DETERMINATION OF YOUNG'S MODULUS BY THE SONIC METHOD

Thesis for the Degree of B. S.
MICHIGAN STATE COLLEGE
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1941

Determination of Young's Modulus
By the Sonic Method

A Thesis Submitted to

The Faculty of

MICHIGAN STATE COLLEGE

of

AGRICULTURE AND APPLIED SCIENCE

by

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Bachelor of Science

June 1941

THESIS

Cop. 1

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The method of testing materials by vibrating them is quite old. Probably one of the most common examples is that of the metallic currency. Coins were struck on the table or bar to prove their genuiness by the ringing tone given off. (Perhaps more so in times past rather than in these days of the F. B. I. and the G-men) Blowers of fine glass tapped their wares to detect flaws by the characteristic sounds produced by the good and the bad specimens. The vibrations of the reed, the drum, the moving column of air in a pipe -- thus whether recognized or not, the characteristic vibration of a body has played some part in men's lives since time immemorial.

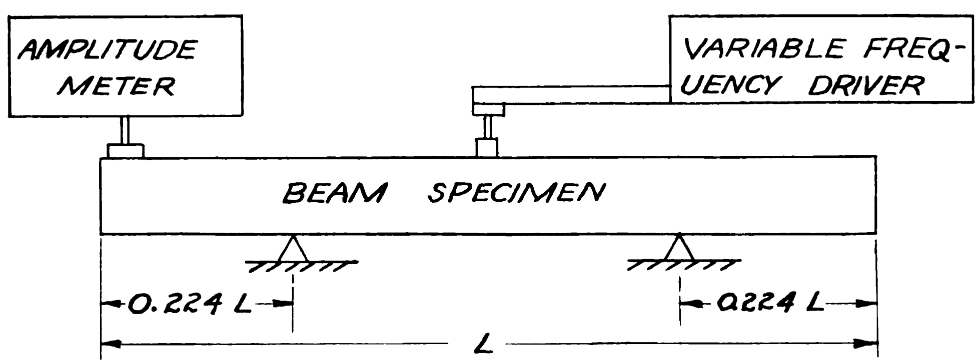
Very likely the natural frequency of vibration of a plucked string was the first to be studied. It was relatively easy to see the longitudinal waves set up in a flexible cord. From this, the next step was the study of the vibration of a rod or rigid body. Of the several methods of holding a rigid body, the three most common are: (a) the fixed-free bar or where one end is built-in at the support and the other is free. (b) the bar built-in at both ends. (c) the free-free, or bar free at both ends. The free-free bar is customarily simply supported at the nodal points. The difference between the rod and string is that in the case of a string, vibrations are maintained by tension externally applied; whereas in the rod, they are maintained by the elasticity of the rod itself. Due to the variation in the forces brought into action, there is a wide variation in the kind and the successive rates of vibration. Generally the sounder, denser body will have the higher rate of fundamental frequency of vibration.

Sir Charles Wheatstone, with the aid of his kaleidophone, studied the action of a vibrating fixed-free bar. A bright silvered bead attached to the free end of a rod was illuminated with a beam of light. When the rod vibrated, the bead showed as an intensely brilliant spot and was easily seen. Thus it was possible to record the great variety and beauty of the designs produced by the moving bead. He found that the rod vibrated simultaneously in two perpendicular directions and the curves produced by the lighted bead due to the combinations of the two rectangular motions.

After Wheatstone came a long line of other searchers for knowledge. The vibration of a body, the importance of the effects of those vibrations on their immediate surroundings became more and more evident. This is in general, the essence of the problem of vibration and for that reason a brief description of method and apparatus used by R. K. Bernhard (*-7) has been included in this paper.

Nearly all materials such as concrete, tile, stone, etc., change internally with age and fatigue. If the condition of these materials could be measured at any time without causing damage to the specimen, a great benefit would result in both economy of time and accuracy of data. This - a particular subdivision of the general problem of vibration - then, is the specific object of this paper: to describe methods of testing materials and to show the reasoning for that procedure.

It will be shown mathematically here, that Young's Modulus of Elasticity is a function of the size and shape of the specimen, its density, and its natural frequency of vibration. Thus if the shape and density of a specimen could be computed and the natural frequency of vibration determined, then the modulus of Elasticity can be calculated. If a specimen is mounted so as to be free to vibrate, (for the fundamental frequency it is simply supported at the nodal points which are 0.224L from each end, where L is the length of the sample) then when struck sharply it will be free to vibrate at its natural frequency. Furthermore, if the specimen is vibrated by some device whose frequency is capable of being varied and which can maintain a continuous frequency, then it will be possible to reach a corresponding resonant frequency in the specimen. If further, a device is attached to specimen (preferably at the ends since the amplitude there is a maximum for the beam) which is capable of amplifying vibrations produced -- then it will be possible to select the fundamental natural frequency of the specimen, since at resonance the amplitude of vibration will be a maximum.



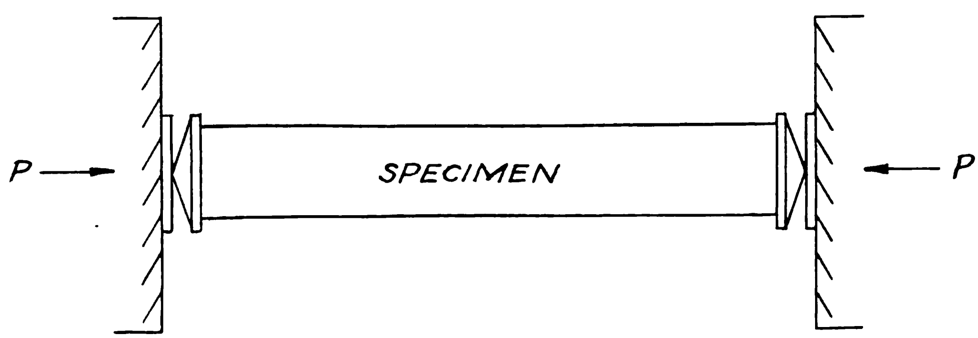
DISCUSSION OF USEFULNESS

Experiments completed have showed the feasibility of calculating E by vibrations without the slow stress-strain measurements. Values of E checked quite closely with those obtained by static methods and were shown to be reliable. Some of the materials which have been tested successfully by the sonic method are tile, slate, brick, lead, ebonite wall board, paraffin wax, rock asphalt, asphalt mastic, quartz bars, steel, nickle, and nickle alloys. (*-11)

Interpretation of observed elastic wave velocities in rock of the earth's crust is the basis for inferring structure at inaccessible depths. Such use requires knowledge of elastic moduli which are characteristic of typical rock when subjected to the minute alternating stresses produced by earthquakes or explosion waves. Most important of the elastic constants to know is Young's Modulus of Elasticity. Studies (*-8) have shown that the statically determined E varies appreciably with the stress at which it is measured. Generally this is attributed to tiny cracks and cavities which tend to increase yielding until they are closed. This uncertainty, inherent in all static methods, raises the question of how accurately seismic values of E are obtained. Hence the present trend toward dynamic methods for determination of E for rocks.

For those materials with a tendency to show a permanent and plastic flow or time-yield, (commonly called creep) the high speed of the dynamic method will prove to give better values for E. This seems to have been verified by the fact that tests by Sonic methods regularly found E to be a higher value than found by the usual static method for the same material. Jensen and Richart (*-9), in some compression studies of concrete, found that measurable creep occurred even at loads for a period of only a few seconds. Hence in slower tests it should be recognized that error enters due to creep.

Sometimes it becomes useful to know E of a material under load and its variation from the E of the unloaded specimen. Toward this end, Leonard Obert (*-10) investigated the E of building materials under pressure. (No work has yet been found to have been done on specimens under any load but compression) First the E was found by the Sonic method using the unloaded free-free bar. Then using the same specimen, it was subjected to pressure (see figure) and its E found again.



The figure just preceding is a sketch of the Riehle Universal testing machine used to produce compression in sample specimen. Ball and socket joints insure a uniform bearing on the ends of beam. Vibrator device and pickup are both located at the center of beam, but are oriented such that the pickup is perpendicular to the plane of vibration and to the axis of beam. With this setup it was possible to determine the E of a material under load, i.e., compression. The work done on stone, with the exception of limestone, showed E to increase rapidly in the region of low pressures and to approach some constant asymptotically. This was quite similar to the change in concrete beams except that the change was much larger. It was also shown that the no-load E of stone did not check with the modulus determined under load. But for concrete the no-load E compared fairly well with the E obtained under load -- the variation not exceeding 10% for normal grades of concrete stressed up to $\frac{1}{4}$ of the ultimate stress.

APPARATUS AND PROCEDURES

Directly following is shown the procedure to establish the relationship (derivation of the working equation.) between the natural frequency of vibration, shape, modulus of elasticity and density of a specimen. These relationships are the basis for calculating modulus of elasticity if the other factors can be determined. Also included at the end is a method of correction for the effects of rotatory and lateral inertia where appreciable.

All methods include some kind of apparatus enabling the determination of the fundamental frequency of the specimen. Generally, all types fall into two main classifications, those vibrating the specimen longitudinally and those vibrating it flexurally. Since most other apparatus is merely some modification of these two types, it will be sufficient to describe these two as representative of their classes.

Apparatus for inducing vibrations in specimens is included in this paper, not because it falls under this particular topic directly, but rather because it is pertinent and has extremely interesting possibilities.

Not very widely known is a quite novel manner, of getting the fundamental frequency, which does not fall into either of the above primary classifications. This is based on the relationship between frequency of vibration and the quality of tone produced. In this method the specimen is struck as to produce an audible tone, then that tone is approximated with one of a set of "orchestra bells", or tuning forks. Thus the natural frequency is easily determined.

TRANSVERSE VIBRATIONS OF ELASTIC BARS

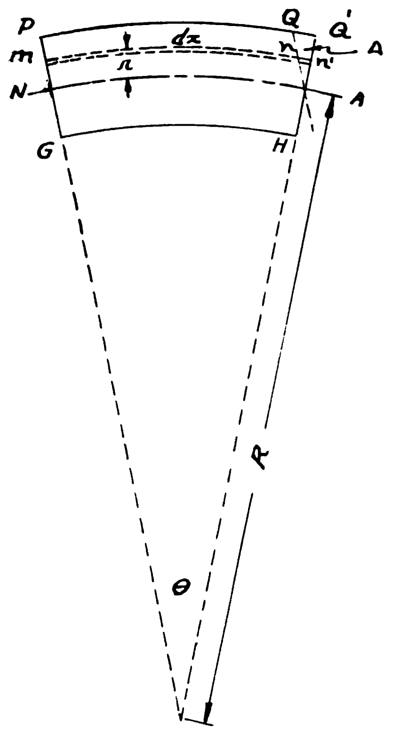


Fig. I

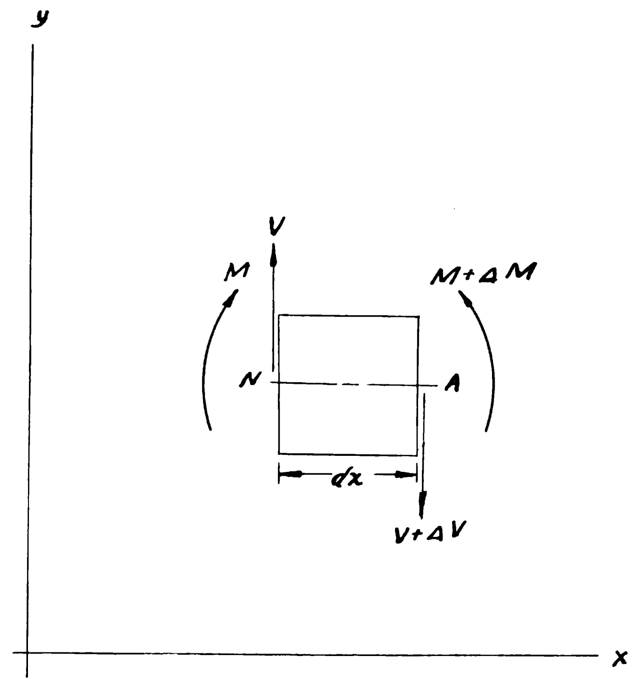
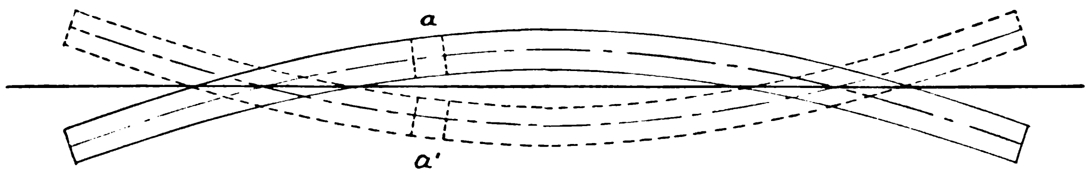


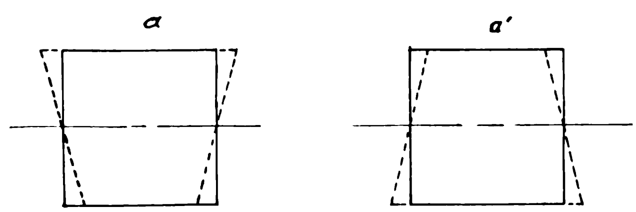
Fig. II

a	-	cross-sectional area of element mn
delta	-	increment of extension mn to mn' when bar is bent
I_m	=	inertia of mass about neutral surface thru NA
	=	$pAdxK^2$
p	=	density of material
K	=	radius of gyration
e	=	deflection from tangent to NA at N at a distance dx from origin
V	=	transverse shearing force
M	=	moment of horizontal and compressive forces acting perpendicular to the cross-section and about NA
I_a	=	inertia of cross-section about horizontal plane thru Na. Remains constant thruout length of bar.
	=	AK^2
E	=	Young's Modulus of Elasticity
$\frac{d^2y}{dx^2}$	=	acceleration in transverse direction

Rotatory and Lateral Inertia



The respective positions of the dotted element a-a' shows that there is not only translatory motion but also rotation during vibration of the beam. The angle of rotation is variable and is equal to the slope of the deflection curve. Thus for a small cross-section-to-length ratio, effects of rotatory motion is small and can be assumed negligible. (See assumption No. 3, page 12. Also see procedure for corrections on page 18.)



Enlarged cross-section view of element a-a'

The effects causing lateral inertia can be seen from the sketch above. Solid lines show bar in neutral position, while dotted lines show actual cross-section of the element a-a' corresponding to both positions. Thus it can be seen that as the cross-section-to-length ratio becomes greater, the effects of lateral inertia become appreciable. (See page 18)

A - ASSUMPTIONS An ideal bar

1. Bar is straight, of uniform density and cross-section
2. The bar is not subject to compression or tension
3. That the amplitude of vibration is sufficiently small that rotatory effects may be neglected.
4. When bar is bent, its curvature is sufficiently small such that it may be represented by $\frac{d^2y}{dx^2} = \frac{1}{R}$
5. Length of bent bar is equal practically to that of bar when straight.

B - Let F = summation of forces tending to stretch element mn (Fig. 1)

1. $$\frac{\frac{F}{a}}{\frac{\text{delta}}{dx}} = \frac{\text{stress}}{\text{strain}} = E \qquad \text{Then } F = \frac{\text{delta}\cdot aE}{dx}$$

2. By geometry $$\frac{dx + \text{delta}}{R + r} = \frac{dx}{R}$$

Then $$\frac{\text{delta}}{dx} = \frac{r}{R} \qquad \text{And } F = Ea\frac{r}{R}$$

3. Moment of F about NA $= -Fr = -Ea\frac{r^2}{R}$

Then summation $M = -\frac{E}{R}\sum ar^2 = -\frac{EI}{R}$

From (A4) Summation $M = -EIa\frac{d^2y}{dx^2}$

And $$-\frac{d^2M}{dx^2} = EIa\frac{d^4y}{dx^4} = EAK^2\frac{d^4y}{dx^4}$$

C - Let diagram represent an indefinitely (Fig. II) small prismatical element cut from (Fig. I) a bar subjected to bending.

Equations of motion

1. $\Sigma F = Ma$ Then $\Sigma V = pAdx\dfrac{d^2y}{dx^2}$

2. $\Sigma T = I\alpha$ or $(\Delta V dx - \Delta M) = I_m\dfrac{d^2\theta}{dt^2}$

Then $\Delta V - \dfrac{\Delta M}{dx} = I_m\dfrac{d^3y}{dx^2dt^2}$

Let $\Delta V = dV$, $\Delta M = dM$, $\theta = \dfrac{dy}{dx}$, $\dfrac{d^2\theta}{dt^2} = \dfrac{d^3y}{dxdt^2}$

3. Combining (1,2) $dV - \dfrac{dM}{dx} = I_m\dfrac{d^3y}{dx^2dt^2} = pAk^2\dfrac{d^3y}{dxdt^2}$

or $\dfrac{dV}{dx} - \dfrac{d^2M}{dx^2} = pAk^2\dfrac{d^4y}{dx^2dt^2}$ Then $pA\dfrac{d^2y}{dx^2} - \dfrac{d^2M}{dx^2} =$

$pAK^2\dfrac{d^4y}{dx^2dt^2}$

4. Combining (B3, C3) $pA\dfrac{d^2y}{dx^2} + EAK^2\dfrac{d^4y}{dx^4} - pAK^2\dfrac{d^4y}{dx^2dt^2} = 0$

Which simplified, becomes $\dfrac{d^2y}{dx^2} + \dfrac{EK^2}{p}\dfrac{d^4y}{dx^4} - k^2\dfrac{d^4y}{dx^2dt^2} = 0$

5. Last term of (4) involves rotary effects and may be neglected (See A3) *-2

Then (4) becomes $\dfrac{d^2y}{dx^2} + \dfrac{EK^2}{p}\dfrac{d^4y}{dx^4} = 0$

6. Let $c^2 = \dfrac{E}{p}$ = velocity of longitudinal elastic wave

Then (5) becomes $\dfrac{d^2y}{dx^2} + K^2c^2\dfrac{d^4y}{dx^4} = 0$

Note: $K^2 = \frac{r^2}{2^2}$ for cylinder of radius = r, $K^2 = \frac{t^2}{12}$ for a rectangular bar of thickness t in plane of vibration.

"The constant Kc cannot be regarded as a velocity unless we divide it by a quantity of the dimensions of a length, viz. λ. As Raleigh points out, the mere admission that harmonic waves can be transmitted at all is sufficient to prove that the velocity must vary inversely as the wavelength. Consequently we must have

$$c(\text{transverse}) = \frac{kc}{\lambda}(\text{long.}) = \frac{K}{\lambda}\sqrt{\frac{E}{p}} \quad " \qquad *\text{-}2$$

7. From (5) $\frac{d^2y}{dx^2} + \frac{EK^2}{p}\frac{d^4y}{dx^4} = 0$ or $pA\frac{d^2y}{dx^2} + EI\frac{d^4y}{dx^4} = 0$

Introducing acceleration of gravity into equation.

Let $I = \frac{AK^2}{g}$ Then $\frac{pAd^2y}{gdx^2} + \frac{EId^4y}{dx^4} = 0$ or

$$\frac{d^2y}{dx^2} + \frac{a^2d^4y}{dx^4} = 0 \qquad \text{where } a^2 = \frac{EIg}{pA}$$

D - To show that Young's Modulus, $E = p\left[\frac{N2\pi L^2}{m^2K}\right]^2$

p = density of the body

N = frequency of vibration

L = length of the specimen

m = bl = roots of equation (D11) and depends on mode of vibration of the specimen.

a. Let Normal mode of vibration be such, that deflection at any point varies harmonically with time.

b. Let $X = f(x)$, where value of abscissa x, determines shape of the normal mode of vibration being studied.

1. From (c7) $\frac{d^2y}{dt^2} + \frac{a^2d^4y}{dx^4} = 0$

2. Let deflection $= y = [X \; A\cos rt + B\sin rt]$ $\qquad r = \frac{\text{radians}}{\text{second}}$

Substituting value of y in (1) which

then becomes $\frac{d^4X}{dx^4} - \frac{r^2X}{a^2} = 0$ Letting $\frac{r}{a} = b^2$, this

can be written $\frac{d^4X}{dx^4} - b^4X = 0$ and is a differential

equation whose solution *-1, is

$$X = C_1e^{bx} + C_2e^{-bx} + C_3\sin bx + C_4\cos bx$$

3. or $X = C_1\cosh bx + C_2\sinh bx + C_3\sin bx + C_4\cos bx$

in which c C_n are arbitrary constants determined in each case by conditions at end of the bar.

Case I Simply supported end - deflection and

bending moment $= 0$ $\quad X = 0, \quad \frac{d^2y}{dx^2} = 0$

Case II Built-in end - deflection and slope of

deflection curve $= 0$. $\quad X = 0, \quad \frac{dy}{dx} = 0$

Case III End free - bending moment and shear = 0

$$\frac{d^2y}{dx^2} = 0 \qquad \frac{d^3y}{dx^3} = 0$$

4. Rewriting (3). constants are not identical.

$C_1[\cos bx + \cosh bx] + C_2[\cos bx - \cosh bx] -$

$C_3[\sin bx + \sinh bx] + C_4[\sin bx - \sinh bx]$

5. Taking origin of coordinates at one end, then by Case III, for a bar with both ends free, there will be four conditions:

(i) $\left.\frac{d^2X}{dx^2}\right|_{x=0} = 0$ (ii) $\left.\frac{d^2X}{dx^2}\right|_{x=L} = 0$

(iii) $\left.\frac{d^3X}{dx^3}\right|_{x=0} = 0$ (iv) $\left.\frac{d^3X}{dx^3}\right|_{x=L} = 0$

6. When $x = 0$, (conditions i, iii), then

$(\mathrm{Cos}\ bx - \mathrm{Cosh}\ bx) = (\mathrm{Sin}\ bx - \mathrm{Sinh}\ bx) = 0$

Hence $C_2 = C_4 = 0$

7. Then $X = C_1(\mathrm{Cos}\ bx + \mathrm{Cosh}\ bx) + C_3(\mathrm{Sin}\ bx + \mathrm{Sinh}\ bx)$

8. When $x = L$
From ii $C_1(\mathrm{Cosh}\ bl - \mathrm{Cos}\ bl) + C_3(\mathrm{Sinh}\ bl - \mathrm{Sin}\ bl) = 0$

9. From iv $C_1(\mathrm{Sin}\ bl + \mathrm{Sinh}\ bl) + C_3(\mathrm{Cosh}\ bl - \mathrm{Cos}\ bl) = 0$

10. Solving 8, 9, when C_1, C_3, are not zero

Then $(\mathrm{Cosh}\ bl - \mathrm{Cos}\ bl)^2 - (\mathrm{Sinh}^2\ bl - \mathrm{Sin}^2\ bl) = 0$

Simplifying $(\mathrm{Cos}^2\ bl - \mathrm{Sin}^2\ bl) - (\mathrm{Cosh}^2\ bl - \mathrm{Sinh}^2\ bl - 2\mathrm{Cos}\ bl\ \mathrm{Cosh}\ bl = 0$

or $1 + 1 - 2\mathrm{Cos}\ bl\ \mathrm{Cosh}\ bl = 0$

11. Then $\mathrm{Cos}\ bl\ \mathrm{Cosh}\ bl = 1$ whose consecutive roots are: *-1

b_1l	Fund-amental b_2l	1st. over-tone b_3l	2nd. over-tone b_4l	b_5l	b_6l
0	4.730	7.853	10.996	14.137	14.279

Using (c (C7) whence $a^2 = \frac{EIg}{pA}$ $\quad \frac{r^2}{a} = \frac{r^2 pA}{EIg} = b^4$

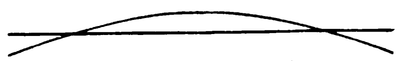
Then $r = b^2K\sqrt{\frac{E}{p}}$

Since frequency $N = \frac{r}{2\pi}$ and $m = bl$

By substitution $N = \frac{m\ K}{2\ L}\sqrt{\frac{E}{p}}$

Now then, $E = p\left(\frac{N2\pi L^2}{m^2 K}\right)^2$

MODES OF VIBRATION



Fundamental

1st. overtone

Fundamental - nodes are 0.2242L from the ends. amplitude at end = 1.645 times that at the center. $m = 4.730$

1st. overtone - nodes are 0.132L from ends and in center. $m = 10.996$ *-5

*-3 CORRECTIONS FOR LATERAL AND ROTATORY INERTIA

In the derivation of the formula for Young's Modulus, the effects of rotary inertia were omitted for simplicity and since it was inappreciable where the cross-section is relatively small in comparison with the length. But where cross-section of beam is a comparatively large fraction of length, rotary inertia becomes appreciably large. Similarly, the effect of neglecting effect of lateral inertia introduces no appreciable error where the cross-section is small but for bars of large cross-section-to-length ratio, the error introduced in neglecting this term becomes sufficiently large as to deserve attention.

Following is an exact solution, by W. P. Mason, (See reference *-5 for material on this subject also) for frequency of vibration where lateral and rotary inertia are taken into account. This solution gives a formula which in itself is very complicated but the curve of the equation (Fig. III) simplifies solution to where it can readily be used.

$$1 - \cos\left\{m\left[\left(1 + \frac{m^4R^4}{24^2}\right) + \frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\}\cosh\left\{m\left[\left(1 + \frac{m^4R^4}{24^2}\right)^{\frac{1}{2}} - \frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\} + \left(\frac{3m^2R^2}{24} + \frac{4m^6R^6}{24^3}\right)\sin\left\{m\left[\left(1 + \frac{m^4R^4}{24}\right)^{\frac{1}{2}} + \frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\}\sinh\left\{m\left[\left(1 + \frac{m^4R^4}{24^2}\right)^{\frac{1}{2}} - \frac{m^2R^2}{24}\right]^{\frac{1}{2}}\right\} = 0$$

This solution by Mason, corrects for rotary inertia when the bar is vibrating in the fundamental mode. From the curve in Fig. III:

Let m_1 = value of m from curve

Then corrected for lateral inertia,

$$E = p\left(\frac{N2\pi L^2}{(m_1)^2 K}\right)^2$$

The inertia effect due to expansion and contraction of beam may also be corrected for, simultaneously with that for rotary inertia. Since the former is proportional to Poisson's ratio, λ, Mason shows that:

Let $R_1 = R(1 + \lambda)^{\frac{1}{2}}$

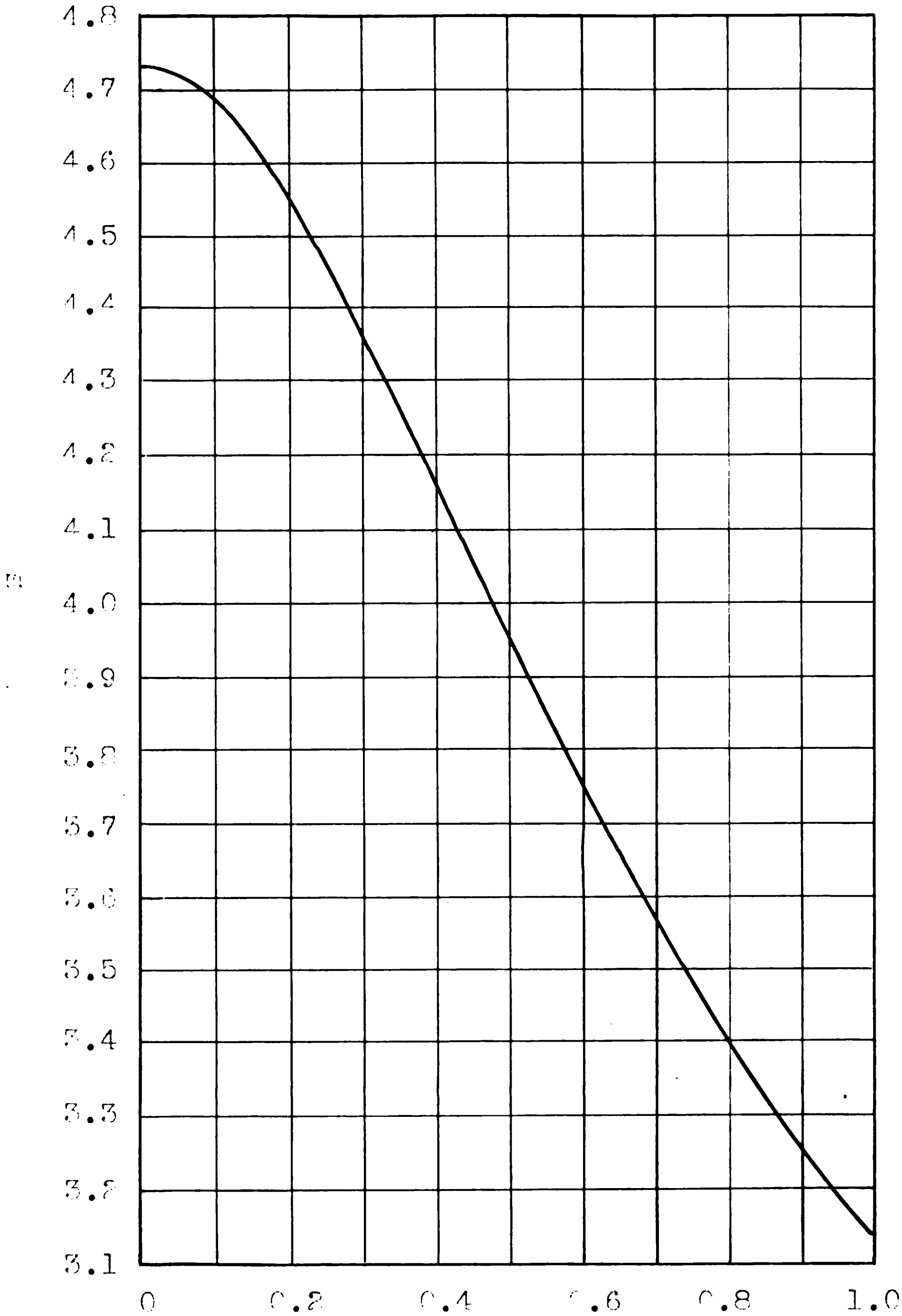
Then let corresponding m from curve = m_2

Now then, corrected for both rotary and lateral inertia,

$$E = p\left(\frac{N2\pi L^2}{(m_2)^2 K}\right)^2$$

Relation between m and R, (Ratio of depth to length) for rectangular bar vibrating laterally in the fundamental mode.

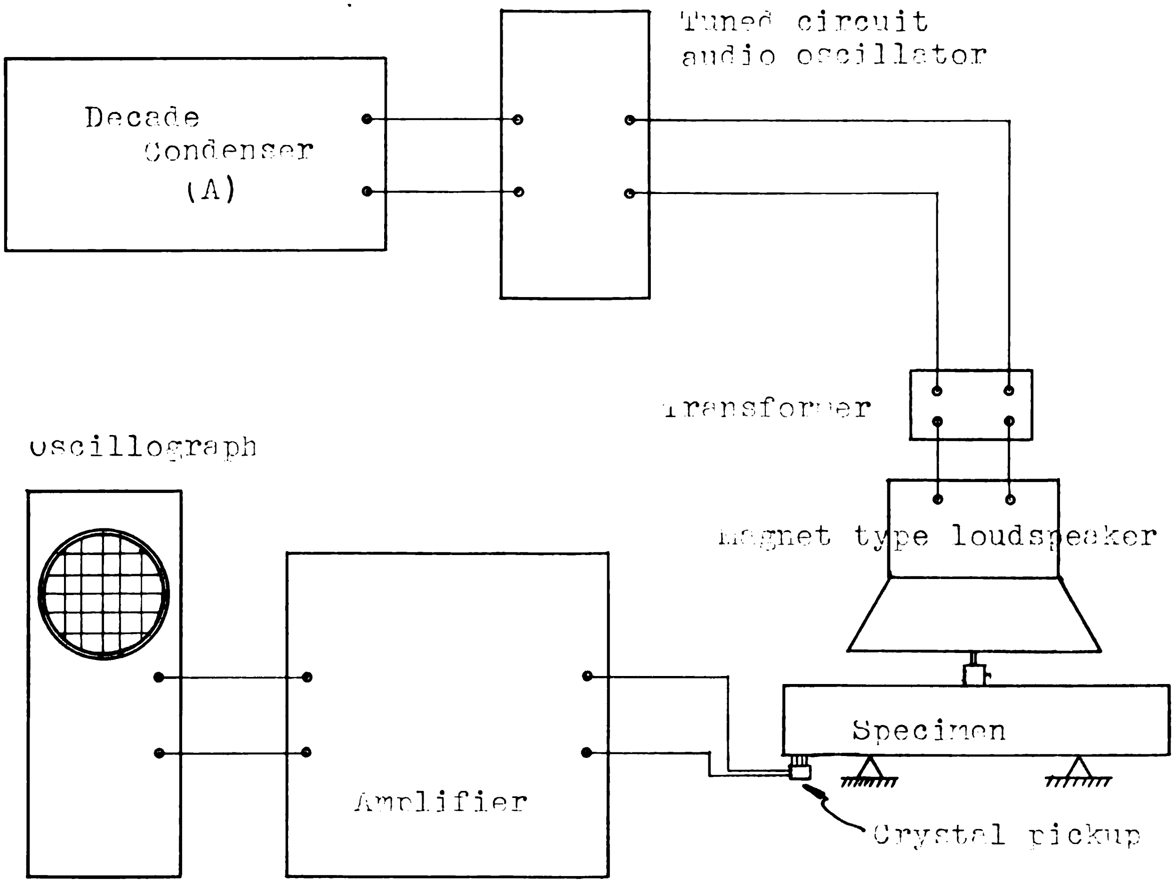
For cylinders, multiply indicated value for m, by 1.07



$R - \frac{t}{L}$ or $\frac{2r}{L}$

Fig. III

Sonic Apparatus used by F. P. Hornibrook in
in freezing and thawing studies of concrete for the
United States Bureau of Standards *-4



(A) Mica decade condenser with capacitance variable in steps of .001 microfarads for a total range of from .001 - 1.11 microfarads. Also contains a variable air condenser having a total capacitance of .003 microfarads.

The oscillator is connected thru a transformer to a six inch magnet-type loudspeaker. Speaker is mounted on a channel-iron frame having standards adjustable for supporting specimens at the nodes. A very light aluminum rod cemented to voice coil of speaker transmitts vibration to specimen thru socket which is screwed into nut cast in the concrete.

Piezo-electric pickup is cemented to a light three-pronged base so that a three-point contact can be obtained. Pickup is held to side of specimen by rubber band. Pickup output is fed thru amplifier to a cathode ray oscillograph.

Frequency output of oscillator was calibrated by synchronizing sweep circuit of oscillograph with the 60 cycle power supply; the sweep being synchronized at 60, 120, 180 cycles per second. The decade condenser of oscillator is then adjusted such that a fixed number of waves is maintained on the oscillograph screen corresponding to a frequency which is an exact multiple of the sweep frequency whose value is given by the number of complete waves which appear. In this way, condenser was calibrated over a range of 200 - 2000 cycles per second. This calibration was checked at various times and was never found to vary more than one-half of one per cent.

Apparatus and procedure used by G. Grime in the determination of Young's Modulus of building materials. *-6

The essence of this method: longitudinal vibration excited in a specimen by tapping one end is recorded photographically with aid of cathode-ray oscillograph and its frequency determined by comparison with a superimposed time scale.

1. Then $E = 4\,l^2n^2p$

2. Precisely, $E = 4l\,n^2p^2\left(1 - \frac{\pi^2\sigma^2K^2}{2l^2}\right)^2$, but for frequency determination, test specimens are generally selected such that the bracketed term approaches unity. Hence (1) is sufficiently accurate for a first approximation.

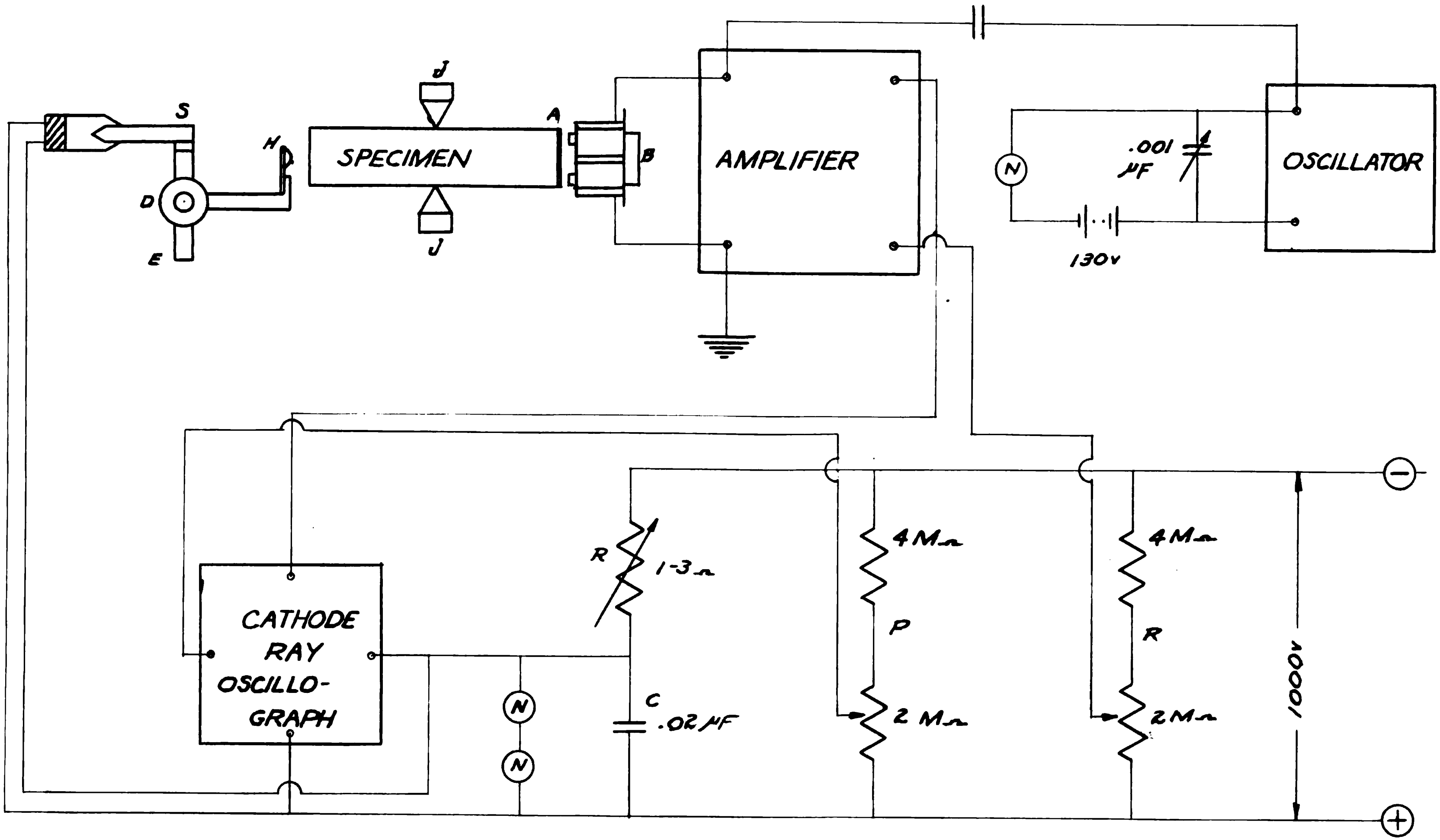
Where l - length of specimen

n - frequency of longitudinal vibration

p - density

σ = Poisson's ratio

K - radius of gyration of cross-section about line of centers of normal sections.



APPARATUS USED BY G. GRIME
LONGITUDINAL VIBRATIONS RECORDED PHOTOGRAPHICALLY
WITH AID OF CATHODE RAY OSCILLOGRAPH

A. - General Arrangement

1. Specimen, rod or cylinder of uniform cross-section, is held at its midpoint and tapped with a light hammer.

2. Electromagnetic detector transforms mechanical vibration into electrical voltage oscillation, which when amplified, controls vertical deflection of fluorescent spot of cathode-ray oscillograph.

3. Single-traverse time-base spreads out vibration pattern horizontally across screen at proper time, while a time-marker superimposes on the photographic record, a break of short duration every millisecond.

4. By comparing vibration pattern with the time marker, frequency is determined.

B. - Detailed Arrangements

1. Exciter and Detector

a. Wooden jaws, J, secure specimen sufficiently firm as to prevent appreciable translatory movement when struck by the light spring hammer, H.

b. Detector - A, a light soft iron armature, about 1/32" thick and cemented to end of specimen is separated by a small air-gap from U-shaped permanent magnet B, wound with coils of the type used in ordinary telephone receivers.

2. Amplifier

   a. Most suitable is a resistance-capacity unit of good response in the upper audible range.

   b. In these experiments, a direct-coupled amplifier of good characteristics, being available, was used.

3. Cathode-ray Oscillograph and Time-base

   a. Oscillograph operated normally at anode voltage of 600-1000, according to intensity requirements. Potentiometers, P, across high-tension supply provide control of initial position of the fluorescent spot.

   b. There being no need of a linear time-base, a very simple type is used, consisting of: variable high resistance, R, and condenser, C, whose plates are connected to horizontally deflecting plates of oscillograph in series with biassing voltage in the usual way. Condenser is charged from oscillograph high tension supply.

   c. When apparatus is prepared for a record, condenser is short-circuited by a switch, S, attached to hammer release, D, and the fluorescent spot is deflected to left side of screen.

   d. Allowing pendulum to strike arm, E, releases hammer, switch opens and voltage across condenser rises, moving the spot horizontally across screen at correct instant to spread out the vibration pattern.

e. At a voltage of approximately 500, when spot has traversed screen, discharge takes place in two small neon lamps, N, connected in series across condenser, preventing further voltage rise.

f. Charging current, and with it the spot velocity decrease exponentially as condenser charges. Rate of decrease for first part of charging process, is quite small. Since first part of charging process is that used to move spot across screen, time-base is, therefore, almost linear.

4. Time-Marker

a. Neon lamp, N, and a battery of voltage just insufficient to cause flashing, are joined in series across output of the fixed-frequency oscillator.

b. With oscillator operating and resistance correctly adjusted, neon lamp is flashed by ½ cycle and extinguished by the next, such that a surge of current takes place in circuit every cycle.

c. Slight capacitative coupling with impulse of amplifier is sufficient to superimpose this on record at regular intervals. Condenser across output of oscillator enables some control of impulse shape and duration.

d. Impulse has a duration of about 50 microseconds and occurs 1024 times per second.

e. Frequency of time-marker was checked over a period of several months, against a 1024 tuning fork by superimposing time marks on record obtained from the fork. Frequency was found to be constant within 0.25%.

5. Camera

a. With a Ross Xpres f-1.9 lens fitted to a ¼-plate camera, records of frequencies up to 15,000 per second can be made without recourse to highly sensitive plates.

b. Six records are taken on each plate by traversing plate-holder vertically.

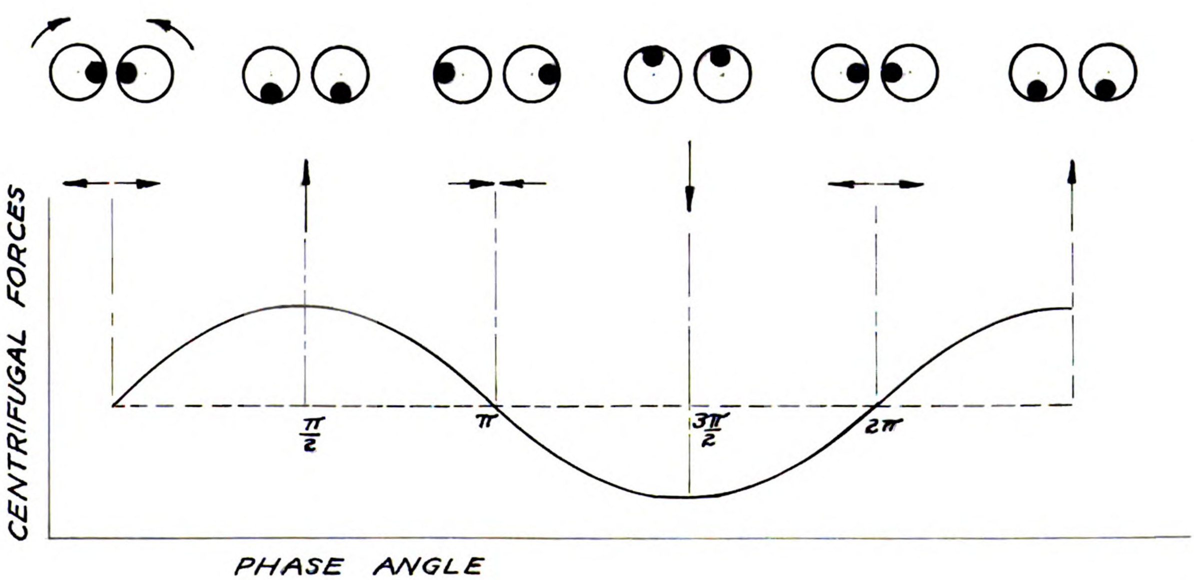
6. Density Determinations

a. Dry porous materials - bulk density by regular procedure.

b. Wet porous materials - These have/not been worked with yet since the part played by water in the transmission of vibration is yet unknown.

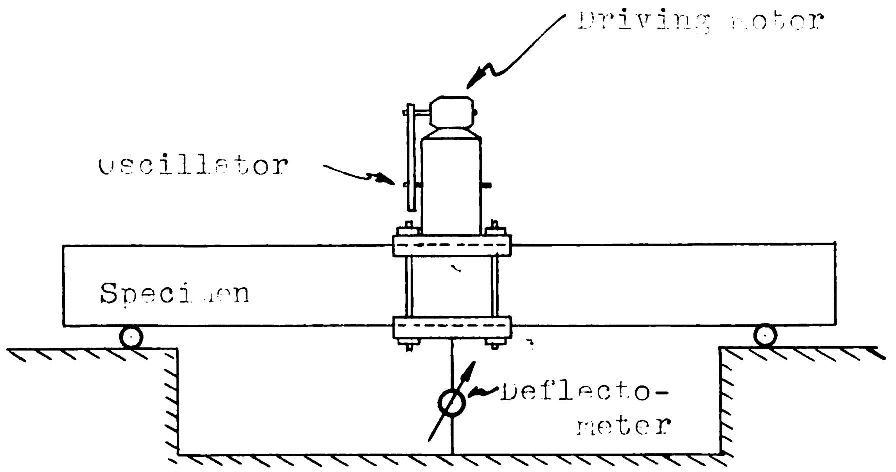
DYNAMIC TESTS BY MEANS OF INDUCED VIBRATIONS *-7

This method of testing is based on the principle that practically all systems in the field of engineering can be set in motion by alternating forces of a Sine form. Such forces are readily reproducible by two revolving masses, eccentrically supported and moving in opposite directions. As shown by the diagram below, (in which the small solid circles are the eccentrically supported revolving masses) at integral values of pi for the phase angle, centrifugal forces act in such a way as to cancel one another. At other values of pi, resultant action is such as to produce a curve of the Sine form.

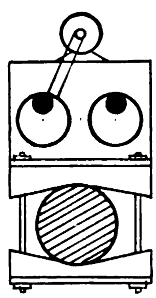


INDUCING VIBRATIONS

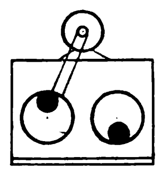
Each unit consists of two revolving disks, an electric motor driving, instruments to measure frequency of disks and to measure power input by motor.



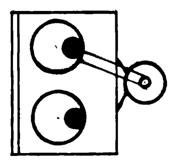
(a)



(b)
Bending Test



(c)
Torsion Test



(d)
Torsion Test

(c) Change in phase of 180°

(d) Change in relative position of revolving masses

The frequency of vibration is variable, oscillators have been built for frequencies up to 100 cycles per second. Centrifugal forces have been produced of from 1 - 50,000 pounds. Since the unit is attached rigidly to the body, the maximum working force produced is proportional to the amplifying factor of the system. For example: in bridges an average factor of amplification is 40, therefore, the maximum alternating force produced will become as high as 40 x 50,000 or 2,000,000 pounds.

Procedure

With the oscillator rigidly fixed to vibrating system, the corresponding power input for each frequency can be determined, and a frequency-power curve plotted. For best results the frequency should go beyond at least one resonance point and the frequency-power curve checked by slowly decreasing speed of oscillator.

Advantages

1. An oscillator is less expensive than a standard fatigue testing machine of equal capacity.
2. Is easily operated in laboratory and in field. Is independent of temperature, weather, and other external conditions.
3. Frequencies and centrifugal forces have a wide range of adjustment. (see top of page)

Advantages continued

4. Dynamic loads can be combined with static loads. Fatigue and aging continued simultaneously to destruction, thus more closely approaching natural conditions.

Disadvantages

1. Very few experience records are available. Until this is remedied, continual checks should be made with standard testing machines.
2. In field tests, it may be difficult to avoid energy losses thru radiation. Example: in vibrating a floating hull, energy was lost thru the churning of the water adjacent to the hull.

Applications of this method are quite numerous. Vibrations produced by machinery might be very dangerous, hence pre-determination of resonant frequency in early stages of construction would prove valuable. By means of this unit, it is possible to reproduce natural vibrations of engines and where resonance was found in surroundings, changes could be made before completion of construction or installation of the machinery. This has been done on ship decks and hulls. One of the very first zepplins failed thru resonant vibration of the crank-shaft. If known before hand, it would likely have been a simple matter to correct. Similarly, horizontal, vertical, and torsional frequencies of ceilings, walls, towers, and foundations could be found.

CONCLUSION

From studies of a number of articles on the subject, "Determination of Young's Modulus by the Sonic Method", the following general statements seem to be indicated.

1. In the derivation of the relationship between E and frequency of vibration, the material is assumed perfectly elastic. By the Sonic method, amplitudes of vibration up to 10^{-6} inches per inch are produced, which seems to be sufficiently small as to remain within the elastic limits for the material and thus verify the accuracy of the assumption.

2. That specimens can be tested without destroying, damaging, or otherwise altering the sample. Hence the same bar can be used for progressive tests at all ages. This is conducive to greater accuracy of results since the number of variables in the test sample can be held to the change in the internal structure of the beam itself.

3. In the freezing and thawing tests of concrete, the largest change in E occurs during the first few cycles and even E of good concrete will show a decrease. Poor specimens can usually be detected within 20 cycles and the E of the good and the bad specimens at that time is quite different.

There seems to be great promise in the building material field where aging occurs thru cycles of freezing and thawing or some such similar cycles.

4. Temperature changes have little effect on results. Small moisture changes in a saturated or near saturated specimen have little effect on the results. Hence a saturated condition is well suited for obtaining comparative measurements. The Sonic Modulus is very sensitive to small changes of moisture in a relatively dry sample.

Yet unexplained - That the difference in E of the saturated and the dry specimen is several times that due to the change in density, the modulus being greater for the wet specimens.

5. That oven drying will cause a considerable change in natural frequency. Obert (*-10) found that his best results were obtained when the sample was cured 28 days in water, 28 days in air and then dried for 24 hours in an oven; this last step being absolutely necessary. For testing of stone, two week drying at ordinary room temperatures was sufficient to give consistent results.

6. As aging process proceeds, specimen begins to disintegrate and causes the resonance peak to spread out, it becomes more and more difficult to select fundamental frequency.

Broken or chipped specimens, or with any other irregularities, tend to increase the probable error of the results.

7. The simplicity, accuracy, and speed of the Sonic test (routine tests can be run in as little as 1-2 min-

utes per test sample) may prove of value in statistical studies where a large group of samples is encountered.

8. There seems to be little if any information available concerning the effects of various sizes of aggregates in concrete specimens. Care should be taken so the maximum size aggregate will not be such as to produce a prism of alternate sections of stone and mortar since the resulting frequency would be difficult to interpret.

9. The high speed of vibrations is such as to eliminate the effects of creep in the material.

10. The added accuracy obtainable may not always be commensurately valuable except for laboratory studies of stress-strain relationships. Accurate values for E will usually prove useful in providing a good foundation or starting point for other studies.

11. As an aid to comparison between results of different investigators, it should be very advantageous to establish a standard routine for testing materials by Sonic methods.

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