x_2}}{P_{x_1}}$. Therefore, the rate of product substitution is equal to the rate of outlay substitution at the point of tangency. This may be expressed in form such as $\frac{MPP_{x_2}}{MPP_{x_1}} = \frac{P_{x_2}}{P_{x_1}}$. This is the optimum combination of resources. If a series of these points are joined together the resultant line (EF shown in Figure 2) is termed the scale line because it indicates the optimum proportions in which to combine $X_1$ and $X_2$ to produce any given output. For more

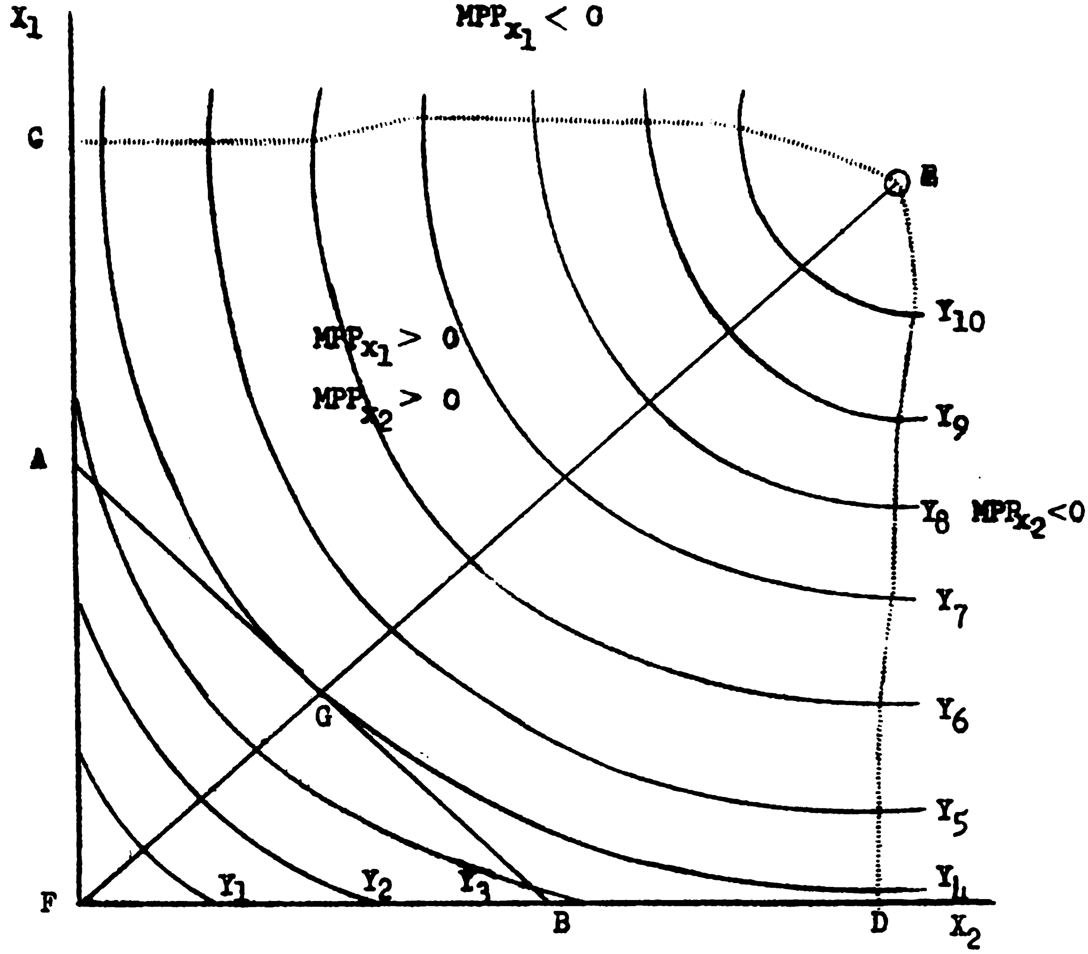


**Figure 2. A Production Surface Showing Iso-Produce Lines, Iso-Cost Line, Ridge Lines and Scale Line.**

than two inputs, they may be combined in scale line proportion. The optimum combination of $X_n$ inputs may be determined by

$$\frac{MPP_{x_1}}{P_{x_1}} = \frac{MPP_{x_2}}{P_{x_2}} = \cdots = \frac{MPP_{x_n}}{P_{x_n}} = 1$$

## Substitutability and Complementarity Relationships

Resources can be either substitutes or complements. They are complements where they must be combined in fixed proportions or where a reduction in input of one factor cannot be replaced by an increase in other factor. Factors are substitutes when output can be maintained as resources are reshuffled, when one factor is reduced in quantity, a second factor must always be increased. Resources which are complements or substitutes can be defined by the marginal rate of substitution and are illustrated in Figure 3.

Resources are complements when the marginal rate of substitution is zero or greater.[1] In Figure 3, the ratio of change, $\Delta X_2 / \Delta X_1$, is zero when output is constant and input of $X_2$ is at the minimum of OT. If $X_1$ is increased beyond OR, none of $X_2$ is replaced.

Resources are substitutes when the marginal rate of substitution is less than zero. In other words, the sign of $\Delta X_2 / \Delta X_1$, the marginal rate at which one factor substitutes for another, must be negative.

A product contour characterizing these relationships is illustrated in Figure 3. If output is maintained at the 100 units

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[1] Heady, Earl O. Economics of Agricultural Production and Resource Use. New York: Prentice-Hall, Inc., 1952, pp. 146-149.

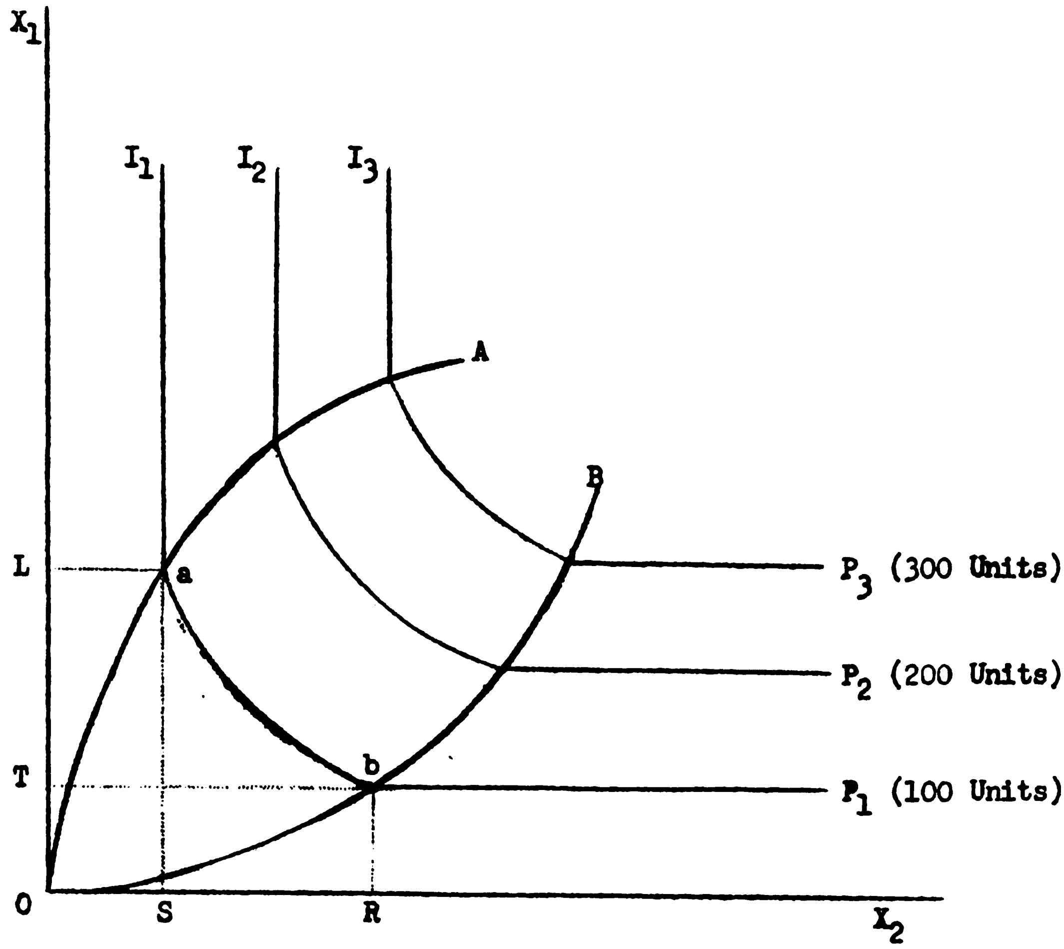


Figure 3. A Production Surface Showing Substitution and Complementary Resources.

indicated by contour $I_1$ $P_1$, at the 200 units by contour $I_2$ $P_2$ and at the 300 units by contour $I_3$ $P_3$, $X_1$ and $X_2$ serve as substitutes only between the extent of the ridge lines as OA and OB. Beyond these extent, increase in input $X_2$ require that inputs of $X_1$ also be increased.

## The Cobb-Douglas Production Function

The Cobb-Douglas production function is an exponential equation, linear in logarithmic form. It was developed by a mathematician, Charles W. Cobb of Amherst College in cooperation with Paul H. Douglas, an economist now in the United States Senate,[2] in 1927-28.

In natural numbers, the equation is written $Y = aX_1^{b_1} X_2^{b_2} \ldots X_n^{b_n}$ where Y is the dependent variable, $X_1$ to $X_n$ are independent categories of inputs and the exponents ($b_i$'s) are elasticities of the dependent variable with respect to their corresponding independent categories of inputs and a is a constant.

In logarithmic form, the equation is linear and is fitted by linear multiple correlation using the least square technique to determine the $b_i$'s. The equation written in logarithmic form becomes:

$$\log Y = \log a + b_1 \log X_1 + b_2 \log X_2 + \ldots + b_n \log X_n$$

A function of this type allows the diminishing physical productivity to be reflected in the estimated relationship between the dependent and independent categories. The sum of the b's indicates the nature of the productivities to scale for the business as a whole. Diminishing

2 Cobb, Charles W. and Paul H. Douglas. "A Theory of Production," The American Economic Review, Vol. XVIII, Supplement, (March, 1928).

physical productivity exists if the sum of the b's is less than one, increasing if greater than one and constant if equal to one.

This function is capable of reflecting any one stage of production, but for any input or the sum of any combination of inputs, it can reflect only one stage for different inputs at the same time.

## Value Productivity and Profit

The physical relationships may be converted to value relationships by multiplying the marginal, average, and total physical products by the price of the product. Thus the value of the marginal product, the value of the average product, and the value of the total product may be obtained. Under conditions of perfect competition assuming homogeneity and divisibility of factors and products it is also true that VMP = MVP, VAP = AVP, and VTP = TVP. The value productivity curves are shown in Figure 4.

The determination of the maximum profit point, when the price of the product and of the variable inputs is known, can be shown graphically as in Figure 4 where at point B the $P_{X_1}$ line intersects the MVP line.

These economic theories provide the analytical framework used in this study to measure input-output relationships for wheat with experimental (Chapter IV) and survey data (Chapter V). This gave a basis for estimating the possibility of getting useful information via the two different methods.

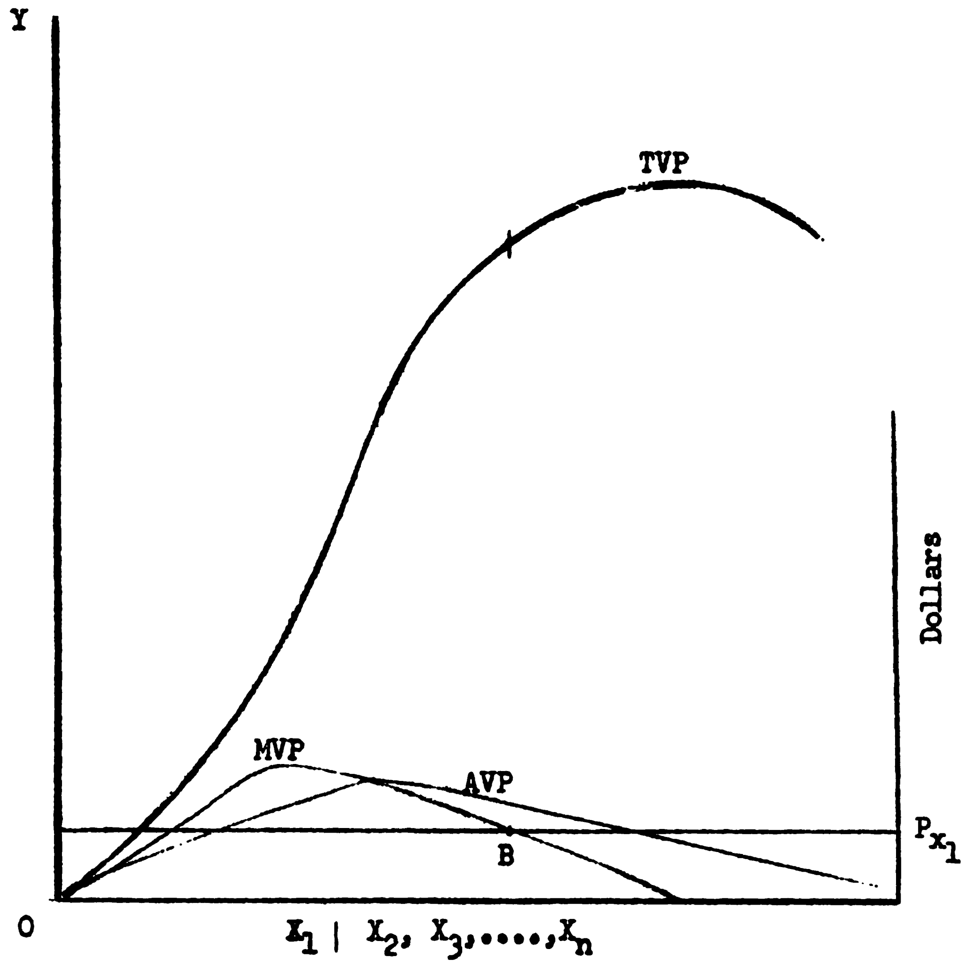


Figure 4. Relationship of Factor Input to Total, Marginal and Average Value Production and Location of the High Profit Point Using Variable Factor $X_1$ with $X_2$, $X_3, \ldots, X_n$ Fixed.

# CHAPTER III

## METHODS OF SAMPLING AND ANALYSIS

### Methods of Gathering Data

The farms visited were selected by the random sampling method and were drawn from the wheat listing sheets of the County Agricultural Stabilization and Conservation Committee. The Sampling data were collected from 411 farms in four areas covering nineteen townships in five counties. The townships within those counties were selected with an attempt to maintain approximately a uniform soil type within each of the areas. Data on the calendar years 1952 through 1954 were collected. The farms that were visited were distributed among those four areas as follows:

Five townships in the southern part of Kalamazoo County were included in area I and 103 farms were interviewed in this area. The major farm enterprises are wheat, corn, dairy, and hogs. The soil type is well drained sandy loam.[1]

In area II, two counties, Gratiot and Isabella, were selected. Three of the townships and 69 farms were in Gratiot County and 40 farms were interviewed in two townships in Isabella County. The

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1 Hill, Elton B. and Russell G. Mawby. Types of Farming in Michigan. Michigan Agricultural Experiment Station, East Lansing, Special Bulletin 206, September 1954.

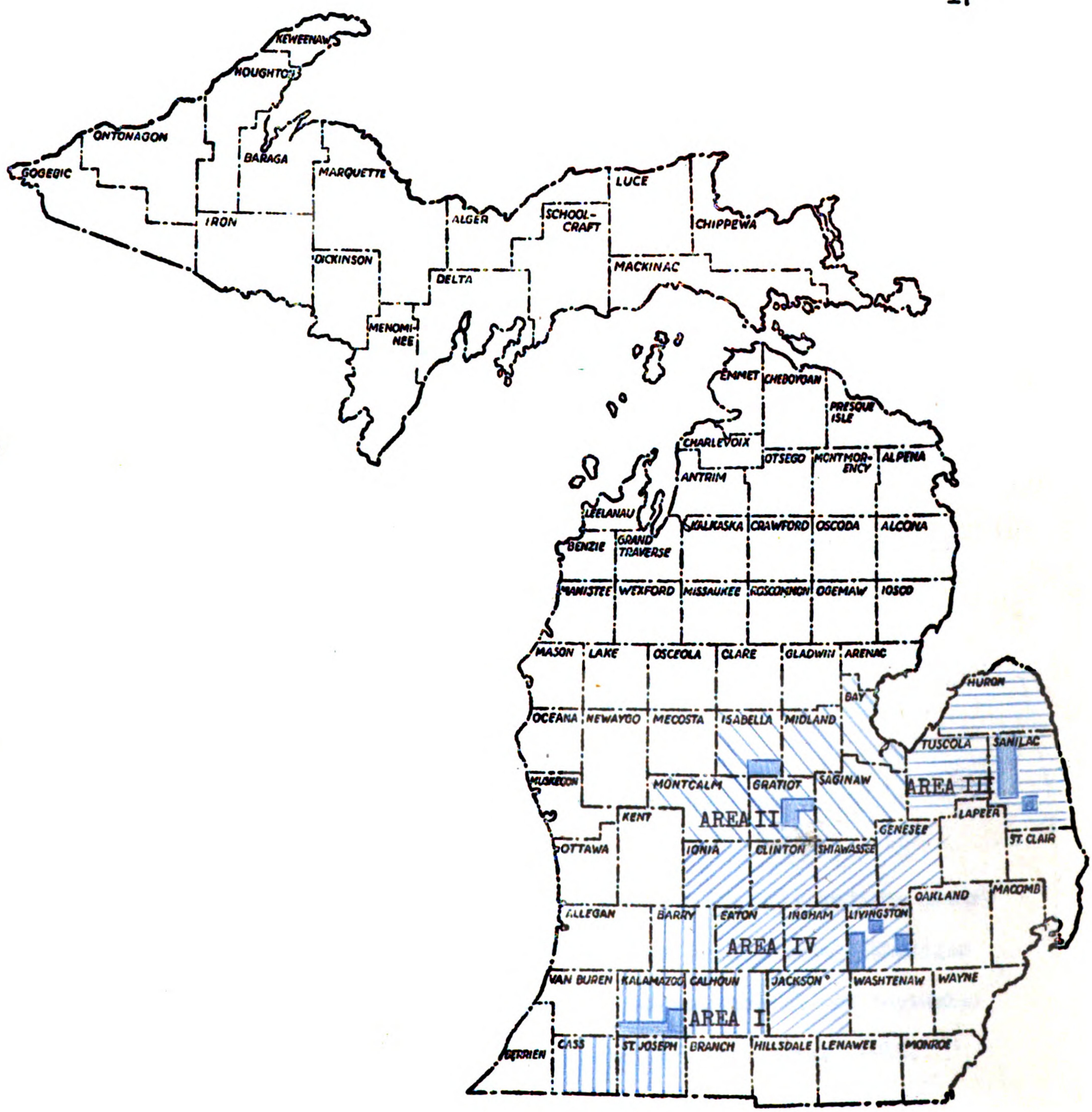


Figure 5. Map of Michigan Showing Counties in Which Surveys Were Taken.

major farm enterprises are cash crops and dairy. The soil type is clay loam to silty clay loam soils.

Five townships in Sanilac County were included in area III and 99 farms were visited. Cash crops and dairy are important in this area. The soil type is poorly drained loam, silt loam and clay loam.

Four townships in Livingston County were included in area IV and 100 farms were selected. The major farm enterprises are dairy and general farming in this area. The soil type is well drained loamy sands and sandy loam.

A portion of the data sheet used in gathering the information is shown in Appendix A. The major contents of the questionnaire are as follows:

1. Acreage of wheat.
2. Yield per acre of wheat.
3. Seed quality and fertilizer practices for wheat.
4. Livestock enterprises on the farm.

This interviewing was done by four graduate students and one undergraduate student in the Department of Agricultural Economics, Michigan State University. The data were gathered between June 24 and September 16, 1954 and were tabulated between October, 1954 and April, 1955.

## Method of Analysis

The estimates for this study were derived by fitting a Cobb-Douglas function. The equation was of the form: $X_1 = aX_2^{b_2} X_3^{b_3} \ldots X_{22}^{b_{22}}$ and appeared in the log form as

$$\log X_1 = \log a + b_2 \log X_2 + b_3 \log X_3 + \ldots + b_{22} \log X_{22}.$$

By converting the basic data to logs, it is possible by multiple linear regression technique to determine a mean relationship between the independent variable factor categories and the dependent variable, physical yield of wheat, which is (1) linear in the logs and (2) exponential in the natural number.

The variables included in the equations are indicated as follows:

$X_1$: is the logarithm of total yields of wheat.

$X_2$: is the logarithm of total pounds of nitrogen plant food including nitrogen top dressing.

$X_3$: is the logarithm of total pounds of phosphorus plant food.

$X_4$: is the logarithm of total pounds of potassium plant food.

$X_5$: is the logarithm of total pounds of nitrogen top dressing.

$X_6$: is the logarithm of total pounds of nitrogen applied at the planting time.

$X_7$: indicates seed quality, where log 1 indicates certified seed was not used while log 10 indicates certified seed was used.

$X_8$: indicates the animal units per acre. Log 10 indicates the presence of no animal units per acre while log 1 indicates some livestock in 1953.

$X_9$: indicates the animal units per acre. Log 10 indicates 0.01--0.19 animal units per acre while log 1 indicates the presence of more or less than 0.01--0.19 animal units per acre in 1953.

$X_{10}$: indicates the animal units per acre. Log 10 indicates the presence of 0.20 or more units per acre while log 1 indicates less than 0.20 units in 1953.

$X_{11}$, $X_{12}$....$X_{22}$: indicates the relationships between four areas in which interviewing was done and three years 1952, 1953 and 1954 as follows:

| Year | Areas | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| 1952 | $X_{11}$ | $X_{14}$ | $X_{17}$ | $X_{20}$ |
| 1953 | $X_{12}$ | $X_{15}$ | $X_{18}$ | $X_{21}$ |
| 1954 | $X_{13}$ | $X_{16}$ | $X_{19}$ | $X_{22}$ |

Log 10 indicates the observed event occurred in a given area and year while log 1 indicates that the event did not occur.

$X_{11}$: Log 10 indicates the observed event occurred in area I in 1952 while log 1 indicates that it did not occur in area I in 1952.

$X_{12}$: Log 10 indicates the observed event occurred in area I in 1953 while log 1 indicates that it did not occur in area I in 1953.

$X_{13}$: Log 10 indicates the observed event occurred in area I in 1954 while log 1 indicates that it did not occur in area I in 1954.

$X_{14}$: Log 10 indicates the observed event occurred in area II in 1952 while log 1 indicates that it did not occur in area II in 1952.

$X_{15}$: Log 10 indicates the observed event occurred in area II in 1953 while log 1 indicates that it did not occur in area II in 1953.

$X_{16}$: Log 10 indicates the observed event occurred in area II in 1954 while log 1 indicates that it did not occur in area II in 1954.

$X_{17}$: Log 10 indicates the observed event occurred in area III in 1952 while log 1 indicates that it did not occur in area III in 1952.

$X_{18}$: Log 10 indicates the observed event occurred in area III in 1953 while log 1 indicates that it did not occur in area III in 1953.

$X_{19}$: Log 10 indicates the observed event occurred in area III in 1954 while log 1 indicates that it did not occur in area III in 1954.

$X_{20}$: Log 10 indicates the observed event occurred in area IV in 1952 while log 1 indicates that it did not occur in area IV in 1952.

$X_{21}$: Log 10 indicates the observed event occurred in area IV in 1953 while log 1 indicates that it did not occur in area IV in 1953.

$X_{22}$: Log 10 indicates the observed event occurred in area IV in 1954 while log 1 indicates that it did not occur in area IV in 1954.

The data from the questionnaires were arranged in the coded sheets and were converted into logs and then punched on International Business

Machine cards. The exponent ($b_i$'s) were computed by using the Doolittle Method. The computation work was carried out partly on the Friden Calculator and partly on the International Business Machine. The five and six variable equations did not include all of the variables and were computed on the Friden Calculator. The seventeen and eighteen variable equations using of the all dummy variables were computed on the International Business Machine.

The fertilizer data were fitted into two different forms. In the first form, fertilizer application were classified into total pounds of nitrogen ($N_2$), total pounds of phosphoric acid ($P_2O_5$), and total pounds of potassium ($K_2O$). The variable for nitrogen in this form included nitrogen applied at planting time and nitrogen top-dressing as one independent variable. The second form of fertilizer application were classified into nitrogen applied at the planting time and nitrogen top-dressing as two independent variables.

The total amounts of fertilizer applied by the wheat farmers range from zero pound to 500 pounds. The nitrogen top dressing range from30 pounds to 250 pounds. In addition, to account for the application of animal manure, the equivalent animal units per acre in 1953 were used to adjust for differences in resulting soil fertility, assuming this would have the same effect on the production of wheat. The conversion rates for the various types of livestock are shown in Appendix B.

The dummy variables were used in the seventeen and eighteen variable equations to allow for differences in weather between years and for differences in soil type between the four areas.

# CHAPTER IV

## ANALYSIS OF INPUT-OUTPUT RELATIONSHIPS FROM EXPERIMENTAL DATA

In order to evaluate the usefulness of survey data, it seemed desirable to compare it with available experimental data. Two controlled experiments using different fertilizer applications were available for this purpose.[1] One fertilizer used was 3-12-12 fertilizer. The applications began with zero pounds and increased at 25 pound increments until they reached a maximum of 550 pounds. The yield ranged from 23 bushels of wheat at zero pound of fertilizer to 42.8 bushels when using 550 pounds of 3-12-12. The second experiment included a combination of 3-12-12 fertilizer and nitrogen top-dressing as before, the 3-12-12 was increased in increments of 25 pounds until a maximum of 550 pounds was reached, but in each case there was an additional application of 20 pounds of nitrogen top-dressing. The yield ranged from 27.0 bushels where zero fertilizer was applied to 43.9 bushels where 550 pounds of 3-12-12 plus 20 pounds of nitrogen top-dressing was applied.

The relationships between the total and marginal physical product in the two experiments are shown in Table I and are illustrated graphically in Figures 6 and 7.

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1 The fertilizer input data is the result of experimental work done by the Department of Soil Science, Michigan State University. Fertilizer applications of differing amounts and various analysis were applied on Fox sandy loam in 1946 to 1947.

TABLE I

THE EFFECT OF INCREASING RATES OF FERTILIZER APPLICATION OF THE TOTAL, MARGINAL, AND AVERAGE PHYSICAL PRODUCT FOR WHEAT

| Pounds of Fertilizer | 3-12-12 Fertilizer | | | 3-12-12 Fertilizer Plus 20 Pounds Nitrogen Top-Dressing | | |
|---|---|---|---|---|---|---|
| | TPP[a] Bushel | MPP[b] Bushel | APP[c] Bushel | TPP[a] Bushel | MPP[b] Bushel | APP[c] Bushel |
| 0 | 23.0 | | | 27.0 | | |
| | | 1.5 | | | 1.4 | |
| 25 | 24.5 | | 1.50 | 28.4 | | 1.40 |
| | | 1.5 | | | 1.1 | |
| 50 | 26.0 | | 1.50 | 29.5 | | 1.25 |
| | | 1.5 | | | 1.1 | |
| 75 | 27.5 | | 1.50 | 30.6 | | 1.20 |
| | | 1.5 | | | 1.4 | |
| 100 | 29.0 | | 1.50 | 32.0 | | 1.25 |
| | | 1.4 | | | 1.2 | |
| 125 | 30.4 | | 1.48 | 33.2 | | 1.24 |
| | | 1.4 | | | 1.2 | |
| 150 | 31.8 | | 1.47 | 34.4 | | 1.23 |
| | | 1.3 | | | 1.2 | |
| 175 | 33.1 | | 1.44 | 35.6 | | 1.23 |
| | | 1.3 | | | 1.2 | |
| 200 | 34.4 | | 1.43 | 36.8 | | 1.22 |
| | | 1.3 | | | 1.2 | |
| 225 | 35.7 | | 1.41 | 38.0 | | 1.22 |
| | | 1.3 | | | 1.1 | |
| 250 | 37.0 | | 1.40 | 39.1 | | 1.21 |
| | | 1.0 | | | 1.1 | |
| 275 | 38.0 | | 1.36 | 40.2 | | 1.20 |
| | | 1.0 | | | 0.8 | |
| 300 | 39.0 | | 1.33 | 41.0 | | 1.17 |
| | | 1.0 | | | 0.8 | |
| 325 | 40.0 | | 1.31 | 41.8 | | 1.14 |
| | | 0.9 | | | 0.4 | |
| 350 | 40.9 | | 1.28 | 42.2 | | 1.09 |
| | | 0.7 | | | 0.4 | |
| 375 | 41.6 | | 1.24 | 42.6 | | 1.04 |
| | | 0.5 | | | 0.4 | |
| 400 | 42.1 | | 1.19 | 43.0 | | 1.00 |
| | | 0.5 | | | 0.3 | |
| 425 | 42.6 | | 1.16 | 43.3 | | 0.96 |
| | | 0.4 | | | 0.3 | |
| 450 | 43.0 | | 1.11 | 43.6 | | 0.92 |
| | | 0.0 | | | 0.2 | |
| 475 | 43.0 | | 1.05 | 43.8 | | 0.88 |
| | | 0.0 | | | 0.2 | |
| 500 | 43.0 | | 1.00 | 44.0 | | 0.85 |
| | | -0.1 | | | 0.0 | |
| 525 | 42.9 | | 0.95 | 44.0 | | 0.81 |
| | | -0.1 | | | -0.1 | |
| 550 | 42.8 | | 0.90 | 43.9 | | 0.77 |

[a] TPP: Total physical product for wheat.

[b] MPP: Marginal physical product.

[c] APP: Average physical product.

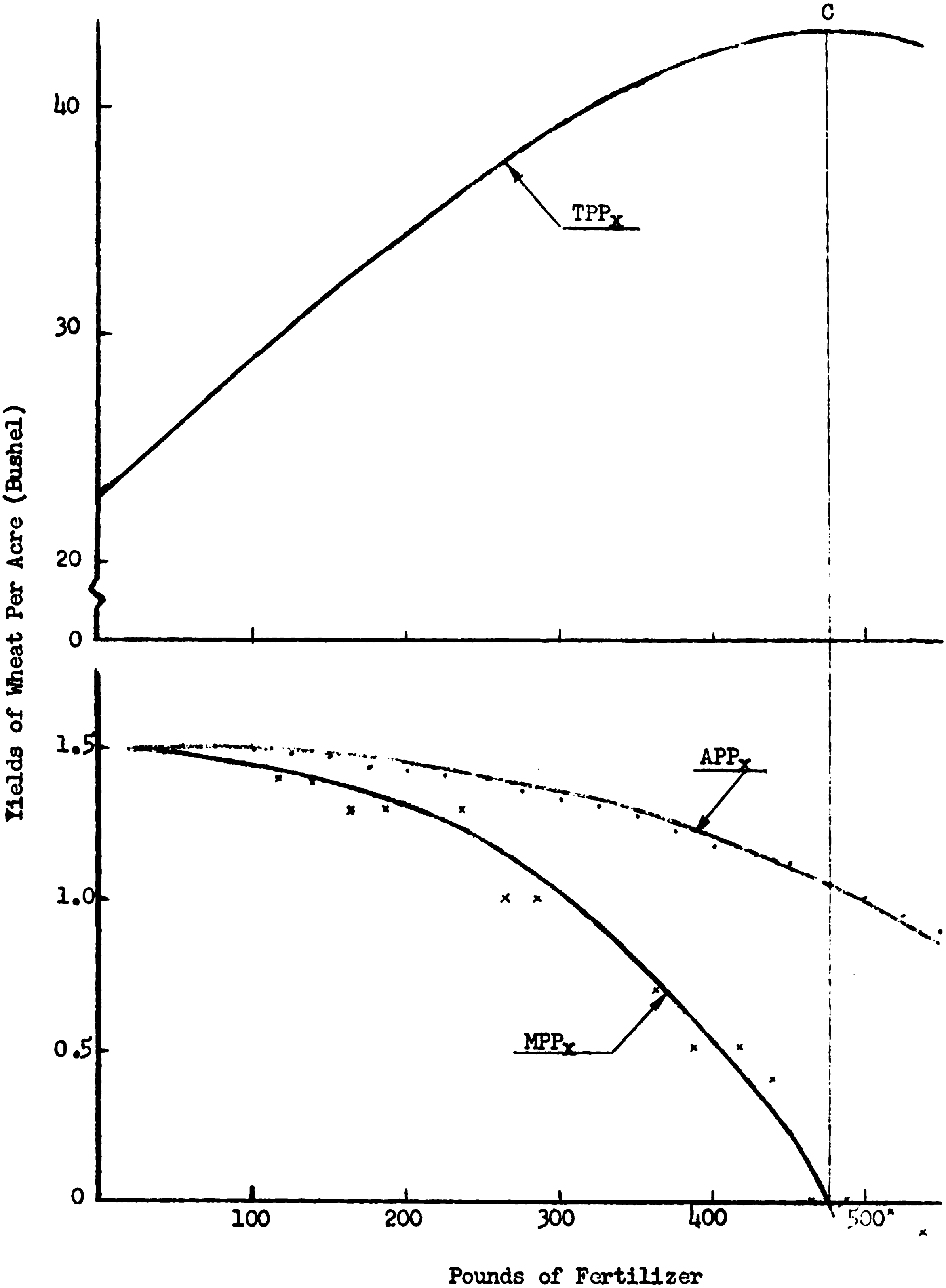


Figure 6. The Effect of Increasing Rates of 3-12-12 Fertilizer Application on the Total, Marginal, and Average Physical Product for Wheat.

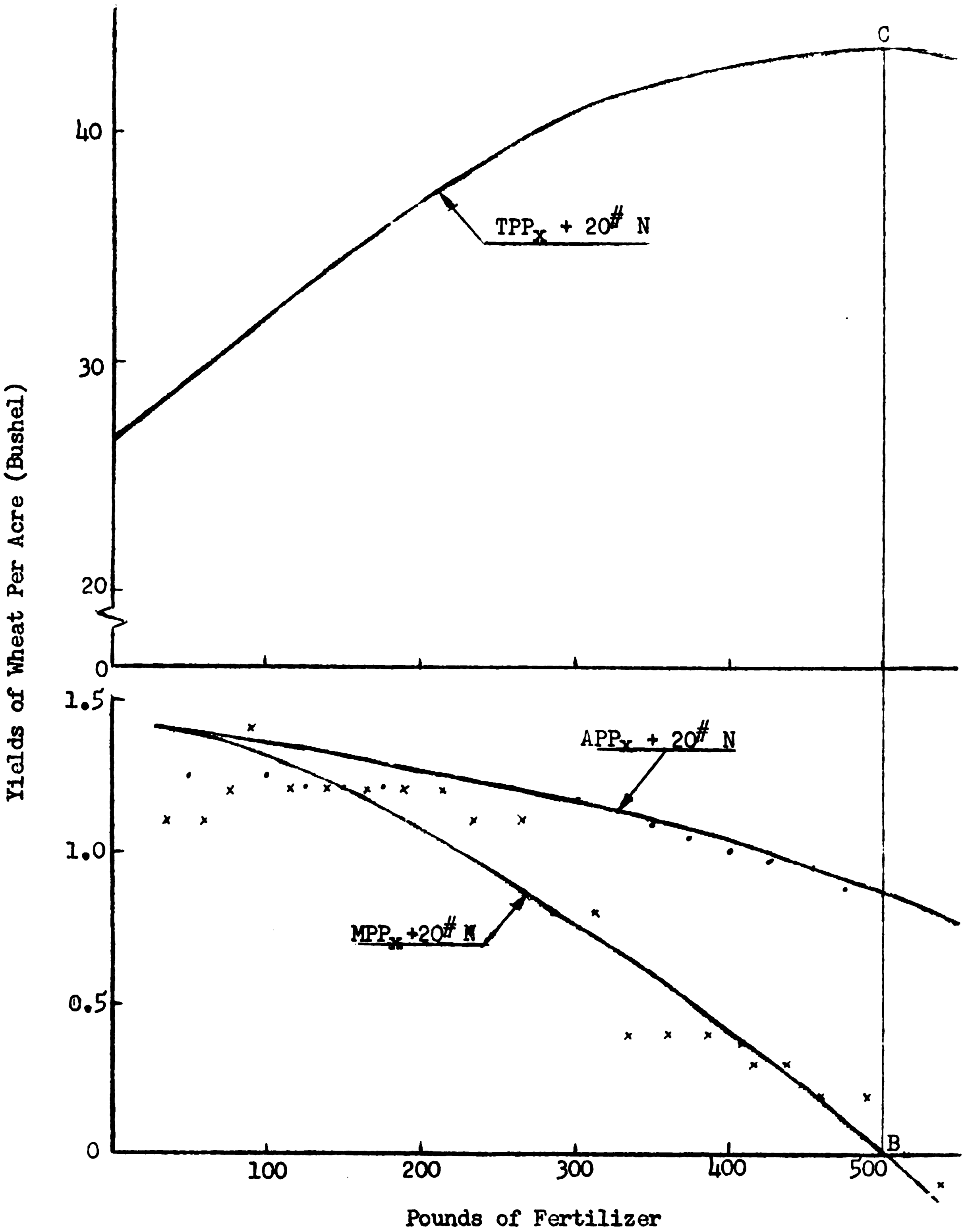


Figure 7. The Effect of Increasing Rates of 3-12-12 Fertilizer Plus 20 Pounds Nitrogen Top Dressing Applications on the Total, Marginal and Average Physical Product for Wheat in Experimental Data.

Where only the 3-12-12 fertilizer was used the $TPP_x$ of wheat continues to increase at a decreasing rate as long as the $MPP_x$ decreases but is greater than zero. It reaches a maximum at a point C in Figure 6, 43.0 bushels, as the $MPP_x$ becomes zero at a point B. The $MPP_x$ decreases continuously and is equal to zero where 19 units of 3-12-12 fertilizer were used. The line BC in the Figure 6 indicates the end of a rational area of production of wheat.

Figure 7 shows the same relationships for the 3-12-12 plus the addition of a 20 pound application of nitrogen top-dressing. Using the same method of locating the end of a rational area of production for wheat, it is reached a point B in Figure 7 where 20 units of 3-12-12 fertilizer plus 20 pounds nitrogen top-dressing is used. The $TPP_x$ + 20 pounds nitrogen reaches a maximum at a point C, 44.0 bushels, as the $TPP_x$ + 20 pounds nitrogen becomes zero at a point B.

In order to determine the point of the highest profit, we must take into consideration the price of the input and the price of the output. Of course, we must realize that recommendation concerning optimum amount of any input which are made under one set of price conditions will change under a different set of price conditions.

We can derive a fundamental condition which will hold true for any problem concerning the optimum use of a variable input. This condition is that the marginal value product (MVP) of any input ($X_1$) must equal the price of the variable input $P_{x_1}$. This equation of this statement can be stated algebraically in the following manner.

$$MVP_{x_1} = P_{x_1} \text{ or } \frac{MVP_{x_1}}{P_{x_1}} = 1$$

This equation tells us several things:[2] (1) If the last unit of $X_1$ does not pay for itself less of $X_1$ should be used, (2) If the last unit of $X_1$ more than pays for itself more of $X_1$ should be used and (3) the use of $X_1$ should be stopped at the point at which $X_1$ just pays for itself.

## The Pricing of Output and Input

If we apply a set of prices to Table II, we can determine the most profitable rate of production. Let the price of each unit of 3-12-12 fertilizer be \$0.50 in 1952, \$0.47 in 1953, and \$0.46 in 1954[3] while the price of each unit of 3-12-12 fertilizer plus 20 pounds nitrogen top-dressing be same as the price of 3-12-12 fertilizer. Because of additional 20 pounds nitrogen top-dressing are kept constant as a fixed cost. The price of each bushel of wheat for 1952 would be \$1.97, for 1953 would be \$2.00 and for 1954 would be \$2.09.[4]

## The Most Profitable Rate of Application of Fertilizer

According to the equation, $MVP_{x_1} = P_{x_1}$ or $\frac{MVP_{x_1}}{P_{x_1}} = 1$, we find the most profitable rate of application in 1952 was 434 pounds of 3-12-12 fertilizer and 436 pounds of 3-12-12 fertilizer plus 20 pounds nitrogen

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[2] Bradford, Lawrence A., and Glenn L. Johnson. Farm Management Analysis. New York: John Wiley and Sons, Inc., 1953, pp. 116-117.

[3] Price of 25 pounds increment of 3-12-12 fertilizer was collected from the Davison Chemical Company, Lansing, Michigan. MFC = \$0.50 in 1952, MFC = \$0.47 in 1953, and MFC = \$0.46 in 1954 for 3-12-12 fertilizer.

[4] Wheat prices are the seasonal average price per bushel for crop years 1952-54 received by Michigan farmers as shown in Michigan Agricultural Statistics.

## TABLE II

### THE EFFECT OF INCREASING RATES OF FERTILIZER APPLICATIONS ON THE MARGINAL VALUE PRODUCT FOR WHEAT IN 1952-54

| Pounds of Fertilizer | 3-12-12 Fertilizer | | | 3-12-12 Fertilizer Plus 20 Pounds Nitrogen Top Dressing | | |
|---|---|---|---|---|---|---|
| | 1952 | 1953 | 1954 | 1952 | 1953 | 1954 |
| | MVP | MVP | MVP | MVP | MVP | MVP |
| | $1.97/bu. | $2.00/bu. | $2.09/bu. | $1.97/bu. | $2.00/bu. | $2.09/bu. |
| 0 | | | | | | |
| | 2.96 | 3.00 | 3.14 | 2.76 | 2.80 | 2.93 |
| 25 | | | | | | |
| | 2.96 | 3.00 | 3.14 | 2.17 | 2.20 | 2.30 |
| 50 | | | | | | |
| | 2.96 | 3.00 | 3.14 | 2.17 | 2.20 | 2.30 |
| 75 | | | | | | |
| | 2.96 | 3.00 | 3.14 | 2.76 | 2.80 | 2.93 |
| 100 | | | | | | |
| | 2.76 | 2.80 | 2.93 | 2.36 | 2.40 | 2.51 |
| 125 | | | | | | |
| | 2.76 | 2.80 | 2.93 | 2.36 | 2.40 | 2.51 |
| 150 | | | | | | |
| | 2.56 | 2.60 | 2.72 | 2.36 | 2.40 | 2.51 |
| 175 | | | | | | |
| | 2.56 | 2.60 | 2.72 | 2.36 | 2.40 | 2.51 |
| 200 | | | | | | |
| | 2.56 | 2.60 | 2.72 | 2.36 | 2.40 | 2.51 |
| 225 | | | | | | |
| | 2.56 | 2.60 | 2.72 | 2.17 | 2.20 | 2.30 |
| 250 | | | | | | |
| | 1.97 | 2.00 | 2.09 | 2.17 | 2.20 | 2.30 |
| 275 | | | | | | |
| | 1.97 | 2.00 | 2.09 | 1.58 | 1.60 | 1.67 |
| 300 | | | | | | |
| | 1.97 | 2.00 | 2.09 | 1.58 | 1.60 | 1.67 |
| 325 | | | | | | |
| | 1.77 | 1.80 | 1.88 | 0.79 | 0.80 | 0.84 |
| 350 | | | | | | |
| | 1.38 | 1.40 | 1.46 | 0.79 | 0.80 | 0.84 |
| 375 | | | | | | |
| | 0.99 | 1.00 | 1.05 | 0.79 | 0.80 | 0.84 |
| 400 | | | | | | |
| | 0.99 | 1.00 | 1.05 | 0.59 | 0.60 | 0.63 |
| 425 | | | | | | |
| | 0.79 | 0.80 | 0.84 | 0.59 | 0.60 | 0.63 |
| 450 | | | | | | |
| | 0.00 | 0.00 | 0.00 | 0.39 | 0.40 | 0.42 |
| 475 | | | | | | |
| | 0.00 | 0.00 | 0.00 | 0.39 | 0.40 | 0.42 |
| 500 | | | | | | |
| | -0.20 | -0.20 | -0.21 | 0.00 | 0.00 | 0.00 |
| 525 | | | | | | |
| | -0.20 | -0.20 | -0.21 | -0.20 | -0.20 | -0.21 |
| 550 | | | | | | |

top-dressing while the most profitable rate of application in 1953 was 435 pounds and in 1954 was 436 pounds of 3-12-12 fertilizer and 441 pounds in 1953 and 445 pounds in 1954 of 3-12-12 fertilizer plus 20 pounds nitrogen top-dressing. These are shown graphically for the year 1952 in Figure 8.

In Figure 8, the following evaluations can be made from a study of the experimental data:

1. The essence of the law of diminishing returns exists in the experimental data.
2. An increase in the price of wheat and a decrease in the price of fertilizer from 1952 to 1954 would increase the amount of fertilizer which it pays to use and the amount of wheat it pays to produce.
3. All changes in the use of fertilizer as a result of changes in price of wheat and fertilizer are limited to stage II in which $MVP < AVP$ and in which $MVP > 0$.

Thus, it appears that the experimental data provides some basis for comparison for the field survey data. Some of the limitations and weaknesses of the experimental data will be discussed at a later point.

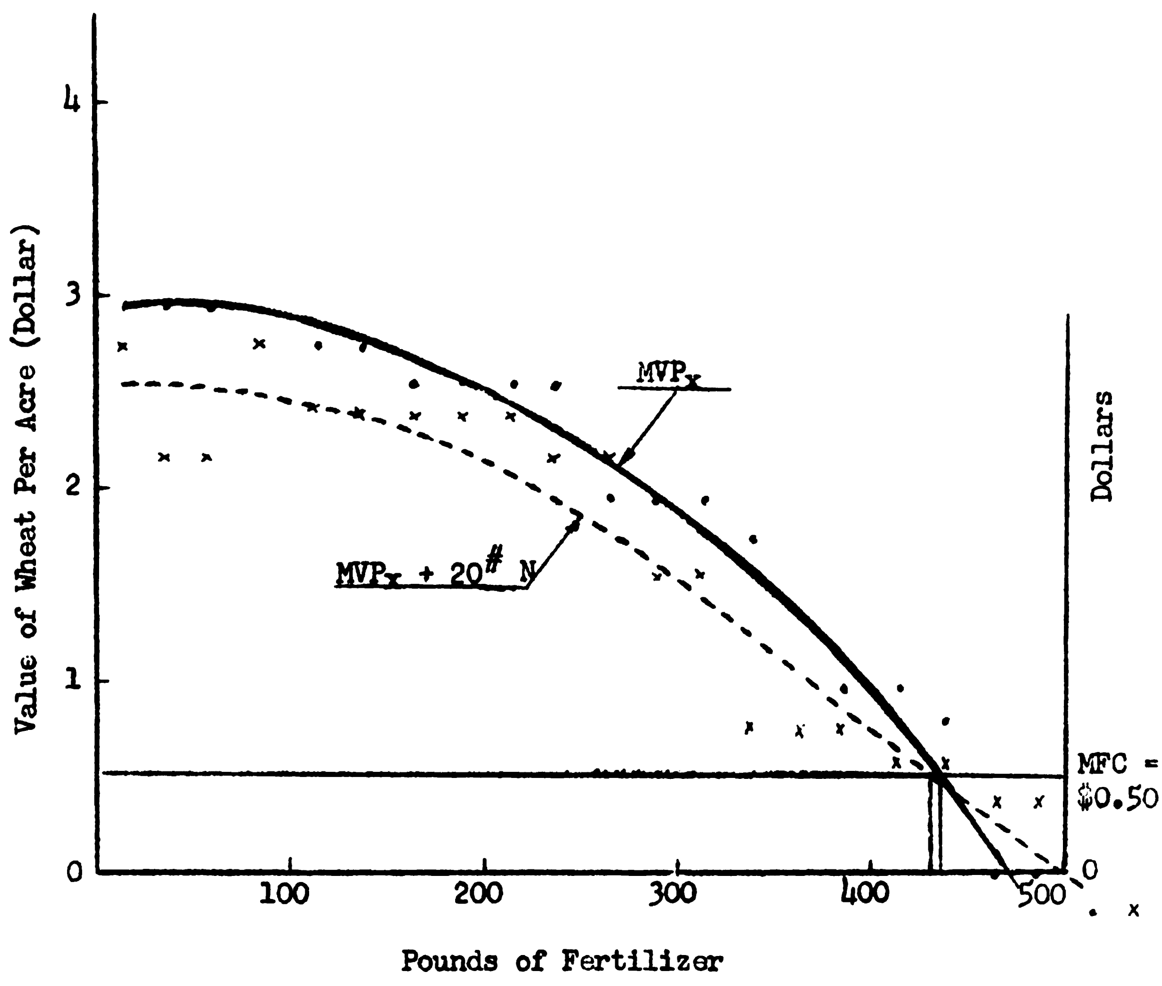


Figure 8. Equating Marginal Value Product of Wheat with Marginal Factor Cost of Fertilizer to Determine Optimum Rate of 3-12-12 Fertilizer and 3-12-12 Plus 20 Pounds Nitrogen Top Dressing Application with 1952 Prices.

# CHAPTER V

## AN EVALUATION OF THE STATISTICAL RESULTS AFTER FITTING THE FUNCTION

The data from the field survey was fitted in four different forms. The first equation included five variables: total nitrogen input, total phosphorus input, total potassium input, seed quality, and livestock intensity. A six variable equation was fitted using the same variables except that the nitrogen input was divided into that applied at planting time and nitrogen applied as top-dressing. Each of these equations were fitted three times using a different dummy variable for livestock intensity per acre to test the effect this had on the coefficients for the inputs of nutritive elements.

The other equations fitted included seventeen and eighteen variables. The seventeen variable equation included the variables of the five variable equation and an additional variable for livestock intensity and eleven additional dummy variables to allow for variations in area and year. The eighteen variable equation was the same as the seventeen variable equation but it included two variables for nitrogen inputs as did the six variable one discussed above.

The method followed in fitting the Cobb-Douglas function was that presented by M. Ezekial[1] for fitting a normal equation and coefficient

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[1] Ezekiel, Mordecai. Methods of Correlation Analysis, 2nd Edition. New York: John Wiley and Sons, Inc., 1949, pp. 208-212.

of multiple correlation. Hence, the normal equation were solved by the Doolittle Method. The resultant regression coefficients (elasticities), multiple correlation coefficients ($\bar{R}$), coefficients of multiple determination ($\bar{R}^2$) for the several independent variables and their corresponding standard error of estimate ($\bar{S}$), standard error of regression coefficients ( $\sigma b_1$) and constant (a) for the function in each case were indicated in Tables III and IX and are discussed in the following sections.

## The Results of the Five and Six Variable Equations

In Table III, the regression coefficient for nitrogen plant food ($X_2$) in the five variable equation was significant at the five percent level while the regression coefficient for nitrogen top-dressing in the six variable equation was significant at one percent level by "t" test. This indicates that increasing quantities of nitrogen top-dressing might increase the yield of wheat per acre. The sign of the regresssion coefficient for nitrogen applied at planting time in the six variable equation was negative. The regression coefficient for nitrogen applied at planting time was not significant by "t" test. According to Tintner and Brownlee,[2] "Negative regression coefficients within the range of inputs on most farms are meaningless."

The regression coefficient for seed quality was significant in the five variable equation at the one percent level of significance

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[2] Tintner, Gerhard, and D. H. Brownlee, "Production Functions Derived from Farm Records," Journal of Farm Economics, Vol. XXVI, (August, 1944).

TABLE III

REGRESSION COEFFICIENTS, STANDARD ERRORS OF REGRESSION COEFFICIENTS, COEFFICIENTS OF MULTIPLE CORRELATION, COEFFICIENTS OF MULTIPLE DETERMINATION AND STANDARD ERRORS OF ESTIMATE IN FIVE AND SIX VARIABLE EQUATIONS

| Cases | a | $N_2$ ($X_2$) | | $P_2O_5$ ($X_3$) | | $K_2O$ ($X_4$) | | Nitrogen Top Dressing ($X_5$) | | Nitrogen Applied at Planting Time ($X_6$) | | Certified Seed ($X_7$) | | Zero Animal Units Per Acre ($X_8$) | | 0.01-0.19 Animal Units Per Acre ($X_9$) | | 0.20 or more Animal Units Per Acre ($X_{10}$) | | $\bar{R}$ | $\bar{R}^2$ | $\bar{S}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | $b_{1.n}$ | $\sigma b_{1.n}$ | | | |
| 1 | 1.38150 | 0.02673* | 0.01103 | 0.04777* | 0.01854 | 0.00967 | 0.02132 | - | - | - | - | 0.01618** | 0.00331 | -0.00922 | 0.00331 | - | - | - | - | 0.21627 | 0.04677 | 0.10646 |
| 2 | 1.37869 | 0.02668* | 0.01088 | 0.04760* | 0.02102 | 0.01029 | 0.02102 | - | - | - | - | 0.01604** | 0.00328 | - | - | -0.00012 | 0.00656 | - | - | 0.21319 | 0.04545 | 0.10552 |
| 3 | 1.37798 | 0.02612* | 0.01103 | 0.04898* | 0.02130 | 0.00815 | 0.02130 | - | - | - | - | 0.01578** | 0.00332 | - | - | - | - | 0.01373# | 0.00941 | 0.21797 | 0.04751 | 0.10641 |
| 4 | 1.38637 | - | - | 0.05338* | 0.02143 | 0.01722 | 0.02133 | 0.02303** | 0.00709 | -0.00521 | 0.01423 | 0.01568# | 0.01035 | -0.00977 | 0.00783 | - | - | - | - | 0.22291 | 0.04969 | 0.10629 |
| 5 | 1.38241 | - | - | 0.05290* | 0.02145 | 0.01769 | 0.02136 | 0.02300** | 0.00712 | -0.00435 | 0.01616 | 0.01563# | 0.01036 | - | - | 0.00143 | 0.00696 | - | - | 0.21955 | 0.04820 | 0.10638 |
| 6 | 1.38264 | - | - | 0.05427* | 0.02145 | 0.01537 | 0.02133 | 0.02248** | 0.00709 | -0.00444 | 0.01402 | 0.01532# | 0.01035 | - | - | - | - | 0.01337# | 0.00946 | 0.22387 | 0.05011 | 0.10627 |

Remarks:

**: 1 percent level of significance for regression coefficients by "t" test.

*: 5 percent level of significance for regression coefficients by "t" test.

#: 32-6 percent level of significance for regression coefficients by "t" test.

-: Variable not included in this equation.

and in the six variable equation at the 32-6 percent level. This indicates that using certified seed might increase the yield of wheat per acre.

It will be noted that negative regression coefficients were obtained for zero animal units per acre in the first case and in the fourth case and for 0.01-0.19 animal units per acre in the second case. The regression coefficient for animal units were not significant. But, the regression coefficient for animal units both were significant in the third case and in the sixth case at 32-6 percent level of significance. This suggests that the presence of 0.20 or more animal units per acre might increase the yield of wheat per acre.

After taking nitrogen top-dressing and nitrogen applied at planting time as two independent variables in the six variable equation, the significance of the regression coefficient for seed quality dropped from one percent level in the five variable equation to 32-6 percent in the six variable equation. The regression coefficients for nitrogen applied at planting time in the six variable equation were not significant. This suggests that the regression coefficients in the five variable equation were affected by the intercorrelation between the use of certified seed and nitrogen top-dressing or between good practices and the use of nitrogen top-dressing.

The coefficient of multiple determination or $\bar{R}^2$ was approximately 0.05 in both five and six variable equations showing that five percent of the variance in the dependent variable or the yield of wheat was associated with two forms of fertilizer applications. The remaining 95 percent of the variation of the dependent variable may have been

due to other input factors such as labor, machinery or other unstudied variable factors such as weather and management which were assumed to be normally distributed.

The logarithm of the estimated yields of wheat E (Y) was approximately 1.3810 in both five and six variable equations, the antilog of which is 24.04 bushels. The standard error of estimate ($\bar{S}$) of the dependent variable was found to be approximately 0.1060.

The sum of the regression coefficients (elasticity) were less than one in both five and six variable equations, and decreasing returns to scale are indicated in the 411 farms.

The multiple correlation coefficients ($\bar{R}$) were between 0.21319 and 0.22387 in the five and six variable equations. With 978 observations and either five or six independent variable and one dependent variable, this high a multiple correlation coefficient would be expected in one sample out of a thousand if the true R containing five variables in five percent level of significance were 0.097 and in one percent level of significance was 0.115.[3] Consequently the correlation is significant.

The TPP per acre was derived by using various rates of fertilizer applications to fit in each of five and six variable functions in the logarithmic form. The MPP was the additional yield of wheat resulting from the last 25 pounds increment of fertilizer.

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3 Ferber, Robert. Statistical Techniques in Market Research, 1st Edition, New York: McGraw-Hill Book Company, Inc., pp. 520-521.

Tables and figures derived from the data from the five and six variable equations illustrate the essence of the law of diminishing returns and show the effect of increasing rate of two forms of fertilizer applications on the total, marginal physical products and marginal value products under different price conditions. These are also related to three different variables representing animal units and indicate the effect of using the different analysis of fertilizer such as 3-12-12 fertilizer and 0-12-12 fertilizer plus ten pounds nitrogen top-dressing on the yield of wheat. The 0-12-12 fertilizer analysis was the least cost combination which was derived from the six variable equation and is discussed in the following sections.

All TPP figures in Table IV were derived by fitting the five variable equation for 3-12-12 fertilizer with three different animal unit variables included. In Table V the TPP of wheat per acre was derived by fitting the six variable equation for 0-12-12 fertilizer plus ten pounds nitrogen top-dressing with the variables for three intensities of animal units per acre included.

The relationships between the total and marginal physical products in two forms of fertilizer applications and the comparison on the yield of wheat per acre with three different animal unit variables included were illustrated as follows:

The TPP per acre was continuing to increase at a decreasing rate in both five and six variable equations as long as the MPP decreased but was greater than zero. The TPP per acre of wheat was slightly greater in 0-12-12 plus nitrogen top-dressing than in 3-12-12 fertilizer.

TABLE IV

THE EFFECT OF INCREASING RATES OF 3-12-12 FERTILIZER APPLICATIONS ON THE TOTAL, MARGINAL PHYSICAL PRODUCTS AND MARGINAL VALUE PRODUCTS PER ACRE FOR WHEAT WITH DIFFERENT INTENSITIES OF ANIMAL UNITS IN THE FIVE VARIABLE EQUATION

| Pounds of Fertilizer | Zero Animal Units Per Acre | | | | | 0.01-0.19 Animal Units Per Acre | | | | | 0.20 or More Animal Units Per Acre | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TPP | MPP | MVP | | | TPP | MPP | MVP | | | TPP | MPP | MVP | | |
| | bu. | bu. | a | b | c | bu. | bu. | a | b | c | bu. | bu. | a | b | c |
| 0 | 0.00 | | | | | 0.00 | | | | | 0.00 | | | | |
| 25 | 25.85 | | | | | 26.24 | | | | | 27.31 | | | | |
| | | 1.56 | $3.07 | $3.12 | $3.26 | | 1.58 | $3.11 | $3.16 | $3.30 | | 1.62 | $3.19 | $3.24 | $3.39 |
| 50 | 27.41 | | | | | 27.82 | | | | | 28.93 | | | | |
| | | 0.95 | 1.87 | 1.90 | 1.99 | | 0.97 | 1.91 | 1.94 | 2.03 | | 0.99 | 1.95 | 1.98 | 2.07 |
| 75 | 28.36 | | | | | 28.79 | | | | | 29.92 | | | | |
| | | 0.69 | 1.36 | 1.38 | 1.44 | | 0.71 | 1.40 | 1.42 | 1.48 | | 0.73 | 1.44 | 1.46 | 1.53 |
| 100 | 29.05 | | | | | 29.50 | | | | | 30.65 | | | | |
| | | 0.55 | 1.08 | 1.10 | 1.15 | | 0.57 | 1.12 | 1.14 | 1.19 | | 0.57 | 1.12 | 1.14 | 1.19 |
| 125 | 29.60 | | | | | 30.07 | | | | | 31.22 | | | | |
| | | 0.46 | 0.91 | 0.92 | 0.96 | | 0.46 | 0.91 | 0.92 | 0.96 | | 0.48 | 0.95 | 0.96 | 1.00 |
| 150 | 30.06 | | | | | 30.53 | | | | | 31.70 | | | | |
| | | 0.39 | 0.77 | 0.78 | 0.82 | | 0.40 | 0.79 | 0.80 | 0.84 | | 0.41 | 0.81 | 0.82 | 0.86 |
| 175 | 30.45 | | | | | 30.93 | | | | | 32.11 | | | | |
| | | 0.35 | 0.69 | 0.70 | 0.73 | | 0.35 | 0.69 | 0.70 | 0.73 | | 0.36 | 0.71 | 0.72 | 0.75 |
| 200 | 30.80 | | | | | 31.28 | | | | | 32.47 | | | | |
| | | 0.31 | 0.61 | 0.62 | 0.65 | | 0.32 | 0.63 | 0.64 | 0.67 | | 0.32 | 0.63 | 0.64 | 0.67 |
| 225 | 31.11 | | | | | 31.60 | | | | | 32.79 | | | | |
| | | 0.27 | 0.53 | 0.54 | 0.56 | | 0.28 | 0.55 | 0.56 | 0.59 | | 0.29 | 0.57 | 0.58 | 0.61 |
| 250 | 31.38 | | | | | 31.88 | | | | | 33.08 | | | | |
| | | 0.25 | 0.49 | 0.50 | 0.52 | | 0.26 | 0.51 | 0.52 | 0.54 | | 0.26 | 0.51 | 0.52 | 0.54 |
| 275 | 31.63 | | | | | 32.14 | | | | | 33.34 | | | | |
| | | 0.24 | 0.47 | 0.48 | 0.50 | | 0.23 | 0.45 | 0.46 | 0.48 | | 0.24 | 0.47 | 0.48 | 0.50 |
| 300 | 31.87 | | | | | 32.37 | | | | | 33.58 | | | | |
| | | 0.22 | 0.43 | 0.44 | 0.46 | | 0.22 | 0.43 | 0.44 | 0.46 | | 0.23 | 0.45 | 0.46 | 0.48 |
| 325 | 32.09 | | | | | 32.59 | | | | | 33.81 | | | | |
| | | 0.19 | 0.37 | 0.38 | 0.40 | | 0.19 | 0.37 | 0.38 | 0.40 | | 0.20 | 0.39 | 0.40 | 0.42 |
| 350 | 32.28 | | | | | 32.80 | | | | | 34.01 | | | | |

[a] Under 1952's price conditions, Wheat price = $1.97 per bushel and MFC = $0.50 for 25 pounds increment of 3-12-12 fertilizer.

[b] Under 1953's price conditions, Wheat price = $2.00 per bushel and MFC = $0.47 for 25 pounds increment of 3-12-12 fertilizer.

[c] Under 1954's price conditions, Wheat price = $2.09 per bushel and MFC = $0.46 for 25 pounds increment of 3-12-12 fertilizer.

TABLE V

THE EFFECT OF INCREASING RATES OF 0-12-12 FERTILIZER PLUS 10 POUNDS NITROGEN TOP DRESSING APPLICATIONS ON THE TOTAL, MARGINAL PHYSICAL PRODUCTS AND MARGINAL VALUE PRODUCTS PER ACRE FOR WHEAT WITH DIFFERENT INTENSITIES OF ANIMAL UNITS IN THE SIX VARIABLE EQUATION

| Pounds of Fertilizer | Zero Animal Units Per Acre | | | | | 0.01-0.19 Animal Units Per Acre | | | | | 0.20 or More Animal Units Per Acre | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | TPP | MPP | MVP | | | TPP | MPP | MVP | | | TPP | MPP | MVP | | |
| | bu. | bu. | a | b | c | bu. | bu. | a | b | c | bu. | bu. | a | b | c |
| 0 | 0.00 | | | | | 0.00 | | | | | 0.00 | | | | |
| 25 | 27.86 | | | | | 27.66 | | | | | 27.69 | | | | |
| | | 1.40 | $2.76 | $2.80 | $2.93 | | 1.39 | $2.74 | $2.78 | $2.91 | | 1.37 | $2.70 | $2.74 | $2.86 |
| 50 | 29.26 | | | | | 29.05 | | | | | 29.06 | | | | |
| | | 0.85 | 1.67 | 1.70 | 1.78 | | 0.84 | 1.65 | 1.68 | 1.76 | | 0.83 | 1.64 | 1.66 | 1.73 |
| 75 | 30.11 | | | | | 29.89 | | | | | 29.89 | | | | |
| | | 0.62 | 1.22 | 1.24 | 1.30 | | 0.62 | 1.22 | 1.24 | 1.30 | | 0.61 | 1.20 | 1.22 | 1.27 |
| 100 | 30.73 | | | | | 30.51 | | | | | 30.50 | | | | |
| | | 0.49 | 0.97 | 0.98 | 1.02 | | 0.48 | 0.95 | 0.96 | 1.00 | | 0.47 | 0.93 | 0.94 | 0.98 |
| 125 | 31.22 | | | | | 30.99 | | | | | 30.97 | | | | |
| | | 0.40 | 0.79 | 0.80 | 0.84 | | 0.40 | 0.79 | 0.80 | 0.84 | | 0.40 | 0.79 | 0.80 | 0.84 |
| 150 | 31.62 | | | | | 31.39 | | | | | 31.37 | | | | |
| | | 0.35 | 0.69 | 0.70 | 0.73 | | 0.35 | 0.69 | 0.70 | 0.73 | | 0.34 | 0.67 | 0.68 | 0.71 |
| 175 | 31.97 | | | | | 31.74 | | | | | 31.71 | | | | |
| | | 0.30 | 0.59 | 0.60 | 0.63 | | 0.30 | 0.59 | 0.60 | 0.63 | | 0.30 | 0.59 | 0.60 | 0.63 |
| 200 | 32.27 | | | | | 32.04 | | | | | 32.01 | | | | |

[a] Under 1952's price conditions, wheat price = $1.97 per bushel and MFC = $1.87.

[b] Under 1953's price conditions, wheat price = $2.00 per bushel and MFC = $1.76.

[c] Under 1954's price conditions, wheat price = $2.09 per bushel and MFC = $1.66.

The profitable rate of fertilizer application indicated by the survey data can be derived by fitting the equation: the MVP of any input ($X_i$) must be equal to the price of the variable input ($P_{x_i}$). The price of wheat and the price of fertilizer used were the same as used previously in Chapter IV for the crop years 1952 to 1954.

The most profitable rate of two forms of fertilizer application and the most profitable yields for wheat per acre with different intensities of animal units per acre in 1952-54 are shown in Table VI. If these figures are compared with the experimental results, they appear to indicate that application rates of 3-12-12 fertilizer derived from the survey data are approximately 100 and more pounds lower than the rates indicated by the experimental data. The most profitable application rate for 0-12-12 fertilizer plus 10 pounds nitrogen top dressing appears to be very low. The TPP and MVP for the experimental and survey data with 1953 prices are shown in Figure 9.

Once the regression coefficients of the various input categories were determined, it is possible to derive estimate of marginal value products from the formula

$$MVP_{x_i} = \frac{b_i \, E\,(Y)}{X_i}$$

where $b_i$ is the regression coefficient (elasticity) of gross income with respect to corresponding inputs, E (Y) is estimated gross income, and $X_i$ is the quantity of the input. E (Y) and $X_i$ are measured in natural numbers. The MVP of each input categories indicates an increase in gross income resulting from an increase in the use of $X_i$.

TABLE VI

THE MOST PROFITABE RATE OF 3-12-12 FERTILIZER AND 0-12-12 FERTILIZER PLUS 10 POUNDS NITROGEN TOP DRESSING APPLICATION AND THE MOST PROFITABLE YIELDS FOR WHEAT WITH DIFFERENT INTENSITIES OF ANIMAL UNITS, 1952-54

| | | 3-12-12 Fertilizer | | 0-12-12 Fertilizer Plus 10 pounds Nitrogen Top Dressing | |
|---|---|---|---|---|---|
| | | The most profitable amounts of fertilizer applied | The most profitable yields of wheat per acre produced | The most profitable amounts of fertilizer applied | The most profitable yields of wheat per acre produced |
| | | (pounds) | (bushels) | (pounds) | (bushels) |
| 1952 | Zero animal units per acre | 244 | 31.30 | 45 | 28.86 |
| | 0.01-0.19 animal units per acre | 254 | 31.92 | 44 | 28.61 |
| | 0.20 and over animal units per acre | 256 | 33.14 | 44 | 28.64 |
| 1953 | Zero animal units per acre | 281 | 31.69 | 48 | 29.01 |
| | 0.01-0.19 animal units per acre | 271 | 32.09 | 48 | 28.81 |
| | 0.20 and over animal units per acre | 288 | 33.46 | 47 | 28.79 |
| 1954 | Zero animal units per acre | 300 | 31.87 | 56 | 29.44 |
| | 0.01-0.19 animal units per acre | 300 | 32.37 | 55 | 29.20 |
| | 0.20 and over animal units per acre | 308 | 33.66 | 54 | 29.18 |

E

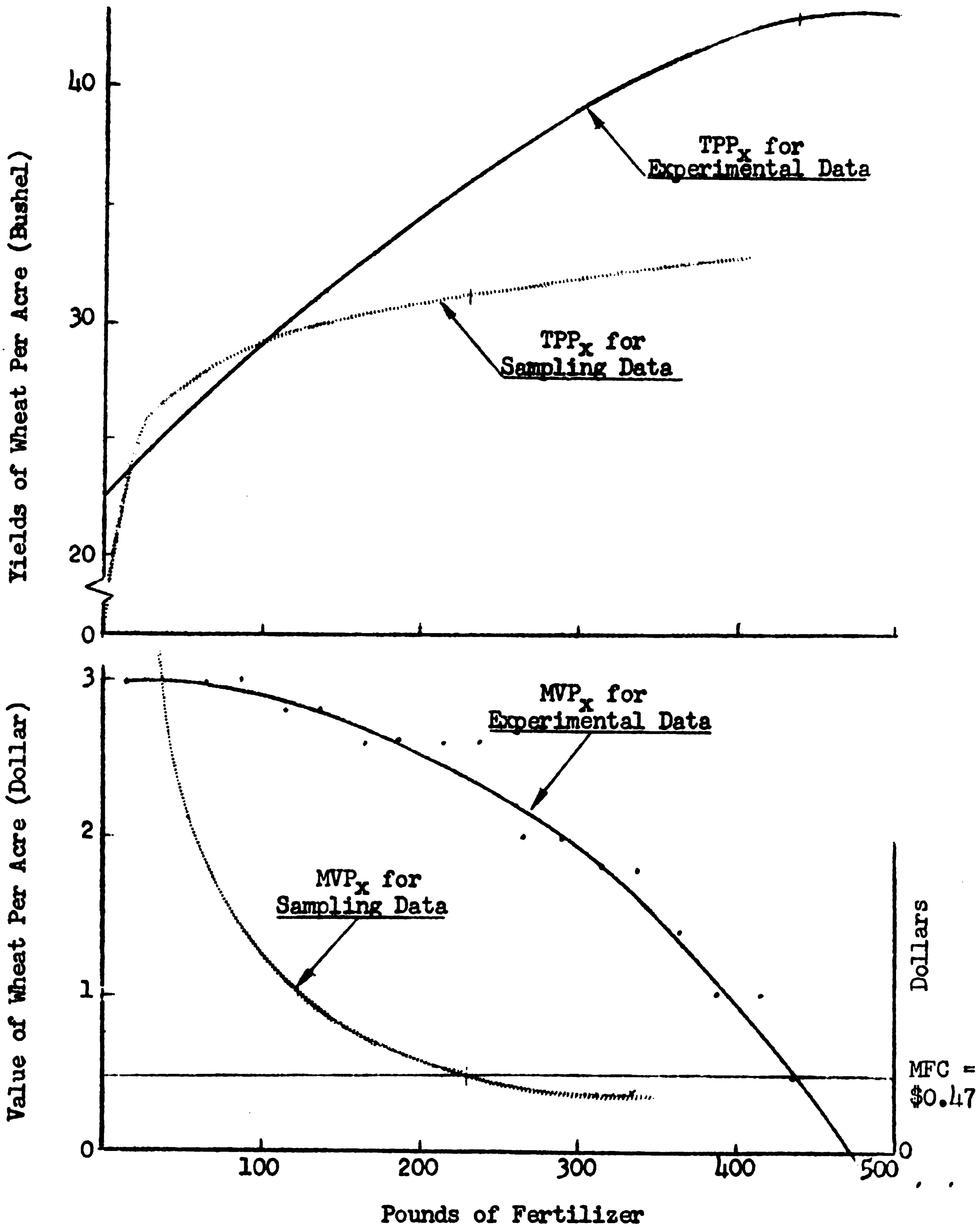


Figure 9. A Comparison between Experimental and Survey Data of the Effect of Increasing 3-12-12 Fertilizer Application on the Total Physical Product and Marginal Value Product for Wheat with 1953 Prices Using the Five Variable Equation.

The MVP for the various input categories in the survey data computed at their geometric mean were shown in Table VII. In the five variable equation, the MVP for one pound of nitrogen plant food was approximately \$0.160 with 1952 prices while the MVP were approximately \$0.161 and \$0.168 with 1953 and 1954 prices. The MVP for one pound of phosphorus plant food and one pound of potassium plant food were approximately \$0.077 and \$0.019 for 1952 prices. They were \$0.078 and \$0.019 for 1953 prices and \$0.082 and \$0.020 with 1954 prices. This indicates that returns for one pound of nitrogen plant food was probably twice that for one pound of phosphorus plant food and eight times that for one pound of potassium plant food.

Under 1952, 1953 and 1954's price conditions, the MVP for one pound of nitrogen applied at planting time was approximately zero or less and for one pound nitrogen top dressing was approximately \$0.68. The MVP for one pound of $P_2O_5$ plant food was approximately \$0.09 and for one pound of $K_2O$ plant food was approximately \$0.03. A negative MVP for one pound of nitrogen applied at planting time is meaningless in this study.

Under a set of price conditions, the least cost combination for N-P-K fertilizer ratio might be two-two-one in the five variable equation and zero-one-one in the six variable equation shown in Table VIII. These were derived by the formula

$$\frac{MVP_{N_2}}{MFC_{N_2}} = \frac{MVP_{P_2O_5}}{MFC_{P_2O_5}} = \frac{MVP_{K_2O}}{MFC_{K_2O}} = 1$$

where $MVP_{X_i}$ is the marginal value products of the $X_i$ input categories and $MFC_{X_i}$ is the marginal factor cost for the corresponding $X_i$ input.

TABLE VII

MARGINAL VALUE PRODUCTS OF $N_2$, $P_2O_5$, $K_2O$, AND NITROGEN TOP DRESSING IN FIVE AND SIX VARIABLE EQUATIONS
(UNIT = DOLLAR)

| Cases | MVP in 1952 | | | | | MVP in 1953 | | | | | MVP in 1954 | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $N_2$ | $P_2O_5$ | $K_2O$ | Nitrogen top dressing | Nitrogen applied at planting time | $N_2$ | $P_2O_5$ | $K_2O$ | Nitrogen top dressing | Nitrogen applied at planting time | $N_2$ | $P_2O_5$ | $K_2O$ | Nitrogen top dressing | Nitrogen applied at planting time |
| 1 | 0.160 | 0.077 | 0.019 | - | - | 0.162 | 0.078 | 0.019 | - | - | 0.169 | 0.082 | 0.020 | - | - |
| 2 | 0.158 | 0.077 | 0.020 | - | - | 0.160 | 0.078 | 0.020 | - | - | 0.168 | 0.081 | 0.021 | - | - |
| 3 | 0.154 | 0.079 | 0.016 | - | - | 0.157 | 0.080 | 0.016 | - | - | 0.164 | 0.083 | 0.016 | - | - |
| 4 | - | 0.086 | 0.033 | 0.678 | -.040 | - | 0.089 | 0.034 | 0.688 | -.040 | - | 0.093 | 0.035 | 0.719 | -.042 |
| 5 | - | 0.086 | 0.034 | 0.671 | -.033 | - | 0.087 | 0.035 | 0.681 | -.033 | - | 0.091 | 0.036 | 0.712 | -.035 |
| 6 | - | 0.088 | 0.030 | 0.656 | -.033 | - | 0.089 | 0.030 | 0.666 | -.034 | - | 0.093 | 0.031 | 0.696 | -.035 |

TABLE VIII

THE LEAST COST COMBINATION FOR $N_2$, $P_2O_5$, AND $K_2O$ IN FIVE AND SIX VARIABLE EQUATIONS[4], 1952-1954

| Cases | 1952 | | | | 1953 | | | | 1954 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $N_2$ | $P_2O_5$ | $K_2O$ | Nitrogen Top Dressing | $N_2$ | $P_2O_5$ | $K_2O$ | Nitrogen Top Dressing | $N_2$ | $P_2O_5$ | $K_2O$ | Nitrogen Top Dressing |
| 1 | 1.064 | 0.857 | 0.466 | - | 1.154 | 0.980 | 0.471 | - | 1.298 | 1.024 | 0.492 | - |
| 2 | 1.053 | 0.850 | 0.491 | - | 1.145 | 0.971 | 0.498 | - | 1.289 | 1.015 | 0.521 | - |
| 3 | 1.029 | 0.874 | 0.388 | - | 1.119 | 0.998 | 0.394 | - | 1.259 | 1.043 | 0.412 | - |
| 4 | 0.000 | 0.956 | 0.825 | 0.452 | 0.000 | 1.113 | 0.850 | 0.491 | 0.000 | 1.163 | 0.875 | 0.553 |
| 5 | 0.000 | 0.956 | 0.850 | 0.447 | 0.000 | 1.088 | 0.875 | 0.486 | 0.000 | 1.138 | 0.900 | 0.548 |
| 6 | 0.000 | 0.978 | 0.750 | 0.437 | 0.000 | 1.112 | 0.750 | 0.476 | 0.000 | 1.163 | 0.775 | 0.535 |

[4] Marginal factor cost for $N_2$ was based on the average price of 20.5 percent Nitrogen for Ammonium Sulphate and 33.5 percent Nitrogen for Ammonium Nitrate, for $P_2O_5$ was based on the average price of 0-20-0 and 0-45-0 analysis of fertilizer, and for $K_2O$ was based on the price of 0-0-60 analysis of fertilizer. The fertilizer price was furnished by Davison Chemical Company, Lansing, Michigan.

Under 1952's price conditions, $MFC_{n_2}$ = \$0.15, $MFC_{P_2O_5}$ = \$0.09, and $MFC_{K_2O}$ = \$0.04.

Under 1953's price conditions, $MFC_{n_2}$ = \$0.14, $MFC_{P_2O_5}$ = \$0.08, and $MFC_{K_2O}$ = \$0.04.

Under 1954's price conditions, $MFC_{n_2}$ = \$0.13, $MFC_{P_2O_5}$ = \$0.08, and $MFC_{K_2O}$ = \$0.04.

It appears that the ratio of the commercial fertilizer used by farmers was weighted too much on potassium and too little on nitrogen. In other words, there might be an increase in nitrogen and a decrease in potassium in the fertilizer ratio used.

The Results of the Seventeen and Eighteen Variable Equations

As presiously mentioned, the seventeen variable equation included nitrogen plant food as one independent variable and dummy variables to represent different soil types and years. The eighteen variable equation was the same as in the seventeen variable equation except nitrogen top dressing and nitrogen applied at planting time were separated as two independent variables.

In Table IX, the regression coefficients for total nitrogen plant food including nitrogen top dressing in the seventeen variable equation was significant at the one percent level while the regression coefficient for nitrogen top dressing was significant at five percent level by "t" test. This indicates that increasing quantities of either the nitrogen plant food or nitrogen top dressing might increase the yield of wheat per acre.

The regression coefficient for seed quality was significant in the seventeen variable equation at the one percent level and in the eighteen variable equation at the 32.6 percent level of significance by "t" test. This indicates that using the certified seed might increase the yield of wheat per acre.

TABLE IX

REGRESSION COEFFICIENTS, STANDARD ERRORS OF REGRESSION COEFFICIENTS, COEFFICIENTS OF MULTIPLE CORRELATION, COEFFICIENTS OF MULTIPLE DETERMINATION, AND STANDARD ERRORS OF ESTIMATE IN SEVENTEEN AND EIGHTEEN VARIABLE EQUATIONS

| Variables and Others | | Seventeen Variable Equation | | | Eighteen Variable Equation | | |
|---|---|---|---|---|---|---|---|
| | | $b_{1.n}$ | $\sigma b_{1.n}$ | | $b_{1.n}$ | $\sigma b_{1.n}$ | |
| Total $N_2$ plant food including N top dressing | $X_2$ | 0.01277** | 0.00001 | | - | - | |
| Total $P_2O_5$ plant food | $X_3$ | 0.06845** | 0.01700 | | 0.07176** | 0.02101 | |
| Total $K_2O$ plant food | $X_4$ | 0.00278 | 0.02596 | | 0.00858 | 0.02068 | |
| Nitrogen top dressing | $X_5$ | - | - | | 0.01482* | 0.00678 | |
| Nitrogen applied at planting time | $X_6$ | - | - | | -.01001 | 0.01314 | |
| Seed quality | $X_7$ | 0.01381** | 0.00001 | | 0.01339# | 0.00972 | |
| 0.01-0.19 animal units per acre | $X_9$ | 0.00985** | 0.00001 | | 0.01142# | 0.00762 | |
| 0.20 or more animal units per acre | $X_{10}$ | 0.01771** | 0.00981 | | 0.01861# | 0.01033 | |
| Area I, 1952 | $X_{11}$ | 0.08262** | 0.00981 | | 0.08290** | 0.01613 | |
| Area I, 1953 | $X_{12}$ | 0.12301** | 0.00981 | | 0.12246** | 0.01562 | |
| Area I, 1954 | $X_{13}$ | 0.08579** | 0.00981 | | 0.08585** | 0.01536 | |
| Area II, 1952 | $X_{14}$ | 0.09463** | 0.00981 | | 0.09485** | 0.01595 | |
| Area II, 1953 | $X_{15}$ | 0.09272** | 0.00981 | | 0.09197** | 0.01519 | |
| Area II, 1954 | $X_{16}$ | 0.13454** | 0.00981 | | 0.13312** | 0.01538 | |
| Area III, 1952 | $X_{17}$ | 0.04938** | 0.00981 | | 0.05031** | 0.01627 | |
| Area III, 1953 | $X_{18}$ | -.00019 | 0.00981 | | - | - | |
| Area III, 1954 | $X_{19}$ | 0.09163** | 0.00981 | | 0.09341** | 0.01570 | |
| Area IV, 1952 | $X_{20}$ | 0.07347** | 0.01388 | | 0.07361** | 0.01652 | |
| Area IV, 1953 | $X_{21}$ | 0.09436** | 0.01388 | | 0.09311** | 0.01586 | |
| Area IV, 1954 | $X_{22}$ | - | - | | -.00055 | 0.01595 | |
| $\bar{R}$ | | | | 0.41455 | | | 0.41795 |
| $\bar{R}^2$ | | | | 0.17185 | | | 0.17468 |
| $\bar{S}$ | | | | 0.09923 | | | 0.09906 |
| a | | | | 1.28426 | | | 1.28686 |

Remarks: **: 1 percent level of significance for regression coefficients by "t" test.

*: 5 percent level of significance for regression coefficients by "t" test.

#: 32-6 percent level of significance for regression coefficients by "t" test.

-: Variable not included in this equation.

After taking nitrogen top dressing and nitrogen applied at planting time as two independent variables in the eighteen variable equation, the significance for regression coefficient for seed quality dropped from one percent level in the seventeen variable equation to 32-6 percent in the eighteen variable equation. The regression coefficient for nitrogen applied at planting time in the eighteen variable equation was not significant. This again suggests that the regression coefficient were affected by the intercorrelation between certified seed and nitrogen top dressing or between good practices and more nitrogen top dressing. But the yield of wheat was not increased by nitrogen applied at planting time.

It will be noted that negative regression coefficients were obtained for Area III in 1953 ($X_{18}$) in the seventeen variable equation and for nitrogen applied at planting time ($X_6$) and for Area IV in 1954 ($X_{22}$) in the eighteen variable equation. These indicate that the negative regression coefficients for nitrogen applied at planting time and for dummy variables of Area III in 1953 and of Area IV in 1954 were meaningless. The regression coefficients for these dummy variables $X_{18}$ and $X_{22}$ and for nitrogen applied at planting time were not significant. It might indicate that weather or other physical conditions for Area III in 1953 and for Area IV in 1954 decreased the yield of wheat per acre.

The regression coefficients were significant for both 0.01-0.19 and 0.20 or more animal units per acre in the seventeen variable equation at one percent level of significance and in the eighteen variable equation at 32-6 percent level of significance by "t" test, indicating a positive relationship between animal units per acre and the yield of wheat per acre.

The coefficient of multiple determination or $\bar{R}^2$ was approximately 0.17 in both seventeen and eighteen variable equations showing that seventeen percent of the variance in the dependent variable or the yield of wheat per acre was associated with the variables included in the equation. The remaining eighty-three percent of the variation in the dependent variable (Y) may have been due to other factors such as labor, machinery or other unstudied variable factors such as weather and management which were assumed to be normally distributed.

The logarithm of the estimated yields of wheat E (Y) was approximately 1.2853 in both seventeen and eighteen variable equations, the antilog of which is 19.29 bushels per acre. The standard error of estimate ($\bar{S}$) of the dependent variable was found to be approximately 0.0991.

The sum of the regression coefficient (elasticities) was greater than one in both seventeen and eighteen variable equations indicating increasing returns to scale on the 411 farms. But, the TPP figures shown in Table X were increasing at a decreasing rate. This may have been influenced by the dummy variables which were classified as fixed variables in the Cobb-Douglas function.

The multiple correlation coefficients ($\bar{R}$) were 0.41455 and 0.41912 in the two equations. Under the conditions of 978 observations with either seventeen or eighteen independent variable and one dependent variable, this high a multiple correlation coefficient would be expected in one sample out of a thousand if the true R containing five variables in five percent level of significance were 0.097 and in one percent level of significance were 0.115. Consequently the degree of correlation is significant.

The TPP per acre was derived by using various rate of fertilizer applications to fit in both seventeen and eighteen variable equations in the logarithm form.

In Table X, the TPP per acre was continuing to increase at decreasing rate in the seventeen variable equation for 3-12-12 fertilizer and in the eighteen variable equation for 0-30-10 fertilizer plus 10 pounds nitrogen top dressing as long as the MPP decreased but was greater than zero. The 0-30-10 fertilizer analysis was the least cost combination which was derived from the eighteen variable equation. The TPP per acre was greater in 0-30-10 fertilizer plus nitrogen top dressing than in 3-12-12 fertilizer. The survey results also show the effect of using small amount of fertilizer in higher analysis would produce higher yields per acre than larger amount of fertilizer with a lower analysis. For instance, 200 pounds of 3-12-12 fertilizer was required to furnish the plant food contained in 100 pounds of 0-30-10 fertilizer plus 10 pounds nitrogen top dressing with smaller yields per acre.

The TPP per acre probably the highest in Area II from 1952-54 while the TPP per acre in Area I was the second highest, in Area IV was third higher and in Area III was the lowest from 1952 to 1954. This indicates that the effect of the good soil types on higher yields per acre assuming other factors studied were constant.

The most profitable rate of 3-12-12 fertilizer application for the seventeen variable equation were 244 pounds with 1952 prices, 269 pounds with 1953 prices, and 288 pounds with 1954 prices. If these figures were compared with the experimental results, it would indicate that an application rate of 3-12-12 fertilizer in the survey data would be about

TABLE X

THE EFFECT OF INCREASING RATES OF 3-12-12 FERTILIZER FOR SEVENTEEN INDEPENDENT VARIABLES AND 0-30-10 FERTILIZER PLUS 10 POUNDS NITROGEN TOP DRESSING FOR EIGHTEEN INDEPENDENT VARIABLES ON THE TOTAL, MARGINAL PHYSICAL PRODUCTS AND MARGINAL VALUE PRODUCTS PER ACRE FOR WHEAT

| Pounds of Fertilizer | 3-12-12 Fertilizer | | | | | 0-20-10 Fertilizer Plus 10 Pounds Nitrogen Top Dressing | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | TPP bu. | MPP bu. | a | b | c | TPP bu. | MPP bu. | d | e | f |
| 0 | 0.00 | | | | | 0.00 | | | | |
| | | - | - | - | - | | - | - | - | - |
| 25 | 25.44 | | | | | 28.68 | | | | |
| | | 1.52 | $2.99 | $3.04 | $3.17 | | 1.64 | $3.23 | $3.28 | $3.43 |
| 50 | 26.96 | | | | | 30.32 | | | | |
| | | 0.93 | 1.83 | 1.86 | 1.94 | | 1.00 | 1.97 | 2.00 | 2.09 |
| 75 | 27.89 | | | | | 31.32 | | | | |
| | | 0.68 | 1.34 | 1.36 | 1.42 | | 0.74 | 1.46 | 1.48 | 1.55 |
| 100 | 28.57 | | | | | 32.06 | | | | |
| | | 0.55 | 1.08 | 1.10 | 1.15 | | 0.58 | 1.14 | 1.16 | 1.21 |
| 125 | 29.12 | | | | | 32.64 | | | | |
| | | 0.44 | 0.87 | 0.88 | 0.92 | | 0.48 | 0.95 | 0.96 | 1.00 |
| 150 | 29.56 | | | | | 33.12 | | | | |
| | | 0.39 | 0.77 | 0.78 | 0.82 | | 0.41 | 0.81 | 0.82 | 0.86 |
| 175 | 29.95 | | | | | 33.53 | | | | |
| | | 0.34 | 0.67 | 0.68 | 0.71 | | 0.37 | 0.73 | 0.74 | 0.77 |
| 200 | 30.29 | | | | | 33.90 | | | | |
| | | 0.30 | 0.59 | 0.60 | 0.63 | | | | | |
| 225 | 30.59 | | | | | | | | | |
| | | 0.27 | 0.53 | 0.54 | 0.56 | | | | | |
| 250 | 30.86 | | | | | | | | | |
| | | 0.25 | 0.49 | 0.50 | 0.52 | | | | | |
| 275 | 31.11 | | | | | | | | | |
| | | 0.23 | 0.45 | 0.46 | 0.48 | | | | | |
| 300 | 31.14 | | | | | | | | | |
| | | 0.21 | 0.41 | 0.42 | 0.44 | | | | | |
| 325 | 31.55 | | | | | | | | | |

[a], [b] and [c] are the same as footnotes [a], [b] and [c] in Table IV.

[d] Under 1952's price conditions, wheat price = $1.97 per bu. and MFC = $2.28.

[e] Under 1953's price conditions, wheat price = $2.00 per bu. and MFC = $2.10.

[f] Under 1954's price conditions, wheat price = $2.09 per bu. and MFC = $2.00.

one-half that indicated by the experimental data. The TPP and MVP for the experimental and survey data with 1953 prices are shown in Figure 10.

The most profitable rate of 0-30-10 fertilizer plus 10 pounds nitrogen top dressing were 44 pounds with 1952 prices, 48 pounds with 1953 prices and 49 pounds with 1954 prices. There were no comparable experimental results, but the indicated application rate of 0-30-10 fertilizer plus 10 pounds nitrogen top dressing appears to be very low.

In Table XI, the MVP for one pound of total nitrogen plant food was approximately $0.06 in the seventeen variable equation under 1952 and 1953's price conditions while the MVP was approximately $0.07 with 1954 prices. Under 1952 to 1954's price conditions, the MVP for both one pound of phosphorus plant food and one pound of potassium plant food were approximately $0.090 and $0.004 in the seventeen variable equation. This appears that returns for one pound of phosphorus plant food was probably one-half higher than for one pound of nitrogen plant food and one pound of nitrogen plant food was probably twelve times greater than for one pound of potassium plant food. In the same table, the MVP for one pound of nitrogen applied at planting time was approximately zero or less under 1952, 1953 and 1954's price conditions and for one pound nitrogen top dressing was approximately $0.35. The MVP for one pound of phosphorus plant food was approximately $0.095 and for one pound of potassium plant food was approximately $0.013. A negative MVP for one pound of nitrogen applied at planting time is meaningless in this study. It appears that returns from one pound of phosphorus plant food was probably much greater than for one pound of

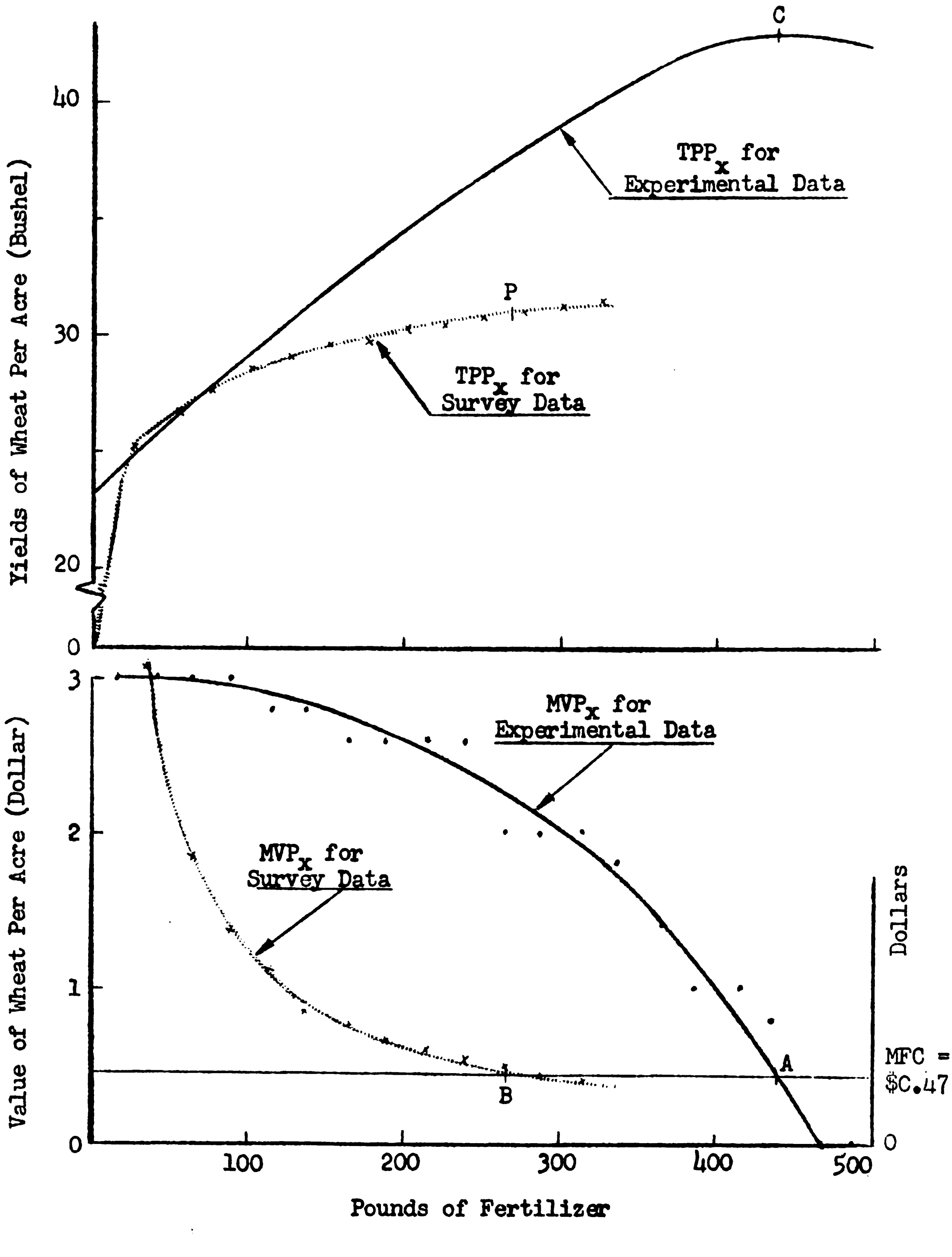


Figure 10. A Comparison between Experimental and Survey Data of the Effect of Increasing 3-12-12 Fertilizer Application on the Total Physical Product and Marginal Value Product for Wheat with 1953 Prices Using the Seventeen Variable Equation.

TABLE XI

MARGINAL VALUE PRODUCTS OF $N_2$, $P_2O_5$, $K_2O$, NITROGEN TOP DRESSING, NITROGEN APPLIED AT PLANTING TIME IN SEVENTEEN AND EIGHTEEN VARIABLE EQUATIONS, 1952-54

| | | Seventeen Variable Equation Marginal Value Products | | | Eighteen Variable Equation Marginal Value Products | | |
|---|---|---|---|---|---|---|---|
| | | 1952 | 1953 | 1954 | 1952 | 1953 | 1954 |
| $N_2$ including Nitrogen top dressing | $X_2$ | $0.061 | $0.062 | $0.065 | - | - | - |
| $P_2O_5$ | $X_3$ | 0.089 | 0.090 | 0.094 | $0.093 | $0.095 | $0.099 |
| $K_2O$ | $X_4$ | 0.004 | 0.004 | 0.005 | 0.013 | 0.013 | 0.014 |
| Nitrogen top dressing | $X_5$ | - | - | - | 0.352 | 0.357 | 0.373 |
| Nitrogen applied at planting time | $X_6$ | - | - | - | -0.060 | -0.061 | -0.064 |

potassium plant food and an increase in gross income would probably be greatest resulting from an increase in use of nitrogen top dressing.

The least cost combination for N-P-K fertilizer ratio under a set of price conditions in 1952-54 might be 4-10-1 for the seventeen variable equation and 0-3-1 in the eighteen variable equation shown in Table XII. This again suggests that the ratio of the commercial fertilizer used by farmers was weighted too much on potassium plant food and too little on phosphorus plant food.

# TABLE XII

## THE LEAST COST COMBINATION FOR $N_2$, $P_2O_5$, AND $K_2O$ IN SEVENTEEN AND EIGHTEEN VARIABLE EQUATIONS, 1952-54

| | | Seventeen Variable Equation | | | Eighteen Variable Equation | | | |
|---|---|---|---|---|---|---|---|---|
| | | $N_2$ | $P_2O_5$ | $K_2O$ | Nitrogen Applied at Planting Time | $P_2O_5$ | $K_2O$ | Nitrogen Top Dressing |
| 1952 | $MVP_{x_1}$ | $0.061 | $0.089 | $0.004 | $-0.060 | $0.093 | $0.013 | $0.352 |
| | $MFC_{x_1}$ | 0.150 | 0.090 | 0.040 | 0.150 | 0.090 | 0.040 | 0.150 |
| | Least Cost Combination | 0.405 | 0.984 | 0.107 | -0.400 | 1.038 | 0.329 | 2.346 |
| 1953 | $MVP_{x_1}$ | $0.062 | $0.090 | $0.004 | $-0.061 | $0.095 | $0.013 | $0.357 |
| | $MFC_{x_1}$ | 0.140 | 0.080 | 0.040 | 0.140 | 0.080 | 0.040 | 0.140 |
| | Least Cost Combination | 0.441 | 1.124 | 0.108 | -0.435 | 1.185 | 0.334 | 2.552 |
| 1954 | $MVP_{x_1}$ | $0.065 | $0.094 | $0.005 | $-0.064 | $0.099 | $0.014 | $0.373 |
| | $MFC_{x_1}$ | 0.130 | 0.080 | 0.040 | 0.130 | 0.080 | 0.040 | 0.130 |
| | Least Cost Combination | 0.496 | 1.174 | 0.113 | -0.492 | 1.238 | 0.349 | 2.872 |

# CHAPTER VI

## SUMMARY AND CONCLUSIONS

This study was to test the feasibility of estimating input-output relationships for wheat in Michigan using sampling data. After the results obtained from experimental and survey data have been compared, several conclusions can be drawn.

In general, the results obtained from an analysis of both the survey data and from the experimental plots appeared to conform to the economic theory outlined in Chapter II.

The essence of the Law of Diminishing Returns existed in both experimental and survey data.

The sum of regression coefficients or elasticities were less than one in all equations derived from the survey data indicating decreasing returns to scale for the 411 farms studied. The elasticity of a Cobb-Douglas production function, however, is constant over all ranges of independent variables. This means that the Cobb-Douglas function can approximate only a segment of the complete function. This appeared to be the lower limit of stage II in all equations. It indicates, on the average, that the farmers that were interviewed ceased application of fertilizer before the upper limit of stage II in the production function was reached. An increase in fertilizer application might be recommended in the future years.

In accordance with static factor-factor analysis, technical complements appeared to exist in N-P-K fertilizer analysis where fertilizer was obtained in a fixed ratio. The least cost combination for 4-10-10 fertilizer ratio in the seventeen variable equation and for 2-2-1 fertilizer ratio in the five variable equation indicate that a reduction in nitrogen cannot be replaced by an increase in element of phosphorus or potassium in a given fertilizer analysis. The animal manure and nitrogen top dressing appear to be substitutes in producing the minimum nitrogen required in wheat production. When animal manure or nitrogen top dressing is increased in quantity, total pounds of fertilizer might be decreased with the same yields of wheat per acre. For instance, 400 pounds of 3-12-12 fertilizer plus 20 pounds nitrogen top dressing substituted for 450 pounds of 3-12-12 fertilizer with the same yields per acre (43.0 bushels per acre) in the experimental data, as shown in Table I, and 250 pounds of 3-12-12 fertilizer with 0.01-0.19 animal units per acre substituted for 300 pounds of 3-12-12 fertilizer with zero animal units per acre on the same Iso-product line (31.87 bushels per acre) as shown in Table IV. The use of more nitrogen top dressing and animal manure on wheat apparently should continue to be encouraged. But nitrogen applied at planting time was not profitable according to the relationships appearing.

The most profitable rate of 3-12-12 fertilizer application obtained from the results of the five variable equation was approximately 220 pounds and from the seventeen variable equation was approximately 260 pounds. The most profitable rate of 0-12-12 fertilizer plus 10 pounds nitrogen top dressing in the six variable equation was approximately

48 pounds and of 0-30-10 fertilizer plus 10 pounds nitrogen top dressing in the eighteen variable equation was approximately 46 pounds. The 0-12-12 and 0-30-10 fertilizers were the least cost combination and were derived from six and eighteen variable equations.

If these figures were compared with the experimental results, it appears that the application rate of 3-12-12 fertilizer in the survey data were slightly more than one-half those indicated by the experimental results. The most profitable rates of 0-12-12 and 0-30-10 fertilizer plus 10 pounds nitrogen top dressing were not available from experiments to compare with results from the survey data. Nitrogen top dressing was used as an independent variable in the survey data and as a fixed variable in experimental design.

Lack of capital and some limitations in the farm operation might be major reasons why the farmers ceased more application of fertilizer before the maximum profit point in their farm business.

The least cost combination for N-P-K fertilizer analysis was approximately 2-2-1 in the five variable equation and 4-10-1 in the seventeen variable equation while the least cost combination for N-P-K fertilizer plus nitrogen top dressing were approximately 0-1-1 in the six variable equation and 0-3-1 in the eighteen variable equation. If these combinations were compared with 1-4-4 in experimental data, it appears that an increase in nitrogen and a decrease in potassium in the fertilizer analysis used by farmers might be recommended in the future years. It also suggests that the future research projects for determining the optimum fertilizer applications should give more attention to least cost combinations than has been the case in the past.

## Limitations of Using Survey Data for Input-Output Relationships

The output response of wheat yields with respect to the corresponding inputs of fertilizer, appears smaller in the survey data than in the experimental data. In other words, the regression line in the survey data was much flatter throughout from the origin than the regression line in the experimental data originate somewhere on the Y axis. This might have been due to several weaknesses in the survey data that was used. It is possible that farmers with low fertilizer inputs and low wheat yields had an upward bias in their yield estimates which would result in underestimating the true coefficients for fertilizer in the statistical results. It is also possible that the intercorrelation between fertilizer use and inherent level of soil fertility could also have resulted in a downward bias in the coefficients.

Biases and errors arising from both the interviewer and the sample also were possible using the random sampling method.

In addition, some shortcoming of the nature of the Cobb-Douglas production function itself which shows yields always starting at the origin and constant elasticities throughout all ranges of $X_1$ may in part explain the difference in results between the survey data and the experimental data. In the future it might be desirable to make some adjustment and modification of the function to partially overcome these objections.[1]

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1 Carter, Harold O. "Modifications of the Cobb-Douglas Function to Destroy Constant Elasticity and Symmetry." Unpublished M. S. Thesis, Department of Agricultural Economics, Michigan State University, 1955.

Different methods of fertilizer applications, different rotation systems and other variations in farm practices would exist in the survey data. These factors might affect the yields of wheat per acre. Other factors such as differences in weather, moisture, and soil types within the four areas in this study might have influenced the yields of wheat per acre.

There was not enough cases in the survey data with larger application of fertilizer. Most of the farmers' fertilizer application appear to have been concentrated in the area of the lower limit of stage II. To get meaningful input-output data, applications over the entire range of inputs are needed.

## Some Advantages of Survey Data

The survey data offers some results of value which are not available from current experimental data. Although they are not quantitative estimates the positive coefficients for seed quality and intensity of animals per acre indicate that these contribute to increased yields of wheat under actual farm conditions. It is also possible to compute least cost combinations of fertilizer under average farm conditions, something that is not possible with the available experimental data.

## Conclusions and Recommendations for Future Studies

In general, it appears to be possible to derive estimates of input-output relationships from survey data that are not inconsistent with economic theory or existing experimental data. The results obtained from this analysis has several questionable aspects which have been mentioned. However, these may be due in part to the particular method of handling the data, particularly in the use of the dummy variables, and may be due to the statistical function that was fitted. It appears that further work should be done with this or similar data to determine the effect of using differing methodology.

The data used in this analysis was not gathered specifically for use in this type of analysis. If data were collected specifically for this purpose several improvements might be achieved in reliability, number of input factors controlled, range of inputs, and other factors which would improve the usefulness of the data.

Thus, while the results of this analysis indicate that the use of survey data to determine input-output relationships is feasible, further testing and methodological research is indicated before it can become a useful research tool.

## APPENDIX A

### A PORTION OF THE QUESTIONNAIRE IN FARM MANAGEMENT SURVEY USED IN THIS STUDY

1. a. How many acres of wheat will you harvest in 1954? ____________ A.
   b. How many acres of wheat did you harvest in 1953? ____________ A.
   c. How many acres of wheat did you harvest in 1952? ____________ A.

   (IF WHEAT ACREAGE WAS REDUCED FROM 1953 TO 1954:)

   d. What were the reasons for your reduction in wheat acreage?

   ________________________________________

   ________________________________________

   e. Were there any other reasons for your reduction in wheat acreage?

   ________________________________________

   ________________________________________

   f. How many acres of wheat would you have planted in 1953 (for the 1954 harvest) if there had been no acreage allotment? ________ A.

2. a. What was your wheat yield per acre in 1954? (expected) ________ bu.
   b. What was your wheat yield per acre in 1953? ________ bu.
   c. What was your wheat yield per acre in 1952? ________ bu.

   Now I'd like to ask you about some of your production practices for wheat:

| 3. On the crop | Use Cert. Seed? | Fertilizer lbs./A. | Fertilizer Anal. | | N. Top Dr. lbs./A. | N. Top Dr. Anal. | |
|---|---|---|---|---|---|---|---|
| a. Planted in 1953 Harvested in 1954 | | | | | | | |
| b. Planted in 1952 Harvested in 1953 | | | | | | | |
| c. Planted in 1951 Harvested in 1952 | | | | | | | |

4. a. Under the most favorable conditions what is the highest wheat yield you think you can get on your farm? ________ bu./A.

(IN ASKING QUESTION 3b INSERT THE ANALYSIS OF FERTILIZER THAT FARMER HAS MOST RECENTLY USED ON HIS WHEAT.)

b. What is the greatest amount of _________ fertilizer that you can profitably apply on wheat on your farm? __________________ lbs./A.

c. Do you believe nitrogen top-dressing for wheat would be profitable on your farm?

Yes _________; How many pounds per acre can you profitably use?

No _________. __________________________________ lbs. of N.

D.K. ________.

5. a. Have you made any changes in your livestock numbers because of the acreage allotments on your crops? Yes _____ No _____

b. IF THE ANSWER IS YES, ASK WHAT KIND OF LIVESTOCK HAS BEEN ADJUSTED, CHECK THIS CATEGORY AND GET THE DATA FROM THEM. THEN COMPLETE THE LIVESTOCK INVENTORY AND LIST THE REASONS FOR ALL CHANGES IN THE APPROPRIATE SPACE BELOW.

| Kind of Livestock | No. on hand July 1, 1954 | No. on hand July 1, 1953 | Direction of change | No. of change |
|---|---|---|---|---|
| 1.__Dairy cows | | | | |
| 2.__Heifers (Dairy) | | | | |
| 3.__Beef cows (Breeding) | | | | |
| 4.__Feeder cattle | | | | |
| 5.__Bred sows | | | | |
| 6.__Hogs on feed | | | | |
| 7.__Laying hens | | | | |
| 8.__Pullets | | | | |
| 9.__Broilers | | | | |
| 10.__Turkey, geese, etc. | | | | |
| 11.__Sheep, ewes | | | | |
| 12.__Feeder lambs | | | | |
| 13.__Other _____ | | | | |

(IF THERE HAVE BEEN CHANGES IN LIVESTOCK NUMBERS IN ANY CATEGORY, ASK WHY FOR EACH ONE AND LIST THE REASONS BELOW BY NUMBER.)

| | | |
|---|---|---|
| 1. ________ | 6. ________ | 11. ________ |
| 2. ________ | 7. ________ | 12. ________ |
| 3. ________ | 8. ________ | 13. ________ |
| 4. ________ | 9. ________ | |
| 5. ________ | 10. ________ | |

# APPENDIX B

## CONVERSION RATES FOR LIVESTOCK TO STANDARD ANIMAL UNITS

The animal units were converted using one cow as a standard unit. It is primarily on manure produced in one year per 1000 pounds of liveweight[1] as follows:

| | Head of animals equal to one animal unit | Manure produced in one year per 1000 pounds of liveweight |
|---|---|---|
| Cow | 1 | 12.0 T |
| Steer | 1 | 8.5 |
| Horse | 1 | 8.0 |
| Hog | 6 | 16.0 |
| Sheep | 8 | 6.0 |
| Chickens | 250 | 4.5 |

[1] Source: Illinois Agriculture Handbook, 1949, pp. 206.

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