IN HIGH VOLTAGE TRANSMISSION LINE

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This is to certify that the

thesis entitled

"THE ECONOMY OF BUNDLING CONDUCTOR IN HIGH VOLTAGE

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ABSTRACT

The subject of Power Transmission by bundle conductor has received considerable attention during the last few years. The bundle conductor is the use of more than one conductor per phase in a three-phase, high voltage transmission line. The Power transmission by use of two conductors per phase in a horizontal configuration in overhead line is the subject matter of this paper. This paper investigates a definite relation to the bundle in comparison with a single conductor per phase with respect to inductance, capacitance, surface voltage gradient, disruptive critical voltage and corona loss.

The characteristics of the lines are studied with different aspects, and appropriate formulas have been established for the bundle conductor of two conductors per phase. Several problems are solved, and a comparative study has been made between the single and two conductor bundle per phase by plotting curves.

This paper reveals that when two overhead conductors, a few inches apart, are used for each phase of a transmission line circuit instead of one of larger cross section, the inductive reactance is reduced by 20% and capacitive increases by 15% for a particular spacing of the conductor. As the inductance of the line decreases and the capacitance increases with greater spacing of sub-conductors, the use of bundle conductor is the gain in the appreciably lower value of surge impedance. It has been shown that the double conductor line has the capacity to carry 1.2 times the allowable load of that of a single conductor line.

This paper shows the gain in lower surface voltage gradient attributed to bundle and the decrease of 5% through use of the bundle has been shown. The maximum surface gradient occurs at the middle phase due to non-uniform charge density on the conductor periphery of the bundle conductor circuit.

A precise computation of its value has been determined by Maxwell's coefficient. The reduction in the maximum surface gradient is based on the value of single conductor surface gradient.

As the surface voltage gradient directly affects the corona-starting voltage, critical voltage is 15% higher for a bundle. With the same amount of material, much higher voltage can be used without corona loss when the transmission line is built with two small conductors per phase properly arranged than with a single conductor of equivalent area. This is a great advantage of the bundle over a single conductor circuit. The economy of the bundle has been established on this point in case of a high voltage transmission line.

The corona losses have been found to be much smaller when the sub-conductor spacing of the two-conductor bundle is less than 15 inches. The multiple shielding effects of a subconductor reduce the electric field intensity on the individual conductors, which in turn reduces corona. According to theoretical calculation, it is found that the corona losses are about the same when the spacing is greater than 15 inches.

THE ECONOMY OF BUNDLING CONDUCTOR IN HIGH VOLTAGE TRANSMISSION LINE

By

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A THESIS

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ABSTRACT

A general survey of the theoretical study has been carried out of the Bundle-conductors for Transmission line use. The different aspects of bundle-conductors are studied and an appropriate formula is obtained for inductance, capacitance, voltage-gradient, corona loss, and disruptive critical voltage.

Several problems are solved for line characteristics, and graphs are shown in every case. A comparative study has been made between the single and two conductor bundle per phase by plotting curves.

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SCOPE OF THIS PAPER

1) The bundle of only two conductors per phase has been studied.

2) Only horizontal flat configuration of the bundle, i.e., all are in the same plane, has been considered and only on overhead lines.

3) For varying m from 8 to 24 inches, all characteristics of the line are obtained.

4) Conductors are considered smooth and round in all cases except for corona calculation.

5) The equivalent cross-section of the conductor is constant: diameter for 1 conductor per phase is 1.41 inch; diameter for 2 conductors per phase is 1.0 inch.

6) Ground-wire effect is neglected in capacitance and surface charge calculation.

7) The formulas for x, b, g, e_o, and P (consult list of symbols) has been established for the two conductor bundle per phase.

8) Each characteristic as a function of m is illustrated by working out one problem in each case.

9) For verification of the theoretical calculation, some examples about the line construction cost is quoted,[1] but the economy regarding cost of line is out of the scope of this paper.

10) Nothing has been stated about the most suitable subconductor spacing as it is beyond the scope of the present paper.

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[1] H. L. Deloney and W. L. Rush, "Bundle Conductor for Transmission Line Capacity Increase, <u>Electircal World</u>, December, 1955.

LIST OF SYMBOLS

f = the frequency in cycles per second

g = the maximum surface gradient in volts/inch

e_o = the disruptive critical voltage in kv

e_n = the applied voltage (working) to neutral in kv

X = the inductive reactance in ohms/mile

b = the capacitive susceptance in mho/mile

I = the total current per phase in amps

E = the potential difference in volts

Q = the charge in coulombs

C = the capacitance in farads

S = the distance between conductors of different phases (position 1, 2, and 3) in feet

m = the distance between conductors of the same phase in inches

h = the height above ground in feet

D_{rs} = the distance between conductor r and image of s in feet

r = the radius of the conductor in inches

d = the diameter of the conductor in inches

D = the flux density in webers/sq. meter

L = the self inductance in henry

M = the mutual inductance in henry

P = the corona loss for 3-phase in kw/mile

M_o = the irregularity factor

b_1 = the barometric pressure in cm

t = the temperature in degrees centigrade

δ = the air density factor

p = Maxwell's coefficient

φ_c' = the corona function

ϵ = the permittivity in farads/m

INTRODUCTION

During the last few years, the subject of Power Transmission by bundle-conductor circuits has received considerable attention. The term bundle conductor, employed in this paper refers to the use of two conductors instead of one for each phase. The two conductors are therefore electrically in parallel and form one effective conductor of large cross-section.

The purpose of this paper is to investigate a definite relation to the bundle in comparison with a single conductor per phase with respect to inductance, capacitance, voltage-gradient, disruptive critical voltage, and corona loss.

This paper reveals when two overhead conductors, a few inches apart, are used for each phase of a transmission line circuit instead of one of larger cross section, that the inductive reactance is reduced by 20% and capacitive increases by 15% for a particular spacing of the conductors. The gain in lower surface voltage-gradient is attributed to bundle and the decrease has been shown by 5% by the use of bundle. The disruptive critical voltage is higher for a bundle because multiple shielding effects of sub-conductors reduce the electric field intensity on the individual conductors which in turn reduces the corona loss.

The scope of this paper does not include any vertical configuration of conductors. Only horizontal two conductors per phase

separated by 8 to 24 inches are studied. Conductors are considered smooth but for stranded and hollow conductors a correction factor is given.

Formulas have been established for the calculation of inductive reactance, capacitive susceptance, surface voltage-gradient, disruptive critical voltage for the three phase horizontal configuration.

Formulas and estimating curves given in this paper may be of interest to the transmission engineer but this only supplements previous work.

CHAPTER I

THE INDUCTIVE REACTANCE OF TWO CONDUCTORS PER PHASE

When a 3-phase bundle two conductor transmission line is completely transposed so that each conductor occupies each of the tower position for equal distances, the phase being rotated in cyclic order, the total inductance to neutral of any conductor will be the sum of the inductances in each position. The average inductance per mile of all conductors to neutral will be the same.

A formula for the average inductance to neutral per mile of each conductor of a perfectly transposed 3-phase transmission line having two conductors per phase has been derived in Appendix 1, by the method of Geometric Mean Distance (G.M.D.)

From Equation (10) Appendix 1

$$L_c = 0.08047 + 0.74113 \log_{10} 2/d \frac{(S_{gmd})^2}{m_{gmd}} \quad \text{millihenry/mile}$$

where

S_{gmd} = The geometric mean distance between conductors of the different phases in feet.

m_{gmd} = The geometric mean distances between the conductor of the same phase in feet.

Geometric Mean Distance for two conductors per phase with sub-conductor spacing m and phase distance S in feet has been derived in equation (28) Appendix 2B.

$(G.M.D.)^2$ for the horizontal flat configuration with two conductors per phase, as given in equation (28):

$$(S_{gmd})^2 = S^2 . \sqrt[3]{4} \sqrt[6]{[1-(\frac{m}{S})^2]^2[1-(\frac{m}{2S})^2]}$$

The total inductance to neutral of any conductor in any position is the sum of self and mutual inductances. The formula for self and mutual inductance of non-magnetic wires in air has been taken from "Bulletin of the Bureau of Standards." From equations (6) and (7) in Appendix 1 of this paper

$$L_1 = 2L' [\log_e \frac{4L'}{d} - \frac{3}{4}]$$

$$M = 2L' [\log_e \frac{2L'}{S} - 1]$$

Now the average 60 cycle reactance per conductor from Equation (11) Appendix 1

$$X_c = 0.3034 + 0.27941 \log_{10} \frac{2}{d} \frac{(S_{gmd})^2}{m} \quad \text{ohm/mile}$$

Since two conductors of the phase are in parallel, the average inductance per phase is the one-half of the inductance per conductor.

$$X = \frac{1}{2} [0.03034 + 0.27941 \log_{10} \frac{24}{d} \frac{(S_{gmd})^2}{m} \quad \text{ohm/mile d in inch}$$

This value of reactance is on the assumption of solid, smooth round wire. The correction for Stranded and hollow conductor are given in Tables I and II.

It has been illustrated by Problem 1 and curve 1 in this paper that an overhead double-conductor line has approximately 20% less

TABLE I

CORRECTION FOR STRANDED CONDUCTORS FOR REACTANCE CALCULATION

Conductors Per Phase	Number of Strands 7	19	39	61
1	0.0086	0.0033	0.0017	0.0010
2	0.0043	0.0017	0.0009	0.0005
3	0.0029	0.0011	0.0006	0.0004
4	0.0021	0.0008	0.0004	0.0003
5	0.0017	0.0007	0.0003	0.0002

TABLE II

CORRECTION FOR HOLLOW CONDUCTORS FOR REACTANCE CALCULATION

Conductors Per Phase	Ratio of Internal to External Diameter 0.2	0.4	0.6	0.8	1.0
1	-0.0022	-0.0075	-0.0145	-0.0222	-0.0303
2	-0.0011	-0.0038	-0.0073	-0.0111	-0.0152
3	-0.0007	-0.0025	-0.0048	-0.0074	-0.0101
4	-0.0005	-0.0019	-0.0036	-0.0056	-0.0076
5	-0.0004	-0.0015	-0.0029	-0.0044	-0.0061

reactance than a single conductor line of the same phase of spacing S and of equivalent cross-section.

It has also been shown in Problem 1(b) that for the same reactance of both bundle and single conductor, the diameter of a single conductor has to increase five times for particular spacing of sub-conductor.

CHAPTER II

CAPACITIVE SUSCEPTANCE OF TWO CONDUCTORS PER PHASE

Assuming a 3-phase bundle of a two conductor line is completely transposed so that each conductor occupies each of the tower position for equal distance, the phase being rotated in cyclic order, the total capacitance to neutral can be found by Geometric Mean Distance method, by means of the approximate average value of the charge when the charges on the line are unequal.

For horizontal flat configuration, a formula for the average capacitance to neutral per mile of each conductor of a perfectly transposed 3-phase transmission overhead line having two conductors per phase has been derived in equation (24) of Appendix 2.

From equation (24)

$$C = \frac{1}{18 \times 10^9 \ l_n \left(\frac{S_{gmd}}{mr}\right)^2} \quad \text{farad}$$

The capacitive susceptance for 60 cycle to neutral from equation (25)

$$b_c = \frac{14.66 \times 10^{-6}}{\log_{10} \left(\frac{S_{gmd}}{mr}\right)^2} \quad \text{mho/mile}$$

Since two conductors are parallel, the average b per phase is two times that per conductor. From equation (27)

$$b \text{ (per phase)} = \frac{29.28}{\log_{10} \frac{24}{d} \left(\frac{S_{gmd}}{m}\right)^2} \quad \text{micromho/mile}$$

where d in inch

The equation for b is computed by assuming uniform charge density. The error due to this approximation is of the order of 1 to 2%.

It has been illustrated by Problem 2 and curve 2 in this paper that an overhead double conductor line has nearly 20% greater susceptance than that of a single conductor line of the same S and of equivalent cross-section.

CHAPTER III

SURFACE VOLTAGE GRADIENT OF BUNDLE CONDUCTOR

With the tendency toward higher a.c. transmission line voltages, due to greater distance of power transmission, the type of line conductor to be used becomes of increasing importance. As conductor surface voltage gradient directly affects corona loss, and corona starting voltage, the ultimate problem is of loss. This problem has a solution if a bundle conductor, i.e., two or three conductors per phase are used instead of one.

A good physical picture of the phenomenon associated with flux conditions in a bundle conductor at the same potential is necessary for understanding the effects of bundle conductor two per phase.

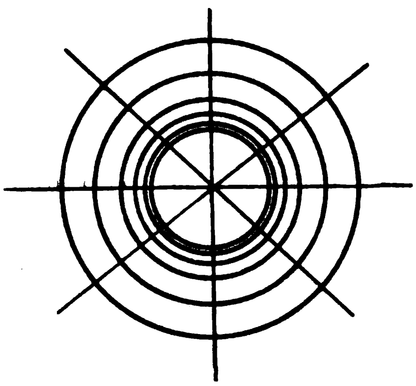

Figure 1

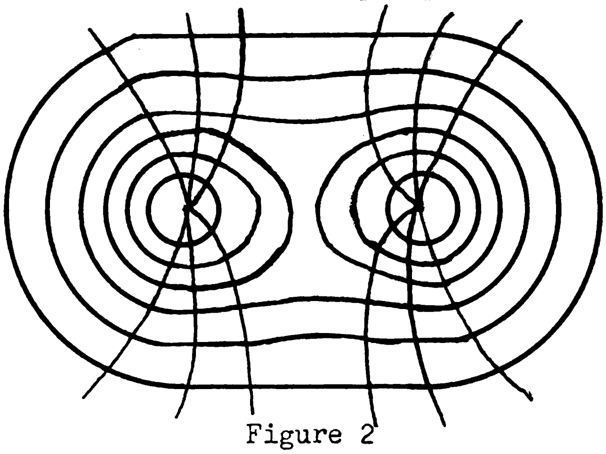

Figure 2

Figure 2 illustrates the lines of force and equipotential lines, two conductors per phase of a 3-phase line. Figure 2 indicates the charge distribution among conductors at the instant of time when the voltage on the center phase is one and that of the end phases -0.5.

Figure 2 also indicates that the lines of force are distorted between conductors, resulting in a variable voltage gradient around the conductor periphery.

The analytical method used in the calculation of conductor surface gradient is outlined in the Appendix 3. The maximum surface gradient occurs at the middle phase, and the expression for g is from equation (38) Appendix 3

$$g = \frac{V_b \left(1 + \frac{2r}{m} - \frac{2r^2}{m^2} + \ldots\right)}{2 \times 2.303 r \left(\log_{10} \frac{S}{\sqrt{mr}} - \frac{1}{3} \log 2\right)} \quad \text{volt/inch}$$

where g is the maximum surface gradient at the mid-phase in volt/inch

m is distance between conductor of the same phase in inches

r is the radius of the conductor in inches

V_b is the applied voltage on the mid-phase.

As the charge density is not uniform around the wire of the bundle conductor circuit, a precise computation of its value is determined by the equation given in a paper[1] by H. B. Dwight. Problem 3 in this paper is an example of g with varying sub-conductor distance. Curve 3 illustrates that the greatest reduction in maximum gradient is with a sub-conductor spacing of 12 inches.

The equation is deduced on the following assumptions:

a) Conductors are arranged horizontally, i.e., they are in the same plane.

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[1]H. B. Dwight, "The Direct Method of Calculation of Capacitance of Conductor," Trans. A.I.E.E., vol. 52, 1933.

b) Conductors are perfectly smooth and round with m spacing for other conductors of the same phase.

c) The distance between phases S is considered from the center of the conductor of one phase to another.

d) The term voltage gradient designates the maximum voltage-gradient at the conductor surface.

e) The ground effect is neglected.

The charge distribution on the conductors of one and two conductors per phase are shown in Figures 3 and 4, with applied voltage +1 on center phase and -0.5 on the end phases.

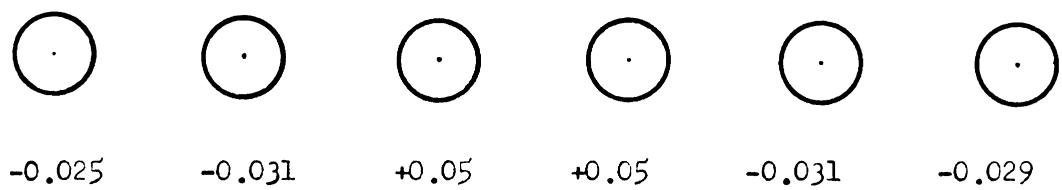

Figure 3

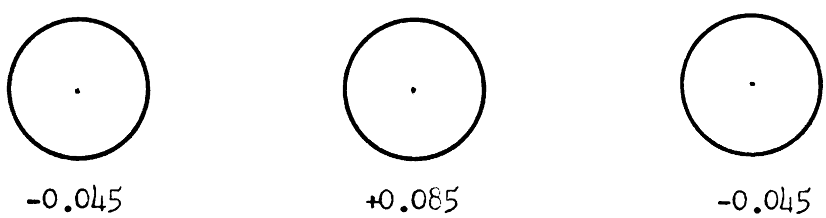

Figure 4

The equations for the calculation of the above charges in bundle conductors each having self and mutual capacitance with each of the other conductors and with certain applied voltage to ground, the only way of calculation is by Maxwell coefficient.[1]

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[1]Two-Dimentional Fields in Electrical Engineering, Bewly & Macmilan, 1948.

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} P_{11} & P_{12} & P_{13} & P_{14} & P_{15} & P_{16} \\ P_{21} & P_{22} & P_{23} & P_{24} & P_{25} & P_{26} \\ P_{31} & P_{32} & P_{33} & P_{34} & P_{35} & P_{36} \\ P_{41} & P_{42} & P_{43} & P_{44} & P_{45} & P_{46} \\ P_{51} & P_{52} & P_{53} & P_{54} & P_{55} & P_{56} \\ P_{61} & P_{62} & P_{63} & P_{64} & P_{65} & P_{66} \end{bmatrix} + \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix}$$

where V is the voltage applied

P is Maxwell's coefficient or proportionality factor

Q is the charge on conductor

For abbreviation the equation can be rewritten:

$$[Q] = [P]^{-1} [V]$$

It may be shown in general, by Green's Theorem that $P_{rs} = P_{sr}$ (Electricity and Magnetism, Jeans) and they are all positive. The numerical value of the potential coefficient has been worked out by the method of image considering ground as zero potential for the bundle of two conductors per phase, in Appendix 3B.

The general equation deduced for P in Appendix 3B is as follows:

$$P_{11} = P_{22} = \ldots = P_{66} = 2 \log \frac{2h}{r}$$

$$P_{21} = \ldots + \ldots P_{56} = 2 \log \frac{D_{rj}}{m}$$

where h is the height above ground, and 2h is the distance from the conductor concerned to its own image.

D_{rj} is the distance between conductor r to the image of conductor j and so on.

CHAPTER IV

DISRUPTIVE CRITICAL VOLTAGE OF THE BUNDLE

The discuptive critical voltage is the voltage at which corona starts to form on transmission line conductors in fair weather. This is the limit of operating voltage on the line. It depends on the potential gradient at the surface of the conductor. In Chapter III, g, the maximum stress at the conductor surface has been derived and g, corresponding to e_o, may be called the disruptive gradient. e_o greatly depends upon the conductor radius, if r is smaller, then e_o will be larger.

As the surface gradient of the center phase is the highest for horizontal configuration, a formula is developed in Appendix 4 for e_o for the middle phase conductor which has the lowest critical voltage.

For a perfectly smooth polished conductor for which there is only one per phase

$$e_o = g \times r \times M_o \times \delta \log_e \frac{S}{r} \qquad \text{kv}$$

where g is the critical gradient

S is the distance between phase conductors

r is the radius of the conductor.

If each phase consists of 2-conductor of spacing m from equation (47) then

$$e_o = \frac{g r M_o \delta}{1 + \frac{2r}{m}} \log_e \frac{S^2}{mr} \qquad \text{kv}$$

g is considered constant for conductors used in H.V. transmission line and value of g for wires taken under a variety of conditions on the outdoor line are given in Table III.[1] According to Peek, g is 53.6 kv/inch.

Since g = 53.6 kv/inch

$$e_e = \frac{123 r M_0 S}{1 + \frac{2r}{m}} \quad \log_{10} \frac{S^2}{mr} \qquad kv$$

e_0 is determined by the equation (47) in Appendix 5, and M_0 is taken as 0.96 for solid wires.

TABLE III

DISRUPTIVE CRITICAL VOLTAGE GRADIENT FOR WIRES
(Values corrected to 76 cm. Barometer and 25°)

Spacing in cm.	Radius in cm.	g_0 kv/cm.max.
152	0.084	31.3
229	0.084	31.6
550	0.084	36.5
122	0.164	28.8
244	0.164	29.0
366	0.164	25.6
488	0.164	25.3
91.4	0.259	28.7
183	0.259	26.5
275	0.259	26.0
397	0.259	26.2
91.4	0.463	28.7
183	0.463	30.4
214	0.463	30.5
275	0.463	31.0

[1] F. W. Peek, Dielectric Phenomenon in High Voltage Engineering, Third edition, McGraw-Hill Book Co., 1929.

CHAPTER V

CORONA LOSS MEASUREMENT

In bundle conductors, the voltage-gradient is not uniform around the conductor periphery. A correlation must be established between corona-loss and the variable voltage gradient. For theoretical calculation, relative losses can be determined by working with maximum conductor surface voltage-gradient or for two conductors per phase grouping; the mean value of maximum and average voltage-gradient is used. The maximum voltage gradient is considered in this paper.

The loss on a transmission line varies depending upon the temperature, and weather conditions. Some various factors like moisture, frost, fog, sleet, rain, and snow have an appreciable effect upon the loss.

A number of formulas has been worked out for estimating the corona loss values of which Peterson's formula is widely used.
A theoretical formula modified by empirical correction was developed by Peterson which is applicable for calculating corona loss for the lower value of the losses as well as higher values.

The corona loss is considered to be the loss due to charging current flowing rapidly through the corona-envelope. The drop in voltage through the envelope is assumed as being the integral of the potential gradient from the point where the gradient in air exceeded 53.6 kv/inch to a point on the conductor at some higher value.

For the portion of the cycle where the voltage is too low to cause air-breakdown, no loss is assumed to occur.

As a result of this theory, the following formula is evolved.[1] From equation (48) Appendix 5

$$P_c = \frac{0.0000337 \, f \, e_n^2}{(\log_{10} \frac{S}{r})^2} \quad (\varphi_c') \qquad \text{kw}$$

where P_c is the corona loss in kw per mile per conductor

f is the frequency

S is the distance between phases

r is the radius of the conductor in inches

e_o is the disruptive voltage in kv

φ_c' is the value taken from the curve A and is a function of the ratio e_n/e_o.

For a smooth round conductor, the value of e_o, as calculated in Appendix 4, can be used with a slight modification for air density.

$$e_o = \frac{123 \, M_o \, \delta^{2/3} \, r}{1 + \frac{2r}{m}} \log_{10} \frac{S^2}{mr} \qquad \text{kv to neutral}$$

The air density $\delta = 1$ at 25°C., 76 cm. barometer.

At any other condition, it may be calculated by the following equation:

$$\delta = \frac{3.92b}{273 + t}$$

[1]Joseph Carroll, "Corona Loss Measurement for the Design of Transmission Line," Trans. A.I.E.E., vol. 52, 1933.

where b = barometric pressure in cm.

t = temperature in degrees Centigrade

M_o is the irregularity factor and varies from 0.98 to 0.93 for roughend wires

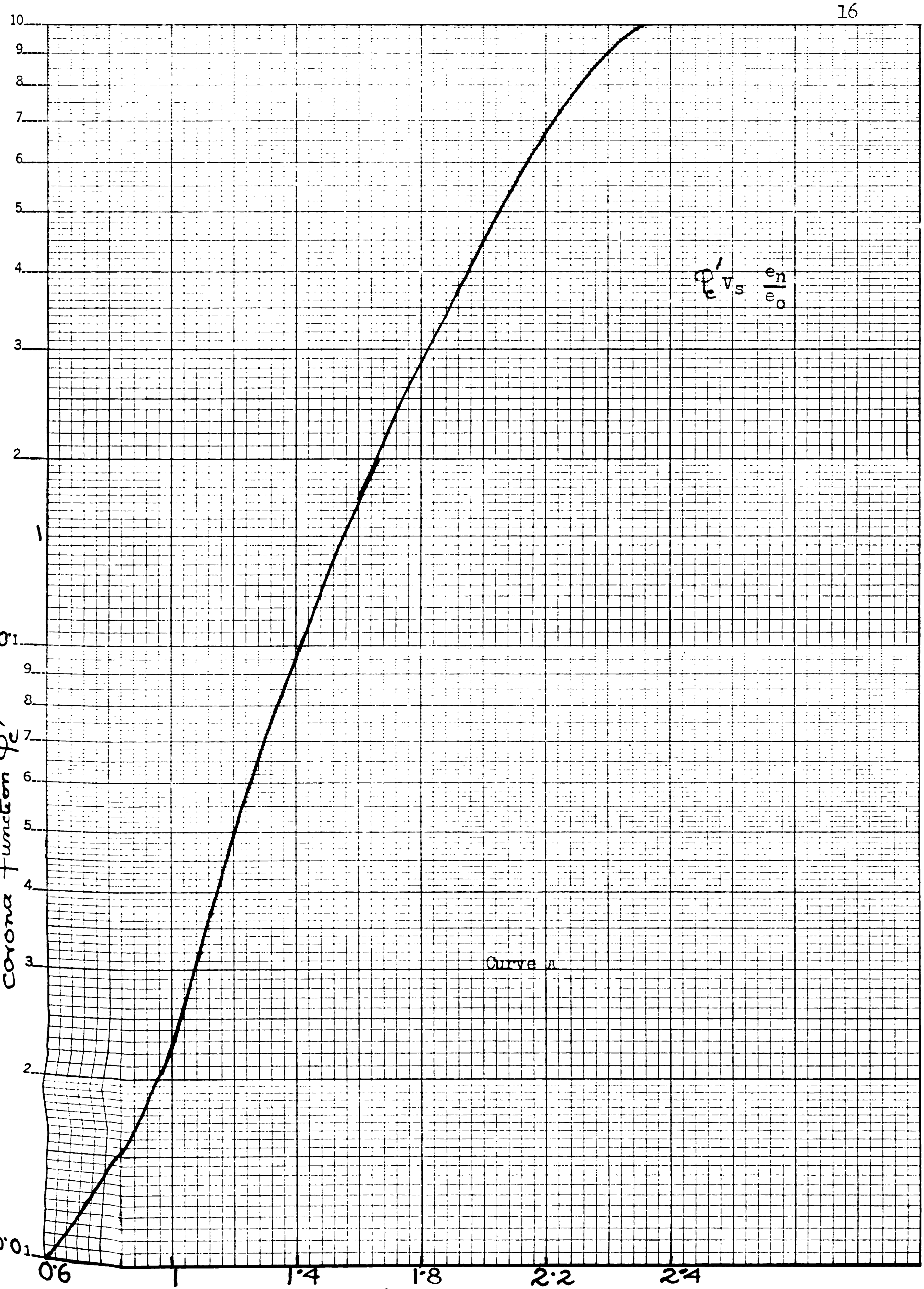
e_n is the voltage to neutral in kv (applied).

The equation (48) in Appendix 5 is true for one or two conductors per phase. For six conductors, the total loss is six times that of P_c. For all practical purposes, the losses on the bundle conductor, however, can be calculated on the basis of those of the single conductor.[1]

According to theoretical calculation in Problem 5, the corona loss are about the same when the spacing is 15 inches. From the test[2] experiment on the bundle of two conductors, it has been observed that losses are much smaller than that of a single conductor per phase. The losses on a single conductor per phase is about ten times as high as those of the bundle.

[1] O. Gerber, "Corona Losses of Single and Bundle-Conductors," C.I.G.R.E., paper 403, 1950.

[2] F. Cahen, "Results of Test Carried out at the 500 kv Experimental Station of Cherilly (France) on Corona-Behaviour of Bundle Conductors," Trans. A.I.E.E., vol. 67, 1948.


φ_e' Vs $\frac{e_n}{e_o}$
Curve A
Corona function φ_e'
10
9
8
7
6
5
4
3
2
1
0·1
9
8
7
6
5
4
3
2
0·01
0·6
1
1·4
1·8
2·2
2·4

CHAPTER VI

APPLICATION

Problem 1

Assume a 3-phase line of spacing between phases 40 feet horizontal flat configuration.

Assuming that each phase conductor consists of two sub-conductors having one inch diameter each and spaced m feet apart horizontally.

a) Calculate the phase inductive reactance as a function of m and plot.

b) Calculate and plot the diameter of a single conductor as a function of m which is equivalent to the bundle in the sense that it has equal inductive reactance.

Solution

a) Assuming m varies from 8 to 24 inches

2) conductor diameter are such as to keep the total conductor area per phase constant.

2-conductor per phase -- diameter one inch of each sub-conductor

1-conductor per phase -- diameter 1.41 inches (for equivalent cross-section)

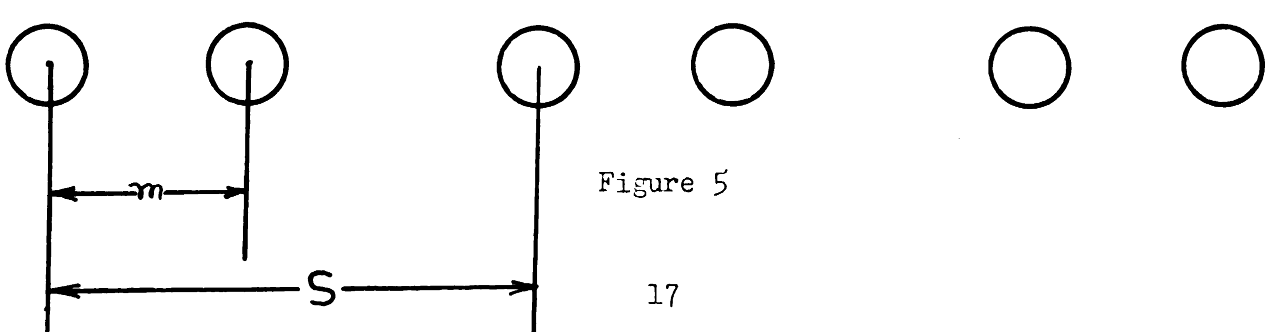


Figure 5

Reactance from Appendix 2B for two conductors per phase

$$X = \frac{1}{2}\left[0.0304 + 0.27941 \log_{10} 24 \sqrt[3]{2} \frac{S}{d}\right]$$

$$+ 0.1397 \log_{10} \sqrt[3]{2} \frac{S}{m} \sqrt[6]{[1-(\frac{m}{s})^2]^2[1-(\frac{m}{2s})^2]}$$

ohms/mile

where S and m are in feet and d is in inches; S/d is constant.

The first part of the X is constant and equal to:

$$\frac{1}{2}\left[0.0304 + 0.27941 \log_{10} 24 \times 1.25 \times \frac{40}{1}\right]$$

$$= 0.44$$

The second part of X is a function of m and can be calculated:

m in inches	m in feet	$\log_{10} \sqrt[3]{2}$ S/m
.8	3/4	1.8325
12	1	1.699
16	5/4	1.6021
20	5/3	1.47771
24	2	1.4176

The value for $\sqrt[6]{[1-(\frac{m}{s})^2]^2 [1-(\frac{m}{2s})^2]}$ is practically constant and equals to 0.955 for m varying from 8 to 24 inches. The value of the second part is now tabulated below:

m in inches	$0.1397 \log_{10} \sqrt[3]{2} \frac{S}{m} \sqrt[6]{[1-(\frac{m}{S})^2]^2[1-(\frac{m}{2S})^2]}$
8	0.243
12	0.225
16	0.214
20	0.196
24	0.182

Now the phase inductive reactance is shown as a function of m.

m = 8 inches	X (reactance) in ohms/mile	0.44 + 0.243 = 0.683
m = 12 inches	X =	0.44 + 0.225 = 0.665
m = 16 inches	X =	0.44 + 0.214 = 0.654
m = 20 inches	X =	0.44 + 0.196 = 0.636
m = 24 inches	X =	0.44 + 0.182 = 0.622

It is found from the result as the spacing between the sub-conductor increases, the reactance decreases. The relation between m and X has been plotted in curve 1.

b) For the single conductor per phase of diameter 1.41 inches reactance for phase spacing 40 feet is calculated as follows:

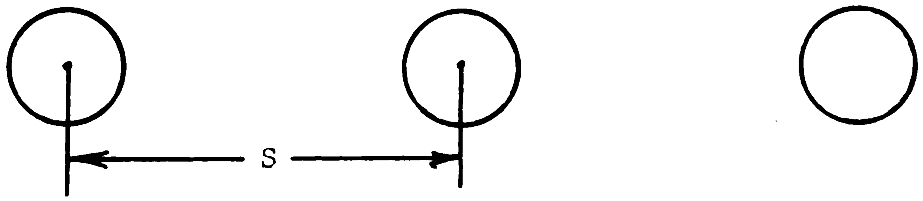


Figure 6

i) As the G.M.D. of the figure is $\sqrt[3]{2}S$, from Appendix 1

$$X = [0.03034 + 0.27941 \log_{10} 24 \sqrt[3]{2} \times \frac{S}{d}]$$

$= 0.851$ ohms/mile

This value will remain constant if S and d are constant.

ii) To calculate the diameter of a single conductor from the result of the part a) considering equivalent reactance in both the cases.

X in ohms/mile from a) $= 0.03034 + 0.27941 \log_{10} 24\ 3/\overline{2}\ \ S/d$

$0.683 - 0.03034 = 0.27941 \log_{10} 1200/d_1$

$0.665 - 0.03034 = 0.27941 \log_{10} 1200/d_2$

$0.654 - 0.03034 = 0.27941 \log_{10} 1200/d_3$

$0.636 - 0.03034 = 0.27941 \log_{10} 1200/d_4$

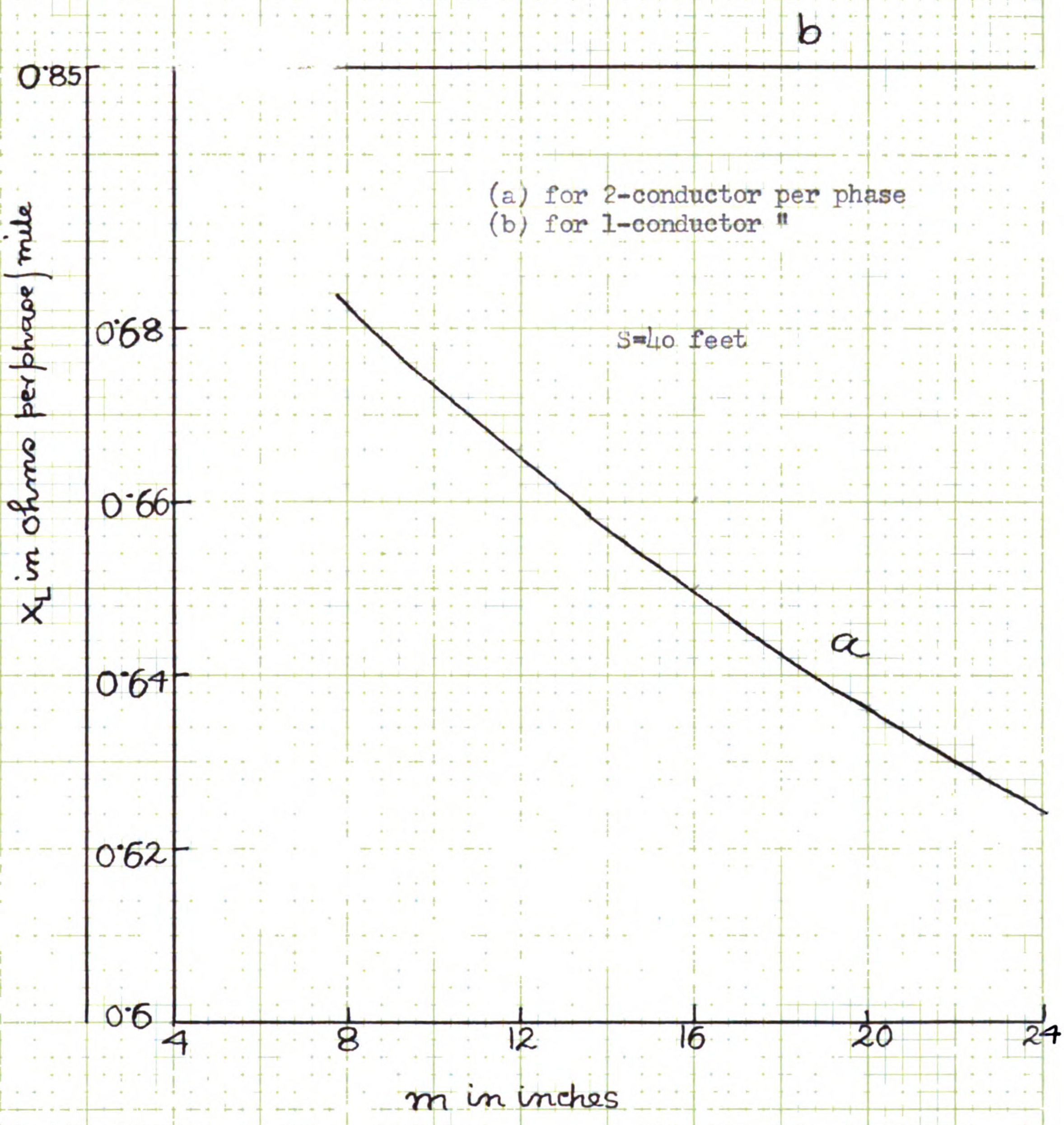
$0.622 - 0.03034 = 0.27941 \log_{10} 1200/d_5$

---

m vs d

m = 8 inches	d_1 = 5.5 inches
m = 12 inches	d_2 = 6.3 inches
m = 16 inches	d_3 = 6.9 inches
m = 20 inches	d_4 = 8 inches
m = 24 inches	d_5 = 8.7 inches

From the results of Problem 1



Curve 1 Inductive reactance in ohms/mile vs m (sub-conductor spacing)

Problem 2

Assume a 3-phase line of spacing 20 feet horizontal flat configuration.

Assuming that each phase conductor consists of two sub-conductors having one inch diameter each and spaced m feet apart horizontally.

a) Calculate the phase capacitive reactance as a function of m and plot.

b) Calculate also for the single conductor per phase of equivalent cross-section.

Solution

a) Assuming m varies from 8 to 24 inches

Conductor diameter are such as to keep the total conductor area per phase constant.

2-conductor per phase -- diameter one inch each

1-conductor per phase -- diameter 1.41 inches

Considering Figure 5 in Problem 1, S is the phase distance and equal to 20 feet.

Capacitive susceptance from Appendix 2 for two conductors per phase

$$b = \frac{29.28}{\log_{10} 24 \sqrt[3]{2}\, \frac{S}{d} + \log_{10} \sqrt[3]{2}\, \frac{S}{m} Z} \quad \text{mho/mile}$$

$$= \frac{29.28}{\log_{10} 24 \sqrt[3]{2}\, \frac{S}{d} + \log_{10} \sqrt[3]{2}\, \frac{S}{m} \sqrt[6]{[1-(\frac{m}{S})^2]^2 [1-(\frac{m}{2S})^2]}}$$

In the above equation, S and m are in feet and d is in inches; S/d is constant.

First part of the denominator of the expression is constant and equal to:

$$\log_{10} \; 24 \; 3/2 \; \frac{S}{d} = 2.6324$$

Second part of the denominator of b is a function of m and can be calculated.

m in inches	m in feet	$\log_{10} 3/2 \; S/m$
8	2/3	log 37.5
12	1	log 25
16	4/3	log 18.75
20	5/3	log 15
24	2	log 12.5

The value for $6/\overline{[1-(\frac{m}{S})^2]^2[1-(\frac{m}{S})^2]}$ is practically constant and equal to 0.96 for m varying from 8 to 24 inches. The value for the second part of the denominator is now tabulated below:

m in inches	$\log_{10} 3/2 \; S/m \; 6/\overline{[1-(\frac{m}{S})^2]^2[1-(\frac{m}{S})^2]}$
8	1.556
12	1.380
16	1.255
20	1.155
24	1.08

Now the capacitive susceptance is shown as a function of m as below:

m in inches	b in μ mho/mile
8	29.28/4.188 = 7.05
12	29.28/4.01 = 7.35
16	29.28/3.88 = 7.6
20	29.28/3.78 = 7.75
24	29.28/3.71 = 7.8

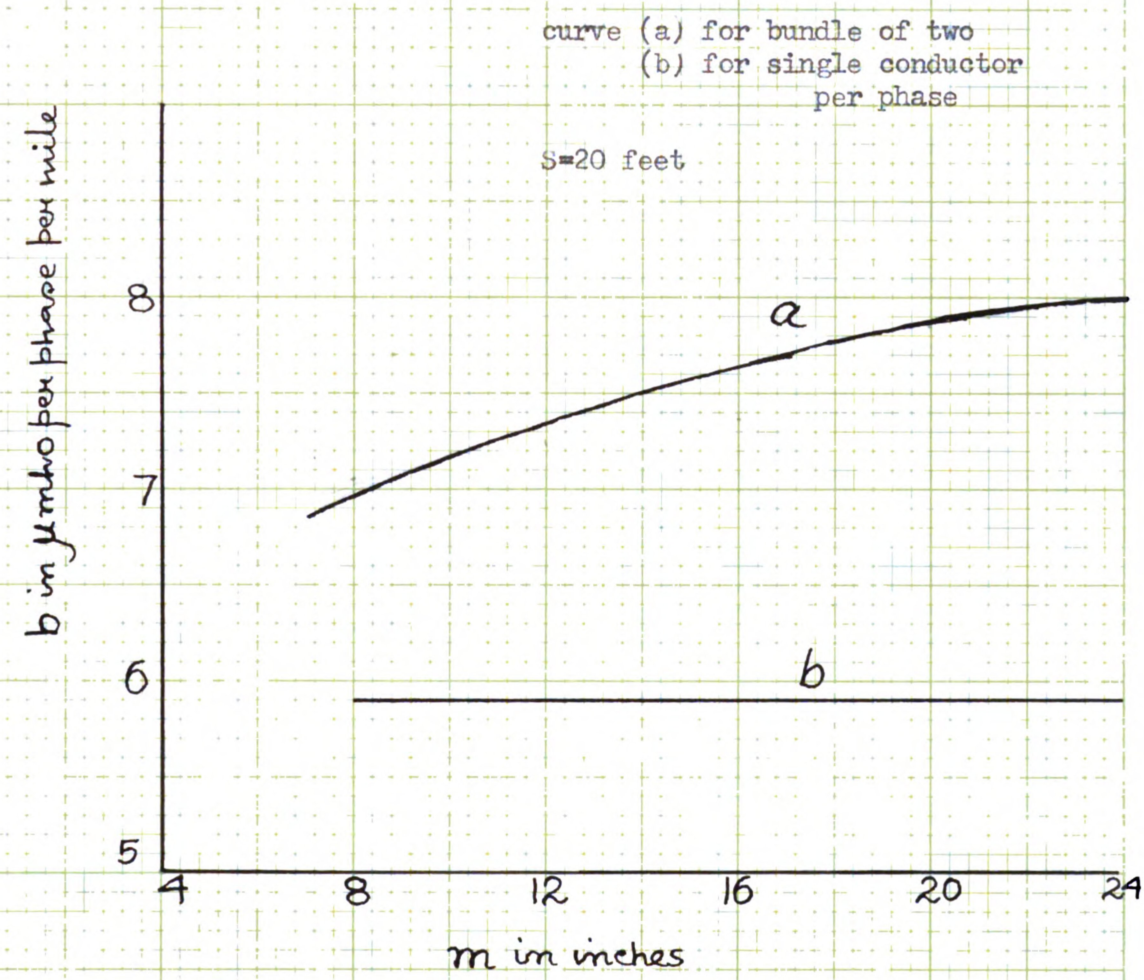
It is found from the above result as the spacing between sub-conductor increases, b increases. Nature of the relation between m and b has been shown in curve 2.

b) Single conductor per phase, diameter of which is 1.41 inches. Reference to Figure 6 G.M.D. of the spacing between phases $\sqrt[3]{2}\ S$

$$\therefore\ b = \frac{14.66}{\log_{10} \frac{24}{d} \cdot \sqrt[3]{2}\ S} = \frac{14.66}{\log_{10} 300}$$

$$= 5.9 \quad \text{mho/mile.}$$

From Problem 2



Curve 2 capacitive susceptance (b) vs subconductor spacing (m)

Problem 3

Assuming a 3-phase transmission line of spacing between phases 32 feet horizontal flat configuration and considering each phase consists of two bundle conductors having one inch diameter of each.

a) Calculate the maximum surface voltage gradient of the middle-phase as a function of m which varies from 8 to 12 inches.

b) Plot the surface voltage gradient reduction in per cent of bundles based on value of single conductor of equivalent area.

Solution

a) Assuming m varies from 8 to 12 inches.

Conductor diameter are such as to keep the total conductor area per phase constant.

2-conductor per phase -- diameter one inch of each

1-conductor per phase -- diameter 1.41 inches

Maximum surface gradient occurs at the middle-phase.

Then from Appendix 3, equation (38)

$$g = \frac{V_b \left(1 + \frac{2r}{m} - \frac{2r^2}{m^2} + \ldots\ldots\right)}{2 \times 2.303\, r \left(\log_{10} \frac{S}{\sqrt{mr}} - \frac{1}{3} \log_2\right)} \quad \text{volt/inch}$$

where m is the intragroup distance in inches

r is the radius of bundle of each in inches

S is the phase-conductor distance in inches

The numerator of the expression g is calculated, taking V_b as one volt, the maximum value at particular instant when the voltages of the other phases are -0.5 and neglecting the higher order term in the parenthesis.

m in inches	$(1 + 2r/m - 2r^2/m^2 + \ldots\ldots)$
.8	1.15
12	1.08
16	1.06
20	1.05
24	1.04

The denominator is again a function of m and can be calculated with the values of S equal to 32 feet. S/d is constant for varying m.

m in inches	$2 \times 2.303r(\log_{10} S/\sqrt{mr} - 1/3 \log_2)$
8	5.04
12	4.82
16	4.7
20	4.55
24	4.42

The maximum surface voltage gradient of the middle-phase as a function of m is given below:

m in inches	g	m	g (maximum gradient) for V_b = 1 volt
8	0.228	16	0.226
12	0.224	20	0.231
		24	0.234

b) g of the middle conductor with flat spacing one conductor per phase, from equation Appendix 3.

$$g = \frac{V_b}{2.303r\,(\log \frac{S}{r} - \frac{1}{3}\log 2)} \qquad \text{volt/inch}$$

The radius of the conductor equals 0.705 inch

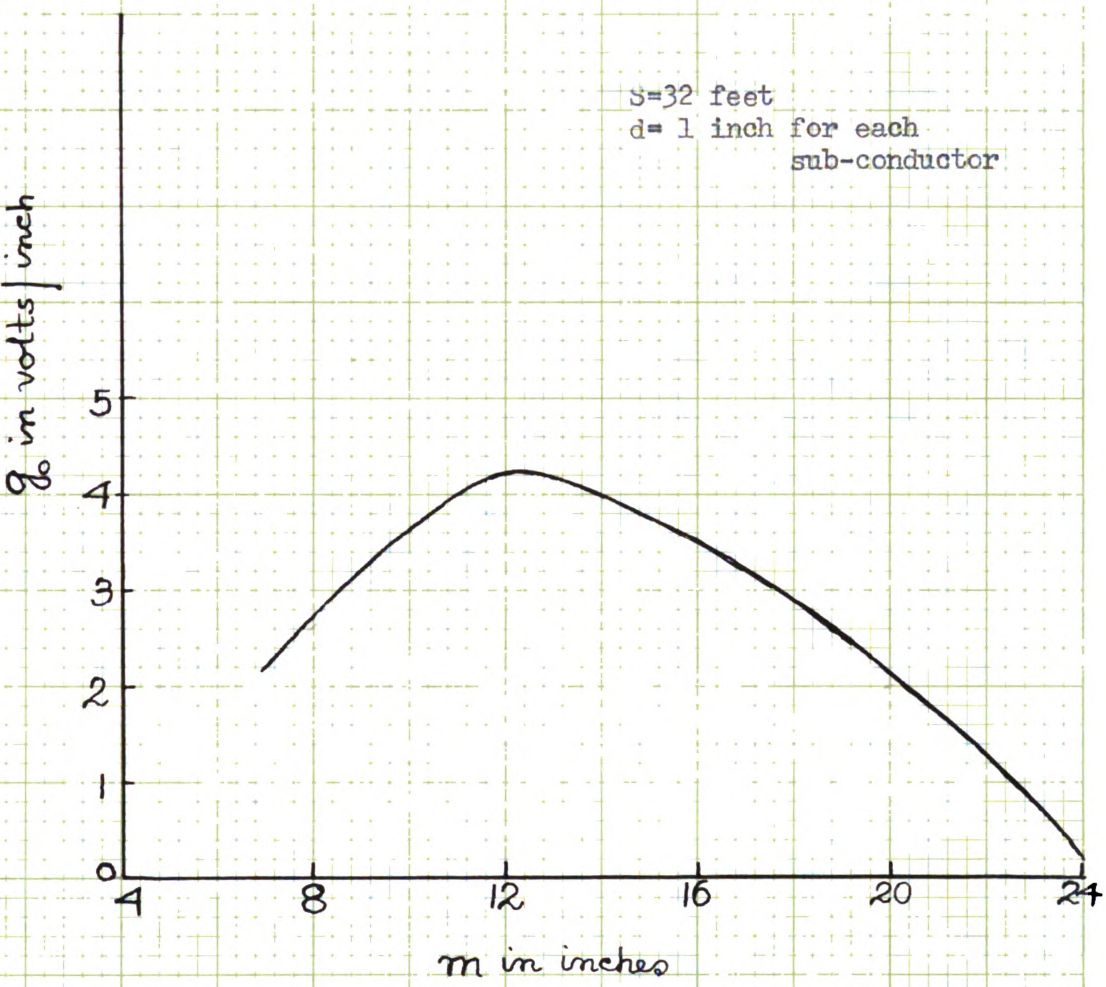
$$g = \frac{1}{2.303 \times 0.705\,(\log \frac{384}{.705} - \frac{1}{3}\log 2)}$$

Considering V_b as 1 kv (1000 v)

m in inches	Maximum gradient for middle phase of bundle in volt/inch	Maximum gradient for middle phase of single conductor in volt/inch	Per cent gradient reduction of bundle based on the value of single conductor
8	228	234	2.6
12	224	234	4.26
16	226	234	3.42
20	231	234	1.28
24	234	234	0

Voltage gradient is decreased by more than 4% for spacing 16 inches based on the value of single conductor per phase of equivalent area.

From the results of Problem 3



Curve 3 Maximum surface gradient reduction (g_o) vs m the sub-conductor spacing

Reduction based on the value of single conductor surface gradient

Problem 4

a) Considering the Problem 1, calculate the disruptive critical voltage of bundle of two conductors per phase for m, the spacing between conductors of same phase, varying from 8 to 24 inches.

b) Plot a curve showing the relation between single and bundle conductor in respect of e_o for varying m.

Given $M_o = 0.96$ $\delta = 0.966$

Datas: $S = 40$ feet, $r = 0.5$ inch for bundle and 0.705 inch for one.

Solution

The lower disruptive critical voltage of mid-phase conductor can be expressed by the equation (47) in Appendix 4.

$$e_o = \frac{123 r M_o \delta}{1 + \frac{2r}{m}} \log_{10} \frac{S^2}{mr} \qquad (1)$$

The equation (1) is a function of m only and S, m, r are in inches

m in inches	$123r M_o / 1 + \frac{2r}{m}$	$\log_{10} S^2/mr$
8	51.5	4.76
12	53.6	4.58
16	54.6	4.46
20	55.4	4.36
24	55.6	4.28

The equation for e_0 is based upon the value of g, the surface gradient of the conductor which Peek[1] took as 53.6 kv/inch. Peek indicated that g is constant for conductors of diameter that would be used for H.V. transmission line. But subsequent investigation[2] raises considerable doubt as to whether g is constant in the range of low losses. The indications are that g might well increase with decreasing radius shown in Problem 3. This factor can be taken into consideration and would be regarded as a factor favorable to bundle conductor.

The calculation of Problem 4 on the basis of 53.6 kv/inch for g

m in inches	e_0 in kv/corona starting voltage
8	245
12	244
16	242
20	241
24	240

e_0 for single conductor per phase is given by the equation (47a)

$$e_0 = 123\, M_0 r \delta \log_{10} S/r \quad \text{kv}$$

$$= 123 \times .96 \times .966 \times 0.705 \log \frac{480}{0.705}$$

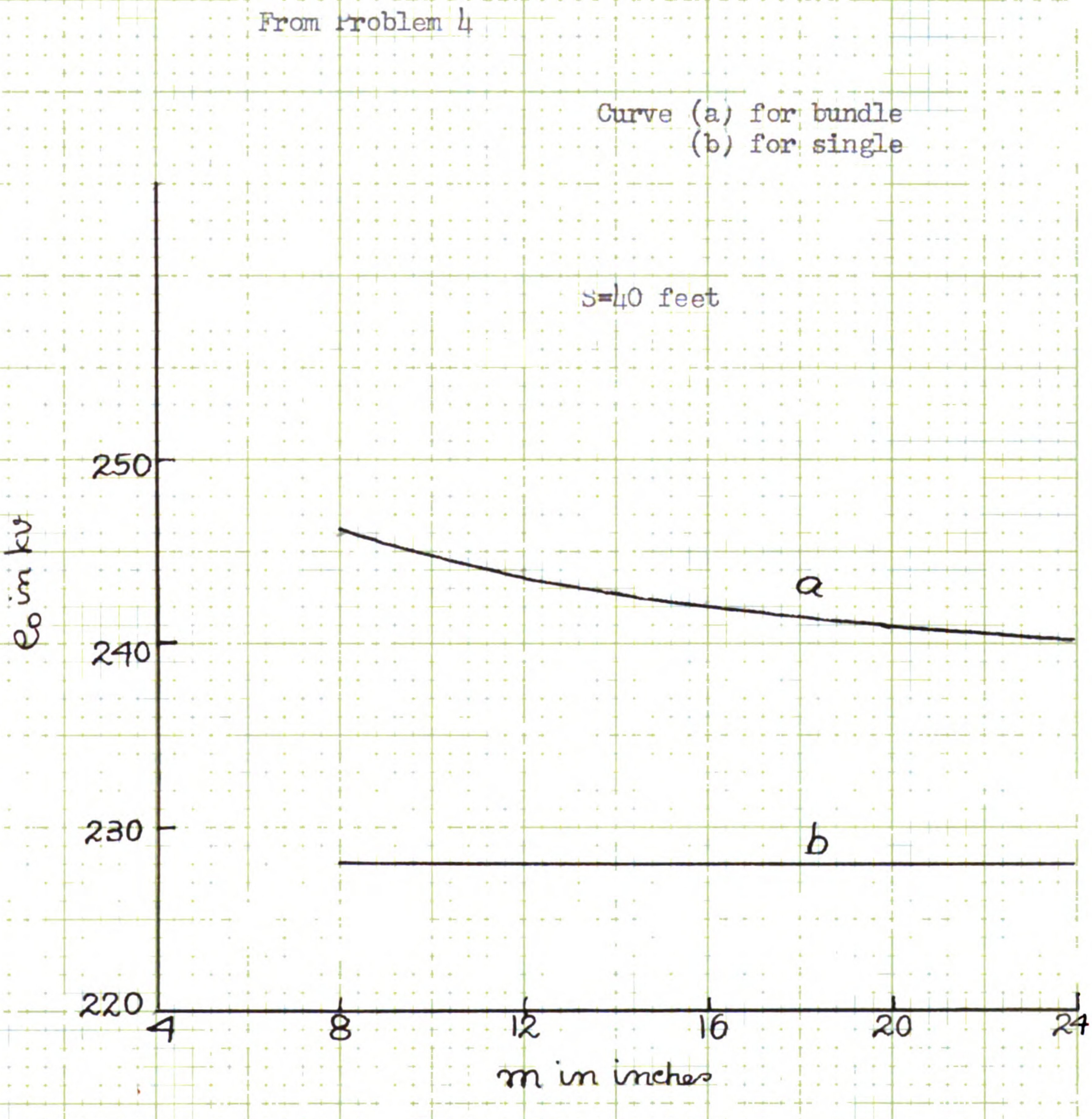
$$= 228 \text{ kv.}$$

The gain on the corona-starting voltage of bundle over single conductor per phase is 8%.

---

[1] F. W. Peek, Dielectric Phenomenon in High Voltage Engineering, Third-edition, McGraw-Hill Book Co., 1929.

[2] M. Themoshok, "Relative Surface Voltage Gradient of Grouped Conductors," Trans. A.I.E.E. vol. 67, 1948.



Curve 4 e_o (corona starting voltage) vs m (sub-conductor spacing)

Problem 5

a) Calculate the corona loss of the bundle of two conductors per phase having one inch diameter of each. Assume 3-phase line, spacing between phases 40 feet, horizontal flat configuration. Voltage between lines is 480 kv and f 60 cycle. Temperature is 20° C. and barometer 72.2 cm. Plot the loss for 3-phase as a function of m, sub-conductor spacing.

b) Calculate the corona loss at the same temperature and pressure of the single conductor per phase of equivalent area.

Solution

a) The loss is given by the equation (48) of Appendix 5 of Peterson formula:

$$P_c = \frac{0.0000337 \, f \, e_n^2}{(\log_{10} \frac{S}{r})^2} \quad (\varphi_c') \qquad \text{kw/mile per conductor}$$

φ_c' = The ratio of e_n/e_o, the value of which is given in the curve 4

e_n is the voltage to neutral and equal to 480/1.73 = 278 kv.

The air density factor

$$\delta = \frac{3.92b}{273 + t} = \frac{3.92 \times 72.2}{273 + 20}$$

$$= 0.966$$

The numerator of the equation is constant for particular f and e_n, and the value is calculated below:

$$0.0000337 \, f \, e_n^2 = 3.37 \times 60 \times (278)^2 \, 10^{-5} = 156$$

The denominator is also constant for particular S and r which equals

$$(\log_{10} \frac{S}{r})^2 = (\log_{10}\frac{40\times12}{0.5})^2 = (\log_{10} 960)^2$$

$$= (2.98)^2 = 8.95$$

$$P_c = \frac{156}{8.9} \varphi_c' = 17.5 \varphi_c'$$

Now for same M_o and δ we can take the data from the Problem 4

m in inches	e_o in kv	e_n/e_o
8	245	278/245 = 1.134
12	244	278/244 = 1.14
16	242	278/242 = 1.15
20	241	278/241 = 1.152
24	240	278/240 = 1.156

The value of e_n/e_o taken from the curve 4 and tabulated

e_n/e_o	φ_c'
1.134	0.053
1.14	0.062
1.15	0.07
1.152	0.078
1.156	0.085

Final results can be calculated thus:

m in inches	P (corona loss for 3-phase $17.5 \times \varphi_c'$ per mile)
8	5.45
12	6.38
16	7.26
20	8.3
24	9.8

The minimum loss in fair weather condition corresponds to a spacing of 8 inches. Beyond 16 inches losses keep on increasing more rapidly. At 24 inches, they are almost double.

b) Calculation of loss for the single conductor per phase is exactly the same as above:

$r = 0.705$ inch

$e_o = 228$ kv from Problem 4; $\left(\log_{10} \frac{480}{0.705}\right)^2 = 19.5$

$e_n/e_o = 278/228 = 1.218$

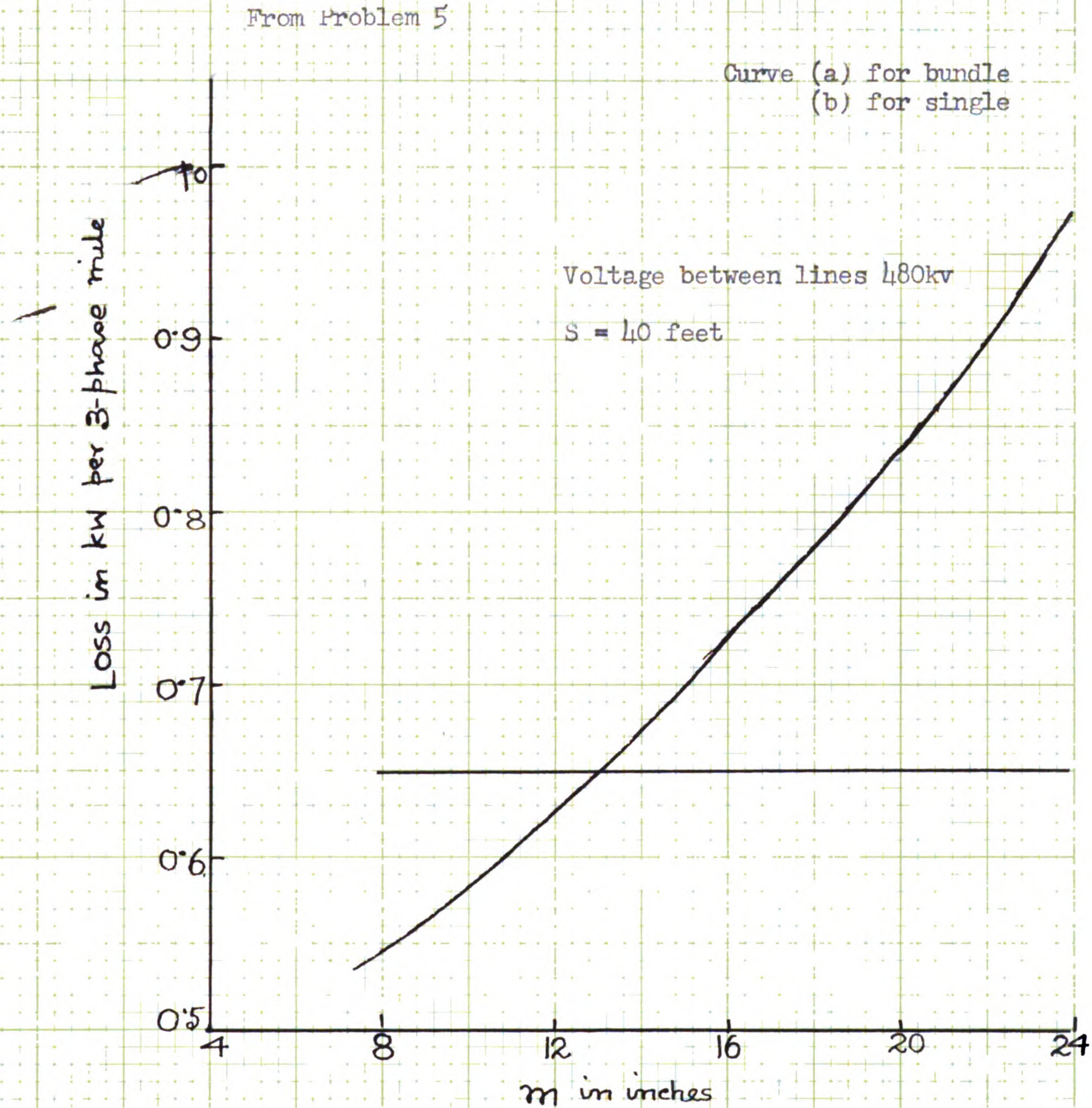
From curve 4, the value of φ_c' corresponding to 1.218 is 0.11

$P_c = 19.5 \times 0.11$

The total loss for 3-phase equals to:

$P = 3 \times 19.5 \times 0.11 = 6.43$ kw per 3-phase mile

According to theoretical calculation, the corona loss is about the same when the spacing is 15 inches. But corona loss is smaller when the spacing is less than 15 inches. Though there is not much difference between single and bundle so far as loss is concerned, but it is an advantage that the corona starting voltage is much higher in case of bundle.



Curve 5 Corona loss Vs m, subconductor spacing

Cost Comparison of Single and Bundle Conductor

The double conductor lines give a lower transmission cost than single conductor line.[1] The gain which increases with the line length lies within the range 2 to 14 per cent at 220 kv and 5 to 11 per cent at 380 kv.

TABLE IV[2]

Line Length in km	220 kv 592 mm² (single)	220 kv 2 x 328 mm² (bundle)
200	100 (base value)	98
400	100	93
600	100	90
800	100	87
1000	100	86

[1]Comparison of Lines Cost with Single and Double-Conductor, C.I.G.R.E., paper 405, 1950.

[2]*Ibid.*

CHAPTER VII

CONCLUSION

An overhead double-conductor line has approximately 20% less reactance than a single conductor line of the same weight of conducting metal. If the voltage control is such that it can operate successfully with a certain per cent of current times reactance, on either 50 or 60 cycles frequency, the allowable load on the double conductor line is 1.2 times that of a single conductor line. In many usual cases, especially where there is not complete control of the voltage by synchronous condensers, the reactance is the most important item in determining the power rating of the line for both the voltage drop and the stability limit of the load which depends principally on the reactance. Therefore, in many instances, without increasing the weight of the conductor metal, a line can be built for about one-fifth greater power rating at very little increase in cost where ice load is absent by using double conductor construction. The "Bundled"[1] 336,400 C.M., A.C.S.R. circuit costs approximately $800 per mile more than the single 666 C.M., A.C.S.R. circuit (which is approximately 5% more based on total circuit cost) whereas it has approximately 30% more actual capacity based on impedance drop limitations.

The capacitance of an overhead double conductor line is 20% greater than that of a single conductor line of the same weight of

[1]H. L. Deloney and W. L. Rush, "Bundle Conductor for Transmission Line Capacity Increase," Electrical World, December, 1955.

conductor metal (Graph 2). This is an advantage in the case of power-networks where synchronous condensers are used almost entirely with strong field currents. An increase of the line capacitance of 20% means a definite saving of the amount for synchronous condensors. Another advantage of the use of bundle-conductor is the gain in the appreciably lower value of surge impedance as the inductance of the line decreases and the capacitance increases with greater spacing of sub-conductors. With the lower surge impedance, there is a possibility of greater stability and a larger capacity of the line.

With the same amount of material, much higher voltage can be used without corona loss when the transmission line conductor is built with two small conductors per phase properly arranged than with a single conductor. It is due to the multiple shielding effects of the sub-conductor which reduce the electric field intensity on the individual conductor. It has been calculated that the surface voltage-gradient in case of bundle conductor reduced by 4% for a particular spacing based on the value of single conductor of equivalent area.

APPENDICES

APPENDIX 1

Inductance and 60 Cycle Reactance of Bundle Conductor Circuit

When a 3-phase bundle two conductor transmission line is completely transposed so that each conductor occupies each of the tower position or its equivalent for equal distances, the phases being rotated in cyclic order, the total inductance to neutral of any conductor will be the sum of the inductances in each position.

The phase position on the tower will be designated by I, II, III and the conductor positions 1,2; 1',2'; 1",2" respectively.

Starting with phase A in position I, the conductor of the phase A which first occupies the position I-1 will in turn occupy each of the six position for equal distances one-sixth of the total length of each conductor.

Let I = current per phase

$l'/2$ = length (total) of each conductor

$l'/6$ = length of conductor in each position

Assuming $\frac{I}{2}\left(-\frac{1}{2} - j\frac{\sqrt{3}}{2}\right) = \frac{I}{2}\,a^2$ current in phase II

$\frac{I}{2}\left(-\frac{1}{2} + j\frac{\sqrt{3}}{2}\right) = \frac{I}{2}\,a$ current in phase III

For sine waves of current of frequency f, the total inductive voltage drop in any conductor will be the sum of the voltage drops in the conductor for 6 positions and may be expressed in terms of self inductance L and mutual inductance M.

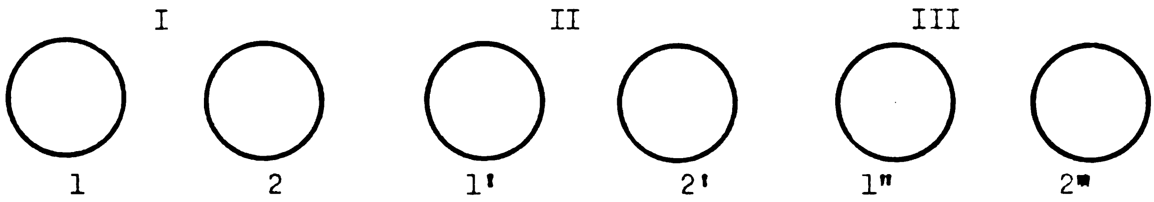


Figure 7

Position I-1

$$V_{I-1} = 2\pi f \left[\frac{I}{2} (L_1 + M_{12}) + \frac{I}{2} a^2 (M_{11'} + M_{12'}) + \frac{I}{2} a (M_{11''} + M_{12''}) \right] \quad (1)$$

There will be two equations one for each of conductor position in phase position I.

There will be also two equations when phase A occupies position II and phase position III.

$$V_{II-1'} = 2\pi f \left[\frac{I}{2}(L_1 + M_{1'2'}) + \frac{I}{2} a^2 (M_{1'1''} + M_{1'2''}) + \frac{I}{2} a (M_{1'1} + M_{1'2}) \right] \quad (2)$$

$$V_{III-1''} = 2\pi f \left[\frac{I}{2}(L_1 + M_{1''2''}) + \frac{I}{2} a^2 (M_{1''1} + M_{1''2}) + \frac{I}{2} a (M_{1''1'} + M_{1''2'}) \right] \quad (3)$$

Adding for 6-positions, the total drop V is obtained

$$V = 2\pi f \frac{I}{2} \left[6L_1 + 2 (M_{12} + M_{1'2'} + M_{1''2''}) - (M_{11'} + M_{1'1''} + M_{1''1} + M_{12'} + M_{1'2''} + M_{1''2} + M_{12''} + M_{1''2} + M_{1''2'} + M_{22'} + M_{2'2''} + M_{2''2}) \right] \quad (4)$$

From equations (1), (2), (3), and (4), it is seen that there will be 12 mutuals between conductors in the same phase positions and total 3 for the 3-phase positions. The number of mutuals between conductors of different phases will be 12.

The total inductance L_t;

$$L_t = \frac{V}{2\pi f \frac{I}{2}} = \left[6L_1 + 2 \sum 3M_{aa} - 12M_{ab} \right] \quad (5)$$

where M_{aa} = mutual inductance between conductor of same phase,

and M_{ab} = mutual inductance between conductor of different phases.

The formula in absolute units for self and mutual inductance of non-magnetic wires in air follows from those given in the "bulletin of the Bureau of Standards" when the length $L' \gg d$ and S

$$L_1 = 2L' \left[\log_e \frac{4L'}{d} - \frac{3}{4} \right] \quad \text{self inductance} \qquad (6)$$

$$M = 2L' \left[\log_e \frac{2L'}{S} - 1 \right] \quad \text{mutual inductance} \qquad (7)$$

Replacing L' by L/6 in (6) and (7) and substituting in (5) and called M_{aa} and M_{ab} by Geometric mean distance of m and S respectively, the final equation becomes:

$$L_t = 6 \times 2 \times \frac{L}{6} \left[\text{Ln} \frac{4L}{6d} - \frac{3}{4} \right] + \frac{6L2}{6} \left[\text{Ln} \frac{2L}{6M_{gmd}} - 1 \right]$$

$$- \frac{12 \times 2L}{6} \left[\text{Ln} \frac{2L}{6\, S_{gmd}} - 1 \right]$$

$$= L \left[2 \ln \frac{\frac{4L}{6d} \times \frac{2L}{6M_{gmd}} \times \frac{6 \times (S_{gmd})^2}{4\, L^2}}{4\, L^2} + \frac{1}{2} \right] \qquad (8)$$

$$L_t = L \left[2 \ln \frac{2}{d} \frac{(S_{gmd})^2}{(M_{gmd})} + \frac{1}{2} \right] \quad \text{abhenries} \qquad (9)$$

Expressing in practical units and d and M in same units, average inductance to neutral per conductor is:

$$L_{con} = 0.08047 + 0.74113 \log_{10} \frac{2}{d} \frac{(S_{gmd})^2}{m} \ \text{mH/mile} \quad \text{as } m_{gmd} = m \qquad (10)$$

and average 60 cycle reactance: per conductor

$$X_{con} = 0.03034 + 0.27941 \log_{10} \frac{24}{d} \frac{(S_{gmd})^2}{m} \ \text{ohms/mile} \qquad (11)$$

Reactance per phase of two conductor

$$X = \frac{1}{2} \left[0.03034 + 0.27941 \log_{10} \frac{24}{d} \frac{(S_{gmd})^2}{m} \right. \qquad \text{ohms/mile} \quad (12)$$

where d in inches

APPENDIX 2

Capacitance and 60 Cycles Reactance of Bundle-Conductor Circuit

Considering a 3-phase transmission line with one conductor per phase, completely transposed so that each conductor occupies each of the tower positions for equal distances, the phase being rotated in cyclic order, the total capacitance to neutral can be found out very easily by Geometric mean distance method, by means of the approximate average value of the charge when the charges on the line are unequal.

A case which occurs in this case is that a symmetrical flat spacing, arranged cab, the middle conductor a has its potential vector along the axis of reference. Assume that the charges q_a, q_b, q_c are 120 degrees apart in phase and they are of the same magnitude in all sections of the line.

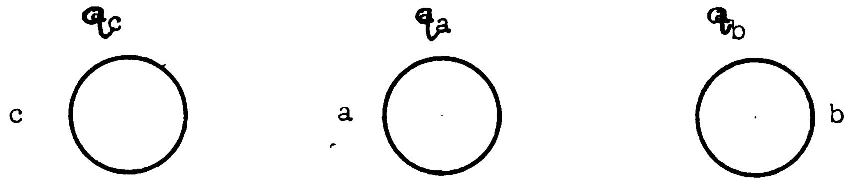


Figure 8

$$q_c = \frac{1}{2} q_a + j\, q_a \qquad (13)$$

$$q_b = \frac{1}{2} q_a - j\, q_a \qquad (14)$$

The potential difference produced by these assumed charges

$$E_{ab} = 18 \times 10^9 \sum_{j=a}^{n} q_j \ \ln \frac{D_{jb}}{D_{ja}} \tag{15}$$

$$= 18 \times 10^9 \left(q_a \ln \frac{S}{r} + q_b \ln \frac{r}{S} + q_c \ln \frac{2S}{S} \right)$$

$$= K \left[q_a \ln \frac{S}{r} + \left(\tfrac{1}{2} q_a + j\, q_q\right) \ln \frac{S}{r} + \left(-\tfrac{1}{2} q_a + j\, q_q\right) \ln 2 \right]$$

where $K = 18 \times 10^9$

$$E_{ab} = K \left[\frac{3}{2} q_a \ln \frac{S}{3/2\, r} + j\, q_q \ln \frac{2S}{r} \right] \tag{16}$$

$$= \frac{3}{2} E + j \frac{\sqrt{3}}{2} E$$

Equating the reals to the reals and imaginaries to the imaginaries and solving

$$q_a = \frac{E}{18 \times 10^9 \ln \frac{S}{3/2\, r}} \quad \text{cou per meter} \tag{17}$$

$$q_q = \frac{\sqrt{3}\, E}{36 \times 10^9 \ln \frac{2S}{r}} \quad \text{cou per meter} \tag{18}$$

In the next transposition sections where phase a replaces b, b replaces c and c replaces a, the charge on a will be equal to the charges on b in the first section considered, shifted forward through an angle of 120 degrees. Likewise in the remaining section q_a will be equal to the value q_c in the first section shifted backward through an angle of 120 degrees. Designating sections 1, 2, 3

$$q_{a1} = \frac{E}{18 \times 10^9 \ln \frac{S}{3/2}} \tag{19}$$

$$q_{a2} = \left(-\frac{1}{2} \frac{E}{K \ln \frac{S}{3/2\, r}} - j \frac{\sqrt{3}\, E}{2K \ln \frac{2S}{r}} \right) \left(-\frac{1}{2} + j \frac{\sqrt{3}}{2} \right)$$

$$= \frac{E}{4K \ln \frac{S}{3/2\, r}} + \frac{3E}{4K \ln \frac{2S}{r}} + j\left(\frac{\sqrt{3}\, E}{4K \ln \frac{2S}{r}} - \frac{\sqrt{3}\, E}{4K \ln \frac{S}{3/2\, r}} \right) \tag{20}$$

$$q_{a3} = \left(-\frac{1}{2} \frac{E}{K\, l_n \frac{S}{\sqrt[3]{2}\,r}} + j \frac{\sqrt{3}\, E}{2K\, l_n \frac{2S}{r}} \right) \left(-\frac{1}{2} - j\frac{\sqrt{3}}{2} \right)$$

$$= \frac{E}{4K\, l_n \frac{D}{\sqrt[3]{2}\,r}} + \frac{3E}{4K\, l_n \frac{2D}{r}} - j \left(\frac{\sqrt{3}\, E}{4K\, l_n \frac{2S}{r}} - \frac{\sqrt{3}\, E}{4K\, l_n \frac{S}{\sqrt[3]{2}\,r}} \right)$$

The average of q_a over the whole transposed line is equal to (21)

q_a (average)

$$= \frac{q_{a1} + q_{a2} + q_{a3}}{3} \quad (22)$$

$$= \frac{E}{10^9} \left[\frac{1}{12\, l_n \frac{S}{\sqrt[3]{2}\,r}} + \frac{1}{12\, l_n \frac{2S}{r}} \right] \div 3$$

$$= \frac{E}{2K} \left[\frac{l_n \frac{2S}{r} + l_n \frac{S}{\sqrt[3]{2}\,r}}{l_n \frac{S}{\sqrt[3]{2}\,r}\; l_n \frac{2S}{r}} \right]$$

$$= \frac{E}{K} \left[\frac{l_n \frac{\sqrt[3]{2}\, S}{r}}{\left(l_n \frac{\sqrt[3]{2}\, S}{r} - l_n 2^{2/3}\right)\left(l_n \frac{\sqrt[3]{2}\, S}{r} + l_n 2^{2/3}\right)} \right]$$

$$= \frac{E}{K\, l_n \frac{\sqrt[3]{2}\, S}{r}} \quad \text{cou per meter} \quad (23)$$

$$C = \frac{1}{18 \times 10^9\; l_n \frac{\sqrt[3]{2}\, S}{r}} \quad \text{Farads/to neutral} \quad (24)$$

The term $l_n \sqrt[3]{2}\, S$ is equal to the geometric mean of the three spacings.

The capacitive susceptance to neutral is

$$b = \frac{2\pi f}{18 \times 10^9\, l_n \frac{S_{gmd}}{r}} \quad (25)$$

$$b_{con} = \frac{2\pi f \times 60 \times 160{,}900}{18 \times 10^{11} \times 2.303 \log_{10} \frac{S_{gmd}}{r}} = \frac{14.66 \times 10^{-6}}{\log_{10} \frac{S_{gmd}}{r}} \quad \text{mho/mile}$$

(26)

For bundle conductor of two per phase, the term is identical with the inductive reactance but reciprocal of the external inductance to neutral.

$$b \text{ (per phase)} = \frac{2 \times 14.66 \times 10^{-6}}{\log_{10} \frac{24}{d} \left(\frac{S_{gmd}}{m}\right)^2} = \frac{29.28}{\log_{10} \frac{24}{d} \left(\frac{S_{gmd}}{m}\right)^2} \text{ micro mho/mile}$$

where d(diameter) in inches

(27)

Note: It is computed by assuming uniform charge density. The error due to this approximately is the order of 1 or 2% in transmission line equation.

APPENDIX 2(B)

Equation for the Geometric Mean Distance of the unsymmetrical 3-phase line with two conductors per phase. Case for the horizontal flat configuration:

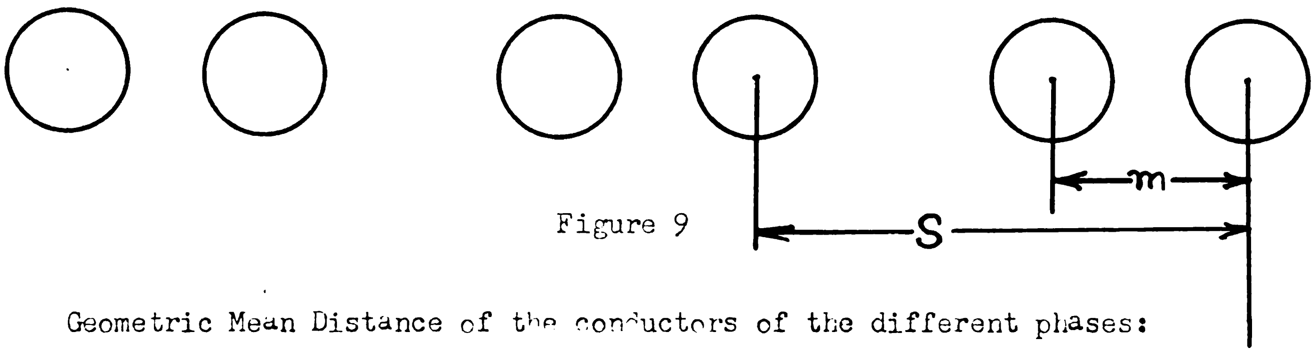


Figure 9

Geometric Mean Distance of the conductors of the different phases:

$$S_{gmd}^2 = \left[\sqrt[12]{S^4(2S)^2(S^2-m^2)^2(2S^2-m^2)} \right]^2$$

$$= \left[\sqrt[6]{S^6.4.(S^2-m^2)^2(4S^2-m^2).S^4.4S^2} \right]$$

$$= S^2.\sqrt[3]{4}\ \sqrt[6]{\left(\frac{S^4-2m^2S^2+m^4}{S^4}\right)\left[1-\left(\frac{m}{2S}\right)^2\right]}$$

$$= S^2.\sqrt[3]{4}\ \sqrt[6]{\left[1-\frac{2m^2}{S^2}+\frac{m^4}{S^4}\right]\left[1-\left(\frac{m}{2S}\right)^2\right]}$$

$$= S^2.\sqrt[3]{4}\ \sqrt[6]{\left[1-\left(\frac{m}{S}\right)^2\right]^2\left[1-\left(\frac{m}{2S}\right)^2\right]} \qquad (28)$$

Let for abbreviation $Z = \sqrt[6]{\left[1-\left(\frac{m}{S}\right)^2\right]^2\left[1-\left(\frac{m}{2S}\right)^2\right]}$

Then from Appendix I,

$$X = \frac{1}{2}\left[0.0304 + .2791 \log_{10} \frac{24S}{d}\ \sqrt[3]{2}\ \right] + 0.1397 \log_{10} \sqrt[3]{2}\ \frac{S}{m}\ Z \ .$$

ohms/mile

(29)

From Appendix 2

$$b = \frac{29.28}{\log_{10} 24\, 3/2 \, \frac{S}{d} + \log_{10} 3/2 \, \frac{S}{m} \, Z} \quad \text{mho/mile} \qquad (30)$$

APPENDIX 3(A)

Surface Voltage Gradient of Bundle of Two Conductor Per Phase

The dielectric flux density at a point x meters from the axis of the charged wire:

$$D = \frac{q}{2\pi x} \quad \text{coulombs/sq.m} \qquad (31)$$

This is true because we have a total flux of q coulombs per meter length of the wire passing radially through the curved surface of a circular cylinder one meter long having radius x meters. The area of the surface is $2\pi x$ xl.

The force per unit charge in the field, at radius x which is the same as the field intensity or voltage-gradient is

$$\frac{dE}{dx} = \frac{1}{\epsilon}\frac{dD}{dx} = \frac{36 \times 10^{9} \text{ x}}{2} \quad \frac{q}{x} \quad \text{volts/m}$$

$$g_o = 18 \times 10^{11} \quad \frac{q}{x} \quad \text{volts/cm} \qquad (32)$$

Therefore, the gradient at the surface of an isolated conductor is $18 \times 10^{11} \quad \frac{q}{r}$ ----------------(32)

where q in cou/
and r radius in inches

From Appendix 2 equation (17)

$$q_b = \frac{E_b}{18 \times 10^{11} \, l_n \frac{S}{\sqrt{3/2}\, r}} \quad \text{Cou/cm}$$

$$= \frac{E_b}{K(l_n \frac{S}{r} - \frac{1}{3} \, l_n \, 2)} \qquad (33)$$

The value of q_b in equation (33) may be substituted in equation (32) giving the followins:

The voltage gradient at the surface of the middle conductor with flat spacing; one conductor per phase to neutral:

$$g_o = \frac{18 \times 10^{11} E_b}{r\,(18 \times 10^{11}(l_n \frac{S}{r} - \frac{1}{3} l_n 2)} = \frac{E_b}{2.303r\,(\log_{10} \frac{S}{r} - \frac{1}{3} \log_{10} 2)} \quad (34)$$

This gradient is in volts per inch when r in inches. For a transmission circuit which has one conductor per phase, the assumption that the potential gradient is uniform around the surface of the conductor is justifiable, when the ratios of the diameter to spacing is very large.

When the charge density is not uniform around the wire in case of bundle conductor circuit, a computation of its value may be needed for precise computation of corona. A convenient way to compute the charge density and the voltage-gradient at any part of the surface is used the following equation from paper[1] by H. B. Dwight, equation 8. For the charge density at angle θ due to q_a, the wire's own charge and q_b, the charge on a neighboring parallel wire at an axial distance p.

$$q_\theta = \frac{q_a}{2\pi r} - \frac{q_b}{\pi r}\left(\frac{r}{p} \cos\theta + \frac{r^2}{p^2} \cos 2\theta + \ldots + \frac{r^n}{p^n} \cos n\theta\right) \quad (35)$$

The angle θ is measured from the line joining the centers of the two wires.

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[1] H. B. Dwight, "The Direct Method of Calculation of Capacitance of Conductor," Trans. A.I.E.E. vol. 43, 1924.

The effect of charges q_c, q_d and so forth, on other parallel wires is given by series involving q_b but angle θ is measured from the approximate line of centers in each case.

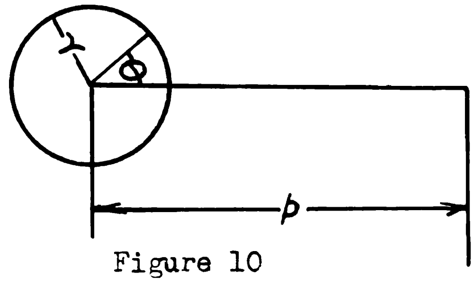

Figure 10

In the case of double conductor line, where there are two conductors per phase, q_b is almost exactly in phase with q_a especially for the middle phase which has the greatest gradient.

For conductors of d inches diameter and m inches apart the effect of the conductor is to multiply the gradient at $\theta = 180$ degrees by

$$1 + 2r/m - 2r^2/m^2 + \quad + \text{(higher power neglected)} \tag{36}$$

The charge on one of the middle conductors of a double-conductor line is one-half of the charge per phase and

$$b = \frac{E_b}{2K(\log \frac{S}{G_{ss}} - \frac{1}{3}\log 2)} \tag{37}$$

Substituting this equation (37) in equation (32) and adding the correction for distortion of the charge density and putting $G_{ss} = \sqrt{mr}$

The maximum gradient obtained for the middle phase

$$gm = \frac{V_b\ (1 + \frac{2r}{m} - \frac{2r^2}{m^2} + \ldots)}{2 \times 2.303r\ (\log \frac{S}{\sqrt{mr}} - \frac{1}{3}\log 2)} \tag{38}$$

APPENDIX 3(B)

Maxwell's Potential Coefficient by Image Method

A transmission circuit consists usually of several conductors and ground. For a three phase line, the two conductors per phase, there is a total of six conductors and ground. For calculation of potential coefficient by image method increases the conductor numbers to 12 if the effect of ground is replaced by the six images of the bundle conductors of two per phase. The equations for the calculation of charges with certain applied voltages to ground can be expressed by the equation

$$[Q] = [P]^{-1}[V] \qquad (38a)$$

where Q_1 is the charge on conductor

P is the Maxwell's coefficient

V is the applied voltage.

By the laws of electrostatics, the gradient or intensity at a distance r from a line charge Q in a medium of permittivity is

$$E = -\frac{\partial v}{dr} = \frac{4\pi Q}{2\pi r \epsilon} = \frac{2Q}{r\epsilon} \qquad (39)$$

The potential reckoned from a distance R is

$$\frac{dv}{dr} = -\int_R^r \frac{2Q}{r\epsilon}\, dr = \frac{2Q}{\epsilon} \log_e \frac{R}{r} \qquad (40)$$

Suppose a conductor of radius r at a height h above ground at a potential v with respect to ground. The ground plane may be regarded as a zero potential surface, and the field will be the same as through the effects of the ground replaced by the image of the conductor at a depth below the ground surface and having a charge $-Q$. From Figure 11 below, and from equation (40)

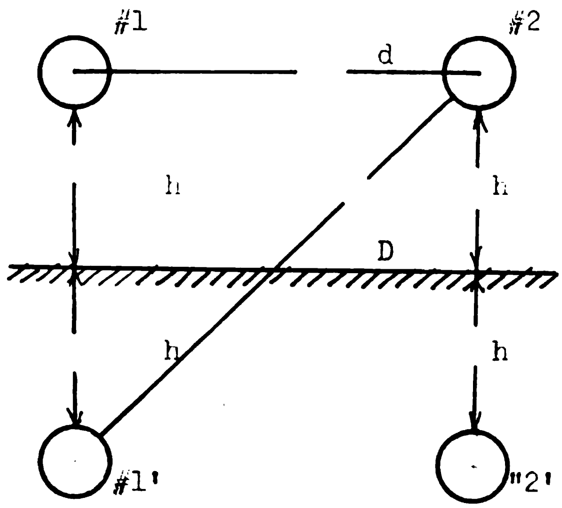

Figure 11

$$\vartheta_1 = p_{11}Q_1 = Q_1\, 2 \log_e \frac{2h}{r} \qquad (40a)$$

$$(\epsilon = 1)$$

Now the potential at a point distance d from #1 and the distance D from the image #1' is again by equation (40)

$$\vartheta_2 = p_{21}Q_1 = Q_1\, 2 \log_e \frac{D}{d} \qquad (40b)$$

The equation (38a) can be rewritten in terms of Q, p, and v.

$$\begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \\ Q_4 \\ Q_5 \\ Q_6 \end{bmatrix} = \begin{bmatrix} p_{11} & p_{12} & --- & p_{16} \\ p_{21} & p_{22} & --- & p_{26} \\ p_{31} & p_{32} & --- & p_{36} \\ p_{41} & p_{42} & --- & p_{46} \\ p_{51} & p_{52} & --- & p_{56} \\ p_{61} & p_{62} & --- & p_{66} \end{bmatrix}^{-1} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \qquad (40c)$$

The equation (40c) directly determines the charges on a three phase transmission line with two conductor per phase. The potential coefficient p_{11}, p_{12}, etc., for a horizontal flat configuration can be taken from equation (40a) and (40b) for self and mutual respectively.

$$\text{Self } P_{11} = 2 \log_e 2h/r$$

$$\text{Mutual } P_{12} = 2 \log_e D/d$$

The Figure 12 shows the arrangement of the conductors under consideration with proper distance-mark and the images are shown for the determination of 36 coefficient.

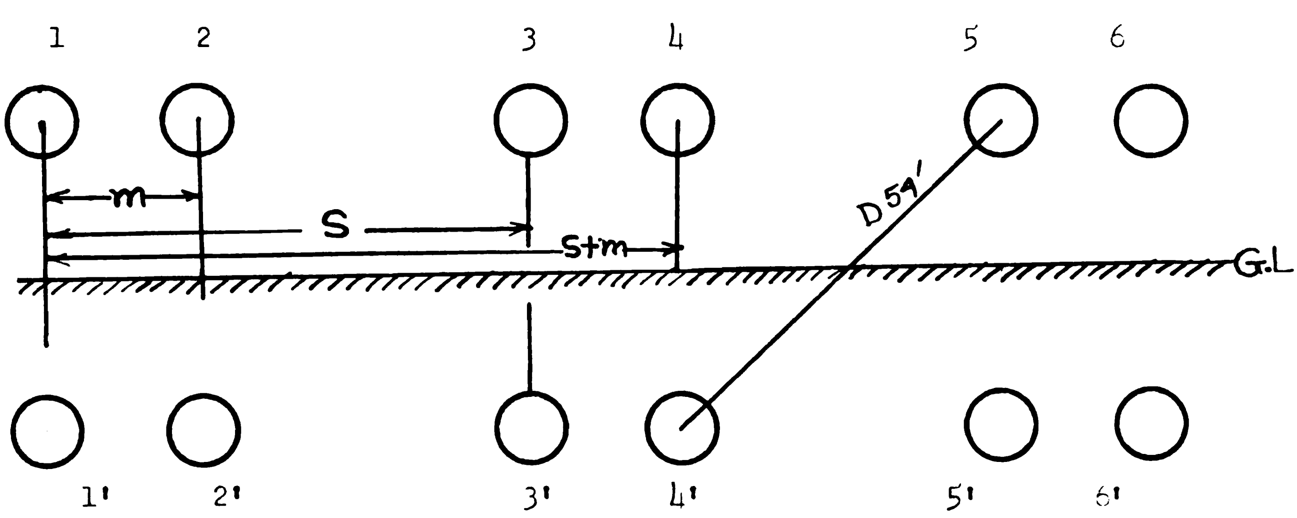

Figure 12

$$P_{11}' = P_{22}' = P_{33}' = P_{44}' = P_{55}' = P_{66}' = 2 \log_e 2h/r$$

$$P_{21}' = P_{12}' = P_{3}'_{4} = P_{4}'_{3} = P_{5}'_{6} = P_{6}'_{5} = 2 \log_e D_{1}'_{2}/m$$

$$P_{3}'_{1} = P_{1}'_{3} = P_{3}'_{5} = P_{53}' = P_{4}'_{6} = P_{6}'_{4} = 2 \log_e D_{1}'_{3}/S$$

$$P_{2}'_{3} = P_{3}'_{2} = P_{4}'_{5} = P_{5}'_{4} = 2 \log_e D_{2}'_{3}/S-m$$

$$P_{1}'_{4} = P_{4}'_{1} = P_{3}'_{6} = P_{63}' = 2 \log_e D_{14}'/S+m$$

$$P_{1}'_{5} = P_{5}'_{1} = P_{2}'_{6} = P_{6}'_{2} = 2 \log_e D_{15}'/2S$$

$$P_{1}'_{6} = P_{6}'_{1} = 2 \log_e D_{16}'/2S+m$$

(40d)

The value of D_{jk}, the distance between the centers of the jth conductor and the image of the kth conductor is given in terms of h, S, and m.

$$D_{12}' = \sqrt{(2h)^2 + m^2}$$

$$D_{13}' = \sqrt{(2h)^2 + S^2}$$

$$D_{14}' = \sqrt{(2h)^2 + (S+m)^2}$$

$$D_{15}' = \sqrt{(2h)^2 + (2S)^2}$$

$$D_{16}' = \sqrt{(2h^2) + (2S+m)^2}$$

$$D_{2}'_{3} = \sqrt{(2h)^2 + (S-m)^2}$$

(40e)

The equation (40e) is substituted in equation (40d) which gives the coefficients (Maxwell's) in electrostatic units per cm. length of line. From equation (40c) the charge on individual conductor of the 6-conductor then calculated out by assuming some applied voltage.

As the two conductors of the bundle are electrically parallel, the voltage

$$V_1 = V_2$$

$$V_3 = V_4$$

$$V_5 = V_6$$

APPENDIX 4

Disruptive Critical Voltage of a Bundle-Conductor (Two) In Transmission Line

For a transmission circuit which has one conductor per phase, the assumption that the potential gradient is uniform around the surface of the conductor is justifiable, when the ratios of diameter to spacing between phases is very small (Appendix 3). However, a conductor of a multiple-conductor circuit is relatively close to the other conductor of the same phase. An appreciable field distortion may result which will lower the surface gradient.

The following analyses determines the disruptive critical voltage assuming that the field due to the conductors of the other phases is negligible and the field produced by the other conductors of the same phase may be considered uniform.

Let E_{01} = The field produced by the other conductors of the same phase in the region of conductor 1, conductor 1 may be any conductor in any phase.

The intensity at a distance m from a line charge of Q per unit length is:

$$E = \frac{2Q}{m} \qquad (41)$$

$$E_{01} = 2Q\sqrt{\left[\frac{1}{m_{12}}\cos\theta_{12}\right]^2 + \left[\frac{1}{m_{12}}\sin\theta_{12}\right]^2} \qquad (42)$$

where m_{12} = the distance between conductor 1 and the second of the same phase.

θ_{12} = the angle between a line joining the centers of conductor 1 and 2 and the reference axis.

At the surface of a conducting cylinder in a uniform field, the maximum intensity is $2E_{01}$ and the total maximum intensity at the surface when the conductor is charged, is obtained by adding $2E_{01}$ and $2Q/r$ and the maximum intensity E_m is,

$$E_{m1} = 2E_{01} + \frac{2Q}{r} \qquad (43)$$

where Q = charge on conductor

r = radius of conductor

Substituting (42) in (43) and replacing r by d/2 the equation (44) is obtained:

$$E_{m1} = 4Q[\sqrt{(\frac{1}{m_{12}} \cos \theta_{12})^2 + (\frac{1}{m_{12}} \text{Sin } \theta_{12})^2} + \frac{1}{d}] \qquad (44)$$

For a three-phase system the charge on the other two phase conductors are $-Q/2$ when the charge on conductor 1 is $+Q$, the potential of conductor 1 of phase 1 becomes:[1]

$$V_1 = 2Q \log_e \frac{2h}{r} = 2Q \log_e \frac{4h}{d} \qquad (45)$$

where h is the height above ground

Substituting the value of Q determined from equation (44) in equation (45) and 53.6kv/inch for E_{m1} from Table III; the following expression is obtained for the disruptive critical voltage in effective kv to neutral of conductor 1,

$$V_1 = \frac{2E_{m1} \times l_n \frac{2\sqrt{(S_{13}S_{14})(S_{15}S_{16})}}{dm_{12}}}{4[\sqrt{(\frac{1}{m_{12}} \cos \theta_{12})^2 + (\frac{1}{m_{12}} \text{Sin } \theta_{12})^2} + \frac{1}{d}]} \qquad (46)$$

$$V_1 = \frac{53.6 \times l_n \frac{2\sqrt{(S_{13}S_{14})(S_{15}S_{16})}}{dm_{12}}}{2[\sqrt{(\frac{1}{m_{12}} \cos \theta_{12})^2 + (\frac{1}{m_{12}} \text{Sin } \theta_{12})^2} + \frac{1}{d}]}$$

[1]Two-Dimentional Fields in Electrical Engineering, Bewly and Macmilan, 1948.

It is clear from equation (46) that in any arrangement, the conductors of the central phase have a lower disruptive critical voltage than those of the two outside phases as the numerator of the argument of the logarithm of equation has a smaller value for the center phase conductors than it has for the outside phase conductors.

Introducing the irregularity and air density factors, M_0 and δ respectively and multiplying numerator and denominator of the equation (46) by d; e_0, the lowest corona voltage of the system

$$e_0 = \frac{d \times 53.6 \times 2.303\, M_0 \delta \log_{10} \frac{2\sqrt{(S_{13}S_{14})(S_{14}S_{15})}}{dm}}{2\left[\sqrt{\left(\frac{d}{m_{12}}\cos\theta_{12}\right)^2 + \left(\frac{d}{m_{12}}\sin\theta_{12}\right)^2} + 1\right]}$$

$$= \frac{2r \times 53.6 \times 2.303\, M_0 \delta \log_{10} 2\frac{S^2}{m \times 2r}}{2\left(1 + \frac{2r}{m}\right)} \qquad S_{13} = S_{14} = S_{15} = S_{16}, \quad \cos\theta_{12} = 1$$

$$e_0 = \frac{123 r\, M_0 \delta}{1 + \frac{2r}{m}} \log_{10} \frac{S^2}{mr} \qquad (47)$$

where e_0 = line to line corona-starting voltage in kv

where S = the distance between phase conductor

m = the distance between the conductor of the same phase

r = the radius of the conductor

M_0 = the irregularity factor

δ = the air-density factor

e_0 for one conductor per phase is given by the following equation[1]:

$$e_0 = 123\, M_0 r \delta \log_{10} \frac{S}{r} \quad \text{kv} \qquad (47a)$$

---

[1] Electric Power Transmission and Distribution by Woodruff.

APPENDIX 5

Corona Loss Measurement in Bundle Conductor

The formulae worked out for estimating corona-loss are all of empirical correlation the test values. Peek's formula is applicable for the higher value of the losses but Peterson formula which is applicable for the lower value as well as higher values, of losses are widely used. The loss can be expressed by the following equation.

$$P_c = \frac{0.0000337 f \, e_n^2}{(\log_{10} \frac{S}{r})^2} \quad (\varphi_c') \qquad (48)$$

where P_c is the corona loss in kw per mile per conductor.

The ratio of e_n/e_o is a function of φ_c' the value of which can be taken from the curve .

The Peek's loss formula is given below for reference:

$$P_c = \frac{390}{\delta} (f + 25) \sqrt{\frac{r}{S}} (e_n - e_o)^2 \times 10^{-5} \quad \text{kw/mile} \qquad (49)$$

For a mile of the whole line all three wires, the loss is three times of the equation (48).

For bundle of two, the total loss is six times the value of the equation (48).

The equations (48) are on the basis of the charge density uniform on the conductor periphery.

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