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EXPERIMENTAL ANALYSIS OF
INDETERMINATE STRUCTURES AS
APPLIED TO RIGID FRAMES

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STRUCTURES AS APPLIED
TO RIGID FRAMES

by
HENRY LOREN THOMPSON

A THESIS

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INTRODUCTION

During the past fifteen years there have been more statically indeterminate structures built than in any other like period in the history of construction. Such construction has been made possible to a great extent by the research worker experimenting with structural models. Engineers have long recognized the economic and aesthetic advantages of the continuous structure. The tie that has held the designer to the old type of structure, consisting of abutments and simple spans, is the simplicity of the mathematical theory. In the highly complicated designs the designing organization is confronted with two main problems—developing or recruiting designers capable of handling the more complicated theory and the high cost of the design.

In order to aid in the solution of the problems of giving the public the "aesthetic structure" the laboratory analysis of the statically indeterminate structure has been developed. Model analysis is a distinct method of approach to the solution of statically indeterminate structures and can be used alone or in conjunction with the theoretical analysis as a check.

PURPOSE

The research to be described in this thesis has been carried out with the following definite objectives in mind: (1) the design and the construction of a sample deformeter gage that will be sufficiently accurate for model analysis and one that can be made cheaply; (2) the testing of the gages as designed with a direct comparison of results obtained with the deformeter gages and a theoretical analysis of a statically indeterminate structure; and (3) the experimental analysis of single, double, and triple span rigid frame bridges to determine the variation in the moment at the left knee due to a variation of the depth of the haunch.

THEORY OF MODEL ANALYSIS

The laws laid down by Clapeyron, Castigliano, Maxwell, Hooke, and others are basic and some knowledge of them is, therefore, necessary for a complete understanding of the mechanical solution. These underlying laws are not superseded by the laboratory methods but modern conditions have simply created a demand for a mechanical short-cut in applying them.

Assume the fixed arch as in Fig. 1 and let it be required to find the reaction produced at point 1 along the line X-X by a load "P" at point 2. First with the arch free of any load, suppose the point 1 to be removed from its anchorage and moved a distance Δ_{1x} in the direction X-X only. To accomplish this it will be necessary to apply not only the distorting force X_0 , but, also, a restraining moment Z_0 to prevent rotation of the arch about the point 1, and a linear restraining force Y_0 to prevent movement at right angles to the line X-X. As

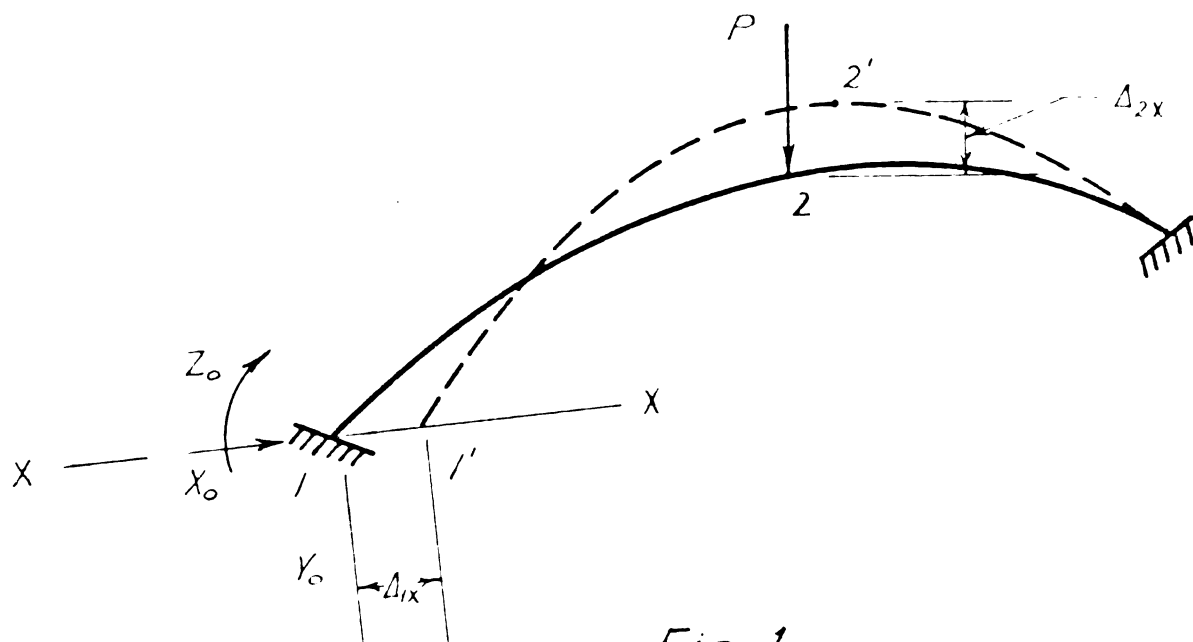


Fig. 1

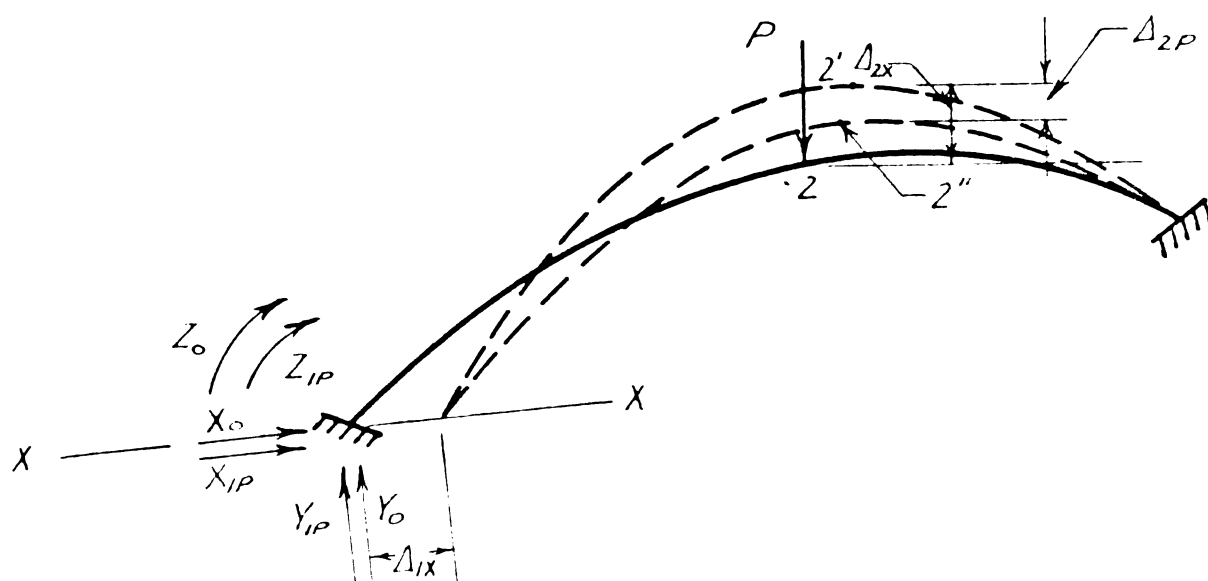


Fig. 2

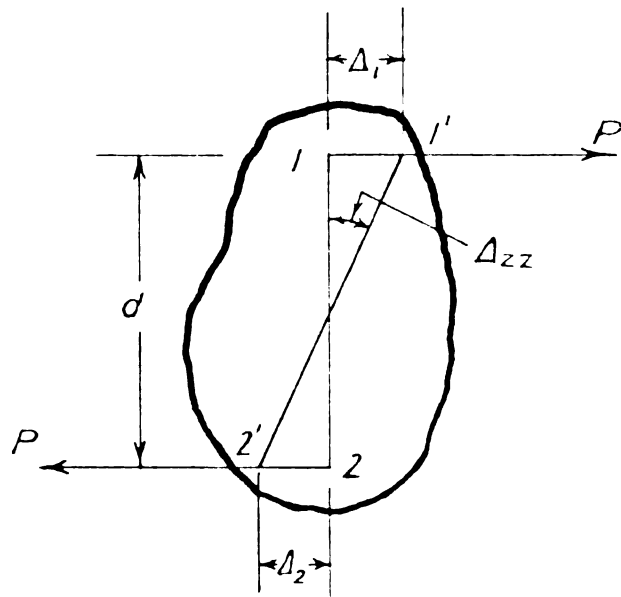


Fig. 3

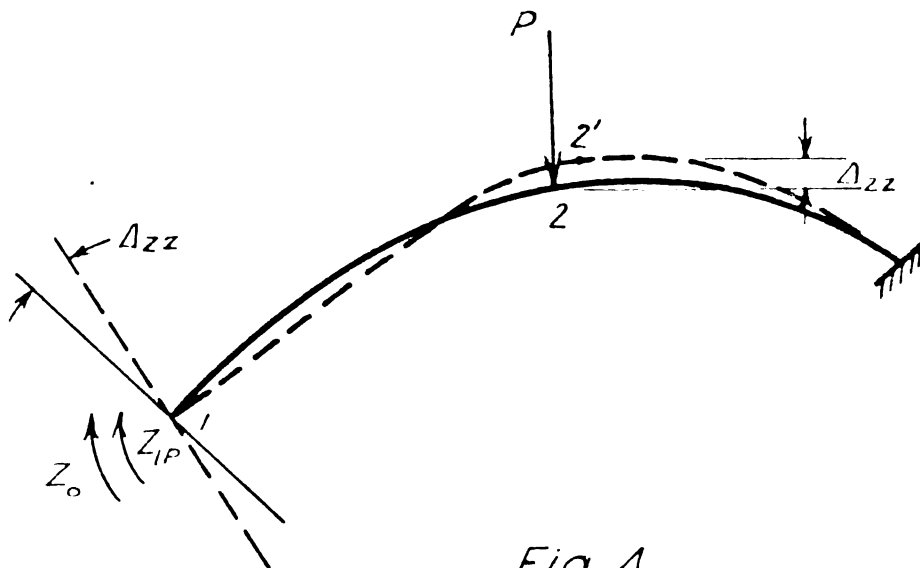


Fig. 4

no Z or Y movement takes place, no work is performed by these restraining forces. Therefore, the external work done or the elastic strain energy is given by the following equation:

$$U_1 = \frac{1}{2}X_0 \Delta_{1x} = W'$$

where W' is the internal work.

Now with the rib in its deflected position add the load "P" at point 2 and its consequent reactions X_{1p} , Y_{1p} , and Z_{1p} . Since these reactions perform no work, the second increment of work is

$$U_2 - \frac{1}{2}P \Delta_{2p} = W''$$

where W'' is the internal work. Δ_{2p} is the corresponding deflection at point 2 produced by the load "P" and Δ_{1x} is the corresponding deflection at point 1 produced by X. All deflections in this discussion have a similar meaning. By corresponding deflection it is meant the measured deflection in the direction of the load or reaction at the point considered. The total work performed on the arch is

$$I \quad W = W' + W'' = \frac{1}{2}X_0 \Delta_{1x} + \frac{1}{2}P \Delta_{2p}$$

The same deflected position may be attained with the same expenditure of energy by applying X_0 , X_{1p} , and "P" simultaneously, in which case the external work is

$$U = \frac{1}{2}X_0 \Delta_{1x} + \frac{1}{2}X_{1p} \Delta_{1x} + \frac{1}{2}P (\Delta_{2p} - \Delta_{2x})$$

Since the external work is equal to the internal work, this equation becomes

$$II \quad U = W = \frac{1}{2}X_0 \Delta_{1x} + \frac{1}{2}X_{1p} \Delta_{1x} + \frac{1}{2}P \Delta_{2p} - \frac{1}{2}P \Delta_{2x}$$

Since equation II is the expression for the total internal work, it is

equal to equation I. Equating I and II and cancelling out similar terms gives

$$X_{lp} \Delta_{1x} = P \Delta_{2x}$$

or

$$X_{lp} = P \frac{\Delta_{2x}}{\Delta_{1x}}$$

This last expression shows that any two related forces are inversely proportional to the coincident displacement of their points of application in the direction of the forces or inversely proportional to their corresponding displacements.

Similarly, by giving the reaction point 1 a Y movement and restraining it from both X and Z movement, the Y reaction may be expressed as

$$Y_{lp} = P \frac{\Delta_{2y}}{\Delta_{1y}}$$

The same method of attack is used in the determination of the angular reactions or moment at the supports. Any moment Z may be expressed as a couple as P times d (Fig. 3). If under the application of this moment, point 1 of Fig. 3 moves to 1' and point 2 to 2', the total work done by the moment is

$$U = \frac{1}{2}P\Delta_1 + \frac{1}{2}P\Delta_2 = \frac{P}{2}(\Delta_1 + \Delta_2)$$

As P is equal to Z divided by d the external work becomes

$$U = \frac{Z}{2d} (\Delta_1 + \Delta_2) = \frac{1}{2}Z\Delta_{zz}$$

where Δ_{zz} is the angular rotation expressed in radians. Therefore, the external or internal work done by a couple is the moment of that couple times the angular rotation of the coincident point divided by two, provided this rotation is small, otherwise, $(\Delta_1 + \Delta_2)$ divided by "d" would not equal Δ_{zz} .

Now consider Fig. 4 which is an arch. First assume no load on the rib and apply a moment Z at the point 1. The total work is

$$U_1 = W' = \frac{1}{2}Z_0\Delta_{1z}$$

With this moment acting apply the load "P" at point 2 restraining point 1 from turning. In order to do this a new moment Z (moment at 1 due to the load "P") must be applied but no work is done by this new moment. The total work done by applying this load is

$$U_2 = \frac{1}{2}P\Delta_{2p} = W''$$

The total internal work done by this body with these loads is

$$\text{III} \quad W = W' + W'' = \frac{1}{2}Z_0\Delta_{1z} + \frac{1}{2}P\Delta_{2p}$$

This same deflected position can be attained by applying all of the loads and reactions simultaneously. The internal work is then

$$W = \frac{1}{2}Z_0\Delta_{1z} + \frac{1}{2}Z_{1p}\Delta_{1z} + \frac{1}{2}P(\Delta_{2p} - \Delta_{2z})$$

$$\text{IV} \quad W = \frac{1}{2}Z_0\Delta_{1z} + \frac{1}{2}Z_{1p}\Delta_{1z} + \frac{1}{2}P\Delta_{2p} - \frac{1}{2}P\Delta_{2z}$$

Where Δ_{2z} is the deflection at point 2 due to the angular rotation at point 1. Equating equations III and IV, since they are equal, Z_{1p} the

moment at the support is given by the following expression after proper cancellation of terms:

$$Z_{1p} = P \frac{\Delta_{2z}}{\Delta_{1z}}$$

For this work equation to be true, the dimensions d , Δ_1 , and Δ_2 of Fig. 3 must be measured in the same units as Δ_{2z} of Fig. 4. Therefore, if Δ_{1z} radians distortion at point 1 produces Δ_{2z} inches deflection at the load point 2, the moment at point 1 due to a unit load at point 2 is $\frac{\Delta_{2z}}{\Delta_{1z}}$ pound-inches. If the displacement Δ_{2z} be measured in feet, then the moment reaction for a unit load is $\frac{\Delta_{2z}}{\Delta_{1z}}$ pound-feet.

HISTORY OF MODEL ANALYSIS

The first practical use of model analysis of indeterminate structures was made by Mr. James R. Griffith, then Assistant Professor of Civil Engineering, Armour Institute of Technology, in the design of the catenary structures and signal bridges for the Illinois Central Railroad's Chicago electrification in 1924. Mr. Griffith was confronted with a three-column structure which was indeterminate by twenty-one variables. The prospective cost of designing these and more complicated structures by any analytical method presented a serious problem.

Apparently the only possibility of avoiding this tedious mathematical analysis was by a method of measuring the deflection in celluloid or cardboard models. This method was only a laboratory experiment

at that time, which had been devised by the late Professor George E. Beggs of Princeton University and described in a paper presented before the American Concrete Institute, in 1922. Professor Beggs loaned his original equipment to Mr. Griffith and by the use of this equipment Mr. Griffith demonstrated that approximately two-thousand dollars per year could be saved in the cost of design alone. The Illinois Central Railroad then purchased one of the first commercial deformeters and used it in their design. By the use of the deformeters and models a considerable saving was obtained in the designing cost and in the actual cost of the structures.

Model analysis started with the late Professor George E. Beggs in the summer of 1916 when his attention was given to the problem of the deflections and stresses of a triple span continuous truss bridge of the B. & L. E. R. R. over the Allegheny River. He found it advantageous to construct the influence lines for the trusses. After having plotted these lines Professor Beggs noticed that they resembled the elastic curves of continuous beams fastened at three supports and deflected a unit distance at the remaining support for whose reaction the influence line was drawn. To determine the accuracy of this observation, a ten-foot pole of uniform section and three nails were used. To draw the influence line for one reaction the pole was restrained by single nails at the other three reactions and deflected a distance to scale of one-thousand pounds at the first reaction. The ordinates of this elastic curve were measured. The values obtained by this crude mechanical apparatus and by the exact theory showed very close agreement.

In January 1920, Professor Beggs checked the theory of mechanical analysis (Maxwell's Theorem) by means of a wood model of a single span bent hinged at the supports and with a horizontal member half the length of the deck placed as shown in Fig. 5. The members were made of wood strips five-eighths by seven-eighths of an inch in cross-section and the columns and main horizontal member were fifty inches in length center to center of connections. Metal gusset plates and screws fastened the bars together. No loads were applied while the deflections were being measured. The horizontal and vertical reactions were found by the use of Maxwell's Theorem and then these loads were applied at the three supports as shown and the pins removed. The supports did not move which proved the correctness of this theory and application. These reactions as found by measuring the deflections are seen to satisfy very well the three laws of static equilibrium.

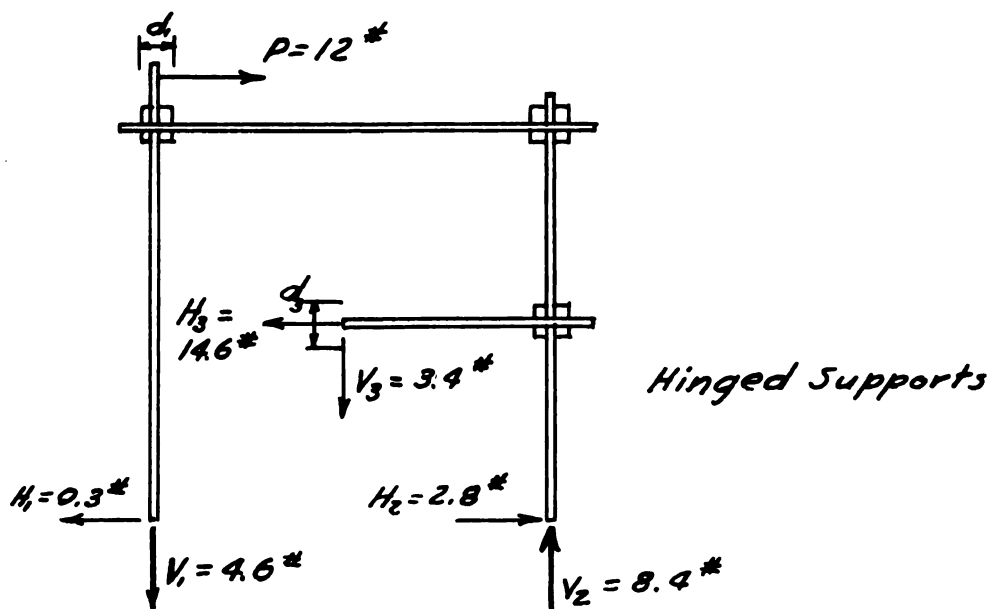


Fig. 5

After feeling definitely that this method could be applied to statically indeterminate structures, Professor Beggs designed some deformer gages to be used in connection with microscopes to measure the deflections. The gages produced a known displacement and the microscopes measured the deflections at the assumed points of application of the loads and in the direction of the loads. These gages as originally designed were somewhat crude in comparison with the gages sold by the late Professor Beggs during the past few years. This equipment is probably the most accurate yet devised but is quite expensive. A set of six gages and three microscopes costs about eight-hundred dollars.

Although Professor Beggs has probably done most for the analysis of structures by models, there are others who have worked independently and have used similar or different methods based on Maxwell's Theorem. Among these is Mr. Otto Gottschalk who has developed the "Continostat Gottschalk". His method and apparatus has been described in various technical literature, the first being a paper entitled "Mechanical Calculation of Elastic Systems" given before the Franklin Institute and appeared in the Journal of the Franklin Institute in July 1926. One of Mr. Gottschalk's latest publications appeared in the Proceedings of the American Society of Civil Engineers for January 1937, under the title of "Structural Analysis Based on Unloaded Models".

The Continostat Gottschalk consists of various mechanical supports and very flexible splines used as the elastic structure. Known deflections are produced at the point at which the reactions are sought and the flexible splines forming the members of the structure bend in the shape of the elastic curve of the member. The ordinates from the

original position to the deflected position are measured by means of the ordinary scale. The reactions at the different points are then found by the ratio of the deflections. The deflection at the point where the reaction is sought is usually made equal to one unit, which makes the elastic curve represent the influence line for that reaction.

An article appeared in the "Engineering News-Record", Volume 99, Number 23, December 8, 1927, by Mr. Anders Bull in which he explains a method of analysis using wire models. As a partial fulfillment of the requirements for the Master of Science Degree in Civil Engineering at the University of Idaho, Mr. Lyman G. Youngs prepared a thesis on the subject of "Stress Determination in Statically Indeterminate Structures by Means of Brass Wire Models". This thesis represents a very good study of the possibilities of wire models. Wire models are useful provided the moment of inertia of each member is constant, but if the members have a variable depth it is almost impossible to obtain a model that truly represents the structure. Therefore, it seems that this method can be used only for the more simple structures.

In order to get around the high cost of the Beggs Deformeters with the microscopes there have been various other gages devised and among these is the "home made" deformer designed by Mr. T. Werner, Engineer for the Arthur G. McKee Company of Cleveland, Ohio. This deformer was developed in the Structural Models Laboratory of the Case School of Applied Science. These gages are inexpensive to make and have proved quite satisfactory for most work and particularly for demonstration work for structural classes in model analysis. A redesign of these gages was used in the preparation of this thesis. The same

fundamental idea was used but an improved method of holding the gages together and still permitting the desired motions was incorporated into the original design.

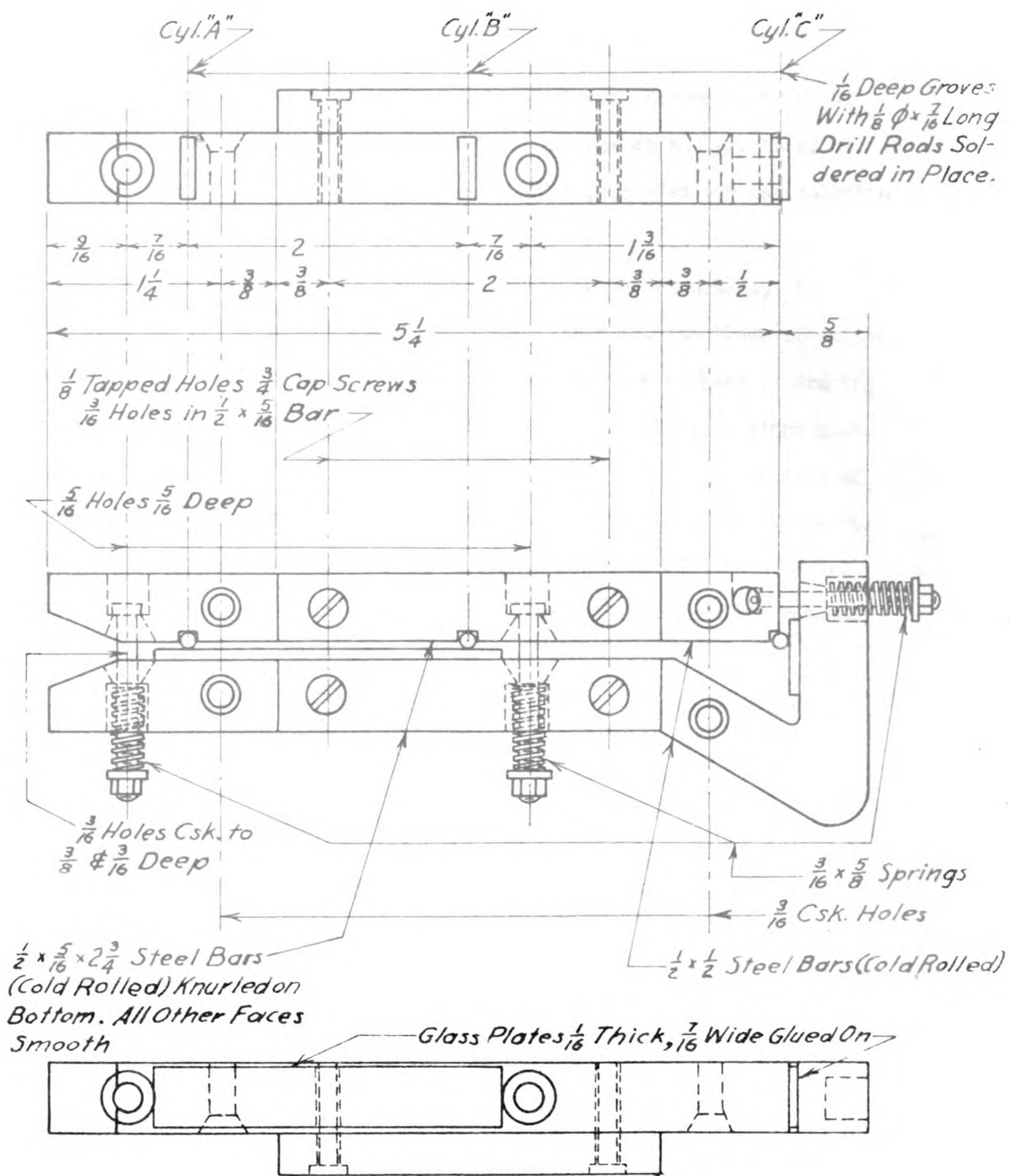
DESIGN AND OPERATION OF THE DEFORMETER GAGES

The redesign of the Werner Deformeter is shown on the print, page 12. The original gage only had two springs which held the bars together and the wire passing through the springs passed through the bars at an angle which caused a push of the small bar against the vertical glass plate of the long bent bar. The three springs and the new method of mounting the springs makes a much stronger gage and one that is more sure in operation. A set of four gages as shown on the print (page 12) was made under the author's direction at the Michigan State College of Agriculture and Applied Science and another set made in the Physics' Machine Shop at the University of Idaho. Both of these sets worked satisfactorily.

The two bars are held in contact by means of the three springs and the cylinders "a", "b", and "c". These cylinders were soldered with extreme care in order that they would be exactly parallel to each other when the bars are placed together since they must be in contact with the glass plates of the opposite bar for their entire length.

By inserting a filler plate of uniform thickness between the cylinders "a" and "b" and the glass plate a definite movement takes place. When a filler plate is inserted between the cylinder "c" and the glass plate another movement will take place at right angles to the first. If a filler plate is inserted only between cylinder "a" and

1



DESIGN OF DEFORMETER GAGE

the glass plate an angular movement takes place. According to Maxwell's Theorem this movement and the deflections at the points of application of the assumed loads is all that is needed for the solution of the structure.

One gage is necessary for each support of the structure. In setting up the equipment all the reaction lines and the lines of action of the loads are drawn as accurately as possible on a sheet of drawing paper either tacked to a table or drawing board. The long glass plate of the gage must be set exactly at right angles to the center line of the model at the support. This is usually the vertical reaction line. The cylinder "b" is placed directly over the intersection of the two reaction lines. The straight bar (one with the cylinders) is screwed to the table or drawing board.

The model is usually made of celluloid and is clamped under the one-half times five-sixteenths of an inch bar on top of the long bent bar. The model should be clamped accurately so the reaction lines on the model will correspond exactly in position with those drawn on the drawing board. To keep the model in a horizontal plane it is necessary to support it by either glass or celluloid plates at different points. Between these plates and the model a small steel ball about one-sixteenth of an inch in diameter is placed. The small steel balls reduce the friction at these points to a negligible quantity. The means of measuring the deflections at the several points are a mirror, mirror bar, focusing flashlight, and a screen.

The mirror consists of a half razor blade with a needle soldered on the back of the blade and a small mirror glued on the front. The

height of the razor blade from the sharp edge of the blade to the top of the needle point should be about one-half of an inch. This will give about the best size deflection on the screen. The mirror bar used is a one-quarter inch copper tube about twelve inches long. In one end of the bar a needle is soldered at right angles to the longitudinal axis of the bar. The other end is flattened for a couple of inches from the end and a piece of sand paper is glued to the same side as the needle. The needle of the mirror bar is placed lightly into the model at a load point and the other end with the sand paper is placed on top of the needle of the razor blade. The center line of the mirror bar should coincide with the action line of the load. A slight deflection of the load point will tip the mirror and cause a very large deflection on the screen.

A focusing flashlight of good quality is used with a small house bell transformer instead of batteries. This gives a permanent light of the same quality. The flashlight is placed about fifteen feet from the mirror and a screen is placed at twenty or more feet in front of the model. The screen consists of a cross-sectional paper about sixty or more inches high and should be at least ten inches wide. The light should be focused on the mirror until a good image appears on the screen.

With the model in a normal position (with no distortion) a reading of the initial position of the image on the screen is taken. A filler plate is inserted in each support giving the whole model a deflection equal to the thickness of the filler plate. A second reading is taken and the difference between the two readings gives the filler thickness as measured on the screen. The filler plates are then removed and one plate is inserted in the gage for which the reaction is

desired. The deflection is again measured on the screen in the same manner. The ratio of these deflections gives the reaction at the support considered for a unit load at the load point, since the ratio of the deflections times the load gives the desired reaction as discussed in the theory of model analysis.

Let D = filler plate thickness as measured on
 the screen

 d = deflection of the load point as measured
 on the screen

 $H, M,$ and V = reactions

 P = Load at load point (considered as
 unity)

From the basic theory of Model analysis the reactions are given as follows:

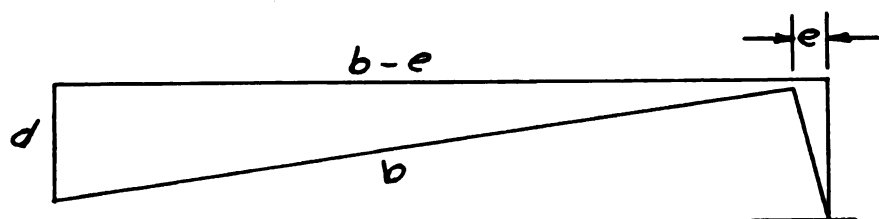
$$V = \frac{d}{D} \quad ; \quad H = \frac{d}{D} \quad ; \quad M = \frac{d}{D} K \quad .$$

K is the distance between the cylinders "a" and "b" measured to the same scale as the model either in inches or feet giving a result in pound-inches or pound-feet.

In measuring the deflections the needle of the mirror bar at the load point moves a very small distance at right angles to the line of action of the load as well as in the vertical direction (the latter is the displacement desired). In making this movement the mirror will be slightly tipped giving a deflection on the screen that is not due to any vertical movement of the load point. If this movement is large it will be necessary to correct for this error, which is given approximately by the following equation:

$$e = \frac{d^2}{2bk}$$

In this equation "d" is the deflection on the screen due to this movement at right angles to the line of action of the load, "b" is the length of the mirror bar, and "k" is the number of times the apparatus is multiplying the actual deflection of the model. ("k" is about 1000.) The diagram of Fig. 6 is a proof of this error.



$$b^2 - d^2 = (b - e)^2$$

$$b^2 - d^2 = b^2 - 2be + e^2 \quad (e^2 \text{ is an error of higher order})$$

$$e = \frac{d^2}{2bk}$$

Fig. 6

CONSTRUCTION OF THE MODEL

The model should be made of well seasoned celluloid of uniform thickness. However, it can be made of other material but celluloid is much better than card board or any other material. As the thickness of the celluloid is constant the depth of the section of the model should

be proportional to the cube root of the moment of inertia of the particular section of the actual structure. This will make the model structural similar to the actual structure.

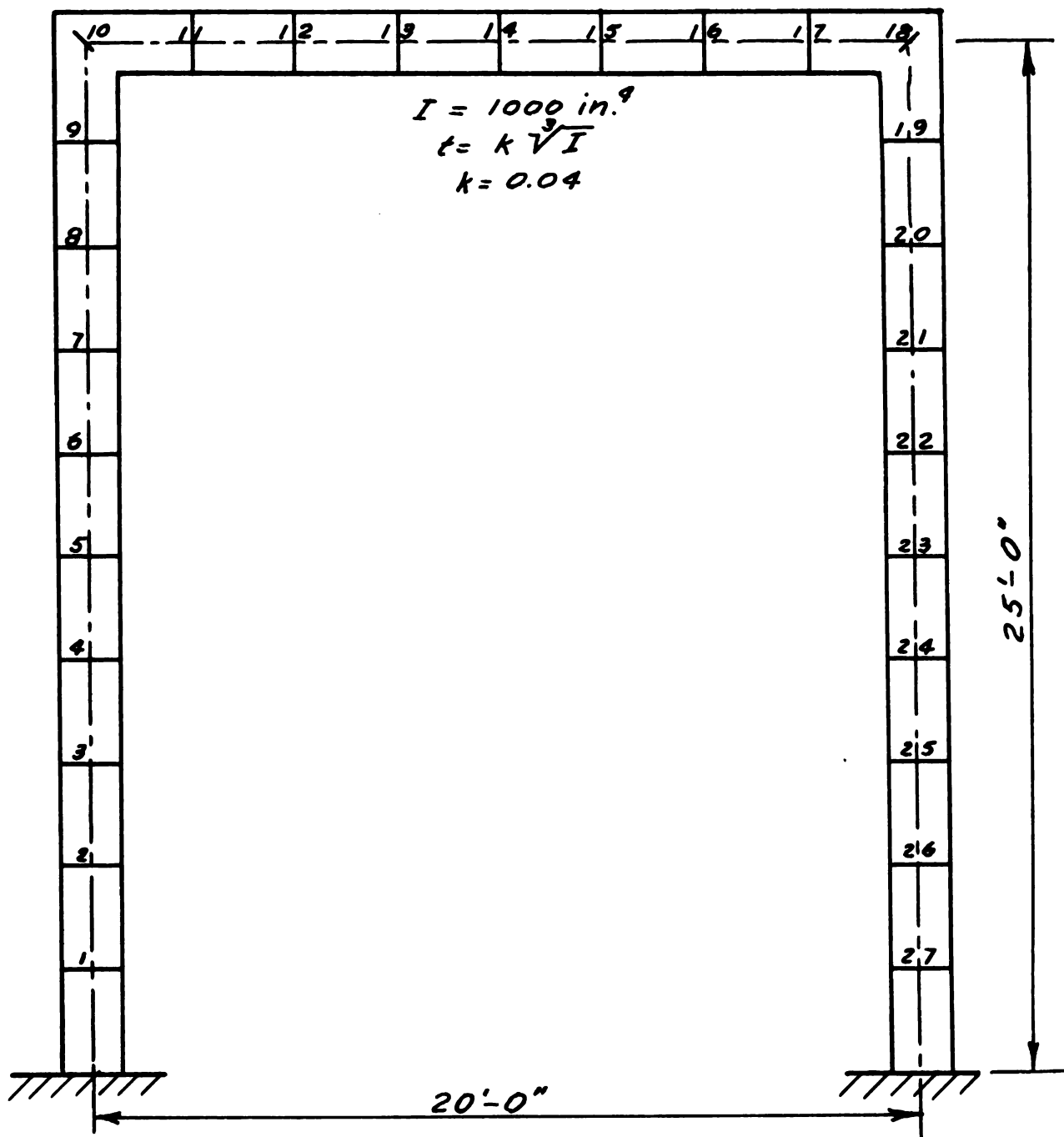
$$d = k \sqrt[3]{I}$$

The proportionally factor should be so chosen as to make the model flexible. The scale of the model should be chosen so the model is of convenient size.

After the scale and "k" have been chosen the centerline of the model is laid out on the celluloid sheet. The outline is drawn and the model is cut out with a coping or band saw. The celluloid is planed down to near the proper dimensions and then sanded by means of sand paper to the exact dimensions. The thickness of the section of the model should be to the nearest 0.001 of an inch of the proper dimension. An error in the thickness of the section will cause an appreciable error in the results.

SINGLE SPAN FRAME BENT

The first structure tested or analyzed by this equipment was a single span frame bent as shown in Fig. 7. The moment of inertia of the bent was assumed constant for the legs and the deck and assumed to be one-thousand units to the fourth power. The height was twenty-five feet and the span twenty feet. The influence ordinates were calculated by means of formulas from "Stresses in Frame Structures" by Hool and Kinne. A unit load was considered throughout the analysis.



SINGLE SPAN BENT

Fig. 7

The mathematical formulas used to analyze this bent with the loads on top of the deck were:

$$M_{AB} = + \frac{P a b}{2L^2} \left[\frac{L}{n+2} + \frac{a-b}{6n+1} \right]$$

$$M_{DC} = - \frac{P a b}{2L^2} \left[\frac{L}{n+2} + \frac{b-a}{6n+1} \right]$$

$$M_{BC} = - \frac{P a b}{2L^2} \left[\frac{2L}{n+2} + \frac{b-a}{6n+1} \right]$$

$$M_{CB} = - \frac{P a b}{2L^2} \left[\frac{2L}{n+2} + \frac{a-b}{6n+1} \right]$$

$$K = \frac{I}{L} = \frac{1000}{20} = 50$$

$$n = \frac{Kh}{I} = \frac{50 \times 25}{1000} = 1.25$$

After the moments were calculated the vertical and horizontal reactions were calculated by means of statics. The following tables show the results as obtained from the mathematical analysis and the results as obtained from the deformer analysis.

Moment at the Left Support

<u>Load Point</u>	<u>Mathematical Value</u>	<u>Deformeter Value</u>
10	0.000	0.000
11	0.240	0.248
12	0.467	0.474
13	0.652	0.646
14	0.769	0.774
15	0.790	0.790
16	0.637	0.674
17	0.433	0.427
18	0.000	0.000

Vertical Reaction at the Left Support

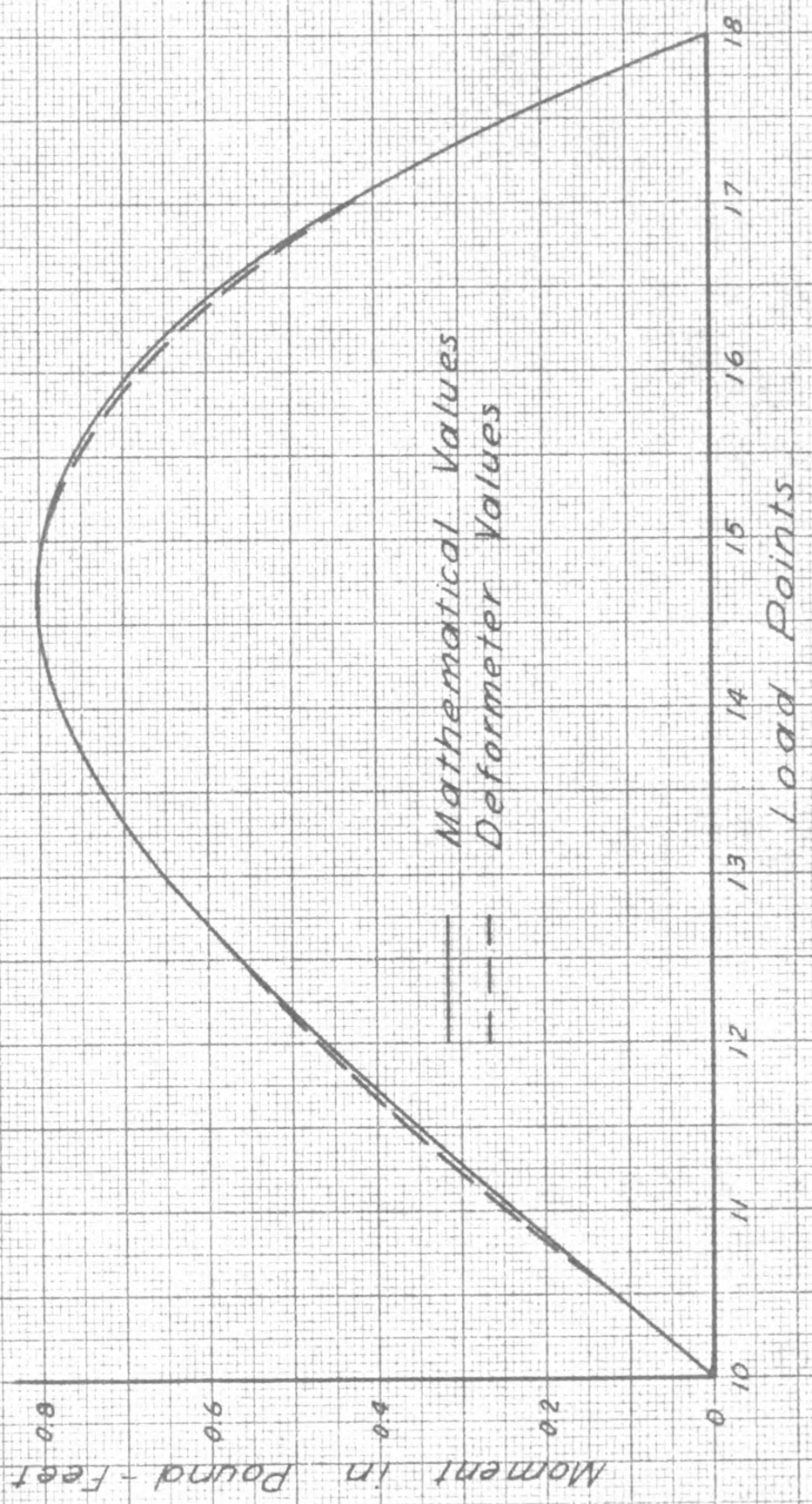
<u>Load</u> <u>Point</u>	<u>Mathematical</u> <u>Value</u>	<u>Deformeter</u> <u>Value</u>
10	1.000	1.000
11	0.884	0.884
12	0.761	0.756
13	0.632	0.636
14	0.500	0.499
15	0.368	0.367
16	0.239	0.238
17	0.117	0.120
18	0.000	0.000

Horizontal Reaction at the Left Support

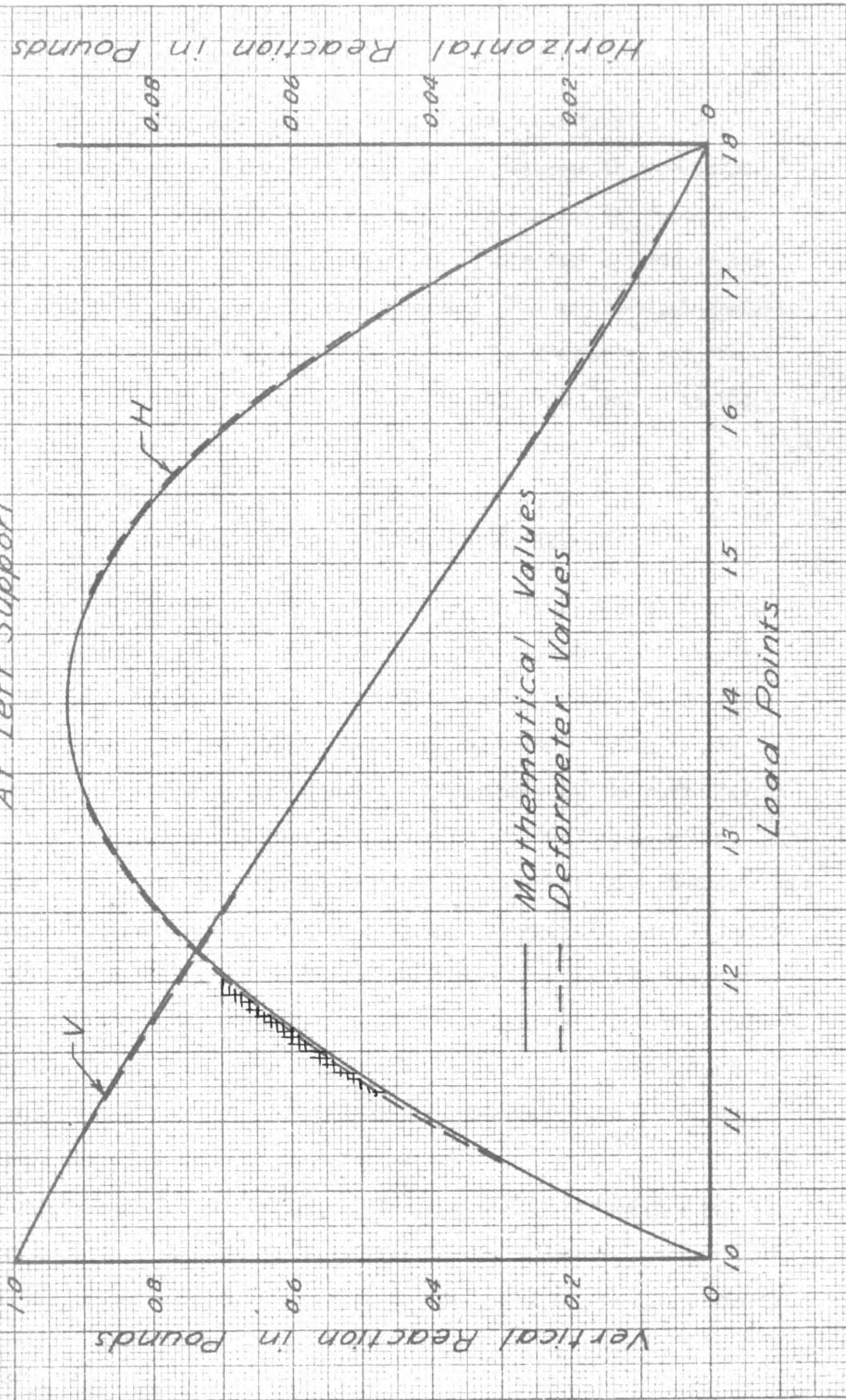
<u>Load</u> <u>Point</u>	<u>Mathematical</u> <u>Value</u>	<u>Deformeter</u> <u>Value</u>
10	0.000	0.000
11	0.040	0.041
12	0.069	0.069
13	0.086	0.087
14	0.092	0.092
15	0.086	0.087
16	0.069	0.069
17	0.040	0.041
18	0.000	0.000

From the above tables and the curves plotted to represent this data on pages 21 and 22, the close agreement of the deformeter values to those calculated mathematically can be seen. With careful handling of the gages little trouble was experienced in obtaining proper values. Each time the filler plates were inserted in the gages care was taken to see that the two bars came together exactly the same manner. A little experience with the gages created confidence in the accuracy of the work.

SINGLE SPAN FRAME BENT Influence Lines For Moment At Left Support



Influence Lines For Vertical And Horizontal Reactions At Left Support



From the experience with the single span frame bent it was satisfactory to assume that the equipment as designed is capable of accurate work. At least, the error will be small and the value one obtains from an experimental analysis approaches very closely to the actual value of the stress in the structure. The mathematical analysis is based on some assumptions and it is doubtful whether or not the values thus derived approach any closer to the actual values in the structure.

ANALYSES OF THE RIGID FRAME BRIDGE MODELS

The data for the bridge constants for the single and double span bridges were taken from "The Rigid Frame Bridge" by Mr. A. G. Hayden, Ch. VI, pp 43-58, and Ch. IX, pp 107-126, respectively. The data for the triple span bridge were assumed. The depth of sections as given in the above text or those assumed were considered as the normal depth. In the first analysis of each bridge model a depth of ten per cent greater than normal was used at the load point next to the abutments or piers. This increased depth varied as the offsets of a parabolic curve to zero at the center of the span. For the second analysis the normal depth of section was used. For the third analysis a depth of section that was ten per cent less than normal was used at the load point next to the abutments or piers. The abutments and piers were of the same section thickness for all three analyses.

The purpose of the variation of the depth of the haunches was to find out how much and in what direction the moment at the left knee varied for this situation. In the design of single span rigid frame

bridges the depth of section next to the abutments and at the center are usually assumed to be one-fifteenth and one-fortieth of the span respectively. This has proved best from experience in the design of such bridges.

The models were made of transparent celluloid and only one bridge model was used for all three analyses in each case. The second and third analyses were made after the models were sanded down to their proper depth of section each time. In this way more comparative results were obtained and any errors in making the models were the same for all three analyses. All of the tabulations and the curves showing the results are in the appendix.

After the values of the vertical, horizontal, and moment reactions were found from the deformeter method they were plotted and smooth curves drawn through the plotted points. Necessary values to compute the moment at the left knees by statics were taken from the curves and not from the tables of values for these forces given in the appendix.

Single Span Model

The curves for the moment at the left knee for fixed supports show a definite increase as the depth of haunch is reduced. The maximum moment occurs at the left knee when a unit load is at a point forty-one-hundredths of the span from the left support. The increase of the second analysis was 0.65 pound-feet or 7.8 per cent over the first at the maximum and the third analysis was 0.38 pound-feet or 4.2 per cent increase over the second analysis. This shows that the increase in moment is not according to a straight line variation.

For hinged supports a definite increase was shown and the maximum value occurs with a concentrated load at the center. The increase of the third analysis over the second was 5.8 per cent. No data for hinged supports were taken for the first analysis as the deformeters were not arranged at that time to analyze the models for hinged supports.

Double Span Model

The curves for the moment at the left knee for the fixed supports show an increase in the value of the moment near the maximum as the depth of the haunch decreases. The percentage increases are 2.9 and 2.6 per cent respectively for the second and third analyses. The maximum value for the moment at the left knee occurs when a concentrated load is at a point thirty-eight-hundredths of the span from the left support.

The results for the hinged supports gave little difference between the values of the moment at the left knee for the three different analyses. The slight difference could be attributed to experimental error. The maximum value occurs at a point thirty-eight-hundredths of the span from the left support, the same as for fixed supports.

Triple Span Model

With the triple span model the deformer analysis showed a definite decrease in the value of the moment at the left knee. However, this decrease was small, being 1.6 and 2.5 per cent respectively for the second and third analyses. For the hinged supports the decrease was 2.2 and 5.4 per cent respectively.

CONCLUSIONS

In the design of the deformer gages the principle of the Werner Deformer has been retained. The principle on which the gages work is the use of filler plates to give displacements at the supports between the three cylinders, "a", "b", and "c", and the glass plates. These displacements can be measured as well as displacements at the load points. The deformers as designed for use in this thesis have an improved method of holding the bars together and give a more sure action of the gages. The results should be more accurate as it is easier to get the bars to come together exactly the same each time which is an important point in the operation of the gages. The gages are very easy to manipulate.

In the operation of the equipment it is best to have one person operating the gages (placing the filler plates in the gages and taking them out as required) and a second person taking the readings at the screen. This saves much time over that required to operate the gages by one person, as well as the walking back and forth to the screen. When two persons are operating the equipment the Werner Deformers are much faster than the more expensive Beggs Deformers and just as accurate. With the Beggs Deformers a microscope must be focused each time a reading is desired, while one focus of the flashlight usually is enough for two to four load points before a change of the position of the flashlight is necessary.

The results as obtained for the single span frame bent with the gages showed a very close agreement with the theoretical analysis.

Also, the results for the single span rigid frame bridge for the second analysis (which was for a normal thickness) checks very closely with the results for the influence lines as given for this same bridge in the text "The Rigid Frame Bridge" by A. G. Hayden, pp 65 and 66. Although the influence line for point 5L is not shown it would be of identical shape to that shown for point 6L except that it would start at zero and go down to a maximum of about 8.8 or 9.0 pound-feet. These are the only curves that can be checked with the theoretical analysis. This close check leads one to feel confident that the other results as obtained with the deformeters are as accurate or will approach as closely to the actual value of the moments and reactions that exist in the structure as the theoretical analysis since the latter is based on some assumptions.

The cost of the four gages to complete a set with the mirror bar was forty-eight dollars. Therefore, the gages are inexpensive, and since they do give accurate results, are recommended for use both as laboratory demonstration work and for research work on statically indeterminate structures. However, because of the method necessary to use to obtain the displacements at the supports, the gages can only be used for structures with horizontal and vertical reaction lines. At least the gages must be placed so that the glass plate is either horizontal or vertical.

No attempt was made to check the experimental results for the three bridge models except as noted above, nor, account for the results for the variation of the depth of the haunch from a theoretical standpoint. The values as given in the tabulations in the appendix were

those as obtained from the work and all readings were taken at least twice and some checked more than once. These results were then averaged and the averaged values plotted to make the curves of the reactions as shown in the appendix. However, a theoretical analysis would seem to indicate that the moment at the knee should be reduced as the depth of the section is reduced as more material at the knee would "draw" the greater amount of moment from a theoretical fixed-end moment. By not changing the abutment or pier sections probably had something to do with the fact that the moment at the knee was not reduced in the single and double span models (an increase was noted). However, in the triple span model a decrease in the moment at the left knee was shown. There is the possibility that a still more accurate method than yet devised would be necessary for research of this nature where small changes in the model would show up better in the results. If this is true, the allowable error in the construction of the model is greater than thought by those who have worked with model analysis. It would not be necessary to sand the sections down to the nearest 0.001 of an inch but possibly the nearest 0.01 of an inch is all that is necessary. This would make the models much easier to construct which is one objection to the use of this type of analysis.

The results show need for further research with the deformer analysis with a theoretical analysis as a direct comparison so as to have a more definite tie-up of the deformer analysis and the mathematical theory.

APPENDIX

Influence Lines Pages

Single Span Model Reactions

Fixed Supports	- - - - -	33-35
Hinged Supports	- - - - -	36-37

Double Span Model Reactions

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Triple Span Model Reactions

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Table I

SINGLE SPAN RIGID FRAME BRIDGE AND MODEL CONSTANTS*

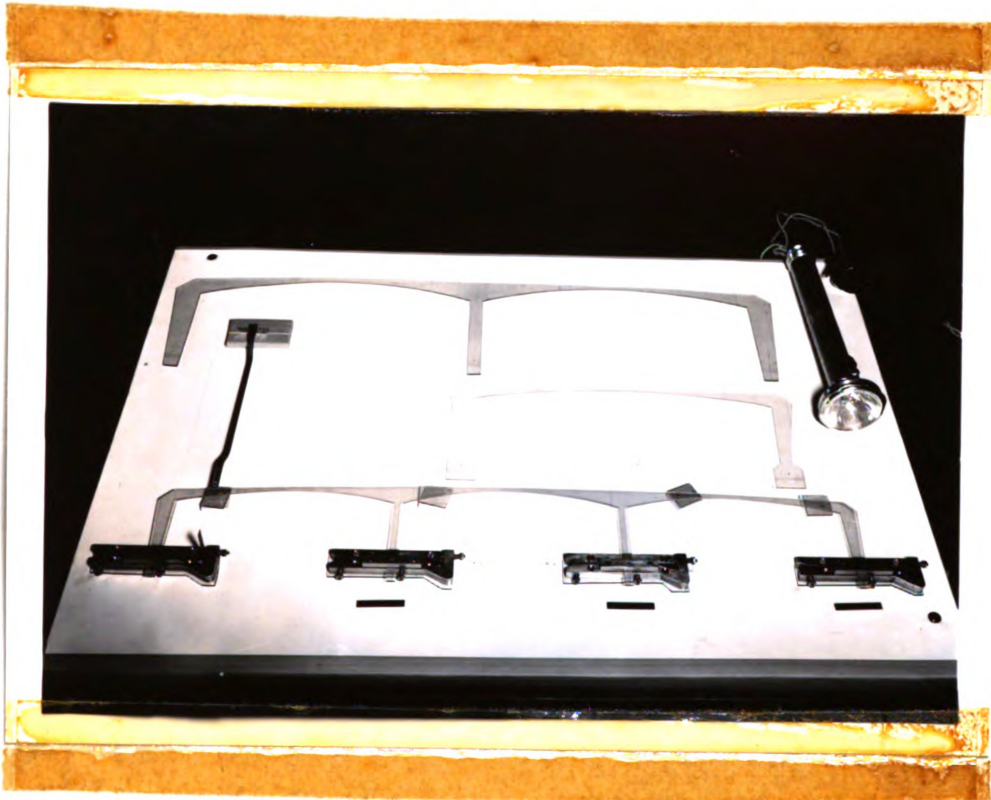
Point	t (ft)	I (ft ⁴)	Y (ft)	X (ft)	Coordinates of Model**	
					x (in.)	y (in.)
1L	6.00	18.00	1.0	0.4	0.13	0.33
2L	2.65	1.55	4.0	0.3	0.10	1.33
3L	3.21	2.75	8.0	0.0	0.00	2.67
4L	3.85	4.75	12.0	-0.3	-0.10	4.00
5L	4.00	5.33	15.9	0.1	0.03	5.30
6L	3.55	3.73	16.9	3.9	1.30	5.63
7L	2.75	1.73	17.3	7.9	2.63	5.77
8L	2.15	0.83	17.7	11.9	3.96	5.90
9L	1.70	0.41	17.9	15.9	5.30	5.97
10L	1.40	0.23	18.1	19.9	6.63	6.03
11L	1.25	0.16	18.2	23.9	7.96	6.07
11R	1.25	0.16	18.2	27.9	9.30	6.07
10R	1.40	0.23	18.1	31.9	10.63	6.03
9R	1.70	0.41	17.9	35.9	11.96	5.97
8R	2.15	0.83	17.7	39.9	13.30	5.90
7R	2.75	1.73	17.3	43.9	14.63	5.77
6R	3.55	3.73	16.9	47.9	15.96	5.63
5R	4.00	5.33	15.9	51.7	17.23	5.30
4R	3.85	4.75	12.0	52.1	17.36	4.00
3R	3.21	2.75	8.0	51.8	17.26	2.67
2R	2.65	1.55	4.0	51.5	17.16	1.33
1R	6.00	18.00	1.0	51.4	17.13	0.33

** Scale of Model 1" = 3' * Bridge Constants taken from "The Rigid Frame Bridge" by Arthur G. Hayden, Ch. VI, pp 43-58.

Table II

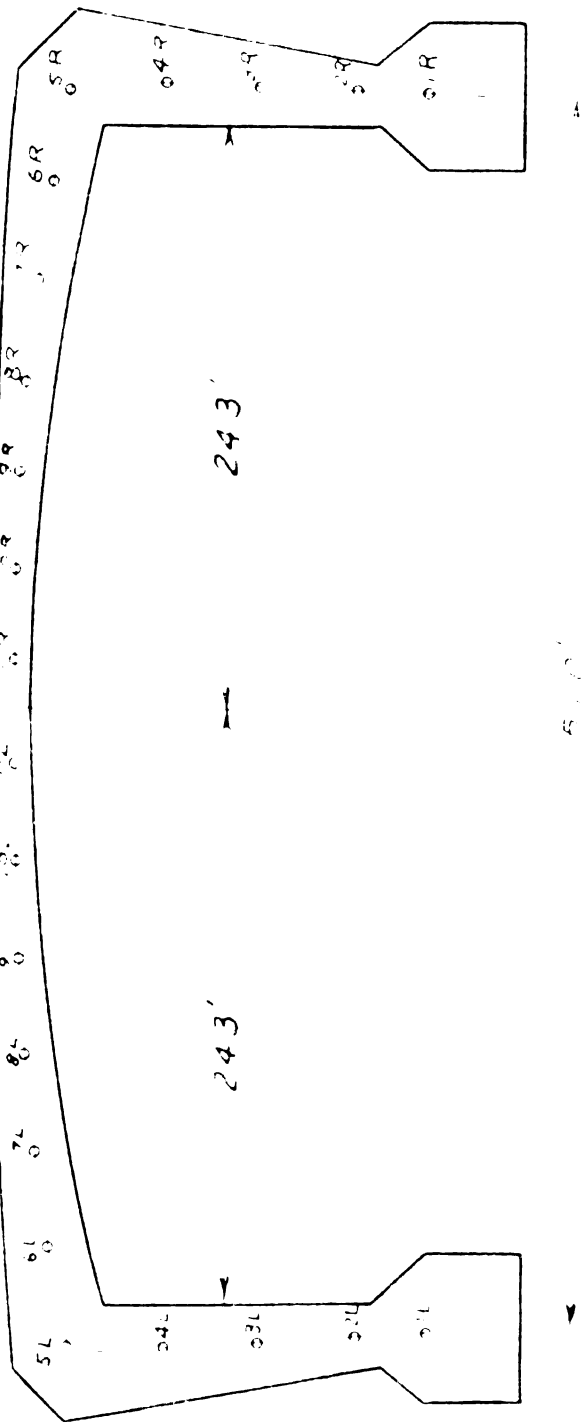
SECTION THICKNESS OF SINGLE SPAN RIGID FRAME BRIDGE MODEL

Point	Thickness		
	First Analysis	Second Analysis	Third Analysis
1L	1.573	1.573	1.573
2L	0.694	0.694	0.694
3L	0.842	0.842	0.842
4L	1.009	1.009	1.009
5L	1.048	1.048	1.048
6L	1.024	0.931	0.838
7L	0.782	0.720	0.658
8L	0.602	0.564	0.526
9L	0.463	0.446	0.429
10L	0.375	0.368	0.361
11L	0.327	0.326	0.325



Deformers and Models

(31A)

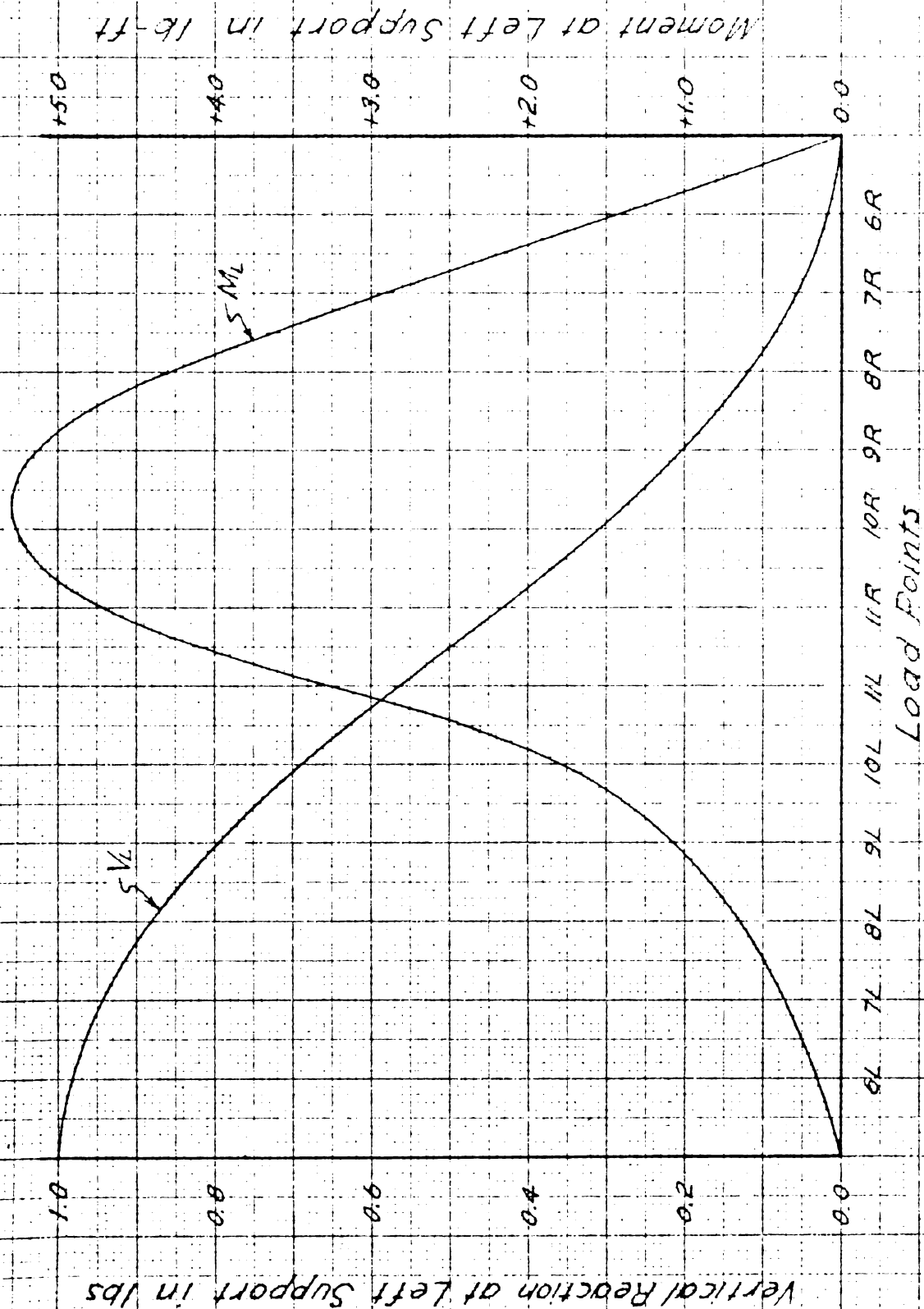


SINGLE SPAN RIGID FRAME BRIDGE

Scale of Model 1/4" = 3'

 $x = 0.60$

SINGLE SPAN RIGID FRAME BRIDGE Influence Lines For V_L and M_L - Fixed Supports



SINGLE SPAN RIGID FRAME BRIDGE Influence Lines For H_L - Fixed Supports

Horizontal Reaction at Left Support in lbs

1.0

0.8

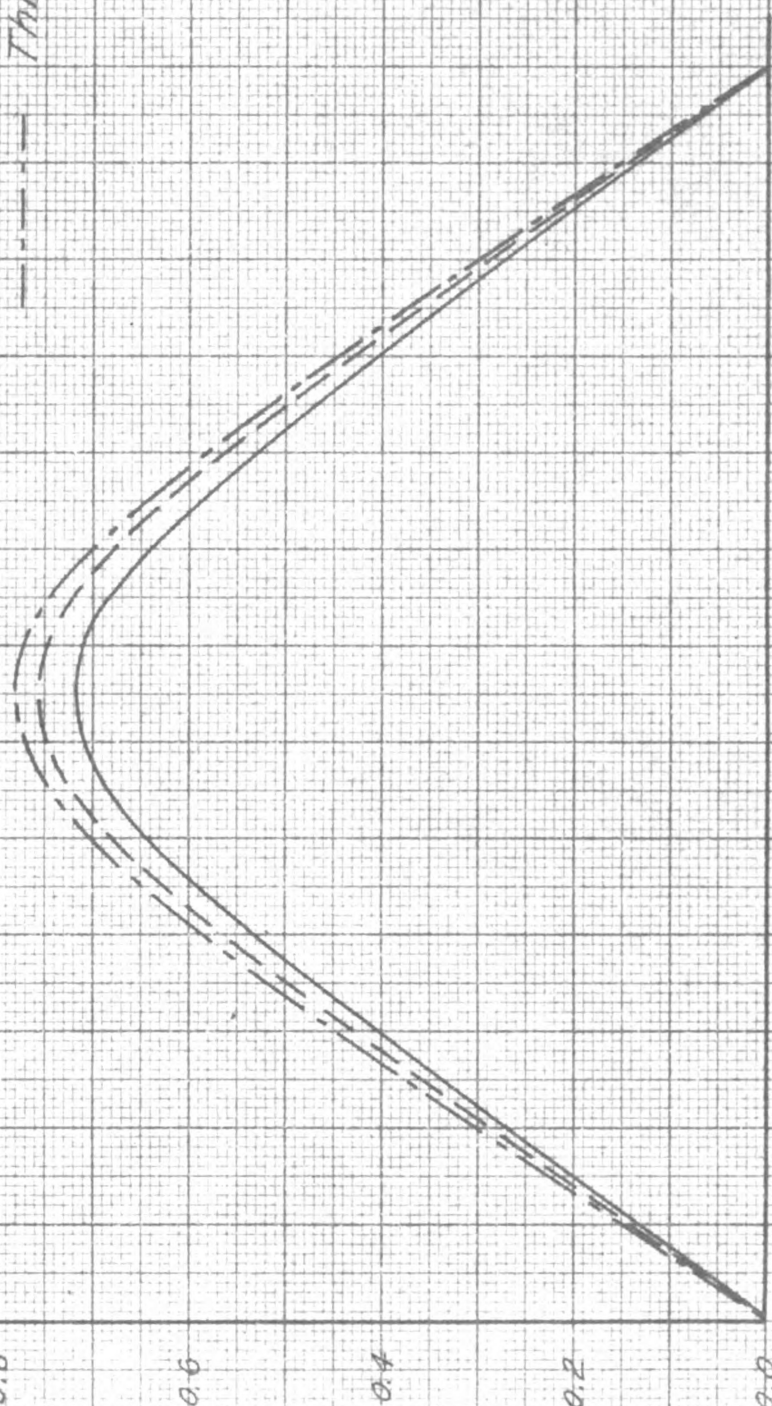
0.6

0.4

0.2

0.0

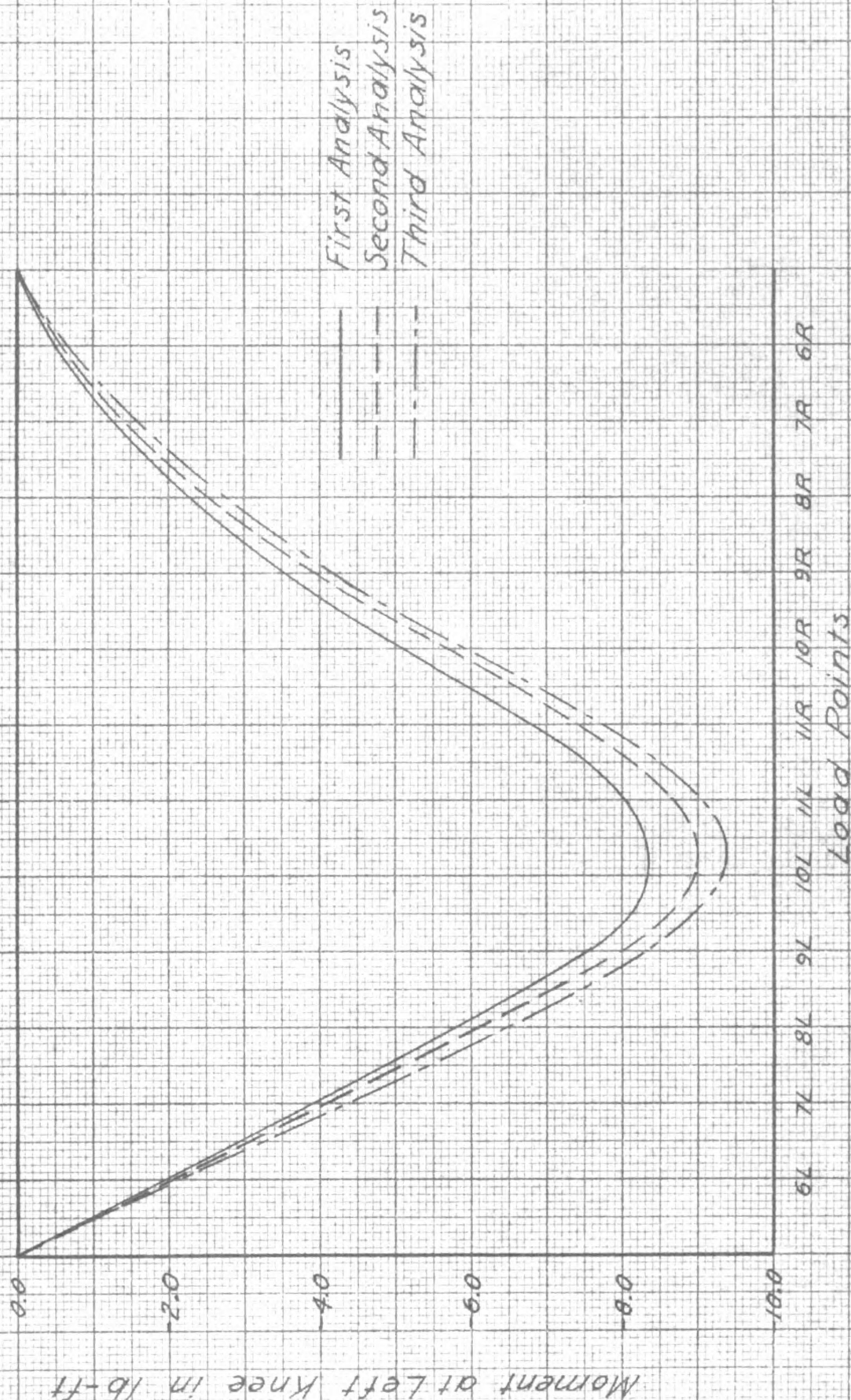
First Analysis
Second Analysis
Third Analysis



Load Points

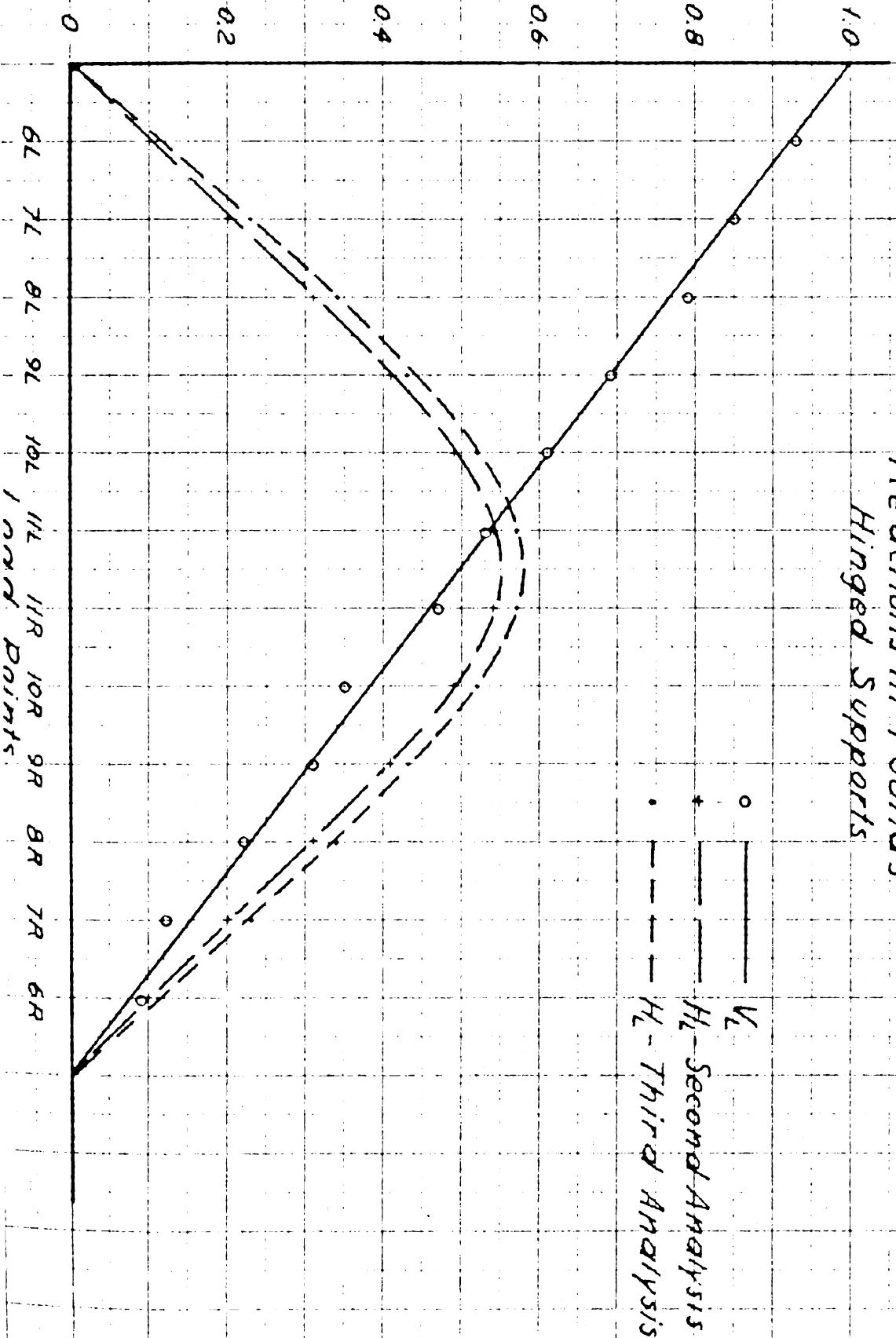
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225L 226L 227L 228L 229L 230L 231L 232L 233L 234L 235L 236L 237L 238L 239L 240L 241L 242L 243L 244L 245L 246L 247L 248L 249L 250L 251L 252L 253L 254L 255L 256L 257L 258L 259L 260L 261L 262L 263L 264L 265L 266L 267L 268L 269L 270L 271L 272L 273L 274L 275L 276L 277L 278L 279L 280L 281L 282L 283L 284L 285L 286L 287L 288L 289L 290L 291L 292L 293L 294L 295L 296L 297L 298L 299L 300L 301L 302L 303L 304L 305L 306L 307L 308L 309L 310L 311L 312L 313L 314L 315L 316L 317L 318L 319L 320L 321L 322L 323L 324L 325L 326L 327L 328L 329L 330L 331L 332L 333L 334L 335L 336L 337L 338L 339L 340L 341L 342L 343L 344L 345L 346L 347L 348L 349L 350L 351L 352L 353L 354L 355L 356L 357L 358L 359L 360L 361L 362L 363L 364L 365L 366L 367L 368L 369L 370L 371L 372L 373L 374L 375L 376L 377L 378L 379L 380L 381L 382L 383L 384L 385L 386L 387L 388L 389L 390L 391L 392L 393L 394L 395L 396L 397L 398L 399L 400L 401L 402L 403L 404L 405L 406L 407L 408L 409L 410L 411L 412L 413L 414L 415L 416L 417L 418L 419L 420L 421L 422L 423L 424L 425L 426L 427L 428L 429L 430L 431L 432L 433L 434L 435L 436L 437L 438L 439L 440L 441L 442L 443L 444L 445L 446L 447L 448L 449L 450L 451L 452L 453L 454L 455L 456L 457L 458L 459L 460L 461L 462L 463L 464L 465L 466L 467L 468L 469L 470L 471L 472L 473L 474L 475L 476L 477L 478L 479L 480L 481L 482L 483L 484L 485L 486L 487L 488L 489L 490L 491L 492L 493L 494L 495L 496L 497L 498L 499L 500L 501L 502L 503L 504L 505L 506L 507L 508L 509L 510L 511L 512L 513L 514L 515L 516L 517L 518L 519L 520L 521L 522L 523L 524L 525L 526L 527L 528L 529L 530L 531L 532L 533L 534L 535L 536L 537L 538L 539L 540L 541L 542L 543L 544L 545L 546L 547L 548L 549L 550L 551L 552L 553L 554L 555L 556L 557L 558L 559L 560L 561L 562L 563L 564L 565L 566L 567L 568L 569L 570L 571L 572L 573L 574L 575L 576L 577L 578L 579L 580L 581L 582L 583L 584L 585L 586L 587L 588L 589L 590L 591L 592L 593L 594L 595L 596L 597L 598L 599L 600L 601L 602L 603L 604L 605L 606L 607L 608L 609L 610L 611L 612L 613L 614L 615L 616L 617L 618L 619L 620L 621L 622L 623L 624L 625L 626L 627L 628L 629L 630L 631L 632L 633L 634L 635L 636L 637L 638L 639L 640L 641L 642L 643L 644L 645L 646L 647L 648L 649L 650L 651L 652L 653L 654L 655L 656L 657L 658L 659L 660L 661L 662L 663L 664L 665L 666L 667L 668L 669L 670L 671L 672L 673L 674L 675L 676L 677L 678L 679L 680L 681L 682L 683L 684L 685L 686L 687L 688L 689L 690L 691L 692L 693L 694L 695L 696L 697L 698L 699L 700L 701L 702L 703L 704L 705L 706L 707L 708L 709L 710L 711L 712L 713L 714L 715L 716L 717L 718L 719L 720L 721L 722L 723L 724L 725L 726L 727L 728L 729L 730L 731L 732L 733L 734L 735L 736L 737L 738L 739L 740L 741L 742L 743L 744L 745L 746L 747L 748L 749L 750L 751L 752L 753L 754L 755L 756L 757L 758L 759L 760L 761L 762L 763L 764L 765L 766L 767L 768L 769L 770L 771L 772L 773L 774L 775L 776L 777L 778L 779L 780L 781L 782L 783L 784L 785L 786L 787L 788L 789L 790L 791L 792L 793L 794L 795L 796L 797L 798L 799L 800L 801L 802L 803L 804L 805L 806L 807L 808L 809L 810L 811L 812L 813L 814L 815L 816L 817L 818L 819L 820L 821L 822L 823L 824L 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1353L 1354L 1355L 1356L 1357L 1358L 1359L 1360L 1361L 1362L 1363L 1364L 1365L 1366L 1367L 1368L 1369L 1370L 1371L 1372L 1373L 1374L 1375L 1376L 1377L 1378L 1379L 1380L 1381L 1382L 1383L 1384L 1385L 1386L 1387L 1388L 1389L 1390L 1391L 1392L 1393L 1394L 1395L 1396L 1397L 1398L 1399L 1400L 1401L 1402L 1403L 1404L 1405L 1406L 1407L 1408L 1409L 1410L 1411L 1412L 1413L 1414L 1415L 1416L 1417L 1418L 1419L 1420L 1421L 1422L 1423L 1424L 1425L 1426L 1427L 1428L 1429L 1430L 1431L 1432L 1433L 1434L 1435L 1436L 1437L 1438L 1439L 1440L 1441L 1442L 1443L 1444L 1445L 1446L 1447L 1448L 1449L 1450L 1451L 1452L 1453L 1454L 1455L 1456L 1457L 1458L 1459L 1460L 1461L 1462L 1463L 1464L 1465L 1466L 1467L 1468L 1469L 1470L 1471L 1472L 1473L 1474L 1475L 1476L 1477L 1478L 1479L 1480L 1481L 1482L 1483L 1484L 1485L 1486L 1487L 1488L 1489L 1490L 1491L 1492L 1493L 1494L 1495L 1496L 1497L 1498L 1499L 1500L 1501L 1502L 1503L 1504L 1505L 1506L 1507L 1508L 1509L 1510L 1511L 1512L 1513L 1514L 1515L 1516L 1517L 1518L 1519L 1520L 1521L 1522L 1523L 1524L 1525L 1526L 1527L 1528L 1529L 1530L 1531L 1532L 1533L 1534L 1535L 1536L 1537L 1538L 1539L 1540L 1541L 1542L 1543L 1544L 1545L 1546L 1547L 1548L 1549L 1550L 1551L 1552L 1553L 1554L 1555L 1556L 1557L 1558L 1559L 1560L 1561L 1562L 1563L 1564L 1565L 1566L 1567L 1568L 1569L 1570L 1571L 1572L 1573L 1574L 1575L 1576L 1577L 1578L 1579L 1580L 1581L 1582L 1583L 1584L 1585L 1586L 1587L 1588L 1589L 1590L 1591L 1592L 1593L 1594L 1595L 1596L 1597L 1598L 1599L 1600L 1601L 1602L 1603L 1604L 1605L 1606L 1607L 1608L 1609L 1610L 1611L 1612L 1613L 1614L 1615L 1616L 1617L 1618L 1619L 1620L 1621L 1622L 1623L 1624L 1625L 1626L 1627L 1628L 1629L 1630L 1631L 1632L 1633L 1634L 1635L 1636L 1637L 1638L 1639L 1640L 1641L 1642L 1643L 1644L 1645L 1646L 1647L 1648L 1649L 1650L 1651L 1652L 1653L 1654L 1655L 1656L 1657L 1658L 1659L 1660L 1661L 1662L 1663L 1664L 1665L 1666L 1667L 1668L 1669L 1670L 1671L 1672L 1673L 1674L 1675L 1676L 1677L 1678L 1679L 1680L 1681L 1682L 1683L 1684L 1685L 1686L 1687L 1688L 1689L 1690L 1691L 1692L 1693L 1694L 1695L 1696L 1697L 1698L 1699L 1700L 1701L 1702L 1703L 1704L 1705L 1706L 1707L 1708L 1709L 1710L 1711L 1712L 1713L 1714L 1715L 1716L 1717L 1718L 1719L 1720L 1721L 1722L 1723L 1724L 1725L 1726L 1727L 1728L 1729L 1730L 1731L 1732L 1733L 1734L 1735L 1736L 1737L 1738L 1739L 1740L 1741L 1742L 1743L 1744L 1745L 1746L 1747L 1748L 1749L 1750L 1751L 1752L 1753L 1754L 1755L 1756L 1757L 1758L 1759L 1760L 1761L 1762L 1763L 1764L 1765L 1766L 1767L 1768L 1769L 1770L 1771L 1772L 1773L 1774L 1775L 1776L 1777L 1778L 1779L 1780L 1781L 1782L 1783L 1784L 1785L 1786L 1787L 1788L 1789L 1790L 1791L 1792L 1793L 1794L 1795L 1796L 1797L 1798L 1799L 1800L 1801L 1802L 1803L 1804L 1805L 1806L 1807L 1808L 1809L 1810L 1811L 1812L 1813L 1814L 1815L 1816L 1817L 1818L 1819L 1820L 1821L 1822L 1823L 1824L 1825L 1826L 1827L 1828L 1829L 1830L 1831L 1832L 1833L 1834L 1835L 1836L 1837L 1838L 1839L 1840L 1841L 1842L 1843L 1844L 1845L 1846L 1847L 1848L 1849L 1850L 1851L 1852L 1853L 1854L 1855L 1856L 1857L 1858L 1859L 1860L 1861L 1862L 1863L 1864L 1865L 1866L 1867L 1868L 1869L 1870L 1871L 1872L 1873L 1874L 1875L 1876L 1877L 1878L 1879L 1880L 1881L 1882L 1883L 1884L 1885L 1886L 1887L 1888L 1889L 1890L 1891L 1892L 1893L 1894L 1895L 1896L 1897L 1898L 1899L 1900L 1901L 1902L 1903L 1904L 1905L 1906L 1907L 1908L 1909L 1910L 1911L 1912L 1913L 1914L 1915L 1916L 1917L 1918L 1919L 1920L 1921L 1922L 1923L 1924L 1925L 1926L 1927L 1928L 1929L 1930L 1931L 1932L 1933L 1934L 1935L 1936L 1937L 1938L 1939L 1940L 1941L 1942L 1943L 1944L 1945L 1946L 1947L 1948L 1949L 1950L 1951L 1952L 1953L 1954L 1955L 1956L 1957L 1958L 1959L 1960L 1961L 1962L 1963L 1964L 1965L 1966L 1967L 1968L 1969L 1970L 1971L 1972L 1973L 1974L 1975L 1976L 1977L 1978L 1979L 1980L 1981L 1982L 1983L 1984L 1985L 1986L 1987L 1988L 1989L 1990L 1991L 1992L 1993L 1994L 1995L 1996L 1997L 1998L 1999L 2000L 2001L 2002L 2003L 2004L 2005L 2006L 2007L 2008L 2009L 2010L 2011L 2012L 2013L 2014L 2015L 2016L 2017L 2018L 2019L 2020L 2021L 2022L 2023L 2024L 2025L 2026L 2027L 2028L 2029L 2030L 2031L 2032L 2033L 2034L 2035L 2036L 2037L 2038L 2039L 2040L 2041L 2042L 2043L 2044L 2045L 2046L 2047L 2048L 2049

SINGLE SPAN RIGID FRAME BRIDGE Influence Lines For M_k - Fixed Supports



Vertical and Horizontal Reactions at Left Support

SINGLE SPAN RIGID FRAME BRIDGE
Influence Lines For V_L and H_L
Reactions in Pounds
Hinged Supports



SINGLE SPAN RIGID FRAME BRIDGE
Influence Lines For M_K for
Hinged Supports

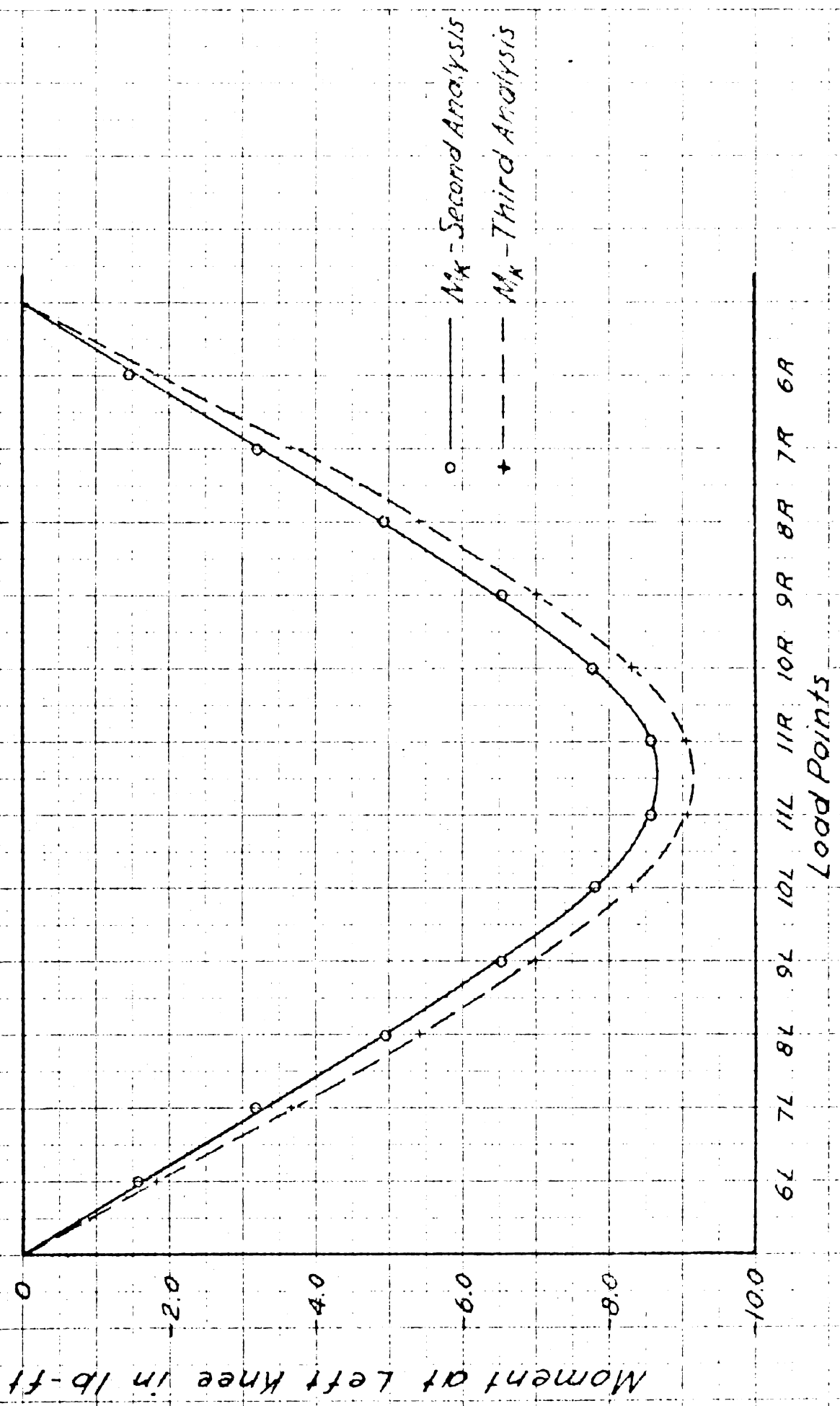


Table III

EXPERIMENTAL RESULTS

SINGLE SPAN RIGID FRAME BRIDGE - FIXED SUPPORTS

Results for First Analysis

<u>Point</u>	<u>Moment at Left Support</u>	<u>Vertical Reaction Left Support</u>	<u>Horizontal Reaction Left Support</u>	<u>Moment at Left Knee</u>
6L		0.96	0.12	-1.90
7L	0.07	0.92	0.27	-3.92
8L	0.26	0.85	0.40	-5.70
9L	0.81	0.78	0.52	-7.52
10L	2.08	0.68	0.64	-8.36
11L	3.41	0.57	0.71	-8.00
11R	4.41	0.42	0.71	-6.62
10R	5.27	0.30	0.64	-4.93
9R	5.02	0.20	0.52	-3.47
8R	4.03	0.11	0.40	-2.06
7R	2.82	0.07	0.27	-1.43
6R	1.42	0.03	0.12	-0.61

Results for Second Analysis

<u>Point</u>	<u>Moment at Left Support</u>	<u>Vertical Reaction Left Support</u>	<u>Horizontal Reaction Left Support</u>	<u>Moment at Left Knee</u>
6L		0.96	0.12	-2.06
7L	0.14	0.92	0.28	-4.07
8L	0.29	0.88	0.43	-6.18
9L	0.85	0.80	0.57	-8.00
10L	2.01	0.69	0.68	-9.01
11L	3.40	0.57	0.75	-8.64
11R	4.57	0.42	0.75	-7.26
10R	5.30	0.30	0.68	-5.58
9R	5.13	0.21	0.57	-3.95
8R	4.39	0.12	0.43	-2.54
7R	2.93	0.08	0.28	-1.58
6R	1.49	0.03	0.12	-0.77

Table III (cont.)

Results for Third Analysis

<u>Point</u>	<u>Moment at Left Support</u>	<u>Vertical Reaction Left Support</u>	<u>Horizontal Reaction Left Support</u>	<u>Moment at Left Knee</u>
6L	0.29	0.97	0.14	-2.22
7L	0.23	0.93	0.30	-4.39
8L	0.50	0.89	0.45	-6.50
9L	1.12	0.80	0.57	-8.31
10L	1.71	0.69	0.69	-9.32
11L	3.49	0.54	0.77	-8.96
11R	4.50	0.42	0.77	-7.58
10R	5.27	0.30	0.69	-5.89
9R	5.03	0.20	0.57	-4.26
8R	4.29	0.12	0.45	-2.86
7R	2.82	0.07	0.30	-1.90
6R	1.48	0.03	0.14	-0.93

Table IV

EXPERIMENTAL RESULTS

SINGLE SPAN RIGID FRAME BRIDGE - HINGED SUPPORTS

Results for the Second Analysis*

<u>Point</u>	<u>Vertical Reaction Left Support</u>	<u>Horizontal Reaction Left Support</u>	<u>Moment at Left Knee</u>
6L	0.93	0.10	-1.59
7L	0.85	0.20	-3.18
8L	0.79	0.31	-4.93
9L	0.69	0.41	-6.52
10L	0.61	0.49	-7.79
11L	0.53	0.54	-8.58
11R	0.47	0.54	-8.58
10R	0.35	0.49	-7.79
9R	0.30	0.41	-6.52
8R	0.22	0.31	-4.93
7R	0.12	0.20	-3.18
6R	0.09	0.10	-1.59

* The deformeter gages were not constructed so as to use hinge supports when the first analysis of the single span bridge was analyzed, and, therefore, only the results for the second and third analyses are shown.

Table IV (cont.)

Results for the Third Analysis

<u>Point</u>	<u>Vertical Reaction</u> <u>Left Support</u>	<u>Horizontal Reaction</u> <u>Left Support</u>	<u>Moment at</u> <u>Left Knee</u>
6L	0.92	0.11	-1.83
7L	0.88	0.23	-3.66
8L	0.79	0.34	-5.40
9L	0.69	0.43	-7.00
10L	0.61	0.52	-8.30
11L	0.55	0.57	-9.06
11R	0.47	0.57	-9.06
10R	0.38	0.52	-8.30
9R	0.29	0.43	-7.00
8R	0.18	0.34	-5.40
7R	0.15	0.23	-3.66
6R	0.07	0.11	-1.83

Table V

DOUBLE SPAN RIGID FRAME BRIDGE AND MODEL CONSTANTS*

<u>Point</u>	<u>t (ft)</u>	<u>I (ft4)</u>	<u>Y (ft)</u>	<u>X (ft)</u>	<u>Dimensions of Model</u>		
					<u>x (in.)</u>	<u>y (in.)</u>	<u>Depth</u>
1L	2.5	1.50	1.8	0.0	0.00	0.600	0.962
2L	2.6	1.81	6.8	0.0	0.00	2.267	1.024
3L	3.3	3.58	11.8	0.0	0.00	3.933	1.286
4L	3.7	4.96	16.7	0.0	0.00	5.567	1.433
5L	2.8	2.23	18.2	5.0	1.67	6.067	1.098
6L	2.0	0.85	19.2	10.0	3.33	6.400	0.796
7L	1.5	0.39	19.9	15.0	5.00	6.633	0.614
8L	1.1	0.15	20.4	20.0	6.67	6.800	0.446
9L	1.0	0.11	20.5	25.0	8.33	6.833	0.403
10L	1.2	0.20	20.4	30.0	10.00	6.800	0.491
11L	1.7	0.56	20.2	35.0	11.67	6.733	0.692
12L	2.6	1.87	19.7	40.0	13.33	6.567	1.035
13L	4.0	6.39	19.1	45.0	15.00	6.367	1.559
14	5.7	17.71	18.1	50.0	16.67	6.033	2.190
15	2.0	0.79	13.1	50.0	16.67	4.367	0.777
16	2.0	0.79	8.1	50.0	16.67	2.700	0.777
17	2.0	0.79	3.1	50.0	16.67	1.033	0.777

Table V (cont.)

Point	t (ft)	I (ft ⁴)	Y (ft)	X (ft)	Dimensions of Model		
					x (in.)	y (in.)	Depth
13R	4.0	6.39	19.1	55.0	18.33	6.367	1.559
12R	2.6	1.87	19.7	60.0	20.00	6.537	1.035
11R	1.7	0.56	20.2	65.0	21.67	6.733	0.692
10R	1.2	0.20	20.4	70.0	23.33	6.800	0.491
9R	1.0	0.11	20.5	75.0	25.00	6.833	0.403
8R	1.1	0.15	20.4	80.0	26.67	6.800	0.446
7R	1.5	0.39	19.9	85.0	28.33	6.633	0.614
6R	2.0	0.85	19.2	90.0	30.00	6.400	0.796
5R	2.8	2.23	18.2	95.0	33.33	6.067	1.098
4R	3.7	4.96	16.7	100.0	33.33	5.567	1.433
3R	3.3	3.58	11.8	100.0	33.33	3.933	1.286
2R	2.6	1.81	6.8	100.0	33.33	2.267	1.024
1R	2.5	1.50	1.8	100.0	33.33	0.600	0.962

* Bridge constants taken from "The Rigid Frame Bridge" by Arthur G. Hayden, Ch. IX, pp 107-126.

Table VI

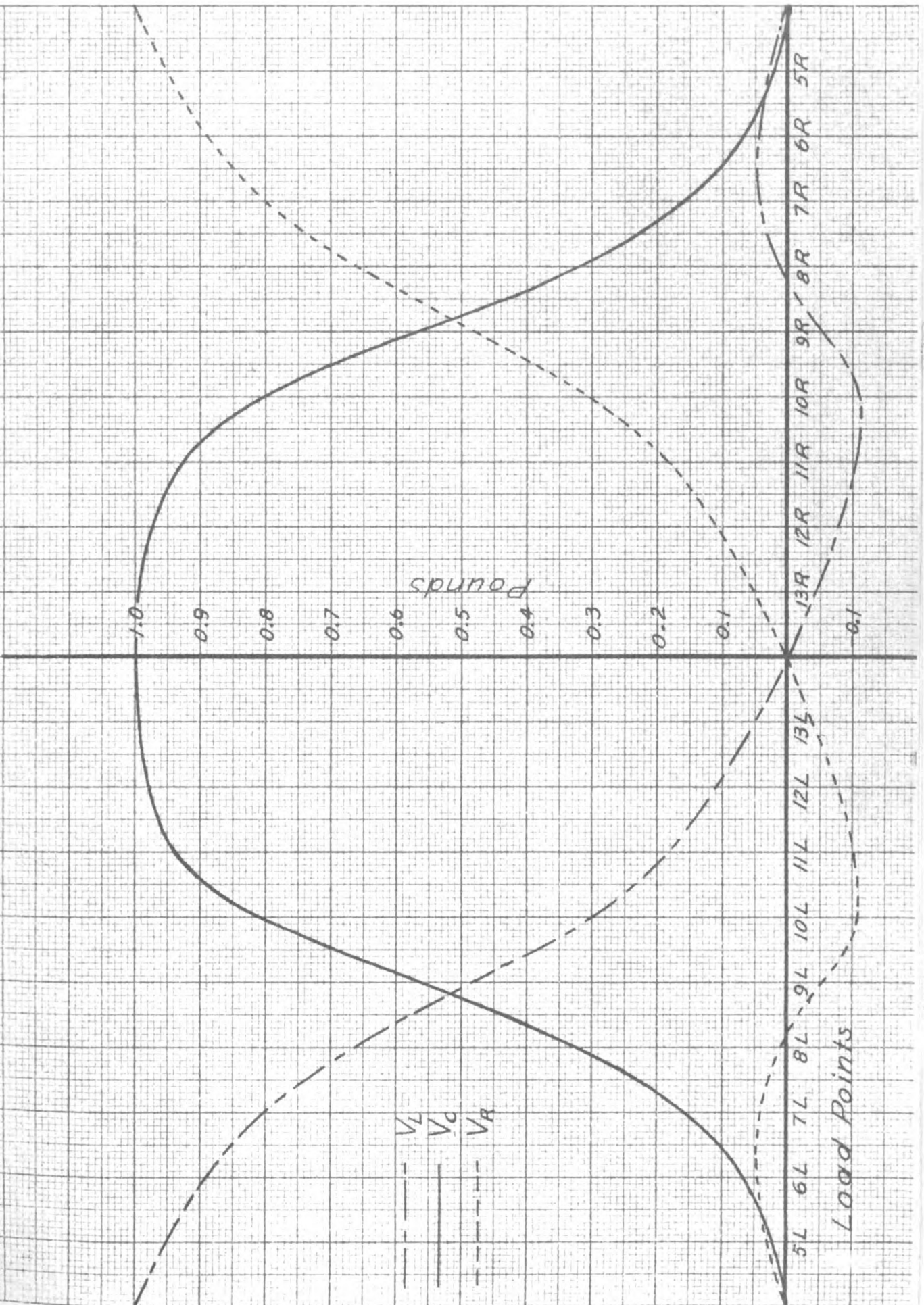
SECTION THICKNESS OF DOUBLE SPAN RIGID FRAME BRIDGE MODEL

Point	Thickness		
	First Analysis	Second Analysis	Third Analysis
5L	1.207	1.098	0.988
6L	0.858	0.796	0.734
7L	0.641	0.614	0.586
8L	0.453	0.446	0.439
9L	0.403	0.403	0.403
10L	0.501	0.491	0.482
11L	0.731	0.692	0.653
12L	1.123	1.035	0.947
13L	1.715	1.559	1.403
13R	1.715	1.559	1.403
12R	1.123	1.035	0.947
11R	0.731	0.692	0.653
10R	0.501	0.491	0.482
9R	0.403	0.403	0.403
8R	0.453	0.446	0.439
7R	0.641	0.614	0.586
6R	0.858	0.796	0.734
5R	1.207	1.098	0.988



TWO SPAN RIGID FRAME BRIDGE
 Scale of Model 1/3
 $n = 0.07$

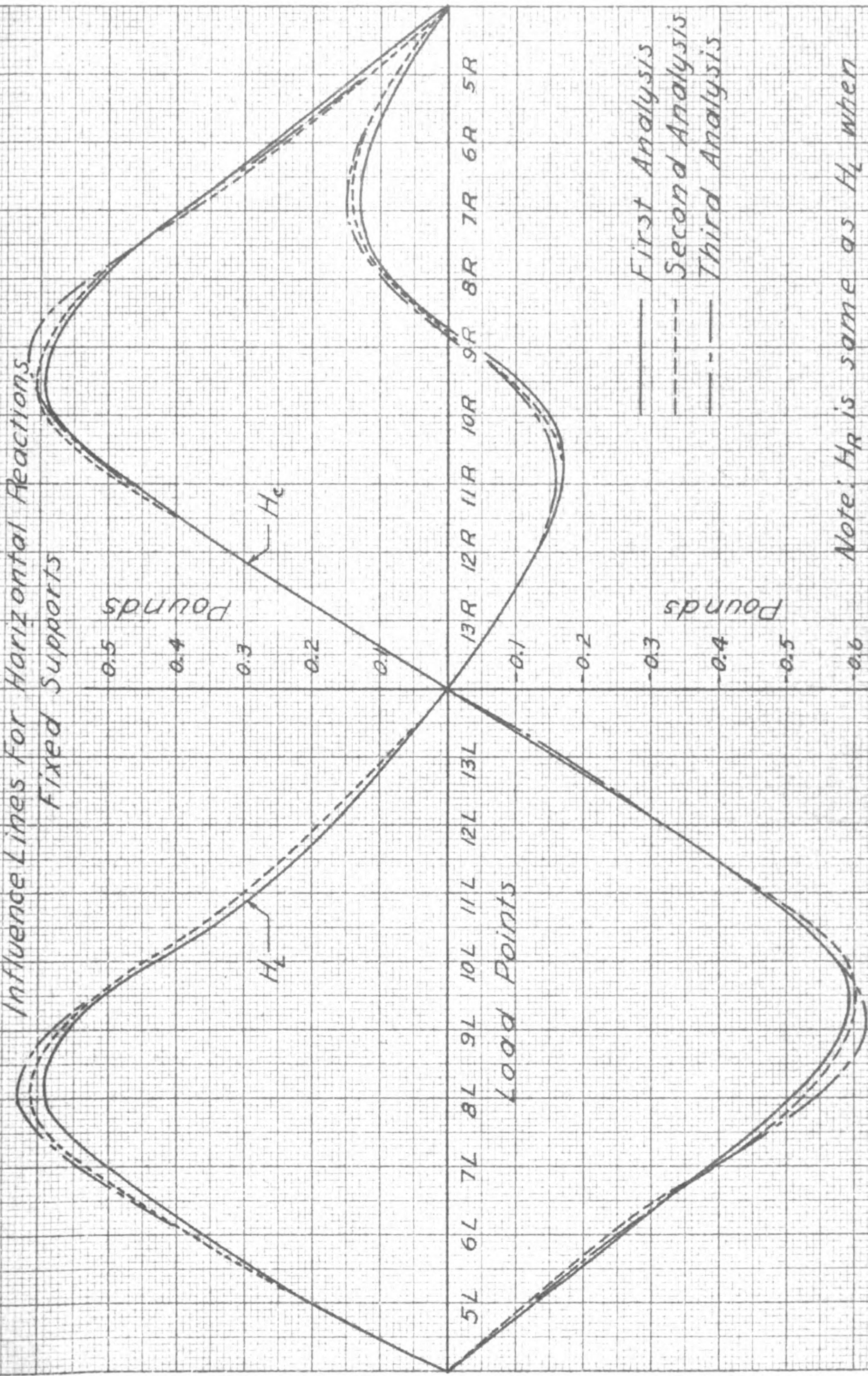
Influence Lines for Vertical Reactions - Fixed Supports



TWO SPAN RIGID FRAME BRIDGE

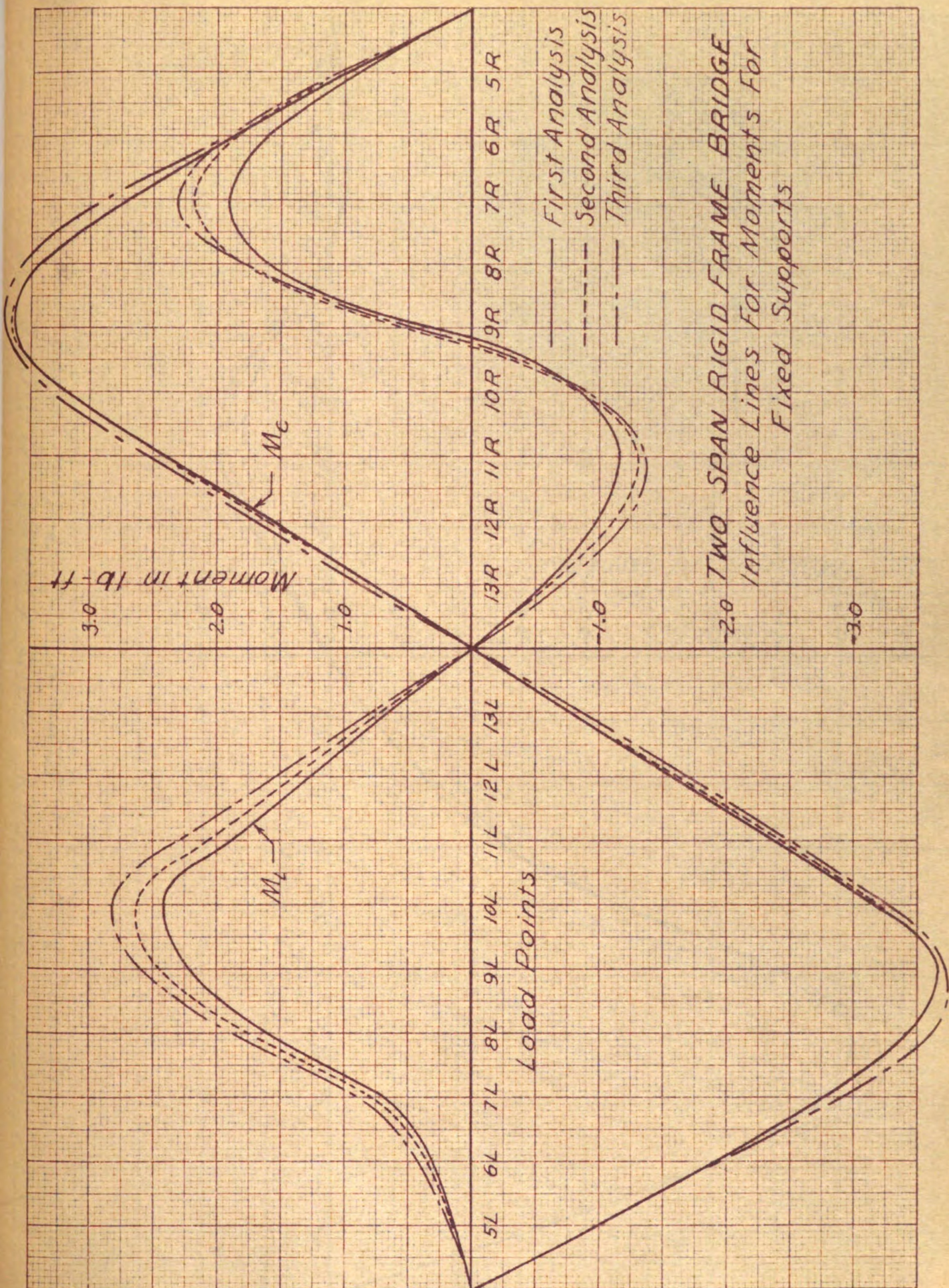
Influence Lines for Horizontal Reactions

Fixed Supports

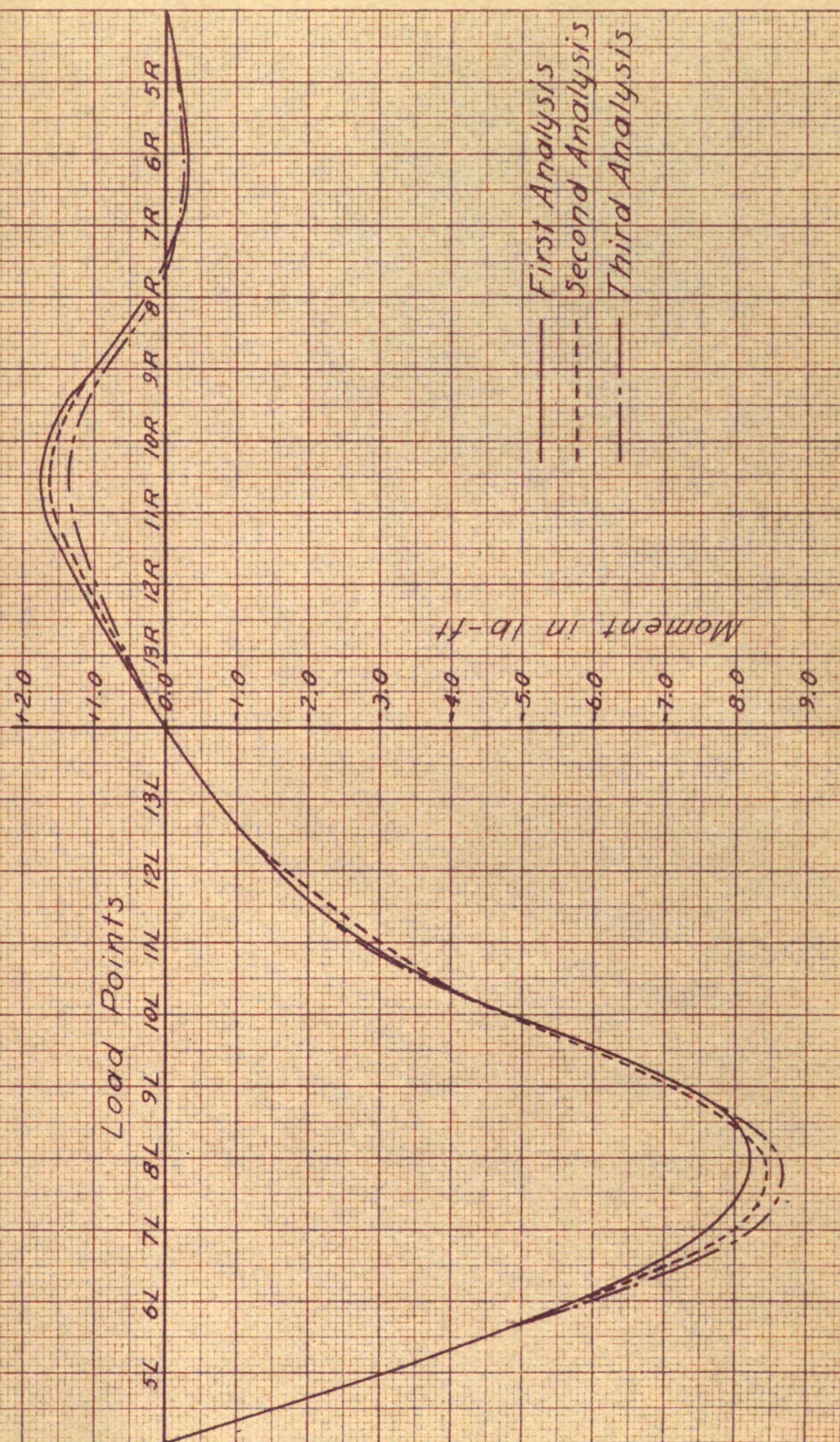


- First Analysis
- - - Second Analysis
- . - Third Analysis

Note: H_R is same as H_L when load is on opposite side of ϕ with reverse sign.

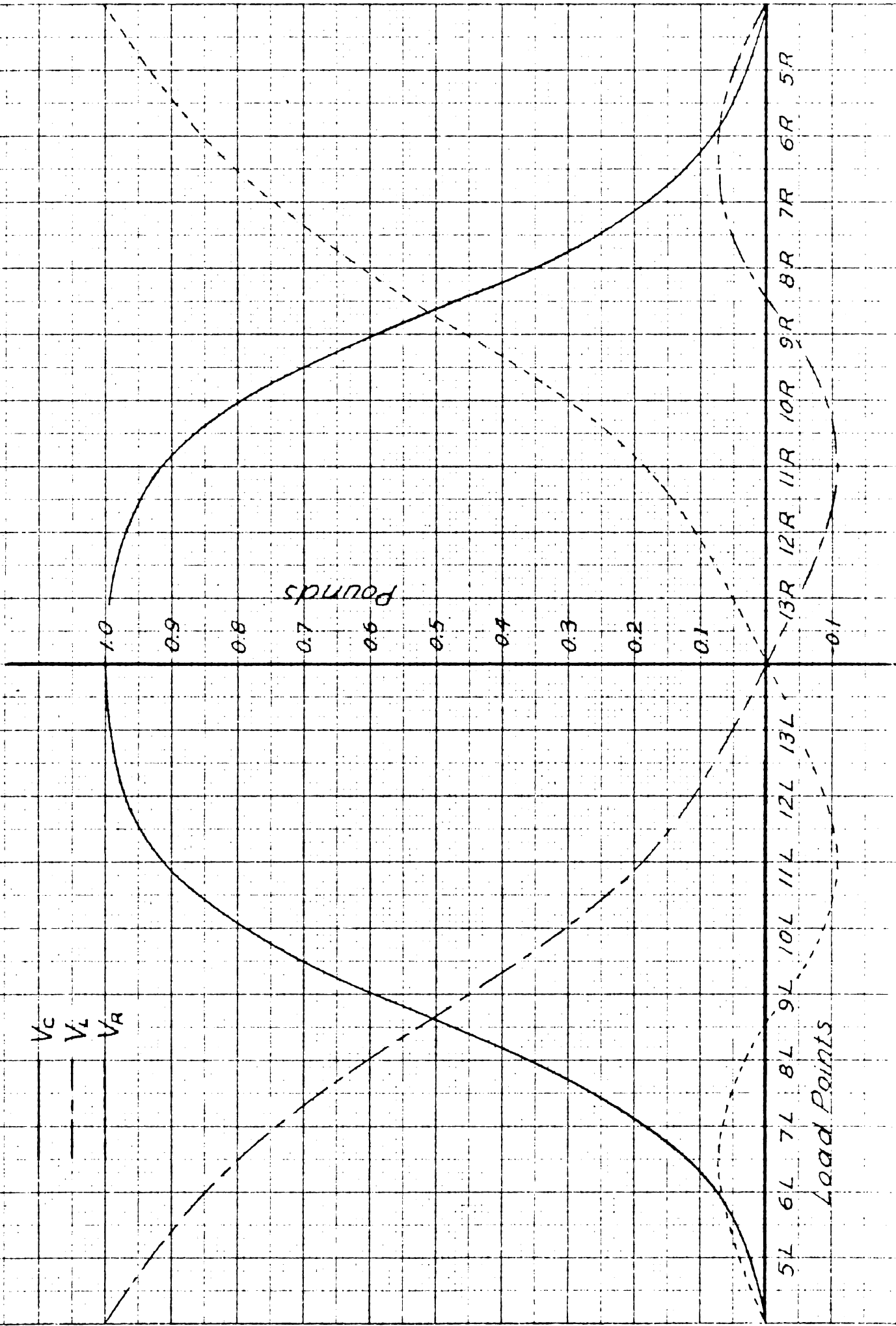


TWO SPAN RIGID FRAME BRIDGE Influence Lines for Moment At Left Knee - Fixed Supports

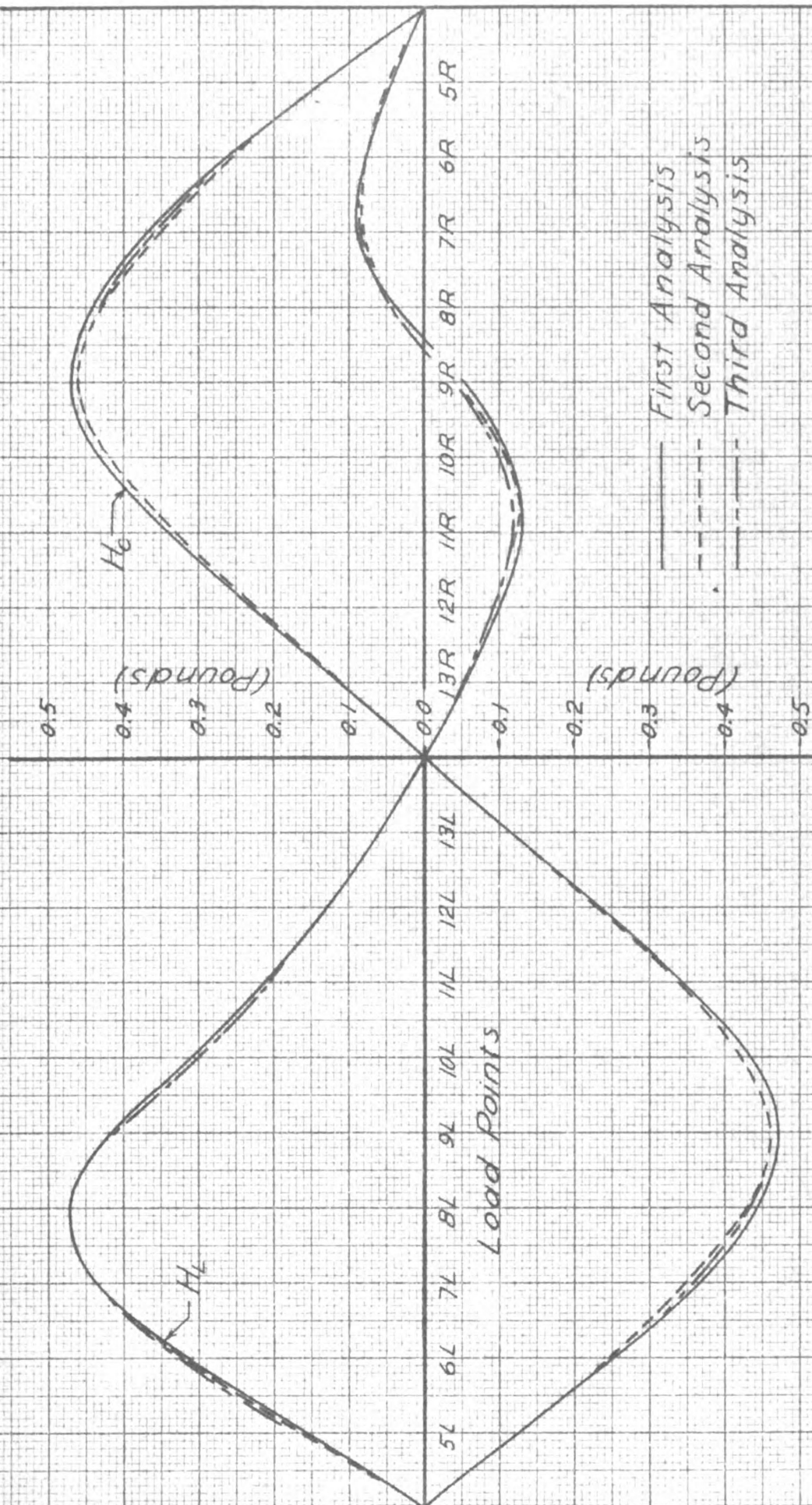


TWO SPAN RIGID FRAME BRIDGE Influence Lines For Vertical Reactions - Hinged Supports

V_C
 V_L
 V_R



TWO SPAN RIGID FRAME BRIDGE Influence Lines For Horizontal Reactions - Hinged Supports



Note: H_R is same as H_L when load is on opposite side of ϕ with reverse sign.

Note: Reactions to the right are positive.

TWO SPAN RIGID FRAME BRIDGE Influence Lines For Moment At Left Knee - Hinged Supports

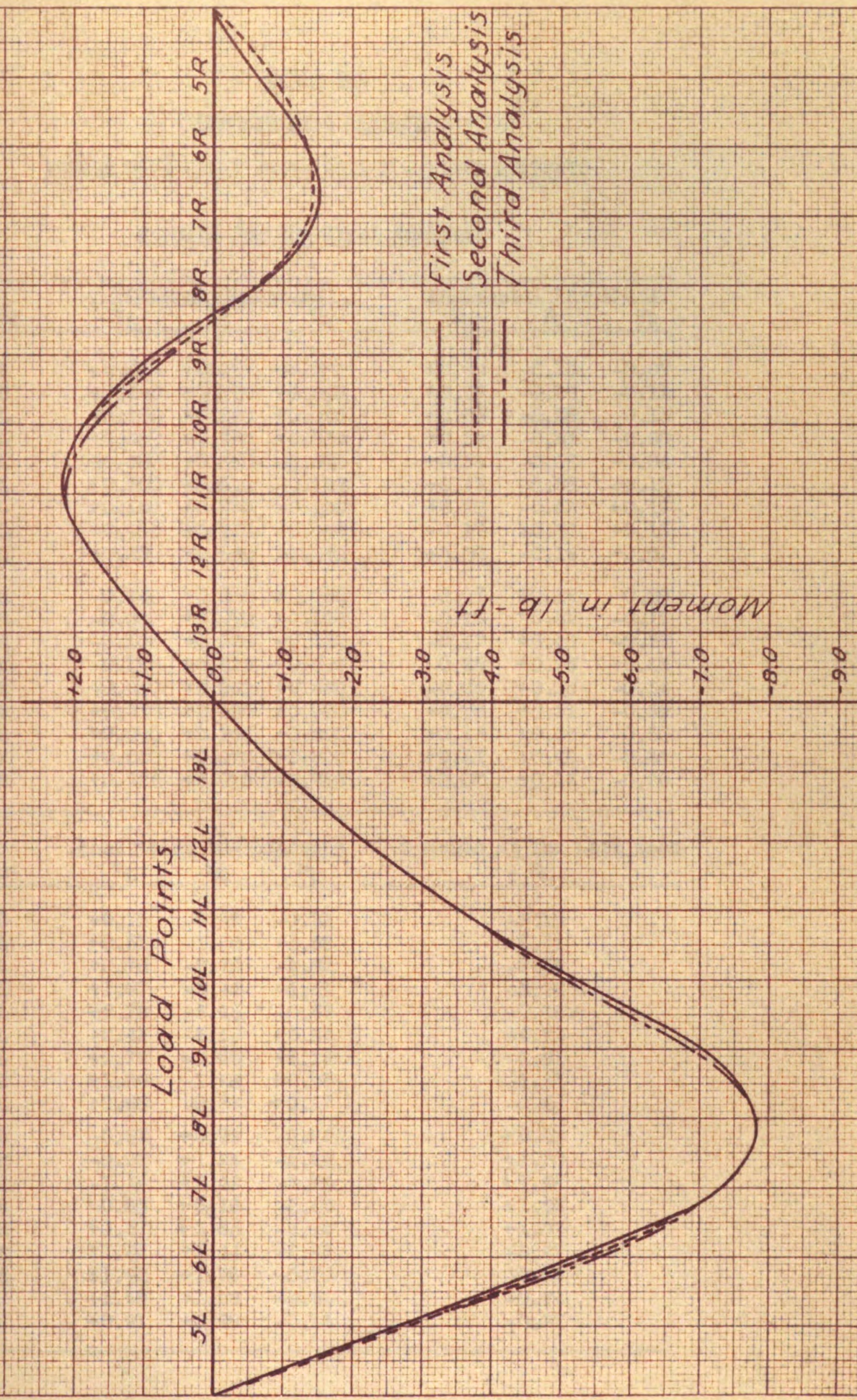


Table VII

EXPERIMENTAL RESULTS

DOUBLE SPAN RIGID FRAME BRIDGE - FIELD SUPPORTS

Results for the First Analysis

Point	<u>Vertical Reactions</u>			<u>Horizontal Reactions</u>		
	V _L	V _C	V _R	H _L	H _C	H _R
5L	+0.950	+0.025	+0.025	+0.20	-0.13	-0.07
6L	+0.890	+0.070	+0.040	+0.36	-0.26	-0.11
7L	+0.800	+0.160	+0.040	+0.50	-0.38	-0.13
8L	+0.660	+0.330	+0.010	+0.59	-0.51	-0.08
9L	+0.490	+0.560	-0.050	+0.54	-0.58	+0.04
10L	+0.300	+0.810	-0.110	+0.43	-0.58	+0.15
11L	+0.180	+0.930	-0.100	+0.28	-0.46	+0.17
12L	+0.110	+0.970	-0.080	+0.17	-0.32	+0.13
13L	+0.050	+0.990	-0.040	+0.09	-0.16	+0.07
13R	-0.040	+0.990	+0.050	-0.07	+0.16	-0.09
12R	-0.080	+0.970	+0.110	-0.13	+0.32	-0.17
11R	-0.100	+0.930	+0.180	-0.17	+0.46	-0.28
10R	-0.110	+0.810	+0.300	-0.15	+0.58	-0.43
9R	-0.050	+0.560	+0.480	-0.04	+0.58	-0.54
8R	+0.010	+0.330	+0.660	+0.08	+0.51	-0.59
7R	+0.040	+0.160	+0.800	+0.13	+0.38	-0.50
6R	+0.040	+0.070	+0.890	+0.11	+0.26	-0.36
5R	+0.025	+0.025	+0.950	+0.07	+0.13	-0.20

Point	<u>Moments at the Supports</u>			<u>Moments at the Left Knee</u>
	M _L	M _C	M _R	M _K
5L	+0.13	-0.98	-0.91	-3.19
6L	+0.31	-1.88	-1.58	-5.71
7L	+0.70	-2.76	-1.92	-7.65
8L	+1.67	-3.44	-1.51	-8.25
9L	+2.18	-3.65	-0.37	-7.15
10L	+2.42	-3.16	+0.83	-4.79
11L	+1.79	-2.43	+1.16	-2.81
12L	+1.29	-1.54	+0.97	-1.57
13L	+0.66	-0.78	+0.57	-0.68
13R	-0.57	+0.78	-0.66	+0.60
12R	-0.97	+1.54	-1.29	+1.20
11R	-1.16	+2.43	-1.79	+1.68
10R	-0.83	+3.16	-2.42	+1.66
9R	+0.37	+3.65	-2.18	+1.02
8R	+1.51	+3.44	-1.67	+0.27
7R	+1.92	+2.76	-0.70	-0.25
6R	+1.58	+1.88	-0.31	-0.24
5R	+0.91	+0.98	-0.13	-0.27

Table VII (cont.)

Results for the Second Analysis

Point	<u>Vertical Reactions</u>			<u>Horizontal Reactions</u>		
	V _L	V _C	V _R	H _L	H _C	H _R
5L	+0.96	+0.03	+0.02	+0.20	-0.12	-0.08
6L	+0.91	+0.06	+0.03	+0.37	-0.24	-0.13
7L	+0.80	+0.16	+0.03	+0.54	-0.39	-0.14
8L	+0.67	+0.33	+0.01	+0.61	-0.52	-0.09
9L	+0.47	+0.57	-0.05	+0.57	-0.59	+0.02
10L	+0.29	+0.80	-0.09	+0.44	-0.59	+0.14
11L	+0.18	+0.92	-0.10	+0.30	-0.47	+0.17
12L	+0.10	+0.97	-0.07	+0.19	-0.32	+0.13
13L	+0.05	+0.99	-0.04	+0.09	-0.16	+0.07
13R	-0.04	+0.99	+0.05	-0.07	+0.16	-0.09
12R	-0.07	+0.97	+0.10	-0.13	+0.32	-0.19
11R	-0.10	+0.92	+0.18	-0.17	+0.47	-0.30
10R	-0.09	+0.80	+0.29	-0.14	+0.59	-0.44
9R	-0.05	+0.57	+0.47	-0.02	+0.59	-0.57
8R	+0.01	+0.33	+0.67	+0.09	+0.52	-0.61
7R	+0.03	+0.16	+0.80	+0.14	+0.39	-0.54
6R	+0.03	+0.06	+0.91	+0.13	+0.24	-0.37
5R	+0.02	+0.03	+0.96	+0.08	+0.12	-0.20

Point	<u>Moments at the Supports</u>			<u>Moments at the Left Knee</u>
	M _L	M _C	M _R	M _K
5L	+0.14	-1.01	-1.07	-3.19
6L	+0.35	-1.87	-1.82	-5.83
7L	+0.79	-2.72	-2.18	-8.05
8L	+1.70	-3.40	-1.76	-8.43
9L	+2.46	-3.67	-0.55	-7.07
10L	+2.60	-3.19	+0.78	-4.73
11L	+2.09	-2.44	+1.30	-2.95
12L	+1.43	-1.60	+1.12	-1.80
13L	+0.72	-0.81	+0.62	-0.70
13R	-0.62	+0.81	-0.72	+0.55
12R	-1.12	+1.60	-1.43	+1.04
11R	-1.30	+2.44	-2.09	+1.54
10R	-0.78	+3.19	-2.60	+1.49
9R	+0.55	+3.67	-2.46	+1.05
8R	+1.76	+3.40	-1.70	+0.30
7R	+2.18	+2.72	-0.79	-0.15
6R	+1.82	+1.87	-0.35	-0.37
5R	+1.07	+1.01	-0.14	-0.28

Table VII (cont.)

Results for the Third Analysis

Point	<u>Vertical Reactions</u>			<u>Horizontal Reactions</u>		
	V_L	V_C	V_R	H_L	H_C	H_R
5L	+0.96	+0.02	+0.02	+0.20	-0.12	-0.08
6L	+0.90	+0.07	+0.03	+0.38	-0.25	-0.13
7L	+0.81	+0.16	+0.03	+0.54	-0.39	-0.15
8L	+0.67	+0.34	-0.01	+0.63	-0.54	-0.09
9L	+0.47	+0.57	-0.04	+0.58	-0.62	+0.03
10L	+0.29	+0.78	-0.08	+0.45	-0.58	+0.13
11L	+0.18	+0.91	-0.09	+0.30	-0.47	+0.17
12L	+0.10	+0.97	-0.07	+0.19	-0.32	+0.13
13L	+0.05	+0.99	-0.03	+0.09	-0.17	+0.08
13R	-0.03	+0.99	+0.05	-0.08	+0.17	-0.09
12R	-0.07	+0.97	+0.10	-0.13	+0.32	-0.19
11R	-0.09	+0.91	+0.18	-0.17	+0.47	-0.30
10R	-0.08	+0.78	+0.29	-0.13	+0.58	-0.45
9R	-0.04	+0.57	+0.47	-0.03	+0.62	-0.58
8R	-0.01	+0.34	+0.67	+0.09	+0.53	-0.62
7R	+0.03	+0.16	+0.81	+0.15	+0.39	-0.54
6R	+0.03	+0.07	+0.90	+0.13	+0.25	-0.38
5R	+0.02	+0.02	+0.96	+0.08	+0.12	-0.20

Point	<u>Moments at the Supports</u>			<u>Moments at the Left Knee</u>
	M_L	M_C	M_R	M_K
5L	+0.16	-1.01	-1.08	-3.18
6L	+0.47	-1.94	-1.92	-5.95
7L	+0.87	-2.91	-2.32	-8.29
8L	+1.82	-3.61	-1.75	-8.63
9L	+2.59	-3.71	-0.48	-7.09
10L	+2.81	-3.30	+0.88	-4.70
11L	+2.27	-2.46	+1.36	-2.71
12L	+1.56	-1.70	+1.22	-1.65
13L	+0.78	-0.89	+0.69	-0.82
13R	-0.69	+0.89	-0.78	+0.47
12R	-1.22	+1.70	-1.56	+0.95
11R	-1.36	+2.46	-2.27	+1.30
10R	-0.88	+3.30	-2.81	+1.27
9R	+0.48	+3.71	-2.59	+0.98
8R	+1.75	+3.61	-1.82	+0.13
7R	+2.32	+2.91	-0.87	-0.19
6R	+1.92	+1.94	-0.47	-0.27
5R	+1.08	+1.01	-0.16	-0.28

Table VIII

EXPERIMENTAL RESULTS

DOUBLE SPAN RIGID FRAME BRIDGE - HINGED SUPPORTS

Results for the First Analysis

Point	<u>Vertical Reactions</u>			<u>Horizontal Reactions</u>			<u>Moment at Left Knee</u>
	V _L	V _C	V _R	H _L	H _C	H _R	M _K
5L	+0.94	+0.03	+0.03	+0.16	-0.13	-0.04	-2.67
6L	+0.86	+0.08	+0.06	+0.31	-0.26	-0.08	-5.18
7L	+0.75	+0.17	+0.07	+0.44	-0.37	-0.09	-7.35
8L	+0.63	+0.33	+0.03	+0.47	-0.44	-0.04	-7.85
9L	+0.44	+0.59	-0.04	+0.42	-0.47	+0.05	-7.01
10L	+0.29	+0.79	-0.10	+0.31	-0.43	+0.11	-5.18
11L	+0.18	+0.93	-0.11	+0.21	-0.34	+0.13	-3.51
12L	+0.11	+0.97	-0.09	+0.13	-0.23	+0.10	-2.17
13L	+0.06	+0.99	-0.05	+0.06	-0.11	+0.05	-1.00
13R	-0.05	+0.99	+0.06	-0.05	+0.11	-0.06	+0.84
12R	-0.09	+0.97	+0.11	-0.10	+0.23	-0.13	+1.67
11R	-0.11	+0.93	+0.18	-0.13	+0.34	-0.21	+2.17
10R	-0.10	+0.79	+0.29	-0.11	+0.43	-0.30	+1.84
9R	-0.04	+0.59	+0.44	-0.05	+0.47	-0.42	+0.84
8R	+0.03	+0.33	+0.63	+0.04	+0.44	-0.47	-0.66
7R	+0.07	+0.17	+0.75	+0.09	+0.37	-0.44	-1.50
6R	+0.06	+0.08	+0.86	+0.08	+0.26	-0.31	-1.33
5R	+0.03	+0.03	+0.94	+0.04	+0.13	-0.16	-0.66

Results for the Second Analysis

Point	<u>Vertical Reactions</u>			<u>Horizontal Reactions</u>			<u>Moment at Left Knee</u>
	V _L	V _C	V _R	H _L	H _C	H _R	M _K
5L	+0.93	+0.03	+0.04	+0.17	-0.13	-0.05	-2.84
6L	+0.86	+0.08	+0.06	+0.32	-0.24	-0.08	-5.35
7L	+0.76	+0.18	+0.06	+0.44	-0.35	-0.08	-7.35
8L	+0.62	+0.34	+0.03	+0.47	-0.43	-0.04	-7.85
9L	+0.45	+0.59	-0.04	+0.44	-0.49	+0.04	-7.01
10L	+0.31	+0.79	-0.10	+0.31	-0.42	+0.11	-5.18
11L	+0.18	+0.93	-0.11	+0.20	-0.33	+0.12	-3.51
12L	+0.11	+0.97	-0.08	+0.13	-0.22	+0.09	-2.17
13L	+0.05	+0.99	-0.05	+0.06	-0.11	+0.05	-1.00

Table VIII (cont.)

Results for the Second Analysis (cont.)

Point	<u>Vertical Reactions</u>			<u>Horizontal Reactions</u>			<u>Moment at Left Knee</u>
	V _L	V _C	V _R	H _L	H _C	H _R	M _K
13R	-0.05	+0.99	+0.05	-0.05	+0.11	-0.06	+0.84
12R	-0.08	+0.97	+0.11	-0.09	+0.22	-0.13	+1.50
11R	-0.11	+0.93	+0.18	-0.12	+0.33	-0.20	+2.17
10R	-0.10	+0.79	+0.31	-0.11	+0.42	-0.31	+1.84
9R	-0.04	+0.59	+0.45	-0.04	+0.49	-0.44	+0.66
8R	+0.03	+0.34	+0.62	+0.04	+0.43	-0.47	-0.66
7R	+0.06	+0.18	+0.76	+0.08	+0.35	-0.44	-1.33
6R	+0.06	+0.08	+0.86	+0.08	+0.24	-0.32	-1.33
5R	+0.04	+0.03	+0.93	+0.05	+0.13	-0.17	-0.84

Results for the Third Analysis

Point	<u>Vertical Reactions</u>			<u>Horizontal Reactions</u>			<u>Moment at Left Knee</u>
	V _L	V _C	V _R	H _L	H _C	H _R	M _K
5L	+0.93	+0.03	+0.04	+0.17	-0.13	-0.05	-2.84
6L	+0.85	+0.08	+0.07	+0.33	-0.24	-0.08	-5.51
7L	+0.74	+0.19	+0.06	+0.43	-0.36	-0.08	-7.35
8L	+0.61	+0.36	+0.03	+0.47	-0.43	-0.05	-7.85
9L	+0.45	+0.59	-0.04	+0.41	-0.46	+0.04	-6.85
10L	+0.30	+0.79	-0.09	+0.30	-0.40	+0.10	-5.01
11L	+0.19	+0.92	-0.11	+0.20	-0.33	+0.12	-3.51
12L	+0.11	+0.97	-0.08	+0.13	-0.22	+0.09	-2.17
13L	+0.06	+0.99	-0.05	+0.06	-0.11	+0.06	-1.00
13R	-0.05	+0.99	+0.06	-0.06	+0.11	-0.06	+0.84
12R	-0.08	+0.97	+0.11	-0.09	+0.22	-0.13	+1.50
11R	-0.11	+0.92	+0.19	-0.12	+0.33	-0.20	+2.17
10R	-0.09	+0.79	+0.30	-0.10	+0.40	-0.30	+1.67
9R	-0.04	+0.59	+0.45	-0.04	+0.46	-0.41	+0.66
8R	+0.03	+0.36	+0.61	+0.05	+0.43	-0.47	-0.66
7R	+0.06	+0.18	+0.74	+0.08	+0.36	-0.43	-1.33
6R	+0.07	+0.08	+0.85	+0.08	+0.24	-0.33	-1.33
5R	+0.04	+0.03	+0.93	+0.05	+0.13	-0.17	-0.84

Table IX

TRIPLE SPAN RIGID FRAME BRIDGE AND MODEL CONSTANTS

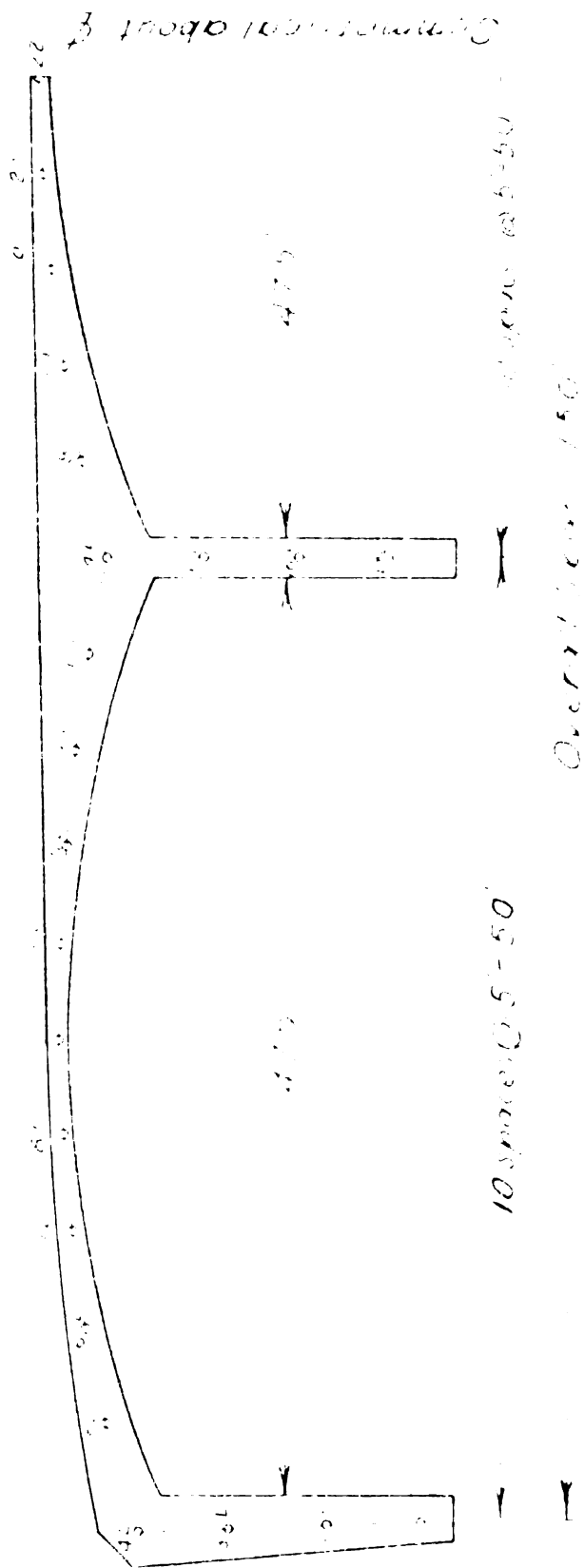
Point	t (ft)	I (ft ⁴)	X (ft)	Y (ft)	<u>Dimensions of Model</u>		
					x (in.)	y (in.)	Depth
1L	2.5	1.50	0.0	1.8	0.000	0.40	0.550
2L	2.6	1.81	0.0	6.8	0.000	1.51	0.585
3L	3.3	3.58	0.0	11.8	0.000	2.62	0.735
4L	3.7	4.96	0.0	16.7	0.000	3.71	0.819
5L	2.8	2.23	5.0	18.2	1.111	4.04	0.627
6L	2.0	0.85	10.0	19.2	2.222	4.27	0.455
7L	1.5	0.39	15.0	19.9	3.333	4.42	0.351
8L	1.1	0.15	20.0	20.4	4.444	4.53	0.255
9L	1.0	0.11	25.0	20.5	5.556	4.55	0.230
10L	1.2	0.20	30.0	20.4	6.667	4.57	0.281
11L	1.7	0.56	35.0	20.2	7.778	4.55	0.396
12L	2.6	1.87	40.0	19.7	8.889	4.49	0.591
13L	4.0	6.39	45.0	19.1	10.000	4.37	0.891
14L	5.7	17.71	50.0	18.1	11.111	4.22	1.251
15L	2.0	0.79	50.0	13.1	11.111	2.91	0.444
16L	2.0	0.79	50.0	8.1	11.111	1.80	0.444
17L	2.0	0.79	50.0	3.1	11.111	0.69	0.444
18	4.0	6.39	55.0	19.4	12.222	4.42	0.891
19	2.6	1.87	60.0	20.3	13.333	4.59	0.591
20	1.7	0.56	65.0	21.0	14.444	4.70	0.396
21	1.2	0.20	70.0	21.4	15.556	4.76	0.281
22	1.0	0.11	75.0	21.5	16.667	4.79	0.230
23	1.2	0.20	80.0	21.4	17.778	4.76	0.281
24	1.7	0.56	85.0	21.0	18.889	4.70	0.396
25	2.6	1.87	90.0	20.3	20.000	4.59	0.591
26	4.0	6.39	95.0	19.4	21.111	4.42	0.891
17R	2.0	0.79	100.0	3.1	22.222	0.69	0.444
16R	2.0	0.79	100.0	8.1	22.222	1.80	0.444
15R	2.0	0.79	100.0	13.1	22.222	2.91	0.444
14R	5.7	17.71	100.0	18.1	22.222	4.22	1.251
13R	4.0	6.39	105.0	19.1	23.333	4.37	0.891
12R	2.6	1.87	110.0	19.7	24.444	4.49	0.591
11R	1.7	0.56	115.0	20.2	25.556	4.55	0.396
10R	1.2	0.20	120.0	20.4	26.667	4.57	0.281
9R	1.0	0.11	125.0	20.5	27.778	4.55	0.230
8R	1.1	0.15	130.0	20.4	28.889	4.53	0.255
7R	1.5	0.39	135.0	19.9	30.000	4.42	0.351
6R	2.0	0.85	140.0	19.2	31.111	4.27	0.455
5R	2.8	2.23	145.0	18.2	32.222	4.04	0.627
4R	3.7	4.96	150.0	16.7	33.333	3.71	0.819
3R	3.3	3.58	150.0	11.8	33.333	2.62	0.735
2R	2.6	1.81	150.0	6.8	33.333	1.51	0.585
1R	2.5	1.50	150.0	1.8	33.333	0.40	0.550

Table X

SECTION THICKNESS OF THE TRIPLE SPAN RIGID FRAME BRIDGE MODEL

Point	Thickness		
	First Analysis	Second Analysis	Third Analysis
5L	0.690	0.627	0.564
6L	0.490	0.455	0.419
7L	0.366	0.351	0.335
8L	0.259	0.255	0.251
9L	0.230	0.230	0.230
10L	0.286	0.281	0.275
11L	0.418	0.396	0.373
12L	0.642	0.591	0.541
13L	0.980	0.891	0.802
18	0.980	0.891	0.802
19	0.642	0.591	0.541
20	0.418	0.396	0.373
21	0.286	0.281	0.275
22	0.230	0.230	0.230
23	0.286	0.281	0.275
24	0.418	0.396	0.373
25	0.642	0.591	0.541
26	0.980	0.891	0.802
13R	0.980	0.891	0.802
12R	0.642	0.591	0.541
11R	0.418	0.396	0.373
10R	0.286	0.281	0.275
9R	0.230	0.230	0.230
8R	0.259	0.255	0.251
7R	0.366	0.351	0.335
6R	0.490	0.455	0.419
5R	0.690	0.627	0.564

1

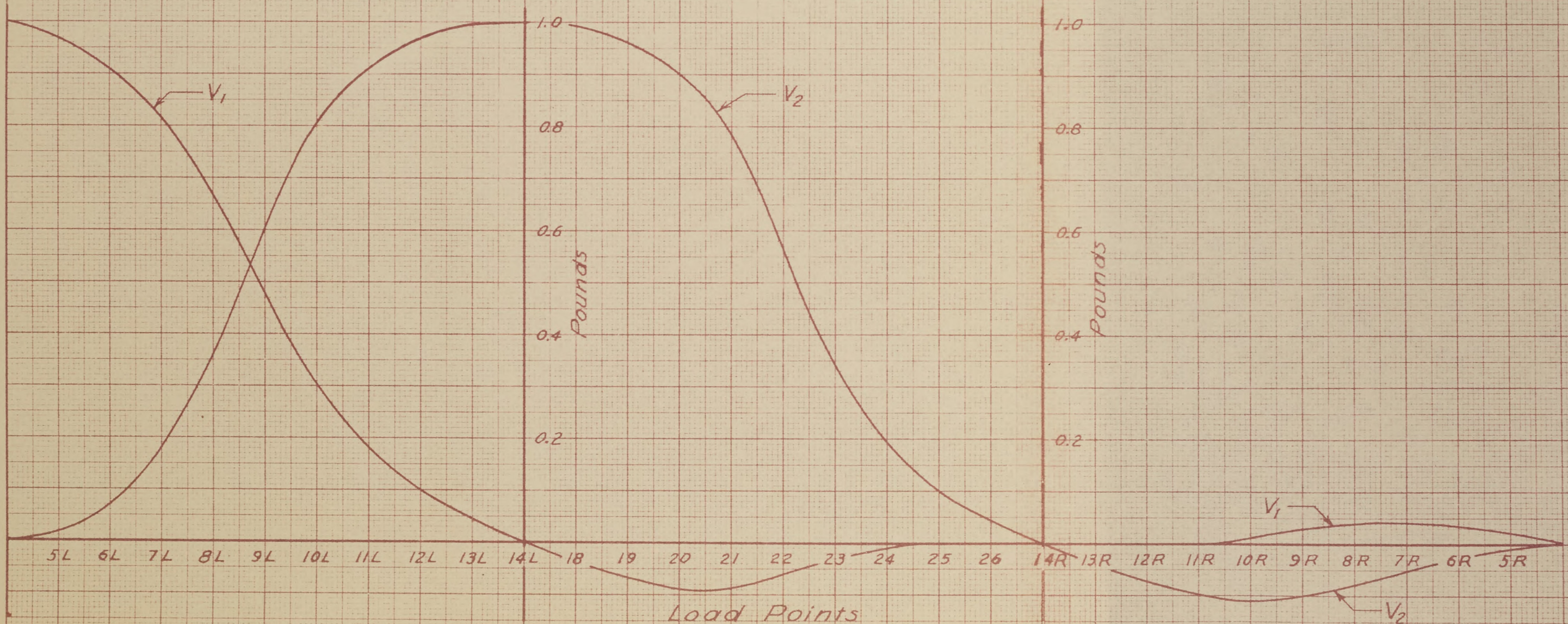


THREE SPAN RIGID FRAME BRIDGE

Scale of Model 1" = 4.5'

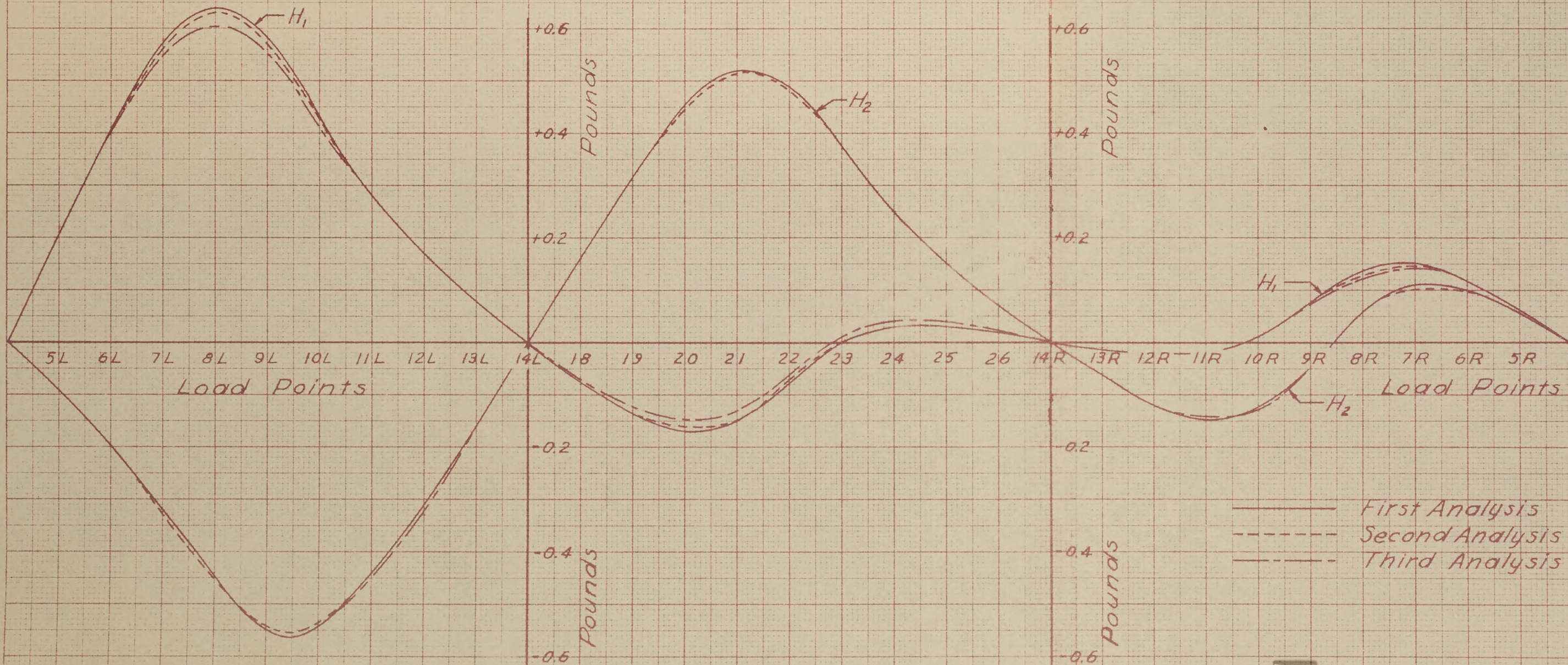
n = 0.04

THREE SPAN RIGID FRAME BRIDGE Influence Lines For Vertical Reactions - Fixed Supports

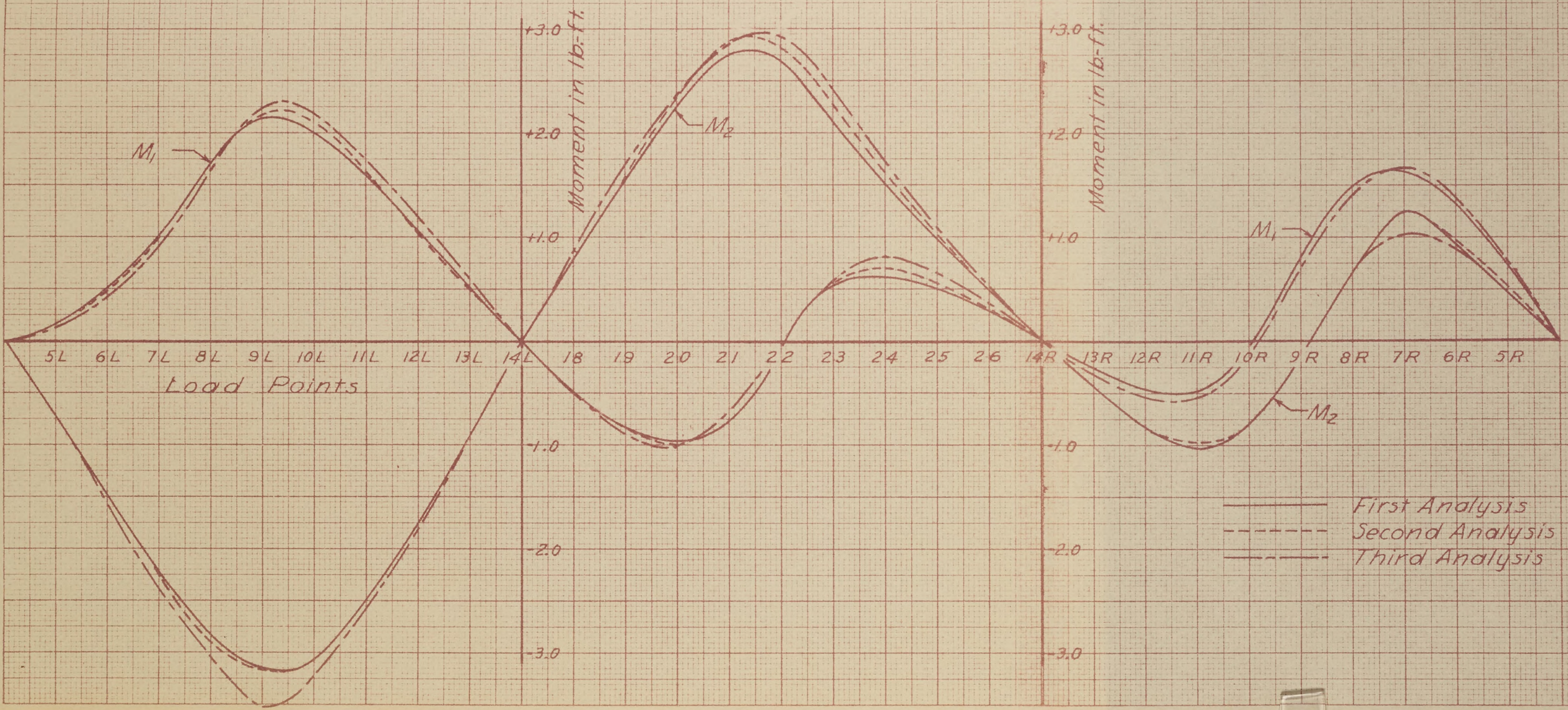


THREE SPAN RIGID FRAME BRIDGE Influence Lines For Horizontal Reactions - Fixed Supports

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THREE SPAN RIGID FRAME BRIDGE Influence Lines For Moments - Fixed Supports



- First Analysis
- - - Second Analysis
- · - Third Analysis

THREE SPAN RIGID FRAME BRIDGE Influence Lines For Moment at Left Knee-Fixed Supports

Load Points

5L 6L 7L 8L 9L 10L 11L 12L 13L

+2.0

+1.0

0.0

-1.0

-2.0

-3.0

-4.0

-5.0

-6.0

-7.0

-8.0

18

19

20

21

22

23

24

25

26

Moment in lb-ft

+2.0

+1.0

0.0

-1.0

-2.0

-3.0

-4.0

-5.0

-6.0

-7.0

-8.0

Load Points

13R 12R 11R 10R 9R 8R 7R 6R 5R

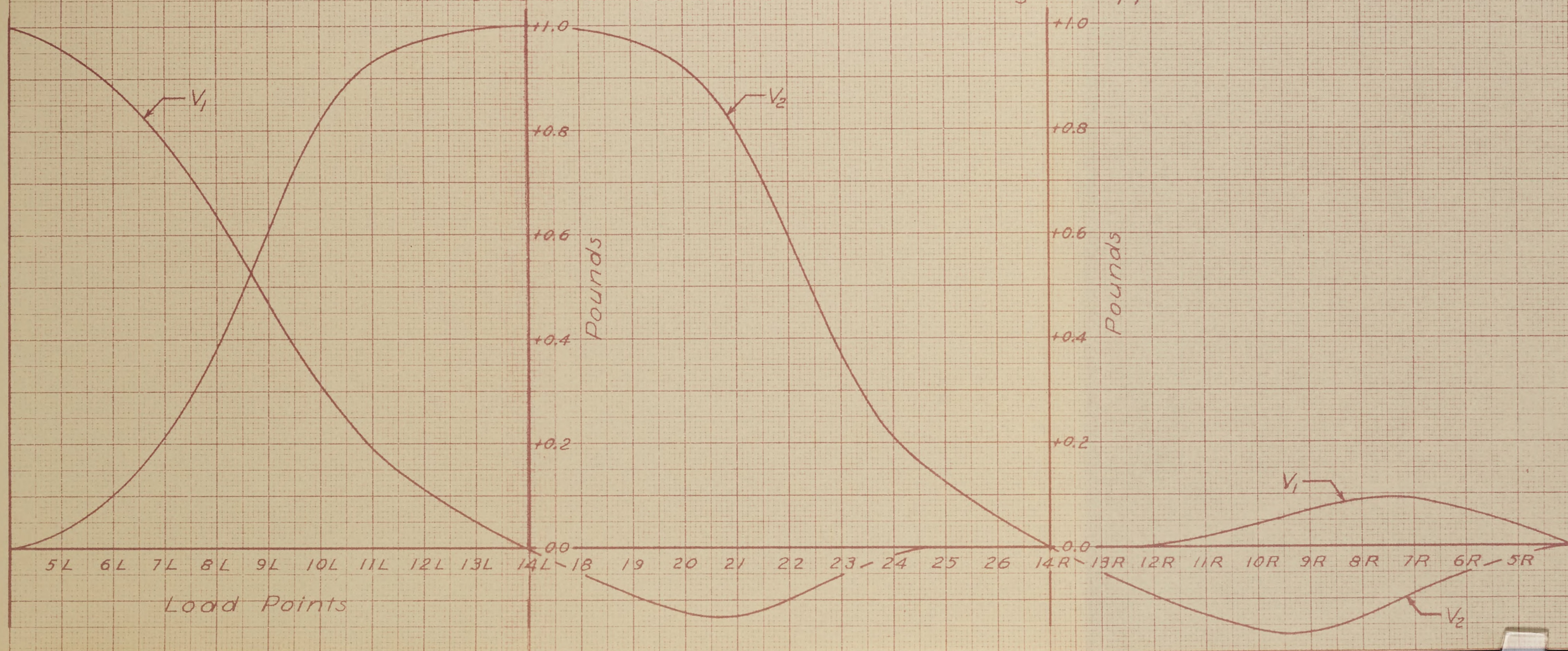
Moment in lb-ft

First Analysis

Second Analysis

Third Analysis

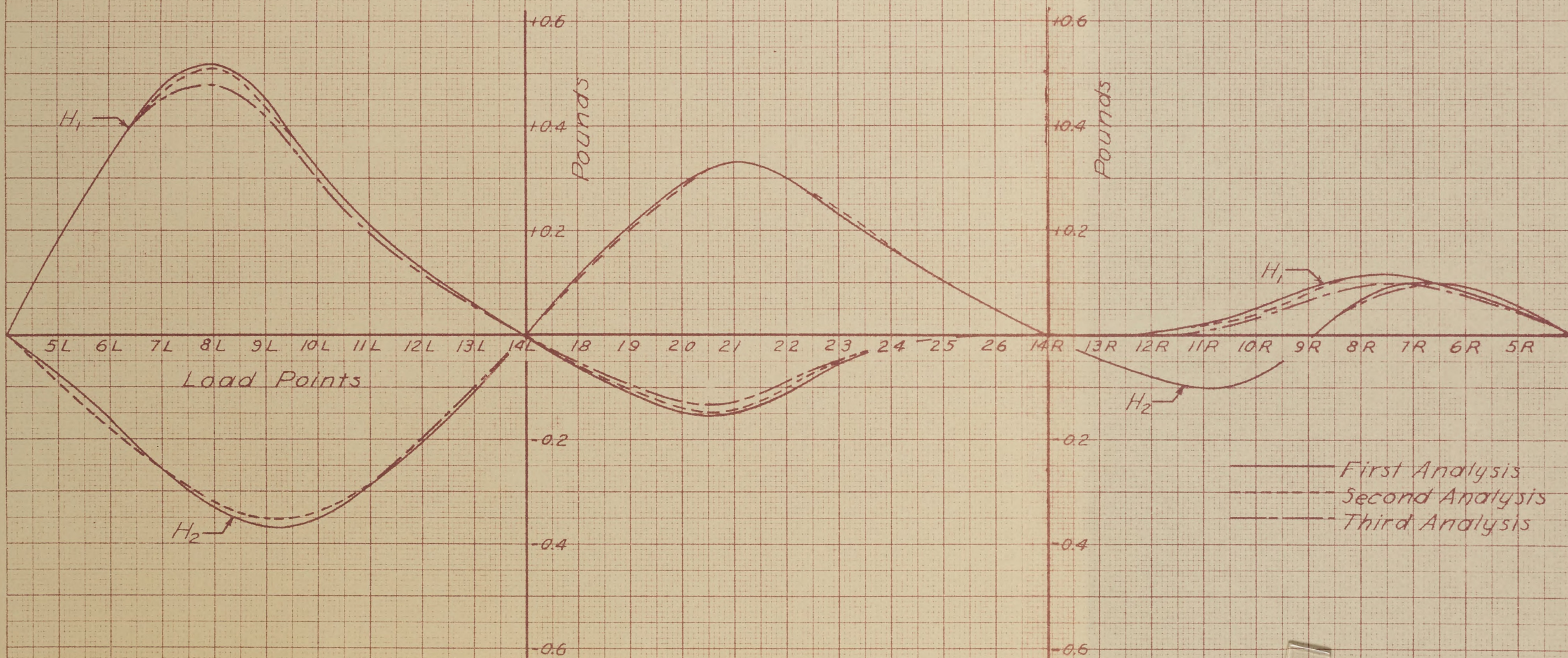
THREE SPAN RIGID FRAME BRIDGE Influence Lines For Vertical Reactions - Hinged Supports



THREE SPAN RIGID FRAME BRIDGE

Influence Lines For Horizontal Reactions - Hinged Supports

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THREE SPAN RIGID FRAME BRIDGE Influence Lines For Moment at Left Knee-Hinged Supports

Load Points

5L 6L 7L 8L 9L 10L 11L 12L 13L

+2.0

+1.0

0.0

-1.0

-2.0

-3.0

-4.0

-5.0

-6.0

-7.0

-8.0

Moment in lb.-ft

18

19

20

21

22

23

24

25

26

Load Points

13R 12R 11R 10R 9R 8R 7R 6R 5R

+2.0

+1.0

0.0

-1.0

-2.0

-3.0

-4.0

-5.0

-6.0

-7.0

-8.0

Moment in lb.-ft

First Analysis
Second Analysis
Third Analysis

Table XI

EXPERIMENTAL RESULTS

TRIPLE SPAN RIGID FRAME BRIDGE - FIXED SUPPORTS

Results for the First Analysis

Point	<u>Vertical Reactions</u>				<u>Horizontal Reactions</u>			
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄
5L	+0.97	+0.02	-0.01	+0.02	+0.21	-0.09	-0.06	-0.06
6L	+0.91	+0.08	-0.03	+0.04	+0.41	-0.20	-0.10	-0.11
7L	+0.83	+0.17	-0.05	+0.05	+0.58	-0.32	-0.11	-0.15
8L	+0.69	+0.34	-0.08	+0.05	+0.64	-0.44	-0.06	-0.14
9L	+0.48	+0.58	-0.11	+0.04	+0.58	-0.55	+0.05	-0.08
10L	+0.29	+0.82	-0.13	+0.02	+0.43	-0.54	+0.12	-0.01
11L	+0.18	+0.93	-0.11	0.00	+0.28	-0.45	+0.15	+0.02
12L	+0.11	+0.97	-0.08	0.00	+0.17	-0.31	+0.12	+0.02
13L	+0.05	+0.99	-0.04	0.00	+0.09	-0.16	+0.06	+0.01
18	-0.04	+0.99	+0.05	0.00	-0.07	+0.16	-0.07	-0.02
19	-0.08	+0.97	+0.10	0.00	-0.13	+0.31	-0.15	-0.03
20	-0.10	+0.92	+0.19	-0.01	-0.17	+0.45	-0.25	-0.03
21	-0.09	+0.80	+0.32	-0.03	-0.15	+0.52	-0.37	0.00
22	-0.06	+0.56	+0.56	-0.06	-0.08	+0.49	-0.49	+0.08
23	-0.03	+0.32	+0.80	-0.09	0.00	+0.37	-0.52	+0.15
24	-0.01	+0.19	+0.92	-0.10	+0.03	+0.25	-0.45	+0.17
25	0.00	+0.10	+0.97	-0.08	+0.03	+0.15	-0.31	+0.13
26	0.00	+0.05	+0.99	-0.04	+0.02	+0.07	-0.16	+0.07
13R	0.00	-0.04	+0.99	+0.05	-0.01	-0.06	+0.16	-0.09
12R	0.00	-0.08	+0.97	+0.11	-0.02	-0.12	+0.31	-0.17
11R	0.00	-0.11	+0.93	+0.18	-0.02	-0.15	+0.45	-0.28
10R	+0.02	-0.13	+0.82	+0.29	+0.01	-0.12	+0.54	-0.43
9R	+0.04	-0.11	+0.58	+0.48	+0.08	-0.05	+0.55	-0.58
8R	+0.05	-0.08	+0.34	+0.69	+0.14	+0.06	+0.44	-0.64
7R	+0.05	-0.05	+0.17	+0.83	+0.15	+0.11	+0.32	-0.58
6R	+0.04	-0.03	+0.08	+0.91	+0.11	+0.10	+0.20	-0.41
5R	+0.02	-0.01	+0.02	+0.97	+0.06	+0.06	+0.09	-0.21

Point	<u>Moments at the Supports</u>				<u>Moment at Left Knee</u>
	M ₁	M ₂	M ₃	M ₄	M _k
5L	+0.18	-0.72	-0.50	-0.70	-3.34
6L	+0.52	-1.45	-0.92	-1.35	-6.33
7L	+1.00	-2.24	-1.28	-1.63	-8.32
8L	+1.70	-2.85	-0.67	-1.49	-8.96
9L	+2.16	-3.15	+0.09	-0.81	-7.56
10L	+1.98	-3.08	+0.77	-0.05	-5.18
11L	+1.61	-2.44	+1.02	+0.44	-3.10
12L	+1.04	-1.74	+0.83	+0.42	-1.77
13L	+0.53	-0.95	+0.42	+0.26	-0.81

Table XI (cont.)

Results for the First Analysis (cont.)

Point	<u>Moments at the Supports</u>				<u>Moment at Left Knee</u>
	M ₁	M ₂	M ₃	M ₄	M _k
18	-0.46	+0.75	-0.48	-0.28	+0.84
19	-0.84	+1.53	-1.00	-0.50	+1.51
20	-0.97	+2.42	-1.53	-0.62	+1.88
21	-0.75	+2.76	-2.19	-0.54	+1.74
22	-0.07	+2.70	-2.70	+0.07	+1.29
23	+0.54	+2.19	-2.76	+0.75	+0.52
24	+0.62	+1.53	-2.42	+0.97	+0.11
25	+0.50	+1.00	-1.53	+0.84	0.00
26	+0.28	+0.48	-0.61	+0.46	0.00
13R	-0.26	-0.42	+0.95	-0.53	-0.08
12R	-0.42	-0.83	+1.74	-1.04	-0.14
11R	-0.44	-1.02	+2.44	-1.61	-0.15
10R	-0.05	-0.77	+3.08	-1.98	-0.22
9R	+0.81	-0.09	+3.15	-2.16	-0.52
8R	+1.49	+0.67	+2.85	-1.70	-0.86
7R	+1.63	+1.28	+2.24	-1.00	-0.88
6R	+1.35	+0.92	+1.45	-0.52	-0.68
5R	+0.70	+0.50	+0.72	-0.18	-0.27

Results for the Second Analysis

Point	<u>Vertical Reactions</u>				<u>Horizontal Reactions</u>			
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄
5L	+0.96	+0.03	-0.01	+0.02	+0.22	-0.09	-0.06	-0.07
6L	+0.91	+0.08	-0.03	+0.04	+0.41	-0.20	-0.10	-0.12
7L	+0.82	+0.18	-0.05	+0.05	+0.56	-0.32	-0.10	-0.14
8L	+0.67	+0.35	-0.08	+0.05	+0.63	-0.44	-0.06	-0.13
9L	+0.47	+0.59	-0.11	+0.03	+0.57	-0.54	+0.05	-0.07
10L	+0.29	+0.81	-0.12	+0.02	+0.41	-0.53	+0.13	-0.01
11L	+0.17	+0.91	-0.11	+0.01	+0.27	-0.44	+0.13	+0.02
12L	+0.10	+0.97	-0.08	0.00	+0.17	-0.31	+0.12	+0.03
13L	+0.05	+0.98	-0.04	0.00	+0.08	-0.16	+0.06	+0.02
18	-0.04	+0.99	+0.05	0.00	-0.07	+0.16	-0.07	-0.02
19	-0.07	+0.97	+0.10	0.00	-0.13	+0.31	-0.15	-0.03
20	-0.10	+0.91	+0.20	-0.01	-0.16	+0.45	-0.25	-0.03
21	-0.10	+0.79	+0.34	-0.03	-0.15	+0.52	-0.37	0.00
22	-0.07	+0.57	+0.57	-0.07	-0.07	+0.48	-0.48	+0.07
23	-0.03	+0.34	+0.79	-0.10	0.00	+0.37	-0.52	+0.15
24	-0.01	+0.20	+0.91	-0.10	+0.03	+0.25	-0.45	+0.16
25	0.00	+0.10	+0.97	-0.07	+0.03	+0.15	-0.31	+0.13
26	0.00	+0.05	+0.99	-0.04	+0.02	+0.07	-0.16	+0.07

Table XI (cont.)

Results for the Second Analysis (cont.)

Point	<u>Vertical Reactions</u>				<u>Horizontal Reactions</u>			
	<u>V₁</u>	<u>V₂</u>	<u>V₃</u>	<u>V₄</u>	<u>H₁</u>	<u>H₂</u>	<u>H₃</u>	<u>H₄</u>
13R	+0.00	-0.04	+0.98	+0.05	-0.02	-0.06	+0.16	-0.08
12R	+0.00	-0.08	+0.97	+0.10	-0.03	-0.12	+0.31	-0.17
11R	+0.01	-0.11	+0.91	+0.17	-0.02	-0.13	+0.44	-0.27
10R	+0.02	-0.12	+0.81	+0.29	+0.01	-0.13	+0.53	-0.41
9R	+0.03	-0.11	+0.59	+0.47	+0.07	-0.05	+0.54	-0.57
8R	+0.05	-0.08	+0.35	+0.67	+0.13	+0.06	+0.44	-0.63
7R	+0.05	-0.05	+0.18	+0.82	+0.14	+0.10	+0.32	-0.56
6R	+0.04	-0.03	+0.08	+0.91	+0.12	+0.10	+0.20	-0.41
5R	+0.02	-0.01	+0.03	+0.96	+0.07	+0.06	+0.09	-0.22

Point	<u>Moments at the Supports</u>				<u>Moment at Left Knee</u>
	<u>M₁</u>	<u>M₂</u>	<u>M₃</u>	<u>M₄</u>	<u>M_k</u>
5L	+0.17	-0.77	-0.54	-0.77	-3.34
6L	+0.46	-1.53	-0.91	-1.36	-6.37
7L	+1.00	-2.26	-0.99	-1.62	-8.37
8L	+1.65	-2.99	-0.65	-1.48	-8.81
9L	+2.19	-3.34	+0.07	-0.73	-7.34
10L	+2.13	-3.07	+0.78	+0.09	-5.08
11L	+1.61	-2.15	+0.98	+0.45	-3.04
12L	+1.09	-1.80	+0.81	+0.46	-1.81
13L	+0.50	-0.94	+0.41	+0.25	-0.84
18	-0.50	+0.72	-0.49	-0.32	+0.84
19	-0.86	+1.60	-1.00	-0.55	+1.50
20	-1.00	+2.34	-1.67	-0.71	+1.67
21	-0.64	+2.91	-2.30	-0.56	+1.74
22	-0.03	+2.84	-2.84	+0.03	+1.12
23	+0.56	+2.30	-2.91	+0.64	+0.55
24	+0.71	+1.67	-2.34	+1.00	+0.19
25	+0.55	+1.00	-1.60	+0.86	+0.05
26	+0.32	+0.49	-0.72	+0.50	0.00
13R	-0.25	-0.41	+0.94	-0.50	-0.08
12R	-0.46	-0.81	+1.80	-1.09	-0.14
11R	-0.45	-0.98	+2.51	-1.61	-0.15
10R	-0.09	-0.78	+3.07	-2.13	-0.22
9R	+0.73	-0.07	+3.34	-2.19	-0.52
8R	+1.48	+0.65	+2.99	-1.65	-0.69
7R	+1.62	+0.99	+2.26	-1.00	-0.88
6R	+1.36	+0.91	+1.53	-0.46	-0.68
5R	+0.77	+0.54	+0.77	-0.17	-0.27



Table XI (cont.)

Results for the Third Analysis

Point	<u>Vertical Reactions</u>				<u>Horizontal Reactions</u>			
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄
5L	+0.97	+0.02	-0.01	+0.02	+0.21	-0.09	-0.06	-0.06
6L	+0.90	+0.07	-0.02	+0.03	+0.40	-0.20	-0.10	-0.11
7L	+0.82	+0.18	-0.05	+0.04	+0.55	-0.34	-0.10	-0.14
8L	+0.67	+0.36	-0.08	+0.04	+0.60	-0.44	-0.05	-0.12
9L	+0.47	+0.60	-0.10	+0.03	+0.56	-0.54	+0.05	-0.07
10L	+0.29	+0.80	-0.11	+0.01	+0.41	-0.54	+0.13	-0.01
11L	+0.18	+0.90	-0.10	0.00	+0.28	-0.45	+0.14	+0.03
12L	+0.10	+0.95	-0.07	0.00	+0.17	-0.33	+0.12	+0.03
13L	+0.04	+0.97	-0.04	0.00	+0.08	-0.16	+0.06	+0.02
18	-0.04	+0.97	+0.04	0.00	-0.07	+0.16	-0.07	-0.02
19	-0.07	+0.96	+0.07	0.00	-0.12	+0.32	-0.15	-0.04
20	-0.09	+0.91	+0.19	-0.01	-0.15	+0.44	-0.25	-0.04
21	-0.09	+0.76	+0.32	-0.03	-0.13	+0.51	-0.36	-0.01
22	-0.06	+0.57	+0.57	-0.06	-0.06	+0.50	-0.50	+0.06
23	-0.03	+0.32	+0.75	-0.09	+0.01	+0.36	-0.51	+0.13
24	-0.01	+0.19	+0.88	-0.09	+0.04	+0.25	-0.44	+0.15
25	0.00	+0.07	+0.96	-0.07	+0.04	+0.15	-0.32	+0.12
26	0.00	+0.04	+0.97	-0.04	+0.02	+0.07	-0.16	+0.07
13R	0.00	-0.04	+0.97	+0.04	-0.02	-0.06	+0.16	-0.08
12R	0.00	-0.07	+0.94	+0.10	-0.03	-0.12	+0.33	-0.17
11R	0.00	-0.10	+0.90	+0.18	-0.03	-0.14	+0.45	-0.28
10R	+0.01	-0.11	+0.80	+0.34	+0.01	-0.13	+0.54	-0.41
9R	+0.03	-0.10	+0.60	+0.47	+0.07	-0.05	+0.54	-0.56
8R	+0.04	-0.08	+0.36	+0.67	+0.12	+0.05	+0.44	-0.60
7R	+0.04	-0.05	+0.18	+0.82	+0.14	+0.10	+0.34	-0.55
6R	+0.03	-0.02	+0.07	+0.90	+0.11	+0.10	+0.20	-0.40
5R	+0.02	-0.01	+0.02	+0.97	+0.06	+0.06	+0.09	-0.21

Point	<u>Moments at the Supports</u>				<u>Moment at Left Knee</u>
	M ₁	M ₂	M ₃	M ₄	M _k
5L	+0.12	-0.77	-0.56	-0.75	-3.34
6L	+0.36	-1.59	-0.90	-1.35	-6.42
7L	+0.92	-2.42	-1.03	-1.67	-8.27
8L	+1.65	-3.05	-0.67	-1.43	-8.53
9L	+2.26	-3.53	+0.12	-0.68	-6.95
10L	+2.19	-3.23	+0.78	+0.14	-4.66
11L	+1.73	-2.57	+1.02	+0.53	-2.93
12L	+1.17	-1.88	+0.85	+0.54	-1.64
13L	+0.57	-0.93	+0.45	+0.32	-0.74

Table XI (cont.)

Results for the Third Analysis (cont.)

Point	<u>Moments at the Supports</u>				<u>Moment at Left Knee</u>
	M ₁	M ₂	M ₃	M ₄	M _k
18	-0.51	+0.79	-0.51	-0.33	+0.67
19	-0.90	+1.73	-1.10	-0.66	+1.10
20	-1.01	+2.37	-1.75	-0.82	+1.48
21	-0.64	+2.86	-2.37	-0.63	+1.50
22	+0.07	+2.97	-2.97	-0.07	+0.95
23	+0.63	+2.37	-2.86	+0.64	+0.42
24	+0.82	+1.75	-2.37	+1.01	+0.15
25	+0.66	+1.10	-1.73	+0.90	0.00
26	+0.33	+0.51	-0.79	+0.51	0.00
13R	-0.32	-0.45	+0.93	-0.57	-0.16
12R	-0.54	-0.83	+1.88	-1.17	-0.21
11R	-0.53	-1.02	+2.57	-1.73	-0.22
10R	-0.14	-0.78	+3.23	-2.19	-0.30
9R	+0.68	-0.12	+3.53	-2.26	-0.45
8R	+1.43	+0.67	+3.05	-1.65	-0.64
7R	+1.67	+1.03	+2.42	-0.92	-0.67
6R	+1.35	+0.90	+1.59	-0.36	-0.62
5R	+0.75	+0.56	+0.77	-0.12	-0.22

Table XII

EXPERIMENTAL RESULTS

TRIPLE SPAN RIGID FRAME BRIDGE - HINGED SUPPORTS

Results for the First Analysis

Point	<u>Vertical Reactions</u>				<u>Horizontal Reactions</u>				<u>Moment at Left Knee</u>
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄	M _k
5L	+0.97	+0.04	-0.03	+0.04	+0.19	-0.08	-0.06	-0.05	-3.01
6L	+0.87	+0.11	-0.05	+0.07	+0.34	-0.13	-0.09	-0.08	-5.68
7L	+0.78	+0.21	-0.09	+0.09	+0.47	-0.27	-0.10	-0.11	-7.85
8L	+0.64	+0.39	-0.13	+0.10	+0.51	-0.32	-0.07	-0.11	-8.69
9L	+0.47	+0.62	-0.17	+0.08	+0.45	-0.37	+0.01	-0.09	-7.69
10L	+0.30	+0.82	-0.16	+0.04	+0.32	-0.35	+0.08	-0.05	-5.35
11L	+0.19	+0.93	-0.14	+0.02	+0.21	-0.29	+0.10	-0.02	-3.51
12L	+0.11	+0.97	-0.10	+0.01	+0.13	-0.20	+0.08	-0.01	-2.17
13L	+0.05	+0.98	-0.05	0.00	+0.06	-0.11	+0.05	0.00	-1.00

Table XII (cont.)

Results for the First Analysis (cont.)

Point	Vertical Reactions				Horizontal Reactions				Moment at Left Knee M _K
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄	
18	-0.05	+0.98	+0.05	0.00	-0.06	+0.11	-0.05	+0.00	+1.00
19	-0.09	+0.97	+0.13	-0.01	-0.11	+0.20	-0.10	+0.01	+1.83
20	-0.12	+0.93	+0.21	-0.02	-0.15	+0.29	-0.16	+0.02	+2.51
21	-0.13	+0.80	+0.36	-0.05	-0.15	+0.33	-0.23	+0.05	+2.51
22	-0.10	+0.59	+0.59	-0.10	-0.11	+0.30	-0.30	+0.11	+1.83
23	-0.05	+0.36	+0.80	-0.13	-0.05	+0.23	-0.33	+0.15	+1.00
24	-0.02	+0.21	+0.93	-0.12	-0.02	+0.16	-0.29	+0.15	+0.33
25	-0.01	+0.13	+0.97	-0.09	-0.01	+0.10	-0.20	+0.11	+0.17
26	0.00	+0.05	+0.98	-0.05	0.00	+0.05	-0.11	+0.06	0.00
13R	0.00	-0.05	+0.98	+0.05	0.00	-0.05	+0.11	-0.06	0.00
12R	+0.01	-0.10	+0.97	+0.11	+0.01	-0.08	+0.20	-0.13	0.00
11R	+0.02	-0.14	+0.93	+0.19	+0.02	-0.10	+0.29	-0.21	-0.33
10R	+0.04	-0.16	+0.82	+0.30	+0.05	-0.08	+0.35	-0.32	-0.84
9R	+0.08	-0.17	+0.62	+0.47	+0.09	-0.01	+0.37	-0.45	-1.50
8R	+0.10	-0.13	+0.39	+0.64	+0.11	+0.07	+0.32	-0.51	-1.83
7R	+0.09	-0.09	+0.21	+0.78	+0.11	+0.10	+0.27	-0.47	-1.83
6R	+0.07	-0.05	+0.11	+0.87	+0.08	+0.09	+0.16	-0.34	-1.34
5R	+0.04	-0.03	+0.04	+0.97	+0.05	+0.06	+0.08	-0.19	-0.84

Results for the Second Analysis

Point	Vertical Reactions				Horizontal Reactions				Moment at Left Knee M _K
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄	
5L	+0.98	+0.05	-0.02	+0.05	+0.18	-0.09	-0.05	-0.04	-3.01
6L	+0.86	+0.11	-0.05	+0.07	+0.34	-0.16	-0.09	-0.08	-5.68
7L	+0.78	+0.21	-0.09	+0.08	+0.46	-0.25	-0.10	-0.10	-7.69
8L	+0.65	+0.39	-0.13	+0.09	+0.51	-0.32	-0.07	-0.10	-8.52
9L	+0.47	+0.64	-0.17	+0.07	+0.44	-0.37	+0.02	-0.08	-7.35
10L	+0.30	+0.82	-0.16	+0.04	+0.31	-0.35	+0.08	-0.04	-5.35
11L	+0.19	+0.92	-0.13	+0.02	+0.21	-0.29	+0.10	-0.02	-3.51
12L	+0.11	+0.98	-0.10	+0.01	+0.13	-0.21	+0.08	0.00	-2.17
13L	+0.05	+0.99	-0.05	0.00	+0.06	-0.10	+0.04	0.00	-1.00
18	-0.04	+0.99	+0.05	0.00	-0.05	+0.10	-0.05	0.00	+1.00
19	-0.09	+0.98	+0.11	0.00	-0.10	+0.20	-0.10	0.00	+1.67
20	-0.12	+0.92	+0.21	-0.02	-0.14	+0.28	-0.16	+0.01	+2.34
21	-0.13	+0.80	+0.37	-0.05	-0.14	+0.33	-0.24	+0.05	+2.34
22	-0.09	+0.59	+0.59	-0.09	-0.10	+0.31	-0.31	+0.10	+1.67
23	-0.05	+0.37	+0.08	-0.13	-0.05	+0.24	-0.33	+0.14	+0.84
24	-0.02	+0.21	+0.92	-0.12	-0.01	+0.16	-0.28	+0.14	+0.33
25	0.00	+0.11	+0.98	-0.09	0.00	+0.10	-0.20	+0.10	+0.17
26	0.00	+0.05	+0.99	-0.04	0.00	+0.05	-0.10	+0.05	0.00

Table XII (cont.)

Results for the Second Analysis (cont.)

Point	<u>Vertical Reactions</u>				<u>Horizontal Reactions</u>				<u>Moment at</u>
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄	<u>Left Knee</u>
									M _K
13R	+0.00	-0.05	+0.99	+0.05	0.00	-0.04	+0.10	-0.06	0.00
12R	+0.01	-0.10	+0.98	+0.11	0.00	-0.08	+0.21	-0.13	0.00
11R	+0.02	-0.13	+0.92	+0.19	+0.02	-0.10	+0.29	-0.21	-0.33
10R	+0.04	-0.16	+0.82	+0.30	+0.04	-0.08	+0.35	-0.31	-0.67
9R	+0.07	-0.17	+0.64	+0.47	+0.08	-0.02	+0.37	-0.44	-1.34
8R	+0.09	-0.13	+0.39	+0.65	+0.11	+0.07	+0.32	-0.51	-1.83
7R	+0.08	-0.09	+0.21	+0.78	+0.10	+0.10	+0.25	-0.46	-1.83
6R	+0.07	-0.05	+0.11	+0.86	+0.08	+0.09	+0.16	-0.34	-1.34
5R	+0.05	-0.02	+0.05	+0.98	+0.04	+0.05	+0.09	-0.18	-0.84

Results for the Third Analysis

Point	<u>Vertical Reactions</u>				<u>Horizontal Reactions</u>				<u>Moment at</u>
	V ₁	V ₂	V ₃	V ₄	H ₁	H ₂	H ₃	H ₄	<u>Left Knee</u>
									M _K
5L	+0.960	+0.035	-0.020	+0.035	+0.19	-0.09	-0.05	-0.04	-3.01
6L	+0.880	+0.100	-0.050	+0.060	+0.34	-0.16	-0.09	-0.08	-5.68
7L	+0.780	+0.220	-0.080	+0.080	+0.46	-0.25	-0.10	-0.09	-7.52
8L	+0.640	+0.390	-0.120	+0.090	+0.48	-0.32	-0.06	-0.09	-8.02
9L	+0.460	+0.620	-0.140	+0.060	+0.41	-0.35	+0.01	-0.07	-7.02
10L	+0.300	+0.820	-0.150	+0.030	+0.30	-0.34	+0.08	-0.03	-5.01
11L	+0.190	+0.920	-0.130	+0.020	+0.20	-0.28	+0.10	-0.01	-3.18
12L	+0.110	+0.970	-0.090	+0.010	+0.12	-0.19	+0.08	0.00	-2.00
13L	+0.050	+0.990	-0.040	0.000	+0.05	-0.10	+0.05	0.00	-1.00
18	-0.040	+0.990	+0.050	0.000	-0.05	+0.11	-0.05	0.00	+0.84
19	-0.090	+0.960	+0.120	0.000	-0.09	+0.20	-0.10	0.00	+1.50
20	-0.110	+0.920	+0.210	-0.020	-0.13	+0.28	-0.17	+0.01	+2.17
21	-0.120	+0.800	+0.370	-0.050	-0.12	+0.33	-0.24	+0.04	+2.17
22	-0.090	+0.580	+0.580	-0.090	-0.09	+0.30	-0.30	+0.09	+1.50
23	-0.050	+0.370	+0.800	-0.120	-0.04	+0.24	-0.33	+0.12	+0.84
24	-0.020	+0.210	+0.920	-0.110	-0.01	+0.17	-0.28	+0.13	+0.33
25	0.000	+0.120	+0.960	-0.090	0.00	+0.05	-0.11	+0.05	+0.17
26	0.000	+0.050	+0.990	-0.040	0.00	+0.05	-0.11	+0.05	0.00
13R	0.000	-0.040	+0.990	+0.050	0.00	-0.05	+0.10	-0.05	0.00
12R	+0.010	-0.090	+0.970	+0.110	0.00	-0.08	+0.19	-0.12	0.00
11R	+0.020	-0.130	+0.920	+0.190	+0.01	-0.10	+0.28	-0.20	-0.17
10R	+0.030	-0.150	+0.820	+0.300	+0.03	-0.08	+0.34	-0.30	-0.50
9R	+0.060	-0.140	+0.620	+0.460	+0.07	-0.01	+0.35	-0.41	-1.17
8R	+0.090	-0.120	+0.390	+0.640	+0.09	+0.06	+0.32	-0.48	-1.50
7R	+0.080	-0.080	+0.220	+0.780	+0.09	+0.10	+0.25	-0.46	-1.67
6R	+0.060	-0.050	+0.100	+0.880	+0.08	+0.09	+0.16	-0.34	-1.17
5R	+0.035	-0.020	+0.035	+0.960	+0.08	+0.05	+0.09	-0.19	-0.84

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